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THE PRINCIPLES OF THE PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

OR AN INQUIRY INTO THE PRINCIPLES of the MODERN PHILOSOPHY;

THAT IS, INTO THE SEVERAL CHIEF RATIONAL SCIENCES, WHICH ARE EXTANT.

IN SEVEN BOOKS.

By *ROBERT GREENE*, M. A. and Fellow of CLARE-HALL.

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MVSEVM
BRITAN
NICVM

To His GRACE

THE

DUKE OF NEW-CASTLE,

One of His MAJESTIES Principal SECRETARIES OF STATE.

MY LORD,

I Do here moſt humbly beg Leave to Preſent to your Grace my Principles of the Philoſophy of the Expanſive and Contractive Forces, or, my Inquiry into the Modern Philoſophy, and the ſeveral Parts of it, which has been the Labour and Study of many Years, and to which no One has ſo Juſt a Title as your Grace, who formerly Honoured this College with your Reſidence amongſt us, and ſince, with your Patronage and Favour; And it would be no Inconſiderable Reward for the Fatigues, I have Undergone in theſe Matters, if your Grace will Pleaſe to Accept of this Addreſs, as a Teſtimony of that Unfeigned Gratitude and Reſpect, I owe to your Grace, for the many Great Condeſcenſions, which I have formerly Received from You, and of which I ſhall always Retain a moſt Faithfull Memory, Who am,

Your Grace's

moſt Obliged, and moſt

Obedient, Humble Servant,

Robert Greene.

PREFACE.

IT is with the utmoſt Pleaſure and Delight, that I am at Length able to Preſent to the World, after the Labour and Pains of many Years, my Principles of the Expanſive and Contractive Forces, or my Inquiry into Philoſophy and the ſeveral Parts of it, which was begun, even before the Noble Perſon's Admiſſion into the College, I have the Happineſs to be of, to whom theſe Sheets are Addreſſed, and was continued during his Reſidence here, to this Time of its Publication; So far, as the Affairs of the Society, the Buſineſs of my Pupils, and the Studies of my Profeſſion, as a Clergy-Man, would permit.

I ſhall only here Add in Reſpect of my ſelf, in order to Obviate any Calumnies, which may have been Raiſed againſt me, as to my Affection to the Preſent Government, That my firſt Oaths were to King William, *my ſecond to Queen* Ann, *and my laſt to his Late* Majeſty, *and as They are with me of a Religious Nature, ſo I have always Acted Conformably to them, and with the utmoſt Duty and Allegiance, in my private Station, to the ſeveral Princes, to whom I had given my Faith, and called God to Witneſs for my Obedience.*

'TIS *True, as I have had ſeveral moſt Good and ſincere Friends, who come under the Character of Whigs, Tho' I could heartily Wiſh thoſe Names of Diſtinction were entirely Sunk amongſt us, I have alſo had a very Juſt and Deſerved Affection, and Eſteem, for ſeveral of my moſt Worthy Friends, who are under the Denomination of Tories, and particularly, for Two Nonjurors in this Univerſity, whom I could Name; Whoſe great Learning, Piety, and Abilities, have claimed and challenged, upon the Account of their own intrinſick Value and Merit, that Reſpect and Regard from me, and as, upon ſome Letters which paſſed betwixt them and me, before I Publiſhed my Demonſtration of the Chriſtian Religion many Years ago, I was ſure were Conſcientious and Learned Men, and were only Unfortunate in taking of Oaths, from which they could not depart, and for whom I am as certain, that the Worthy* Dr. Laughton, *Late Fellow of this College, had a very ſingular Regard too; The One of theſe Learned and Good Men is Dead, and the Other, who Survives him, is Reſpected and Eſteem'd by every one in this Univerſity, of what Denomination ſoever, who has not loſt all Senſe of Religion and Humanity. But my own Principles and Perſuaſions have been ever on the Side of the Oaths, which I have taken, and Conſequently on the Side of the Glorious and Happy Revolution, and againſt the Maxims of an Hereditary and Indefeiſible Right, as will be ſeen in a Latin Oration, Annexed to this Inquiry into Philoſophy, Spoken by* *me, when Bachelor of Arts of this College, and of which the Noble Perſon's Good Tutor, to whom theſe Papers are Inſcribed, was pleaſed to give thoſe Encomiums and Commendations, which, I am afraid, it did not deſerve.*

ONE of my firſt Lectures to my Pupils, after his Late Majeſty's *Happy Acceſſion to the Throne, was concerning the ſeveral Rights, by which Princes can be any ways ſuppoſed to Govern, which I told them muſt either be, by Conqueſt, or by Primogeniture, or by Election, or by a Recognition of the States of any Kingdom: That the firſt his Majeſty Renounced, as every Engliſh-Man Abhors, that he had a Legal and Juſt Claim upon the Second, ſince Primogeniture was reſtrained by the Laws to thoſe, who were Proteſtants, as likewiſe upon the Third, ſince the Voice of the People was on his Side, and alſo upon the Fourth, ſince the States in Parliament had evidently Recogniz'd him; That, as to ſtrict Primogeniture or Hereditary Right, Half of the Kings upon the Engliſh Throne, or more, ſince the Times of the Conqueror, had ſat there by Recognition of Parliament, who were not the Immediate Heirs to the Crown, and that the Conqueror himſelf, from whom they deriv'd, had ſet aſide* Edgar Atheling, *the True and Rightful Heir according to the Hereditary Scheme, as* Harold *before him had done; That on the other Hand the* Pretender *to the Imperial Crown of theſe Kingdoms had not one of theſe Rights; His Primogeniture was not only Diſputed, but there were great Reaſons to Think, that in Fact he was not a Legitimate Prince, nor had he the Voice of the People, or the Recognition of Parliament to ſupport his Pretenſions, and that it was Impoſſible for him to come in, any other Way, than by the firſt Title, Namely, that of Conqueſt; Which, as it was Odious in it's ſelf, muſt be alſo Maintain'd and Aſcertain'd thro' Seas of Blood, and at length by making all Engliſh Men, without Diſtinction, a Prey to Foreign Troops, and Foreign Maſters, either* Germans, Ruſſians, French, *or* Spaniards, *to the Total Ruin and Subverſion of our Religion, our Laws, and our Liberties.*

IN the Buſineſs of my Profeſſion, as a Clergy-Man, I have likewiſe taken all Occaſions, both at the Univerſity-Church, and in thoſe of the Pariſhes, where I am, or have been, Miniſter, to ſhew the Happineſs, We enjoy under his Late Majeſty's Excellent Adminiſtration, when the Times were Proper for ſuch Diſcourſes, as upon his Late Majeſty's Happy Acceſſion to the Throne; Nor have I been wanting in this College, as Dean, and Fellow of it, upon ſuch Publick Solemnities to Mark and Diſtinguiſh them by Appointing thoſe Orders and Rules to be Obſerved, which ſhould Expreſs a Becoming and Decent Feſtivity and Joy. But I Treſpaſs perhaps too much upon my Reader's Patience, and I rather, having this ſingle and only Opportunity in my Life to do it in ſo Publick a Manner, Recount this, to Declare to the World my unreſerv'd Sentiments in theſe Matters, under the Patronage of this Noble Perſon, than to Convince any one of my Loyalty and Affection to his Preſent, and to his Late moſt Excellent Majeſty, of which every Honeſt and Good Man, I dare ſay, has been always Satisfied.

PREFACE.

AS to my Principles of the Philoſophy of the Expanſive and Contractive Forces, or my Inquiry into the Modern Schemes and Syſtems of Nature, which I have humbly Preſumed to Addreſs to the Noble Perſon above Named, They are Principles, which I had Entertain'd concerning it very Early, and upon my firſt Application to the Studies of Philoſophy, and even before my Acquaintance with Sir Iſaac Newton's, which, in many Particulars, rather Juſtifies, than Contradicts, what I Propoſe: and when I was an Under-Graduate in this Univerſity, and could not ſee the Reaſon of the Abſtractions, which Mathematicians and Philoſophers made Uſe of in their ſeveral Sciences, I Applied my ſelf to the Late Reverend Dr. Laughton, who was then Eminent in thoſe Studies, was in Part my Tutor and my Friend, and who very kindly Endeavour'd to Inſtruct me, ſo far as he could, in thoſe Matters, but without giving me any farther Information, which I have ſince found, it was Impoſſible for him to do; Becauſe thoſe Abſtractions, which are at preſent Eſpous'd, are Wrong and Erroneous, and are the only Obſtruction and Hindrance to a farther Progreſs in Truth, and this kind of Knowledge, which I have Laboured to Remove, by Propoſing a Geometry and Philoſophy, which ſhall be Adequate to, and which ſhall reach, the whole Extent of Being.

I can only Say, that theſe Principles of the Expanſive and Contractive Forces have carried me much farther in the Explanations of Nature, and the Phenomenons of it, than I could at firſt have poſſibly Imagin'd; For I at firſt Thought, if I could give any tolerable Account of Matter, and what the Eſſence of it was, which all others Deſpaired of, I ſhould make a Notable Advance in that Science; And this was what I originally Intended; But Purſuing the ſame Principles, I perceiv'd they would give an Account not only of That, but of the various Properties and Affections of it, as Colour, Sound, Taſte, Smell, &c. Sir Iſaac Newton's Gravitation, and even of the Surprizing Effects in Chymiſtry, of the Vegetation of Plants, of the Conſtitution and Formation of the Parts of an Human Body, of the Reaſon of the Revolution of the Planets, of the Capacities and Powers of the Mind, and of the Union of the Material Syſteme with it; Problems, which all Philoſophers have hitherto Reſigned as not to be Solved, and which yet ſeem to be fairly Explained upon the ſame Principles of the Expanſive and Contractive Forces. My Reader may eaſily Suppoſe, that ſuch Contemplations, as theſe, did therefore Afford me Matter both of Pleaſure and Aſtoniſhment, and I do humbly beg Leave to Profeſs to the World, that I believe there is ſcarcely any Phenomenon in Nature, which may not be Accounted for from the ſame Principles.

AS to my Reaſonings upon theſe Subjects, I am not much Apprehenſive, that they are Inferiour to thoſe of the Philoſophers, whom it is my Misfortune not to Agree with; But Prejudice and Partiality and a Fondneſs for Great Names, without ever Inquiring into Things themſelves, are Hard and Obſtinate Buſineſſes to Encounter, which yet I would hope might be Diminiſhed, if Men would but Conſider that ſeveral Opinions and Syſtems of Philoſophy were Patronized and Defended in the Free-State of Athens, without any great Re-

proach to any Sect of them, and why may not the ſame Liberty be Indulged to the Free-State of Great-Brittain?

THE *Famous and Celebrated Univerſity of* Oxford *at this Time adheres to the* Ariſtotelian *Scheme, and our own of* Cambridge, *which is no leſs Renowned for its Studies and its Learning, and the Great Men it has Produced, ſeems to be too much Addicted to the* Carteſian, *or, which is much the ſame as to the Principles of a Similar and Homogeneous Matter, to the* Newtonian; *On the Contrary, as* Ariſtotle *Affirmed Four Principles,* Des-Cartes *Three, and Sir* Iſaac Newton *One, which is that of Gravitation or Attraction, I beg the Favour, under the Patronage and Protection of the Noble Perſon I have Mentioned, to Intrude my ſelf amongſt theſe Philoſophers, and to Declare there are neither More nor Leſs than Two; Namely, the Expanſive and Contractive Force. I cannot but here Acquaint the World, that the Preſent Philoſophers Derive all their Notions of Nature from* Italy *and* Galileus, *or from* Des-Cartes *and* France, *Excepting what* Kepler *a* German *has done in Reſpect of thoſe Sciences, and from whom Sir* Iſaac Newton *is ſaid to have taken his Principle of Gravitation, and who is Eſteem'd to have been the moſt Learned and Sagacious Man of his Age; But in all other Reſpects, our Philoſophy, as it is now Received and Embraced, is the Product of Popiſh Countries, Imported to us from* Italy *and* France, *and I will leave the World to Judge, whether Superſtition and Foppery in Religion in any Nation, will not make an Impreſſion of Atheiſm or Deiſm upon the Philoſophy of it, as it moſt certainly has done.*

ALL *therefore, which I Deſign and Intend, is to Propoſe a Philoſophy, which is truly* Engliſh, *a* Cantabrigian, *and a* Clarenſian *one, as it was Born and Educated and Studied in thoſe Places, and which it is not Impoſſible may be as Juſt in its Reaſonings, and as Good in its Religion, as That which is brought over to us from Countries, which abound either in Superſtition or in Atheiſm; And as my Name is not much Worſe in the Letters, which belong to it, than thoſe of* Galileus, *or* Des-Cartes, *and for Reaſons Alledged at the End of this Philoſophy, by which it is Evident, there are only Four Ways of Explaining Nature, One of which I have Taken, and no One elſe has done, I ſhall Venture to call the* GREENIAN.

THE *Original Occaſion of my Purſuing theſe Inquiries with that Vehemence and Application, which I have Uſed, was upon the Account of my Dear and Good Pupils, whom I Love, and whom, I thought, I was to Inform, as well as I could, and not to Indulge them in Errors, which I could not Subſcribe to; and which, if they are ſuch, as Philoſophy is the Parent of Phyſick, muſt have coſt Millions of Lives in the Practice of it, upon Corpuſcularian or Atomical Principles, or, which is the ſame, upon thoſe of a Similar and Homogeneous Matter; Beſides, the Impreſſion ſuch Notions have made upon Chriſtianity it ſelf, to its great Prejudice, as may be ſeen in the following Sheets, will Sufficiently Juſtify me in the Oppoſing of them.*

PREFACE.

IF there are any Miſtakes in this Scheme of Philoſophy, as Perhaps there may be many, for it is a moſt difficult Thing to Explain, what Exiſtence or Being is, I may yet Say, that I have uſed all the Integrity, Diligence and Capacity, which it is Poſſible for me to do, and that it is much Eaſier to Underſtand an Hundred Volumes in the Greek *or* Latin *Authors, than That ſingle One of Nature. I muſt however beg Leave to Plead in Excuſe to the World, if I am Miſtaken in the Main, as I believe, and am pretty well Aſſured, I am not, That this Inquiry, as Large as it is, has been amidſt ſeveral other Studies, Buſineſſes and Employs, which have had a Relation to the College, to my Pupils, or my Pariſhes, all of whom I moſt Exceedingly Love, Value, and Eſteem; And, if it had not been upon their Accounts, and for the Charge and Expenſe of Printing, and Creating too great a Fatigue and Trouble to my Reader, I could have Extended theſe Inquiries to five Times the Number of Sheets, which I here Preſent to the World, and which any One will See and Perceive upon a Curſory Peruſal of them only.*

I have one farther Apology to make to the World, that many of theſe Papers were Written ſeveral Years ago, even before the Printing of my laſt Book of Philoſophy in 1712, *to which the Noble Perſon, I have ſo often Referred to, gave me the kind Encouragement of his Subſcription, and which has not been Confuted, nor can be, by any of our Modern Reaſoners, however Great their Names may be, and however High their Reputations may Stand, in the Philoſophick or Mathematick World. On the Contrary, the Celebrated Sir* Iſaac Newton, *in ſome Meaſure, falls into my Notions, and now Confeſſes a Plenum, or ſomething like it, who had in the firſt Edition of his Principles Propoſed to Demonſtrate, that there muſt of Neceſſity be a Vacuum, againſt which I Argued in the ſaid Book, and of which I did my ſelf the Honour to make a Preſent to him upon the firſt Publication of it. But I ſhall Tire my Reader with theſe Preliminary Explanations, and would only Humbly beg the Favour of him at his Leiſure-Hours, that he will Pleaſe to Inquire with me, whether the Philoſophers are True or not, in their Solutions of the ſeveral Problems in Nature, which, I think, I may punctually Affirm, they are not; and I am Satisfied, that whenever any One of Common Sagacity and Penetration will take that Province upon himſelf, he will find all the Reaſons in the World to enter into the ſame Sentiments; That is, if he firſt Reads thoſe Authors in Philoſophy, whom I Quote, and then Sits down to Conſider of them; I only ſhall Add, that I Eſteem and Look upon this, together with my Philoſophy of* 1712, *as my Second Demonſtration of the Chriſtian Religion, by ſhewing the Fallacies of thoſe Reaſonings in a Philoſophy, which ſeems to Oppoſe it, or to have very little Regard for it; My farther Demonſtrations of the ſame Excellent Religion will be, with his Allowance and Permiſſion, Addreſſed to the Good* Marqueſs of Blandford, *for whom, during his Reſidence here, as I Received from him the moſt Signal Kindneſſes and Condeſcenſions, and as his Piety, Charity, Humility, and Goodneſs, were beyond Example, I cannot but always have the higheſt Eſteem and Veneration; Which Demonſtrations of the Chriſtian Religion Lie Written by me, and are ready, in ſome ſhort Time, if he will Pleaſe to Allow them that Honour, for his Lordſhip's Patronage.*

THE

PRINCIPLES

OF THE

PHILOSOPHY

OF THE

EXPANSIVE and CONTRACTIVE FORCES.

Book I.

CONCERNING the Principles of the Mechanical Philoſophy of *Galileus*, *Chriſtian Hugens*, Sir *Iſaac Newton*, and Dr. *Keill*.

Book II.

Concerning Thoſe of the Phyſical Aſtronomy of *Kepler*, Sir *Iſaac Newton*, and Dr. *Gregory*.

Book III.

Concerning the Principles of Anatomy, Chymiſtry, Pneumaticks, and Hydroſtaticks, of Dr. *Gibſon*, Dr. *Keill*, Dr. *Drake*, *Tauvry*, *Blancard*, and *Alphonſus Borellus*; Of *Lemery*, Mr. *Boyle*, and Dr. *Freind*; and of Dr. *Wallis*, Dr. *Halley*, and Others.

Book IV.

Concerning Thoſe of the Opticks, Dioptricks, and Catoptricks, of *Des-Cartes*, *Rohault*, *Molyneux*, Dr. *Gregory*, and *Hugens*.

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In all which *Sciences*, Compared with the ſeveral Demonſtrations of the *Chriſtian Religion*, is Shewn,

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ERRATA.

Pag. 56. Line Penultim. dele *and*, Pag. 104. Title to §. 34. Lege *in the Æquinoctial Syzygiæ than in the Solſtitial*, Pag. 109. Line Penultim, Lege *Diſperſed*, Pag. 111. Lege *Paradiſe*, Pag. 807. Lege *Injure*, Pag. 824. Line 35. Lege *it will be Said not*; *If* Pag. 904. Lege §. 7. *The Reaſon of this Procedure*, §. 7. Pag. 974. Lege εὕρηκα.

THE PRINCIPLES OF THE PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK I.

Concerning the Principles of the MECHANICAL PHILOSOPHY.

INTRODUCTION.

BOOK I.

CHAP. I.

Concerning Similar Matter, a Vacuum, and the New Theory of Light, and Colours, and of Sounds, with a Solution of the Rainbow.

The Difference betwixt us, and the preſent Philoſophers.

§. 1. BEFORE we Proceed to an Inquiry into, and an Explanation of the *Principles* of the *Mechanical Philoſophy*, and the *Phyſical Aſtronomy*, it will be neceſſary to clear our Way to it, by Conſidering what has been alledg'd againſt us in Relation to our *Principles of Natural Philoſophy*; in which our Principal Affirmations were, that there is no ſuch Thing as a Vacuum in Nature, that Matter is not Similar and Homogeneous, but that Light is ſo, that the Qualities or Properties of Matter, which occaſion the Senſations in us of Taſting, Smelling, Feeling, Hearing, and Seeing, are not deriv'd from different Modifications of the ſame Homogeneous Matter, nor from any Figures of the Corpuſcles of it, or Actions in it; but from certain innate Forces which are Inherent therein, which Forces we propos'd to Explain: On the contrary the preſent Philoſophy informs us, that there is a Vacuum, that Matter is Similar and Homogeneous, but that Light is not, that the Properties of Matter, by which one Portion of it is diſtinguiſh'd from Another, are owing to the particular Figure of it's Parts, and to it's Variety of Pores, and to the Motions of ſuch Parts one amongſt another.

Concerning a Vacuum, and how that Controverſy ſtands at Preſent.

Principles of Natural Philoſophy, Chap. 5. §. 43. Part. 1.

Sir Iſ. Newt. Princip. Prop. 6. Corol. 3. Lib. 3. Edit. 1. & 2.

§. 2. As to a Vacuum, we ſaid, that the Proofs of it turn'd upon a Suppoſition, that all Matter was Homogeneous, which was the very Thing in Queſtion, for if all Matter was alike, and the World was full of it, it was own'd, there could be no Specifick Gravity, but that all Matter was alike, was diſputed and in Terms deny'd, and this Subject we proſecuted thro' our whole fifth Chapter of the Principles of Natural Philoſophy, and §. 43. we ſhew'd the Inconcluſiveneſs of the Argument drawn from Specifick Gravity to evince a Vacuum: On the contrary the preſent Philoſophy in it's firſt Edition had affirm'd from thence *the Neceſſity of a Vacuum*, which whether convinc'd by what I had offer'd in Contradiction to that Demonſtration, or upon a farther Canvaſſing and Conſidering the Matter, it, in the ſecond Edition, alter'd to this Aſſertion, that, *all Spaces were not equally full*, if by that is meant, that ſome Spaces are not full, but on the Contrary are void of all Matter, 'tis the ſame as the former Propoſition, that *there is a Vacuum*, and all the Advantage there is in the Change, is, that it is more doubtfully and ambiguouſly, and therefore more oraculouſly Expreſs'd; if by it is meant, that *all Spaces are full of Matter, but not equally ſo*, the firſt Proſelyte we have had to our Reaſoning and Arguments, is the Perſon, whoſe Reaſoning and Arguments we oppos'd, ſince that is what we maintain, that there is a Plenum, but that Matter being Various and Diſſimilar, Spaces are not full of the ſame Matter, but are repleniſh'd with different Sorts and

Kinds of it, and therefore are not *equally full*; but this is an Honour ſo mean a Perſon, as I, cannot expect.

Sir Iſ. Newt. Optical Queries 18, 19, 20, 21, 22.

That this Philoſophy yet has ſome Inclinations to embrace this Opinion which I have defended, is manifeſt likewiſe from a new Edition of it's Light and Colours, and the Optical Queries annext to it, where an Æthereal Medium is introduc'd, which may have little or no Reſiſtance to the Motions of the Planets and Comets, which may be the Occaſion of the Refractions and Reflections of Light, and by whoſe Vibrations Light may communicate Heat to Bodies, and be put into Fits of eaſy Reflection, and eaſy Tranſmiſſion; and by whoſe Elaſticity Gravity is thought to be accounted for: Theſe are all Notions which tend to the Confirming of what I originally advanc'd, that there is an Æthereal Plenum, and which the Great and Divine Dr. *Barrow* has long ſince patroniz'd and aſſerted, and the Learned and Judicious Dr. *Pearſon* adopted into his own Perſuaſions and Opinions.

Sir Iſ. Newt. Optical Query 28.

But notwithſtanding theſe Conceſſions, this Philoſophy ſtill relapſes into it's old Doctrine of a Vacuum, and a Similar Matter in the *28th* Query, and eſteems all Hypotheſes to be Erroneous in which Light is ſuppos'd to conſiſt in Preſſion or Motion propagated thro' a Fluid Medium, and all that is argu'd in that Query concerning the Rarity and Denſity of a Medium is upon the Suppoſition, that there is no other Rarity or Denſity, but what proceeds from the Minuteneſs or Groſsneſs of the Parts of the ſame Similar Matter, and the leſs or greater Number of them in the ſame Space, from whence it concludes, that a Plenum or a Denſe Fluid can be of no uſe for explaining the Phænomena of Nature, and therefore may be juſtly rejected; for the doing of which, it is ſaid, we have the Authority of the oldeſt and moſt eminent Philoſophers of *Greece* and *Phœnicia*, who made a Vacuum and Atoms, and the Gravity of Atoms, the firſt Principles of their Philoſophy: It is farther here told us, that the main Buſineſs of Natural Philoſophy is to argue from Phænomena without feigning Hypotheſes; and to deduce Cauſes from Effects, till we come to the firſt Cauſe, which certainly is not Mechanical, and not only to unfold the Mechaniſm of the World, but chiefly to reſolve theſe and ſuch like Queſtions; What is there in Places *Empty of Matter*, and whence is it that the Sun and Planets gravitate towards one another *without denſe Matter* between them? Whence is it that Nature doth nothing in vain? And whence ariſes all that Order and Beauty which we ſee in the World? To what end are Comets, and whence is it that Planets move all one and the ſame way in Orbs Concentrick, while Comets move all manner of ways in Orbs very Excentrick; and what hinders the fixt Stars from falling upon one another, *&c.* for we ſhall take notice of no more Queſtions here, becauſe the following relate to the Animal and Intelligent World, which do's not come under our preſent Conſideration.

A Vacuum muſt be defended by this Philoſophy, or elſe it muſt Renounce it's Principles.

§. 3. **From** whence it is evident, that this Philoſophy ſcarcely knows what to aſſert, ſometimes it ſeems to think there is a Plenum, but that *all Spaces are not equally full*, that there is an Æthereal Medium ſpread and diffus'd thro' the whole Solar Syſteme, which may occaſion the Refractions and Reflections of Light, and the Fits of eaſy Reflection and eaſy Tranſmiſſion, by whoſe Vibrations Light may communicate Heat to Bodies, and Gravity be poſſibly ſolv'd by it's Elaſticity; at other Times it is ſaid, a Plenum can be of no uſe for explaining the Phænomena of Nature, and is therefore to be rejected, and for the doing of which we have the Authority of the ancienteſt and beſt Philoſophers of *Greece* and *Phœnicia*.

To clear up this Matter a little therefore to the preſent Philoſophy, we muſt take the Liberty to inform it, that however glad we might be to have ſo great an Authority to juſtify our Reaſonings upon this Subject, it cannot come into the acknowledgement of a Plenum, without Diſcarding and Diſavowing it's own Principles, and therefore it might as well have retain'd it's Aſſertion of the *Neceſſity of a Vacuum*, according to the firſt Edition of them, as alter'd it to another, that *all Spaces are not equally full*, which is done in the ſecond; for tho' one who maintains a Diſſimilar and Heterogeneous Matter in it's own Nature, may agreeably and conſiſtently with ſuch a Notion affirm a Plenum of a thin and ſubtle Æther, which is as much and as ſubſtantially Matter as the groſeſt we can Name, yet the Caſe is not the ſame with thoſe, who define Matter to be one Similar, Paſſive, Solid, Extended and Invariable Thing, ſince if Spaces are full of ſuch a Matter, it is evident it will be all one, as if it was full of the groſseſt, for Inſtance of Lead or Gold, ſeeing there is no Difference in Matter, as it is ſuch, and conſequently it is truly and juſtly concluded, that there would be no Specifick Gravity, and Bodies would no more deſcend in Air, than they do in Gold.

The Objections againſt a Vacuum and a Similar Matter farther conſider'd and enforc't, with a Deſcription of this Similar Matter.

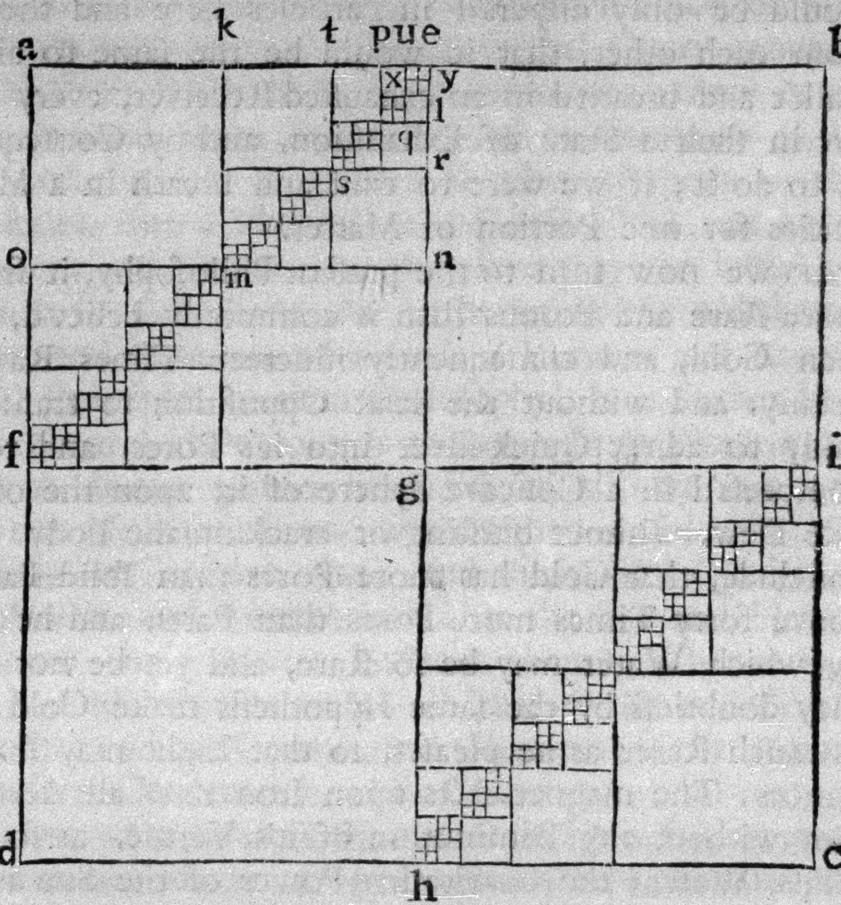
§. 4. Since then a Vacuum is what the preſent Philoſophy muſt defend, however it may appear to moderate and qualify it's Aſſertions concerning it; we ſhall firſt conſider ſome farther Arguments alledg'd for the Proof of it, which we did not particularly inquire into, tho' they were in General anſwer'd in our *Principles of Natural Philoſophy*, and afterwards examine into the pretended Authority of the Philoſophers of *Greece* and *Phœnicia* for the rejecting a Plenum, and eſpouſing a Vacuum, and it's Atoms and the Gravity of them.

As to the firſt, one principal Objection againſt a Vacuum and a Similar Matter, for they muſt ſtand and fall together, is, that in the infinite Rarity of the Air, according to this Doctrine, where there is nothing but Similar Matter or Pores and Vacuities, there would be ſo much Vacuum, and ſo little Matter in Proportion, which would be only diſpers'd in Particles here and there at very conſiderable Diſtances from each other, that it would be the ſame to all Intents and Purpoſes, as if we walk't and breath'd in an exhauſted Receiver, every one knows, that no Animals can live in ſuch a State of Exhauſtion, and by Conſequence it wou'd be impoſſible for us to do it; if we were to exiſt and breath in a Medium, which has a thouſand Vacuities for one Portion of Matter.

Sir Iſ. Newt. Opticks Prop. 8. Part 3. Book 2.

If we now turn to the preſent Philoſophy, it frankly owns, that Bodies are much more Rare and Porous than is commonly believ'd, Water is nineteen Times lighter than Gold, and conſequently nineteen Times Rarer, and Gold is ſo rare, as very readily, and without the leaſt Oppoſition to tranſmit the magnetick Effluvia, and eaſily to admit Quick-ſilver into it's Pores, and to let Water paſs thro' it, when compreſs'd in a Concave Sphere of it, upon the outſide of which it ſtands in Drops like Dew, without burſting or cracking the Body of Gold, from all which we may conclude, that Gold has more Pores than ſolid Parts, and therefore that Water has above forty Times more Pores than Parts, and he that ſhall find out an Hypotheſis by which Water may be ſo Rare, and yet be not capable of Compreſſion by Force, may doubtleſs by the ſame Hypotheſis make Gold and Water, and all other Bodies as much Rarer as he pleaſes, ſo that Light may find a paſſage thro' tranſparent Subſtances: The magnet Acts upon Iron thro' all denſe Bodies, not magnetick nor red Hot without any Diminution of it's Vertue, as for inſtance, thro' Gold, Silver, Lead, Glaſs, Water; the Gravitating Power of the Sun is tranſmitted thro' the vaſt Bodies of the Planets without any Diminution, ſo as to act upon all their Parts to their very Centers with the ſame Force, and according to the ſame Laws, as if the Part upon which it Acts were not ſurrounded with the Body of the Planet; the Rays of Light, whether they be very ſmall Bodies projected, or only Motion or Force propagated, are mov'd in Right Lines, and whenever a Ray of Light is by any obſtacle turn'd out of it's Rectilinear Way, it will never return into the ſame Rectilinear Way, unleſs perhaps by very great Accident, and yet Light is tranſmitted thro' Pellucid ſolid Bodies in Right Lines to very great Diſtances. How Bodies can have a ſufficient Quantity of Pores for producing theſe Effects is very difficult to conceive, but perhaps not altogether Impoſſible: For the Colours of Bodies ariſe from the Magnitudes of the Particles, which reflect them, as was explain'd above; now if we conceive theſe Particles of Bodies to be ſo diſpos'd amongſt themſelves, that the Intervals or empty Spaces between them may be equal in Magnitude to them All, and that theſe Particles may be compos'd of other Particles much ſmaller, which have as much empty Space between them, as Equals all the Magnitudes of theſe ſmaller Particles, and that in like manner, theſe ſmaller Particles are again compos'd of Others much ſmaller, all which together are equal to all the Pores or empty Spaces between them, and ſo on Perpetually till you come to the ſolid Particles, ſuch as have no Pores or empty Spaces within them. [Why if ſo, what then? truly nothing at all, for this Philoſophy here, as in what follows, is ſo Wiſe as to leave us without a Concluſion:] And if in any groſs Body, there be for Inſtance, three ſuch Degrees of Particles, the leaſt of which are Solid, this Body will have ſeven Times more Pores than ſolid Parts: But if there be four ſuch Degrees of Particles, the leaſt of which are Solid, the Body will have fifteen Times more Pores than ſolid Parts; if there be five Degrees, the Body will have one and thirty Times more Pores than ſolid Parts; if ſix Degrees, the Body will have ſixty and three Times more Pores than ſolid Parts, and ſo on perpetually; and there are other ways of conceiving how Bodies may be exceeding Porous, but what is really their inward Frame is not yet known to us.

THUS in Reſpect of the Calculation above, let *abcd* repreſent the Square Section of a Cubical Particle of Matter, divided firſt into four Square Areolæ, and let one be Matter, and the other Vacuity alternately; Then let *aegf* be divided into four other Square Areolæ, the one Matter, and the other Vacuity alternately, and let the ſame be done by the other Particle of Matter in the firſt Diviſion: Afterwards let *kemn* be divided into four other Square Areolæ, the one Matter, and the other Vacuity alternately; and let the ſame be done by every of the Particles of Matter in the ſecond Diviſion; The Matter in this third Diviſion will be to the Vacuities or Pores as 1 to 7, becauſe the Matter in *kemn* will be to the Vacuity, as 1, Matter, to 1, Vacuity; and *akmo* will be 2, Vacuity; and therefore the Matter in *kemn* will be to the Vacuity in *kemn*, and *akmo* as 1, Matter, to 3, Vacuity; and the ſame may be ſaid of the reſt of the Matter in *aegf* to it's Vacuity, that it will be, as 1, Matter, to 3, Vacuity, which conſiſts therefore of four Parts, but *egbi* will conſiſt likewiſe of four Parts, all of which will be Vacuity, and conſequently the Portion of Matter in the Space *abif*, will be to the Vacuity in the ſame Space, as 1 to 7, and the ſame may be ſaid of all the other Portions of Matter and Vacuity in the Space *abcd*: Let now the leaſt Part of Matter in *kemn*, viz. *erst* be divided into four other Square Areolæ, the one Matter, the other Vacuity; the leaſt Part of Matter therefore in *kemn* will be divided into Two, one of which will be Matter, and the other Vacuity, and the leaſt Part of Vacuity will likewiſe conſiſt of two Parts, and the Matter in the leaſt Particle of it in *kemn* will be to Vacuity as 1 to 3, and the ſame may be ſaid of all the Particles of Matter and Vacuity in *kemn*, which conſequently will conſiſt of four Parts, one of which will be Matter, and the other three Vacuity, therefore *akmo* will conſiſt of four Parts, which will be all Vacuity, and the Portion of Matter in *aeno* will be to it's Vacuity as 1 to 7, and the ſame may be ſaid of the reſt of the Parts of Matter, and Vacuity in *aegf*, which therefore conſiſts of eight Parts, one of which will be Matter, and ſeven Vacuity, but *ebig* (= to *aegf*) which will be all Vacuity, conſiſts of eight Parts, and therefore there will be in *abif* 1, Matter, and 15, Vacuity, and the ſame may be ſaid of all the other Square Areolæ in *abcd*; and by the ſame Method we may proceed to a fifth or ſixth Diviſion, from whence we ſhall perceive that the Body will have 31 or 63 Times more Pores than ſolid Parts, as may be ſhewn by taking *pqle*, *uxye* for the leaſt of ſuch Parts.



That the foregoing Deſcription and Calculation do's not ſerve the Purpoſe for which it was contriv'd, but the Contrary.

§. 5. THIS is a true Pattern or Delineation of that Matter, which is now embrac'd and receiv'd for Authentick in Nature, which therefore in this Scheme we have deſcrib'd, and which explains the utmoſt Stretch of Genius, which the preſent Philoſophy makes uſe of, to get quit of a Difficulty ſo uneaſy to it, that is, to find out a ſubtle Matter, whilſt it affirms all Matter to be alike, which ſhall be continu'd in it's Parts, and yet infinitely Rare and Porous, for if that is not done, if Air, for inſtance, has one Part only of Matter for a thouſand Vacuities, and thoſe Parts of Matter are ſeparated and at a Diſtance from each other, for one Part of Matter we breath in, we ſhall meet with a Thouſand of Vacuum; and how it would be poſſible for us to breath at all in ſuch a Medium, any more than in an exhauſted Receiver, is not, nor can be accounted for: To evince therefore how a continu'd Subſtance, as Air, or Water, or Glaſs, or Gold, may have an infinite Number of Pores in it, ſo that the Space they take up ſhall be infinitely Greater than the ſolid Parts of Matter in them, this Calculation is contriv'd, which with what Succeſs it anſwers the Intention of it, we ſhall now Examine.

AND firſt, it is manifeſt, that in the Square *a b c d* the Parts of Matter are disjoin'd from each other by the intermediate Areolæ of Space or Vacuum, and that the Areolæ of Matter in the firſt Diviſion touch each other only in four ſingle Points, in *i*, *g*, *h*, and *c*, ſuppoſing this Alternation of Space and Matter to be continu'd to ſixteen Square Areolæ, in which the Reaſoning above will be the ſame.

SECOND, it is manifeſt, that in the ſecond Diviſion, the Areolæ of Matter recede farther from each other, and have more Vacuity betwixt them, and that they do not touch excepting in two Points, and many of them in one Point only.

THIRD, it is manifeſt, that in the third Diviſion the Areolæ of Matter ſtill farther recede from each other, and ſtill have more Vacuity betwixt them, and that their Points of Contact are fewer and rarer than in the Former.

FROM whence it is plain, that the farther the Diviſion is carry'd on, the ſmaller is the ſolid Part of Matter, which is ſuppos'd to be without any Vacuity, and the greater is the Vacuum round it, by which it is more remov'd from a Contact with any other Matter, that is, you diminiſh the Matter and increaſe the Vacuum, or Space about it.; and is not this a worthy and very extraordinary Solution of the Difficulty, which it in Fact confirms? We ſay it is a great Difficulty againſt a Vacuum and a Similar Matter, that there ſhould be ſo much void Space and ſo little Matter in Proportion, becauſe it do's not ſeem probable we could breath in ſuch a Medium; This Philoſophy anſwers that Difficulty by ſhewing, that there is ſo much void Space, and ſo little Matter in Proportion, and demonſtrates at the ſame Time, that theſe ſmall Parts of Matter in an infinitely rare Medium, may be infinitely remov'd from each other, for tho' the whole Square *a b c d* ſhould be infinitely ſmall, yet ſince the Particles of Matter in it are infinitely ſmaller, there will be the ſame Proportion betwixt Matter and Vacuity: And this is to ſolve Waters, not being compreſs'd in a Concave Sphere of Gold, and it's paſſing, as well as Quick-ſilver, thro' the Pores of it; This is to ſhew us how Light may be tranſmitted thro' Glaſs, how the Magnet may act upon Iron thro' all denſe Bodies, not Magnetick nor red Hot, as Gold, Silver, Lead, Glaſs, Water, without any Diminution of it's Vertue: And laſtly, how the gravitating Power of the Sun may be tranſmitted thro' the vaſt Bodies of the Planets without any Diminution, ſo as to act upon all their Parts to their very Centers with the ſame Force, and according to the ſame Laws, as if the Part, upon which it acted, were not ſurrounded with the Body of the Planet; Namely, becauſe all Bodies are more Porous than we imagine, and becauſe even in Gold it ſelf, there are more Pores and Vacuities than there is Matter, which Pores and Vacuities are before explain'd, and their Proportion to the ſolid Parts in Bodies, by the preceding Calculation.

AND now I ask could any Authority, beſides that of the preſent Philoſophy, ever ſupport ſo abſurd an Aſſertion as this, that Gold and conſequently all other denſe Subſtances here mention'd have more Pores than ſolid Parts? Why, we muſt bid adieu to our Senſes, and to all our Notices communicated from thence, if we muſt acknowledge this for a Truth, and affirm that Solidity, Plain, Viſible, and palpable Solidity is nothing but mere Space, this is a Philoſophical Tranſubſtantiation, which requires as much Faith to believe it, and as much Infallibility to maintain it, as the Popiſh or Religious one; if there was ſo much more Space and Vacuity in denſe Subſtances than Matter, as this Philoſophy repreſents, wou'd it not be as manifeſt, as we find it to be in the foregoing Scheme, which is made uſe of to eſtabliſh this Doctrine? Should we not perceive little Parcels of Matter diſpers'd with vaſt Spaces ſurrounding them, and not ſmall Spaces and Vacuities, and almoſt inviſible ſurrounded with vaſt Portions of Matter? And wou'd not ſuch a Matter as the preſent Figure exhibits

Principles of Natural Philoſophy Chap. 7. §. 8. Part I.

exhibits to us be eaſily compreſs'd, and the Particles of it ſoon cruſh'd and conſtipated into a cloſer Connexion? Wou'd Water ſweat out in ſmall Drops like Dew, or pour out in large and handſome Streams? Wou'd not ſuch a Conſtitution of Matter be as leaky and perfluous as a Sieve, and do's it not give us the Picture of one? Theſe Queſtions carry their own Anſwers with them, which is, that they need none; but we have argu'd againſt this kind of porous Matter in our *Principles of Natural Philoſophy*, when we conſider'd this very Experiment of the Impoſſibility of Compreſſing Water in a Concave Sphere of Silver, an Experiment made by the Academy *Del Cimento*. As to the Magnet, it is ſomething ſtrange when there are ſuch large Pores contriv'd for the Paſſage of it's Force, that it cannot get thro', if the ſolid Body is Magnetick, or Heated; one wou'd be apt to imagine there might be Room enough for it, if nothing elſe was neceſſary, but Vacuum, to it's Tranſmiſſion, but the Caſe is otherwiſe, as we ſhall afterwards ſhew.

As to the gravitating Power of the Sun acting upon every Particle of Matter in the vaſt Bodies of the Planets to their very Centers without any Diminution, it will be afterwards prov'd, that ſuch an Action is not certain in Fact, and is only a Creature form'd in a great Meaſure from the Principles of this Philoſophy, into which we are now inquiring.

The Difficulties endeavour'd to be ſolv'd by a Vacuum, and a Similar Matter, are better accounted for by a Plenum, and a Matter which is Heterogeneous.

§. 6. On the Contrary ſuppoſing a Plenum and an Heterogeneous Matter, moſt of theſe Difficulties obtain an eaſy Solution.

Sir Iſ. Newt. Princip. Prop. 19. Theorem. 14. §. 5. Lib. 2.

First, Water and even Air cannot be compreſs'd, the firſt little or not at all, and the laſt, excepting to a certain Degree, becauſe all Spaces according to the Philoſophy we propoſe are full, tho' not equally ſo, or rather, becauſe the Action of Water is Homogeneous in it ſelf, and Heterogeneous to all others, and therefore ſupports it ſelf in this Caſe of Compreſſion againſt any external Force, as much as any ſolid Body or Action will do, but if there is any Weaker or Heterogeneous Action as that of Air intermixt with that of Water, it may be compreſs'd, till the Action of the included Air becomes equal to that of Water; and thus the Subſtance or Action of Air, being intermingled with Heterogeneous Subſtances or Actions, may be compreſs'd till thoſe Heterogeneous Subſtances or Actions become equal to that of Air, but when any Subſtance or Action is Homogeneous it will be as impoſſible to Compreſs it, be it weaker or ſtronger in it's own Nature, as to make it what it is not; ſince if you afterwards Compreſs it, it being equally balanc'd in all it's Parts, it cannot admit of any Preponderancy, Alteration, or Motion in them, as is evidently prov'd in this Philoſophy, where it is ſaid, that, the Parts of an Homogeneous, and unmov'd Fluid, which is incloſ'd in an unmov'd Veſſel, and is compreſs'd on all Hands (ſetting aſide the Conſideration of Condenſation, Gravity, and all Centripetal Forces) are preſs'd equally on every ſide, and without any kind of Motion ariſing from that Preſſion, remain and continue in their own Places.

Second, Gold or Silver, upon the Compreſſion of the Water included, may give a Paſſage to it, without either of them having any Pores, in the ſame Way and Manner, as Paper or Parchment to a Needle, that is, where the Force impreſs'd is Superior to the reſiſting; thus tho' the Subſtance or Action of Gold or Silver is ſuperior to that of Water, yet ſince there is an additional Force by Compreſſion to Water, which, as we have ſaid, can have no effect to diſorder it, which is equally Balanc'd and Homogeneous, that additional Force makes the Water recoil upon the Heterogeneous Action or Subſtance of Gold or of Silver, and being made by that means Superior to it, it over-balances and penetrates thro' it, and forms, and points out a Paſſage to it ſelf, which was not made before by any imaginary Pores or Vacuities.

Third, Quick-ſilver enters into the Subſtance or Action of Gold by it's own innate Force and Volatility, by which it's Action is Superior to it, in the ſame manner, as Water into Salt, or Water compreſs'd into Gold, or *Aqua fortis* into Iron, that is, it makes it's own way into the Subſtance or Action of it, without being beholden to any Pores or Vacuities for it's Ingreſs; and the ſame may be ſaid of Light in it's Tranſmiſſion thro' Glaſs, or of the magnetick Force thro' the denſeſt Bodies, and the Reaſon, that the magnetick Force will not have the ſame effect upon Bodies which have a magnetick Force, and which are heated red Hot, is, becauſe thoſe Forces are more than a Balance to it, and repel it, by a ſuperior Degree of the Expanſive, which will be afterwards Explain'd.

Concerning the ancient Authorities relating to a Plenum and a Vacuum drawn from Plutarch *and* Laertius.

§. 7. As to what is ſaid concerning the Authority of the *Grecian* and *Phœnician* Philoſophers in the preſent Caſe, the Ancienteſt and moſt Eminent of whom made a Vacuum and Atoms, and the Gravity of them, the firſt Principles of their Philoſophy, and rejected a Plenum, this is ſo far from being True, that the Contrary is Certain and Evident., and what we affirm'd, True, *Princip. Philoſ. Pref.* §. 5.

Plutarch in his Account of the Opinions of Philoſophers in five Books, Chap. 18. Book the firſt, aſſures us, that all the Natural Philoſophers from *Thales* to *Plato* rejected a Vacuum, and that *Empedocles* allow'd of no Vacuity in Nature; that *Lucippus, Democritus, Demetrius, Metrodorus,* and *Epicurus* affirm'd an infinite Number of Atoms, and that a Vacuum was infinite in it's Extenſion, that *Ariſtotle* and alſo the *Stoicks* acknowledg'd a Vacuum beyond the World, but that all was a Plenum within it, *Laertius* alſo is a full and unconteſted Witneſs to the ſame Thing.

Concerning Pherecydes *and* Thales.

§. 8. *Diogenes Laertius* in his Lives of the Philoſophers Lib. I. Segm. 116. *Menagius*'s Edition, in the Life of *Pherecydes* the *Syrian* or *Phœnician*, tells us, that *Theopompus* affirms this *Pherecydes* was the firſt who writ concerning Nature and the Gods, in reſpect of which we are inform'd from another Author, that tho' he did not ſacrifice to any, yet he us'd to ſay, that he led as pure and vertuous a Life, as if he had ſacrific'd Hecatombs, and that there was a ſmall Book extant of his, when *Diogenes* writ of which he juſt mentions the Beginning, he alſo gives us an Epiſtle, which he ſent to *Thales*, and this is the Chief of what we have of *Pherecydes*, excepting that *Thales* the Author of the *Ionick* Sect in Philoſophy, and *Pythagoras* of the *Italick* were both of them his Auditors and Scholars, from whom therefore we may poſſibly be inſtructed in the Sentiments of the Maſter.

In the Life of *Thales*, Lib. I. Segm. 22. of the ſame Edition, it is ſaid that he was a *Phœnician* by Family, and that he apply'd himſelf to the Studies of Nature, and follow'd them chiefly in *Ægypt* as *Plutarch* writes, but left no Monument of his Parts, or his Sagacity in theſe Matters behind him, as ſome affirm, and that according to *Callimachus* he was the firſt who obſerv'd the leſſer Bear; but others think he writ concerning the Tropicks, and the Æquinoxes, and moſt are of Opinion, if *Eudemus* may be credited in his Hiſtory of Aſtrology, that he was the firſt, who ſearch'd into the Secrets of that Science, and who foretold the Eclipſes of the Sun, and the Changes of the Air, that *Xenophanes* and *Herodotus* admir'd him for it, and *Heraclitus* and *Democritus* atteſted the ſame, that ſome alledg'd he was likewiſe the firſt, who Taught, that the Soul was Immortal, that the Sun's Courſe was from Tropick to Tropick, and that the Lunar Orb was the ſeven hundred and twentieth Part of the Sun's Magnitude, that according to *Ariſtotle* and *Hippias* he attributed *Souls* to *inanimate Beings*, from what he obſerv'd of the Attractions of the Load-ſtone, and of Amber, that he learn'd the Precepts of Geometry in *Ægypt*, and was the firſt, who form'd a Rectangle Triangle by deſcribing it in a Semicircle, whoſe Baſis was the Diameter, and the Angle in the Circumference, as likewiſe who advanc'd the Doctrine of Triangles and Rectilineal Deſcriptions, tho' *Callimachus* aſcribes it in his Iambicks to *Euphorbus* the *Phrygian*, that he was a ſincere Promoter of the Intereſts of his Country, and preſerv'd it from Ruin by diſſuading the *Miletians* from joining with *Crœſus* againſt *Cyrus*, that he lov'd a private and retir'd Life, prefer'd Celibacy to Marriage, and contemn'd Riches, and planting of Olive-yards, ſaying, that *Water was the Principle of all Things, and that the World was animate and full of that Animation*, or of *Spirits*, that is by what forego's concerning the Magnet and Amber, of *Forces* which were unaccountable, or of Matter ſuppoſing it to conſiſt of ſuch Forces.

Concerning Anaximander *and* Anaximenes.

§. 9. In the Life of *Anaximander*, who as *Thales*, was alſo a *Mileſian*, and of the *Ionick* Sect, was Contemporary with him, and his Acquaintance Lib. 2. Segm. 1. of the ſame Edition, we are inform'd, that his Opinion was, that the Principle and Element of all Things was immenſe, and infinite, and therefore that it was *full*, not yet determining whether it was Air, or Water, or any Thing elſe, however, he eſteem'd it to be Changeable in it's Parts, tho' Immutable in the Whole, that the Earth was plac'd in the mid'ſt and Center of this infinite Plenum, and that it was Globular and Round, that the Moon was an opake Body, and deriv'd it's Light from the Sun, and that the Sun was at leaſt as big as the Earth, and conſiſted of the pureſt, and ſubtleſt, and moſt refin'd Fire; He firſt found out the Gnomon, and plac'd it according to *Phavorinus* in the Dials at *Lacedæmon*, in order to obſerve the Æquinoxes and

Solſtices, and was the firſt Perſon who meaſur'd the Ambitus of the Earth, and who made a Sphere.

Anaximenes, Lib. 2. Segm. 3. of the ſame Edition, was a *Mileſian* likewiſe, the Scholar of *Anaximander,* and a Correſpondent of *Pythagoras,* there being extant two Letters of his to him, in one of which he highly extols *Thales,* gives an Account of his falling down a Precipice, when he went out one Morning with his Maid, as his Cuſtom was, to make Obſervations in the Heavens, not minding the Situation of the Place, which was the Occaſion of his Death, according to the *Mileſians,* ſays, he acquieſces in his Doctrine, and believes him to have found out the Principle of the Univerſe; in the other he complains of the Tyrannical Government amongſt the *Mileſians,* and that they were in a continual Fear of an Invaſion from the *Medes,* upon which account he had but little Heart for Aſtronomical Obſervations, who was under conſtant Apprehenſions of Death or of Slavery, and he commends *Pythagoras*'s Prudence for removing in ſuch hazardous Times from *Samos* to *Croton* and *Italy,* where he was regarded and eſteem'd. This Perſon taught that Air was the Principle of all Things, that it was Infinite, and that therefore Space was *full* of it, and that the Stars did not move over the Earth, but about it.

Concerning Anaxagoras *and* Archelaus.

§. 10. *Anaxagoras,* Lib. 2. Segm. 6. was of *Clazomene* in *Ionia* on the ſame Continent with *Miletus,* and the Scholar of *Anaximenes,* and whereas all before him had affirm'd Matter or the ὕλη to be the firſt Principle, that is the Infinite or the Plenum, he added an Intelligence and a Mind to it, and was therefore call'd the Mind or the Intelligence, and he begins his Philoſophy thus; all Things exiſted together, and at once, that is, there was an Univerſal Plenum of Being, which exiſted of it ſelf, and together, and in one time, or always, and afterwards the Mind or Intelligence diſpos'd it into Beauty, and order, and form'd, and conſtituted the World; he is ſaid to have been Noble by his Family, and Rich in his Poſſeſſions, but particularly eminent for the Greatneſs of his Mind, in quitting his whole Patrimony and reſigning it to his Relations, his farther Opinions were, that the Sun was a *red hot Iron,* heated quite thro', and Greater than the *Peloponneſus,* that the Moon was Inhabited, and had it's Hills and it's Valleys, ἀρχὰς δὲ τὰς ὁμοιομερείας, καθάπερ γὰρ ἐκ τῶν ψηγμάτων λεγομένων τὸν χρυσὸν συνεστάναι, οὕτως ἐκ τῶν ὁμοιομερῶν μικρῶν σωμάτων τὸ πᾶν συγκεκρίσθαι, namely, that the Principles of Bodies, of which they conſiſt, were Homogeneous to the whole, that is, that the Parts were of a like Nature with the Bodies which they compos'd, as Gold is made up of the ſmall Raſures or Fileings into which it may be divided, and in like manner, that the whole Univerſe was made up of Bodies Homogeneous to the whole, which conſiſts of them, and into which it may be likewiſe divided, and as to Motion, that the Mind or Intelligence was the Origine of it; that heavy Bodies tended downwards, as Earth, and that light ones, as Fire, aſcended upwards, but that Water and Air affected a middle Situation, for thus Water lies above the Earth, and is eaſily converted into Vapours by the Sun; that the Pole was originally in the Vertex, or directly over our Heads, and the Stars were carry'd round it's Axis, but that it afterwards receiv'd it's Inclination, that the Galaxy or *Via Lactea* was owing to a Reflection of the Sun's Light, and not to innumerable Stars ſhining and diſpers'd in it, τοὺς δὲ κομήτας σύνοδον πλανητῶν, φλόγας ἀφιέντων, τούς τε διάττοντας οἷον σπινθῆρας ἀπὸ τοῦ ἀέρος ἀποπάλλεσθαι, that the Comets were the Congreſſion or Concourſe of wandering Stars, which emitted Flames, and that theſe Emications, like the Twinkling in the Stars, were ſtruck by the Vibrations of the Air, that Winds were cauſed from the Rarefaction of the Air by the Sun, that Thunder aroſe from the Daſhing together and the Colliſion of Clouds, and that Lightenings were deriv'd from the Attrition of them; that Earth-quakes were made by the Airs being included in the Cavities of it, and endeavouring to return to the Exterior Parts, and to the Air above them; that Animals were firſt produc'd from Moiſture and Heat, but afterwards from Male and Female; that all the Heavens were form'd of Stones, and that by their Revolution they were kept in their Orbit, which otherwiſe wou'd fall from thence, [which poſſibly might be no more than that the Planets were ſolid and heavy Bodies, and were revolv'd in their Orbits, and that they wou'd ſink to the Center, if not kept up by ſuch a Revolution, and it is not improbable, that the Story of his foretelling a Stone falling down from the Heavens or the Sun about the River *Ægos,* was nothing more than a Fiction of ſome, who did not underſtand his Principles;]

Sir Iſ. Newt. Princip. Prop. 41. Lib. 3. Pag. 466. Edit. 2.

Deſcartes's Princip.

Sir Iſ. Newt. Princip.

This is however certain, that his Philoſophy was charged with Impiety, that he was accus'd by *Cleon* for aſſerting, that the Sun was a red hot Iron, and defended by his Scholar *Pericles* the famous *Athenian*, that he was notwithſtanding fin'd five Talents and condemn'd to Baniſhment, and afterwards, as a Betrayer of his Country, ſentenc'd to Death, all which he bore with an invincible Patience and Conſtancy.

Archelaus, Lib. 2. Segm. 16. was alſo a *Mileſian*, the Scholar of *Anaxagoras*, and the Maſter of *Socrates*, and the firſt who introduc'd Natural Philoſophy into *Athens* from *Ionia* (ſo that *Anaxagoras* do's not ſeem by this to have publickly taught it in *Athens*) and it is ſaid to have ended with him, by *Socrates*'s advancing Morality and Ethicks in it's Room, the Foundations and Principles of which he is thought to have learn'd from this Philoſopher, who reaſon'd in his Lectures, concerning Laws, and what was Vertuous and Commendable, and what was Juſt; His Opinions were, that there were two Cauſes of Productions or Generations, Hot and Cold, that Animals and Men alſo were form'd from a Mud or ſlimy Matter, that Law and Preſcription, and not Nature was the Meaſure of what was Juſt, or Baſe and Diſhoneſt, that Voice and Sound was from the Percuſſion of the Air, that the Sea ſtood in a Concave Baſon, and the Water ouz'd thro' the Earth into it, that the greateſt of the Stars was the Sun, and that the τὸ πᾶν or the Univerſe, or the ὕλη, (for we hitherto find no Mention of Space) was Infinite.

Concerning *Socrates, and* Plato, *and* Ariſtotle.

§. 11. *Socrates* turn'd all his Philoſophy upon Morality, but no doubt had ſome Opinions and Notions concerning Nature, and as he was a Diſciple of the *Ionick* Sect, and his Maſter *Archelaus*, who all affirm'd a Plenum, and as *Plato*, who was *Socrates*'s Auditor at twenty Years old, and join'd the Morality of his Maſter with the Philoſophy of *Anaxagoras* and *Thales*, affirm'd the ſame Doctrine, it is reaſonable to think that *Socrates* acquieſce't in it, tho' his Genius and Studies led him to inquire more into the Mind and Intelligence of *Anaxagoras*, and the Vertue and Vice of *Archelaus*, than into their Diſputes concerning Matter, and the Conſtitution of the Univerſe.

FROM *Socrates* we have a long Series of Philoſophers, namely, *Xenophon*, *Æſchines*, *Ariſtippus*, *Phædon*, *Euclides*, *Diodorus*, *Stilpo*, *Crito*, *Simon*, *Glaucon*, *Simmias*, *Cebes* the Author of the Table, and *Menedemus*, till we come to *Plato*, who all buſy'd themſelves in Ethicks, Logicks, Oeconomicks, or Politicks, and which we may juſtly ſuppoſe were only Efforts towards a Reformation of Government, and Manners againſt Polytheiſm and Paganiſm, in which Cauſe *Pherecydes* firſt appear'd, a *Syrian* or *Phœnician*, inſtructed, it is likely, by the *Ægyptians* or the Books of *Moſes*, and after him *Thales*, *Anaximander*, *Anaximenes*, and *Anaxagoras* by aſſerting Philoſophy and one infinite Being, in Oppoſition to all the Vanities of the Heathens, and their Plurality of Gods, in the Courſe of which Doctrines, and the aſſerting of them, *Anaxagoras* and *Socrates* loſt their Lives.

Plato, Lib. 3. Segm. 69. δύο τῶν πάντων ἀπέφηνεν ἀρχὰς θεὸν ϗ ὕλην, ὃν ϗ νοῦν προσαγορεύει ϗ αἴτιον, εἶναι δὲ τὴν ὕλην ἀσχημάτιστον ϗ ἄπειρον, ἐξ ἧς γίνεσθαι τὰ συγκρίματα, ἀτάκτως δέ ποτε αὐτὴν κινουμένην, ὑπὸ τοῦ Θεοῦ φησὶν εἰς ἕνα συναχθῆναι τόπον, τάξιν ἀταξίας κρείττονα ἡγησαμένου, τραπέσθαι δὲ τὴν οὐσίαν ταύτην εἰς τέτταρα στοιχεῖα, πῦρ, ὕδωρ, ἀέρα, γῆν, ἐξ ὧν αὐτόν τε τὸν κόσμον ϗ τὰ ἐν αὐτῷ γεννᾶσθαι, That is, *Plato* held, and it was his Doctrine, that there were two Principles of all Things, God and Matter, whom he alſo call'd the Mind or Intelligence, and the Cauſe; that Matter was without any Form and Infinite, and from it the ſeveral Concretions, and particular Bodies or Subſtances of it were made, that originally there were Motions in this Matter but very Diſorderly and Irregular, and that by God it was reduc'd into one Place, (that is, according to the Account given us by *Moſes*, each Portion of Matter ſeparated to it's own Kind or Species) God, or the Mind or Intelligence, preferring Beauty and Order to the Contrary, and therefore that this Eſſence, or Subſtance, or Plenum was converted into four Elements, Fire, Water, Air and Earth, of which Elements the World, and all Things in it were form'd and made: As to the Character of *Plato* and his Writings, they are ſo well known, that there is no occaſion for me to tranſcribe his other Opinions, but it is very manifeſt and plain, that he exactly agrees with *Anaxagoras* in his Matter, and his Mind or Intelligence, and the Diſpoſition of the Former, by the Latter, into the Frame of the World in which we now ſee it, and therefore it is very reaſonable to believe, that *Socrates*, who was a Diſciple of

Archelaus, the Scholar of *Anaxagoras*, and who was the Maſter of *Plato*, enter'd into the ſame Notions.

FROM *Plato* Lib. 4. there runs on an Order of Names, *Speuſippus*, *Xenocrates*, *Polemo*, *Crates* the *Athenian*, and Scholar of *Polemo*, *Crantor*, *Arceſilaus*, *Bion*, *Lacydes*, *Carneades*, *Clitomachus*, till we come to *Ariſtotle*, who ſpent their Time in writing of Commentaries and Dialogues on Ethical Subjects moſtly, in Diſputing, and in Eloquence, without any great Advantage to the Knowledge of Nature, in which we may ſuppoſe they held much the ſame Tenets with *Anaxagoras* and *Plato*.

Ariſtotle, Lib. 5. Segm. 1. was of *Stagira* on the *Strymonian* Bay, and the Scholar of *Plato*, Segm. 32. he held that God was Incorporeal, as *Plato* before him, that his Providence extended to the Heavens, and that he was Immoveable, he affirm'd four Elements, Air, Earth, Fire, and Water, that the Heavens, and the Heavenly Bodies conſiſted of a Matter different from all theſe, which was his *Quinta Eſſentia*, and that it had a peculiar Motion belonging to it, namely, an Orbicular one, from whence it is Evident, that as *Anaxagoras* and *Plato*, ſo he aſſerted a Plenum of Matter, and a Mind or Intelligence which agitated and mov'd it, beſides it is confeſs'd on all Hands, that he maintain'd the Material World was Infinite and Eternal, and conſequently, that it was *full*, excepting that *Plutarch* denies it, who it is evident was much more Converſant in the Actions and Sayings of Eminent Men, than Wiſe or Judicious in the Sentiments and Opinions of Philoſophers.

Theophraſtus was the Scholar of *Plato* firſt, and afterwards of *Ariſtotle*, Lib. 5. Segm. 36. and therefore it may be reaſonably preſum'd that his Opinion was the ſame in this Point, in which he found both his Maſters to agree, he writ an immenſe Number of Books upon all Subjects, tho' we have nothing remaining of him excepting a ſmall Manual of Characters, and his Volumes are ſaid to contain in the Whole two Hundred and thirty Thouſand eight Hundred and eight Verſes or Diſtinctions.

After him, Lib. 5. Segm. 58, &c. come *Strato*, *Lycon*, *Demetrius*, *Heraclides*, and Lib. 6. Segm. 1, &c. *Antiſthenes*, *Diogenes*, *Monimus*, *Oneſicritus*, *Crates* the *Theban* and Scholar of *Diogenes*, *Metrocles*, *Menippus*, *Menedemus*, amongſt whom we find no mention of a Vacuum, but it is very probable they acquieſce't in the Doctrine of a Plenum, ſuitable to what had been taught by *Anaxagoras*, *Plato*, and *Ariſtotle*, from whom they deriv'd their Philoſophy.

Concerning Zeno *and* Cleanthes.

§. 12. *Zeno*, Lib. 7. Segm. 1. was of *Cittium* in *Cyprus* a ſmall *Greek* Town, and inhabited by a Colony of *Phœnicians*, and from thence was call'd a *Phœnician*, and by ſome an *Ægyptian* for the ſame Reaſon, he was the Father of the *Stoick* Philoſophy, as *Plato* of the *Academick*, *Ariſtotle* of the *Peripatetick*, and *Diogenes* of the *Cynick*, and his addicting his Mind to the Studies of Philoſophy, was from his bringing ſome Merchandize in a Ship from *Phœnicia* to *Athens*, where he loſt all by a Wreck which he ſuffer'd at the *Piræus*, he was thirty Years old when he came into *Athens*, and ſiting down in a Bookſeller's Shop, and Reading the ſecond Book of *Xenophon*'s Commentaries, he was ſo much pleas'd with it, that he inquir'd where he might meet with ſuch Men, *Crates* the *Cynick*, who had been *Diogenes*'s Scholar, accidentally paſſing by at that Time, the Bookſeller pointed to him, and bid him follow him, which he accordingly did, and became for ſome time his Auditor, whom afterwards he left, and for Twenty years from thence was a Diſciple of *Stilpo*, *Xenocrates*, and *Polemo*, and from this Philoſopher it is, and his Followers, that we firſt hear of a Vacuum amongſt thoſe of the *Ionick* Sect, and it is not impoſſible, but this may be one of the *Phœnicians* whom the preſent Philoſophy intends, when it mentions them as Patrons of that Doctrine; If it were ſo, there wou'd be no great advantage of Authority to this Philoſophy from a Sect of Men, as the *Stoicks*, who were ſo far from explaining Nature, that they went farther from it, and embrac'd more Romantick and Extravagant Notions concerning it, than any other Denomination of Philoſophers; Thus their making the World a real Animal, and aſſerting that there was no Evil in Pain, are ſufficient Evidences of it.

BUT it is alſo to be conſider'd, that the Vacuum they contended for was nothing like that, which the preſent Philoſophy eſpouſes, and ſeems to be little more than an Æthereal Matter, and the Quinteſſence of *Ariſtotle*. For Segm. 134. to 141. we find, that there are two Principles of all Things affirm'd, τὸ ποιοῦν ϗ τὸ πάσχον, the

Efficient, and the Patient; That which is Paſſive is the ὕλη or Matter, ἄποιος οὐσία, a Subſtance in general without any particular Quality or Property, ſo that it is not Gold, or Air, or Water, or Earth, or Fire, but an univerſal Matter, or the *Prima Materia* of the *Logicians*, out of which the *Materia Secunda* was form'd; And that which is Active is the Wiſdom and Reaſon obſervable in it, which diſpoſes it into Order, and to ſeveral Uſes and Ends, which Wiſdom and Reaſon is God; Now this ſeems to be the very Doctrine of *Anaxagoras* and *Plato*, who affirm'd a Plenum, and a Mind or Intelligence belonging and annex't to it; This will farther appear from what follows in this Segment, where it is ſaid, that there is a Difference betwixt the Principles and Elements of Things, ſince the firſt, the ἀρχαὶ are ἀγέννητοι ϗ ἄφθαρτοι, they are unproduc'd and incorruptible, the laſt, the στοιχεῖα, κατὰ τὴν ἐκπύρωσιν φθείρονται, that is, by the Influences of the Sun, Water is rais'd into Vapours, the Earth is parch'd, the Air heated, and Elementary Fire extinguiſh'd, and therefore theſe Elements are Corruptible, but God and Matter are unproduc'd and Incorruptible, if Matter therefore is unproduc'd and Incorruptible, as God himſelf, how is it poſſible, in the Notion of the *Stoick* Philoſophy, that it ſhould not be Infinite, and Eternal, and therefore that the World ſhould not be full of it? Another Diſtinction, which is ſaid to be betwixt Principles and Elements, is στοιχεῖα σώματα εἶναι ϗ μεμορφῶσθαι, τὰς δ' ἀρχὰς ἀσωμάτους ϗ ἀμόρφους, that the Elements, as Air, Water, Earth, and Fire are Bodies, and have a certain Form affixt to them, by which they become ſuch a Kind of Matter, and of a definite Shape and Subſtance, but that the Principles, as God and Matter, are Incorporeal, and without Form or any definite Shape or Subſtance; If therefore Matter is Incorporeal, it is plain the Vacuum of the *Stoicks* was Matter, tho' not ſuch as Corporeal Beings, or any Syſtems or Maſſes of groſs Matter, conſiſt of, it was the *Quinta Eſſentia* of *Ariſtotle*, or the *Prima Materia* of the *Logicians*, as we before obſerv'd; If farther this Matter was ἄμορφος, undefin'd, without any particular Shape or Subſtance, it muſt be Infinite, and by Conſequence the *Stoicks* muſt have held a Plenum of a moſt ſubtle, confus'd, and undiſtinguiſh'd Matter, out of which the Mind or God, they afterwards tell us, produc'd the four Elements above-mention'd; they likewiſe taught, that the World was properly a Quality of the univerſal Subſtance, ἐστι κόσμος ὁ ἰδίως ποιὸς τῆς τῶν ὅλων οὐσίας, that the World was finite, rul'd and govern'd by the Providence of God, that it was one, and of a Globular Figure, ἐν δὲ τῷ κόσμῳ μηδὲν εἶναι κενὸν, ἀλλ' ἡνῶσθαι αὐτὸν, and that in it there was no Vacuum, but all Things in it were united and cemented together, but that beyond it, there was an immenſe Vacuum, which was Incorporeal, and that Incorporeal was what contain'd Bodies, but not contain'd by them, from whence it is evident, that they allow'd no Vacuum in the World, which yet the preſent Philoſophy aſſerts, but thought the Fire, which they plac'd in the Heavens, that is in the uppermoſt Region, Air, which they diſpos'd immediately under it, Water, which was the next, and Earth, which was the loweſt, were united and connected together, ſo as to make one continu'd tho' Difform and Diſſimilar Plenum, as conſiſting of Heterogeneous Matters and Subſtances; nor is it manifeſt, that the Vacuum beyond this World was meer Space, only an univerſal Subſtance, out of which the particular Ones of the World were made, it was Incorporeal, that is, had none of the Bodies or diſtinct Syſtemes of Matter in it, which were found in the World, and it was what contain'd Bodies as ſurrounding and inveſting them, but was not contain'd by them, becauſe it was ἄπειρος and Immenſe.

Cleanthes, famous for his indefatigable Labour and his Study, was a Scholar of *Zeno*, Lib. 7. Segm. 168, &c. and was entirely attach't to his Doctrines.

Sphærus ſeems to have been an Auditor of *Zeno* firſt, and after of *Cleanthes*; and *Chryſippus*, tho' an Auditor of each, diſſented from both in many Particulars, he was a celebrated Logician and Diſputant, but it do's not appear, that he made any great Advances in Nature and Philoſophy, and here ends the *Ionick* Sect, who it is very plain were unanimouſly for a Plenum, and even *Zeno* in the World, and in the Senſe which I have explain'd, beyond it: We ſhall now proceed to the *Italick* Sect, which was another Branch of the *Syrian* or *Phœnician* Philoſophy taught by *Pherecydes*, of which his Scholar *Pythagoras* was the Chief, as *Thales*, and after him *Anaxagoras* of the *Ionick*.

Concerning Pythagoras *&* Empedocles.

§. 13. *Pythagoras,* Lib. 8. Segm. 1, 2, 3, &c. whether a *Syrian, Samian,* or a *Tuſcan,* is a Queſtion, was the Scholar of *Pherecydes,* and after his Death, of *Hermodas* at *Samos;* when he was young, out of a great Affection and Inclination to Knowledge, and the being inform'd, he left his native Country, and was initiated almoſt in all the Myſteries of the *Greeks* and *Barbarians,* at length he came to *Ægypt,* and learn'd their Language, and from thence into *Chaldæa,* where ſome affirm he had *Zoroaſtres* for his Maſter, he afterwards return'd to *Samos,* and finding his Country oppreſs'd by the Tyranny of *Polycrates,* he went to *Croton* in *Italy,* where he was Celebrated for his Number of Scholars, and the Wiſdom of his Laws, which he gave to the *Italians,* he had Brothers, *Eunomus* and *Tyrrhenus,* his Father, *Mneſarchus,* and a Servant, whilſt he continu'd at *Samos,* named *Zamolxis,* who being releas'd from his Servitude, taught the Immortality of the Soul amongſt his Country-men the *Getæ,* and was worſhip'd by 'em as a God; Segm. 25. his Doctrine was, ἀρχὴν μὲν τῶν ἁπάντων μονάδα, ἐκ δὲ τῆς μονάδος, ἀόριστον δυάδα ὡς ἂν ὕλην τῇ μονάδι αἰτίῳ ὄντι ὑποστῆναι, ἐκ δὲ τῆς μονάδος, καὶ τῆς ἀορίστου δυάδος, τοὺς ἀριθμούς, ἐκ δὲ τῶν ἀριθμῶν, τὰ σημεῖα, ἐκ δὲ τούτων, τὰς γραμμάς· ἐξ ὧν τὰ ἐπίπεδα σχήματα, ἐκ δὲ τῶν ἐπιπέδων τὰ στερεὰ σχήματα, ἐκ δὲ τούτων τὰ στερεὰ σώματα, ὧν καὶ τὰ στοιχεῖα εἶναι τέτταρα, πῦρ, ὕδωρ, γῆν, ἀέρα, ἃ μεταβάλλειν δὲ καὶ τρέπεσθαι δι᾽ ὅλων, καὶ γίνεσθαι ἐξ αὐτῶν κόσμον, ἔμψυχον, νοερόν, σφαιροειδῆ, μέσην περιέχοντα τὴν γῆν, καὶ αὐτὴν σφαιροειδῆ καὶ περιοικουμένην, that the Principle of all Things was one, that is, that the Univerſe was one individual Thing, which contain'd every Thing in Nature, that this one Nature was divided into Two, into Matter, and into the Intelligence or the Mind the Author of it, and that this Duality, God and Matter, were Interminate, or Infinite, and from this Duality aroſe Numbers, from thence Points, and from thence Lines, and from thence Surfaces, and from Surfaces, Solids, according to the Mathematick Way of Generation, from Solids, Bodies, or Corporeal Subſtances, and from them the four Elements, Fire, Water, Earth, and Air, which poſſeſs'd and paſs'd thro' all Things, that is, which, one or other, were every where diſpers'd thro' the vaſt Immenſum, and conſtituted the World, which had a Soul, and a Mind, or Underſtanding belonging to it, was Globular, contain'd the Earth in the middle of it, which was it ſelf Globular and Inhabited all round, ſo as to have Antipodes; *Plutarch* likewiſe affirms this to be the Notion of *Pythagoras,* and *Aldobrand* and *Menage* very truly Remark that all Philoſophers, as well as *Pythagoras,* maintain'd that the World was animated, and had an Intelligence annex'd to it, excepting *Democritus* and *Epicurus;* ſo that all the Philoſophers, who affirm'd a Plenum, acknowledged an infinite Mind and Intelligence, which actuated it, and the two Philoſophers, and their Followers, who deny'd this Plenum, and aſſerted a Vacuum, were thoſe, who deny'd ſuch a Mind or Intelligence; and we may obſerve that theſe were Men ſtrict in their Reaſon, & ſerious in their Philoſophy, Vertuous in their Lives, and Pious towards God, and the *Pythagoreans* in particular are celebrated for their Precepts of Piety, Morality and Vertue; and in this ſtrain went Philoſophy, till Laugh and Grimace, Irreligion and Atheiſm began to be ſubſtituted in it's Room, by *Democritus* and *Epicurus,* the firſt Promoters of Atoms and a Vacuum.

I ſhall only juſt mention the Names of the Philoſophers, who came after *Pythagoras,* and then proceed to the Sentiments of *Democritus.*

Empedocles, Lib. 8. Segm. 54. was the Auditor of *Pythagoras,* and Segm. 56. of *Parmenides,* at the ſame Time as *Zeno,* he afterwards was a Scholar of *Anaxagoras,* and *Pythagoras,* and propos'd the Gravity of Life, and of Habit of the firſt, and the Reaſonings upon Nature of the laſt for his Imitation, and according to *Ariſtotle,* he was the firſt Inventor of Rhetorick or Oratory, as *Zeno* was of Logicks or Dialecticks.

Epicharmus of *Coos,* Segm. 78. was an Auditor of *Pythagoras,* and *Archytas* the Geometer mention'd by *Horace,* Segm. 79. was a *Pythagorean,* and had an Intercourſe with *Plato,* by Letter, as is there recited, and the Letters betwixt them ſet down at large.

Alcmæon of *Croton,* Segm. 83. was alſo an Auditor of *Pythagoras;* and *Hippaſus* of *Metapontium,* and *Philolaus* of *Croton,* Segm. 84. were *Pythagoreans; Eudoxus,* Segm. 86. learn'd his Geometry from *Archytas,* and was an Auditor of *Plato.*

Heraclitus, Lib. 9. Segm. 1. was of *Epheſus,* it is not ſaid of what Sect he was, he blam'd many Things, but did not advance much, he had ſome Sayings, and ſmall Stories are told of him, which is the Caſe of the other Philoſophers, in which their

Hiſtory many Times chiefly conſiſts, according to *Laertius*; from him follow Lib. 9. Segm. 18, &c. *Xenophanes*, *Parmenides*, *Meliſſus*, another *Zeno*, *Leucippus* an Auditor of the *Stoick*, till we come to *Democritus*.

Concerning Democritus *and* Epicurus.

§. 14. *Democritus*, Lib. 9. Segm. 34, &c. was of *Abdera* in *Thrace*, on the Shore of the *Ægean*, betwixt the Rivers *Neſſus* and *Hebrus*, as ſome, and as others, of *Miletus*, it is ſaid he learn'd Theology and Aſtrology of the *Chaldeans*, when a Boy, and afterwards join'd himſelf to *Leucippus* and *Anaxagoras*, being younger than the latter about forty Years; he us'd to ſay, that what *Anaxagoras* had writ of the Sun and Moon was old, that his Opinions concerning them were not his own, but ſtol'n and purloin'd from Philoſophers, that were much ancienter; he likewiſe endeavour'd to detract from his Philoſophy in Relation to his Compoſition of the World, and his Mind or Intelligence annext to it, being implacably bent againſt, and an adverſary to him, becauſe he wou'd not ſuffer him to continue in the Number of his Scholars; it is ſaid, that he went into *Ægypt* to the Prieſts there to learn Geometry, and into *Perſia* to the *Chaldeans*, and as far as the Red-Sea, and that he had an Interview and Conference with the *Gymnoſophiſts* in *India*, he was the younger Brother of Three, and when a Diviſion was made of their Father's Subſtance into three Portions, he took the leaſt, which conſiſted all of Money, for which he ſhou'd have occaſion in his Travels, which was eſteem'd by his Brothers to have been a piece of Art and Craft in him, he was very much addicted to his Studies, and ſhut himſelf up in a ſmall Cell in a Houſe encompaſs'd with a Garden to proſecute them with more Vigour and leſs Interruption, he came to *Athens*, but was not willing to be known out of Modeſty, and a contempt of Glory, ſo that he knew *Socrates* tho' unknown by him, yet ſome affirm he never came to *Athens*; he was perfectly vers'd in all Natural, Moral, and Mathematick Knowledge, and in the Liberal Arts and Sciences, *Thraſyllus* relates, that he was a Follower of the *Pythagoreans*, and a great Admirer of *Pythagoras*; he liv'd in a very retir'd and ſolitary Manner, and us'd to be frequently amongſt the Sepulchers, which he might almoſt ſeem to inhabit; After his Return from his Travels, he appear'd in a very mean Condition, having ſquander'd away his whole Patrimony, till he foretold certain Matters, whoſe Event verify'd his Predictions, for which by ſeveral he was thought worthy of divine Honours, and to avoid the Ignominy of wanting a Grave, which by the Laws he was depriv'd of, for ſpending his Patrimony, he recited his Diacoſm, which was much the Beſt of all his Works, and for which he was honour'd with a reward of five Hundred Talents, and with Images of Braſs erected to him, he liv'd to his Hundred and ninth Year, and dy'd with great Complacency of Mind, and with little Pain; *Ariſtoxenus*, in his Hiſtorical Commentaries, ſays, that *Plato* wou'd have burnt what Writings he cou'd meet with of *Democritus*'s, which he collected for that Purpoſe, but that he was hinder'd by *Amyclas* and *Clinias* who were *Pythagoreans*, and who told him, that it wou'd ſignify little, ſince Copies of them were preſerv'd in ſeveral Hands; his Doctrines were, ἀρχὰς εἶναι τῶν ὅλων ἀτόμους ϗ κενόν· τὰ δ' ἄλλα πάντα νενομίσθαι δοξάζεσθαι, ἀπείρους τε εἶναι κόσμους ϗ γενητούς, ϗ φθαρτούς· μηδέντε ἐκ τοῦ μὴ ὄντος γίνεσθαι, μηδὲ εἰς τὸ μὴ ὂν φθείρεσθαι, ϗ τὰς ἀτόμους δὲ ἀπείρους εἶναι κατὰ μέγεθος ϗ πλῆθος· Φέρεσθαι δ' ἐν τῷ ὅλῳ δινουμένας, ϗ οὕτω πάντα τὰ συγκρίματα γεννᾶν, πῦρ, ὕδωρ, ἀέρα, γῆν· εἶναι γὰρ ϗ ταῦτα ἐξ ἀτόμων τινῶν συστήματα, ἅπερ εἶναι ἀπαθῆ ϗ ἀναλοίωτα διὰ τὴν στερρότητα. That the Principles of all Things were Atoms and a Vacuum, and that all other Things reſted in Opinion only, that there were innumerable Worlds, which were form'd and diſſolv'd by the Concourſe or Diſunion of theſe Atoms, that nothing could be produc'd from that which was not in Being, nor diſſolv'd into that which never had an Exiſtence, that therefore every Thing was made from Atoms or ſmall Particles of Matter, which always exiſted, and was ultimately reſolv'd into them, that theſe Atoms were infinite both in their Number and their Magnitude, or in the Extenſion of them all together, and that they were carry'd and whirl'd about in the Univerſe one amongſt another, from whence ſeveral Concretions of them were diſpos'd into Fire, Water, Air, and Earth, and that many Syſtemes which were compounded of certain of theſe Atoms or Particles of Matter, were cemented ſo ſtrongly, and conſiſted of ſuch ſolid Atoms and Particles, that they were not liable to Corruption, or were in the leaſt affected by any External Impreſſions upon them; he farther taught, that the Sun and Moon were form'd from the Motion, and at length the Concretion of theſe ſolid and obſtinate

Particles of Matter, or Atoms, and that the Soul, by which the Body or the Univerſe was animated, was ſo likewiſe, and the Mind or Intelligence of *Anaxagoras* he took to be no other, and that Viſion was cauſ'd by the Images of Things being impreſs'd upon our Eyes; alſo, that all Things were neceſſarily form'd and made by this Concourſe of Atoms without any voluntary or intelligent Agent; and that the End, which every one ſhould propoſe to himſelf, ſhould be a quiet and ſerene State of Mind, not as ſome have interpreted him Pleaſure, but a Mind continually happy in a perfect Tranquillity and Conſtancy of it's Thoughts, and not diſturb'd by any Fear or Superſtition, or any other Paſſion which may create it's Diſorder. And this is the firſt Syſteme of Atoms, and a Vacuum, and of all Things being produc'd by Neceſſity, and of a World without an Intelligence, which we find amongſt the Philoſophy of the Ancients, and from whence it is manifeſt, that Atheiſm, and Atoms, and a Vacuum had their Riſe and their Birth together.

Protagoras, Lib. 9. Segm. 50, &c. was of *Abdera*, and an Auditor of *Democritus*; he affirm'd his Principles in a very confus'd Manner, that Man was the Meaſure of all Things, that the Soul was nothing but Senſation, and that whether there were any Gods or not, he could not determine, for which Aſſertions he was baniſh'd *Athens*, and his Books publickly burnt in the *Forum*, the *Præco* or Cryer making Proclamation thereupon, and ſearch was every where order'd for them in what Hands ſoever they ſhould be found; he was a mercenary Philoſopher, and ridiculous Stories are told of him upon that Account.

Anaxarchus, Lib. 9. Segm. 58, &c. was likewiſe of *Abdera*, whoſe chief Doctrine was, that he knew, that he knew nothing, which might not be improbable.

The next is *Pyrrho*, Lib. 9. Segm. 61. the famous Sceptical Philoſopher, he was at firſt a Painter, and afterwards was the Auditor of *Dryſo*, the Son of *Stilpo*, and then of *Anaxarchus*, to whom he adher'd in all his Notions, ſo that he went to the *Gymnoſophiſts* in *India*, and to the *Magi* in *Perſia*, with whom he confer'd; his Doctrine was, that he cou'd not make a peremptory Deciſion upon any Thing whatſoever, nor was there any Thing which we could fully Comprehend, that there was no ſuch Thing as Commendable or Baſe, or Juſt or Unjuſt, that Truth was a meer Chimæra, and that the Rules by which Men judg'd and acted were thoſe of Law and Cuſtom, which were the Meaſure of Virtue and Vice, of Right and Wrong, and that there was not any one Thing which was really preferable, or one ſhould ſooner chuſe than another; agreeably to theſe Sentiments he form'd his Life and Manners, avoiding nothing which came in his way, whether Chariots, or Carriages, or Dogs, or Precipices, becauſe his Opinion was, that you ſhould not truſt your Senſes, and therefore not reſign your ſelf to the Conduct of 'em, and he was often preſerv'd from great Dangers and Inconveniences, to which he was expos'd upon this Account, by his Friends, that follow'd him; he was mighty deſirous of being, or being thought to be a good Man, and he liv'd to the Age of Ninety, and he us'd to ſay, that it was very hard to put off the Man, but Endeavours were to be made to do it to the utmoſt of our Power, in our Actions, and if that could not be, by our Reaſon even againſt Things, as they might appear to us.

Mr. Lock's Eſſay on Human Underſtanding.

Deſcartes's Meditat. and Mallebranche Recherche de la Verité.

After him come *Timon*, and *Sextus Empyricus*, Lib. 9. Segm. 109. and from them we are brought to *Epicurus*.

Epicurus, Lib. 10. Segm. 1, &c. was an *Athenian*, had his Education in *Samos* amongſt a Colony of his Country-men ſent thither, and return'd to *Athens*, when he was about eighteen Years old, at the time that *Xenocrates* preſided over the Academy, and *Ariſtotle* had form'd a School in *Chalcis*, he at firſt philoſophiz'd in Common with others, but afterwards eſtabliſh'd a Sect apart from the reſt, and call'd it after his own Name, he began to ſtudy Philoſophy when he was no more than twelve or fourteen Years of Age, he inveigh'd againſt the *Grammarians* for not being able to explain what *Heſiod* meant by his *Chaos*, for he taught Grammar firſt, but reading the Books of *Democritus*, he turn'd his Mind to Philoſophy, from whom he is ſaid to have deriv'd all his Opinions concerning Nature; The *Stoicks* were his great Adverſaries, and handed about no leſs than fifty Epiſtles writ in a looſe and laſcivious Strain, which they affirm'd to be his, he made a Pathick of one of his Brothers, and kept *Leontium* as a Miſs, he was guilty of very baſe and ſervile Flatteries in his Epiſtles to *Lyſimachus*, giving him the Appellations of *Pæan*, and King, he practic'd the ſame mean Arts towards *Idomeneus*, *Herodotus*, and *Timocrates*, and in his Epiſtles to *Leontium* he makes uſe of the ſame Titles of *Paan*, and King, and writing to *Pythocles*, a beautiful young Man, he ſays, he will ſtay there, in Expectation of his moſt lovely and divine Preſence, nor was *Leontium* his only Miſs, for he writ to ſeveral others, and in his Book περὶ τέλους, or, *de fine*, that is concerning the *Summum Bonum*, or, the chief Good which a Man ſhould purſue, his Words are; For I truly have no Notion of what is Good, if we take away all the Pleaſures of Taſt, and the Enjoyments of Luſt, the Delicacies of Sounds and of Muſick, and the Satisfaction which ariſes from beholding Comelineſs and Beauty, οὐ γὰρ ἔγωγε ἔχω τί νοήσω τἀγαθὸν, ἀφαιρῶν μὲν τὰς διὰ χυλῶν ἡδονὰς, ἀφαιρῶν δὲ καὶ τὰς δι' ἀφροδισίων, καὶ τὰς δι' ἀκροαμάτων, καὶ τὰς διὰ μορφᾶς. *Epictetus* alſo reprehends him for his laſcivious Way of Diſcourſing, and has many very ſevere Expreſſions againſt him, not much to the Honour of his Character; *Timocrates*, the Brother of *Metrodorus*, the great Admirer of *Epicurus*, and once one of his Scholars, but left him, relates, that he us'd to vomit twice a day from his exceſſive Luxury, and that he himſelf had much ado to eſcape theſe nocturnal Cuſtoms, and theſe ſecret Conventions of Riot and Philoſophy, that *Epicurus* was very Ignorant in his Language, and his Converſation, but more ſo, as to the Conduct of Life, and the Knowledge of the World, that he was grievouſly afflicted with Diſtempers in his Body, ſo that he could not riſe from his Seat for many Years, and yet that he ſpent at the ſame Time a Mina, or above three Pounds every day in Eating and Drinking, which is alſo acknowledg'd by him in his Epiſtle to *Leontium*, and the Philoſophers of *Mitylene*; in his Epiſtles concerning *Nauſiphanes* his own Maſter, and the Scholar of *Pyrrho*, he reviles him with the Names of Lungs, or the ſenſeleſs Prater, an Illiterate, an Impoſtor, and a Whore-Maſter; he calls the Followers of *Plato* the Flatterers of *Dionyſius*, and *Plato* himſelf was, for his Pride, or his Riches, or his Politeneſs, and Elegance of Language, which *Epicurus* condemn'd, term'd the Golden; *Ariſtotle*, he ſays, was a Spendthrift, and went for a Soldier upon the conſuming of his Patrimony, and ſold Drugs for his Support, or perhaps turn'd Mountebank; *Protagoras* was a Porter, writ for *Democritus*, and became a poor Retailer of Learning in the *Trivia*, and the common Streets; *Heraclitus* was a Puzler, and a Diſturber, and Confounder of Philoſophy, without knowing any Thing certainly and clearly; *Democritus* was capable of Judging of Trifles; and *Antidorus* was a fooliſh and inſignificant Flatterer; the *Cyzicenes* were Enemies of *Greece*, and the *Logicians* were a ſet of Men full of Malice and Envy, and *Pyrrho* was one, who had neither Learning nor Education; The Hiſtorian yet tells us, that theſe were Strange and Romantick Stories forg'd of him, that there were a great many, who atteſted his Extraordinary and Incredible Candor towards all, and that the Statues of Braſs, which were erected by the *Athenians* in Honour of him, were a Proof of it; that he had ſo many Friends, that whole Cities could not contain them, that all his Diſciples were ſo fully attach't to his Doctrine, that none of them ever left him excepting *Metrodorus* of *Stratonice*, who went over to *Carneades*, and that there was a perpetual Succeſſion of Diſciples in his School, which continu'd when all Others fail'd, he had likewiſe the greateſt Regard for, and profeſs'd the utmoſt Duty to his Parents, expreſs'd the moſt intimate Love and Affection for his Brothers, and us'd all the Lenity and Mildneſs, which was poſſible towards his Servants, and to comprehend the Sum of his Character in one ſhort Deſcription, he was a Perſon of the moſt exceeding Humanity, and of an univerſal Benevolence, he was a devout Worſhipper of the Gods, whom therefore he did not deny, but only the God of *Anaxagoras*, and *Plato*, that is the Mind and Intelligence of the other Philoſophers, and an hearty and ſincere Lover of his Country, he us'd to live with his Followers and Friends in a Garden, and according to *Diocles*, upon very plain and ordinary Meats, a little Wine, and with Water for their common Drinking, he was not willing that all Things ſhould be common amongſt his Diſciples, which *Pythagoras* preſcrib'd amongſt his, becauſe he thought that ſhew'd a diſtruſt of each other, which ought to be baniſh'd all real Friendſhip, which he was deſirous to eſtabliſh amongſt thoſe who retain'd to him; As to his living upon Water, and Bread, and ſuch kind of Food, he is his own Witneſs in his Epiſtles, in which he likewiſe deſires a little Cheeſe may be ſent to him, that he may, if he has a mind to it, make a more ſumptuous Banquet, than he generally allows himſelf; of the ancient Philoſophers he had the higheſt Opinion of *Anaxagoras*, tho' he

This Paſſage Athenæus alſo makes mention of, and Tully in his Third Book of the Tuſculan Queſtions.

contradicted him in ſome Things, and of *Archelaus* the Maſter of *Socrates*; he was very ſimple and plain in his Diction and Stile, and in his Book of Oratory he adviſes that nothing elſe ſhould be regarded, but how to expreſs your ſelf with Clearneſs and Perſpicuity, and in his Epiſtles where others us'd to write χαίρειν, [tho' the Epiſtles mention'd in his Life, to *Herodotus*, *Pythocles*, and *Menœceus* do all begin with χαίρειν] he choſe the Form of εὖ πράττειν, and it was his Notion that to live Vertuouſly was the beſt Thing one cou'd do; he began to ſtudy Philoſophy when he was twelve Years old, and was at the Head of a School which he inſtituted and form'd at Thirty, he was born ſeven Years after *Plato* dy'd, he taught at *Mitylene* and *Lampſacus* for the firſt five Years, and then remov'd to *Athens*, where he continu'd to his Death, which was in his two and ſeventieth Year, and was cauſ'd by a Stone in the Bladder, from whence a Stoppage of Urine enſu'd, he labour'd under this Calamity for fourteen Days, and being worn out with Pain, he went into a hot Bath, deſir'd a draught of ſtrong Wine to be given him, adviſ'd his Followers and Friends to remember the Inſtructions, and the Precepts he had taught them, and ſo expired.

I have been more Particular in the Life of this Philoſopher, becauſe it is ſo variouſly Reported, and it is not Impoſſible, but he might be a better Man than his Enemies, and a worſe Man than his Friends, repreſented him; it is plain, however, that his Doctrines tended to Vice, and to Atheiſm; to Vice, by the Paſſage before alledg'd, and to Atheiſm, by thoſe which follow; His Doctrines Segm. 29. were comprehended in three Epiſtles, the firſt to *Herodotus*, concerning Natural Philoſophy, the ſecond to *Pythocles*, concerning the Celeſtial Bodies, or Aſtronomy, and the third to *Menœceus*, concerning Morality and the Conduct of Life, and what in it we ought to chuſe or avoid.

In that to *Herodotus* he tells us, Segm. 37. and 38. πρῶτον μὲν οὖν τὰ ὑποτεταγμένα τοῖς φθόγγοις δεῖ εἰληφέναι, &c. that the firſt Thing we are to do in Philoſophy, is to conſider, and to aſcertain, what Ideas are annext to the Words we make uſe of, ſo that our Diſputes ſhould not be about Names or Words, but about the Things expreſs'd by them. Segm. 38, 39, 40, 41, 42. 44. he tells us, ὅτι οὐδὲν γίνεται ἐκ τοῦ μὴ ὄντος, καὶ μὴν καὶ τὸ πᾶν ἀεὶ τοιοῦτον ἦν οἷον νῦν ἐστι, καὶ ἀεὶ τοιοῦτον ἔσται, οὐθὲν γάρ ἐστιν εἰς ὃ μεταβάλλει, παρὰ γὰρ τὸ πᾶν οὐθὲν ἐστὶν, εἰς ὃ ἂν εἰσελθὸν αὐτὸ τὴν μεταβολὴν ποιήσαιτο, τὸ πᾶν ἐστὶ σῶμα, καὶ εἰ μὴ ἦν ὃ κενὸν, καὶ χώραν καὶ ἀναφῆ φύσιν ὀνομάζομεν, οὐκ ἂν εἶχε τὰ σώματα ὅπου ἦν οὐδὲ δι' οὗ ἐκινεῖτο, παρὰ δὲ ταῦτα οὐθὲν οὔτε ἐπινοηθῆναι δύναται, τῶν σωμάτων τὰ μέν ἐστι συγκρίσεις, τὰ δ' ἐξ ὧν αἱ συγκρίσεις πεποίηνται, ταῦτα δέ ἐστιν ἄτομα καὶ ἀμετάβλητα, εἴπερ μὴ μέλλει πάντα εἰς τὸ μὴ ὂν Φθαρήσεσθαι, ἀλλ' ἰσχύοντα ὑπομένειν, ἐν ταῖς διαλύσεσι τῶν συγκρίσεων, πλήρη τὴν φύσιν ὄντα, καὶ οὐκ ἔχοντα ὅπῃ ἢ ὅπως διαλυθήσεται, ὥστε τὰς ἀρχὰς ἀτόμους ἀναγκαῖον εἶναι σωμάτων φύσεις, ἀλλὰ μὴν καὶ τὸ πᾶν ἄπειρόν ἐστι τῷ πλήθει τῶν σωμάτων, καὶ τῷ μεγέθει τοῦ κενοῦ, πρός τε τούτοις τὰ ἄτομα τῶν σωμάτων, καὶ μεστὰ, ἐξ ὧν καὶ αἱ συγκρίσεις γίνονται καὶ εἰς ἃ διαλύονται, ἀπερίληπτά ἐστι ταῖς διαφοραῖς τῶν σχημάτων, μηδὲ ποιότητά τινα φησὶ περὶ τὰς ἀτόμους εἶναι πλὴν σχήματος καὶ μεγέθους καὶ βάρους. Namely, that Nothing can be produc'd from what is not in Being, and that the Univerſe always was ſuch as it now is, and always will be, ſince there is not any Thing, into which it can be transform'd or chang'd, for beſides the Univerſe there is nothing, which entering into it, can cauſe ſuch Alteration or Change, this Univerſe is Corporeal or Body, and if there was not what we call Vacuum, or Space, or a Nature which is not Solid, Bodies would neither have a Place to exiſt in, nor Room to move in; of more than theſe, that is than of Space and Bodies, we can have no Apprehenſion or Notion, and of Bodies there are two Sorts, either they are Compoſitions and Concretions, or they are ſuch of which thoſe Compoſitions or Concretions are made, the latter of which are Atoms, or the leaſt Portions of Matter, of which Bodies conſiſt, which Atoms are immutable, for if they could be diſſolv'd, or chang'd, they would be diſſolv'd into nothing, which is impoſſible, ſince as ſomething cannot be produc'd out of nothing, ſo neither can it be reſolv'd into it, theſe Atoms therefore are firm, and ſtable, and full of their own Subſtance, without any Vacuity, and cannot, upon the Diſſolution of Compoſitions or Concretions, be themſelves diſſolv'd by any Force without them, or within them, ſo that the firſt Principles of all Things are theſe ſolid, conſiſtent Atoms, ſince all Things are Corporeal; moreover the Univerſe is ſaid to be infinite in the Multitude of Bodies, or of Atoms the Compounders of them, and in the largeneſs and extent of the Vacuum, theſe Atoms are likewiſe affirm'd to be incom-

Sir Iſ. Newt. Optical Query 31. towards the End. It ſeems probable to me, that God in the Beginning form'd Matter in Solid, Maſſy, Hard, and Impenetrable Particles &c.

prehenſible, and infinite in the Difference of their Figures, and to have no Quality belonging to them, excepting that of Figure, Magnitude, and Gravity.

The Principles of the foregoing Philoſophers compar'd with one another, and with thoſe of the Moderns.

§. 15. AND thus we have a ſhort Epitome of the preſent Philoſophy of a Similar Matter, and a Vacuum from the famous and renown'd *Epicurus*, and all that it has wiſely and prudently done, is to annex the Mind or Intelligence of *Anaxagoras* to that Syſteme, which diſcarded and renounc'd it, but whatever prudential Reaſons it might have for ſuch a Conduct, which are Evident, it will be proper to conſider the Philoſophy of ſuch a Procedure.

ANY one, who affirms an infinite Vacuum, and an infinite Number of Atoms unchangeable in their Nature, which were always ſuch, and always will be ſuch, by whoſe Concourſes and Concretions all Things are and were form'd, and excludes a God, or a Mind and Intelligence from the Univerſe, propoſes Things which are conſiſtent; for as Almighty God is an infinite Agent, what Exertion of Power or Action is there in an infinite Vacuum? An infinite Space without any Thing in it entirely deſtroys an infinite Agency, but is well enough ſuited for the Rambling of Atoms about in it in order to form their own Productions; Beſides, what Influence can there be ſuppos'd from Almighty God, or an infinite Mind, or Intelligence, in the making or conſtituting of ſolid Atoms of Matter, which always were and ever will be the ſame? They are Principles of Nature independent of any Creating Power, and therefore it is abſurd to annex an Infinite and Superintending Being to ſuch a Syſteme.

Pherecydes, *Thales*, *Anaximenes*, *Anaximander*, *Anaxagoras*, *Plato*, *Ariſtotle*, *Zeno*, and *Pythagoras* on the Contrary, who affirm'd a Plenum, might juſtly and conſiſtently affirm a God, a Mind, and an infinite Intelligence, from whom all that Plenitude and Fullneſs of Being was deriv'd, who actuated and inform'd the Whole, and diverſify'd the τὸ πᾶν according to his own Will and Pleaſure, without being oblig'd to a neceſſary and infinite Vacuum, and an infinite Number of neceſſary and ſolid Atoms to be the Inſtruments of his Productions, which are nothing more than the Machinery of the Philoſophers. To take therefore the Mind or Intelligence of the one Sect of Philoſophers, and the Vacuum and Atoms of the other, is a great want of Underſtanding in the Principles of the ancient Philoſophy, and ſhews no leſs an Ignorance in the Nature of Things.

WHAT Side the beſt and wiſeſt *Romans* took in this Controverſy of the Philoſophers concerning *Anaxagoras*, and *Plato*'s, and *Zeno*'s Scheme of the Univerſe, and *Democritus*, and *Epicurus*'s is not unknown; *Cato*, and *Tully*, and *Brutus*, and all the ſober Part of *Rome*, and the Patrons of the Commonwealth, and of the ancient Conſtitution amongſt them, eſpous'd the former, and *Catiline*, *Cæſar*, and *Anthony*, and the Followers of them, the Licentious and Vicious, the Advancers of Tyranny and Arbitrary Power, and the Over-turners of that celebrated and noble Polity, contended for the latter.

Concerning Ocellus Lucanus *and* Sanchoniathos

§. 16. *Ocellus Lucanus*, who is affirm'd to have been Contemporary with *Moſes*, ſays thus, ἀγένητον τὸ πᾶν καὶ ἀνώλεθρον, ἄναρχον καὶ ἀτελεύτητον, ἀεὶ ὄντος τοῦ κόσμου ἀναγκαῖον καὶ τὰ μέρη αὐτοῦ συνυπάρχειν, τὸ ἀεὶ κίνητον θεῖον μὲν καὶ λόγον ἔχον καὶ ἔμφρον, συνέχει τὸν κόσμον ἁρμονία, ταύτης δ' αἴτιος ὁ Θεός, from whence it is manifeſt, that he with *Anaxagoras*, and *Plato*, affirm'd an Eternity and Infinity of the World, and therefore that there was a Plenum in it, but that it was animated and rais'd into an Energy by the infinite Mind and Intelligence, who was the Author of the Beauty, Order, and Harmony we obſerve thro' the whole Expanſe of Nature.

Sanchoniathos in his Collections of the moſt ancient Hiſtories of the *Phœnicians*, and who himſelf was a *Phœnician*, the Prieſt of *Belus* or *Baal*, and not much later than *Moſes*, according to *Philo Byblius*'s Verſion of him, has this Account of the World's Origine. Φοινίκων θεολογία τὴν τῶν ὅλων ἀρχὴν ὑποτίθεται ἀέρα ζοφώδη καὶ πνευματώδη, ἢ πνοὴν ἀέρος ζοφώδους, καὶ χάος θολερὸν ἐρεβῶδες, ταῦτα δὲ εἶναι ἄπειρα, καὶ διὰ πολὺν αἰῶνα μὴ ἔχειν πέρας, ὅτε δέ, φασίν, ἠράσθη τὸ πνεῦμα τῶν ἰδίων ἀρχῶν, καὶ ἐγένετο σύγκρασις, ἡ πλοκὴ ἐκείνη ἐκλήθη πόθος, αὕτη δὲ ἀρχὴ κτίσεως ἁπάντων, αὐτὸ δὲ οὐκ ἐγίνωσκε τὴν αὐτοῦ κτίσιν, καὶ ἐκ τῆς αὐτοῦ συμπλοκῆς τοῦ πνεύματος ἐγένετο Μώτ, τοῦτο τινές φασιν ἰλύν, οἱ δὲ ὑδατώδους μίξεως σῆψιν καὶ ἐκ ταύτης ἐγένετο πᾶσα σπορὰ κτίσεως καὶ γένεσις τῶν ὅλων. That is, the *Phœnician* Theology ſuppos'd a dark and ſpirituous and active Air, or the breathing or blaſt of a dark Air, and a tumultuous and dark Chaos, to be the Principle and

Origine of all Things, and that theſe were Infinite, and for a long Tract of Time without Bounds or Limits, but when the Wind or Spirit was in Love with it's own Principles, and a Mixture of the one with the other enſu'd, this Union and Conjunction of them was call'd, Appetite, or Inclination, by which Bodies became endu'd with certain Propenſities, which was the Principle of the Formation of all Things, but the Spirit it ſelf knew no Cauſe of it's Production; from the Concourſe of this Spirit with, and it's Influences upon the *Chaos*, Μῶτ or the ἰλὺς or Mud, or a Mixture of Water and Earth aroſe, from whence were deriv'd the Seeds of every created Being, and of all Things whatſoever.

In which Account we have evidently contain'd an infinite and paſſive Subſtance, and of a *Chaos*, which fill'd the Univerſe, or of a Plenum and of an active and intelligent Principle, which diſpos'd every Thing into the Order we now ſee, which was Self-exiſtent and Eternal.

Concerning Moſes *and* Juſtin Martyr *and the Chriſtian Philoſophy.*

§. 17. And this ſo exactly Correſponds with what *Moſes* relates in the ſacred Writings, of God's creating the Heaven and the Earth, of the Earth's being without Form, and Void, and of Darkneſs being upon the Face of the Deep, and of the Spirit of God moving upon the Face of the Waters, and his dividing Light from Darkneſs, and ſeparating the Waters, Earth, Sun, Moon, and Stars to their ſeveral Places, his Creating Animals, and the Herbs of the Field, that we may juſtly think and imagine *Sanchoniatho's*, and the *Phœnician* Philoſophy was deduc'd from that of *Moſes*, and conſequently that the Moſaick do's ſuppoſe an univerſal Plenum, or *Chaos* actuated and influenc'd into Order by the Energy and Influence of the divine Intelligence, eſpecially ſince Waters, and Darkneſs, and the Deep, are all affirm'd by him to exiſt before this Creation, and therefore we may reaſonably conclude that the moſt ancient and current Philoſophy amongſt the *Ægyptians* likewiſe, in whoſe Learning *Moſes* was ſo particularly converſant, was in the ſame Strain, as well as that all the Eaſtern Nations, that *Abraham*, and *Melchiſedec*, or *Sem*, and conſequently that the Antediluvian Patriarchs taught the ſame Doctrine till we come to *Adam* himſelf; and this is the more probable ſince *Pherecydes*, who was a *Syrian*, or *Phœnician*, *Thales*, *Anaxagoras*, *Plato*, and *Pythagoras*, ſome of whom travel'd into *Ægypt*, ſtill brought from thence the ſame Opinions of the World's Production, and conſtantly propos'd an infinite Plenum of undigeſted Matter, and an infinite Mind or Intelligence, which acted upon it for their Principles of all Things, till *Democritus*, and *Epicurus* firſt invented their Neceſſary and Self-exiſtent Vacuum, and Atoms, and the Gravity of them, without any wiſe or intelligent Cauſe to direct or guide them; and for the ſame Reaſon, I do not doubt, but the Aſtronomy of *Thales*, *Anaxagoras*, and *Pythagoras*, which is now the *Copernican*, was profeſs'd by the Antediluvians up to our firſt Parents, and the beginning of the World.

And as to the Chriſtian Philoſophy, we find *Juſtin Martyr* in his firſt Apology for the Chriſtians propoſing their Sentiments concerning Nature exactly conformable to thoſe which *Moſes* had taught, ϗ πάντα τὴν ἀρχὴν ἀγαθὸν ὄντα δημιȣργῆσαι αὐτὸν ἐξ ἀμόρφȣ ὕλης δι' ἀνθρώπȣς δεδιδάγμεθα.

Another way of defending a Vacuum, and the Confutation of it. Dr. Keill Introduct. ad Veram Phyſ. Lect. 5. Edit. 3.

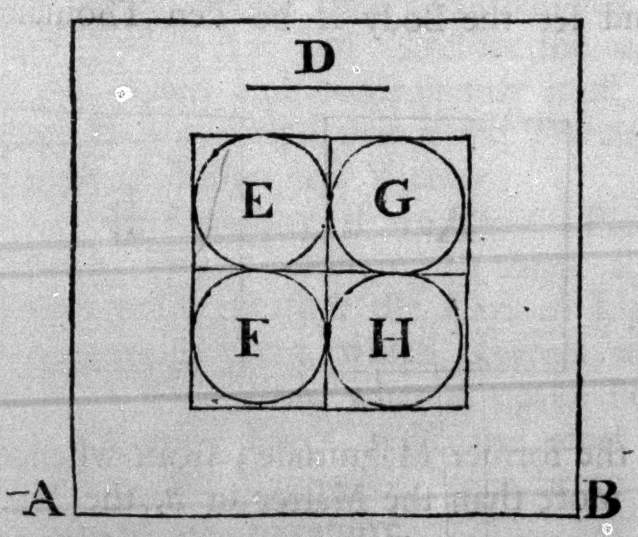
§. 18. Notwithstanding Matters ſtand thus in Relation to a Plenum and a Vacuum, that all the moſt ancient and celebrated Philoſophers aſſert the firſt, and reject the laſt, yet the preſent Philoſophy is ſo fond of the latter, that it has contriv'd another Solution of the Difficulty before-mention'd, which lies againſt a Vacuum, that in an exceeding rare Medium as the Air, there muſt be ſo much Vacuity, whilſt we affirm Matter to be Similar, that it wou'd be as impoſſible for us to breath in it, as in an exhauſted Receiver, Water likewiſe might be compreſs'd, *&c.*

In order to evade this ſort of Arguments, it is endeavour'd by the obtaining Philoſophy to attenuate and ſubtilize Matter as far as poſſible, when it do's not at the ſame time conſider, that, let us diminiſh the Parts of a Similar Matter as much, and as Infinitely as we pleaſe, all thoſe Parts will take up no more Space than the Whole out of which they were divided.

Thus it is premis'd by way of Lemma, that any Quantity of Matter being given, out of it, or any Part of it may be form'd a Concave Sphere, whoſe Semidiameter is equal to a right Line given. Let the Particle of Matter be a^3, and the right Line given b, let the Ratio of the Periphery of the Circle be to the Radius as p to r, let the Semidiameter of the Concavity be x, and the Thickneſs of the ſmall ſkin encompaſſing the Concavity of the Sphere will be $b-x$, and the Cylinder circumſcrib'd to the Sphere, whoſe Radius is b, will be $\frac{p\times b^3}{r}$, from whence the Sphere inſcrib'd in the Cylinder will be $\frac{2\times pb^3}{3r}$, and by the ſame Reaſoning the Sphere, whoſe Radius is x, will be $\frac{2\times px^3}{3r}$, and the Difference of theſe two Spheres, namely $\frac{2p}{3r}\times b^3-x^3$ will be equal to the Spherical Lamella or to the Particle of Matter given a^3, that is, $\frac{2p}{3r}\times b^3-x^3=a^3$, and by the common Reduction of Equations $b^3-x^3=\frac{3ra^3}{2p}$ and $x^3=b^3-\frac{3ra^3}{2p}$ and $x=\sqrt{3.\,b^3-\frac{3ra^3}{2p}}$, and conſequently the Thickneſs of the Spherical Lamella or $b-x$ will be $=b-\sqrt{3.\,b^3-\frac{3ra^3}{2p}}$.

AND by the ſame Method of arguing from a given Quantity of Matter concave Cubes, and concave Cylinders, or Bodies of any other concave Figure may be form'd, whoſe Sides are equal to a right Line given; 'Tis very well, therefore according to this Demonſtration, the ſmalleſt Particle of Matter may be form'd into the largeſt Cube, or Sphere, or Cylinder, whoſe Concavity may be infinitely great, and the Thickneſs of the Lamella which encompaſſes it infinitely ſmall, it is own'd and acknowledg'd, but will it from thence follow that Space is more fill'd than before? Will not there be ſtill the ſame Proportion betwixt Matter and Vacuum, as there was? And can it be imagin'd that a Matter blown up into the Bubbles of a School-Boy cannot be compreſs'd? Water is not in it ſelf compreſſible or very little, nor even Air to any extraordinary Degree, but if either the one or the other conſiſted of ſuch Globes, whoſe Concavity was great, and the Film which encompaſs'd it infinitely ſmall, wou'd they not be more eaſily broken and diſſolv'd by the extreme Force of Screws, and other Violences us'd to deſtroy their Dimenſions, than the Bubbles we nam'd by the Force of the Air? There is nothing can be more certain.

HOWEVER, this Philoſophy goes on to tell us, in it's firſt Theorem conſequent upon the preceding Lemma, that any Quantity of Matter, however ſmall being given, and any Space which is finite, however large, being alſo given, which Space for Inſtance, is ſuppos'd to be a Cube circumſcrib'd to the Sphere of Saturn, it is poſſible that ſuch a little Grain of Matter ſhould be diffus'd thro' all that Space, and ſhou'd fill it ſo, that there ſhould be no Pore or Vacuity in it, whoſe Diameter ſhould exceed a right Line given, that is, it is propos'd to prove that the leaſt Particle of Matter may fill the greateſt Space without any ſenſible Interſtices in ſuch a Repletion. Let the given Space be a Cube, whoſe ſide is AB equal to the Diameter of *Saturn*'s Orbit, let the Particle of Matter given be b^3, and the Diameter of the Pores D, which is in the preſent Caſe ſuppos'd exceeding ſmall; Let AB be imagin'd to be divided into Parts equal to the right Line D, whoſe Number therefore will be Finite, ſince neither AB is ſuppos'd to be infinitely great, not yet D infinitely ſmall, let that Number be n, and conſequently, $nD=AB$ and $n^3D^3=\overline{AB}|^3$; Let us now conceive the given Space to be divided into Cubes, the ſides of each of which are equal to D, and the Number of the Cubes will be n^3, and let theſe Cubes be expreſs'd by E, F, G, H in the Figure; Farther, let the Particle of Matter b^3 be conceiv'd to be divided into Parts, whoſe Number is n^3 and in every Cubical Space, let one of theſe Parts be placed, by which means the Particle of Matter b^3 will be diffus'd into all that Space, every Particle likewiſe of b^3 being diſpos'd into the little Cell belonging



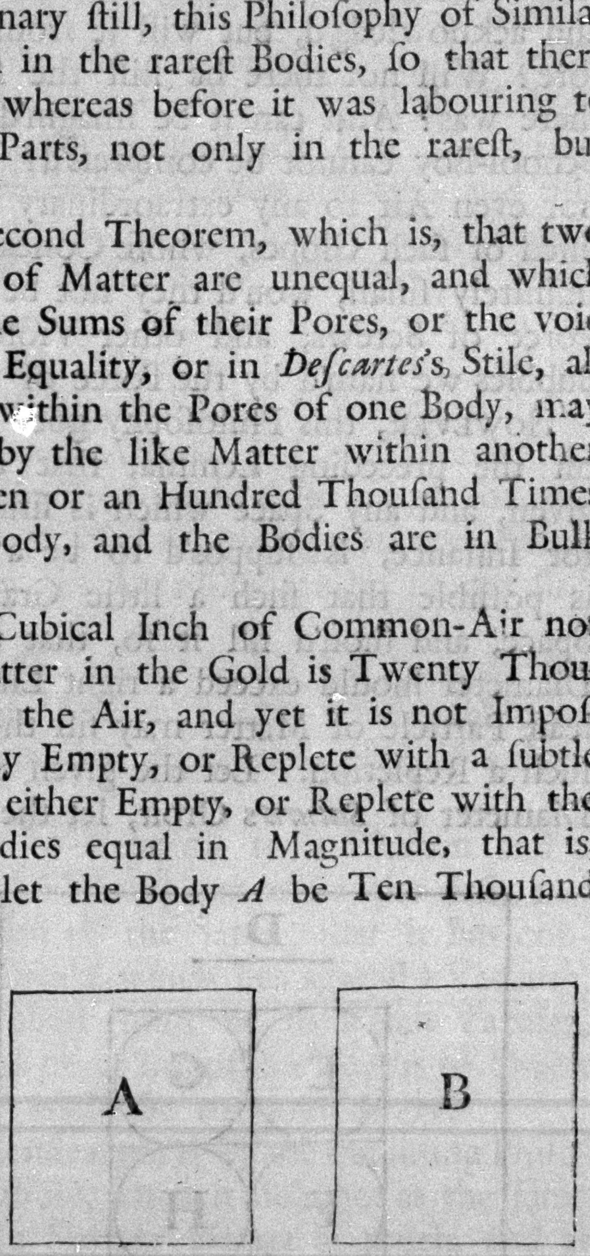
to it, may be form'd into a concave Sphere, whoſe Diameter is equal to the right Line given *D*, from whence it will come to paſs, that every Sphere will touch that which is next it, and a given Particle of Matter, however ſmall, as b^3, will fill a Space given however large, ſo that there ſhall be no Pore in it, whoſe Diameter ſhall exceed the right Line *D*, which is ſuppos'd extremly, tho' not infinitely ſmall.

To which we again anſwer, that ſuppoſing the Particle of Matter, whilſt we conſider it as Similar, is b^3 or 1, and imagining the Space to be $\overline{AB}|^3$ or 10 Millions of Millions of Millions *&c.* and let the Number of Cubes of this Space be n^3, and the Number of Parts into which b^3 is divided be alſo n^3, we ſay every Part of the Particle of Matter b^3 will be to every Part of Space $\overline{AB}|^3$, as 1 to 10 Millions of Millions of Millions, *&c.* and conſequently every Particle of Matter, if form'd into a Sphere, wou'd either ſurround, or be ſurrounded, by 10 Millions of Millions of Millons, *&c.* of Space, agreeable to what we have affirm'd in our *Principles of Natural Philoſophy*, Chap. 7. §. 8. Part 1. againſt which this Theorem ſeems to be level'd, This therefore cannot but be an odd way of filling Space with Matter, which leaves it to be compreſs'd by the ſmalleſt Force into a 10 Millionth of Millonth of Millionth, *&c.* of the Dimenſions into which this Philoſophy endeavours to extend it.

BESIDES, it is ſomething Marvellous and Strange, that it ſhould endeavour with ſuch Labour and Pains to ſubtilize Matter into a Plenum, which it is ſo unwilling to own, and can be no otherwiſe accounted for excepting it's being Conſcious, that a kind of Plenum, or what was approaching to it, was neceſſary in Nature, both for the Explanation of the ſmall Compreſſion of Water or Air, and of the way by which we live and breath: And what is more extraordinary ſtill, this Philoſophy of Similar Matter is here filling up all the Interſtices it can in the rareſt Bodies, ſo that there ſhould be no Vacuum, or a very inſenſible one, whereas before it was labouring to prove, that there was more Vacuum than ſolid Parts, not only in the rareſt, but even in the groſſeſt and denſeſt.

NOTWITHSTANDING which, it proceeds to a ſecond Theorem, which is, that two Bodies may be equal in Bulk, whoſe Quantities of Matter are unequal, and which obtain a given Reaſon to each other, and yet the Sums of their Pores, or the void Spaces betwixt them come near to a Reaſon of Equality, or in *Deſcartes's* Stile, all the Space which is poſſeſt by the ſubtle Matter, within the Pores of one Body, may be almoſt equal to the Space which is poſſeſs'd by the like Matter within another Body, altho' the proper Matter of one Body is Ten or an Hundred Thouſand Times more than the proper Matter of the other Body, and the Bodies are in Bulk equal.

For example, a Cubical Inch of Gold, and a Cubical Inch of Common-Air not condens'd; 'Tis certain, that the Quantity of Matter in the Gold is Twenty Thouſand Times more than the Quantity of Matter in the Air, and yet it is not Impoſſible, that the Spaces in the Gold, either abſolutely Empty, or Replete with a ſubtle Matter, may be equal to the Spaces in the Air, either Empty, or Replete with the ſame ſubtle Matter. Let *A* and *B* be two Bodies equal in Magnitude, that is, let each of them be a Cube of one Inch, and let the Body *A* be Ten Thouſand Times heavier than the Body *B*, and conſequently, include Ten Thouſand Times more Matter; Let us now ſuppoſe the Quantity of Matter in *A* to be reduc'd into a Space abſolutely full, which let it be the Hundred Thouſandth Part of a Cubical Inch, for that may be done by the Corollary of the preceding Theorem, which is, that a Body may be given, whoſe Matter, if reduc'd to an abſolute Plenum, that Plenum may be any Part given of the former Magnitude; from whence, ſince the Matter in *A* is Ten Thouſand Times more than the Matter in *B*, the Matter in *B*, if compreſs'd into an abſolute Plenum, will poſſeſs only $\frac{1}{1000000000}$, or one Ten Thouſandth Hundred Thouſandth Part of a Cubical Inch, and therefore the remaining 999999999 Parts will be either abſolute Vacuum, or Space replete with ſubtle Matter; farther, ſince the Quantity of Matter in *A* fills only the Hundred Thouſandth



ſandth Part of an Inch, the remaining 99999 Hundred Thouſandth Parts will be either abſolute Vacuum, or Space replete with the ſame ſubtle Matter, that is, by reducing the Fraction to the Denomination of the former, the void Spaces in *A* will be 999990000, Ten Thouſand Hundred Thouſandth Parts, and conſequently, the Vacuities in *A* will be to the Vacuities in *B*, as 999990000 to 999999999, or nearly in a Reaſon of Equality; But this whole Reaſoning depends upon the Corollary of the preceding Theorem, that a Body may be given, whoſe Matter, if reduc'd to an abſolute Plenum, that Plenum may be any Part given of the former Magnitude, for if ſo, there is no Queſtion we may make Choice of ſuch a Part, which ſhall leave juſt as much Vacuity, in one Body as the other, and in a Cubical Inch of Gold, as in a Cubical Inch of Common-Air; If therefore this Corollary is not true, nor the Theorem from whence it is deduc'd, which we have ſhewn, the preſent Theorem, and the Demonſtration of it will be alike Inconcluſive.

Besides which, let us conſider the very abſurdity of the Propoſition, which is endeavour'd to be prov'd, that tho' there are Twenty Thouſand more Parts of Matter in a Cubical Inch of Gold, than in a Cubical Inch of Common-Air, yet there are as many Vacuities in the one as in the other, ſuppoſing then, there are 20000 Parts of Matter, and 1 of Vacuity in Gold, there will be 1 of Matter, and 20000 of Vacuity in Common-Air, and yet by the Strength and Force of the foregoing Mathematicks, either 20000 Vacuities are prov'd equal to 1, or Matter is transform'd into Space, and Space into Matter, which is a Faculty hitherto unknown to that Science, and ſeems to explain an Artifice and Sophiſtry which is unworthy of it.

As to what is ſaid farther in this Place, that all Bodies are exceeding Rare, and that they contain very little Matter in Proportion to their Bulks, and that this is manifeſt from Diaphanous Bodies, and the Tranſmiſſion of the Rays of Light thro' Glaſs or Water as well as Air every way in right Lines, and therefore that there are Rectilineal Pores in all Directions thro' thoſe Bodies, by which the Light paſſes, and that this wou'd be impoſſible, if the Matter of the Diaphanous or Tranſparent Body was not inconſiderable in Reſpect of it's Bulk, which perhaps may be no more than a Grain of Sand to the whole Earth, and that therefore Gold, which is eight Times denſer than Glaſs, bears a very ſmall Proportion of it's Matter to it's Bulk, and that upon this infinite Porouſneſs of Bodies the Magnetick Effluvia may be accounted for in their Paſſing as eaſily thro' denſe Gold as the thinneſt Air; all theſe are Notions equally abſurd with the foregoing Demonſtrations, and are deriv'd from, and connected with them, they proceed from a great Inclination which this Philoſophy has to a Similar Matter, to Pores, and a Vacuum, and which Propenſity to them makes it aſſert very manifeſt and palpable Contradictions; Thus, beſides what we have before expreſs'd, Glaſs and Water are heavier than Air, and therefore, according to this Philoſophy, has more Matter in an equal Bulk of them, and yet they are equally Porous, becauſe they equally tranſmit the Rays of Light, that is, they have more Matter and leſs Vacuities, and they have more Matter and equal Vacuities, which is in Terms a Contradiction; ſo likewſe Gold is eight Times denſer than Glaſs, and Twenty Thouſand Times denſer than Common-Air, and yet the Magnetick Effluvia paſs as eaſily thro' Gold as Air, and therefore the one, according to this Doctrine, muſt be as Porous as the other, and conſequently muſt have as little Matter, and as much Vacuity, when it is confeſs'd that it has Twenty Thouſand Times more Matter, and Twenty Thouſand Times leſs Vacuity.

On the Contrary if we allow a Plenum, and a diſſimilar Matter, with various and differing Forces belonging to them, all theſe Difficulties are eaſily ſolv'd, Diaphanous Bodies eaſily tranſmit the Rays of Light, becauſe they have an expanſive Force appertaining to them, which concurs with the Expanſive of Light, and permits a Liberty to it's Action, whereas opake Bodies being endu'd with a greater Contractive than expanſive Force, reſiſt and repel the Rays which are Incident on them, Gold admits of the Magnetick Force being of a Contractive Nature, but repels it, when heated, by the Expanſive.

This Philoſophy yet thinks, from this infinite and imaginary Porouſneſs of Bodies, and from the vaſt Celerity of Light, it can give an Account, why the Rays of Light proceeding from ſeveral Objects, and tranſmitted thro' a ſmall Foramen, do not hinder each other's Progreſs but preſerve their Motion in a direct Line, which is ſaid cannot be explain'd by an Impulſe upon a fluid Plenum, ſince any Body, as a Plenum, having Impreſſions made upon it by ſeveral Forces in different Directions, receives and admitts of one certain and determinate Direction only, compounded and made up of all of them.

THERE are therefore two Things here to be conſider'd, the Velocity of Light, and the Tranſmiſſion of it thro' the Foramen in different Directions.

As to the Velocity of Light this Philoſophy ſeems to entertain a wrong Opinion of it, whilſt it makes it to come from the Sun to us in ſeven or eight Minutes of Time, by an actual Motion of Particles of Matter emitted from thence, and which I think has been in another Place conſider'd, what I ſhall here ſay is, that if the Emiſſions of Particles from the Sun are according to what this Philoſophy repreſents, or thoſe of Light from a Lamp or a Candle, thoſe luminous Particles muſt fill all the Space round them, which is Incredible, and the Aſſertion of ſuch a Doctrine is only owing to the Imagination, that one Particle of Similar and Homogeneous Matter may fill Ten Millions of Space, which is Impoſſible and Inconceiveable as we have already ſhewn.

AND yet again, if we will but acknowledge that Matter is Diſſimilar, and that all the World is full of it, and that Diſſimilar Matter has Diſſimilar Forces, the Propagations of Light from the Sun, from a Lamp, or a Candle, is readily ſolv'd, by the expanſive Force of the Sun, of a Lamp, or a Candle, diffuſing it's Action, and not emitting it's Particles thro' the ambient Medium, which receives their Impreſſions, not barely from a Communication of them, but as the Medium has an expanſive Force agreeable to the Expanſive impreſs'd, which it Promotes and Advances.

As to the next Thing, that the Tranſmiſſion of Light thro' the Foramen in different Directions is eaſily accounted for by the Rectilineal Pores every way, but not by bare Impulſes of Light upon a Plenum for the Reaſon above-mention'd, we anſwer.

FIRST, To ſuppoſe the Rays of Light tranſmitted every way thro' Rectilineal Pores, is to require us to imagine a very hard Thing, namely, that the Diaphanous Body, whether Glaſs or Air, is Omniporous, or which is the ſame, that it conſiſts altogether of Pores, or Vacuity, for in that infinite Croſſing of Pores at all the minuteſt Angles, which is here preſum'd, there muſt be of neceſſity a Deſtruction of all Matter in the Body, and nothing left but mere Space and Vacuum, and conſequently, the Diaphanous Body will not only be depriv'd of it's Solidity, but even of the Configuration of it's Pores too, by being reſolv'd into one continu'd Vacuity, and this Solution will meet with the Difficulty it objects to the other, ſince the Actions of Light by this means will be blended together, which the Pores were to ſeparate, and conſequently compound an Action of them all united into one.

SECOND, Granting theſe Rectilineal Pores tranſverſe to each other in an infinite Number of Angles, and the Rays of Light to paſs thro' them, wou'd not theſe Rays meet and interfere with each other in a Thouſand different Directions, and would not ſuch mutual Occourſes and Impactions of them as much interrupt and hinder their Progreſs in Directum, as if they were to meet each other in the Foramen, without ſuch Rectilineal Pores to defend and aſſiſt their Progreſſion?

THIRD, But what is more in this Doctrine, it is preſum'd, that the Action of Bodies, as the Rays of Light, is beſt promoted by contriving Pores and Vacuities for them, and the ſame is affirm'd in reſpect of the Magnetick Force, and therefore Vacuities are aſſign'd for it's paſſing thro' Gold; whereas it is wonderful that we ſhould want Vacuities and Pores here for the Propagation of the Action of Light, when to propagate the Action of other Bodies, Bodies not Vacuities are requir'd; Thus Fire and Flame is not propagated in Vacuum, but in Bodies, which are inflameable, Motion is not communicated to Vacuum but to Bodies, which are moveable, the Action of Sound is not propagated thro' Vacuum, but thro' Bodies which are Springy and Elaſtick, and why therefore ſhou'd we not think that Light is not promoted thro' Pores and Vacuum, but thro' Bodies which partake of the Nature of Light, as Diaphanous or Tranſparent Bodies, that is, which have, as Light, an expanſive Force appertaining to them?

ON the other hand, there is no Difficulty in apprehending, how a Medium, as Air or Glaſs, ſhould tranſmit an Action, as that of Light, in the ſame Direction, in which it is impreſs'd, without a Compoſition of thoſe different Forces into one, ſince that Medium, whatever it is, is equally balanc'd in it's Action of it's Parts one amongſt another, if therefore a foreign Action, as that of Light, invades it in one Direction, it muſt ſubmit to that Action, that is, there muſt be a Preponderancy, if we may call it ſo, that way, and if a Second, or a Third, or Fourth, *&c.* Aſſaults it in different Directions, it muſt likewiſe ſubmit to thoſe Actions, and in thoſe Directions and accordingly tranſmit them, ſince it holds it ſelf indifferently to all, and therefore muſt tranſmit them all in the ſame Directions, in which they are incident, allowing ſtill for the Difference of Action in Light, and in the Body, thro' which it is tranſmitted, and from whence the Refractions of it ariſe; and the ſame may be ſaid of the Magnetick Force, and of Gold, or any other Body thro' which it paſſes.

Having therefore now ſaid what we intended concerning a Vacuum, we ſhall proceed to inquire into ſome farther Arguments made uſe of for a Similar Matter and Pores, which we did not conſider in our Principles of Natural Philoſophy.

§. 19. It is ſaid in Axioms, the *14th*, *15th*, and *16th*, that Matter is every where of the ſame Nature, and has the ſame Eſſential Attributes, whether in the Heavens or in the Earth, whether it appears under the Form of a Fluid, Hard, or any other Body; that is, the Matter of any Body, of Wood for Example, do's not Eſſentially differ from the Matter of Iron, or a Stone, but that the different Forms and Appearances of Bodies are only different Modifications of the ſame Matter, and ariſe from the Various Magnitude, Figure, Texture, Situation, *&c.* of the Particles, which compoſe thoſe Bodies, and that the Qualities, or Actions, or Powers of any Bodies upon others, depend only upon the foregoing Modifications of Matter in Conjunction with the Motion of ſuch Particles.

Another way of Defending a Similar Matter, and Pores, and a Confutation of it, with a Propoſal of the Doctrine of the Expanſive and Contractive Forces.

Dr. Keill Introd. ad Ver. Phyſic. Lect. 8.

These are call'd Axioms or Self-evident Principles in Philoſophy, which yet with me are ſo far from being Self-evident, that I think the contrary to all this is manifeſt and certain, that Matter is not Homogeneous in it's own Nature, and that it's apparent Difference do's not ariſe from the Difference of the Situation, Figure, and Magnitude of it's Parts, and that it's Powers and Qualities are not deriv'd from them in Conjunction with Motion, but from the peculiar Expanſive and Contractive Forces of it; by an Expanſive Force I mean, that which tends from a Centre of Action, and by a Contractive, that which tends towards one.

Notwithstanding which Axioms or Self-evident Principles of Philoſophy, it thinks good to prove the Truth of them, and from that Endeavour ſhews, that thoſe Principles are not Self-evident ones.

But how is this endeavour'd? It is by giving us an Hiſtory of the Tranſmutation of Sea-Water into Clouds and Vapours, from thence into Rain, after that into Plants and Trees, according to the Nature of the different Seeds, in which it Lodges, which alſo conſiſt of Various and Heterogeneous Parts, as the Root, Stalk, Fibres, Flowers, *&c.* for Inſtance, let us take Flax or Hemp, this beaten or pounded is ſpun into Thread, then wove into Cloth, the Shreds of which ſteep't in Water, reduc'd to a Pulp by wooden Peſtles, and ſpread upon Wires and dry'd, is what we call Paper, this thrown into the Fire vaniſhes into Smoke and Aſhes, all which Phænomenons are affirm'd to proceed from a Change in the Texture, Situation, Figure, and Magnitude of the Parts of Sea-Water. Thus Metals are fus'd by the force of Fire, and by a change of Situation in the Parts are diſſolv'd from their Coheſion into a Fluid Subſtance. The Diſſolution of Salts and Metals in their proper Menſtruums ariſes from a Separation of Parts by Fermentation, and from a Diviſion of them into the leaſt Atoms, and an Agitation of them by Motion, from whence they become Fluid; and from the Figures and the other Modifications of the Parts of Bodies, diverſe Qualities, which are peculiar to them, are caus'd, which are deſtroy'd, if that Conſtitution of Parts is alter'd.

In like Manner from the ſame Matter, that is, Iron, Keys, Knives, Files, Saws, and an hundred other Inſtruments are made, whoſe Qualities depend only upon their Figures: Keys have the power of a Lever to remove the Obſtacle, and Knives of a Wedge to ſeparate the Parts of Matter to which they are apply'd, and it is prov'd by the Writers in Mechanicks, that their Moment or Force is owing to their make: Clocks derive their Powers and Motions from their Wheels, and the Machines, which produce ſuch ſtupendous Effects we Experience, obtain thoſe Faculties from their Compoſition and Fabrick: The Roughneſs in the Surfaces of Bodies is the occaſion of Light's being reflected every way in various and different Angles, and conſequently it's Reflection is owing to the Configuration of the Parts of ſuch Bodies, this is confirm'd from Ice or Glaſs, which when whole, and ſmooth, and poliſh'd, is not endu'd with any Colour, but when Ground and broken into Pieces, and by that means acquiring a rough and angular Surface, appears White, and it is the ſame in Water when work'd and fermented into a Froth.

In order to juſtify this Philoſophy of Similar Matter ſtill farther, Sir *Iſaac Newton's* Doctrine of Light and Colours is alledg'd, of which we ſhall ſay no more here, and even Opticks and the Phænomenons from Plane, Concave, and Convex Speculums are introduc'd to prove the Truth of the Corpuſcular Syſteme, which are all ſaid to proceed from a Change of Diſtance or Situation.

THE Earth is revolv'd about the Sun with it's Axis inclin'd to the plane of the Ecliptick in an Angle of 66½ Degrees, but always carry'd Parallel to it ſelf, from whence in it's Period, firſt one of it's Poles is obverted to the Sun, and then another, by which all the different Seaſons of the Year are occaſion'd, and the various Productions of them, which are only effects conſequent upon a Change of Situation; What therefore may not Situation, Diſtance, Magnitude, Figure, and the Structure of Parts, and the infinite Combinations of all theſe perform join'd with the Motion and Subtlety of Effluviums? The magnetick Attractions are owing to the Structure of the Parts, for if the Magnet receives a good ſtrong Blow from an Hammer, it changes it's Poles, and if thrown into the Fire, looſes it's Vertue, which muſt be from an Alteration in the Conſtitution and Poſition of the internal Parts, but how and in what Manner the Magnet operates is a myſtery not yet explain'd by Philoſophers, tho' endeavour'd by ſeveral; This is a ſhort Account of this new Defence of a Similar and Homogeneous Matter.

FIRST, As to Sea-Water being transform'd into Clouds and Vapours by rarefying the Matter and enlargeing the Pores, that is much better accounted for by the expanſive Force of the Sun changing the Action of the Sea-Water, by which from a Salt, Heavy, and Contractive Body, it becomes a Sweet, Volatile, Light, and Expanſive one, afterwards by the contractive Force of Cold theſe Clouds and Vapours coagulate into Rain, and fall in Showers, or into Snow and Hail and deſcend in that Form, which is not done by altering the Poſition of the Parts, but changing the intrinſick Force of them, namely, from an Expanſive into various Degrees of the Contractive; Rain moiſtening the Seeds of the Earth, and thereby exerciſing a kind of expanſive Force upon the ſolid Matrixes which receive it, and the Sun afterwards ſupervening with a ſtronger and more vigorous Expanſive, Vegetation is cauſ'd, and the Seeds, thus impregnated with the Expanſive of the Sun and the Rain, ſhoot forth into Plants and Trees, which do's not proceed from an Alteration of the Figure and Texture of the Parts, for that is only conſequent, but from new Forces impreſs'd and induc'd upon them, by which from a contractive Nature they riſe into a Degree of the Expanſive from the Rain and Sun; The Parts of Plants and Trees are Root, Stalk, Fibres, and Flowers, that is, more or leſs Solid, as they have a greater or leſs Proportion of the Contractive, and a leſs or greater of the Expanſive; amongſt the Plants, becauſe it admits of moſt Variations, let us, as in the Inſtance before, take Flax or Hemp, which when beaten or pounded, that is, from an Expanſive Force it becomes diſſipated and diffuſ'd into a ſilky and pliant Subſtance, by a Contractive of the Fingers it is ſpun into Thread, and wove into a cloſe Contexture by that of the Loom, it is by the Expanſive Force reduc'd into Shreds and a Pulp, and then from the Contractive of the Air, and from the Diſſipation of the Expanſive of Water, which is equal to a Contractive, cemented into Paper, and laſtly by the Expanſive of Fire it is diſſolv'd into Smoke, and a Light, and Volatile Aſhes, from whence it is apparent, that theſe different Effects are produc'd from different Forces, and that the Effects, which are produc'd, conſtantly partake of the Nature of the Forces, which produce them, and that the Alterations of the Poſition of Parts are not the Cauſes of theſe Effects, nor are the Effects themſelves, but the Forces acting are the Cauſes, and the Forces impreſs'd are the Effects, and the Changes of Situation in the Parts of theſe Bodies do only follow, and are conſequent upon ſuch Effects, and ſuch Actions producing them.

The Fuſion of Metals is the next Example, which this Philoſophy alledges to prove the different Situation of Parts is the cauſe of the different Forms of Bodies, which is an Argument for the Doctrine we have advanc'd of the Expanſive and Contractive Forces, for Metals are Solid, and have their Coheſion from the Contractive Force in them, and they become Fluid by the Expanſive Force of Fire, which if in an equal Balance with the Contractive, the Parts of the fluid Metal are Quieſcent, and there is no change in the Situation of them, but if in this Equilibrium we put an Iron-ſpatula into the fus'd Metal, the Force, which we give to it, will be imparted to the Fluid, and will agitate it which way we pleaſe; if the Expanſive Force of the Fire is likewiſe increas'd, by which it becomes Superior to the Contractive of the Metal, it will boil up, and the Situation of the Parts will be chang'd, but this is not Eſſential to the Fluidity, but to the Motion of it.

And thus we may imagine that where the Expanſive Force of the Sun or the ambient Medium and the Contractive of any Matter are in an equal Balance, Water is produc'd, which tho' Fluid in it ſelf is perfectly Quieſcent in a Stagnation of it, and which we find the leſs Power the Sun has upon it, as in Winter, the more it's Contractive prevails, ſo far as at certain Seaſons it is chang'd into a perfect Solid; Theſe Phænomenons are not therefore owing to a difference of Situation in the Parts, but to a different Force, which is induc'd upon them, and if what we have ſaid is true in Reſpect of Water, it is no more impoſſible that it ſhould be produc'd in Perennial Fountains, where the Expanſive Force of the Sun, and ambient Medium are equally balanc'd with the Contractive of an Earthy Matter, than that Subterraneous Fires ſhou'd be caus'd by the Expanſive of the Sun being Superior to the inflammable Matter it ſets a-Burning, and which may continue for ever in that State.

The Diſſolution of Salts and Metals in their proper Menſtruums, is another Inſtance made uſe of by this Philoſophy to juſtify it's Aſſertions of Bodies, owing their particular Forms to the different Situation and Poſition of their Parts, whereas Diſſolutions are more rationally ſolv'd by an Expanſive Force in the Menſtruum acting upon the Contractive of the Salts or Metals; beſides it is abſurd to ſay, that a Diſſolution is cauſed by a various Situation of Parts, ſince that is cauſed by a Diſſolution, and it is ridiculous to ſay, that a Diſſolution is caus'd by an Agitation, that is a Diſſolution of Parts, without aſſigning the Cauſe of ſuch an Agitation or Diſſolution.

As to the Inſtance of Keys, Knives, Files, Saws, and various other Inſtruments made out of the ſame Matter, Iron, which obtain their particular Effects from the Peculiarity and Diſtinction of their Figures, and of Mechanical Engins deriving their Powers from their ſeveral Structures, even their Effects we have ſhewn in the following Sheets are owing to the Expanſive and Contractive Forces, without which their different Configurations wou'd ſignify no more than the drawing Lines and Figures upon a plain piece of Paper; 'tis the Force and Energy with which they are endu'd, and not the mere Figure of the Parts, which gives them that Importance of Action, and that Ability to produce the Effects we experience from them; what wou'd a Key perform, which, as we have ſaid, has the Nature of a Lever to remove the Obſtacle, which oppoſes it, without the Expanſive Force apply'd to it, in order to ſurmount the Contractive of the Obſtacle? Or a Knife, which has the nature of a Wedge, without a like Expanſive Force actuating and envigorating it againſt the Contractive it is deſign'd to ſubdue? Wou'd a File or a Saw by it's Parts and the Diſpoſition of them, without the Expanſive Force of the Mind, and the Hand, which obeys it's Directions, diſſolve the Contractive of Matter? Or wou'd the Teeth of either of them ſignify much without the Reciprocation?

That the Reflections of Light, and that Colours, and particularly White, are owing to the Roughneſs of the Surface, on which it is Impingent, and conſequently to the Diſpoſition and Situation of it's Parts is not manifeſt, ſince Glaſs or Water may be Colourleſs or Tranſparent from their Expanſive Force aſſiſting and promoting the Progreſs or Tranſmiſſion of Light, which has alſo an Expanſive Force, thro' them, from a kind of Similarity and Congruency of Action, thus the Expanſive of Fire paſſes more eaſily thro' Air or Water, or a Body heated, which are Expanſive, than thro' Glaſs, or Solid, or cold Bodies, which are of a Contractive Nature, but when this Expanſive Force of Glaſs or Water is interrupted and broken, the Light is of Courſe obſtructed and ſtop'd, and where it do's not find a Paſſage, returns it's Force upon the ambient Plenum, which cauſes the Senſation of White in us, that is, it is not reflected in Rays to the Eye, but reſts Glowing in a pale and languid Flame on the Surface, where the Impaction is made, and by that means affects the Medium, which ſurrounds it, with that Action, which produces the above-mention'd Senſation in us.

SIR *Iſaac Newton's* Doctrine of Light and Colours we have in another Place conſider'd, and ſhall add ſomething farther concerning it in the following Section, and therefore will here only ſay, that it is not from thence prov'd, that Colours depend upon the different Figures and Poſitions of the Parts of Bodies, ſince thoſe Colours obtain a truer Solution from ſuppoſing there are various Compoſitions of the Expanſive and Contractive Forces in Bodies, upon which, when the ſame Similar and Homogeneous Light is incident, it receives different Impreſſions and Actions which communicated to the ambient Plenum ſtrike us with that Variety of Colours we behold in them; and from theſe Expanſive and Contractive Forces in Bodies it is, that we feel as well as ſee them, and that in various Ways and Manners as will afterwards appear.

OPTICKS are alſo alledg'd to ſhew, that diverſe Effects are produc'd from a change of Diſtance and Figure, as in plain, concave, and convex Speculums, we ſhall have occaſion to examine into the Foundations of this Science in another Place, and ſhall here only anſwer, that the Expanſive Force of Light, the Expanſive and Contractive of Glaſs and Quick-ſilver have no Inconſiderable, becauſe the chief Influence in the Production of theſe Phænomenons, without which, Figure and Diſtance would ſignify juſt nothing.

As to the Revolution of the Earth about the Sun with it's Axis parallel to it ſelf, and inclin'd to the Plane of the Ecliptick, by which all the Seaſons of the Year are produc'd, and the ſeveral Effects ariſing from them caus'd, they ſo manifeſtly depend upon the various Degrees of the Sun's Expanſive Force, and of the Earth's Contractive, that it wou'd be trifling to confute this Pretence for a Similar Matter, and the Parts, and Pores of it, with their different Situations and Textures, betwixt which, and the Inclination and Paralleliſm of the Earth's Axis, there is no imaginable Connexion.

THE laſt Argument to prove this Doctrine is drawn from the magnetick Attractions, which are alter'd by the Blow of an Hammer, and by the Heat of the Fire are annihilated and deſtroy'd, which will be a very good one, when it can be evinc'd, that there is no Expanſive Force in Percuſſion or in Fire, till then thoſe Forces will aſſume to themſelves the Prerogative of the Cauſe, and any change in the Situation of the internal Parts, if there is any, will be only an humble and paſſive Effect, deriv'd from it.

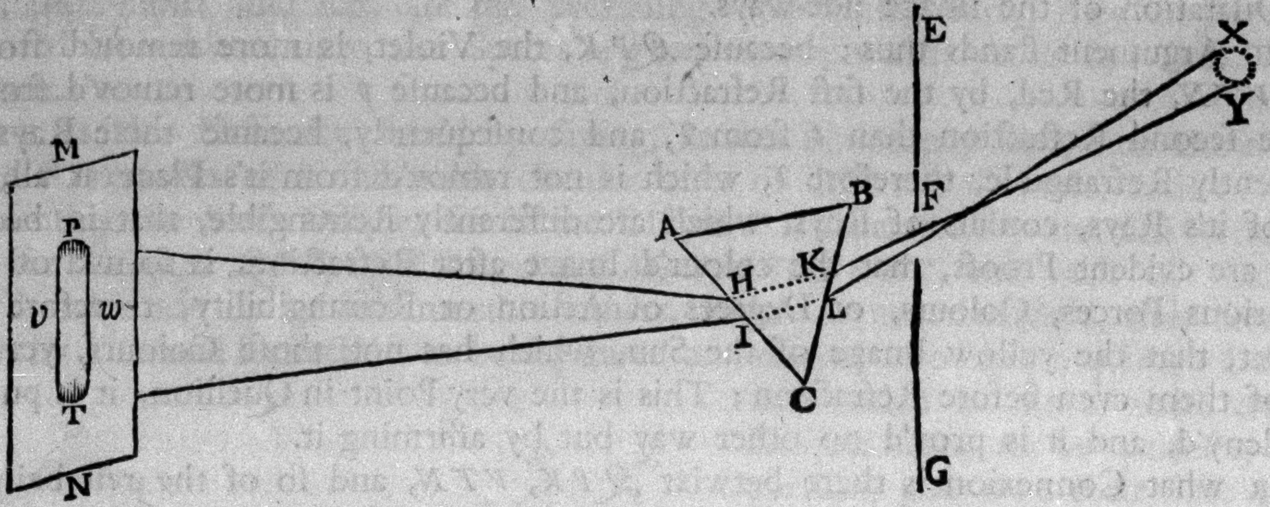
What I have formerly objected againſt the new Doctrine of Light, and Colours, and Sounds, farther Explain'd, and the true Theory of the Rainbow propos'd.

§. 20. IN Relation to the new Doctrine of Light and Colours, which was the laſt Thing I propos'd to ſpeak to, in the beginning of this Chapter, we ſaid in our *Principles of Natural Philoſophy*, Part 2. Chap. 3. Sect. 40. That the next Sort of Experiments, to prove the Rays of Light are differently refrangible Originally and before their primary Refraction, are ſuch as at the moſt demonſtrate them only to be ſo afterwards, *&c.* I ſhall therefore now, as a Supplement to what I have before offer'd on this Subject, go thro' the ſeveral Experiments, and ſhew that what I have there alledg'd is true.

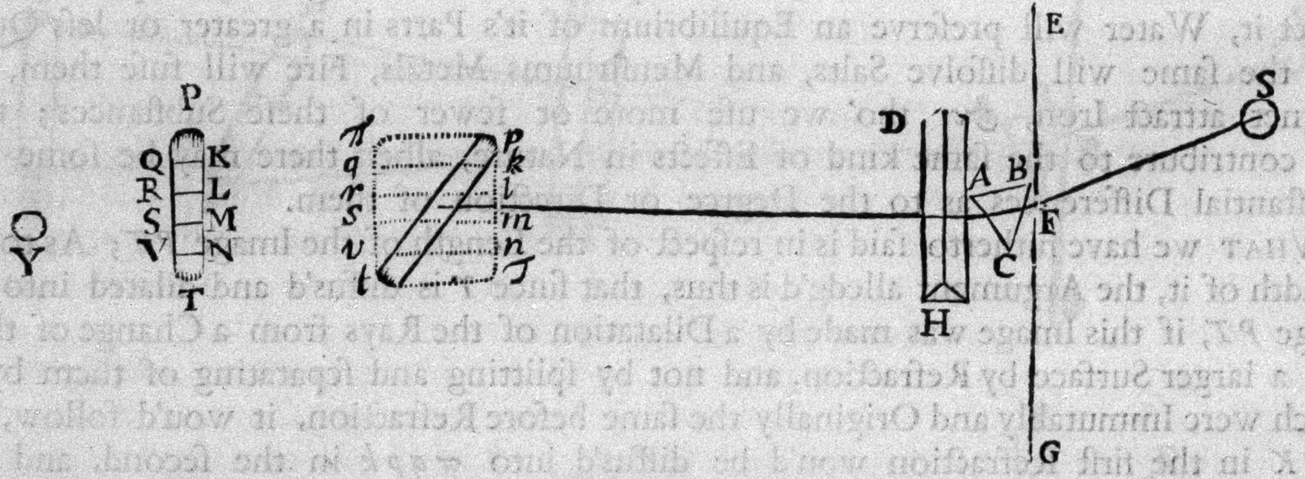
EXPERIMENTS the 1*ſt*, 2*d*, and 3*d*. we have already conſider'd in thoſe Principles, and ſhall now proceed to the 4*th*, it will be too Tedious for me to recount all the Experiments, and deſcribe them in Order, I ſhall therefore for the moſt Part only refer to them, with their Schemes, as they are deliver'd and explain'd to us by this Philoſophy, and reaſon and argue upon them from thence.

Sir *Iſ.* Newt. Opticks Prop. 2. Part 1. Book 1. Experiments 4, 5, 6, 7, 8, 9, and 10. Fig. 13, 14, 15, 16, 17, 18, 19, 20, 21, 22. Plate 3 & 4.

Experiment the *4th.* Fig. 13. which is by looking at the Foramen *F* in the Window-ſhut thro' the Priſm *ABC* is confeſs'd to be of the ſame Nature with the third, it ſhews the refracted Image of the Foramen to be Oblong, and not Circular, which the common and receiv'd Laws of Opticks wou'd require, and therefore, that, in equal Incidences of the Rays, there is a conſiderable Inequality of Refractions, but whence this Inequality ariſes do's not yet appear, and conſequently it is thus far acknowledg'd, that ſuch Inequality may be as well caus'd, by an Alteration of Light by the Priſm into variouſly colour'd Shadows, that is, by an Enervation and Diffuſion of the Rays in different Degrees, as by a Separation of Rays differently refrangible in their own Nature before their Tranſmiſſion thro' it.



Experiment the *5th.* Fig. 14. *S* is the Sun, *F* the Foramen of the Window-ſhut, *ABC* the firſt Priſm, *DH* the ſecond, *PT* the oblong Image of the Sun made by the firſt Priſm, when the ſecond is taken away, *pt* the Image made by the Croſs Refractions of both Priſms together, and *Y* the Image of the Sun made without any Priſm, or any Refraction at all; if therefore (it is ſaid) the orbicular Light *Y* is by the Refraction of the firſt Priſm drawn out and dilated into a long Image *PT* by a Diffuſion of Rays, the Light *PQK*, which is ſuppos'd to take up a Space of the ſame Length and Breadth with the Light *Y*, ought to be, by the Refraction of the ſecond Priſm, dilated and drawn out into the long Image $\pi q k p$, and the Light *KQRL*, into the long Image *kqrl*, and the Lights *LRSM*, *MSVN*, *NVT*, into ſo many other long Images *lrsm*, *msvn*, $n v t \tau$, and all theſe long Images wou'd compoſe the four Square Image $\pi \tau$, for the ſecond Refraction wou'd ſpread the Rays one way, as much as the firſt doth another, and ſo dilate the Image in Breadth, as much as the firſt doth in Length; But the event is otherwiſe, for the Image was not made broader by the ſecond Refraction, only became more Oblique, as repreſented at *pt*, it's upper end *P* being, by the ſecond Refraction, tranſlated to a greater Diſtance than it's lower end *T*; ſo then the Light, which went towards the upper end *P* of the Image, was, at equal Incidences, more refracted in the ſecond Priſm, than the Light, which tended towards the lower end *T*, that is the Blue and Violet, than the Red and Yellow, and therefore was more refrangible; The ſame Light was, by the Refraction of the firſt Priſm, tranſlated farther from the place *Y*, to which it tended before Refraction, and therefore ſuffer'd as well in the firſt Priſm, as in the ſecond, a greater Refraction, than the Reſt of Light, and by conſequence was more Refrangible



gible than the reſt even before it's Incidence on the firſt Priſm, and the ſame was try'd by a third and fourth Priſm, but the Rays, which were more refracted than the reſt in the firſt Priſm, were alſo more refracted in all the reſt, and that without any Dilatation of the Image ſide-ways.

THE Argument ſtands thus; becauſe *QPK*, the Violet, is more remov'd from *Y* than *VTN*, the Red, by the firſt Refraction, and becauſe *p* is more remov'd from *P* in the ſecond Refraction than *t* from *T*, and conſequently, becauſe theſe Rays are differently Refrangible, therefore *Y*, which is not remov'd from it's Place at all, nor any of it's Rays, conſiſts of Rays, which are differently Refrangible, that is, becauſe there are evident Proofs, that the colour'd Image after Refraction, is form'd of Rays of various Forces, Colours, or Degrees of Action or Refrangibility, therefore it is evident, that the yellow Image of the Sun, which has not thoſe Colours, yet conſiſts of them even before Refraction; This is the very Point in Queſtion, it is punctually deny'd, and it is prov'd no other way but by affirming it.

FOR what Connexion is there betwixt *QPK*, *VTN*, and ſo of the reſt being remov'd from their Places, and *Y* and it's Parts continuing in the ſame Place? betwixt *PT* being painted with Variety of Colours, and *Y* and it's Parts repreſenting only one? Is it not more natural to conclude from thence, that therefore *PT* is compos'd of different Sorts of Rays, and differently Refrangible, and that *Y* is not? Beſides what Connexion is there betwixt *QPK*, being more remov'd from *Y* than *VTN*, and *p* being more remov'd from *P*, than *t* from *T*? In the firſt Caſe two different Colours, Violet and Red, are produc'd, in the ſecond the ſame Colours are only preſerv'd, and therefore there cannot be the ſame Argument from *p* and *t* to *P* and *T*, as from *P* and *T* to *Y*.

IT is plain, the ſame Colours are preſerv'd thro' two, three, or more Priſms, and by conſequence their different Degrees of Refrangibility, but the Impreſſion upon Light is made by the firſt Priſm, and the Contorſion of the Image by the ſecond, without any Alteration of the Colours or different Actions of Light produc'd by the firſt; For this ſeems to be a general Law in Nature, that when once the Actions betwixt any two different Bodies are balanc'd and adjuſted, ſo as to produce a certain Effect, an addition of the ſame Kind of Action on one hand or another do's not change the Phænomenon as to the Nature of it, tho' it may as to the Degree, or Direction, or ſome other Circumſtance; Thus one, or two, or three Concave or Convex Glaſſes will have the ſame Effect to diſperſe or collect Light, and Speculums to reflect it, Water will preſerve an Equilibrium of it's Parts in a greater or leſs Quantity, the ſame will diſſolve Salts, and Menſtruums Metals, Fire will fuſe them, the Magnet attract Iron, *&c.* tho' we uſe more or fewer of theſe Subſtances; they will contribute to the ſame kind of Effects in Nature, albeit there may be ſome Circumſtantial Differences as to the Degree or Direction of them.

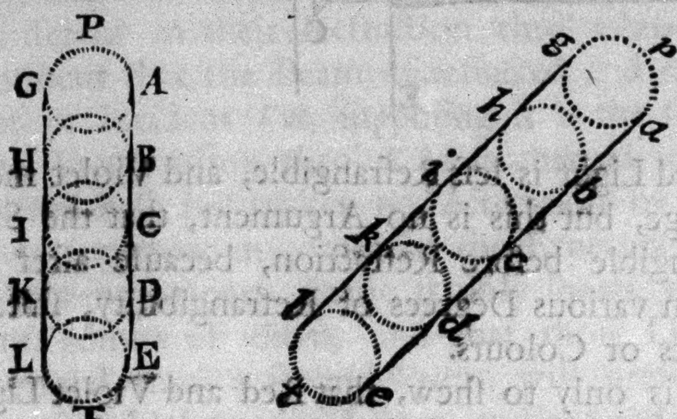
WHAT we have hitherto ſaid is in reſpect of the Length of the Image *PT*; As to the Breadth of it, the Argument alledg'd is thus, that ſince *Y* is diffus'd and dilated into the Image *PT*, if this Image was made by a Dilatation of the Rays from a Change of them into a larger Surface by Refraction, and not by ſplitting and ſeparating of them by it, which were Immutably and Originally the ſame before Refraction, it wou'd follow, that *QPK* in the firſt Refraction wou'd be diffus'd into *πqpk* in the ſecond, and that *QKRL*, *RLSM*, *SMVN*, *VNT*, in the firſt Refraction, wou'd be reſpectively diffus'd into *qkrl*, *rlsm*, *smvn*, *vntτ*, in the ſecond.

IN anſwer to which we muſt ſay, that altho' there is a Diffuſion of *Y*, or unrefracted Light into *PT*, the refracted thro' a Priſm, it is not from thence conſequent, that there ſhould be a Diffuſion of refracted Light or *PT* thro' a ſecond Priſm into *πτ*, for the Reaſons before offer'd.

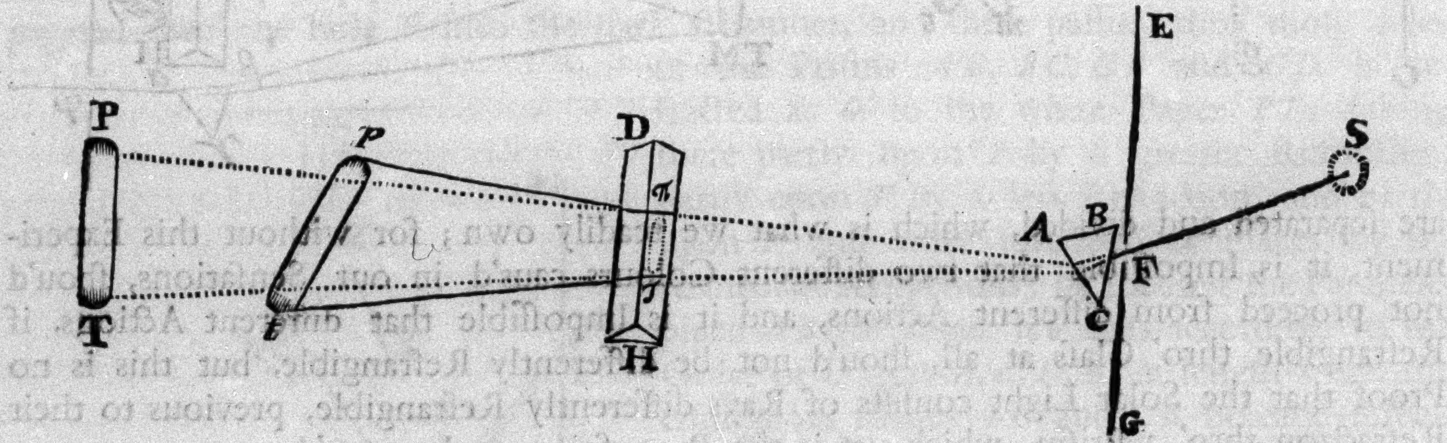
Farther, if *QPK*, *VTN*, and the reſt are to obſerve the ſame Laws as *Y*, in their Refraction, Violet and Red are ſuppos'd to have the ſame Properties as the pure Rays of the Sun, and *πpqk*, *vntτ*, ought not only to be of the ſame Length, but of the ſame Colours with *PT*, which wou'd be very abſurd to argue; we on the contrary, who affirm a Dilatation of the Image *Y* into *PT*, by an Alteration of the Properties of the Sun's Rays into Colours, deny, when they are thus alter'd, that they will obtain the ſame Effects of Dilatation or Change, becauſe Violet, Red, *&c.* are not the ſame as the Sun's Rays, but Shadows and Enervations of them.

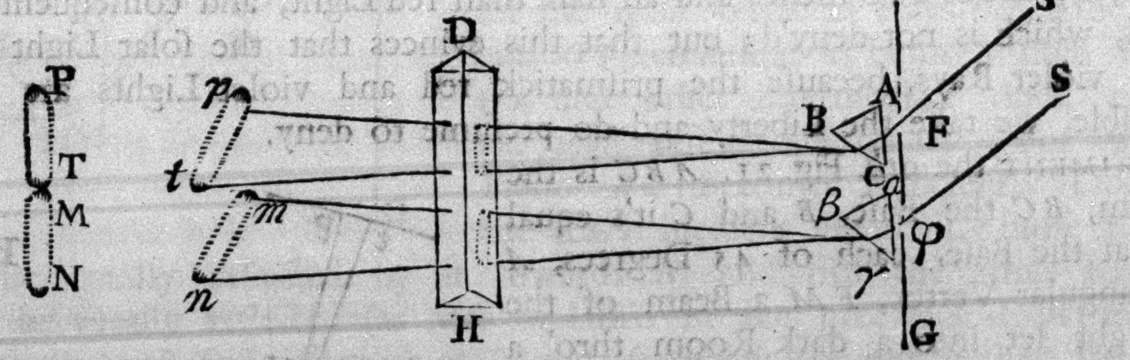
On the other hand, wou'd not this Objection be very Inconvenient to the preſent Hypotheſis of Diſſimilar Rays being ſplit and ſeparated by Refraction? Since *QPK*, *VTN*, and the reſt, each of them are blended with a thouſand different kind of Rays, tho' Violet and Red are the prevailing, and conſequently, they ought to be ſeparated and dilated into the Spaces *πpqk*, *vntτ*, which yet is not Fact or Experience.

Nor is this Difficulty ſolv'd by ſaying, as this Philoſophy do's, that the oblong Image *PT* conſiſts of innumerable circular ones of different Colours, each of them repreſenting the Sun's Diſque, for if the Diſque *Y* is ſeparated by the firſt Refraction into the Diſques *QPK*, *QKRL*, *RLSM*, *SMVN*, *VNT*, and each of theſe are form'd and made up of innumerable others, which is acknowledg'd, they certainly ought by a ſecond Refraction to be ſeparated and dilated into *πpqk*, *qkrl*, *rlsm*, &c. But which is more, theſe circular Diſques ſeem to be perfectly Chimerical, and owe their Riſe to the Breadth of the Image anſwering to the Sun's apparent Diameter, and to the Image being terminated by Semicircular Ends, which are very uncertain and precarious Proofs of it, eſpecially ſince Experiment the 11*th*, we find theſe Images of an Oblong and Triangular Form, ſuitable to the Figure of the Foramen; The Scheme Annex't expreſſes the imagin'd Faſhion of the Circular Diſques.

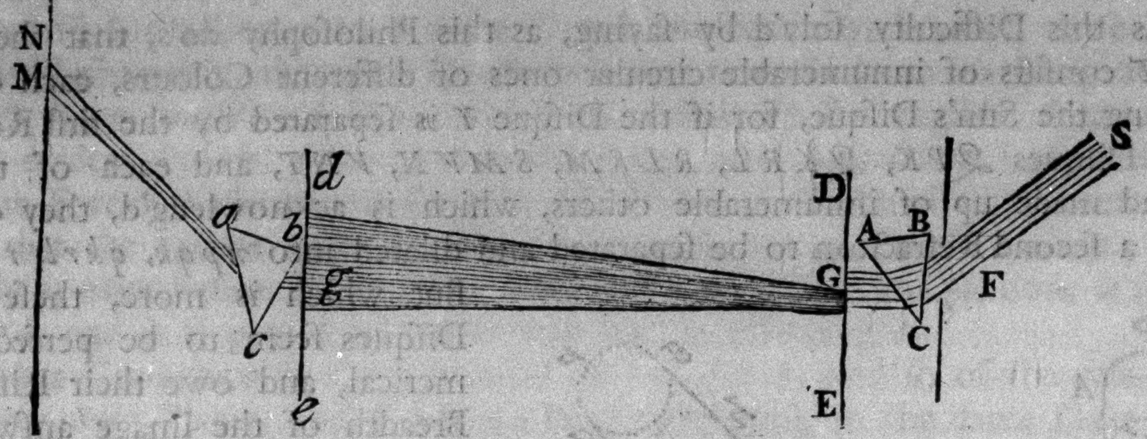


As to caſting the Image *Y*, by placing a Lens at the Foramen, without a Penumbra, from whence the Penumbra of the Images *PT*, *pt* wou'd likewiſe diſappear, that will equally agree with what we affirm, that the Refractions are perform'd exactly and regularly thro' the firſt Priſm, and continu'd and preſerv'd thro' a Second, or a Third; nor do Figures the 16*th*, where the ſecond Priſm is remov'd to ſome Diſtance from the firſt, and 17*th*, where two Images are painted upon the Wall in a direct Line by two diſtinct Priſms, and transfer'd from thence to an oblique Situation by the Interpoſition of a Third, evince any more than what we have already conſider'd and confuted.

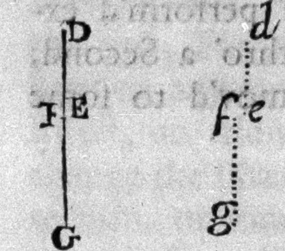


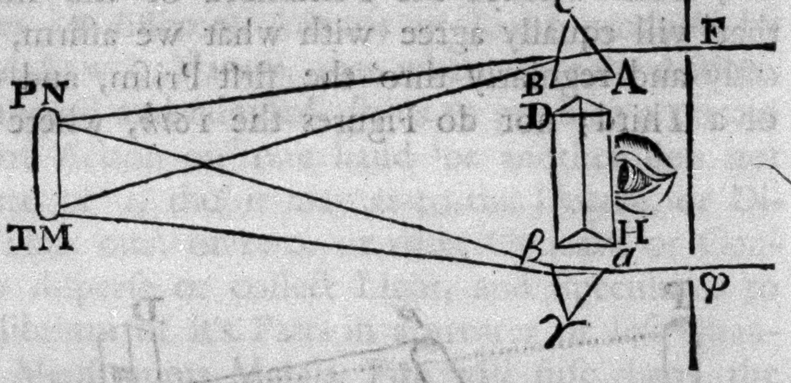


Experiment the *6th*. Fig. 18. only ſhews, that red Light paſſing thro' a ſecond Priſm *a c b*, is always refracted to a lower place *M*, in the oppoſite Wall, and Purple or Violet, paſſing thro' the ſame Priſm, and in the ſame Situation, is always refracted to an upper *N*, and the intermediate Colours to intermediate Places betwixt *M* and *N*, which is a Conviction, that red Light is leſs Refrangible, and Violet more, and ſo of the reſt, which we acknowledge, but this is no Argument, that the Sun's Light conſiſts of Rays differently Refrangible before Refraction, becauſe after Refraction different Lights or Colours obtain various Degrees of Refrangibility, ſuitable to the Nature and Action of thoſe Lights or Colours.

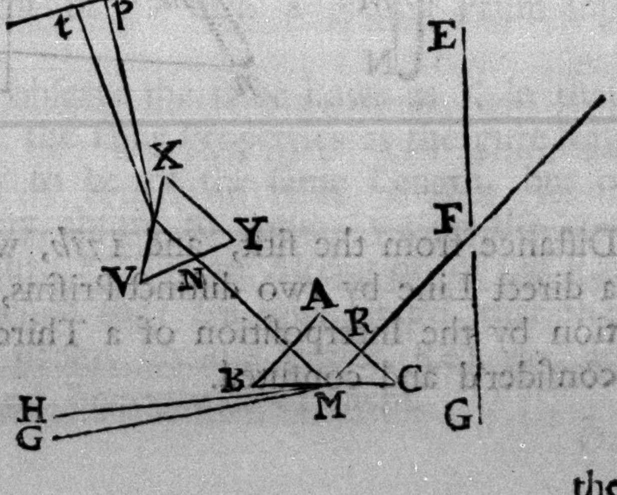


Experiment the *7th*. Fig. 17. 19, 20. is only to ſhew, that Red and Violet Lights produc'd by two Priſms, and view'd thro' a Third, are differently refracted by it, that is, tho' the red and violet Images are continu'd to each other, yet by Refraction they are ſeparated and divided, which is what we readily own; for without this Experiment, it is Impoſſible that two different Colours caus'd in our Senſations, ſhou'd not proceed from different Actions, and it is Impoſſible that different Actions, if Refrangible thro' Glaſs at all, ſhou'd not be differently Refrangible, but this is no Proof that the Solar Light conſiſts of Rays differently Refrangible, previous to their Refraction thro' a Priſm, which yet is the Propoſition to be prov'd.





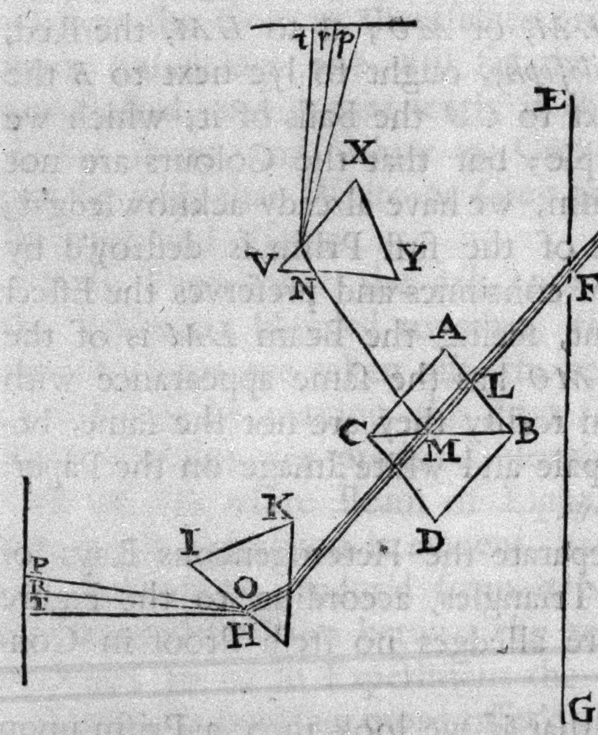
Experiment the *8th*, is of the ſame kind with the Former, namely, to ſhew that the violet priſmatick Light tranſmitted thro' a Lens, converges ſooner and nearer to the Lens, by about two Inches and an half than red Light, and conſequently is more refracted, which is not deny'd; but that this evinces that the ſolar Light conſiſts of red and violet Rays, becauſe the priſmatick red and violet Lights are differently Refrangible, we take the Liberty and do preſume to deny.



Experiment the *9th*. Fig. 21. *ABC* is the firſt Priſm, *BC* the Baſe, *B* and *C* it's equal Angles at the Baſe, each of 45 Degrees, *A* it's rectangular Vertex, *FM* a Beam of the Sun's Light let into a dark Room thro' a Hole *F*, one third Part of an Inch broad, *M* it's Incidence on the Baſe of the Priſm, *MG* a leſs refracted Ray, *MH* a more refracted Ray, *MN* the Beam of Light reflected from the Baſe, *VXY* the ſecond Priſm, by which this Beam, in paſſing thro' it, is refracted, *Nt* the leſs refracted Light of this Beam, and *Np* the more refracted Part of it; when the firſt Priſm *ABC* is turn'd about it's Axis according to the Order of the Letters *A, B, C,* the Rays *MH* emerge more and more obliquely out of that Priſm, and at Length after their moſt oblique Emergence are reflected towards *N,* and going on to *p* do increaſe the Number of the Rays *Np*; afterwards by continuing the Motion of the firſt Priſm, the Rays *MG* are alſo reflected to *N,* and increaſe the Number of the Rays *Nt,* and therefore the Light *MN* admits into it's Compoſition, firſt the more refrangible Rays, and then the leſs refrangible Rays, and yet, after this Compoſition, is of the ſame Nature with the Sun's immediate Light *FM,* the Reflection of the ſpecular Baſe *BC* cauſing no Alteration therein.

THIS Experiment therefore is an Argument for what we ſtill confeſs, that *MG* and *MH* are differently Refrangible and differently Reflexible, becauſe Violet and Red being different Colours have different Actions, and conſequently their Effects muſt be different in their Refraction thro' Glaſs, or their Reflexion from it, but it is no Argument that the Beam of refracted and reflected Light *MN,* is the ſame with the Solar or Incident *FR,* ſuppoſing it is the ſame with *FM,* ſince the Solar Beam *FR* in it's Tranſmiſſion thro' *RM* is chang'd into the Colours *MG, MH,* and then indeed the Baſe *BC* makes no Alteration by it's Reflexion; this therefore is no more convincing than any other of the Experiments to prove, that the Solar Light conſiſts of different Colours, ſeeing it only ſhews, that the Solar Light, when refracted by a Priſm, admits of them, and that thoſe Lights, when reflected from *M* to *N,* and tranſmited thro' a ſecond Priſm, will preſerve their Degrees of Refrangibility, and the purple Light proceed to *p,* and the red to *t,* nothing of which is conteſted, but affirm'd, at the ſame Time, that we cannot allow *FR* to be the ſame Light as *RM* or *MH, MG,* or *MN,* and ſo to *p* and *t*; for if it were ſo, any Glaſs Baſis, as well as *BC* the Baſis of the Priſm, ſhould have the ſame Effect, and reflect *MH, MG* in the ſame manner to *N,* and from thence thro' the Priſm to *p* and *t,* if *MH, MG* were originally in *FR,* before it's Ingreſs into the Priſm *ABC.*

EXPERIMENT the *10th.* Fig. 22. *ABC* and *BCD* are two Priſms ty'd together in the Form of a Parallelepiped, their ſides *BC* and *CB* being Contiguous, and their ſides *AB* and *CD* parallel; *HIK* is a third Priſm, by which the Sun's Light propagated thro' the hole *F* into the dark Chamber, and there paſſing thro' thoſe ſides of the Priſms *AB, BC, CB* and *CD* is refracted at *O* to the white Paper *PT,* falling there partly upon *P* by a greater Refraction, partly upon *T* by a leſs Refraction, and partly upon *R* and other intermediate Places by intermediate Refractions; by turning the Parallelepiped *ABCD* about it's Axis according to the Order of Letters *A, C, D, B,* at length when the contiguous Planes *BC* and *CB* become ſufficiently oblique to the Rays *FM,* which are incident upon them at *M,* there will vaniſh totally out of the refracted Light *OPT,* firſt of all the moſt refracted Rays *OP,* (the reſt *OR* and *OT* remaining as before) then the Rays *OR* and other intermediate ones, and laſtly the leaſt refracted Rays *OT*; For when the Plane *BC* becomes ſufficiently Oblique to the Rays incident upon it, thoſe Rays will begin to be totally reflected by it towards *N,* and firſt the moſt Refrangible Rays will be totally reflected (as was explain'd in the preceding Experiment) and by conſequence muſt firſt diſappear at *P,* and afterwards the reſt as they are in order totally reflected to *N,* they muſt diſappear in the ſame order at *R* and *T*; ſo then the Rays, which at *O* ſuffer the greateſt Refraction, may be taken out of the Light *MO,* whilſt the reſt of the Rays remain in it, and therefore that Light *MO* is compounded of Rays differently Refrangible, and becauſe the Planes *AB* and *CD* are parallel, and therefore by equal and contrary Refractions deſtroy one another's Effects, the incident Light *FM* muſt be of the ſame Kind and Nature with the emergent Light *MO,*



and therefore doth alſo conſiſt of Rays differently Refrangible; Theſe two Lights *FM* and *MO*, before the moſt refrangible Rays are ſeparated out of the emergent Light *MO*, agree in Colour and in all other Properties ſo far as the Obſervation of this Philoſophy reaches, and therefore are deſervedly reputed of the ſame Nature and Conſtitution, and by conſequence the one is compounded as well as the other; But after the moſt refrangible Rays begin to be totally reflected, and thereby ſeparated out of the emergent Light *MO*, that Light changes it's Colour from White to a dilute and faint Yellow, a pretty good Orange, a very full Red ſucceſſively, and then totally Vaniſhes; For after the moſt refrangible Rays which paint the Paper at *P* with a purple Colour are, by a total Reflexion, taken out of the Beam of Light *MO*, the reſt of the Colours, which appear on the Paper at *R* and *T*, being mixt in the Light *MO*, compound there a faint Yellow, and after the Blue and part of the Green, which appear on the Paper between *P* and *R*, are taken away, the reſt, which appear between *R* and *T* (that is the Yellow, Orange, Red, and a little Green) being mixt in the Beam *MO* compound there an Orange, and when all the Rays are by Reflexion taken out of the Beam *MO*, except the leaſt Refrangible, which at *T* appear of a full Red, their Colour is the ſame in that Beam *MO* as afterwards at *T*, the Refraction of the Priſm *HIK* ſerving only to ſeparate the differently refrangible Rays without making any Alteration in their Colours, as is ſaid will be more fully prov'd hereafter; The Reaſoning in the *Scholium* which follows is of a like Kind, and therefore we ſhall not here ſubjoin it.

In anſwer to the way of arguing upon this Experiment (for the Experiment we do not deny, nor any other of this Philoſophy) we ſay, that this is much the ſame Caſe with the Former, and that altho' the Beam *LM* may be of the ſame compounded Light with *MO*, as having been tranſmitted thro' the Priſm *ABC*, it is not manifeſt, that *FL* the incident and unrefracted Beam is thus compounded before ſuch Tranſmiſſion; and a moſt convincing Argument, that it is not, is, that if it were, by the turning about of the ſide *AB* of the Priſm, when the blue Rays of the Beam *FL* come to fall with a certain Obliquity, they ought to be reflected from thence, and after them the Green, Yellow, Orange, and Red ſucceſſively, in like Manner, as the Rays of the Beam *LM* or *MO* are reflected to *N*, and by that means transfer'd from *P*, *R*, and *T* to *p*, *r*, and *t*, one after another, but this do's not happen, from whence it is evident to a Demonſtration, that *FL* the incident Beam is not of the ſame compounded Nature with the refracted *LM*, or *MO*; as to *LM*, the Red, as we have ſhewn in our *Principles of Natural Philoſophy*, ought to lye next to *B* the refracting Angle of the Priſm, and the Purple next to *CA* the Baſis of it, which we find in fact, ſince *OT* is the Red, and *OP* the Purple; but that the Colours are not alter'd by a Tranſmiſſion thro' a ſecond or third Priſm, we have already acknowledg'd; It is ſaid, that *AB*, *CD* being parallel, the Effect of the firſt Priſm is deſtroy'd by the ſecond, which we deny, ſince the ſecond Priſm continues and preſerves the Effect of the firſt, as is plain from this very Experiment, ſeeing the Beam *LM* is of the ſame Kind and Nature with that of *MO*; that *MO* has the ſame appearance with *FL* is not Material, ſince we have evinc'd, that in reality they are not the ſame, beſides it is not certain, that *MO* do's not cauſe a pale and white Image on the Paper, whereas *FL* paints on it a ſtrong and vivid Yellow.

Experiment the 11*th*. Teaches us how to ſeparate the Heterogeneous Rays of compound Light into Circles, Parallelograms or Triangles, according to the Figure of the Foramen, which tranſmitts it, and therefore alledges no freſh Proof in Confirmation of the foregoing Doctrine.

Experiments the 12*th*, 13*th*, and 14*th* ſhew, that if we look thro' a Priſm upon a Circular Object illuminated by the Sun's Light, it appears oblong, confus'd, and indiſtinct, but if we look thro' a Priſm on the ſame Object illuminated with Red, Green, Blue or any other Homogeneal Light, it appears circular and perfectly defin'd, but this is no more than in other Terms to tell us, what we are inform'd of in the third Experiment, that the Sun's Light by a Priſm is refracted into various Colours; but that Homogeneal Light, or Colours produc'd by Refraction are not, this is no Argument therefore that ſuch Colours are originally in the Sun's Light, or that they are not made and produc'd by Refraction, tho' after their Production they become unalterable, and are never from thence Metamorphos'd or Chang'd.

Another Series of Experiments is to prove, that the Phænomena of Colours, in refracted or reflected Light, are not caus'd by new Modifications of the Light variouſly impreſs'd according to the various Terminations of the Light and Shadow, which is to demonſtrate the Falſhood of the *Carteſian* Hypotheſis, and ſignifies nothing to the evincing the Point in Queſtion, namely, that the Solar Light conſiſts of diſſimilar and differently colour'd Rays.

Sir Iſ. Newt. Opticks Prop. 1. Book 1. Part 2.

There are again other Experiments to ſhew, that all Homogeneal Light has it's proper Colour anſwering to it's Degree of Refrangibility, and that Colour cannot be chang'd by Reflexions and Refractions; that is Homogeneal Red, Green, or Blue Light, for Inſtance, after they are produc'd by the Priſm, are unchangeable, and each has it's peculiar Action belonging to it, or it's proper Degree of Refrangibility; but this is no Proof, that the Solar Light conſiſts of ſuch various Homogeneal Lights, only that the Priſm is capable of producing them, and which when produc'd, that there is no Alteration of Them.

Sir Iſ. Newt. Opticks Prop. 2. Book 1. Part 2.

As to what is ſaid, that Colours may be produc'd by Compoſition, which ſhall be like to the Colours of Homogeneal Light, as to the Appearance of Colour, but not as to the Immutability of Colour and Conſtitution of Light, and that thoſe Colours by how much they are more compounded by ſo much are they leſs full and intenſe, and that by too much Compoſition they may be diluted and weaken'd, till they ceaſe, and the Mixture becomes White or Grey, that alſo whiteneſs and all grey Colours may be compounded of Colours, and that the whiteneſs of the Sun's Light is compounded of all the primary Colours mixt in a due Proportion; a great Part of this is own'd, that the Lights produc'd by the Priſm may be variouſly mix't and blended, from whence new Lights will ariſe, which may be again ſeparated and diſtinguiſh'd, alſo that theſe colour'd Lights, produc'd by the Priſm, will conſtitute a White or Grey, which White or Grey is again reſolvible into thoſe colour'd Lights, of which they are form'd; but that the Solar Beam is either White or Grey, which is plainly Yellow, or that it is made up of differently colour'd and Homogeneal Lights is deny'd, and what is alledg'd in theſe two Propoſitions do's in Part confirm it; for it is ſaid, that the Colours, by how much they are more compounded, by ſo much are they leſs full and intenſe, and by too much Compoſition they may be diluted and weaken'd till they ceaſe, and the Mixture becomes White or Grey, therefore the White or Grey proceeds from a Weakneſs or Dilution of the Colours, whereas the Colours themſelves proceed from a Weakneſs and Dilution of the Sun's Rays, unleſs any one will ſay the colour'd Image is ſtronger than the Sun's Beam unrefracted, and conſequently if the Colours are an Enervation of the Sun's unrefracted Beam, and White or Grey is an Enervation of the Colours, I hope it will not be ſaid, that White or Grey is the ſame as a Beam of the Sun acting in it's full Strength and Vigour.

Sir Iſ. Newt. Opticks Prop. 4. & 5. Book 1. Part 2. Experiments 9, 10, 11, 12, 13.

And Experiments the 9*th*, 10*th*, 11*th*, &c. only ſhew, that, when the Colours of the Priſm are blended together, they produce a white Image, and that if any of thoſe Colours are taken out, the remaining repreſent the Colour of the Compoſition, as alſo that by making the Rays fall unequally upon the Paper, which receives the Rays, Colours are produc'd, nothing of which is contradicted by us; but that the Caſe of this white Beam of Light after Refraction is not the ſame with the Solar before it's Refraction is evident, becauſe the Solar Beam cannot be ſo manag'd, nor can Colours be produc'd from it by any Inclination of the Paper to it, and there is the ſame Diſtinction betwixt the refracted and unrefracted Beam, as there was before evidenc'd to be in Experiment the 9*th* and 10*th*, Part the firſt.

There is nothing more offer'd in this Theory of Light and Colours to prove the Truth of it, and it's fundamental Propoſition, namely, that the Solar Light conſiſts of differently colour'd Rays originally to it's Refraction thro' the Priſm, unleſs what we have already conſider'd in our *Principles of Natural Philoſophy*, and our Inquiry there, into this Doctrine, ſo that till this is anſwer'd we may be juſtly excus'd any farther trouble on this Head.

A Solution of Colours, and of the Rainbow, and concerning Sounds.

§. 21. I ſhall only here add, that the Colours of the Priſm ſeem to ariſe from what I there mention'd in thoſe *Principles*, namely Red from a Tranſmiſſion thro' the thinneſt Part of the Priſm or the very Edges of it, by which it's Colour is neareſt to the Sun's, and it's Rays the moſt vigorous and lively, and therefore the leaſt Refrangible;

gible; Orange, Yellow, Green, Blue and Violet paſs thro' thicker Portions of the Priſm, by which they loſe of their Action, become leſs and leſs vigorous, and therefore are more Refrangible and Weak, and depart more from the Life and Activity of the vivid Beams of the Sun, and accordingly impreſs upon our Senſations Colours, which are darker and more gloomy; for the Rays of Light, when they impinge upon the Body of the Priſm, will yet, it is probable, be tranſmitted thro' the Edges of it, where there is the leaſt Reſiſtance;

THAT the ſame is the Caſe of thin Plates of Glaſs, Air, or Water, which, when thin, repreſent theſe various Colours to us from a certain Proportion requiſite betwixt the Action of the Sun's Rays, and of Glaſs, Air, or Water thro' which they are tranſmitted;

THAT the Action of the Sun's Rays is of a moſt ſubtle and expanſive Nature, and that it cannot have any Effect to produce Colours by Tranſmiſſion, unleſs it is incident upon Bodies, which are very Minute and Thin, which have ſome Degree of the Expanſive to aſſiſt it's Tranſmiſſion, and but a ſmall Degree of the Contractive to impreſs an Alteration and change upon the Expanſive of the Sun's Rays, which is the Caſe of ſuch thin Plates;

THAT other groſſer Bodies, as colour'd Fluids and Solids, derive thoſe Colours from the Impreſſions, which they give to the Sun's Light diſpers'd thro' the Plenum from their own proper Expanſive and Contractive Forces, by which it receives a new Action, and thereby communicates to us thoſe Senſations of Colours; That therefore a Lamp or a Candle, enlightening the ſaid Plenum, cauſes the Colours of Natural Bodies, whoſe Expanſive and Contractive Forces are the ſame, to appear much the ſame, yet with ſome little Diſtinction ariſing from the different Nature of the Light of the Sun and a Lamp, which illuminate the ſame Plenum;

Sir Iſ. Newt. Opticks.

THAT the Experiment of Dr. *Halley* at ſeveral Fathoms deep in the Sea, where the upper part of his Hand was illuminated with Red, and the lower with Green, is a Conviction, that the Sun's Rays were only weaken'd to a Red in the firſt Caſe, and by penetrating deeper and being reflected, to a Green;

THAT the various Colours of the Rainbow ariſe from a different Enervation of the Sun's Rays by thin Plates of a watry Medium of different Thickneſſes or Conſtipations, the uppermoſt being the thinneſt, cauſe the leaſt Enervation, and the loweſt being the groſſeſt, occaſion the greateſt, from whence Red in the firſt caſe is produc'd, and Violet in the laſt, and ſo the intermediate Degrees of Colours in the intermediate Spaces;

THAT when the watry Medium is too Thin, as above the Red, the Rays paſs without any Alteration, and when it is too Groſs, as below the Violet, there is no due Proportion of Action betwixt the Sun's Light and the Medium by which an Alteration can be produc'd, and therefore we only ſee the plain watry Medium, in the ſame Manner, as it is in the Priſm, which when we come to a certain Thickneſs of it no Colours are produc'd, and we only perceive the Tranſparency of the Priſm it ſelf; or elſe, which I rather believe, it is becauſe the Expanſive Force of the Sun is too great for the Contractive of the watry Medium, and by that means diſſipates and ſurmounts it;

THAT the Eye is made ſenſible of theſe Alterations in the Sun's Rays, incurring upon different Mediums, not by the imaginary Refractions and Reflections of Light, propos'd by *Deſcartes*, but by thoſe different Actions communicated to the Plenum, which conveighs theſe Notices to us, and by the Intervention of which, and our Union with it, we are inform'd of all the various Changes we obſerve in Nature, which when differently affected we are ſo too, not only in our Sight, but in our Hearing, Smelling, and Feeling, which Impreſſions yet were never thought to be imparted to us by ſuch fancy'd Reflections and Refractions;

To explain this Theory ſtill farther, It is known and confeſs'd, that a Rainbow never appears, but when the Air is moiſt and dewy, after Rain, and when the Sun ſhines out directly oppoſite to it, in this Situation of the Sun, the Centre of Action is from thence, which by it's Expanſive Force, to a large Circuit round it, overcomes and diſſipates any counter Action againſt it from the Dew or Vapors, but the dewy or watry Medium preſſing and contracting againſt this Expanſive Force, at length, when the Rays of the Sun become more Oblique, is capable of making ſome Reſiſtance to it, and the greateſt Preſſure or Reluctance to the Expanſive Force of the

Sun being from the innermoſt Circle, becauſe it do's not only preſs it ſelf againſt it, but is back'd and enforc'd by all the ambient Circles which ſurround it, and whoſe Preſſure tends the ſame way, the innermoſt Circle for that Reaſon will be the groſſeſt, from whence the Expanſive Force of the Sun being there moſt reſiſted, it's Rays become of a blackiſh or purple Colour;

THE next Circle being leſs conſtipated, and denſe, and preſſing leſs againſt the ſame Expanſive Force of the Sun's Rays, becauſe it has fewer ambient Circles to ſtrengthen it's Preſſure, changes the Sun's Rays to a Blue, the next to a Green, and the ſubſequent ones to Yellow and Orange, till at length where the outermoſt Circle becomes the weakeſt in it's Preſſure, Reſiſtance and Contraction by the dewy Medium being now Rarer, and having fewer ambient Circles to ſupport it, the Rays of the Sun and the Expanſive Force of them being leaſt reſiſted are form'd into a Red, which compoſes the *Primary Iris*; After which the Expanſive Force of the Sun's Rays meeting with no conſiderable Oppoſition, they paſs on without any remarkable Phænomenon from them for ſome Space, till now becoming more oblique, and more weak, the Rarity of the dewy Medium is capable of making a new Impreſſion upon them by changing them into Red tho' a fainter one than before, becauſe the Action on both Hands, both of the Sun and of the Medium, is more languid, after which the Rays of the Sun and their Expanſive Force growing ſtill more oblique and weaker and the dewy Medium being much the ſame, as being now upon it's Termination, the Sun's Light is vary'd into Orange, Yellow, Green, Blue, and Purple by ſo many Degrees of Enervation, till the dewy Medium being ended the Rays of the Sun take their Progreſs as uſual without any Impediment or Oppoſition to them, which ſeems to be the true Reaſon of the *Secondary Iris*;

AND from hence may be given a full Account for the different Heighths of the *Iris* from 41 to 45 Degrees, which is not done in any other Hypotheſis, with many Phænomenons of it beſides, which have not been hitherto explain'd, It alſo appears at ſuch a certain Depth of the Atmoſphere, and at ſuch a Diſtance from the Sun, and on the oppoſite Side to it, becauſe the Rays ſurmount all Oppoſition upon their firſt Ingreſs into the watry Medium, but afterwards penetrating farther, and growing weaker, they are chang'd by the Contractive Force of the ambient Dew into the Colours, and in the Manner we have mention'd;

THAT the Expanſive Force of the Sun acts in the Manner I have recounted is not difficult to apprehend, and I believe will be readily own'd, but that the dewy Medium ſhou'd act by a Contractive Force againſt it, will not be ſo ſoon imagin'd, till we come to be convinc'd, which will be ſhewn in the following Sheets, that all Nature conſiſts of theſe two Forces, and that all the Phænomenons in it ariſe from their mutual Conteſts and Conflicts with each other;

THE Account of the Rainbow above-mention'd is farther confirm'd by what every one may obſerve of a Candle burning in the Dark, and in an open and moiſt Air, which if you look upon at ſome Diſtance is encompaſs'd with an illuminated Medium, and afterwards with Purple, Blue, Green, Yellow, Orange, and Red Circles, repreſenting the Colours of the Rainbow, which can be no otherways ſolv'd than upon the foregoing Hypotheſis.

Our Principles of Philoſophy and Chapter upon Sounds, and the ſecond Edition of Sir Iſ. Newt. Principles upon that Head.

As to the Doctrine of Sounds, which we had in our *Principles of Natural Philoſophy* argu'd againſt, the Preſent has been ſo complaiſant to thoſe Reaſonings and to the *London* and *Paris* Experiments, as to alter it's Calculations from betwixt 920 and 1085 Feet, which Sound performs in a ſecond, according to it's firſt Edition, to 1142 in it's ſecond Edition; ſo eaſy it is for this Philoſophy to change and vary it's Principles, as new Matters of Fact ariſe, to which it as readily accommodates it ſelf, as the Chamæleon to the Air it breaths in; but beſides the Velocity of Sound, which in Eccho's is exceeding great and impoſſible to be aſcertain'd, from which yet theſe Calculations are fram'd, the Expanſive and Contractive Forces of Bodies are ſo infinitely various, from which Sounds are really and in Nature produc'd, which will be afterwards ſhewn, that it is Vain and Idle to imagine they can be exacted to any Mathematical Rule of their Progreſſion.

Concluſion upon the Epicurean *Philoſophy.*

§.22. Having therefore thus far conſider'd the *Corpuſcular* Philoſophy, or the Philoſophy of Similar and Homogeneous Matter, and the Atoms of it, which I call the *French* Philoſophy being taught indeed originally by *Epicurus*, but reviv'd by *Deſcartes*, and patroniz'd by *Gaſſendus*, who writ the Life of that Philoſopher very much in favour of him; and having likewiſe conſider'd the Improvements upon it from our own Nation, who have added a Vacuum to it, which *Deſcartes* ſhou'd have done, it being Impoſſible to maintain a Plenum, and a Similar and Homogeneous Matter together, I ſhall now take my final leave of it, and proceed to what may be juſtly term'd the *Italick* Philoſophy, or the *Mechanical*, being principally deriv'd from *Galileus*, taken up by *Borellus*, and eſpous'd by *Kepler*, *Chriſtian Hugens*, and Sir *Iſ. Newton*, and which Philoſophy likewiſe agrees ſo far with the former, as to affirm a Similar Matter, and the Atoms of it, to which they have, with ſome kind of Senſe and Conſiſtency with themſelves, annex't a Vacuum, by which it becomes the entire Syſteme and Doctrine of *Epicurus*; but then again, they have with ſome Abſurdity join'd a Deity to this Similar Matter, Atoms, and a Vacuum, which their Maſter more truly, tho' perhaps not more prudently rejected from it, and left his Similar Matter very honeſtly, and the Atoms or Particles of it, to their own Actions, after he had contriv'd Space and Room enough for them, in which they were at Liberty to perform their random Motions and Vagaries.

Sir Iſ. Newt. Princip. Lib. 3. Pag. 482. Edit. 2.

HOWEVER this Philoſophy has been ſo Indulgent to what we had mention'd in our *Principles*, Chap. 1. §. 16. That we ſhould make the Demonſtration of *Religion* the *End* and *Concluſion* as well as *Deſign* of all our Speculations, as to annex an Explication of what it means by a God, tho' not a Demonſtration of one, at the End of it's ſecond Edition, which was wanting in the firſt; but it's Atoms, which were Immutable, and it's Vacuum, which was Infinite, Eternal and Unproduc'd, wou'd not allow it to acknowledge, that the Intelligence was an *Anima Mundi*, or had any Connexion with Nature, or any Union with it, or was in any Senſe a ſource from which it was deriv'd, only that it was a Being, which was an *Imperator Univerſalis*, or Lord Paramount to theſe Atoms, and guided and conducted them as he pleas'd; which is ſaying nothing at all in the Proof of a God, but forming a Creature of our own Fancies, which we ſet at the Head of our imaginary Atoms, and is, in Reality and Truth, a Fineneſs, which *Epicurus* had not the Wit, or too great a Simplicity in his Notions, to think of; for when he had made all to depend upon Atoms, and the Gravity of them, and upon a Vacuum, he juſtly concluded, that he had effectually diſcarded *Anaxagoras*'s Mind or Intelligence out of the World, and therefore, for which the *Athenians* valu'd him, that he had nothing remaining to profeſs of Religion, but to expreſs himſelf hearty for the Gods of his Country, without having any real Eſteem for them, and which is pretty evidently the Caſe of ſome of our Modern Philoſophers.

IF any one ſhall now ask me, whether therefore we maintain the Plenum of the Ancients, and that God and Matter are the Principles of all Things, and that Matter, in Conſequence, is Eternal and Unproduc'd;

WE anſwer firſt, That we are under no Obligation from what we have hitherto alledg'd to aſſert any ſuch Doctrine, all that we have endeavour'd to ſhew and evince is, that the current Opinions of the oldeſt and moſt Celebrated Philoſophers of *Greece* and *Phœnicia* were on the Side, not of a Vacuum, but a Plenum, and that the Wiſeſt and Beſt Men eſpous'd the Latter, whilſt the Former was defended by none but by the Atheiſtical and the Vicious.

SECOND, But to ſatisfy this Queſtion more directly, we anſwer, That with the Ancients we affirm a Plenitude of Corporeal Being or Matter, which yet we ſay and ſhall prove, is not a Self-exiſtent Principle, but is deriv'd from and created and form'd by God, and that it could not ſubſiſt one Moment without him; whether this Production of the τὸ πᾶν, or the Univerſe, or the Material Plenum was in Time or from Eternity, Reaſon cannot determine, and therefore muſt be ſubmitted to Revelation, becauſe in an Eternal Succeſſion of Ages, there is no point of Time, in which the Infinite Mind was not equally capable of exerting it's Almighty Faculties into Creation, and conſequently what admitted of no Limitation from his Power, muſt receive it from his Will, which it is Impoſſible for us to know without ſuch Revelation.

CHAP. II.

Concerning the Principles of the Mechanical Philoſophy, and the Matter and Motion aſſerted in it, as alſo of Space, Place, Time or Duration, and Celerity.

§. 1. WHAT has hitherto engag'd our Enquiries has been the certainty of an exiſting Nature in general, the Opinions concerning a Vacuum and a Plenum, and the Philoſophy of Matter with the ſeveral Properties of it, it's being Extended, Solid, and Impenetrable, Diviſible, Moveable, *&c.* Cold, Heat, Hardneſs, and Softneſs, as alſo the Sounds, Taſts, Smells, Colour and the like, which we in fact Experience reſult from it, and the Theory of the Rainbow, which concluded the firſt Part of our Examination into the Principles of natural Knowledge; we have from thence proceeded in the Courſe of our Studies, to ſhew the Infirmities and Weakneſs in the various Hypotheſes of the Reflection and Refraction of Bodies, and in conſequence, of that of Light, which carry'd our Searches into the Principles of Opticks, and the Solutions they give of Viſion, of concave and convex Glaſſes, the Conſtruction of Microſcopes and Teleſcopes, and the numerous Phænomenons in that Science, all which we ſhall at preſent omit, and we ſhall now therefore go on to conſider, the Philoſophy of Matter with that of Motion, and the Laws and Hypotheſes which have been rais'd to account for them, and as Opticks was one Part of what may be term'd the active Philoſophy in oppoſition to the Corpuſcular, ſo this is another.

Two Parts in Philoſophy, the Corpuſcular and the Active, the former of which has been conſidered, and the latter is reſolv'd Principally into that of Light, and Opticks, and Mechaniſm, or Motion.

§. 2. IN the Theory of Mechaniſm or Motion, there are therefore four Things to be conſider'd; Firſt, what Motion is, Second, the Nature of that which is mov'd, Third, the Place which that which is mov'd poſſeſſes, as well as the Space thro' which it moves, and laſtly, the time which it takes up in it's Motion; Conſequently to determine the Motion of Body, and what Laws it obſerves, we are to inquire in general; Firſt, what Motion is, Second, what Body or Matter is, Third, the Meaning of Place, and what real Space is, and Fourth, the Notion of Time.

In the Theory of Mechaniſm or Motion, four Things to be conſidered.

§. 3. As to what reſpects Motion it will not I am apt to think be neceſſary to prove there is ſuch an appearance in Nature, becauſe it is as plain and manifeſt as any thing in it, and thoſe who have rais'd Difficulties againſt it, as *Diodorus* or *Zeno*, or whoever elſe, argue againſt it upon a Suppoſition of Space and Body exiſting, which are no more demonſtrable or apparent than Motion it ſelf is, for it is only by our Senſes that we know there is Space or Body, and by the ſame we are convinc'd, that there is Motion likewiſe.

Motion as plain and evident as any other Phænomenon in Nature.
Dr. Keill Lect. 6. Int. ad Ver. Phyſicam p. 57.

§. 4. WHAT Motion is therefore we are next to examine; if it is a Tranſlation of Body out of the Vicinity of a certain Similar Homogeneous Matter, which ſurrounds it, into another, be it ſubtle or otherwiſe, whilſt the Notion of Matter continues the ſame, 'tis a Tranſlation or Motion of Body out of it ſelf into it ſelf, which is impoſſible, unleſs it be allow'd that a Penetration of Dimenſions is not ſo, or it is the ſame as to ſay, that Gold can move in it's own Subſtance; if Motion is affirm'd to be a Tranſlation of Matter out of one Part of Vacuum into another, then it is evident a Vacuum is taken for granted as before a Similar and Homogeneous Plenum was, neither of which are prov'd according to what has been alledg'd in the firſt part of our Examination into the preſent Philoſophy, that there is Motion is evident, but that it is in a Vacuum or a Similar and Homogeneous Plenum is not ſo evident, till the one or the other is demonſtrated, which as we have ſhewn cannot be done.

Neither the Carteſian *nor* Newtonian *account of Motion agreeable to Nature.*

§. 5. FROM what has been ſaid in the foregoing Section, it is plain that the Definition of Motion in general, is ſuited to the ſeveral Hypotheſes of a Vacuum, or a Similar and Homogeneous Plenum, neither of which are true in the Senſe which the Modern Philoſophers affirm, as we have in the firſt Part of this Philoſophy endeavour'd to evince, namely, that the Vacuum of Sir *Iſaac Newton*, and the Plenum of *Deſcartes* are equally Chimerical and Abſurd.

Becauſe the Carteſian *Plenum and the* Newtonian *Vacuum are equally abſurd.*

§. 6. REAL

Motion is to be defin'd by the ſeveral Particulars which enter into the complex Idea of it.

§. 6. REAL Motion is not therefore to be ſolv'd or defin'd in general by the Hypotheſes of a Similar and Homogeneous Plenum or a Vacuum, but by conſidering the ſeveral Particulars which enter into our complex Senſation of it, which are recounted in §. 2. as Matter, Space, Place, and Time.

The Definition of Motion not clearer than the Idea which is endeavour'd to be explain'd, and what is meant by Philoſophers in Defining it is only to eſtabliſh the neceſſity of their own Hypotheſes of a Similar Plenum on the one Hand, or of a Vacuum on the other.

§. 7. As for Motion abſtracted from all it's Circumſtances, it cannot be better defin'd than by naming it a continual Alteration or Change of Diſtance, altho' any Definition of it is in ſome Meaſure improper, for the ſame Reaſon, as one of Sound or Colour wou'd be, our Senſes inform us of Motion (as they do of Sound and Colour,) and we cannot deſcribe it by any other Characters than ſuch as we receive from our Senſes, and therefore the Explanation will be no clearer than the Idea which is endeavour'd to be explain'd; This is very evident, which ſince it is ſo, ſome may be apt to wonder upon what account Philoſophers have taken ſuch Pains in defining Motion; But the Admiration will ſoon ceaſe, if we obſerve that under that pretence they lay a Foundation for their own Hypotheſes, Thus where one Sect has eſpous'd the Notion of a Similar Plenum, there Motion is a Tranſlation of Body out of the Vicinity of the Corpuſcles which ſurround it, into the Vicinity of others, that is, in ſhort, Motion is the moving of a Body in a Similar Plenum; On the contrary where another Sect has embrac'd the oppoſite Opinion of a Vacuum, with them Motion is a change of a Body's Primary Place, that is, Motion is the moving of a Body in an abſtracted Space or a Vacuum, and ſince Diſtance is alike, whether there is a Vacuum or a Plenum, they have not thought fit to make uſe of that Term, becauſe it wou'd be of no Service to the promoting their particular Doctrines, ſo that what we gain by ſuch Definitions is not a diſtincter Apprehenſion of Motion, but the advantage only of knowing what Opinion the Philoſopher is of, who propoſes them, and how careful he is to make us believe his own Hypotheſis is neceſſary to a right Conception of it.

The Explanation of what real Motion is and not an Hypothetick requires us to conſider what Matter is, which is mov'd. Deſcartes and Rohault, Sir Iſ. Newt. Def. Princ. Math. Dr. Keill Lect. 2. Intr. ad Ver. Phyſ.

§. 8. SETTING aſide therefore the ſeveral Hypotheſes introduc'd not to explain Motion, but to be affixt and made Eſſential to it, we ſhall conſider what real Motion is, and what is requiſite to the perfect underſtanding of it; The firſt thing requiſite (ſince we only at preſent enquire into external Nature, and not that of our Minds in which there is a Kind or Species of Motion obſervable, tho' different from that into which we now examine,) The firſt thing requiſite is Matter, for in real Motion there is always ſome material Being which is mov'd, the Definition of which, if not truly given, is one Argument againſt all the Reaſonings made uſe of in the preſent Philoſophy of Motion; It is defin'd to be a Subſtance, Extended, Solid and Impenetrable, Indifferent either to be mov'd or not to be mov'd, that is, it is entirely Sluggiſh and Paſſive, as well as ſuſceptible of any Motion, Figure or Form, which is conſequent upon it's being Paſſive.

The Definition of Matter ſuited to the Hypotheſes of Philoſophers but not agreeing with Nature either as to its Subſtance, Extenſion, or Solidity. Chap. 6, 7. Part 1. of our Principles of Philoſophy.

§. 9. 'TIS very certain ſuch a Definition of Matter, as this, is extremly well adapted to the Purpoſes of Philoſophers, which are that they may uſe it as they pleaſe, that they may divide, mangle, and ſhape it, as they ſhall find occaſion, which is done in the Corpuſcular Philoſophy, or ſubmit it to what Laws of Motion they ſhall think fit or agreeable to their Explanations of Nature, which is done in the Mechanical; on the other Hand, Matter is not ſo Tame and Submiſſive a Being, as they are willing to apprehend it; for firſt, we have before prov'd that it's Subſtance, as they term it, which is the Baſis and Foundation of the other Properties of it, is unexplain'd, and yet is as neceſſary to a real Knowledge of Matter, as any thing elſe which belongs to it, being that which connects and cements the ſeveral Qualities of it together, and from which they all reſult; Second, in reſpect of it's Extenſion, Solidity, Impenetrability, theſe have all been before conſider'd, and ſhewn to be different from what this Philoſophy imagines or conceives.

Not agreeable to Nature as to it's Inertia.

§. 10. THE next Property affirm'd in this Definition is the Inertia of Matter, in Relation to it's being mov'd or not, and that it is perfectly indifferent to Motion or Reſt in it's own Nature, than which nothing can be more falſe, ſince if we take any ſolid, conſiſtent, and heavy Body, as Lead, it has not that Indifference, becauſe 'tis very manifeſt it makes a Reſiſtance to Motion, whereas on the contrary in other Matter, as Fire, there is a Reſiſtance made to Reſt, ſo that Matter is ſo far from being in general Indifferent either to Motion or Reſt, that ſome Parts of it do moſt violently affect the one, and others the direct oppoſite, inſomuch as very difficulty to be put out of that State of Activity or Quieſcence which they have embrac'd, as is notorious to any one who knows how hard a thing it is to extinguiſh an outrageous Fire, or to move a Weight of ſeveral Millions of Tuns.

§. 11. If it is ſaid the Reaſon, why, in the laſt caſe, Matter, which is in it's ſelf Paſſive, makes a Reſiſtance to any Impreſſion which is ſtruck upon it, is, becauſe it has a Force of Gravitation; that Force is either in the Matter which Gravitates, or in that to which it Gravitates, if in the former, Matter is Active at the ſame time that it is ſuppos'd to be Paſſive, if in the latter, ſince that is Matter too, the Argument is ſtill Valid, If the gravitating Force is neither in the Matter which Gravitates, nor in that to which it Gravitates, it has a gravitating Force, and it has not, which is a Contradiction, for we do not now enquire what is the Cauſe of Gravitation, but whether there is not ſuch a Property or Effect in Matter, which is as plain and evident to our Senſes, as Extenſion or Solidity is.

Gravitation in Matter an Argument for it's not being endu'd with an Inertia or a Perfect Inactivity.

§. 12. We cannot but here farther obſerve one thing more in this Philoſophy, which ſeems to be very Strange and Surprizing, that whereas in the beginning of it, it ſuppoſes Matter to be perfectly Sluggiſh and Unactive, by that time it comes to the Concluſion, Matter is left with a Force and Activity able to keep the whole Planetary Syſtem within the Bounds of it's Orbits, by it's immenſe and prodigious Gravitation; from whence it is manifeſt, that however it might be neceſſary to make Matter an uſeleſs and paſſive Thing in order to turn it about, and project it this way or the other at Pleaſure, yet Truth wou'd prevail, and in the Proceſs of theſe Philoſophers arguings upon Nature, they were oblig'd to introduce a Force into Matter, which they wou'd not at firſt acknowledge, and own at the laſt that it was endued with a Power of Gravitation; So that if we compare the original Hypotheſis of this Philoſophy with the Concluſions of it, the one ſeems to be little leſs than a Confutation of the other, Matter is ſuppos'd to be entirely Paſſive, The Sum of all the Mathematick Reaſoning upon that Hypotheſis, is, that it has in every Part of it a Force of Gravitation, that is, that it is entirely Active, and if ſuch a Concluſion is not a Contradiction to ſuch an Hypotheſis, I cannot tell what is.

The Suppoſition of an Inertia in Matter, and the Concluſion from ſuch an Hypotheſis that all Matter Gravitates or is Active, is a Contradiction.

§. 13. Allowing however the Reſiſtance of Matter to Motion cou'd be accounted for by Gravitation, ſo as Matter ſhou'd, notwithſtanding that, be ſtill Paſſive, we ſhall yet want a freſh Solution for the oppoſite Quality obſervable in Fire, or in the Rays of the Sun, which do not make a Reſiſtance to Motion, but to Reſt; if this is done by any Æthereal Particles aſſembling together, ſince thoſe Æthereal Particles are ſuppos'd to be Matter, Matter will ſtill be Active; Whence that Action proceeds is not now the Queſtion, only whether there is ſuch an Action, which it is very certain there is, and which we can with no greater Reaſon deny to belong to the real Nature of that Matter, than Brightneſs or Heat, or Extenſion, or any other Quality which we obſerve in it. If any one ſhou'd argue with us, that Fire, which is endu'd with Motion, is Matter, and Lead, which is at Reſt, is Matter, and therefore Matter is Indifferent to Motion or Reſt, or has a perfect Inertia, the Sophiſtry is eaſily anſwer'd, by ſaying that Fire is Matter, and yet is not Indifferent to Reſt, and Lead is Matter, and yet is not Indifferent to Motion, and therefore Matter is not Indifferent to Motion or Reſt, the Caſe is, They who argue thus, ſpeak of a General, Fancy'd, and Abſtracted Matter, but we of a True and a Real.

The Activity of Fire and of the Rays of the Sun, an Argument for Matters not being endu'd with a perfect Inertia.

§. 14. Another Property which is ſaid to enter into the Definition of Matter is, that it is ſuſceptible of any Figure or Form which we can imagine; This likewiſe is well contriv'd by Philoſophers, to make Matter obedient to their Carving and Cutting it out into what Shapes or Figures they judge convenient to their ſeveral Purpoſes, but that Matter will yield to them in this is not ſo very certain, becauſe tho' we can by Superior Forces apply'd give what Impreſſion we think fit to Gold or a Diamond, yet we are not able to ſubdue Flame, and put it into what Figures we are deſirous it ſhou'd receive; The Reaſon of which is, that quieſcent Bodies may be conquer'd by an active Force, but Fire and Flame can't, and therefore to form a Notion of Matter's being ſuſceptible of any Figure, from our being capable of imparting it to ſome Portions of it, is at the beſt only a Partial defining of it, and even in ſolid Bodies this Property is not true, excepting where the Force is ſuperior to the Reſiſtance; as to any other Form of Matter beſides the Figure of it, we have no

Matter not ſuſceptible of whatever Form or Figure we pleaſe, becauſe ſome Parts of it may be ſo divided.

occaſion to conſider it, ſince, in this Philoſophy, the various Forms of Matter are owing to the various Figures and Textures of it's Parts.

Nor yet becauſe we can imagine Matter thus diviſible into certain Forms and Figures.

§. 15. If it is not upon the foregoing account that we ſuppoſe Matter to be ſuſceptible of any Figure, that is, becauſe we actually experience we can divide it at our pleaſure, which as we have ſaid is not true, it muſt be upon this, namely, becauſe we can conceive Matter to be ſo divided; but if, becauſe we can mentally divide Matter into any Figures, it is therefore really diviſible into them, it will follow, becauſe we can mentally imagine Matter not to be divided, but one continued Subſtance, therefore it is really from ſuch a Conception not diviſible into any; and in general, if Things exiſt according to our Imaginations, there needs nothing more to the Explaining of Nature, than for the Mind to imagine Nature is ſo, and from thence to conclude that it is.

If Matter is not juſtly defin'd by Philoſophers, real Motion is not, and as for abſtracted Motion it can ſignify nothing to the Explaining of Nature.

§. 16. Moreover we have ſufficiently conſider'd the Definition of Matter in the firſt Part of our *Principles of Natural Philoſophy*, and endeavour'd to ſhew the Notion, which all the Philoſophy, that has been hitherto propos'd, has entertain'd of it, is nothing elſe than a Creature of the Mind, and not any ways conformable to the real Nature of that Subſtance; which, if true, whatever is advanc'd concerning the Laws of Motion is falſe, if by that we mean real Motion, into which the Nature of Matter, or of the Being which is mov'd muſt be computed; if abſtracted Motion is intended, thoſe Laws, which are form'd from it, may indeed furniſh us with Principles to argue and build Theorems upon, as Extenſion do's in Geometry, but can ſignify nothing to the Explaining of real Motion, no more than the proving the three Angles of a Triangle are equal to two Right ones, or the Square of the Hypotenuſe of a Right-angled Triangle is equal to the Squares of the Sides, is of any Service to the informing us what real Extenſion is in Water, or Gold, or the peculiar Properties of it, as it exiſts in Material Beings,

If the Theory of Mechaniſm or Motion was true when apply'd to Pendulums, it yet wou'd not be ſo when apply'd to Fire and Flame.

§. 17. Lastly, if the whole Theory of Motion was Rational and Exact, as it is apply'd to conſiſtent Bodies, which are mov'd in Pendulums, or made to meet each other by a mutual Concourſe, ſuch a Theory could not explain the Action of Fire or Flame, ſince it is impoſſible to reduce it to a Pendulum, or to any Concourſe of hard and reſiſting Bodies, unleſs it can be prov'd, as we before obſerv'd, that a Reſiſtance to Motion, as in heavy Bodies, and a Reſiſtance to Reſt, as in Fire, are the ſame.

Space and Place the next Ingredient to be conſider'd in Motion which is either Abſolute or Relative.

Sir Iſ. Newt. Princip. Mathem. Defin. Dr. Keill Introd. ad veram Phyſ. Lect. 6.

§. 18. Having examin'd into the Idea of Matter, as it reſpects the Mechanical Philoſophy, the next Thing to be conſider'd, is the Space, which the Body, which is mov'd, either deſcribes, or poſſeſſes, the firſt is term'd Space, the latter Place, both which are diſtinguiſh'd into Abſolute and Relative; Abſolute Place or Space is call'd Primary, and ſuppos'd to be Similar and Immoveable, which continues one fixt and unalterable Expanſion, and do's not depend upon any References to certain Points, by which we judge of the Motion or Reſt of Body; Relative Space or Place on the contrary is determin'd by ſuch References. Thus, if we are in a Ship under Sail, whilſt we continue in the ſame Situation to all the Parts of it, there is a Relative Reſt to thoſe Parts, tho' there is an Abſolute Motion in Reſpect of there being a Change of Abſolute Space or Place, tho' not of the Relative, by the Ship's Moving.

Relative Space, Place or Motion of no Conſideration in Philoſophy, tho' uſeful in reconciling Religion to it.

§. 19. From whence it follows that, in actual Motion, or in a true Philoſophy of it, Relative Place or Space is of no Conſideration, it being only a Reference made by the Mind without any Reſpect to the Reality of Things; however one Advantage may be made from this Diſtinction of Abſolute and Relative Quantities, to wit, of Place and Space, and conſequently of Motion, that it gives a full Account of thoſe Paſſages in Holy Scripture which Philoſophy ſeems to contradict, as that the Earth ſtands ſtill, and the Sun moves round us, which tho' it may not poſſibly be true in an Abſolute Senſe, yet is ſo in a Relative, and it is more agreeable to think that the Scriptures ſpeak in a Common, rather than in a Philoſophick or a Mathematick Stile, ſince the Sun may with as great a Propriety of Speech be ſaid to move, if it only do's ſo Relatively, as if it Really did.

Abſolute Space or Place, as conſider'd in the Mechanical Philoſophy, an

§. 20. As to what concerns therefore Abſolute Space or Place, which is requiſite to real Motion, we have before ſhewn that Primary or Abſolute Place or Space, or Vacuum which is the ſame, is nothing but an abſtracted Idea of the Mind, and do's not furniſh us with any real Knowledge of the Nature of that Space which exiſts, from whence our abſtracted Notion of it is form'd by a partial Conſideration only of Length, Breadth, and Thickneſs, and neglecting the other Properties, which belong to it; and therefore upon this account likewiſe the preſent Doctrine of Mechanicks propoſes to us nothing more than a Theory or Contemplation of abſtracted Motion, which by §. 16. will be far from Informing us in any thing that is Real, and which is found in fact Exiſting.

abſtracted Notion and do's not exiſt in Nature.

§. 21. THE laſt Thing to be conſidered in Relation to Motion is the Time, which it takes up in it's Progreſs, which is in like manner, as Space, diſtinguiſh'd into Abſolute and Relative; the firſt is ſaid to flow uniformly without any Alteration, and the ſecond to depend upon the Meaſure we make uſe of to determine it, as the Motion of the Sun and Moon, which this Philoſophy takes notice of, or the Succeſſion of Ideas, which is alledg'd by another, which we may call the Metaphyſical.

Time another Ingredient of Motion is likewiſe defin'd to be Abſolute or Relative.
Sir Iſ. Newt. Princ. Math. Definit.
Dr. Keill Introd. ad ver. Phyſicam.
Mr. Locke.

§. 22. As to what Reſpects Abſolute Time, which is Uniform, and paſſes one Minute after another without the leaſt Acceleration or being Retarded, it is manifeſt, as in Abſolute Space, that this is alſo a Notion form'd by the Mind, and not agreeable to Nature it ſelf; for how is it poſſible for us to know what Time or Duration is in Experience and Fact, that is, what it Really is, and as it Exiſts, excepting by the Obſervation of that External Motion, as of the Sun or Moon, which is without us, or that Internal one, which we perceive in our own Minds? for ſuppoſing Time to flow uniformly without any Experience on our Part, that it do's, it is the ſame as to ſuppoſe an uniform, uninterrupted Space, without any ſuch Experience that it really Exiſts; and therefore, as the Latter has only it's Exiſtence in the Mind, ſo we may reaſonably think the Former only has; beſides which, we have before ſhewn, that our Conception of Matter's being diviſible is no Proof that it is, and for the ſame Reaſon, unleſs it is juſtify'd by Experience, our Conception of Time being uniformly continu'd, is no Proof of it's being ſo; What I mean in general, and which may be concluded from what has been ſaid, is, that we muſt form our Notions of Time, as well as of Space, not from any arbitrary Abſtractions of the Mind, but from our real Experience and Obſervations, and our juſt and plain Reaſonings upon them.

Abſolute Time in the Senſe of this Philoſophy, as much a Creature of the Mind as Abſolute or Primary Space is.

§. 23. FROM what has been ſaid it is evident, that, as in Space, the Abſolute or Primary, which is an abſtracted Idea, was only conſidered, in this Philoſophy's Account of Motion, ſo in Time or Duration, the Abſolute or Abſtracted, not the Real, is likewiſe aſſum'd, and is ſuppos'd to flow uniformly by a continual Succeſſion of Moments or Inſtants without any Change or Variation, which we affirm to be a Creature of the Mind, and that real Time or Duration, and our Notion of it muſt be deriv'd from ſomething, which falls under our Experience and Obſervation, as exiſting in Nature; for if we our Selves did not exiſt, nor any Thing elſe beſides us, Time or Duration wou'd not, But as we can have no Idea or Imagination of Nothing, and as Time or Duration is neceſſarily connected with Being, ſo we can have no Conception of no Time or no Duration.

The Notion of Time or Duration ought to be deriv'd from Obſervation and Experience, and not from our Abſtractions.

§. 24. SINCE therefore our Notion of Time or Duration muſt be deriv'd from Exiſtence, which it always accompanies, and from which it is never ſeparated, it muſt be from thence, and from our Obſervations upon it, and from the Nature of it, that we are to fix and ſettle our Apprehenſions of real Time and Duration, both as to it's Being, and as to it's Meaſure; in reſpect of the firſt of which, we are ſufficiently inform'd of it from our own Exiſtence, and from the Succeſſion of our Ideas in our Minds, as likewiſe from all the Beings, whether Quieſcent or in Motion, which exiſt without us; the principal and only Enquiry therefore is, what is the real Meaſure of Time or Duration, and whether it flows uniformly and by a Succeſſion of certain and determin'd Inſtants and Moments or not.

Time or Duration, and the Meaſure of it, Two Things.

§. 25. IN order to examine into this, the juſteſt way for us is to conſider the ſeveral Beings in Nature, which come under our Senſes or Obſervations, and from thence to know, whether any of them can afford us a Solution of what is requir'd, that is, can give us a Standard or Meaſure of Time or Duration; and here we are not at firſt to take it for granted, that Time or Duration do's flow uniformly by a Succeſſion of determin'd Moments or Inſtants, but to enquire whether it do's ſo, either from what we obſerve in Bodies, which are Quieſcent, or which are in Motion, or from the Succeſſion of Ideas in our own Minds; and if we cannot prove

The Meaſure of Duration to be aſcertain'd from conſidering the ſeveral Beings which exiſt, and how far they are ſufficient for that End.

ſuch a Time or Duration in fact from Beings, which exiſt, it is plain and evident, that it is a Creature of the Underſtanding, form'd and made by it without any Affirmation from Nature, or from Things as they really Exiſt.

Any certain or determin'd Moments or Inſtants of Time following one another in an abſolute and unvary'd Succeſſion not concluded from Beings as they exiſt.

§. 26. But before we come to enquire what Beings, as they really Exiſt, do preſcribe to us to think, in reſpect of the Meaſure or Standard of Duration, it will be neceſſary for us to ſhew, in Order to obviate the Prejudices, which have been hitherto receiv'd, that a Succeſſion of certain and determin'd Inſtants of Time or Duration is a mere Chimæra, and a thing of Imagination only, for there is not any real Being, which do's actually meaſure out Time or Duration in that Manner; Bodies which are Quieſcent, as this Table or Paper, on which I write, do exiſt Moment after Moment, but they do not inform us, by their Quieſcence, what Duration that Moment has, or what the Meaſure of it is, or whether one Moment or Inſtant of Time may not be different from another, they only aſſure us, that they do exiſt, and that by Conſequence they have Time and Duration in general belonging to them; Bodies which are in Motion, tho' we ſhou'd know the preciſe Moments, in which they move, as of the Earth and Moon in their ſeveral Orbits, ſo as from thence to be able to calculate the Solar and Lunar Eclipſes to a Minute, Theſe cannot yet aſſure us, that Time or Duration flows any more according to the uniform Inſtants, which are proportionate to their Revolutions, Namely of the Earth about it's Axis in 24 Hours, and about the Sun in 365 Days, and of the Moon about the Earth in 27 Days and 7 Hours, than according to any other uniform Inſtants, which may be computed and eſtimated from any different Revolutions; For ſuppoſing the Earth had Revolv'd round it's Axis, in half the Time, in which it now do's, and we were to divide that Duration into 24 equal Parts, and were to name them Hours, as we have done the 24*th* Parts of the Preſent Revolution, every Hour wou'd be in ſuch a Caſe as ſwift, that is, as ſhort again, as what we now Experience, and conſequently every Minute, or the ſmalleſt Inſtant of ſuch a Duration, wou'd be Proportionably Accelerated. Thus, if the Computations of Aſtronomers are true, *Venus* performs it's Rotation round it's Axis in 23 Hours, *Mars* in 24 Hours and 40 Minutes, and *Jupiter* in 9 Hours and 56 Minutes, and if thoſe Planets ſhou'd receive any Inhabitants, and they ſhou'd divide the Time of theſe ſeveral Gyrations into 24 equal Parts, and ſhou'd likewiſe call thoſe Parts Hours, it is manifeſt that in the firſt Inſtance, ſuch Hours wou'd be quicker and ſhorter than ours, and in the third, that they wou'd be more than as ſhort again, and in the ſecond Inſtance, that they wou'd be ſomething longer, and ſlower than thoſe by which we meaſure Duration, and the ſeveral Moments and Inſtants of ſuch Time wou'd be therefore in Proportion vary'd; and yet, it is as evident, that there is no more reaſon to aſſert the Equability of the flux of Time, from the Revolution of the Earth round it's Axis, than from that of *Venus*, or *Mars*, or *Jupiter* about Theirs; from whence it is plain, that real Time or Duration may be Quicker or Slower, according to the Greater or Leſs Celerity of the Moving Body, which we take to compute it by.

Dr. Greg. Aſtron. Prop. 15. Lib. 1. Dr. Keill's Aſtron. p. 37. 96. Edit. 1.

Dr. Gregory's Aſtr. Prop. 30. Lib. 1. Dr. Keill's Aſtron. p. 48, 49. Edit. 1.

Another kind of Moving Bodies, by which we meaſure Time or Duration, are thoſe of Pendulums, whoſe Motions are computed by the Hours or Minutes, which they move in, and which Hours or Minutes are determin'd by the Earth's Revolution about it's Axis.

Succeſſion of Ideas no Meaſure of Duration.

§. 27. As to the Succeſſion of Ideas in our own Minds, that can never give us the leaſt Notion of a uniform Duration at all, much leſs of a uniform Duration, which conſiſts of certain and determin'd Inſtants, becauſe ſometimes we have no ſenſible Succeſſion of our Ideas, as in a Revery, in a Sleep without Dreaming, or when our Minds are fixt upon an Object intently without paſſing to any other; and when the Succeſſion of our Ideas is in the fulleſt Rapidity of them, as in Pleaſure, or in the utmoſt Languor of them as in Pain, we are far from thinking that Time flows Uniformly, ſince the Duration of an Hour in the laſt Caſe is ten Times longer than what we Experience and Imagine it to be in the firſt; It muſt be alſo acknowledg'd, that the Paces of our Ideas are Different in different Perſons, and in the ſame Perſons at different Times; Thus one of a quick and ready Apprehenſion will find his Ideas to move with more Life and Celerity than another, who perceives a kind of Torpor and Dullneſs, which clogs and lies heavy upon his Underſtanding, ſo likewiſe, the ſame Perſon, upon different Circumſtances of his Mind, as he is more Chearful or Debonair, or more Dejected, will proportionably find his Ideas to move with a more or leſs Intenſe Velocity; Beſides which, ſuppoſing a continual and uniform Velocity of our Ideas, ſince yet there is a Succeſſion of them, which is not continu'd and uninterrupted, as Time and Duration is, it will be Impoſſible for diſtinct and ſeparate Ideas, let them follow one another as uniformly, as they will, to give us a Notion of that Time or Duration, which intercedes their Succeſſion.

From all which it is very Evident, that the Succeſſion of our Ideas has nothing to do with the Meaſure of Time or Duration, that it can only inform us, that there is Time or Duration in general, (and which is no more than what Bodies, which are Quieſcent, do) and that it is longer or ſhorter, as it's Ideas move with a greater or leſs Celerity, correſpondent to what we find from Bodies, which are in Motion.

§. 28. Having therefore conſider'd all the Beings, which can poſſibly give us any Notion or Apprehenſion of a Duration or Time, which conſiſts of an uniform Progreſſion of certain and determin'd Inſtants, and ſince there are no Beings that can do it, and ſince we can have no Notion or Apprehenſion of Time or Duration, unleſs from ſome Being, which exiſts, according to §. 23 and 24. it is manifeſt, that a uniform Progreſſion of certain, and fixt, and determin'd Inſtants of Time, is nothing elſe than an abſtracted Idea, and as much ſo, as univerſal and abſtracted Space is, and that therefore thoſe Philoſophers, who proceed upon ſuch Abſtractions, as the preſent Mechanical Philoſophy do's, are not juſtify'd in ſuch Abſtractions, by what we find and Experience in Nature.

Since the uniform Progreſſion of certain, fixt and determin'd Inſtants is not juſtify'd by Nature, it is a Creature of the Mind.

§. 29. Since therefore the Meaſure of Time and Duration is ſo little underſtood by the preſent Philoſophers, even in reſpect of a temporary or finite Duration, and ſince they all of them think in the abſtracted Way of a uniform Progreſſion of certain, fixt and determin'd Inſtants of Time, for which they have no Proof from any thing in Being, it is a little ſurprizing, that ſome Men ſhou'd ſpeak or write ſo poſitively about an Infinity of Duration, when they know nothing of a Finite one, unleſs of an Abſtracted, which, as we have before ſhewn, is not in fact exiſting, and therefore is a Creature of the Imagination only.

If we have not a true Notion of Finite Duration much leſs have we of an Infinite one.

§. 30. It is now Incumbent upon us to ſhew, what the real Meaſure of Time or Duration is, which is ſome real Motion or other, which we obſerve in Nature; For thus we Argue, that if there is any certain Body, which will perform it's Motion in one determin'd Portion of Time, the ſame Body will, we conclude, with all it's Circumſtances continuing the ſame, perform a like Motion in another equal Portion of it, becauſe the Principle and Cauſe ſtill ſubſiſting, without any Variation, muſt of Neceſſity produce the ſame Effect in the ſame Time; from hence it is, that ſince we ſee no Reaſon, why the Earth, in it's Revolution about it's Axis, ſhou'd not be Uniform and Conſtant, and it's ſeveral Revolutions therefore equal to one another, becauſe we do not ſee or know any thing, which ſhou'd diſturb it's Periodical Gyrations, from thence it is, that we take ſuch Revolutions, for our Meaſures of Days, and Hours, and Minutes, and conſequently of Time and Duration, which, whether we Sleep or Wake, and whether our Ideas ſucceed or not, will ſtill be the ſame and unvary'd; and from the Determination of theſe Days, and Hours, and Minutes, we determine the Equation of Time, and know how many of ſuch Minutes are ſpent in the Earths revolving round in the Ecliptick, that is, how many of them belong to the Year; And according to this Meaſure of Duration, a Pendulum, which has 39 Inches and $\frac{1}{5}$ in Length, compleats it's Oſcillation, made up of it's Progreſs and Return, in two Seconds of Time, or in two *60th* Parts of a Minute, which is the *60th* Part of an Hour, and which is the *24th* Part of the Earths Revolution about it's Axis.

How we fix our Meaſure of Time or Duration.

Our Principles of Natural Philoſ. p. 229.

On the contrary, we might take the Oſcillations of Pendulums, upon the Maxim in Philoſophy before mention'd, that Bodies which are in Motion will have equal Effects in equal Times, where the Circumſtances are the ſame, and ſuppoſing the Circumſtances to be the ſame, for a Meaſure of the Earth's Diurnal Revolution.

But yet, in both Caſes, if the Revolutions of the one, or the Oſcillations of the other are not Uniform and Conſtant, our Days, and Hours, and Minutes may be Double or Triple, or in any other Proportion of what we account and eſteem them to be, that is Uniform and Equable, and by conſequence all the Moments and Inſtants of ſuch a Meaſure of Time or Duration wou'd be Double or Triple, or in any other Proportion of what we reckon them.

THUS, ſuppoſing the Earth, in it's Revolution about it's Axis, ſhou'd, in one Part of it, quicken it's Motion, from ſome Cauſes, which are unknown to us, as it certainly do's in it's Revolution about the Sun from Cauſes, which are imagin'd to be known to us, we have divided it's Revolution indeed about it's Axis into 24 equal Hours, upon the Suppoſition of it's Equable Motion, but if that Motion is not Equable, it is plain that the Duration of one Hour is not equal to that of another, but that one may be much longer or ſhorter than another.

AND the ſame may be ſaid in the Oſcillations of Pendulums, in Clocks or Watches, that if the Weight or Spring, by which they move, do's not keep the ſame conſtant Tenor, (which it is impoſſible it ſhou'd do, when it is wound up to the Height, and therefore is more Vigorous and Strong, and when it has not the ſame utmoſt Elaſtick Force, and therefore is more Weak and Languid, and which is ſenſible from the Vibrations, which in the former Caſe are Quicker and Smarter, and in the latter, more Faint and Duller) the Meaſures of Duration, from ſuch Pendulums, will be unequal, and one Minute or Inſtant of it will be different from another, and the ſame, in ſome degree, may be affirm'd of Common Pendulums, which are made to Swing from a Center of Suſpenſion only, Becauſe the Body, which is plac'd in the Center of Oſcillation, cannot have exactly the ſame Force of Gravitation in all the Points of it's Periods, nor the ſame Relation to the Center of Suſpenſion.

Dr. Keill's Aſtronom. De Temporis Æquatione Lect. 26.

FROM all which it is apparent, that Time or Duration muſt be eſteem'd to be quicker or ſlower, as the Body which moves, and which we take for a Meaſure of Duration, is ſo in it's Motion; and ſince the Motion of Pendulums is uncertain, which are the Contrivances of Men, and the Revolution of the Earth about it's Axis, which is the Effect of a Superior Agent, is not ſo, which is confirm'd from the Calculations made of Eclipſes to a Minute, upon the Suppoſition of the Middle or Aſtronomical Days, Hours, and Minutes, which we make uſe of in ſuch Calculations, being Equable, it is likewiſe evident that ſuch a Revolution is only the proper Meaſure of Time or Duration, to which all others will be the moſt juſtly and truly Exacted.

WHAT yet we maintain, is, that the Meaſure of Time or Duration muſt be had from the Motion and Celerity of the Body, which moves, and which we take for the Meaſure of Duration; the Revolution of the Earth round it's Axis is that, by which we compute, and if we had been plac'd in *Venus*, in *Mars*, or in *Jupiter*, ſuppoſing thoſe Planets to be inhabited, and had, as we do here, divided their Revolutions about their Axes into 24 Parts, and had nam'd them Hours, they wou'd, as we before obſerv'd, have been ſhorter or longer than thoſe we now make uſe of, and we might have reduc'd the Revolution of the Earth about it's Axis to ſuch Days, Hours, and Minutes, which thoſe Revolutions preſcribe, with as much Reaſon and Juſtice, as we do now their Revolutions to the Days, Hours, and Minutes, which the Revolution of our Earth has preſcrib'd to them; That is, we take the Revolution of the Earth round it's Axis for the Meaſure of Time or Duration, as being ſuppos'd to be Uniform and Equable in it's Motion, but then the Inſtants and Moments of this Meaſure are no more the proper, peculiar and diſtinguiſhing Meaſure of the Flux of Time or Duration, than any quicker or ſlower Moments or Inſtants are, and if we proceed any farther to abſtract the Moments or Inſtants of Time or Duration from any Celerity, with which they flow, or from the Revolutions of Bodies round their Axes, be they One or Another, we depart from Nature and real Exiſtence, which can only give us a Notion of Duration, and come to Fancies and Chimæra's of our own Minds.

Celerity the Meaſure of Time or Duration.

§. 31. THUS, to ſpeak in general, Real Time or Duration, and not an Abſtracted one, is to be meaſur'd by real Motion, for if the ſame Body moves twice as far, as before, with the ſame Degree of Motion or Force, the Time will be double, in which it moves, if the ſame Body moves as far, as before, with double the Force, it will be in half the time, that is, the Time or Duration, will be as quick and ſhort again in the laſt Caſe, as in the former; and therefore to compute real Time or Duration, as it exiſts in Nature, and according to our Obſervations, without which we cou'd have no Apprehenſion or Notion of it at all, the knowledge of ſome uniform Motion or other muſt be neceſſary to our meaſuring of it, to which uniform Force, which we conſider, whether quicker or ſlower, the Flux of Real Time and Duration, and not an Abſtracted will be Correſpondent, and accordingly Accelerated or Retarded.

From whence it again follows, that the Succeſſion of our Ideas can give us no kind of Meaſure of Duration or Time, unleſs their Force, with which they move or their Celerity is known, and that ſuch Force or Celerity of their Motion, or Succeſſion is Equable and Uniform; and for the ſame Reaſon, the Revolution of the Earth about the Sun in the Ecliptick, or of the Moon about the Earth in the Zodiack, can be no Meaſure of Time or Duration, becauſe they Move unequally in their Orbits, and have different Celerities in different Parts of their Periods, and which Revolutions are therefore reduc'd to Days, and Hours, and Minutes, computed from the Revolution of the Earth round it's Axis, and which cou'd not be known to be unequal, unleſs it was preſum'd that ſuch Days, and Hours, and Minutes were Uniform and Equal; as is Evident from the Doctrine of the Equation of Time before Cited.

Abſtracted Time or Duration no Meaſure of the Real.

§. 32. From the foregoing Sections it is conſequent, that in all the Arguments or Demonſtrations concerning Motion, where an Abſtracted Time is us'd, upon ſuch an Account likewiſe, the Reaſonings will be Abſtracted, and therefore not Anſwerable to Nature, ſince the Meaſure of Time or Duration is originally and previouſly to be aſcertain'd by the Force and Degree of ſome Real, Equable, and Uniform Motion, which is aſſum'd, in order for us to judge of the Periods of Time, and of the Moments or Inſtants, which belong to it.

So likewiſe, if the Time or Duration (which is Correſpondent to, and is the Reſult of one uniform Motion or Celerity) is confounded with that, which anſwers to another, before the one is reduc'd to the other by comparing them together, the Reaſoning, from ſuch a Confuſion of Time into one and the ſame uniform and abſtracted Time, will be wrong and inconcluſive, as if we take the Revolution of the Earth about it's Axis, and divide it's Time or Duration into 24 Parts or Hours, and again the Revolution of *Venus*, *Mars*, or *Jupiter*, about their Axes, and divide their Time or Duration into 24 Parts or Hours, we cannot argue or reaſon upon theſe Times or the Inſtants of them alike, becauſe they are apart Equable, ſince, compar'd with one another, they will be extreamly different, and we muſt reduce the one to the other, before we can ſpeak with any certainty of the Meaſure of Time or Duration, that is, we muſt exact our Notions of Time or Duration to ſome fixt Standard or other of it, to which all others are to ſubmit; and yet that Standard may be the Time or Duration of the Revolution, of one Planet, or another, about it's Axis divided into the ſame Number of Parts, namely of Hours, Minutes, and Moments, which may have different Degrees of Celerity, according to the various Celerities of ſuch Revolutions.

And thus it is in Space, the abſtracted Idea of it is that of one uniform Extenſion, which a Foot, or a Yard, or any other ſtated Length may Meaſure, whereas the Meaſure of Real, and not Abſtracted Space, is the intrinſick or native Force of that Body, which poſſeſſes it, according to what we have ſhewn, in our *Geometry of Solids*, or *of Matter*, and in our *Principles of Philoſophy*; and conſequently, as Time or Duration is meaſur'd by the Force or Celerity of that, which we make our Standard, when that Force or Celerity is finite, be it quicker or ſlower, the Time or Duration is accordingly finite, and is ſhorter or longer; if therefore there is a Being, which has an Infinite Force and Agency, it's Force and Agency muſt be the Meaſure of it's own Duration (thro' all Time) which muſt be likewiſe Infinite, and can be no more adjuſted with a finite Duration, than one finite Duration, whoſe Meaſure is a finite Celerity, can be that of another finite Duration, whoſe Meaſure is a different finite Celerity; What we ſay is, that Time or Space, in the Abſtract, are undefin'd, and are capable of all Quantities, which can be aſſign'd to them.

From what has been hitherto alledg'd from §. 2. to the preſent Section, it enſues, that the firſt Definition in the Mechanical Philoſophy, of Motion's being a continual and ſucceſſive Change of Place, is not true, becauſe it do's not conſider all the Ingredients, which Appertain to Motion, that is, neither what real Motion is, nor what Body or Matter is, nor what real Space or Place is, nor yet what Time or Duration is, all which yet are neceſſary to give us an Idea of a Motion in Nature, and therefore we may conclude, that the firſt Definition of Motion is only a Thing of Abſtraction, and a mere Creature and Imagination of the Mind.

Definition of Celerity Imperfect. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 7. Def. 2.

§. 33. THE Second Definition is that of Celerity, which is ſaid to be an Affection of Motion by which the Body, which is mov'd, paſſes thro' a given Space in a given Time, againſt which there are two Objections.

FIRST, that Motion is introduc'd into this Definition, which was not juſtly defin'd before, according to §. 32. and the preceding ones.

SECOND, that a given Time is ſuppos'd, whereas Time or Duration, by what foregoes, is to be meaſur'd by ſome uniform Motion or Celerity, and therefore Motion or Celerity is not to be meaſur'd by Time, thus the Motion or Celerity of the Earth's Revolution about it's Axis is the Meaſure of Time or Duration to us, and therefore that preciſe Time or Duration is aſcertain'd by that Celerity and Motion, and Time is not previouſly aſſum'd, but determin'd by it; if therefore now there was no Revolution of the Earth about it's Axis, nor any other fixt Meaſure or Standard of Time, the firſt thing, we cou'd do, wou'd be to take a Body, as a Pendulum for Inſtance or any other Body, which from it's Nature we might conclude wou'd, as much as poſſible, deſcribe the Spaces it paſs'd thro' regularly, and with Uniformity; from it's Motion thro' a double Space, we ſhou'd infer there was a double Time, thro' a triple Space, a triple one, and ſo on, and the Time wou'd flow in Proportion to the Motion or Celerity of ſuch a Body; if again we ſhou'd make uſe of a Pendulum, or any other moving Body, which deſcrib'd twice the Space, whilſt the former deſcrib'd only one half of the ſame Space, and we made this latter Celerity our Standard of Duration, it is plain, ſince Time wou'd flow in Proportion to this latter Celerity, it wou'd flow as quick again, as in the former Inſtance, and ſince we might, as it happen'd, make uſe of a quicker, or ſlower Body, in ſuch a Caſe, for our Meaſure of Duration, our Time or Duration wou'd accordingly move quicker, or ſlower.

AND this we more particularly find in Muſick, where the Time is accounted quicker or ſlower, as the Celerity of the Notes is ſo, and the Time, which is kept, and to which they are adjuſted, is meaſur'd by an equable and uniform Pulſation, which yet is no more a proper Meaſure of Time, than any other quicker, or ſlower Pulſation is, than what is us'd on thoſe Occaſions; from all which, and from what has been before ſaid, it appears plain and evident, that a Duration, conſiſting of uniform, certain, and determin'd Inſtants, is an abſtracted Idea, and that the Meaſure of all Duration to us is Celerity and Motion, and which as it is different, Time and Duration is different to us; and the ſame we obſerve in the Succeſſion of our Ideas, for as they move ſwifter or ſlower, Time or Duration appears to us either ſhorter or longer, but yet for the Reaſons above, and upon the Account of what is here ſaid, ſuch a Succeſſion cannot be a fixt Standard of Duration.

How we Meaſure Celerity.

§. 34. SINCE therefore the Celerity of the moving Body is the Meaſure of Time in real Nature, ſo far as our Apprehenſions or Notions of Nature can go, it may be enquir'd, what is the Meaſure of Celerity, which cannot be a uniform or abſtracted Time or Duration, becauſe it ſelf is meaſur'd by ſome certain Celerity or other, whether greater or ſlower is not determin'd, but by a Celerity, whatever it is, which is yet Certain and Uniform; in this Perplexity, what is the Mind's Procedure? namely, to aſſume one kind of Celerity, or Motion, as that of the Revolution of the Earth about it's Axis, which Meaſures out ſo much Time and Duration, which it takes up in the Performance of it's Circuit, and from thence to determine by comparing other Celerities with it, what Proportion of Time or Duration belongs to them, without declaring or knowing, what Time or Duration the Revolution of the Earth round it's Axis takes up, beſides that which it's own Celerity meaſures and aſcertains, and which Time wou'd have a different Fluxion to us, if meaſur'd by a different Celerity, as that of *Venus*, *Mars*, or *Jupiter*'s Revolution about their Axes.

Infinite Time, or Duration an Abſtraction and Creature of the Mind, if an Infinite Being is not prov'd previous to ſuch a Duration.

§. 35. As to an infinite Time or Duration therefore, which proceeds from an abſtracted Notion of it, from all moving Bodies or Celerities whatſoever in Nature, 'tis a pure Metaphyſical Buſineſs, and has no Foundation in the Reality of Things, and all the Arguments, which are form'd upon ſuch an Abſtraction, are Creatures and Fancies of the Mind, in which Philoſophers may, if they pleaſe, indulge themſelves, but without any Advantage to Truth or to Reaſon; for if there were no Being or Exiſtence in the World, there cou'd be no Time or Duration, and if there were no Motion in it, there cou'd be no Meaſure of ſuch Duration, and where that Motion once was not, there was to us no Meaſure of Time, and whatever Meaſure of Time or Duration, we extend beyond it, it is a Supplement made by our own Minds to fill up the vaſt Chaſm, which lies betwixt Eternity and the World's Creating, and thus the *Julian* Period extends it ſelf 730 Years beyond that Time, which can be nothing elſe than a mere Chimæra as to the Meaſure of Time, if there was nothing then Exiſting; it is an Imagination, which can carry it ſelf into Time and Duration, which was not, if there was then no Exiſtence, ſince we have already ſhewn, that Time and Duration is neceſſarily connected with Exiſtence, and cannot be or ſubſiſt in Nature without it.

On the contrary we have ſaid, that Exiſtence and Duration go together, and cannot be ſeparated the one from the other, that an abſtracted Notion of Time or Duration proceeding in uniform Inſtants, which are not meaſur'd by ſome real and moving, and therefore exiſting Force, or other, is a Fancy of the Philoſopher, and not a Thing in Nature, and that all the Arguments upon ſuch Abſtractions are not nor can be juſtified or defended.

And therefore as to infinite Duration, we muſt firſt prove there is an infinite Being, which when we have done from the Works of Nature, and from what we obſerve in the World, (as there is no other way of Proof, excepting that which reſults from a Conviction of the Truth of our Religion) we are from thence to ſhew the Infinity of Duration, that is, from the Infinite Force, Power, and Agency of the Supreme Mover, we are to prove the Infinity of Duration and Time, and not prepoſterouſly from the Infinity of Time or Duration, which lies in our own Romantick Eſtimations and Opinions of it, to prove the Infinite Duration and Eternity of the Supreme Mover; The firſt way is that of a true Philoſophy, and the laſt that of Metaphyſicks and Trifling in the Knowledge of Nature; what we mean is, that a God and Self-exiſting Being, that is a Neceſſary one, muſt be firſt evinc'd, which is eaſily done from a Conſideration of Nature, and then Eternity will follow ſuch a Conviction, but Eternity is not firſt to be preſum'd, and then an Eternal Exiſtence infer'd from it.

The Definition of Reſt is, that it is the Continuance of any Body in the ſame Place, againſt which the ſame Objections lye, as againſt Motion, of it's being only an abſtracted Idea; The remaining Definitions in this Philoſophy, unleſs thoſe which depend upon what we have already examin'd, ſeem to concern only the Explanation of Words, which therefore we omit and ſhall proceed to the Axioms alledg'd.

Dr. Keill's Intr. ad Ver. Phyſ. Lect. 7.

§. 36. The firſt Axiom we ſhall take notice of is, that any Change, ſuperinduc'd to a Natural Body, proceeds from an External Agent, ſince all Matter is unactive, and can produce no Alteration in it ſelf; this therefore depends upon the Inertia of Matter, which has been already conſider'd and refuted.

Axiom the Third. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 8.

§. 37. The next, which we ſhall mention is, that all Matter is of the ſame Nature and has the ſame Eſſential Attributes, whether fluid or hard, *&c.*

The laſt, that the different Forms of Bodies are only different Modifications of the ſame Matter, and depend on the Various Figure, Texture, Poſition, *&c.* of their Parts; both which Axioms have likewiſe been ſufficiently confuted, in the firſt Part of our *Principles of the Modern Philoſophy* examin'd, and in the *Introduction* to the Preſent.

Axioms the Fourteenth and Fifteenth. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 8.

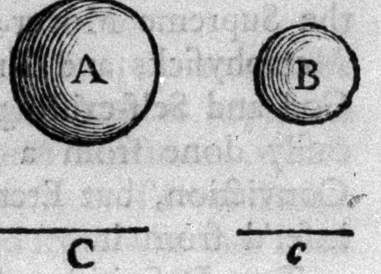
CHAP. III.

Concerning the Principal Theorems of the Mechanical Philoſophy, with a Propoſal of a new Syſteme of Mechanicks.

Theorems which are form'd upon abſtracted Principles, and Notions, of no Service to the Underſtanding of Nature.

§. 1. FROM what has been ſaid concerning the Definitions and Axioms, made uſe of in the Modern Philoſophy, it follows, that if all of them were acknowledg'd and admitted, Since they are abſtracted Ideas, they wou'd be of no greater Service to the Explaining of Nature, and a real Philoſophy, than the Elements of *Euclid,* to the Informing us, what real Extenſion is, as it exiſts in Bodies; for, as in the laſt, if we knew never ſo perfectly, that the Diagonal of a Square is incommenſurate with it's Sides, or that Parallelograms of equal Heights and Baſes are Equal, we ſhou'd not from thence be told, what the Extenſion of Wood was, in Diſtinction to that of Ivory, or Gold, ſo, if, in the firſt, we cou'd demonſtrate a Hundred Theorems concerning Motion, we ſhou'd not, by that, be let into the Knowledge of the real Properties of Motion, in Fire, Air, Water, or perhaps in Mechanicks themſelves, as they are found in Nature, which we ſhall now however more particularly conſider upon the Principles, which have been laid down.

Theorem the Firſt explain'd; That, in the Motions of Bodies, if the Quantities of Matter are Equal, the Moments will be, as the Celerities.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9. Dr. Wallis de Motu, Cap. 1. Prop. 22. Part 1. Vol. 1.



§. 2. The Firſt Theorem is, That in comparing the Motions of Bodies, if the Quantities of Matter, in the Bodies which are mov'd, are Equal, the Moments or Quantities of their Motions will be as their Celerities; The Demonſtration of which, without the Ambages here us'd, as in the following Theorems, in ſhort is; Let *A* and *B* be two Bodies, which have equal Quantities of Matter, and let their Celerities be *C* and *c*, the whole Quantity of Motion in *A* is to that in *B*, as *C* to *c*; becauſe, if *A* is = to *B* $AC.Bc::C.c$, or $\frac{AC}{Bc}=\frac{C}{c}$, i. e. $C.c::C.c$, and from whence this Corollary is deriv'd, if $AC.Bc::C.c$, *A* is = to *B*.

The foregoing Theorem conſiders Matter only as a mere Magnitude or Extenſion of Parts.

§. 3. THE foregoing Propoſition, and the Demonſtration of it, is true only upon this Suppoſition, that all Matter is Similar and alike, and that there is nothing in ſuch Similar Matter, excepting it's Magnitude and the mere Number of Parts, for if either Matter is various and different in it's own Nature, or if it has any Properties belonging to it, as Gravity or any other Force whatſoever, tho' we take equal Portions of it, the Moments or Quantities of their Motions will not be, as the Velocities, which are Impreſs'd; But whoever knew any Matter in the whole Univerſe, which only conſiſted in mere Magnitude and the Number of it's Parts? and we might as well take Number and Extenſion, and demonſtrate this Theorem concerning them, ſuppoſing the Units or Parts to be mov'd, as ſuch a Matter, which, what it wou'd ſignify to the Explaining Matter, as it really exiſts in Nature, cannot be eaſily Imagin'd.

Second Theorem explain'd and examin'd; That, in the Motions of Bodies, if the Celerities are Equal, the Moments will be, as the Quantities of Matter.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9. Dr. Wallis de Motu, Cap. 1. Prop. 22, &c. Part 1. Vol. 1.

§. 4. THE Second Theorem is, That in the Compariſon of Motions, if the Celerities are Equal, the Quantities of the Motions will be, as the Quantities of Matter in the Bodies, which are mov'd, or, if the Bodies, which are mov'd, are Homogeneous, as their Magnitudes; Let *A* and *B* be unequal, if $C=c$, $AC.Bc::A.B$, or $\frac{AC}{Bc}=\frac{A}{B}$, i. e. $A.B::A.B$; or if *M* and *m* are the Magnitudes, and $A.B::M.m$, it will be $MC.mc::M.m$, and the Corollary is, if $AC.Bc::A.B$, $C=c$. This Demonſtration proceeds, as before, upon the Suppoſition, Firſt, of Matter and a certain Sum of Units or Parts being the ſame, ſo that nothing elſe is conſider'd in Matter, but the Number of Parts or Units in it, Second, that each Part of both Bodies moves no farther, the one than the other, Third, that Homogeneous Bodies and Heterogeneous ones conſiſt of the ſame kind of Matter, tho' differently modify'd.

As to the Firſt Suppoſition, it has been conſider'd in the preceding Theorem; Second, as to each Part of the Body being mov'd, the one as far as the other, that is impoſſible, unleſs we imagine the whole Body as a Point, and then the Quantities of Motion will not be different, but the ſame as their Celerities; if we do not imagine the Bodies, as Points, then the Sum of the Celerities will not be, as the Bodies, which are accelerated, ſince ſuch Celerities will be different, becauſe one Part is really carried farther than another, and therefore it is mov'd with a greater Celerity; ſo that, in the preſent Caſe, it is manifeſt this Philoſophy conſiders not the actual Motions of Bodies, but the Sums of abſtracted Celerities, Independent of the Bodies, which are mov'd, which is an abſtracted Reaſoning, and is of no uſe to the Explaining of Nature, ſeeing the Sums of the Celerities, and of the Parts of Body, are both abſtracted Ideas; Laſtly, as to Homogeneous and Heterogeneous Bodies being the ſame in their Matter, that has been already ſufficiently confuted; not to mention, that where the Sums of the Parts or Units in Body are only conſidered, there can be no real Diſtinction betwixt the Magnitude of a Body, and the Sum of the Parts of that Body, or the Quantity of Matter in it, which yet, in this Theorem, are diſtinguiſh'd.

§. 5. FROM whence it is apparent, that in this Propoſition, and the Demonſtration of it, There are three Things requir'd, the firſt of which is a Suppoſition, which cannot be granted; the ſecond is an Impoſſibility in Nature; and the laſt an Abſurdity, which is That of the Corpuſcular Philoſophy, in which all Matter is preſum'd to be the ſame.

The foregoing Theorem requires more than can be allow'd or granted.

§. 6. A Third Theorem is, That, in the Compariſon of the Motions of Bodies, the Ratio or Proportion of their Moments to each other, is compounded of the Ratio's or Reſpects of their Quantities of Matter and their Celerities. Let A and B the Bodies which are moved be equal or unequal, the Celerities C and c; the Moment of A will be $= AC$, and the Moment of $B = Bc$, or M, m; therefore $M . m :: AC . Bc$, and $\frac{M}{m} = \frac{AC}{Bc} = \frac{A}{B} \times \frac{C}{c}$.

FROM which Theorem, the two former are only Corollaries, for if $A = B$, then will $\frac{AC}{Bc}$ be $= \frac{C}{c}$; if $C = c$, then $\frac{AC}{Bc}$ will be $= \frac{A}{B} = \frac{M}{m}$, as alſo it follows, that if $M = m$, $A . B :: c . C$ or $AC = Bc$, that is, if the Quantities of Matter in the Bodies, which are moved, are reciprocally Proportional to their Celerities, their Moments will be equal.

The Third Theorem explain'd and examin'd; That the Ratio of the Moments of Bodies is compounded of the Ratio's of their Quantities of Matter, and their Celerities.

Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9. Dr. Wallis de Motu, Cap. 1. Prop. 27. Part 1. Vol. 1.

IN all which Demonſtrations, ſince the Sums of the Celerities, and thoſe of the Parts of Matter are only conſider'd, as Sums of Homogeneous Units, multiply'd into each other, it is little more than proving, that 3×4 is $= 12$, or $4 \times 5 = 20$, which Theorems in Arithmetick cannot but be of very conſiderable Importance to the Knowledge of Nature.

§. 7. THE Fourth Theorem is of the like kind with the former, That, in the Compariſon of Motions, the Ratio of the Celerities is compounded of the Direct Reaſon or Proportion of the Moments, or Quantities of Motion, and of the Reciprocal Reaſon or Proportion of the Quantities of Matter.

THAT is, $\frac{AC}{Bc} \times \frac{B}{A} = \frac{C}{c}$, which is manifeſt upon the Principles before preſum'd, and which are here again deny'd, and for the Reaſons alledg'd. The Corollaries of this Theorem are, that $\frac{AC}{A} = C$, and $\frac{Bc}{B} = c$, That is, that the Moments of Bodies, divided by their Matter, are as their Celerities; It may be likewiſe ſaid, by the ſame way of Reaſoning, that the Ratio of the Quantities of Matter is compounded of the Direct Reaſon or Proportion of the Moments or Quantities of Motion, and of the Reciprocal Reaſon or Proportion of the Celerities, that is, $\frac{AC}{Bc} \times \frac{c}{C} = \frac{A}{B}$; alſo $\frac{AC}{C} = A$, and $\frac{Bc}{c} = B$, or the Quantities of Matter are, as the Moments of Bodies divided by their Celerities; all which Demonſtrations admit of the ſame anſwers as were before given.

The Fourth Theorem explain'd and examin'd; That the Ratio of the Celerities is compounded of the Direct Reaſon of the Moments, and of the Reciprocal of the Quantities of Matter.

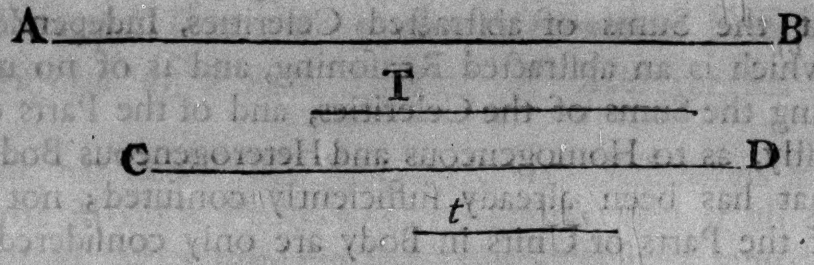
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9. Dr. Wallis de Motu, Cap. 1. Prop. 27. Part 1. Vol. 1.

§. 8. THE Fifth Theorem is, That if the Celerities of the Bodies, which are moved, are equal, the Spaces, deſcrib'd by them, will be in a Direct Reaſon or Proportion of the Times, in which they are perform'd. Let C, c, repreſent the Celerities, L, l, the Lengths or Spaces, and T, t the Times, if $C = c$, $L . l :: T . t$, and the Corollary is, if $L . l :: T . t$, $C = c$.

Theorem the Fifth explain'd and examin'd; That, if the Celerities of Bodies are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times of their Deſcription.

Dr. Keill's Intr. ad ver. Phyſ. Lect. 9. Dr. Wallis de Motu, Cap. 1. Prop. 23. Part 1. Vol. 1.

Thus let the Body, which is moved, be carried thro the Space AB, in the Time T, and thro' the Space CD, in the Time t, with the ſame Celerity, the Line AB will be to the Line CD, as T to t; becauſe, if the Time T is double or triple of the Time t, it may be divided into 2 or 3 Parts, each of which will be equal to t, and therefore each of the Spaces, which are deſcrib'd in theſe equal Parts of Time, and with the ſame Celerity, will be equal to the Space deſcrib'd in the Time t, and conſequently AB will be to CD, as 2 or 3, to 1, as T is double or triple of t.



Several Things in this Demonſtration taken for granted, which are not prov'd.

§. 9. In this Demonſtration the Matter, which is mov'd, is ſuppoſ'd to have a perfect Inertia, which we have before refuted; the Celerities are preſum'd Equal, which perhaps in Reality cannot be ſaid of any two actual Motions in the Univerſe; the Spaces likewiſe deſcrib'd are nothing elſe but an abſtracted Idea; upon all which Accounts, ſuch a Reaſoning can be of no Service or Import to the Science of Nature: but that which we are at preſent to take notice of is, that the Times of theſe equal Celerities are ſuppoſed Proportional to the Spaces, which ſuch Celerities deſcribe, which was the Thing to be prov'd; for to evidence this, it is ſaid, that if T is the double of t, it will be equal to $2t$, and whatever Space is deſcrib'd in t, there will be a Double deſcrib'd in $2t$; 'tis true, if $1t$ has 1 Length, $2t$ will have 2 Lengths, or *Vice verſâ*, becauſe it is a plain Axiom, but this is ſuppoſing and not proving, for how is it manifeſt, that $1t$ will have 1 Certain and Determin'd Length? if it is anſwer'd, becauſe $2t$ will have 2 Lengths, that is impoſſible, becauſe $2t$ has 2 Lengths upon the Account, that $1t$ has 1 Length; if it is told us $1t$ has 1 Certain Length, becauſe $1t$ has 1 Certain Length, that is affirming what was to be ſhewn; this I ſay, becauſe $1t$ may not only have 1 Determin'd Length, but 2, 3, or a 100 Various Lengths, where the Celerities are the ſame, that is, if the Denſities of the Spaces are different.

Abſtracted Time made uſe of, which yet is determin'd by the Celerity, which Meaſures it.

§. 10. If it is farther alledg'd, that $1t$ muſt have ſome Length or other belonging to it, and $2t$ will have double of that Length, the ſame Celerity continuing, it is ſtill ſuppoſing Time to be a Meaſure of the Celerity, and the Space deſcrib'd by it, that is, it is a Notional Time, which therefore reſts entirely in our own Minds, and is not conformable to Nature, ſince Real Time is the ſame, in it's Meaſure, as the Celerity or the Space deſcrib'd by it, and therefore the Times will be as the Spaces deſcrib'd, and the Spaces deſcrib'd, as the Celerities, or $T.t::L.l::C.c$; for the Real Times, in which any Spaces are perform'd, are commenſurate to thoſe Spaces which are perform'd, but the Spaces which are perform'd, are not commenſurate to an abſtracted Idea of Time, which is made up of Uniform and Unvary'd Points or Moments of it; That is, the Times in which the Celerities are perform'd, are to be meaſur'd by the Celerities which are perform'd, and not the Celerities by the Times (which reſult from them, and are conſequent on them) unleſs that, which is Meaſur'd, can Meaſure; What we Intend is, that Time do's not Conſiſt of any Certain and Determin'd Inſtants, which yet are here ſuppoſ'd, and have been before Confuted.

See Corol. 11. Theorem 5. of the Doctrine of Mechanicks ſubjoin'd.

But that which is ſtill more is, that there may be Celerities ſo perfectly different from what this Philoſophy conſiders, that where the Celerities are Equal, and the Lengths or Spaces Different, the Times may be the ſame, that is, tho' $C=c$, and L greater than l, yet $T=t$, For ſuppoſing AB and CD not to be mere abſtracted Lengths or Spaces, but to be Real ones, and the Firſt to be a more Thin or Diluted, and the Laſt to be more Conſtipated and Denſe, the ſame Celerity or Force will deſcribe the one no ſooner than the other; and in this way of conſidering Extenſion, where $C=c$, L may be to $l::t.T$, which is quite the reverſe of the Proportion aſſign'd by the preſent Theorem, that is, if the Space or Extenſion of CD is ſo much Rarer than it was before, as the Time t is leſs than that of T; ſince in ſuch a Caſe, and upon ſuch a Suppoſition, $TL=tl$; For the ſame Force will deſcribe a leſs Denſe Space in a leſs Time, as it will a more Denſe Space in a Greater.

AND the Advantage of conſidering Extenſion in this Manner, I have already ſhewn in the *Geometry* of *Solids* or of *Matter*, and in which it was propos'd from hence to reduce all Material Beings to Mathematicks, in as full a Senſe, as mere Extenſion now is, nor can it be juſtly deny'd, that we may not as well conſider Lines to be more Denſe or Diluted, as Imagine them to have one Uniform and Abſtracted Extenſion; Eſpecially, if we ſuppoſe thoſe Lines to conſiſt in a more diffus'd or a more compacted Action, which is the Caſe in Nature.

v. Geometria ſolidorum ſive Materiæ.

§. 11. THE Sixth Theorem is, That in the Comparing of Motions, if the Times of the Motions are Equal, the Spaces deſcrib'd will be as the Celerities. That is, if $T = t$, $L.l :: C.c$, and the Corollary is, if $L.l :: C.c$, $T = t$.

THE ſame Anſwer may be given to this Theorem, as to the Former, and farther, that there may be Celerities ſo entirely Different from what this Philoſophy conſiders, that where $T = t$, and C greater than c, yet L may be equal to l, that is, if CD is ſo much Rarer than AB, as the Celerity c is leſs than C; and conſequently L may be $.l :: c.C$, which is the Reverſe of the Proportion aſſign'd in this Theorem, that is, if CD is ſo much Rarer than it was before, as the Celerity c is leſs than that of C.

Theorem the Sixth explain'd and examin'd; That, if the Times are Equal, the Spaces deſcrib'd will be as the Celerities.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9.
Dr. Wallis de Motu, Cap. 1. Prop. 23. Part 1. Vol. 1.

§. 12. The Seventh Theorem is, That the Lengths or Spaces perform'd by the Motions of Bodies are in a Ratio compounded of the Ratio's of their Times and their Celerities. That is, $L.l :: TC.tc$, becauſe the Effects L, l will be proportionable to their Cauſes TC, tc. But $\frac{TC}{tc} = \frac{T}{t} \times \frac{C}{c} = \frac{L}{l}$, which was the thing to be prov'd.

FROM this Theorem, the two Former may be deriv'd, as plain and manifeſt Corollaries from it; for ſince $\frac{L}{l} = \frac{TC}{tc}$, If $C = c$, $\frac{L}{l} = \frac{T}{t}$, or $L.l :: T.t$; If $T = t$, $\frac{L}{l} = \frac{C}{c}$, or $L.l :: C.c$. Alſo, if $L = l$, $TC = tc$, or $C.c :: t.T$, that is, if the Spaces or Lengths perform'd are Equal, the Celerities will be reciprocally as the Times.

ALL which Proportions, or Theorems, depend upon the ſame Principles of a Similar and Homogeneous Matter, which is perfectly Paſſive and Unactive, and, in which, nothing elſe is conſider'd, excepting it's Parts, which are imagin'd not to be different from ſo many Abſtracted Units, ſo that Moving Numbers, wou'd as well ſerve the Turn of theſe Demonſtrations, as Moving Bodies.

Theorem the Seventh explain'd; That the Spaces deſcrib'd are in a Ratio compounded of the Ratio's of the Times, and the Celerities.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9.
Dr. Wallis de Motu, Cap. 1. Prop. 25. Part 1. Vol. 1.

§. 13. THE Eighth Theorem is, in Comparing of Motions, the Ratio of the Times is Compounded of the Direct Reaſon of the Lengths, and the Reciprocal of the Celerities. For, by the preceding Theorem, $\frac{L}{l} = \frac{TC}{tc}$, multiply both into $\frac{c}{C}$, and $\frac{Lc}{lC} = \frac{TCc}{tcC} = \frac{T}{t} = \frac{L}{l} \times \frac{c}{C}$. And by the ſame way of Reaſoning it may be ſhewn, that the Celerities are in a Direct Reaſon of the Spaces or Lengths, and a Reciprocal of the Times; For $\frac{L}{l} = \frac{TC}{tc}$, by the preceding Theorem, multiply both into $\frac{t}{T}$, and $\frac{Lt}{lT} = \frac{TCt}{tcT} = \frac{C}{c} = \frac{L}{l} \times \frac{t}{T}$.

FROM whence alſo it follows, if $\frac{L}{l} = \frac{TC}{tc}$, Firſt, that $\frac{T}{t} = \frac{Lc}{lC}$, and $T.t :: \frac{L}{C}.\frac{l}{c}$; Namely, the Time is, as the Space perform'd, divided by the Celerity; Second, that $\frac{C}{c} = \frac{Lt}{lT}$, or $C.c :: \frac{L}{T}.\frac{l}{t}$, that is, the Celerity is, as the Space perform'd, divided by the Time; All which Theorems and Proportions terminate, as before, in the ſame Principles, and do not need to be farther conſider'd here.

Theorem the Eighth explain'd; That the Ratio of the Times is compounded of the Direct Reaſon of the Lengths, and the Reciprocal of the Celerities.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 9.
Dr. Wallis de Motu, Cap. 1. Prop. 25. Part 1. Vol. 1.

§. 14. I have thus far given a ſhorter Demonſtration of theſe Theorems, than is any where Extant, and farther ſhall here ſubjoin a Doctrine of Mechanicks, which has not been yet propos'd by any Philoſophy, and which is Infinitely more Extenſive than the Preſent, as it is alſo more Agreeable to Nature.

THEOREM the Firſt is, That the Ratio of Moments in Bodies Elaſtick, that is, ſo far as they have an Expanſive Force, is compounded of the Reaſons of the Intrinſick or Expanſive Forces of Matter, of the Quantities of thoſe Intrinſick Forces, and, if put into Motion, of their Velocities. Let A and B be Two Bodies, M, m the Mo-

Theorem the Firſt of the New Syſtem explain'd; That the Ratio of Moments of Bodies, which are Elaſtick or Expanſive, is compounded of the Reaſons of the Expanſive

Forces, of the Quantities of thoſe Forces, and of their Velocities.

ments, the Intrinſick or Expanſive Forces E and e, the Quantities or Sums of thoſe Forces A, B, the Velocities V, v. It will be, $M.m::AEV.Bev$, or $\frac{M}{m}=\frac{AEV}{Bev}=\frac{A}{B}\times\frac{E}{e}\times\frac{V}{v}$. For in any Two Bodies, which have an Expanſive Force or a Propenſity to Motion, the Moment of the one muſt be Greater than that of the other, as the Velocity of the one, the Expanſive Force, or Propenſity to Motion, and the Bulk or Magnitude is Greater than That of the other; and therefore, if we Multiply the Expanſive Force into the Magnitude, and both into the Velocity, we ſeem to have the whole Moment of ſuch Bodies in their Motions.

Corol. 1. If therefore $A=B$, That is, if the Sums or Quantities of the Forces are Equal, the Moments will be, as the Intrinſick or Expanſive Forces, multiply'd into the Velocities, or in a Ratio compounded of them, or $M.m::EV.ev$, or $\frac{M}{m}=\frac{E}{e}\times\frac{V}{v}$.

Corol. 2. If $E=e$, which is the Caſe of the Mechanical Philoſophy, in which the Intrinſick or Expanſive Forces are Equal, that is, none at all, the Moments will be in a Ratio compounded of the Quantities, and the Velocities, or $M.m::AV.Bv$, or $\frac{M}{m}=\frac{A}{B}\times\frac{V}{v}$.

Corol. 3. If $V=v$, That is, if the Velocities are Equal, the Moments will be in a Ratio, compounded of the Intrinſick or Expanſive Forces, and the Quantities of them, or $M.m::AE.Be$, or $\frac{M}{m}=\frac{A}{B}\times\frac{E}{e}$.

Corol. 4. If $A=B$ and $E=e$, That is, if the Intrinſick or Expanſive Forces, and their Quantities are Equal, the Moments will be as their Velocities, or $M.m::V.v$, or $\frac{M}{m}=\frac{V}{v}$. This likewiſe is a Theorem in the Mechanical Philoſophy.

Corol. 5. If $A=B$ and $V=v$, That is, if the Sums or Quantities of the Intrinſick or Expanſive Forces, and the Velocities are Equal, the Moments will be as the Intrinſick or Expanſive Forces, or $M.m::E.e$, or $\frac{M}{m}=\frac{E}{e}$.

Corol. 6. If $E=e$ and $V=v$, That is, if the Intrinſick or Expanſive Forces, and Velocities are Equal, the Moments will be, as the Sums or Quantities, of the Intrinſick or Expanſive Forces, or $M.m::A.B$, or $\frac{M}{m}=\frac{A}{B}$. Which is alſo a Proportion affirm'd in the Mechanical Philoſophy.

Corol. 7. If $M=m$, $AEV=Bev$, and $V.v::Be.AE$, i. e. If the Moments are Equal, the Velocities will be in a Reaſon compounded of a Reciprocal, of the Intrinſick, or Expanſive Forces, and the Quantities of them.

Corol. 8. If $M=m$, $AEV=Bev$, and $E.e::Bv.AV$, i. e. If the Moments are Equal, the Intrinſick or Expanſive Forces, will be in a Reaſon compounded of a Reciprocal of the Velocities, and the Quantities of the Intrinſick or Expanſive Forces.

Corol. 9. If $M=m$, $AEV=Bev$, and $A.B::ev.EV$, i. e. If the Moments are Equal, the Quantities of the Intrinſick or Expanſive Forces, will be in a Reaſon compounded of a Reciprocal one of the Velocity, and Intrinſick or Expanſive Forces.

Corol. 10. If $M=m$, and $A=B$, $E.e::v.V$, and $EV=ev$, That is, if the Sums or Quantities of the Intrinſick or Expanſive Forces are Equal, the Intrinſick or Expanſive Force of the Firſt Body, is to that of the Second Reciprocally, as the Velocity of the Second, is to that of the Firſt, and the Moments on both Sides will be Equal.

Corol. 11. If $M=m$, and $E=e$, $A.B::v.V$, and $AV=Bv$, That is, if the Intrinſick or Expanſive Forces are Equal, the Quantities of them are Reciprocally, as their Velocities, and the Moments on both Sides will be Equal, which is another Caſe in the Mechanical Philoſophy.

Corol. 12. If $M=m$, and $V=v$, $A.B::e.E$, and $AE=Be$, That is, if the Velocities are Equal, the Quantities of the Intrinſick or Expanſive Forces are Reciprocally, as the Intrinſick or Expanſive Forces, and the Moments will be Equal.

So that in all the Caſes, which we have mention'd, there are only Four, which the Mechanical Philoſophy conſiders, The Second, Fourth, Sixth, and Eleventh, and theſe only upon the Suppoſition, that E and e are nothing, when they are Equal; As for theſe Intrinſick Forces, we ſhall Reſerve them for a Farther Explanation, and ſhall only at preſent in general name them the Expanſive and Contractive, by the Former of which, Bodies are previouſly diſpos'd to Motion, and are more eaſily put into it by any Impreſſion made, and by the Latter, are oppoſite and averſe to it, and by conſequence, more difficultly obey the like Impreſſions; of the Firſt Kind, are all Elaſtick Bodies, and of the Laſt, all the Groſs and Heavy ones: And when I ſpeak of Forces in Matter, I muſt here declare ſo far againſt the Inertia of it, that I do take upon me to Pronounce Action to be the Subſtratum, and the very Eſſence of it; The denial of which has been the Occaſion of all the Abſurdities in the Corpuſcular Syſtem, and the Reaſon of making ſo ſmall a Progreſs in the accounting for the Phænomenons in Nature. The Expanſive Forces, by which Bodies are more or leſs diſpos'd to Motion, and a Deſcription of Space, are only conſider'd in this Theorem.

Theorem the Second of the New Syſtem explain'd; That the Ratio of the Velocities is compounded of a Direct Reaſon of the Moments, and of a Reciprocal of the Expanſive Forces, and the Quantities of Them.

§. 15. **Theorem** the Second is, That the Ratio of the Velocities is compounded of a Direct Reaſon of the Moments, and of a Reciprocal of the Intrinſick or Expanſive Forces, and the Sums or Quantities of them. For by the preceding Theorem, if M and m are the Moments, $\frac{M}{m} = \frac{AEV}{Bev}$, multiply $\frac{M}{m}$ into Be, and divide it by AE, and $\frac{V}{v} = \frac{MBe}{mAE} = \frac{M}{m} \times \frac{B}{A} \times \frac{e}{E}$. From whence by forming other Equations, other Theorems, or Corollaries may be deduc'd, as in the foregoing.

Theorem the Third of the New Syſtem explain'd; That the Ratio of the Expanſive Forces is compounded of a Direct Reaſon of the Moments, and of a Reciprocal of the Velocities, and of the Quantities of the ſaid Forces.

§. 16. **Theorem** the Third is, That the Ratio of the Intrinſick or Expanſive Forces, is compounded of a Direct Reaſon of the Moments, and of a Reciprocal of the Velocities, and of the Quantities or Sums of the Intrinſick or Expanſive Forces. For $\frac{M}{m} = \frac{AEV}{Bev}$, multiply $\frac{M}{m}$ into Bv, and divide it by AV, and $\frac{E}{e} = \frac{MBv}{mAV} = \frac{M}{m} \times \frac{B}{A} \times \frac{v}{V}$.

Theorem the Fourth of the New Syſtem explain'd; That the Ratio of the Quantities of the Expanſive Forces is compounded of a Direct Reaſon of the Moments, and of a Reciprocal of the ſaid Forces, and of the Velocities.

§. 17. **Theorem** the Fourth is, That the Ratio of the Quantities, or Sums of the Intrinſick or Expanſive Forces, is compounded of a Direct Reaſon of the Moments, and of a Reciprocal of the Intrinſick or Expanſive Forces, and of the Velocities. For $\frac{M}{m} = \frac{AEV}{Bev}$, multiply $\frac{M}{m}$ into ev, and divide it by EV, and $\frac{A}{B} = \frac{Mev}{mEV} = \frac{M}{m} \times \frac{e}{E} \times \frac{v}{V}$. And from theſe a Thouſand Caſes may be offer'd, and Equations form'd upon them, and a great part of Material Nature, in Conjunction with the following Theorems, explain'd by them, if we carefully examine what theſe Intrinſick or Expanſive Forces are, which I ſhall reſerve, as was before ſaid to a farther Expoſition.

Theorem the Fifth of the New Syſtem; That the Spaces deſcrib'd by Two Bodies are in a Ratio compounded of the Reaſons of the Times, the Velocities, the Expanſive Forces, and the Denſities of the Spaces.

§. 18. **Theorem** the Fifth is, That the Spaces deſcrib'd by Two Bodies, which are mov'd, are in a Ratio compounded of the Reaſons of the Times, the Velocities, the Intrinſick or Expanſive Forces, and the Denſities of the Spaces. Let S, s repreſent the Spaces, T, t the Times, E, e the Intrinſick or Expanſive Forces, D, d the Denſities of the Spaces, V, v the Velocities. The Theorem will be, $S . s :: TVED . tved$, or $\frac{S}{s} = \frac{TVED}{tved} = \frac{T}{t} \times \frac{V}{v} \times \frac{E}{e} \times \frac{D}{d}$. For one Space deſcrib'd muſt be Greater than another, as the Time and Velocity, and the Diſpoſition to Motion, or the Expanſive Force, and the Denſity of the Space, which is deſcrib'd, is Greater.

Corol. 1. If $T = t$, $S . s :: VED . ved$, or $\frac{S}{s} = \frac{V}{v} \times \frac{E}{e} \times \frac{D}{d}$, That is, if the Times are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Velocities, of the Intrinſick or Expanſive Forces, and the Denſities of thoſe Spaces.

Corol. 2. If $V=v$. $S.s::TED.ted$, or $\frac{S}{s}=\frac{TED}{ted}=\frac{T}{t}\times\frac{E}{e}\times\frac{D}{d}$, That is, if the Velocities are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times, the Intrinſick or Expanſive Forces, and the Denſities of thoſe Spaces.

Corol. 3. If $E=e$, $S.s::TVD.tvd$, or $\frac{S}{s}=\frac{TVD}{tvd}=\frac{T}{t}\times\frac{V}{v}\times\frac{D}{d}$, That is, if the Intrinſick or Expanſive Forces are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times, the Velocities, and the Denſities of thoſe Spaces.

Corol. 4. If $D=d$, $S.s::TVE.tve$, or $\frac{S}{s}=\frac{TVE}{tve}=\frac{T}{t}\times\frac{V}{v}\times\frac{E}{e}$, That is, if the Denſities of the Spaces are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times, the Velocities, and Intrinſick or Expanſive Forces.

Corol. 5. If $T=t$, and $V=v$, $S.s::ED.ed$, or $\frac{S}{s}=\frac{ED}{ed}=\frac{E}{e}\times\frac{D}{d}$, That is, if both the Times, and the Velocities are Equal, the Spaces deſcrib'd are in a Direct Reaſon of the Intrinſick or Expanſive Forces, and the Denſities of thoſe Spaces.

Corol. 6. If $T=t$, and $E=e$, $S.s::VD.vd$, or $\frac{S}{s}=\frac{VD}{vd}=\frac{V}{v}\times\frac{D}{d}$, That is, if the Times, and the Intrinſick or Expanſive Forces are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Velocities, and the Denſities of thoſe Spaces.

Corol. 7. If $T=t$, and $D=d$, $S.s::VE.ve$, or $\frac{S}{s}=\frac{VE}{ve}=\frac{V}{v}\times\frac{E}{e}$, That is, if the Times, and the Denſities of the Spaces are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Intrinſick or Expanſive Forces, and the Velocities.

Corol. 8. If $V=v$, and $E=e$, $S.s::TD.td$, or $\frac{S}{s}=\frac{TD}{td}=\frac{T}{t}\times\frac{D}{d}$, That is, if the Velocities, and the Intrinſick or Expanſive Forces are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times, and the Denſities of thoſe Spaces.

Corol. 9. If $V=v$, and $D=d$, $S.s::TE.te$, or $\frac{S}{s}=\frac{TE}{te}=\frac{T}{t}\times\frac{E}{e}$, That is, if the Velocities and the Denſities are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times, and the Intrinſick or Expanſive Forces.

Corol. 10. If $E=e$, and $D=d$, $S.s::TV.tv$, or $\frac{S}{s}=\frac{TV}{tv}=\frac{T}{t}\times\frac{V}{v}$, That is, if the Intrinſick or Expanſive Forces, and the Denſities are Equal, the Spaces deſcrib'd will be in a Direct Reaſon of the Times, and the Velocities; which is the only Caſe, which the Preſent Mechanical Philoſophy conſiders, and is the Proportion of Theorem the Seventh in it, which is nothing more than one Corollary of the Ten, already deriv'd from this Fifth Theorem.

Corol. 11. If $T=t$, $V=v$, and $E=e$, $S.s::D.d$, That is, if the Times, the Velocities, and Intrinſick or Expanſive Forces are Equal, the Spaces deſcrib'd will be as the Denſities.

Corol. 12. If $T=t$, $V=v$, $D=d$, $S.s::E.e$, That is, if the Times, the Velocities, and Denſities are Equal, the Spaces deſcrib'd will be as the Intrinſick or Expanſive Forces.

Corol. 13. If $T=t$, $E=e$, $D=d$, $S.s::V.v$, That is, if the Times, the Intrinſick or Expanſive Forces, and Denſities are Equal, the Spaces deſcrib'd will be as the Velocities.

Corol. 14. If $V=v$, $E=e$, $D=d$, $S.s::T.t$, That is, if the Velocities, the Intrinſick or Expanſive Forces, and the Denſities are Equal, the Spaces deſcrib'd will be as the Times. Theſe Two laſt Corollaries are the Fifth and Sixth Theorems of the Mechanical Philoſophy.

Corol. 15. If $S=s$, $TVED=tved$, or $TV.tv::ed.ED$, That is, the Time and Velocity of the Firſt, will be to the Time and Velocity of the Second, as the Intrinſick or Expanſive Force, and Denſity of Space of the Second, is to the Intrinſick or Expanſive Force, and Denſity of Space of the Firſt, and the Spaces deſcrib'd on both Hands will be Equal. As alſo $T.t::ved.VED$, and $V.v::ted.TED$, and $E.e::tvd.TVD$, and $D.d::tve.TVE$.

Corol. 16. If $T=t$, $VE.ve::d.D$, That is, if the Times are Equal, the Velocity, and Intrinſick or Expanſive Force of the Firſt, will be to the Velocity and Intrinſick or Expanſive Force of the Second, as the Denſity of Space of the Second, is to the Denſity of Space of the Firſt, and the Spaces deſcrib'd on both Hands will be Equal.

Corol. 17. If $V=v$, $TE.te::d.D$, That is, if the Velocities are Equal, the Time and Intrinſick or Expanſive Force of the Firſt, will be to the Time and Intrinſick or Expanſive Force of the Second, as the Denſity of Space of the Second, is to the Denſity of Space of the Firſt, and the Spaces deſcrib'd on both Hands will be Equal.

Corol. 18. If $E=e$, $TV.tv::d.D$, That is, if the Intrinſick or Expanſive Forces are Equal, the Time and Velocity of the Firſt, will be to the Time and Velocity of the Second, as the Denſity of Space of the Second, is to the Denſity of Space of the Firſt, and the Spaces on both Hands will be Equal.

Corol. 19. If $D=d$, $TV.tv::e.E$, That is, if the Denſities of the Spaces are Equal, the Time and Velocity of the Firſt, will be to the Time and Velocity of the Second, as the Intrinſick or Expanſive Force of the Second, is to the Intrinſick or Expanſive Force of the Firſt, and the Spaces deſcrib'd on both Hands will be Equal.

Corol. 20. If $T=t$, and $V=v$, $E.e::d.D$, That is, if the Times and Velocities are Equal, the Intrinſick or Expanſive Force of the Firſt, will be to the Intrinſick or Expanſive Force of the Second, as the Denſity of Space of the Second, is to the Denſity of Space of the Firſt, and the Spaces deſcrib'd on both Hands Equal.

Corol. 21. If $T=t$, and $E=e$, $V.v::d.D$, That is, if the Times, and Intrinſick or Expanſive Forces are Equal, the Velocity of the Firſt, will be to the Velocity of the Second, as the Denſity of Space of the Second, is to the Denſity of Space of the Firſt, and the Spaces deſcrib'd on both Hands will be Equal.

Corol. 22. If $T=t$, and $D=d$, $V.v::e.E$, That is, if the Times and Denſities are Equal, the Velocity of the Firſt, will be to the Velocity of the Second, as the Intrinſick or Expanſive Force of the Second, to the Intrinſick or Expanſive Force of the Firſt, and the Spaces on both Hands Equal.

Corol. 23. If $V=v$, and $E=e$, $T.t::d.D$, That is, if the Velocities, and Intrinſick or Expanſive Forces are Equal, the Time of the Firſt, will be to the Time of the Second, as the Denſity of Space of the Second, is to the Denſity of Space of the Firſt, and the Spaces on both Hands will be Equal.

Corol. 24. If $V=v$, and $D=d$, $T.t::e.E$, That is, if the Velocities and Denſities are Equal, the Time of the Firſt, will be to the Time of the Second, as the Intrinſick or Expanſive Force of the Second, is to the Intrinſick or Expanſive of the Firſt, and the Spaces on both Hands Equal.

Corol. 25. If $E=e$, and $D=d$, $T.t::v.V$, That is, if the Intrinſick or Expanſive Forces, and Denſities are Equal, the Time of the Firſt, will be to the Time of the Second, as the Velocity of the Second, is to that of the Firſt, and the Spaces deſcrib'd on both Hands will be Equal, which is the only Caſe, which the Preſent Mechanicks conſiders.

Theorem the Sixth of the New Syſtem; That the Ratio of the Times is compounded of a Direct Reaſon of the Spaces, and a Reciprocal one of the Velocities, the Expanſive Forces, and the Denſities of thoſe Spaces.

§. 19. Theorem the Sixth is, That the Ratio of the Times is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Velocities, of the Intrinſick or Expanſive Forces, and the Denſities of thoſe Spaces. For, by the foregoing Theorem, $\frac{S}{s}=\frac{TVED}{tved}$, multiply $\frac{S}{s}$ into ved, and divide it by VED, and $\frac{T}{t}=\frac{Sved}{sVED}=\frac{S}{s}\times\frac{v}{V}\times\frac{e}{E}\times\frac{d}{D}$.

Theorem the Seventh of the New Syſtem; That the Ratio of the Velocities is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Times, the Expanſive Forces, and the Denſities of thoſe Spaces.

§. 20. Theorem the Seventh is, That the Ratio of the Velocities is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Times, of the Intrinſick or Expanſive Forces, and the Denſities of thoſe Spaces. For, by Theorem the Fifth, $\frac{S}{s}=\frac{TVED}{tved}$, multiply $\frac{S}{s}$ into ted, and divide it by TED, and $\frac{V}{v}=\frac{Sted}{sTED}=\frac{S}{s}\times\frac{t}{T}\times\frac{e}{E}\times\frac{d}{D}$

§. 21.

Theorem the Eighth of the New Syſtem; That the Ratio of the Expanſive Forces is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Times, the Velocities, and the Denſities of thoſe Spaces.

§. 21. THEOREM the Eighth is, That the Ratio of the Intrinſick or Expanſive Forces is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Times, of the Velocities, and the Denſities of thoſe Spaces. For, by Theorem the Fifth, $\frac{S}{s} = \frac{TVED}{tved}$, multiply $\frac{S}{s}$ into tvd, and divide it by TVD, and $\frac{E}{e} = \frac{Stvd}{sTVD} = \frac{S}{s} \times \frac{t}{T} \times \frac{v}{V} \times \frac{d}{D}$.

Theorem the Ninth; That the Ratio of the Denſities of the Spaces is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Times, the Velocities, and the Expanſive Forces.

§. 22. THEOREM the Ninth is, That the Ratio of the Denſities of the Spaces is compounded of a Direct Reaſon of the Spaces, and of a Reciprocal one of the Times, the Velocities, and the Intrinſick or Expanſive Forces. For, by Theorem the Fifth, $\frac{S}{s} = \frac{TVED}{tved}$, multiply $\frac{S}{s}$ into tve, and divide it by TVE, and $\frac{D}{d} = \frac{Stve}{sTVE} = \frac{S}{s} \times \frac{t}{T} \times \frac{v}{V} \times \frac{e}{E}$. Upon which Equations a Multitude of others might be form'd, but I forbear now to Enlarge, and Proceed to thoſe Theorems and Corollaries, which reſpect the Contractive Forces.

WE have here made uſe of Time and Velocity in the ſame way, as has been hitherto done, but as Time is different in it's Flux, according to the Meaſure we make uſe of, and as there may be a Velocity in an Infiniteſimal Space, Equal to a Velocity, which ſhall deſcribe an Infinite one, in a Time given, ſo we, at preſent, abſtract from them, which, if we were to take into the Preſent Theorems, there wou'd be another Compoſition of Forces, namely, by making T and τ to Repreſent the Intrinſick and Real Fluxes of Time, and C, c to Repreſent the Intrinſick Forces of Velocity; Thus, tho' we make uſe of Portions of Time, which flow Equably, yet we do not now determine according to what Meaſure of Celerity they Flow, and tho' we take Velocities, which are meaſur'd by the longer or ſhorter Spaces, which they deſcribe in the ſame Time, we do not conſider thoſe which are different in the ſame Time, and in the ſame Points, or in the ſame Spaces, which they deſcribe, for Example, if all that Force of Velocity, which is ſpent in deſcribing Three Foot, was contracted within the Compaſs of one, every Part of ſuch a Velocity wou'd be very different, from that which we now conſider; If ſuch Times and Velocities therefore were taken into the Computation,

THE Firſt Theorem wou'd be $\frac{M}{m} = \frac{AEVC}{Bevc}$, That is, the Moments of Moving Bodies, wou'd be as their Intrinſick or Expanſive Forces, the Sums of thoſe Forces, their Velocities meaſur'd by the Lengths, which they deſcribe in the ſame Times, and as the Intrinſick or Expanſive Forces of thoſe Velocities, in every Point of the Lengths deſcrib'd by them,

THE Fifth Theorem wou'd be $\frac{S}{s} = \frac{T\tau VCED}{tTvced}$, That is, the Spaces deſcrib'd wou'd be compounded of the Times Directly, and the particular Flux or Celerities of thoſe Times Reciprocally, of the Velocities meaſur'd by the Lengths, which they deſcribe in the ſame Time, and the Intrinſick or Expanſive Forces of thoſe Velocities, in every Point of the Lengths deſcrib'd by them, of the Intrinſick or Expanſive Forces of Matter, and the Denſities of the Space perform'd, Directly. Hitherto we have ſpoke of thoſe Intrinſick Forces only, which we nam'd the Expanſive, we are now come to conſider the Contractive, concerning which the following Theorem may be form'd.

Theorem the Tenth; That the Ratio of Moments of the Expanſive Forces in Heavy Bodies is compounded of the Direct Reaſon of the Velocities, and of a Reciprocal of the Contractive Forces, and of the Sums of thoſe Forces.

§. 23. THEOREM the Tenth is, That the Ratio of Moments of the Expanſive Forces, in Heavy Bodies, is compounded of the Direct Reaſons of the Velocities, and of a Reciprocal of the Intrinſick or Contractive Forces, and of the Sums of thoſe Forces. Let A and B be two Heavy Bodies, M, m the Moments, the Intrinſick or Contractive Forces, C, c, the Quantities, or Sums of thoſe Forces, A and B, the Velocities, V, v; $M . m :: VcB . vCA$, and $\frac{M}{m} = \frac{VcB}{vCA} = \frac{V}{v} \times \frac{c}{C} \times \frac{B}{A}$. That is, one Body will be Quicker and more Expanſive, and in it's Motion, than another, as it's Velocity is Greater, and it's Contractive Force, and the Sum or Quantity of it is Leſs.

Corol. 1. If therefore $A=B$, $\frac{M}{m}=\frac{Vc}{vC}$, and $M.m::Vc.vC$, That is, if the Sums or Quantities of the Intrinſick or Contractive Forces are Equal, the Moments will be in a Ratio, compounded of a Direct one of their Velocities, and a Reciprocal of the Intrinſick or Contractive Forces.

Corol. 2. If $C=c$, $\frac{M}{m}=\frac{VB}{vA}$, and $M.m::VB.vA$, That is, if the Intrinſick or Contractive Forces are Equal, the Moments will be in a Ratio compounded of a Direct one of their Velocities, and a Reciprocal one of the Sums or Quantities of the Intrinſick or Contractive Forces.

Corol. 3. If $V=v$, $\frac{M}{m}=\frac{cB}{CA}$, and $M.m::cB.CA$, That is, if the Velocities are Equal, the Moments will be in a Ratio compounded of a Reciprocal one, both of their Intrinſick or Contractive Forces, and the Quantities of them.

Corol. 4. If $A=B$, and $C=c$, $\frac{M}{m}=\frac{V}{v}$, and $M.m::V.v$, That is, if the Intrinſick or Contractive Forces, and the Quantities of them are Equal, the Moments will be in a Direct Reaſon of the Velocities.

Corol. 5. If $A=B$, and $V=v$, $\frac{M}{m}=\frac{c}{C}$, and $M.m::c.C$, That is, if the Quantities of the Intrinſick or Contractive Forces, and the Velocities are Equal, the Moments will be in a Reciprocal Reaſon of the Intrinſick or Contractive Forces.

Corol. 6. If $C=c$, and $V=v$, $\frac{M}{m}=\frac{B}{A}$, and $M.m::B.A$, That is, if the Intrinſick or Contractive Forces, and the Velocities are Equal, the Moments will be in a Reciprocal Reaſon of the Quantities of the Intrinſick or Contractive Forces.

Corol. 7. If $M=m$, $VcB=vCA$, and $V.v::CA.cB$, That is, if the Moments are Equal, the Velocities will be in a Reaſon compounded of a Direct one, both of the Intrinſick Forces, and the Quantities of them.

Corol. 8. If $M=m$, $VcB=vCA$, and $C.c::VB.vA$, That is, if the Moments are Equal, the Intrinſick or Contractive Forces will be in a Reaſon compounded of a Direct one of the Velocities, and of a Reciprocal of the Quantities of thoſe Forces.

Corol. 9. If $M=m$, $VcB=vCA$, and $A.B::Vc.vC$, That is, if the Moments are Equal, the Quantities of the Intrinſick or Contractive Forces, will be in a Reaſon compounded of a Direct one of the Velocities, and of a Reciprocal of thoſe Forces.

Corol. 10. If $M=m$, and $A=B$, $Vc=vC$, and $V.v::C.c$, That is, if the Moments and the Quantities of the Forces are Equal, the Velocities will be in a Direct Reaſon of the Intrinſick or Contractive Forces.

Corol. 11. If $M=m$, and $C=c$, $VB=vA$, and $V.v::A.B$, That is, if the Moments and the Intrinſick or Contractive Forces are Equal, the Velocities will be in a Direct Reaſon of the Quantities of the Intrinſick or Contractive Forces.

Corol. 12. If $M=m$, and $V=v$, $cB=CA$, and $C.c::B.A$, That is, if the Moments and the Velocities are Equal, the Intrinſick or Contractive Forces will be in a Reciprocal Reaſon of their Quantities.

§. 24. Theorem the Eleventh is, That the Ratio of the Velocities is compounded of a Direct Reaſon of the Moments, the Intrinſick or Contractive Forces, and the Quantities of thoſe Forces. For, by the preceding Theorem, $\frac{M}{m}=\frac{VcB}{vCA}$, therefore $\frac{V}{v}=\frac{MCA}{mcB}=\frac{M}{m}\times\frac{C}{c}\times\frac{A}{B}$. From whence by Forming other Equations, other Theorems, or Corollaries may be deduc'd, as in the Former.

Theorem the Eleventh is, That the Ratio of the Velocities is compounded of a Direct Reaſon of the Moments, the Intrinſick or Contractive Forces, and the Quantities of thoſe Forces.

Theorem the Twelfth of this New Syſtem.

§. 25. **Theorem** the Twelfth is, That the Ratio of the Intrinſick or Contractive Forces, is compounded of a Direct Reaſon of the Velocities, and of a Reciprocal one of the Moments, and of the Quantities of ſuch Intrinſick Forces. For, by Theorem the Tenth, $\frac{M}{m} = \frac{VcB}{vCA}$, therefore $\frac{C}{c} = \frac{VBm}{vAM} = \frac{V}{v} \times \frac{m}{M} \times \frac{B}{A}$.

Theorem the Thirteenth of this New Syſtem.

§. 26. **Theorem** the Thirteenth is, That the Ratio of the Quantities of the Intrinſick or Contractive Forces is compounded of a Direct one of the Velocities, and of a Reciprocal of the Moments and Intrinſick or Contractive Forces. For $\frac{M}{m} = \frac{VcB}{vCA}$, therefore $\frac{A}{B} = \frac{Vmc}{vMC} = \frac{V}{v} \times \frac{m}{M} \times \frac{c}{C}$.

Theorem the Fourteenth is, That the Moments of the Contractive Forces in Bodies, which are Heavy, are in a Reaſon compounded of a Direct one of their Intrinſick or Contractive Forces and of the Quantities of Them, and of a Reciprocal of their Velocities.

§. 27. **Theorem** the Fourteenth is, That the Moments of the Contractive Forces in Bodies, which are Heavy, are in a Reaſon compounded of a Direct one of their Intrinſick or Contractive Forces and of the Quantities of them, and of a Reciprocal of their Velocities, viz. $\frac{M}{m} = \frac{CAv}{cBV} = \frac{C}{c} \times \frac{A}{B} \times \frac{v}{V}$, ſuppoſing C and c, as before, to be the Intrinſick or Contractive Forces.

From whence the ſame Number of Corollaries may be drawn, as in the preceding Theorems of Expanſive Forces,

1. If $C=c$, $M.m::Av.BV$.
2. If $A=B$, $M.m::Cv.cV$.
3. If $V=v$, $M.m::CA.cB$.
4. If $C=c$, and $A=B$, $M.m::v.V$.
5. If $C=c$, and $V=v$, $M.m::A.B$.
6. If $A=B$, and $V=v$, $M.m::C.c$.
7. If $M=m$, $C.c::BV.Av$.
8. If $M=m$, $A.B::cV.Cv$.
9. If $M=m$, $V.v::CA.cB$.
10. If $M=m$, and $C=c$, $A.B::V.v$.
11. If $M=m$, and $A=B$, $C.c::V.v$.
12. If $M=m$, and $V=v$, $C.c::B.A$.

Theorem the Fifteenth of this New Syſtem.

§. 28. **Theorem** the Fifteenth is, That the Moments of the Contractive Forces in Bodies, which are Elaſtick or Expanſive, are in a Reaſon compounded of a Reciprocal one of the Intrinſick or Expanſive Forces, of the Quantities of them, and their Velocities, or $\frac{M}{m} = \frac{Bev}{AEV} = \frac{B}{A} \times \frac{e}{E} \times \frac{v}{V}$. From whence may be deriv'd as many Corollaries as before, which I here omit, ſince the Way is ſhewn, and it is eaſily done.

Theorem the Sixteenth, and which is more General than the Preceding ones.

§. 29. **Theorem** the Sixteenth is, That in General, the Moments of Bodies, where the Expanſive Forces are only conſider'd, may be Varied according to the following Method, for $\frac{M}{m}$ may be Equal to $\frac{A}{B} \times \frac{E}{e} \times \frac{V}{v}$, or $\frac{A}{B} \times \frac{E}{e} \times \frac{v}{V}$, or $\frac{B}{A} \times \frac{E}{e} \times \frac{V}{v}$, or $\frac{A}{B} \times \frac{e}{E} \times \frac{V}{v}$, or $\frac{B}{A} \times \frac{e}{E} \times \frac{V}{v}$, or $\frac{A}{B} \times \frac{e}{E} \times \frac{v}{V}$, or $\frac{B}{A} \times \frac{E}{e} \times \frac{v}{V}$, or $\frac{B}{A} \times \frac{e}{E} \times \frac{v}{V}$, &c.

And the Moments of Bodies, where the Contractive Forces are only conſider'd, will be as follows, $\frac{M}{m} = \frac{A}{B} \times \frac{C}{c} \times \frac{V}{v}$, or $\frac{A}{B} \times \frac{C}{c} \times \frac{v}{V}$, or $\frac{B}{A} \times \frac{C}{c} \times \frac{V}{v}$, or $\frac{A}{B} \times \frac{c}{C} \times \frac{V}{v}$, or $\frac{B}{A} \times \frac{c}{C} \times \frac{v}{V}$, or $\frac{B}{A} \times \frac{c}{C} \times \frac{V}{v}$, or $\frac{A}{B} \times \frac{c}{C} \times \frac{v}{V}$, or $\frac{B}{A} \times \frac{C}{c} \times \frac{v}{V}$, &c. in which Caſes, M is not ſuppos'd Greater than m, as in the Former, but They are left at large to Expreſs the Moments, as they ſhall be determin'd by the Forces, Quantities, and Velocities which compound them, each of which Equations wou'd make a diſtinct Theorem, with a like Number of Corollaries with the Former.

Hitherto we have conſider'd the Expanſive and Contractive Forces ſeparately, but as they may be, and really are united together in all Material Beings whatſoever, the Moments of them will admit of Various and Differing Theorems.

Theorem the Seventeenth is, That the Moments of Bodies, in which the Intrinſick Forces conſiſt of the

§. 30. **Theorem** the Seventeenth is, That the Moments of Bodies, in which the Intrinſick Forces conſiſt of the Expanſive and Contractive, and which are in Motion, as they have hitherto been ſuppos'd, muſt be computed from their Intrinſick Expanſive and Contractive Forces, from the Quantities of them, and from their Velocities.

Expanſive and Contractive, and which are in Motion, muſt be Computed from their Intrinſick Expanſive and Contractive Forces, from the Quantities of them and their Velocities.

Therefore Bodies endu'd with ſuch Forces, have their Moments compounded of all of them, That is, $\frac{M}{m}$ may be Equal to $\frac{AECV}{Becv} = \frac{A}{B} \times \frac{E}{e} \times \frac{C}{c} \times \frac{V}{v}$, or $\frac{AECv}{BecV} = \frac{A}{B} \times \frac{E}{e} \times \frac{C}{c} \times \frac{v}{V}$, or $\frac{AEcV}{BeCv} = \frac{A}{B} \times \frac{E}{e} \times \frac{c}{C} \times \frac{V}{v}$, or $\frac{AeCV}{BEcv} = \frac{A}{B} \times \frac{e}{E} \times \frac{C}{c} \times \frac{V}{v}$, or $\frac{BECV}{Aecv} = \frac{B}{A} \times \frac{E}{e} \times \frac{C}{c} \times \frac{V}{v}$, or $\frac{AEcv}{BeCV} = \frac{A}{B} \times \frac{E}{e} \times \frac{c}{C} \times \frac{v}{V}$, or $\frac{AecV}{BECv} = \frac{A}{B} \times \frac{e}{E} \times \frac{c}{C} \times \frac{V}{v}$, or $\frac{BeCV}{AEcv} = \frac{B}{A} \times \frac{e}{E} \times \frac{C}{c} \times \frac{V}{v}$, or $\frac{Aecv}{BECV} = \frac{A}{B} \times \frac{e}{E} \times \frac{c}{C} \times \frac{v}{V}$, or $\frac{BecV}{AECv} = \frac{B}{A} \times \frac{e}{E} \times \frac{c}{C} \times \frac{V}{v}$, or $\frac{Becv}{AECV} = \frac{B}{A} \times \frac{e}{E} \times \frac{c}{C} \times \frac{v}{V}$, or $\frac{BeCv}{AEcV} = \frac{B}{A} \times \frac{e}{E} \times \frac{C}{c} \times \frac{v}{V}$, or $\frac{BEcv}{AeCV} = \frac{B}{A} \times \frac{E}{e} \times \frac{c}{C} \times \frac{v}{V}$, or $\frac{BECv}{AecV} = \frac{B}{A} \times \frac{E}{e} \times \frac{C}{c} \times \frac{v}{V}$, &c. Each of theſe Equations is a diſtinct Theorem, and has a like Number of Corollaries with Theorem the Fifth; Which is a manifeſt Conviction of the wonderful Extenſiveneſs of this Doctrine.

And here again likewiſe M is not ſuppos'd Greater than m, but they are meant to Expreſs the Compoſition of Forces, in their Compariſon with one another, whatever ſuch Compoſition may be. Whenever we find the Quantities Inverted, the Theorems and Corollaries are much the ſame.

Theorem the Eighteenth is, That the Ratio of Spaces deſcrib'd by Bodies, which have an Expanſive and Contractive Force, may be differently compounded of the Expanſive and Contractive Forces, of their Velocities, of the Times, and the Denſities of Spaces.

§. 31. Theorem the Eighteenth is, That the Ratio of Spaces deſcrib'd by Bodies, which have an Expanſive and Contractive Force, may be differently compounded of the Expanſive and Contractive Forces, of their Velocities, of the Times, and the Denſities of Spaces; Each of which will obtain an Influence in Determining the Proportion of them: Thus, $\frac{S}{s} = \frac{ECVDT}{ecvdt}$, or $\frac{ECVDt}{ecvdT}$, or $\frac{ECVdT}{ecvDt}$, or $\frac{ECvDT}{ecVdt}$, or $\frac{EcVDT}{eCvdt}$, or $\frac{eCVDT}{Ecvdt}$, or $\frac{ECVdt}{ecvDT}$, or $\frac{ECvdT}{ecVDt}$, or $\frac{EcvDT}{eCVdt}$, or $\frac{ecVDT}{ECvdt}$, or $\frac{ECvdt}{ecVDT}$, or $\frac{EcvdT}{eCVDt}$, or $\frac{ecvDT}{ECVdt}$, or $\frac{eCVdt}{EcvDT}$, or $\frac{eCvdT}{EcVDt}$, or $\frac{eCvDt}{EcVdT}$, or $\frac{ecVdT}{ECvDt}$, or $\frac{ecvdT}{ECVDt}$, or $\frac{ecvDt}{ECVdT}$, or $\frac{Ecvdt}{eCVDT}$, or $\frac{ecvdt}{ECVDT}$, &c. That is, by all the Various Ways, by which theſe Forces, Times, Velocities, and Denſities can be poſſibly combin'd, there will be different Spaces, and S, s, will be greater or leſs, according to ſuch Combinations; each of theſe will alſo ſupply us with ſo many new Theorems, and the Corollaries conſequent upon them; and wherever we find the Quantities Inverted, the Theorems and Corollaries will be much the ſame.

We have thus far ſuppos'd theſe Intrinſick Forces, and the Quantities of them to be in Motion, and endu'd with a certain Velocity, to ſhew the preſent Mechanical Philoſophy, how Short and Defective it is in giving us an Account of the Real Motions of Bodies, and at the ſame time, to explain in ſome Sort, the Notion we have of the Expanſive and Contractive Forces, in Compariſon with Velocity, the Former of which we make to concur with Velocity, and the Latter to act Counter to it, and with Gravity; So that, As Impreſs'd Velocity is a Force directly contrary to Gravity, as is not only manifeſt, from projecting a Body Perpendicularly upwards, from the Horizon or Parallel to it, or in any other Inclination, but even Perpendicularly downwards, Since, by how much, it is accelerated by the Velocity impreſs'd, by ſo much it is hindred from producing it's own proper Effect; So Poſitive Reſt, which is oppoſite to Velocity, is Poſitive Gravitation; Suitable to theſe Forces of Velocity and Gravitation, which are Obſervable in Material Beings, which Gravitate and Move, are thoſe which I call the Intrinſick Expanſive, and Contractive Forces, in Bodies, Diſtinct from ſuch Velocity or Gravitation, by which in their own Nature with which they were originally endow'd, they obtain theſe two Qualities of Expanſion, or as much as poſſible dilating themſelves, and by which they ſhew a conſtant Conatus of Receding from the Centres of their Actions, whilſt, by another Force of Contraction, they are Compreſs'd and Condens'd towards them; And from a various Mixture of a greater or leſs Proportion of theſe Forces, all the Qualities of Bodies ariſe, as we ſhall ſhew in the Preſent and following Chapters, and that they wou'd preſerve theſe Forces, tho' they were ſet in the midſt of an Infinite Space, without any other Bodies, as the Earth or Sun to affect them; Inſomuch, that if the Earth, or any other Planet, were plac'd at an Immenſe Diſtance from all other Bodies, it wou'd remain fixt and immoveable, and if Put into Motion, it wou'd not continue that Motion in Infinitum, but wou'd return to it's Reſt by it's own Contractive Force, when that Impreſs'd Velocity was ſpent; as the Sun continues in it's Station by it's Expanſive.

SINCE therefore we reckon ſo much depends upon theſe Intrinſick Forces, in accounting for Nature, we ſhall now give the Theory of them ſeparate from Velocity, in Conjunction with which they were before conſider'd.

Theorem the Nineteenth is, That the Moments of Bodies, a-part from the Quantities or Velocities of Them, conſiſt of their Expanſive and Contractive Forces.

§. 32. THEOREM the Nineteenth is, That the Moments of Bodies, a-part from the Quantities or Velocities of them, conſiſt of their Expanſive and Contractive Forces, $\frac{M}{m} = \frac{EC}{ec}$, or $\frac{Ec}{eC}$, or $\frac{eC}{Ec}$, or $\frac{ec}{EC}$.

Caſ. 1. If $\frac{M}{m} = \frac{EC}{ec}$, i. e. If the Moments of Bodies are Equal to a Direct Reaſon of the Expanſive Force, to the Expanſive, and of the Contractive, to the Contractive, i. e. If they are as the greateſt Expanſive Force, into the greateſt Contractive Force, to the leaſt Expanſive Force, into the leaſt Contractive, it will probably be the Caſe of Gold, which has the greateſt Contractive by it's Weight, and the greateſt Expanſive by it's Colour, which comes the neareſt to that of the Suns, which has an entirely Expanſive Force, as that of Fire and Flame has, and of black Aſhes or Dirt, which has the leaſt Contractive and Expanſive, by it's Weight and it's Colour: And from this Theorem, ſeveral Years ago, I had an Opinion, that if to the Weight of Quick-Silver or Mercury, there cou'd be communicated by Fire the Expanſion of it, Gold might be made; But whilſt you endeavour to impart the Expanſiveneſs of Fire or Flame, to the Weight or Contractive Force of Mercury, you diſſipate the one without communicating the other; On the contrary, this is Effectually done in Nature by the Sun's, or the Subterraneous Heats continual Influence upon the ſame Mineral, where the Expanſive Force of the Former, is not ſo violent, as to diſſolve the Contractive of the Latter, but adds it's Expanſive to it.

I try'd, at that Time, to obviate this Difficulty, and, as it appear'd to me, by Rational Methods enough, namely, by encloſing the Mercury, ſo as it ſhou'd not, if poſſible, be able to evaporate, to wit, in a Strong, Thick, Iron Shell, and putting all thoſe Ingredients to it, which might, as I imagin'd, contribute to the Effect deſir'd, and which was plac'd in a large Fire, and made Red-Hot; upon opening the Iron Shell the Mercury had forc'd it's Way thro', but had left a ruddy Dew or Powder upon the outmoſt Sides of it; and I yet do not Queſtion, but Gold might be made, if we cou'd communicate Gradually the Expanſive Force of Fire to the Weight of Mercury, which by ſo many Degrees, as it was communicated, wou'd likewiſe by ſo many Degrees increaſe the Weight or Contractive Force of it, becauſe, as it ſeems, no Expanſive Force can prevail, the Contractive continuing, till it has made Room for it ſelf, that is, by condenſing the Contractive or Exciting it to Exert it's own Force in Oppoſition to it, and thereby Making it unite the Cloſer; and which in all Probability is the Reaſon, why ſeveral Heavy Bodies, as Lead, receive an Addition to their Weight by being Calcin'd, whilſt others, as Bones, Wood, Coal, *&c.* which are of a Lighter Nature, and have a more Expanſive Force find a Diminution of it, becauſe the Contractive is increas'd, in the former Caſe, by being Excited, and the Expanſive, not being able to Diſſipate it, lodges it ſelf in it, which gives the Red-Colour, which is again chang'd to a White by a Contractive Force, as Vinegar; and, in the latter, the Expanſive is increas'd by a Diſſolution of the Contractive, and an Addition to the Expanſive.

THE Experiment I have mention'd, I ſuppoſe, is ſomething Dangerous to be try'd, in Caſe the Shell ſhou'd burſt, by the Expanſion of the Quick-Silver, tho' I did not find any ſuch Effect; But if it were made by a more Gradual, and by a Quick and Conſtant Fire, as that of Charcoal, and not too Violent, or by a Sand-Heat, it is not Impoſſible but it might ſucceed better. As to the preſent Theorem, it extends to all other Bodies, which have intermediate Degrees of the Expanſive and Contractive Forces, betwixt the Greateſt and Leaſt, and which are in the Direct Proportion here aſſign'd of the Expanſive to the Expanſive, and of the Contractive to the Contractive.

I ſhall here add, what I find Written, many Years ago, in ſome Looſe Papers, but will not Vouch the Certainty of the Experiment, becauſe I have not ſince then try'd it, and perhaps there may be ſome Miſtake in it, but this I find, that on *Friday* the *16th* of *October*, the Year is not nam'd, I pour'd out two or three Drops of Quick-Silver into a Copper Veſſel, and laid upon it thrice the Quantity of Antimony Powder'd, then pour'd upon it ſome Lime-Water, with a little Lime, and cover'd it with ſmall Gravel Stones infus'd in Brine, and there ris' a Yellow Ductile Metal not unlike Gold; I alſo made ſeveral other Experiments, as I perceive by thoſe Papers, with the ſame Intention of Imparting the Expanſive Force of Fire, or Flame, or Heat to Quick-Silver, and of fixing, and increaſing it's Contractive, and tho' I did not Succeed, I had the Pleaſure of Thinking, it was not Impoſſible but I might, and little elſe, as I well Remember, ran in my Thoughts for ſome Time, but Building of Theaters, and Raiſing of Colleges and Chappels, and other Noble and Magnificent Structures, till the Golden Dream at length vaniſh'd, and left me juſt as Rich, as it found me.

Corol. 1. If $E=e$, $M.m::C.c$, i. e. The Moment of one Body will be to that of another, as the Contractive Force of the one is to that of the other, which is the Caſe of Heavy Bodies, and whoſe Expanſive or Contrary Force is Equal; Thus if Quick-Silver and Water have an Equal Degree of being thrown into Fumes or Expanſiveneſs, Velocity or Volatility, they will be as their Contractive Forces, which are Analogous to their Forces of poſitive Reſt or Gravitation; and the ſame may be ſaid of all other Bodies, where their Inclinations and Propenſities to Expanſion are Equal, and their Inclinations and Propenſities to poſitive Reſt or Gravitation different.

Corol. 2. If $C=c$, $M.m::E.e$, i. e. If the Contractive Forces are Equal, the Moments of Bodies will be in a Direct Reaſon of the Expanſive Forces; The Bodies, which have the leaſt Contractive Force, and the greateſt Expanſive, are thoſe of the Sun, and of Common Fire and Flame, which if their Contractive Forces are Equal, will be in a Direct Reaſon of their Expanſive, and one will be ſo much Hotter, and more Vivid and Strong in it's Action than the other, as it's Expanſive Force is greater; All lively and volatile Bodies, ſo far as they are ſo, come under this Corollary, as all Groſſer and more Heavy ones, ſo far as they are ſuch, under the Former.

Corol. 3. If $M=m$, $EC=ec$, and $E.e::c.C$, i. e. as the Expanſive Force of one Body is to that of the other, ſo the Contractive of the Latter, is to that of the Former, or a greater Expanſive Force into a leſſer Contractive, will be Equal to a leſſer Expanſive Force into a greater Contractive.

THUS Gun-Powder, which has a vaſt Expanſive Force, and but a ſmall Contractive one, is Equal in it's Moment to whole Tuns in Weight, which have a great Degree of the Contractive Force, and but a ſmall one of the Expanſive.

Caſ. 2. If $\frac{M}{m}=\frac{Ec}{eC}$, The Moments of Bodies, in Reſpect of their Intrinſick Forces, will be in a Direct one of the Expanſive, and a Reciprocal of the Contractive; i. e. $M.m::Ec.eC$, or the Moments of the Intrinſick Forces, in ſuch a Caſe, are as the greateſt Expanſive Force multiply'd into the leaſt Contractive, to the leaſt Expanſive into the greateſt Contractive, and ſo of any intermediate Degrees of theſe Forces, where they hold the ſame Reciprocal Proportion, that is, as the Moment of Fire to that of Lead, or of Air to Earth, *&c.*

Corol. 1. If $E=e$, $M.m::c.C$, i. e. The Moments of Bodies will be Reciprocally as their Contractive Forces, or a leſſer Moment into a greater Contractive Force, is Equal to a greater Moment into a leſs Contractive; Thus a leſſer Portion of Lead, will be Equal in it's Moment, to a greater of Iron.

Corol. 2. If $C=c$, $M.m::E.e$, i. e. The Moments will be in a Direct Reaſon of the Expanſive Forces, according to *Corol.* 2. *Caſ.* 1.

Corol. 3. If $M=m$, $Ec=eC$, or $E.e::C.c$, i. e. The Expanſive Force of the firſt Body, will be to the Expanſive of the Second, as the Contractive of the Second, to the Contractive of the Firſt, which may be the Caſe of Gun-Powder and Heavy Bodies, for if we multiply the greateſt Expanſive Force into the leaſt Contractive, it will be Equal to the greateſt Contractive into the leaſt Expanſive, which is the Caſe before, in *Corol.* 3. *Caſ.* 1. of Gun-Powder and Heavy Bodies.

Caſ. 3. If $\frac{M}{m} = \frac{eC}{Ec}$, The Moments will be in a Direct Reaſon of the Contractive Forces, and in a Reciprocal one of the Expanſive, or $M.m::eC.Ec$, which is that of Lead to Fire, or Earth to Air, *&c.* The Corollaries are much the ſame with the Former.

Caſ. 4. If $\frac{M}{m} = \frac{ec}{EC}$, The Moments will be in a Reciprocal Reaſon of the Expanſive and Contractive Forces, that is, as the leaſt Expanſive to the greateſt Expanſive, and the leaſt Contractive to the greateſt Contractive, which may be the Caſe of Black Dirt or Aſhes to Gold.

And the Corollaries are much the ſame with thoſe of *Caſ.* 1. ſo that all theſe Four Caſes are ultimately Reducible to the Firſt and Second, and Bodies, compar'd with one another, will be either thoſe which have a greater Expanſive or Contractive Force, in Reſpect of others, which have a leſs; or elſe thoſe, which have a greater Expanſive, and a leſs Contractive, in Reſpect of others, which have a leſs Expanſive, and a greater Contractive.

From the Various and Infinite Mixtures and Combinations of theſe Forces in differing Proportions, all the ſeveral Forms and Species of Material Beings ſeem to ariſe, and which will much better ſolve, as we ſhall afterwards ſhew, the apparent Properties and Qualities of Bodies, than the Imaginary Figures and Textures of Parts, introduc'd by the Corpuſcular Syſtem, for that Purpoſe and End; That there are ſuch Forces in Material Beings is very Evident, ſince ſome have a greater Diſpoſition to Motion, Velocity, and Expanſion, which I call the Expanſive Force, and others to Reſt, and to what is Contrary to Action or Motion, Velocity and Expanſion, which I call the Contractive: The firſt is ſeen in Fiery, Luminous, Volatile, Light, and Elaſtick Bodies; and the laſt in Groſs, Heavy, Viſcous, Torpid and Dull ones.

That theſe Forces likewiſe Exiſt together, and in Conjunction, is manifeſt from all the firſt Kind, having Gravity, and from all the laſt, having Colour, and a Capacity of being mov'd, which we ſhall farther Explain in the following Chapter; After having Premis'd, that we make all the World to be full of an Ethereal Matter, Quick, Subtle, and Elaſtick, which is capable of communicating the leaſt Impreſſions or Actions of Bodies to our Organs of Senſation, and by which our Souls, from the Mediation of thoſe Organs, become united to all the Beings which ſurround us.

That a Vacuum has not been prov'd by any Philoſophers, I have already evinc'd, from their ſuppoſing Matter to be Similar and Homogeneous, in Order to ſuch a Demonſtration; I have alſo ſhewn, that ſuch a Similar and Homogeneous Matter is a mere Hypotheſis of the Mind, and wholly Incapable of ſolving any one Appearance we can name, or any one Quality of that Matter, which it is aſſum'd and taken to Account for; On the contrary, we propoſe from theſe Intrinſick Forces in Matter, and from an Univerſal Action in all Nature, thro' the vaſt Expanſe of it, and which Forces and Action we can Evidence, muſt be deriv'd from an Infinite and All-wiſe Agent, we propoſe, I ſay, from them to Explain, the Various Phænomenons which occur to us, and which we are fully ſatisfied may be better and more Rationally done, than from fanciful and humorous Abſtractions of a void Space, of a mere Length, Breadth, and Thickneſs, and of an Hypothetick, Extended, Solid, Diviſible, and Moveable Matter, without one Real Force belonging to it.

In the foregoing Theorems, we have made the Compoſition of Forces to conſiſt in the Multiplication of Them into each Other, tho' Nature may poſſibly, in uniteing of various Forces together, draw out other Effects from ſuch Compoſitions, than what will reſult from a bare Multiplication of Them, to aſcertain which, the Genius and Temper of the Expanſive and Contractive are to be Inquir'd into, and what follows from their Mixture or Union.

CHAP.

CHAP. IV.

Concerning the Five Mechanical Powers, and the Demonſtrations of Them.

Gravity Inquir'd into, as it reſpects the Mechanical Philoſophy. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Theorem 9. Dr. Wallis de Motu, Cap. 1. Prop. 13. Part 1. Vol. 1. Sir Iſ. Newt. Princ. Def. 8. Lib. 1. & Prop. 7. Lib. 3.

§. 1. HAVING conſider'd Matter as an Unactive Being, and the Theory from thence of Motion, and the ſeveral Circumſtances, which enter into it, as Space, Time, and Celerity, we are now to Examine into another Property of Matter, namely, it's Gravity or Weight; the Laws of which this Philoſophy endeavours to eſtabliſh, and from them to account for the Mechanick Powers, which are imagin'd to be ſolv'd by them: The firſt Theorem in this Doctrine is, that the Weights of all ſenſible Bodies, near the Superficies of the Earth, are Proportional to the Quantities of Matter contain'd in them.

The Notion of Matters, being Proportional to it's Weight already confuted.

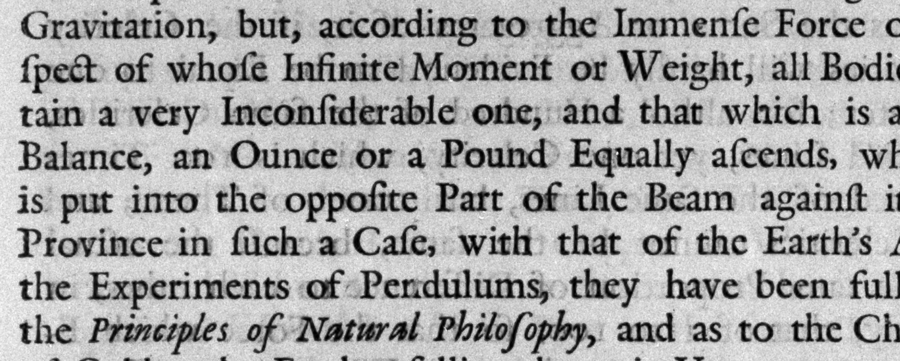
§. 2. We have abundantly, as we hope, confuted this Notion in our firſt Part of the Modern Philoſophy examin'd, and therefore it might be needleſs to give a particular Anſwer to it's being aſſerted here; however, ſince the only Reaſon aſſign'd for the Truth of this Theorem is, that the Bodies by the Force of their Gravitation falling downwards (if we take off or allow for the Reſiſtance of the Air) deſcribe Equal Spaces in the ſame Times, which is ſaid to be Evident from ſeveral Experiments made with Pendulums, ſince this is the only Reaſon aſſign'd, we ſhall not think much to conſider the Validity of it.

Why Matter is not really Proportional to it's Weight farther ſhewn.

§. 3. That therefore this is no good Argument is evident, becauſe near the Earth it is impoſſible Bodies ſhou'd Gravitate according to their own Intrinſick Force of Gravitation, but, according to the Immenſe Force of Attraction in the Earth, in Reſpect of whoſe Infinite Moment or Weight, all Bodies, which deſcend to it, muſt obtain a very Inconſiderable one, and that which is altogether Inſenſible; juſt as in a Balance, an Ounce or a Pound Equally aſcends, where a Million Weight or a Tun is put into the oppoſite Part of the Beam againſt it, and which performs the ſame Province in ſuch a Caſe, with that of the Earth's Attraction in the Former; As to the Experiments of Pendulums, they have been fully conſider'd in the firſt Part of the *Principles of Natural Philoſophy*, and as to the Chief Experiment, beſides, of a Piece of Gold, and a Feather falling down in Vacuo at once, it is not yet prov'd, that ſuch a Heavy Body, as Gold, and ſo Light a one, as a Feather, may not have the ſame Effect, when ſet together, as all other Bodies have, of being Obedient to, and following the Action of the Stronger; it is not yet ſhewn, that the Gravity of the Gold may not carry the Feather along with it, by it's Superior Influence and Action; or if not, it is not yet evinc'd, nor ever will be, that the Matter is of the ſame Sort in the Gold, as in the Feather, if it is not, Matter will not be proportional to it's Weight, but different Kinds of Matter will Gravitate alike, and therefore the ſame Kind of Matter will Gravitate differently, contrary to this Hypotheſis.

Problem the Firſt explain'd and examin'd. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10.

§. 4. The next Thing propos'd in this Philoſophy is, to find the Velocity, with which a Body given is to be mov'd, ſo that it may have a Moment, or Quantity of Motion, Equal to any Moment given. Let the Body given, be *A*, whoſe Moment, or whole Quantity of Motion, ought to be Equal to the Moment of *B*, mov'd with the Celerity *c*; let it therefore be, *A . B* :: the Celerity *c*, to another Celerity *C*, which will be the Velocity enquir'd or ſought, namely, that, with which, if *A* is mov'd, it's Moment will be Equal to the Moment of *B*, as is evident from what goes before; ſince the Moments of Bodies are Equal, if their Celerities are Reciprocally Proportional to thoſe Bodies; but by the Hypotheſis *c . C* :: *A . B*, from whence the Moment of *A* will be = to the Moment of *B*, or $CA = cB$, which was to be found.



§. 5. But

This Demonſtration proceeds upon Matter being conſider'd, as entirely unactive, and conſiſting only of mere Magnitude.

§. 5. But it is evident, that this Demonſtration proceeds upon Matter's being perfectly unactive, excepting as Motion or Celerity is impreſs'd upon it, that Matter, conſider'd in it's ſelf, is nothing but Bulk or Magnitude, without any other Properties belonging to it, and therefore, the Moment of a Body is no other than a Sum of Celerities, which are counted up and computed by the Number of Points in the Magnitude, and if ſo, we may throw Matter entirely out of the Queſtion, and only conſider a Globe of Space, according to whoſe Dimenſions we may aſcertain the Moment of the Celerities; for ſince this Philoſophy takes Matter to be nothing in it's ſelf, the Demonſtration will amount to this, that 100 Equal Celerities, for Inſtance, are the ſame as 1 Celerity, which is 100 Times greater than the Former, which abſtracted Axiom, ſuppoſing it to be one, can be of no more Importance, in the Explaining of Nature, than that of 100 Lines, which are plac'd Parallel to each other, as in the firſt Caſe, being = to 100 of the ſame Lines plac'd in Directum, as in the Latter, or that of 100 being Equal to it's ſelf; but can any one argue, that, becauſe 3 is = 3, or 100 to 100, in the Abſtract, therefore the Body or Corporeal Subſtance $A \times C$ the Celerity is Equal to B the other Material Subſtance $\times c$ the Celerity, which Matter and which Celerity are Real, and not Abſtracted, as they exiſt in Nature?

The Celerities not ſo fully explain'd by this Philoſophy as they ſhould be.

§. 6. We have in the laſt Section conſider'd the Notion, which this Philoſophy aſſerts of Matter, and which will as well agree to Abſtracted Spaces, or Abſtracted Numbers, and inſtead of Matter, we may as juſtly take either the one or the other, in order to aſcertain the Moment of the Celerities; let us now examine the Celerities themſelves, and we ſhall find it difficult to prove, that they are Reciprocal to their Sums or Bulks to make their Moments Equal; for if the Celerity is the ſame, the Moments of Celerity will be as the Sums or Aggregates of it, if the Celerity is different, 100 of the ſame Celerities will hardly be Evidenc'd to be Equal to one Celerity, which is 100 Times greater; for altho' a Hundred of the ſame Celerities, deſcribe a 100 certain and determin'd Lines, yet one Celerity, which is 100 Times greater, do's not deſcribe a Hundred of the ſame Lines, but 10000 of Them; and the Moment or Force of ſuch Celerities cannot be the ſame, becauſe the actual Force of each will always keep the ſame Proportion of Difference to each other, in a longer or a ſhorter Line, namely, that of 1 to 100, ſo that the Force which Equably deſcribes 100 Lines, will never amount to that 100 Times greater Force, which deſcribes 10000: Since the Progreſs of the ſame Motion cannot give any Addition to it, ſo as to change it into a Greater; that is, from 100 into 10000, no more than 1 can riſe to 100; that is, if A is 100, and B is 1, and it is as $A.B::c.C$, the Celerity of c will be 100 Times greater than that of C, and where C deſcribes 1 Space, c will deſcribe 100; if therefore, AC deſcribes 100 Spaces, and Bc 100 with a Celerity 100 Times as great, the Celerity, in every Point of the laſt Line, will be 100 Times greater than in every Point of the Firſt, and the laſt Line, or Force of which it is the Effect, will be to the Firſt, as 10000 to 100, or 100 to 1; If c and C are only Sums of the ſame Celerities, the Proportion will hold of $A.B::c.C$, that is, as the Sum of the Parts of A, to the Sum of the Parts of B, ſo the Sum of the ſame Celerities c, to thoſe of C; or if A and B have contranitent Forces to c and C, as thoſe of Gravity are to thoſe of Velocity, and $A.B::c.C$, AC will be $= Bc$, but this will not be by conſidering A and B as mere Magnitudes, but as conſiſting of certain Gravitating Forces.

Matter and Celerity to be determin'd before we affirm any Thing of the Moments of Bodies.

§. 7. Notwithstanding which, it is affirm'd from this Propoſition, that the ſmalleſt Body may have a Moment Equal to the Greateſt, which is mov'd with any given Velocity; but by what foregoes, that will not be certain, till the Notion of Matter and Celerity is a little better determin'd.

From this Principle the Five Mechanical Powers are explain'd by this Philoſophy.

§. 8. It is farther from this Principle, that all the Mechanical Forces are explain'd, which are made uſe of, or contriv'd to Draw or to Raiſe Heavy Bodies, namely, if the Machines are ſo adjuſted, that the Celerity of the Power is to the Celerity of the Weight, as the Weight is to the Power, or ſince, if $c.C::A.B$, $AC = Bc$, ſuppoſing C is the Celerity of the Power, and c is the Celerity of the Weight, or Gravity to be mov'd, P the Power, and G the Weight or Gravity, if $C.c::G.P$, $C \times P = c \times G$, and in ſuch a Caſe the Power will ſuſtain the Weight; or, which is the ſame, the whole Moment of the one will be Equal to the whole Moment of the other, and this is endeavour'd to be exemplified by the Five Mechanical Powers.

Mr. Deſaguliers. Philoſph. Tranſact. Numb. 375.

Which we ſhall conſider, after we have made one ſhort Digreſſion to what has been lately Aſſerted in Proof of this Doctrine; Namely, that the Quantity of the Force of moving Bodies is to be Eſtimated, by multiplying the Maſs of the Bodies by their Velocity, and not by the Square of it, as it ſeems, *John Bernoulli*, *Wolfius*, *Hermannus*, and *Polenus* of *Padua* have affirm'd, in Defence of Monſieur *Leibnitz*, who firſt advanc'd this Opinion, which we are far from Maintaining, and do believe it to be more Abſurd than what is Avow'd and Patroniz'd by the Preſent Mechanicks; that is, the Principles of Nature, which we contend for, do reject both as Chimerical, and which have only their Riſe in the Mind and Fancy of the Philoſopher; The Firſt Opinion, which we ſhall only think it worth our Time to Examine into, and which if not True, it is Impoſſible the other ſhou'd be, becauſe our Arguments equally obtain againſt Determining the Moments of Bodies, by multiplying the Maſs of them into their Velocities, or the Square of their Velocities, or any other Power, Degree, or Proportion of Them whatſoever, ſince we ſay their own Intrinſick Forces are to be taken into the Computation, The Firſt Opinion is thus Defended;

If a Man, it is ſaid, with a certain Force, can Move a Weight of Fifty Pounds, thro' a Space of Four Feet in a determinate Time, it is certain he muſt employ twice that Force to Move one Hundred Pounds Weight, thro' the ſame Space in the ſame Time; But if he uſes but the ſame Force, he will Move the one Hundred Pounds Weight, but Two Feet in the ſame Time; and hence we are told it appears, that the Force is Proportionable to the Maſs multiply'd into the Velocity: And is it not Plain, that Matter is conſider'd, as Similar and Homogeneous, in this Demonſtration, and that there is nothing Imagin'd in it but Gravity or Weight? On the contrary, ſuppoſing we do not take Different Weights of the ſame Matter, but Different Portions of Different Matters, as for Inſtance, of Lead and Copper, or Wood, will it be affirm'd, that, if a Man with a certain Force, can Move Fifty Parts of Lead, thro' a Space of Four Feet, in a determinate Time, it is Evident, he muſt employ twice that Force to Move a Hundred Parts of Copper or Wood, thro' the ſame Space, in the ſame Time, and if he uſes the ſame Force, that he will Move the one Hundred Parts of Copper or Wood, but Two Feet in the ſame Time? If it is affirm'd, all the Experience in the World contradicts the Affirmation; If not, what becomes of this Argument to prove, that the Moments of all Bodies are Eſtimated, by multiplying their Velocities into their Parts or their Maſſes? If it is ſaid, that the ultimate Parts of Lead are the ſame, and have the ſame Weight, as the ultimate Parts of Copper, or Wood, that is what we have ever deny'd, and of which we ſtill require a Proof, which I am aſſur'd neither This, nor any other Philoſophy can give.

Of the ſame Nature is the Argument made uſe of from the Balance, which ſhews, that if the Brachium or Arm, on one Side the Fulcrum, is one Fourth of That on the other, a Weight of one Hundred Pounds, ſuſpended at the Extremity of the Former, will be Equal to Twenty Five Pounds, ſuſpended at the Extremity of the Latter, becauſe the Velocity of the Latter, if it were to Move, wou'd be four Times Greater than the Velocity of the Former, and therefore, ſince 1 Velocity, into 100 Weight, is = to 4 Velocity, into 25 Weight, the Moments of Bodies are to be Computed, from Multiplying their Maſſes into their Velocities, and not into the Squares of Them, for, if ſo, the Weight 25 ſhou'd be ſuſpended at half the Diſtance, ſince then the Square of 2 = 4, multiply'd into 25, wou'd be = to 100.

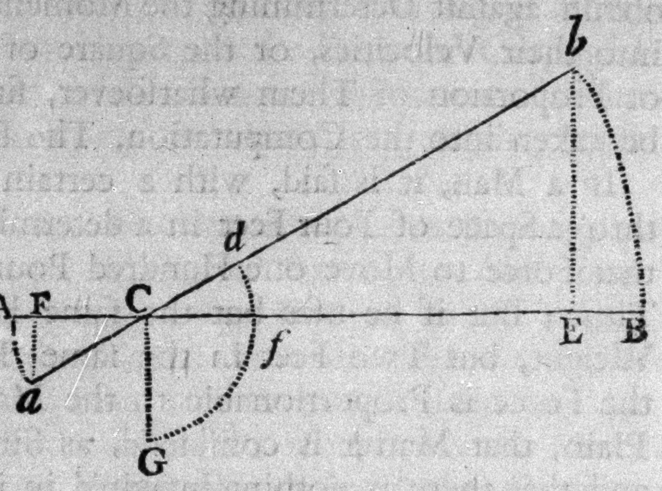
Let us allow this therefore to be a juſt Argument againſt *Leibnitz*, and his Followers, do's it yet Prove, what it wou'd Evince, that the Moments of Bodies are to be Eſtimated, by multiplying their Maſſes into their Velocities? Nothing leſs; it Proves it indeed, ſuppoſing Matter is Similar, and every Portion of it has the ſame Weight; but let us, as before, take Parts of Different Matter, and not the Weights of them, and the Fallacy will ſoon appear, For 100 Parts of Lead, ſuſpended at the Extremity of the ſhorter Arm of the Balance, will not be Equal, in their Moment, to 25 Parts of Copper or Wood, plac'd at the Extremity of the Longer.

Nor do the other Experiments of a Weight in a Scale at one End of the Beam, being rais'd by the Shock or Blow given by a Weight falling on the other, differ from what forego's, for if inſtead of Weights, we were to make uſe of Parts of

Different Matter, all this Reaſoning, as above, wou'd be of no Force to Prove, that the Moments of Bodies are to be Eſtimated from multiplying their Maſſes into their Velocities, but wou'd, with the Weight it alledges, fall to the Ground, ſince an Hundred, or a Thouſand Parts of Cork, or a Feather, will not by their Shock or their Blow raiſe 10 Parts of Lead, unleſs we again ſuppoſe, that the ultimate Parts of the one are the ſame, and have the ſame Gravity with thoſe of the other, which we humbly beg Leave once more to profeſs our Diſſent to, till we are ſo happy as to Meet with a farther Conviction of it; And now we ſhall Return to what we have many Years ago writ, and proceed to the Mechanical Powers.

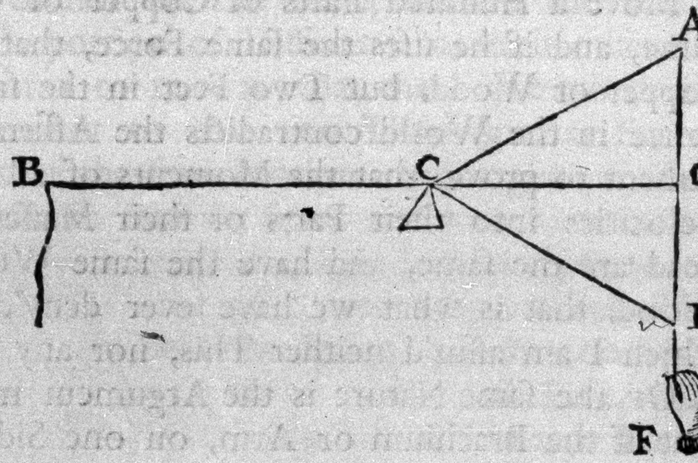
The Vectis or Firſt Mechanical Power, as explain'd by the Preſent Mechanicks. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Dr. Wallis de Motu, Cap. 6. Prop. 1, &c. Part 3. Vol. 1. Sir Iſ. Newt. Princip. Leg. Corol. 2. & 6. & Schol.

§. 9. The Firſt is that of the Vectis, and in Order to give an Account of it, AB is ſuppos'd to be ſuch a one, whoſe Fulcrum is C, on which it moves; if ſo, it is ſaid, that the Space, which is deſcrib'd by each Point in it's Revolving, will be as it's Diſtance from C.



Caſ. 1. For let the Vectis be mov'd from it's Situation ACB, to that of aCb, the Point A will deſcribe the Periphery Aa, and B the Periphery Bb, but the Sectors ACa, BCb, being Similar, it will be, as $Aa . Bb :: AC . BC$, that is, the Spaces deſcrib'd by the Points, A and B, will be as their Diſtances from the Fulcrum C.

Caſ. 2. If to the Points, A and B, the Powers are affixt ſo, as to draw the Arms of the Vectis perpendicularly, the Spaces deſcrib'd will be as aF, bE, and ſince the Triangles aCF, bCE are Similar, it will be as $aF . bE :: aC = AC . bC = BC$.



Caſ. 3. If the Direction of the Power is not a Right Line, which is Perpendicular to the Vectis, but is inclin'd, as AF to CA, draw CG Perpendicular to the Line of Direction, and the Space deſcrib'd by the Power, as in Caſe the 2*d*, will be proportional to the Diſtance from the Fulcrum C, for it is the ſame Thing, whether the Thread AG, by which the Power acts, is affixt to A, G, or D; from whence it follows, that in all theſe Three Caſes, the Space deſcrib'd by any Power, according to it's proper Direction, is Proportional to the Diſtance of the Line of Direction from the Fulcrum.

All that is prov'd by this Demonſtration is only, that Different Radius's, drawn from the ſame Center, will deſcribe Similar Archs at the ſame Time.

§. 10. All therefore, which is Demonſtrated in this Theorem, is only this, that if the Line ACB, *Fig.* 1, is ſuppos'd to be mov'd on it's Center C, A will arrive at a, at the ſame time, that B comes to b, which is no more than to ſay, that in the Line CfB, where Cf is Equal to AC, f will come to d, at the ſame time, as B will to b, that is, in Concentrical Circles, Similar Archs of them will be perform'd in the ſame Time, and be in the ſame Proportion with the Radius's of them; for ſince the Motion of the Line ACB is only conſider'd, and what the Conſequence of ſuch a Motion will be, all that we can ſay in General is, that whether on this Side the Center C, or the other, Aa or df will have the ſame Proportion to Bb, as AC or Cf to CB, but what it will ſignify to explain the Mechanical Power of the Vectis, to ſay df is to Bb, as Cf to CB, we cannot readily apprehend; for CA is no more determin'd to deſcend, than Cf to riſe, that is, neither the one nor the other wou'd do it, excepting by ſuppoſing it, and the Line ACB will in Reality keep it's Situation, for all that has been yet Demonſtrated in this Theorem.

No Motion one way or other, or any Inclination to Motion prov'd.

§. 11. We conſequently acknowledge, that if the Line ACB is mov'd round it's Center C, it will deſcribe Aa, df, and Bb, at the ſame Time, for the like Reaſon, that in deſcribing a Circle, all the Concentrical Peripheries will be perform'd by the ſame Motion of the Radius; But which way are we from thence made to underſtand any more, how *Cb* ſhou'd actually, and not in Suppoſition only deſcend, for inſtance to *CB*, at the ſame Time, that *Ca* riſes to *CA*, than how *Cdb* ſhou'd deſcend to *CfB* in fact, by ſuppoſing an imaginary Motion only, which preſumes, in the Deſcription of Concentrical Circles, that *CfB* may aſcend, or *Cdb* deſcend, and therefore, that there is no real Inclination to move one way more than another?

§. 12. Besides, if *ACB* is an Abſtracted and Mathematick Line, which has no Weight, nor no Dimenſions, except that of Length, *CB* will weigh no more than *AC*, and therefore, notwithſtanding this Geometrick Reaſoning, *ACB* will continue in the ſame Poſture, without any Propenſity to Move one way or other, if it is Mov'd by Suppoſition, there can be nothing more collected from thence than that Concentrical Circles are deſcrib'd in the ſame Time, and that the Celerities of their Deſcription, or their Spaces, are Proportional to their Diſtances from their Center; but this is no Explanation how the Power and the Weight perform ſuch Spaces, ſince, altho' *Aa*.*Bb*::*AC*.*BC* in this Figure, where *B* and *A* are indiviſible Points, and *AC* and *BC* Geometrick Lines, yet, other Things being the ſame, as in the preſent Scheme, if two Bodies of Equal Matter and Bulk are affixt to *B*, and *A*, theſe Spaces will not be deſcrib'd by them, becauſe *B* and *A* being Equal, and the Scheme continuing the ſame, there will no real Motion enſue; and whatever Motion is introduc'd, muſt be, as before, by Suppoſition, unleſs *B* and *A*, when conſider'd as Equal Points, are different from *B* and *A*, when conſider'd as Equal Congeries of the ſame kind of Points, which is impoſſible, for all Equal Parts, equally multiply'd, will have the ſame Relation to each other, as their Simples.

If two Equal Magnitudes are affixt to the Extremities of the Vectis, conſider'd only as a Geometrick Line, nothing will from thence enſue.

§. 13. If the Parcels of Matter affixt to the Extremities of the Vectis are Unequal, ſo are likewiſe the Lines *AC*, *CB*, which yet will continue in the Poſition *ACB*, unleſs they are ſuppos'd to be mov'd; if therefore *B* and *A* (which, tho' Portions of Matter, are nothing more than Portions of Abſtracted Space in this Philoſophy, by §. 5.) are unequal Portions of Space (as *AC* and *CB* are), there can be no Reaſon aſſign'd, why ſuch unequal Portions of Space ſhou'd any more cauſe a Motion in the Vectis, than why *AC* and *CB* ſhou'd, which are unequal Portions of Abſtracted Space, and therefore *ACB* notwithſtanding theſe Appendages of Abſtracted Matter, at the Extremities of the Vectis, will not be put into Motion, without a Suppoſition of it.

If two Unequal Magnitudes are affixt to this Geometrick Line, nothing will be conſequent from thence.

§. 14. Farther, if *B* is of the ſame Sort of Matter with *A*, and it's Bulk is Reciprocal to it's Diſtance from *C*, with that of *A* from *C*, for the ſame Reaſon, as in the former Section, *ACB*, with theſe Reciprocal Appendages of their Diſtance from *C*, will yet ſtill be unable to move out of the Situation *ACB*, to that of *aCb*, without a Suppoſition of Motion to aſſiſt it's Inactivity, for if Unactive and an Abſtracted Matter is affixt to an Abſtracted and Unactive Line, it is impoſſible any Action ſhou'd reſult from it.

If the Magnitudes are Reciprocal to the Diſtances from the Center, all will continue without any Motion or Inclination to it.

§. 15. If, moreover, *B* is not of the ſame kind of Matter with *A*, and they have different Forces, it is a Contradiction to the fundamental Hypotheſis of this Philoſophy, which ſuppoſes all Matter to be the ſame; and if ſuch an Hypotheſis is not true, as we have endeavour'd to prove it is not, The Motion of *A* and *B*, as it is propos'd in this Philoſophy, will be much leſs aſſerted by the preſent Theorem than before, as will be evident, if Lead is plac'd at one End of the Vectis, and Fire or Light, or even an Equal Portion of Wood or Cork, at the other; But this Argument we have urg'd more at Large, §. 8.

If Matter has Forces, and they are different in themſelves, this Theorem will not be true.

§. 16. Supposing yet all Matter to be the ſame, and ſome kind of Action, as Gravity, is introduc'd into it, which we will likewiſe preſume Proportionable to the Matter which Gravitates, the contrary to which we have notwithſtanding prov'd, both in this Part of our *Principles of Philoſophy*, and more particularly in the Firſt; let now the Gravitation of *A* be Equal to that of *B*, if ſo, for ought, that has been hitherto Demonſtrated, it do's not appear, that *ACB* will change it's Situation by this Acceſſion of a perpendicular Action or Gravity, for, if the Abſtracted Diſtance of *CB* and *CA* can ſignify nothing to make *B* or *A* incline one way or the other, as has been before ſhewn, the Addition of Equal Weights cannot be of any great Importance to the Depreſſing the one Part of the Vectis, or Raiſing the other.

If Matter is the ſame, and has a Force of Gravitation Proportional to the Matter which Gravitates, this Theorem not prov'd.

If the Portions of Similar and Gravitating Matter are Reciprocal to their Diſtance from the Center, they are not prov'd by this Theorem to be in an Equal Poiſe.

§. 17. For the Reaſon, which is alledg'd in the foregoing Section, if the Weight of *A* is Greater than that of *B*, or the contrary, and they are in a Reciprocal Proportion to their Diſtances from the Fulcrum, it will not be neceſſary, that the Moment of *A* ſhou'd be Equal to that of *B*, becauſe, if, where *A* and *B* are Equal Weights, or have an Equal Force of Gravitation, the Line or Vectis *ACB* is unmov'd, it will follow, that where *A* and *B* are unequal Weights, and in a Reciprocal Proportion to their Diſtances from the Fulcrum, they will not be Equally pois'd, ſince the Diſtances *CA* and *CB* have no Influence or Moment (as they are Abſtracted Lines) in the aſſiſting the Gravitation of the one Body or the other.

If the Line ACB *is not a Geometrick but a Gravitating Line, nothing can be concluded from this Abſtracted Reaſoning.*

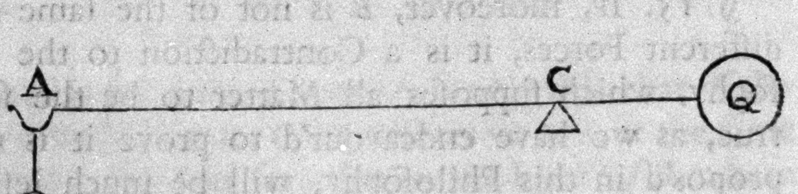
§. 18. Lastly, if we do not conſider the Diſtances, *AC* and *BC*, from the Fulcrum, as Abſtracted Lines, but as one Gravitating continued Line, the Caſe will be ſtill more perplext in Reſpects of the Weights, *A* and *B*, which have been hitherto imagin'd to exert their Gravitation or Action at the Extremities of the Abſtracted Line *ACB*, ſince, if now the Abſtracted Line *ACB* is ſuppos'd to conſiſt of Real Matter, which Gravitates, all the Parts of it will be as ſo many Gravitating Bodies, *A* and *B*, in their ſeveral Diſtances from the Fulcrum, and therefore, in this Abſtracted Reaſoning, will be of no greater Significance to the Weighing down the one Part of the Vectis than the other.

What ought to have been Demonſtrated in the foregoing Theorem.

§. 19. In fine, what is wanting in the whole Demonſtration is to ſhew, that ſuppoſing *C*, *f*, *E*, and *B*, to be four Gravitating Points, acting Downwards in a Perpendicular, with an Equal Force of Gravitation, and yet detain'd by the Point *C* from Receding from thence, what the Spaces will be which they deſcribe, which at *C* can be none, at *f*, *E*, and *B*, will be ſo many Quadrants, made by the Radius's *Cf*, *CE*, and *CB*, which will each of them Coincide with *CG* at the ſame Time, upon this Suppoſition, that at *f*, *E*, and *B*, the Celerities will be Greater, the farther thoſe Points are from the Center of their Detention *C*, tho' their Perpendicular Gravities are ſuppos'd to be the ſame, which is nothing more than a Reaſonable Preſumption, if it is not an Axiom; becauſe, if, at *C*, the Gravitation is entirely hindred from Exerting it's Action into the Deſcribing any Space, and only preſſes in a Point, the Action of a Similar Gravity, which is here ſuppos'd, will ſuitably obtain it's own Force in Proportion more fully, and ſtrongly, the farther it is remov'd from that Point, where it's Effect in Celerity was Equal to nothing, that is, was wholly Obſtructed; and by Conſequence ſuch an Equal Action, as that of Gravitation, will be more forcible, and be carried with a greater Celerity in a greater Diſtance, from the Center of it's Detention, than in a Leſſer; and if ſo, one Gravitating Point at *B*, may be Equal in it's Force to Ten at *A*, or *f*; but of This we ſhall ſay more at the End of this Philoſophy of the Mechanical Powers.

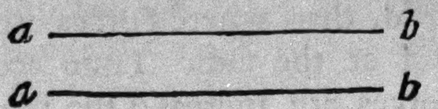
Theorem the Eleventh explain'd; That, in a Vectis, which is in an Æquilibrium, The Power is to the Weight, as the Diſtance of the Weight from the Fulcrum, to the Diſtance of the Power from it. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Dr. Wallis de Motu, Cap. 6. Prop. 1, &c. Part 3. Vol. 1. Sir Iſ. Newt. Leg. Corol. 2. & 6. & Schol.

§. 20. The next Theorem, which is founded upon the foregoing Principles, is, that in a Vectis, the Vis Motrix, or Power, as *P*, which has the ſame Proportion to the Weight *Q*, which the Diſtance of the Line of Direction of the Weight from the Fulcrum, namely, *CQ*, has to the Diſtance of the Direction of the Power from the Fulcrum, to wit, *CA*, will ſuſtain the Weight; that is, if $P . Q :: CQ . CA$, $Q \times CQ$ will be $= P \times CA$, or the Power of *P*, in ſuch a Circumſtance of it's Diſtance from *C*, will be Equal to the Weight *Q*, conſider'd at the Diſtance of *CQ*; becauſe the Ratio of the Moments of any Body is compounded of the Ratio's of the Quantities of Matter, and of the Celerities, that is for Example, 3 Portions of Diſtance into 4 of Matter are Equal to 3 of Matter into 4 of Diſtance, and in the preſent Caſe, ſince the Celerities are Proportional to the Diſtances, 3 Portions of Celerity into 4 of Matter are Equal to 3 of Matter into 4 of Celerity, that is, 1 Portion of Celerity, and, by Conſequence, of Diſtance in the Vectis, is always to be computed to have the ſame Force as 1 Portion of Matter, and ſo on the Contrary, or, let *C* be the Celerity of the Power, *c* of the Weight, *P* the Power, *G* the Gravity or Weight, it will be, $P . G :: c . C$, and $PC = Gc$.



Three Anſwers to the forego-ing Abſtracted Reaſoning.

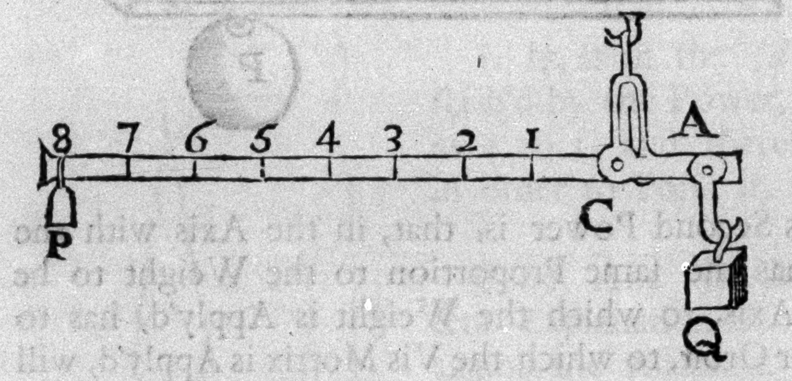
§. 21. In Anſwer to which we may ſay, that Firſt, according to what we have already ſhewn, it is not neceſſary to ſuppoſe Matter alike, and to be no other than a Congeries of ſo many Points of Abſtracted Space or Number, which ſerve only to Count up the particular Ingredient Celerities, that make up the whole Moment or Quantity of Motion: Second, That if Matter was this Similar Being, which ſhou'd only conſiſt of ſo many Points, by which we ſhou'd reckon the Sum of the whole Motion, yet it is not Evident that a Sum of the ſame kind of Celerities, is Equal to any Sum of Celerities, which are of a different Nature and Activity, as ſuppoſing, *Q* is 4, and *P* is 3, and the Celerity of *Q* is 3, and that of *P* is 4, it is no more prov'd, that theſe Moments are Equal, than that a double Celerity has the ſame Force, that two Equable ones have; thus let Two Points, *a*, *a*, move in the ſame Time to *b*, *b*, the Force or Activity of ſuch a Celerity will be expreſs'd by either of theſe Lines, which will never riſe to the Force of a double Celerity, which deſcribes *a b* in half the Time, or twice *a b* in the ſame Time, ſince *a*, *a*, moving together in the ſame Time, is, as if *a* ſhou'd move twice the Length of *a b* in double the Time, but *a* moving, where the Celerity is double, thro' *a b* in half the Time, or thro' twice *a b* in the ſame Time, cannot be Equal to *a* moving thro' *a b* in the ſame Time, or twice *a b* in double the Time: Third, If Matter was the Abſtracted Being, which it is repreſented to us, and Unequal Celerities had Equal Forces, or Moments with the Sums of the ſame Celerities, yet, by what has been ſaid concerning the Vectis, it wou'd not be Equally pois'd by ſuch a Reciprocal Matter and Celerity, any more than it wou'd by an Equal Matter affixt at both the Extremities of it, which we have Evidenc'd, §. 16 and 17, will be as entirely in an Æquilibrium, as any Reciprocal Matters or Portions of it, which we can name, and therefore deſtroys this Reaſoning, which ſuppoſes the Reciprocal Diſtances neceſſary to the Equal Balance; For it is ſhewn in thoſe Sections, and in others Preceding, that there will be no Motion of *A C Q* in any Caſe, unleſs upon a Suppoſition of it; that is, to make theſe Celerities Reciprocal, the Vectis is put into Motion by the Mind, which Motion is not prov'd or deduc'd from any Principles alledg'd in this Philoſophy.



The Roman Balance made uſe of to confirm this Theorem.

Dr. Keill's Intr. ad ver. Phyſ. Lect. 10.
Dr. Wallis de Motu, Cap. 3. Prop. 25. Part 1. Vol. 1.
The Counterpoiſe like a Pomgranate, Hebr. *Rimmon*, Arab. *Romman*, and us'd in *Turky*, as here.

§. 22. 'Tis true the Roman, or according to Dr. *Wallis* the *Romman*, Balance is made uſe of, as an Inſtance, to juſtify this Reaſoning, where *C* is the Fulcrum, *C A* the Diſtance of the Pondus to be examin'd, *P C* is the other Part of the Beam, which is diſtinguiſh'd into 8 ſeveral Portions or more, as there ſhall be occaſion, *P* is a Weight (ſuppoſing a Pound) already known, if therefore at *P*, where the Beam is 8 Times longer than *C A*, *P* is in an Equal Balance with *Q*, *Q* is 8 Pounds, as *P* is 1, that is, the Weights are in a Reciprocal Proportion of their Diſtances from the Fulcrum *C*.



Not Accounted for by this Theorem.

§. 23. On the contrary, this Example of the Roman Balance is ſo far from confirming the former Reaſoning, that the former Reaſoning ſeems to be contriv'd to Account for this Balance, the Principles of which, ſince they are not Capable of Explaining the Propoſitions, which are deſign'd to be Preparatory to it's Solution, cannot be any ways ſignificant to an Expoſition of it.

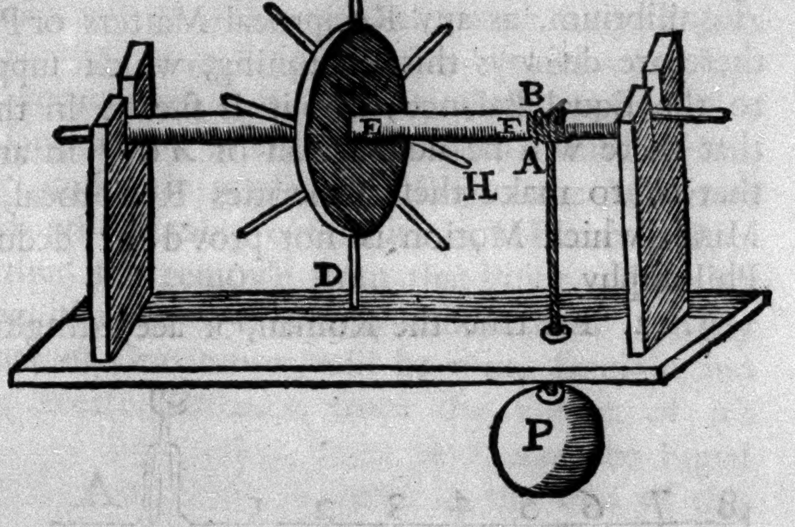
Not Accounted for by this Theorem.

§. 24. Besides which, if all, that has been hitherto confuted, was true in Reſpect of the Balance not moving at all, unleſs by Suppoſition, it wou'd not prove, that *P* and *Q*, in the preſent Circumſtance of their Diſtances from the Fulcrum being Reciprocal to their Weights, wou'd be in an Equal Poiſe with each other: Firſt, Becauſe the Weights of the Bodies *P*, *Q*, are here ſubſtituted inſtead of the Celerities, whereas it is not manifeſt, from this Philoſophy, that Forces, which Equally deſcend, and are only diſtinguiſh'd by the different Sums or Aggregates of them, are Anſwerable or Analogous to different Forces of Celerity, according to Argument the ſecond, §. 21; Second, Becauſe, if that was allow'd, the Point *Q*, and that at *P*, wou'd be in an Equal Poiſe without any Appendage of Reciprocal Portions of Matter to their Diſtance from the Fulcrum; ſince if *Q* moves with 1 Celerity, and *P* moves with 8, and ſince at the ſame Time, that *Q* moves with it's Celerity about the Fulcrum *C*, *P* will obtain it's own Celerity likewiſe, it will be impoſſible for *Q* or *P* to Preponderate, from whence it follows, that the Point *Q* and the Point *P*, without the Addition of the Weight *Q* on the one Hand, or *P* on the other will be in an Æquilibrium; if therefore, the Weight *Q* is added to one Extremity, and *P* at the other, the firſt of which is ſuppos'd 8 Times Greater, in it's Force or Gravitation, than the laſt, the Preponderancy will be on the Side of *Q*, according to this Reaſoning, which, by Experiment, do's not Preponderate at all.

FROM all which it is manifeſt, that, inſtead of proceeding in this Abſtracted Way, the Preſent Philoſophy, according to §. 19, ſhou'd have prov'd, that, where Forces are ſuppos'd to Gravitate Equally, and in a Perpendicular, and at the ſame Time are detain'd ſo, as not to recede from the Fulcrum *C*, thoſe Forces are increas'd the farther they are remov'd from that Fulcrum, or the Centre of their Detention, and in Proportion to their Removal from it; which will ſolve the Properties of the Vectis, and the Roman Balance, and will anſwer the Difficulties, which I have rais'd againſt the Preſent Reaſoning, becauſe, in ſuch a Caſe, *AP* is not conſider'd as an Abſtracted Line, and the Celerities are not conſider'd as Abſtracted ones, but Different in themſelves, and ſuch as may Balance a contrary Force of Gravitation oppos'd to them, for by that Means *P* will have a Force of Percuſſion or Celerity Impreſs'd upon it, which will be Equal to the Gravitating Force in *Q*.

The Axis in Peritrochio deſcrib'd, which is the ſecond Mechanical Power.

§. 25. THE next Mechanical Power after the Vectis, which we are to enquire into, is the Axis in Peritrochio, the Axis is the Cylinder ſupported by two Fulcrums at the Extremities of it, and the Peritrochium is the Circular Body affixt to the Axis, to which ſeveral Scytalæ, or Handles, are ſuppos'd to be join'd, and by a Force apply'd to one of theſe Scytalæ, the Pondus *P* is to be rais'd. This is the Deſcription of the preſent Mechanical Power, and the Theorem, which is form'd upon it, we ſhall explain in the following Section.



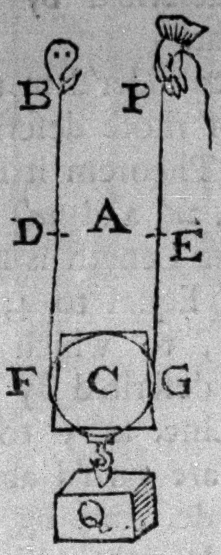
The Theorem form'd upon this Mechanical Power; That the Power is to the Weight, as the Perimeter of the Axis to the Ambitus of the Scytala. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Theorem 12. Dr. Wallis de Motu, Cap. 7. Part 3. Vol. 1. Sir Iſ. Newt. Princip. Leg. Corol. 2. & 6. & Schol.

§. 26. THE Theorem to Explain this Second Power is, that, in the Axis with the Peritrochium, the Vis Motrix, which has the ſame Proportion to the Weight to be ſuſtain'd, which the Perimeter of the Axis, to which the Weight is Apply'd, has to the Perimeter of the outmoſt Ambitus or Orbit, to which the Vis Motrix is Apply'd, will be Equal to the Weight; if therefore ſuch a Vis Motrix is never ſo little augmented, it will be ſufficient to raiſe the Weight, to which it was before Equal; For the Weight *P* is drawn up in Proportion, as the Line, from which it depends, wraps about the Axis, and therefore in one Revolution of the Axis, the Weight will be anſwerably rais'd the Length of the Axis's Ambitus, and at the ſame Time the Power, or Vis Motrix, apply'd to the Scytala, or Handle of the Peritrochium, will deſcribe the Circumference made by the entire Revolution of it, that is, the outmoſt Orbit, or Ambitus perform'd by the Vis Motrix; from whence it follows, that the Celerities of the Power, and the Weight, which are as the Spaces they deſcribe, will be as the Perimeter of the outmoſt Orbit to the Perimeter of the Axis, wherefore, if the Weight is to the Power, as the Perimeter of the outmoſt Orbit to the Perimeter of the Axis, the Celerity of the Power will be to that of the Weight Reciprocally, as the Weight to the Power, which, by what foregoes, is the Condition requiſite to make the Power and the Weight in an Equal Balance; that is, let *C* be the Celerity of the Power, *c* of the Weight, *P* the Power, and *G* the Gravity or Weight, it will be as $P . G :: c . C$, and $PC = Gc$.

The Axis with the Peritrochium exactly

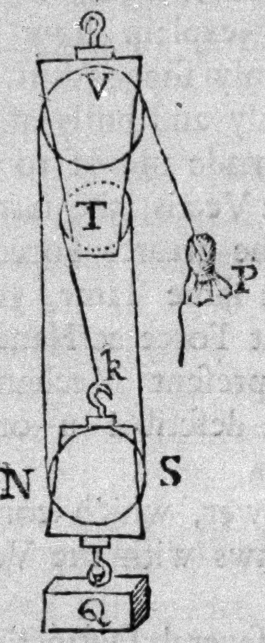
§. 27. IF we conſider the Circumferences of the Axis *AB*, and of the Scytalæ *CD* to be drawn from the ſame Center *E* or *F*, and whilſt *C* moves from thence to *H* and *D*, we imagine *A* to deſcribe an Oppoſite Arch, from thence to *B*, the Caſe will be exactly the ſame, as that of *Fig.* 1. §. 9. where *CB* will repreſent *EC* in the Figure before us, and *AC* that of *AF*, ſo that there is nothing more to be prov'd in this Mechanical Power than that any equally Gravitating Point has a greater Force in Proportion to it's Removal from the Center of it's Detention.

the ſame with the Vectis and the Roman Ballance, and therefore admits of the ſame Anſwer.

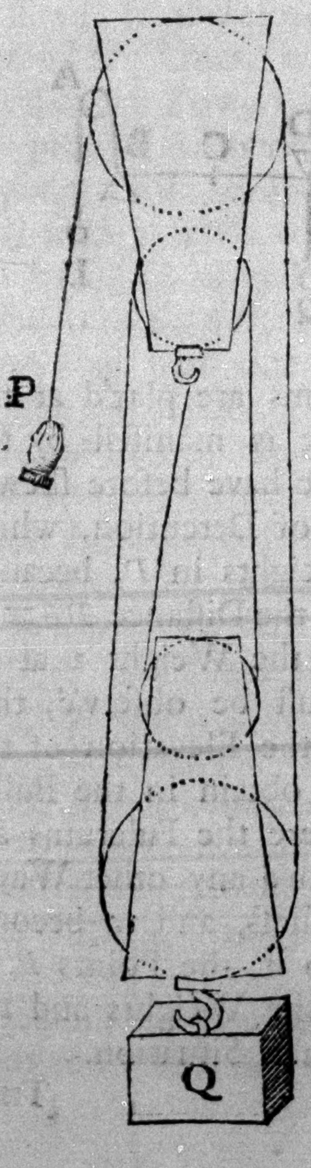


§. 28. The Third Mechanical Power is that of the Trochlea, Polyſpaſt, or Pully, which is accounted for by the following Theorem, That the Force, which is to the Weight, as 1 to the Number of Lines by which the Weight is ſuſpended, will be ſufficient to bear up that Weight, and if increas'd, will ſurmount it; Let there be a Line, one of whoſe Extremities is affixt to the Uncus *B*, in the Fold or Doubling of which let the Pully be plac'd with the Weight annext to it, and the other Extremity to *P*; in order to raiſe the Weight *Q* one Foot from *C* to *A*, there will be two Foot of the Lines *BF*, *PG* carry'd upwards at the ſame Time, namely, from *F* to *D*, and from *G* to *E*, ſuppoſing the Diameter of the Pully *FCG* to be inconſiderable; or, which is the ſame, the Uncus remaining fixt, whilſt *P* is mov'd upwards, to cauſe *C* to aſcend to *D*, *D* and *C* will be Coincident, ſo that *DCE* or *DFGE* will have mov'd twice as far as *CA*, that is, by that Time *C* arrives at *A*, *D* will come to *E*, and *E* will riſe as far above *P*, as *G* is below *E*, in both which Caſes, the Line of the Force or Power will deſcribe Double the Space of what the Weight do's; from whence it follows, that the Celerity of the Power will be Double the Celerity of the Weight, and conſequently, if the Power is to the Weight, as 1 to 2, it's Moment will be Equal to that of the Weight, and therefore will ſuſtain it.

The Third Mechanical Power explain'd, which is That of the Trochlea or Pully, and the Theorem, that the Power or Force is to the Weight, as 1 to the Number of Lines, by which the Weight is ſuſpended. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Dr. Wallis de Motu, Cap. 8. Part 3. Vol. 1. Sir Iſ. Newt. Princ. Leg. Corol. 2. & 6. & Schol.



2. If, as in the 2*d* Figure, Three Lines are us'd to raiſe the Weight one Foot, the Space, which the Power deſcribes, will be to that, which the Weight do's, as 3 to 1, and therefore the Power ought to be to the Weight, as 1 to 3.



3. If, as in the 3*d* Figure, Four Lines are us'd, the Space deſcrib'd by the Power, to that deſcrib'd by the Weight, will be as 4 to 1, and therefore the Power muſt be to the Weight in order to Poiſe it, as 1 to 4, and ſo on in Infinitum.

§. 29. All the Difficulties before alledg'd againſt the Preſent Explanation of the Mechanical Powers, are alike Valid againſt the Account, which is given of this, namely, that the Matter to be rais'd is conſider'd as Similar and Homogeneous, that Gravity is likewiſe the ſame in all Bodies, *&c.* What we have to ſay here, in particular, is, that the Center of Detention is *B*, *Fig.* 1. of the foregoing Section, and if *P* is ſuppos'd to be a fixt Point, it will be another Center of Detention likewiſe, which will together ſuſtain the Body *Q*, whether the Lines are Longer or Shorter, or whether *Q* has a Greater or Leſs Weight; and let the Line or Space, which *P* is ſuppos'd to deſcribe to Balance the Weight, be what it will, it ſignifies nothing to the Support of it, if *P* is not ſuppos'd to be a Center of Detention, and if the Power lies in the Space to be deſcrib'd, without an Actual Force in *P*, Equal to that in *B*, to ſupport the deſcending Body, ſince it will certainly deſcend, notwithſtanding any Imaginary Lines or Spaces, which are deſcrib'd in the Fancy to ſuſtain it; For a double Line, which is to be deſcrib'd by the Power in *P*, can never ſupport a double Weight in *Q*, it muſt be an actual Force in *P*, which, together with the Uncus *B*, will be Equivalent to the Weight in

The Objections which lye againſt the Explanation of this Third Mechanical Power.

in *Q*, and not a Space to be deſcrib'd by *P*, which will be capable of obtaining that Effect; the Trochlea therefore in this Scheme is nothing elſe but a Vectis, which has a Fulcrum at each Extremity, with this Difference, that the Trochlea has it's Diſtances from the Fulcrums Perpendicular to the Horizon, whereas thoſe of the Vectis are Parallel to it. Moreover $DFGE$ are not Lines Double of AC, and, whilſt *C* moves to *A*, there will be more than Double that Space deſcrib'd by the Line round the Pully, in order to Elevate the Weight to that Height.

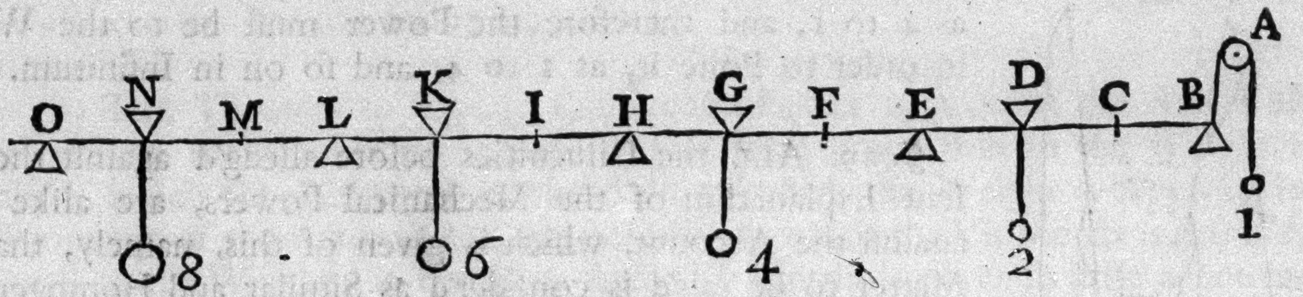
Other Objections againſt this Explanation.

§. 30. We muſt farther obſerve, that the Length of the Lines is conſider'd in this Explanation, to make the Spaces deſcrib'd by the Power, Double of thoſe deſcrib'd by the Weight, that is, AC is half of $DF+GE$, *Fig.* 1, and yet in the Theorem it ſelf, the Number of Lines is only conſider'd; on the other Hand, in *Fig. 2d*, and 3*d*. the Length of the Lines is not conſider'd, nor the Number of them; the Length is not, unleſs VN is Equal to $ST=TR$, nor the Number, if 3 is not Equal to 4, or $NV+ST+TR=$ to $NV+ST+TR+VP$, for, in the 2*d* Figure, to which we Refer, the Power is to the Weight, as 1 to 3, becauſe the Spaces, deſcrib'd by the 3 Lines of the Power, are to the Elevation of the Weight in one Line, as 3 to 1, when the Lines are neither Equal, nor have an Equal Direction, nor are turn'd about Equal Pullies, all which are at leaſt neceſſary to make the Spaces deſcrib'd by the Power 3 Times as much, and no more, than that deſcrib'd by the Weight; If this yet were true, ſince VP is alſo added, it will be Impoſſible to explain how the Action of 4 Lines, ſhou'd have the ſame Effect as 3, which are only ſuppos'd in deſcribing 3 Times the Space, which 1 Line do's, unleſs we can truly and juſtly affirm that 3 is Equal to 4; and it is manifeſt there are only 2 Lines made uſe of to deſcribe a double Space in *Fig.* 1. But we ſtill Inſiſt, that tho' in the Vectis, the farther any Gravitating Point is Diſtant from the Center of Detention, the greater Force of Velocity it has, and therefore will deſcribe a greater Space in the ſame Time, yet a greater Velocity to be deſcrib'd, is no Argument, that there is that Force at Hand to deſcribe it, till ſuch Force is prov'd, which is not done in the preſent Mechanical Power, but aſſum'd, becauſe a greater Velocity is Requiſite to be deſcrib'd in order to Elevate the Weight.

A True Account of this Mechanical Power, by which it is reduc'd as the Former, to that of the Vectis.

§. 31. The Trochlea or Pully therefore is that Mechanical Power, which conſiſts of ſeveral Centers of Detention, and which obſerves the ſame Laws with the Vectis with Two Fulcrums at the Extremities of it.

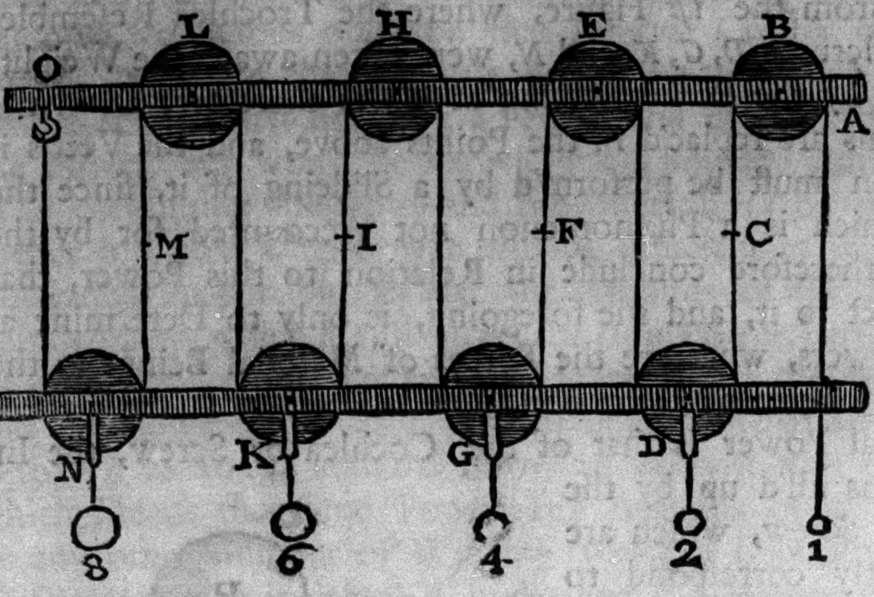
Thus let BO repreſent a compound Vectis, divided into it's ſeveral Diſtinctions, *B, C, D, E, F, G, H, I, K*, &c.



In the firſt Part of it, BE expreſſes a Vectis, whoſe Fulcrums are plac'd at the Extremities of it in *E*, and *B*, with an Oppoſite one at *D*. It is manifeſt by the Laws of the Vectis, and it's Forces, which, according to what we have before ſhewn, are increas'd in Proportion to their Diſtance from the Center of Detention, which here is the Fulcrum *E*, that 1 Weight in *B*, is Equal to 2 Weights in *D*, becauſe, as the Diſtance $ED=1$, from the Center of Detention *E*, is to the Diſtance $DB=2$, from the Center of Detention *D*, ſo the Weight 1 at *B*, is to the Weight 2 at *D*, that is, $ED\,.\,DB::W1\,.\,W2$, and $W1\times DB=W2\times ED$. It muſt be obſerv'd, that we, at preſent, ſuppoſe the oppoſite Fulcrum *D*, not to hinder the Elevation of the Weight 2, and ſo of the other oppoſite Fulcrums, which only obtain in the Pully, and are the Occaſion of the Slideing Motion in it, ſince, where the Fulcrums are oppoſite to each other, it is Impoſſible Motion ſhou'd be perform'd any other Way.

Let now the Vectis be ſuppos'd to looſe it's Rigor and Stiffneſs, and to become Pliant and Yielding to the Weights, and the Fulcrums annext to it, the Points *E, D, C, B, A* will retain the ſame Forces, as before, in Reſpect of the Weights and the Fulcrums, and will only change their Horizontal to a Perpendicular Situation.

Thus



THUS $ED2CBA1$, in *Fig.* 1. will fall into $ED2CBA1$, *Fig.* 2. in which the Lengths of the Perpendicular Lines are of no Conſideration, and only the Relation, which they have to their Fulcrums, is Regarded; $BA1$ is nothing, $BCD=2$, and $DE=1$.

THE ſecond Portion of the Vectis *Fig.* 1. is EH, Let $ED2$ be turn'd round a Pully in E, by the Former Vectis, or Pully, $2W$ in D, $=1W$ in B, therefore the whole Weight or Power, in the Former Vectis or Pully, will be $=$ to $2W$ in E; and for the ſame Reaſon, as before, the Weight 2 in E, will be Equal to the Weight 4 in G, becauſe $HG=1$, the Diſtance from the Center of Detention H, is to $GE=2$, the Diſtance from the Center of Detention G, as the Weight 2 to the Weight 4, that is, $HG.GE::W2.W4$, and $HG\times W4=GE\times W2$, but $W2$ in D, or E, $=W1$ in B, or A, and therefore $W1$ in B, or A, $=W4$ in G.

THE third Diviſion of this Compound Vectis is HL, in which let $HG4$ be thrown round a Pully in H, and the Weight 4 in H will be Equal to the Weight 6 in K, for the whole Moment or Weight of the Two former Vectes or Pullies were $1W$ in B, or A, $=2W$ in D, $=4W$ in G, and all of them $=3W$ in H, but $LK=1.KH=2::3W$ in $H.6W$ in K; or $1W$ in A, or B, $=6W$ in K.

THE laſt Part of this Compound Vectis is LO, in which $1W$ in A, or B, $=2W$ in D, $=4W$ in G, $=6W$ in K, and all of them $=4W$ in L, but $ON=1.NL=2::W4$ in $L.W8$ in N, and 1 Diſtance $\times\ 8W=2$ Diſtance $\times\ 4W$.

LET the whole Vectis, which may be continu'd in Infinitum, become Limber and Pliant to the Weights, which depend from it, and the Three laſt Diſtinctions of it, as well as the Firſt, will Sink into Pullies, and $EFG4H$, in the 1*ſt* Figure, will fall into $EFG4H$, in the 2*d*, $HIK6L$ into $HIK6L$, and $LMN8O$ into $LMN8O$.

A Confirmation of the Former Account of the Pully.

§. 32. IF therefore we now make uſe of the Number of Pullies themſelves, and not of the Lines, which encompaſs them, for a Solution of the Properties of this Mechanical Power, we ſhall eſcape all the Difficulties, which we have urg'd againſt the preſent Account, and ſhall find here the Forces, which are wanting there, to explain thoſe Properties; Thus with 2 Pullies B and D, and a Fulcrum E, $1W$ will ſuſtain $2W$; with 4 Pullies B, D, E, G, and a Fulcrum H, $4W$; with 6 Pullies B, D, E, G, H, K, and a Fulcrum L, $6W$; with 8 Pullies B, D, E, G, H, K, L, N, and a Fulcrum O, $8W$, &c. From hence alſo it is manifeſt, that in Pullies, which are not connected with one another, but are ſuſtain'd by ſeveral Diſtinct Fulcrums, the Force of the 2*d* will be Double of that of the 1*ſt*, that of the 3*d* Double of that of the 2*d*, and that of the 4*th* Double of that of the 3*d*, &c. as alſo, that $1W$ will ſuſtain $5W$ with five Pullies connected with one another without any Diſtinct Fulcrums; From hence it is farther Evident and Plain, that every Center of Detention weakens the Force of Gravitation, as in the other Mechanical Powers, every Removal from it, Increaſes it, that is in General, according to the Notion before Explain'd of Expanſive and Contractive Forces, the Center of Detention has in this Caſe a Contractive Force, and hinders Expanſion and Velocity, and the Gravitating Force in Reſpect of ſuch a Center is an Expanſive one, tho' in other Reſpects and Relations, as in Fire or Flame, 'tis a Contractive. So in the Trochlea or Pully, nothing can be more certain than, if we hang two Equal Weights upon it, they will be detain'd by it from Deſcending, and there will be one Center of Detention, if we make uſe of 2 Pullies, there will be 2 Centers of Detention, if of 3, there will be 3 Centers of Detention, *&c.* according to the Figures preceding.

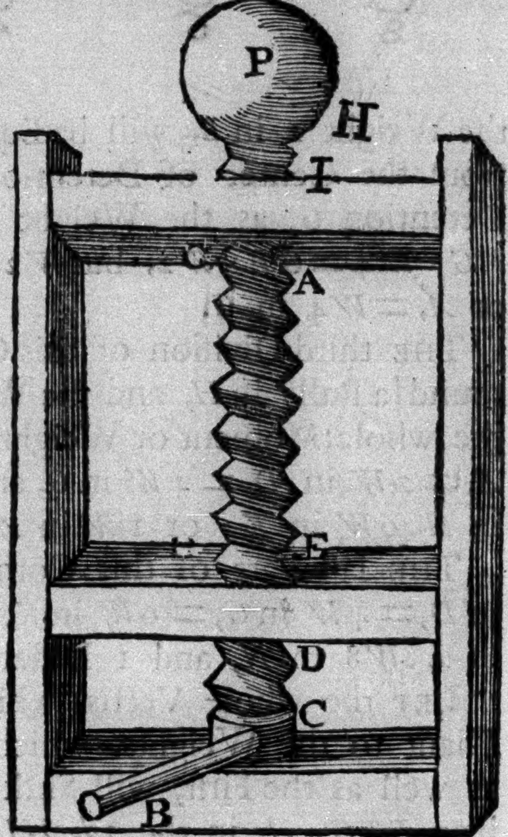
Mr. *Whiſton's* & Mr. *Hauksbee's* Experiments. Plate. 3, & 4.

AND to ſhew that this Account of the Trochlea, is the moſt Rational of any, which has been yet given, it ſupplies us with a Reaſon, why this Mechanical Power

Deviates from the Others in it's Proportions, for whilſt the Others are as the Diſtances from the ſame Center of Detention, this is as the Number of the Different Centers of it. Moreover it is manifeſt from the *1ſt* Figure, where the Trochlea Reſembles a Vectis, that if the oppoſite Fulcrums *D*, *G*, *K*, and *N*, were taken away, the Weights wou'd be rais'd at thoſe Points by the Power Acting at *B*; if therefore, as in the Pully, *Fig.* the *2d*, thoſe Fulcrums are replac'd in the Points above, and the Vectis is chang'd into a Cord, the Action muſt be perform'd by a Slideing of it, ſince the Fulcrums forbid it's Aſcent; which is a Phænomenon not Accounted for by the Preſent Mechanicks. We ſhall therefore conclude in Relation to this Power, that what we have to do with Reſpect to it, and the foregoing, is only to Determine at the End of theſe Mechanical Powers, what are the Forces of Material Beings at the ſeveral Diſtances from the ſame Center of Detention.

The Fourth Mechanical Power explain'd, which is that of the Cochlea or Screw.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10.
Dr. Wallis de Motu, Cap. 9. Part 3. Vol. 1.
Sir Iſ. Newt. Princip. Leg. Corol. 2. & 6.

§. 33. The Fourth Mechanical Power is that of the Cochlea or Screw, the Interior has it's Spiral Indentations fill'd up by the Prominent Parts of the Exterior Screw, which are made Spiral likewiſe, and exactly correſpond to the Indentations of the Inward; or, which is the ſame in Effect, the Spiral Convexities of the Inward Screw anſwer to the Spiral Concavities of the Outward, and either the Outward Screw is fixt, and the Inward paſſes thro' the ſeveral Spirals of it, or elſe the Inward is fixt, and the Outward moves upon the Spirals of the Inward.



The Theorem made uſe of to Account for this Power.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Theorem 14.

§. 34. The Theorem Introduc'd to Explain this Power is, that if the Ambitus, which the Force or Power apply'd Deſcribes in one Entire Circuit or Circumvolution of the Screw, is to the Interval of the two next Spirals, which Interval is to be computed according to the Length of the Screw, as the Weight or Reſiſtance to the Power, the Power and Weight will be Equally pois'd, and if there is the leaſt Addition to ſuch a Power, it will be capable of Moving the Weight; Let *CA* be the Interior Screw, *DE* and *AI* the Exterior ones, thro' which the Interior is forc'd by the Scytala or Handle *CB*, to which the Power is Apply'd, and by the Rotation of which the Weight *P* is rais'd; it is manifeſt by Obſerving the Motion of the Screw, that in one Revolution of it, the Weight is ſo much Elevated, as the Interval of the Two firſt Spirals, Expreſs'd by *HI*, Meaſures in a Perpendicular Length, and that the Power deſcribes at the ſame Time a Space Equal to the Ambitus, which it Delineates in one Revolution; that is, the Way or Space deſcrib'd by the Weight, is to that deſcrib'd by the Power, as the Interval of the Two Spirals, to the Ambitus of the Power, and conſequently, if the Power is to the Weight, as the Interval of the Spirals to the Ambitus of the Power, the Power and Weight will be Equal in their Forces, and if an Addition be made to the Power, it will be Superior to the Weight; that is, as before, if *P* is the Power, *G* the Weight, *V* the Velocity of the Power, *v* of the Weight, $PV = Gv$, or, $P.G::v.V$.

The First Objection againſt this Account.

§. 35. The whole Reaſoning of the preceding Section depends upon this, that whilſt *CB* performs it's Revolution, the Two Spirals *IH* appear above the Exterior Screw *IA*; Two Things therefore, will be neceſſary to be prov'd, to make this Demonſtration Juſt. Firſt, that Two Spirals of the Interior Screw, will always in their Elevation be exactly Correſpondent to the Ambitus, deſcrib'd by *CB*, which do's not ſeem to be certain, ſince, if the Spirals are made Cloſer to, or more Remote from each other, whilſt the Ambitus of the Scytala *CB* continues the ſame, they cannot have the ſame Relation to it; for Inſtance, if inſtead of 8 Spirals betwixt *A* and *E*, there ſhou'd be Double the Number, as 16, or half, as 4, it do's not appear to be Probable, that whilſt *CB* perform'd it's Ambitus, there ſhou'd be only Two Spirals, as *HI*, rais'd above the Exterior Screw, whether the Number of the Spirals in the Screw are 8, as in the Preſent Caſe, or 4 or 16, as in thoſe, which are ſuppos'd.

The Second Objection againſt it.

§. 36. Second, if this were Certain, that all Spirals were alike in their Proportion of Two of them to the Ambitus made by the Power, that is, that the Interval of Two Spirals only, be that Interval Greater or Leſs, was to be barely conſider'd, yet, ſince the Intervals wou'd be Different, as the Spirals were Cloſer or Wider, *HI* wou'd not have a certain and determin'd Proportion to the Ambitus of the Power, but wou'd be varied in a Thouſand Inſtances, and if ſo, the Ambitus of *B* continuing the ſame, and the Interval *HI* being Different, we cannot be from thence inform'd what Power will be ſufficient to raiſe the Weight *P*, when there is no ſettled Proportion betwixt the Ambitus of the Power, and the Interval of the Spirals. Nay, which is ſtill more, the ſame Power, performing the ſame Ambitus, wou'd be capable of raiſing Different Weights, for if the Ambitus of *B* is to *HI*, as 20.2, and the Power of *B*, as 1, it will be 2.1::20.10 the Weight, if the Ambitus *B* is to *HI*, as 20.4, and the Power of *B*, 1, it will be 4.1::20.5 the Weight, by which Means the ſame Power and Ambitus of it, will be able to Raiſe 10 or 5, or any other Number of Weights, which is Impoſſible. Beſides, *HI* is not the Way, or Space, which is deſcrib'd in Raiſing the Weight, but that which is perform'd by the Circumvolution of the Two Spirals, and therefore it cannot be, *B.P*::*HI*. Ambitus of *B*, which this Doctrine ſuppoſes.

Another Objection againſt this Account.

§. 37. Farther, it ſeems to be very Manifeſt, whatever the Proportion is betwixt the Ambitus of the Power, and the Interval of the Spirals, that whether there is a Weight, as *P*, plac'd at the Top of the Screw or not, upon one Revolution there will be the ſame Interval of the Spirals, and conſequently, that the Phænomenon of Raiſing the Weight do's not depend upon the Spaces deſcrib'd, but the Forces apply'd at *B*, which, if we likewiſe conſider, it will be impoſſible to adjuſt any Proportion betwixt them, and the Weight which is mov'd, unleſs we know what Proportion of a Man's Strength is Equal to ſo much Weight or Gravity.

A True Account of the Cochlea, or Screw.

§. 38. The neareſt to this Mechanical Power, if it may be term'd One, is the Axis in Peritrochio, excepting that the Circumvolutions are Spiral and Horizontal, and by the Screws not Retiring backward from the Nature of Heavy Bodies, Moving within one another, that ſuch Circumvolutions may be ſtopt and advanc'd at Pleaſure, and are not perform'd by Weights, but by Human Force and Strength.

Whereas the Gyrations of the Axis in Peritrochio are Circular, and Perpendicular to the Horizon, and Return back upon any Remiſſion of the Force apply'd to it, and tho' Human Strength is ſometimes us'd in this Mechanical Power, yet it may be Balanc'd by mere Weight and Gravity, as has been ſhewn in Reducing it to the Vectis. On the Contrary, let the Cochlea or Screw be plac'd in an Horizontal Poſition, what Weight or Weights will turn it round with any kind of Conformity to the Axis in Peritrochio, which Mechanical Power yet it moſt Reſembles? There is the Reſiſtance of Wood, or Iron, or Steel, Moving within it ſelf to be ſubdu'd, the Revolution not Circular but Spiral, and mere Gravity can only act in a Direct Line, which will be ſo far from Promoting the Revolution of a Spiral, that it will be an Impediment to it's Revolving.

From all which it ſeems to be Evident to me, that the Cochlea, or Screw, is to be Exterminated from the Number of Mechanical Powers, and that it is not Reducible to any of them; for let us now replace it in it's Proper Situation, that is, in a Perpendicular one to the Horizon, it is plain no Mechanical Force of Gravity, or any other Natural Power, beſides what is Imported from the Will and Strength of the Mind, can have any Effect upon it; but what the World has generally Eſteem'd Mechanical Power, is that which lies in Matter, Motion and Gravity, without any Interpoſition of an Animal or Arbitrary Agency.

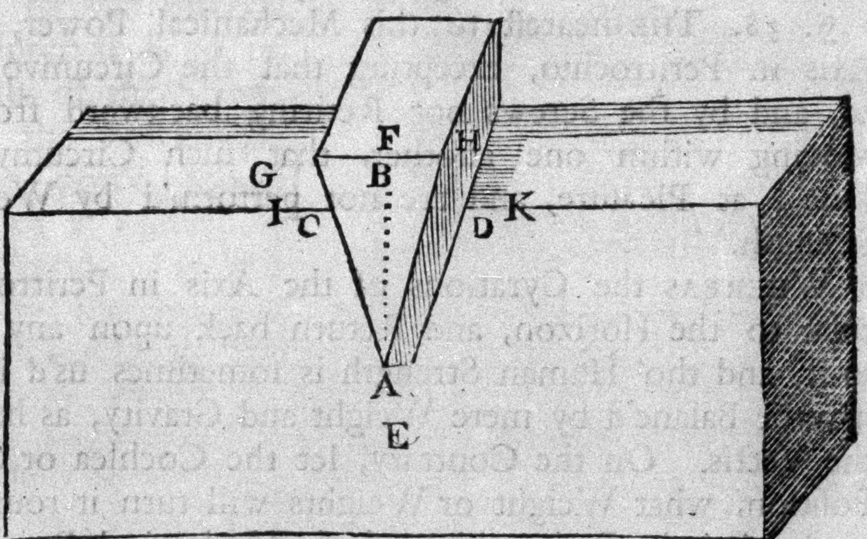
The Notion and Opinion therefore, which we may juſtly have of the Cochlea, or Screw, is this, that it is ſo far a Mechanical Power, as it has a Center of Detention with the Reſt, which is the Center of the Ambitus of the Scytala, but the Power made uſe of to the Moving of the Screw, is not Mechanical, unleſs a Human Force, of which there is no Eſtimation or Meaſure, may be accounted ſuch; ſo likewiſe the Paſſing of the Interior Screw, thro' the Exterior, meets with a Various Reſiſtance ariſing from the Nature of the Matter, thro' which it is Carry'd, and the Compreſſion made by it Differs, as the Human Force Apply'd, or the Renitency of the

Matter compreſs'd, Changes, Nothing of which has any Thing to do with Gravity or Mechaniſm, or is Reducible to any Laws of it.

AND as for the Screw it ſelf, it has no Power at all, excepting that of being mov'd Upwards or Downwards, by the means of the Scytala, and according to the Way, which lies open for it's Motion in a Spiral, and of ſuſtaining any Weight or Preſſure after it has acquir'd it's Poſition, by the Application of Human Force; the Reaſon of which is, becauſe the Gravity or Preſſure Acting in a Perpendicular upon the Screw, which can only move in a Spiral, faſtens and cloſes the Indentations of it, inſtead of putting them into Motion: But, if the Firſt entitles it to the Being a Mechanical Power, that is, it's Capacity of Moving Upwards and Downwards in a Way, which is Carv'd out on Purpoſe for it, a Cord drawn up and down thro' a Hole made for it's Motion, may be as well call'd a Mechanical Power; If the Laſt, that is, it's Capacity of ſuſtaining any Weight or Preſſure, after a Poſition acquir'd by the Application of Human Force, any Hard or Conſiſtent Body may be Term'd a Mechanical Power, which, when any Weight is laid upon it by the Application of Human Force, will ſuſtain it's utmoſt Preſſure, as fully as the Teeth of a Screw will do. So that we can ſee nothing more in the Cochlea, than a Center of Detention at one End of the Scytala, or Vectis, and the Application of a Human Force at the Other, by which the Screw will neceſſarily Riſe, and which therefore is all, which we need to conſider in this Power, or according to the Notion of Expanſive and Contractive Forces, there will be a Center of Detention or Contraction at the one Extremity of the Vectis, and an Expanſive Force at the Other.

The Cuneus, or Wedge, explain'd, which is the Fifth Mechanical Power. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 10. Theorem 15. Dr. Wallis de Motu, Cap. 12. Part 3. Vol. 1.

§. 39. THE laſt Mechanical Power to be Reſolv'd is, that of the Cuneus, or Wedge. The Theorem made uſe of to unfold this Power is, that the Power perpendicularly Apply'd to the Back of the Wedge, which ſhall be to the Reſiſtance to be ſurmounted by the Wedge, as the Thickneſs of the Wedge, to it's Heighth, will be of Equal Force with that of the Reſiſtance, and if ſuch a Power is increas'd, it will be Superior to, or an Over-balance for the Reſiſtance. For let the Reſiſtance to be over-comn by the Wedge, be the Firmneſs or Tenaciouſneſs of the Wood, or any other Body, whilſt the Wedge is Drove in as far as *A*, the Length, which the Power deſcribes is *B A*, and *D C* is the Space deſcrib'd by the Impediment, which is vanquiſh't and made to yield to the Force of the Power; for the ſame Reaſon alſo, whilſt the whole Length of the Wedge *F A*, penetrates to *E*, and deſcribes *B E*, the whole Breadth of it *G H*, deſcribes *I K* in the Impediment, and proportionally in it's Progreſs, as we take a greater or leſſer Part of the Cuneus, which is Evident from the Nature of the Triangle *G A H*, from whence the foregoing Theorem is ſuppos'd to be Manifeſt.



The Firſt Objection againſt the foregoing Account of the Cuneus.

§. 40. IN Anſwer to which we muſt affirm, that *F A* to *G H*, is ſaid, in the Theorem, to be as the Reſiſtance to the Power, when the Power and Reſiſtance are in an Equal Poiſe; whereas, this is prov'd, by ſuppoſing the Power to be Superior to the Reſiſtance, which is a Contradiction; for, if the Reſiſtance is Equal to the Power, it is altogether Impoſſible, that the Wedge ſhou'd ever Penetrate to *A*, or ſo much as enter the Impediment, altho' the preſent Demonſtration proceeds upon a Suppoſition, that it has enter'd the Impediment; if it be ſaid the Wedge may enter the Impediment, and after it is fixt in a certain Situation there, we are then to conſider it in an Equal Poiſe with the Reſiſtance, we muſt ask which way it came into that Situation, if by a Power Superior to the Reſiſtance, then the Power and Reſiſtance are not Equal. If it is farther told us, that the Power is ſuppos'd to be Augmented ſomething above the foregoing Proportion, to cauſe the Cuneus to move againſt the Impediment, that can have no Conſideration here, ſince the Forces, in the preſent Situation of the Wedge, are ſuppos'd to be Equal, having the Proportion aſſign'd by the Theorem, and yet this Situation of the Wedge, we ſay, cou'd not be, unleſs the Power was Superior to the Reſiſtance.

A Second Objection.

§. 41. MOREOVER, if we admit the Cuneus to be plac'd in the Poſition we now ſee it, if P and R repreſent the Power and Reſiſtance $P . R :: CD . BA$, but P and R by Suppoſition are Equal, therefore BA and CD are ſo likewiſe, which is Impoſſible in the preſent Figure, where the Height of the Triangle is not Equal to the Baſis; or which is the ſame to make $P = R$, they are to be as $CD . BA$, that is, one Leſs than the other; but if the Reſiſtance it ſelf is Greater than the Power, I don't ſee how the Way of the one or the other, will help the Latter.

A Third Objection.

§. 42. ANOTHER Objection againſt this Reaſoning is, that the Proportion of $P . R :: CD . BA$ is not Explain'd, that is, that the Power is to the Reſiſtance Reciprocally, as the Way of the Reſiſtance to that of the Power, for as we have juſt now ſaid, if the Power is Leſs than the Reſiſtance, it ſeems Inconceivable, how the longer the Way, which the Power is to go, ſhou'd make it Equal in it's Force to a greater Reſiſtance, which proceeds thro' a leſs Space; The Power and Reſiſtance will ſtill be, the one leſs than the other, and not Equal, ſo far as I am able to Imagine. If this Proportion is Aſſerted, becauſe it is preſum'd to be prov'd in the other Mechanical Powers, and particularly in the Vectis (for there is Nothing offer'd to explain it from the Power it ſelf), we firſt ſay, it is only Preſum'd, and not Prov'd, and in the ſecond Place, that if it were Prov'd in the Vectis, ſince there is no Analogy betwixt it and the preſent Power, it wou'd not be Conſequent, that ſuch a Reciprocal Proportion obtain'd here.

A Fourth Objection.

§. 43. IF there is a Similitude betwixt the Vectis and the Cuneus, the Center betwixt the Power and Reſiſtance deſerves to be fixt, and then the Reciprocal Proportion of Diſtance from ſuch a Center, which is ſo Intricate a Problem, as will Require a more accurate Reaſoning, than is here made uſe of, to ſolve.

A Fifth Objection.

§. 44. BESIDES the Three Firſt Powers are thoſe, which conſider the Forces, which Gravitate againſt them, but in the Preſent, not the Gravitation, but the Reſiſtance of a Body is to be overcomn; if therefore, by the ſame Fundamental Maxim of the Velocity of the Power being to the Velocity of the Weight, as the Weight to the Power, Reſiſtance is alſo to be Explain'd, and the Velocity of the Power is to the Velocity of the Reſiſtance, if we may ſo call it, as the Reſiſtance is to the Power, The Forces of Gravitation and Reſiſtance are the ſame, which no one has hitherto affirm'd, who do's not declare the Hardneſs of Marble, and it's Weight, to be Equivalent Terms.

A Sixth Objection.

§. 45. LASTLY, it is Notorious in Fact, that Bodies, which are Cleſt by a Wedge, by their own Force and Elaſticity, will ſometimes throw it out, when it has Penetrated to ſome Depth as A, and therefore, tho' the Wedge is Forc'd thither, after it has conquer'd it's Reſiſtance, and conſequently ſhou'd be in an Æquilibrium with it by this Demonſtration, yet is not, but is repuls'd by the Elaſtick Power of the Body; On the other Hand, where a Force is us'd to make the Wedge penetrate into other Bodies, as in ſoft or brittle Ones, it ſhall go farther into them by it's own Greater, or by the Leſs of the Bodies, which it Penetrates, than in Proportion to what was impreſs'd upon it; And laſtly, ſometimes Bodies will Cling to the Wedge with that Cloſeneſs, that the Reſiſtance will be Superior to the Power which forc'd it in; ſo little is there to depend upon ſuch Abſtracted Reaſonings, as theſe, to Explicate the full Powers of Nature.

AN Example of Each of theſe Phænomenons in Bodies we have, of the Firſt, in Metals, of the Second, in Clay, of the Third, in Wood, which eaſily ſeparates, and of the Fourth, in That, which has a more than ordinary Toughneſs.

A True Account of the Cuneus, or Wedge.

§. 46. WE have hitherto conſider'd the Cuneus, as a Mechanical Power, but this, as the Cochlea, or Screw, ſeems not properly to fall under that Denomination, and conſequently, that, truly ſpeaking, there are only Three, the Vectis, the Axis in Peritrochio, and the Trochlea.

FOR, as the Cochlea or Screw, the Cuneus likewiſe depends upon Forces, which are not Mechanical, and cannot be reduc'd to them, as the Human Force, and the

Force of Reſiſtance in Various Bodies; All therefore that can be ſaid, in Reſpect of this Power, ſo far as it can be thought to be Mechanical, is, that if a Force is apply'd Perpendicularly to *F*, or the Dorſum of the Wedge, *G*, *H*, and *A*, will obtain a greater Effect, the farther they are from *F*, againſt any Reſiſtance; ſo that in this Caſe, *F* is to be conſider'd as the Center of Detention, and *G*, *H*, and *A*, as the ſeveral Removals from that Center, which therefore will act with a Greater Force, as they are more Diſtant from it; and in this Property it agrees with the Vectis, which is the Standard of all Mechanical Powers; Thus, if *FA* repreſented a Lever, the farther the Power *F* is from *A*, the greater Moment it wou'd have, and if *FH*, or *FG*, repreſented a Vectis of Protruſion, or a Lever Protruding inſtead of Elevating, the more Remov'd *G* and *H* are from *F*, their Influences and Forces wou'd be the Greater, as the Line *FA*, on the contrary, wou'd have Little or None, being in this Reſpect a Line of Quieſcence, Contraction, or Detention.

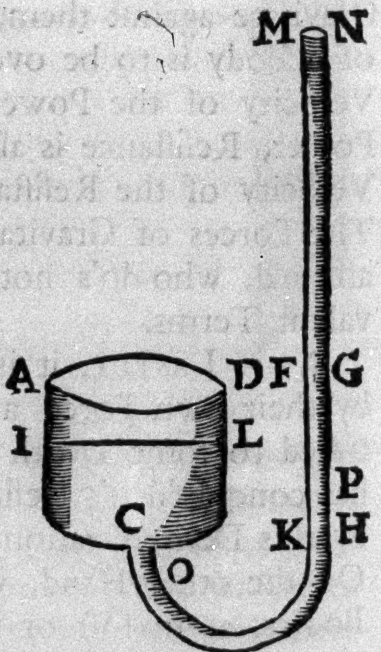
THUS far the Cuneus may be conſider'd as a Mechanical Power, but then it is not a Mechanical Power, that Gives an Action to the Cuneus, which is, what we Eſteem Eſſential to ſuch a Power; no Gravity, no Mechanical Force, or Action, has any Effect in the Wedge, and it's Power is wholly owing to a Human, Animal, and Expanſive Force.

WHAT therefore muſt be determin'd in Reſpect of this Mechanical Power is, that it is an Expanſive Force, acting in a Center of Detention or Contraction *F*, which Force of Expanſion, the Farther it is from the Center of Contraction, the Greater Influence and Power it has, conformable to what was before ſhewn in the Cochlea, or Screw.

THIS Power, conſequently, has not ſo great a Force, as the Former, becauſe, the one Acts in the very Center of Contraction, by which it is Enervated and Weaken'd, and the other at ſome Diſtance from it, by which it is more Vigorous and Strong.

The Inflected Syphon explain'd from the preſent Doctrine of Mechanicks. Dr. Keill's Intr. Theor. 15. Schol. Lect. 10. Rohault. Phyſ. Cap. 10. §. 11. Part 1. & Annotat.

§. 47. FROM the preſent Doctrine of Mechanicks is alſo Infer'd the Reaſon, why in the Inflected Syphon, whoſe Canals, or Arms, are of Unequal Capacities, the Water ſtands at an Equal Height, becauſe, if the Cylinder of Water *AILD* Deſcends, there muſt be an Equal Cylinder of Water Aſcend in the contrary Tube, as *MFGN*, but by the 15 Prop. El. 12. *Euclid.* the Baſes and Altitudes of Equal Cylinders are in a Reciprocal Proportion to each Other, that is, as *FM*. *AI* :: Orifice *AD*. Orifice *MN*, but *FM* is to *AI*, as the Velocity of the Water's Aſcent in the Leſſer Tube, is to the Velocity of the Water's Deſcent in the Larger, and the Orifice *AD* is to the Orifice *MN*, as the Water in the Cylinder *ACD* to that in *FKHG*, Becauſe Cylinders of Equal Heights are as their Baſes; from whence it follows, that the Velocity of the Aſcending Water *MF*, will be to that of the Deſcending *AI*, as the Water in the Canal *ACD* to that in *FKHG*, that is, the Velocities of the Waters in Each will be Reciprocally Proportional to their Quantities, and therefore their Moments Equal, but their Moments are Contrary, and therefore will produce an Æquilibrium, or Quieſcence.



The Firſt Objection againſt this Reaſoning.

§. 48. BUT in the firſt Place we have no Account, why the Water Stagnates in *COKH*, which is ſuppos'd to have no Part or Share in the Preſent Demonſtration, and yet *CO* may be as well preſum'd to riſe to *HP*, as *AI* to *FM*, whereas it is not thought neceſſary to prove, that the Water in *COKH* will be otherwiſe than in an Æquilibrium, and conſequently there is the ſame Reaſon, why *AI* ſhou'd not want any ſuch Proof; for if it is manifeſt, that *CO* will not riſe to *HP* an Equal Cylinder, it is as Evident, that *AI* will not riſe to *FM* an Equal Cylinder, or, which is the ſame, if the Surface or infinitely ſmall Cylinder *C* will not riſe in *KH*, it is as plain, that *AD* the Baſis of the Cylinder *AI*, or an infinitely ſmall Portion of it, will not riſe in *FG*, ſince *AD* is ſuppos'd to move at once, and by one uniform Depreſſion towards *IL*, and will therefore keep it's Station, as well as *FG*;

as therefore the true Cauſe, why the Surface *C* do's not Deſcend, nor *KH* Aſcend, is, that the Parts of Water are in an Equal Balance with one another, and obtain no Force or Moment in Relation to each, ſo the ſame Reaſon is Valid with Reſpect to the Surface *AD*, which will not Deſcend, not becauſe *FG* is, in ſuch a Caſe, to Aſcend to *MN*, but becauſe the Surface *AD* has really no Moment or Force, either in Reſpect of it's Parts with one another, or of any Subjacent ones, where *AD* and *FG* have their laſt Adjuſtments of their Gravities to the Earth by their Equal Diſtance from it.

A Second Objection.

§. 49. **Second**, the Aſcent and Deſcent of the Water is ſuppos'd to be Equal in this Demonſtration to the Surface, or ſmall Cylinder, *AD*, whereas, if one Part or Portion of *AD* cou'd be ſuppos'd to Deſcend, the Cylinders of Reciprocal Heights and Baſes wou'd be deſtroy'd, and the Reaſoning upon them, for tho' *AD* cou'd not Deſcend, by this Demonſtration, it wou'd not be from thence Evinc'd, that no Part of it cou'd; but the Suppoſition, which keeps one Part of the Cylinder *AD* from Deſcending, unleſs the Whole do's, namely, becauſe one Part or Portion has not a greater Power or Force of Deſcending than another, will keep the Whole from Deſcending in Reſpect of the other Surfaces, which are in an Æquilibrium with it, and conſequently, if *AD* do's not Deſcend, it is Manifeſt, that *FG* will not only not Aſcend to *MN*, but that it will not Aſcend at all; We may here add, that ſuppoſing the Velocities of the Parts in the Cylinders *ACD*, *FKHG* were Reciprocal to the Quantities of Matter in them, and therefore their Moments Equal, they wou'd not yet be Contrary, unleſs *FKHG* was carry'd in an Oppoſite Velocity to *ACD*, whereas it is ſuppos'd to have none, but what is Communicated from *ACD*, and therefore, if *ACD* is mov'd, muſt Tend the ſame Way.

A Third Objection.

§. 50. **Third**, let the Canals be never ſo Irregular, and let us contrive all the Imaginable, and moſt Diſagreeing and Different Shapes for them which we can, the Water in all the Canals will ſtand at the ſame Height, where it is permitted to do it, and yet it will be Impoſſible to reduce ſuch Figures to the Cylinders of *Euclid*, and the Reciprocal Proportions of thoſe that are Equal, upon which this Demonſtration depends, and which can be only accounted for from the Equal Moment, which the Parts of Water have amongſt one another; ſo little is there in ſuch Kind of Reaſonings, which yet carry with them the Appearance of Demonſtrations, becauſe certain Propoſitions in the Mathematicks are introduc'd to ſupport them, which ſtand much better in the Elements, from whence they are taken, than in the Philoſophy to which they are often, with too much Haſt, and too little Deliberation Apply'd.

Dr. Keill's Introduct. Theorem 15. Schol. Lect. 10.

As to Water or any other Fluid's paſſing from a Larger Channel, into a Smaller, with a Greater Celerity, that proceeds from a known Law in Mechanicks, that, where Water flows with a certain Force in a Larger Channel, if that Force is compreſs'd into a Narrower Space, it will be increas'd; and from which Law Sir *Iſ. Newton* argues againſt the *Vortices* of *Des-Cartes*, according to what we have before ſhewn in our *Principles of Philoſophy*.

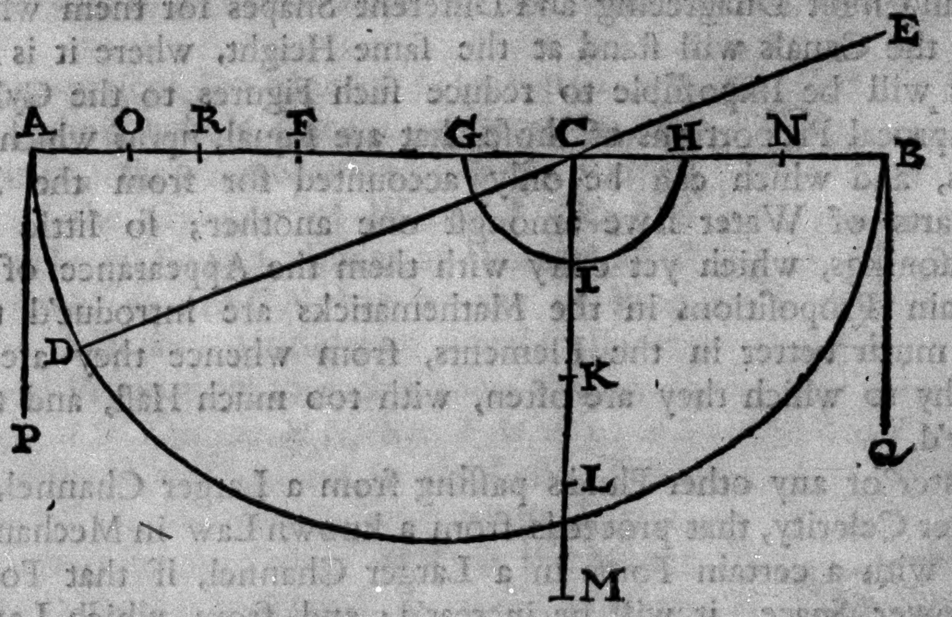
CHAP. V.

Concerning the Expanſive and Contractive Forces, Acting in a Center of Detention, or in any Diſtances or Removals from it.

The Doctrine of the Expanſive And Contractive Forces, Acting in a Center of Detention, or in any Removal from it, Explain'd, and the ſeveral Caſes Recounted.

§. 1. HAVING now gone thro' the Mechanical Powers, and ſhewn, that the Two Firſt, the Vectis and Axis in Peritrochio, are to be Eſtimated from their Diſtance from their Center of Detention, that is, that their Powers will be the Greater, the farther they are Remov'd from it, that the Third, which is the Trochlea or Pully, conſiſts of ſeveral Centers of Detention, by which the Force of that Power is weaken'd, that the Fourth and Fifth are not properly Mechanical Powers, but ſo far as they are, that the Cochlea or Screw conſiſts of a Center of Detention, with an Expanſive Force at Different Removals from it, and the Cuneus or Wedge of a Center of Detention or Contraction with an Expanſive Force Acting in that Center; We ſhall proceed, more particularly, to conſider theſe Centers of Detention, with the Various Diſtances from them, and the Expanſive and Contractive Forces plac'd in thoſe Diſtances, as alſo, what will reſult from a Complication of theſe Centers of Detention at Different Diſtances, with the Expanſive or Contractive Forces Acting in, or at a Diſtance from them.

LET AB be an Inflexible, ungravitating Line, and capable of being mov'd on the Center C, if any Weight, or other Force, is apply'd to A or B, it is manifeſt a Motion will be produc'd round the Point C, which I call the Center of Detention, becauſe it is not a Center of Gravity, on which Forces are Equally pois'd, which is only one Caſe of the Center of Detention, but a Center, by which the Forces depending at A or B, are kept and detain'd from Receding from it, and which, if the Forces at A and B, computing their Diſtances, are Equal, will be, what is call'd a Center of Gravity.



Caſ. 1. IF therefore any Point, or little Body, as a Minute Globe of Similar and Homogeneous Matter, which has a certain Force of Gravitation, be plac'd in the Center C, it will be hinder'd from Deſcending, and will remain Quieſcent in the Point of Detention; Let the ſame Gravitating Point, or Force, be remov'd one Point or Line towards A or B, ſuppoſing to G or H, it will then Deſcend, but ſince it cannot Recede from the Center of Detention C, it will deſcribe the Arch of a Circle round it, which Arch will be Greater or Leſs, according to the Force of Gravitation; thus, if it has the Force of CI, it will Deſcend from G to I, if of CK, or double the Force of the Former, ſince it cannot Deſcend to K, it will ſpend the Remainder of it's Force IK, above that of CI, in Riſing towards H, and Deſcending from thence to I, and then Riſing towards G, till it's Force IK is conſum'd in Reciprocal Oſcillations.

IF the Gravitating Force is CL, or Triple of the Former, or CM, which is Quadruple of it, the Oſcillations will riſe higher towards H and G, and if this Gravitating

Force is ſtill increas'd to a Centuple or Millecuple Proportion of the Former, the Oſcillations will be chang'd into entire Revolutions, which will not Ceaſe about the Center of Detention, till the whole Gravitating Force is Conſum'd and Exhauſted, and if ſuch Gravitating Force is Permanent and Conſtant, the Revolutions will be Uninterrupted and Continual, from whence it is Evident, that there is nothing more Requiſite to ſuch Revolutions, than the Gravitating Force only, without any Projectile one, That is, than an Expanſive, which the Gravitating here Repreſents, and a Contractive, which Exerts it ſelf in the Center of Detention; or *Vice Verſâ.*

Caſ. 2. If now the Gravitating Point, or Little Body, of Similar Matter, ſhou'd be remov'd from the Center of Detention, Two, Three, Four, or a Hundred, or a Thouſand, or a Million of Points, or ſmall Lines, from the Center of Detention, all Things wou'd happen as before, in Reſpect of the Oſcillations or Revolutions of it, excepting that the Force of it wou'd be increas'd, the farther it retir'd from ſuch a Center, in which it's Force was entirely Hinder'd and perfectly Quieſcent; For, if the ſame Gravitating Force, which in the Center of Detention is Quieſcent, is apply'd to one Diſtance from it, it is Reaſonable to think, that it will have one Effect more than it had before, which was none, that, at Two Diſtances, it will have a double Effect, and ſo on; that is, any certain Gravitating Force will be increas'd in it's Energy or Expanſive, in Proportion, as it departs from the Center of Detention or Contraction, which prohibits any Effect of it; and we may the more eaſily acquieſce in this Reaſoning, becauſe it is confirm'd by Experience, and by the Preſent Mechanicks, for if the ſame Force of Gravitation is Greater at *A*, than it is at *B*; in Proportion to their Diſtances from the Center of Detention, it will neceſſarily follow, that the ſame Gravitating Force at the Diſtance *AC*, will be Equal to that at the Diſtance *CF*, or *BC*, ſuppoſing the Sum of the ſame Gravitating Forces at *B*, to that of *A*, is as the Diſtance of *A* from the Center of Detention *C* to that of the Diſtance of *F* or *B* from the ſame Center.

Caſ. 3. If now an Expanſive Force, which is impreſs'd, for hitherto we have only conſider'd the Gravitating or Contractive, is ſuppos'd to Act in the ſame Point *C*, which is a Center of Detention, as if we ſtrike with a Violent Force upon it, it's Effect will be Greater than that of a Gravitating, and may ſpread and diffuſe it's Action towards *H* and *G*, ſince it's Progreſs is ſtopt in *C*, by it's being ſuppos'd a Center of Detention, tho' without any actual Motion of the Line *AB*; Let the Force of Percuſſion be ſtill increas'd in the Point of Detention *C*, and the Action will be the Greater, which is communicated along the Lines *CF*, *CB*, but yet without any Motion of them.

Let now the Force of Percuſſion or Expanſion exert it ſelf, one Remove from the Center of Detention, in *G* or *H*, upon the Gravitating Point, and it will give an Additional Force to it, and aſſiſt to the producing Greater Oſcillations, or Entire Revolutions, and let the Force of Percuſſion be ſtill Increas'd, and the Effect will be ſtill the Greater; For the Contractive Force or the Gravitating, in Reſpect of *C* the Center of Detention, is an Expanſive one, by which it Endeavours to retire from the ſaid Center, and conſequently, the Force of Percuſſion acts, in this Caſe, in Conjunction with it, being likewiſe an Expanſive one Impreſs'd, and therefore promotes and advances the ſame Effect, and is the Projectile of this Philoſophy.

Caſ. 4. If the ſame Expanſive Force impreſs'd, or Force of Percuſſion is remov'd, Two, Three, Four, or a Hundred, or a Thouſand, or a Million of Points, Lines, or Diſtances from the Center of Detention *C*, it will be in Proportion augmented, and for the ſame Reaſon, as the Gravitating or Contractive, which in the preſent Circumſtance has the Nature of an Expanſive. We here call the Force of Percuſſion an Expanſive Impreſs'd, to diſtinguiſh it from the Intrinſick Expanſive, which we ſhall afterwards Conſider, for of the Two Intrinſick Forces of Matter, the Contractive or Gravitating is here only alledg'd.

From what foregoes it is Evident, that the Forces, or Moments, or Powers of Bodies, which have a Contractive or Gravitating Force, and one of Percuſſion, and which Act from a Center of Detention are Greater, the Greater that Gravitating Force is, the Greater the Sum or Quantity of it is, the farther it is remov'd from the Center of Detention, and the Greater the Expanſive Force which is Impreſs'd, and the farther it's Diſtance is from the ſame Center. If therefore, *M*, *m* are made to repreſent the Moments of ſuch Bodies acting upon a Center of Detention; *C, c* the Contractive or Gravitating Forces; *A, B* the Sums or Quantities of them; *R, r* the Diſtances or Removals from the Center; and *P, p* the Percuſſions or Expanſive Forces Impreſs'd; the following Theorems will be certain.

Theorem the Firſt of this Doctrine, and it's Corollaries, That the Moments of Bodies, which have a Force of Contraction or Gravitation and Percuſſion, and which Act from a Center of Detention, are in a Direct Reaſon of the Forces of Gravitation or Contraction, of the Quantities or Sums of thoſe Forces, of the Forces of Percuſſion, and of the Diſtances from the Center of Detention.

§. 2. THEOREM the Firſt is, That the Moments of Bodies, which have a Force of Contraction or Gravitation, and Percuſſion, and which Act from a Center of Detention, are in a Direct Reaſon of the Forces of Gravitation or Contraction, of the Quantities or Sums of thoſe Forces, of the Forces of Percuſſion, and of the Diſtances from the Center of Detention, that is, $\frac{M}{m} = \frac{ACPR}{Bcpr} = \frac{A}{B} \times \frac{C}{c} \times \frac{P}{p} \times \frac{R}{r}$; From whence it Enſues.

Corol. 1. IF $A = B$, $\frac{M}{m} = \frac{CPR}{cpr} = \frac{C}{c} \times \frac{P}{p} \times \frac{R}{r}$, or $M.m::CPR.cpr$, or, if the Quantities of the Gravitating or Contractive Forces are Equal, the Moments of ſuch Bodies will be as the Forces, the Percuſſions, and the Diſtances or Removals from the Center of Detention.

Corol. 2. IF $C = c$, $\frac{M}{m} = \frac{APR}{Bpr}$, or, if the Gravitating or Contractive Forces are Equal, the Moments of ſuch Bodies will be as the Sums or Quantities of the Forces, the Percuſſions, and the Diſtances or Removals from the Center.

Corol. 3. IF $P = p$, $\frac{M}{m} = \frac{ACR}{Bcr}$, or, if the Percuſſions are Equal, the Moments of theſe Bodies will be as the Gravitating or Contractive Forces, the Sums of them, and their Removals from the Center.

Corol. 4. IF $R = r$, $\frac{M}{m} = \frac{ACP}{Bcp}$, or, if the Diſtances from the Center are Equal, the Moments of the preſent Bodies, will be as the Gravitating or Contractive Forces, the Sums or Quantities of them, and the Percuſſions.

Corol. 5. IF $A = B$, and $C = c$, $\frac{M}{m} = \frac{PR}{pr}$, or, if the Gravitating or Contractive Forces are Equal, and the Sums or Quantities of them, the Moments of theſe Bodies will be as the Percuſſions, and their Removals from the Center.

Corol. 6. IF $A = B$, and $P = p$, $\frac{M}{m} = \frac{CR}{cr}$, or, if the Quantities or Sums of the Gravitating or Contractive Forces, and the Percuſſions are Equal, the Moments of ſuch Bodies will be as the Gravitating Forces, and their Removals from the Center; and if $C.c::R.r$, $M.m::R^2.r^2$. ſee *Corol.* 8.

Corol. 7. If $A = B$, and $R = r$, $\frac{M}{m} = \frac{CP}{cp}$, or, if the Sums or Quantities of the Forces, and the Removals from the Center are Equal, the Moments of the Bodies will be as the Gravitating or Contractive Forces, and the Percuſſions.

Corol. 8. IF $C = c$, and $P = p$, $\frac{M}{m} = \frac{AR}{Br}$, or, if the Gravitating or Contractive Forces, and the Percuſſions are Equal, the Moments of the Bodies will be as the Sums of thoſe Forces, and their Removal from the Center, which is the Caſe of the Vectis, and of the Mechanical Powers, ſo far as they are Reducible to it, and which from what has been ſaid are but illy Explain'd, and Accounted for, by an Abſtracted Reaſoning, and without the Real Forces here aſſerted. And if *A* and *B*, as in *Corol.* 6. are Equal, and the Forces of Gravity are increas'd in Proportion to their Diſtance from the Center of Detention, $\frac{M}{m} = \frac{R^2}{r^2}$, or the Gravitating Forces will be as the Squares of the Diſtances from the Center of Detention, and therefore, ſince they they have the Nature of an Expanſive from it, their Forces towards that Center will be Reciprocally, as the Squares of the ſaid Diſtances, which is a Celebrated Propoſition in the preſent Phyſical Aſtronomy, and which is affirm'd to be the Law, by which the Planets Gravitate towards the Sun.

Corol. 9. If $C=c$, and $R=r$, $\frac{M}{m}=\frac{AP}{Bp}$, or, if the Gravitating or Contractive Forces, and their Removals from the Center are Equal, the Moments of the Bodies will be as the Sums or Quantities of thoſe Forces, and as the Percuſſions.

Corol. 10. If $P=p$, and $R=r$, $\frac{M}{m}=\frac{AC}{Bc}$, or, if the Percuſſions and the Removals from the Center are Equal, the Moments of the Bodies will be as the Gravitating or Contractive Forces, and the Sums or Quantities of them.

Corol. 11. If $A=B$, $C=c$, and $P=p$, $\frac{M}{m}=\frac{R}{r}$, or, if the Gravitating or Contractive Forces, the Sums or Quantities of them, and the Percuſſions are Equal, the Moments of the Bodies will be as the Diſtances or Removals from the Center.

Corol. 12. If $A=B$, $C=c$, and $R=r$, $\frac{M}{m}=\frac{P}{p}$, or, if the Gravitating or Contractive Forces, the Sums or Quantities of them, and their Removals from the Center are Equal, the Moments of the Bodies will be as the Percuſſions.

Corol. 13. If $C=c$, $P=p$, and $R=r$, $\frac{M}{m}=\frac{A}{B}$, or, if the Gravitating or Contractive Forces, the Percuſſions, and the Removals from the Center are Equal, the Moments of the Bodies will be as the Sums or Quantities of thoſe Forces.

Corol. 14. If $A=B$, $P=p$, and $R=r$, $\frac{M}{m}=\frac{C}{c}$, or, if the Sums or Quantities of the Gravitating or Contractive Forces, the Percuſſions, and the Removals from the Center are Equal, the Moments of the Bodies will be as the Gravitating or Contractive Forces.

Corol. 15. If $M=m$, $ACPR=Bcpr$, and $A.B::cpr.CPR$, or, if the Moments of the Bodies are Equal, the Sum or Quantity of the Gravitating or Contractive Forces will be in a Reciprocal Proportion of thoſe Forces, of the Percuſſions, and the Removals from the Center.

Corol. 16. If $M=m$, $ACPR=Bcpr$, and $C.c::Bpr.APR$, or, if the Moments of the Bodies are Equal, the Contractive or Gravitating Forces will be in a Reciprocal Proportion of the Sums or Quantities of ſuch Forces, of the Percuſſions, and Removals from the Center.

Corol. 17. If $M=m$, $ACPR=Bcpr$, and $P.p::Bcr.ACR$, or, if the Moments are Equal, the Percuſſions will be in a Reciprocal Proportion of the Gravitating or Contractive Forces, of the Quantities or Sums of them, and of their Removals from the Center.

Corol. 18. If $M=m$, $ACPR=Bcpr$, and $R.r::Bcp.ACP$, or, if the Moments are Equal, the Diſtances or Removals from the Center are in a Reciprocal Proportion of the Gravitating or Contractive Forces, the Sums of them, and the Percuſſions.

Corol. 19. If $M=m$, that is, $ACPR=Bcpr$; $AC.Bc::pr.PR$, and $AP.Bp::cr.CR$, and $AR.Br::cp.CP$.

Corol. 20. If $M=m$, and $A=B$, $CPR=cpr$, and $C.c::pr.PR$, or $P.p::cr.CR$, or $R.r::cp.CP$; That is, if the Moments are Equal, and the Sums of the Gravitating or Contractive Forces are ſo too; The Forces will be in a Reciprocal Proportion of their Removals from the Center, and of the Percuſſions; The Percuſſions, in a Reciprocal one of the Forces, and of their Removals from the Center; and the Removals from the Center, in a Reciprocal one of the Percuſſions, and of the Forces.

Corol. 21. If $M=m$, and $C=c$, $APR=Bpr$, and $A.B::pr.PR$, and $P.p::Br.AR$, and $R.r::Bp.AP$.

Corol. 22. If $M=m$, and $R=r$, $ACP=Bcp$, and $A.B::cp.CP$, and $C.c::Bp.AP$, and $P.p::Bc.AC$.

Corol. 23. If $M=m$, $A=B$, $C=c$, $PR=pr$, and $P.p::r.R$, or, if the Moments are Equal, the Sums of the Contractive Forces, and the Forces, the Percuſſions will be in a Reciprocal Proportion of their Diſtances from the Center.

Corol. 24. If $M=m$, $A=B$, $P=p$, $CR=cr$, and $C.c::r.R$, or, if the Moments are Equal, the Sums of the Contractive Forces, and the Percuſſions, the Forces will be in a Reciprocal Proportion of their Diſtances from the Center.

Sir Iſ. Newt. Princ. Prop. 8. Theorem 8. Part 3. Edit. 2. Corol. 3. & 4.

Thus a Globe of pure Lead, Iron, Wood, or Wool, without any Pores or Vacuities, will be Equal in it's Moment and Weight to a Globe of Gold of the ſame Diameter, without any Pores or Vacuities, if ſo much, as the Gold is Heavier, by ſo much farther the Globe of Lead, Iron, Wood, or Wool, be Remov'd from the Center of Detention; Thus alſo the Planets, the Nearer they are to the Sun, the Denſer they are, and the Farther they are Remov'd from it, the Rarer; *Jupiter* is Denſer than *Saturn*, the *Earth* than *Jupiter*, the *Moon* than the *Earth*, and *Mercury* than Either; but this Denſity ſuppoſes all Matter to have the ſame Gravitating Force, and only the Parts of it to be more Conſtipated, whereas the preſent Corollary ſuppoſes the Gravitating Forces to be Different, and that the Matter of *Jupiter* is intrinſically Heavier than that of *Saturn*, that of the *Earth* than that of *Jupiter*, and that of *Mercury* than that of the *Earth*, or the *Moon*, by which, according to this Corollary, their Moments will be Equal, and the Æquilibrium of the Planetary Syſtem will be preſerv'd.

Corol. 25. If $M=m$, $A=B$, $R=r$, $CP=cp$, and $C.c::p.P$, or, if the Moments are Equal, the Sums of the Forces, and the Diſtances from the Center, the Forces will be in a Reciprocal Proportion of the Percuſſions.

Corol. 26. If $M=m$, $C=c$, and $P=p$, $AR=Br$, and $A.B::r.R$, or, if the Moments are Equal, the Forces, and the Percuſſions, the Sums or Quantities of the Forces will be in a Reciprocal Proportion of the Diſtances or Removals from the Center, which is the Caſe of the Vectis, when the Weights are in an Equal Balance, that is, 1 *l.* of Lead at 5 Removes from the Center, will be Equal to 5 *l* at 1 Remove from it; And if we take in the Intrinſick Forces Themſelves, that is, if $M=m$, $P=p$, ACR will be $=Bcr$, according to which it is not Impoſſible, but it may have pleas'd Almighty God to adjuſt the Situations and Poſitions of the Heavenly Bodies, ſo that the Intrinſick, Innate, and Different Gravities of *Mercury*, *Venus*, the *Earth*, of *Mars*, *Jupiter*, and *Saturn* into their Diſtances and Bulks, ſhou'd be Equal, by which there wou'd be a perfect Balance amongſt them, and if the leaſt Increas'd on the one Hand, or the other, there wou'd be a Preponderancy, and conſequently, a Revolution; but of this we ſhall ſay more at the End of theſe Corollaries.

Corol. 27. If $M=m$, $C=c$, and $R=r$, $AP=Bp$, and $A.B::p.P$, or, if the Moments are Equal, the Contractive Forces, and the Removals from the Center, the Sums of the Forces will be in a Reciprocal Proportion of the Percuſſions.

Corol. 28. If $M=m$, $P=p$, $R=r$, $AC=Bc$, and $A.B::c.C$, or, if the Moments are Equal, the Percuſſions, and the Removals from the Center, the Sums or Quantities of the Forces will be Reciprocally as the Forces; thus, if Gold is 19 Times Heavier than Water, 19 Parts of Water will be Equal, in their Moment, to 1 of Gold, ſuppoſing no Pores or Vacuities in them; and thus, if the Planets revolv'd at Equal Diſtances from the Sun, and their Moments were Equal, their Gravitating Forces wou'd be Reciprocally, as their Quantities or Sums.

The Laws of Percuſſion hitherto conſider'd in Conjunction with the Gravitating Force, in Obedience to the Preſent Philoſophy, which joins a Gravitating with a Projectile.

§. 3. I have hitherto conſider'd the Laws of Percuſſion, in Conjunction with the Force of Gravitation, in Obedience to the Preſent Philoſophy, which unites a Projectile with it, which indeed it found neceſſary to do, becauſe, it was by Means of the Projectile, that the Gravitating Force was ſuſtain'd from falling to the Center, and by the Gravitating Force, that the Projectile was detain'd from flying off in a Tangent; The Projectile therefore ſeems to have been a mere Invention only, to keep the Gravitating Forces in Action, which otherwiſe wou'd have immediately Tumbled to the Center, and to ſupport the whole Syſtem, which wou'd have been Ruin'd at once, rather than any Force, which is Obſervable in Nature, or has it's Foundation in the Exiſtence of Things.

The Revolutions of Bodies accounted for by a Gravitating Force without a Pro-

§. 4. On the Contrary, in the Way, which we go, of Conſidering Bodies, Acting with a Force of Gravitation, round a Center of Detention, we have no Occaſion for ſuch Projectile Forces, or of any Percuſſions, which, as we have ſhewn, only Tend to promote and increaſe the Effect of Gravitation, and which we ſhall therefore now Diſmiſs, nor do we make uſe of any other Forces, than what, on all *jectile, which, when ſpent, wou'd leave them Quieſcent.*

Hands, are acknowledg'd, and are apparent in every Common Vectis or Pendulum, all that we require is, that, if there are Gravitating Forces, which are Capable of deſcribing Oſcillations round a Center of Detention, there may be ſuch a Proportion of thoſe Forces in themſelves, and by their Removal from the Center of Detention, which ſhall carry the Body round in Entire Revolutions; The Tendency of all Bodies to the Earth hinders this in the Vectis, and in Pendulums; But let us now ſuppoſe a Body, which has a Contractive or Gravitating Force in it ſelf and which it wou'd exert if it approach'd near to other Bodies, Plac'd at an Immenſe Diſtance from all Material Beings whatſoever; It cou'd not Deſcend, nor cou'd it Aſcend, nor Move one Way or Another, becauſe there is nothing to determine it to any Side or Part of Space, and it wou'd remain perfectly Quieſcent in the vaſt Expanſe, Immoveable in it ſelf, and obey the Power of it's own Contractive and Reſiſting Force to Motion; If any Projectile Force ſhou'd be Communicated to it, the Effect wou'd be, that it wou'd Roll on in Space that Way, to which the Projectile Force determin'd it, and when that Impreſs'd Force was ſpent, wou'd be again Quieſcent; which wou'd be the Caſe of the Earth and Planets, Acted by a Temporary, Finite, and a Projectile Force, and as for an Infinite one, we ſhall afterwards ſhew there is no ſuch Affirm'd, when we come to Inquire into the Phyſical Aſtronomy.

Yet let this Contractive, Quieſcent, and Gravitating Force be plac'd at the End of an Inflexible Line or Balance, namely, at *A* in the preceding Figure, and another Equal to it at *B*, at Different Diſtances from the Center of Detention *C*, the Firſt wou'd Preponderate the Laſt, and deſcend to *D*, whilſt the other ris' to *E*, upon the very Account that they have Moments and Forces at Different Diſtances from the ſame Center, and ſince there is no Tendency to the Earth, or any other Body to hinder this Motion, it wou'd continue for Ever in a Conſtant Revolution, For *D* has the ſame Preponderancy to *E*, in this Situation, as *A* had to *B*, in the Former, having the ſame Relation to the Point or Center of Detention *C*, as before.

If therefore now the Force at *B* and *E* is taken off, *A* and *D* will ſtill have a Greater Force of Revolving round the ſame Center, there being nothing to prohibit or retard it's Motion, either from *B* or *E*, or from the Earth, or any other Body, it being only conſider'd with Relation to it's Center of Detention, from which by it's own proper Gravitating Force it wou'd Recede, but is confin'd to it, by the imagin'd and inflexible Line *AC*; It muſt be farther obſerv'd, that as the Forces at *A* and *B*, are the ſame in reſpect to their Center of Detention, whether they act from *A* or *B*, or from *P* and *Q*, in a Common Balance, where the Force acts Perpendicularly to the Earth, that is, at the ſame Diſtances from that Center, ſo is it in Reſpect of the Celeſtial one, for the Forces will be the ſame in every Part of the ſeveral Orbits, where they are Equidiſtant from it.

Why the Planets Revolve the ſame Way.

§. 5. Let *H* repreſent *Mercury*, *N*, *Venus*, and *B*, the *Earth*, on one Side of the Balance, *R*, *Mars*, *O*, *Jupiter*, and *A*, *Saturn*, on the other, and let *C*, the Center of Detention, repreſent the *Sun*.

It is Manifeſt, if theſe Planets are with their Diſtances from the Center, and their Innate Forces in an Æquilibrium, at *A*, *O*, *R*, *H*, *N*, *B*, they will be ſo in every other the ſame Diſtance from *C*, and therefore wou'd be Quieſcent, but ſince they are not Quieſcent, the Planets at *A*, *O*, and *R*, that is, *Saturn*, *Jupiter*, and *Mars*, do Preponderate thoſe at *B*, *N*, and *H*, namely, the *Earth* with it's *Moon*, *Venus* and *Mercury*, and by Conſequence, all of them revolve the ſame Way, from Weſt to Eaſt, that is, from *A* to *D*, and from *B* to *E*, a Phænomenon, which has not been yet ſolv'd, no more than that of their Revolutions.

That Revolutions ſhou'd be perform'd, and that they ſhou'd be perform'd the ſame Way, is Manifeſt from this Doctrine, as alſo that a Revolution may be perform'd by one Gravitating Body only about a Center of Detention, as that of the *Moon* about the *Earth*, which therefore is a Conviction of the Truth of ſuch a Doctrine, that we find it ſo in Fact, and of the Comprehenſiveneſs of it, that it ſolves thoſe Phænomenons, an Account of which has been hitherto ſo much Deſpair'd of, that it has ſcarcely been Attempted, and which yet we ſhall now Propoſe another Way.

Another Way of Explaining the Revolutions of the Planets.

§. 6. If *A* Preſſes, Acts, or Gravitates towards *B*, with a Force or Moment, which is compenſated by an Equal Action, Gravitation, or Preſſure of *B* towards *A*, *A* and *B* will remain Quieſcent, their Mutual and Equal Moments being ſupported and

balance'd

balance't by each other; But, if *A* Acts or Preſſes with a Greater Force or Moment towards *B*, than *B* towards *A*, the Exceſs of the one Force above the other, will cauſe a Conatus; or endeavour in *A* to approach towards *B*, which ſince it cannot be Effected in the Direct Line *A B*, which is ſuppos'd Inflexible, it will (as all other Actions of Bodies vent themſelves that Way, where there is the leaſt Reſiſtance to them) Exert it ſelf Side-ways, for Inſtance, towards *D*, whilſt *B* recedes to *E*, ſo as to preſerve the ſame Diſtance from *C*, which Action ſtill Continuing the Effect will be Continu'd, and Revolutions Perform'd by it; And if *B* is taken away, *A* Preſſing or Acting towards *C*, and not finding a Paſſage for it's Action, will Exert it ſelf Side-ways to *D*, and by that Means revolve round the Center, which it Endeavours to approach.

THIS cannot obtain in Bodies which Gravitate to the Earth, ſo as, being ſtopt at the Surface, they ſhou'd revolve round it, by an Exertion of their Action Side-ways, becauſe they have not an Exceſs of Gravity above other Bodies, but are nearly in an Equal Balance with them, and therefore muſt be Quieſcent, however, that there is ſome Action Side-ways upon their being ſtopt at the Surface, is Evident from almoſt all Bodies dropt Perpendicularly to it, which, more or leſs, are thrown, from the Place where they fall, into a Lateral Direction.

As alſo the Surface of the Earth it ſelf (and the ſame may be ſaid of the other Planets) recedes Laterally by this Side-Action or Preſſure, where the Direct cannot obtain, and by that Means it and all the Planets are Revolv'd about their Axes the ſame Way, and upon the ſame Principles, as their ſeveral Gyrations in their Orbits are Perform'd, which is another Phænomenon, not in the leaſt Explain'd, by the Phyſical, or any other Aſtronomy.

Why the Orbits of the Planets are Elliptical.

§. 7. FROM hence likewiſe it is Manifeſt, That the Orbits of the Planets will be Elliptical, and that they are deſcrib'd in the Periheliums with a Greater, and in the Apheliums with a Leſs Velocity; For the Force or Gravitation at *A*, ſtill Preſſing more and more towards *C*, ſuppoſing the Line *A C* not to be wholly Inflexible, becomes continually ſhorter, till the Planet arrives at the Perihelium *B*, where it obtains it's utmoſt Preſſure and Approach towards *C*, and from thence is again Weaken'd and Diminiſh't to it's Return in *A*, and conſequently, where the Force is Stronger, it will be carry'd with a more Intenſe, and where Diminiſh't and Weaker with a more Remiſs and Languid Velocity.

THUS, if inſtead of the Inflexible Line, we ſubſtitute an Expanſive or Repulſive Force in the Sun, which may be as juſtly allow'd, from the Planets not falling Directly to it by their proper Gravitation, as that there is a Gravitating Force in Bodies to each other, tho' at ſeveral Thouſand Miles Diſtance, and from the very Nature of that Body, which Exerts it ſelf in an Expanſive Force of Light and Heat, the Repulſive or Expanſive Force from *C*, falling directly upon *A*, prohibits any nearer Approach to it, which therefore Preſſing towards *D*, and meeting with an Obliquity of the Expanſive Force from *C*, and which is thereby weaken'd, it Approaches nearer to the Center, till at Length, by it's utmoſt Efforts, it arrives at *B*, the Neareſt it can to *C*, but there being a Direct Action from *C*, the Contractive or Gravitating removes to a Weaker, and more Oblique of the Expanſive *E*, and ſo Revolves to *A*. The Center of this Expanſive Force is the Center of Detention, by the Endeavours of the Gravitating to Approach to it.

Dr. Greg. Aſtron. Lib. I. Prop. 66. 67, 68.

HE who will Impartially weigh, and conſider what I have writ, and compare it with *Kepler*'s Phyſical Account of theſe Matters, will perhaps find the Principles, I have propos'd, are more Rational, and leſs Romantick than his, who yet was the firſt Author of this Doctrine, into which we now Examine, nay, I may farther add, are juſtify'd by Nature, and by Fact, and a fair and ſincere Reaſoning upon them.

BUT if what I have offer'd may be more liable to Exception, upon the Expanſive Force introduc'd in this Account, to ſupply the Place of a Vectis or Inflexible Line, it will come to the ſame, If we ſuppoſe a Force of Attraction in the Sun, or in the Center of Detention in a Vectis, and a Centrifugal one in *A*, by Endeavouring to depart from that Center, by it's Proper Force of Contraction and Gravitation, which Terms, perhaps, will be more Eaſily admitted, according to the preſent Current Opinions in Philoſophy, tho', in Truth and Reality, they are not Different from what I have mention'd. Having therefore given an Account, as far as we cou'd in ſo ſhort a Compaſs, of the Cauſes of the Revolutions of the Planets, under the Direction and Providence of God, who Eſtabliſhes his own Laws, by which all Nature ſhall Move and be Acted, We now Return to our Proportions of theſe Forces.

A Computation of the Gravitating Forces of the ſeveral Planets from this Doctrine.

§. 8. THE Proportions of theſe Forces are, what we are next to Determine, in the Computing of which we conſider C to be a Center of Detention, and the Moments of the Planets to be $\frac{M}{m} = \frac{ACR}{Bcr}$, and M to be $= m$, and therefore $ACR = Bcr$. See *Corol.* 8. 24. 26. preced. Theorem.

THUS, ſuppoſe the Diſtances of *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn*, from the *Sun* to be, as 4. 7. 10. 15. 52. 95. and their Diameters to be, as 3. 5. 8. 3. 52. 44. which yet are Uncertain, Different Aſtronomers giving very Different Calculations of them, *Mercury* in it's Balance wou'd be Equal to *Venus*, if the Bulk of *Mercury* $= 3 \times 3 \times 3 = 27 = A$, $\times 4 = R$ the Diſtance, $\times 1 = C$ the Gravitating or Contractive Force, that is, if $ACR = 108$ is Equal to $5 \times 5 \times 5 = 125 = B$ the Bulk of *Venus*, $\times 7 = r$ the Diſtance, $\times \frac{108}{875} = c$ the Force, that is, if $Bcr = 875 \times \frac{108}{875}$, or, if the Contractive or Gravitating Force of *Mercury* is to that of *Venus*, as 1 to $\frac{1}{8}$, or thereabout; But *Venus* is near Twice the Diſtance from the *Sun*, that *Mercury* is, and ſince the proper Forces are Increas'd, the farther they are Remov'd from the Center of Detention, we muſt take half of the Force in *Venus* for it's proper Contractive one, and therefore the Real Contractive Force in *Mercury*, to that in *Venus*, will be as 16 to 1: And when we conſider the near Approach, which *Mercury* makes to the *Sun*, we need not wonder, that it's Conſtitution has ſo great a Proportion of the Phlegmatick and Contractive in it, ſince otherwiſe, in all Likelihood, it wou'd be Diſſipated into Fumes and Vapours; But theſe Diſtances muſt be Alter'd, if what follows, §. 9. is True.

BY the ſame Way of Reaſoning, the Moment of the *Earth* = to that of *Mercury*, will be, ſuppoſing $z =$ to the Contractive or Gravitating Force of the *Earth*, $5120z = 108$, and the Contractive Force at the *Earth*'s Diſtance from the *Sun* $= z = \frac{108}{5120} = \frac{1}{51}$, or thereabout; therefore, ſince the Diſtance of the *Earth* from the *Sun*, is to that of *Mercury* from the *Sun*, as 10 to 4, or as $2\frac{1}{2}$, or, ſuppoſing, 3 to 1, the proper Contractive Force of the *Earth*, in Compariſon with that of *Mercury*, will be $\frac{1}{3}$ of $\frac{1}{51}$, or $\frac{1}{153}$, or the Contractive Force of *Mercury* will be 153 Times Greater than that of the *Earth*'s.

THUS alſo *Mars*'s Moment will be $= 405z = 108$ the Moment of *Mercury*, and $z = \frac{108}{405} = \frac{1}{4}$, or thereabout, but the Diſtance of *Mars* from the *Sun* is to that of *Mercury* from the *Sun*, as 15 or 16 to 4, or as 4 to 1, the proper Contractive or Gravitating Force, therefore, of *Mars* in Compariſon with that of *Mercury*, will be $\frac{1}{4}$ of $\frac{1}{4}$, or $\frac{1}{16}$, or the Contractive Force of *Mercury* will be 16 Times Greater than that of *Mars*.

LIKEWISE the Moment of *Jupiter* will be $= 7311616z = 108$, the Moment of *Mercury*, and $z = \frac{108}{7311616}$, or $\frac{1}{73116}$, or thereabout; But *Jupiter*'s Diſtance from the *Sun* is to that of *Mercury* from it, as 52 to 4, or 13 to 1, therefore $\frac{1}{13}$ of $\frac{1}{73116}$, is Equal to the proper Gravitating or Contractive Force of *Jupiter* in Compariſon with that of *Mercury*, or the Contractive Force of *Mercury* is to that of *Jupiter*, as 950508 to 1.

LASTLY, The Moment of *Saturn* is Equal to $8292680z = 108$ the Moment of *Mercury*, or $z = \frac{108}{8292680} = \frac{1}{82926}$, or thereabout; But the Diſtance of *Saturn* from the *Sun* is to that of *Mercury* from it, as 95 to 4, or, as 24 to 1 nearly, therefore the proper Force of Gravitation or Contraction of *Saturn* will be, in it's Compariſon with that of *Mercury*, $\frac{1}{24}$ of $\frac{1}{82926}$, or the Contractive or Gravitating Force of *Mercury* will be to that of *Saturn*, as 2090224 to 1. And the ſame Kind of Calculation may be Extended to the *Moon*, and the *Secondary Planets*; which *Calculus* of the Forces of the Planets, I dare Affirm and can Prove, requires no more Precarious Things than what are Demanded by Sir *Iſ. Newton* in his Determining the Denſities of Them.

Sir Iſ. Newt. Princip. Prop. 8. & Corol. Edit. 1. & 2.

Concerning the Periodical Times, and the Diſtances

§. 9. IF therefore *Mercury* by it's Force, according to the Method we have before Explain'd, is capable of Revolving round the *Sun*, all the Planets at their ſeveral Diſtances, and whoſe Moments are ſuppos'd Equal to that of *Mercury*, will admit of

Revo-

of the Planets, and how we may Aſcertain the Latter from the Former.

Revolutions at thoſe Diſtances; *Mercury* Performs it's Period in about 3 Months, *Venus* in $7\frac{1}{2}$, or 8 Months, the *Earth* in 12, *Mars* in 24, *Jupiter* in 144, and *Saturn* in 360. Conſequently, If the Moments of the Planets are Equal, as there is all the Reaſon in the World to think they are, in their Revolutions about the *Sun*, becauſe there can be no Imaginable Account given, why, if *Mercury*, for Inſtance, revolves round it, by a certain Moment made up of it's particular Force, the Quantity of that Force, and the Diſtance, *Venus* ſhou'd not do it by an Equal Moment, made up of it's particular Force, the Quantity of that Force, and it's Diſtance; ſince, whatever the Action is betwixt theſe Bodies and the *Sun*, the Relative Moment, which they have to each other, muſt be the ſame, unleſs we ſuppoſe the *Sun*, or theſe Bodies, can Exert themſelves Differently, all Circumſtances continuing the ſame, or alike and in the ſame Manner, when the Circumſtances of 'em, as thoſe of the Moments of Force, Quantity, and Diſtance, are alter'd; which is as much as to affirm, that the *Sun* can Burn up the *Earth*, at the Diſtance of *Mercury*, and yet be no Hotter than it is with us, at the ſame Diſtance. If then the Moments of the Planets are Equal, and *Mercury* at the Diſtance of 3, and with it's proper Force, and the Bulk or Quantity of it, deſcribes it's Period or Circle, which will likewiſe be as 3, in 3 Months, *Venus* at the Diſtance of $7\frac{1}{2}$ or 8, will perform it's Period, which will be as $7\frac{1}{2}$ or 8, in $7\frac{1}{2}$ or 8 Months, the *Earth* at the Diſtance of 12, it's Period, which will likewiſe be as 12, in 12 Months, *Mars* at the Diſtance of 24, in 24 Months, *Jupiter* at the Diſtance of 144, in 144 Months, *Saturn* at the Diſtance of 360, in 360 Months. But theſe are not the Diſtances Aſſign'd by Aſtronomers, which are of *Mercury*, as 4, which is 1 More than theſe Periods and theſe Equal Moments require, of *Venus*, as $7\frac{1}{2}$ or 8, which is Agreeable and Correſpondent to them, of the *Earth*, as 10, which is 2 Leſs, of *Mars*, as 15, which is 9 Leſs, of *Jupiter*, as 52, which is not half ſo much, of *Saturn*, as 92, which is not one Third ſo much, as theſe Periods and theſe Equal Moments aſſert. Is it therefore Juſt and Reaſonable for us to aſcertain the Diſtances by the Periods, and Equal Moments, or the Periods by the Diſtances? If the Former, we proceed according to Fair and Rational Deductions, if the Latter, we ſtand by very Fallible, and very Uncertain Obſervations.

Dr. Keill's Aſtronom. Lect. 27.

As to the Periodick Times, They are Determin'd, by finding Two Heliocentrick Places of a Superior Planet, from Two Oppoſitions of it with the *Sun*, and of an Inferior, from Two Conjunctions with it, or from it's Greateſt Elongations, for the Arch, betwixt thoſe Places, will be to the Circumference, as the Time, in which it is deſcrib'd, to the Periodick Time; or elſe by the Planet's being Twice in the ſame Node, for the Time betwixt the Two Obſervations is the Periodick Time; ſuppoſing then the Equal Moments of the Planets, if one Planet, with a Certain Moment, deſcribes one Certain Space in one Time, Another, with an an Equal Moment, will deſcribe Twice the Space in Double that Time, and Thrice the Space in Treble the Time, *&c.* and conſequently, ſince the Circles are Proportional to their Radius's, or the Orbits of the Planets to their Diſtances from the *Sun*, the Radius's will be Determin'd from the Circles, or the Diſtances from the Revolutions, but not the Contrary.

The Modern Way of Aſcertaining the Diſtances of the Planets, Precarious and Fallible.

Rohault. Phyſ. Cap. 13. Part 2. Annotat.

Sir Iſ. Newt. Princip. Phænom. 4. Lib. 3.

§. 10. For, on the Contrary, in the firſt Place, the Diſtances in Fact are Variouſly recounted by Aſtronomers, ſome making the *Earth* to be 12000 of it's Diameters Remov'd from the *Sun*, others 10000, and the Lateſt, and, it is ſaid, the moſt Exact, only 5000, and this vaſt and immenſe Difference ariſes from the great Difficulty and Labour of Determining the *Sun*'s Parallax; If therefore the Diſtance of the *Earth* from the *Sun* is thus unſettled, we cannot ſuppoſe that thoſe of the other Planets are much better aſcertain'd. And even thoſe, who pretend to compute them, from the Squares of the Periodick Times being as the Cubes of the Diſtances, which Diſtances yet are thus Precarious, make them Differ from what is ſaid to be found by Obſervation.

The Middle Diſtances of the Planets and the *Earth* from the *Sun*,

	Saturn.	*Jupiter.*	*Mars.*	The *Earth.*	*Venus.*	*Mercury.*
According to *Kepler*,	951000.	519650.	152350.	100000.	72400.	38806.
Bullialdus,	954198.	522520.	152350.	100000.	72398.	38585.
The Periodick Times,	953806.	520116.	152399.	100000.	72333.	38710.

IN which the Diſtances, Calculated from the Periodick Times, are Different, from what are affirm'd by Aſtronomers, ſeveral Hundred Parts, and ſometimes more than a Thouſand, each of which Parts is one Hundred Thouſandth Part of the *Earth's* Diſtance from the *Sun*, which, according to the moſt moderate Computation, will be 54 Millions of Miles, and therefore one of thoſe Parts will be Equal to 540 Miles, and a Thouſand of them to 540 Thouſand Miles, which is above Twice as far, as the Moon is from us, and therefore no Minutia in Nature.

BESIDES which, let us now Examine into the Way and Method, by which theſe Diſtances are Computed, which is, by firſt finding the Parallax of the Phænomenon or Planet, whoſe Diſtance is Requir'd, and then it will be, as the Sine of the Parallax, to the Sine of the Apparent Diſtance of the Phænomenon or Planet from the Vertex, ſo the Semidiameter of the Earth, to the Diſtance of the Phænomenon or Planet from the Earth's Center. But this Obſervation of the Parallax is acknowledg'd to be a very ſlippery and uncertain one, and the Propoſitions, which are afterwards made uſe of, to make it leſs liable to Error, depend upon the Theory of the Planets before deliver'd and the Ratio of their Diſtances to one another, whereas their Diſtances, and their Ratio to one another are firſt to be determin'd by Previous Obſervations, and from thence the Theory of the Planets is to be form'd and deriv'd, and not aſſum'd to aſcertain the Parallax, and from it the Diſtances. In the foregoing Proportion therefore, there are Two Things, which render the Diſtance very Precarious, which is Concluded from it, Firſt the Semidiameter of the *Earth*, and Second the Parallax.

Dr. Gregory's Aſtronom. Prop. 46. Lib. 2.

Dr. Keill's Aſtronom. Lect. 22.

Dr. Gregory's Aſtron. Sect. 6. Prop. 43, 44. Lib. 3.

As to the Firſt, ſome make the whole Diameter to contain 8202 Miles, and others only 7935, in which there is a Difference of above 200 Miles; but that which Contributes the moſt to this Uncertainty, is the Parallax, where the Angles are ſo Minute, that the leaſt Error in Obſervation, which is eaſily committed, will occaſion a great Diſproportion in the Diſtances aſſerted from them. And this is ſo entirely own'd, in Reſpect of the fixt Stars, that it is thought their Diſtances can be no ways determin'd, for if the Parallax of the Orbis Magnus, which, according to Mr. *Flamſteed*, is 42″, ſhou'd be 25″ more than it Really is, which is an Error, in taking ſuch ſmall Angles, which can ſcarcely be avoided, the Diſtance of the fixt Stars wou'd be Double of what is affirm'd from Obſervation. And in the Planets, if any Phænomenon is farther Diſtant than 15000 Semidiameters of the Earth from it's Center, which the Sun is ſaid to be, even it's Horizontal Parallax becomes Inſenſible.

Dr. Keill's Aſtronom. Lect. 8. & 22.

THUS alſo it is aſſerted, that, in each of the Superior Planets, *Mars*, *Jupiter*, and *Saturn*, the Parallax of the Orbis Magnus is as the Angle, under which the Semidiameter of that Orb, paſſing thro' the Earth, is ſeen from the Planet, and by how much Nearer the Planet is to the *Earth*, and to the *Sun*, by ſo much will that Angle be Greater, the Parallax therefore of the Orbis Magnus being given in every Planet, it's Diſtance from the *Sun*, it is ſaid, will be Determin'd, in Reſpect of the Earth's Diſtance from it; For the Elongation of the Planet from the *Sun* is known by Obſervation, or by a Calculus, And, as the Sine of the Parallax of the Orbis Magnus, is to the Sine of the Elongation of the Planet from the *Sun*, ſo is the Diſtance of the *Earth* from the *Sun* to that of the Planet from it.

Dr. Keill's Aſtronom. Lect. 16. p. 217.

BUT here again are the Two Recurring Difficulties, how the Parallax of the Orbis Magnus from each Planet, and the Diſtance of the *Earth* from the *Sun*, ſhall be accurately and ſufficiently defin'd, the laſt as we have ſhewn is not done, becauſe the Parallax of the *Sun* cannot be, *Jupiter* and *Saturn* are ſtill farther Diſtant from it, and therefore their Parallaxes will be more Minute, *Venus* is the neareſt to it, and therefore it's Parallax the moſt ſenſible and the beſt determin'd, and conſequently we find §. 9. it's Diſtance, and it's Periodick Time to Agree, and that of the *Earth* and *Mercury* not much to Differ, *Mars* ſomething more, and *Jupiter* and *Saturn* the moſt of all. Beſides one Fundamental Maxim in this Reaſoning about the Diſtances of the Planets is, that the Apparent Diameters of them are increas'd in Proportion to their Approximation to us, which is falſe by what we Experience in other Objects, and is unprov'd in Theſe; Alſo the Heights of any Phænomenons are ſo hard to be determin'd, that I have known very good Obſervers miſtake One or Two Yards in Fifty, which will be no Wonder to thoſe who underſtand, how Nice and Critical Trigonometry is in the Practice, and conſequently the Error in the Meaſure of many Millions of Millions cannot be Inconſiderable.

Dr. Keill's Aſtronom. Lect. 1. & 27. P. 440.

WHAT therefore we wou'd Infer from hence is, that ſince it is ſo difficult a Matter to fix the Diſtances of the Planets from the Parallaxes, and the Obſervations of them, and ſince, where the Parallaxes are the moſt ſenſible, the Periodick Times come the neareſt to a Proportion with their Diſtances, we may juſtly attribute their Receding from that Proportion to the fallibleneſs and uncertainty of Obſervations, and may therefore Conclude, that the Squares of the Periodick Times are not as the Cubes of the Diſtances, or, which is the ſame, that the Periodick Times are not as the Square-Roots of thoſe Cubes, or in a Seſquiplicate Proportion, but as the Cube-Roots of thoſe Cubes, that is, as their Diſtances.

Concluſions from the foregoing Doctrine.

§. 11. IF this is ſo, which I ~~only~~ Propoſe, and leave to the Inquiries of Others, the Moments of the Planets, with reſpect to the *Sun*, will be the ſame, and their Forces ſuch as I have before mention'd, §. 8. and it will be from thence moreover Conſequent.

1. THAT the Nearer or Farther any Planet is from the *Sun*, it's proper Contractive or Gravitating Force will be Greater or Leſs, excepting in *Mars* and *Venus*, which have the ſame, and are compenſated by the greater Bulk or Quantity of the one near the *Sun*, as in *Venus*, and by the Leſs in the other, which is more Remote from it, as in *Mars*.

2. THAT the Matter therefore in *Mercury* is very near, as Intrinſically Different from that of *Venus* or *Mars*, as Gold with us is from Water, and that it is very near ſo much Heavier and Denſer; that the Matter of *Mercury*, in Reſpect of that of the *Earth*, is 8 Times Heavier and Denſer than Gold is in Reſpect of Water with us, *&c.*

3. THAT the Matter of *Jupiter* is above 6000, and that of *Saturn* above 10000 Times leſs Denſe and Heavy than that of the *Earth*'s, which will not therefore, according to the Notions of the preſent Philoſophy of Similar Matter, be Infinitely Porous, but will be of a Thinner and more Diluted Nature, will fill the ſame Dimenſional Spaces, but the Forces will be Different, leſs Contractive, and more eaſily excited into Action by the *Sun*'s Rays, ſo that the Nature of it may be as juſtly proportion'd to all the Influences of the *Sun* at that Diſtance, as that of the *Earth* is to them at it's Diſtance, or *Mercury* in it's particular Proximity to it. If any one objects other Calculations of the Denſities of the Planets, or the Proportions of them, they proceed upon a Suppoſition of Matter's being Homogeneous and Similar, which we here again Deny, as having been already Confuted.

Sir Iſ. Newt. Princip. Prop. 8. Lib. 3. Theorem 8.

4. THAT hence it is Manifeſt, how Different Matters, and their having in themſelves Different Forces and Moments, may at various Diſtances proceed in an Equable Motion; If the Matter of *Saturn* was the ſame, as that of *Mercury*, at ſuch an Immenſe Diſtance from the *Sun* it's Center of Detention, it wou'd be carry'd with an infinitely Greater Velocity, but it's Force being leſs, and it's Diſtance greater, and the Quantity of thoſe Forces greater, ſo as to be in an Equal Moment with *Mercury*, it will deſcribe the ſame Space in the ſame Time, and therefore it's Periodick Time will be in a Proportion to it's Diſtance; the Preſent Aſtronomers ſay, that it is in the Ratio of the Square of the Time to the Cube of the Diſtance, or as the Periodick Time to the Square-Root of that Cube, which, if we will allow for the uncertainties of Obſervation, may poſſibly be reduc'd to the Cube-Root of that Cube, or the Periodick Time will be as the Diſtance, which we here Account for, but how it ſhou'd be ſo nearly in that Proportion, as the Square-Root of the Cube, is not in the leaſt Explain'd by the Phyſical Aſtronomy, and by that Similar and Homogeneous Matter, which it has thought fit to adopt.

Theorem the Second explain'd.

§. 12. THEOREM the Second is, That the Sums or Quantities of the Contractive or Gravitating Forces are in a Reaſon compounded of a Direct one of the Moments, and of a Reciprocal of the Forces, the Percuſſions, and the Removals from the Center of Detention; For, by the Preceding Theorem, $\frac{M}{m} = \frac{ACPR}{Bcpr}$, therefore $\frac{A}{B} = \frac{Mcpr}{mCPR}$.

§. 13. **Theorem** the Third is, That the Contractive or Gravitating Forces are in a Reaſon compounded of a Direct one of the Moments, and of a Reciprocal of the Sums or Quantities of the Forces, of the Percuſſions, and the Removals from the Center of Detention; For, by the Firſt Theorem, $\frac{M}{m} = \frac{ACPR}{Bcpr}$, therefore $\frac{C}{c} = \frac{MBpr}{mAPR}$.

Theorem the Third explain'd.

§. 14. **Theorem** the Fourth is, That the Percuſſions are in a Reaſon compounded of a Direct one of the Moments, and of a Reciprocal of the Forces, the Quantities of the Forces, and the Removals from the Center of Detention; For, by the Firſt Theorem, $\frac{M}{m} = \frac{ACPR}{Bcpr}$, therefore $\frac{P}{p} = \frac{MBcr}{mACR}$.

Theorem the Fourth explain'd.

§. 15. **Theorem** the Fifth is, That the Removals from the Center of Detention are in a Reaſon compounded of a Direct one of the Moments, and of a Reciprocal of the Forces, the Quantities or Sums of the Forces, and of the Percuſſions; For, by the Firſt Theorem, $\frac{M}{m} = \frac{ACPR}{Bcpr}$, therefore $\frac{R}{r} = \frac{MBcp}{mACP}$.

Theorem the Fifth explain'd.

§. 16. **Theorem** the Sixth is, That the Spaces perform'd by Bodies, which obtain a Contractive or Gravitating Force, and a Force of Percuſſion, and which Act round a Center of Detention, are in a Direct Reaſon of the Forces, the Percuſſions, and their Diſtances from the Center of Detention, where ſuch Forces and Percuſſions act in Conjunction, and ſpend their Moment forwards, and do not return back in Oſcillations, as alſo of the Times and the Denſities of the Spaces perform'd, That is, $\frac{S}{s} = \frac{CPRTD}{cprtd}$. Thus ſuppoſing a Point at *G*, Fig. preced. has a certain Force of Percuſſion and Gravitation, by which at the Diſtance *CG*, it wou'd be carry'd round it's Orbit in a Day's Time, a Gravitation and Percuſſion of a Hundred Times the Force at *G* wou'd deſcribe a Space a Hundred Times Greater, that is, an Orbit whoſe Semidiameter wou'd be a Hundred Times Longer than *CG*; and if the Diſtance *CA* is a Hundred Times Greater than that of *CG*, the Force at *A* wou'd be ſo much Increas'd, and conſequently by that Means wou'd deſcribe 100 of thoſe Spaces, or Orbits, in a Day's Time, and in 100 Days 10000 of them; and ſuppoſing the Spaces, thro' which theſe Revolutions are made, are a Hundred Times Denſer than the others, they will be Equal to 1000000 of the former Revolutions in which the Spaces were a Hundred Times Thinner and more Diluted. Where we ſpeak of the Denſities of Spaces, we do not conſider the Reſiſtances or Actions of them, but only their Cloſer or more Conſtipated Space and Extenſion; If therefore likewiſe the Actions of theſe Spaces were Included in the preſent or former Reaſoning about Space, there wou'd ſtill other Proportions ariſe to the Moments of Bodies, Moving in Them, than what we have mention'd. We ſhall not at Preſent conſider, what Effect Different Percuſſions wou'd have, for the Reaſons alledg'd above, nor yet, what wou'd be the Caſe of differing Denſities, which, with their proper Forces and Actions, yet, in Real Space, and not an Abſtracted and Imaginary one, we may juſtly think will obtain ſome Moment in the Motions of Bodies, and even in the Revolutions of the Planets.

Theorem the Sixth explain'd with it's Corollaries. That the Spaces perform'd by Contractive Bodies Revolving round a Center of Detention are in a Direct Reaſon of the Contractive Forces, the Percuſſions, the Diſtances from the Center, as alſo of the Times and the Denſities of the Spaces.

Corol. 1. If therefore $P = p$, and $D = d$, $\frac{S}{s} = \frac{CRT}{crt}$, That is, one Space, which is perform'd, will be Greater than another, in Proportion, as the Gravitating or Contractive Force, the Removal from the Center of Detention, and the Time is Greater, in which the Greater Force Acts.

Corol. 2. If now $C = c$, $\frac{S}{s} = \frac{RT}{rt}$, or the Spaces perform'd will be Greater, if the Forces are Equal, as the Diſtances from the Center, and the Times, in which Equal Forces Act, are Greater.

Corol. 3. If $R=r$, $\frac{S}{s}=\frac{CT}{ct}$, or, if the Diſtances from the Center are Equal, the Spaces perform'd will be Greater, as the Forces, and the Times, in which Greater Forces Act, are Greater.

Corol. 4. If $T=t$, $\frac{S}{s}=\frac{CR}{cr}$, or, if the Times, in which the Forces Act, are Equal, the Spaces perform'd are Greater, as the Forces and the Diſtances from the Center of Detention are Greater.

Corol. 5. If $C=c$, and $R=r$, or $CR=cr$, $\frac{S}{s}=\frac{T}{t}$, or, if the Forces and Diſtances from the Center, or the Moments of them together, are Equal, the Spaces perform'd will be as the Times, in which thoſe Equal Forces Act.

Corol. 6. If $C=c$, and $T=t$, $\frac{S}{s}=\frac{R}{r}$, or, if the Forces and Times are Equal, the Spaces perform'd will be as the Diſtances from the Center, ſince therefore, where the Forces or Moments of them computed with their Diſtances are Equal, the Spaces, which they deſcribe, will be as the Times, by *Corol.* 5. in ſuch a Caſe $\frac{T}{t}=\frac{R}{r}$, or the Periodick Times will be as the Diſtances; and conſequently, we may juſtly conclude, that this is the Circumſtance of the Heavenly Bodies, ſince that of their Periodick Times being in a Seſquiplicate Reaſon of their Diſtances is not juſtify'd by any Laws of Mechanicks, as we ſhall afterwards ſhew.

Corol. 7. If $R=r$, and $T=t$, $\frac{S}{s}=\frac{C}{c}$, or, if the Diſtances from the Center, and the Times are Equal, the Spaces perform'd will be as the Forces.

Corol. 8. If $S=s$, $CRT=crt$, and $C.c::rt.RT$, or $R.r::ct.CT$, *&c.* for we ſhall go no farther, ſince the Remaining Corollaries may be eaſily deriv'd from the preſent Equation.

Theorem the Seventh explain'd.

§. 17. Theorem the Seventh is, If $\frac{S}{s}=\frac{CRTD}{crtd}$, $\frac{C}{c}=\frac{Srtd}{sRTD}$, $\frac{R}{r}=\frac{Sctd}{sCTD}$, $\frac{T}{t}=\frac{Scrd}{sCRD}$, and $\frac{D}{d}=\frac{Scrt}{sCRT}$, on which we ſhall not Enlarge, but go on to the Expanſive.

Concerning the Expanſive Forces, and their Compariſon with the Contractive.

§. 18. We ſhall now proceed to a Conſideration of the Expanſive Force, and the Theorems which reſult from it, and tho' we made Percuſſion, as well as the Contractive or Gravitating Force to be an Expanſive in Reſpect of the Center of Detention *C*, Fig. preced. becauſe it endeavour'd to recede from it, yet the proper Expanſive Force is that which acts directly Contrary to the Contractive, ſo that as the Contractive, if plac'd in the Center of Detention *C*, wou'd ſtrongly unite it ſelf to it, and preſs towards it, and the farther it retir'd from it the more Languidly and Faintly, or, which is the ſame, more Vigorouſly recede from it; On the Contrary, the Expanſive in the Point *C* will act the moſt forcibly to the Diſſipation of the Contractive, or, any other Force of Detention, which obſtructs it's Action, and more Languidly and Faintly the farther ſuch Force is from the Center of it's Action, or, which is the ſame, leſs violently recede from it. And as to the Laws of the Former or Contractive Force, all Gravitating and Heavy Bodies, ſo far as they are ſo, Cold, Poſitive Reſt, Attraction, Chryſtalization, Coagulation, are to be reduc'd, ſo Fire and Flame, Heat, Vegetation, Fluidity, and Softneſs, Taſt, Smell, and Sound, Light and Colours, Motion, Elaſticity, Ebullition upon the Mixture of an *Acid* and *Alkali*, as they are term'd, will naturally fall under the Laws of the Latter or the Expanſive, as alſo all Animal Force and Action, tho' in all theſe there is a Mixture in Different Proportions of both theſe Forces. The Two Great Principles therefore, which we take to be in Nature, ſo far as it is Material, are thoſe of the Expanſive and Contractive Forces, which Act directly oppoſite to one another, and which are the moſt fully exerted in the utmoſt Heat or Fire, as that of the Sun, and in the utmoſt Cold, as Froſt, and Gravitation it ſelf is no other than a Mixture of the Contractive and Expanſive, in which the Contractive prevails; but ſince we have no exact Meaſures of theſe Forces, we have hitherto made uſe of Heavy Bodies to explain in ſome Degree the Contractive Force from a Center of Detention, and as for the Expanſive, it do's not ſeem to be capable of being reduc'd to any, for there is no Balance of Fire and Flame; we ſhall therefore omit any farther Proſecution of the Centers of Detention and proceed to the Centers of Expanſion and Contraction.

§. 19. **Theorem** the Eighth is, That the Moments of Bodies, which have a Force of Expanſion, and which Act from a Center of it, are in a Reaſon compounded of a Direct of the Expanſive Forces, of the Sums or Quantities of them, and of a Reciprocal of the Diſtances; And if we ſuppoſe a Force acting towards a Center, as the Contractive, and not from a Center of Detention, as before, The Moments of Bodies, which have a Force of Contraction, and which Act towards a Center of it, will be in a Reaſon compounded after the ſame Manner.

Theorem the Eighth explain'd with it's Corollaries; That the Moments of Bodies which have a Force of Expanſion or Contraction, and Act from a Center of it, are in a Reaſon compounded of a Direct one of the Forces, the Sums or Quantities of them, and of a Reciprocal of the Diſtances.

Let us therefore put F, f for the Expanſive or Contractive Forces, and the Moments of Bodies, thus Acting, will be $\frac{M}{m} = \frac{AFr}{BfR}$, That is, the Forces will be Greater as they are Intrinſically ſo, as the Quantities or Sums of them are Greater, and as the Diſtances of their Actions from the Center of them are Leſs; Thus we find it in Fire and Heat, in the Action of the Sun, Percuſſion, Vegetation, and other Expanſive Forces; Thus in Gravitation and Attraction, in Froſt and Cold, which are weaken'd the farther they are from the Center of Contraction, and in other the Contractive Forces.

Corol. 1. If $A = B$, $M . m :: Fr . fR$. *Corol.* 2. If $F = f$, $M . m :: Ar . BR$.
Corol. 3. If $R = r$, $M . m :: AF . Bf$. *Corol.* 4. If $A = B$, and $F = f$, $M . m :: r . R$.
Corol. 5. If $A = B$, and $R = r$, $M . m :: F . f$. *Corol.* 6. If $F = f$, and $R = r$, $M . m :: A . B$.
Corol. 7. If $M = m$, $AFr = BfR$, and $A . B :: fR . Fr$, or $F . f :: BR . Ar$, or $R . r :: AF . Bf$. *Corol.* 8. If $M = m$, and $A = B$, $F . f :: R . r$.
Corol. 9. If $M = m$, and $F = f$, $A . B :: R . r$. *Cor.* 10. If $M = m$, and $R = r$, $A . B :: f . F$.
Corol. 11. $A . B :: MfR . mFr$, and $F . f :: MBR . mAr$, and $r . R :: MBf . mAF$, or $R . r :: mAF . MBf$, &c.

§. 20. We ſhall Firſt therefore conſider the Proportions of Expanſion, the Quantities and Diſtances of it aſſerted in this Theorem, from whence all the Corollaries, which follow it, are deriv'd, and afterwards thoſe of Contraction. Thus in Reſpect of Expanſion it is affirm'd, that it's Force is Greater, as it is Intrinſically ſo, as it's Quantity is Greater, and as it's Diſtance is Leſs.

Inſtances in Nature of the Expanſive Forces.

In Reſpect of the Intrinſick Forces of Expanſion, there is no one will deny, but that the Intrinſick Force of the Flame of Wine, when Burnt, or any other Spirituous Liquor, is leſs in it's Force, than that of the Flame of a Lamp, or a Candle, or any other Oleous Body; that the Flame of an Oleous Body is leſs in it's Force, than that of Common Fire, where a Groſſer Matter is Inflam'd; that Common Fire is ſtill leſs in it's Force, than that of the Immenſe Heat and Fire of the Sun; all Operations in Chymiſtry, where Flame or Ebullitions are produc'd, come under this Head of Expanſive Forces. In Reſpect of the Quantities of theſe Forces, every one knows, that a Spark of the ſame kind of Fire or Flame is leſs in it's Force than a whole Maſs of it, and that one Ray of the Sun is of leſs Power than a Thouſand, or a Million of them Collected together. In Reſpect of the Diſtances, it is alſo Notorious, that the nearer an Approach is made to a Flame, to a Fire, or the Sun, the more violent the Action is, and the more ſenſible is the Heat, and the Expanſive Force of it; and if the *Earth* was plac'd at the Diſtance of *Mercury*, it wou'd in all Probability repel it to the Station, in which it now is, in which it exerts a Gentler and more Moderate Action upon it, to Warm it only into Vegetation, and to keep it in it's Period; and if *Mercury* was where the *Earth* is, it wou'd ſink to it's own Station by the Exceſs of it's Contractive, above the Expanſive of the Sun, and ſo of the other Planets.

Inſtance 1. *The Intrinſick Force of Fire and Flame.*

The Quantity.

The Diſtance.

Thus alſo in Animal Action, as That which exerts it ſelf in the Force of Percuſſion, the Stronger that Action is, the Greater the Quantity of the ſame Force of Percuſſion is, and the Nearer it is to the Center of Percuſſion, that is to the Point, where

Inſtance 2. *The Intrinſick Force of Animal Action, as Percuſſion. The Quantity. The Diſtance.*

where the Stroke is made, upon all theſe Accounts, the Expanſive Force will be Increas'd, and upon the Contrary Diminiſh'd.

Inſtance 3. Vegetation.
The Intrinſick Force.
The Quantity.
The Diſtance.

LIKEWISE in Vegetation, in which there is a Principle of Expanſion contrary to that of Contraction or Gravitation, by which the Stronger it is in it ſelf, and the Greater Quantity there is of the ſame Force, and the Nearer it is to the Center of it's Exertion, the more Power and Moment it will have. For there is more Expanſive Force in one Plant, than there is in Another, as one Riſes higher than another, and this is but a weak Expanſion, and a ſmall Degree of it in all of them; The Force or Principle of Expanſion is ſtill Greater in Shrubs, and the Greateſt in Trees, ſuppoſing the Contractive Force is Equal. And as the Quantities or Sums of theſe Forces are Increas'd, the Moment of them is Greater, thus the Moment of the Expanſive Force in an Oak, is Greater than that in a Walnut or a Willow, in a Walnut or a Willow, than in a Thorn or a Haſle, and in them, than in Plants and in Moſſes; and in the ſame Tree it is Greater in the Root and Trunk, than in the Shoots of the Root or in the Branches, Greater in the Shoots, than in the Strings and Fibres of them, and Greater in the Branches than in the Leaves. Thus alſo the Expanſive Force in Vegetation is Greater in leſs Diſtances from the Center of it's Action, and Leſs in greater Diſtances from it, from whence it is, that in the Root or Center, where this Expanſive Force is the Greateſt, the Colour is ſometimes Rediſh, or a Kind of Orange, or Yellow, or a Yellowiſh White, whereas in the Foliage, or in the Extremities, or greateſt Diſtances of it's Action from the Center, the Colour is generally Green; but, from what we have ſaid in the Doctrine of Light and Colours, Red, Orange, and Yellow are produc'd in us by the Stronger Actions of the Sun's Rays, and Green by the Weaker, Suitably theſe Expanſive Forces, ſtrikeing upon the Ethereal Plenum, with a Greater Percuſſion in the firſt Caſe, give us the Senſations of Red, or Orange, or Yellowiſh, or Pale Yellow, and in the laſt with a Weaker, the Senſations of Various Greens; and the Colours of Flowers proceed from Different Proportions of the ſame Expanſive: Fluidity, and Softneſs, and Taſt and ſome other Qualities in Bodies have a Degree of Expanſion, but we ſhall not conſider them here, becauſe the Inſtances we ſhall at Preſent make uſe of, ſhall be only of thoſe Expanſive Forces, which diſplay themſelves the moſt ſenſibly, that we may thereby the better judge of the Nature and Manner of their Action.

Inſtance 4. Odorous Bodies.
The Intrinſick Force.
The Quantity.
The Diſtance.

SMELL is the next we ſhall Select, which is accounted for, by ſuppoſing, that ſeveral Effluviums are ſent out from the Odorous Body to a certain determin'd Sphere round it, which yet ſeems to be nothing more than this Expanſive Force, Acting upon the Ethereal Plenum, which environs it, to a certain Diſtance; And the Odour is Stronger, as the Expanſive Force is Greater, or as the Quantity of it is Greater, and the Diſtance from the Center of it's Action is Leſs.

Inſtance 5. Sound.
The Intrinſick Force.
The Quantity.
The Diſtance.

ANOTHER Expanſive Force we ſhall mention is, that of Sound, which is Greater, as the Expanſive Force is Greater, as the Quantity of it is ſo, and as the Diſtance is Leſs; a quicker and a more lively Expanſion ſtrikes upon the Ethereal Plenum, and conſequently, the Ear with a ſtronger Percuſſion, all other Things being Equal, than a Flatter, and a Duller, and a Shrill Sound pierces more than a Grave or an Hoarſe one. A Greater Quantity of this Force adds likewiſe to the Moment of it, and a Shrill Bell will make a Greater Impreſſion upon us, than the String of a like Note in a Harp, or a Violin, and a Larger Bell than a Smaller. A leſs Diſtance alſo from the Sonorous Body Increaſes the Sound, and a farther Removal from it Diminiſhes it.

Inſtance 6. Colour.
The Intrinſick Forces.
The Quantity.

THE Sixth kind of Expanſive Force we ſhall Name, ſhall be that of Colour, which by what we have ſhewn in our Philoſophy of Light and Colours, conſiſts of Different Intrinſick and Expanſive Forces, Red has the greateſt, Orange the next, Yellow is after Orange, Green, Blue, and Violet ſucceed the Yellow, and deep Violet has the leaſt Expanſive Force of all, in the Sun's Rays tranſmitted and alter'd thro' a Priſm. And the ſame we may aſſert in Reſpect of Colour'd Bodies, ſtrikeing upon the Ethereal Plenum, with their particular and diſtinct Forces of Expanſion, that Red Bodies have the greateſt Force of Expanſion, Orange the next, and ſo Yellow, Green, Blue, and Violet, and deep Violet in order have a Weaker. And all theſe Differing Colours are more eaſily ſeen, the Larger and Greater the Quantity is of them, a colour'd Point is not ſo readily diſtinguiſh't, as a colour'd Body, the Mind is more

Dubious to what Colour to aſſign a ſmall Atom of Matter, than a whole Maſs of it, and ſtill the Larger that Maſs is, and the Greater the Uniformity of the Impreſſion upon the Senſes, the Mind is left leſs to doubt of the Certainty of the Colour exhibited to it; This perhaps may not be ſo fully apprehended, unleſs after a juſt and perfect Conſideration of it. We may farther ſay, that the Nearer we approach to any colour'd Body, the more Fully and Diſtinctly we perceive it's Colour, and the Farther we are remov'd from it, it is more confuſed and undetermin'd, and therefore it's Expanſive Force Weaker.

The Diſtance.

The Seventh Sort of Expanſive Force we ſhall conſider, is that of Motion, which is the Second Inſtance of the Expanſive Force in Animal Action, Thus it is maniſeſt, we can ſtrike or throw a Ball with a Greater or Leſs Force, and the Moment of it will accordingly be ſo. If we uſe a Finger to communicate this Expanſive Force, it will be Leſs, if a Hand, Greater, and if we exert the whole Moment of all the Muſcles ſubſervient to that End, it will be the Greateſt of all, that is, if the Quantity of the Expanſive Force is the Greateſt. And ſuch a Motion Communicated will be diminiſh'd (as all other Forces are) the Farther it is from the Center of it's Action.

Inſtance 7. Motion impreſs'd.
The Intrinſick Force.
The Quantity.
The Diſtance.

Another Force of Expanſion is that of Elaſticity, which has been ſufficiently exemplify'd in that of Sonorous Bodies.

Inſtance 8.

The laſt Inſtance, we ſhall make uſe of, is that Expanſive Force, which is obſervable in Animal Bodies, which is the Third Inſtance in Animal Action, The Expanſive Force is Greater in the Animal Syſtem of an Eſtrich or an Elephant, than that of an Inſect or a Fly, in a Greater Brute, than in a Leſſer, in a Man, than in a Child, and in one of a Greater, than in one of a Leſſer Stature, all other Things being Equal. The Quantity of the Intrinſick Forces may be exemplify'd the ſame way, for in the foregoing Inſtances, as the Intrinſick Forces are Greater in the one than the other, ſo are likewiſe the Quantities of them. And in all theſe Caſes, the Expanſive Forces are the ſtrongeſt at the Centers, and the weakeſt at the Extremities of the Syſtems, there is more Expanſive Force in the Heart, than there is in the Finger, more in the Blood than in the Veſſels, which contain it, more in the Muſcles, than there is in the Skin, but to fix the preciſe Center of this Expanſive Force, will perhaps be difficult to do, and if we take the Heart, it may be as Probable a one, as any other Point in the Animal Frame, but that we ſhall not now Determine, having given Inſtances enough of the Expanſive Force to the purpoſe of the preſent Theorem, and when we come to our Inquiry into the Principles of Anatomy, we ſhall more fully Explain our ſelves upon that Subject.

Inſtance 9. The Animal Syſtem.
The Intrinſick Force.
The Quantity.
The Diſtance.

§. 21. The Expanſive Force we make to be that, which retires by it's own Energy from a Center, and which therefore is adapted to the Diſpelling and Diſcuſſing of Bodies, which lye in it's Way, on the other Hand the Contractive Force Tends and Acts towards a Center, and therefore Cements, Unites, and Precipitates them to each other, of which laſt Force we ſhall give the following Inſtances.

Inſtances in Nature of the Contractive Forces.

The Contractive Force of Common Salt imparts a Coldneſs to Water, which has ſome Degree of the Expanſive in it from it's Fluidity, and it is known to have this Effect, *viz.* to Cement a Pewter Veſſel, in which the Salt and Water is contain'd, to a Piece of Wood, on which it ſtands;

Inſtance 1. Cold.
The Intrinſick Forces.

The Salts, which mix with the Aqueous Particles in the Air, are ſtill ſtronger by Cementing the Particles of Water it ſelf into Snow and Hail; the Salts, which are infus'd into Water in Froſt, obtain a Greater Force, by which they change it into a Solid and Conſiſtent Body, Ice;

Inſtance 2. Salts.

The Stupendous Globe of Solid and Compacted Salt, the Moon, is yet ſtronger in it's Efficacy and Force, and has it's Influences at 60 Semidiameters of the Earth, or about 240000 Miles from us, ſo that as the Different Kinds of Fire and Flame amongſt us, are deriv'd from that Immenſe Source of it, the Sun, by it's Actions upon our Planet, we may with the ſame and equal Juſtice think, that all Cold and the Various Salts, which are Productive of it, are deriv'd from the Moon, and which therefore is plac'd by the Infinite Wiſdom of Almighty God, at that Diſtance from the Earth, both to Reflect Light to us, and to be a Principle of the Contractive Force, as the Sun is of the Expanſive, which are ſo Evident and Manifeſt in all the Beings we converſe with: That the Moon has this Influence of Cold, is as Evident, as the Sun has that of Heat, ſince in Winter, when the Sun has the leaſt Power by

Inſtance 3. The Moon which ſeems to be an Immenſe Globe of Rock-Salt or Metallick Ice, the Reaſons.

From hence in Winter Clear Moon-ſhine Nights the Froſtieſt.

it's Expanſive Force to Obſtruct the Contractive of the Moon, and when the Air is clear from Vapours, and the Moon obtains it's full Contractive Force, the Nights are always the Sharpeſt and the Froſtieſt, and it has the very ſame Effect in Reſpect of Cold, as the Sun has in Reſpect of Heat, if we compare the Circumſtances of the one with the other; And all Nights are more Cold, when the Moon, as all Days more Hot, when the Sun Prevails.

From hence the Exceſſive Cold in the Northern Climates.

WHAT farther confirms this Aſſertion is, that the nearer we approach to the Northern Pole, and the leſs Effect the Expanſive Force of the Sun has, and the Greater the Contractive of the Moon, the Colder thoſe Parts are, being oppreſs'd with a continual Froſt and Snow, which cannot proceed from any Different Intrinſick Force of the Earth it ſelf, and therefore muſt be owing to a Foreign Impreſſion.

From hence the Higheſt Mountains, and the Higheſt Parts of the Atmoſphere are the Coldeſt.

IT may be alſo alledg'd in Reſpect of the Higheſt Mountains, and the Higheſt Parts of the Atmoſpere, all other Things being Equal, that the Higher they are, the Colder they are, which can be attributed to nothing elſe than their nearer Approximation to the Moon, which will have a much Greater Influence upon them, by ſuch an Elevation than the Sun can have, becauſe the Sun is, at the leaſt Computation, 54 Millions of Miles from us, whereas the Moon's Diſtance is only 240 Thouſand at the moſt, a ſmall Approximation therefore towards the Moon, will have a conſiderable Effect in Compariſon with that towards the Sun, which will have Little or None at all.

From hence the Pale and Silver Light of the Moon deriv'd.

A farther Argument, which we have for the Truth of this, is, That, in fact, Salts Reflect Light with a Greater Force than Earth do's, and which therefore we may ſuppoſe to be the Caſe of the Moon, which, as all other Contractive Bodies, has a contrary Action to the Expanſive of the Sun, and therefore will more Vigorouſly repel it; Salts do likewiſe reflect a Pale and Silver Light, which is the known Property of this Luminary; Nothing of which ſo fully Agrees with a Reflection of it from an Earthy or Graſſy Surface.

From hence the Rays Reflected from the Moon not capable of being Collected to a Burning Focus.

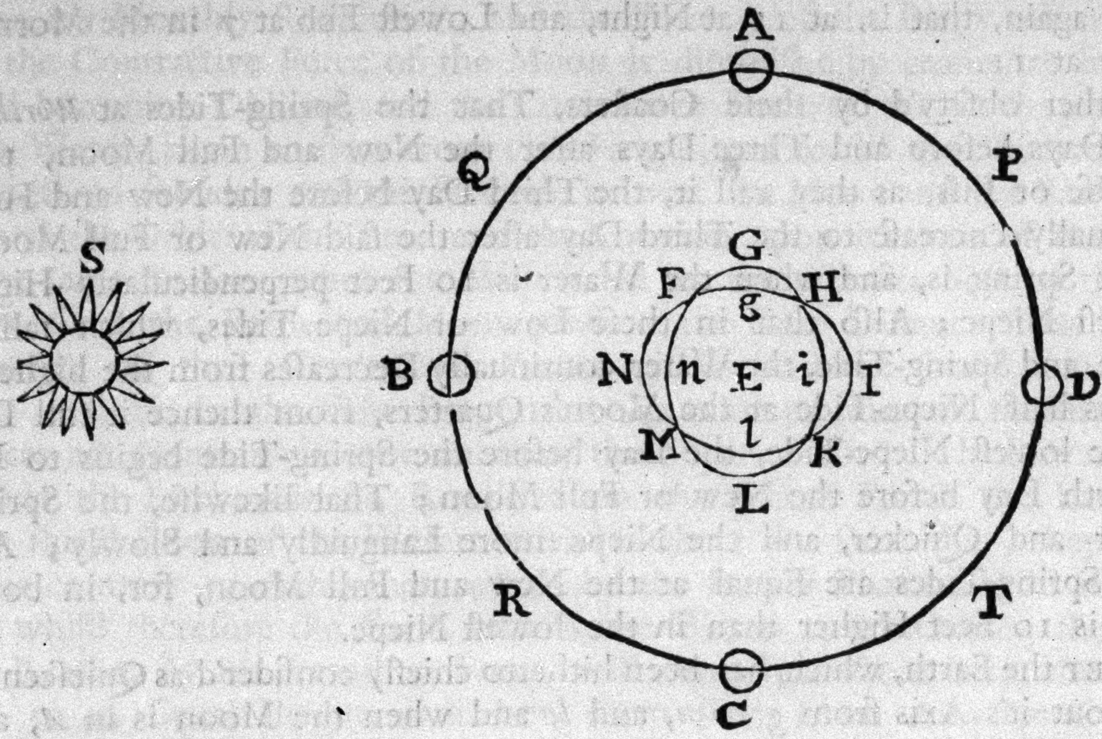
AND laſtly, in the Old or New Moon immediately before or preſently after it's Conjunction with the Sun, the Solar Rays which are reflected from it, are not much diverted from a Rectilinear Progreſs to the Earth, and yet are ſo far from having the Heat and Warmth of the Direct ones, that they can by no Means, nor by any Glaſſes be Collected to an Inflammatory Focus, which can ariſe from nothing elſe, but from the Impreſſions of Cold, which they receive from the Moon, and which they tranſmit with them to the Earth, and for the ſame Reaſon, as the Earth is a Cold and Contractive Body, the Sun's Rays are ſuitably weaken'd by the Reflections from it; There is alſo obſervable a Glowing Brightneſs in the Moon, which is the Property of Light ſtrongly Reflected from Bright, Hard and Icy, or Metalline Bodies, but not from Earth and Graſs.

FROM all which we may Rationally believe and think, that the Moon is a Vaſt Globe or Craggy Rock of Solid and Compacted Salt, Chryſtal, or Metallick Ice, Infinitely Colder and Stronger than that which is produc'd here, by it's Influence only, at ſo many Thouſand Miles Diſtance, that it is Deſolate and Uninhabited, without Atmoſphere, as is Evident from the ſudden Immerſion of Stars into, and their Emerſion from behind it, and plac'd in the Heavens, as the Scriptures inform us, to Govern the Night, that is, to Communicate Light and Cold to us. The Face of the Moon is variouſly diſtinguiſh'd into Parts, ſome of which are Brighter, and the others more Obſcure, the Former of which are compar'd by Philoſophers to Rocks of Adamant, and large Prominences of Pearl and Chryſtal, and tho' ſome have fancy'd the Obſcurer Parts to be Seas and Lakes and Marſhes ſcatter'd about upon it's Surface, as in our Earth, yet, by the better Kind of Teleſcopes, they are found to be Huge Caverns, and void Cavities with the Shadows falling within them, both which Accounts perfectly agree with that Cold, Solid, Icy, and Chryſtal Nature, and the Prerupt and Cragged Figure, which we have aſſign'd it. And it is not certain but the Planets may be of the ſame Species, or at leaſt the Secondary ones.

Dr. Keill's Aſtronom. Lect. 10. Pag. 117.

An Account from hence of the Flux and Reflux of the Sea.

§. 22. FROM the Contractive Force of the Moon ariſes the Flux and Reflux of the Sea, which, by drawing the Waters towards it, makes them Lift and Heave from the Bottom, and from thence, by their proper Gravity, or the Contractive Force of the Earth, they fall downwards, and roll in Waves and Surges to the Shores, or deſcend into ſuch Rivers, which Communicate with the Ocean, and by the Moon's receding, and the Contractive Force being thereby diminiſh'd, the Waters ſink to their former Ebb; by which means the Flux and Reflux of them is Perform'd. Thus, let *S* Repreſent the Sun, *ABCD* the Moon in it's Orbit, *E* the Earth, which we at preſent ſuppoſe to be Quieſcent; When the Moon is in *A*, *g*, *l* are under the perpendicular or ſtrongeſt Influence of it's Contractive Force in *A*, and the Waters will accordingly ſwell to the utmoſt Height in *G*, and *L*; when the Moon is in *B*, *n*, *i*, will riſe to the utmoſt Height in *N*, *I*; when in *C*, from *l*, *g*, again to *L*, *G*; and when in *D* from *i*, *n* to *I*, *N*; in all which Caſes, when the Waters are rais'd in *G*, they will by their own Gravity, or the Contractive Force of the Earth, deſcend towards the Parts *F* and *H*, in *N* towards *F* and *M*, in *L* towards *M* and *K*, and in *I* towards *K* and *H*, which makes the whole Flux of the Waters, by which the Ocean is fill'd to the Shores of it, to the Height of ſeveral Feet, which may be eaſily conceiv'd, if we make the Parts lying above *H* and *F*, *F* and *M*, *M* and *K*, *K* and *H* to repreſent thoſe Shores; But as the Moon recedes from *A* to *B*, by which it's Contractive Force, becoming more Oblique to the Waters in *G*, will be continually Diminiſh'd and in *B* Inſenſible, the Waters in *FGH* will continually ſubſide by the Contractive Force of the Earth, till they fall to *FgH*, where they will be in their utmoſt Ebb, which is, what is call'd, the Reflux of them; and the ſame will happen to the Waters in *N*, when the Moon is in *C*, to the Waters in *L*, when the Moon is in *D*, and to thoſe in *I*, when the Moon is in *A*.



From what has been ſaid it is Manifeſt, that in the Moon's Revolution there will be Two Fluxes, and as many Refluxes to the ſame Part of the Earth, that is, Two Fluxes from *g* to *G*, when the Moon is in *A* or in *C*, and Two Refluxes, when in *B* and *D*; Two Fluxes from *n* to *N*, when the Moon is in *B* or in *D*, and as many Refluxes, when it is in *C* or *A*; Two Fluxes from *l* to *L*, when the Moon is in *C* or in *A*, and as many Refluxes, when in *D* and *B*; and laſtly, Two Fluxes from *i* to *I*, when the Moon is in *D* or in *B*, and as many Refluxes when in *A* and in *C*. Conſequently, every New and Full Moon, when, as in *B* and *D*, the Moon is in Conjunction with or in Oppoſition to the Sun, there will be Spring-Tides, or a Flux of the Sea from *n* to *N*, and when it is in it's Quadratures with it, the Tides will be Niepe, or upon the Reflux from *N* to *n*, which is affirm'd by thoſe, who are Coaſters upon the Sea, and have conſtantly obſerv'd them. And what has been ſaid of *n*, may be ſaid of *g*, *i*, and *l*, &c. when by their Diurnal Rotation they come to the ſame Syzygiæ.

An Hiſtory of the Tides on the Suſſex-*Shore.*

§. 23. Thus at *Worthing-Point* in *Suſſex*, in the Pariſh of *Broadwater*, where the Ocean lies Free and Open, and the Shore is one continu'd Flat for ſeveral Miles together, the Greateſt Flux or Full Sea is at 11 of the Clock in the Morning every New and Full Moon, and the Greateſt Ebb or Reflux 6 Hours after, namely, at 5 a Clock in the Afternoon; it is Full Sea every Day $\frac{3}{4}$ of an Hour and $\frac{1}{2}$ a Quarter later, namely, The firſt Day after New or Full Moon $\frac{3}{4}$ and $\frac{1}{2}$ a Quarter after 11 a

Clock in the Morning, The ſecond Day 1 Hour and $\frac{3}{4}$ after 11, and ſo on, ſo that in ſeven Days after New and Full Moon, Full Sea is about 6 Hours later than it is at the New or Full Moon, or 6 Hours and $\frac{1}{8}$; And in the Quadratures or Quarters of the Moon, or the Middle Time betwixt the New and Full Moon, the Greateſt Ebb is at 11 a Clock in the Morning, and the Greateſt Flow or Full Sea is at 5 in the Afternoon, and the Greateſt Ebb and the Greateſt Flow are always at 6 Hours Diſtance from one another.

At *Shoreham-Ferry*, which is upon the ſame Coaſt, but is an Inlet from the Sea into the Land, the Flow of the Sea is ſaid to be 4 Hours, and the Ebb 8, ſo that it is Full Sea at 11 a Clock in the Morning, every New and Full Moon, as before, and 8 Hours after, the Greateſt Ebb, or at 7 a Clock at Night, and 4 Hours after, Full Water again, that is, at 11 at Night, and Loweſt Ebb at 7 in the Morning, and Full Water at 11.

It is farther obſerv'd by theſe Coaſters, That the Spring-Tides at *Worthing-Point* are Three Days before and Three Days after the New and Full Moon, that they begin to Riſe or Lift, as they call it, the Third Day before the New and Full Moon, and continually encreaſe to the Third Day after the ſaid New or Full Moon, when the Greateſt Spring is, and when the Water is 10 Feet perpendicularly Higher than in the loweſt Niepe; Alſo that in theſe Low or Niepe Tides, which fall betwixt Spring-Tide and Spring-Tide, the Water continually Decreaſes from the higheſt Spring-Tide, and is half Niepe-Tide at the Moon's Quarters, from thence it ſtill Decreaſes, till it is the loweſt Niepe-Tide, the Day before the Spring-Tide begins to Lift, that is, the Fourth Day before the New or Full Moon; That likewiſe, the Spring-Tides move Faſter and Quicker, and the Niepe more Languidly and Slowly; And that, Laſtly, the Spring-Tides are Equal at the New and Full Moon, for, in both Caſes, the Water is 10 Feet Higher than in the loweſt Niepe.

Why there are Two Fluxes and Two Refluxes every 24 Hours.

§. 24. Let the Earth, which has been hitherto chiefly conſider'd as Quieſcent, be now revolv'd about it's Axis from *g* to *n*, and *l*, and when the Moon is in *A*, as before, the Waters will Riſe to *G* and *L*, and thoſe at *N*, and *I* will ſubſide to *n* and *i*, Let *i* by it's Revolution arrive at *g*, under the perpendicular Influence of the Moon in *A*, and the Waters will Riſe or Flow in *I* and *N*, and thoſe at *G* and *L* will ſubſide to *g* and *l*, and ſo of the Reſt; From whence it will follow, that in the ſame Place there will be every Day Two Fluxes and Two Refluxes of the Waters, namely, when *g* is directly under *A*, and in Conjunction with it, and when *g* comes to *l*, and is in Oppoſition to it, there will be a Flux of them, to *G* and *L*, and when *g* comes to *n* or to *i*, in it's Revolution, there will be a Reflux, to wit, when the Moon is in the Meridian above or below the Horizon of the Place, that is, in a perpendicular Situation to it, there will be the Greateſt Flow, and when it is in the Eaſt or Weſt, that is, in the Horizon, or in a Quadrate Situation with the Place, there will be the Greateſt Reflux.

And thus according to the Account, which the Coaſters give at the above-mention'd *Point*, When the Moon is North and South, as they Term it, that is, when it is in the Meridian below the Horizon, or in the Meridian above it, 'tis always High Water; when it is Eaſt or Weſt, that is, when it Riſes or Sets, it is always Low Water; When the Moon Riſes it is Low Water, which continually Increaſes till it comes to the Meridian, when it is High Water, and from thence Decreaſes till the Moon Sets, and then it is Low Water, after which it Increaſes till the Moon is in the Meridian below the Horizon, when it is again High Water, and from thence Decreaſes to the Riſing of the Moon, when it is Low Water; And the Water is equally High, when the Moon is in the Meridian Above or Below the Horizon.

Why the Water is equally High, when the Moon is in the Meridian Above or Below the Horizon.

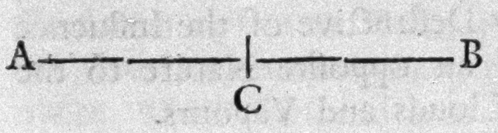
§. 25. The Main and Principal Difficulty here is to aſſign a Reaſon, why the Water Riſes to the ſame Height, when the Moon is in the Meridian above the Horizon, or in that Below it, by the ſame Contractive Force, that is, how it comes to paſs that whether the Moon is in *A* or *C*, the Water in the Diurnal-Tides always Riſes from *g* to *G*, ſince one wou'd think, that the ſame Contractive Force, which in *A* draws the Water from *g* to *G*, in *C* wou'd draw it from *G* to *g*; So likewiſe in the Spring or Monthly Tides, the ſame Effect is obſervable, when the Moon is in Conjunction with the Sun, or in Oppoſition to it, that is, when it is in *B* or in *D*, name-

D, namely, on the ſame Side of the Earth with the Waters, which are raiſ'd, or on the Oppoſite to them.

THE Solution of which can be no other than this, That when the Moon is in *A* in the Diurnal, or in *B* in the Monthly Tides, the Contractive Force of the Moon acting againſt the Contractive Force of the Earth, each of them Endeavours to draw the Waters to them, but by how much more the Contractive Force of the Moon is Superior to that of the Earth, by ſo much do's it prevail againſt it, and Elevates and Raiſes the Waters above that Situation or Poſition, which the Contractive Force of the Earth wou'd give them, and if the Moon and the Earth were to remain Quieſcent in *A* or *B*, and in *E*, the Waters wou'd continually ſtand at the Height of *G* or *N*, by the Superior Force of the Moon's Contraction, but as the Moon in *B* moves towards *C* in it's Monthly Revolution, and the Point *g* of the Earth towards *n* in it's Diurnal, the Contractive Force of the Moon is diminiſh't by ceaſing to be Perpendicular, and becoming Oblique, and the Contractive Force of the Earth prevails, upon which the Waters begin to flow from their Height, and to fill the Places below them, and at length to ſubſide; and this Struggle betwixt the Moon and the Earth continues quite round the Orbit of the one, and the Rotation upon it's Axis of the other.

FOR the ſame Reaſon, when the Waters lye on the oppoſite Side of the Earth to the Moon, ſince the Earth and Moon preſerve the ſame Diſtance from each other, the Difference of their Contractive Forces will be ſtill the ſame, namely, that of the Moon will be in the ſame Proportion Superior to that of the Earth, but as theſe Forces act Contrary to one another, the Superior Force, which was Attractive on this Side the Earth, will be Repulſive on the other; For if the Earth and the Moon, in this Poſition of the Waters, attracted them the ſame Way, they wou'd not be Actions contrary to each other, which by the former Phænomenon it is Evident they are, whilſt therefore the Earth attracts the Waters to it's Center, the Moon for the ſame Reaſon, by it's prevailing Force, will repel them from it on the other Side the Earth, to an Equal Height, as it drew them from it on this; for if theſe Two Actions are contrary to each other, it will be the ſame, whether the Earth draws the Waters one Way, and the Moon another, which is the firſt Caſe, or whether the Earth draws them the ſame Way, as the Moon did before, and the Moon the Contrary, that is, Repels them, which is the Second.

FOR thus, in any Two Equal Forces acting with contrary Directions, it will be the ſame, whether the Firſt Acts towards the Right Hand, and the Second towards the Left, or the Firſt to the Left, and the Second to the Right, for it is the ſame, whether *A* preſſes to the Right Hand upon the Point *C*, and *B* to the Left on the ſame Point, or whether *A* acts towards the Left, and *B* to the Right by Attracting the Point *C* Different Ways with Equal Forces, ſo that Two Contractive Forces, as thoſe of the Moon and Earth, Acting againſt each other, may be Repreſented by Two Forces Attracting one another with contrary Directions, as when the Moon is in *A*, and the Waters in *g*, or by Two Forces propelling each other with contrary Directions, as when the Moon is in *A*, and the Waters in *l*, or, which is ſtill the ſame, the Moon's Force may be conſider'd as a Propelling or Expanſive one from the Center *E* in the Earth, and the Earth's as a Gravitating or Contractive one towards the ſame Center, for one Contractive Force Acting in a contrary Direction to another, is the ſame as an Expanſive one in Reſpect of that Contractive.



Why the Spring or Monthly-Tides are the Greateſt in the Moon's Conjunction and Oppoſition with the Sun, and Niepe in the Quadratures.

§. 26. HAVING conſider'd the Moon in it's Revolution about the Earth, and the Earth's Revolution about it's Axis, and what Effect they have upon the Flux and Reflux of the Sea, it is Neceſſary to know whether the Sun has any Influence in theſe Tides, becauſe in the Conjunction and Oppoſition of the Moon with it, that is, when the Moon is in *B* or in *D*, the Spring or Monthly Tides or Fluxes are the Greateſt, and in the Quadratures of the Moon with the Sun, the Tides are Niepe, and which it is Certain it has for that very Reaſon, ſince this Increaſe of the Tides cannot proceed from the Moon's being in *B* or in *D*, that is in Conjunction or Oppoſition with the Waters, which are Raiſ'd, which it is every Day in the Diurnal ones, and yet no Spring-Tides are conſequent from thence; it muſt be therefore affirm'd, that the Sun has a Force and Efficacy in Raiſing the Waters, both when the Moon is in Conjunction, and when in Oppoſition with it, or rather that it is Aſſiſting to and Concurrs with the Force of the Moon in Raiſing of them, by whoſe Power that Effect is principally and properly produc'd.

THE Solution of which Phænomenon ſeems to be this, That as the Sun has Evidently a Great Expanſive Force, which always acts Contrary to the Contractive, ſo ſuppoſing the Moon in *B*, whilſt it by it's Contrary and Superior Contractive Force to that of the Earth Raiſes the Waters to *N*, the Expanſive Force from the Sun in *S*, Acting upon the the ſame Waters, Diminiſhes their Contractive Force, tho' it is impreſs'd with the ſame Direction, that is, Tends towards *E*, the ſame Way as the Contractive Force of the Earth or Waters do's, but notwithſtanding it's Tendency of Exertion is the ſame Way, yet ſince it's Force is of a Different Nature, it will weaken that Force which is of a Contrary one; thus tho' we ſtrike a Ball to the Earth, and the Force of Percuſſion acts the ſame Way with that of Gravitation, yet the Effect of Gravitation obtains Leſs, as the Force of Percuſſion is Greater, becauſe Percuſſion is an Expanſive Force Different from that of Contraction or Gravitation; ſo likewiſe in Fire, which exerts it ſelf all Ways, and conſequently Downwards, that is the ſame Way which Bodies Gravitate, the Force of Gravitation or Contraction is yet weaken'd by it, as Solid and Heavy Bodies are by it chang'd into a Light Cinder.

THE Expanſive Force therefore of the Sun concurrs with the Moon's Contrary and Contractive Force to the Earth, which, as has been ſhewn, may be conſider'd as a kind of Expanſive, in Raiſing the Waters and Diminiſhing the Force of the Earth's Contractive, That is, an Expanſive Force, as of the Sun, Acting upon a Contractive, as of the Earth, the ſame Way, and in the ſame Direction of it's Force, may be eſteem'd a Repulſive or Expanſive one from the Center *E*, in Compariſon of the Earth's Contractive, which Tends and Preſſes towards it.

AND for the ſame Reaſon, when the Moon is in *D*, or in Oppoſition with the Sun, the Effect will be ſtill the ſame, the Sun continuing in the ſame Situation as before, in Reſpect of the Waters in *n*, and it has been already prov'd, that the Moon's Power will be alike in *D* or in *B*.

LASTLY, when the Moon is in *D*, and the Waters are rais'd at *I*, the Expanſive Force of the Sun will have the ſame Effect, whether it Acts in the ſame Tendency and Direction with the Earth's Contractive Force, that is, from *n* to *E*, or in a Contrary one to it, that is, from *E* to *i*, for they will be both Repulſive of the Earth's Contraction, the firſt will be repreſented by a Percuſſion to the Center *E*, and the other by a Percuſſion from it, and both equally Deſtructive of the Influence and Power of Gravitation, or that Force, which is of an oppoſite Nature to the Expanſive, for by this it is, that Waters are rais'd into Clouds and Vapours.

The Laws of the Expanſive and Contractive Forces.

AND from what we have ſaid, we may lay down theſe General Rules in Reſpect of the Expanſive or Contractive Forces.

1. THAT a Contractive Acting in Oppoſition to a Contractive, is the ſame, as an Expanſive Force Acting from a Center, and a Contractive Acting towards one.

2. THAT an Expanſive Force Acting the ſame Way with or in an Oppoſite to a Contractive, is the ſame as a Contractive Acting in Oppoſition to a Contractive, or an Expanſive Force Acting from a Center, and a Contractive Acting towards one.

3. THAT an Expanſive Force Acting in Oppoſition to an Expanſive Force, is the ſame, as a Contractive Acting againſt a Contractive, or as an Expanſive Force Acting from a Center, and a Contractive towards one.

4. THAT an Expanſive Force Acting the ſame Way with another Expanſive Force, or a Contractive the ſame Way with another Contractive, is the ſame, as Increaſing the Expanſive Force from the ſame Center, or Increaſing the Contractive towards one.

How the Sun's Expanſive Force, and the Moon and Earth's Contractive ſolve all the Phænomenons of the Tides.

§. 27. WHAT Remains now is to ſhew, how this Theory of the Sun's Expanſive and the Moon's Contractive Force is anſwerable to the Obſervations in Fact, before recounted; And firſt it is from hence Evident, that on every Full and New Moon, the Tides will be the Greateſt, that is, when the Moon is in Conjunction or Oppoſition with the Sun, and that they will be Niepe or not up, when it is in the Quadratures with it; They will alſo be at the Greateſt Ebb, 6 Hours after they are at the Fulleſt, by the Earth's Revolution about it's Axis, for if the Waters are at their Height in *N* or in *I*, when *n* by it's Revolution comes to *l*, or *i* to *g*, which will be in 6 Hours, and, when the Sun and Moon will have the leaſt Influence to raiſe them by their Oblique Actions, they will accordingly ſubſide to the loweſt Standard; It will likewiſe be Full Sea $\frac{3}{4}$ of an Hour and $\frac{1}{2}$ a Quarter later every Day from the Full or New Moon, ſo as in 7 Days after, it is 6 Hours or 6 Hours $\frac{1}{8}$ later than in the Full or New Moon;

HENCE it is, that the Monthly and Diurnal Tides do agree, for as Full Sea is at *N*, when the Moon is in *B* or in *D* at thoſe Tides, ſo it is at the loweſt Ebb 6 Hours after, that is, when by the Revolution of the Earth about it's Axis *n* arrives at *l*, or *i* at *g*. And thus it is in the Spring and Niepe or Monthly Fluxes and Refluxes, for in the Conjunction of *B* with the Sun or in the New Moon, and it's Oppoſition at *D* or in the Full, the Spring-Tides are obſerv'd, and 7 Days after the half Niepe, which is nearly the ſame in the Moon's Revolution about and Receſs from the Sun, as 6 Hours in the Diurnal Revolution of the Earth about it's Axis, and the Receſs of the Parts of the Earth from the Moon; that is, in both Caſes, the Moon is Perpendicular to the Earth in the Greateſt Diurnal Tides, and in $\frac{1}{4}$ Part of the Earth's Revolution about it's Axis, that is, in 6 Hours, the loweſt Ebb is, and in the Monthly or Spring and Niepe-Tides, the greateſt Spring Flow is, when the Waters are under the perpendicular Influence of the Sun and Moon, and the leaſt Spring Flow or half Niepe, in $\frac{1}{4}$ Part of the Moon's Revolution in it's Orbit or Receſs from the Sun, which is in 7 Days nearly, it being 29 Days and $\frac{1}{2}$ in paſſing from the Sun to it's following Conjunction with it; from thence they fall Gradually, till they are at the loweſt Niepe the fourth Day before the New or Full Moon, and, the Day after, the Spring-Tides begin to riſe.

THAT the Full Tide is $\frac{3}{4}$ and $\frac{1}{8}$ of an Hour later every Day after the Full or New Moon, ſeems to be from hence, that, the Moon paſſing about $\frac{3}{4}$ and $\frac{1}{8}$ of an Hour every Day from Weſt to Eaſt in it's Menſtrual Revolution, it poſſeſſes a Meridian, which is $\frac{3}{4}$ and $\frac{1}{8}$ of an Hour Eaſtward of the former, and therefore, whilſt the Earth is carry'd Round upon it's Axis from Weſt to Eaſt likewiſe, it will be $\frac{3}{4}$ and $\frac{1}{8}$ of an Hour later, before the ſame Part of the Earth comes to the Moon's Meridian, and thus in 7 Days Full Sea is about 6 Hours later than in the Full and New Moon, for $7 \times \frac{3}{4} + \frac{1}{8} = 6$ Hours $\frac{1}{8}$; The Moon farther Proceeds 13° to the Eaſt every Day, which Reſolv'd into Time, Meaſur'd by the Earth's Revolution about it's Axis, which Performs 15° in an Hour, is $\frac{13}{15}$ of an Hour $= 3\frac{7}{15}$ divided by $4 = \frac{3}{4} + \frac{1}{8}$ nearly.

THAT at *Shoreham-Ferry* the Sea Flows 4 Hours and Ebbs 8, is, becauſe the Waters there, which Deſcend into the Sea, lie Higher than the Ocean, ſo that Two Hours are ſpent in raiſing the Ocean to a Level with them, which from thence begins to Flow and Deſcend upon them for the Remaining 4 Hours of it's Flux; On the Contrary, the Waters at the *Ferry* will Return more ſlowly than thoſe in the Ocean, becauſe, when rais'd by the Flux, they do not ſtand ſo High and Steep above their Natural Situation, as the Waters of the Ocean, and therefore will not Precipitate ſo faſt, and likewiſe, becauſe they are met in their Regreſs by the Sea, which is now a ſecond Time upon it's Flow to reduce it ſelf to a Level with them; And this will give a Solution of all the like Phænomenons obſervable in the Fluxes and Refluxes of Rivers Different from thoſe of the Ocean. See alſo §. 29.

FROM hence it is alſo manifeſt, why the Waters begin to Riſe or Lift three Days before the Full or New Moon, and are continually Increas'd to the third Day after, when they are at the utmoſt Height, becauſe the Actions of the Sun and Moon are too oblique to obtain any conſiderable Influence till that Time, and their Force is Increas'd even after they are the moſt Perpendicular, that is, in the Full or New Moon, becauſe thoſe Impreſſions ſtill remain, to which are added the Subſequent ones; Thus the Sun's Force in Summer, is more ſenſible ſome Time after it has been in the Tropick, than when it is in the Tropick it ſelf.

FOR the ſame Reaſon, as the Moon recedes from that Perpendicular Influence, or from three Days after the Full or New Moon, the Waters will continually Fall, and when the Moon is in it's Quadratures, or in the Middle way betwixt Full and New Moon, they will be half Niepe, and the loweſt Niepe will be the Day preceding the third Day before the Full or New Moon, that is, the fourth Day before it, See §. 31.

ing the Sun and Moon have little or no Effect till they are nearly in Conjunction with or Oppoſition to one another, not becauſe they deſtroy each others Effects, ſince they concur in their Actions, but becauſe the Sun has little Power to raiſe the Waters unleſs in Concurrence with the Moon; And that the Spring-Tides will be Equal at the New or Full Moon has been already ſhewn, becauſe their Expanſive and Contractive Forces will be in both Caſes Equal.

LASTLY, the Waters move Faſter in the Spring than in the Niepe-Tides, becauſe the Forces of the Sun and Moon, which Actuate them are Greater, by which they are rais'd Higher and Quicker, and deſcend with more Violence from ſuch Elevation to the Shores. What has been ſaid here, in Reſpect of the Tides at *Worthing-Point* and *Shoreham*, will be Applicable to any other, allowing for the Different Circumſtances of the Place or Ocean. Thus far we had Writ, before we had ſeen Sir *Iſ. Newton* on this Head.

An Examination of the preſent Celebrated Philoſophy of Tides, by the foregoing Explanation of Them. Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

§. 28. FROM the Doctrine of Tides before deliver'd, and which is confirm'd by the conſtant Obſervation of the Coaſters upon the Sea, which waſhes the Shores of *Suſſex*, we may be enabled in ſome Meaſure to Judge of what is propos'd to us from the preſent Celebrated Philoſophy, which very much ſupports our Opinion.

AND firſt it is plain from hence, which is there Affirm'd, that the Sea, every Lunar and Solar Day, or every Revolution of any Part of the Earth from the Moon or Sun to the ſame Luminaries, ought to have and really has Two Fluxes and Two Refluxes, but then we muſt farther add, that ſo it wou'd have, if the Moon was only conſider'd in this Revolution, tho' they wou'd not be altogether ſo Great, the Sun only Aſſiſting by it's Expanſive Force, what is principally perform'd by the Moon's Contractive, according to what has been already Explain'd, and if that Expanſive Force of the Sun was Abſent, the Contractive of the Moon wou'd raiſe the Waters to nearly the ſame Height, ſince the Expanſive Force of the Sun only Facilitates the Moon's Operation.

SECOND, It is not yet ſo Evident, what the preſent Philoſophy likewiſe Aſſerts, that the Greateſt Height of the Waters in Seas, which are Free and Open and Deep, follows upon the Appulſe of the Luminaries to the Meridian of the Place in Leſs than 6 Hours, and that in the *Atlantick*, the *Æthiopick*, and *Pacifick* Ocean, the Greateſt Tides or Fluxes at the Shores are about the Third Hour after ſuch Appulſe, Reckoning an Hour to be the 24*th* Part of the Moon's Apparent Revolution from to the ſame Meridian.

BECAUSE we take the Diurnal Tides, according to the Account foregoing, to proceed from the Appulſe of the Moon to the Meridian, and that they are then or thereabout the Greateſt, that is a Little after ſuch Appulſe, that 3 Hours after they Decline, and in 6 Hours entirely Sink, as is plain in Fact from what is affirm'd on the Coaſts of *Suſſex*, and for which there has been a Reaſon given.

THIS cannot be attributed to the Streightneſs of the one Sea in Reſpect of the others mention'd, ſince tho' it is not ſo Large, it is yet as unreſtrain'd and unconfin'd as any other can be, it lies within it's Chanel to a great Diſtance from the Shore, and has a Plain, Flat, and Even Strand for ſeveral Miles together, it do's not Riſe or Move, but as the Moon comes to the Meridian of the Place above or below the Horizon, and do's not owe the Flux of it's Waters to any others pouring in upon it.

Why on the Shores of the Atlantick, *the* Æthiopick, *and* Pacifick *Oceans, the Greateſt Tides or Fluxes are about the Third Hour, after the Appulſe of the Moon to the Meridian.* Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

§. 29. ON the other Hand, what can be ſaid, when it is as Punctually maintain'd, that upon the Shores of the *Atlantick*, the *Æthiopick*, and the *Pacifick* Ocean, the Greateſt Tides or Fluxes are about the Third Hour, after the Appulſe of the Moon to the Meridian, that is, when it is in the Mid-way betwixt the Meridian and Horizon? The Reaſon of which is Probably this, that the Greateſt Height of the Waters in the Ocean is to be diſtinguiſh'd from the Greateſt Height of them at the Shores, the firſt proceeding from the Appulſe of the Moon to the Meridian, and the laſt from their Flowing down from that Height by their proper Gravity to the Shores below them, the Elevation of the Waters, in the former Caſe, is Common to the Ocean every where, which is conſtantly rais'd to the utmoſt Height upon the Moon's Appulſe to the Meridian, or preſently after, but the Elevation of the Waters, in the ſecond Caſe, is owing to the Deſcent of them from that Height, and being yet in Part ſuſtain'd by the Moon's Contractive Force, which Deſcent will be in a Greater or Leſs Time, as the Shores are Nearer or Farther Remov'd from the above-mention'd Height; And this will give an Account why the utmoſt Tides and Fluxes in Rivers, which communicate with the Ocean, may be at Different Times after the Elevation of the Waters by ſuch an Appulſe. This Philoſophy ſeems only to conſider the Former, namely, the Riſe of the Waters in the Ocean, and not to make any Diſtinction of it from the Latter, or the Riſe of them at the Shores, whereas they proceed from very Different and Contrary Cauſes, the firſt from the Moon's Contraction, and the laſt from the Earth's. It may be added, that as the Sun's Expanſive Force is Greateſt betwixt the Tropicks, ſo is the Contractive of the Moon, and about 5° beyond Them, under which Tract of the Heavens the aforeſaid Oceans lie, for our Winters ſeem to be caus'd by frequent Returns of the Moon to our Tropick, whilſt the Sun Retires to the other; and therefore, the Waters, being ſuſpended by a Greater Contractive Force of the Moon, are not permitted to Deſcend to the Shores, till that Force is Impair'd ſome Time after the Departure of the Moon from the Meridian, that is, at ſome Diſtance from it.

How our Winters are produc'd from the Contractive Force of the Moon, as our Summers by the Expanſive of the Sun.

§. 30. The ſecond Thing aſſerted in this Philoſophy is, That the Two Motions, which are Excited by theſe Two Luminaries, are not ſeparately Diſcernible, but produce a certain mixt Motion, and that in the Conjunction or Oppoſition of them, or their Syzygiæ, their Effects will be join'd and united by which the Greateſt Flux and Reflux will be Caus'd, which has been already ſufficiently explain'd from the Expanſive Force of the Sun, and the Contractive of the Moon, and that the Former is Inconſiderable compar'd with the Latter; and ſo it is likewiſe in Reſpect of it's Heat, which is not able to diſſolve, with all it's Force, the Ice and Snow, which lie in the *Northern* Seas and on the Tops of the higheſt Mountains, produc'd, as was before ſhewn, by the Moon's Influence.

The Action of the Sun Inconſiderable in Raiſing the Tides in Reſpect of that of the Moon.
Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

§. 31. The third Aſſertion is ſomething Extraordinary, and which is found in both the Editions of this Philoſophy, that in the Quadratures the Sun will Raiſe the Water, where the Moon Depreſſes it, and will Depreſs it, where the Sun Raiſes it, and from the Difference of their Effects, the leaſt Flux and Reflux will ariſe; which, in a Grammatical Conſtruction, is as much as to ſay, that the Sun will Raiſe the Water, where the Moon Depreſſes it, and the Sun will Depreſs it, where the Sun Raiſes it, which is a Contradiction, or elſe in a leſs Grammatical one, the Sun will Raiſe the Water, where the Moon Depreſſes it, and the Moon will Depreſs it, where the Sun Raiſes it, which is a Tautology.

The Elevation of the Waters in the Syzygiæ, and the Depreſſion of them in the Quadratures illy explain'd by the Modern Philoſophy.
Sir Iſ. Newt. Princip. Prop. 24. Lib. 3. Edit. 1, & 2.

What I ſuppoſe is meant, is this, that as the Greateſt Flux and Reflux is in the Syzygiæ, ſo the Leaſt is in the Quadratures, and that, where each of theſe Luminaries Act perpendicularly, they will have the Greateſt Force to Raiſe the Waters, either jointly, as in the Syzygiæ, or ſeparately, as in the Quadratures, and where the moſt Obliquely the Leaſt, as the Sun in Reſpect of the Waters, which lie under the Moon in it's Quadrature, and the Moon in Reſpect of the Waters, which lie under the Sun in that Quadrate Situation, which is a perfect Axiom, and has been before Accounted for. As to what is farther alledg'd concerning the Forces of theſe Luminaries, betwixt the Syzygiæ and the Quadratures, it do's not need to be here conſider'd, ſince it entirely Depends upon what foregoes.

§. 32. And the Reaſon, why the ἀκμὴ of the Waters is later in Rivers, which Communicate with the Sea, than in the Shores of it, do's not ſo much proceed from their Shallows, nor from the Streightneſs of ſuch Paſſages, thro' which the Waters are convey'd, as from the Diſtance they have from the Height of the Waters rais'd in the Ocean by the Appulſe of the Moon to the Meridian, and from the Height of thoſe Rivers in Reſpect of the Ocean, which being Different, the Times from the Appulſe of the Moon to the Meridian, will be Various, in which the Waters of the Ocean deſcend into the Rivers, in order to produce betwixt them one uniform Level firſt, and after, a Flux of the one upon the other.

Why the ἀκμὴ of the Waters is later in Rivers, which Communicate with the Sea than on the Shores of it.
Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

§. 33. 'Tis true the Effects of theſe Luminaries do alſo depend upon their Diſtances from the Earth, ſo that in leſſer Diſtances their Effects are Greater, and in Greater Leſs, but perhaps not in that Reaſon and Proportion, which is here aſſign'd, that is, in a Triplicate one of the Apparent Diameters, ſince we have before ſhewn what little Reaſon there is to depend upon a Computation from ſuch Diſtances, where the Diſtances themſelves are ſo Precarious and Uncertain, however we may in General ſay,

The Greateſt Tides will be when the Two Luminaries are in their Syzygiæ, and in their Perigeums.
Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

ſay, that in Winter, when the Sun is in it's Perigeum, it produces Greater Effects, and makes the Tides riſe Higher, tho' very little, than in Summer, becauſe of it's ſmall Expanſive Force in Reſpect of the great Contractive one of the Moon; as alſo, that the Moon every Month, in it's Perigeum, excites Greater Tides than fifteen Days before and after, when it is in it's Apogeum; From whence it comes to paſs, that the Two Greateſt Tides or Fluxes of all, cannot follow one another in any Two ſubſequent Syzygiæ: For ſuppoſing the Sun and Moon to be in their Perigeums, when they are in their Syzygiæ, the Greateſt Æſtus or Tide will enſue, but in the next Syzygiæ it is plain, that both of them will not be in their Perigeums, nor in the Syzygiæ following the former, and ſo on; that is, the Greateſt Tides will be, when the Sun and Moon are in their Syzygiæ, and when both of Them are in their Perigeums, and not otherwiſe.

The Reaſon why the Tides are Greater about the Æquinoxes than the Solſtices. Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

§. 34. If the Moon, whoſe Effect is Chiefly and Principally to be conſider'd, were plac'd in the Pole, and without any Motion, it is certain, by what we have alledg'd in the Doctrine of Tides, that by it's Contractive Force, which is Superior to that of the Earth's, it wou'd draw the Waters towards it, and they wou'd not Deſcend at all, and conſequently, there wou'd be no Reciprocation in the Motion of the Waters, or any Flux or Reflux of them; But, when this is confeſs'd by the preſent Philoſophy, it muſt likewiſe acknowledge, which we contend for, that the Contractive Force of the Earth and the Motion of the Moon from the Meridian are the Cauſe of the Flux of the Waters to the Shores, by the firſt of which they are made to Deſcend from the Height, to which the Moon in it's Meridian had rais'd them, and by the laſt the Perpendicular and Contractive Force of the Moon is weaken'd and diminiſh'd ſo far, as to allow the Waters to Deſcend from their Height, and to give the Contractive Force of the Earth the Prevalence; and when the Moon is ſtill farther remov'd from the Meridian, the Waters Sink from the Shores and return to the Ocean by the Contractive Force of the Earth ſtill more Prevailing; and the Reciprocation of the Waters proceeds from the Mutual and Reciprocal Attraction, by the Moon and Earth, of the Waters from the Earth and Shores towards the Moon, and from the Moon towards the Earth and Shores.

It is, notwithſtanding, but illy infer'd from a Suppoſition of the Moon's being fixt in the Pole, and obtaining no Reciprocation of the Waters there, that it's Effect and the Sun's is diminiſh'd, the farther they Decline from the Æquator, and the nearer they Approach to the Pole, and therefore, that the Tides are leſs in the Solſtitial than in the Æquinoctial Syzygiæ; For it wou'd be the ſame, if the Moon was ſuppos'd to remain Immoveable in the Æquator, and the Earth not to be Revolv'd, that is, there wou'd be no Fluxes or Tides, but the Water wou'd continue ſuſpended without any Reciprocation or Motion; It wou'd yet be a hard Inference from thence to affirm, that the Flow of the Waters wou'd be the Greater, the farther the Moon was Diſtant from the Æquator, and therefore, that the Tides are Greater in the Solſtices than in the Æquinoxes; notwithſtanding which, this may be as well Infer'd as the other, ſince we may as well ſuppoſe the Moon to ſtand ſtill in the Æquator, and the Earth to be Quieſcent upon it's Axis, as Imagine the Moon to be fixt in the Pole.

In Truth, there is no Connexion at all betwixt the Suppoſition and the Inference, betwixt the Waters being ſuſpended, and their being ſuſpended to a Higher Degree (which they muſt be in Greater Fluxes) the farther they are from ſuch a Suſpenſion; For where is the Force of this Concluſion? The Waters will be ſuſpended, if the Moon is ſuppos'd to be in the Pole, therefore they will be ſuſpended Higher in the Solſtices, and yet Higher in the Æquator; The Waters are Quieſcent, where the Moon is ſo in the Pole, therefore the Fluxes will be Leſs in the Solſtices, and Greater in the Æquator, becauſe the Moon is not there Quieſcent; How are the Degrees of the Fluxes aſcertain'd from the Moon's not Moving at all in the Pole, and Moving in the Æquator?

The Reaſon therefore, why the Tides are Greater about the Æquinoxes than the Solſtices, is; First, Becauſe when the Earth or Sun is in the Plane of the Æquator, the Moon and it's Orbit will more nearly Coincide with that Plane, than when the Earth and it's Moon is in the Greateſt Declination from it, that is in the Solſtices, and conſequently in the firſt Caſe, the Moon will be in a more Exact Conjunction and Oppoſition with the Sun, than in the Laſt, by which the Waters will be proportionably raiſ'd, in like Manner, as they are in the Monthly Syzygiæ; or the Luminaries, Acting more perpendicularly in the Æquator than in the Solſtices, will produce a Greater Effect upon the Waters in the one Circumſtance than in the other.

Second, Becauſe the Waters at the Æquinoxes are in the fitteſt Temper and Diſpoſition to be raiſ'd, as being then in the utmoſt Fluidity; In Winter they are Dull and Torpid, and in ſome Seas to that Degree, as to be Frozen into Ice, and as Ice cannot be raiſ'd into a Flux by the Moon, ſo any Approaches of the Waters to that Phlegmatick State will render them in ſome Meaſure Averſe and Reluctant to the Impreſſions of it; In Summer (ſince the Moon can no more raiſe Fire than Ice into a Flux) they are too Heated and Fiery, and by that Means likewiſe are leſs Obedient to the ſame Contractive Impreſſions; but in Spring and Autumn both theſe Obſtructions are Remov'd, and they in Conſequence riſe with Greater Freedom and Eaſe upon the Application of the Moon's Force to them.

Why the Tides are Greater in the Solſtitial Quadratures than in the Æquinoctial ones.
Sir Iſ. Newt. Princip. Prop. 24. Lib. 3.

§. 35. On the Contrary, the Tides are Greater in the Solſtitial Quadratures than in the Æquinoctial ones, becauſe in the Solſtitial Quadratures, the Sun is in the Tropicks, and the Moon in the Æquator, by which the Former will have the Leaſt, and the Latter the Greateſt Influence, and in the Æquinoctial Quadratures, the Sun will be in the Æquator, and the Moon in the Tropicks, by which the Former will have the Greateſt Influence, and the Latter the Leaſt, and therefore ſince the Moon's Contractive Force has the Greateſt Effect in Raiſing the Waters, the Tides will be the Greateſt in the Solſtitial Quadratures, and the Leaſt in the Æquinoctial ones: The Sum of All is, The Tides are the Greateſt in the Æquinoctial Syzygiæ, becauſe the Sun and Moon are then in the Æquator, or the neareſt to it, and Leſs in the Solſtitial Syzygiæ, becauſe they are then in the Tropicks, in which they have the leaſt Force; and the Tides are Greateſt in the Solſtitial Quadratures, becauſe the Sun is in the Tropicks, and the Moon in the Æquator, and Leſs in the Æquinoctial Quadratures, becauſe the Sun is in the Æquator, and the Moon in the Tropicks; which exactly Correſponds and Agrees with what we have before affirm'd, and is a full Conviction of our Doctrine of the Expanſive and Contractive Forces. From whence it follows, as is aſſerted in this Philoſophy, that the Greateſt Tides fall on the Syzygiæ, and the Leaſt on the Quadratures about the Times of the Æquinoxes, and that the Greateſt Tides in the Syzygiæ are follow'd by the Leaſt in the Quadratures, which is alſo found by Experience.

Why the greateſt and leaſt Tides oftner precede the Vernal Æquinox than follow it, and oftner follow the Autumnal than precede it.
Sir Iſ. Newt. Princip. Lib. 3. Prop. 24.

§. 36. That the Greateſt and Leaſt Tides oftner precede the Vernal Æquinox than follow it, and oftner follow the Autumnal than precede it, is not owing ſo much to the leſs Diſtance of the Sun from the Earth in Winter than in Summer, which has an Inconſiderable Effect in Raiſing the Waters, and is only Concurring and Aſſiſting to the great Action of the Moon, as firſt to the fuller Power and Force, which the Moon has in Winter than in Summer, by which it Harden's and Contracts Seas into Ice, which ſhews a Greater Force than Raiſing of them in the Flux; and ſecond, becauſe the Waters are more diſpos'd to be Elevated in the Colder than in the Hotter Seaſons of the Year, which is the Reaſon that there is more Rain in Winter, and in Spring, and Autumn, than in the Summer, that is, they are more Fluid, excepting in Froſt, than in the Summer, when their Parts are rather in a Tumult and Ferment and heated into a Degree of Fire, than paſſive and yielding to any Force which ſhall be impreſs'd upon them; And it is not impoſſible or unlikely but rather certain, that the Quantities of Waters raiſ'd, and which fall in the Year, are not only to be Attributed to the Sun's, but to the Moon's Action, by which it is capable of Raiſing whole Mountains of them in the Ocean, and that the Expanſive Force of the Sun, by diſſolving the Contractive of the Moon, or the Moon's Contracting what the Sun has Diſſipated and Exhal'd, is the Occaſion of their Falling, and which may be the Reaſon, why there is the Greateſt Plenty of Showres in Spring and Autumn. As to the ſecond Cauſe alledg'd for heightening the Waters in the Æquinoxes, namely, the Temper and Diſpoſition of them, that do's not obtain in the Solſtitial Quadratures, becauſe the Waters may be made too Fiery and Expanſive from the Solſtice for an Elevation by the Moon's Contractive.

If the Modern Doctrine of Attractive Forces was

§. 37. The Doctrine we have propos'd concerning the Contractive Power of the Moon and the Expanſive of the Sun, if carefully conſider'd, may be apply'd to what follows in this Philoſophy, and by which we may judge, where it Deviates

True, the Sun, Moon, Earth, and Planets, wou'd fall to one another. Sir Iſ. Newt. Princip. Prop. 25. 36, 37. Lib. 3. Sir Iſ. Newt. Optical Queries.

from the Truth, and where it Agrees with it; As to the Calculations of the Sun's and Moon's Force in Raiſing the Waters, they depend upon Principles, which ſeem to be falſe in themſelves, and therefore I ſhall not particularly conſider them, that is, of an univerſal Attraction of Bodies to each other, which, if true, if the Sun attracted the Earth, and the Earth the Sun, if the Moon attracted the Earth, and the Earth the Moon, as is ſuppos'd in this Philoſophy, what ſhou'd hinder the Moon's Deſcending to the Earth, and the Earth to the Sun? what ſhou'd obſtruct all the Planets falling together, and uniting in one Body with it, unleſs we introduce an Imaginary and Projectile Force to keep them up from ſuch a Tendency? On the Contrary, the Sun has an Expanſive and Diſſipating Force, as is Evident in Fact, the Earth has a Contractive, as is Evident from Gravitation, and the Moon has a Contractive, but which acts in a quite contrary Direction to that of the Earth, as is manifeſt from the Tides produc'd by their Contranitent Forces.

The Revolutions of the Moon about the Earth, and the Planets about the Sun, accounted for from theſe Principles.

§. 38. As therefore we plainly find, that a Motion or Revolution of Waters is effected by Two contrary Forces contending with each other, and as the Waters obey the Contractive Force of the Moon or the Earth, as the one or the other becomes Superior, ſo it is neceſſary that the Moon it ſelf, as it's Contractive Force is Superior, ſhou'd either draw the Earth to it, or Revolve round it, for that Superiority of Force muſt be ſpent ſome Way, and ſince it cannot attract the Earth to it, it muſt therefore move about it; thus in any Two contrary Forces, as that of a Sling thrown round upon a Center, if the Centrifugal Force, which proceeds from the Expanſive of the Mind, is Greater than the Centripetal of the Stone, a Revolution will be perform'd about the Center which detains it, and we have ſhewn, that Two Contractive Forces Acting in Oppoſition to each other are the ſame as a Centrifugal and a Centripetal. For the ſame Reaſon the Planets will Revolve round the Sun, the one having an Expanſive Force and the other a Contractive, that is Two contrary ones, if the Contractive is Superior to the Expanſive; from whence ariſes that Equal Poiſe we before aſſerted amongſt the Heavenly Bodies of the Moon with the Earth, and the Planets with the Sun, the Exceſs of their Forces being ſpent in their Revolutions. And from whence is Evident, the Infinite Wiſdom of the Almighty Diſpoſer, who weighs the Heavens as in a Balance, and Adjuſts Millions of Syſtems, beſides our own, according to the ſtricteſt Laws of Proportion and Reaſon.

Why River and Spring Waters are not rais'd into Tides as the Ocean is.

§. 39. From the Contractive Force of the Moon an Account may likewiſe be given, why Rivers are not Elevated in Fluxes, but yet may be Frozen by it; They cannot be rais'd in Tides or Fluxes for Two Reaſons, becauſe the Contractive Force of the Earth encloſes them too nearly, and becauſe the Vivacity and Spring of thoſe Running Waters, by which they Approach nearer to a Fiery or Expanſive Force, makes it, that they are leſs Obedient to, or capable of the Impreſſions from the Contractive Force of the Moon, whereas the Ocean lies at large from the Shores, and is of a Saltiſh and Contractive Nature, and is therefore more liable to the Actions of a Body, as the Moon, to whoſe Nature it more Approaches; For this muſt be acknowledg'd as an univerſal Principle, that the nearer any Bodies Approach to each other in the Quality of their Forces, the more Ready they are to Unite, and the more Contrary thoſe Forces are, as the Expanſive and Contractive, the more Averſe they are to ſuch an Union, as may be prov'd by a Thouſand Inſtances.

Why Ice is Lighter than Water, and Rivers or Seas, which are Frozen, riſe in Height, and are Contracted from the Banks or Shores, and why ſome Winters are Sharper than others.

§. 40. From the Contractive Force of the Moon it is alſo Evident, why Rivers are rais'd and ſwell'd by being Frozen, that is, why Ice is Lighter and takes up a Greater Space than Water, becauſe the Influence and Contractive Force of the Moon being Superior to that of the Earth, in the utmoſt Efforts of it in Froſt, both Contracts the Water from the Banks of Rivers, or the Shores of Seas, which are Frozen, and Heightens and Raiſes them from the Earth; and it is not Impoſſible but very Probable, that the Moon in it's Libration or Æquiponderance with the Earth, ſometimes comes much nearer to us than what is allow'd by the Calculations of Aſtronomers, who do not nor have conſtantly obſerv'd it's Motions, in which Caſes harder and ſharper Winters will be produc'd.

Why Lunaticks are more particulary ſubject to the Moon's Influence.

§. 41. From the ſame Contractive Force of the Moon it is, that Melancholy Perſons are more liable to it's Influences than thoſe whoſe Animal Spirits act with Greater Fire, Life, and Vivacity; the Dull and Languid of the one being more capable of being Affected by the Moon than the Brisk, Fiery, and Expanſive ones of the other; and thus no one ever knew a Man Diſtracted with Joy and with Chearfulneſs, where the Spirits and Blood move Briskly, but a Thouſand have been with Grief and with Sorrow, where their Force has more of the Contractive than the Expanſive; Which ſeems to be the Caſe alſo of the Hydrophobia.

Different Contractive Forces in Nature, and the Inſtances of Them.

§. 42. Having thus far confirm'd the Contractive Force of the Moon from ſeveral Inſtances, it will be now neceſſary to proceed to other Contractive Forces; That of the Earth is leſs than that of the Moon, as we have already evinc'd from the Tides occaſion'd by their Mutual Conflicts; the Contractive Force of one Planet we may reaſonably conclude is Greater than that of another; the Contractive Force of Snow is Greater than that of Rain, of Hail than that of Snow, of ſharp Froſt than that of Hail; the Contractive Force of Gold is Greater than that of Silver, of Silver Greater than that of Copper or of Iron, ſuppoſing the Expanſive Equal; and if theſe Bodies were plac'd at any Diſtance from each other, without the Earth's Intervention, they wou'd for the ſame Reaſon ſhew and exert their Particular Forces of Contraction, as much as we Experience the Earth and Moon do; and if their Contractive Forces act with a contrary Direction, they will revolve about each other, if in the ſame, they will mutually attract each other, and unite and cloſe together, which is the moſt likely, as they all Gravitate to the Earth, and therefore have the ſame Contractive Force with it.

Quantities of the Contractive Forces.

The Greater Quantity of any Contractive Force there is, the Force will by ſo much be Increas'd; a Ball of Snow has a Greater Contractive Force, and is Colder than a ſmall Plume, Flower, or Iceele of it; a large Heap or Congeries has a Greater Contractive Force than a Ball of it; a Mountain or Ocean of Ice has a Greater Contractive Force than a little Chryſtal of it; a Globe of Gold, Lead, Iron, *&c.* of 10, 20, or 100, or 1000 Inches Diameter has a Greater Contractive Force than one of 2, 3, or of 5; The Earth has a Greater Contractive Force than any Greater or Leſs Portion of it; and every Planet has a Greater Contractive Force than it's Half or any other Part.

Diſtances of the Contractive Forces.

As to the Diſtances of theſe Contractive Forces, the Forces are Greater the Nearer they are from the Centers of their Action; the Nearer the Moon comes to the Earth, the Greater Effects it has in Freezing Waters into Ice, or Raiſing them into Tides, ſo that if the Earth was ſome Miles nearer to it, it wou'd be Cover'd with an univerſal Froſt, and the whole Seas drawn into Vaſt and Stupendous Mountains from the Boſom of the Ocean; if we touch Ice or Snow, it is Colder than if we are at ſome Diſtance from it, if we are in the Coldeſt Countries, as in *Greenland*, *Norway*, or *Ruſſia*, we are more ſenſible of it's Effects, than if we are in more Diſtant Regions from them; the Nearer any Planets approach to each other, the Greater is their Force of Contraction, that is, they more obſtinately and ſtrongly maintain their own Particular Forces of Drawing and Attracting Bodies to their own Centers, thus alſo the Nearer any Gravitating Body is to the Earth, it is more forcibly drawn and impell'd by it.

Inſtance 2. Air, Water, Earth, Stones, Minerals, Metals.

Air, which is the fartheſt from the Center of the Earth's Contraction, has a leſs Contractive Force than Water or Earth; Water and Earth, which are upon the Surface, than Marl, Clay, Stones, Minerals, and Metals, which lie within the Bowels of it; and ſtill Nearer to the Center, there may be a Matter for all we know, which is ſtill Denſer, Groſſer, and more Impenetrable than any we meet with here.

Inſtance 3. Vegetation.

The Contractive Force of Plants and Trees, which are farther from the Center, is leſs than that of Earth or Water, by which Wood becomes Lighter than it, 'tis leſs in the Leaves than in the Boughs, in the Branches than in the Trunk, and in the Trunk than at the Root the Particular Center of ſuch Contractive Forces.

It may be farther obſerv'd, in Reſpect of Vegetation, that the Moon likewiſe in all Probability, as well as the Sun, has here it's Influence, the Sun by it's Expanſive, and the Moon by it's Contractive Force, both Conſpiring as to raiſe the Tides, ſo to draw forth Plants and Trees from the Earth; From whence it will follow, that as the Fluxes in ſuch certain Circumſtances are Elevated only to a determin'd Height, ſo Plants and Trees, according to their ſeveral peculiar Expanſive and Contractive Forces, acting from and towards the Root and aſſiſted by the Expanſive and Contractive of the Sun and Moon, have their ſtinted Growth, and do not riſe in Infinitum, but agreeably to the Strength and Proportion of the Acting Forces; and as the

Sun and Moon are the Two Grand Sources of the Expanſive and Contractive, ſo ſuitably we find all Earthly Bodies, under whoſe Influences they are, are compounded of Them.

Inſtance 4.
OdorousBodies.

ODOROUS Bodies have alſo a Contractive Force, by which they are hinder'd from being diſſipated by their Expanſive, and which therefore only Exert themſelves in one continu'd Pulſe upon the Ethereal or Airy Plenum, which ſurrounds them; This Contractive Force is the Greateſt at the Center, and Leſs in any Removals from it, which is the Reaſon, that the Flowers and Leaves of Trees Smell more than the Branches, the Stock, or the Root.

Inſtance 5.
Sound.

BODIES likewiſe, which are Sonorous, have a Contractive Force, which is the Greateſt at the Center, and Leſs at various Diſtances from it, thus the String of a Harp or a Violin, is more Sonorous in the Middle of it than in the Center of it's Contraction or Tenſion; A Bell is more Sonorous towards the Extremities than in the Center of it's Suſpenſion or Contraction.

Inſtance 6.
Colour.

COLOUR'D Bodies have a Contractive Force, by which their Parts are kept and detain'd together, and the Nearer thoſe Parts are to each other, their Contractive Force is the Greater, all other Things being Equal, and the more firmly they cohere.

Inſtance 7.
The Animal Syſtem.

LASTLY, The Animal Syſtem has a Contractive, as well as an Expanſive Force belonging to it, as is Evident from the Syſtole and Diaſtole of the Heart, from the Pulſe or Contraction and Dilatation of the Arteries, and from the Expanſion and Contraction of the Muſcles. The Source of Expanſion ſeems to be that of the Mind, by which the Body is transfer'd from one Place to another, and all it's Actions are perform'd according to the Direction of the Will, and the Source of Contraction that of Matter, in Reſpect of the Mind; and as the Expanſive and Contractive Forces are united, according to what we have ſhewn, in all Material Beings, ſo the Expanſive Force of the Mind, may be united with the Contractive of Matter; but yet, as the Expanſive Force of the Sun is perfectly Contrary to and Different from the Contractive of the Earth, or the Moon, or the Planets, ſo likewiſe the Nature of the Mind, which is Expanſive, is entirely Different from that of Matter, which is Contractive; and laſtly, as the Expanſive Force, as that of the Sun, can ſubſiſt of it ſelf without the Contractive, ſo likewiſe can the Mind, which is of an Expanſive Nature, without Matter, tho' at preſent it is united to it; which, if duly attended to, will give an Account, how the Mind may be united as it really is to the Body, and how it may yet Exiſt in a State of Separation from it.

From whence proceeds the Difference and Variety of the Expanſive and Contractive Forces.

§. 43. IF it be now Inquir'd, what is the Cauſe of theſe Expanſive and Contractive Forces, of which all Material Nature conſiſts, and what is the Reaſon of the Difference and Variety of them? The laſt Queſtion may be eaſily anſwer'd, that they proceed from the Different and Various Combinations of their Forces, Quantities, and Diſtances alledg'd in the Preſent Theorem; Thus, ſuppoſing the Sun to have it's particular Expanſive Force, the Moon to have it's Particular Contractive, and the Earth another Contractive peculiar to it ſelf, and contrary to the Former, and each of them their ſeveral determin'd and certain Quantities and Diſtances, we may eaſily conceive what Effects will ariſe from Differently Combining them. Let us firſt take the Sun and it's Expanſive Force, at an Infinitely ſmall Diſtance from it, the Intrinſick Force, by which it Heats, Diſſipates, and Repels, conſider'd together with the vaſt Quantity of it, will be Prodigious and Immenſe, ſo that if any Planet were to approach near it, it wou'd not only Heat and Diſſipate the Parts of it to an Inexpreſſible Degree, but wou'd Repel it with the utmoſt Force and Violence; and which may be a Reaſon, why the Planets move with a Greater Velocity in their Periheliums than in their Apheliums, as their Contractive Forces may be another, which have been before Explain'd.

THIS vaſt Expanſive Force to us, who are in the Northern Climates, at the Diſtance of our Earth and in our Tropick, Heats, and in the Winter Solſtice only Warms, and to thoſe, who are under the Æquinox, it Scorches and Burns, and it's Diſſipating and Repulſive Power is accordingly leſs or more Forcible, by which it drives us into Shades, raiſes Water into Vapours, and propels all Things from it, which are too Weak for it's Action; and a Million of Intermediate Degrees of this Expanſive Force lie at Various Diſtances betwixt us and the Sun, as here it is Different in Different Climates, that is, at Different Diſtances from it; It is alſo vary'd at the ſame Diſtance, by taking Different Proportions and Quantities of it; It is diminiſh'd by being Reflected and Diſſipated, and increas'd by being Collected in Greater Plenty to a Focus, the Firſt ariſing from the Superiority of the Contractive Force in the Glaſs, above that of the Expanſive in the Rays, and the Laſt from the Superiority of the Expanſive in the Rays, above the Contractive in the Glaſs. This Expanſive Force mixing in Various Proportions with Earthly Bodies, or which have a Contractive Force, cauſes Different Effects, and in all of them has a Warming, Diſſipating, and Repulſive Quality; Where a great Proportion of it is join'd with a light and leſs Contractive Force, as with Wood or Hay, or any other Combuſtible Matter, it produces Fire and Flame, which has Heat by it's Warming of us, has a Force of Diſſipation by it's throwing Fuel into Smoke, and Water into Vapours, and Metals into Fuſion, and has a Repulſive Power, by it's Power of Diſſipation, as alſo by making us retire from it, when it's Action is too Violent, and by Raiſing Liquors into Waves and Ebullitions in the Boiling of them; The ſame Expanſive Force mixing ſtill in leſs Proportions with Bodies, which have a like Contractive Force, or in like Proportions with Bodies, which have a Greater, do's not Riſe into Flame, but Sinks into Taſt, Smell, Colour, Sound, Vegetation, *&c.* which all, as has been ſhewn, have a Degree of the Expanſive, by which they Exert themſelves againſt the Contractive, and Endeavour to Diſſipate and Repel it's Force; This conſtant Conatus of the Expanſive ſtrikes as conſtantly upon the Medium, which ſurrounds it, by which means thoſe Senſations are convey'd to us, which alſo have a kind of Life and Warmth in them by the Vivacity and Chearfulneſs which they impart to us, and is another Property of the Expanſive.

As we have ſhewn therefore, that the Expanſive Force of the Sun may be Infinitely diverſify'd, by it's Quantity and Diſtance, and by the Nature of the Contractive with which it Mixes, ſo likewiſe it wou'd be eaſy to Explain how the Contractive Forces of the Moon and Earth, which are Contrary in themſelves to each other, by their Various Diſtances and Quantities, may compound Variety of Solid, Cold, Heavy, and Contractive Bodies, which, mingling and blending their Forces with the Expanſive ones of the Sun, are the Occaſion of the numberleſs Appearances, which occur to us in this Earth or Planet; in which we may juſtly ſuppoſe there is no ſuch Thing as a Central Fire, as has been Imagin'd by ſome, but rather that the Nearer we come to the Center, the Solid and Contractive Force will be the Greater, without any Expanſive at all, which we Originally and Principally Derive from the Sun, and which it is not unlikely is the Central Fire, meant and intended by Philoſophers, as being the Center, tho' not of the Earth, yet of the Earth's Syſtem.

The Expanſive and Contractive Forces themſelves cannot be Self-exiſtent, but are deriv'd from an Infinite and All-wiſe Agent.

§. 44. THUS we ſay it is eaſy to Anſwer the ſecond Queſtion, *viz.* from whence proceeds the Variety of Expanſive and Contractive Forces obſervable in Nature, and perhaps this is as far, as it is poſſible for Human Minds to reach in Philoſophy, but as to the Firſt, what is the Cauſe of them, we can ſay no more than that they muſt be either Self-Exiſting, and no Cauſe can be aſſign'd of them, or elſe, that they muſt be deriv'd from the Will and Pleaſure of God; To imagine or think the Firſt is Abſurd, for whatever Exiſts of it ſelf, will have a neceſſary, certain, and unvary'd Principle of it's Exiſting, becauſe there is nothing, which can change the Nature of a Neceſſary-Exiſtence, but here are a Thouſand and a Million of Expanſive Forces, Different in themſelves, and Acting contrary to each other, and from Different Centers; for what we have alledg'd of the Sun will be equally Applicable to the ſeveral fixt Stars in the Univerſe, which may be juſtly deem'd to be ſo many Suns, Acting with Various Expanſive Forces, Intrinſically Different in themſelves, and of Various Quantities and Magnitudes, and which Actuate and Inform ſo many Syſtems round them; Beſides which, the Contractive Forces of the Earth, the Moon, and Planets, which Conſtitute this Solar Syſtem are Forces contrary to the Expanſive, and Act with a contrary Direction amongſt one another, and are Intrinſically Different in the Nature and Degree of them; Add to this, that the Contractive Forces of Planets, which belong to a Million of other Solar Syſtems diſper'd thro' the Grand and Univerſal Expanſe, and which lie ſo Deep in it as perfectly to eſcape our Sight and Obſervation, may be Infinitely Different from thoſe which appertain to This, which Variety of Forces cannot agree with one certain Self-Exiſting Principle of all their Actions, unleſs a Directing and Intelligent one. Moreover, where we obſerve Expanſive and Contractive Forces Acting all Manner of Ways, and in all Kinds of Direction, and which are ſo adjuſted as to produce Regular and Uniform Syſtems, we always find an Intelligent and Thinking Being annext to them, or is the Cauſe of them, thus it is in the Human and Animal Frame, and thus it is in thoſe Various Pieces of Mechaniſm and Art, contriv'd by the Invention and Sagacity of Men; when therefore we perceive the ſame in Nature, excepting in a more Infinite Degree, it is Reaſonable and Juſt to ſuppoſe, that there is an Infinite Mind affixt to it, and that it Proportions all the Actions and Forces of it, ſo as to produce thoſe Regular and Orderly Syſtems we ſo conſtantly behold; If a Balance was hung up, and Different Portions of Matter plac'd in it, by which there wou'd be an Equal Poiſe of them, it wou'd be Madneſs not to believe, and think this was done by ſome Intelligent Being; And is it poſſible for us, when we ſee whole Worlds, and their Various Forces, and the Quantities of them, Calculated with that Exactneſs, and plac'd at ſuch Diſtances, which ſhall Conſerve the Order, Poiſe, and Balance of them, ſhall Perpetuate their Revolutions for Ages together, and be the Occaſion of the Numberleſs Productions of Metals, Plants, and Trees, and the other Phænomenons of Material Beings, not to acknowledge the All-wiſe and Intelligent Contriver, Orderer, and Diſpoſer of them?

This to me is the chief and only Proof of the Being of a God, to wit, the Works of Nature; and when we have Found what Nature is, we are then the beſt Capable of Judging of the Author of it, not by Metaphyſical Notions and Fancies of our own Inventing, but by conſidering what really Exiſts, and from thence Inferring and Collecting the neceſſary Cauſe of ſuch an Exiſtence; We have in Conſequence of this ſhewn, that all Nature conſiſts of an Infinite Number of Actions Different in their Forces, and Contrary in their Directions, that ſuch cannot be Self-Exiſtent, and therefore muſt neceſſarily lead us to ſome Agent; that theſe Actions are Infinite, and Infinitely vary'd, that they are Extended thro' Infinite Space, and therefore this Agent muſt be Infinite and Almighty; that they are adjuſted and diſpos'd according to the Exacteſt Calculations of their Intrinſick Force, their Quantities and their Diſtance, and therefore this Infinite and Almighty Agent muſt be All-wiſe and Infinitely Intelligent; that theſe Calculations are made to produce that Exact, Beautiful, and Excellent Order and Diſpoſition of Things we at Preſent ſee, and for the Uſe and Service of his Creatures, and therefore he muſt be Infinitely Juſt and Good; From hence alſo, we are Inform'd of the Way and Manner, by which all Matter and the whole World was Created, and may be Annihilated, for ſince all Matter and the whole World conſiſts of certain Actions or Forces, it is eaſy to Comprehend how an Infinite Agent, by the Energy of his Power, may produce ſuch Forces into Being, and when he Surceaſes his Action, may Reduce them to Nothing.

Tho' all True Philoſophy muſt at length have Recourſe to a Supreme Agent and Director, yet the Principles we have advanc'd are the moſt Extenſive, which have been hitherto propos'd in any Syſtems of it, and beyond which no Philoſophy can go.

Sir Iſ. Newt. Queries, at the End of his Theory of Light and Colours.

§. 45. If any one ſhall object againſt this Philoſophy, that we are at length oblig'd to recur to the Will and Pleaſure of the Infinite and All-wiſe Diſpoſer for a Solution of all, we ſhall only in Anſwer to ſuch an Objection ſhew the Difference, which lies betwixt what others have propos'd in Philoſophy, and what we have done in this, and demonſtrate that we have advanc'd ſeveral Steps beyond any of them, and have more Effectually prov'd that the ultimate Principle of all is God. As to what is call'd the Corpuſcularian, it is known, that after, as we have ſhewn, it had given a Lame and Imperfect Account of the Accidental Properties of Matter, as Heat, Cold, Colour, Sound, Taſt, Smell, *&c.* upon the Suppoſition of it's being Similar and Homogeneous, it did not pretend to Explain the Primary Qualities of it, Extenſion, Solidity, Impenetrability, and the like, which it own'd were Properties not to be medled with, they conſtituted the firſt Eſſence of Matter, and which was no farther to be conſider'd than as ſuch, which Almighty God had Created, or which Exiſted ſo of it ſelf without any other Cauſe; and yet we have gone farther, and given a Reaſon and Explanation of thoſe Properties.

As to the Preſent Mechanical, it reſolves all ultimately into Gravitation, but did not at Firſt offer at any Solution of Gravitation it ſelf, but either ſuppos'd it was Inherent in Matter or Impreſs'd by God, and left it to the Piety and Reaſon of it's

Followers to conclude the one or the other, and afterwards in it's Queries, at the End of the Light and Colours, when it had Invented an Imaginary and Elaſtick Fluid to account for that Force, it was oblig'd to ſtop there, and we were as much to ſeek from whence that Elaſticity was deriv'd, as the Gravitation aſſerted before, and all that cou'd be Inſer'd from this New Addition to that Philoſophy, was only this, that it appear'd to affirm a Plenum, which it had formerly deny'd, and in ſo many Terms recanted it's Demonſtration of a Vacuum; and yet notwithſtanding this, we have endeavour'd to give a Reaſon of Gravitation and of Elaſticity both, the laſt Proceeding from an Expanſive or Repulſive, and the firſt from a Contractive Force, whoſe Laws and Proportions of Action we have aſſign'd, and by them ſhewn how Nature may be in a great Meaſure accounted for; but if after this we wou'd ſtill proceed farther without a Supreme and Superintending Agent, we ſhall be miſtaken; and it will not be poſſible, I believe I may foretel it, for all the Great Wits and Genius's in the Preſent or any Future Ages, to extend their Enquiries beyond theſe Two Principles of Expanſion and Contraction; there we muſt Stand, and there we muſt Acknowledge, that as they cannot ſubſiſt of themſelves without an Agent, ſo we muſt Confeſs, that Almighty God is that Agent.

A True Philoſophy do's not Weaken but Confirm the Evidences, which we have of the Chriſtian Religion, and is an Atteſtation to the Truth of the Rewards and Puniſhments which it propoſes.

§. 46. And ſince we are Enter'd into the Aſſertion of Religion, which was one Deſign and Intention of this Philoſophy, as the other was to Enquire into what was Truth and Reaſon, I ſhall here add, that in Reſpect of the Happineſs promis'd to the Good and Vertuous in another Life, and the Torments and Miſeries Threaten'd to the Wicked and the Vicious, we have Nothing in Nature to contradict or to weaken thoſe Perſuaſions, but on the Contrary have abundant Arguments to confirm them. Everlaſting Fire is Threaten'd to a Diſobedience to Religion, if therefore, we cou'd find and obſerve no ſuch Thing in Nature, it might make us leſs Apprehenſive of the Threat, but when we perceive not only that Immenſe Fire, the Sun, but a Thouſand and a Million of others, as we may juſtly believe in the vaſt Expanſe of the Univerſe, ready Lighted up, and Prepar'd for what uſe the Infinite and Great Creator ſhall pleaſe to aſſign them, it wou'd make any Conſiderate Man very careful how he provok'd the Diſpleaſure of a Being who Created all of them, and for more Intents and Purpoſes than ſome Fond and Fooliſh Men may be apt to Imagine; it wou'd make him at leaſt Inquire with Seriouſneſs, whether that Chriſtian Religion, which is ſaid to be a Revelation from him, is, or is not ſo; and if it is, as there are a Thouſand Arguments for it's being ſo, it wou'd make him very Cautious how he offends againſt it, either in Reſpect of it's Doctrines or it's Morals.

On the other Hand, Everlaſting Happineſs is promis'd to the Believing and to the Obedient, and here we may likewiſe ſay, if ſuch a Thing appear'd to be Impoſſible in Nature, we might reject the offer perhaps with Indignation, and look upon it as a Paradice propos'd to the Fooliſh and the Credulous, that is, if there were not ſufficient Reaſons to aſſure us of the Truth of Chriſtianity; But when in Conjunction with thoſe Reaſons, we find in Nature nothing but what is agreeable to, and juſtifies ſuch Propoſals of Happineſs to us, this likewiſe wou'd make us Careful, how we Deſpiſe and Contemn ſuch a Religion.

We obſerve in our own Earth, that we are here Mortal, but tho' we are ſo, we Experience a great Deal of Pleaſure and Satisfaction, whilſt we Live, from that Diſpoſition of the Sun and Moon, which obtains with us, and from the Various Productions, which they Cauſe in Nature, of Plants and Herbs and Trees, of Minerals and Metals, and the other Material Beings, which ſurround us. Suppoſing now, as in *Jupiter* and *Saturn*, ſeveral Moons were to revolve about our Earth, with Various Situations and Diſpoſitions, wou'd it not Increaſe our Pleaſure, by the wonderful Variety of ſuch Luminaries, and their Different Forces and Situations to us? And as the Conſtitution of Things is entirely Different in other Planets from that of our own, why may we not think, that there may be Rational and Intelligent Agents in them, whoſe Bodies are of a Different Make and Conſtitution from ours, and who live ſeveral Years or Ages longer than we do? Eſpecially, when in our own Planet, we are fully Inform'd, that in the Times before the Deluge, and preſently after, Men were more Gigantick in their Stature, and more Strong and Laſting in their Stamen than what was Experienc'd in after Times, or is in the Preſent; The Reaſon of which can be no other than the Elevation of the Antediluvian

Earth towards the Sun and Moon above our own, before the falling of the Cruſt or Surface of it, by which means the Expanſive Force of the one, and the Contractive of the other, wou'd have a Greater Influence and Power, ſo as to raiſe both the Animal and Vegetable World to a Greater Height than what we at preſent find, and as their Forces were Stronger, their Effects wou'd be more Permanent and Laſting.

Let us now paſs to other Syſtems, ſuch, which lie the Deepeſt in the vaſt Expanſe and the Remoteſt from us, and of which we have no Knowledge; Who yet with all his Reaſon and Philoſophy can ſay, that there may not be ſuch contriv'd by the Infinite Wiſdom and Goodneſs of God, which ſhall be fitted and adapted to glorify'd Spirits and Bodies, by which their Pleaſure ſhall be Immenſe, and their Duration Eternal, and which he has deſtin'd to be the Seats and Manſions of the Bleſſed? And Who can contradict, what our Saviour affirms, that in his Father's Houſe, in the Great and Spacious Palace of the Univerſe, there are many of them? Let us Imagine then this farther, that as there are even in our Syſtem Reaſons to think, that there may be a Thouſand Occaſions of Delight to us, more than what we at preſent Experience, ſo in the Innumerable Syſtems, which there are in the Great Expanſe, we may find, when we are Immortal, and when the Expanſive Force of our Souls is ſeparated from that of the Contractive of the Body, an Incredible and Everlaſting Pleaſure, by Removing to and Contemplating every Syſtem, according to the Direction and Pleaſure of the Mind. This I only mention to ſhew, that there is nothing in Nature, which Contradicts our Religion, but which ſtrongly Enforces it, and that therefore our firſt Wiſdom is to be Good and Real Chriſtians, and in the next Place to underſtand as much of Reaſon and Philoſophy, as the Preſent Attainments in it or our own Studies will allow us to do.

CHAP. VI.

Concerning the Mutual Concourſe of Bodies, Hard, Soft, or Elaſtick.

Propoſition the Firſt Explain'd, Relating to the Progreſs or Quieſcence of the Center of Gravity. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 13. Theorem 20. Sir Iſ. Newt. Princip. Leg. Corol. 4.

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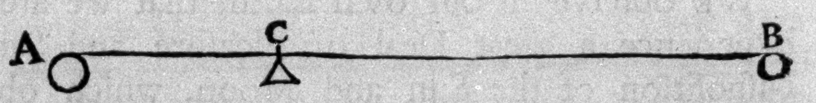
The Firſt Propoſition therefore, in Order to furniſh us with a Solution of theſe Matters, is, That if Two or More Bodies are carry'd with an Equable Motion to the ſame or contrary Sides or Parts, their Common Center of Gravity before their Mutual Occourſe will be either Quieſcent, or will be mov'd Uniformly in Directum.

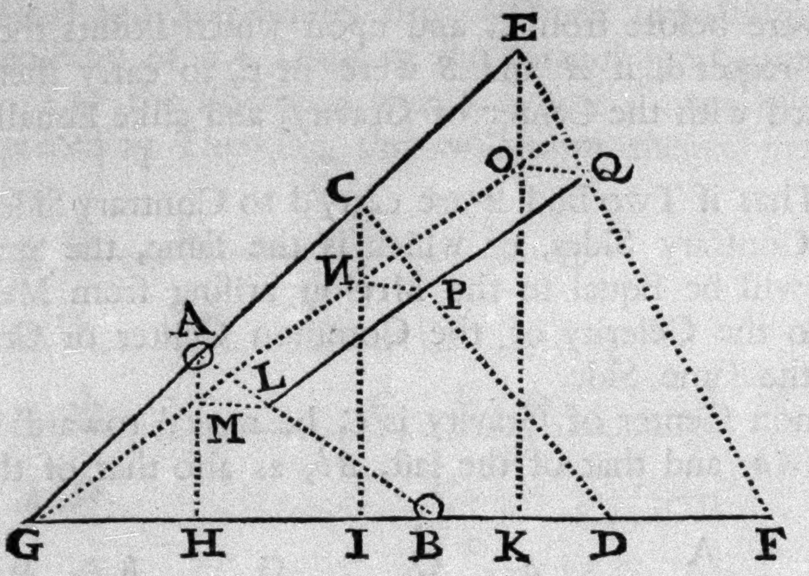
Caſ. 1. Supposing Two Bodies are mov'd to Contrary Parts, that is, let A and B be mov'd towards C, or from C, ſince the Nature of the Center of Gravity is, that $A . B :: BC . AC$, or that the Bodies, in their Weights, ſhou'd be Reciprocal to their Diſtances from their Center C, upon which they are Pois'd, and are in an Equal Balance with each other, it follows, if A and B are mov'd towards C, or from it, ſo as to preſerve the ſame Reciprocal Proportion of their Diſtances from C to their Matter, that they will ſtill be in an Æquilibrium, which will be Effected, if their Velocities are as BC to AC, or as A to B, that is, if their Motions are Equal, for if A the Matter $. B$ the Matter $::$ BC the Velocity $. AC$ the Velocity, the Motion $A \times AC$ is $= B \times BC$; and conſequently, A and B may be carry'd from C in Infinitum, or meet in C if carry'd towards it, whilſt the Center of Gravity will be always Quieſcent in C.



Caſ. 2. If now the Two Bodies A and B are carry'd to the ſame Side with a like Motion as before, or to Contrary Sides with an unequal one, their common Center of Gravity will be mov'd in Directum, becauſe the Velocities AC, CB will be Reciprocal to the Quantities of Matter A and B, and thoſe Velocities are as the Diſtances from the Center of Gravity; as therefore A and B move the ſame Way, it is neceſſary that C ſhou'd do ſo too, that the Proportion of the Velocities and Diſtances from C may be preſerv'd Reciprocal to A and B.

On the other Hand, if A and B are carry'd to Contrary Sides with an Unequal Motion, ſince the Velocities are not Reciprocal to the Quantities of Matter A and B, nor the Diſtances from the Center of Gravity AC, BC, the Bodies A and B will be no longer in an Equal Poiſe upon C, and which muſt therefore change it's Situation to make them ſo; that is, C muſt be remov'd to ſome other Point in the Line AB, where $A.B::BC.AC$.

Caſ. 3. Supposing the Bodies A and B move in Two Lines AC and BD inclin'd to each other, and that in Equal Times they ſeparately perform Equal Spaces, ſo that AC is $=CE$, and $BD=DF$, we ſay, that the Center of Gravity will be mov'd uniformly in the Line LQ, and that LP will be Equal to PQ; Let the preſent Figure and Conſtruction be Imagin'd as here Deſcrib'd, that is, let $AC.BD::AG.GH$, draw the Line AH, and from C and E, CI and EK Parallel to it; Join alſo A and B, C and D, E and F, and to the Line BD let ML, NP, OQ be likewiſe Parallel, and in the firſt Situation of A and B, let L be the Center of Gravity, and from the Point G thro' M draw the Right Line GO.



First, P and Q in the Line LQ will upon the Motion of A to C and E, and of B to D and F, be the Centers of Gravity, for by Elem. 6. Prop. 4. Corol. 2. Edit. Tacquet, $AM.MH::CN.NI::EO.OK$; but by Prop. 2. Elem. 6. $AM.MH::AL.LB$, and $CN.NI::CP.PD$, and $EO.OK::EQ.QF$, therefore $AL.LB::CP.PD::EQ.QF$.

Second, $LP=PQ$, for by the Conſtruction $AC.BD::AG.GH$, by Elem. 6. Prop. 2. $::AC.HI$, therefore $BD=HI$, add IB to both, and $ID=HB$; alſo by the Conſtruction, $CE.DF::AG.GH$, by Elem. 6. Prop. 2. $::CE.IK$, therefore $DF=IK$, add KD to both, and $KF=ID$, that is, the Baſes of the Three Triangles BAH, DCI, FEK are Equal; But by Elem. 6. Prop. 4. $AM.AH::ML.HB::CN.CI::NP.ID$, ſince therefore $HB=ID$, it will follow that $ML=NP$, and by the ſame way of Reaſoning $NP=OQ$; farther by Elem. 6. Prop. 2. Corol, $AC.CE::MN.NO$, But $AC=CE$, therefore $MN=NO$; and conſequently, ſince a Line drawn from L thro' P and Q will be Parallel to MO, becauſe ML, NP, OQ are Parallel and Equal, LP will be $=$ to PQ, ſince MN and NO are Equal; and what has been Demonſtrated of P and Q, may be ſhewn of any other Points in LQ.

Caſ. 4. If the Bodies are not mov'd in the ſame Plane, but in Different ones, their Progreſſes, as well as that of the Center of Gravity, are to be reduc'd to the ſame Plane, by letting down Perpendiculars to it from each Point of their Progreſs, by which means this Caſe will be brought under the Former, and the Demonſtration will be the ſame.

Caſ. 5. Supposing now there are more Bodies than Two, by placing the Center of Gravity of the Two Bodies ſo, as the Diſtance of it from a New Center of Gravity, ſhall be to the Diſtance of a Third Body from the ſame Center, as the Third Body is to the Sum of the Two, all Things will be in the ſame Circumſtances as before, excepting that the Two Bodies, poiſ'd upon their Center of Gravity, are now conſider'd as one; and the like will obtain, if we ſuppoſe Three, Four, Five, or a Hundred Bodies all Balanc'd upon a Center of Gravity, for in that Caſe, they may be conſider'd, in their Sum and their Weight, as one Body only.

Earth towards the Sun and Moon above our own, before the falling of the Cruſt or Surface of it, by which means the Expanſive Force of the one, and the Contractive of the other, wou'd have a Greater Influence and Power, ſo as to raiſe both the Animal and Vegetable World to a Greater Height than what we at preſent find, and as their Forces were Stronger, their Effects wou'd be more Permanent and Laſting.

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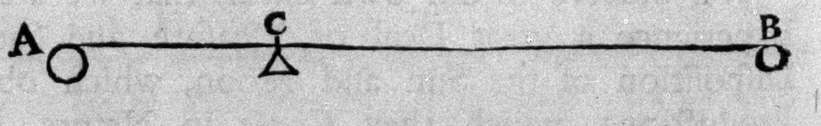
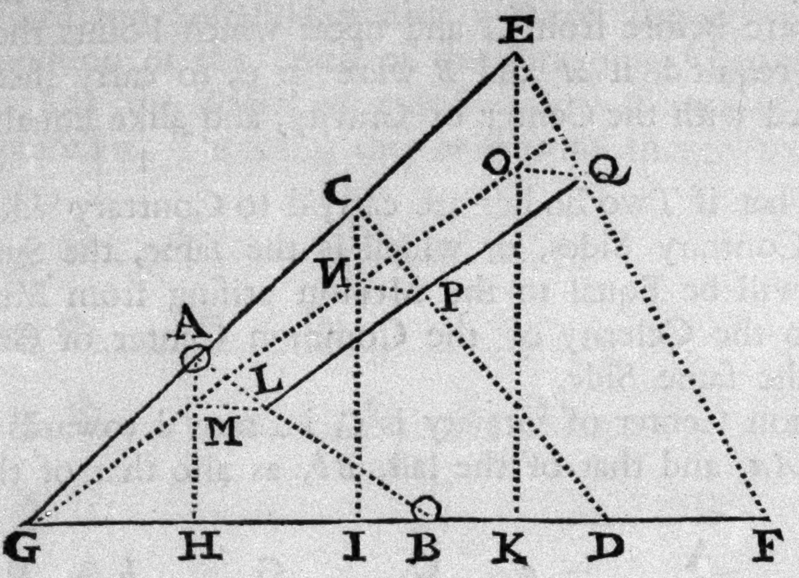
Caſ. 1. Supposing Two Bodies are mov'd to Contrary Parts, that is, let A and B be mov'd towards C, or from C, ſince the Nature of the Center of Gravity is, that $A.B::BC.AC$, or that the Bodies, in their Weights, ſhou'd be Reciprocal to their Diſtances from their Center C, upon which they are Pois'd, and are in an Equal Balance with each other, it follows, if A and B are mov'd towards C, or from it, ſo as to preſerve the ſame Reciprocal Proportion of their Diſtances from C to their Matter, that they will ſtill be in an Æquilibrium, which will be Effected, if their Velocities are as BC to AC, or as A to B, that is, if their Motions are Equal, for if A the Matter $.B$ the Matter $::BC$ the Velocity $.AC$ the Velocity, the Motion $A \times AC$ is $= B \times BC$; and conſequently, A and B may be carry'd from C in Infinitum, or meet in C if carry'd towards it, whilſt the Center of Gravity will be always Quieſcent in C.



Caſ. 2. If now the Two Bodies A and B are carry'd to the ſame Side with a like Motion as before, or to Contrary Sides with an unequal one, their common Center of Gravity will be mov'd in Directum, becauſe the Velocities AC, CB will be Reciprocal to the Quantities of Matter A and B, and thoſe Velocities are as the Diſtances from the Center of Gravity; as therefore A and B move the ſame Way, it is neceſſary that *C* ſhou'd do ſo too, that the Proportion of the Velocities and Diſtances from *C* may be preſerv'd Reciprocal to *A* and *B*.

On the other Hand, if *A* and *B* are carry'd to Contrary Sides with an Unequal Motion, ſince the Velocities are not Reciprocal to the Quantities of Matter *A* and *B*, nor the Diſtances from the Center of Gravity *AC*, *BC*, the Bodies *A* and *B* will be no longer in an Equal Poiſe upon *C*, and which muſt therefore change it's Situation to make them ſo; that is, *C* muſt be remov'd to ſome other Point in the Line *AB*, where *A.B*::*BC.AC*.

Caſ. 3. Supposing the Bodies *A* and *B* move in Two Lines *AC* and *BD* inclin'd to each other, and that in Equal Times they ſeparately perform Equal Spaces, ſo that *AC* is = *CE*, and *BD* = *DF*, we ſay, that the Center of Gravity will be mov'd uniformly in the Line *LQ*, and that *LP* will be Equal to *PQ*; Let the preſent Figure and Conſtruction be Imagin'd as here Deſcrib'd, that is, let *AC.BD*::*AG.GH*, draw the Line *AH*, and from *C* and *E*, *CI* and *EK* Parallel to it; Join alſo *A* and *B*, *C* and *D*, *E* and *F*, and to the Line *BD* let *ML*, *NP*, *OQ* be likewiſe Parallel, and in the firſt Situation of *A* and *B*, let *L* be the Center of Gravity, and from the Point *G* thro' *M* draw the Right Line *GO*.



First, *P* and *Q* in the Line *LQ* will upon the Motion of *A* to *C* and *E*, and of *B* to *D* and *F*, be the Centers of Gravity, for by Elem. 6. Prop. 4. Corol. 2. Edit. Tacquet, *AM.MH*::*CN.NI*::*EO.OK*; but by Prop. 2. Elem. 6. *AM.MH*::*AL.LB*, and *CN.NI*::*CP.PD*, and *EO.OK*::*EQ.QF*, therefore *AL.LB*::*CP.PD*::*EQ.QF*.

Second, *LP*=*PQ*, for by the Conſtruction *AC.BD*::*AG.GH*, by Elem. 6. Prop. 2. ::*AC.HI*, therefore *BD*=*HI*, add *IB* to both, and *ID*=*HB*; alſo by the Conſtruction, *CE.DF*::*AG.GH*, by Elem. 6. Prop. 2. ::*CE.IK*, therefore *DF*=*IK*, add *KD* to both, and *KF*=*ID*, that is, the Baſes of the Three Triangles *BAH*, *DCI*, *FEK* are Equal; But by Elem. 6. Prop. 4. *AM.AH*::*ML.HB*::*CN.CI*::*NP.ID*, ſince therefore *HB*=*ID*, it will follow that *ML*=*NP*, and by the ſame way of Reaſoning *NP*=*OQ*; farther by Elem. 6. Prop. 2. Corol, *AC.CE*::*MN.NO*, But *AC*=*CE*, therefore *MN*=*NO*; and conſequently, ſince a Line drawn from *L* thro' *P* and *Q* will be Parallel to *MO*, becauſe *ML*, *NP*, *OQ* are Parallel and Equal, *LP* will be = to *PQ*, ſince *MN* and *NO* are Equal; and what has been Demonſtrated of *P* and *Q*, may be ſhewn of any other Points in *LQ*.

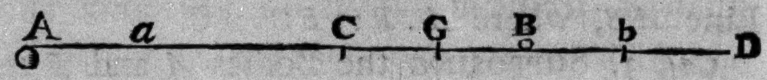
Caſ. 4. If the Bodies are not mov'd in the ſame Plane, but in Different ones, their Progreſſes, as well as that of the Center of Gravity, are to be reduc'd to the ſame Plane, by letting down Perpendiculars to it from each Point of their Progreſs, by which means this Caſe will be brought under the Former, and the Demonſtration will be the ſame.

Caſ. 5. Supposing now there are more Bodies than Two, by placing the Center of Gravity of the Two Bodies ſo, as the Diſtance of it from a New Center of Gravity, ſhall be to the Diſtance of a Third Body from the ſame Center, as the Third Body is to the Sum of the Two, all Things will be in the ſame Circumſtances as before, excepting that the Two Bodies, pois'd upon their Center of Gravity, are now conſider'd as one; and the like will obtain, if we ſuppoſe Three, Four, Five, or a Hundred Bodies all Balanc'd upon a Center of Gravity, for in that Caſe, they may be conſider'd, in their Sum and their Weight, as one Body only.

The Second Propoſition relating to the Motion of Bodies, and their Center of Gravity explain'd. Dr. Keill's Introduct. Lect. 13. Theorem 21.

§. 2. THE Second Propoſition in this Doctrine is, That if Two Bodies, however Equal or Unequal, are carry'd to the ſame Side, with Celerities however Equal or Unequal, the Sum of the Motions in both Bodies will be Equal to the Motion ariſing from Multiplying the Sum of the Bodies into the Celerity of the Common Center of Gravity.

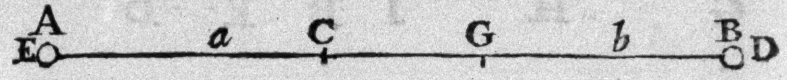
THUS, let *A* and *B*, whoſe Common Center of Gravity is *C*, be mov'd towards *D*, let the Celerity of *A* be *Aa*, That of *B*, *Bb*, and That of the Center of Gravity, *CG*, it is ſaid, that $A \times Aa + B \times Bb$ is $= \overline{A+B} \times CG$; becauſe it muſt be the ſame Thing, and the ſame Moment and Force muſt be requir'd, whilſt *C* moves to *G*, to carry *A* to *a*, and *B* to *b*, where *a* and *b* are at the ſame Diſtance from *G*, as *A* and *B* were before from *C*, and upon which Points they remain Equally Pois'd, as wou'd be requir'd, if *A* and *B* were in *C*, to carry them to *G*, where they are always Connected with the Center of Gravity, and alike Equally Balanc'd upon it.



The Third Propoſition explain'd. Dr. Keill's Introduct. Theorem 22.

§. 3. THE Third Propoſition is, That if Two Bodies are carry'd to Contrary Sides, the Difference of their Motions to Contrary Sides, or which is the ſame, the Sum of their Motions to the ſame Side, will be Equal to the Motion ariſing from Multiplying the Sum of the Bodies into the Celerity of the Common Center of Gravity, by which they are carry'd to the ſame Side.

THUS, let *A* and *B*, whoſe Common Center of Gravity is *C*, be mov'd towards *C*, and let the Celerity of the firſt be *Aa*, and that of the laſt, *Bb*, as alſo that of the Center of Gravity, *CG*, it is ſaid, that $A \times Aa - B \times Bb$, is $= \overline{A+B} \times CG$; becauſe the Difference of their Motions to Contrary Sides, is the ſame as the Sum of their Motions to the ſame Side, which has been before prov'd to be Equal to $\overline{A+B} \times CG$; that is, by the Nature of the Center of Gravity $BC . AC :: bG . aG$, and therefore that Force or Moment, which will carry *A* to *a* and *B* to *b*, whilſt *C* moves to *G*, upon which Points *A* and *B* are Equally Balanc'd, the ſame Force or Moment will carry *A* and *B* in *C*, from thence to *G*, where they are Connected with the Center of Gravity; and from hence it follows, that if the Difference of the Motions to Contrary Sides is Equal to Nothing, that is, if their Motions are Equal, the Center of Gravity is Quieſcent, as is Evident in it ſelf, and from the foregoing Equation, for if $A \times Aa - B \times Bb$ is $= \overline{A+B} \times CG$, and $A \times Aa - B \times Bb$ is $= 0$, $\overline{A+B} \times CG$ will be alſo Equal to 0, and conſequently *CG* Nothing, or *C* will be Quieſcent.



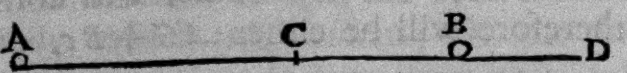
The Fourth Propoſition explain'd. Dr. Keill's Introduct. Theorem 23.

§. 4. THE Fourth Propoſition is, That if any Bodies Impinge or Act upon each other, in any Manner whatſoever, the State and Condition of their Common Center of Gravity, whether it be that of being Quieſcent or Moving uniformly in Directum, is not from thence alter'd. For, by what has been prov'd before in Theorems the 18*th* and 19*th* of this Philoſophy, and which we have afterwards conſider'd and examin'd, if Bodies Impinge upon each other, the Sum of the Motions, before and after Impaction, will be the ſame, and by what has been now Demonſtrated, the Sum of the Motions, before and after Impaction, will be Equal to the Motion ariſing from Multiplying the Sum of the Bodies into the Celerity of the Common Center of Gravity, therefore the Celerity of the Common Center of Gravity will be the ſame before and after Impaction.

The Firſt Propoſition explain'd in Reſpect to the Concourſe of Hard and Soft Bodies. Dr. Keill's Introduct. Lect. 13. Theorem 24.

§. 5. THE next Thing, which this Philoſophy proceeds to, is the Concourſe of Hard or Soft Bodies, the firſt Propoſition of which is, that if a Hard or a Soft Body Impinges directly upon another Hard or Soft Body, whether That, on which the Impaction is made, is Quieſcent or Moves more ſlowly to the ſame Side, or even to Contrary Sides, and ſuppoſing the Motions are Unequal, both the Bodies after Impaction will be mov'd together, and be carry'd along with their Common Center of Gravity.

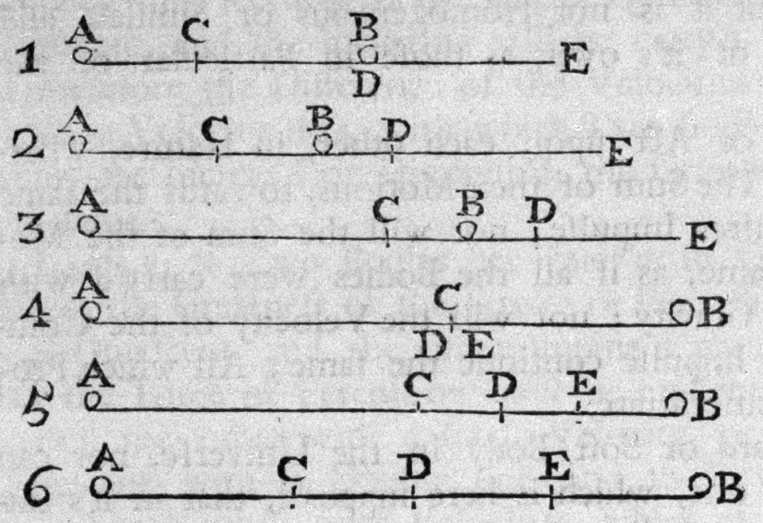
FOR let *A* Impinge upon *B*, according to the Conditions above-mention'd, ſince *B* has no Impediment from any Bodies, which ſurround it, it will, by the Force Impreſs'd from *A*, be mov'd towards the ſame Side, to which the Direction of that Force Tends, and it will be mov'd together, and in Conjunction with *A*, becauſe it cannot move ſlower than *A*, which is behind it, and preſſes it forward, nor can it move faſter than *A*, ſeeing it derives all the Force and Celerity, which it has, entirely from it, there being *ex Hypotheſi* no other Cauſe of this Motion, but the Superior Moment of the Impellent; and as to the Center of Gravity, that muſt Evidently be, where the Bodies are united and move together.



The Second Propoſition explain'd. Dr. Keill's Introduct. Lect. 13. Probl. 2.

§. 6. THE Second Propoſition, in this Doctrine, is to Determine the Motions of Hard and Soft Bodies after a Direct Impaction, which is eaſily done upon the Suppoſition of the Truth of the foregoing Theorem.

LET the Bodies *A* and *B*, whoſe Center of Gravity is *C*, concurr in *D*; by the preceding Theorem, they will from thence move together, and will deſcribe after Impaction a Line $DE=CD=$ to the Velocity of the Center of Gravity before Impaction, becauſe it's Velocity, by what has been already evinc'd, is the ſame before and after Impaction, and therefore the Velocity of *A* before Impaction, will be to it's Velocity after, as AD to DE, and the Velocity of *B* before Impaction to it's Velocity after, as BD to DE, ſince *A* and *B* will move with the Velocity of the Center of Gravity after Impaction.



Corol. IF *B Fig.* 1. is Quieſcent, the Points *B* and *D* will Coincide, and ſince $B.A::AC.BC=DE$ by the Nature of the Center of Gravity, it will be, by Compoſition, $A+B.A::AC+BC=AD.BC=DE$; that is, the Velocity of *A* before Impaction $=AD$ will be to it's Velocity after $=DE$, as $A+B$ the Sum of the Bodies to *A* the Impingent one.

THUS, 1*ſt*, If the Impingent *A* is = to the Quieſcent *B*, $A+B.A::2.1$. 2*d*. If $A.B::1.9$, $A+B.A::10.1$. 3*d*. If $A.B::0.1$, $A+B.A::1.0$, and ſuitably the Velocity of *A* before Impaction will be to it's Velocity after, in theſe ſeveral Caſes, as 2.1, or 10.1, or 1.0, which laſt Caſe is That of a Body Incurring upon a Firm and Immoveable Obſtacle, when, after Impaction, the Impingent *A* becomes Quieſcent.

AGAIN, if the Impingent *A Fig.* 2. is = to *B*, which is not now Quieſcent, but Moves the ſame way with *A*, tho' more ſlowly, ſince the Center of Gravity is in the Middle betwixt them, $AC=BC$, and therefore $CD,=DE$, will be $=\frac{AB}{2}+BD=\frac{AB+2BD}{2}=\frac{AD+BD}{2}$, or the Velocity after Impaction will be Equal to half the Sum of the Velocities before Impaction.

IF farther *A* and *B Fig.* 4. tend to Contrary Sides with Equal Motions, the Points *D* and *C* will Coincide, and $CD=DE$ will be $=0$, that is, after Impaction both the Bodies will be Quieſcent; From whence it is Infer'd, that the Law of the *Carteſians* is falſe, which affirms, that there is always the ſame Quantity of Motion Preſerv'd in the Univerſe, ſince, where Unelaſtick Bodies meet, with Equal Motions, thoſe Motions mutually deſtroy each other.

LASTLY, If Equal Bodies *Fig.* 5. are carry'd to Contrary Sides with Unequal Motions, $DE=CD=CB-BD=\frac{AB}{2}-BD=\frac{AB-2BD}{2}=\frac{AD-BD}{2}$, or the Velocity after Impaction will be Equal to half the Difference of the Velocities before Impaction.

The General Calculation by Algebra. Dr. Keill's Introduct. Theorem 19. Lect. 12.

§. 7. THE General Calculation of what forego's by *Algebra* is as follows; Let the Velocity of the Body *A* be *C*, of *B*, *c*; if the Bodies are mov'd the ſame Way, the Sum of their Motion will be $AC+Bc$, if Contrary Ways $AC-Bc$, but by a Theorem we ſhall ſee afterwards, the Sum of the Motions, to the ſame Side, will continue the ſame before and after Impaction, and therefore will be either $AC+Bc$, or $AC-Bc$, and $\frac{AC+Bc}{A+B}$ or $\frac{AC-Bc}{A+B}$ will Expreſs the Velocity of the Bodies *A* and *B* after Impulſe, and conſequently, if the Body *B* is Quieſcent, that is, if *c* is Equal to Nothing, the Velocity of the Bodies *A* and *B* will be Repreſented by $\frac{AC}{A+B}$.

The ſeveral Objections which lie againſt this Doctrine of the Mutual Concourſe of Hard or Soft Bodies.

§. 8. BUT againſt this Account of the Mutual Concourſe of Hard and Soft Bodies, ſo far as to explain any Thing in Nature by it, it is to be Objected.

FIRST, That the ſeveral Theorems concerning the Center of Gravity are Demonſtrated upon the Suppoſition, that Matter is in it ſelf Homogeneous and alike, nay farther, that it is Endu'd with no Active Properties of it's own, for if it is, tho' it ſhou'd be Granted, that it was Similar, the foregoing Demonſtrations wou'd not ſtand; On the Contrary, we affirm, that it is not Homogeneous or Similar, and that it is Endu'd with Active Properties of it's own, as thoſe in Particular of the Expanſive and Contractive.

SECOND, If therefore Bodies Impinge or Act upon each other, in Nature, whatever they may do, in any one's Fancy, The Sum of their Motions, towards the ſame Side, will not be the ſame before and after Impulſe; nor will the Sum of the Motions before and after Impulſe be the ſame, as if all the Bodies were carry'd with the Velocity of the Common Center of Gravity; nor will the Velocity of the Common Center of Gravity before and after Impulſe continue the ſame; All which Propoſitions this Philoſophy pretends to Demonſtrate.

THIRD, Neither is there any ſuch Hard or Soft Body in the Univerſe, nor can there be Imagin'd or Conceiv'd ſuch an one, which is here ſuppos'd, that in it's Impaction upon another, which is Quieſcent, ſhall proceed exactly along with it, ſo as to divide it's Incurring Velocity betwixt both; Eſpecially, if there is a Peculiar Action belonging to each, and which if you take away, you deſtroy the Matter or the Body it ſelf, which is, what I take upon me to Pronounce and Defend; and that, where we reſt in ſuch Imaginary Abſtractions as theſe, we may as well Propoſe the Logicks of *Burgerſdicius*, or the Eſſay of *Locke*, or the Elements of *Euclid*, which are all mere Abſtractions of the Mind, for Principles to Explain Nature.

FOURTH, On the Contrary, all which is here endeavour'd to be explain'd in the Concourſe of Bodies, and which we find in Fact, may be eaſily Accounted for by the Expanſive and Contractive Forces, which is, that a Body Incurring upon a firm Obex or Obſtacle may be Quieſcent after ſuch Impulſe, that is, if the Body, upon which the Impaction is made, has a Contractive Force Superior to the Expanſive, as in Lead or Clay, which therefore will not Repel the Impinging Body; and proportionably as the Contractive Force more or leſs prevails, the Impinging Body will leſs or more reſult from it, whether the Body, upon which the Impaction is made, is Quieſcent or Moves more ſlowly to the ſame Side; but This we have more fully Diſcours'd of in another Place.

Concerning the Doctrine of Percuſſion. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 13. Theorem 25, 26, 27, 28.

§. 9. IN order to Account for the Concourſe of Elaſtick Bodies, it is firſt Preſum'd, that if a Body, which is mov'd, directly Impinges upon another, which is either Mov'd or Quieſcent, the Greatneſs of the Stroke or Percuſſion will be Proportional to the Moment, which is loſt upon their Mutual Occourſe; for the Greatneſs of the Stroke will be Equal to the Force, which is Impreſs'd from the Percutient on the Body, which receives the Percuſſion, but the Force which is Impreſs'd upon the Body, which receives the Percuſſion, is loſt to the Percutient, by Law the Third of Re-action being Contrary and Equal to Action, and therefore the Motion, which is loſt to the Percutient, will be Proportional to the Force, which is Impreſs'd upon the Body, which receives the Percuſſion, that is, to the Greatneſs of ſuch Stroke or Percuſſion.

FARTHER, If a Body, which is Given, directly Impinges upon another Quieſcent Body, which is Given, the Greatneſs of the Stroke or Percuſſion will be always

Proportional to the Velocity of the Impinging Body; This ſcarcely wants any other Demonſtration upon the Preſent Principles, than Saying, that if the Body is the ſame, which Incurrs upon another, which is Quieſcent, the Greater the Force or Velocity is, with which it Incurrs, the Greater muſt be the Stroke or the Percuſſion receiv'd by the Body, which is Quieſcent; Or, on the Contrary, if the Body, which was before ſuppos'd Quieſcent, Incurrs upon that, which before was ſuppos'd the Incurrent one, and it loſes the ſame Motion, which the Incurrent one loſt before, the Magnitude or Greatneſs of the Stroke or the Percuſſion will be the ſame.

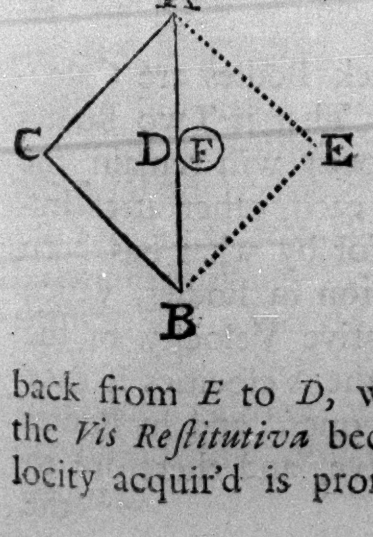
Again, It is told us, that if one Body Impinges directly upon another, which is carry'd more ſlowly to the ſame Side, the Greatneſs of the Stroke or Percuſſion will be the ſame, as if the Preceding Body were Quieſcent, and the Incurring were carry'd upon it with the Difference of their Velocities; Becauſe, by how much more Velocity the Preceding Body is mov'd with to the ſame Side with the Incurring, by ſo much more it declines or avoids it's Stroke or Percuſſion, by which means an Equal Portion of Velocity in the Incurrent will be ſpent without any Effect, and conſequently it will be the ſame, if we ſubſtract an Equal Portion of Velocity from each; from whence the Preceding Body will be Quieſcent, and will now ſtand the Shock of the Incurring, and the Incurring will be carry'd with the Difference of their Velocities; If therefore the Difference of the Velocities continues the ſame, that is, if the Reſpective Velocity do's, with which Two Bodies approach each other, howſoever their Sums are Increas'd or Diminiſh'd, the Greatneſs of the Stroke or Percuſſion will be the ſame.

Lastly, If Two Bodies are carry'd in Contrary or Oppoſite Motions to each other, the Greatneſs of the Stroke or Percuſſion will be the ſame, as if one of them was Quieſcent, and the other Imping'd upon it with the Sum of their Velocities; For the Force of Percuſſion muſt be Evidently the ſame, whether each Body ſtrikes againſt the other with a ſingle Velocity, or one ſtrikes againſt the other, which is Quieſcent, with a Double one, and ſo it will be the ſame, whether they Incurr againſt each other, with Velocities, which are as 2 and 3, 5 and 7, 6 and 9, *&c.* or whether the one Impinges upon the other, which is Quieſcent, with a Velocity of 5, 12, or 15; and therefore, if the Sum of the Velocities continues the ſame, that is, the Reſpective Velocity, with which the Bodies approach each other, whatſoever is the Difference of the Velocities, or however the Velocity is divided betwixt the Two Concurring Bodies, the Greatneſs of the Stroke or Percuſſion will be the ſame; Alſo conſequently in Bodies, which are Given, the Greatneſs of the Stroke or Percuſſion will be Proportional to their Reſpective Velocity.

§. 10. After the Premiſing theſe ſeveral Propoſitions concerning the Force of Percuſſion upon the Mutual Impulſe of Bodies, This Philoſophy proceeds to explain what is meant by Elaſticity.

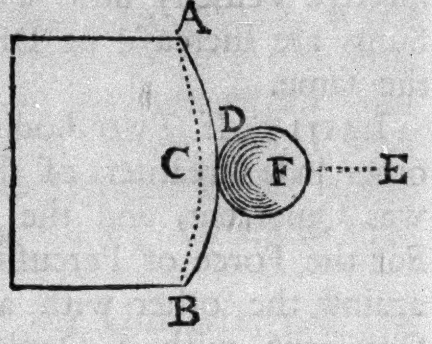
The Doctrine of Elaſticity, and the Cauſe of it explain'd. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 14. Theorem 29. and Probl. 3. Dr. Wallis de Motu, Cap. 13. Part 3. Vol. 1.

And Firſt, it is ſaid, that there are very few Bodies, in which there is not a ſmall Degree of it; for Clay, Wax, and the Like, contain certain Particles of Air in them, by which they become in ſome Meaſure Elaſtick, and from thence it is, that Bodies after Impulſe do not move together, but reſult from each other, and ſometimes are carry'd to the ſame Side, and ſometimes to Contrary ones, with a Different and Various Velocity.



Second, The Cauſe and Manner of Elaſticity or Reſilition is thus Illuſtrated; Let *A B* be a String fixt at the Extremities *A* and *B*, after it is drawn to the utmoſt Stretch, and then let the middle Point *D* be forc'd, from it's Situation there, to *C*, and from thence permitted and reſign'd to it's own Activity; it is Plain the String will not continue in *A C B*, but will endeavour to reſtore it ſelf to it's former Situation in *D*, and having acquir'd a Velocity by the continual Action of the *Vis Reſtitutiva*, it will not reſt in *A D B*, but be carry'd forwards to *E*, till the *Vis Reſtitutiva*, which acts contrary to this acquir'd Velocity, and at length is Equal to it, brings it back from *E* to *D*, which by the acquir'd Velocity is again carry'd towards *C*, till the *Vis Reſtitutiva* becomes Equal to it, and reduces it again to *D*, but by the Velocity acquir'd is promoted towards *E*, till the *Vis Reſtitutiva* again becomes Equal to it, and forces it back to *D*, *&c.* by which means the ſucceſſive Vibrations of the String are produc'd, till it becomes Quieſcent. Let us now imagine the Body *F* to Incurr upon the ſame String; by the Force in the Incurrent Body *F*, the Point *D* in the String will be carry'd to *C*, together with the Body *F*, which Motion will be continu'd, till the *Vis Reſtitutiva*, Acting contrary to the Motion of the Body *F*, is Equal to it, for then all the Motion towards *C* will be deſtroy'd, and the *Vis Reſtitutiva* will reduce the String to it's firſt Situation in *D*, and *F* will be brought back with the ſame Velocity with which the String returns; But upon the Account of the Extreme Tenſion of the String, it will reſtore it ſelf with the ſame Force, with which it was Inflected to *C*, and the Force, with which it was Inflected to *C*, is Equal to the whole Moment of the Impinging Body, which is ſuppos'd to be entirely ſpent upon the Flexion of the String; conſequently, the String will Recommunicate to the Body the Force, which was Imparted to it from it, and the Body *F* will return with the ſame Velocity, with which it Incurr'd, and thus Reflexion is perform'd.

LET us now farther, inſtead of a String, ſuppoſe ſome Elaſtick Body *A B*, and in the firſt Place, that it is Fixt and Immoveable, and let it's Surface *A D B* be compreſs'd Inwards by the Force of the Incurring Body *F*; As ſoon as the Force, which Compreſſes, that is, as ſoon as the Motion of the Body *F* ceaſes, the *Vis Reſtitutiva* will exert it ſelf, and reſtore the Body compreſs'd to it's Priſtine Figure, and urge and repel the Body *F* towards *E*; and if either of the Bodies is perfectly Elaſtick, the *Vis Reſtitutiva* of that Body, which is Elaſtick, will be Equal to the Force, which compreſſes it, that is, to the Moment of the Body *F*, and therefore Re-acting upon the Body *F* with that Force, it will oblige it to Retire with the ſame Velocity, with which it Imping'd: If the Body *A D B C* is not Fixt, but Moveable, the Elaſtick Force will act Equally in both Bodies, and produce Equal Changes of the Motions; For if the Body *A D B* urges the Body *F* towards *E*, it likewiſe will be Equally urg'd to the Contrary Side by the Body *F*, and the Bodies will be mutually Reflected from each other; and thus, it is ſaid, an Account is given, how Bodies after Impulſe are not jointly Quieſcent or Mov'd, but by Reſulting from each other with Different Velocities, ſometimes take Contrary Ways and ſometimes the ſame.



THIRD, The *Carteſians* affirm, that the Reflexion of Bodies is deriv'd from a Continuation of Motion, which cannot be deſtroy'd, the Direction of it being only alter'd, which is endeavour'd to be Refuted by this Philoſophy.

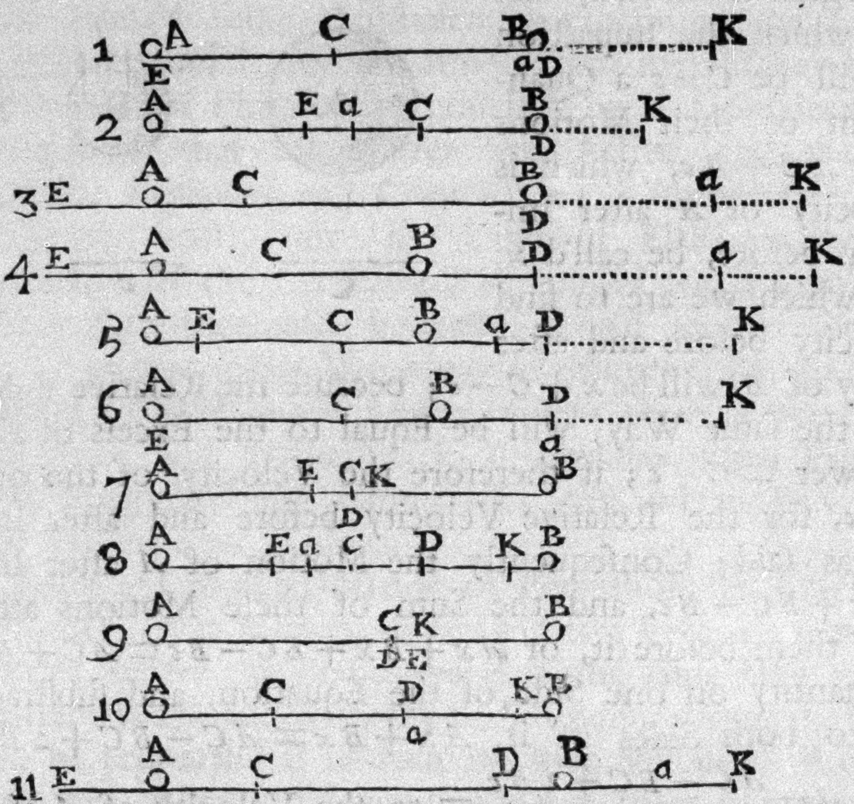
FOURTH, To prove there is ſuch an Elaſticity in Bodies here contended for, by which the Surface of the Elaſtick Body is compreſs'd Inwards by the Incurring Force, and then reſtores it ſelf to it's former Situation, and repels the Incurrent, an Experiment is alledg'd of Two Glaſs or Ivory Globes; of which let one be Quieſcent, and Painted with Ink or any Colour, and the other Impinge upon it, the Impingent, inſtead of being Ting'd in one Point of it's Surface only, which it ought to be, by the Nature of Spherical Bodies, which touch each other in a ſingle Point, will have a Broad Black or Colour'd Spot Impreſs'd upon it, and which is thought cannot poſſibly proceed from any Thing elſe, but the Depreſſion of it's Surface upon Impaction, and which is not afterwards perceiv'd, becauſe it's Elaſticity has reſtor'd it to it's firſt Figure.

The Theorem upon which the Rules of Congreſs in Elaſtick Bodies are calculated and defin'd. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 14. Theorem 29.

§. 11. UPON theſe Principles the Rules of Motion for Elaſtick Bodies are Calculated and Defin'd, and the Twenty-Ninth Theorem is Propos'd; That if Two Bodies perfectly Elaſtick Impinge upon each other, their Relative Velocity will remain the ſame before and after Impaction, that is, they will retire from each other after Impaction with the ſame Celerity, with which they Imping'd; For by what has been ſhewn, the Compreſſive Force, or the Greatneſs of the Percuſſion in Bodies, which are Given, that is, which are the Same, ariſes from the Relative Velocity of the Bodies, and is proportional to it, namely to the Difference of the Velocities if they proceed the ſame Way, and to the Sum of them if Contrary ones, and in Bodies perfectly Elaſtick the *Vis Reſtitutiva* is Equal to the Compreſſive, and therefore to the Relative Velocity or that Force, with which the Bodies were carry'd to each other before Impaction, But by the ſaid *Vis Reſtitutiva* the Bodies are forc'd to depart from each other, and conſequently it's Acting upon them will produce that Relative Velocity in their Return, which they had upon their Acceſs, or, which is all one, will cauſe them to Retire with the ſame Velocity, with which they Imping'd; From whence it is Infer'd by way of Corollary, that if we take Equal Times before and after Impaction, the Diſtances of the Bodies from each other will be Equal, and by conſequence in the ſaid Times, the Diſtances of the Bodies from the Common Center of Gravity will be Equal.

The Rules of Congreſs in Bodies perfectly Elaſtick.

§. 12. FROM the preceding Corollary are the Rules of Congreſs in Bodies, which are perfectly Elaſtick, determin'd. Let A and B be Two Bodies perfectly Elaſtick, whoſe Common Center of Gravity is C, and let the Bodies be ſuppos'd to meet in D, let CE be made Equal to CD, it is affirm'd; That after Impaction EA will repreſent or expreſs the Velocity of the Body A, from E towards A, and EB the Velocity of the Body B, from E towards B; for the Common Center of Gravity, by what has been ſhewn, before and after Impulſe, proceeds uniformly with the ſame Velocity, and therefore, ſince CD is the Velocity of the Center of Gravity before Impulſe, being deſcrib'd by C in the ſame Time as AD is by A, DK, which is made Equal to CD, will be the Velocity of the Center of Gravity after Impulſe; but, by the Corollary juſt now explain'd, in Equal Times before and after Impulſe the Diſtances of the Bodies from the Common Center of Gravity are Equal, and conſequently, if Ka is made Equal to AC, the Body A will be in a after Impulſe, and it's Velocity Da; Farther, as Ka is $=AC$ and $KD=EC$, $Ka-KD=Da$ will be Equal to $AC-EC=EA$, that is, the Velocity of the Body A after Impulſe, will be $=Da$ from D to $a=EA$ from E to A; Alſo the Relative Velocity of the Bodies being the ſame before and after Impulſe, and EA being the Velocity of A, the Velocity of B after Impulſe will be EB from E to B; becauſe whether A and B move the ſame Way, or Contrary ones, the Relative Velocity will be as the Differences, or as the Sums of the Velocities of both the Bodies, or ſuppoſing the Quieſcence of one of them, as the Velocity of the other. *Q.E.D.*



Corol. 1. FROM this Demonſtration it is Infer'd, Corollary the Firſt, that if the Body B is Quieſcent, D and B will be Coincident, as in the Three firſt Figures, and ſince $B.A::AC.CB$ by the Center of Gravity, by Compounding, $B+A.A::AC+CB=AB.CB$, and by doubling the Conſequents, $B+A.2A::AB.2CB=EB$, or the Sum of the Bodies will be to Twice the Impinging Body, as the Celerity of the Impinging Body before Impaction, to the Celerity of the Quieſcent Body after it.

Corol. 2. THEREFORE if $A=B$, *Fig.* 1. $A+B=2A$, and EB the Celerity of B after Impaction $=AB$ the Celerity of A before it, and therefore A and E Coinciding, AE the Celerity of A after Impaction $=0$.

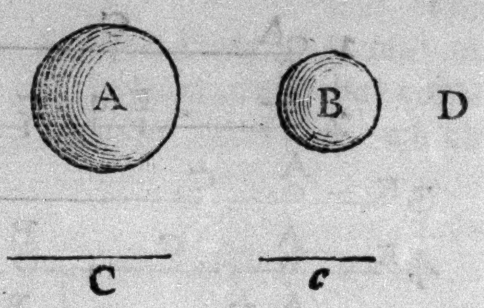
Corol. 3. IF $A=B$, *Fig.* 4. and they are carry'd to the ſame Side, they will after Impaction be carry'd to the ſame Side, and exchange Celerities with each other; For ſince $CE=CD$ by Conſtruction, and $AC=CB$ by the Center of Gravity, $CE-AC=EA=CD-CB=BD$, and therefore EA the Celerity of A after Impaction $=BD$ the Celerity of B before it; Farther, ſince $EA=BD$, $EB=AD$, becauſe the Relative Celerity, both before and after Impaction, continues the ſame, and therefore EB the Celerity of B after Impaction will be Equal to AD the Celerity of A before it.

Corol. 4. If $A = B$, *Fig.* 8. and they are carry'd to Contrary Sides, after Impaction they will recede to Contrary Sides and exchange their Celerities; For ſince $AC = CB$ by the Center of Gravity, and $EC = CD$ by Conſtruction, $AC - EC = AE = CB - CD = BD$, and therefore EA the Celerity of A after Impaction $= BD$ the Celerity of B before it; Farther, ſince $EA = BD$, $AD = EB$, becauſe, the Relative Celerity before and after Impaction continues the ſame, that is, EB the Celerity of B after Impaction will be Equal to AD the Celerity of A before it.

The foregoing Rules Algebraically Demonſtrated.

§. 13. This Doctrine Algebraically is thus Demonſtrated.

1. Let us firſt ſuppoſe the Bodies A and B, whoſe Bulks are known, to be carry'd to the ſame Side, and let C, a Quantity known, be the Velocity of the Impinging Body A, and c, alſo a Quantity which is known, that of the preceding Body B, upon which the Impaction is made, the Relative Velocity will be $C - c$ a Quantity likewiſe known, and the Sum of their Motions towards the ſame Side will be $AC + Bc$, which is therefore known; let the Velocity of A after Impaction, towards the ſame Side as before, be call'd x, being a Quantity unknown, and which we are to find out, and ſince the Relative Velocity before and after Impaction is the ſame, the Velocity of B will be $x + C - c$; becauſe the Relative Velocity of Bodies, which are mov'd the ſame Way, will be Equal to the Exceſs of the Swifter Velocity above the Slower $= C - c$; if therefore the Velocity of the one is x, the other muſt be $x + C - c$, for the Relative Velocity before and after Impaction continues the ſame, as was ſaid; Conſequently the Motion of A after Impaction is $= Ax$, and of $B = Bx + BC - Bc$, and the Sum of theſe Motions after Impaction is Equal to the Sum of them before it, or $Ax + Bx + BC - Bc = AC + Bc$, and by leaving the unknown Quantity on one Side of the Equation, and ſubſtracting BC from and adding Bc to both Sides of it, $Ax + Bx = AC - BC + 2Bc$, and by dividing each by $A + B$, $x = \frac{AC - BC + 2Bc}{A + B}$ = to the Velocity of A after Impaction; but the Velocity of B is $= x + C - c$ after Impaction $= \frac{AC - BC + 2Bc}{A + B} + C - c = \frac{AC - BC + 2Bc + AC + BC - Ac - Bc}{A + B} = \frac{2AC - Ac + Bc}{A + B}$. If therefore BC is Greater than $AC + 2Bc$, x or $\frac{AC - BC + 2Bc}{A + B}$ will be a Negative Quantity, and conſequently, the Velocity of the Body A will be towards the Contrary Side, and it's Motion towards D will be Negative, that is, from it. If B is Quieſcent, or $c = 0$, the Velocity of A after Impaction is $= \frac{AC - BC}{A + B}$, either forwards or backwards, as $+$ or $-$ prevails.



2. Let us now ſuppoſe A and B to be carry'd with the Velocities C and c to contrary Sides, the Sum of their Motions to the ſame Side will be $AC - Bc$, and their Relative Velocity, where they meet each other, $C + c$; let the Velocity of A after Impaction be x, and it's Motion, towards the ſame Side as before, will be Ax, the Velocity alſo of B after Impaction will be $x + C + c$, becauſe the Relative Velocity $C + c$ continues the ſame after Impaction, and the Motion of B towards D will be $Bx + BC + Bc$, and the Sum of the Motions towards the ſame Side $Ax + Bx + BC + Bc$ after Impaction $= AC - Bc$ the Sum of 'em before it, and therefore $Ax + Bx = AC - BC - 2Bc$, and $x = \frac{AC - BC - 2Bc}{A + B}$ the Velocity of the Body A after Impaction, and $x + C + c = \frac{AC - BC - 2Bc}{A + B} + C + c = \frac{AC - BC - 2Bc + AC + Ac + BC + Bc}{A + B}$

$= 2AC$

$= \frac{2AC + Ac - Bc}{A + B}$ the Velocity of B after Impaction. If therefore $BC + 2Bc$ is Greater than AC, x will be a Negative Quantity, or A after Impaction will be carry'd backwards and retire from D; The Expreſſions above of the Velocities of A and B after Impaction are exactly Analogous to what we find in Sir *Iſaac Newton*'s Algebra, Problem. 12. Pag. 91. and in my worthy Friend the Reverend Mr. *Morgan*'s Comments upon *Rohault*; and in theſe Laws of perfectly Elaſtick Bodies, both *Chriſtian Hugens* and Sir *Chriſtopher Wren* did alſo agree, tho' calculated ſeparately and diſtinctly by them; and no wonder, ſince they all depend upon the ſame Principle of a perfect Elaſticity, a Notion originally deriv'd from Dr. *Wallis*, and which many Years ago I particularly examin'd, and wrote ſeveral Papers in Confutation of it, which are ſome where by me, but not occurring, I ſhall here repeat that Trouble.

Rohault's Phyſ. Cap. 11. Par. 1. Edit. 3.

An Examination of the foregoing Doctrine of Percuſſion.

§. 14. I once was in Hopes, that from theſe Laws of Elaſtick Bodies I ſhou'd have receiv'd ſome Aſſiſtance in explaining the Laws of Nature; for having been long ſince, and I may ſay almoſt from my Infancy in this Univerſity and from my firſt Study of Philoſophy, poſſeſs'd, by a Conſideration of Material Beings, with a Perſuaſion, that all Matter was Active, and that it's Forces might be reſolv'd into the Expanſive and Contractive; I immediately concluded, that when I came to ſearch ſtrictly into the Laws of Elaſticity, which ſeem'd to have ſome Reſemblance of the Expanſive I contended for, I ſhou'd there find ſomething pertinent to my Enquiries, and at leaſt I ſhou'd receive ſome Light and Information from thence of the ſecret Workings of Nature; But to my great Diſappointment, when I came to look more narrowly into them, I perceiv'd they were nothing elſe but Abſtracted and Mathematick Reaſonings upon Principles and Suppoſitions, which were Imaginary and Chimerical, and which had no Foundation in the Reallity and Exiſtence of Things.

THUS, As to the Doctrine of Percuſſion, in which it is affirm'd, that the Force or Greatneſs of it is Equal to the Relative Velocity, that is, to the Difference of the Velocities, where the Bodies proceed the ſame Way, and to the Sum of them, where contrary Ways, and that, the Bodies being given, the Force or Greatneſs of Percuſſion will be proportional to ſuch Relative Velocity, We ſay, that the Demonſtrations of This depend upon a Suppoſition, that there is nothing to be conſider'd in Matter but the Force of it's Velocity, that it is Similar and has no peculiar Forces of it's own, for if it had, theſe Computations of the Force of Percuſſion, from the Difference or Sum of the Velocities of the Incurring Bodies, wou'd be quite diſorder'd and confounded, and it wou'd be Impoſſible for us ever to know, what the Force of Percuſſion wou'd be, without underſtanding the Nature of thoſe Forces, which reſide in the Bodies themſelves; For Inſtance, let Fire and Water meet, I ſuppoſe the Force of Percuſſion will not be readily determin'd by the Rule of Relative Velocity, and ſomething more muſt be firſt Explain'd, that is, the Nature of the Force of theſe Two Bodies, before we can pronounce, what will be the Reſult and Effect of their Congreſs.

The Doctrine of Elaſticity examin'd.

§. 15. As to Elaſticity, in the Beginning of the Tenth Section it is confeſs'd, that there are few, if any Bodies, which are not endu'd with ſome Degree of it, as even Clay and Wax have ſome; from whence it is plain, that the Doctrine, Deliver'd before, concerning Bodies, which are perfectly Hard or Soft, is acknowledg'd to be a Creature of the Mind, without any ſuch Bodies Exiſting in Nature; and therefore it is ſaid, that after Impulſe they do not move together, but reſult from each other with varying Velocities; whereas this Elaſticity yet is aſcrib'd to Particles of Air Inſinuating themſelves into, and reſting in the Pores of theſe Bodies, it ſeems to be a mere Fiction, and we may as well attribute the Tranſparency of Glaſs to Tranſparent Particles of Air diſpers'd in it's Pores, as the Elaſticity of Bodies to Elaſtick Particles of Air lodg'd in Them; For why may not Bodies be in themſelves, in ſome Degree or other, Elaſtick, as well as Air is, unleſs becauſe the Preſent Philoſophy of Paſſive Matter cannot account for it's Elaſticity without Introducing ſuch extraneous Particles? Which is no Objection to us, who aſſert an Active Matter, and that all Matter has a Proportion and Degree of the Expanſive; Beſides, this Fancy of the Elaſticity of Bodies, being deriv'd from Elaſtick Particles of Air mixt with them, is

Contrary to the following Explanation of it, which reſigns it to the *Vis Reſtitutiva* in Bodies themſelves.

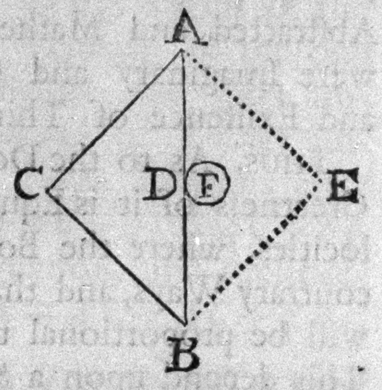
The Vis Reſtitutiva *gives no Solution of what Elaſticity is.*

§. 16. That there is ſuch a Phænomenon or Effect in Nature, as Elaſticity, or a Force of Reſilition, by which Bodies are Driven and do Recoil from each other, is beyond Diſpute, being confirm'd by daily Experience, but that the *Vis Reſtitutiva* is not an adequate Cauſe of ſuch an Effect, I ſhall now Evince.

For there is no Satisfaction given us, how this *Vis Reſtitutiva* comes to be inherent in ſome Bodies, and not in Others, or at leaſt not in the ſame Degree, whence it is deriv'd to, and how it ſubſiſts in them; there is nothing Explicitly ſaid of it, which ſhou'd make us not conclude it a Fictitious Force, and to have it's Being in the Imagination only; which is not the Caſe of the Expanſive and Contractive we Defend, which we have already ſhewn to Exiſt in all Nature in Various and Different Proportions, and have explain'd the mediate Sources, from whence they are deriv'd, tho' ultimately from God himſelf.

Besides, that the *Vis Reſtitutiva* is an Imaginary Force invented by Philoſophers, on purpoſe to account for this particular Difficulty in Nature, is Evident from hence, that it is not made uſe of upon any other Occaſion, nor is it connected with the Explanation of any other Phænomenon whatſoever; whereas the ſame Expanſive and Contractive Forces, which we alledge for the Solution of this, are connected with and extended to all other Appearances, we can name, of Solidity and Fluidity, Hardneſs and Softneſs, Gravity and Levity, Cold and Heat, Opacity and Tranſparency, &c.

Farther, the Explication, which is given of the *Vis Reſtitutiva*, ſhews it to be an Imaginary Force, a mere Invention of the Philoſopher, without any Ground or Foundation in Nature or Reaſon; ſince, altho' this *Vis Reſtitutiva* ſhou'd be allow'd to the String *ADB*, when drawn by an External Force to *ACB*, by which it might reduce it to it's former Situation in *ADB*, how the ſame *Vis Reſtitutiva* ſhou'd propel and carry it from thence to *AEB* is inconceivable, unleſs the Force, which reſtores it to it's firſt Situation, has the Power to repel it from that Situation, that is, unleſs the *Vis Reſtitutiva* to a certain Situation, is a *Vis Repulſiva* from it, which is a Contradiction; and for the ſame Reaſon, granting it Poſſible for the *Vis Reſtitutiva* to propel the String from *ADB* to *AEB*, how the ſame *Vis Reſtitutiva* ſhou'd bring it back from *AEB* to *ADB* cannot be Imagin'd, and ſo it may be argu'd in it's Second Progreſs to *ACB*, and Regreſs to *ADB*, or it's Second Progreſs to *AEB*, and Regreſs to *ADB*, or in it's Third, Fourth, Fifth, &c.



This Philoſophy ſeems to have been aware of the foregoing Difficulty, and therefore aſcribes the Progreſſes to the Velocity acquir'd from the Motion of the String, and the Regreſſes to the *Vis Reſtitutiva*; But can there be any Velocity acquir'd, which is not owing to ſome Cauſe? And is there any Cauſe aſſign'd excepting that of the *Vis Reſtitutiva?* And can a Velocity acquir'd from the *Vis Reſtitutiva* act againſt and in Oppoſition to the *Vis Reſtitutiva* from whence it is deriv'd? It is altogether abſurd to ſuppoſe it; and we may as well affirm, that if we Impel a Body one Way, it will move a Contrary one from that Impulſe without any other Direction.

Elaſticity ſolv'd from the Doctrine of the Expanſive and Contractive Forces.

§. 17. On the other Hand, the Expanſive and Contractive Forces give a plain Account of this Phænomenon, for *A* and *B* being Two fixt Points, upon which the Tenſion of the String is made, they are Two Centers of Contraction, towards which the Intermediate Parts of the String are urg'd, and betwixt which they are Attracted and Drawn; Again, the Parts about *D*, which are the fartheſt from the Centers of Contraction, will be the leaſt Affected by their Influences, and conſequently will have leſs of the Contractive and more of the Expanſive Force, to which if we add the Expanſive Force of the Finger drawing the String from it's Contractive Situation, or the Expanſive Force of the Velocity of the Incurring Body *F*, here will be Two Contrary Forces, the Contractive and Expanſive, Acting againſt each other, by which means a Conteſt will be rais'd and continu'd betwixt them, till the one or the other prevails, and mutual Reciprocations excited, till the String becomes Quieſcent.

Thus, when the String is in *C*, the Expanſive Force, tho' Inferior to the Contractive, ſtruggles and acts againſt it, and if it ſhou'd be Increas'd ſo far as to break it, the Expanſive wou'd then prevail and be Superior, and the Contractive Force from *A* and *B* wou'd obtain no farther Effect; but the Expanſive at *C* being Inferior to the Contractive in *A* and *B*, it is forc'd to yield to the Contractive, which reduces the String to *ADB*, notwithſtanding which, a Portion of the Expanſive ſtill ſubſiſting in the String, and being not Entirely vanquiſh'd and ſubdu'd by the Contractive, it now exerts it ſelf towards *E*, ſtill in Oppoſition to the Contractive in *A* and *B*, to which it is always Contrary, till it is again made to yeild to the Contractive, and the String is brought back to *D*; and thus the Progreſſes and Regreſſes are perform'd, the firſt by the Power of the Expanſive Force, and the laſt by that of the Contractive, till the impreſs'd Inferior Expanſive is ſpent and ſuccumbs, and the Contractive prevails, and therewith the String obeys the Superior Force, and at length becomes Quieſcent betwixt the Two Centers of Contraction *A* and *B*.

So likewiſe, for the Caſe is much the ſame, ſuppoſing Two Bodies, which are in an Equal Balance with each other, to be ſuſpended upon their Common Center of Gravity, which being the Point, by which they are detain'd in their Æquilibrium, is a Center of Contraction, and that Force, which is Impreſs'd upon them from the Earth's Contraction, is a Force of Expanſion, by which they tend from their Center of Gravity towards the Earth; if therefore by Impulſe a new Expanſive Force is Communicated to the Balance, the Bodies will riſe and fall by Turns from this Expanſive, till the Force is ſpent and the Contractive prevails, whereby the Bodies become Quieſcent; that is, *D*, by an Expanſive Force, Oſcillating upon *A* and *B* the Contractive ones, will not be much Different (as to the Manner of theſe Forces acting) from *A* and *B*, by an Expanſive Force, Oſcillating upon *D* a Contractive; In both Inſtances, the Expanſive Force will carry the String or the Balance out of the Line *ADB*, and the Contractive will reduce them to it, till the Expanſive is overpower'd and over-comn, and in the End ſubmits to the Contractive.

§. 18. The Expanſive and Contractive Forces are not only capable of ſolving the preſent Phænomenon, which the *Vis Reſtitutiva* cannot do, but alſo, as will be ſhewn afterwards, all the Properties of Matter whether Eſſential or Accidental, and upon which Account we affirm Action or Force, which is divided into an Expanſive and Contractive one, to be the very Eſſence of Matter; What I mean, is, that Matter is Action or Force it ſelf, and not the Support, Subſtratum, or Seat of it; but this we ſhall perhaps have another Opportunity of ſpeaking more largely to.

A Summary Account of the Doctrine of the Expanſive and Contractive Forces, and of their Extenſiveneſs in ſolving the Effects of Nature.

What we ſhall take Notice of at Preſent, by way of Anticipation, is, that this Doctrine makes only Two Principles in Nature, the Expanſive and Contractive, both Contrary to each other, and both reſolvible into one Principle of Action; which, in Reſpect of the Being of a God, is agreeable to our Notions of one Supreme and Superintending Agent, and indeed is an Evident Proof of one, and alſo explains the Reaſon, why ſome have imagin'd Two Gods, the one the Author of Good, and the other of Evil, in Compliance with theſe Two diſtinct and oppoſite Forces; And in Reſpect of Nature, what is more Evident than that the Whole of it, as to the Material or Immaterial Part, is diſtinguiſh'd into Effects, which are either Contrary to each other, or mixt and compounded of ſuch as are?

Thus, as to the Material Part, there are Light, Heat, Tranſparency, Fluidity, Levity, Elaſticity; and in Relation to Colours, Red, Orange, Yellow; to Taſts, Sweet and Luſcious; to Smells, Fragrant; to Sounds, Harmonious and Soft, which fall under the Expanſive, in which yet there is ſome Mixture of the Contractive; whilſt the Contrary Qualities, as Darkneſs, from the Earth's Contraction of the Medium which the Sun Expands, Cold, Opacity, Solidity, Gravity, Unelaſtickneſs; and in Relation to Colours, Blue, Violet, and Black, (for Green Lies Intermediate to both the Orders); to Taſts, Acid and Sower; to Smells, Fœtid; to Sounds, Harſh and Dull, ſeem to derive their Origin chiefly from the Contractive, tho' not without ſome Degree of the Expanſive in their Compoſition.

Also, as to the Immaterial World, and that Thinking Subſtance, which Animates and Informs us, tho' it is Different and Diſtinct from Matter, yet has it's Expanſive and Contractive Forces; Thus Gaiety, Wit, Memory, and Recollection, Levity, Courage, Raſhneſs, and Fool-Hardineſs, Fierceneſs, Hatred, Haughtineſs, and Inſolence, Anger, Revenge, Ill-Nature, and Covetouſneſs, Appear to be deriv'd from the Expanſive Force of our Minds, by which we Repel others from us; On the Contrary, Gravity and Judgment and Reaſoning, a Generoſity and Fear of Offending others and a Love of them, Dejection of Spirit, Clemency and Compaſſion, and the Like, are from the Contractive Force of them, by which we Draw and Attract others to us; and all the Various Paſſions and Affections of our Minds have a Greater or Leſs Degree and Proportion of the Expanſive or Contractive Force in them, or are compounded of them; And by this Means, by the Mind and Matter agreeing in theſe Forces, we ſuppoſe their Union is effected, tho' Thought and the Mind it ſelf is manifeſtly Different from theſe Forces, and in the ſame Manner, as Almighty God is diſtinct from the Univerſe, which yet he has been pleas'd to Fill and Actuate with them.

A Continuance of Motion is produc'd, ſo long as there is a Conteſt maintain'd betwixt the Two Oppoſite Forces, the Expanſive and Contractive.

§. 19. BUT to leave theſe Diſquiſitions at Preſent, which we may perhaps ſome other Time reſume, ſince we only intended to ſhew in General here the Extenſiveneſs of theſe Principles of the Expanſive and Contractive Forces, we ſhall farther obſerve from this Explanation of Elaſticity by them, that a Continuance of Motion is produc'd, ſo long as there is a Conteſt maintain'd betwixt the Two oppoſite Forces, the Expanſive and Contractive, but when the one or the other entirely Prevails, the Effect of the other Ceaſes; thus if, in the Tenſion of a String, the Expanſive Force is leſs than the Contractive, there is a Reciprocation of the Progreſſes and Regreſſes of it, till the Expanſive is Subdu'd and the Contractive is Predominant, by Reducing it at length to a Quieſcence in *ADB*; if therefore now the Expanſive Force, Acting towards *C*, was Conſtant and Regular, and Equal to the Contractive from *A* and *B*, there wou'd be an endleſs Reciprocation of Motion, and the Progreſſes and Regreſſes wou'd be continu'd for ever, from whence a Perpetual Motion wou'd ariſe; but if the Expanſive ſhou'd be Superior to the Contractive, the String wou'd Break and be Diſſolv'd, and the Effects of the Contractive Forces from *A* and *B* wou'd be Deſtroy'd.

Where Two Forces are Equal and Contrary, there will be a Perpetual Motion and Conteſt betwixt them, and if the one or the other prevails, the Effect of the Oppoſite Force will be ſo far ſuperſeded, and this is a General Law of Nature.

§. 20. FROM hence therefore we may lay down this for a Law of Nature, that where Two Forces are Equal, and are Contrary in their Genius and Temper, and Act againſt each other, as the Expanſive and Contractive, there will be a Perpetual Motion and Conteſt betwixt them, neither Prevailing againſt the other; but if the one or the other Prevails, the Effect of the oppoſite Force will be Superſeded, and Vacated in ſome Degree, tho' not entirely Deſtroy'd and Annull'd.

THUS alſo, if *ADB* is conſider'd as a Balance, in which the Bodies *A* and *B* are Equally Pois'd upon *D*, if the Expanſive Force, from Impulſe upon *A* or *B*, is Leſs than the Contractive in *D*, there will be a Reciprocation of Motion till the Balance reſts in *ADB*, and ſubmits to the Contractive in *D*; On the Contrary, if the Expanſive Force, from Impulſe upon *A* or *B*, is Greater than the Contractive in *D*, it will ſtrike off the Body *A* or *B*, and free it from it's Center of Contraction in *D*; But if the Expanſive Force in *A* or *B* is Equal to the Contractive in *D*, ſuch a Body muſt be Perpetually mov'd backwards and forwards about it's Center *D*, by Continual Oſcillations, without being ever Quieſcent in the Line *ADB*; and if *A* or *B* is now taken off, which is an Equal Poiſe to the other, the Remaining Body muſt be carry'd in a Continual Revolution round the Center of Contraction; For ſince neither the Contractive nor Expanſive Force are ſuppos'd to Prevail, but are Equal, there muſt a Perpetual Motion enſue, and ſince it cannot be to the Contractive Force nor from it, for then the one or the other wou'd Prevail, nor yet backwards and forwards by Way of Oſcillation, becauſe one of the Bodies is now taken off from the Æquilibrium which caus'd ſuch Oſcillation, it muſt be in an Orbit round it.

THAT this Law of Nature obtains in all Material Beings, is Evident; Thus in the Contractive Force of Fuel and the Expanſive of Fire, there is a Motion produc'd and continu'd ſo long, as there is a Contention betwixt thoſe Two Forces; If the Expanſive of the Fire is Superior, the Contractive is ſubdu'd, and the Fuel reduc'd to a Light and Expanſive Aſhes, with little or no Contraction belonging to them; If the Contractive Prevails, the Expanſive Ceaſes, and the Fire is Extinguiſh'd; And if an Equal Balance is kept up betwixt the Expanſive of the Fire, and Contractive of the Fuel, by a continual Addition of the Latter, in a juſt Proportion to the Former, the Motion and the Fire becomes Perpetual, the Air, which is always neceſſary to the Being of Fire, having the Nature of the Expanſive, and Concurring with it.

§. 21. **From** what foregoes, the Reaſon, why a Perpetual Motion cannot be found out in our Earth is, becauſe the Contractive Force of the Earth and the Bodies which belong to it, is Infinitely Superior to the Expanſive, deriv'd from the Sun, or from any Subterraneous Fires; which, in all Probability, were originally owing to ſome Accenſion of Inflammable Matter, and that Accenſion to the Sun, or to the Artificial Methods we have of Producing Fire, by the Means and Aſſiſtance of it; and therefore all the Interior Parts of the Earth, we may Reaſonably think, are of a Strong and Contractive Force, and that, the Nearer we come to the Center of it, the Matter is the more Compact and Denſer.

Why a Perpetual Motion cannot be found out, and why yet there is one in the Heavens.

On the Contrary, the Reaſon, why we have a Perpetual Motion in the Heavens, is, becauſe the Expanſive Force of the Sun and the Contractive of the Planets are in their ſeveral Diſtances nearly Equal, and from which Continual Struggle and Conteſt betwixt the Sun and them, the Conſtant Periods, which they keep, do Ariſe. And, as theſe Forces of the Sun and Planets vary, They come Nearer to, or retire Farther from it.

§. 22. **This** Law of Nature is likewiſe manifeſt from the Mixture of what the Chymiſts call *Acid* and *Alkali*, which we have more particularly Diſcours'd of in another Place, and ſhall here only conſider them, ſo far as they juſtify the Preſent Law; for, whilſt neither the Expanſive nor the Contractive Force prevail, there is an Effervеſcence and an Ebullition produc'd, but ſo ſoon as the Contractive has the Aſcendant, a Coagulum or a Precipitation is cauſ'd, and when the Expanſive of Spirit of Nitre, a Bright and Tranſparent Fluid ſucceeds; Thus alſo Winds, Tempeſts, and Hurricans in the Air, Follow from a Contention betwixt the Expanſive of the Sun, and the Contractive of the Earth and Cold, which therefore with us Generally happen betwixt Summer and Winter, that is, in Spring or Autumn, when the Expanſive and Contractive Forces are the neareſt to an Æquilibrium, and before the Expanſive is Predominant in Summer, and the Contractive in Winter.

The foregoing Law farther Evinc'd.

Why Winds and Tempeſts are rather in Spring or Autumn, than in Summer or Winter.

Farther, in Natural Bodies, which are all of them produc'd by a Preceding Fermentation, as Liquids from Fruits, which are evidently Raiſ'd from the Expanſive Force of the Sun, and from the Contractive of the Earth and Soil in which they Grow, there is firſt a Motion and Vegetation ariſes from theſe Two Contending Powers, this Conteſt is afterwards continu'd by Fermentation, which is owing to the Mixture of the Expanſive and Contractive Force, till at length the Expanſive prevails and ſettles in a Fluid, in which the Contractive has it's Effect, which is Evident from it's Continuity and Gravitation, but is Inferior to the Expanſive by it's Fluidity, Lightneſs, and Tranſparency.

How Liquid Bodies are Form'd.

On the other Hand in the Formation of Solid Bodies, as Wood and Metals, there is a Fermentation from the Expanſive and Contractive Force, till at length the Contractive prevails, and makes the Body, like the Coagulum before mention'd in Chymiſtry, Opake, Solid, and Heavy, but yet not ſo, as entirely to Deſtroy or Annull the Expanſive, which Shows and Exerts it ſelf in Elaſticity, Colour, and in a Greater or Leſs Degree of Levity, as the Expanſive is leſs or more ſubdu'd by the Contractive.

How Solid Bodies are Made.

§. 23. **And** from hence we may have a juſt Account of what Earth, Fire, and Water is, the Former being that Matter, in which the Contractive moſt prevails, and the Latter in which the Expanſive; that is, where the Expanſive Force of the Sun has juſt ſo much Power, as to Raiſe the Contractive Force of our Planet into a Fluidity, it becomes Water, where into an Inflammability, it becomes Fire, where into Fermentation, it becomes a Vegetable or a Metal, and where into none of theſe, it becomes Earth; ſo that all theſe Phænomenons in Nature ſeem only to be Deriv'd from the Various Proportions, Mixtures, and Degrees of the Expanſive and Contractive Forces, which Exiſting in Separate and Diſtinct Portions conſtitute all the Bodies we behold, and in one General and Unlimited Diffuſion, that Plenum we contend for, by which all the Material Beings in the Univerſe are Connected into one continu'd Active Subſtance, Originally Deriv'd and Produc'd from one Infinite and Almighty Agent, the Supreme Orderer and Governor of all Things, God; From whence it will Appear, that tho' Space is Full, yet that it is not Equally ſo, there being Different Forces and Actions, ſome

What Earth, Water, Air, Fire, Matter, and Plenum is, and how reduc'd to Geometry.

Sir Iſ. Newt. Princip. Prop. 6. Corol. 3. Lib. 3. Edit. 2. Compar'd with Edit. 1.

Thicker and Denſer, and Others more Feeble, Diluted, and Languid, as thoſe of Gold on the one Hand, and Water or Air, or Æther on the other, in the ſame or Equal Portions of the ſaid Space: This was firſt Affirm'd by us in the Philoſophy, which we ſome Years ago Publiſh'd in Oppoſition to a Vacuum, which has been ſince Subſcrib'd to by the Modern Philoſophy, but without Explaining how Space is not *Equally full*, which we have here done. From hence alſo is Deriv'd a True and Proper *Geometry* of *Matter* hitherto unknown to Philoſophers, becauſe where they make Matter an Abſtracted and Homogeneous Subſtance, it is Impoſſible it's Nature ſhou'd admit of Quantities and Proportions; On the Contrary, we who affirm Matter to conſiſt of Various Degrees of Forces in the ſame Space, it is no Wonder, that we may be allow'd to Diſcourſe of the Mathematicks and Quantities of it.

Geometria
Solidorum
ſive Materiæ.

The ſame Law of Nature Confirm'd from the Immaterial or Thinking Part of it.

§. 24. But to proceed upon the Preſent Law of Nature, which is alſo Confirm'd from the Immaterial and Thinking Part of it; We plainly Experience, that all Quarrels and Diſſenſions, which are properly the Motions and Agitations of Men's Minds, Ariſe from Oppoſite and Different Qualities in them, that is, from the Expanſive and Contractive Forces which belong to 'em; Thus Wit, Gaiety, and Mirth, are conſtantly at Variance with Judgment, Sedateneſs, and Gravity; Vice Combats with Vertue; Inſolence and Pride with Modeſty and Humility; Cruelty with Clemency and Compaſſion; Extravagance and Licentiouſneſs with Sobriety and Goodneſs; and Covetouſneſs and Ill-Nature with Generoſity; and as theſe Principles of our Minds are Diametrically Contrary to each other, and as the Firſt ſeem to be of the Expanſive Sort, or the Repulſive, by which others are Alienated from us, and the Laſt of the Contractive, by which They are Allur'd or Drawn towards us, ſo where-ever we find Men of theſe Tempers, which are Reluctant to each other, we as certainly find they Diſagree, and that the Contention laſts till the one Force or the other, the Expanſive or Contractive, Prevails, and then all is Quiet and Compos'd, tho' the Force which ſubmits to the other is not entirely Deſtroy'd or Annull'd.

Why the Good and Vertuous Qualities of Men's Minds Generally in the Iſſue prevail againſt the Ill, in Bodies and Societies of Men.

§. 25. And ſince, as we have before ſhewn, the Expanſive Force on the Earth is but Small and Inconſiderable in Reſpect of the Contractive, therefore, tho' the Expanſive may ſometimes prevail, yet Generally ſpeaking, and at length the Contractive will; and conſequently from hence it is Evident, that tho' Wit, Gaiety, and Mirth may have it's Proſelytes, and Vice, Pride, Cruelty, Extravagance and Licentiouſneſs, Covetouſneſs and Ill-Nature may prevail for a Time, yet Judgment, Gravity, Sedateneſs and Seriouſneſs, Modeſty, Humility, Clemency and Compaſſion, Sobriety, Goodneſs and Generoſity will have the Strongeſt Force, and be in the End too hard for the other; and thus it was, in part, that the Chriſtian Morals became Superior to all the Ill-Qualities and Affections of Men, which Exerted themſelves in Oppoſition to them, and thus the Vertues of Human Society have been ſtill maintain'd and preſerv'd to it, notwithſtanding the Continual Struggles of the Vices of Men to Extirpate and to Eradicate them.

Why the ſame Obtains in ſingle Perſons, and why they ſeem Contrary to Themſelves.

§. 26. If now we come to the Minds of ſingle Perſons, without Comparing them with thoſe of others, there are always Two Forces obſervable in them, the Expanſive and Contractive, by which the ſame Perſon ſeems Contrary to himſelf in a Thouſand Qualities, and is really Different as the one or the other Prevails; and whilſt there is a kind of Equal Balance betwixt them, there is a continual Conteſt, Agitation or Struggle in a Man's own Breaſt, till the one or the other, is ſubdu'd; Thus every Man has the Seeds in his own Nature of Vertue and Vice, and of all the oppoſite Qualities and Affections we can Imagine, but they are in ſuch an Æquilibrium, and ſo ordain'd by the Almighty and All-wiſe Agent, that it is in every ones Will and Power to determine the Balance on the one Side or the other, that is, to the Expanſive or Contractive; Whilſt this Conteſt laſts, the Mind is in a Ferment and Uneaſy, that is, in it's Paſſage from the Vitious Qualities of the Mind to the Vertuous, or on the Contrary from the Vertuous to the Vitious, but when Either of them Prevails, the Mind is in a State of Compoſure and Quieſcence, and Satisfaction in the firſt Caſe, and in the Laſt in a State, which may be Agreeable for a Time; but when the Contractive renews the Conteſt, as it will do, when the Expanſive is Weaken'd, as in Sickneſs or near Death, there is a Reluctance and Uneaſineſs, a Motion, a Ferment and Agitation in the Mind, which cauſes the Greateſt Pains, which can Appertain to it; And from hence, that is, from the Different Degrees and Proportions

From hence a Pſycheometria

portions of the Expanſive and Contractive Forces in Men's Minds are their Natures and Identities aſcertain'd, and therefore the Powers and Faculties of it are as much Reducible to Mathematicks and Proportions, or a *Pſycheometria*, as any other Quantities whatſoever, The Elements of which I promiſ'd in the *Principles of Philoſophy*.

From hence the Foundation of Morality, and the Reaſon of Religion.

This will alſo give us an Account of what Morality is, and what the Foundation of it, ſince Almighty God has been pleaſ'd to Implant thoſe Original Principles in us, by which we are Diſpos'd to acquire all our Good or Ill-Qualities from the Contractive and Expanſive, and at the ſame Time has Endu'd us with a Faculty and Power to determine our ſelves to the one or the other; but leſt this Faculty and Power ſhou'd be Ill-Imploy'd by us, that is, leſt the Principles of Natural Religion ſhou'd not Sway and Incline us to that which is Good, he has alſo given us a Reveal'd one, whoſe Doctrines and Precepts tend the ſame Way with the Contractive Force of our own Minds, which is to Incline them to all the Vertuous Affections and Properties of our Natures, which are Contrary to the Expanſive and Repulſive of them, and which fall in with Love and Benevolence, Charity and Goodneſs, and all the Contractive Forces which belong to them.

Why Sobriety of Thought and Judgement prevails againſt Gaiety and Wit.

§. 27. Let us farther Conſider, what theſe Expanſive and Contractive Forces of the Mind are Abſtracted from Morality, or the Vertuous and Vitious Inclinations it Poſſeſſes, and we ſhall ſtill find the ſame Law of Nature obtains; Thus Memory, Recollection, Wit, and a Quickneſs of Thought and Apprehenſion is deriv'd from the Expanſive Force of the Mind, as Perception, Judgment, Reaſoning, and Solidity is owing to the Contractive Force of it; for in the firſt, we may obſerve a kind of Elaſticity in the Production of our Ideas, and a Volatility as well as Incoherence of them, and in the ſecond a Contraction by their Cloſeneſs, Connexion, and Gravity; and as in the other Caſes, ſo in this, the Mind, when it is Agitated betwixt theſe Two Forces, is in a ſort of Conteſt with it ſelf, and uneaſy, till it is ſettled in the Contractive.

Why Men of Wit are Inclin'd to Vice, and Men of Judgment to Vertue.

We may farther take Notice here, that as we have made the Vitious Propenſities of our Minds to be chiefly deriv'd from the Expanſive Force, and the Vertuous from the Contractive, and Memory, Wit, Quickneſs and Gaiety of Thought to be the Reſult of the Former Force, and Judgment or Reaſoning the Effect of the Latter, ſo we likewiſe in Fact and Experience find, that thoſe who are the Wittieſt and the Gaieſt are generally the moſt likely to be carry'd into Vice, and thoſe who are of the beſt Reaſon and Judgment to be determin'd to Vertue, and in Conſequence of it to Religion, which is the Grand Promotor and Aſſertor of it.

How the Mind is united to Matter, and yet Diſtinct from it.

§. 28. I wou'd not be here thought to make the Nature of the Mind to conſiſt in the Expanſive and Contractive Forces, or Action in General, ſince That we have ſaid is the Eſſence of Matter, and which will ſolve all the Phænomenons and Senſations we perceive from it, but Action will not give a Solution of Thought and Intelligence; we therefore affirm the Mind to be a Thinking Immaterial Subſtance, to which yet there are a certain Kind of Expanſive and Contractive Forces belonging, and which are Indications and Characteriſticks of the Genius and Temper of the Mind, in which they Reſide, and by the Intervention of which Forces it ſeems Probable, that it is united with Matter.

How the foregoing Law of Nature is juſtify'd by the Union of the Mind with Matter.

§. 29. To which, if we now Extend this Law, that is, to the Union of the Mind with the Body, we ſhall find it ſtill the ſame; for thus whilſt the Mind and Body are in their Natural State, the Contractive Force of the Mind joins it with the Contractive of Matter, and the Expanſive of the Mind, being in an Æquilibrium with the ſame Contractive of Matter, continually keeps the Fluids in Motion, and Circulates the Blood and Spirits, and all other the Juices, which are Requiſite to the Animal Oeconomy.

The Cauſe of Lethargies, Comas, Jaundice, Dropſies, &c.

But Firſt, If the Contractive of Matter is too Powerful for the Expanſive of the Mind, the Æquilibrium is broken, and the Expanſive of the Mind ſinks beneath the Contractive of the Body, as in Lethargies, Comas, Jaundice, and Dropſies, and the other Diſtempers, which proceed from a Viſcidity, Flaccidneſs, Thickneſs or Languor of the Blood and Serum, or too Great a Quantity of them, whilſt yet the Contractive of the Mind remains, by which it can Reaſon, Perceive, and Judge as at other Times; ſo that the very ſame Principle, namely an Æquilibrium betwixt the Expanſive and Contractive Force, which ſolves the Revolutions in the Heavenly Syſtem, accounts likewiſe for the Circulation of the Blood and the other Fluids in a Human one, excepting that there it is the Expanſive and Contractive Force of Matter, whereas here it is the Expanſive of the Mind or of the Juices of the Body, which is Analogous to it, and the Contractive of Matter;

The Cauſe of Fevers and Phrenzy.

Second, If the Expanſive of Matter is too Powerful for the Expanſive of the Mind, the Contractive of the Mind ſtill continues, ſo far as to preſerve it's Union with the Body, but is diſturb'd by the Superior Expanſive of Matter joining with the Expanſive of the Mind, from whence an Extreme Volatility and Diſſipation of Thought ariſes, an Incoherence of Ideas, and a ſurprizing Quickneſs in their Motion, as in Drinking Spirituous Liquors, in Fevers, and in a Phrenzy: And in both theſe Caſes, when the Contractive Force of Matter in the Firſt is too Great, and the Expanſive Force in the Laſt, the Mind Reſults from the Body, a Diſſolution of the Union Enſues, and it Departs with it's own proper Expanſive and Contractive Forces from the Body, which it had hitherto Actuated and Inform'd;

The Cauſe of Melancholy.

Third, If the Contractive Force of the Mind by any Ways or Means is ſo far Increas'd, as to break the Æquilibrium betwixt the Forces of the Mind and of Matter, as by Great and Intenſe Study, by an Extreme Concern for the Good of others, by Afflictions in the Cauſe of Religion and Vertue, or too Mighty a Zeal, Concern, and Sollicitude for them, which we have ſhewn are all of a Contractive Nature, the Mind may be caſt into a Depth of Melancholy, and tho' it's Reaſoning and Judgment is ſtill Preſerv'd, the Body may be brought into an Ill and Languid State, and the Mind grow Heavy, Dull, and Unactive.

The Cauſe of Madneſs and Diſtraction.

Lastly, If the Expanſive Force of the Mind is likewiſe Increas'd beyond an Æquilibrium with the Forces of Matter in the Body, by too Exceſſive a Pride, Covetouſneſs, Ambition, Revenge or Cruelty, which are of an Expanſive and Repulſive Force, or by any other Vices of the like Selfiſh Complexion, a too Great Expanſion ariſes in the Mind, and by Conſequence a Madneſs and Diſtraction, by which the Ideas of it are Interrupted and Incoherent, Acquire an Exceeding Swiftneſs and Velocity, and the Fluids of the Body are put into a Wild Ferment, and the Whole Maſs and Compoſition into the Higheſt Diſorder, and from hence Anger and Fury, which owes it's Origine to theſe Expanſive Qualities of the Mind, has been juſtly Term'd and Eſteem'd a ſhort Madneſs.

How Good Men are Happy after Death, and Ill Men Miſerable.

In theſe Two laſt Caſes, if the Diſturbance is very Great, the Union betwixt the Mind and Body is Diſſolv'd, and Melancholy or Madneſs Equally deſtroy the Harmony betwixt both, whilſt the Mind departs Compos'd and Satisfy'd in the Firſt Caſe, and in the Laſt, Roil'd and Uneaſy, Full of Wretchedneſs, Pain, Anxiety, and Miſery. But whilſt the Mind and Matter continue United, it is more Difficult to Cure the Former Diſturbance of the Æquilibrium than the Latter, becauſe it is harder to Raiſe a Contractive Force into an Expanſive, than to Depreſs an Expanſive to a Contractive, ſince, all the Expanſive Forces, of what kind ſoever in the Earth we Inhabit, bear a very ſmall Proportion to the Contractive, and for which Reaſon we ſaid, tho' there might be a Perpetual Quieſcence, there cou'd not be a Perpetual Motion; thus likewiſe all Animate Beings are but few in Proportion to the Inanimate, and the Aſſiſtances in Nature to the Raiſing the Contractive Force into the Expanſive, are much Fewer than thoſe which we have to Depreſs the Expanſive to the Contractive; And from hence, it is much Eaſier to Extinguiſh a ſmall Fire, than to Throw into Fumes and Vapour a like Quantity of Water.

Why Melancholy more Difficult to Cure than Madneſs.

Concerning the Diſtempers Incident to this Union, and the Rational Methods of Curing them in Conſequence of this Law.

§. 30. From what we have alledg'd, it may be eaſily Infer'd, that, in theſe Four Caſes, there are Different Ways in Phyſick to be purſu'd to Reſtore the Æquilibrium, and that the Forces of the Mind, in all Preſcriptions in that Science, ought to be as Carefully, and Punctually Examin'd and Attended to, as thoſe of the Blood or of the Fluids of the Body;

In the Firſt, the Chief Intention ought to be to Raiſe the Expanſive Force of the Blood and Animal Juices, by Spirituous Liquors moderately us'd, or by Exerciſe, or by a Diminution of the Quantity of them by Abſtinence, or by all of them together;

In the Second, the Principal Intention ſhou'd be to Depreſs the Expanſive Force of them, by Cooling and Contractive Medicines, and by a Regular and Thin Diet, and all other Methods, which are Fitted to Allay the Heat and Efferveſcence of the Blood and Animal Juices;

In the Third, Medicines will be of little Force, but Action, Mirth, and Company, and all the Innocent Diverſions of Muſick, and the Agreeable Entertainments of Wit and Gaiety, will prove the moſt Effectual to raiſe the Contractive Force of the Mind into a juſt Expanſive; and Riding and Travelling into Diſtant and New Scenes is not a little Conducive to the ſame End.

In the Laſt, Precepts of Vertue and Religion, and the Impreſſions of Reaſon, Sobriety, and Judgment, and Exerciſing the Mind in that Way, will in all Probability have more Influence, than all the Drugs which can poſſibly be Invented or Preſcrib'd; and I have ſometimes thought, that the Chriſtian Religion, which, as we have ſhewn, is of a Contractive Nature, may probably have as Real and Neceſſary an Effect in Curing Demoniacks, or in theſe latter Ages Madmen, as it has had in Reducing Men from Vice, Immorality, Ill-Nature, and Extravagance, which likewiſe, as Madneſs, are of an Expanſive Nature, to Sobriety, Vertue and Goodneſs, which are of a Contractive one.

How the Mind is Diſtinct from the Body, and will Survive it.

From all which we may reaſonably Conclude, that as there are theſe ſeveral Maladies Incident to our Natures, which have Diſtinct Origins, ſo that the Mind is a Subſtance Different from the Body, and which will Survive it; ſince, according to this Law, which we have Evinc'd to be a General one in Nature, where one Force prevails againſt another, the Force, which ſubmits, is not Deſtroy'd or Annull'd, but ſtill continues; and as it is Evident, that an Expanſive and Contractive Force of Matter is preſerv'd even to the Body diſſolv'd from the Mind, ſo it is Equally Rational to think, that the Expanſive and Contractive Force of the Mind is Conſtant and Permanent upon it's Diſſolution from the Body; beſides which, the Contractive Force of the Mind, it's Reaſoning and Judgment, is not much Affected by the Contractive Force of the Body being too Great, as in the firſt Caſe; in the Second, it is, by the Expanſive Force being ſo, of which therefore when it is acquitted by a Diſſolution from it, it muſt be unaffected by it; in the third Caſe, the Contractive of the Mind is Increas'd, and in the laſt, the Expanſive; from whence it is Manifeſt, that the Forces of the Mind in any of theſe Caſes are not, nor can be Diminiſh'd, much leſs Annihilated by it's Separation and Diſſolution from the Matter, with which it is United.

The Nature of Dreams, and the Reaſon of Them.

In the Union of the Mind with the Body, we ſhall only farther Inſtance in Dreams, that in a Natural State, where the Forces of the Mind and of Matter are in an Æquilibrium, the Blood and Fluids circulate Regularly, and Various Ideas are Exhibited to the Mind, according to the Various Temper, Craſis, or Action of ſuch Fluids, as they paſs thro' the Brain, the Place of the Mind's Reſidence; but if the Contractive Force of the Blood and Juices is Increas'd beyond the Natural State, if they become Colder and Chiller than ordinary, Repreſentations of Water are produc'd, as in ſome ſmaller Degrees of Melancholy, if the Expanſive is Increas'd, as in Fevers, they Change into thoſe of Fire; In the firſt Caſe, the Mind is more Compos'd, and in the laſt, more Diſturb'd and Uneaſy, and the Images in the Mind carry an Analogy with the Expanſive or Contractive Forces of the Blood and Fluids, which Impreſs them; and our not Dreaming ſometimes at all ſeems to ariſe from the Contractive Force of the Blood and Fluids being too Powerful for the Forces of the Mind, ſo long as it is united with them, as in an Exceſs of Eating, or from too Great a Quantity of the Expanſive, as in taking in too Much of Spirituous Liquors, both which Oppreſs and Over-turn the Forces of the Mind for the Time, that is, During the Union of the Mind with Matter; and if the Exceſs is carry'd to too Great a Degree, the Mind, when it cannot over-come, Reſults and Rebounds from theſe Forces, by which it is Diſunited and Separated from them.

Concerning the Diſtempers Incident to the Body conſider'd by this Law; with the Cauſes of Colds, Agues, Small-Pox, Fevers, and Peſtilence.

§. 31. If we now conſider the Body it ſelf, we ſhall find it Incident to thoſe Diſtempers, which cou'd not be otherwiſe Accounted for, than upon this Law and the Forces of it; for how comes it to paſs that Fevers, Colds, and Agues are the Common Ailments in theſe Climates, whilſt in Hotter ones Plagues and the moſt Peſtilential Diſtempers are the Produce of them? And why is the Small Pox in Particular almoſt every where Allotted to our Natures, unleſs in Countries of the moſt Exceſſive Heats? Fevers are Evidently from too Great an Expanſive Force in our Blood and Juices, and Colds from too Great a Contractive one, and in Agues there is a Reciprocation of and a Conteſt betwixt both; and it is no Wonder in our

Climate, where theſe Principles of the Expanſive and Contractive prevail in their Turns, in Summer and Winter, that they ſhou'd Induce the ſame Qualities and Properties into the Blood and Fluids, and by Conſequence Excite Fevers, or Impreſs a Chillneſs, a Damp and Cold upon them; and as in Spring and Autumn, the Expanſive and Contractive are in a kind of Conflict by the Winds and Tempeſts we obſerve at thoſe Times, ſo it is not ſtrange that ſuch an Effect is Propagated to the Fluids of the Body, and that Agues are caus'd, in which the Expanſive and Contractive Forces in the Blood and Juices Reciprocally Triumph and Yield; But that which occaſions the Original Diſturbance and Diſorder, in this laſt Diſtemper, is the Contractive, as Agues proceed from Cold, and the Burning Fit, Experience't in them, ariſes from the Exaſperation of the Expanſive by the Oppoſition of the Contractive, which we have Evidenc'd is the Law of theſe Forces, and therefore the Hot Paroxyſm ſucceeds the Cold one, as is known.

How the Bark of Peru, *and other Medicines Operate in the Cure of Aguiſh Diſtempers.*

THE Way therefore to Prevent or to Cure ſuch Aguiſh Diſtempers is to Aſſiſt the Expanſive either to Repel the Contractive before it's Admiſſion into the Blood and Juices, or to Drive it out afterwards, which is done by Riding and Exerciſe, by Hot Vegetables, Roots, and Herbs, by Bitters, and eſpecially by the Bark of *Peru*, or other Barks of a like Nature, and by a Moderate uſe of the moſt Spirituous Liquors, all which have an Expanſive Force, and tend to the ſubduing of the Contractive; and this ſeems to be the real Method, by which the ſaid Bark Operates, which, if ſo, will be a Confutation of a Thouſand Fancies concerning it, and it's Power of Locking up Pores, or Abſorbing the Febrifick Matter, and of doing Many more Feats of Activity, which no one ever Dream't of, Except the Phyſician or the Philoſopher; but above all, an Employment of the Mind in all the Senſations and Faculties of it, and Raiſing it's Expanſive by Study, or by Wit and Diverſion, or any Way, which is the moſt Agreeable to it, ſeems to Carry the Greateſt Force towards the ſubduing the Contractive of the Blood and Fluids of this Diſtemper.

Why not to be Given in the Paroxyſm.

FROM hence it is alſo Evident, that theſe Remedies are not to be apply'd in the Paroxyſm it ſelf, becauſe, when the Contractive is in it's full Power, the Expanſive will only add to it's Triumphs, and render it more Pertinacious and Obſtinate to be Over-comn according to the preſent Law; that is, the Conteſt will be the Greater by an Addition of the Expanſive, in like Manner, as the more Expanſive Force we give to the String or ſtretch'd Wire, whilſt the Contractive Force continues fixt at the Centers of the Tenſion, the ſtronger will be the Vibrations betwixt the Expanſive and Contractive; Beſides which, ſince the Expanſive Force is the Cauſe of Digeſtion, and ſince any Aliments, which we take in, are in ſome Meaſure of a Contractive Nature, that is ſuch, as want to be Digeſted, and ſince the Expanſive Force always Acts in Oppoſition to the Contractive, the Expanſive Force in the Animal Oeconomy is weaken'd, and taken off from ſubduing the Contractive of the Blood and Juices, by Exerting it ſelf in the Digeſtion of the Aliment or Medicine Receiv'd.

Why Agues in Spring more Eaſily Cur'd than Thoſe in Autumn.

WE muſt likewiſe ſay, from hence it may be Infer'd, which, as the Phænomenons before, no Philoſophy has yet Accounted for, that an Ague, which conſiſts in a Conteſt betwixt theſe Expanſive and Contractive Forces, is more eaſily Cur'd, when it begins in the Spring than in the Autumn; becauſe the Expanſive Force in the Summer, Experience't from the Sun's Rays, very much Contributes to the ſubduing of the Contractive Force, whereas the Contractive Force of the Winter-Colds ſupports and continues that of the Blood and Juices, and conſequently falls on the Side of the Force, which wants to be Vanquiſh'd and Subdu'd.

Why Riding and Exerciſe Beneficial in Colds and Conſumptions.

THUS alſo in Colds and Catarrhs the Contractive Force Prevails in the Blood and Fluids, and Requires the Expanſive of Riding and Exerciſe, and a Moderate uſe of Spirituous and Warm Liquors to Diſſipate and Diſſolve it; and if, notwithſtanding, the Contractive Increaſes, a Conſumption many Times enſues, and a Languor and Inactivity is Induc'd upon the Blood and Juices; Nature ſinks and droops, by means of the Contractive, and a Hectick, or ſlow and ſilent Fever accompanies the Diſorder; becauſe there is a continual Conteſt betwixt the Contractive, which ſpreads it ſelf all over the Body, and the Expanſive of the Blood and Fluids, by which they Circulate; and conſequently Soft and Warm Liquors, as Aſſes Milk, which are Nouriſhing and Contribute to the Expanſive, as alſo Gentle Riding and Exerciſe, are alike Beneficial, as in Colds.

Wherein the Difficulty of Phyſick Lies.

IT muſt here be Obſerv'd, that when the Intention is to Conquer and Surmount the Contractive, the Expanſive muſt be Proportion'd in ſome Meaſure to it, and the ſame may be ſaid of the Contractive, when the Expanſive is to be Curb'd and Allay'd, and herein ſeems to conſiſt the chief Skill in Phyſick, Namely, Firſt, to know of what Nature and Degree the Expanſive or Contractive Force is, which is to be Reduc'd into Order, which requires the Sagacity and Penetration of one well vers'd in the ſeveral Diagnoſticks of Diſtempers Incident to the Human Frame and Compages; and Second, to be fully Acquainted with the Expanſive and Contractive Forces of Medicines, and to be Able to Determine what Degree of the Expanſive or Contractive Force is Proper to be Apply'd upon the Preſent Emergence.

THUS in Agues, where the Contractive Force is Great, as is Evident by it's Shaking the whole Animal Syſtem, and Obtaining the ſame Effect in the Blood and Fluids, as an Extreme Cold in the outward Parts of the Body, Strong Bitters, and Spirituous Liquors are found Neceſſary, which, in a Conſumption, wou'd be Prejudicial, where the Contractive being leſs is to be prevail'd upon by more Gentle Expanſives and Milder Applications; for ſo a vaſt Weight is to be Remov'd by a Prodigious Degree of Velocity, which, if Apply'd to a Smaller, wou'd produce an Exceſſive Velocity likewiſe, and, in the Animal Syſtem, ſo Great a one, as might Endanger the Diſſolution of it, as much as the Contractive will, if not ſubdu'd, Depreſs and Sink it.

From whence Fevers and Peſtilential Diſtempers, and How Cur'd.

ON the Contrary, Fevers and Peſtilential Diſtempers ariſe from the Prevalence of the Expanſive, and are the Natives of Hot Seaſons and Countries, and therefore it is Reaſonable to Think, that Cold and Contractive Medicines are what we ſhou'd have Principally Recourſe to in ſuch Caſes, as Water, Lemmons, *&c.* but this is to be done with ſome Prudence and Caution, becauſe, if a Contractive Force is added to the Blood from ſuch Applications, the Fever may be Increas'd by Exciting and Exaſperating the Expanſive, which always acts in Oppoſition to the Contractive, and conſequently it may not be Improper to Infuſe theſe Contractive Forces into the Blood with a Mixture of the Expanſive. And ſince the Writing of this I find, that *Aromatick Acids* were of the Greateſt Service at the Plague of *Marſeilles.*

Why the Small-Pox a General Diſtemper Excepting in Hot Countries.

THE Small-Pox manifeſtly is deriv'd from a Prevalence of the Contractive, and conſequently, Cordials and Spirituous Liquors, which are of an Expanſive Force, if Nature cannot do it, are Aſſiſtant to the Expelling of it; Every one is Subject to this Diſtemper, becauſe Every one has more of the Contractive than the Expanſive, in his Blood, excepting in Exceeding Hot Countries, where the Expanſive prevails, and for the ſame Reaſon Some never have Them.

Concerning other Animate Beings, and the Expanſive and Contractive Forces of them.

§. 32. LET us now Deſcend to the Creatures, which are below us, and amongſt thoſe, whoſe Senſes have moſt of the Expanſive Force, we may reckon the Volatile Kind, and amongſt thoſe, whoſe Senſes have moſt of the Contractive, the Aquatick; the one Inhabiting that Elaſtick Medium, the Air, and the other the Phlegmatick one of Water, from each of which their Senſitive Faculties derive their ſeveral Tinctures, and from whence we find the utmoſt Life, Quickneſs, and Vivacity, and almoſt even Wit and Humour in the Former, and in the Latter, an Eminent Torpor and Dullneſs; in the Land-Animals likewiſe, which partake of Earth, Air, and Water, we ſee a Degree of Senſe ſomething Intermediate to them both; And by the ſame Method we might Diſtinguiſh all the Sundry Species of Animals from each other, by their Expanſive and Contractive Forces, but it is at preſent ſufficient for us to ſhew, that theſe Forces obtain and prevail in all Nature, and that the Law of 'em before mention'd is Eſtabliſh'd in the ſeveral Beings of it.

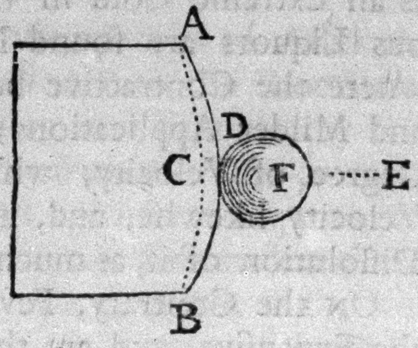
From whence is Concluded the Infinite Superiority of the Human Syſtem, and the Immortality of the Soul.

§. 33. WHAT therefore we ſhall farther ſay here is only This, that the Great Diſtinction betwixt Human Minds and the Senſitive Faculties of Animals is, that the Firſt have a Contractive Force by which they Reaſon and Judge, whereas the Laſt, at the Moſt, have only an Expanſive in the Compariſon; that is, their Faculties are ſuch, from which Ideas are ſtruck out by the Expanſiveneſs or Elaſticity of them, and by the ſame Force remember'd, but are not ſtrong enough to Retain thoſe Ideas, ſo as to Reaſon, Conſider, and Judge of Them; The Mind therefore has a Force Infinitely Superior to theſe Small and Momentany Vibrations of the Faculties of other Animals, and conſequently Infinitely more Durable and Laſting, as the Vibrations of the Heavenly Bodies upon their Center, that is, their Revolutions round it, are more Permanent than the Oſcillations of the Wire of a Virginal, or as the Solid Strokes of the

the Expanſive of Marble or Gold upon the Æthereal and Elaſtick Plenum are of a more Eternal Nature, (for, unleſs thoſe Subſtances are Annihilated, thoſe Pulſes will always Remain and Continue), than the Temporary Glances of the Expanſive Force of a Glow-Worm, or of the Flaſhes of Lightening; and it is from theſe Grand and Noble, theſe Strong, Sublime, and Magnificent Forces of the Mind, that is, from the Contractive, by which it is Capable of Reaſon, Morality, Vertue, and Religion, of which no other Beings are, that we may juſtly Conclude it's Immortality, whilſt it's Expanſive is Common to it, in ſome Meaſure and Degree, with other Animate and Senſitive Creatures.

The Modern Doctrine of Elaſticity, as apply'd to Natural Bodies Explain'd and Examin'd.

§. 34. Having thus Finiſh'd what we Intended at preſent to propoſe concerning this Law of Nature and of the Expanſive and Contractive Forces, we ſhall Return to our Examination of the Modern Doctrine of Elaſticity; in Order to Explain which in Natural Bodies, the Surface *ADB* is ſuppos'd to be Compreſs'd, by the Incurring Body *F*, Inwards to *ACB*, and by the *Vis Reſtitutiva* to be Reſtor'd to it's Firſt Situation in *D*, with the ſame Celerity with which it is Compreſs'd, if the Bodies are perfectly Elaſtick; But we ſhall ſee now the Shortneſs and Defectiveneſs of ſuch Principles, as well as the Falſhood and Untruth of them;



For Firſt, the *Vis Reſtitutiva*, as we have ſhewn, is an Imaginary Thing, and has no Foundation in Nature for it's Support, ſince the Inſtance of the String or a Wire, Stretch'd betwixt two Fixt and Immoveable Points, is no Proof or Eviction of it;

Second, Granting this *Vis Reſtitutiva*, by the foregoing Explanation of it from a ſtretch'd Wire or String, the Surface *ADB* ought to be Compreſs'd to *ACB*, by the Incurring Body *F*, the *Vis Reſtitutiva* ought likewiſe to reſtore the Compreſs'd Surface to it's Former Situation in *D*, and by the Velocity acquir'd to go beyond it towards *E*, then the *Vis Reſtitutiva* is to draw it back from thence to *D*, and by the Velocity acquir'd, the Surface is again to be carry'd towards *C*, and ſo Alternately the Progreſſes and Regreſſes of the Surface are to be perform'd, till it becomes Quieſcent in *D*; which if ſo, ſuppoſing any other Body, after the Repulſion of *F* to *E*, is Apply'd to the Surface *ADB* thus agitated, and ſuppoſing ſuch Body is barely Apply'd to it, without any Incurring Velocity, it muſt be in ſome Degree Repell'd from it towards *E*, and nearly with the ſame Force as *F* is, which is Contrary to all Fact and Experience;

Third, Granting likewiſe this Imaginary *Vis Reſtitutiva*, by which the Surface *ADB*, when Compreſs'd by the Incurring Velocity to *ACB*, Reſults with the ſame from *C* to *D*, what wou'd be the Conſequence of it? not the Repulſion of *F* from *D* to *E*, but from *C* to *D*, which wou'd prove a very ſlender Account of the Reflection of Bodies at ſeveral Diſtances from the Surfaces, on which they Incurr'd, when, by this, they wou'd not recede one Tittle from them;

Fourth, As to the Experiment, which is made uſe of to Confirm this Compreſſion of the Surfaces of Bodies and the Reſtitution of them to their Firſt Situation, namely, that if you take Two Globes of Glaſs or Ivory, and Paint the Quieſcent one with Ink or any other Colour, and let the Impingent one fall upon it, the Impingent Globe will be Colour'd in a Broad Spot, tho', according to the Nature of Spherical Figures, It ought only to be Ting'd or Colour'd in a Single Point; The Reaſon of which Broad Spot upon the Impingent Globe is thought therefore to be this, that the Surface of the Quieſcent Body is Compreſs'd upon Impaction, and conſequently, that the Impingent is by that means Colour'd in a larger Circuit, than otherwiſe it wou'd be, but ſince we find the Body, upon which the Impaction is made, ſtill Spherical after ſuch Concourſe and Compreſſion, it muſt have Reſtor'd it's Surface to it's former Situation; As to This Experiment We ſay;

That It by no means proves the *Vis Reſtitutiva*, or the Compreſſion of the Surface, for which it is alledg'd; Becauſe it is not Evinc'd, that this Broad Spot of Ink, which is Impreſs'd upon the Impinging Globe, may not be cauſ'd by an Electricity or Attraction of the Impingent to a certain Circuit, or by a Repulſive Force in the Globe upon which the Impaction is made, which Qualities may be

Excited in the one or the other by the mutual Action betwixt them; it being moreover a known Fact, that a Body falling upon a Fluid, as the Ink here is ſuppos'd to be, Daſhes it in a Circuit upwards, and makes it Riſe towards the Body which is the Aggreſſor, and ſeems to be owing to the Law before mention'd, where the Contractive of the Ink acts in Oppoſition and Contrary to the Expanſive of the Incurring Velocity.

It may yet be Diſputed, whether this Experiment will ſucceed in all Elaſtick Bodies, for by what Trials I have made, it do's not betwixt Two Globes of Marble; and conſequently, if ſo, there may be an Elaſticity, where there is no Alteration of the Surface, and therefore no *Vis Reſtitutiva* in it, and which can be no other ways accounted for, than by the Expanſive Forces Reſident in Bodies in Various Proportions and Degrees, according to what we have already Mention'd and Explain'd.

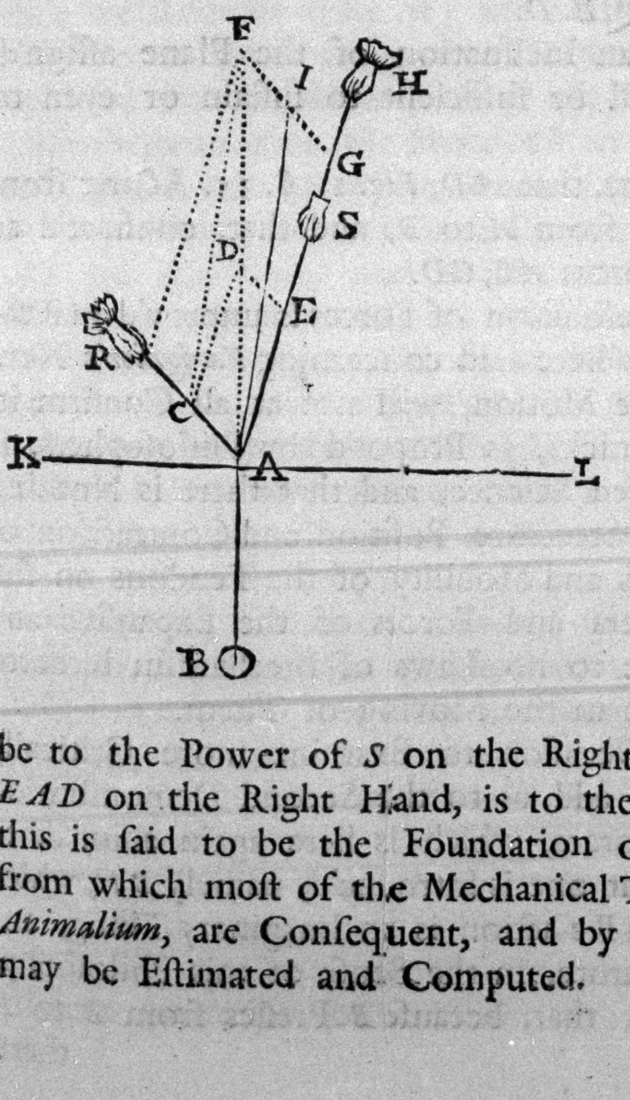
Fifth, As to what is urg'd againſt the *Carteſians*, who make Elaſticity or the Reflection of Bodies to conſiſt in the Continuation of the ſame Motion, tho' in an oppoſite Direction, it is a ſufficient Confutation of ſuch a Fancy to ſhew them, that according to them, all Bodies upon Impaction ought to be Reflected, whereas in Fact and Experience ſome Bodies are not, as thoſe which are Eſteem'd to be Unelaſtick; but at the ſame Time it is no Confirmation of the Preſent Doctrine, which Introduces a Compreſſion of Surfaces and a *Vis Reſtitutiva* of them, as much unknown in Nature, and which has as little Foundation in it, as the Continuation of Motion, where no Motion is Continu'd.

Sixth, From hence therefore all which is ſaid, in Theorem the *29th*, That if Two Bodies perfectly Elaſtick Impinge upon each other, their Relative Velocity before and after Impaction will remain the ſame, that is, after Concourſe, that they will retire from each other with the ſame Celerity with which they approach'd, is without any Grounds in Nature and Fact, it is an Abſtracting Reaſoning, and all the Rules of Congreſs determin'd from thence, in Problem the *3d*, and the Algebraick Calculations form'd thereupon, are mere Notions and Chimeras, as are alſo Theorem 31, 32. and Problem 4.

See §. 11, 12, 13.

Dr. Keill's Introduct. Lect. 14.

§. 35. The next Doctrine we ſhall Enquire into in this Philoſophy, is concerning the Forces and Powers of Bodies, which act againſt each other, ſo as to produce an Æquilibrium in them. Let B be a Weight ſupported by Two Powers R and S, which draw it, by the Mediation of Two Strings, according to the Right Lines AR and AS; The Point A is conſequently urg'd by Three Powers, Two of which act in the Direction of AR and AS, and the Third is the Force of Gravity in B, Tending in the Right Line AB Perpendicular to the Earth; The Power R on the Left Hand will be to the Force of Gravity, as AC to AD, or, as the Sine of the Angle DAE on the Right Hand to the Sine of the Angle CAE, comprehended betwixt both the Forces, and = to AB the Repreſentation of the whole Force; and the Power S on the Right Hand will be to the Force of Gravity, as EA to AD, or, as the Sine of the Angle CAD on the Left Hand, to the Sine of the Angle CAE, comprehended betwixt both the Forces, = AB the Repreſentation of the whole Force; and the Power of R on the Left Hand, will be to the Power of S on the Right Hand Reciprocally, as the Sine of the Angle EAD on the Right Hand, is to the Sine of the Angle CAD on the Left; And this is ſaid to be the Foundation of the New Mechanicks Publiſh'd by *Varignion*, from which moſt of the Mechanical Theorems, contain'd in *Borellus*'s Treatiſe *de Motu Animalium*, are Conſequent, and by which the Forces and Powers of the Muſcles may be Eſtimated and Computed.



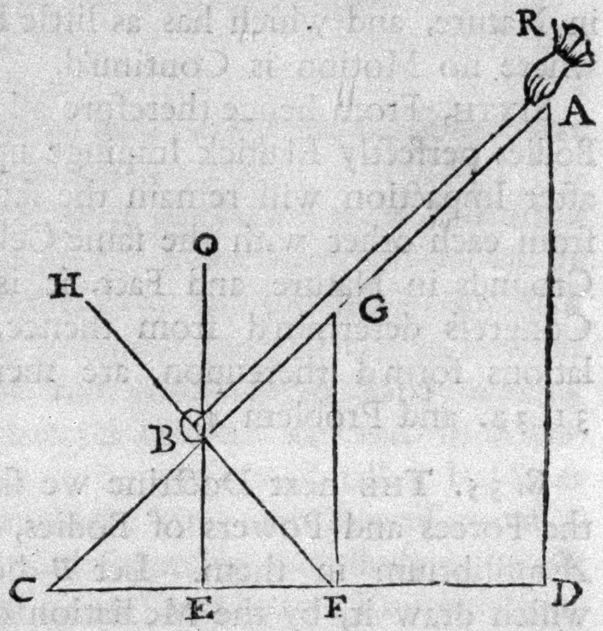
Concerning the Powers of Bodies Drawing againſt each other, and in oppoſite Directions.

Dr. Keill's Intr. ad Ver. Phyſ Lect. 14. Theorem 33. and 34. and Corollaries.

THE Demonſtration of which Theorem is upon this Suppoſition, that *AD*, Acting from *A* to *D*, is Equal to the Force of Gravity in *B*, and which is Repreſented by *AB*, and that *AD* the Diagonal Force is Reſolv'd into the Lateral *AC*, *CD*, but ſince, in every Triangle, the Sides are as the Sines of the Angles oppos'd to thoſe Sides, $AC.CD = AE$:: Sine of the Angle $ADC = DAE$. Sine of the Angle *DAC*; from whence it follows, that any Two Powers will be to each other Reciprocally, as the Sines of the Angles, which the Lines of the Directions contain with the Line of Direction of the Third Power; Moreover *AD*.*AC*:: Sine of the Angle *DCA*, or it's Complement $DCR = CAE$. Sine of the Angle $CDA = DAE$; and in like Manner, the Power Acting according to the Right Line $AB = AD.AE$:: Sine of the Angle *CAE*. Sine of the Angle $ADE = CAD$. *Q. E. D.*

AFTER this comes Theorem the 34*th*, which affirms, That if a Heavy Body *B* is Incumbent upon an Inclin'd Plane, and is ſupported and kept from Deſcending by a Power *R*, which acts Parallel to the ſaid Plane, the Power *R* will be to the Weight of the Body *B*, as the Sine of the Angle of Inclination to the Radius, or *R*.*B*::*AD*.*AC*.

FOR *B* is acted or urg'd by Three Powers, from *B* towards *E* by it's Gravity, from *B* towards *A* by the Power *R*, and from *B* towards *H* by Re-action, ſince *B* Preſſes on the Plane *AC* towards *F*, and is therefore by Re-action Repreſs'd towards *H* in a Line Perpendicular to the Plane; Conſequently, if we Draw the Line *OB*, we ſhall have the Caſe of the Preceding Theorem, where Three Powers act againſt each other ſo, as to produce a Quieſcence or an Æquilibrium in *B*; and *R* the Power on the Right Hand is to *B* the whole Force or Sum of the Forces, as the Sine of the Angle on the Left Hand *HBO* is to the Sine of the Angle Comprehended betwixt both the Forces *HBR*, that is, as *BG* to *GF*; for *BG* is the Sine of the Angle $BFG =$ to the Angle *HBO*, becauſe *FG* and *BO* are Parallel, alſo *FG* is the Sine of the Angle *GBF*, which is a Right one = to the Angle *HBR*, But by 8. Elem. 6. *BG*.*FG*::*FG*.*GC*, by 4. Elem. 6, ::*AD*.*AC*, and therefore *R*.*B*::*AD*.*AC*. *Q. E. D.*



Hence it follows, That there may be ſuch an Inclination of the Plane aſſign'd, that, upon it, the leaſt Power Imaginable ſhall be ſufficient to ſuſtain or even to raiſe the Greateſt Weight whatſoever.

The Preceding Doctrine Examin'd.

§. 36. THE foregoing Demonſtrations preſume, that *AD*, *Fig*. 1. §. 35. Acting from *A* to *D*, is Equal to *AB*, or to a Force Acting from *A* to *B*, and that, conſider'd as a Diagonal, it is Reſolvible into the Lateral Forces *AC*, *CD*.

BUT we ſhall afterwards ſhew, that ſuch a Reſolution of Forces is unprov'd in Reaſon, and unjuſtify'd by Experience, and what is here ſaid concerning *Varignion's* New Mechanicks, and *Borellus's* Account of Muſcular Motion, will not at all Confirm it, till it is Evinc'd, that the Doctrine of Mechanicks, as Propos'd by Philoſophers, is ſomething Real in Nature, and not an Abſtracted Science, and that there is Nothing more to be Conſider'd in the Muſcles, than the Structure, Poſition and Contraction of the Fibres on the one Hand, and the Paſſiveneſs and Mobility of the Tendons on the Other, and that there are not Certain Powers and Forces of the Expanſive and Contractive in the Mind, which are Reducible to no Laws of Mechaniſm hitherto Known, which have their Influence and Effect in the Moving of Them.

Borellus de Motu Animalium Propoſ. 4 & 7.

WE ſhall however poſſibly take another Occaſion to Examine more Critically into the Reaſonings of that Author, and only add, as to the Second Demonſtration here alledg'd, that beſides the Reſolution of Forces, which is here again aſſum'd, to make *B* conſiſt of Three Powers, a very Pleaſant one is Introduc'd, namely *BH*, which is ſuppos'd to be deriv'd from Re-action, which Re-action is an Imaginary Thing, and wants a Proof of it's Exiſting any where in Nature, in the Senſe of this Philoſophy; for farther, is it not very abſurd to Conclude, that, becauſe *B* Preſſes from *B* to *F*, therefore, and for that Reaſon it Preſſes from *B* to *H,* a Way which is the Direct Contrary? Not to Inſiſt, that it is altogether Inconceivable how *B* ſhou'd Preſs perpendicularly upon the Plane in the Direction *BF,* when by Confeſſion it Preſſes Obliquely upon it in the Direction *BE*; Since, if it's Force and Tendency is from *B* to *E,* I cannot have any Apprehenſion, how it's Force and Tendency ſhou'd be from *B* to *F,* and yet upon this Contradiction the Re-action *BH* depends.

Besides, If this Proportion is True, that *R.B::AD.AC,* if *AD* is = 0, or Infinitely Small, that is, if *AC* approaches Infinitely near to *CD,* the Force *R* will likewiſe be Infinitely Small, and therefore will be able to Support the Weight in *B,* and preſerve it in an Æquilibrium, and conſequently the leaſt Addition of Force will Move *B* towards *R,* or Elevate it, which is Impoſſible; for let *B* be a Hundred Pounds Weight of Lead, it can neither be ſtaid from Deſcending, if there is any Deſcent at all, by an Infinitely Small Force, nor much leſs can it be Rais'd towards *R,* but by a very Great one; which ſhews, that this Demonſtration is a Thing of mere Abſtraction, and *B* muſt not be conſider'd to have any Forces in it ſelf, but ſuch as *R* may Conquer in the Proportion of *AD* to *AC,* which is ſaying juſt Nothing at all, for by the Inſtance alledg'd it is Manifeſt, that when *AD* is Infinitely Small, and *R* is ſo, yet *B* ſhall Remain unaffected by it, and can neither be kept in an Æquilibrium nor be Elevated towards *R*.

But to Enquire a little farther into *Varignion*'s New Mechanicks, Let us conſider, that, Granting the Force *AD,* which acts in Oppoſition to *AB,* may be reſolv'd into *AC, CD,* and conſequently, that every Diagonal Force may be reſolv'd into it's Lateral ones, it is True, that according to this Reaſoning, *Fig.* 1. §. 35. the Force *AC*. Force *AE* :: Sine of the Angle *DAE*. Sine of the Angle *DAC*; and if the Force to be ſupported in *B* is ſtill Greater, and *AF* is an Oppoſite one Equal to it, *AR.AG*:: Sine of the Angle *DAE*. Sine of the Angle *DAC*; and therefore, *AC.AE::AR.AG,* or *AC.CD::AR.RF,* which is certain, by Elem. 6. But *AG, AR,* Acting at a Greater Diſtance from their Center *A,* than *AE, AC,* and in the ſame Inclination to *AD,* will be leſs Forces and will ſupport a leſs Weight in *B,* tho' they are here made the Repreſentations of Greater Forces, as *AF* is Greater than *AD*.

Thus it will be Harder to Raiſe the Body *B* to *A* at the Diſtance of *AL,* by a Force apply'd in *L,* than at the Diſtance *AK,* by a Force apply'd in *K*; that is, the Force of Raiſing Bodies is Increas'd in Proportion, as the Center of Action approaches Them, as the Force of Depreſſing Bodies in a Lever or Balance is Increas'd the farther we Remove from it; and conſequently *AE, AC* are Stronger Forces, and have a Power to Raiſe a Greater Weight in *A,* than *AG* and *AR* have, and yet *AG* and *AR* are the Repreſentatives of the Stronger Forces.

Supposing now the Force *AF* to be Reſolv'd into *AC, CF,* or *AC, AI,* it is Manifeſt, that ſince *AF* is the ſame, *AR, AG* ought to be Equivalent to *AC, AI,* which is Impoſſible; for *AC* is a Greater Force than *AR*, becauſe the Inclinations to *AF* are the ſame, and *R* acts at a Greater Diſtance from *A* than *C* do's, which Weakens and Enervates it; On the other Hand, *AI* is a Greater Force than *AG* is, becauſe, at much the ſame Diſtance of *I* and *G* from *A,* it acts nearer to *AF,* that is, in a perfect Oppoſition to *B,* for thus it is much Eaſier, that is, a Force has more Power, to Raiſe a Weight in *A* when it is apply'd Directly, than when Obliquely to it; if therefore *AC* is a Greater Force than *AR,* and *AI* than *AG,* it is Plain, that *AC, AI* cannot be Equal to *AR, AG,* and yet by this Doctrine they ought to be, being both Equal to *AF*.

Farther, *AE.AC*:: Sine of the Angle *CAD*. Sine of the Angle *DAE,* and *AI.AC*:: Sine of the Angle *CAD*. Sine of the Angle *IAF*; As therefore the Sine of the Angle *IAF* is Leſs in Compariſon with the Sine of the Angle *CAD,* than the Sine of the Angle *DAE* is, ſo ought *AC* to be Leſs in Compariſon with *AI* than with *AE,* that is, *AI* will be Greater than *AE*; and by the ſame Reaſoning and Enlarging the Angle *FAI,* till it comes to *FAG,* any Line betwixt *AI* and *AG* will Repreſent a Force Greater than *AE,* whereas we have ſhewn *AE* to be Greater in it's Force than *AG,* and yet *AG,* or the next Line to it, by what foregoes, will be in it's Force Greater than *AE,* which is Abſurd.

Lastly, If we conſider only the Fundamental Proportion of this Doctrine, that $AE . AC ::$ Sine of the Angle CAD. Sine of the Angle DAE, it will be liable to Objections; for as the Sine of the Angle CAD is Greater than the Sine of the Angle DAE, ſo AE muſt be Greater in it's Force than AC, which is not Evident; becauſe, tho' AC is more Oblique than AE in it's Action and therefore Weaker, yet Reciprocally C acts nearer to A, than E do's, and therefore upon that Account is Stronger; and conſequently AC and AE may be ſo diſpos'd in Reſpect of AD, if not in this Scheme, yet in an Aſſignable one, where the Obliquity of AC may be Compenſated by the Length of AE, in which Caſe, the Forces AC, AE may be Equal, notwithſtanding the Sine of the Angle DAE may be Leſs than the Sine of the Angle CAD.

CHAP. VII.

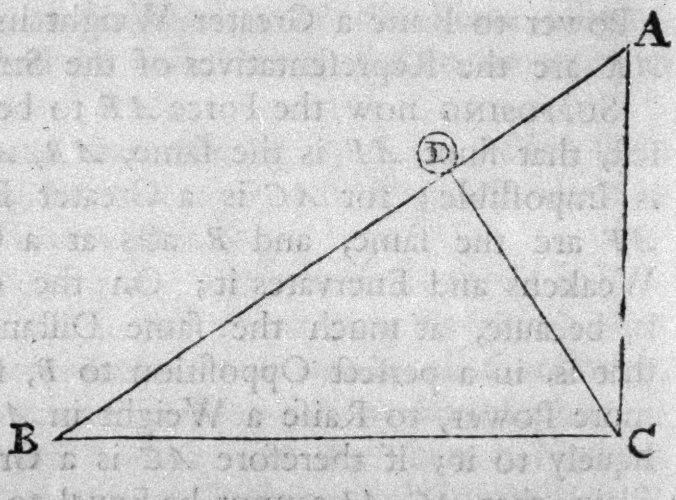
Concerning the Deſcent of Heavy Bodies in Inclin'd Planes, and the Oſcillation of Pendulums.

Concerning the Deſcent of Heavy Bodies in Inclin'd Planes. Theorem the Firſt. Dr. Keill's Intr. ad Ver. Phyſ. Lect. 15. Theorem 35. Hugen. Horolog. Oſcillator. Prop. 7. Par. 2.

§. 1. IN Order to Explain and Aſcertain this Doctrine, it is Suppos'd and Preſum'd, that the Force of Gravity is every where the Same and Alike, altho' in Different Diſtances from the Center of the Earth, it is ſaid to be Reciprocally Proportional to the Squares of the Diſtances; the Reaſon of which Suppoſition is, becauſe the Differences of the ſeveral Diſtances of Projectiles from the Center of the Earth are very Small and Inconſiderable in Compariſon with their Diſtances from the ſaid Center, and therefore may be Eſteem'd as Nothing. It is farther told us, that the Motion of Heavy Bodies is here ſuppos'd to be Perform'd, either in Planes Inclin'd to the Horizon, or in Curve Superficies, ſuch as Spherical and Cycloidical ones, and in Spaces perfectly Free and Unreſiſting.

And the Firſt Theorem in this Doctrine is, That the Deſcent of an Heavy Body upon any Inclin'd Plane is a Motion Equably Accelerated; Alſo, That the Velocity, which an Heavy Body, Falling down from it's Reſt upon an Inclin'd Plane, in any Given Time Acquires, is to the Velocity Acquir'd in the ſame Time, by an Heavy Body falling Perpendicularly, as the Height of the Plane to the Length of it. Let the Inclin'd Plane be AB, upon which let the Heavy Body D Deſcend;

First, by Corol. 1. Theorem 34, the Force, with which a Body endeavours to Deſcend upon any Inclin'd Plane, is to the Abſolute Force of Gravity, with which it wou'd Deſcend in a Perpendicular, in a Conſtant Reaſon of the Sine of the Inclination of the Plane to the Radius, or as the Height of the Plane to it's Length; and Conſequently, ſince the Abſolute Force of Gravity of the Body D continues the Same and Unalter'd, the Force will Remain the ſame, with which it Endeavours to Deſcend upon the Plane AB, That Force therefore will always act with the ſame Uniformity upon the Heavy Body D, and, by Law the Second, will continually ſuperadd Equal Increments of Velocities, in the ſame Manner, as is done in Heavy Bodies falling in a Perpendicular. From whence it follows, that the Deſcent of Heavy Bodies in an Inclin'd Plane is a Motion Uniformly or Equably Accelerated. *Q. E. D.*



Second, The Increments of the Velocities of Heavy Bodies, falling in a Perpendicular, and in an Inclin'd Plane, which are produc'd in the ſame Indefinitely ſmall Time, are to each other, as the Forces, by which they are produc'd, But the Forces are in a Conſtant Reaſon, namely, as the Length of the Plane AB to it's Height AC, and therefore the Increments of the Velocities, ariſing from thence, will be in the ſame Reaſon, and by 12. Elem. 5. the Sum alſo of the Increments of the one will be to the Sum of the Increments of the other, in the ſame Reaſon; that is, the Velocity of the Heavy Body falling in a Perpendicular is to the Velocity of an Heavy Body Deſcending, in the mean Time, upon an Inclin'd Plane, as the Length of the Plane to the Height of it. *Q. E. D.*

Corol. 1. THE Velocities of an Heavy Body falling in an Inclin'd Plane are as the Times, in which they are Acquir'd.

Corol. 2. WHATEVER has been Demonſtrated in Theorem 17. concerning a Motion uniformly Accelerated will be likewiſe True concerning the Deſcent of Bodies in Inclin'd Planes; Namely, That the Space deſcrib'd by an Heavy Body falling in an Inclin'd Plane in a given Time, Computed from the Beginning of it's Motion, will be the half of That, which is deſcrib'd by it, in the ſame Time, with the Velocity laſt Acquir'd; Likewiſe, That the Spaces, deſcrib'd and computed from the Beginning of the Motion, are in a Duplicate Reaſon of the Times or the Velocities, and the Velocities and Times in a Subduplicate Reaſon of the ſaid Spaces.

See Principles of Philoſoph. Chap. 5. Part 1. §. 33.

Corol. 3. HENCE the Aſcent of an Heavy Body up any Inclin'd Plane is a Motion uniformly Retarded, in like Manner, as in the Aſcent of a Body in a Perpendicular, and it is accompany'd with the ſame Properties and Symptoms.

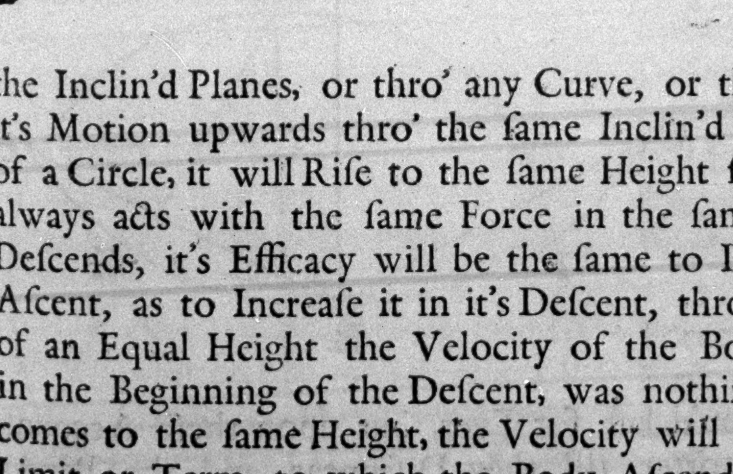
Schol. IF we have Recourſe to Experiments, we ſhall find them Conformable to the foregoing Reaſonings, which it will be Eaſy to Try in Planes, which are not very much Inclin'd; ſince the Motions, being perform'd with a leſs Degree of Velocity, may be Exactly and Critically Meaſur'd, which do's not happen in Perpendicular Deſcents, where the Exceeding Swiftneſs of the Motion leaves no Room for any Accurate Obſervations. It is here to be Noted, that the Planes are ſuppos'd to be perfectly Poliſh'd, and that the Motion is not Obſtructed by any Uneveneſs or Roughneſs of the Surface.

§. 2. AND from hence, Theorem *36th*, It is Demonſtrated, *Fig.* §. 1. That the Time, in which the Inclin'd Plane *AB* is deſcrib'd, is to the Time, in which the Perpendicular *AC* is deſcrib'd, as *AB* the Length of the Plane to *AC* the Length of the Perpendicular;

The other Theorems Recounted, which Relate to the ſame Doctrine.

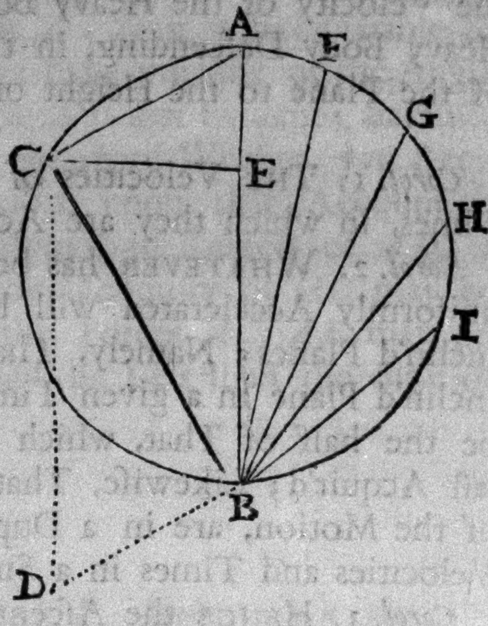
THE *37th*, *Fig.* §. 1. That the Celerities of Heavy Bodies, upon any Inclin'd Plane, and in a Perpendicular, are Equal, when the Heavy Bodies, falling from the ſame Height, come to the ſame Horizontal Line, that is, from *A* to *B* and *C*.

Dr. Keill's Intr. ad Ver. Phyſ. Lect. 15. Theorem 36, 37, 38, 39, 40. Hugen. Horolog. Oſcillator. Prop. 6, 7, 8, 9, &c. Par. 2.

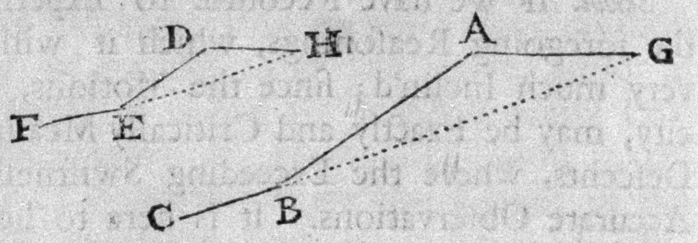


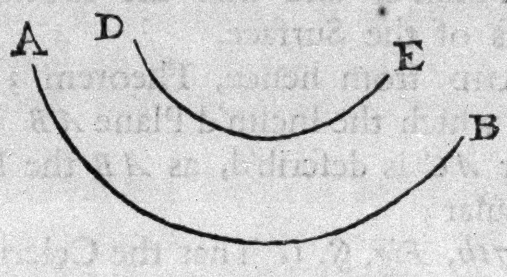
THE *38th*, That if a Body Deſcends in a continu'd Motion from the ſame Height, thro' any Contiguous Inclin'd Planes, *AB*, *BC*, *CD*, or *GC*, *CD*, or *ED*, it will always Acquire the ſame Velocity at the End in *D*, which is Equal to That, which is Acquir'd by it's falling Perpendicularly from a like Height, or from *E* to *F*; From whence it is Infer'd, that, if a Body Deſcends thro' the Circumference of a Circle, or thro' any Curve Lines, which are ſuppos'd to Conſiſt of an Infinite Number of Strait ones, it always Acquires the ſame Velocity, as if it had Deſcended in a Perpendicular from the ſame Height; and if, after it's Deſcent thro' the Inclin'd Planes, or thro' any Curve, or the Circumference of a Circle, it Changes it's Motion upwards thro' the ſame Inclin'd Planes, or Curve, or the Circumference of a Circle, it will Riſe to the ſame Height from which it Fell; becauſe, ſince Gravity always acts with the ſame Force in the ſame Plane, whether the Body Aſcends or Deſcends, it's Efficacy will be the ſame to Diminiſh the Velocity of the Body in it's Aſcent, as to Increaſe it in it's Deſcent, thro' the ſame Line or Space, and in Points of an Equal Height the Velocity of the Body will be the ſame; But the Velocity, in the Beginning of the Deſcent, was nothing, and therefore, in it's Aſcent, when it comes to the ſame Height, the Velocity will be 0, or nought, which Point will be the Limit or Term, to which the Body Aſcends; and conſequently, if a Body Deſcends thro' the Arc of a Circle, or a Curve, to the loweſt Point of that Arc, and afterwards, by the Velocity there Acquir'd, Aſcends thro' an Equal Arc, it will Aſcend and Deſcend thro' Equal Arcs or Spaces in Equal Times;

THE 39*th*, That, if a Circle is plac'd Perpendicular to the Horizon, and from the Higheſt and Loweſt Point of it Two Inclin'd Planes, *AC, BC*, be Drawn to the ſame Point of the Circumference, the Times of the Deſcents thro' either of the ſaid Planes will be Equal to the Time, in which any Heavy Bodies fall Perpendicularly thro' the Diameter *AB*; and therefore all Planes, which are Terminated by the Upper or Lower Point of the Diameter and by the Circumference of a Circle, however Inclin'd, are deſcrib'd in the ſame Time; As *AC, BC, BF, BG, BH*, &c.



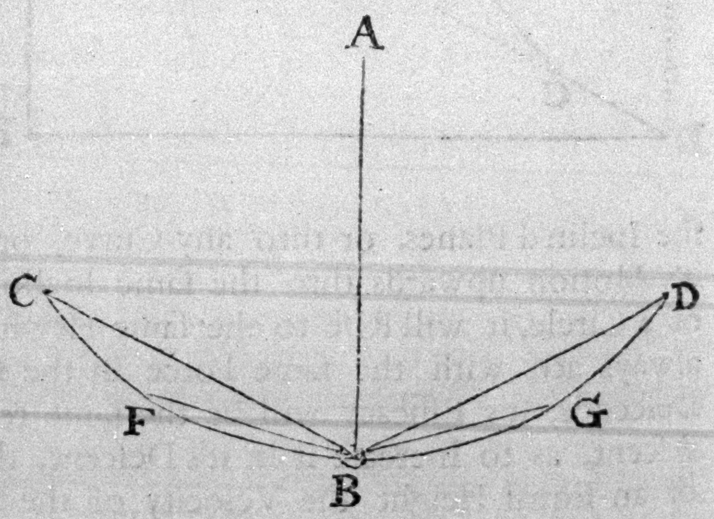
THE 40*th*, That, if Two Heavy Bodies Deſcend upon Two or More Planes, *GABC, HDEF*, that are Similarly Inclin'd and are Proportional, The Times, ſpent in Deſcribing of 'em, are in a Subduplicate Reaſon of the Lengths of the Planes; And the ſame may be ſaid of Two Curve Superficies, *AB, DE*, which are Similar, and have a Similar Poſition, conſider'd as conſiſting of an Infinite Number of Inclin'd Planes, namely, that the Times, ſpent in Deſcribing of them, are as the Roots of the Curves or Lengths.





The Vibrations of the Pendulum from hence explain'd. Dr. Keill's Intr. ad Ver. Phyſ. Defin. Theorem 41.

§. 3. THESE ſeveral Theorems being prov'd by a Geometrick Reaſoning, and Depending upon the Principles above, in the Definition preceding the 41*ſt* Theorem they are apply'd to the Pendulum, or any Heavy Body Swinging by a ſmall Thread upon a fixt Center, which is here given as the Definition of one; If therefore ſuch a Pendulum Oſcillates about ſuch a Center, and deſcribes the Arc of a Circle by that Oſcillation, there will the ſame Motion Arrive to a Body thus Oſcillating, and it will be the ſame Caſe, as if a Body ſhou'd be Mov'd in a Spherical Surface perfectly Hard and Poliſh'd; Becauſe a Motion is ſuppos'd about the Center entirely Free and Abſtracted from the Air's Reſiſtance, which in Heavy Pendulums is very Small and Inconſiderable; If therefore the Pendulum is now drawn to *C* from it's Perpendicular Situation in *AB* and permitted to Swing, it will deſcribe an Arc of a Circle in it's Fall or Deſcent to that Situation, and there Acquire a Velocity, which wou'd Carry it off in a Tangent from *B*, but being Retain'd in a Circle by the Thread, it Riſes in an Arc to the ſame Height in *D*, from which it fell, according to the foregoing Theorems, where, loſing all it's Velocity, it again Deſcends by it's Gravity to it's Perpendicular Situation, and by the Velocity there Acquir'd mounts to it's firſt Height in *C*, and ſo, by Aſcending and Deſcending, it's continual Vibrations are perform'd in the Circumference of a Circle; and if the Motion of Pendulums was not Obſtructed by the Air, and by the Friction in the Center of Rotation, theſe Vibrations wou'd laſt for Ever, but upon theſe Accounts the Arcs deſcrib'd Grow Shorter and Shorter, till they Diſappear, and the Pendulum becomes Quieſcent.



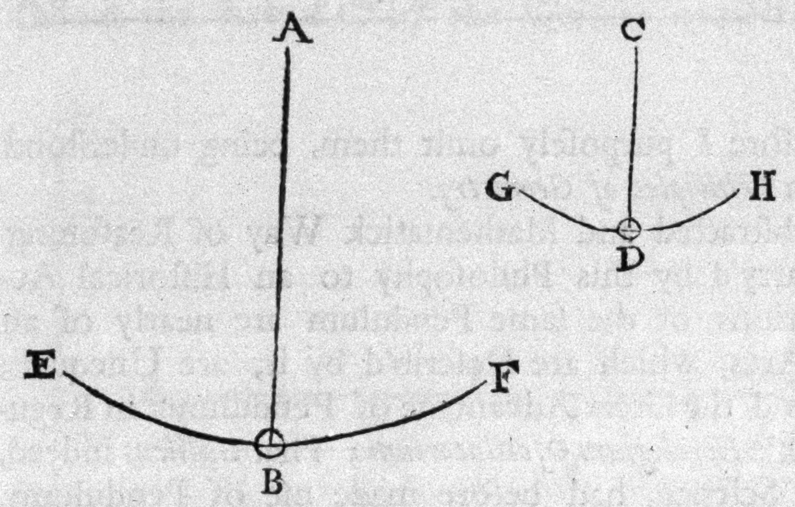
The Vibrations of Pendulums in Unequal

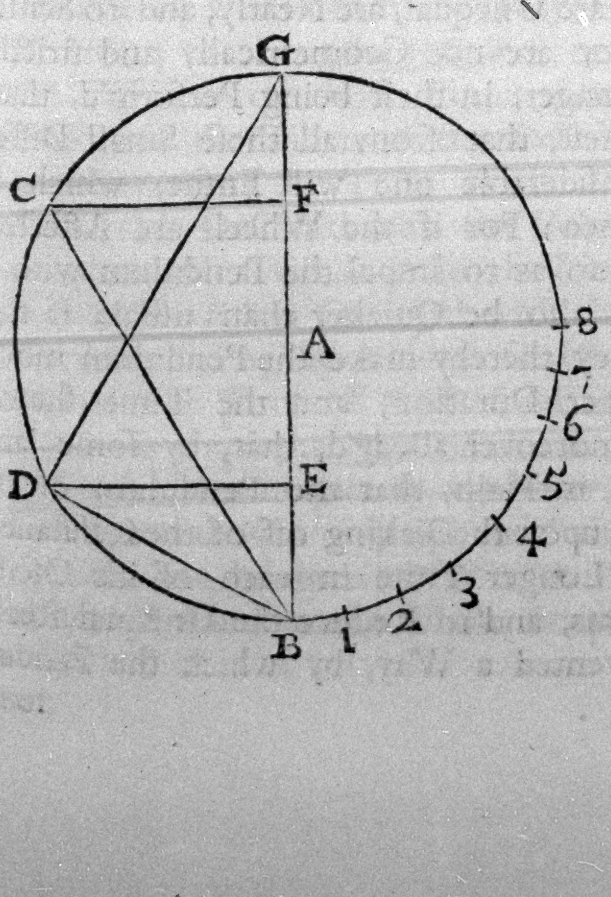
§. 4. AFTER which it is affirm'd, Theorem the 41*ſt*, That the ſmall Vibrations of the ſame Pendulum, however Unequal they are, very Nearly, and to Senſe, are perform'd in the ſame Time, or, which is the ſame, the Oſcillations in Unequal Arcs, if the Arcs are not too Large; becauſe, in ſuch a Caſe, the Arcs and Subtenſes do not much Differ, but, by the foregoing Theorems, the Subtenſes, drawn from any Point of the Circumference to the Bottom of the Diameter, are perform'd in Equal Times; And this Theorem is ſaid to be juſtify'd by Experience, for if Two Pendulums of an Equal Length are ſet a-Swinging, ſo that the one ſhall Deſcribe much Greater Arcs than the other, the Times of the Oſcillations ſhall be almoſt Equal, for in an Hundred Oſcillations there will ſcarcely be the Difference of the Time of One.

Arcs perform'd very nearly in Equal Times. Dr. Keill's Intr. ad Ver. Phyſ. Theorem 41.

§. 5. The 42*d* Tells us, That the Times or Durations of the Oſcillations of Two Pendulums, which deſcribe Similar Arcs, *EF, GH*, are in a Subduplicate Reaſon of the Lengths of the Pendulums *AB, CD*; Becauſe, by the foregoing Theorems, ſince the Arcs are Similar, the Time of the Deſcent thro' *EB* will be to the Time of Deſcent thro' *GD*, in a Subduplicate Reaſon of *EB* to *GD*; that is, half the Times of the Oſcillations will be in a Subduplicate Reaſon of half the Arcs, and therefore the Times of the whole Oſcillations will be in a Subduplicate Reaſon of Them; but the Arcs or half Arcs being Similar are as the Semidiameters, which deſcribe them, or as the Lengths of the Pendulums, and conſequently the Durations or Times of the whole Oſcillations will be in a Subduplicate Reaſon of thoſe Semidiameters or Lengths; From whence it follows, that, on the Contrary, the Lengths of the Pendulums are in a Duplicate Reaſon of the Times or Durations, in which the Oſcillations are Perform'd; And ſince the Durations or Times of the Vibrations are Reciprocally, as the Number of Vibrations perform'd in the ſame Time, it will be Eaſy, from the Number of Vibrations Given, which are made by one Pendulum *P* of a known Length in a Time Given, to Determine the Number of Vibrations, which are made in the ſame Time by another Pendulum *p* of a known Length; for as *P, p* Repreſent the Lengths of the Pendulums, Let *N, n* Expreſs the Numbers of the Vibrations of the ſaid Pendulums, and *T, t* the Times; *N* therefore is here Given, and *n* is the Number to be found; By the Preſent Theorem $T . t :: \sqrt{P} . \sqrt{p}$, and by what has been now alledg'd, $T . t :: n . N$, conſequently, $n . N :: \sqrt{P} . \sqrt{p}$, or $\sqrt{p} . \sqrt{P} :: N . n$, but $\sqrt{P}$, $\sqrt{p}$, and N, being known, *n* likewiſe will be Aſcertain'd; On the Contrary, from the Number of Vibrations Given, *N, n*, which are perform'd by the Two Pendulums in the ſame Time, and from the Given Length of one of Them *P, p* the Length of the other may be Defin'd, for $P . p :: T^2 . t^2 :: n^2 . N^2$, by what forego's, therefore $n^2 . N^2 :: P . p$.

The Times or Durations of the Vibrations of Pendulums in a Subduplicate Reaſon of the Lengths of the Pendulums. Dr. Keill's Introduct. Theorem 42.



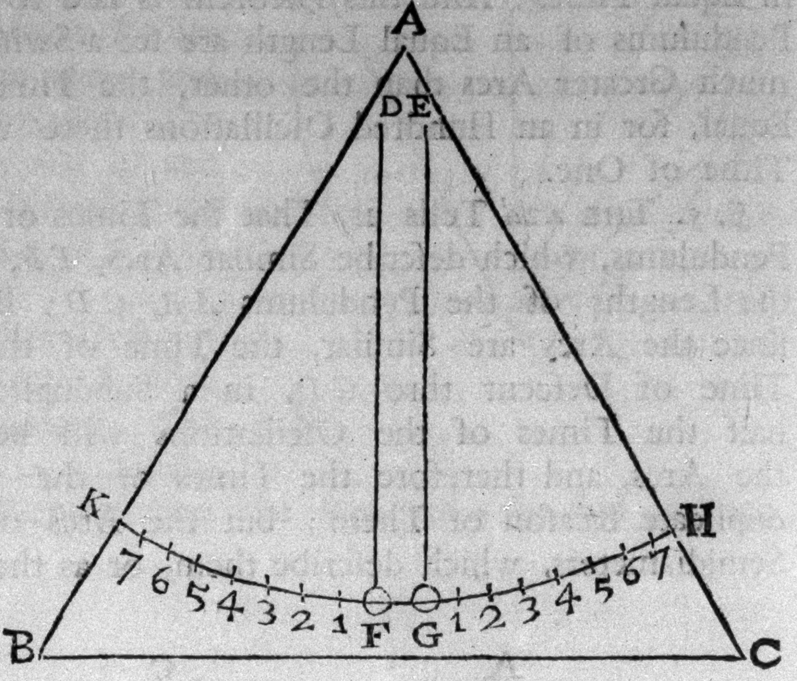


§. 6. The 43*d* Informs us, That the Velocity of a Pendulum, in it's Loweſt Point *B*, is as the Subtenſe of the Arc, which it Deſcribes in it's Deſcent; which Depends upon the Truth of the preceding Theorems; if therefore, from the ſaid Loweſt Point, ſuch Arcs be taken upon the Circumference of a Circle, deſcrib'd by the Pendulum, that their Subtenſes ſhall be, as 1, 2, 3, *&c.* Reſpectively, and ſome Acting Force Impels the Pendulum upwards, thro' the Firſt, Second, or Third Arc, *&c.* the Velocities of the Pendulum, mov'd by theſe Forces, will, in the Loweſt Point, be as 1, 2, 3, *&c.* Reſpectively; by which means, we may Impart Different Velocities to a Pendulous Body in any Reaſon aſſign'd, and thoſe, which are deriv'd

The Velocity of a Pendulum in it's Loweſt Point, as the Subtenſe of the Arc, which it Deſcribes in it's Deſcent. Dr. Keill's Introduct. Theorem 43.

riv'd from the Percuſſion of another Body, may be compar'd with one another, as well as with others, which are Originally Given.

Thus, if Two Pendulums of the ſame Length, *DF*, *EG*, hang Cloſe together, ſo that the Pendulous Bodies touch, and the one is drawn from it's Perpendicular to the Point 5 in one Arc, and the other to the Point 3 in the Oppoſite, and are then let Looſe; by the Theorems above, they will come to the Perpendicular at the ſame Time, and the Velocities with which they ſtrike each other will be as 5 and 3; and if, after Percuſſion, the Firſt Riſes to 3, and the Laſt to 5, their Velocities will be Exchang'd, and will be as 3 and 5 to Contrary Sides; And by a like Method, the Rules of Motion, both in Hard and Elaſtick Bodies, before Explain'd, may be Reduc'd and Submitted to Experience; The Demonſtrations of theſe ſeveral Theorems are Celebrated and Known, and therefore I purpoſely omit them, being underſtood by any one, who knows the Common *Principles of Geometry.*



The Hiſtory of Pendulums, and the Advances made in that Science, and by Whom.

§. 7. Thus far we are got in the Abſtracted and Mathematick Way of Reaſoning upon Pendulums, and now we are carry'd by this Philoſophy to an Hiſtorical Account of Them; That the leaſt Vibrations of the ſame Pendulum are nearly of an Equal Time or Duration, altho' the Arcs, which are Deſcrib'd by it, are Unequal; That *Chriſtian Hugens*, from hence, ſhew'd the Great Advantage of Pendulums in Regulating the Motions of Automatons, in his *Horologium Oſcillatorium*; That *Galileus* indeed, the firſt Author and Inventor of this Science, had before made uſe of Pendulums, in his Obſervations in Aſtronomy and Nature, which require an Exact Meaſure of Time; That *Hugens*, notwithſtanding, was the Firſt, who Introduc'd Automatons with Pendulums, and Prov'd them, by Experiments, far to Exceed thoſe, which before Obtain'd, whoſe Librators were Horizontal; That from thence Theſe Automatons were generally Receiv'd, ſome of which have been ſo Artificially made, as to give a Meaſure of Time, much Juſter and more Exact than the Motion of the Sun, which ſhews the Apparent or Relative Time only, but not the Real and Abſolute; From whence it comes to Paſs, that Pendulums, at Certain Times, ſhew the Hour Different from the Apparent one, and ſometimes Fifteen or Sixteen Minutes more or above what is Pointed out by the Sun-Dial, and at others, ſo many Leſs, and it is not, Excepting Four Times in a Year, that the Pendulum and the Sun Agree;

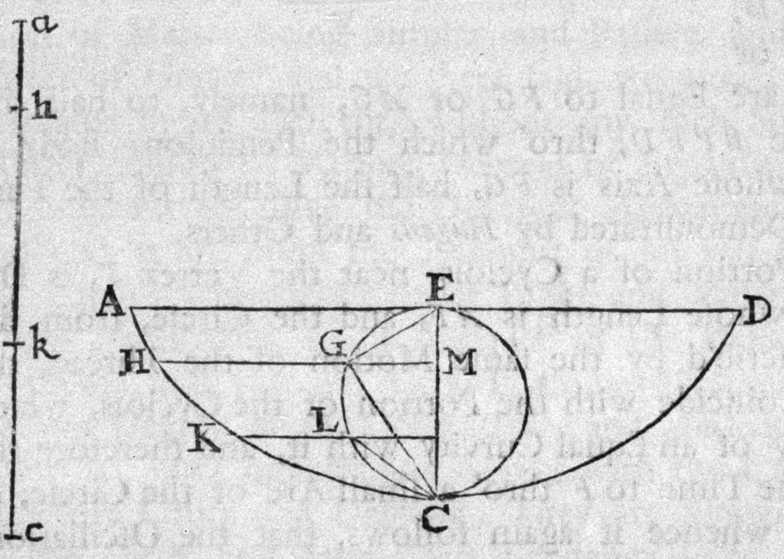
Dr. Keill's Aſtronom. Lect. 26. Pag. 413. Edit. 1.

It is farther ſaid in this Hiſtory of Pendulums, that, altho' the Vibrations of the ſame Pendulum, where the Arcs deſcrib'd by it are Unequal, are Nearly, and to Senſe, of an Equal Time and Duration, yet, ſince they are not Geometrically and ſtrictly ſo, but the Greater Vibrations are ſomething Longer, in their being Perform'd, than the Leſs, and by that means Differ, it is manifeſt, that from all theſe Small Differences taken together, at Length a very Conſiderable one will Enſue, which is Matter of Fact, and what we find by Experience; For if the Wheels are Affected with a Kind of Stiffneſs, as in Cold Weather, ſo as to Impel the Pendulum with a Leſs Force, the Oſcillations being Shorter will alſo be Quicker than uſual; If the Wheels have too Great a Lubricouſneſs, and they thereby make the Pendulum move in Larger Arcs, the Oſcillations are of a Longer Duration, and the Time ſhewn by the Pendulum proceeds Slower; It is alſo moreover alledg'd, that, by ſome late Experiments in the *Philoſophical Tranſactions*, it is Plain, that the Pendulum of an Automaton, Performing it's Vibrations in Vacuo, upon the Taking off of the Reſiſtance of the Air, deſcrib'd Greater Arcs, and was a Longer Time in each of it's Oſcillations; To Remedy this Inaccuracy of Pendulums, and to Reduce the Unequal Reciprocations of them to Equal Times, *Hugens* Invented a Way, by which the Pendulous Body ſhou'd be carry'd in the Arc of a Cyclois, but, by the Nature and Properties of that Figure, the Times of the Deſcents of Heavy Bodies, thro' any Arcs to the Loweſt Point or Vertex of it, are Equal, and therefore, if the Pendulous Body always Moves in the Arc of a Cyclois, the Times of it's Oſcillations will be exactly Equal, whether the Pendulum Vibrates in Greater or Leſs Arcs.

§. 8. In order therefore to Demonſtrate this Property of the Cyclois, Theorem the 45*th.* it is propos'd to be Prov'd, That, if a Body is urg'd in a Right Line towards a Center, by Forces which are Proportional to the Diſtances from the ſaid Center, Let it fall from any Height, It will always come to the Center in the ſame Time; and that Time is to the Time, in which the Body can Deſcribe the ſame Path with a uniform Velocity and Equal to That, which is Acquir'd at the End of it's Fall, as the Semiperiphery of a Circle to it's Diameter.

The Cyclois Introduc'd by Hugens, *in which all the Vibrations of a Pendulum are Equal, tho' the Arcs are Unequal.*

The 46*th* Shews, That, in a Cyclois, whoſe Axis ſtands Perpendicular to the Horizon, and whoſe Vertex is Inverted, the Times, in which a Body, urg'd by the Force of Gravity and let fall from any Point in the ſaid Cyclois, arrives at the Vertex, are Equal, and, to the Time of it's Perpendicular Deſcent thro' the Axis, are in the Proportion of the Semiperiphery of a Circle to it's Diameter. Thus let ACD be a Cyclois, whoſe Axis is CE and it's Generating Circle ECG; ſince the Tangent in any Point H is Parallel to the Chord CG drawn in the Generating Circle Form'd about the Axis EC, by the Writers upon this Figure it is manifeſt, that the Body in it's Deſcent is Accelerated with the ſame Force in H, as if it had Deſcended thro' the Right Line GC; But the Force, with which it is Accelerated in GC, is to the Force of Gravity, as MC to GC, by what foregoes, and $MC.GC::GC.CE$, by Elem. 6. Prop. 8. Therefore the Force, with which the Body is Accelerated in H, is to the Force of Gravity, as GC to CE; by the ſame Reaſoning, the Force of Gravity is to the Force, with which the Body is Accelerated in any other Point K, as CE to CL, and conſequently, *ex Æquo*, the Force, with which the Body is Accelerated in H, is to the Force, with which it is Accelerated in K, as GC to LC, or as the Double of GC to the Double of LC, that is, by the Writers upon this Figure, as the Curve of the Cyclois HC to the Curve KC; The Forces therefore, with which a Body is Accelerated in it's Deſcent upon a Cyclois, are as the Lengths of the Curve to be Deſcrib'd; Let us now ſuppoſe the Right Line ac Equal to the Length of the Curve AC, and let a Body be Conceiv'd to be urg'd with the ſame Forces in the Right Line ac towards c, with which the Body is urg'd in it's Deſcent thro' the Curve AC; But the Forces, with which the Body is urg'd in any of the Points of the Cyclois H and K, are as the Lengths HC, KC, or hc, kc, that is, the Forces in any Places, are as the Diſtances of thoſe Places from the Point c, and therefore, by the Preceding Theorem, the Times of the Deſcents from any Height are Equal.



Dr. Keill's Intr. ad Ver. Phyſ. Lect. 15. Theorem 45, 46.

Hugen. Horolog. Oſcillator. Prop. 25. Par. 2.

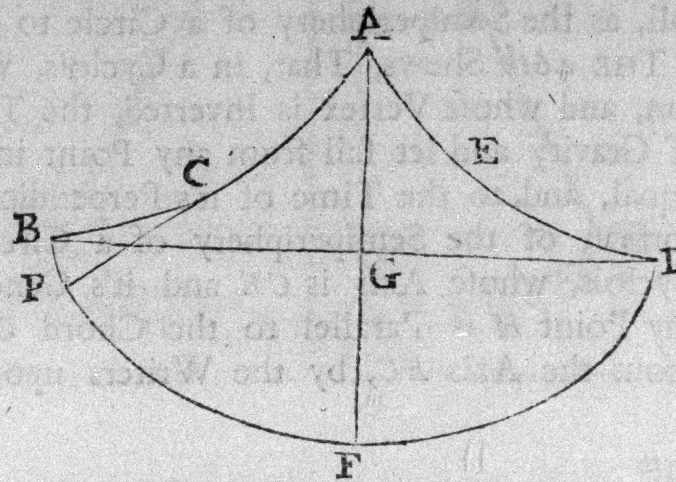
Moreover, the Time of the Deſcent from a to c is to the Time, in which ac or $2EC$ is Deſcrib'd with the Velocity Acquir'd at the End of the Fall, as the Semiperiphery of a Circle is to the Diameter, by the Preceding Theorem; but the Time, in which $2EC$ is Deſcrib'd with the ſame Velocity, is Equal to the Time, in which the Body falling by it's own Gravity Deſcends thro' EC the Axis of the Cyclois; from whence the Time of the Deſcent thro' ac, or AC is to the Time, in which the Body Deſcends thro' the Axis of the Cyclois, as the Semiperiphery of the Circle to the Diameter; Conſequently, the Time, in which the Body Deſcends in the Cyclois thro' the Arc AC, and Aſcends thro' that of CD, that is, the Time of the Motion thro' the whole Cyclois ACD is to the Time of the Perpendicular Deſcent thro' the Axis of the Cyclois, as the whole Periphery of the Circle to it's Diameter.

§. 9. If

How to make a Pendulum Vibrate in a Cyclois. Dr. Keill's Intr. ad Ver. Phyſ. Theorem 46. Hugen. Horolog. Oſcillator. Definit. & Prop. 6. Par. 3.

§. 9. If therefore a Pendulous Body Vibrates in a Cyclois, whether it Performs Greater or Leſſer Arcs, the Oſcillations will be always made in Equal Times; but how to Effect, that a Pendulous Body ſhou'd Vibrate in a Cyclois, is the Difficulty, which *Hugens* has ſhewn the Way to Surmount, in his *Horologium Oſcillatorium*, Part the Third.

Thus, a Curve is to be found, by whoſe Evolution the Curve Given is Deſcrib'd, and Two Plates are to be Bent into the ſame Curvature, betwixt which the Pendulous Body, Hanging by Threads of a Certain and Determin'd Length, do's not Deſcribe a Circle, but ſome other Curve; Let Two Plates, *ACB*, *AED*, be Bent into Similar and Equal Figures, and from the Point *A* let the Thread of the Pendulum be ſuſpended, which, whilſt the Pendulum Oſcillates, Wraps it ſelf Round the Plates, *ACB*, *AED*, to which it always Adheres and Adapts it ſelf; by this Application of the Thread to the Plates, the Motion of the Pendulum in a Circle is continually Interrupted and Hinder'd, and the Pendulous Body is carry'd thro' the Curve *BPFD*; the Curve *ACB* or *AED* is call'd the Evolute, and the Curve *BPFD* is ſaid to be Deſcrib'd by Evolution; If the Curves, *ACB*, *AED*, are two Semicyclois's, whoſe Axes or Diameters of the Generating Circles are Equal to *FG* or *AG*, namely, to half the Length of the Pendulum, the Curve *BPFD*, thro' which the Pendulous Body is carry'd, becomes an entire Cyclois, whoſe Axis is *FG*, half the Length of the Pendulum, according to what has been Demonſtrated by *Hugens* and Others.



The ſmalleſt Arcs of the Cyclois and the Circle nearly Coincide.

§. 10. Furthermore, Since the Portion of a Cyclois, near the Vertex *F*, is Deſcrib'd by the Motion of a Thread, whoſe Length is *AF*, and the Circle, from the Center *A* at the Interval *AF*, is Deſcrib'd by the ſame Motion of the Thread, the Circle, Paſſing thro' *F*, will nearly Coincide with the Portion of the Cyclois, which is next to the Vertex *F*, and is nearly of an Equal Curvity with it, and therefore the Pendulous Body is carry'd in the ſame Time to *F* thro' a ſmall Arc of the Circle, as thro' an Arc of the Cyclois; From whence it again follows, that the Oſcillations of a Pendulum, thro' the ſmall Arcs of a Circle, will be perform'd in Equal Times, ſince thoſe in a Cyclois, with which they nearly Coincide, are ſo.

The Time therefore, in which a Pendulum performs the leaſt Oſcillation in a Circle, is Equal to the Time, in which an Oſcillation is perform'd thro' the Arc of a Cyclois, whoſe Axis is half the Length of the Pendulum; But the Time, in which the Oſcillation in a Cyclois is perform'd, is to the Time of the Perpendicular Deſcent thro' the Axis of the Cyclois, that is, thro' half the Length of the Pendulum, as the Periphery of the Circle to the Diameter; and from hence it is Conſequent, that the Time of Every the leaſt Oſcillation is, to the Time of the Deſcent thro' the Length of the Pendulum, in a Conſtant and Certain Proportion of the Periphery of the Circle to the Diameter of it, Multiply'd into the Square-Root of Two.

The Vibrations of a Pendulum Slower, when Nearer to the Æquator, than when Farther from it.

§. 11. If, in Different Climates and Regions of the Earth, the ſame Pendulum performs it's Oſcillations in Unequal Times, the Times of the Deſcents thro' the Length of the Pendulum in theſe Different Climates will be likewiſe Unequal, and where the Oſcillations Proceed Slower, the Body will Deſcend Slower in a Perpendicular, and in a Given Time will Deſcribe a leſs Space in it's Fall; but it is Certain by Experiment, that in thoſe Regions, which are Neareſt to the Æquator, the Oſcillations of the ſame Pendulum are Slower than in other Places, whoſe Latitude is Greater, and therefore Heavy Bodies, in thoſe Countries, Deſcribe in their Fall a leſs Space in a Given Time, and are leſs Accelerated in their Motion than in Ours, which is farther Remov'd from the Æquator; ſo that by Experiments it is Evinc'd, that the Action of Gravity is Leſs in thoſe Places, whoſe Latitudes are Leſs, than in Places, which are Nearer to the Pole;

Which Decreaſe of Gravity is ſaid to ariſe from the Centrifugal Force, deriv'd from the Earth's Rotation about it's Axis, which is Greater, as the Circuits of the Bodies Revolving are ſo, and is always therefore as the Sine of the Diſtance of the Place from the Pole, which under the Æquator is Greateſt, and under the Pole none; and conſequently, the Force of Gravity, in the Æquator, is the Leaſt, and, in the Pole, the Greateſt. This is a full and ſuccinct Account of the Doctrine of Pendulums, and of the Deſcent of Heavy Bodies in Inclin'd Planes, into both of which we ſhall now Inquire.

The Reaſon of the Deſcent of Heavy Bodies, and their Acceleration to the Earth.

§. 12. As to the Deſcent of Heavy Bodies in Inclin'd Planes, §. 1, we muſt obſerve, that, in Order to juſtify this Reaſoning, the Force of Gravity being Reciprocally Proportional to the Squares of the Diſtances, a Favorite Notion of the Preſent Philoſophy, is Diſcarded, which is it's Firſt Abſtraction, and tho', upon the Account there mention'd, where the Difference of the Diſtances is very Small, the Difference of the Force of Gravity may not be Conſiderable, yet it is ſomething, and takes from the Geometrick Exactneſs of theſe Demonſtrations, but this we Paſs over; And the next and moſt Material Objection againſt this Doctrine is, that it ſuppoſes Matter to be Similar and Homogeneous, to have no Intrinſick Forces of it's own, neither Expanſive nor Contractive, to be entirely Paſſive in it ſelf, and only Acted with one uniform Force of Gravity, and that this Force of Gravity urges it by Continual and Succeſſive Impulſes, by which it is Equably Accelerated; On the Contrary, we have, in our firſt Part of the *Principles of Philoſophy*, endeavour'd to confute this Opinion of Matter being Similar and Paſſive, and being Acted upon by uniform Impulſes of Gravity, and we there ſaid, we wou'd give a Solution of Matter's being Accelerated in it's Deſcent, without any ſuch Suppoſitions; We therefore affirm, that Matter conſiſts of Various Degrees of Expanſive and Contractive Forces, which will be ſufficient to ſolve the Different Appearances we find in it; that Fire do's not Deſcend at all, upon the Account of it's vaſt Expanſive Force, which is oppoſite to the Earth's Contractive; that Light and Volatile Bodies Deſcend leſs than others, which are Heavier, upon Account of the ſame Expanſive Force being more ſenſible in them; and that the Groſſeſt Bodies Deſcend the moſt, as being endu'd with a Greater Degree of the Contractive, and conſequently with a Force, which is not Oppoſite to, but Concurrs and Agrees with the Contractive of the Earth, to which they therefore Unite and Deſcend; They are likewiſe Accelerated, the Nearer they come to the Earth, and have a leſs Tendency to it, the Farther they are from it, for the ſame Reaſon, that the Expanſive Force of Fire obtains a Greater Effect upon Inflammable or Expanſive Matter, when Near, than when at a Diſtance, and for the ſame Reaſon, that all other Forces are ſtronger, in their Actions upon each other, when leſs Retir'd, than when more ſo; And from hence it is manifeſt, that Various Bodies are more or leſs Accelerated to the Earth, according to the Different Degrees, which they have, of the Contractive or Expanſive, and that the ſame Bodies are Accelerated towards it, if of a Contractive Nature, rather in a Reciprocal, than in a Reciprocally Duplicate, Proportion to their Diſtances; for thoſe of an Expanſive Retire actually from it, as Fire, or Endeavour, in ſome Meaſure, to do it, as Light and Volatile Bodies.

Principles of Philoſ. Chap. 5. Part 1. §. 18.

See Chap. 5. of this Book.

See Schol. §. 1.

As to what is here ſaid of Experiments, It is Confeſs'd, that the Perpendicular Deſcents of Bodies cannot be Determin'd by Them, and Thoſe in Inclin'd Planes are own'd not to Agree Exactly with the Doctrine Aſſerted from Abſtraction and Reaſon, which therefore may be as well Aſcrib'd to ſomething in the Expanſive and Contractive Forces, as to the Uneveneſs of the Surface, on which the Experiments are made. Farther, it is not Evident, that Bodies are more Accelerated, Preciſely, as They come Nearer to the Earth, and Leſs, as they Retire from it, ſince Forces do not always Exert themſelves more or leſs, as we Approach to them, or Recede from them; many Times they Exert themſelves to a Great Degree in a near Approximation to 'em, and not at all at ſome farther Diſtance from them, as in Fire, which is of an Expanſive Nature, and in Snow, which is of a Contractive one; and thus it may be, in Reſpect of the Earth, that tho' Heavy Bodies, near it, are vehemently Urg'd and Accelerated towards it, yet, at a Greater Diſtance from it, they may not be Gradually Diminiſh'd in ſuch their Acceleration;

celeration; but this we have Diſcours'd of in another Place, where we conſider'd the Propagation of Action from a Center.

Why the ſeveral Theorems Recounted, §. 2. *are not True, and that Bodies do not Riſe to the ſame Height from which they Fell.*

§. 13. THE ſeveral Theorems, in §. 2, Depending upon theſe Principles of a Similar and Paſſive Matter, without any Intrinſick Forces belonging to it, and upon uniform Impulſes from Gravity, which are all of them Chimerical Fancies and Mere Suppoſitions, all the Demonſtrations, which belong to them, will be only a Mathematick and Abſtracted Reaſoning upon Properties and Things, which have no where an Exiſtence, but in the Mind, which Creates them; Beſides which, in the 38*th* Theorem, mention'd in the ſame Section the 2*d*, It cannot be Conceiv'd, how a Body ſhou'd Aſcend, thro' ſeveral Inclin'd Planes or thro' a Curve, to the ſame Height, from which it fell, by the ſame Action and Impulſe of Gravity, and then Deſcend, and ſo alternately in continual Vibrations; becauſe, tho', ſuppoſing the Body to be Accelerated and to Acquire a conſiderable Velocity by a large Deſcent, that Velocity wou'd be Gradually Diminiſh'd in the Bodies Aſcent in a Spherical Surface, till it was Deſtroy'd, when it came to the ſame Height from which it fell, and then it wou'd again Deſcend and be Accelerated, the Contrary Way, by the Perpetual Action and Impulſe of Gravity, if Matter was at the ſame Time Paſſive, and had no Internal Force of it's own, yet this is not Fact; ſince a Globe of Lead will not riſe to near the Height from which it fell, nor is it Reaſonable it ſhou'd, if it has in it ſelf a Force, Reluctant to a Velocity Impreſs'd, which it has, by being of a Contractive Nature, whereas other Bodies, which are of a more Expanſive and Elaſtick, will riſe to a Greater Height by their concurring with the Force of Velocity Acquir'd, which is likewiſe of an Expanſive one: Moreover, where the Deſcent of Heavy Bodies is Small and Inconſiderable, which is the Caſe of Pendulums, we cannot Imagine, that there is any ſenſible Acceleration, ſo as by a Velocity Acquir'd it ſhou'd Aſcend at all, or at moſt, to a very little Height; We think, that they are rather carry'd down by their Gravity with a moſt Minute Velocity imparted to them, and therefore, that they are ſtop't and hinder'd from Riſing far by the ſame Force, by which they Deſcend. This therefore appears to be the Real Theory of Deſcending Bodies in a Spherical Surface, and that the Expanſive and Contractive Forces, which Reſide in them, Enter into the Account, and are the Reaſon of their Riſing to a Greater or Leſs Height, after their Deſcent, which will but Illy Agree with their Riſing to the ſame Height, from which they fell, by the Paſſiveneſs of Matter and the uniform Impulſes and Action of Gravity; and hence will be ſeen the Vanity of that Proportion, that the Times, ſpent in Deſcribing Similar Curves or Planes Similarly Inclin'd, are in a Subduplicate Reaſon of the Curves or Lengths.

The Doctrine of Pendulums Conſider'd, and the True Theory of them Explain'd.

§. 14. FROM the Deſcent of Bodies, in Curves or Inclin'd Planes, we are Led to the Vibrations of a Pendulum, for which the preceding Doctrine was Principally Contriv'd; and the firſt Step, which is made, is to let us know, §. 3, that the Caſe of Deſcending Bodies, in Curves or Inclin'd Planes, is the ſame with that of the Oſcillations in a Pendulum; that the Center of Suſpenſion, from which the Pendulum Oſcillates, performs the ſame Office, in Retaining the Body in it's Circular Vibrations in the one Circumſtance, as the Spherical Surface do's in the other; that they are alike Accelerated by Gravity and Deſcend by it, and Aſcend by the Velocity Acquir'd, till that Velocity is Loſt by the Action of Gravity, when they again Deſcend, *&c.* But here is the Diſtinction, that, in Bodies Deſcending in Inclin'd Planes, the Gravity of them, and the Velocity ariſing from thence, and the Determination of their Motion from the Inclination of the Planes, are only to be Conſider'd; whereas, in the Pendulum, the Two Former do not only Enter into the Account, and the Latter from the Detention of them in a Circle, but alſo the Center of Suſpenſion has an Effect peculiar to it, and to which there is nothing Analogous in the mere Deſcent of Bodies thro' Inclin'd Planes, and conſequently, the Demonſtrations of the one cannot be Reaſonably Extended to the Properties of the other; for it is not the ſame Thing for a Body to be carry'd Round in a Sling, where there is a Perpetual Centrifugal Force Exerted, and for a Body to Roll upon a Table, whether Plane or Concave, without any Center, from which it Endeavours to Recede.

The Nature of Pendulums, and the Reaſon of their Oſcillation.

The Nature of Pendulums and of their Vibrations therefore ſeems to be this, which will ſufficiently Diſtinguiſh them from the Deſcent of Bodies only upon Inclin'd Planes, that, when any Heavy Body hangs, by a String, from the Center of Suſpenſion, there are Two Contrary Forces, which act againſt each other, the one in the ſaid Center, which is a Force of Contraction, by which it Endeavours to bring the Pendulous Body towards it, the other an Expanſive Force in Reſpect of that Center, by which it Endeavours to Recede from it, by it's Gravitation to the Earth, Theſe, Acting againſt each other and the Contractive prevailing, a Quieſcence enſues; but, if to the Expanſive Force of the Pendulous Body is added the Expanſive of Percuſſion by the Hand or by any other Inſtrument, or of a Greater Weight and Gravity, ſo as to Exceed the Contractive of the Center, the Thread is Broken and the Body Deſcends to the Earth; If by this Percuſſion the Expanſive do's not Exceed the Contractive, there is a Conteſt betwixt the one and the other, by which the Oſcillations are Produc'd; for the Pendulous Body cannot Recede from the Center, and it is leſs from Approaching towards it, than before this New Addition of the Expanſive; and conſequently, the Force muſt be ſpent in Circulating Backwards and Forwards, till at length by the Prevalence of the Contractive in the Center, it becomes again Quieſcent.

Why Heavier Pendulums will Oſcillate longer than Lighter ones.

And First, From hence it is manifeſt, that Heavier Pendulums will Oſcillate longer than Lighter ones, becauſe the Expanſive Force in Heavier Bodies, in Reſpect to the Center of Detention, is Greater than that of Lighter ones, and if there is an Addition of the Expanſive Force by Percuſſion, it will be longer before the Contractive Force of the Center prevails in the one Caſe, than the other; which is Contrary to the Deſcent of Bodies in Inclin'd Planes, where Lighter and more Elaſtick ones will Aſcend and Deſcend to a longer Time than Lead will, which is ſoon Quieſcent.

On the other Hand, this Phænomenon is Explain'd by the Difference of the Air's Reſiſtance, which is leſs in Proportion to Heavier Bodies than to Lighter, but if that is a true Solution, it ſhou'd be told us, why the Reſiſtance of the Air has not the ſame Effect in the Deſcent of Heavier Bodies thro' Inclin'd Planes, which it Apparently has not, and therefore the Solution I have given ſeems to be the moſt Rational.

Why the leaſt Vibrations of the ſame Pendulum are nearly of an Equal Time.

Second, From hence likewiſe is Evident, the Reaſon, why the leaſt Vibrations of the ſame Pendulum are nearly of an Equal Time or Duration, but yet that it will Deſcribe longer Arcs, in ſomething a longer Time, than ſhorter ones; becauſe the Expanſive Force of the Pendulum, which Acts againſt the Contractive of the Center of Detention, is chiefly owing to the Gravitation of the Pendulous Body, by which it Endeavours to Recede from that Center, but that Force being always the ſame and conſtant, and the Gentle Percuſſion, by which it is put into Motion, adding Little to it or Equably doing it, and the Arcs being ſmall, it is Plain, that ſuch Unequal Arcs will be nearly Perform'd in the ſame Time, tho' not Exactly ſo, becauſe Really the ſame Force will Deſcribe a longer Arc in a longer Time, and a ſhorter one in a Shorter; for this Phænomenon the preſent Philoſophy gives no Solution, but takes it for Matter of Fact, that the Vibrations are Perform'd Exactly and Geometrically in the ſame Time, Raiſes Theorems and Proportions upon it, and then Confeſſes, that they are not perform'd Geometrically in the ſame Time, but that longer Arcs Require a longer Time to Deſcribe them, and the Shorter, a Leſs.

Why Pendulums Oſcillate Slower, when nearer to the Æquator, and Quicker, when nearer to the Poles.

Third, From hence laſtly is Explain'd, why Pendulums Oſcillate Slower, when nearer to the Æquator, and Quicker, when nearer to the Poles, becauſe the Expanſive Force of the Sun's Heat Acts contrary to and Enervates the Contractive Force of the Pendulum, by which it Acts leſs againſt the Contractive Force in the Center of Detention, as Bodies, turn'd into Fire and Flame, Gravitate leſs, and put on the Force of the Expanſive; On the other Hand, the Contractive Force of Froſt and Cold, near the Poles, concurrs with and ſtrengthens the Contractive Force of the Pendulum, by which it is Envigorated in it's Action againſt the Contractive Force in the Center of Detention, and thereby the Vibrations are Quicken'd; and for the ſame Reaſon, Pendulums Oſcillate Faſter in Winter than in Summer, and in Colder Seaſons than in Hotter; whereas the Reaſons alledg'd for the Solution of theſe Phænomenons, by the Philoſophy before us, are Various and do not Confirm, but Contradict each other;

for the firſt is ſaid to Proceed from the Earth's Rotation, by which Gravity is Diminiſh'd more in the Æquator than in the Poles, which can be of no Service to Account for Pendulums, whoſe Oſcillations Differ in the ſame Climate, purely upon Account of the Difference of Heat and Cold; and the laſt is ſaid to Proceed from a Difference of the Air's Temperature, which cannot Explain thoſe Phænomenons, which are affirm'd to ariſe from a Different Situation to the Pole and the Æquator.

However, notwithſtanding theſe Miſtakes in the Nature of the Pendulum, and this Philoſophy's being oblig'd to own, that the Oſcillations of it are, in Unequal Times, in Unequal Arcs, tho' it had hitherto Reaſon'd upon the Suppoſition, that they were perform'd, in Equal Times, in Unequal Arcs, it can Paſs all this Over, and thinks to Correct all Irregularities by the Help of the Cyclois, albeit, by ſo many Arguments, as the Cyclois is Introduc'd, by ſo many is the former Doctrine Diſcarded, and yet, which is ſtrange, it Retains and Makes uſe of it; Let us therefore now Conſider, what farther is advanc'd in Relation to that Figure;

And Firſt, in order to Demonſtrate the Property of the Cyclois, which is Wanting, the ſeveral Abſtractions of a Similar, Paſſive, and Gravitating Matter are Preſum'd, which have been before Diſputed; alſo that Gravity Acts by Conſtant and Uniform Impulſes, which is either Falſe, or remains a Queſtion; and that the Motion of Pendulums is the ſame as, and may be Explain'd by, the mere Gravity and Deſcent of Bodies in Inclin'd Planes, which is not Granted; Theſe Points, being not Explicite and Clear, hinder us from allowing *Hugens* to be the moſt Accurate Philoſopher, how much Sagacity and Penetration ſoever we may be willing to Give to his Geometry.

In the ſecond Place, if all theſe Abſtractions and Preſumptions were acknowledg'd and ſubmitted to, the Way, which is preſcrib'd for the Oſcillation of a Pendulum betwixt Two Cyclois's, in order to make the Pendulous Body Deſcribe a Third, is Impracticable and Romantick; for whatever Schemes we may Form to our ſelves, or whatever Mathematick Reaſonings we may Advance, it will be ſtill Certain, that the Oſcillations are Perform'd very nearly in the Arcs of a Circle, ſince, tho' the String of the Pendulum ſhou'd Wrap it ſelf about the Cyclois's, betwixt which it Vibrates, the very Inſtant it departs from them, the Body will be found in the Arc of a Circle, from it's ſtretching the String, by it's Gravitation, to a Radius of it, unleſs we can Imagine a Heavy Body to Oſcillate at the End of a String not ſtretch'd, but bent into a Curve; which muſt be the Reaſon, that Artificers, not perceiving the Advantage, from Experiment, of theſe Cyclois's, uſe no ſuch String, but a Stiff and Inflexible Iron-Rod, at the End of which the Body is affix'd, and which therefore can Oſcillate in no other Figure, than that of a Circle; and from hence it is, that Dr. *Keill* Labours to ſhew, (for no Curves of a Different Nature will), that the ſmall Portions of a Circle and Cyclois Coincide, which is as much as to Confeſs, that, after all theſe Fine Demonſtrations concerning the Properties of the Cyclois's in the *Horologium Oſcillatorium*, the Pendulum will do as well without them, by Oſcillating only in the Plain Arcs of a Circle; It is true, there is ſomething ſubſtituted in the Room of the Cyclois, that is, a Flat, Thin, Iron-Plate, Tranſverſe to the Plane of Oſcillation, by the bending of which, in the Vibrating, the Pendulum is Shorten'd, and by which the ſame Effects are ſuppos'd to be Produc'd, as if the Pendulum had Oſcillated betwixt Two Cyclois's; However this may be found in Practice, no Mathematician can ever Prove, that the ſhortening of the Pendulum, by drawing the Flat Plate up into the Arc of a Circle, is the ſame, in a Geometrick Figure, as the ſhortening of the Pendulum, by ſtriking againſt a Cyclois; and if not, either this Doctrine of the Cyclois is unneceſſary to the Explanation of the Pendulum, or elſe another ought to be Contriv'd, by which it may be Geometrically Evinc'd, that the Shortening of a Pendulum, by the Contraction of the Plate into the Arc of a Circle, is the ſame as a Pendulum, Oſcillating betwixt Two Cyclois's.

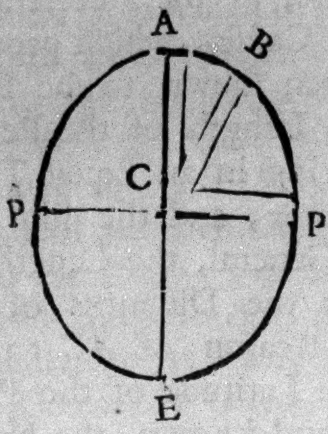
The Doctrine of Pendulums, as apply'd to Determine the Figure of the Earth, which is ſaid to be

§. 15. To conclude this Doctrine of Pendulums, we ſhall conſider, what is alledg'd concerning them, in the preſent Celebrated Philoſophy, to Determine the Figure of the Earth, that is, what Proportion it's Axis bears to the Diameter, Tranſverſe or Perpendicular to it;

Let

that of a Spheroeid, Explain'd and Conſider'd. Sir Iſ. Newt. Princip. Prop. 19, 20. Lib. 3. Dr. Gregory's Aſtronom. Prop. 52. Lib. 3.

Let the Figure *APEp*, whoſe Center is *C*, Repreſent the Earth, *Pp* it's Axis, *AE* any Diameter of the Æquator, Perpendicular to the Axis; Let the Lengths of the Pendulums be Meaſur'd, which Oſcillate in the Places *A* and *P*, in a Cyclois, in a Given Time, and *AC*, *PC*, will be Reciprocally as the ſaid Lengths of the Pendulum, and therefore in a known Proportion; ſo likewiſe in any Intermediate Place, *AC*, *BC*, will be Reciprocally, as the Lengths of the Pendulums, which Oſcillate in a like Space of Time in the Places *A* and *B*, and therefore in a known Proportion.

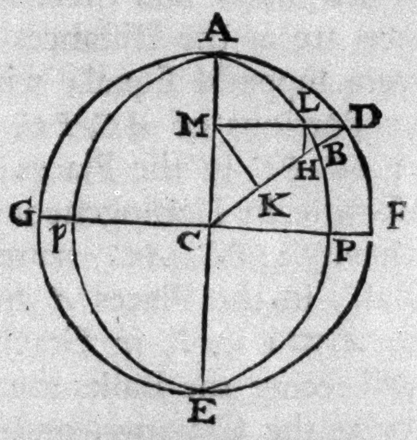


For let *ACP* be conceiv'd to be an Inflected Siphon or Canal, full of a Fluid and Homogeneous Matter; from the Quieſcence of the Fluid it is Evident, that it is in an Æquilibrium in both the Crura, *AC*, *PC*, the Centrifugal Force from the Motion of the Earth about it's Axis, and the Greater Diſtance from the Center of the Earth, Raiſing and Supporting the Fluid, in the Crus *CA*, to a Greater Height, than in that of *CP*; from whence any Portion of the Fluid in *CA* is in an Æquilibrium, or is Equally Heavy, with a like Portion, and alike Situated, of the Fluid in *CP*, which therefore is True of any other Homogeneous Bodies, tho' they are not Fluid; The uppermoſt Points *A* and *P* of the Crura are alike Situated, and conſequently the Homogeneous Bodies, in *A* and *P*, which are as *AC*, *PC*, have an Equal Gravity towards the Center of the Earth; But the Gravity of a Body in *A*, which is as *PC*, is to the Gravity of another Body in *A*, that is Homogeneous to the Former, and which is as *AC*, as *PC* to *AC*, that is, the Weights of Homogeneous Bodies, in a near Situation to each other, are as the Bodies or the Bulks of them, and therefore the Gravities of Homogeneous Bodies, which are Equal and Situate in *A*, and *P*, are as *PC* to *AC*, that is Reciprocally, as the Diſtances from the Center; for ſuppoſing *AC*.*PC*::100.1, ſince *AC*, *PC*, or *A* and *P*, Gravitate Equally, a Part of *AC* Equal to *PC*, or a Part of *A* Equal to *P*, will Gravitate a Hundred Times leſs, or it's Gravity in Reſpect of *P*, will be as 1 to 100; By a like Reaſoning it may be ſhewn, that the Gravity of a Body in *B*, is to the Gravity of an Equal and Homogeneous one in *P*, as *CP* to *CB*; from whence *ex Æquo* the Gravities of Equal and Homogeneous Bodies, whereſoever they are on the Superficies of the Earth, are Reciprocally, as their Diſtances from the Center; But the Accelerating Gravity of a Body is, as it's Abſolute Gravity divided by it's Bulk, and therefore the Accelerating Gravities of Bodies on the Earth's Surface are as the Diſtances of the Places from the Center Inverſely, becauſe the Bulks were ſuppos'd Equal; which may be thus Compriz'd in ſhort; Let *A* and *P* be Similar Portions of *AC*, *PC*, ſince *AC* = *PC* in Gravity, *A* = *P* in Gravity, but the Bulks :: *AC*.*PC* in the Places *A* and *P*, (and what is ſaid of Homogeneous Fluids obtains likewiſe in Homogeneous Solids;) On the other Hand in the ſame Place *A*, *P*.*A* in Gravity :: *PC*.*AC*, namely, the Weights are as the Bulks, if therefore, *A* = *P* in Bulk, in the Places *A* and *P*, *A*.*P* in Gravity :: *PC*.*AC*; for Inſtance, if 100 Parts in *A*, = 1 in *P*, in Gravity, 1 Part in *A* will Gravitate 100 Times leſs than 1 in *P*; and ſeeing the Bulks are ſuppos'd Equal, the Accelerating Forces, in *A* and *P*, will be as the Gravities, and therefore :: *PC*.*AC*, or Reciprocally, as the Diſtances from the Center.

Farther, Since the Oſcillation of the Pendulous Body at *A*, in a Cyclois, is ſuppos'd, from Obſervation, to be perform'd in the ſame Time with the Oſcillation of another Body at *P*, in a Cyclois, and by Prop. 25. Part 2. *Horolog. Oſcillator.* The Time of the Deſcent, thro' the Axis of the Cyclois, is in a Given Proportion to the Time of Oſcillation in the ſaid Cyclois, namely, as the Diameter of a Circle to it's Circumference, and by Prop. 6, 7. Part 3. The Axis of the Cyclois, which the Pendulous Body Deſcribes, is in a Given Proportion to the Length of the Pendulum, Namely, as 1 to 2, and therefore the Time of the Deſcent, thro' the Axis of the Cyclois, is in a Given Proportion to the Time of the Deſcent, thro' the Length of the Pendulum, that is, in a Subduplicate Reaſon of the Former, the Times or Celerities of Deſcent being in a Subduplicate Reaſon of the Spaces Deſcrib'd; It will be Conſequent, that the Time of Deſcent, thro' the Length of the Pendulum at *A*, will be Equal to the Time of Deſcent, thro' the Length of the Pendulum at *P*, ſince the Times of Oſcillation in both the Pendulums are Equal, and they have, both of them, the ſame Given Reaſons to the Times of Deſcent thro' the Lengths of 'em.

But the Spaces perform'd at *A* and *P* in Equal Times are, as the Accelerating Forces or Gravities in the ſaid Places, or the Effects are, as the Cauſes, which produce them, and the ſaid Accelerating Forces or Gravities have been ſhewn to be Reciprocally, as the Diſtances from the Center, and therefore the Length of the Pendulum at *A*, is to the Length of the Pendulum at *P*, Oſcillating in an Equal Time, as *PC* to *AC*, or Reciprocally, as the Diſtances from the Center; and the like may be Demonſtrated of any other Point, as *B*, and therefore in General, the Lengths of Pendulums, Oſcillating in Equal Times, are Reciprocally, as the Diſtances of the Places from the Center of the Earth; Since therefore the Reaſon of *AC* to any other Diſtance *BC*, and the Inclination of *BC* to *AC*, or the Latitude of the Place, and the Reaſon of *AC* to *PC*, in Particular, are found out and known, the Nature of the Figure *APEp* is Determin'd, to wit, the Section of the Earth with the Plane of the Meridian, which Revolv'd upon the Axis *Pp* will produce the Earth's Figure.

If we have a Mind to Determine the Figure of the Earth by the foregoing Method, it is not neceſſary to have a Meaſur'd Length of a Pendulum, Oſcillating in a Given Time, at the very Pole it ſelf *P*, becauſe, by ſeveral Lengths obſerv'd in Different Latitudes, it may be ſafely concluded, what the Length of the Pendulum will be, which Oſcillates in the ſame Given Time at the Pole; Again, in order to ſolve this Problem, it is not neceſſary in Different Places ſo to adjuſt the Pendulum, that it ſhou'd perform it's Oſcillations in the ſame Time; 'tis enough to take a Pendulum, and adjuſt it ſo to one Place, that it ſhall perform it's Oſcillations in a Time Given, and then Transfer it into other Places, and Mark how much the Clock, to which the ſaid Pendulum is affix't for a Balance, is Retarded or Accelerated in the Time of one Revolution of a fixt Star to the ſame Place, as the Places, into which it is Transfer'd, are nearer to the Æquator or the Pole, than that to which the Automaton or Clock is adjuſted, and from that Retardation or Acceleration to compute what Length of the Pendulum is Requiſite to perform the Oſcillations in the ſame Time, in which they were perform'd in the Place, to which the Pendulum was originally Adapted.



From what is above Demonſtrated, it is Collected, ſuppoſing all Things, as before, and *AGEF* to be a Circle drawn Round the preceding Figure, that *DB* is, as the Increment of the Accelerating Gravity, as we Remove from the Æquator towards either of the Poles, or, as the Increment in the Place *B* above the Length of the Pendulum, which Oſcillates at the Æquator in a Given Time; for *AC* Repreſents the Accelerating Gravity, or the Length of the Pendulum, at the Pole *P*, and *CP* the Accelerating Gravity, or the Length of the Pendulum, Oſcillating in an Equal Time, at the Æquator *A*, and therefore *PF* will Repreſent the Increment of the Accelerating Gravity, or of the Length of the ſaid Pendulum, at the Pole *P*; In like Manner, *CB* Repreſents the Accelerating Gravity, or Length of the Pendulum, at *A*, and $AC = DC$, the Accelerating Gravity, or Length of the Pendulum, at *B*; and conſequently, *DB* will Repreſent the Increment of the Accelerating Gravity, or of the Length of the Pendulum, at *B*, above the Accelerating Gravity, or Length of the Pendulum Oſcillating in the ſame Time at *A*.

And from hence, by a Geometrick Reaſoning, it is Prov'd, that ſuppoſing *APEp*, a Section of the Earth with the Plane of the Meridian, to be an Ellipſis, the Increments of the Accelerating Gravity, in Different Places of the Earth's Surface, above the Accelerating Gravity, or the Length of the Pendulum, at the Æquator, is very nearly in a Duplicate Reaſon of the Right Sines of the Latitudes of thoſe Places, that is, for Inſtance, *PF* is to *BD* in a Duplicate Reaſon of *CF* to *MD*, or *PF*.*BD*::*CFq*.*MDq*, and the ſame may be ſhewn of any other Points or Places, or Latitudes, as well as of *P* and *B*; The Demonſtration is too Tedious here to Inſert, but is eaſily Apprehended by Mathematicians.

It muſt be yet Obſerv'd, that what foregoes is True upon the Suppoſition, and *ex Hypotheſi*, that the Earth conſiſts of an Equally Denſe and Uniform Matter, ſince the Fluid, with which the Canals, *ACP*, *BCP*, Fig. 1. were Repleniſh'd, was preſum'd to be Uniform and Homogeneous; but if the Matter, of which the Earth conſiſts, is Denſer towards the Center, than nearer to it's Surface, the aforeſaid Increments of the Diſtances from the Center, by Retiring from the Poles to the Æquator, and Proportional to theſe Diſtances the Increments of the Accelerating Gravity, and the Length of the Pendulum, Oſcillating in a Given Time, by Retiring from the Æquator to the Poles, will be Greater, than what is Aſſerted in the above-mention'd Computation, and the Axis of the Earth will be Leſs, in Reſpect of the Diameters Perpendicular to it; for, if that Matter towards the Center, by which the Earth is there Denſer, is taken away and conſider'd apart, ſo that the Remaining Matter ſhou'd be Equally Denſe, it is Evident, by what has been alledg'd, that the Accelerating Gravity in Different Places of this Equally-Denſe Earth's Surface is Reciprocally Proportional to the Diſtance from the Center; But the ſaid Accelerating Gravity is farther Increas'd by the Addition of the Redundant Matter about the Center, and very nearly, by what has been before Demonſtrated in this Philoſophy, in a Reciprocal Duplicate Proportion of the Diſtance from it.

Since therefore, where we ſuppoſe the Earth Uniformly Denſe, it's Surface is Elevated towards the Æquator, and Depreſs'd towards the Pole, and ſince the Accelerating Gravity at the Æquator, is Leſs than at the Pole, in Proportion of the Diſtance of the Pole from the Center to the Semidiameter of the Æquator, and moreover, ſince the Accelerating Gravity to the Redundant Matter about the Center is Leſs at the Æquator than at the Pole, in a Duplicate Reaſon of the Diſtance of the Pole from the Center to the Semidiameter of the Æquator, which is Leſs than the Former ſimple Reaſon, being a Reaſon of a leſs Inequality, it follows, that, where we ſuppoſe the Earth Denſer towards the Center, the Accelerating Gravity, compounded of both the Former, is Leſs at the Æquator than at the Pole, in a leſs Reaſon, than that of the Diſtance of the Pole from the Center to the Semidiameter of the Æquator; and conſequently, from this leſs Accelerating Gravity at the Æquator, in Reſpect of that at the Poles, the Earth will be more Elevated at the Æquator, than what the preceding Doctrine of an Earth Equally Denſe wou'd Require, which, by the Calculations of this Philoſophy, and ſuppoſing a Mile conſiſts of 5000 Feet, wou'd be $17\frac{1}{6}$ Miles, but if the Earth is Denſer towards the Center, $31\frac{7}{12}$, and therefore the Earth will be nearly Spherical, but Properly and Truly a Spheroeid; The ſame likewiſe obtains in the Length of a Pendulum, Oſcillating in a Given Time, which has been ſhewn to be Proportional to the Accelerating Gravity; namely, where the Earth is ſuppos'd to be Denſer towards the Center, the Length of the Pendulum Oſcillating in a Given Time, is Leſs at the Æquator than at the Pole, in a leſs Reaſon, than that of the Diſtance of the Pole from the Center to the Semidiameter of the Æquator.

§. 16. And from hence this Philoſophy proceeds to Determine the Proportion, which the Axis of the Sun or any Given Planet has to the Diameters, which are Perpendicular to it.

How the ſame Doctrine is Extended to the Sun and the Planets.

First of all let the Proportion be found of the Centrifugal Force in the Middle Circle, betwixt the Poles of the Sun or the Planet Given, to the Centrifugal Force in the Æquator of the Earth, which is done by this Propoſition, that the Centripetal or Centrifugal Forces are Reciprocally, as the Squares of the Periodick Times divided by the Radius's of the Circles; But the Proportion betwixt the Centrifugal Force in the Æquator of the Earth, to the Gravity on the Earth's Surface, is Given, which is, as the Exceſs of the Earth's Diameter at the Æquator to it's Axis, ſince, by the preceding Propoſition, that Exceſs ariſes from the Centrifugal Force alone, and therefore is Proportional to it; and the Proportion of Gravity on the Earth's Surface to the Gravity on the Sun's Surface, or of the Planet Given, is known by that Propoſition, which aſcertains the Proportion of the Quantity of Matter in the Sun, to the Quantity of Matter in any Primary Planet; that is, if the Matter of Two Globes, Gravitating to each other, is Homogeneous in Places Æquidiſtant from the Centers,

Sir Iſ. Newt. Princip. Prop. 4. Corol. 1. Lib. 1.
Sir Iſ. Newt. Princip. Prop. 8. Lib. 3.
Dr. Gregory's Aſtronom. Prop. 48. Lib. 3.

Centers, the Weight of one Globe to the Other will be Reciprocally, as the Square of the Diſtance betwixt the Centers, ſuppoſing them to be Remov'd at Various Diſtances from each other; and conſequently, the Proportion, which is Compounded of theſe Given ones, will be likewiſe Given, to wit, the Proportion, which the Centrifugal Force in the Middle Circle betwixt the Poles of the Sun or Planet Given, has to the Gravity on it's Surface, and is the ſame, as the Exceſs of the Diameter of the Æquator of the Sun or Planet above it's Axis, to the ſaid Axis, and therefore the Proportion betwixt the Sun's or Planet's Axis, and it's Diameter Perpendicular to that Axis, will be Determin'd.

Sir Iſ. Newt. Princip. Prop. 18. Lib. 3.

AND in General it is form'd into a Propoſition, that the Axes of the Planets are Leſs than the Diameters, which are Perpendicular to them; Becauſe, tho' the Planets ought to be of a Spherical Figure, Abſtracting from their Circular Motion Round their Axes, upon the Account of the Equal Gravitation of the Parts on all Hands to the Center, yet by that Circular Motion it comes to Paſs, that the Parts Retiring from the Axes, endeavour to Aſcend towards the Æquators, and therefore the Matter, if it is Fluid, by ſuch Aſcent will Increaſe the Diameters, at the Æquators, and by it's Deſcent Diminiſh them at the Poles; Thus the Diameter of *Jupiter*, by the Agreement of the Obſervations of Aſtronomers, is found to be Shorter betwixt the Poles, than from Eaſt to Weſt; and by the ſame Argument, unleſs our Earth was Higher at the Æquator than under the Poles, the Seas wou'd ſubſide at the Poles and Aſcend towards the Æquator, and there Cauſe an Univerſal Inundation, namely, by the Earth's Revolution, and by the Parts of it thereupon Retiring from it's Axis towards the Æquator.

The Table Conſtructed upon the foregoing Doctrine. Sir Iſ. Newt. Princip. Prop. 20. Lib. 3. Edit. 2.

§. 17. FURTHERMORE, in Reſpect of Pendulums, Oſcillating in a Second of Time, a Table is Calculated of the ſeveral Lengths of them in Various and Different Latitudes, from no Degrees Latitude under the Æquator to 90 under the Poles, upon the foregoing Hypotheſis, that the Earth is uniformly Denſe, and from the Reaſonings upon that Hypotheſis, which have been before Explain'd, and particularly from this Theorem, that the Increment of Gravity, by Proceeding from the Æquator to the Poles, is as the Square of the Right Sine of the Latitude very nearly, as alſo, that the Lengths of Synchronal Pendulums, that is, which Oſcillate, in Equal Times, in Different Places or Latitudes, are as the Accelerating Gravities in thoſe Places or Latitudes; From whence the Following Table is Computed, in which a Line is the *12th* Part of an Inch.

Latitude of the Place.	Length of the Pendulum.		Latitude of the Place.	Length of the Pendulum.	
Degrees.	Feet .	Lines.	Degrees.	Feet .	Lines.
0	3	. 7,468	46	3	. 8,461
5	3	. 7,482	47	3	. 8,494
10	3	. 7,526	48	3	. 8,528
15	3	. 7,596	49	3	. 8,561
20	3	. 7,692	50	3	. 8,594
25	3	. 7,811	55	3	. 8,756
30	3	. 7,948	60	3	. 8,907
35	3	. 8,099	65	3	. 9,044
40	3	. 8,261	70	3	. 9,162
41	3	. 8,294	75	3	. 9,258
42	3	. 8,327	80	3	. 9,329
43	3	. 8,361	85	3	. 9,372
44	3	. 8,394	90	3	. 9,387
45	3	. 8,428			

FROM this Table it is ſaid to be manifeſt, that in Geographical Matters the Earth may be Eſteem'd as Spherical, and that the Inequality of it's Diameters, which is but ſmall, may be more Eaſily and Certainly found out, and Determin'd by the Experiments of Pendulums than by the Eclipſes of the Moon or any other Way. It is likewiſe Conſtructed upon the Hypotheſis, that the Earth is uniformly Denſe, but if it's Matter is Denſer nearer the Center than at the Surface, for the Reaſons alledg'd above the Differences of the Pendulums will be ſomething Greater, than what are Expreſs'd in the foregoing Table; becauſe, tho' Gravity is Reciprocally, as the Diſtance of the Weight from the Center, where the Earth is uniformly Denſe, it will yet be compounded of another Proportion, which will be Reciprocally, as the Square of the Diſtance from the Redundant Matter, and therefore Gravity under the Æquator is leſs to that Redundant Matter, than in the preceding Computation, and the Earth will riſe Higher, and it's Surface be more Elevated, by this Defect of Gravity, and the Exceſs of the Lengths of the Pendulums at the Poles will be Greater, than what is before Defin'd.

The Experiments alledg'd to Prove the Truth of the Preceding Table.

§. 18. Thus far is Reaſon'd and Hypotheſes are Invented, and to Confirm this from Experience, it is told us, that certain Aſtronomers, who have been ſent into Foreign Parts, on purpoſe to make Aſtronomical Obſervations, have found, that the Pendulums of Clocks move Slower, nearer the Æquator, than the Poles, and that the firſt Perſon, who took Notice of this, was Mr. *Richer* in the Year 1672, and in the Iſland *Cayenne*, for, whilſt he obſerv'd the Paſſage of the fixt Stars by the Meridian, in the Month of *Auguſt*, he found the Pendulum of his Clock to move Slower, than according to the Middle Motion of the Sun, with a Difference of $2'.28''$ every Day; he afterwards made a Single Pendulum, which ſhou'd Oſcillate in a Second of Time, by the Beſt Pendulum Clock that cou'd be had, and mark't the Length of this Single Pendulum, and this he did frequently every Week for Ten Months together; Then Returning to *France*, he compar'd the Length of this Pendulum with the Length of an Iſochronal Pendulum at *Paris*, which was Three *Paris* Feet, and Eight Lines, with Three Fifth Parts of a Line, and found it Shorter with the Difference of one Line, and one Fourth of a Line; But by the Slowneſs of the Pendulum-Clock in *Cayenne*, the Difference of the Pendulums is Inſer'd by Computation to be one Line and an Half.

Dr. *Halley* in the Year 1677, at the Iſland of St. *Helen's*, found his Pendulum-Clock to move Slower than at *London*, but did not obſerve the Difference, he yet made his Pendulum Shorter by more than an Eighth Part of an Inch, or one Line and an Half.

In the Year 1682, Mr. *Varin* and Mr. *Des Hayes* found the Length of the Pendulum, Oſcillating in a Second of Time, to be, in the Royal Obſervatory at *Paris*, 3 Feet and $8\frac{5}{9}$ Lines, and in the Iſland of *Goreè*, by the ſame Method, they found the Length of a Synchronal Pendulum to be 3 Feet and $6\frac{5}{9}$ Lines, and the Difference of the Lengths therefore 2 Lines; And in the ſame Year, at the Iſlands *Guadaloupe* and *Martinico*, they found the Length of a Synchronal Pendulum to be 3 Feet and $6\frac{1}{2}$ Lines.

Afterwards in the Year 1697, in the Month of *July*, Mr. *Couplet* the Son Adjuſted his Pendulum-Clock to the Middle Motion of the Sun in the Royal Obſervatory at *Paris*, in that Manner, that the Clock Exactly Agreed with the Sun's Motion for a long Space of Time: Then Sailing to *Lisbon*, he found, in *November* following, that his Clock went Slower than before, the Difference being $2'.13''$ in 24 Hours, and, in the Month of *March* after, Sailing to *Paraiba*, he there found his Clock went Slower, than at *Paris*, $4'.12''$, in 24 Hours; and he Affirms, that the Pendulum, Oſcillating in Seconds of Time, was at *Lisbon* $2\frac{1}{2}$, and at *Paraiba* $3\frac{2}{3}$ Lines Shorter than at *Paris*, tho' it is ſaid, he ſhou'd Rather have made the Differences of the Lengths of the Pendulum $1\frac{1}{3}$ and $2\frac{5}{9}$ Lines, as better Correſponding with the Differences of the Times of Oſcillation, $2'.13''$ and $4'.12''$, But, it is told us, theſe Obſervations are too Groſs to be Rely'd on.

In the Years 1699 and 1700, Mr. *Des Hayes*, Sailing again to *America*, Determin'd the Length of the Pendulum, Oſcillating in Seconds of Time, in the Iſlands of *Cayenne* and *Granada* to be ſomething Leſs than 3 Feet and $6\frac{1}{2}$ Lines, in the Iſland of St. *Chriſtopher's* to be 3 Feet and $6\frac{3}{4}$ Lines, and in the Iſland of St. *Dominick* 3 Feet and 7 Lines.

In the Year 1704, *Feuellè* found the Length of the Pendulum, Oſcillating in Seconds of Time, in *Portobel* in *America*, to be 3 *Paris* Feet and only $5\frac{7}{12}$ Lines, that is, 3 Lines Shorter than at *Paris*, but there was an Error in the Obſervation, for afterwards Sailing to the Iſland *Martinico*, he found the Length of an Iſochronal Pendulum to be only 3 *Paris* Feet and $5\frac{10}{12}$ Lines.

But the Latitudes of the Places above-mention'd are as follow, of *Paraiba* 6°.38′ Southern, of *Portobel* 9°.33′, and of the Iſlands *Cayenne*, *Goreè*, *Guadaloupe*, *Martinico*, *Granada*, St. *Chriſtopher*'s, St. *Dominick*'s, Reſpectively 4°.55′, 14°.40′, 14°.0′, 14°.44′, 12°.6′, 17°.19′, and 19°.48′ Northern, and we may add *Lisbon* about 40°.

The Exceſſes of the Length of the Pendulum at *Paris* above the Lengths of Iſochronal Pendulums, obſerv'd in theſe ſeveral Latitudes, are ſomething Greater than according to the foregoing Table, and therefore the Earth is ſomewhat Higher under the Æquator, than that Calculation makes it, which proceeds upon the Earth's being uniformly Denſe; and conſequently, the Earth is Denſer towards the Center, than nearer to the Surface, unleſs the Heats in the Torrid Zone may give ſome Addition to the Length of the Pendulums.

For Mr. *Picart* obſerv'd, that the Iron-Rod of the Pendulum, which, in Winter-Time and Froſty Weather, was in Length one Foot, being Warm'd and Heated at the Fire, became one Foot and one fourth of a Line.

And Mr. *De la Hire* afterwards obſerv'd, that an Iron-Rod, which, in Winter-Time and Froſty Weather, was ſix Feet in Length, when Expos'd to the Sun's Heat, was ſix Feet and two Thirds of a Line: In the Former Caſe, the Heat was Greater than in the Latter, and in the Latter, Greater than the Heat of the External Parts of an Human Body; For Metals grow exceeding Warm by the Sun's Heat in Summer; But the Pendulum of a Clock is never wont to be Expos'd to the Heat of the Summer's Sun, and therefore never admits of an Heat, Equal to that of the External Parts of an Human Body, and conſequently, the Rod of a Pendulum, in a Clock, three Foot Long, will be ſomething Longer in Summer than in Winter, tho' the Exceſs will ſcarcely be more than one fourth Part of a Line; from whence it is ſuppos'd to Follow, that the whole Difference of the Length of Iſochronal Pendulums, in Various Latitudes, cannot be Aſcrib'd to a Difference of Heat; Nor can this Difference be Attributed to the Errors of theſe Aſtronomers in their Obſervations; for albeit, they do not perfectly Agree, yet the Errors are ſo Minute, that they may be juſtly Neglected and Slighted; and in this they Entirely Conſent, that Iſochronal Pendulums are Shorter under the Æquator than at the Royal Obſervatory in *Paris*, the Difference being about Two Lines or the ſixth Part of an Inch; By the Obſervations of Mr. *Richer* in *Cayenne*, the Difference was one Line and a Half, the Error of Half a Line is eaſily Committed; Mr. *Des Hayes*, by his Obſervations in the ſame Iſland, Corrected that Error and found the Difference to be $2\frac{1}{18}$ Lines; Moreover by the Obſervations made in the Iſlands of *Goreè*, *Guadaloupe*, *Martinico*, *Granada*, St. *Chriſtopher*'s, and St. *Dominick*, and Reduc'd to the Æquator, the Difference was not Leſs than $1\frac{19}{20}$ Line, nor Greater than $2\frac{1}{2}$ Lines, and the Middle Quantity betwixt theſe Two Limits is $2\frac{9}{40}$ Lines, Take away the $\frac{9}{40}$ of a Line on Account of the Heats in the Torrid Zone, and there will Remain the Difference of 2 Lines.

Wherefore, ſince that Difference by the Preceding Table, which is Calculated upon the Suppoſition of the Earth's being uniformly Denſe, is only $1\frac{87}{1000}$ Line, the Exceſs of the Earth's Height at the Æquator above that at the Poles, which was $17\frac{1}{6}$ Miles, being now Increas'd by an Increaſe of the Differences of the Lengths of the Pendulums, will be $31\frac{7}{12}$ Miles; for the Slowneſs of the Pendulum under the Æquator Evinces and Proves a Defect of Gravity, and by how much Lighter the Matter of the Earth is, by ſo much Greater ought to be it's Height, that it may ſupport by it's Weight the Matter under the Poles in an Æquilibrium; And from hence the Figure of the Earth's Shadow, Determin'd by the Eclipſes of the Moon, will not be exactly Circular, but it's Diameter from Eaſt to Weſt will be Greater than that from North to South, by an Exceſs of about 55″, and the Greateſt Parallax of the Moon in Longitude will be ſomething Greater than it's Greateſt Parallax in Latitude, and the Greateſt Semidiameter of the Earth will be 19767630 *Paris* Feet, the Leaſt 19609820, and the Middle 19688725 Nearly.

We ſhall here ſubjoin a Table of the Differences of the Lengths of Iſochronal Pendulums, according to what they were found by the foregoing Obſervations, and then Compare them with the Preceding Table, which was form'd from Calculation, after we have Inquir'd into the Hypotheſes and Principles, upon which it was Made and Contriv'd.

	Northern Latitude.		Length of the Pendulum in Feet and Lines.
	Cayenne	4°·55′	*Richer.* *Newton.* *Des Hayes.* $3 \cdot 7\frac{7}{20}$. $3 \cdot 7\frac{1}{10}$. $3 \cdot 6\frac{1}{2}$. ſomething Leſs. 3·7,350. 3·7,100. 3·6,500.
South. Latitude.	*Paraiba*	6·38	*Couplet* Groſs Obſerver. *Newton.* $3\frac{2}{3}$ Lines Leſs than at *Paris*, $2\frac{5}{9}$ Lines Leſs.
	Portobel	9·33	*Feuellè.* $3 \cdot 5\frac{7}{12}$. 3·5,583.
	Granada	12·6	*Des Hayes.* $3 \cdot 6\frac{1}{2}$. ſomething Leſs. 3·6,500.
	Guadaloupe	14·0	*Varin, Des Hayes.* $3 \cdot 6\frac{1}{2}$. 3·6,500.
	Goreè	14·40	*Varin, Des Hayes.* $3 \cdot 6\frac{5}{9}$. 3·6,555.
	Martinico	14·44	*Varin, Des Hayes.* *Feuellè.* $3 \cdot 6\frac{1}{2}$. $3 \cdot 5\frac{10}{12}$. 3·6,500. 3·5,833.
South. Latitude.	St. *Helen's*	15·0	*Halley.* $1\frac{1}{2}$ Line Leſs than at *London.*
	St. *Chriſtopher's*	17·19	*Des Hayes.* $3 \cdot 6\frac{3}{4}$. 3·6,750.
	St. *Dominick*	19·48	*Des Hayes.* 3·7.
	Lisbon	40·0	*Couplet* Groſs Obſerver. *Newton.* $2\frac{1}{2}$ Lines Leſs than at *Paris*, $1\frac{1}{3}$ Line.
	Paris	48·45	*Richer.* *Varin, Des Hayes.* *Couplet.* $3 \cdot 8\frac{3}{5}$. $3 \cdot 8\frac{5}{9}$. $2\frac{1}{2}$ Lines More than at *Lisbon*, 3·8,600. 3·8,555. $3\frac{2}{3}$ More than at *Paraiba.*
	London	51·30	*Halley.* $1\frac{1}{2}$ Line More than at St. *Helen's.*

§. 19. In Anſwer to all this Doctrine of Meaſuring the Diameters of the Earth by the Lengths of the Pendulum, and thereby Determining the Figure of the Earth; We muſt First ſay, that the Principles are Wrong and Falſe upon which this Philoſophy Proceeds, and Then the Concluſions from the Experiments brought to Confirm them; And First, the Demonſtration, that the Lengths of Pendulums Oſcillating in Equal Times are Reciprocally, as the Diſtances from the Earth's Center, is by no Means rationally Evinc'd; Becauſe First, it is ſuppos'd, that the Earth Conſiſts of an Homogeneous Matter, which is not Prov'd; And Second, that whatever is ſhewn of an Homogeneous Fluid, Equally obtains in Homogeneous Solids, in the Preſent Caſe, which is likewiſe far from being Evident; And Third, that the Preſent Doctrine of Hydroſtaticks is Unqueſtionable, which again is not Manifeſt.

The Demonſtration it ſelf of the Lengths of Pendulums, Oſcillating in Equal Times, being Reciprocally as the Diſtances from the Earth's Center not prov'd upon the Suppoſition, that the Earth is Equally-Denſe.

As to the First, if the Matter of Mold, Clay, Stone, Metals, and a Thouſand Bodies more, of which the Earth Conſiſts, is Various in it's Proper and Internal

Forces, and cannot be Conſider'd as a Similar and Uniform Subſtance, all this Reaſoning falls to the Ground, and till that Point is Clear'd, remains Diſputable and Doubtful; Since, if Matter is Different in it's Forces, the Weights of Bodies will not be, as the Bulks of them, nor the Gravities of Bodies, which are Equal and Situate in *A* and *P*, as *PC* to *AC*, that is Reciprocally, as the Diſtances from the Center, upon which Maxim yet this Whole Demonſtration Depends. See §. 15. Fig. 1.

As to the Second, That Fluids and Solids come under one Denomination in the preſent Caſe, becauſe they Gravitate a-like to the Center of the Earth, and therefore, that a Canal of Solid Matter Extended to the Center, or one half Fluid and the other half Solid, is the ſame, as a Canal of Fluid Matter Extended to the Center, is to aſſume a great Deal without any Proof or Eviction of the Thing; becauſe, tho' all Matter was Homogeneous and had an Equal Force of Gravitation in it ſelf, which is not True, yet it muſt be Confeſs'd, that there is a wide Difference betwixt Matter's being left at Liberty to Exert that Force, as in Fluids, and being Reſtrain'd from doing it by a firm Adheſion of Parts, as in Solids; For whoever Imagin'd before, that a Column of Solid Matter might be Balanc'd by a Column of Fluid, in the Way, and according to the Laws of Hydroſtaticks? When it is Plain, that a Column of Solid Matter, drawn from the Surface of the Earth to the Center, beſides it's Force of Gravitation, has alſo a Force of Coheſion, which can be Reduc'd to no Rules of a Balance, ariſing from mere Gravitation, which is the Caſe of Fluids, and which, it is ſuppos'd, may be ſo Reduc'd.

As to the Third, That the Common Hydroſtaticks are Preſum'd to be Demonſtrated, we muſt ſay, that neither is That in fact True; becauſe the Laws and Rules of Hydroſtaticks are Prov'd upon the Suppoſition, that Nothing is to be Conſider'd in Fluids, excepting their Specifick Gravities, whereas it is Certain, that there are ſome Fluids, which are Specifically Heavier than others, and which yet, by the Spirituouſneſs or the Expanſive Force of them, will Riſe in thoſe, which are Specifically Lighter; Beſides which, we muſt affirm, that, in all Fluids, their Proper Expanſive and Contractive Forces are to be Conſider'd, and not their Specifick Gravities only.

Moreover Fourthly, Granting every Thing, which this Philoſophy Requires or Demands, it is ſomething Extraordinary, that it ſhou'd here Endeavour to Demonſtrate, that the Accelerating Gravities are Reciprocally, as the Diſtances from the Earth's Center, whereas Accelerating Gravity in General, and every other Action, is ſhewn to be Reciprocally, as the Squares of thoſe Diſtances, by this Philoſophy, and is a Truth afterwards aſſum'd, as Prov'd, in this very Propoſition, when it comes to Treat of the Redundant Matter, ſuppoſing the Earth not to be uniformly Denſe; upon all which Accounts it is not Prov'd, that the Accelerating Gravity in *A* is to that in *P*, as *PC* to *AC*. See §. 15. Fig. 1.

See §. 15. Fig. 1. Again, If now we take Two Bodies and place them in *A*, it is ſaid, *P . A* : : *PC . AC*, that is, the Gravities will be as the Bulks; which, as before, ſuppoſes, that in Matter nothing is to be Conſider'd, excepting an uniform Gravity, and the Number of Parts in each Body, which are Accelerated by it, and conſequently, that Matter has no peculiar Expanſive or Contractive Forces, which is once more Deny'd, and till it is Prov'd, it has not, This Demonſtration is, for that Reaſon, as we have already Declar'd, Imperfect and Defective.

See §. 15. Fig. 1. The next Part of this Demonſtration concerns the Nature of Pendulums, which we have before particularly Examin'd, and therefore the Objections, which lie againſt the Explanation of that Doctrine, are Equally Valid againſt the Truth of the Preſent Propoſition; Namely, that it is nothing more than an Abſtracted Reaſoning, is not juſtify'd by Fact and Experiment, and that, what is ſo, Admits of a Different Solution, from what is Given by theſe Principles; what we ſhall farther obſerve here is this, that the Lengths of the Pendulums are affirm'd to be, as the Accelerating Forces or Gravities, which are made to be Reciprocally, as the Diſtances from the Earth's Center; we have ſhewn, there is a Difference betwixt Bodies Deſcending by Gravity, and Oſcillating in a Pendulum, and conſequently, this alſo is a Notion not Prov'd, but Aſſum'd; As to what Follows, it ſo entirely depends upon what foregoes, that there is no Occaſion to take any particular Notice of it, Namely, that the Increments of the Accelerating Gravity, in Different Places of the Earth's Surface, above the Accelerating Gravity, or the Length of the Pendulum at the Æquator, are very nearly in a Duplicate Reaſon of the Right Sines of the Latitudes of thoſe Places, becauſe it ſhows more of the Skill of the Geometer, than the Sagacity or the Truth of the Philoſopher; I ſhall only add, that, as any one wou'd be Aſtoniſh'd to Think, how it was poſſible to contrive a Way, to find out the Diameters of the Earth, by the Oſcillations of a Pendulum, ſo he may leave off that Admiration, when he ſees the Strange and Unaccountable Methods taken to do it; he will rather Wonder, that Men ſhou'd enter into ſuch Conceits, than be ſurpriz'd at the Greatneſs of the Performance; as will yet farther Appear in the Progreſs of this Examination.

See §. 15. Fig. 2.

That the Earth is Denſer towards the Center not Prov'd, nor the Reaſonings upon it Concluſive.

See § 15. Fig. 2.

§. 20. Thus far is ſaid to be Demonſtrated, ſuppoſing the Earth is uniformly Denſe, but if it be not ſo, and it is Denſer near the Center, and Rarer at the Surface, as is Generally Conceiv'd, the foregoing Proportion muſt be Alter'd, and the Accelerating Gravities, and the Lengths of the Pendulums, Analogous to Them, at the Æquator, and Poles, will be in a leſs Reaſon, than that of the Diſtance of the Pole from the Center to the Semidiameter of the Æquator, and this leſs Reaſon is Thought more Accurately to agree with the Experiments upon Pendulums than the Former and Greater: Let us then now ſee how the Caſe ſtands; Canals of Homogeneous Matter were Invented, in which Fluids and Solids ſhou'd be all one; Gravity was affirm'd to be Reciprocally, as it's Diſtance from the Center; Pendulums were made to Oſcillate according to the Will and Pleaſure of the Geometer, without any Proof that they did ſo; and the Earth was Imagin'd to be Equally Denſe, in order to Reduce the Diameters of it to ſome Proportion with the Lengths of the other; but all this not Availing, the Earth is now ſuppos'd not to be Equally Denſe, and to Compenſate for that Inequality, a Certain Maſs of Redundant Matter is Thruſt and Ramm'd to the Center, the Gravitation to which is pronounc'd to be Reciprocally, as the Squares of the Diſtances, whereas before Gravity was Reciprocally as the Diſtances: On the other Hand, that the Earth is Denſer at the Center than Nearer the Surface, whatever we may Surmiſe, is as much Unprov'd, as that it is Equally Denſe, or the Matter of it Homogeneous; ſome have affirm'd a Central Fire, and produc'd Arguments for it, and for what this Philoſophy Alledges, which is Nothing but it's own Authority, it may ſtill Remain a Queſtion; Allowing yet this Earth was Originally a Fluid, by which it might be Imagin'd, that Bodies ſubſided to the Center, according to their Specifick Gravities, and that the Earth therefore became Gradually Denſer from the Surface to the Center, Nothing ſtill of this is Certain or Demonſtrable; we find Bodies Promiſcuouſly plac'd, the Heavier above the Lighter, as well as the Lighter above the Heavier; and it is not Plain, that they may not be ſo Mixt and Confounded together, that there may be more Lighter Bodies, towards the Center, than Heavier, by mutual Coalitions and Coagmentations with one another, from whence their Natural Specifick Gravities might be Impeded in their Action; It is not farther Certain, taking all of Them together, the Gravity of the one with the Levity of the other, that they may not Conſtitute a Force of Gravitation, which may be pretty much Uniformly the ſame thro' the Whole; Likewiſe it is not prov'd, ſuppoſing a Gradual Increaſe of Denſity towards the Center, that a Maſs of Matter heap't about it, will have the ſame Effects as ſuch a Gradual Increment; and laſtly, there is Nothing allow'd to the Peculiar Forces of Matter in it's Expanſion and Contraction, which wou'd entirely Diſturb and Annull every Part of this Calculation, and yet, amidſt this vaſt Uncertainty of Things, a Matter uniformly Denſe or Increaſing in it's Denſity, in it's Approach to the Center, is Aſſum'd, as an Evident and Irreſiſtible Truth, and is accordingly made uſe of in this Demonſtration.

The Figure of the Sun and Planets not from hence Determin'd, nor yet their Denſities.

§. 21. Having from theſe Principles, however Precarious, Determin'd the Diameters of the Earth, and conſequently the Figure of it, This Doctrine goes on to Define the Figures and Diameters of the Sun or of any Planet Given.

See §. 16.

First, By Finding the Proportion of the Centrifugal Force in the Middle Circle, betwixt the Poles of the Sun or the Planet Given, to the Centrifugal Force in the Æquator of the Earth, from this Propoſition, that the Centripetal or Centrifugal Forces are Reciprocally, as the Squares of the Periodick Times divided by the Radius's of the Circles, which Propoſition we have afterwards Examin'd and Conſider'd, and ſhewn, it can be no Foundation, to build any certain Reaſonings in Nature upon.

SECOND, By Affirming, that the Proportion betwixt the Centrifugal Force, in the Æquator of the Earth, to the Gravity on the Earth's Surface, is Given, and that it is, as the Exceſs of the Earth's Diameter at the Æquator to it's Axis, ſince, by the preceding Demonſtration, that Exceſs is Produc'd from the Centrifugal Force alone, and therefore is Proportional to it; But ſeeing that Demonſtration is not Juſt, as we have now ſhewn, the Inference will be Equally Invalid.

THIRD, By Aſſerting, that the Proportion of Gravity on the Earth's Surface, to the Gravity on the Sun's Surface, or of the Planet Given, is known, by a Propoſition, which Aſcertains the Proportion of the Quantity of Matter in the Sun, to the Quantity of Matter in any Primary Planet, which is, if the Matter of Two Globes, Gravitating to each other, is Homogeneous in Places Æquidiſtant from the Centers, the Weight of one Globe to the Other will be Reciprocally, as the Square of the Diſtance betwixt the Centers, ſuppoſing them to be Remov'd at Various Diſtances from each other; which therefore depends upon this Preſumption, that all the Matter of the Earth, Sun, and Planets is Homogeneous and Similar, a Notion, which we have abundantly Confuted, in our Inquiry into the Principles of the Corpuſcular Syſtem, and in our Aſſertion of the Expanſive and Contractive Forces of Matter; and, for the ſame Reaſon, we cannot but Reject all the Subtle and Delicate Calculations of the ſeveral Denſities of the Sun and Planets, which are form'd upon that Hypotheſis, without any Demonſtrations, in Fact and Experience, of any of theſe Matters.

See Principles of Philoſ. and Chap. 5. of this Book.

That it is not Prov'd to be Matter of Fact and Experiment, that the Axes of the Sun, and of the Earth and Planets are Shorter than the Diameters Perpendicular to them.

§. 22. FROM hence, notwithſtanding, it is Concluded, that the Axes of the Sun, the Earth, and the Planets are Leſs than the Diameters, which are Perpendicular to them, and conſequently, that their Figures are not Spherical, but Spheroeidical; it is farther ſaid, that this Proceeds from the Revolution of their Bodies about their Axes, by which the Matter of them is made to Recede from the Poles and Aſcend to the Æquator; that in Particular, the Earth is Higher in the Latter Place than in the Former, by $31\frac{7}{12}$ Miles, and that, if it were not Higher, there wou'd be an Inundation of all the Parts and Regions under the Æquator, by an Aſcent of the Water upon them, in it's Retiring from the Poles; that the Slowneſs of the Pendulum under the Æquator Evinces this, for that proves a Defect of Gravity, and by how much Lighter the Matter of the Earth is, by ſo much Greater ought to be it's Height, that it may ſupport, by it's Weight, the Matter under the Poles, in an Æquilibrium; that the Figure therefore of the Earth's Shadow, Determin'd by the Eclipſes of the Moon, will not be exactly Circular, but it's Diameter from Eaſt to Weſt will be Greater than that from North to South, by an Exceſs of about 55″, and the Greateſt Parallax of the Moon in Longitude will be ſomething Greater than it's Greateſt Parallax in Latitude, and the Greateſt Semidiameter of the Earth will be 19767630 *Paris* Feet, the Leaſt 19609820, and the Middle 19688725 nearly; and to juſtify this Doctrine, it is farther alledg'd, that all Aſtronomers agree in their Obſervations, that the Diameter of *Jupiter* is Shorter betwixt the Poles than from Eaſt to Weſt.

See §. 16.

BUT FIRST, That the Axes of the Sun, the Earth, and the Planets are not prov'd to be Shorter than the Diameters Perpendicular to them, by the Preceding Demonſtrations, we have already ſhewn.

SECOND, To Evidence this to be Matter of Fact, there are only Two Inſtances pretended to, that of the *Earth* and of *Jupiter*; Granting which, is it neceſſary that *Mercury*, *Venus*, *Mars*, and *Saturn* ſhou'd obtain the ſame Figure, and that the *Sun* in Particular ſhou'd? If it is ſaid they all Conſiſt of the ſame Homogeneous and Similar Matter, and are exactly in the ſame Circumſtances, it is Deny'd, and wants to be Prov'd; We affirm on the Contrary, that the *Sun* conſiſts of an Expanſive Force, with ſome Contractive; and the Others of a Various Contractive, Mixt and Blended with a Various and Different Expanſive, as the Planets are more or leſs Remov'd from the *Sun*.

THIRD, But how is it Prov'd, that *Jupiter*'s Diameter is Shorter betwixt the Poles than from Eaſt to Weſt? It is ſaid by the Agreement in the Obſervations of Aſtronomers; And cou'd not the ſame Aſtronomers obſerve the ſame in the *Moon*, in *Mars*, in *Mercury*, in *Venus*, or in *Saturn*, or the *Sun*? If not, why ſhou'd the Obſervations upon *Jupiter* be a Standard for the Reſt? If they cou'd or did, why are not thoſe Obſervations here Expreſs'd? It is too Evident, that in this, as in all other Parts of the preſent Celebrated Syſtem, Nature is not Endeavour'd to be Explain'd by it, but ſubmitted and ſubjected to it; beſides which, we have given Reaſons, why the Obſervations of Aſtronomers are not too much to be Rely'd on, upon Account of the Fewneſs of Obſervers, and the Uncertainty and Difficulty of Obſerving.

The Earth in Particular not Prov'd to be Higher at the Æquator than at the Poles, from Calculation, or from it's Revolution.

FOURTH, How farther is it yet Prov'd, that the Earth's Diameter is Shorter betwixt the Poles than from Eaſt to Weſt, or in the Æquator? Namely, by a Calculation, which makes the Earth Higher in the one Caſe than the other, by 31$\frac{7}{12}$ Miles; but ſince that Calculation depends upon a Precarious Propoſition, which is not allow'd, it can be of no Service to our Conviction: Nor is that Argument more Valid, which is drawn from the Revolution of the Earth round it's own Axis, by which it's Parts are ſaid to recede from the Poles to the Æquator; becauſe, this will be Impoſſible, excepting the Earth was once a Fluid, and the ſame may be ſaid of the other Planets, but whatever our Faith, in the *Moſaick* Hiſtories, may Induce us to believe of the Earth, and may Juſtly require of us, it is not Philoſophically Evinc'd, which, in a Matter of mere Reaſoning, is always Expected; Granting yet, that the Earth and the Planets were originally in a Fluid State, it is not Demonſtrated, that they were at that Time Circulated about their Axes; And laſtly, ſuppoſing they were, a Spherical Fluid, Revolv'd upon an Axis, wou'd not produce a Spheroeid, ſince the Centrifugal Force do's not, in ſuch a Caſe, exert it ſelf in any Manner from the Poles towards the Æquator, but in Tangents to the ſeveral Revolving Circles of the Globe; and conſequently, if the Parts of the Fluid were not by their Gravity Retain'd in thoſe Circles, they wou'd not fly up towards the Æquator, but fly off in Tangents, or in ſome Lines or other Parallel to it, ſo little Reaſon is there to Amuſe us with Imaginary Inundations at the Æquator, if the Earth was not Higher there than at the Poles, Eſpecially, as the Earth is now Conſtituted, unleſs we cou'd ſuppoſe Mountains of Ice, Rolling from thence, ſhou'd overflow the Regions betwixt the Tropicks; Beſides, ſince the Æquator is the Largeſt Circle, and it's Parallels are continually Diminiſh'd, as they Approach to the Poles, and ſince their Circuits are perform'd at once, the Parts under the Æquator will be carry'd Round with a Greater Velocity, and by conſequence will obtain a Greater Centrifugal Force than the Parts, which are nearer the Poles, ſo that it do's not appear at all Reaſonable to Think, that a Leſs Centrifugal Force, nearer the Poles, ſhou'd Prevail over and Surmount a Greater, nearer the Æquator, which yet theſe Fictitious Inundations muſt Suppoſe and Preſume.

Nor from the Slowneſs of the Pendulum at the Æquator.

THE next Endeavour to Prove, that the Earth is Higher under the Æquator than under the Poles, is from the Slowneſs of the Pendulum in the Former Place, which is ſaid to Argue a Defect of Gravity, and by how much Lighter the Matter of the Earth is, by ſo much Greater ought to be it's Height, that it may ſupport, by it's Weight, the Matter under the Poles, in an Æquilibrium.

BUT, if the Slowneſs of the Pendulum ariſes from the Heat under the Æquator, as we have before ſhewn, and ſhall farther prove, this Argument is of no Force; Moreover, tho' Matter or the Pendulum and Pendulous Body, Heated by the Expanſive Force, Impreſs'd at the Æquator, may Weigh leſs than at the Poles, and therefore Deſcend Slower, that Defect of Gravity will not be owing to the one being farther Remov'd from the Center of the Earth than the other, till the foregoing Propoſition of the Lengths of the Pendulums and the Accelerating Gravities, being Reciprocally Proportional to the Diſtances from the Earth's Center, is Prov'd, but we have Evidenc'd, that there is no Demonſtration of ſuch a Propoſition; Nor yet can it be juſtly affirm'd, if the Matter is Lighter at the Æquator than at the Poles, ſuppoſing by the Expanſive Force of Heat, or any other Cauſe, (except that of Reciprocal Diſtances, which is taking for Granted, what is to be Prov'd,) that therefore by how much Lighter the Matter of the Earth is, by ſo much Greater ought to be it's Height, that it may ſupport, by it's Weight, the Matter under the Poles, in an Æquilibrium; unleſs we will allow, that the Solid Matter, under the Æquator, and the Poles, is to be conſider'd as a Fluid, and that it is Reducible to the Common Laws of Hydroſtaticks, which we have already ſufficiently Confuted; for, if Heat and Cold have a Force to produce theſe Effects of making the Pendulous Matter Lighter at the Æquator and Heavier at the Poles, there will be no Reaſon, I hope, from thence to make the Column of Solid Matter from the Surface to the

Center Longer at the Æquator than at the Poles, becauſe there is no kind of Connexion in ſuch a Reaſoning.

AND yet upon theſe Unprov'd and Uncertain Premiſes it is Concluded, that therefore the Figure of the Earth's Shadow, Determin'd by the Eclipſes of the Moon, will not be exactly Circular, but it's Diameter from Eaſt to Weſt will be Greater than that from North to South, by an Exceſs of about $55''$; Whereas this ſo Minute Obſervation is not True in Fact, for, according to Aſtronomers, that Shadow cannot be Determin'd within $12''$ at leaſt; that the Greateſt Parallax of the Moon in Longitude will be ſomething Greater than it's Greateſt Parallax in Latitude, which alſo is ſomething Minute; and that the Greateſt Semidiameter of the Earth will be 19767630 *Paris* Feet, and the Leaſt 19609320; all which is affirm'd, upon it's being ſuppos'd to be Prov'd, that the Earth is $31\frac{7}{12}$ Miles Higher at the Æquator than at the Poles, which is Determining Things again to a wonderful Minuteneſs; ſo happy is this Philoſophy in Explaining Minutiæ, which never were in Nature, as it is very Singular in Neglecting others, as the Expanſive Forces, which certainly are.

Dr. Keill's Aſtronom. Lect. 22. Pag. 321. Edit. 1.

How the Earth is prov'd to be a Spheroeid, from the Regreſſion of the Æquinoctial Points, by this Philoſophy.

Sir Iſ. Newt. Princip. Prop. 66. Corol. 20, 21, 22. Lib. 1. Prop. 21. Lib. 3. Dr. Gregory's Aſtronom. Prop. 64. Lib. 1. See Book 2. of this Philoſophy, Chapt. 5.

§. 23. AND upon this Foundation it is, that it Accounts for the Regreſſion of the Æquinoctial Points, and for the Axis of the Earth, every Annual Revolution, Changing it's Inclination to the Ecliptick twice, and twice Returning to the ſame Inclination; Becauſe it has been already prov'd, if this Philoſophy is True, that if a Body, as the Moon, Revolves about the Earth, there will be a Regreſſion of the Nodes, or a Motion of them in *Antecedentia*, and that the Inclination of it's Orbit to the Plane of the Ecliptick will be Chang'd twice in the Moon's Revolution, and will twice Recover it's Priſtine Poſition; Let therefore ſeveral Bodies be thus Mov'd round the Earth, and they will obtain like Laws with thoſe of the Moon; Let theſe ſeveral Bodies be Multiply'd in *Infinitum*, and there will be a Ring produc'd, which will be Subject to the ſame Changes of Inclination to the Plane of the Ecliptick, and to a Regreſſion of the Nodes; Let now the Globe of the Earth Swell and Extend it ſelf to this Ring; In the Flux and Reflux of the Sea, the ſaid Ring is ſuppos'd to be Fluid, and the Water to lie in an Hollow Channel, within the Surface of the Earth, but in the Preſent Solution the Ring is Imagin'd to become Solid, and to riſe above the ſame Surface, or, which is all one, the Fluid Ring is ſuppos'd to become Solid, and the Globe of the Earth to be diminiſh'd and fall below it; which is ſaid to be the Caſe of the Earth, Conſider'd as a Spheroeid, which has the Nature and Figure of a Sphere with a Circle or Ring of Redundant Matter round it, and adhering to it, and which therefore, having the ſame Axis with the Earth, will Oſcillate together with it, and the Nodes Retire in *Antecedentia*; That the Globe of the Earth will Oſcillate with the Ring is afterwards ſhewn, but take the Ring apart from ſuch Globe, and it's Greateſt Angle of Inclination will be, when the Nodes are in the Syzygiæ; From thence, as the Nodes proceed to the Quadratures, the Ring Endeavours to diminiſh it's Inclination, and Impreſſes the ſame Conatus upon the whole Globe; The Globe Retains this Motion Impreſs'd, till the Ring by a Contrary Conatus Deſtroys it, and Impreſſes a New one to a Contrary Side, by which Means, the Greateſt Motion of the Decreaſing Inclination is in the Quadratures of the Nodes, and the leaſt Angle of Inclination in the Octants after the Quadratures, becauſe the Motion Impreſs'd is ſtill Retain'd, till a Contrary Conatus alters it, which is in thoſe Octants; Then the Greateſt Motion of Reclination or Increaſing Inclination is in the Syzygiæ, and the Greateſt Angle of Inclination in the Octants following, for the ſame Reaſon; And the Caſe is the ſame of a Globe, which has no Ring, which is ſomething Higher at the Æquator than the Poles, or which Conſiſts of ſomething a Denſer Matter; for that Exceſs of Matter at the Æquator ſupplies the Place of a Ring, and altho', the Centripetal Force of this Globe being Increas'd, all it's Parts may be ſuppos'd to Tend downwards, in the Manner of Gravitating Parts of the Earth, yet there will be little Alteration from thence in the foregoing Reaſoning.

AND from hence it is ſaid, by the ſame Reaſon, that the Redundant Matter of the Globe, at the Æquator, cauſes, that the Nodes ſhou'd have a Regreſſion, and therefore, that ſuch a Regreſſion ſhou'd be Increas'd by an Increment of ſuch Matter, and Diminiſh'd by a Diminution of it, and Deſtroy'd by an Ablation of it, if more Matter be taken away than the Redundant, that is, if the Globe is more Depreſs'd, or Rarer at the Æquator than at the Poles, the Motion of the Nodes will be Progreſſive, or in *Conſequentia*.

ALSO, on the Contrary, from the Motion of the Nodes, we may Determine the Conſtitution and Nature of the Globe; Namely, if the Globe always Preſerves the ſame Poles, and the Motion is in *Antecedentia*, the Matter at the Æquator will be Redundant, if in *Conſequentia*, it will be Deficient; Suppoſe an uniform Globe, and made perfectly Round, to be firſt Quieſcent in Unreſiſting Spaces, and then, by ſome Oblique Force Acting upon it's Surface, to be Propell'd forwards, and to Admit of a Motion from thence, partly Circular, and partly in *Directum*; but, ſince the Globe is Indifferently Affected towards all it's Axes, and has not a Greater Propenſity to one Axis, or one Situation of the Axis, than to another, it is manifeſt, that it will never, by any Power of it's own, Change it's Axis, or the Inclination of it; Let there be now a Second or Third Oblique Impulſe Communicated to the ſame Globe, and at the ſame Part of it's Surface, as before, that is, in the Æquator of it's Motion, and the like Motion will be produc'd without any Alteration of the Axis, or it's Inclination; An Homogeneous and Perfect Globe therefore do's not Retain ſeveral Diſtinct Motions, but Compounds them all, and Reduces them to one, and, as to it ſelf, is always Revolv'd in a Simple and Uniform Motion about one Axis only, which is Invariable in it's Inclination; nor can the Centripetal Force change the Inclination of the Axis, or the Velocity of the Revolution Round it; for, if a Globe is conceiv'd to be divided into Two Hemiſpheres by any Plane, which paſſes thro' it's Center, and the Center, to which the Centripetal Force Tends, that Force will Equally urge both Hemiſpheres, and therefore will not Incline the Globe to either Side in Reſpect of it's Revolution; But now, if any Freſh Matter is heapt up, like a Mountain, in any Place betwixt the Pole and the Æquator, it will Diſturb the Motion of the Globe, and Cauſe it's Poles to Shift their Place and Wander upon it's Surface, and to Deſcribe Circles about ſuch Pole, and it's Oppoſite Point, by the Perpetual Conatus, which there is in this New Matter of Receding from the Center of it's Motion, in like Manner, as is done by the Poles of the Ecliptick about the Poles of the Æquator; if this Mountain of New and Acceſſory Matter is Situate in either of the Poles, the Nodes of the Æquator will be Progreſſive, as has been already obſerv'd, if in the Æquator, Regreſſive; if the Mountain of this Freſh Matter is plac'd on one Side of the Æquator, and another Maſs of New Matter is plac'd on the other, ſo that there is a Kind of Libration betwixt them, the Nodes will be Progreſſive or Regreſſive, as theſe Maſſes are Nearer to the Pole or the Æquator, becauſe they may be conſider'd in the Nature of one Maſs, which in the Poles will cauſe a Progreſſion of the Nodes, and in the Æquator a Regreſſion of Them.

AND this is the Sum of the preſent Doctrine concerning the Regreſſion of the Æquinoctial Points, and it's proving the Figure of the Earth to be a Spheroeid.

The foregoing Doctrine Refuted from ſeveral Arguments.

§. 24. IN Anſwer to which, we ſhall FIRST ſay, That this Philoſophy, as we have afterwards ſhewn, Book 2, has not given an Account of the Errors and Irregularities of the Moon, nor of the Preceſſion of the Nodes of it's Orbit, to which this Ring of Redundant and Elevated Earth is Compar'd, and Made Analogous.

See Chapt. 5. Book 2.

SECOND, Tho' ſeveral Diſtinct and Single Bodies, Anſwerable to the Moon, wou'd be ſubject to the ſame Laws of Revolving with it, yet United together and Compacted into one Solid Ring, it is not ſo Plain and Evident, that they wou'd; becauſe in a Solitary or Single Body, and ſo in ſeveral, they Paſs ſucceſſively thro' the Syzygiæ, Octants, and Quadratures, and every Point of their Orbit, and receive Impulſes from the Sun's Action, ſuitable to the Places, where they are; but when theſe ſeveral Diſtinct Bodies are Cemented into one Solid Ring, the Sun Acts upon Bodies, which Communicate with each other, and are at once in the Syzygiæ, Octants, and Quadratures, and all the Parts of the Orbit, in which a Solitary, or ſeveral Solitary Bodies, which do not Communicate with each other, ſucceſſively Revolve; and conſequently, the Effects of ſuch Action of the Sun muſt be Extremely Different in the Firſt Caſe, from what it is in the Laſt.

THIRD, But that which will Puzzle and Embarraſs this Analogy betwixt a Solitary or ſeveral Single Bodies, and a Solid Ring the more, is, that there is nothing, excepting the Action of the Sun, to diſturb the Revolution of the Former, but in the Latter, ſince it is ſuppos'd immediately to Encompaſs the Earth, the Sun and Moon have, each of them, their Particular Influences, and alſo very Oppoſite ones; ſeeing the Sun may be in one Syzygia of the Ring, when the Moon is in the Other, or in Different Quadratures or Octants, which muſt neceſſarily produce Impreſſions upon the Ring, not any ways Conſonant or Agreeable to what are made upon a Single, or ſeveral Single and Diſtinct Bodies.

FOURTH, We are farther to take Notice, that the Earth Revolves about the Sun, and conſequently, the Solid Ring with it, whilſt the Moon Revolves about the Earth, and therefore about the ſame Solid Ring, and that the Axis of the Earth and of the Ring is carry'd always Parallel to it ſelf in it's Annual Circuit; which likewiſe will occaſion a Material Difference betwixt this Caſe, and a Body's being only Revolv'd about the Earth and Sun, without it's being Carry'd with an Axis Parallel to it ſelf about the Sun, but in very Irregular and Enormous Motions about the Earth, as the Moon is.

FIFTH, Another Inſtance, in which the Solid Ring, and a Single or ſeveral Single Bodies Diſagree is, that the one Adheres to the Earth, whereas the other Circulates in Free and Unreſiſting Spaces Round it; For, if it is the ſame Thing, let one Body of the ſeveral be affixt to the Earth, and it will be Subject to the ſame Laws of it's Revolving, as if it perform'd it's Revolution at a certain Diſtance from it, but the whole Solid Ring is ſaid to be Subject to the ſame Laws; therefore the Earth will Revolve in the ſame Manner, if one Maſs of Matter, or a large Mountain is plac'd under the Æquator, as if a Ring of Elevated Matter, or a Circle of Mountains were Continu'd under it, which is too Groſs for this Philoſophy to Affirm.

SIXTH, But farther it is not Evident, nor is it Prov'd, that the Sun at that Immenſe Diſtance, which it has from the Earth, ſhou'd cauſe by it's Action any Particular Alteration by it's Impreſſions upon a Matter, which at the moſt, and by the higheſt Calculation in this Philoſophy, is only $31\frac{7}{12}$ Miles Elevated above the Reſt of it's Surface; Can there be much Difference in the Action of the Sun, which is above 50 Millions of Miles, and others make it 80, Remov'd from us, upon Two Portions of Matter, which are only Diſtant from each other about 31 Miles? And yet ſhall the whole Diameter of the Earth's Orbit about the Sun be conſider'd, as a Point in Reſpect of the fixt Stars, and the whole Earth it ſelf in Reſpect of it's Diſtance from the Sun? This ſhews, how Conſiderable the preſent Philoſophy makes it's own *Minutiæ*, and how fondly it Advances them above thoſe, which it Acknowledges in Nature, and which are yet much more Significant and Important.

SEVENTH, Granting, notwithſtanding, that the Sun do's obtain a Diſtinguiſhing Effect upon Portions of Matter, which are at ſo ſhort a Diſtance from each other, it cannot well be Imagin'd, that the Coheſion and firm Union of the Parts of the Earth ſhou'd not hinder any Impreſſions, which cou'd be made by way of Attraction, more upon one Part than another; at leaſt it is not likely, that they wou'd owe their Oſcillation to a ſmall Prominence or Protuberancy, and not be Determin'd from an Impreſſion upon the whole Globe; and therefore the Ring ſhou'd rather obey the Action of the Sun upon the whole Globe, which is not Pretended to produce any Oſcillation, than the Globe obey the Action of the Sun upon the Ring, which is ſuppos'd to do it.

EIGHTH, Beſides, tho' this Philoſophy ſays, that the Globe holds it ſelf Indifferently to all it's Axes, and therefore Conforms it ſelf to the Oſcillations of the Ring, yet a Weighty and Ponderous one, as Indifferent as it may be to it's Axes, will not be Roll'd any Way, which we Pleaſe, by the Application of a Feather; it is therefore to be ſhewn, that the Action of the Sun upon the Ring, in Diſtinction from the Reſt of the Globe, is any more to it. See Book 2.

NINTH, This is all Argu'd upon the Suppoſition of an Attractive Force in the Sun to draw the Earth towards it, but, if that Attractive Force ſhou'd prove to be an Expanſive one, by which the Earth is Repell'd from it, and which we have, Book 2, Endeavour'd to Evince, the Caſe will be alter'd, and this Philoſophy muſt have ſome New Contrivance to make ſuch a Force Square with it's preſent Reaſonings.

TENTH, But what is ſtill Worſe than all this, is, that the Earth, neither by what forego's, nor by what follows concerning Pendulums, is prov'd to be a Spheroeid, upon which yet the whole Argument Depends, and the Solution of the Preceſſion of the Æquinoctial Points.

Lastly, The ſaid Preceſſion may be Accounted for, by the Sun's Expanſive Force, in which the Earth continually Revolving, and being continually Repell'd by it, Moves thro' a Force, which Retards in ſome Meaſure it's Motion, by which it will, every Annual Revolution, be carry'd or forc'd, in ſome ſmall Degree, in *Antecedentia*, and conſequently, the Interſections of the Æquator with the Ecliptick will be ſo too.

The Different Lengths of Pendulums in Various Latitudes, no Proof of the Earth's being a Spheroid.

§. 25. **It** will be yet ſaid, that it may be ſufficiently Demonſtrated, that the Figure of the Earth is that of a Spheroid, from the Different Lengths of Pendulums in Various Climates and Latitudes, into which we ſhall now Particularly Inquire; in order to do which, it will be neceſſary for us to Examine and Compare the Two preceding Tables, the One made from a Calculation upon the Principles of this Philoſophy, the Other from Experiments.

See §. 17, 18.

And in the Firſt Place, in the Table of Experiments, *§.* 18, there are only Thirteen Latitudes of Places mention'd in all, which, ſhou'd the Lengths of the Pendulums, in each of them, have been Agreeable to what was found by Calculation, *§.* 17, it had given us a Preſumption, but not a Demonſtration that the Hypotheſis was True, namely, that the Lengths of Iſochronal Pendulums are Reciprocally, as the Semidiameters of the Earth, where they Oſcillate; But when of theſe Thirteen Latitudes, there are Two, namely, *London* and St. *Helen*'s, to which there is no Length of the Pendulum Aſſign'd, excepting in Groſs, that at St. *Helen*'s, the Pendulum is $1\frac{1}{2}$ Line Shorter than at *London*, obſerv'd by Dr. *Halley*; when farther there are Two more, *Paraiba* and *Lisbon*, to which alſo there is no Length of the Pendulum Aſſign'd, excepting in Groſs, that at the firſt Place it is $3\frac{2}{3}$ Lines, and at the Laſt $2\frac{1}{2}$ Lines Shorter than at *Paris*, according to the Obſervations of *Couplet*, which are Condemn'd and Corrected by Sir *Iſ. Newton* to $2\frac{5}{9}$, and $1\frac{1}{3}$ Lines; when likewiſe there are Three others, that is, *Cayenne*, *Martinico*, and *Paris*, to which Different Lengths of the Pendulum are Aſſign'd by Different Obſervers, namely, by *Richer* and *Des Hayes* at *Cayenne*, which Obſervations yet Sir *Iſaac Newton* Condemns and Corrects, by *Varin* or *Des Hayes* and *Feuellè* at *Martinico*, and by *Richer* and *Varin* or *Des Hayes* at *Paris*; when moreover there are only Two Obſervers, *Varin* and *Des Hayes*, who Agree in the Lengths of the Pendulums, and in only Three Latitudes, which are near to each other, and do not differ a Single Degree, namely, *Guadaloupe*, *Goreè*, and *Martinico*; and when laſtly there are Four Latitudes, *Portobel*, *Granada*, St. *Chriſtopher*'s, and St. *Dominick*, where there is only one Obſerver of the Lengths of the Pendulum, to wit, *Feuellè* or *Des Hayes*; If we conſider the ſmall Number of Latitudes obſerv'd, the Paucity of Obſervers, the Variety and Diſſonance in their Obſervations, it will be hard for us to Imagine, how any Theory or Hypotheſis can be juſtify'd from hence, and eſpecially how the Diameters of the Earth ſhou'd be aſcertain'd from ſuch Obſervations of the Lengths of the Pendulums in Different Latitudes; ſince, at the moſt, there are only Six Latitudes of the whole Number, the Lengths of whoſe Pendulums are Unconteſted, *Portobel* Obſerv'd by *Feuellè*; *Granada*, by *Des Hayes*; *Guadaloupe*, by *Varin* and *Des Hayes*; *Goreè*, by the ſame; St. *Chriſtopher*'s and St. *Dominick*, by *Des Hayes*; and of theſe Six, Four of them have only one Obſerver, which, if others were to try the ſame Experiments apart, it may be Reaſonably thought wou'd Diſagree as much as the Former.

Besides which, let us ſuppoſe theſe Obſervations True and Exact, even thoſe, which Diſagree, the Lengths of the Pendulums Gradually Increaſe, as they are carry'd from the Æquator to the Poles, in the Table form'd from Calculation; whereas in the Succeeding one, made from Experiments, the Length of the Pendulum at *Cayenne*, according to *Richer*, *Des Hayes*, and Sir *Iſ. Newton*, which is only $4^{\circ}.55'$ Latitude, is Greater than, according to *Feuellè*, at *Portobel*, which is $9^{\circ}.33'$ Latitude, and Equal, according to *Des Hayes*, to the Length of it at *Granada*, which is $12^{\circ}.6'$ Latitude, but Greater according to *Richer* and Sir *Iſ. Newton*, alſo Equal, according to *Varin* and *Des Hayes*, to the Length of it at *Guadaloupe*, which is 14° Latitude, but Greater according to *Richer* and Sir *Iſ. Newton*, again almoſt Equal, according to *Varin* and *Des Hayes*, to the Length of it at *Goreè*, which is $14^{\circ}.40'$ Latitude, but Greater according to *Richer* and Sir *Iſ. Newton*, Farther, exactly Equal, according to *Varin* and *Des Hayes*, to the Length of it at *Martinico*, which is $14^{\circ}.44'$ Latitude, but Greater according to *Richer*, Sir *Iſ. Newton*, and *Feuellè*; And yet on the Contrary, if theſe Experiments any way Correſponded with the abovesaid Calculations, the Length of the Pendulum at

Cayenne ought to be Leſs than at *Portobel*, *Granada*, *Guadaloupe*, *Goreè*, and *Martinico*, tho' they unanimouſly Agree to make it Equal or Greater: Thus likewiſe the Length of the Pendulum ſhou'd be Leſs at *Granada* than *Guadaloupe*, and at *Guadaloupe* than *Martinico*, albeit *Varin* and *Des Hayes* make them Equal; Nay, which is ſtill more Extraordinary, *Richer*, and Sir *Iſ. Newton*'s Correction upon him, makes the Length of the Pendulum Greater at *Cayenne*, which is only 4°. 55′ Latitude, than at St. *Chriſtopher*'s, which is 17°. 19′ Latitude, or at St. *Dominick*, which is 19°. 48′ Latitude.

Moreover, According to the Table of Calculations, the Greateſt Difference betwixt the Length of the Pendulum at the Æquator, and at the Poles, namely, betwixt 3 . 7,468, and 3 . 9,387 Feet and Lines, is 1,919 Line, that is, not quite 2 Lines, but as thoſe Calculations were made upon the Suppoſition of the Earth's being uniformly Denſe, if it's Matter is Denſer nearer the Center than at the Surface, the Difference of the Pendulums will be ſomething Greater, and therefore, in the preſent Caſe, let that Difference be a little more than 2 Lines.

But *Couplet* at *Paraiba*, which is 6°, 38′ Latitude, obſerv'd the Length of the Pendulum to be $3\frac{2}{3}$ Lines leſs than at *Paris*, which is 48°. 45′ Latitude, and which Sir *Iſ. Newton*, upon a Correction of this Groſs Obſervation, as he Eſteems it, makes $2\frac{5}{9}$ Lines leſs, Either of which will be a Confeſſion, that the Difference of the Lengths of the Pendulums is Greater, or about as Great, at the Diſtance of 42°. 7′, as of 90°, which is Impoſſible; Eſpecially, when according to the Table of Calculations, the Difference betwixt the Lengths of the Pendulums at 5° Latitude and 48°, which comes the neareſt to the preſent Caſe, namely, the Difference betwixt 3 . 7,482 Feet and Lines, and 3 . 8,528, is only 1,046, that is, ſomething more than a Line, and not $1\frac{1}{4}$ of a Line, allowing for a Greater Difference upon Account of the Earth's being Denſer at the Center than at the Surface.

At *Liſbon*, which is 40° Latitude, the ſame *Couplet* found the Length of the Pendulum $2\frac{1}{2}$ Lines leſs than at *Paris*, which is 48°. 45′ Latitude, and according to Sir *Iſ. Newton*'s Correction it is $1\frac{1}{3}$, but the Difference betwixt 40° and 48°. 45′ is 8°. 45′, So that here is a Difference of the Lengths of the Pendulums, within $\frac{2}{3}$ of a Line, almoſt Equal to what belongs to a Difference of 90° Latitude, which is Impoſſible; Eſpecially, when we farther conſider, that the Difference of the Length of the Pendulum, which belongs to 40° Latitude, which is 3 . 8,261 Feet and Lines, and of that, which belongs to 48° Latitude, which is 3 . 8,528 Feet and Lines, is no more than 0,267, or not much above $\frac{1}{4}$ of a Line in the Table of Calculations.

Dr. *Halley* obſerv'd at St. *Helen*'s, which is 15° Latitude, that the Length of the Pendulum was $1\frac{1}{2}$ Line leſs than at *London*, which is 51°. 30′ Latitude, ſo that, according to this Experiment, the Difference of only 36°. 30′ Latitude will require almoſt as Great a Difference in the Lengths of the Pendulums, as the Difference of 90° Latitude, which is again Impoſſible; Eſpecially, when we farther conſider, that the Difference of the Length of the Pendulum, which is over againſt 15° Latitude in the Table of Calculations, and of that, which ſtands oppoſite to 50° Latitude in the ſame Table, Namely, of 3 . 7,596 and 3 . 8,594 Feet and Lines, is no more than ,998 of a Line, that is, not one entire Line.

According to *Des Hayes*, the Length of the Pendulum at *Guadaloupe* and *Martinico*, which are 14° and 14°. 44′ Latitude, is $\frac{1}{2}$ a Line leſs than at St. *Dominick*, which is 19°. 48′ Latitude, which is the Difference of $\frac{1}{2}$ a Line for a Difference of a little more than 5° Latitude, and yet there is but about 2 Lines Difference for a Difference of 90° Latitude; and in the Table of Calculations, the Length of the Pendulum at 15° is 3 . 7,596, and at 20°, 3 . 7,692, from whence the Difference of the Lengths, for a Difference of 5°, will be only ,096 of a Line, which will fall very much ſhort of ,500 = $\frac{1}{2}$ of a Line, and cannot be repair'd by ſuppoſing the Earth Denſer at the Center.

Again, according to *Richer*, the Length of the Pendulum at *Cayenne*, which is 4°. 55′ Latitude, is $1\frac{1}{4}$ Line leſs than at *Paris*, which is 48°. 45′ Latitude, but according to *Des Hayes* $2\frac{1}{18}$, and by the Table of Calculations the Length of the Pendulum at 5° Latitude is 3 . 7,482, and at 48° Latitude 3 . 8,528, from whence the Difference will be 1,046, which is much Leſs than either $1\frac{1}{4} = 1{,}250$, according to *Richer*, or than $2\frac{1}{18}$ according to *Des Hayes*; and neither of 'em bear any Proportion to the Difference of the Pendulum's Lengths for the Difference of 90° Latitude; ſince the Difference, in the Preſent Caſe, is 43°.50′ Latitude, to which *Des Hayes* aſſigns a Difference of the Length of the Pendulum almoſt Equal to that, which belongs to a Difference of 90°, *Richer* is ſomething too High, and both Diſagree very much with what the Calculation makes it.

And by the ſame Method of Examining we ſhall find, that the few Remaining Experiments bear no Analogy, in the Differences of the Lengths of the Pendulums at any Intermediate Latitudes, with that ſuppos'd Difference, which is betwixt them under the Æquator and Poles; nor yet, that they in any Meaſure Correſpond with the Differences of the Lengths, aſcertain'd in the Table of Calculations; which cannot but be a full Confutation of the preſent Theory of Pendulums, and the Computed Lengths of them, in Various Latitudes, deriv'd from it.

From whence the Different Lengths of Pendulums in Various Latitudes and Climates, as well as in the ſame do Really ariſe.

§. 26. But to ſtrike at the Foundation of all this Reaſoning we farther affirm, that the Shortneſs of Synchronal Pendulums at the Æquator, and the Increaſe of their Lengths towards the Poles, do's not ariſe from the Elevation of the Earth in the one Place, and the Depreſſion of it towards the others, but from the Various Temperature of the Air and Climate, and particularly with Relation to the Heat and Cold Experienc'd in them, which from Different Cauſes may be Different in the the ſame Climates or Regions, and nearly the ſame in Different ones; ſince we do not find all Countries Equally Hot or Cold, which are in the ſame Latitude, nor Differently Hot or Cold, which are in Different ones; And from hence there may be ſome Account given of the Variety and Diſagreement of Obſervations in the ſame Latitudes, as alſo of the Length of the Pendulum not being Increas'd or Diminiſh'd in Proportion to it's Removal from the Æquator or Approach to it.

Thus at *Cayenne*, the Reaſon, why the Length of the Pendulum was leſs according to *Des Hayes*'s Obſervations, by ſomething more than $\frac{1}{2}$ a Line, than according to *Richer*'s Obſervations, might probably be, that the Experiments of the Former were made in an Hotter or Dryer Seaſon or Air, and thoſe of the Latter in a Colder or Moiſter; for it is known, that betwixt the Tropicks their Rain is their Winter.

The Length of the Pendulum at *Paraiba*, according to *Couplet*, was $3\frac{2}{3}$ Lines leſs than at *Paris*, becauſe the Temperature of the Air is much Hotter in the one Place than the other.

Feuellè obſerv'd the Length of the Pendulum to be leſs at *Portobel*, which is 9°.33′ Latitude, than *Richer* or *Des Hayes* did at *Cayenne*, which is only 4°.55′ Latitude, becauſe the Heats were Greater, it is likely, at *Portobel* than at *Cayenne*, when thoſe Obſervations were made, or the Weather and Seaſon Dryer at the firſt Place, and Moiſter at the Laſt.

In like Manner, the Length of the Pendulum, according to *Varin* and *Des Hayes*, was much the ſame at *Granada*, *Guadaloupe*, *Goreè*, and *Martinico*, whoſe Latitudes are 12°.6′, 14°.0′, 14°.40′, 14°.44′, becauſe we may juſtly Imagine, the Heats of thoſe Places are nearly the ſame, or the Seaſons of the Year, in which the Obſervations were made, were ſo far Different, as to Compenſate for the Difference of the Climates, and Render the Temperature of the Air very much Equal; and *Feuellè*'s Pendulum at *Martinico* was Shorter than *Varin*'s and *Des Hayes*'s, becauſe he obſerv'd in an Hotter or Dryer Seaſon.

Dr. *Halley*'s Pendulum at St. *Helen*'s was Shorter than at *London*, becauſe the Heats were more Extraordinary at 15° Latitude than at 51°.30′.

Des Hayes's was Shorter at St. *Chriſtopher*'s than at St. *Dominick*'s, *Couplet*'s at *Lisbon*, and *Des Hayes*'s at *Cayenne*, than at *Paris*, for the ſame Reaſon.

And laſtly, *Varin*'s and *Des Hayes*'s was Shorter at *Paris* than *Richer*'s by $\frac{2}{45}$ of a Line, becauſe the Firſt obſerv'd in an Hotter or Dryer Seaſon, and the Laſt in a Colder and Moiſter.

This Account therefore depends upon the Suppoſition, that the Lengths of the Pendulum are to be Diminiſh'd in an Hotter or Dryer Climate and Temperature of the Air, and to be Increas'd in a Colder or Moiſter. But it is Matter of Fact and Experiment, and ſo acknowledg'd by this Philoſophy, that Pendulums move Slower in the ſame Climate, in Summer than in Winter, in an Hotter than a Colder *Craſis* or Temperature of the Air; and, as Dryneſs ariſes from Heat and Moiſture from

Cold, there is the ſame Reaſon to affirm, that in ſome Degree, a Pendulum moves Slower in a Dry, if ſomething Hot, than in a Moiſt Seaſon, if Cold, and we have Endeavour'd from the Expanſive and Contractive Forces to give a Solution of this Phænomenon; For it is Plain, that an Expanſive Force makes a Pendulous Body Lighter, and to Deſcend Slower, and a Contractive Heavier, and to Deſcend Quicker; It is therefore Manifeſt, that if a Pendulum moves Slower in an Hotter or Dryer *Craſis* or Temperature of the Air than in a Colder or Moiſter, to make it Oſcillate in the ſame Time, that is, in the Second of a Minute for Inſtance, the Pendulum muſt be ſhorten'd in the firſt Caſe, and lengthen'd in the Laſt; becauſe, if the Force of the Pendulous Body is Diminiſh'd, ſo that it moves Slower, it muſt be carry'd Nearer to the Center of Suſpenſion, that it may Deſcribe a Similar Arc of a leſs Circle in it's Oſcillation in the ſame Time, that a Greater Force Deſcribes a Similar one of a Larger; and on the Contrary, if the Force of the Pendulous Body is Increas'd, ſo that it moves Faſter, it muſt be carry'd to a farther Diſtance from the Center of Suſpenſion, that it may Deſcribe a Similar Arc of a larger Circle in the ſame Time, that a Leſſer Force Deſcribes a Similar Arc of a Leſs.

Besides which, ſince the Oſcillations of a Pendulum, which we have before ſhewn, Proceed from the Conteſt, which there is betwixt the Expanſive Force of the Pendulous Body and the Contractive Force of the Center of Suſpenſion; As the Expanſive Force of the Pendulous Body, that is, in the Preſent Caſe, it's Gravitating, is Diminiſh'd by the Expanſive of Heat, it muſt approach Nearer to the Contractive Force of the Center of Suſpenſion or Rotation, and as the ſame Expanſive Force of the Pendulous Body, that is, it's Gravitating, is Increas'd, it muſt Retire farther from the Contractive Force of the Center of Suſpenſion, to Produce the ſame Effect, that is, to Meaſure the ſame Time; becauſe the Nearer any Force is to a Contrary, the more it Partakes of and Communicates with that Force, and is Govern'd by it, and the farther it is from it, it is the leſs Affected with it, whilſt both Exert their Effects in the ſame Time; Thus in the Common Deſcription of a Circle, the Expanſive Force is Slower, the Nearer it is to the Center, which is Quieſcent, and Quicker the more it is Remov'd from it, in Order to produce Two Concentrical Circles, whoſe Times of Deſcription are notwithſtanding the ſame.

The Objections againſt the preceding Explanation Anſwer'd.

See §. 18.

§. 27. But it is objected to this Explanation of the Lengthening and Shortening of Pendulums from Heat and Cold, that, from the Obſervations of *Picart*, the Iron-Rod of the Pendulum, which in Froſty-Weather was one Foot Long, being heated at the Fire, was only one Foot and $\frac{1}{4}$ of a Line, and that, from the Obſervations of *De la Hire*, the Iron-Rod, which in the like Froſty-Weather was Six Feet Long, in the Summer-Heats was only Six Feet and $\frac{2}{3}$ of a Line, that indeed the Summer's Sun do's make Metals exceeding Warm, which are Expos'd to it, but that the Rod of a Pendulum in a Clock is never Expos'd to the Heat of the Summer's Sun, and never has an Heat Equal to that of the External Parts of an Human Body, becauſe, the Pendulum will always be perceiv'd Cold to them; and therefore the Rod of a Pendulum, which is Three Feet Long, will be ſomething Longer in Summer-Time than in Winter, by an Exceſs of little more than $\frac{1}{4}$ of a Line, and conſequently can be ſhorten'd no more, which cannot poſſibly give an Account, why the Pendulum ſhou'd be ſhorten'd at the Æquator, 2 Lines or more; from whence it is Infer'd, that the whole Difference of the Length of Pendulums, which in Various Climates are Iſochronal, cannot be aſcrib'd to the Different Heat of thoſe Climates, nor yet to the Errors of the Obſervers, which are Small and Inconſiderable; and in this they all Agree, that the Iſochronal Pendulums are Shorter under the Æquator, than at the Royal Obſervatory at *Paris*, by a Difference of about 2 Lines.

In Anſwer to which we ſay, that, becauſe the Length of the Pendulum is only Increas'd about $\frac{1}{4}$ of a Line by Heat, and therefore can be ſhorten'd no more upon that Account, is no Argument, that it may not be ſtill ſhorten'd upon the Account of it's Slowneſs, and that ſuch Slowneſs is not owing to Heat; Since the Expanſive Force of Heat Diminiſhes the Contractive, or the Weight, of the Pendulum, and what is Lighter Deſcends Slower; The Obſervations therefore of *Picart* and *De la Hire* do not Overthrow what we have ſaid, but Confirm it, and Evince, that the Pendulum is to be ſhorten'd, by Reaſon of Heat's Swelling and Increaſing the Length of it by it's Expanſive, and we add, that it is likewiſe to be ſhorten'd, becauſe Heat by it's Expanſive Increaſes the Slowneſs of it.

'Tis alſo acknowledg'd, that Metals do admit of a Great Degree of Heat or Warmth from the Rays of the Sun in Summer, and tho' the Pendulum is not Expos'd to the open Violence of it, it partakes of the Effects, which are Communicated to the whole Air, by which it becomes of a Sultry Temper and Diſpoſition, even in Places, which are not Obnoxious to the Impreſſions of the Direct Rays from the Solar Subſtance; and albeit the Heat of the Pendulum may not Exceed the Heat of the External Parts of an Human Body, yet conſidering, that the Heat of thoſe External Parts is mightly Increas'd in Summer, in Reſpect of what they have in Winter, the Heat of the Pendulum will receive a Proportionable Augmentation, compar'd with what it has in Froſty Weather, and ſuch a one as may be ſufficient to produce it's apparent Slowneſs.

As to the Errors of Obſervers, we do not Charge any upon them, but have Endeavour'd to ſhew, that the Variety of Obſervations, even in the ſame Region and Climate, may be juſtly Accounted for in the Way we have Propos'd; On the Contrary, *Couplet* is made to be a Groſs Obſerver, and other Obſervations are Eſteem'd Erroneous by this Philoſophy, becauſe they do not ſo exactly Square and Agree with it's Suppoſitions, and the Calculations upon them; which is a very unfair Method of Judging and Determining concerning the Truth of Experiments, and is really Endeavouring to make Nature ſubmit to our Speculations and Fancies, whereas our Reaſonings and Imaginations about it ought to be Accommodate and Conformable to Nature; but this is not the only Inſtance of this Way of Proceeding in the Preſent Philoſophy, I have ſhewn in my *Principles of Natural Philoſophy*, that it takes the ſame Courſe in Accounting for the Propagation of Sounds, and that it has done the like in Aſtronomy I have alſo Evinc'd, which was the chief Reaſon, or elſe I am Miſinform'd, of the Complaints of Mr. *Flamſteed*, and his Burning the Copies of his Aſtronomical Obſervations, which were Printed under other Men's Directions, becauſe they had Alter'd and Mangled them, and made them Different from what he obſerv'd, on Purpoſe to Confirm the New Notions Introduc'd in the Phyſical Aſtronomy.

For in the Preſent Caſe, why ſhou'd one Obſerver be Prefer'd to Another, why ſhou'd *Couplet* be a Groſs one, and *Varin* and *Des Hayes* more Accurate? Is there any Thing alledg'd in the Genius of the Men, or in the Pendulums which they us'd, which ſhou'd make the one more to be Depended upon, than the other? *Couplet*'s Obſervation is Corrected; but how? Namely, by ſuppoſing the Principles of this Philoſophy True, upon which it Calculates, whereas thoſe Principles ought to be prov'd True by thoſe Experiments it Preſumes to Correct; Either we muſt fix the Certainty of Experiments, or elſe entirely Diſcard them, and if thoſe Experiments ſeem to Claſh and Interfere with each other, we muſt find out Principles, which will Anſwer thoſe ſeeming Contradictions, or Reject the Whole as a Fallacious Buſineſs; but to take ſome Experiments, which Agree with our Hypotheſes, and to Slight and Diſregard, or Tamper with others, which do not, is a Cunning and Artificial Procedure, but not an Honeſt and a True one, and is more ſuitable to the Character of a Sophiſter and Jugler in Reaſon, than of a Philoſopher.

As to the Agreement of Obſervers in this, that Iſochronal Pendulums are Shorter under the Æquator than at the Royal Obſervatory at *Paris*, there is an Evident Reaſon for it, if Heat and Cold are the Grand and Principal Regulators of them, as we have hitherto Endeavour'd to Shew and to Demonſtrate.

Concluſion upon this Whole Doctrine.

§. 28. I Shall Conclude this Doctrine of Pendulums with Obſerving, that in my *Principles of Natural Philoſophy*, which, upon the Publiſhing of them, I immediately ſent to Sir *Iſ. Newton*, Chapt. 5. §. 16. I had Affirm'd, that the Reaſon, why Pendulums did not Oſcillate Equably in Different Climates, was the Difference of the Medium, in which the Oſcillations were made, what that Difference of the Medium was, I had no occaſion then to Conſider; I added, Referring to Sir *Iſaac Newton*'s Princip. Prop. 20. Lib. 3. and Dr. *Gregory*'s Aſtronom. Prop. 52. Lib. 3. That we cou'd not Reaſonably Attribute this Inequality of the Motion of the Pendulum to an Alteration in the Figure of the Earth in Various Parts of it, however Ingenious ſome Calculations might be in order to Prove it; Upon a Second Edition of his Principles, this Point was Labour'd with much more Accuracy than in the Former, but whether he has Prov'd what I there Deny'd, I now leave Others to Judge, and whether I had not ſome Grounds to Affirm, that it was an Ingenious Calculation only.

Dr. Keill's Intr. ad Ver. Phyſ. Lect. 16. Theorem 47, 48, &c. See Principles of Philoſ. Chap. 5. §. 6. Book 1.

The Nature of Projectiles.

As to the Doctrine of Projectiles, which follows in this Philoſophy, and that the Line Deſcrib'd by them is a Parabolick Curve, we have already taken Notice of it in our *Principles of Natural Philoſophy*, and ſhewn, that it entirely Depends upon the Suppoſition of Matter's being Similar and Homogeneous, and that it is Acted by uniform Impulſes of Gravity, neither of which Hypotheſes we Can or are Willing to Allow.

ALL that we ſhall farther ſay is, that Bodies, when Projected, perform Various and Different Figures, according to the Greater or Leſs Reſiſtance of the Medium, and the ſeveral Degrees of the Expanſive and Contractive Forces, with which they are Endu'd, as alſo according to the Proportion of the Expanſive Force, which is Communicated to them from the Projection, and of the Contractive Force of Gravitation, by which they are Impell'd to the Earth; and Caſes, diſtinctly put in Reſpect to each of theſe Forces, wou'd ſupply us with many New Theorems unknown to the Preſent Doctrine, which only conſiders one Caſe, namely, where the Matter is the ſame, and a-like, and where the Force of Gravitation is ſo too, and Acts by Continual and Invariable Percuſſions upon it, by which it is Accelerated towards the Earth's Center.

THE
PRINCIPLES
OF THE
PHILOSOPHY
OF THE
EXPANSIVE and CONTRACTIVE
FORCES.

BOOK II.

CONCERNING THE
PHYSICAL ASTRONOMY.

THE PRINCIPLES of the PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK II.

Concerning the PHYSICAL ASTRONOMY.

CHAP. I.

Concerning the Laws of Nature and Motion.

§. 1. WE have in ſome Chapters of our Former Book Examin'd into the Principles, by which the Mechanical Powers have hitherto been Accounted for; We might now Proceed to the New Hypotheſis rais'd to Explain them, but ſince That is deriv'd from a Law of Motion, concerning the Compoſition of Forces, aſſerted in the Preſent Philoſophy, it will be Neceſſary for us to defer the Conſideration of it, till we come to Enquire into the Law it ſelf, and the Foundations and Reaſons, on which it is Eſtabliſh'd; In Order to which we ſhall conſider the Laws of Nature in the Method they are Propos'd to us, and which, together with the Opinions concerning Matter and Motion, which it Eſpouſes, and were before Conſider'd, are the Baſis of all the ſucceeding Philoſophy.

The Laws of Nature and Motion conſider'd.
Sir Iſ. Newt. Princip. Mathem. Corol. 2. Leg. Mot.

§. 2. THE Firſt Law of Nature or Motion affirm'd is, that every Body perſiſts in it's State of Motion or Reſt Uniformly in Directum, unleſs ſo far as it is oblig'd to change that State by ſome Force or other Impreſs'd upon it; Thus, it is ſaid, Projectiles continue in their Motion, unleſs ſo far as they are Retarded by the Reſiſtance of the Air, or Thrown to the Ground by the Force of Gravitation; A Globe or Cylinder, which Revolves upon it's own Axis, continues to Move, till it is made to Ceaſe, by ſome External Renitency to it; And the Large and Immenſe Bodies of the Planets and Comets preſerve their Progreſſive and Circular Motions to an unlimited Time, becauſe their Spaces have little or a very Inſenſible Reſiſtance in them.

Firſt Law of Nature or Motion explain'd.
Mr. Whiſton Phyſico-Math. Prælect. 5. Art. 1.
Dr. Keill's Intr. ad Phyſ. Lect. 11. Lex. 1.
Sir Iſ. Newt. Princip. Math. Lex. 1.

§. 3. WE ſhall not conſider the Reaſon, at preſent, of a Stone Falling to the Ground, or a Body Revolving upon it's Axis, or the Planets and Comets Preſerving their ſtated Periods, or whether Comets are Mov'd at all in a Circuit, or not; what we ſhall obſerve is, that for a Proof of the Law of Nature, it is ſuppos'd, that Nothing hinders a Projectile from Moving on for ever, excepting the Air, and it's own Proper Gravitation; That Nothing reduces a Body Revolv'd round it's Axis to a Quieſcence, excepting the Air, *&c*; That Nothing obſtructs the Circumgyration of the

The Inſtances alledg'd improper to Evince this firſt Law of Nature or Motion.

the Planets, unleſs an Infinitely ſmall Reſiſting Medium, and therefore they may move on to Innumerable Ages; And laſtly, That Planets and Comets are carry'd Round in a Circular and Progreſſive Motion, that is, in a Motion which is Compounded of a Direct, and a Centripetal; All which are Problems in Philoſophy, which require an Accurate, and Juſt Solution, and therefore are Improperly brought to juſtify the Firſt Law of Nature, before it is more certain than from an Hypotheſis only, that Motion is Stopt or Continued from theſe Cauſes; Since, for all that is yet Prov'd, Bodies may Ceaſe to Move after they are put into Motion by their own Innate and Contractive Forces, according to what has been before Explain'd, and others may continue to Move longer than in Proportion to the Force Impreſs'd by their own Proper Expanſive.

Whether this Law of Nature or Motion can be juſtify'd from a True Idea of Matter and Motion.

§. 4. SINCE therefore theſe Inſtances of Fact can ſignify nothing to the Aſcertaining the preſent Law of Nature or Motion, till they are Evidenc'd to proceed from the Cauſes aſſign'd to them, which cannot be done, in Reſpect of the Motion of Projectiles in particular and of the Planets, excepting upon the Suppoſition of this Law of Nature, it is manifeſt there is no Proof of it to be expected from them; The next Thing we ſhall conſequently inquire into, is, whether ſuch a Law of Nature or Motion can be deriv'd from a True Idea of Matter and Motion, or even from that, which this Philoſophy Propoſes.

If there are Innate Expanſive and Contractive Forces in Matter this Law will not be True.

§. 5. THAT it cannot from a True Conception of Matter, as it really Exiſts, is Evident, ſince we find ſome Matter to Gravitate, as Lead, and other to Aſcend, as Fire, whereas this Law ſuppoſes a perfect Inertia in Matter, conſider'd in it ſelf, without any other Force belonging to it; for if Matter, as was juſt now mention'd, has an Innate Force of Gravitation and Contraction, or of Aſcenſion and Expanſion, tho' a Motion is Impreſs'd upon it, it will not be neceſſary, that it ſhou'd Continue in that Motion, till ſome External Force puts a Stop to it, ſince it's own Innate Force of Gravitation may give a Check, or it's Expanſive, as in Flame, an Addition to it.

If there is a Perfect Inertia in Matter, the Leaſt Force Impreſs'd will be Equally capable of Moving the Greateſt, or the Minuteſt Body.

§. 6. THE Idea of Matter, which this Philoſophy aſſerts, is, that it is Endu'd with a Perfect Inertia, and therefore, if it is in Reſt or in Motion, it muſt continue ſo, unleſs we ſuppoſe it can Move it ſelf, when at Reſt, or Stop it ſelf, when in Motion; But if there is an Abſolute Inertia in Matter, ſo that it has no Force or Activity of it's own, it will be conſequent, that the Leaſt Impreſs'd Force will be capable of Moving the Greateſt Body Equally with the moſt Minute; for whilſt Matter is Nothing elſe but a Maſs of Something, which neither Acts nor Reſiſts, if we take a Greater or a Leſſer Portion of it, it will be the ſame; And for a Parallel Reaſon, that 3 Nothings are Equal to 5 or 100, ſince a Sum or Aggregate of 3 Portions of Inaction in Reſpect of Motion, will be all one with 5 or 100.

Motion and Reſt may as well be affirm'd of an Abſtracted Idea, as of Matter, which has an Entire Inertia.

§. 7. THE Truth is, as we have before obſerv'd, Matter without any kind of Force, whether of Activity or Reſiſtance, that is, when we make it no more than an Extended and Impenetrable Subſtance, is not any thing elſe than an Abſtracted Idea, it is a Notion of the Mind, and we may as well affirm a Mathematical Surface, or Solid to perſiſt in it's State of Motion or Reſt, as Matter to do it, whilſt we conſider Nothing in Matter but Space, and without any kind of Force belonging to it; and it will be eaſy for us to band a Thought about of our own Forming, as we pleaſe, tho' it will be ſcarcely a Queſtion, whether an Abſtracted Idea, any more than Space which is one, is capable of any Real Motion at all, becauſe it certainly is not.

The Vis Inertiæ, *as defin'd by* Sir Iſ. New*ton, ſeems to be little elſe than a Contradiction.*
Sir Iſ. Newt. Princip. Math. Defin. 3.

§. 8. IF the foregoing Sections are True, it will be farther Difficult to account for another Property, which is aſſerted concerning the Inertia, that it is an Innate Force, by which Matter is very hardly, and with Reluctance, made to change it's State of Reſt into Motion, or the Contrary, and indeed ſo Difficult, as will require a Sagacity capable to Reconcile Contradictions; for Matter, by the foregoing Property of it's Inertia, will continue in the Motion or Reſt Impreſs'd upon it, becauſe it has no Force of it's own, but in this Second Property it will be hard to put it out of the State in which it is, becauſe it has a Force of it's own; that theſe Philoſophers may therefore be conſiſtent with themſelves, we muſt tell them, that their Notion of Matter is well Adapted to ſolve both, being an Abſtracted one, which may, on that Account, be with or without Force, according to the Humour they are in, of making an Inſignificant and Trifling, or a very Weighty and Important Buſineſs of it.

§. 9. To Conclude, all that can be ſaid for the Inertia of Matter is, That it is Repugnant to our Notions of it, that Lead ſhou'd Move it ſelf, when in Reſt, or that Fire ſhou'd Determine it ſelf to Reſt, when in Motion, but this does not Evince, that Matter may not have ſome kind of Active Principle in it, becauſe it is ſuppos'd to have none in both theſe Caſes, but the Contrary; for therefore Lead will Determine it ſelf to Reſt, when put into Motion, for this Reaſon only, that it was Difficult to put it into Motion, and Fire wou'd Determine it ſelf to Motion, tho' an External Force was us'd to Quench it, the Firſt from it's Innate Contractive Force, and the Laſt from it's Innate Expanſive. That is, the Contractive Force is Reluctant to the Expanſive, and the Expanſive to the Contractive, but this is not an Inertia, but Force or Action.

The contractive Force has a Reluctance to the Expanſive, and the Expanſive to the Contractive, which is not an Inertia, but Reſiſtance and Action.

§. 10. A Second Law of Nature affirm'd is, that all Motion in it ſelf is Rectilinear, or directed from one Point to another in a ſtrait Line, which is ſuppos'd to follow from the Nature of Motion, ſince it cannot be conceiv'd without ſome Determination from the Point, from which it Commences, and according to the Former Law it muſt Perſevere in that Determination, till it is Chang'd by ſome Foreign Impreſſion; from whence it follows, that where any Body is Mov'd in a Curve, it is made to do it from the Direction and Influence of an External Force, and where ſuch External Force Ceaſes, it will Relinquiſh the Curve, and Proceed again in a ſtrait Line, which is a Tangent to it, at the Point of the External Force's Ceaſing; Thus a Stone whirl'd Round in a Sling deſcribes a Circle, whereas, when it is once quit of it, it flies off in a Tangent.

The Second Law of Nature or Motion Explain'd.

Mr. Whiſton. Prælect. Phyſ. Math. 5. Art. 2. Dr. Keill's Introduct. Sir Iſ. Newt. Princip.

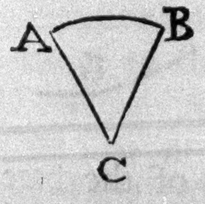
§. 11. That Motion of what kind ſoever muſt have ſome Determination or other is manifeſt, but that a Rectilinear is more Natural to it than a Circular, we deny, ſince it is only offer'd to be Prov'd by the preceding Law of Bodies Perſevering in that Form or Condition of Motion, which is firſt Impreſs'd; Since therefore, that Law is either Uncertain or Falſe, it will be Impoſſible for us with any Safety to depend upon what is Concluded from it; The Motion of Fire Acts in all Kind of Directions, as is Evident from the Undulations of the Flame of it; Water Spilt upon the Ground makes a Thouſand Irregular Forms, *&c.* and ſo it is in Hard Bodies Dropt upon it, which acquire Various Determinations, according to their Different Nature and Action.

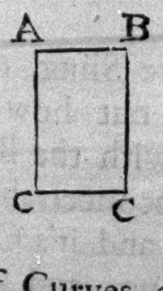
A Rectilinear Motion not neceſſary, or the moſt Natural to Bodies.

§. 12. But that which is more Unfortunate to this Philoſophy is, that a Motion from a Perſeverance of it in one and the ſame Direction, in which it firſt begins, is no more Prov'd to be naturally Rectilinear than Circular, if there may be a Circular Determination, as well as a Rectilinear; and that there in Fact is, is Evident from the very Generation of a Circle, which does not nor can conſiſt of an Infinite Number of ſtrait Lines, as is generally Imagin'd, but the firſt Progreſs it makes is in a Curve; ſince Two Points in a Circle, which are Equidiſtant from a Third, muſt neceſſarily lie in a Curve, according to *Euclid*, and if they approach Infinitely near to each other, will make an Infinitely Small Convexity; But in Deſcribing a Circle there is nothing more to be obſerv'd than the Flux of the Point of the moveable Extremity of the Radius, as in Deſcribing a Right Line the ſame Point is to be obſerv'd, whilſt the whole Radius is carry'd Parallel to it ſelf, which Motions are Expreſs'd in the Figure annext; which if Impartially conſider'd, we ſhall find the Curve *AB* to be as Natural a Determination, where *C* is fixt, and *A* is mov'd, as the Right Line *AB*, where *A* and *C* are both Equably Mov'd. Nor, in this Deſcription, is *AB* in *ACB* any more Retain'd in a Curve, and kept from performing a Strait Line, by a Force in *C*, than *AB* in *ABCC* is preſerv'd in a Direct Line, and kept from performing a Curve, by a Force in *CC*; that is, it is not more Natural, that *C* ſhou'd be moved at the ſame Time that *A* is, than that *C* ſhou'd be fixt whilſt *A* is mov'd, or, which is all one, a Parallelogram is not a more Natural Figure than a Circle.

A Rectilinear not more Natural than a Circular Motion.

Elem. 3. Prop. 2.





§. 13. It is very plain in Mathematicks, that the Circle, the Parabola, Hyperbola, Ellipſis, the Cycloïs, and the whole Order of Curves, are as much deſcrib'd by Determinations peculiar and diſtinct to themſelves, as a Strait Line is, and if we begin in any one of thoſe Determinations, we

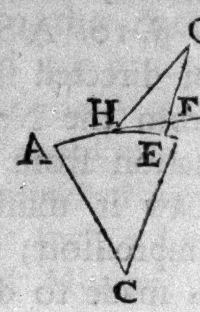
This Rectilinear Motion is aſſum'd as the moſt Natural in Order to Eſtabliſh the

ProjectileForce, whereas the Projectile Force ought to be prov'd to Eſtabliſh the Naturalneſs of the Rectilinear Motion.

always end with it; why therefore a Rectilinear Determination ſhou'd be more Proper to Motion than any other will not be eaſily Comprehended, unleſs by entering into the Deſign of Affirming it, which is to Introduce the Neceſſity of a Projectile and a Gravitating Force in the Elliptick Orbits, which are deſcrib'd by the Heavenly Bodies according to this Philoſophy; That Projectile and Gravitating Force therefore is to be prov'd to Confirm this Law, whereas this Law is on the Contrary Eſtabliſh'd to prove ſuch a Gravitating and Projectile Force.

The Portion of a Circle by this Philoſophy may be as well Reſolv'd into a Centripetal Force and a Secant, as a Centripetal Force and a Tangent.

§. 14. Besides even according to a Law of Motion, which is afterwards alledg'd in the Preſent Philoſophy, where it is ſaid the Diagonal Force is Equal to that of the Sides of a Parallelogram, it is not Neceſſary, that a Body, which deſcribes a Curve, ſhou'd be drawn from the Tangent, or it's firſt Rectilinear Determination, towards the Center of it's Motion; ſince *HE* the Diagonal, which is ſuppos'd to be an Infinitely Small and Strait Line, that is, an Infiniteſima of the Circle, may as well be Expreſs'd by *GE* and *HG* a Secant, as by *FE* and *HF* a Tangent.



The Projectile and Centripetal Direction a Creature of the Mind.

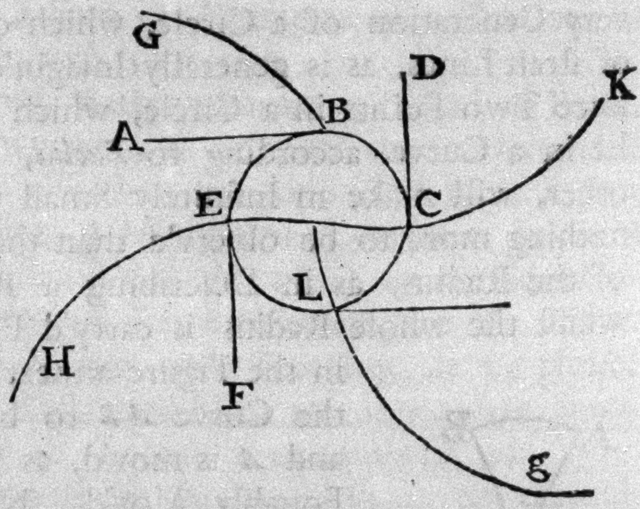
§. 15. That this Projectile or Tangent, and Centripetal Motion is therefore a Creature of the Mind, whilſt it is ſuppos'd neceſſary to Form a Curve, is Evident, ſince, as in §. 12, *A* and *B* may as juſtly be Imagin'd to Tend to *C* and *C* in the ſecond Figure, which yet will not be acknowledg'd, as *A* and *B* to *C* in the Firſt, which conſequently is not prov'd; On the other Hand, *H* may be as Agreeably preſum'd, when the Centripetal Force Ceaſes, to go off in a Secant *HG*, as in a Tangent *HF*, according to the Preceding Section.

A Rectilinear Motion as Compounded, as a Curvilinear, and therefore not more Natural.

§. 16. Farther it is confeſs'd in the Preſent Philoſophy, according to §. 14, that a Rectilinear Motion is Reſolvible into Ten Millions of Determinations, why therefore that ſhou'd be of a Simpler and Plainer Nature than a Motion in a Curve, which cannot well be Reſolv'd into More, is unaccountable; But what is more ſo is, That this Law ſuppoſes a Body to Relinquiſh the Curve, and to Move again in a ſtrait Line, when the External Force Ceaſes, which is to make it Perſiſt in a Rectilinear Motion according to a Force Impreſs'd, but not in a Curvilinear, or to give it an Inertia in the one Caſe, and to take it away in the other; for the Impreſſion is Made, tho' the Force is Diſcontinu'd.

The Inſtance of the Sling Conſider'd.

§. 17. The Inſtance in Fact to prove this Law, is that of a Stone or any other Body mov'd in a Sling, which, when it's Reſtraint is taken off, is ſaid to Proceed in a Tangent, which is ſo far from being true in Reality, that I believe no one ever Found or Experienc'd a Stone to be Slung in a Tangent; For let *CBEL* be the Circle, in which the Sling is Revolv'd from *C* to *B*, *E*, and *L*, if the Projectile goes off in a Tangent, It muſt do it, if we take the chief Points in the Circle, either at *C*, *B*, *E*, or *L*; in the Firſt and Third Caſe, it will be therefore Projected Perpendicular from or to the Horizon; and in the Second and Fourth, Parallel to it, with contrary Directions; whereas in the Second Caſe it is Notorious, that it Proceeds in a Curve not below the Tangent *AB*, which might be ſolv'd by the Gravity of the Projectile, from which it is Determin'd into a Parabolick Circuit, but above it, and by the Force of the bare Projection; and the ſame is very Probable in the other Caſes, in ſome Meaſure and Proportion.



A Conatus to a Motion in a Direct Line from the Center, not the ſame as a Conatus to a Motion in a Tangent.

§. 18. It ſeems to be very plain, that, in this Circular Motion of the Sling, the Body detain'd in it Recedes from the Center by it's Tenſion of it, but how it ſhou'd in every Point retire from the Center, in the ſame Direction with the Radius, and yet depart from it, in a Line Perpendicular to the Radius, is Inconceivable, unleſs it can be affirm'd, that it's Conatus to a Motion in the Tangent, and it's Conatus to a Motion in a Direct Line from the Center, is the ſame Thing; and yet the Conatus to Motion in a Tangent is term'd the Centrifugal Force.

§. 19. We

The Motion of the Sling which is Circular, is not Reducible to any Rectilinear Motions.

§. 19. We cannot but farther ſay, that the Motion of the Sling is to be judg'd of by it's own proper Determination, that is, a Circular, which is no more Reſolvible into any other more Simple ones, as of a Tangent and a Direct Force to the Center, than a Rectilinear Force is, which is ſaid, when conſider'd as a Diagonal, to be Reſolv'd into the Sides of a Parallelogram, of which it is the Diagonal, concerning which we ſhall ſay more afterwards; The Curve of a Parabola is not to be Meaſur'd by any Portion of it's Diameter, it is not Reducible to any Strait Lines, nor is the Circle or rather the Perimeter of it; and as the Curves are Inexpreſſible by any Right Lines, ſo are the Forces likewiſe, ſince the Cauſes and Effects muſt be Proportional; from whence it Enſues, that all Determinations of Curves are in their Motion Peculiar to themſelves, and can be no more Expreſs'd by Right Lines, than Right Lines can be Expreſs'd by them.

The Centrifugal Force of a Sling, as here apply'd, not proper to Explain the Centripetal of the Planets.

§. 20. It is alſo very much to be Queſtion'd, how a Centrifugal Force, as in the Sling, ſhou'd be anſwerable to a Centripetal in the Real Effects of it, and yet the Centrifugal of the Sling is brought to explain the Centripetal of the Planets; for tho' ſo much as it endeavours to Recede from, ſo much it is Kept to the Center, yet the Actual Force is not the ſame, whether it Retires from or Tends to the Center, ſince the one is in a Contrary Direction to the other. If therefore a Body in the midſt of it's Motion is ſet looſe from the Reſtraint it lies under from the Sling, it will not find it ſelf in the ſame Conditions (ſetting aſide the Tangent or Projectile Force) as if it really Tended to the Center, ſeeing the one in an Abſtracted Reaſoning, without conſidering any Foreign or External Action, wou'd make it fall below the Tangent, and the other carry it above it; Not to mention the Peculiar Forces, by which Different Planets tend Differently to their Centers, which cannot be Expounded by the Abſtracted one of a Sling.

Matter is ſuppos'd to be Entirely Paſſive in order to make Motion Rectilinear, which Suppoſition cannot be Granted.

§. 21. Lastly, a Rectilinear Motion is ſuppos'd to be the moſt Natural, becauſe Matter is preſum'd to be perfectly Paſſive and Obedient to the Impreſſions made upon it, and the moſt Simple Impreſſion is Imagin'd to be that of a Right Line; but if Matter is not entirely Paſſive, which has been Prov'd, or if a Curve Line is as much a Single Determination (as it is Peculiar to it ſelf) as a Right Line is, which has been likewiſe Evidenc'd, or laſtly, if a Right Line is or may be Compounded of ſeveral Determinations, as well as a Curve, which this Philoſophy affirms, it is far from being True or Demonſtrable, that Motion is in it ſelf Rectilinear, and conſequently is no more a Law of Nature or Motion, than if we ſhou'd aſſert that Motion in it ſelf is always perform'd in a Curve.

The Third Law of Nature or Motion Explain'd.
Mr. Whiſton's Phyſ. Math. Prælect. 5. Art. 3.

§. 22. A Third Law of Nature or Motion is, That all Bodies, which are Revolv'd upon a Center of Motion, endeavour to Recede from that Center, and the Quicker the Circumvolution is, the More they ſtrive to Retire from it; for ſince Bodies in themſelves affect a Rectilinear Motion, they wou'd proceed in a Tangent of the Curve, which they Deſcribe, if they were not drawn from thence by the Centripetal Force, which Attracts them; which therefore is Contrary and Equal to the Projectile, or the Force Expreſs'd by the Tangent; And conſequently, if the Gyration or Revolution in the Curve is Swifter, that is, if the Central Force, by which it is Retain'd in the Curve, is ſo, the Projectile Force or that, which is Expreſs'd by the Tangent and Repreſents the Centrifugal, muſt be Swifter likewiſe, ſeeing it is exactly Equal to the Former.

Curves are affirm'd to be actually Deſcrib'd by a Centrifugal and a Centripetal Force.

§. 23. As to what concerns Bodies in themſelves Proceeding in a Strait Line, we have in an Abſtracted Reaſoning conſider'd, what Arguments there are to Evince it, and the Inſtance of the Sling made uſe of to Explain it; and from the Whole we cou'd not find any juſt Conviction of the Law of Nature, which was propos'd upon that Principle, and upon the ſame Account alſo, this Third Law is Precarious and Uncertain; we ſhall omit all that has been ſaid before, and at preſent Examine, what is farther Advanc'd in this Law, more than that, which has been already Confuted; for tho' Abſtracted Determinations, ſuch as a Curve Line might be Reſolv'd into by the Mind, were before only modeſtly Introduc'd, we are now plainly told, that they are Actual Forces, which Deſcribe theſe Curves, without any other Proof or Demonſtration of it, excepting that of Pronouncing it.

The Centrifugal and Centripetal Forces are aſſum'd and not prov'd.

§. 24. Since therefore the Motion in a Curve is Actually ſaid to conſiſt in a Centripetal and a Centrifugal Force, which are Equal, the one of which is Expreſs'd by a Direction to the Center, and the other by that of the Tangent to the Curve, we muſt Inquire by what Methods this Law is Prov'd; It is not by conſidering the Curve in an Abſtracted Senſe, as a Mathematical Figure, by what has been before alledg'd; If it is by Real Forces in Nature, thoſe ought to be firſt Prov'd, before they are Aſſum'd, or otherwiſe we cannot but think they are ſuch, which the Philoſopher is pleas'd Arbitrarily to Contrive and make Uſe of, as ſhall beſt Serve his own Purpoſes; For the Attraction to the Center in Bodies, which Revolve in a Curve, is far from being Prov'd by this Law, becauſe this Law, and thoſe, on which this Depends, are Introduc'd to Prove an Attraction, or a Force, which draws the Body from a Tangent into a Curve, that is, towards the Center; and on the other Hand, a Real Projectile or a Rectilinear Force is Equally Unprov'd, becauſe it is Aſſum'd to ſhew, how Curves are deſcrib'd by a Gravitating Force, which is not as yet Demonſtrated.

The Projectile or Centrifugal Force not more Natural than that of Gravitation.

§. 25. In Reſpect of theſe Projectile and Gravitating Forces, it will alſo be juſtly Inquir'd, why the Projectile Force, in the Tangent, ſhou'd be the Natural one, and the Centripetal, in the Curve, ſhou'd be the Impreſs'd; for altho' a Rectilinear Motion were obſerv'd by Bodies left to themſelves after they are once depriv'd of their Reſt, it is yet plain, that they muſt be put into Motion by an External Impulſe, and conſequently, the Effect which Reſults from it, according to the Direction of the Tangent, is as much owing to a Foreign Cauſe, as their being Drawn from that Determination into a Circular or Elliptical, the only Difference being, that the Effect from Impulſe is continued, when the Cauſe Ceaſes, but that from Attraction is Conſtant, as well as the Cauſe which produces it, which can make no Alteration, as to the Forces being Equally Unnatural, and Extraneous to the Bodies which are Mov'd by them; If therefore both the Projectile and Centripetal Force is Impreſs'd upon Bodies mov'd in a Curve, it is well if it is not a Contrivance in the Philoſopher, rather than of Nature, to Project firſt and then Attract the Bodies, whoſe Revolutions are endeavour'd to be Explain'd; eſpecially, ſince they are both Occult Qualities, and no Account is given of them, how they come to Reſide in the Bodies, which they Agitate, and by which they are ſuppos'd to be Retain'd and Continu'd in their Orbits.

The Sling no Proof of a Body's being kept in it's Orbit by a Centripetal Force in the Senſe of this Philoſophy, but by a Centrifugal or Expanſive.

§. 26. We muſt likewiſe add, that the only Inſtance of a Body's being kept in it's Orbit by a Centripetal Force is that of a Sling, which Really is a Centrifugal, and, as we in part before obſerv'd, cannot be Compounded of a Tangent or Projectile Force, and a Centripetal, ſince the Projectile is ſaid to be Natural to a Body once put into Motion, and conſequently that Force, namely the Centripetal, which draws the Body from a Tangent into a Curve, and therefore is Superior to the Projectile, ſhou'd be ſenſible to the Hand, which Revolves the Sling, whereas it perceives no ſuch Force, but the Contrary, that is, a Conatus of the Body to Recede from the Center; This is alſo confirm'd by the Revolution of a Globe round it's own Axis, which, if in the Interim of it's Motion any Liquid is pour'd upon it, will not throw it off in Tangents, but diſperſes it every Way, and, inſtead of caſting it in Horizontal Lines, makes ſome parts of it Aſcend into the Air it ſelf, which is an Evident Argument, that the Circular Motion of a Body is neither Centripetal, whilſt it has an Oppoſite Direction, nor Projects in a Tangent, whilſt it Repells what comes in it's Way in a Perpendicular Direction, as well as an Horizontal, and in all other Intermediate ones.

The Centrifugal Force cannot be the ſame as the Centripetal.

§. 27. If it is ſaid, that the Force of Retiring from the Center is the ſame, as that of Tending to it, upon the Account of Action and Re-action being Equal, whatever that Law is, which we ſhall afterwards Conſider, no Law in the World or in Nature can make Two oppoſite Directions of Motion from a Point and to a Point to be the ſame.

If the Centrifugal and Centripetal Forces are Equal no Motion in a Curve will Enſue.

§. 28. There is another Thing affirm'd in this Law of Nature or Motion, which is, that the Centripetal or Attractive Force to the Center is exactly Equal to that of the Centrifugal or Projectile; If ſo, it wou'd be Impoſſible there ſhould be any Motion in a Curve at all; For if the Projectile Force endeavours to Proceed from the Point of Contact of the Tangent with the Curve with the ſame Energy or

Action, with which the Attractive Force inclines to the Center, the Attractive cannot draw the Body, which is in a Projectile Motion, from that Point, becauſe it is not Superior to it, and all that can follow will be only a perfect Quieſcence; Since if contrary Forces, which Differ in nothing excepting in their Direction, are in an Equal Balance, the Reſult muſt be that of no Motion at all.

The Centrifugal Force or the Projectile in a Tangent being at a Right Angle with the Centripetal cannot account for their Forces being in an Æquilibrium.

§. 29. **THAT** which makes this Doctrine more Unaccountable is, that the Centrifugal and Centripetal Forces, which in Deſcribing a Curve are ſuppos'd to be in an Æquilibrium, and which it is Impoſſible they ſhou'd Deſcribe, if they were in an Æquilibrium, according to the Preceding Section, that we ſay, which makes it more Unaccountable is, that the Centrifugal and Centripetal Forces are not in a Direct Oppoſition to each other, but in a Right Angle, the Projectile going off in a Tangent, and the Centripetal or Attractive Tending to the Center in a Radius Perpendicular to it; for where Forces are Oppoſite or Contrary, it is eaſy to Imagine an Equal Balance betwixt them, if the Forces are Equal, but where they are in a Right Angle the one to the other, it is not, nor can be Prov'd, that they are in an Equal Balance; Becauſe if the Projectile Force is Equal to the Attractive, according to the Suppoſition of this Philoſophy, and that Equality is had and found in Forces which are Contrary to each other, it is Impoſſible it ſhou'd be in Forces, which are not Contrary to each other, and are in a Right Angle only in their Mutual Direction; If an Equality of Forces may be affirm'd without Contrary Directions, it will lie upon this Philoſophy to Prove, what Proportion of the Projectile Force will be exactly Agreeable and Analogous to the Attractive, which it has not done; ſince it Aſſumes both of them, and after that does not Inform us, what Quantity of Motion there is in the Projectile or Attractive to make them Equal, where their Directions are at Right Angles.

From what Forces the Circular Motion of a Stone in a Sling ariſes.

§. 30. **THE** Circular Motion of a Stone in a Sling Ariſes therefore, not from a Centripetal Force, by which it tends to the Center of it's Revolution, and from a Centrifugal, which Acts in the Direction of a Tangent, but from a Centrifugal or Expanſive, Communicated to it by the Expanſive Force of the Hand's Motion, which is Diametrically Oppoſite to the Centripetal of the Stone, and by which it is overcome, and made to Revolve in the Orbit preſcrib'd to it by ſuch Expanſive Force.

THE Stone in this Caſe, tho' it is Endu'd with a Power of Gravity, or of Deſcending to the Earth, yet finds that Inclination and Propenſity actually Deſtroy'd by the Contrary Force of the Expanſive Impreſs'd upon it, as if a Body is Projected Perpendicularly from the Earth with a certain Velocity, the Force of Gravitation or Deſcending downwards Ceaſes, till the Velocity is Spent; and if the Expanſive Force by the Revolution of the Hand is imparted in a Circle, for the ſame Reaſon the Stone will not actually Deſcend, nor will have any Inclination to it, ſo long as that Force Continues, and ſo far is it from being True, that the Centripetal and Centrifugal Forces are Equal, that the Centripetal Force is ſuperſeded and ſurmounted by the Centrifugal.

THUS in any Two Forces acting Contrary to each other, whilſt the one Prevails, the other Submits; If a Body is carry'd by a Superior Velocity one way of $+12$, and another is carry'd with a Contrary, but with a leſs Force of -5, the contrary Force will be Deſtroy'd, and there will be an Exceſs of the Superior Velocity of $+7$, and if there were an Æquilibrium of Two oppoſite Forces, there wou'd not be a Revolution, as in the Sling, or a Motion, as in contrary Velocities, but a perfect Quieſcence.

LIKEWISE, the Centrifugal or Expanſive Force has not it's Direction in a Tangent, but as the Centripetal Force is entirely Vanquiſh'd and Subdu'd, and the Stone is only Detain'd by the Obſtacle of the Sling from Retiring from the Expanſive Force Imparted by the Motion of the Hand, ſo the Expanſive Force, which it acquires from thence, is Directed by the ſame Circular Motion, and is therefore Circular, and we may as well affirm, that the Hand Moves in a Tangent, as that the Stone in the Sling affects ſuch a Motion, we have ſhewn that the Stone in a

Sling has no ſenſible Centripetal Force, and we may add, that if it had one to the Earth, it can with no Manner of Reaſon be ſuppos'd to obtain one to the Center of the Sling's Motion, but if it has neither of theſe Centripetal Forces, what ſhou'd give the Centrifugal the Direction of the Tangent? Effects are not produc'd without their Cauſes, and there is no Imaginable Cauſe of the Expanſive or Centrifugal Acting in that Direction, unleſs we will ſuppoſe the Hand acts in a Tangent, when it acts in a Circle, and that a Tangent and a Circle are the ſame; 'Tis known from whence all this Doctrine proceeds, in Reſpect of the preſent Philoſophy, Namely, from it's eſpouſing Projectile and Centripetal Forces, and the firſt Hint, which was given of Reaſoning in this way of Tangents to a Circle, was from *Des-Cartes*, whoſe other Notions have been either Deſpis'd or Confuted by this very Philoſophy, but the Preſent retain'd for the Valuable Conſideration we have mention'd.

Des-Cartes's Princip. Philoſ. Part. 2. §. 39.

The Application of this Doctrine to the Revolution of the Planets round the Sun by Means of the Expanſive and Contractive Forces, as well as about their own Axes.

§. 31. LET us now apply this to the Revolution of the Planets Round the Sun, and if what we have before propos'd upon that Subject is not Satisfactory and Convincing, where we alledg'd the Balance for an Explanation of it, and made the Sun to be the Center of Detention, and the Planets to have a Greater Force of Expanſion and Velocity, the farther they remov'd from it, perhaps what we here offer may be a farther Confirmation of ſuch a Doctrine.

IT is acknowledg'd therefore, that the Sun has an Immenſe Expanſive Force, by which it diffuſes it's Light and Heat to a Prodigious Diſtance, much beyond our Earth and to the very Orb of *Saturn*; we have ſaid, that the Reaſon why the Solar Heat is not ſo Intenſe here, ſeems to proceed from the Moon's being plac'd ſo near us, which we take to be a vaſtly Cold and Contractive Body, and therefore qualifies that Exceſſive Heat, which wou'd otherwiſe Burn up the whole Globe; and therefore, notwithſtanding the Moderate Heats, which are perceiv'd in this Earth, in Reſpect of the Sun's Immenſe Expanſive Force, ſuch Force may yet be propagated with a ſufficient Degree of Heat, as it is of Light, to *Saturn*.

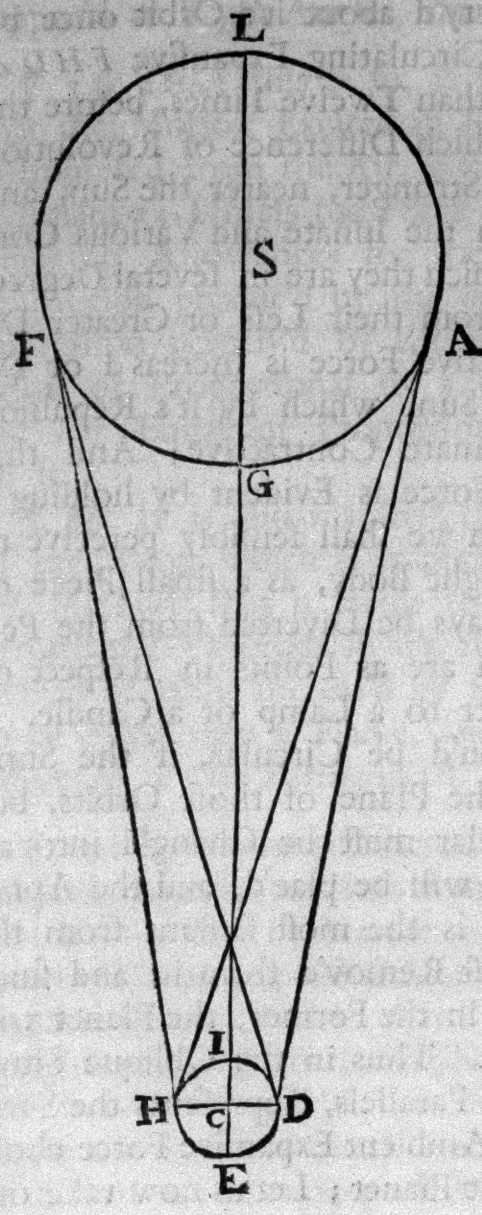
'TIS alſo confeſs'd, that the Sun is Revolv'd about it's Axis, and that the Earth and the Planets, in Reſpect of the Sun as their Center, are but as ſo many Points; If therefore the Expanſive Force of the Sun is ſo Great, and the Earth and Planets are but as Points to the Body of it, we may juſtly think, that they are carry'd Round it, like a Stone in a Sling, and that their Contractive or Gravitating Forces are oblig'd to obſerve the Directions of the Sun's Expanſive, by which means they will all Revolve the ſame Way, and Perform their Gyrations according to the Proportion, which there is betwixt the Expanſive of the Sun and the Gravitating or Contractive of the Planets, which are Revolv'd, and that the Infinite and Almighty Agent, by Impreſſing this Force upon the Sun, Directs and Guides the Motion of the whole Solar Syſtem; That this Expanſive Force of the Sun Diffuſes and Diſperſes it ſelf thro' the whole Ambient Plenum, as far as *Saturn*, and Affects it in like Manner, as it ſelf is Affected, not only in Reſpect of it's Light and it's Heat, which is Evident and Confeſs'd, but in Reſpect of it's Revolution, by which all the Planets will Revolve round their own Axes the ſame Way, and at the ſame Time Deſcribe their Periods about the Sun.

THUS alſo it is acknowledg'd, that the Atmoſphere is Revolv'd with the Earth; that all Bodies obey the Motions of their Syſtems; that Contractive Bodies are diſſolv'd in a Syſtem of the Expanſive Force, as Metals and Minerals in Spirit of Nitre; and Expanſive Bodies reduc'd to a Syſtem of the Contractive Force, as in Chryſtallizations, and in the Gravitation of all Bodies to the Earth, tho' they are never ſo Light and Expanſive.

Why the Planets are Revolv'd from Weſt to Eaſt, and not the Contrary. Dr. Gregory's Aſtronom. §. 5. Prop. 30. Lib. 1. Dr. Keill's Aſtronom. Lect. 5. P. 47.

§. 32. THE Sun is obſerv'd to Revolve from Weſt to Eaſt about it's Axis, from the Rotation of it's Maculæ in 25 or 27 Days, which Axis is inclin'd to the Plane of the Earth's Orbit, in an Angle of about 7 Degrees, If therefore the Sun Attracted the Planets, it would make them Revolve from Eaſt to Weſt contrary to Experience. Thus in the Figure annex't, If *AH*, *AD*, *FD*, *FH*, and *GC*, and all the Intermediate Lines ſhou'd Repreſent the Sun's Attractions in *S* of the Earth or any other Planet in *C*, ſuppoſing now the Sun to be Revolv'd round it's Axis from Weſt to Eaſt, or from *F* to *A*, the Attraction of *D* to *A* wou'd immediately Ceaſe, or be continually

Diminiſh'd,



Diminiſh'd, but there wou'd ſtill remain that of *H* to *A*, and *H* to *F*, which, as *A* mov'd towards *L*, and *F* towards *G*, wou'd Draw and Impell *H* towards *D*; and ſince the ſame wou'd happen in the next Point to *H* towards *E*, and to all the following ones from *H* to *E* and *D*, and ſo round the Earth's Circumference, it is manifeſt, that the Earth or Planet wou'd be Revolv'd from *H* to *I* and *D*, and from *D* to *E* and *H*, or in a contrary Direction to that of the Sun, which is ſuppos'd to be carry'd from *F* to *G*, and from *G* to *A*; for if it was carry'd in the ſame Direction, it would move from *H* to *E*, and from *E* to *D*.

On the other Hand, if we ſuppoſe the Sun to be endu'd with an Expanſive or a Repulſive Force, it will cauſe the Planets to Revolve round their Axes the ſame Way, that is, from Weſt to Eaſt. Thus, as before, whilſt the Repulſive Force of the Sun is mov'd from *A* towards *L*, it's Effect Ceaſes, or is Diminiſh'd in the Repelling of *D*, but Continues and Exerts it ſelf from *A* to *H*, by which *H* is Thruſt and Protruded towards *E*; The Repulſive Force from *F* to *D* is more Oblique than that from *F* to *H*, and therefore Weaker, and moreover is continually Diminiſh'd, as *F* comes to *G*, and conſequently the Repulſive from *F* to *H*, will ſtill Forward and Promote the Revolution of *H* towards *E*; and ſince, in the Motion of *F* towards *G* or *A*, the ſame will happen to any Point next to *H* towards *I* and *D*, it is plain, that the Revolution of the Planet will be from *H* to *E*, and from thence to *D* and *I*, and ſo to *H*, that is, from Weſt to Eaſt, and in the ſame Direction with the Sun's Gyration about it's Axis, which is a Problem, that has not been hitherto ſolv'd; and particularly, *Venus* is Revolv'd round it's Axis in 23 Hours from Weſt to Eaſt, the *Earth* in 24 Hours, *Mars* in 24 Hours 40 Minutes, and *Jupiter* in 9 Hours 56 Minutes, *Mercury*'s and *Saturn*'s Revolutions are not aſcertain'd, the one being too near the Sun, and the other too diſtant from us, for an Obſervation of it's Maculæ, whilſt the Sun is 25 or 27 Days in Revolving; The Reaſon of which is, becauſe the Sun may Propell *HEDI* quite round it's Axis, which is but a ſmall Body in Compariſon of it, whilſt the Sun ſhall perform only $\frac{1}{27}$th of it's Revolution, as in *Venus*, the *Earth*, and *Mars*, and not half ſo much, as in *Jupiter*, if *Caſſini*'s Obſervations are True, which perhaps may be Uncertain.

Sir Iſ. Newt. Optic. Quer. 28. Pag. 334. Octav.

But beſides, this Expanſive Force of the Sun, by which it throws the Planets round upon their Axes in the ſame Way and Direction, in which it is Revolv'd about it's own, will alſo carry the whole Planet round the ſame Way, in which it is Mov'd, for if all the Lines *FH*, *FD*, *GC*, *AH*, *AD* are ſuppos'd to be carry'd together in a Revolution from *F* to *A*, and from thence to *L*, the Planet *HEDI*, which is in the midſt of ſuch a Moving or Revolving Action, will be Determin'd the ſame Way, like a Stone in a Sling, as we before mention'd, and which has been the Inſtance, made uſe of by Philoſophers, to Explain the Imaginary Projectile and Centrifugal Force of the Heavenly Bodies. Here is therefore an Account given, why the Planets Revolve about their Axes and in their Orbits the ſame Way, which will agree with what we have already offer'd concerning them.

§. 33. The only Difficulty, which remains to be ſolv'd, is, why their Periods about their Axes and in their Orbits are Different, and why thoſe Orbits are Excentrical and Elliptical, and not Circular; As to the Firſt, *Mercury* Revolves about the Sun in 3 Months, *Venus* in ſomething more than Eight, and the *Earth* in Twelve or one Year, *Mars* in 2 Years, *Jupiter* in 12, and *Saturn* in 30; in Reſpect of which it is Evident, that the Nearer the Planets are in General to the Sun, the Quicker is their Revolution, and the Farther they are Remov'd from it, the Slower, which perfectly Correſponds

Why the Periods of the Planets are Various and are Perform'd in Ellipſes, and how the Revolutions of the Secondary Planets are ſolv'd.

Sir Iſ. Newt. Princip. Par. 3. Dr. Gregory's Aſtronom. Dr. Keill's.

ſponds with what we affirm, that the Expanſive Force is Stronger, nearer to the Sun, and Weaker, at greater Diſtances from it; ſo that the Sun ſhall Revolve more than three Times round it's Axis, before *Mercury* ſhall be carry'd about it's Orbit once by a Continual, Conſtant, and Uniform Impulſe of the Circulating Expanſive *FHDA*, and more than Eight Times, before *Venus* is, and more than Twelve Times, before the *Earth* is, and ſo on of *Mars, Jupiter,* and *Saturn*; and which Difference of Revolution does not only proceed from the Expanſive Force being Stronger, nearer the Sun, and Weaker, the farther it Acts from it, but likewiſe from the Innate and Various Contractive Forces of the Planets, which are Revolv'd, by which they are in ſeveral Degrees Renitent and Reluctant to the Expanſive, as well as from their Leſs or Greater Diſtances from the Sun, by which their proper Contractive Force is Increas'd or Diminiſh'd; 'Tis alſo the ſame Expanſive Force of the Sun, which by it's Repulſion keeps the Planets from Gravitating to it by their Innate Contractive; And that Fire and Flame Obtain this Expanſive and Repulſive Force is Evident by holding a Feather over the Flame of a Lamp or a Candle, which we ſhall ſenſibly perceive to be Agitated and Repell'd by it, and by Letting any Light Body, as a ſmall Piece of Paper, fall Perpendicularly over it, which yet will always be Diverted from the Perpendicular by this Repulſion; But the Planets, which are as Points in Reſpect of the Sun, will not be much more to it, than a Feather to a Lamp or a Candle.

As to the Second, theſe Orbits of the Planets wou'd be Circular, if the Sun's Axis, on which it is Revolv'd, was perpendicular to the Plane of thoſe Orbits, but being Inclin'd, what otherwiſe wou'd have been Circular muſt be Chang'd into an Ellipſis; In the Focus or Umbilicus of which the Sun will be plac'd, and the Aphelium of the Orbit will be in that Part of it, which is the moſt Diſtant from the Sun's Horizon, and the Perihelium, where it is the leaſt Remov'd from it, and ſince the Expanſive Force is Greater in the Latter Caſe than in the Former, the Planet will move Faſter in the Perihelium than in the Aphelium. Thus in the Oblique Situation and Rotation of the Globe, the Æquator, and it's Parallels, Repreſents the Sun's Revolution about it's Axis, together with that of the Ambient Expanſive Force about the Sun, the Ecliptick expreſſes the Inclin'd Orbit of the Planet; Let us now take one Point of it, namely *Capricorn*, and place it under the Meridian, which wou'd be carry'd round in the Diurnal Circle belonging to that Tropick, by the Revolution of the Sun's Expanſive, if the Planet was not oblig'd to perform it's Orbit in a Plane about the Sun, which the Tropical Circle is not, from whence the Oblique Diurnal Circle Projected on the Vertical Circle, which ſtands at Right Angles with the Meridian, or any other Circle Parallel to it, becomes an Ellipſis, whoſe Perihelium is in *Capricorn*, as being neareſt to the Sun's Horizon, and whoſe Aphelium is in the Point oppoſite to it, and which is the fartheſt from the ſame Horizon, ſuppoſing the Tropick of *Capricorn* is brought to the Meridian; On the Contrary, the Tropical Circle of *Cancer*, Projected on the aforeſaid Vertical, Forms an Ellipſis, whoſe Aphelium is in *Cancer*, and the Perihelium in the Point oppos'd to it, and ſince all the Intermediate Diurnal Circles are to be Projected on the ſame Vertical, they will be Elliptical, and the *Earth*'s Orbit or the Ecliptick will be an Ellipſis, whoſe Perihelium is in *Capricorn*, and the Aphelium in *Cancer*, according to what is Experienc'd by Aſtronomers.

The Revolutions of the Secondary Planets about their Axes, and in their Orbits round the Primary, as the Moon and the Satellites of *Jupiter* and *Saturn*, may be accounted for the ſame Way, excepting, that here Contractive Forces, which Act oppoſite to each other, are to be Aſſum'd inſtead of the Expanſive of the Sun and the Contractive of the Planets; For an oppoſite Contractive has the Force of an Expanſive, as we have already ſhewn.

The Fourth Law of Nature or Motion Explain'd. Mr. Whiſton. Prælect. 5. Phyſ. Math. Art. 4. Dr. Keill's Introduct. Lect. 11. Lex. 2. Sir Iſ. Newt. Princ. Lex. 2.

§. 34. The Fourth Law is, that the Change of Motion is Proportional to the Force of Motion, which is Impreſs'd, and is made according to the Right Line, by which the Force is Communicated; Thus if any Motion is Generated by a given Force, a double Motion will be produc'd by a double Force, and a Triple by a Triple, *&c.* whether ſuch Force is Impreſs'd at once, or Succeſſively, and by Degrees; and ſince Motion is determin'd to that Part, towards which the Force directs it, if a Body is in Motion before, the Force Impreſs'd, and which conſpires with it, will make an Addition to the Former, or, if it is contrary, will make a Subſtraction from it, and in the firſt Caſe will Increaſe the Motion, and in the laſt Diminiſh it; if therefore the Force more or leſs Conſpires with or Acts contrary to the former Motion, in different Degrees of Obliquity it will more or leſs Increaſe or Diminiſh it, and, if they are Oppos'd at Right Angles, there will be no Increaſe or Diminution of it.

§. 35. **That** this is ſo, is far from being True, for we plainly find in Experience, that the Motion Excited in Fire is not Equal or Proportional to the Force, which is Impreſs'd from the Air, either Naturally or by any Artificial Methods, by which it is Inflam'd; Unleſs the Force of Air, Impreſs'd upon the Hand, has the ſame Effect (making Allowances for the Original Motion of the Fire) with the Difference of Motion produc'd by the ſame Force of Air Impreſs'd upon Fire, and after ſuch Impreſſion perceiv'd by the Hand, that is, Unleſs a certain Degree of Coolneſs is Equal or Proportional to a certain Degree of Heat, and therefore the Force of the Air Equal or Proportional to the Motion or Force produc'd in the Fire by it.

Objection the Firſt againſt this Law, that the Motion of Fire is not Proportional to the Motion of Air Impreſs'd.

§. 36. **It** is alſo manifeſt, that, in Fire and Flame, it ſo little obſerves the Direction of the Force, which is Impreſs'd upon it from the Air, Naturally or by any Artificial Methods, that it ſeems to be Reluctant to and to oppoſe it, where it is not too Strong and Vigorous in it's Action, and where it is, to Die and Languiſh under it, inſtead of being Increas'd by it; And thus we find ſmall Sparks, by a Gentle Fanning, to be rais'd into a Flame, and, on the Contrary, by an Exceſs of Force in the Air Impreſs'd, to be Extinguiſh'd.

Objection the Second, That the Motion of Fire by a leſſer Force of Air is Increas'd, and by a Greater Diminiſh'd.

§. 37. **If** ſuch an Inſtance, as that of Fire, which we have mention'd, is Rejected as Abſurd, becauſe we know nothing of the Philoſophy of it, we muſt at leaſt ſay, that it is a Part of Nature and Deſerves as much to be Explain'd and Taken into the Laws of it, as any other Material Being whatſoever; For it will, in Spite of all our Hypotheſes contriv'd to ſolve the Grand Appearances of the Revolutions of the Heavenly Bodies, be an Invincible Objection againſt them, if they cannot Account for Effects in Nature, which are Near us, which are Conſtantly under our Eyes, and are Open and Plain to Every one's Obſervation; It will only be Evident, that ſuch Laws of Nature are Mere Hypotheſes and Suppoſitions, Invented in Order and with a Deſign to explicate, if poſſible, ſome Mighty Phænomenons, which the Philoſopher has in View, without Regarding, and in Contempt of thoſe others, and as he is pleas'd to Think Meaner, with which we are daily Converſant, and perhaps, for that Reaſon, becauſe they are ſo vulgarly known.

The Motion of Fire ought to be Explain'd in a true Philoſophy, as well as the Motion of Heavy Bodies, which the Sun and Planets are ſuppos'd to be.

§. 38. **Albeit** the Deſign and Intention of the preſent Law of Nature is really nothing elſe, than what we have Declar'd in the foregoing Section, and Fire or Flame is excluded from Nature, becauſe in Reality it is too Active for a Dull and Phlegmatick Philoſophy to Explain, whilſt other Inertial Bodies are only Conſider'd, Which it is Preſum'd we can Manage and Put into Motion as we pleaſe, I ſay, notwithſtanding this, we ſhall Enquire farther, whether this Law is Expreſsly Applicable to any other Matter Different from Fire, which we can Name, and which may, perhaps, ſeem more Tractable and Obedient to the Laws, which Philoſophers ſhall Think fit to Impoſe.

Inertial Bodies only conſider'd in this Philoſophy.

§. 39. **If** we take an Elaſtick Body, or a Hard, or a Soft, or a Viſcous, which is Endued with ſome particular Force or other of it's own, it is Impoſſible the Change of Motion ſhou'd be Proportional to the Force, which is Impreſs'd, becauſe that Occult and Unknown Force, at leaſt in the Preſent Philoſophy, by which a Body becomes Elaſtick, or Hard, or Soft, or Viſcous, *&c.* will have a Power of making ſome Alteration or other in the Force, which is Impreſs'd, if ſo, then, altho' a ſingle Force produces a certain Motion, a Double will not produce the Double of it, ſince that Alteration, which is made in the Force Impreſs'd by the peculiar Power of the Natural Body, will unavoidably cauſe the Motion not to be Proportional to the Force; For let the Single Force be 2, and the Double 4, and the Alteration made in the Forces by the Power of the Body it ſelf 1, The Effect or Motion, in the firſt Caſe, will be as 1, and, in the Second as 3, which evidently are not as 2 to 4; When we ſay the Alteration is as 1, and Subſtract it from the Force Impreſs'd, we only put it as an Inſtance, for let the Alteration be what it will, it muſt be Expreſs'd by ſome Quantity, which is either to be Added to or Subſtracted from the Forces Impreſs'd, ſo as to make a real Alteration in them, and let this be done any Way, the Reaſoning is

If Bodies have any kind of Force belonging to them, as all Elaſtick, Hard, Soft, and Viſcous Bodies have, the Motion Impreſs'd will not be Proportional to the Motion Receiv'd.

the ſame againſt Motion being Proportional in Natural Bodies to the Forces, which are Impreſs'd.

If Matter is unactive, we may as well take a Cypher as Matter, and argue upon the Motions of it, with as much Penetration and Sagacity.

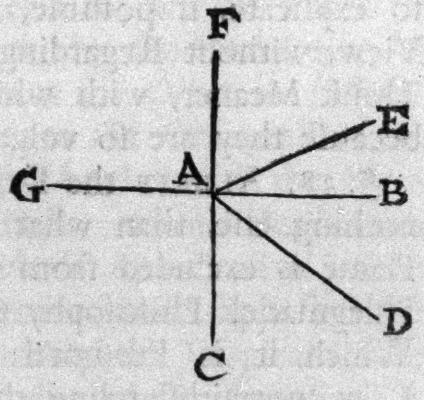
§. 40. Lastly, If we ſuppoſe Matter or Body to be Anſwerable to the Abſtracted Idea of a Solid and Extended Subſtance, which we have Examin'd in the Firſt and this Second Part of our Philoſophy, and ſhewn the Abſurdity and Falſity of ſuch a Notion, that is, if we conſider Matter to be an Idle and Inſignificant Being, which we can Toſs about and Generate juſt as much Motion in it, as we judge convenient to Impreſs, 'tis true, Motion will be Proportional to the *Vis Motrix* which produces it; becauſe this is nothing more than an Identical Propoſition, that the Force which produces any Motion will be Equal to it ſelf, ſince the Effect or the Motion Produc'd, is Equal to the Force or Cauſe which Produces; But of what Service can ſuch an Identical Propoſition be to the true Knowledge of Things, where inſtead of Matter we may as well take a Cypher, and affirm, the Cauſe and Effect being Reciprocal, that Two Acting upon Nothing, the Reſult will be Nothing but the Action of Two? which is wonderfully Sagacious and Penetrating, as well as very Philoſophical and Inſtructive.

The Oblique Directions of Motion not accounted for.

§. 41. Moreover, Granting even this Abſtracted Idea of Motion ſeparate from all Real, the Law of which, as it is here Eſtabliſh'd, being found in no Material Being, which can be mention'd, yet, which is extreamly Unfortunate, this Philoſophy does not account for the Oblique Directions of the Force Impreſs'd, and the Motions, which Reſult from them, altho' no Active Matter is introduc'd to diſturb them, for §. 34. it is ſaid, that if any Motion proceeds in a Direct Line, and a Force acts upon it at Right Angles, there will be no Diminution or Increaſe of the Motion, that in an oppoſite Direction there will be a Diminution of it, and where the Direction of the Force and the Motion Conſpire, an Increaſe, and ſuitably, in all the Intermediate Degrees of Obliquity, the Force and the Motion are variouſly compounded of the Concurring and the Oppoſite.

A Force acting in Right Angles upon a Motion cannot be ſaid neither to give an Increaſe nor a Diminution to it.

§. 42. But in Relation to the foregoing Section muſt we not ask, how all this is Prov'd? It is Plain, if *C* Acts in *A* with the Force *CA* Equal to the Motion *AF*, it will make an Addition of *CA* to *AF*, if they Conſpire, or if they are Oppoſite, there will be a Quieſcence in *A*; if therefore *C* Acts in or upon *A* with the Force *CA* Equal to the Motion *AB* or *AG*, ſuppoſing the Motion on both Hands to proceed from *A* to *B* or *G* at Right Angles, the Force will have an Equal Influence, that is, ſo far as not to determine the Motion from *A* to *B*, or *G*, notwithſtanding which, it wou'd be Ridiculous to Affirm, that an Action in or upon *A*, from *C* to *A*, makes no Alteration in the Motion, from *A* to *B* or *G*, no more than if *A* ſhou'd proceed to *B* or *G* by it's Motion without any Impreſſion upon it in *A*, which yet this Philoſophy does Affirm.



The foregoing Section farther Explain'd.

§. 43. We are conſequently farther to take Notice, that *CA* the Force may be Oppoſite to *FA* the Motion or in the ſame Direction with *AF*, a contrary Motion to the Former, in the firſt Caſe there is an abſolute Privation of Motion, and in the Second it is made Double of what it was before, it will therefore be Impoſſible to ſay with any Reaſon or Truth, that the Force from *C* Acting on *A* is the ſame, whether *A* moves to *B*, or *B* to *A*, that is, *A* to *G*; For if there is this Difference, that whilſt *C* Acts on *A* and *A* moves to *F*, there is a double Force or Motion, whereas on the Contrary, where *F* moves towards *A* there is a perfect Quieſcence, why *C* Acting towards *A* (whether *B* moves to *A* or *A* to *B*) ſhou'd produce no Influence at all upon ſuch oppoſite Forces, as thoſe from *A* to *B*, and *B* to *A*, or *A* to *G*, where it obtain'd ſo conſiderable a one before, is Inconceivable; And the ſame Reaſoning will hold in the Oblique Force of *CA* in Reſpect of the Motion *AE* or *AD*.

The Caſe is, if *F* moves to *A* and *C* to *A*, they will deſtroy each other, ſuppoſing their Forces Equal; If *CA* moves againſt *EA*, it will be Superior to it, if againſt *BA*, it will be ſtill more ſo, if againſt *DA*, it will be Increas'd, till at length a double Force unites in *CA* by an Infinitely ſmall Obliquity of *AD* to *AC*; The Contrary of which will happen, if *C* moves to *A* and *A* to *F*, which will make a double Force, and, in the Infinite Obliquity of *A D*, *A* will move to *D*, whilſt *C* moves or acts to *A*, by which means both their Motions or Forces will be deſtroy'd; ſo that, in the firſt Caſe, there are all the Intermediate Degrees of Motion according to the Various Obliquity of the Forces from 0 to 2, and in the laſt, from 2 to 0.

§. 44. A Fifth Law is, that Re-action is always Equal and Contrary to Action, that is, the Actions of Two Bodies, betwixt one another, whether they are Actions of Impulſe or Attraction, are always Equal, and have contrary Directions; Thus whatever Preſſes or Draws a Thing, it is ſo much Preſs'd or Drawn back by that Thing: For Firſt, If any one Preſſes a Stone with his Finger, there is an Equal Repreſſure of the Stone upon the Finger. Second, If a Horſe Draws a Stone by the Mediation of a Cord, the Stone Draws Back the Horſe with the ſame Force, ſince the Cord, being ſtretch'd on both Hands, will Equally Urge the Horſe towards the Stone, as the Stone towards the Horſe, and will hinder the Horſe from going any farther, than the Stone is mov'd forward. Third, If any Body is carry'd againſt another, and induces upon it a Change of it's Motion, the ſame Change in like manner will alſo be Impreſs'd back upon the Incurring Body, which cauſes the Alteration, and in a contrary Direction, upon the Account of the preſent Law of Action being Equal and Contrary to Re-action; that is, the Repulſe of the Body, whoſe Motion is Chang'd, is Equal to the Impulſe of the Body, which Changes it.

The Fifth Law of Nature and Motion Explain'd.

Mr. Whiſton's Prælect. 5. Phyſ. Math. Art. 5.

Dr. Keill's Intr. ad Ver. Phyſ. Lect. 12. Lex. 3.

Sir Iſ. Newt. Princ. Math. Lex. 3.

It is in this Philoſophy annex't by Way of Caution, that we do not confound Velocity with Motion, for tho' by this Law the Changes of the Motions in Bodies are Equal, thoſe of the Velocities are not, which will be in a Reciprocal Proportion to their Bodies; becauſe the Changes of Motion are Equal, and any Motion is Equal to another, where 10 of Velocity for Inſtance into 3 of Matter is on one Hand Equal to 10 of Matter into 3 of Velocity on the other, and therefore, altho' the Motions are Equal, the Velocities may be as 3 to 10.

§. 45. As to the firſt Inſtance, it can ſignify nothing to the Proof of ſuch a Law, for altho' when I Preſs my Finger upon a Stone, which Reſiſts it, there is a Repreſſure at leaſt Equal to it, it is not certain there is not a Greater, unleſs, becauſe I Preſs with a certain Force, it is taken for Granted, that the ſame Force is Return'd upon that Preſſure; for in Fact it is Evident, that I can Preſs with my Finger, with a Greater or Leſs Force upon a Reſiſting Body, as a Stone; if therefore the Force of Repreſſure is Equal to it, the Stone will have a Power in it ſelf of Repreſſing in Proportion to the Preſſure of my Finger, or not; if it has, it's Proper Reſiſtance is not certain, but varies, as the Action or Impulſe of my Will varies, which is Impoſſible; if it has not, then the Preſſure, which is deriv'd from my Mind, is not always Equal to the Repreſſure of the Body, upon which my Finger by the Direction of it Acts; if it is neither the one, nor the other, but an Abſtracted Idea of Action and Re-action, that ſo much as I Act, ſo much is the Effect of that Action in Reſpect of that, which Acts, and therefore the Cauſe muſt be ſenſible of the Effect, as well as the Effect of the Cauſe, ſuch an Action and Re-action is a Creature of the Mind, and is of no Importance to the Solution of Real Nature; becauſe it only comes to this, that if one Thing has an Influence upon another, that Thing, which is Influenc'd, muſt always have an Influence upon, or a neceſſary Connexion with and Relation to that, which Influences; which is true thus far, that when we Feel we Re-feel, if it may be call'd ſo, that is, when the Mind Acts upon a Body by it's Organs of Senſation, it Perceives what it does, that is, Body Re-acts upon the Mind; when we Preſs a Finger upon a Stone, we feel that Preſſure from without, that is, the Stone Repreſſes, but this Information from without tells us no more, than that we Feel, and not at all, what the Real Action of that Body is, which we Feel; if therefore we take only ſo much Action of the Body, which we Perceive by our Preſſure upon it, we make the Real Nature of Bodies to conſiſt in the Perceptions of our Minds, which is again to affirm External Nature to be a Creature of the Mind, or our own Fancies; for it is Impoſſible, moreover, to ſuppoſe, that the Action of the Finger, directed by the Mind, ſhou'd be the ſame, or have the ſame Effect, in the Return of the Action to the Mind, whether the Preſſure be made by it, upon a Stone, or upon Water, or on Fire; Since it is altogether Abſurd to think, that the Action of the Finger is Equal to

Re action not Equal and Contrary to Action from the Finger preſſing upon a Stone, and being Equally and Contrarily Repreſs'd by it.

that, which Reſiſts it, as in a Stone, or which Yields to it, as in Water, or which Reacts, as in Fire, with a Power Superior to the Preſſure.

The Second Inſtance conſider'd of a Horſe drawing a Stone.

§. 46. The Second Inſtance is; that of a Horſe Drawing a Stone by an Intermediate Cord betwixt them, by which the Equality of Action and Re-action is ſuppos'd to be Prov'd; but who does not ſee, that, if the Stone is Drawn by the Horſe, the Action of the Horſe is Superior to the Re-action of the Stone, or, on the Contrary, if the Stone is not Drawn by the Horſe, the Re-action of the Stone is Superior to the Action of the Horſe? If it is ſaid, that the Horſe and Stone move together, and that if the Horſe does not move, the Stone does not, and if the Stone does not move, the Horſe does not, it is a trifling Way of Proving, that Action and Re-action are Equal, If the Force of the Horſe is Greater than that of the Stone, when he Draws it forwards, or that of the Stone is Greater than that of the Horſe, when it is Impoſſible for him to ſtir it; for, as in the former Caſe of Preſſure in the Finger and Repreſſure from the Body, it is no more than to ſay, we Feel, what we Feel, ſo in the Preſent, if we ſet aſide the Real Forces of the Horſe and the Stone, it is nothing more than to affirm, that the Horſe Draws, when the Stone is Drawn.

The Third Inſtance conſider'd of Motion being Equal in the Change of it, in the mutual Concourſe of Bodies.

§. 47. The Third Inſtance to Juſtify this Law of Action and Re-action being Equal, is, that of Motion being Equal in the Change of it in Bodies, in their Mutual Concourſe, according to §. 44. whereas in any Material Beings, we can mention, it is not certain, and is far from being Prov'd by any Experience, that they obſerve this Law; It is not obſerv'd in the Concourſe of Fluids and Solids, as in Water and Marble, not in Fluids with Fluids, as in Fire and Water, nor yet in Solids compar'd with Solids, as in Steel with Lead, or Wood with Stone; If we will allow theſe ſeveral Bodies to have any Diſtinct and Peculiar Forces of their own.

Sir Iſ. Newt. Optic. Query 31. towards the End.

And from hence it is, that the Quantity of Motion is not always the ſame, betwixt Two Bodies in their Mutual Concourſe, but Motion may be got or loſt, and is more apt to be loſt than got, and is always upon the decay; for Bodies, which are either abſolutely Hard, or ſo Soft, as to be void of Elaſticity, will not Rebound from one another; if the Pendulums be of Lead or Soft Clay, they will loſe all, or almoſt all, their Motions, if of Elaſtick Bodies, they will loſe all, but what they recover from their Elaſticity.

The Doctrine of Expanſive and Contractive Forces, which I have Affirm'd, gives an Account of this Phænomenon, which is here propos'd from Experience and Obſervation, that Motion is more apt to be loſt than got, and is always upon the Decay, (which will not be ſolv'd by the Inertia of Matter or by Gravitation,) becauſe, as the Expanſive Force in Bodies is much leſs than the Contractive, the Latter will be continually prevailing againſt the Former, and the Elaſtick Bodies will loſe all their Motion, but what they Recover from their Elaſticity, that is, from their Expanſive.

Matter ſuppos'd to conſiſt of nothing but Bulk or Magnitude, which is abſurd.

§. 48. If Matter is an Abſtracted Subſtance, which only conſiſts of Bulk or Magnitude, and ſerves by the Parts, of which it is made up, to Number the ſeveral Velocities, or to give the Sum of them in each Motion, it is the ſame as to ſay, that if 5 Velocities, for Inſtance, are carry'd againſt 1 Velocity, which is Five Times Greater than one of the Former, they will be Equal in their Moments, and whatever Change is made by ſuch a Concourſe, it will be Equal in both the Bodies, which Concurr, computing $\frac{1}{5}$th of the Greater Velocity to be anſwerable to 1 of the Leſſer, which, if true, yet ſuppoſes Matter to be nothing but Bulk and Magnitude, which is a falſe and a wrong Notion of it.

Five Velocities not Equal to one Velocity which is Five Times Greater, which this Doctrine ſuppoſes.

§. 49. On the Contrary, that $\frac{1}{5}$th of a Greater Velocity is Equal to 1 Velocity of a different Kind, we have already ſhewn there is ſome Reaſon to Conteſt, which we ſhall here farther Exemplify in the Caſe of Sounds; where ſuppoſing by one Kind of Vibration a Baſe Sound to be produc'd, and by another, that is 5 Times Quicker, a Treble, and the String of the Harp in the Former to be 5 Times Groſſer than that of the Latter; can any one Imagine, that $\frac{1}{5}$th of the Vibration of a Treble will be Equal to 1 of the Baſe, or, if the Treble deſcribes never ſo ſmall a Vibration, and the Baſe never ſo large, that the Velocity or the Sound of the one, will ever be made Equal to that of the other? It ſeems to be Impoſſible, and in General it is abſurd to believe, that 1 Velocity, which is Perform'd in 1 Time, can be Equal to $\frac{1}{5}$th of another Velocity, which Velocity is Perform'd in the ſame Time, unleſs 1 is Equal to $\frac{1}{5}$th: The Caſe therefore is this, that a Force or Velocity, Exerting it ſelf in a Point or a Line, may be 5 Times Greater than another, and will not be Reducible to it, nor will be Equal to the Sum of 5 Forces, each of which are 5 Times leſs, becauſe a Force or Velocity, which is Greater than any one of them, will be likewiſe Greater than all their Velocities put together in their Real Force or Velocity; Thus alſo Fire by the ſame Force, that it ſubdues one Quantity of Force againſt it, is ſufficient to overcome Two, Three, or a Hundred Times of the ſame kind of Force Acting againſt it, and tho' Spaces, deſcrib'd by Bodies which Move, are, according to Mechanicks, compounded of the Times and the Celerities of their Deſcription, and 1 Time into 5 Degrees of Celerity is Equal to 5 Times into 1 Degree of Celerity in the Production of Lines Equal in Length, yet the Intrinſick Value of thoſe Lines will not be Equal, the one being deſcrib'd by a 5 Times Greater Celerity than the other.

Equal Changes of Motion not prov'd upon any Suppoſition.

§. 50. But, that the Changes of Motion are Equal in both Bodies, is leſs Explain'd; for, as in the former Section, if the Velocities are Unequal in the Bodies or Magnitudes, which we will now ſuppoſe Equal, ſince a Unit in one Velocity is not Equal to a Unit in another, and ſince, by what has been ſaid, they are not really Reducible to each other, it will follow, that, upon their Mutual Concourſe, the Changes in each cannot be Equal; if the Velocities are Equal, and the Magnitudes, by which their Sums are Computed, are ſo too, Two Equal Forces, in their contrary Action, will produce Nothing, or, which is the ſame, a Mutual Deſtruction of each other, and therefore there will be no Equal Changes of Motion; if the Magnitudes are Unequal, and the Velocities the ſame in their Degree or their Force, 5 Equal Velocities will be Superior to 1 by 4, and therefore there will not be an Equal Change in the Motions, but 1 will be deſtroy'd and the other 4 proceed without Oppoſition, in all which here is nothing of Action and Re-action, but a Mutual Oppoſition of Forces and no Equal Change of Motion.

If any Thing more is conſider'd than Velocity either Equal or Unequal, and Magnitude or the Sums of ſuch Velocities, that is, if Reſiſtance or Elaſticity, or any other Quality in Matter is, we muſt have Recourſe to the 47th Section, to anſwer the Proof of this Law of Action and Re-action, ſince it is plain from thence, it does not obtain, where Matter is taken in Groſs, and with all it's Qualities; and by the 48, 49, 50, it is as Evident, that it does not, where taken in the moſt Abſtracted Senſe we can Propoſe, Solidity, by Part the firſt of this Philoſophy, being only an Excluſion of one Portion of Matter out of that Space, which another Poſſeſſes, which is alſo ſuppos'd in Magnitude.

We have hitherto endeavour'd to ſhew, that the Real and Abſolute Re-action of Bodies is not always Equal to the Action of them, and if they are not in their Real and Abſolute Forces, they are only ſo in Abſtraction, which therefore is nothing elſe than a Creature and a Chimæra of the Mind. We muſt likewiſe add, that in the Real Forces of Bodies, as Re-action is not always Equal to Action, ſo that neither are they always Contrary; for the Expanſive Force Acts with the Expanſive, and the Contractive with the Contractive, Fire promotes the Expanſive Force of Bodies, which are Inflammable, and Lead the Contractive of Bodies, which are Heavy, by being Laid upon them, and yet neither the one nor the other are Imagin'd to Loſe as much Force, as they Communicate, but Increaſe it in other Bodies, whilſt they Retain it in their own.

This Law not prov'd from Attractions.
Mr Whiſton's Phyſ. Math. Prælect. 5. Art. 5.

§. 51. As for this Law Obtaining in Attractions, Attraction is the very Property in Matter to be Prov'd from it, and therefore to alledge it as an Inſtance for the Proof of that Law, by which it ſelf is to be Prov'd, as it is not done by the Great and Learned Author of the Preſent Philoſophy Sir *Iſ. Newton*, ſo it is no diſtinguiſhing Mark of the Judgment or Penetration of thoſe, who wou'd ſeem to ſupply the ſuppos'd Deficiency of the Perſon they undertake to Explain; we ſhall not therefore enter into the Reaſoning made uſe of by them in Reference to this Phænomenon, which at the beſt depends upon Law the Firſt, and which has been already conſider'd, but proceed to the Experiment, produc'd from the aforeſaid Author, of the Load-ſtone and Iron, which being plac'd in Veſſels peculiar to each, fill'd with Water and touching one another, floated on the Stagnating Fluid, neither of them Propelling the other, but by the Equality of their Attraction kept in and reſtrain'd their Mutual Endeavours to unite, and at length being reduc'd to a perfect Æquilibrium they became Abſolutely Quieſcent.

This Law not prov'd from the Experiment of the Load-ſtone and Iron.

§. 52. But if the Iron and Load-ſtone, plac'd in Water, are in an Æquilibrium, and therefore their Mutual Actions upon each other Equal, when the ſame Bodies are plac'd in Air and come together by their Mutual Actions, thoſe Actions for the ſame Reaſon cannot be Equal; But why a Law of Action and Re-action, and of Nature in General, ſhou'd be formed upon theſe Bodies plac'd in Water, rather than in Air, will be Impoſſible to determine, ſince they are only both of them particular Phænomenons of it, which Reſult from different Circumſtances of the ſame Bodies, and conſequently have neither of them a Right in Preference to the other, to Preſcribe to the Actions of all Nature, no more, than, becauſe Fire Burns in Air, and is Extinguiſh'd in Water, we have therefore juſt Reaſon to Conclude, that the proper Nature of Fire is not to Burn, but to be Extinguiſh'd; It were eaſy to ſhew the Fallacy of Reaſoning from this Experiment more particularly, but we have not Time at Preſent to Enlarge, unleſs to take notice, that both this Experiment, and another, which is Analogous to it, only prove, that the Iron and the Load-ſtone have, each of them, Diſtinct Forces of Action belonging to them, by which they in ſome Caſes Attract each other, and in ſome become Quieſcent, but do not Evidence, that the one is only a Re-action upon the other, Reſulting from the Action of the Former; Thus Fire and Water have Peculiar Forces appertaining to them, and it is not neceſſary, that Water ſhou'd Re-act in the ſame Proportion as Fire Acts, in the Application of the one to the other, only, that Water ſhou'd obtain it's Proper Force, which is independent of the Fire, and Reſiſt or Submit to it, as the Proportion of Action is betwixt them; which Actions, and the Proportion or Quantity of them, it is the Buſineſs of Philoſophy to Enquire into, and Aſcertain.

Dr. Keill's Intr. ad Ver. Phyſ. Lect. 12. Lex. 3. Art. 5.

CHAP. II.

Concerning the Mutual Concourſe of Bodies, and the Laws Relating thereto, as alſo of the Compoſition of Forces.

The Firſt Law concerning the Mutual Concourſe of Unelaſtick Bodies. Mr. Whiſton's Phyſ. Math. Prælect. 5. Art. 6.

§. 1. AFTER Examining into the General Laws of Nature, we are to Inquire into thoſe, which concern the Motion of Bodies in their Mutual Concourſe; the Firſt of which is, that of Two Equal Bodies, which are wholly void of Elaſticity, If the one is mov'd and carry'd againſt the other, which is perfectly Quieſcent, in the Occurſe they will proceed together to the ſame Side and in the ſame Direction, each with half the Velocity of the Body, which is mov'd; becauſe the Body, which is in Motion, will Impell and Accelerate the other, till it can do it no farther, that is, when the Motion of the Quieſcent becomes Equal to that of the Moving Body, and conſequently, when the whole Motion is divided between them both.

Objection the Firſt againſt this Law.

§. 2. Against which there will be this Objection, That Matter, as before, is ſuppos'd to be an Abſtracted Subſtance, without any Property, excepting it's Magnitude, and Preſerving inviolably it's own Dimenſions, and in that Senſe only, Solid or Impenetrable, it is not Hard nor Reſiſting, but merely Paſſive, that is, has no other Action, but the Motion Impreſs'd; This therefore, in the Real Actions of Bodies one amongſt another, is far from being a Law of Nature, ſuch a Law will neither account for the Motion in the Concourſe of Equal Quantities, of Fire and Water, of Oil of Vitriol and Tartar, of Marble with Lead, or common Wood with Clay, or any other Bodies, even of the ſame Kind, as Fire with Fire, or Lead with Lead, *&c.*

Objection the Second againſt this Law.

§. 3. It is not much more happy in proving it's Truth, even upon the Suppoſition of the Preſent Abſtraction; for if a Magnitude is endu'd with a certain Force of Velocity, and is carry'd againſt an Equal Magnitude, which is Quieſcent, that is, where no Powers or Faculties of Matter are conſider'd on either Hand, and no ways Interfere with this Velocity and Quieſcence, the Abſtraction will be the ſame, as if we ſuppos'd 1 Velocity Acting againſt 1 Inertia; but if 1 Velocity can excite 1 Inertia into $\frac{1}{2}$ a Velocity, there is the ſame Reaſon it ſhou'd excite it into 1 Velocity, and by conſequence expend it's whole Force upon it, and become it ſelf 1 Inertia; unleſs an Inertia is more Capable of being Excited out of it's Inertia than an Inertia, which already is Excited, is Capable of being more Excited out of it's Inertia.

§. 4. MOREOVER, how a Mere Velocity can Act upon a Mere Inertia, or one Abſtracted Magnitude or Quantity upon another, any more than Action upon no Action, or a Mathematick Figure upon a Mathematick Figure, or Space upon Space in the Abſtract, is as yet Unexplain'd, to which and no more the Preſent Law Amounts.

Objection the Third againſt this Law.

§. 5. LASTLY, if Re-action is not Equal to Action, it will be Impoſſible, that 1 Velocity ſhou'd not move 1 Inertia with the ſame Velocity, which it brings, without loſing any of it's own; that is, if there is no Return or Repulſe of the Inertia upon the Velocity, it will preſerve it's own Force, and carry on the Inertia with the ſame; but if Re-action is Equal to Action, that Re-action ſhou'd ſubſiſt or be Imagin'd to be in an Inertia, or Non-action, is altogether Abſurd and a direct Contradiction. If therefore this is not a Law of Real Matter according to §. 2, nor of Abſtracted Matter according to the following Sections, of what Matter it is a Law, will be difficult to Affirm; This certainly muſt be ſaid, that what is the Real Reſult from the Concourſe of Bodies can be no otherwiſe determin'd, than from knowing the Peculiar Forces of ſuch Concurring Bodies, which the Preſent Philoſophy ſeems to be little Acquainted with.

Objection the Fourth againſt this Law.

§. 6. A Second Law in Relation to the Mutual Concourſe of Unelaſtick Bodies is, that if Two ſuch Bodies, which are Equal, are carry'd againſt each other with the ſame Velocity, after their Colliſion they will be both Quieſcent, for the ſame Reaſon, that $+a-a$ in the Literal Arithmetick is Equal to 0; For where one Force or Quantity Affirms it ſelf, and another is Negative of it, the Reſult muſt be the Deſtruction or Annihilation of both: It is plain, Oppoſite and Equal Forces will be in an Equal Balance, and ſince neither of them can poſſibly Prevail a Quieſcence muſt ſucceed.

The Second Law concerning the Mutual Concourſe of Unelaſtick Bodies. Mr. Whiſton Phyſico-Math. Prælect. 5. Art. 7.

§. 7. THIS therefore is an Axiom, that Two Equal Forces, Acting in oppoſite Directions, will produce nothing in Reſpect of ſuch Forces, becauſe they Deſtroy each other, that is, their Mutual Progreſs will be abſolutely Hindred; But how, or which way, is ſuch an Axiom a Law of Nature, any more than, 2 Subſtracted from 2 leaving Nothing, is a Law of it? Will Either of theſe Axioms Aſſiſt us in the Laws of Motion, betwixt the Parts of Flame, of a Fluid, or any other Body whatſoever, or betwixt a Fluid and a Solid, or betwixt a Solid and a Solid? We may as juſtly ſay, that, becauſe $2+2$ is Equal to 4, or $2-2$ is Equal to 0, therefore the Fire Burns, or Nature Acts, in all Reſpects, Agreeably to what we Obſerve in the ſeveral Circumſtances of it.

Objection the Firſt againſt this Law.

§. 8. BUT which is more, tho' one Force, Acting againſt another, Produces no Progreſſive Force on either Hands, yet how will this be Reconciled with the Law of Action and Re-action, where the Change of Motion is Equal, ſince, in the Preſent Caſe, there is no Motion at all? For it ſeems to be plain, that if one abſolute Action *A*, carry'd againſt another abſolute Action *B*, is Equal, according to the Law which we now conſider, the ſame abſolute Action *A* cannot be Equal to the Re-action of *B*; becauſe *B* is only Equal to *A*, when it is carry'd with a Poſitive Force againſt it, and therefore cannot be Equal to it by a mere Re-action only, that is, when it is Quieſcent, and is not carry'd with ſuch a Force againſt it.

Objection the Second againſt this Law.

§. 9. BESIDES, as this Law of Motion, in the Concourſe of Bodies, little Agrees with the Fundamental one of Action and Re-action, ſo does it as little with the Firſt, juſt now mention'd; For if a Velocity, Acting upon an Inertia, gives that Inertia half of it's own Action, one Velocity with one Inertia Acting upon another Velocity, which is contrary to it, with the ſame Inertia, ſhou'd cauſe $\frac{1}{2}$ a Velocity on contrary Sides or Directions; For if one Velocity, on one Hand, will give an Inertia $\frac{1}{2}$ its Velocity, the other Velocity, on the other Hand, will give the contrary Inertia $\frac{1}{2}$ of it's Velocity, and the Remainder of the Velocities, on both Hands, will be in an Æquilibrium; Whereas, according to this preſent Law, the whole of the Velocities will be Mutually transfer'd to each other, and from thence will ariſe a Deſtruction of both.

Objection the Third againſt this Law.

§. 10. THE Third Law is, that if Two Unequal Bodies, which are not Elaſtick, ſhou'd meet each other with ſuch a Velocity, that as much as the one Body exceeds the other in Magnitude, ſo much the other ſhou'd exceed the Former in Velocity,

The Third Law of Unelaſtick Bodies.

Mr. Whiſton. Prælect. 5. Art. 8.

or, which is the ſame, if the Velocities are Reciprocal to the Bodies, after their Mutual Concourſe ſuch Bodies will be Quieſcent; Becauſe the Quantities of Motion will be on both Sides Equal, and being Contrary will Deſtroy each other.

This Law already anſwer'd in the foregoing Sections.

§. 11. THE Foundations of this Law have been already Examin'd and Conſider'd, and Firſt in Reſpect of Equal Quantities of Motion, being calculated by making Five Degrees of Velocity, for Inſtance, Equal to the Sum of Five Equal Velocities, which this Law ſuppoſes, and is not prov'd. Second, in Reſpect of Equal Forces deſtroying each other, which is an Axiom, but of little Service to the Explaining of Nature, according to what has been alledg'd in the foregoing Sections.

The Fourth Law of Unelaſtick Bodies Conſider'd. Mr. Whiſton. Prælect. 5. Art. 9.

§. 12. THE Fourth Law concerning Unelaſtick Bodies is, that if a Body, which is mov'd, is carry'd againſt another, which is Quieſcent, however Unequal in their Bulk or their Quantity of Matter, both of them after their Colliſion will proceed with a Common Velocity to the ſame Parts, as in Law the Firſt, and the Common Velocity will be ſo much Diminiſh'd, as both the Bodies taken together are Greater than the Body, which was originally mov'd. Thus, if *AB* is mov'd againſt *C*, which is Quieſcent, with a Certain Velocity, that is, Two Particles of Matter againſt one, the whole Velocity, which appertain'd to *AB*, will after Colliſion be diſtributed and divided into the whole Maſs *ABC*, and therefore, ſince the Moments muſt be the ſame, as before Colliſion, the whole Velocity muſt be Diminiſh'd in Proportion, as the whole Maſs is Increas'd.

FROM whence it is Evident, that this Law depends upon the Truth of the Firſt, which has been already Examin'd, and therefore the Preſent does not want any particular Confutation here; unleſs we farther add, that even upon the Suppoſition of the Truth of that Law, if *A* one Particle of Matter is carry'd againſt *C*, it communicates $\frac{1}{2}$ it's Velocity to it, and ſo it muſt be ſaid of *B*, another Particle of it; how therefore, when they are mov'd together againſt *C*, they ſhou'd ſtop, before they have each of them communicated $\frac{1}{2}$ their Velocity to it, is no otherwiſe to be Explain'd, but by Curbing and Reſtraining the Communication of Velocity at Pleaſure, in order to make the whole *ABC* to Proceed with one and the ſame Celerity; which therefore ſhou'd have been Prov'd, namely, that the Quieſcent Body can never have a Greater Force Impreſs'd upon it from the Action of Two Bodies, or Particles of Matter *A* and *B*, than from that of one as *A*; If it can, ſince, in the Action of *A* upon *C*, they are ſuppos'd to proceed together, in the Action of *A* and *B* upon *C*, they will not, but *C* will proceed with a Greater Velocity than *AB*; that is, if *A* communicates $\frac{1}{2}$ it's Velocity to the Quieſcent Body *C*, *B* will likewiſe do it, from whence *C* will have 1 Velocity, whilſt *A* and *B* Retain only 1 betwixt them, that is, *C* will be carry'd with a Greater Velocity than *AB*, and therefore not with a Common one to Both.

The Fifth Law of Unelaſtick Bodies Conſider'd. Mr. Whiſton. Prælect. 5. Art. 10.

§. 13. THE Fifth Law is, that if Two Unelaſtick Bodies, which are Unequal, are mov'd with an Equal Velocity againſt each other, the Quantity of Motion in both, after Colliſion, will be the Difference of their Motions, before their Mutual Concourſe; becauſe, if *AB* and *C* are carry'd againſt each other, the Equal Moments of both will be deſtroy'd, and there will only Remain the Difference of them, which is to be Impended or Spent upon the whole Maſs, *ABC*; in which Law, Firſt, the Mutual Deſtruction of Equal Moments is not accounted for, by what foregoes; nor Second, according to the laſt Section, the whole Maſs proceeding with one and the ſame Velocity after Colliſion; and in Reality, this Philoſophy of the Mutual Concourſe of Bodies is no more, than Moving *AB* and *C* backwards and forwards with a Uniform Velocity, juſt as we pleaſe, and taking it for granted, that after Colliſion *AB* and *C* muſt certainly, one way or other, move together.

The Sixth Law of Unelaſtick Bodies. Mr. Whiſton. Prælect. 5. Art. 11.

§. 14. THE Sixth Law is, that if Two Unelaſtick Bodies, being Equal, are mov'd with an Unequal Velocity to the ſame Side, after their Colliſion the Quantity or Sum of Motion will continue the ſame, but the Common Velocity will be the half of the Velocity of both taken together; Thus, if *A* or *B* moves with 4 Velocity, and *C* with 2 towards the ſame Side, after *A* or *B* arrives at *C*, they will both proceed with 3 Velocity each, ſo as to make the Sum of Motion the ſame with the Former, and the Common Velocity the half of the whole Motion.

Objection the Firſt againſt this Law.

§. 15. This Law therefore, and the Explanation of it, is as Inconceivable as the Former; ſince, if *A* or *B*, when carry'd againſt *C* conſider'd as Quieſcent, ſpends $\frac{1}{2}$ it's Velocity upon it, according to Law the Firſt, why it ſhou'd communicate none to it, if the Velocities are Equal, and *C* is in Motion, is altogether Inexplicable; But this it muſt not do, if the Preſent Doctrine is True, becauſe after Colliſion they are to obtain one Common Velocity, that is, *A* or *B* and *C* are to have one Common Velocity, which is 2 after Colliſion, therefore *A* or *B* communicates no Velocity to *C*, tho' in Motion, and yet, by Law the Firſt, it wou'd Impart $\frac{1}{2}$ of it's Velocity to it, if it were Quieſcent, which is Abſurd: Or we may put it in this Way, *A* or *B*, by Law the Firſt, will Impart $\frac{1}{2}$ it's Velocity to *C*, if it were Quieſcent, therefore an Unequal Velocity, = to Two Equal Velocities of *A* and *B*, will, according to that Law, communicate $\frac{1}{2}$ of the Velocity of *A*, and $\frac{1}{2}$ of the Velocity of *B* to *C*, when Quieſcent, that is, 2 Velocity, whereas, according to the Preſent Law, it will have only 1 Velocity communicated to it, when in Motion, which is alſo Abſurd.

Objection the Second againſt this Law.

§. 16. Another Objection againſt this Law is, that, whereas *A* is ſuppos'd to have an Unequal Velocity with *C*, yet, when they come together, they have the ſame Velocity, if *A* or *B* has 4, and *C* has 2, it ſeems, by an amicable Compoſition of the Matter, they will each of them proceed with the Velocity of 3; On the Contrary, if the Degrees of Velocity are Really Different, ſo that one is twice as Swift as the other, or as 4 to 2, it will be Impoſſible to ſay, how, upon their Meeting, they ſhou'd obtain the ſame Velocity, according to the Preſent Law; for if a Swifter Velocity, as of *A* or *B*, is Reſtrain'd by a Slower, as of *C*, it ſo far deſtroys that Velocity, and therefore their Sums cannot be the ſame before, as after their Colliſion, which is affirm'd; if the Degrees of Velocity are not Different, but Equal, that is, if the Velocity of one Body, is not Swifter than that of the other, and *A* or *B* has only the Sum of 4 Velocities, each of which is Equal to one of thoſe, of which *C* has 2, there will be different Sums of Velocities, where the Sums or Quantities of Matter are Equal, by which the Sums of the Velocities are Computed; which is again Impoſſible.

The Seventh and Laſt Law of Unelaſtick Bodies Conſider'd.
Mr. Whiſton's Prælect. 5. Art. 12.

§. 17. The laſt Law of Unelaſtick Bodies is, that, if of Two Unequal ones, which are moved the ſame Way, the Greater Impinges upon the Leſs, the Common Velocity after Colliſion will be Greater than half of the Sum of the Velocities; On the Contrary, if the Leſs overtakes the Greater, the Common Velocity will be Leſs after Colliſion, than that half of the Sum of the afore-mentioned Velocities. For Example, if *AB* has 6 Velocities, and *C* has 2, the Common Velocity after Colliſion will be more than 4; becauſe, by the preceding Law, if the Bodies were Equal, *A* and *C* wou'd proceed together with the Common Velocity of 4, but *AB* being Greater than *A*, and conſequently than *C*, it will ſpend more of it's Velocity upon *C*, than *A* did, before they come to an Æquilibrium, and therefore the Common Velocity, with which *ABC* proceeds, will be Greater than 4; and for a Contrary Reaſon, if *C* incurrs upon *AB*, the Common Velocity will be Leſs than 4.

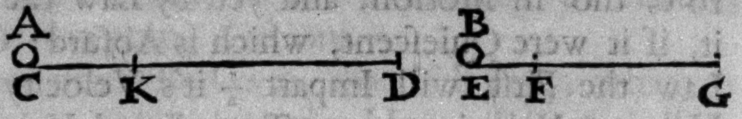
This Law therefore is Examin'd in the Former, and does not require any farther Arguments, or Convictions of it's Uncertainty or Falſhood. The Solution of all this Buſineſs is, that Bodies, which have a Contractive Force Unite, and Adhere to each other by that Force, or Move together, upon their Velocities, which are of an Expanſive one, Deſtroying their Particular Contractive Forces or Gravitation to the Earth.

Two Theorems in Relation to the Impaction or Colliſion of Bodies Demonſtrated.
Sir Iſ. Newt. Princip. Leg. Coroll. 3.
Dr. Keill's Introd. ad Ver. Phyſ. Theorem 18. & 19. Lect. 12.

§. 18. But, before I leave this Subject of the Impaction and Concourſe of Bodies, I ſhall ſubjoin the following Theorem from a Celebrated Philoſopher; That the Quantity of Motion, which is computed by taking the Sum of the Motions, which are carried to the ſame Side, or the Difference of them carry'd to Contrary ones, is not Chang'd by the Action of Bodies amongſt one another. Which Theorem has Furniſh'd an Occaſion to another Excellent and Ingenious Philoſopher, in the Preſent Mechanicks, to Form the Two Subſequent ones, of which I ſhall Give the following Demonſtrations.

Theorem the Firſt is, If one Body Impinges upon Another, which is Quieſcent, or which is Moved Slower to the ſame Side, or in the ſame Direction, the Sum of the Motions in both Bodies, to the ſame Side, will remain the ſame after their Impaction or Colliſion, as before; Let the Body *A* be moved according to the Direction of *CD*, from *C* to *D*, and let it Impinge or Fall upon another Body *B*, which is ſuppos'd either to be Quieſcent, or to be moved more ſlowly in the ſame Direction; It is Affirm'd, that the Sum of the Motions in both Bodies towards the ſame Side, that is, from *C* to *D*, will, before and after their Impulſe, Impaction, or Colliſion, Remain the ſame.

Let therefore *CD*, the Motion of the Body *A*, be $=a$, *EF*, the Motion of the Body *B* to the ſame Side, be $=b$, the Sum of their Motions will conſequently be $CD+EF=a+b$; Let *FG* repreſent the Motion, which is Impreſs'd from *A* on *B*, which we will ſuppoſe $=c$; Since therefore Action and Re-action are Equal and Contrary, *B* by Re-acting upon *A* will take ſo much from *A*, as is Imparted to it; therefore let *KD* be Equal to $FG=c$, and the Motion of *A* after the Impaction will be $=a-c$, and the Motion of $B=b+c$, but $a-c+b+c=a+b$, the Sum of their Motion before ſuch Impaction.



Corol. 1. If $FG=CD$, or $c=a$, *C* and *K* will Coincide, and $CK=a-c=0$, and *A* after Impaction will remain Quieſcent, and *B* will proceed with the Motion of $b+c$; that is, if $a-c=0$, $b+c$ after Impaction $=a+b$, or the Sum of their Motions before it.

Corol. 2. If *FG* is Greater than *CD*, the Point *K* will fall to the Left Hand of *C*, and the Motion of *A* will be Negative, or to a contrary Side; that is, if *c* is Greater than *a*, let *d* be the Difference, therefore $a=c-d$, and the Sum of their Motions after Impaction will be $=c-d-c+b+c=-d+b+c$, that is, $-d$ will belong to *A*, which will be therefore Negative, and $b+c$ to *B*; and $-d+b+c=a+b$, becauſe $c-d=a$.

Theorem the Second is, If Two Bodies Move to Contrary Sides, and Meet Each other, the Sum of their Motions to the ſame Side, which is the Difference of their Motions to Contrary Sides, both before and after their Concourſe will Continue the ſame towards the ſame Side.

Let *a* Repreſent the Motion of *A*, and $-b$ the Motion of *B* to a Contrary Side, the Sum of their Motions to the ſame Side, which is the Difference of their Motions to Contrary Sides, will be $a-b$, let *c* farther be $=$ to the Motion communicated to the ſame Side; therefore $a-c=$ to the Motion of *A* after their Concourſe, and the Motion of *B* after it will be $-b+c$; but $a-c-b+c$ after Concourſe $=a-b$ the Motion of theſe Bodies before it.

Corol. 1. If $b+c=a$, $a-b-c$ the Motion of *A* after Concourſe $=0$, and *A* will be Quieſcent after ſuch Concourſe, and *c* the Motion of *B* after Concourſe $=a-b$ the Sum of both their Motions to the ſame Side, or the Difference of them to Contrary ones.

Corol. 2. If $b+c$ is Greater than *a*, and *d* is the Difference, then $b+c=a+d$, and $b+c-a=d$, and $a-b-c=-d$; that is, $a-b-c$, the Motion of *A* after Concourſe, will be Negative, and to a Contrary Side, and the Motion of *B* will be $+c$, but $-d+c=a-b-c+c$, the Motion of the Bodies after Concourſe, $=a-b$, the Motion of them before it.

The foregoing Theorems and the Demonſtration of Them, Inquir'd into.

§. 19. In Reſpect therefore of theſe Two Theorems, whatever we have alledg'd againſt Action and Re-action being Equal and Contrary, as alſo concerning the Communication of Motion, and Matter being conſidered only as Conſiſting of mere Bulk and Magnitude, is Equally Valid here; And particularly, *c* is ſuppoſed to be the Motion communicated to *B*, and loſt to *A*, if ſo, it is manifeſt, that the Quantity of Motion after Contact, or Impaction, or Impulſe will be the ſame as before it; But how is this Prov'd, that *c* is Loſt to *A* and Gain'd to *B*? Not from the Nature of Matter, which is ſuppos'd to have nothing, but Bulk or Magnitude, which can neither Act or Re-act; If it is ſaid this Force of Re-action is ſuppos'd, then that very Thing is ſuppos'd, which is to be Prov'd; and the foregoing Theorem, which is Reſolv'd into the Two Succeeding ones, is Demonſtrated by Preſuming that to be True, which it propos'd to Evince to be ſo; that is, $a+b=a-c+b+c$, becauſe *a* Loſt *c* and *b* Gain'd it, and *a* Loſt *c* and *b* Gain'd it, becauſe they are ſuppos'd to do it. If there is any Real Force in Matter, by which this is Prov'd, it ſhould be ſhewn us, and how it is deriv'd from mere Bulk and Magnitude, and how it is a Conſtant and Unvary'd

Force in all Material Beings, in Bodies Elaſtick and Unelaſtick, in Fire, Water, Wood, Stone, Metals, and Minerals; for till then, as we have before ſaid, we can conclude it to be no other than a Creature of the Mind, and a Force, which Reſts in the Imagination, without Reſiding in any Part of Nature; and yet, which is ſomething Wonderful, this Property or Theorem of Motion is brought to Prove Action and Re-action to be Equal and Contrary, which Theorem is notwithſtanding Prov'd from ſuppoſing Action and Re-action Equal and Contrary; On the other Hand, we ſhall in this Chapter Give the Laws of the Mutual Congreſs of Bodies from Real Forces in Nature, the Expanſive and Contractive.

Elaſtick Bodies Conſider'd, and the Firſt and Primary Law of Them Explain'd.
Mr. Whiſton. Prælect. 5. Art. 13.

§. 20. After the Laws of Unelaſtick Bodies follow thoſe of Elaſtick, that is, of Bodies, which Reſtore themſelves with the ſame Force, with which they are Compreſs'd; The Firſt is, That in Bodies perfectly Elaſtick and Equal, if the one is Mov'd againſt the other, which is Quieſcent, upon their Impaction that, which is Mov'd, will be Quieſcent, and the Quieſcent will Proceed with the ſame Celerity, which was originally in the Impellent: Becauſe, by the firſt Law of Unelaſtick Bodies, or of Mere Impulſe, it will communicate half it's Celerity, but the Elaſtick Force is Equal to that Celerity, which upon Impulſe the one Body Gains, and the other Loſes, and Acts Contrary ways, ſo as to be Added to the Motion of the Body Impell'd, and Subſtracted from the Impellent; from whence it will follow, that ſince half is Communicated by Simple Impulſe, and the other half by the Elaſtick Force, the whole will be Transfer'd from the Impellent to the Body Impell'd, and the Former will remain Quieſcent, ſeeing the firſt half is loſt by Communication, and the laſt by Re-action; Thus, if *A* is 4 Velocity, and *B* 0 Velocity, after Impulſe *A* is 2, and *B* 2, and the Elaſticity Adds 2 more to *B*, and Gives 2 leſs to *A*, ſo that, in the Reſult, *A* will be 0, and *B* 4 in Velocity.

Objection the Firſt againſt this Primary Law of Elaſticity.

§. 21. Against this Doctrine and Hypotheſis we are therefore Firſt to Obſerve, that That of Simple Impulſe is neceſſary to This, which has been Conſider'd, and which, if not true, Involves the Preſent in the ſame Error; Second, That, tho' this Law of Elaſticity, and the others, which depend upon it, were certain, and did obtain in Nature, they are yet far from being any General Laws of it, ſince Heat and Cold, Hardneſs and Fluidity, *&c.* are not Reducible to them.

Objection the Second againſt this Law.

§. 22. To Omit what we have ſaid in the foregoing Section, the very Hypotheſis of this Kind of Elaſticity (for Simple Impulſe is a ſort of one by it's Action and Re-action) is, in the firſt Place, the moſt Arbitrary, that can be poſſibly Imagin'd, and in the Second, is not Anſwerable to it ſelf: It is Arbitrary, for why the Elaſticity ſhould be 2, in the Inſtance above, rather than 4 or 8, or any other Number, is not determin'd, by which means *A* wou'd be -2 and *B* 6 Velocity, or *A* -6 and *B* 10 Velocity, and Laws might be form'd, and Calculations made, upon ſuch Hypotheſes, as well as that, which is propoſed to us; and ſo on the Contrary, inſtead of 4 and 8 we might likewiſe ſubſtitute 1, or $\frac{1}{2}$, or any Fraction whatſoever, and in the firſt Caſe, *A* wou'd be 1, and *B* 3, and in the Second, *A* wou'd be $1\frac{1}{2}$, and *B* $2\frac{1}{2}$, which wou'd give an abundant Scope to theſe Abſtracted Reaſoners to Compute the Laws and Rules of ſuch a ſuppos'd Elaſticity, whenever They ſhall be at Leiſure.

Objection the Third againſt this Law.

§. 23. That this Proportion or Degree of Elaſticity is farther Arbitrarily aſſum'd, is Evident, ſince to the Nature of Elaſticity it is only required, that the Repreſſure ſhou'd be Equal to the Force, which Compreſſes, which is as well obſerved in $-4+4$, or $-\frac{1}{2}+\frac{1}{2}$, as in $-2+2$, where the Velocities of both Bodies are before ſuppos'd, by Law the firſt, to be Equal; for if one Body Impinges upon another, and after Contact they proceed with the ſame Celerity, any Force ſuperinduc'd of Acting and Re-acting may be Equally call'd an Elaſtick, as that which is Commenſurate to the Force, which one Body Gains, and the other Loſes, in ſimple Impulſe; becauſe there may be ſuch an Innate Elaſtick Force preſum'd in Bodies, which does not depend upon Impulſe.

Objection the Fourth againſt this Law.

§. 24. Besides which, if the Preſſure and Repreſſure of Bodies were to be Meaſured by the Compreſſion Reſulting from Impulſe, what wou'd that be? Not what is here ſuppoſed, the ſame, which one Body Gains by Impulſe and the other Loſes, becauſe it is Impoſſible, that the Celerity Communicated ſhou'd Re-act with the ſame Force, with which it is Progreſſive on the one Hand, or that the Celerity on the other, which remains and is not Communicated, ſhould Act with a Progreſſive Force, which it has loſt by the Preceding Communication; Thus, if *A* is 4, and *B* 0, in their Velocities, and upon Colliſion let *A* be 2, and *B* 2, there is no Reaſon from thence to Affirm, that by Introducing Action and Re-action, *A* ſhall Communicate 2 more of Velocity, becauſe it has loſt 2, or, that *B* ſhall Re-act with the Force of 2, becauſe it has Acquir'd the Force of 2 in Progreſſion; It is farther altogether Unaccountable, how Elaſticity, which conſiſts in Re-action, ſhou'd promote a Propagation of Motion, as will be afterwards Explain'd.

Objection the Fifth againſt this Law.

§. 25. MOREOVER, If this could be Granted, that by Impulſe 2 Velocity was Communicated from *A* to *B*, and by the Elaſtick Force 2 more, ſo that the Whole ſhould be Transfer'd to *B*, yet this will not be eaſily accounted for, why *A* ſhould be Quieſcent, unleſs upon this Suppoſition, that a bare Communication of Motion, and a Re-action againſt it, ſhall have Equal Effects in deſtroying it in the Impellent *A*, which is Impoſſible, unleſs to Communicate is to Re-act, or a mere Tranſlation of it from one Body to another is the ſame, as to Hinder and Reſiſt it's Communication; if therefore 2 Velocity was Communicated by Impulſe from *A* to *B*, and another 2 Velocity from *A* to *B* by Elaſticity, it wou'd not follow, that *A* ſhou'd be Quieſcent; becauſe if Impulſe Communicates 2 Velocity, and is Quieſcent in Reſpect of that Velocity, which it Communicates, if Elaſticity Communicates 2 Velocity, it's Re-action ſhou'd give an Impreſſion upon the Impellent, ſo as to make it Retire with 2 Velocity, or otherwiſe Elaſticity will be the ſame as Simple Impulſe, which yet are here Diſtinguiſh'd; Nor farther is it Evident, that by Simple Impulſe, what Velocity one Body Gains, the other Loſes, becauſe, if one Body Acts upon another, which has no Force of Re-action, it may Impreſs it's own Action upon that other Body without Diminiſhing it ſelf, as in the Caſe of Fire Acting upon Fuel, or Motion Acting upon Non-Reſiſtance in the Abſtract.

Objection the Sixth againſt this Law.

§. 26. ANOTHER Thing, which we muſt obſerve, is, that this Hypotheſis does not Agree with the Definition of an Elaſtick Body, which is ſaid to be ſuch, when the Preſſure and Repreſſure are exactly Equal, which if they were, it would be altogether Impoſſible, that any thing elſe ſhould Reſult from them, but a Perfect Quieſcence in both Forces; for if *A*, Abſtracted from Impulſe, ſhould Preſs upon *B*, and there is a Force of Re-action Equal to that Preſſure, *A* cannot Move *B* forwards, becauſe *B* Preſſes backwards, nor *B* Move *A* backwards, becauſe *A* Preſſes forwards with a Force on both Hands, which is Æquipollent; if it is ſaid, the Force of Repreſſure is ſuppoſed to be ſuch, as can actually Reſtore it ſelf upon any Preſſure made upon it, then Repreſſure is Superior to the Compreſſion, Contrary to the Definition of a Perfect Elaſticity, and would be ſo far from Promoting of Motion forwards, that it would Repell the Incurrent Preſſure to the Oppoſite Side, and give it a Direction that Way.

Objection the Seventh againſt this Law.

§. 27. WE are farther to take Notice, that if Impulſe is made one Part of the Account in the Action of *A* upon *B*, yet even then this Law will not be True; ſince *A* 4 Velocity, and *B* 0 Velocity, if the Elaſticity of *B* is Equal to the whole Impulſe and Elaſticity of *A*, will upon their Congreſs deſtroy each other; if *B* is Equal to the Impulſe only of *A*, it will deſtroy that and leave the Elaſticity of *A* to exert it ſelf with it's Proper Force; if *B* is Equal to the Elaſticity of *A*, *A* and *B* will be in an Æquilibrium in reſpect of their Elaſticity, and conſequently, the Impulſe will not Prevail, which will be ſpent upon Both, if Law the Firſt of Unelaſtick Bodies is true, that is, they will both proceed with the Velocity of 2, Contrary to the preſent Aſſertion of *A* Transferring it's Whole Velocity to *B*; for by Elaſticity I here mean the Expanſive Force, or the Force of Repulſion, and which is the only true Notion of it, as will be afterwards ſhewn.

Objection the Eighth againſt this Law.

§. 28. BESIDES how *A* 4 Velocity ſhou'd transfer, by it's Elaſticity, 2 Velocity to *B*, and deſtroy 2 in the Impellent *A*, is Inconceivable, if Compreſſion and Repreſſure are Equal; becauſe 2 Velocity, Communicated forward from *A*, is not a Force Equal to no Motion or Velocity at all, Impreſs'd from *B*, and therefore, if the Effects are not Equal, the Cauſes cannot be; that is, Preſſure and Repreſſure are not ſo; but there is no Re-action Impreſs'd from *B*, but what is ſuppos'd to Ariſe from *A*, which can be None, becauſe it's Tendency is forwards.

Objection the Ninth againſt this Law.

§. 29. We muſt Add, it is ſtill leſs Anſwerable to the Definition of perfectly Elaſtick Bodies, that, what one Body Gains and the other Loſes by Impulſe, is the Meaſure of this Elaſticity, becauſe in Equal Bodies A may have 4, or 6, or 10 Velocity, when B is Quieſcent; and conſequently, the Meaſure of Elaſticity may be 2, or 3, or 5, or any other Number, whilſt Elaſtick Bodies, conſider'd in their Innate Force of Repulſion or Expanſion, may not be Proportional to ſuch Numbers.

Objection the Tenth againſt this Law.

§. 30. If, what one Body Gains and the other Loſes, was the Meaſure of Elaſticity, it wou'd not yet be Agreeable to the Definition of it's Nature, or of Compreſſion being Equal to Repreſſure; ſince Velocity of Impulſe, and the Communication of it, has no Connection with Acting and Re-acting, unleſs becauſe A, which has 4 Velocity Communicates 2 to B, which has none, therefore A Acts with the Velocity of 2 by it's Elaſtick Force, and B Re-acts with the ſame, betwixt which Two Propoſitions, it is Impoſſible, there ſhould be any Dependence in Reaſon.

Objection the Eleventh againſt this Law.

§. 31. If all this was true, againſt which we have been Diſputing, theſe Laws of Elaſtick Bodies, as well as thoſe others of Unelaſtick, wou'd be far from giving us any juſt Notions of Things, as they Exiſt; for, tho' they were Adapted to the Solution of ſome kind of Material Beings, as of certain Solid Bodies in their Mutual Concourſe, yet, ſince they would not Comprehend all, nor one Millionth Part of them, of what Importance or Significance can ſuch Laws be to the Explaining of Nature, where ſo ſmall a Part of it is conſider'd? For, if theſe Laws could give an Account of the Mutual Congreſs of certain Hard Bodies, which they cannot do, as thoſe of Bowls meeting each other, which are of the ſame Matter and Force, they can ſtill leſs Inform us of the Properties, which Reſult from the Mutual Congreſs of Marble and Lead, of Steel and Marble, or in General of Stone, or Wood, or Metals, and the various Kinds of them in their Concourſe with each other; and if theſe Laws cannot do this, they can ſtill leſs furniſh us with a Solution of Heat or Cold, of Hardneſs or Fluidity, of Coagulation or Chryſtalizing, All which are entirely unaccountable by ſuch Laws as theſe, which therefore are to be Rejected from a True Philoſophy.

Theſe Reaſonings obtain their due Value, when conſider'd only as Mathematick and Abſtracted ones.

§. 32. Notwithstanding, for the ſeveral Reaſons before Alledg'd, we cannot but be of Opinion, that theſe Laws of Elaſtick Bodies do not ſerve us in the Explanation of Nature, we muſt yet confeſs, that the Demonſtrations form'd upon the Preſent Hypotheſis of Simple Impulſe and Elaſticity are ſuch, as ought to give us a Due Value and Eſteem for the Contrivers of them, as they advance our Abſtracted and Mathematick Reaſoning, amongſt whom none appear to have treated this Subject with Greater Clearneſs and Brevity than the Famous Sir *Iſ. Newton*, and my Loving Friend the Reverend and Learned Mr. *Morgan* before-Mention'd.

Sir Iſ. Newt. Algeb. P. 91. Probl. 12.
Mr. Morgan upon Rohault Chap. 11. Part 1.

But having already Conſider'd the Two Principles, upon which theſe Computations depend, I ſhall now Propoſe the Laws of the Mutual Congreſs of Bodies, deriv'd from their Innate Expanſive and Contractive Forces, after having obſerv'd upon the Great Philoſopher and Geometer here Refer'd to, that, when he comes to give us an Account of Bodies, which meet each other, he Tells us, by Changing the Sign of b, the Celerities of A and B after Reflection will be $\frac{aA-aB-2bB}{A+B}$ and $\frac{2aA+bA-bB}{A+B}$, which is Evident, becauſe, when a tends one way, and b another, if the one is Expreſs'd by $+a$, the other will be Expreſs'd by $-b$, and $Aa-x$, and $-Bb+x$ will be the Motion after Concourſe; The Celerities therefore will be $\frac{Aa-x}{A}$, and $\frac{-Bb+x}{B}$; But $\frac{-Bb+x}{B}-\frac{Aa+x}{A}=a+b$, and $-ABa-ABb+Ax+Bx=ABa+ABb$, and $x=\frac{2ABa+2ABb}{A+B}$; Therefore $\frac{Aa}{A}-\frac{2ABa-2ABb}{AA+AB}=\frac{Aa-Ba-2Bb}{A+B}$ = to the Celerity of A; Likewiſe $-\frac{Bb}{B}+\frac{2ABa+2ABb}{AB+BB}=\frac{2Aa+Ab-Bb}{A+B}$ the Celerity of B.

But then, Firſt, when we come to Inſtances and Examples of this Doctrine, Bodies are ſuppoſed, as alſo in the Calculation, to be Homogeneous in their Matter, and the Moments of Them to Conſiſt of Mere Bulk and Celerity, which we

Deny; And Second, ſometimes *A* is Imagin'd to Incurr with a Negative Force, which ſhews it is nothing, but an Abſtracted Reaſoning; Thus, Let *A, a, B, b,* be 3 Bulk, 8 Celerity, 9 Bulk, 2 Celerity, Reſpectively; If *A* and *B* Proceed the ſame Way, the Celerity of *A* after Reflection is $= \frac{aA - aB + 2bB}{A + B} = -1$, that of $B = \frac{2aA - bA + bB}{A + B} = 5$; that is, the Whole Moment of *A* is 3 Bulk × 8 Celerity, of *B*, 9 Bulk × 2 Celerity before Reflection; but *B* = 9 Bulk Acquires 3 Celerity after Reflection, and therefore there muſt have been 9 × 3 Celerity Communicated to it from *A*, which yet had only 8 × 3 Celerity, but how *A* ſhould Communicate 1 × 3 Celerity more than it had, and by that Means Return with the Celerity of − 1, is ſomething Inconceivable, unleſs in ſuch an Abſtracted Reaſoning.

BUT to proceed to the Laws of the Congreſs of Bodies in a Real Philoſophy.

The Laws of the Congreſs of Bodies in a Real Philoſophy and the Theorems of Them, Eſtabliſh'd upon the Doctrine of the Expanſive and Contractive Forces.

§. 33. LET *A* Repreſent the Quantity of Matter, or the Number of it's Parts, in one Body, and *B* the Quantity of Matter, or the Number of it's Parts, in the other; Let *E* and *C* Denote the Expanſive and Contractive Forces in the Former, and *e* and *c* thoſe of the Latter; And let *V* and *v* Expreſs the Velocities in Each.

THEOREM the Firſt; The whole Moment of *A* will be therefore *AECV*, and that of *B*, *Becv*, and if they tend the ſame Way, the whole Moment may be Expreſs'd by *AECV* + *Becv* after Concourſe; in which we ſuppoſe, that *A* and *B* are only the Numerators of the Forces *ECV*, *ecv*, and theſe the Denominators of the Degrees of them; that the Innate Expanſive and Contractive Forces in the ſame Bodies, *E* and *C* in the Firſt, or *e* and *c* in the Laſt, Act contrary to each other, and that, in Different Bodies, *E* Acts to the ſame Part with *c*, and *C* with *e*, but that *E* and *e* the Expanſive Act contrary to each other, as alſo *C* and *c* the Contractive; that likewiſe the Velocities Impreſs'd *V* and *v* in the ſame Bodies Promote and Act with the Expanſive againſt the Contractive, tho' not in the ſame Proportion, as the Expanſive againſt the Contractive, the one being an Expanſive Force and a Stronger deriv'd from the Mind, and the other from Matter only, that is, that *V* Advances and Promotes the Force of *E* againſt *C*, and *v* that of *e* againſt *c*; and laſtly, that in Different Bodies the Velocity of *V* Acts contrary to the Velocity of *v*, by how much the Laſt is ſlower than the Former, and likewiſe *V* Acts with *c* againſt *e*, and *v* with *C* againſt *E*.

THEOREM the Second; If they Act in Contrary Directions, which is the Caſe of the Moment *AECV* − *Becv*, *A* and *B*, as before, are conſidered only, as the Numerators of theſe Forces, which are ſuppos'd to ſubſiſt in every Part of Matter, which they Number; The Innate Expanſive and Contractive Forces in the ſame Bodies, *E* and *C* in the Firſt, or *e* and *c* in the Laſt, Act likewiſe, as before, Contrary to each other, and in Different Bodies *E* Acts to the ſame Part with *c*, and *C* with *e*, but *E* and *e* the Expanſive Act Contrary to each other, as alſo *C* and *c* the Contractive; Farther, the Velocities *V* and *v*, in the ſame Bodies, Promote and Act with the Expanſive againſt the Contractive, tho' not in the ſame Proportion, as the Expanſive againſt the Contractive, and for the Reaſon Alledg'd above, that is, *V* Promotes and Advances the Force of *E* againſt *C*, and *v* that of *e* againſt *c*; and laſtly, in Different Bodies *V* Acts againſt *v* and with *c*, and *v* againſt *V* and with *C*; alſo *V* Acts againſt *e*, and *v* againſt *E*. And in both theſe Theorems + and − are not Marks or Signs of Addition and Subſtraction, but of the Moments Acting the ſame, or contrary Ways.

FROM whence it follows, that theſe Two Theorems only differ in + or −, but that the Acting Forces fall exactly under the ſame Laws, and which, before we proceed, we ſhall Endeavour to Explain.

FIRST, As for *A* and *B*, they being only the Quantities of the Acting Forces, there is no farther Occaſion to ſpeak of them.

SECOND, *E* and *C*, or *e* and *c*, being the Expanſive and Contractive Forces in the ſame Bodies, Act contrary to each other; the Firſt Acting from a Center, or by Repulſion; and the Laſt towards one, or by Contraction.

THIRD, In Different Bodies *E* is Expanſive or Repells the ſame Way from it's Center, as *c* is Contractive or Tends towards it's Center, and *e* is Expanſive or Repells the ſame Way from it's Center, as *C* is Contractive or Tends towards it's Center;

and therefore E Acts with c in another Body, againſt C in it's own, and, *vice verſâ*, e Acts with C in another Body, and againſt c in it's own; Thus likewiſe E the Expanſive or Repulſive Force in one Body Acts Contrary to e the Expanſive in another, they Acting by Expanſion or Repulſion from different Centers; and for the ſame Reaſon, C the Contractive in one Body Acts Counter to c the Contractive in another, becauſe they Tend to Different Centers.

Fourth, The Velocities V and v in the ſame Bodies, V in $ACEV$, and v in $Becv$, Act with E and e againſt C and c, becauſe C and c Tend to a Center, and conſequently to Quieſcence and Reſt, which is oppoſite to Velocity; whereas Velocity has an Expanſive Force, as Tending from the Center of it's Communication, which Center is yet Different from the Center of the Body's Expanſive.

Fifth, The Velocities V and v in different Bodies Act againſt each other, or not at all upon each other; for, if they are Equal, and proceed the ſame Way, there is no Impaction; if they are Superior the one to the other, ſo much as the Difference is betwixt them, ſo far they are Contrary to each other; and, if they Act in Oppoſite Directions, the Caſe is Manifeſt.

Sixth, Laſtly the Velocity V Acts with c, and v with C, in different Bodies; becauſe V Acts with E, and v with e, againſt C and c in the ſame Bodies, by *Art.* 4, and E Acts with c, and e with C, in different Bodies, according to *Art.* 3; Therefore V Acts with c, and v with C, in different Ones; In like Manner, V Acts againſt C, and v againſt c, in the ſame Bodies, by *Art.* 4, but C Acts with e, and c with E, in different Ones, according to *Art.* 3; Therefore V Acts againſt e, and v againſt E, in different Ones.

It muſt be obſerv'd, that $> <$ are Notes, or Signs of Greater or Leſs, in the following Caſes; Being the ſame, which are made uſe of by Dr. *Wallis* in his Limits of Æquations.

The ſeveral Caſes deriv'd from the fore-going Theorems.

§. 34. Let us therefore now ſuppoſe the Quantities of the Forces, or the Numerators of them, A and B, to be Equal, as likewiſe the Velocities, with which they Incurr upon each other, which therefore muſt be the Caſe of the Second Theorem $AECV - Becv$, that is, where theſe Forces are directed to Contrary Sides; Becauſe otherwiſe, having the ſame Velocities, they could not meet, nor would there be any Concourſe or Impaction: And this we at preſent ſuppoſe, that we may Enquire, what the Real Forces of Bodies are.

Why Two Equal and Equally Unelaſtick Bodies, Carry'd againſt each other with Equal Velocities, will upon their Concourſe be Quieſcent.

Caſ. 1. If $E=C$, and $e=c$, in the ſame Bodies, and $E=e$, and $C=c$, in different Ones; that is, if the Expanſive is Equal to the Contractive in the ſame Body, and the Expanſive is Equal to the Expanſive, and the Contractive to the Contractive in different Ones, and the Sums or Quantities of the Forces, and the Velocities in both, are the ſame, a perfect Quieſcence will follow in both Bodies upon their Mutual Concourſe; Becauſe, in the ſame Bodies, the Expanſive and Contractive Forces are in an Equal Balance, being Equal in their Forces, and Acting Contrary to each other; and in different Ones, E, which Propells one Way, is $=$ to e, which Propells another, and C, which Contracts one Way, and Acts with e, is $=c$, which Contracts another, and Acts with E: And this may poſſibly be the Caſe of Two Globes of Lead, or other Matter of the like Nature, where the Expanſive, or Reſilient, or Elaſtick Force is ſo Balanc'd by the Contractive, that little or no Reſiliency Enſues, but a Perfect Quieſcence is Caus'd upon their Mutual Impaction; If the Velocity is Greater in the one than the other, they will proceed to the Side, from which the Weaker Velocity Acts, with the Exceſs of the Greater above the Leſs divided betwixt them; for the Body, which is Acted with the Greater Velocity, cannot move Faſter, than that, which was Acted with the Slower, becauſe this Latter lies before it, and it cannot move Slower, becauſe all the Velocity lies on the Side of the Former, therefore they will move Equably and together, and conſequently, the Exceſs of the Greater Velocity will be divided Equally betwixt them; Nor will this be altered, if A or the Sum of the Velocities is Greater than that of B; But, if A is leſs than B, ſo that, tho' the Velocity is Great, the Moment is too Small to move B, the Velocity of A will be Spent, either Backwards or Sideways. And the Preſent Caſe will be a Poſſible one, becauſe $E=c$, and $C=e$, and are not Æquations, which Contradict each other.

Caſ. 2. If $E=C$, and $e=c$, in the ſame Bodies, and $E=c$, and $e=C$, in different Ones, that is, if the Expanſive is Equal to the Contractive, in the ſame Bodies,

and

and, in Different Bodies, the Expanſive of one is Equal to the Contractive of the other, and the Expanſive of the Latter Equal to the Contractive of the Former, the Velocities, and the Quantities of the Forces continuing, as in Caſe the Firſt, Equal, there will likewiſe Ariſe a perfect Quieſcence from ſuch a Concourſe of Forces: Becauſe, ſince $E=c$ in different Bodies, and $e=c$ in the ſame Bodies, $E=e$; and ſince $e=C$ in different Bodies, and $e=c$ in the ſame, $C=c$; and therefore, all the Forces being Equally Balanc'd, as before, this Caſe will be Reſolv'd into the Former, tho' it is differently Expreſs'd; and it will be a Poſſible one, becauſe all the Æquations are Conſiſtent with each other.

Caſ. 3. If $E=C$, and $e=c$, in the ſame Bodies, and $E=e$, and $C>c$, in different Ones, it will be an Impoſſible Caſe, and cannot obtain in Nature; for $E=e$, and $e=c$, therefore $E=c$; likewiſe $E=e$, and $E=C$, therefore $e=C$; but $C>c$, and $e=c$, therefore $C>e$; and $C>c$, and $E=C$, therefore $E>c$; that is, $E=c$, and $E>c$, $e=C$, and $C>e$, which is Impoſſible.

Caſ. 4. If $E=C$, and $e=c$, in the ſame Bodies, and $E>e$, and $C=c$, in different Ones, it will be alſo an Impoſſible Caſe; becauſe $E=c$, and yet $>c$, and $C=e$, and yet $>e$, by a like Reaſoning with the foregoing.

Caſ. 5. If $E=C$, and $e=c$, in the ſame Bodies, and $E=c$, and $e>C$, in different Ones, this likewiſe will be an Impoſſible Caſe; becauſe $e=E$, and $>E$, and $c=C$, and $>C$, by the foregoing Reaſoning, which is Impoſſible.

Caſ. 6. If $E=C$, and $e=c$, in the ſame Bodies, and $E>c$, and $e=C$, in different Ones, the Caſe, as before, will be Impoſſible.

Why Two Equal and Differently Unelaſtick Bodies, Carry'd againſt each other with Equal Velocities may upon their Concourſe be Quieſcent; as alſo, Why an Unelaſtick Body, Deſcending upon another, do's not Rebound, but if the Velocity is Increas'd of Two Incurring ones, there is a Repulſion.

Caſ. 7. If $E=C$, and $e=c$, in the ſame Bodies, and in different Ones $E>e$, and $C>c$, this will be a Poſſible one; becauſe $E>e$, and $e=c$, and $C>c$ and $E=C$, therefore on both Hands $E>c$; On the Contrary $E>e$, and $E=C$, and $C>c$, and $e=c$, therefore on both Hands $C>e$; that is, A being $=B$, and $V=v$, E will Repell e with a Greater Force to a contrary Side or Direction, than e will Repell E to a contrary One, if nothing elſe was to be conſider'd; but C Acting againſt E with a Force Equal to it, it's Repulſive Force is Balanc'd with the Contractive, as likewiſe e with c, and tho' E Acts with c, yet C alſo Acting with e, there is nothing can Enſue from the Concourſe of ſuch Forces, but the Quieſcence of them, which is the Caſe of Two Unelaſtick Bodies, yet whoſe Intrinſick Forces differ from each other, as probably of Two Equal Globes of Pure and Unmix't Gold, (which is Soft and Unelaſtick) and of Lead, or of Lead, and Clay, and the like. It muſt alſo be obſerved of Unelaſtick Bodies, as Lead, that, if one is Quieſcent, and the other is made to Deſcend upon it, it will not Rebound from it, but if the Velocity of Two Leaden Bodies, Meeting each other, is Increas'd, there will be a Repulſion; which proceeds from the Expanſive of the Velocity being Greater than the Contractive of the Lead, by which means they Propell each other to Contrary Directions, and yet if one Leaden Body Deſcends upon another, which is Quieſcent, and the Velocity is Increas'd, it will not Rebound, becauſe the Expanſive of the Velocity Acts with the Contractive of the Quieſcent Body, and Tends the ſame Way, and there is no Expanſive of the Quieſcent to Repell it.

Caſ. 8. If $E=C$, and $e=c$, in the ſame Bodies, and $E>c$, and $e>C$, in different Ones; therefore $C>c$, and yet $c>C$, as alſo $E>e$, and yet $e>E$, which is again an Impoſſible One.

Caſ. 9. If $E=C$, and $e=c$, in the ſame Bodies, and $E=e$, and $C<c$, in different Ones; therefore $C=e$, and $<e$, $E=c$, and $<c$, which is Impoſſible.

Caſ. 10. If $E=C$, and $e=c$, in the ſame Bodies, and $E<e$, and $C=c$, in different Ones; therefore $E=c$, and $<c$, $C=e$, and $<e$, which is Impoſſible.

Caſ. 11. If $E=C$, and $e=c$, in the ſame Bodies, and $E=c$, and $e<C$, in different Ones; therefore $e=E$, and $<E$, $c=C$, and $<C$, which is Impoſſible.

Caſ. 12. If $E=C$, and $e=c$, in the ſame Bodies, and $E<c$, and $e=C$, in different Ones; therefore $E=e$, and $<e$, $C=c$, and $<c$, which is Impoſſible.

Caſ. 13. If $E=C$, and $e=c$, in the ſame Bodies, and $E<e$, and $C<c$, in different Ones; therefore $E<c$, and $C<e$, which is a Poſſible Caſe, and will exactly Correſpond with *Caſ.* 7. Excepting, that the Greater Forces will be in *Becv*, and the Leſſer in *AECV*, whereas the Greater Forces, in that Caſe, were in *AECV*, and the Leſſer in *Becv*.

Caſ. 14. If $E = C$, and $e = c$, in the ſame Bodies, and $E < c$, and $e < C$, in different Ones; therefore $E < e$, and $e < E$, as alſo $C < c$, and $c < C$, which is Impoſſible.

Caſ. 15. If $E = C$, and $e > c$, in the ſame Bodies, and $E = e$, and $C = c$, in different Ones; therefore $E = c$, and $> c$, and $e = C$, and $> C$, which is Impoſſible.

Caſ. 16. If $E = C$, and $e > c$, in the ſame Bodies, and $E = c$, and $e = C$, in different Ones; therefore $e = E$, and $> E$, $C = c$, and $> c$, which is Impoſſible.

Why an Unelaſtick Body, falling upon a Quieſcent Elaſtick, Rebounds from thence, but an Elaſtick falling upon a Quieſcent Unelaſtick, is not Reflected, but Unites and Cloſes with it, and if they meet with an Equal Velocity, the Elaſtick is Carry'd to the ſame Side with the Unelaſtick.

Caſ. 17. If $E = C$, and $e > e$, in the ſame Bodies, and $E = e$, and $C > c$, in different Ones; therefore $E > c$, and $C = e$, which is a Poſſible Caſe; That is, if the Expanſive Force in one Body is Equal to it's Contractive Force, and the Expanſive Force in the other Greater than it's Contractive, and in different Bodies, if the Expanſive Forces are Equal, and the Contractive Force of the Firſt Body is Greater than the Contractive of the Laſt, It will from thence follow, that the Expanſive of the Firſt is Greater than the Contractive of the Laſt, and the Contractive of the Firſt Equal to the Expanſive of the Laſt.

This will be the Caſe therefore of an Unelaſtick Body EC, whoſe Expanſive and Contractive Forces are in an Equal Balance, as ſuppoſing Lead, and of an Elaſtick Body ec, whoſe Expanſive is Equal to the Expanſive of the Former, but Greater than the Contractive of the Latter, and therefore Elaſtick, as ſuppoſing Marble is ſo, and which will have theſe Properties, which are found in Experience, and are Accounted for, by theſe Forces, as the Concourſe of Lead with Lead, or Gold with Lead, was, by *Caſ.* 1, 2, and 7.

1. That a Globe of Lead, Dropt upon a Marble or Stone Pavement, will be Repuls'd or Rebound from it, or, which is the ſame, a Globe of Lead, Incurring upon a Quieſcent, and Fixt Globe of Marble, will be Reflected from thence; becauſe the Expanſive Force of the Marble, which is Greater than it's Contractive Force, Acts Counter to the Expanſive and Contractive Force of the Lead, which are ſuppos'd to be Equally Balanc'd, and therefore Repells it, but as this Expanſive Force is Weak, and the Contractive of the Lead is Great, the Repulſion is Small; and it is not Impoſſible, but the Contractive Force of the Lead may be Greater than it's Expanſive, which will contribute ſtill more to the making the Repulſion from the Marble Leſs.

2. That a Ball or Globe of Marble, on the Contrary, Dropt upon a Pavement of Lead, or Incurring upon a Quieſcent, and Fixt Globe of it, is not Reflected, but rather ſeems to Tend downwards, if there is any thing below ſuch a Pavement, to which it can Tend, as in the firſt Caſe, or in the Second, Endeavours to Cloſe, and Unite with ſuch Globe; becauſe in this Caſe the Expanſive Force of the Marble Acts the ſame Way with the Contractive of the Lead, and therefore they rather Tend to an Union with, than a Separation from each other.

3. If neither of them are Quieſcent, the Globe of Lead, Meeting with a Globe of Marble with an Equal Velocity, and being Equal in their Bulk and Dimenſions, will Repell it, and Carry it to the ſame Side, to which it ſelf Moves; becauſe the Globe of Lead Acts with the Expanſive Force of it's Velocity, which is Equal to that of the Velocity of the Globe of Marble, but the Contractive Force of the Lead being Greater than that of Marble, for in the Preſent Caſe $C > c$, the Moment of ſuch Expanſive Force of the Velocity of Lead muſt be Greater than that of Marble, and therefore muſt Repell it to the ſame Side, ſince the Expanſive Forces E and e are Equal, but the Marble will not Rebound from the Lead, and will therefore proceed along with it, for the Reaſon alledg'd in *Art.* 2. Namely, becauſe e the Expanſive Force of the Marble Acts the ſame Way with C the Contractive of the Lead. We only bring theſe Two Bodies of Lead and Marble, as Inſtances of Unelaſtick and Elaſtick Bodies in General, tho' perhaps they may not have exactly the Conditions of this Caſe, that is, probably the Expanſive Force of the Lead may not be preciſely Equal to the Contractive of it, but much leſs, nor to the Expanſive of the Marble, for Silver, or Steel, or Iron, or other Metallick Subſtances, may perhaps come nearer to ſuch an Equality of the Expanſive in Lead, than Marble do's.

Caſ. 18. If $E = C$, and $e > c$, in the ſame Bodies, and $E > e$, and $C = c$, in different Ones; $E = c$, and $> c$, and $C > e$, and $e > C$, which is Impoſſible.

Why upon Concourſe the Elaſtick Body may not Rebound from the Unelaſtick, but the Unelaſtick from it.

Caſ. 19. If $E=C$, and $e>c$, in the ſame Bodies, and $E=c$, and $e>C$, in different Ones; therefore $e>E$, and $C=c$, which is a Poſſible Caſe, and will be that of an Unelaſtick and an Elaſtick Body, as in *Caſ.* 17; but, as there the Expanſive of the Firſt was Equal to the Expanſive of the Second, and the Contractive of the Firſt Greater than the Contractive of the Second, ſo here the Expanſive of the Firſt is Equal to the Contractive of the Second, and the Expanſive of the Second Greater than the Contractive of the Firſt; which may Poſſibly be very nearly the Caſe of an Unelaſtick and Elaſtick Body, which have the ſame Weights, where $e>E$, but $C=c$; But in this Caſe, ſomething Different from what was Obſerv'd on *Caſ.* 17,

1. The Unelaſtick Body, Impinging upon a Quieſcent, and Fixt Elaſtick, will Rebound or be Repell'd from it.

2. The Elaſtick Body, Impinging upon a Quieſcent, and Fixt Unelaſtick one, will not Rebound or be Repell'd from thence; And both theſe are Affirm'd, for the Reaſons, before Alledg'd, in *Caſ.* 17.

3. If they meet each other with an Equal Velocity, the Elaſtick will not Rebound from the Unelaſtick, but the Unelaſtick from it; becauſe, their Velocities being not only Equal, but the Moments of them, the Greateſt Force will lie on the Side of the Expanſive e againſt EC, whoſe Forces are in an Equal Balance; And This may be the Caſe of Marble and Dry'd Clay; ſince $E=C$ in the Clay, and $e>c$ in the Marble; likewiſe E in the Clay $=c$ in the Marble, and e in the Marble $>C$ in the Clay; and therefore e in the Marble $>E$ in the Clay, and C in the Clay $=c$ in the Marble, from whence it is neceſſary, that e in the Marble ſhould Prevail, and Propell the Clay from it.

Caſ. 20. If $E=C$, and $e>c$, in the ſame Bodies, and $E>c$, and $e=C$, in different Ones; therefore $E=e$, and $C>c$, in different Ones; which likewiſe is a Poſſible Caſe, where the Unelaſtick Body is EC, whoſe Expanſive and Contractive Forces are in an Equal Balance, and the Elaſtick ec, in which e is Greater than c, as in all the foregoing Caſes, from *Caſ.* 15; And here $E=e$, and $C>c$, as alſo $E>c$, and $e=C$, makes it Exactly the ſame with *Caſ.* 17.

Why an Elaſtick Body may not Repell the Unelaſtick, but be Carry'd to the ſame Side with it.

Caſ. 21. If $E=C$, and $e>c$, in the ſame Bodies, and $E>e$, and $C>c$, in different Ones; therefore $E>c$, and $C>=<e$; if $C<=e$, it is an Impoſſible Caſe; if $C>e$, it will be a Poſſible One: EC, as before, is the Unelaſtick Body, and ec the Elaſtick, but the Expanſive of the Firſt is Greater than the Expanſive of the Second, and the Contractive of the Firſt, than the Contractive of the Second; likewiſe the Expanſive of the Firſt is Greater than the Contractive of the Second, and the Contractive of the Firſt, than the Expanſive of the Second; which may be the Caſe of Lead and Glaſs, where the Firſt is more Expanſive, but is Balanc'd by it's Greater Contractive, and the Laſt is Leſs Expanſive and Leſs Contractive, but the Expanſive is Greater than the Contractive: Upon the Concourſe therefore of theſe Two Bodies,

1. The Unelaſtick Body, Impinging upon a Quieſcent, and Fixt Elaſtick, will not Rebound, or be Repell'd from it; becauſe the Expanſive of the Elaſtick is too Weak to Produce that Effect, and the Moment of the Expanſive and Contractive in the Lead too Strong to admit of ſuch a Repulſion.

2. The Elaſtick Body, Impinging upon a Quieſcent, and Fixt Unelaſtick, will not Rebound, or be Repell'd from thence, for the Reaſons, before Alledg'd, in *Caſ.* 17. *Art.* 2.

3. If they meet each other with an Equal Velocity, the Elaſtick Body will not Repell the Unelaſtick, but the Moment of the Unelaſtick, being Greater, will carry the Elaſtick to the ſame Side with it.

Why a Body which is Extremely Elaſtick Fuſes and Raiſes into an Ebullition an Unelaſtick one.

Caſ. 22. If $E=C$, and $e>c$, in the ſame Bodies, and $E>c$, and $e>C$, in different Ones; therefore $C>c$, and $C<=>c$, and $e>E$, and $E<=>e$; which Caſe likewiſe will be Poſſible, if $C>c$ and $e>E$ on both Hands, and may probably be that of Lead and Fire; Since the Contractive Force of Lead is Greater than the Contractive of Fire, and the Expanſive Force of the Fire is Greater than that of Lead, as alſo, the Expanſive Force of the Lead is Greater than the Contractive of the Fire, and the Expanſive of the Fire Greater than the Contractive of the Lead; From whence it comes to Paſs, that the Expanſive Force of the Fire, being Greater than that of Lead, Repells it, by which it's Ebullition is cauſed upon the Fuſion of it, and being Greater than the Contractive Force of Lead Fuſes it, for they Act the ſame Way; and E being $>c$, and $C>c$, that is, the Expanſive Force of the Lead, being Greater than the Contractive of the Fire, and the Contractive of the Lead, Greater than the Contractive of the Fire, will have no Effect, becauſe E and C are in an Equal Balance with each other, and as E Acts with c, C Acts againſt it. And this is, ſuppoſing Lead is Quieſcent, and Fire Acts upon it. For, as to Velocity in this Caſe, the Expanſive Force of the Fire is ſo Great, that no Communication of the Expanſive Force, as that of Velocity, can give any Moment to it, by which means it will Act upon Lead with it's own Expanſive, which is Peculiar to it ſelf, and by it Diſſolve Firſt, and then Repell it, as has been before ſaid; And if Lead is carry'd againſt it with a certain Velocity, the whole Moment of the Expanſive of the Lead, join'd with it's Great Contractive, will be ſo much Superior to the Expanſive of the Fire, which has but a Small Contractive belonging to it, that it will Paſs through it, with Little or No Oppoſition. And here we muſt obſerve in General, in Reſpect of theſe Concourſes, that Groſſer Expanſive Bodies Require a Velocity to Diſplay their Particular Actions, by ſuſpending their Gravitation, which Hinders and Suppreſſes Them; But it is not ſo in Fire, whoſe Vaſt Expanſive Force ſurmounts all Oppoſition to it from the Earth's Attraction.

Caſ. 23. If $E=C$, and $e>c$, in the ſame Bodies, and $E=e$, and $C<c$, in different Ones; therefore $E>$, and $<c$, and $e=C$, and $>C$, which is Impoſſible.

Caſ. 24. If $E=C$, and $e>c$, in the ſame Bodies, and $E<e$, and $C=c$, in different Ones; therefore $E=c$, and $E=><c$, and $C\leq e$, ſo that ſuppoſing $E=c$, and $C<e$, this will likewiſe be a Poſſible Caſe.

The Remaining Caſes I ſhall only juſt Mention, and leave to others to Inquire, which are Poſſible, and which not, and to what Kind of Elaſtick, and Unelaſtick Bodies they are Applicable, which is eaſily done by Proceeding in the Method before ſet down.

Caſ. 25.	$E=C$, $e>c$		$E=c$, $e<C$	
Caſ. 26.	$E=C$, $e>c$		$E<c$, $e=C$	
Caſ. 27.	$E=C$, $e>c$		$E<e$, $C<c$	
Caſ. 28.	$E=C$, $e>c$		$E<c$, $e<C$	
Caſ. 29.	$E=C$, $e<c$		$E=e$, $C=c$	
Caſ. 30.	$E=C$, $e<c$		$E=c$, $e=C$	
Caſ. 31.	$E=C$, $e<c$		$E=e$, $C>e$	
Caſ. 32.	$E=C$, $e<c$		$E>e$, $C=c$	
Caſ. 33.	$E=C$, $e<c$		$E=c$, $e>C$	
Caſ. 34.	$E=C$, $e<c$	in the ſame Bodies; and	$E>c$, $e=C$	in different Ones.
Caſ. 35.	$E=C$, $e<c$		$E>e$, $C>c$	
Caſ. 36.	$E=C$, $e<c$		$E>c$, $e>C$	
Caſ. 37.	$E=C$, $e<c$		$E=e$, $C<c$	
Caſ. 38.	$E=C$, $e<c$		$E<e$, $C=c$	
Caſ. 39.	$E=C$, $e<c$		$E=c$, $e<C$	
Caſ. 40.	$E=C$, $e<c$		$E<c$, $e=C$	
Caſ. 41.	$E=C$, $e<c$		$E<e$, $C<c$	
Caſ. 42.	$E=C$, $e\leq c$		$E<c$, $e<C$	

In the ſame Bodies. In different Ones.

$E>C$, $e>c$		$E=e$, $C=c$
		$E=c$, $e=C$
		$E=e$, $C>c$
$E>C$, $e=c$		$E>e$, $C=c$
		$E=c$, $e>C$
		$E>c$, $e=C$
$E>C$, $e\leq c$	Each of the Diviſions in the Firſt Column may be Combin'd with Every one of the Diviſions in the Second.	$E>e$, $C>c$
		$E>c$, $e>C$
		$E=e$, $C<c$
$E\leq C$, $e<c$		$E<e$, $C=c$
		$E=c$, $e<C$
		$E<c$, $e=C$
$E\leq C$, $e=c$		$E<e$, $C<c$
		$E<c$, $e<C$
$E\leq C$, $e>c$		

Caſes 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, &c.

From

This Doctrine ſufficient for all the Infinite Variety of Caſes in the Mutual Concourſe of Bodies.

FROM whence will Ariſe 84 Caſes more, which Added to 42, before Set down, will Make 126, Poſſible and Impoſſible Ones, ſo that here cannot but be a ſufficient Latitude for the Explaining of the Mutual Concourſe of all Bodies; Eſpecially, when we likewiſe Conſider, that theſe Forces may be Equal, Greater or Leſs than each other in an Infinite Number of Degrees, thus $3 = 3$, $4 = 4$, $100 = 100$, *&c.* $10 > 7$, or 5, or 3, or any Number below it, and $5 < 10$, or 50, or 1000, or any Number above it; from whence it will follow, that if the Peculiar Expanſive and Contractive Forces of Bodies are vary'd to Infinity, as it is Reaſonable to think they are, which muſt cauſe Effects very Different upon their Meeting each other, the foregoing Theory will Account for all of them; whereas the Modern and Receiv'd Explication of ſuch Concourſe will be Infinitely Defective and Imperfect, which ſuppoſes no Innate Forces in Bodies, but that they are Similar and Homogeneous, and are only Acted by Two Imaginary Ones, Simple Impulſe and Elaſticity.

WHAT has been hitherto ſaid is upon the Suppoſition of the Velocities being Equal, but, if they are Various, there will be new Combinations Produc'd, and a Series of Caſes ſtill Different from the Former.

What Problems we ſhall be able to Solve, when we have Determin'd the Preceding Caſes.

§. 35. WHEN we have gone thro' this Theory of the Expanſive and Contractive Forces, and have Determin'd all the Poſſible Caſes, it will be then eaſy for us to Reſolve the following Problems.

1. WHAT will be the Effect of Two Unelaſtick Bodies, one falling upon the other, which is Quieſcent, and Fixt, with a Certain, a Greater, or a Leſs Velocity.
2. WHAT will be the Effect of Two Unelaſtick Bodies, one falling upon the other, which is not Fixt, but is Quieſcent in a Certain, a Greater, or Leſs Degree, with a Certain, a Greater, or a Leſs Velocity.
3. WHAT will be the Effect of Two Unelaſtick Bodies, one falling upon the other, which is in a Certain, a Greater, or Leſs Motion the ſame Way, with a Certain, a Greater, or Leſs Velocity.
4. WHAT will be the Effect of Two Unelaſtick Bodies, one falling upon the other, which is in a Certain, a Greater, or Leſs Motion the Contrary Way, with a Certain, a Greater, or a Leſs Velocity.
5. WHAT will be the Effect of Two Unelaſtick Bodies, falling the one upon the other, whoſe Unelaſticity is Greater, or Leſs than the Former, with their ſeveral Variations of Quieſcence, and Velocity, mention'd above.
6. WHAT will be the Effect of Two Bodies, the one falling upon the other, the one Elaſtick, and the other Unelaſtick, in Equal or Different Degrees of thoſe Forces, with their ſeveral Variations of Quieſcence, and Velocity, mention'd above.
7. WHAT will be the Effect of Two Elaſtick Bodies, in Equal or Different Degrees of thoſe Forces, the one falling upon the other, with their ſeveral Variations of Quieſcence, and Velocity, mention'd above.

Some Problems ſolv'd in General, upon which the others Depend.

§. 36. IN Relation to all theſe Problems, I ſhall only take Notice (and which we have in Part already done) of Two Unelaſtick Bodies, Two, which are Elaſtick, and Two, one of which is Elaſtick, and the other Unelaſtick, and where one falls upon the other, which is Quieſcent and Fixt; becauſe all the other Caſes are only Differenc'd by the ſeveral Combinations of theſe.

Firſt, Why an Unelaſtick Body falling upon another Unelaſtick Cloſes and Unites with it; farther Explain'd.

AND Firſt, if an Unelaſtick Body falls upon another (which is Equally Unelaſtick, and is Quieſcent and Fixt) with a Certain Velocity Ariſing from it's Gravitation, it will Seem to Cloſe and Unite with it; becauſe there is an Expanſive Force in the Velocity, which Acts the ſame Way with the Contractive of the Quieſcent, and is Superior to the Contractive of the falling Body, as the Contractive Force of the Earth is Superior to the Contractive Force of ſuch Body, and which, Acting contrary to it, has the Nature of the Expanſive; from whence it is, that a Bullet of Lead, Dropt upon a flat Piece of it, does not Rebound from it, but Endeavours to Cement and Join with it.

AND if there is a Greater Expanſive Force or Velocity, Communicated to the Bullet by the Hand, it ſtill leſs Rebounds, but makes Impreſſions into it, and Indentations in it, or Tends to a Situation below the Lead, on which it falls; becauſe the Expanſive Force of the Lead falling, is Increaſed, and ſtill Acts with the Contractive of the Quieſcent, and in the ſame Direction: If the Lead is not Fixt, as in Pendulums,

lums, or is in Motion the Same, or the Contrary Way, with the falling Bullet, tho' Allowances are to be made for the Expanſive Force of Velocity in the Lead, on which the Impaction is made, yet what foregoes holds true, when we have made thoſe Allowances; Thus Two Equal Leaden Bullets, Meeting with an Equal Velocity, will loſe their Motion, becauſe the Contractive Force of each, Acting towards it's own Center cauſes no Repulſion, and the Equal Expanſive Forces of the Velocities deſtroy each other.

Sir Iſ. Newt. Optic. Queries.

It muſt be alſo obſerv'd, that as the Unelaſtick Force of Lead is Different from that of Clay, ſo the Effect of one Piece of Clay falling upon another, which is Quieſcent and Fixt, will be likewiſe in Degree Different, but the Solution of the Adheſion of one to the other, will be the ſame as before; And the like may be ſaid of Lead and Clay in their Mutual Concourſe.

Second, Why Elaſtick Bodies are Repell'd from each other.

Second, If an Elaſtick Body falls upon another, which is Elaſtick, with a Certain Velocity Ariſing from it's Gravitation, as a Ball of Marble upon a Piece of Marble, which is Quieſcent and Fixt, it will be Repell'd from thence; becauſe the Expanſive Force of the Marble, which falls, Acts with the Expanſive Force or Velocity proceeding from Gravitation, but the Expanſive Force of the Marble, on which the Impaction is made, Acts Contrary to the Former Force, and being not only Quieſcent, but Fixt, the Superior Incurring and Expanſive Force, not being able to ſpend it ſelf that Way, muſt neceſſarily Reſult from it, to which the Repulſive Force of the Fixt Marble will alſo Contribute.

If this Expanſive Force is Increas'd by an Acceſſion of Velocity Impreſs'd from the Hand, the Impinging Marble will Rebound the Higher; If Two Balls of Marble Meet each other, they will mutually Repell each other from their Expanſive Force Acting Contrary Ways, and deriv'd from their Elaſticity and Velocity; and if the Velocity is Increas'd, the Repulſe will be the Stronger: Farther, if a Marble is let fall upon a Flint Glaſs, which is a more Elaſtick Body than Marble, it will Rebound more, than when it fell upon Marble, becauſe the Reſiliency is Greater from the Greater Expanſive Force of the Glaſs; and if Two Glaſs Balls Meet, they will be more forcibly Repell'd than Two Balls of Marble; or, if the Glaſs Balls Meet each other with a Greater Velocity, they will be more ſtrongly Repell'd, than if they met with a Leſs; becauſe on both Hands the Expanſive Force is Increas'd.

Mr. Hauksbee's Experiments, Pag. 106.

I ſhall only here add, that upon the ſame Principles are ſolv'd the Experiments, made by the moſt Judicious and Ingenious Mr. *Hauksbee* upon the Preſent Subject of Repulſion, even in Reſpect of Different Mediums, (which yet is the leaſt Thing to be Conſider'd in the Rebounding of Different Bodies, where their Innate and Intrinſick Forces are to be Examin'd) for a Thicker Medium, being more Contractive, hinders the Expanſive, by which Reſilition is Perform'd, and a Thinner Medium, being leſs Contractive or more Expanſive, Promotes it.

Why an Unelaſtick Body, falling upon an Elaſtick, is Repell'd from it, but if an Elaſtick falls upon an Unelaſtick, it Cloſes and Unites with it; farther Explain'd.

Third, If an Unelaſtick Body falls upon an Elaſtick, which is Quieſcent and Fixt, with a Certain Velocity, Ariſing from it's Gravitation, it will be Repuls'd from thence, thus a Ball of Lead, falling upon Marble, will Rebound from it; On the Contrary, If an Elaſtick Body falls upon an Unelaſtick, as a Ball of Marble upon Lead, there is no ſuch Repulſion, but the Marble Endeavours to Cloſe and Unite with the Lead, and ſtrikes off in a Lateral Direction.

The Reaſon of which is, becauſe in the Former Caſe, the Contractive of the Lead, and the Expanſive of the Marble Act Upwards, and in the Latter Downwards in the ſame Direction; For the Contractive of the Lead Acts towards it ſelf, and the Expanſive of the Marble from it ſelf, in the Firſt Inſtance, that is, the ſame Way Upwards; and in the Second, the Expanſive of the Marble Acts from it ſelf, and the Contractive of the Lead towards it ſelf, that is, the ſame Way Downwards.

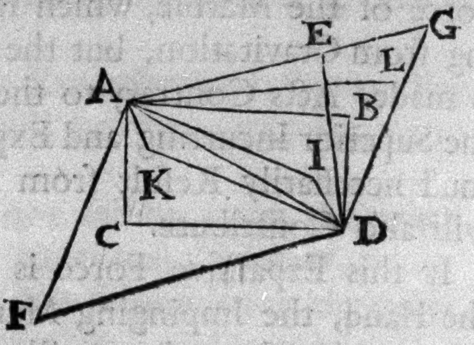
And in the Firſt Caſe, if the Velocity is Increas'd, the Repulſion is likewiſe; becauſe Acting againſt the Expanſive of the Marble, and it's Progreſs being there ſtopt, it is Return'd backwards in the Direction of the Expanſive, and the Contractive of the Lead has Leſs Force, by how much the Expanſive of the Velocity is Greater.

But in the Second, If the Velocity is Increas'd, the Marble Tends with a Greater Force towards an Union with the Lead, on which it Impinges, and is ſtill farther from Rebounding; becauſe the Velocity Acts the ſame Way with the Expanſive of the Marble, and with the Contractive of the Lead, that is, Downwards.

I might here farther Enlarge, but ſhall leave the Remaining Particulars in this Doctrine, to be Traced out and Examin'd by others, who ſhall think it worth their Leiſure and their Inquiry, being Satisfied in having Pointed out the way for Greater Advances and Improvements in this kind of Knowledge, than has been hitherto Attempted by any of the Mechanick Philoſophers: I ſhall only here Add, that, ſince Hydroſtaticks are the Laws, which Ariſe from the Concourſe of Solid and Fluid Bodies, They likewiſe are Explicable by theſe Expanſive and Contractive Forces, as we ſhall ſhew, when we come to Treat of Them.

The Compoſition of Forces Explain'd. Sir Iſ. Newt. Princ. Math. Lex. 3. Corol. 1. Mr. Whiſton's Phyſ. Math. Prælect. 6. Art. 22.

§. 37. HAVING conſider'd Action and Re-action, and the ſeveral Laws of Unelaſtick and Elaſtick Bodies, we are next to Inquire into the Compoſition of Forces; in Reſpect of which, the Diagonal Force is ſaid to be Equal to that of the Sides of a Parallelogram, drawn from the ſame Point with it; Thus *A D* is Equal, in it's Joint Force, to *A B* and *A C*, the Separate Ones: Becauſe *A*, in it's Progreſs to *B*, is not Interrupted by the Force, which Directs it from *A* to *C*, nor *A* to *C*, by that, which carries it from *A* to *B*, ſo that it ſhou'd not Proceed ſo far, as *B* on the one Hand, and as far as *C* on the Other; that is, both theſe Forces are Conſiſtent, and by Conſequence muſt meet in *D*; I call theſe Forces Equal, the Diagonal to the Lateral, becauſe the Firſt are Reſolv'd into the Laſt, and the Laſt Compound the Firſt, according to this Philoſophy, in it's uſe of this Compoſition of Forces.



This Compoſition of Forces contrary to Mathematick Demonſtration. Mr. Whiſton. Prælect. 6. Corol. 5.

§. 38. IN Anſwer to which we muſt Firſt ſay, that, by *Euclid*, Elem. 1. Prop. 47. Schol. *Tacquet*, if *ABDC* is a Square, *AD* is Incommenſurable with the Side *AB* or *AC*, how therefore both together ſhould be Equal to *AD*, is not to be Explain'd, ſince the Forces are as the Spaces, which they Deſcribe in the ſame Time; therefore there muſt be a Thicker and more Compreſs'd Force in a *AD*, and a more Thin and Diluted one in *AB* and *AC*, and which will be Equal the one to the other; but this is Impoſſible, according to the Demonſtration Refer'd to, becauſe theſe Quantities are Incommenſurable, and have no Common Meaſure, by which their Equality, or any other Proportion betwixt them, can be Aſſerted; I muſt own this may be, that a Thicker and a more Conſtipated, or a Stronger Force, may be Reſolv'd into Two Thinner, and more Diluted or Weaker, and that they may be Expreſs'd by theſe Lines, ſuppoſing *AD* a more Conſtipated one, and *AB*, *AC* leſs ſo; but this is what, neither the Preſent Mathematicks, nor the Preſent Philoſophy, or Compoſition of Forces conſiders; and therefore, I argue againſt this Abſtracted Reaſoning in the Compoſition of Forces, from another Abſtracted Reaſoning in the Demonſtrations of Geometry, which I ſay are Inconſiſtent, as Abſtracted Reaſonings, which are the Mere Creatures of Men's Minds will generally be found to be; and not only in theſe, but in all other Studies, as in Logicks, Metaphyſicks, and even in Critique, and the Explanations of Authors, in which there are very Singular Inſtancesof Thoſe, who delight more in Abſtracted Fancies of their own Minds, than in any Real and Solid Reaſoning.

Another Objection againſt it. Mr. Whiſton. Prælect. 6. Corol. 1.

§. 39. A Second Objection is, that as the Angle *BAC* is Greater or Leſs, the Velocity of *A* to *D* will be Leſs or Greater, when yet the Velocities given of *A* to *B* and *C* are ſuppos'd to be Unalter'd; Becauſe where the Angle is Leſs, as in *IAK*, the Direction of the Velocities, which were before in *AB*, *AC*, Tends the ſame Way, but where the Angle is Greater, as in *GAF*, contrary Ways, and therefore the Velocity is Increas'd in *AD* in the Firſt Caſe, and Diminiſh'd in the Laſt; This, we ſay, is a Second Objection againſt the Preſent Compoſition of Forces, ſince, if *AD* is not certain and determin'd in the Force, which it Expreſſes, and ſince ſeveral Forces are the moſt naturally Repreſented by ſeveral Lines, *AB* and *AC* in their various Inclinations and their differing Degrees of Velocity cannot be truly Expreſs'd by one and the ſame Diagonal *AD*, nor can *AD* be juſtly Reſolv'd into them.

§. 40. BESIDES,

§. 40. Besides which, ſuppoſing the Forces, *AB*, *AC*, cou'd be Expreſs'd by the Diagonal *AD*, it is yet Evident, that *AD* cannot be Reſolv'd into the Forces, *AB*, *AC*, at Pleaſure, that is, we may not draw Lines with what Inclination we pleaſe to *AD*; or, which is the ſame, altho' the Forces, *AB*, *AC*, being known, their Compound is determin'd by the Diagonal *AD*, yet, the Diagonal being known it is not Conſequent, that the Forces, which form it, ſhou'd be, becauſe there may be a Thouſand Different Sides of a Parallelogram drawn to the ſame Diagonal; notwithſtanding which, there is nothing more Common in the Philoſophy, which this Notion is brought to Eſtabliſh, than the Reſolving the Diagonal into any Lines, which it finds the moſt convenient for it's Purpoſe, as is Evident in the Explanation it gives of the Doctrine of Mechanicks, and in the Solutions it propoſes of the Revolution of the Secondary Planets about their Primary.

A Third Objection againſt this Compoſition of Forces.

Sir If. Newt. Princip. Lex. 3. Corol. 2.

§. 41. Thus far we have conſider'd the Force repreſented by the Diagonal, and from thence ſhall Proceed to thoſe of the Sides of the Parallelogram, which were ſaid to be vary'd according to *AB* and *AC*, more or leſs, oppoſing each other in the Direction of thoſe Forces, that is, as the Angle *BAC* was Greater or Leſs; but if this will not be Allowed, and if the Velocity from *A* to *B* is given, and that of *A* to *C*, and it is ſaid, that, in whatever Direction they are, they will continue the ſame, becauſe if *A* Acts towards *B* with a certain Force, and towards *C* with another certain Force, ſince the Fountain or Source of Action is ſuppoſed to be in *A*, whatever Inclination the Lines, *AB*, *AC*, have to each other, it can be of no Importance to the Altering of the Velocities propagated from thence, let this Reaſoning then be Allow'd;

This Compoſition of Forces not True upon another Account.

Mr. Whiſton. Prælect. 6. Art. 22.

If therefore *AB* and *AC* have their fixt and determin'd Velocities in any Inclination we can mention, it will be Impoſſible, that *AD* the Diagonal ſhould expreſs them; becauſe, if *AE* is made Equal to *AB*, *ED* will be one Side of the Parallelogram, to which it is Inconceivable *AC* ſhould be Equal, which is ſuppoſed to be the Expreſſion of it's Velocity, and to be Unalter'd.

§. 42. Another Objection is, that *AB* and *AC* muſt, each of them, Act with the ſame Velocities, in whatever Inclination they are, or not; If they do, the foregoing Section is Valid in it's Argument againſt the Preſent Compoſition of Forces, if they do not, ſince the Velocities are Diminiſh'd by Oppoſing each other, or Increas'd by Concurring in differing Inclinations, it will be the ſame, as Contracting or Prolonging the Sides of the Parallelogram in the ſame Inclination, and conſequently, the Diagonal *AD* will be ſuitably made Shorter or Enlarg'd, which is far from agreeing with this Doctrine, which ſuppoſes, that the Diagonal may continue the ſame, altho' the Sides of the Parallelogram are Infinitely vary'd; thus, if *AB*, *AC* are Equal, and recede from each other, till they are perfectly Oppoſite, the Forces, Acting from *A* in contrary Directions, will mutually hinder each others Progreſs, if they approach each other, till they Coincide, the Velocity will be the Sum of both, and therefore all the intermediate Effects, produced in the ſeveral Inclinations of *AB*, *AC*, ſhould not be Expreſs'd by one and the ſame Diagonal Line *AD*, but by an Infinite Number of different Lines of various Lengths betwixt 0, repreſented by a Point, and *AB* + *AC*, which repreſents the Sum of the Forces.

A farther Difficulty Explain'd, which lies againſt this Compoſition of Forces.

§. 43. There is likewiſe a farther Thing to be conſider'd in this Doctrine, namely, that *AF*, *AG*, Parallel and Equal to *GD*, *FD*, are conceiv'd to be the ſame in all Reſpects with *AC* and *AB*, in their Relation to the Diagonal *AD*, and all the Parallelograms, drawn to the ſeveral Parts of *AD* from the ſeveral Parts of *AC* and *AB* on the one Hand, or from *AF* and *AG* on the other, exactly Correſpond in their Diagonal Portions of *AD*, from whence it follows, that the Reſult of the Forces on both Hands are Equal, if the ſame Lines or Parts of a Line are Expreſſive of the ſame Forces, which they ought to be; whereas it is confeſs'd, that, when the Forces Concurr, that is, when *AB* Approaches to *AC*, or when they are Oppos'd, as in their Retiring from each other, the Reſult of the Forces are not Equal, and conſequently cannot be Expreſs'd by the ſame Diagonal Portions; that is, the Preſent Doctrine ſuppoſes the Reſult of the Forces to be Equal, and not Equal at the ſame Time, which is an Abſurdity and a Contradiction.

A Sixth Objection.

Mr. Whiſton. Prælect. 6. Corol. 1.

A Seventh Objection. Prælect. 6. Art. 22,

§. 44. We muſt alſo add, that even the Foundation of this Doctrine of the Compoſition of Forces is not certain, for if a Force is propagated Two Different and even Contrary Ways from *A*, it cannot be altered from thence, becauſe Two ſuch Forces will have no Effect or Influence upon each other, and therefore *AB* and *AC* may be both deſcribed without any Diagonal at all, that is, if we conſider only the Action of Two Forces from the ſame Point, and that any thing more is Prov'd from this Figure is not Certain, ſince the Sides of a Parallelogram may be deſcribed by Two Forces, without ſuppoſing them to be Compounded into a Diagonal; if Two Forces are not ſuppoſed to Act from the ſame Point, but upon it, what has been Alledg'd in §. 42 and 43 will be Valid, if the other, §. 41 will.

Another Objection againſt this Compoſition of Forces.

§. 45. Farther, If *A* is Acted by Two Different Forces repreſented by *AB*, *AC*, it will not yet follow, that the Compound of them ſhould Proceed in the Diagonal *AD*; becauſe after the Action at *A* they are ſuppoſed to preſerve their Original Velocities, *AB* and *AC*, otherwiſe they would not meet in the Point *D*, which is upon the Hypotheſis, that *A* has the Force to carry it to *B*, and to *C*, in which the Parallel Lines to them *CD* and *BD* Concurr; if therefore *A* preſerves it's Velocity to *B* and to *C*, and conſequently it's Direction, for by thoſe Lines and thoſe Directions both the Velocities are Expreſs'd, how they ſhou'd preſerve them in the Diagonal *AD*, where the Direction and the Line is Different, will be Impoſſible to Explain.

The Ninth Objection.

§. 46. On the Contrary, If *AB* and *AC*, after the Action upon *A*, do not preſerve their Velocities, that is, if *A*, by being Acted upon, goes in the Line *AD*, ſo as the Velocities of *AB* and *AC* are Chang'd, *AD* will not be neceſſarily the Diagonal of a Parallelogram, becauſe the Sides of it, and the Velocities repreſented by them, will be Deſtroy'd; that is, ſuitable to what we ſaid before, in §. 44, the Sides of the Parallelogram will continue without a Diagonal, and in this, the Diagonal or rather the Line *AD*, without any Sides, to which it is Refer'd, and Half, or a Quarter, or any other Part, of *AD* may repreſent the Compound of *AB*, *AC*.

What the Truth is in this Caſe.

§. 47. The Truth is in any Compoſition of Motion, or of Forces, as of *AB*, *AC*, it goes a Greater or Leſſer Length in *AD*, not as *AB* or *AC* are the Sides of a Parallelogram, but according as the Forces are Greater or Leſs which Act; becauſe if *AB* and *AC* were to approach Infinitely near to Each other, *A* wou'd be carried with the Force of *AB* + *AC* in a Direct Line, which wou'd be Greater than *AD* the Diagonal; and where *AB* and *AC* are as far from Each other as poſſible, the Compoſition of Forces will be much Leſs than the Diagonal, being reduc'd to a Point, and the ſame may be ſaid of all the intermediate Degrees of Compoſition, with the Allowance of Greater and Leſs in Proportion, according to what has been before mention'd.

A Tenth Objection. Mr. Whiſton. Prælect. 6. Corol. 3.

§. 48. If nothing of what, we hitherto have argu'd, was certain, this is acknowledg'd, that, the Forces being given, the Diagonal is known, but, the Diagonal being known, the Forces Expreſs'd by the Sides of the Parallelogram are not, ſince, according to this Doctrine, there may be ſeveral Sides of a Parallelogram drawn to the ſame Diagonal; but if, upon the Diagonal being known, the Sides are yet Latent and Undiſcover'd, this Philoſophy will want it's main Support, both in the Explanation of Mechanicks, and of the Revolutions of the Secondary Planets, in which it takes it for granted, that, the Diagonal being known, the Sides are no longer a Secret.

The Eleventh Objection.

§. 49. We ſhall laſtly ſubjoin, that it is Impoſſible in the preſent Caſe, if the Forces, *AB*, *AC*, are Compounded into *AD*, but they ſhou'd again be Reſolv'd into them in Reſpect of their Length, unleſs *AD* can be ſhorter than *AB* and *AC* taken together, and yet Expreſs the ſame Force; that is, where Forces are Reduced to Lines, thoſe Lines are Expreſſive of them, or not; if they are, it is abſurd to ſay, that *AB* + *AC* is = *AD*, if they are not, *AB* and *AC* are not the Expreſſions of the Forces, which they are, notwithſtanding, made to repreſent, which is a Contradiction. If it be ſaid, in Reſpect of theſe Two laſt Sections, and of this whole Doctrine, That *AD* may be Reſolv'd into *AI* and *AK* or *AID*, into *AB* and *AC* or *ABD*, into *AG* and *AF* or *AGD*; becauſe they are all the ſame, that is, becauſe they are Forces perfectly Equivalent; and if it be farther ſaid, that theſe Forces are the Same or Equivalent, becauſe, by the Law of Action and Re-action, to which this Compoſition of Forces is made a Corollary, where the Force *AI* is

Leſs than *AD*, the Concurrent Force *AK*, in the Inclination *IAK* will make it juſt Equal to it, or the Force *AID* = *AD*, on the Contrary, where the Force *AG* is Greater than *AD*, by the ſame Law of Action and Re-action, the Oppoſite Force *AF*, in the Inclination *GAF*, will take away ſo much from it, as to make it exactly Equal to it, or *AGD* = *AD*, and ſo the Nearer *AI* and *AK* Approach to Each other, they will come Nearer to an Equality, that is, *AID* will come Nearer to a Coincidence with *AD*, and the farther *AG* and *AF* Retire from Each other, the Longer they will be, that is, their Forces will be the Greater, as their Oppoſition is ſo, in Order to make them Equal to the ſame Force *AD*; If this is ſaid, We Anſwer,

1. That it is not Prov'd, that the Concurrence of *AI* with *AK*, in the Inclination *IAK*, or that the Oppoſition of *AF* to *AG*, in the Inclination *GAF*, adds juſt ſo much to the one, and takes off juſt ſo much from the other, as to make them, on both Hands, Equal to *AD*; Eſpecially, ſince betwixt *AG*, which is Greater than *AD*, and *AB* which is Leſs, there muſt be a Line, which is Equal to it, as for Inſtance *AL*, and conſequently it's Oppoſite Force, which Acts againſt it, muſt either have no Force at all, and then there is an End of theſe Oppoſite Forces, or if it has, *AL* together with it will be Leſs than *AD*, contrary to what was propos'd to be Demonſtrated.

2. Therefore, where *AI* or *AB* are Leſs, and *AG* Greater than *AD* or *AL*, their Concurrent or Oppoſite Forces will not make them Equal to *AD*.

3. Since *AI* and *AK*, *AG* and *AF*, are Velocities perform'd in the ſame Time, they will be in themſelves Different, but how the Addition of Two Different Velocities, which Concurr, or the Subſtraction of Two from Each other, which are Oppoſite, ſhou'd be Equal to one Uniform Velocity in *AD* will be hard to Explain.

4. Granting yet all, which can be deſir'd, ſince there may be a Thouſand Different Velocities and Forces in *AD*, for it may be deſcrib'd by any Force or Velocity Given, how the ſame Sides of a Parallelogram, as *AB*, *AC* or *BD*, ſhould be Equal to all thoſe Varying Forces, cannot be ſhewn; Thus ſuppoſing *AD* to Expreſs the Force of Gravitation of one Body to another, and that *AB* and *BD* will likewiſe do it, which is Preſum'd in the Theory of the Secondary Planets, if, notwithſtanding, there are Different Forces of Gravitation Expreſſible by the ſame Line *AD*, *AB* and *BD* conſider'd only as Sides of a Parallelogram, of which *AD* is the Diagonal, will not be ſufficient to Repreſent them; that is, this Reaſoning proceeds upon Abſtracted Lengths and Lines, whereas, according to the *Geometry* of *Solids*, there may be very Different Forces or Quantities in the ſame Abſtracted Lengths, as we have there endeavour'd to Evince.

§. 50. After conſidering the Foundation, on which this Compoſition of Forces ſtands, we might next Examine, what uſe is made of it, but if the Foundations of a Doctrine are not True, the Applications of it muſt be Falſe, and therefore we may be juſtly excuſed the Trouble of a farther Inquiry on this Head.

The Application of this Doctrine not neceſſary to be Conſider'd.

However to confirm this Law, the Firſt Phænomenons, which are Produc'd, are thoſe of Mechanicks; the Explanation of which, from the preſent Lemma, if any One is deſirous to ſee, will be fully Inform'd from the Reverend and Ingenious Perſon before Refer'd to, in his Comments upon *Rohault*, and from the Celebrated Author of this Compoſition of Forces; It was indeed my Intention, according to the Method preſcrib'd in the Preface to theſe Philoſophical Tracts, to have gone thro' this, as well as the ſeveral other Parts of the Modern Philoſophy; but if we can have no Satisfaction in theſe Fundamental Maxims of it, there is no Reaſon to Expect the Arguings from them will be more Concluſive; We have likewiſe already conſider'd the Principles of Mechanicks, and ſhewn them to be Precarious or Falſe; and what the Preſent Account of them alledges depends upon much the ſame Principles, and all the Sagacity, which is Evident in it, is only this, Namely, the Collecting of ſeveral Caſes, and Uniting them into one, to wit, the Reſolution of the Diagonal Force into the Lateral; whereas thoſe ſeveral Caſes have been Prov'd to be wrongly Accounted for, and this Principle of the Reſolution of Forces to be wholly Chimerical.

Sir Iſ. Newt. Princ. Lex. 3. Coroll. 2.
Rohault. Chap. 14. Art. 9.
Preface to the Principles of Natural Philoſophy.

Concluſion upon the Whole.

§. 51. ALL therefore, which we ſhall farther do in the way of Confutation is only this, to conſider the Phyſical Account given of the Revolutions of the Heavenly Bodies, of the Primary and Secondary Planets in their Ellipſes, and the Comets in their Parabola's, as alſo of the Flux and Reflux of the Sea; we ſhall then Examine into the Doctrine of Hydroſtaticks, and into the Solutions concerning the Barometer, and if any Thing more is deſir'd of us, we are ready to do what lies in our Power, to ſatisfy all Impartial Inquirers into Nature, that the Principles, upon which they Proceed, in any Caſe Propos'd, are Equally inſufficient to the Explaining of it; The next Thing therefore, which will Engage our Enquiries, will be the Method of *Naſcent* and *Evaneſcent Reaſons*, which is the Baſis and Foundation of the *Phyſical Aſtronomy*.

CHAP. III.

Concerning the Method of Naſcent and Evaneſcent Reaſons.

The Firſt Lemma Explain'd. Sir Iſ. Newt. Princip. §. 1. Lem. 1. Lib. 1.

§. 1. THE Firſt Aſſertion, in the Method of Naſcent and Evaneſcent Reaſons, by which the whole following Philoſophy is Demonſtrated, is; That Quantities, or the Proportions or Reaſons of them, which in a Time given continually Tend to an Equality, and by that Means are capable of Approaching nearer to each other than according to any Difference Given, at Length become Equal.

IF it is deny'd, let their Laſt Difference be *D*, therefore they cannot come nearer to an Equality, than according to that Given Difference, contrary to the Hypotheſis, which ſuppoſes, that they may Approach nearer to each other than in any Given Difference.

This Lemma contrary to the Infinite Diviſibility of Extenſion.

§. 2. FIRST, In Reply to which we ſay, that, what Quantities are, ſhou'd be Determin'd, before we can Affirm any thing concerning them, or the Reaſons and Proportions of them; for if by Quantities are meant and intended Quantities in General, this Lemma will not be true; ſince it is thought, at leaſt by the Mathematicks now receiv'd, to be Demonſtrated, that, tho' the Curve of an Hyperbola and the Aſymptote of it continually Approach to Each other, yet they will never Meet; that, in the Extraction of the Root of 2, or of any other Surd or Ineffable Root, there is a Conſtant Approximation to Truth, but no Arriving at it; that, in Two Parallel Lines with a Perpendicular Drop't betwixt them, if Lines are drawn from any one Point, in either of the Parallel Lines, Tranverſe the Perpendicular, to the Oppoſite Parallel, they will never Exhauſt the Points of the Perpendicular, tho' they continually come nearer to the doing of it; and that the Nature of Quantities in General is, that they are Diviſible to Infinity; which if ſo, this Lemma cannot be true, becauſe it ſuppoſes the Laſt Difference to be nothing, or which is the ſame, that in all Quantities there is a Point, which is Indiviſible.

This Lemma depends upon the Preſent Abſtractions in Mathematicks, which are Arbitrary and Precarious.

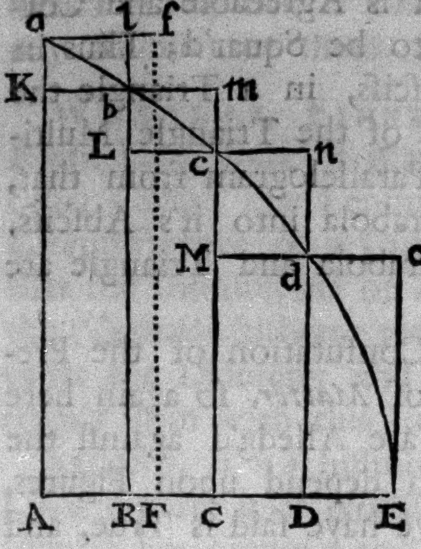
§. 3. SECOND, Another Argument againſt the Univerſal Truth of this Lemma is, that it Preſumes in Quantities it's own Abſtractions of them, that is, that all Quantity in Figures is Reſolved into Length, Breadth, and Thickneſs; whereas in the *Geometry* of *Solids*, or of *Matter*, and in the *Principles* of *Philoſophy*, *Chap.* 4. §. 11. *B.* 1. We have Endeavour'd to ſhew, that a Line of the ſame Length, or a Surface of the ſame Length and Breadth, or a Solid of the ſame Length, Breadth, and Thickneſs, may, and often really do's Expreſs Different Quantities; And yet this Lemma only conſiders the ſame Length, Breadth, or Depth, as Expreſſive of the ſame Quantity.

The Application of this Lemma to Curvilinear Figures not Defenſible.

§. 4. THIRD, The Preſent Lemma, by what follows, is meant of Right-Lin'd Figures, Inſcrib'd and Circumſcrib'd to a Curve, which by diminiſhing of them Infinitely will be at Length Equal to it; The Contrary of which we have Prov'd in the ſaid *Principles* and *Geometry*, to wit, that no Part of any Right Line, Inſcrib'd or Circumſcrib'd to a Curve, can ever be a Meaſure of it, and into which we ſhall farther Examine in the Succeeding Sections.

The Second Lemma Explain'd. Sir Iſ. Newt. Princip. §. 1. Lem. 2. Lib. 1.

§. 5. The Second Lemma or Aſſertion in this Philoſophy, Previous to it's Demonſtrations, is; That if in any Figure, as *AacE*, there ſhou'd be Inſcribed as many Parallelograms, as we pleaſe, *Ab*, *Bc*, *Cd*, &c. contain'd under Equal Baſes *AB*, *BC*, *CD*, &c. and Sides *Bb*, *Cc*, *Dd*, &c. Parallel to the Side of the Figure *Aa*, and the Parallelograms *aKbl*, *bLcm*, *cMdn*, &c. ſhou'd be Fill'd up or Compleated, and afterwards the Latitude of theſe Parallelograms ſhou'd be Diminiſh'd, and the Number of them Increaſed to Infinity; What then? It then peremptorily Affirms and Pronounces, That the Laſt or Ultimate Reaſons or Proportions, which the Inſcrib'd Figure *AKbLcMdD*, the Circumſcrib'd *AalbmcndoE*, and the Curvilinear *AabcdE* obtain to each other, are thoſe of an Equality.



Because the Difference of the Figure Inſcrib'd and Circumſcrib'd is the Sum of the Parallelograms *Kl* + *Lm* + *Mn* + *Do*, that is, as their Baſes are all Equal, the Rectangle under the Baſis *Kb* and the Sum of the Altitudes *Aa* will be Equal to ſuch Difference, namely, the Rectangle *ABla*, but this Rectangle is Leſs than any given, by the Latitude *AB* being Diminiſh'd to Infinity, and therefore, according to the Firſt Lemma, the Figure Inſcrib'd and Circumſcrib'd, and much more the Curvilinear Figure, which is Intermediate to them, will be at Length Equal. *Q. E. D.*

This Demonſtration not Valid, becauſe it either ſuppoſes what is to be Prov'd, or elſe it ſuppoſes a Contradiction.

§. 6. But this Demonſtration fails upon ſeveral Accounts; First, becauſe the Nature of Infinity is not ſo clearly and fully Explain'd, that we ſhou'd be certain, that *AalB*, upon an Infinite Diminution, will be Equal to Nothing, or that the Laſt Difference will be ſo; for to ſay, according to Lemma the Firſt, that the Laſt Difference is Equal to Nothing, becauſe the Quantities are ſuppos'd to Approach nearer to each other, than in any Given Difference, is either to Affirm, that the Laſt Difference is Equal to Nothing, becauſe it is ſuppos'd not to be Something, or elſe it is to Affirm, that the Laſt Difference is Equal to Nothing, becauſe there is no End of any Finite or Given Difference; If the Firſt is Intended, it is Proving a Thing by ſuppoſing the Thing to be Prov'd; If the Laſt, it is a Contradiction; ſince, if there is no End of the Finite Difference, there cannot be a Laſt Difference, nor can that Laſt Difference be Equal to Nothing.

This Demonſtration Leſs Certain, becauſe the Abſtractions of Quantity, on which it proceeds, are not True.

§. 7. Second, As *AalB* is not Evidenc'd to be Equal to Nothing, even upon the Suppoſition of the Preſent Abſtractions in Mathematicks, ſo it will be much leſs Prov'd, if we ſhow, that the Quantities, hitherto Receiv'd in that Science, and Affirm'd in this Lemma, are falſly Stated and Defin'd; For, in this Demonſtration, it is ſuppos'd, that the Lines, *AE*, *Aa*, whether Apply'd to the Curve *aE*, or to the Parallelograms, whoſe Baſes are *AB*, *BC*, *CD*, *DE*, and the Sides Meaſur'd by Portions of the Line *aA*, are the ſame; whereas it has been already Demonſtrated in the *Principles of Philoſophy*, *Chap.* 4. §. 11. *B.* 1, and in the *Geometry of Solids*, or of *Matter*, that Lines, which are the ſame in Reſpect of their Length, Receive Different Values, and are Quantities Infinitely Different in their Nature; Thus, if *aE* is the Curve of a Parabola, *aA* will be a Line of which *AE* is as it's Square Root, if of a Cubical Paraboloeid, *AE* will be as it's Cube-Root, if of a Biquadratical Paraboloeid, as it's Biſquare-Root, *&c*; And conſequently, if *AE* is Multiply'd into *aA*, both of them being conſider'd as Simple Lengths, a Common Parallelogram will Ariſe of the ſame Nature and Conſtitution with thoſe, which are Inſcrib'd and Circumſcrib'd to the Curve, where the bare Lengths of the Lines, and the Multiplication of them only Enter into the Compoſition of the Figure; but if *AE* is the Ordinate of a Parabola, it will be as the Square-Root of *aA*, and Multiply'd into it, a Parallelogram will Ariſe, which, in Reſpect of the Former, will be as the Root of *aA* to *AE*, which was before as *aA*; If *AE* is the Ordinate of a Cubical Paraboloeid, it will be as the Cube-Root of *aA*, and Multiply'd into it, will form a Parallelogram, which, in Reſpect of the Firſt, will be as the Cube-Root of *aA* to *AE*, which was originally as *aA*, &c; From whence it is Evident, that the Reaſoning in this Lemma cannot be juſt, which Imagines, that the Parallelograms Inſcrib'd or Circumſcrib'd to the Curve are of the ſame Nature and

Conſtitution with thoſe, which are form'd from the Multiplication of the Parts of the Lines, conſider'd not as bare Lengths, but as Abſciſſes and Ordinates of ſuch Curves; From whence it follows, that when the Square of a Curve Figure is Compar'd with the Area of it, ſuch a Square is to be made uſe of, which is Agreeable and Correſpondent to the Nature of the Figure, which is Propos'd to be Squar'd; Thus in the Parabola the Ordinate is as the Square-Root of the Abſciſs, in a Triangle the Ordinate is as the Abſciſs, and conſequently, the Ordinate of the Triangle Multiply'd into it's Abſciſs, will Produce a Different Square or Parallelogram from that, which is form'd from Multiplying the Ordinate of the Parabola into it's Abſciſs, altho', as to Length, the Ordinates and Abſciſſes of the Parabola and Triangle are Equal.

Geometriæ Solidorum ſive Materiæ, Pars. 1.

We have taken the Liberty to add this, not only in Confutation of the Preſent Lemma, but as we had before done in our *Geometry* of *Matter*, ſo again here to ſhew of what little Force thoſe Arguments are, which are Alledg'd againſt the *Square* of the *Circle* Propos'd in that *Geometry*, and which depend upon Figures, which are Inſcrib'd or Circumſcrib'd to it; ſince, if what we have ſaid is True, and ſince all Figures, which are Inſcrib'd or Circumſcrib'd, are Reſolv'd into Triangles, whoſe Square will be that of a Triangle, and farther, ſince the Square of a Curve is not of the ſame Nature or Conſtitution with the Square of a Triangle, it is Manifeſt, that no Arguments can Affect us, which are deriv'd from ſuch Ways of Reaſoning.

The Third Lemma Explain'd, Sir Iſ. Newt. Princip. §. 1. Lem. 3. Lib. 1.

§. 8. The Third Aſſertion in this Method of Naſcent and Evaneſcent Reaſons is, That the ſame Ultimate Reaſons, before-mention'd, are likewiſe thoſe of Equality, even where the Latitudes of the Parallelograms *AB*, *BC*, *CD*, *DE* are Unequal, and where they are all Diminiſh'd to Infinity.

For let *AF* be Equal to the Greateſt Latitude, and the Parallelogram *FAaf* Compleated, This will be Greater than the Difference of the Inſcrib'd and Circumſcrib'd Figure *ABla*, but it's Latitude *AF* being Diminiſh'd to Infinity, it will become Leſs than any Given Rectangle.

From whence theſe Corollaries are Drawn;

1. That the Ultimate Sum of the Evaneſcent Parallelograms Coincides with the Curvilineal Figure.

2. And much more the Rectilineal Figure, which is Comprehended within the Chords of the Evaneſcent Arcs *ab*, *bc*, *cd*, at laſt Coincides with the Curvilineal.

3. As alſo the Rectilineal Figure, which is Circumſcrib'd by the Tangents of the ſame Arcs.

4. And therefore theſe Ultimate Figures, as to the Perimeters *acE*, are not Rectilineal, but the Curvilineal Limits of Rectilineal Figures.

The Deſign of this Lemma farther Explain'd, and the Truth of it Examin'd.

§. 9. As to the Deſign of this Lemma, it ſeems to be nothing more than to make that Univerſal, which was Partially Demonſtrated before, ſince *AE* was only conſider'd, as divided into Equal Parts; Let us therefore now ſuppoſe it to be Diſtinguiſh'd into Unequal, and the Caſe in Reſpect of the Ultimate Reaſons will be the ſame, becauſe, if *AFfa* is Greater than *ABla*, and conſequently, the Difference *Kf* Greater than *Kl*, which upon an Infinite Diminution Vaniſhes, it is Manifeſt, that any Difference, whoſe Latitude is Equal to or Leſs than *AB* or *AF*, will alſo be Exhauſted by ſuch an Infinite Diminution, and *Vice Verſâ*. What therefore has been Objected againſt the Preceding Lemma will be Equally Valid againſt This; And moreover we may ſay, that, if *FA* upon an Infinite Diminution is reduced to Nothing, as well as *BA*, either there are an Equal Number of Parts in each, or elſe Infinity is an Equivocal Term, and is thought able to deſtroy Quantities, whoſe Number of Parts is Different; if yet Infinity is Affirm'd to be the ſame Thing, and not an Undetermin'd and Wandering Notion, and if the Number of Parts in *AF* and *AB* are the ſame, all the Difference, which can be betwixt them, muſt be reſign'd to the Difference of Magnitude in the Parts; If the Firſt is defended, That the Number of Parts is Equal, that is Impoſſible, unleſs the Parts are of a Different Magnitude; If the Second, That Infinity is ſomething Undetermin'd, we don't know what, Any Demonſtrations from it will be as Juſt as the Idea is, that is, will Signify Nothing; If the Third is Profeſs'd and Maintain'd, That the Ultimate Part of *AF* is Greater than that of *AB*, they cannot be both Equal, namely, to Nothing.

§. 10.

§. 10. IF therefore the Preceding Lemmas are not Certain or True, the Corollaries from them cannot be. It is, notwithſtanding, ſaid *Corol.* 4, That the Ultimate Figures (as to their Perimeters acE) are not Rectilineal, but Curvilineal Limits of Rectilineal Figures, that is aE is a Curve Line, and not Compounded of Right ones, or any Number of them.

Corollary the Fourth particularly Inquir'd into, and the Nature of Curve Lin'd Figures Conſider'd.

WHEREAS, FIRST, if ab Infinitely Diminiſh'd is not the Portion of any Right Line, ſince it may be as well conceiv'd to be a Part of a Circular, as any other Curve, this Lemma, and the Corollary from it, will Equally prove, that the Circumference of a Circle is not a Polygone of an Infinite Number of Sides, and that the Infiniteſima of ab is not a Right Line; upon which, notwithſtanding, all the Methods hitherto receiv'd of *Squareing* the *Circle*, by finding the Proportion betwixt the Diameter and Circumference do depend, and which Methods, we ſhall readily Join with this Lemma, in Refuſing and Condemning.

SECOND, If ab Infinitely Diminiſh'd is a Curve, and not any Portion of a Right-Lin'd Figure, it is Nothing according to this Reaſoning, for aK, al Diminiſh'd to Infinity are ſuppos'd to be Nothing, what therefore the Point a can be more is not Conceivable; If a is Something, the Queſtion will be, whether it is the Part of a Curve Line, or of a Right one, aK is common to both Figures, that is, to $Kalb$, and Kab, and therefore at length the Inquiry will be, whether a is a Part of the Curve ab, or of the Right Line al; if the Laſt, it is Evident, that a in the Curve ab is a certain Portion of the Right Line al, which yet is here deny'd; If the Firſt, it is Prov'd by Affirming it, and cannot be made Clearer, or more Demonſtrable, by Lines al or Kb, Circumſcrib'd or Inſcrib'd to it, which it is own'd do not any ways Belong or Appertain to it, nor are any the leaſt Portion of it.

THIRD, It is not Evinced, but a, b, c, d, &c. may be Parts of Right-Lin'd Figures, ſince the Point at a is the Concourſe of Two Right Lines Ka, la, which Concourſe cannot produce Curveneſs, but may the Rectitude of each Line Expreſs'd in the ſame Point, and the like may be Affirm'd of b, c, d, &c; that is, there is no Curvity in the Nature of each Point, from the manner of it's Production; But if the Curvity proceeds from the Poſition of the Points in Reſpect of aA, that will not prove the Points themſelves are Curvilineal, or that they may not be of a Rectilineal Conſtitution, we only here Argue with this Philoſophy in it's own Way, becauſe we have ſhewn, that the Parts of Curve Lines are Curves, and are not Reſolvible into Right ones.

FOURTH, If it cou'd be Prov'd, that a, b, c, d, &c. were Curvilineal Parts of aE, by this way of Reaſoning; according to the *Geometry* of *Solids*, they may be of the ſame Nature with Rectilineal Parts, ſuppoſing their Velocities to be alike.

Geometriæ Solidorum ſive Materiæ, Pars. 1.

FIFTH, Beſides all which, ſince AaE may poſſibly be of Different Dimenſions, as to Length, Breadth, and Depth, from the Parallelograms Inſcrib'd and Circumſcrib'd, the Compariſon betwixt them can ſignify no more to the Demonſtrating any Thing from it, than if we ſhou'd compare a Line with a Surface, or that with a Solid, according to the ſame *Geometry*.

LASTLY, The Nature of a Curve, whether of a Circle or any other, when the Abſciſſes and Ordinates are ſuppos'd to Generate it, is Compounded of the Terminations of the Joint Velocities of aK and Kb, as the Diagonal of a Square, or a Parallelogram, is Compounded of the Terminations of the Velocities of the Parts of the Side in Conjunction with thoſe of it's Ordinates; Thus, if aA is the Side of a Square, and aE is ſuppos'd to be the Diagonal of it, whilſt a proceeds with the Velocity of 1 to K, K proceeds with the ſame Velocity of 1 to b, and Lodges or Expreſſes the Firſt Point in the Diagonal, if aK and Kb are Imagin'd to be Infinitely Small; and in like manner, the Second, Third, Fourth Points, *&c.* from a Velocity of 2 and 2, 3 and 3, 4 and 4, *&c.* are produced in the Diagonal, till the whole is form'd.

FROM whence it is Manifeſt, that each Point in the Diagonal is only the Limit or Termination of both the Velocities after they are actually Perform'd, and therefore the Diagonal, which this Philoſophy Propoſes, and which we have Confuted, is not made up of the ſame Forces Conjoin'd, which are ſeparately Deſcrib'd in the Sides of the Parallelogram, ſince the Diagonal is no other than a Termination of thoſe Forces, and is not in any Manner or Way of Conſidering an Union or Conjunction of them.

Second, From this likewiſe it is plain, that the Diagonal is Incommenſurable with the Side of the Square; Becauſe the bare Termination of Forces cannot be Meaſur'd by thoſe Forces, or the Contrary; Thus *b* is the Firſt Point from *a* of the Diagonal, which being only a Termination of the Velocities *aK*, *Kb* is no ways Capable of a Compariſon with them, no more than a Quieſcent Point is with an Actual Force, which it Bounds and Determines.

In like Manner, if *aA* is the Abſciſs of a Parabola, *b* the Point in the Parabolick Curve will be the Limit or Termination of the Velocities *aK*, *Kb*, and the ſame Reaſoning will obtain in Reſpect of the Points *c*, *d*, &c, which will be the Terms or Boundaries of the ſeveral Velocities Expreſs'd by the Abſciſſes and Ordinates, and, as before was ſaid of the Diagonal of the Square, will be Incommenſurable with them.

Dr. Wallis Arithmetica Infinitorum. Geometria Solidorum ſive Materiæ.

We ſhall therefore only farther Add, that ſince in the Triangle, where the Velocities of the Abſciſs and the Ordinate are Equal, or as Equal Velocities, the Diagonal is a Right Line, and ſince in the Parabola and in Paraboloeids, where the Velocity in the Abſciſs is Greater, and in the Ordinates Leſs, the Diagonal is a Curve, which is Concave to the Abſciſs, and ſince in the Complements to the Parabola or Paraboloeids *alb*, where the Velocity of the Abſciſs is Leſs, and that of the Ordinate Greater, the Diagonal is a Curve, which is Convex to the Abſciſs, we may from hence Form a juſt Idea of the Nature of Curves; In which, where the Velocities are Greater in the Abſciſs, and Leſs in the Ordinates, the Curve is Leſs Repell'd from the Abſciſs, and Approaches or is Attracted towards it, by how much it's Force is Greater, and that of the Ordinate Leſs, which hinders it's Approach; On the Contrary, where the Force is Leſs in the Abſciſs, and Greater in the Ordinate, the Curve is Thruſt and Protruded from it, or is Leſs Attracted to it, by how much the Force, which Protrudes is Greater, and the Force, which Acts or Directs it, Leſs; Thus *aE* Converges towards *aA*, and Diverges from *al*. And if we Take any Abſciſs whatſoever, and Divide it According to the Square Numbers 1. 4. 9. 16. 25, *&c.* and Place Ordinates at the ſeveral Diviſions, According to the Square-Roots of thoſe Numbers, namely, a Series of Natural ones, 1, 2, 3, 4, 5, *&c.* we ſhall find the Curve of the Parabola, which Connects the Extremities of the Ordinates, to Converge to the Abſciſs, but if, on the Contrary, the Portions of the Abſciſs are 1, 2, 3, 4, 5, *&c*, and the Ordinates 1. 4. 9. 16. 25, *&c.* the Curve, which Connects Them, will Diverge from it; The ſame may be ſaid of an Abſciſs Divided Cubically, Biquadratically, *&c.* and where the Ordinates are as the Roots of thoſe Cubes or Bi-Squares, or where the Portions of the Abſciſs are as the Roots, and the Ordinates as the Cubes or Bi-Squares, for in the Firſt Caſe the Curve of the Paraboloeid will Converge to the Abſciſs, and in the Laſt Diverge from it; Hence alſo, if we Compare the Space, Produc'd by theſe Ordinates, with that of Paraboloeids and the Complements of them, it will Appear, that Different Spaces may be Effected by the ſame Forces, as we before ſhew'd in the *Geometry* of *Solids*, that the ſame Spaces might be Conſtituted from Different ones.

From this General Theorem it ſeems Plain and Evident, if *aE* is the Portion of a Circle, and *aA* the Radius of it, ſince the Curve in this Caſe Converges to the Abſciſs, that it is in the Number of Paraboloeids, and which, in the *Geometry* of *Solids*, or of *Matter*, we have Endeavour'd to ſhew is a Biquadratical one, and which very well agrees with the General Theorem mention'd.

For let *aK*, and the ſeveral Parts of *aA*, be Repreſented by Numbers in an Arithmetical Progreſſion to Infinity, the Ordinates in the Parabola will be as the Roots of thoſe Numbers, and in the Biquadratical Paraboloeid, as the Roots of the Roots of them; from whence it will follow, that there is a much Greater Protruſion of the Limits of the Velocities from the Abſciſs in the Firſt Caſe than in the Laſt, and conſequently, that the Curve will Approach nearer to a Convexity in the one Inſtance than the other; which is Manifeſt, ſince the Parabola comes nearer to a Right Line, than a Circle do's, or, which is the ſame, a Circle is more Concave to the Abſciſs than the Parabola, but a Right Line is Firſt to be Arriv'd at, before a Convexity to the Abſciſs can be Produced; Thus the $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, *&c.* which are the Ordinates of a Parabola, are in a nearer Proportion to 1, 2, 3, 4, *&c.* which wou'd make the Ordinates of a Triangle, and the Diagonal a Right Line, than the $\sqrt{\sqrt{1}}$, $\sqrt{\sqrt{2}}$, $\sqrt{\sqrt{3}}$, $\sqrt{\sqrt{4}}$, *&c.* which are the Ordinates of a Circle.

Geometriæ Solidorum ſive Materiæ; Pars. 2.

FROM what has been ſaid it is likewiſe Evident, that the *Hyperbola* is alſo in the Number of *Paraboloeids*, but the Proportion, which it has to it's Rectangle, we ſhall Leave to others to Determine, tho' we have it Aſcertain'd in Writings by us, upon the ſame Principles, by which we have *Squar'd* the *Circle*.

A Second Way of Conſidering Curves, and the Generation of Them.

§. 11. HAVING thus far Conſider'd the Nature of Curves, in Reſpect to their Ordinates and Abſciſſes, and their being only Limits or Terminations of their Velocities; we ſhall now ſhew, how they may be Conſider'd in another Way, that is, by the Generation of them from their Centers, as in the *Circle*, or from their Focus's, as in the *Ellipſis*, the *Parabola*, and *Hyperbola*.

As to the Circle, it's Geneſis is Manifeſt, by the Revolution of a Line upon a Center; but the Queſtion, which lies in the *Doctrine* of *Velocities*, or the *Geometry* of *Solids*, is, what the Nature of ſuch a Curve is, which cannot be any other than that of a Right Line; For, if we ſuppoſe ſeveral Circles to be drawn from the ſame Center, their Circumferences will be to each other, as their Radius's, by the Extremities of which they are Deſcrib'd; that is, they will be as Right Lines, and differ Nothing from them, but in their being bent into a Poſition, in which all the Points of the Line are Equidiſtant from the Center, whereas a Right one, as the Radius, has it's Points diſpos'd in a Direct Receſſion from it.

BESIDES, the Velocities in each Circumference are Equable, without any Acceleration or Retarding of them in the ſame Circumference, which is likewiſe the Nature and Property of a Right Line, which is Produc'd by the Equable Fluxion of a Point.

AND in Different Circumferences, where the Velocities are Greater in Proportion to the Greater Lengths of the Radius, the ſame Tenour of Velocity is Preſerv'd, whilſt a Larger Space is Perform'd by a Larger Velocity in the ſame Time, that a Leſſer Space is Perform'd by a Leſſer Velocity; For that, which makes an Alteration from a Right Line into a Curve, is a Different Velocity in the ſame Space and the ſame Time, according to the aforeſaid *Geometry* of *Solids*.

FROM whence it is Apparent, FIRST, that the Circumference of a Circle, Conſider'd in this way of Generation, is perfectly Different from that, which is Produced by the Termination of the Forces Expreſs'd in the Sines and Vers'd Sines of it, (the Laſt of which are it's Abſciſſes, and the Firſt it's Ordinates) ſince the Circumference in the one Caſe is no more than a Right Line, or one Kind of Force, continually Bent, and in the other a Termination of Forces, which continually Differ.

SECOND, The ſame Curve Line to Appearance may therefore in Reality receive very Different Values and Denominations, and be as Diſtinct from what it Appears, as any other Quantity whatſoever, and conſequently, the ſame *Circle*; ſince where the Circumference of it obtains the Nature of a Right Line, it will be a *Plane* Figure, and where that of a Curve, or of a Termination of Forces, which continually Vary, it will be a *Solid* one.

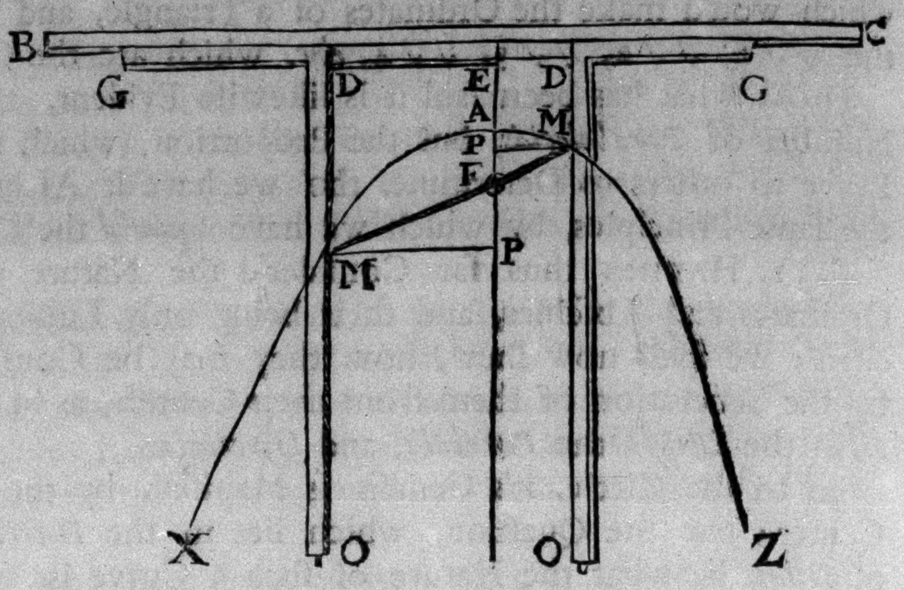
THIRD, If we then Conſider a *Circle* as a *Plane* Figure, the *Square* of it will be as Eaſy, as of any other *Plane* Figure we can Mention; becauſe there is nothing elſe Requir'd, than to make the Radius the Perpendicular of a Right-Angled Triangle, and the Circumference the Baſis of it, which will be half of the Rectangle under the ſame Radius and Baſis, and Equal to the *Circle*; nor would there be any Greater Reaſon to find out the Proportion betwixt the Baſis, *i. e.* the Circumference of the *Circle*, and it's Radius, in order to Obtain the *Square* of the *Circle*, than betwixt the Two Sides of any Triangle, in order to Aſcertain the *Square* of it; for to find the *Square* of the *Circle* is to Aſſign a certain Rectilineal Figure, to which the Area of the *Circle* has a Known Proportion, which is done as Above.

Pardies Geometriæ Elementa, Lib. 4. Art. 24. & 31.

IF the *Circle* is not Conſider'd as a *Plane* Figure, but a *Solid*, we have Endeavour'd to Determine it's Proportion to the *Square* in the *Geometry* of *Solids*, or of *Matter*, namely, that it is as 4 to 5, and is no other than a *Biquadratical Paraboloeid*.

L'Ho'pital:
Sections
Conique,
Livr. 1.
Definit. 1.
Livr. 2. Def. 1.
Livr. 3. Def. 1.

As to the *Parabola*, the Generation of it is Perform'd by ſuppoſing a Rule BC, upon which the Square GDO is mov'd, whoſe Side GD is carry'd along the Rule BC, from E towards B, the other Side of the Square DO has a Line affixt to it, one of whoſe Extremities is faſten'd at O, and the other in ſome Point or other of the Plane, in which it Moves, as F; Let a Style be held always at M, the Point where the Line Tears off from the Side of the Square DO by it's Motion from F, and it will Deſcribe the Parabola AMX; By the ſame Methods AMZ will be likewiſe Deſcrib'd by Inverting the Square towards C, and Moving it that way from the ſame Focus F; And ſince the Deſcriptions of the Ellipſis and Hyperbola depend upon the like Compound Motions, which are Refer'd in ſome Meaſure or other to their Focus's, as their Centers, it will not be Neceſſary to conſider, in this General Idea of the Nature of Curves, the particular Circumſtances of thoſe Deſcriptions, Eſpecially, as the *Circle* and the *Parabola* are the Figures, which we at preſent are Deſirous to Examine; The *Square* of the Laſt being Confeſs'd, and *That* of the Firſt being here Defended.



In this Generation of the *Parabola* it is therefore Evident, that it Differs from that, which Reſults from a bare Termination of the Velocities Expreſs'd by the Abſciſſes and their Ordinates; ſince the Generation of it by a Line, and the Mediation of a Square, and the Focus, is by an Actual Velocity, whereas, when it is Produc'd by a Termination of Actual Forces, in the Abſciſſes and Ordinates, it is made up of ſo many Quieſcent Points, as there are Terminations of ſuch Forces; from whence it will follow, that the Parabola, in the one Caſe, is of a Different Nature from that in the other, unleſs Quieſcence and an Actual Velocity are the ſame.

Second, Suppoſing the Forces were Actual in both the Deſcriptions, yet, ſince AMX is Deſcrib'd from the Focus F, by FA, FM, FX, as the Circle from a Center, they ſo far Agree, and all the Difference ſeems to be, that whilſt the Radius of Deſcription continues Immutable in the Circle, in the Parabola it as conſtantly Varies, from FA to FX.

But the Variation of the Length of the Radius does not hinder the Equableneſs of it's Motion, or at leaſt it's being Equably Increas'd; ſince in Concentrical Circles the Radius's may be of Different Lengths, and the Swiftneſs of their Deſcription Different, and yet the Velocities in the ſeveral Circumferences of the ſame Nature with the Equable ones in Right Lines, according to what has been before obſerv'd; And that this is the Caſe of the Changing Radius's of the Parabola from the ſame Focus is Plain.

For, if we ſuppoſe AP to be the Axis of a Cone, on the Surface of which the Parabola XAZ is drawn, the Point A in the Vertex of it may be Deſcrib'd by FA, the Radius of a Circle Parallel to the Baſis of the Cone, and whoſe Center is in the Axis of the Cone AP; and by the ſame Methods FM, FX, or any other Intermediate Lines from the Focus, may be Repreſented by Radius's of the ſame Lengths, whoſe Circles are Parallel to the Baſis of the Cone, and whoſe Centers lie in the Axis of it; For if, for Inſtance, PM is Imagin'd Equal to FM, it will be the ſame Thing, if we take Different Points P and F, and make the one the Center of a Circle Parallel to the Baſis of the Cone, and the other the Focus of the Parabola, becauſe in both Caſes the Point M has the ſame Diſtance from the Axis of the Cone; And conſequently, if we make all the ſeveral Centers P of the Circles, Parallel to the Baſis of the Cone, unite and fall in with F, and the Circles, which were Parallel to the Baſis, Coincide with the Plane of the Parabola, they will then become one Focus, and the Radius's of the Parallel Circles will be Radius's of the Parabola drawn from the ſaid Focus.

If therefore theſe Parallel Circles are all Preſs'd into one and the ſame Plane, they will become Concentrical, and the Curve of the Parabola will be Reſolv'd into a Line of the ſame kind with Circumferences Deſcrib'd by the Radius's of them, and conſeqnently of the ſame Conſtitution with Right Lines, Deſcrib'd by Equable Velocities; From whence it will Enſue, that the Parabola, by this way of Deſcription, is a Plane Figure, being Reducible to Triangles of Different Heights, as the Circle is to thoſe of the ſame Height, and therefore has the Nature of all other Triangles.

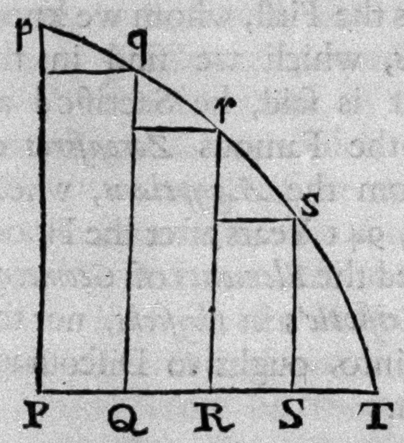
On the Contrary, if the Forces of the Abſciſs and Ordinates are made uſe of to determine the Parabola, the Curve will not be Conſider'd, as a Right Line, but every Point of it will Vary in it's Velocity, or the Termination of it, according to the Proportion of the Forces in the Abſciſſes and Ordinates, with which they End or are Concluded; and conſequently both the Curve and the Plane, which it Comprehends, will be of a Different Nature, and of Different Dimenſions, than when the one is conſider'd as a Right Line, and the other as a Triangle.

Having mention'd the Reduction of a Parabola to Portions of Concentrical Circles, we ſhall here ſhew, how we may Reduce the Angles of Contact to the Aſymptotes of an Hyperbola, albeit, they have been hitherto Miſtaken by Mathematicians for Diſtinct Arguments to Prove the Infinite Diviſibility of Extenſion, whereas they are in Effect the ſame.

For all the Circles, which are Deſcrib'd by the Ordinates of the Hyperbola, and which conſtitute the Hyperbolick Cone, ſince they are Parallel to ſo many Circles in the Baſis, may be Expreſs'd in the Baſis, and therefore will become Concentrical Circles; and as the Baſis, by ſuch Repeated Expreſſions of Circles in it, will be Equal to the Cone, that is, a Plane to a Solid, ſo alſo will the Circumferences be Concentrical, and all lie in a Common and Plane Surface; but every Point in the Aſymptote may be likewiſe Expreſs'd in the Baſis, at the ſame Diſtance from each Circumference in it, which it obſerves in the Aſymptote from the Circles in the Hyperbolick Cone, and at the ſame Time may Touch each Circle; ſince the Point in the Aſymptote is Trajected, by Right or Tangent Lines, thro' each Point of the Hyperbola, whoſe Circumferences are Deſcrib'd in the Baſis; and the whole Figure, by ſuch a Diſpoſition, will amount to the ſame with a Tangent, and the Various Circles annext to it, by whoſe Intervention the Angles of Contact are made.

The Fourth Lemma Explain'd.
Sir Iſ. Newt. Princip. Lem. 4. Lib. 1. §. 1.
See alſo Fig. §. 5.

§. 12. The Fourth Lemma is, That if, in Two Figures *AacE*, *PprT*, there ſhou'd be Two Series of Parallelograms Inſcrib'd, as before, and the Number of both ſhou'd be the ſame, and where the Latitudes are Diminiſh'd to Infinity the Ultimate Reaſons of the Parallelograms in one Figure to the Parallelograms in the other, each to each, ſhou'd be the ſame, it is Pronounc'd, That the Two Figures *AacE*, *PprT*, are to one another in that ſame Reaſon.



This Lemma ſo evidently depends upon the Former, that there is not the leaſt Occaſion to conſider it.

The Fifth Lemma Conſider'd and Examin'd.
Sir Iſ. Newt. Princip. Lem. 5. Lib. 1. §. 1.

§. 13. The Fifth Lemma is, That all the Sides of Similar Figures, which Anſwer to each other or are Homologous, are Proportional, as well Curvilineal as Rectilineal, and their Areas are in a Duplicate Reaſon of their Sides.

That Curvilineal Figures obtain this Property is upon the Suppoſition, that, according to the Preceding Lemmas, Right-Lin'd Figures are Ultimately Reſolv'd into Curvilineal ones, or the Contrary, which has not been Prov'd;

Elem. 6. Prop. 4. 19. 20.

Granting yet this, it is not Evident, that the Homologous Sides of Rectilineal Figures are Proportional, unleſs upon the Abſtractions and Demonſtrations of *Euclid*, in which the Length of the Line is only Conſider'd, and ſuppoſed to be one Conſtant and Invariable Idea; Thus, if I perceive Two Triangles to be Similar, I Judge them to be ſo upon the Equality of the Angles, which Equality is Prov'd, upon the Hypotheſis of one conſtant Uniform Space Abſtracted by the Mind, whereas if Space is not Uniform, and that Abſtraction is not Juſt, the Demonſtration will be in Proportion Defective; And ſince we have ſhewn, in the *Geometry* of *Solids*, and in the

Firſt Part of the *Principles* of *Natural Philoſophy*, that ſuch Abſtractions of a Simple and Fixt Space are not Juſt, neither this Propoſition, nor any other in all *Euclid*, will be Univerſally True.

We do not yet here Affirm, that this Propoſition as well as the others, in the *Geometer* Mention'd, are Abſolutely Falſe, it is Confeſs'd they are True, and that the Three Angles of a Triangle are Equal to Two Right ones, upon the Suppoſition of a Fixt, Abſtracted, and Invariable Space, but ſuch a Space we ſay is Arbitrary, and in Terms deny to Be, or to Exiſt in Nature.

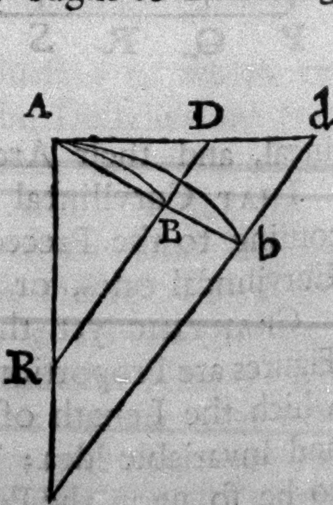
Nor by ſuch an Aſſertion do we deſtroy *Trigonometry*, and all the *Mathematick Sciences*, which are Built upon it; Since Abſtracted Space, which only Conſiſts of Length, Breadth, and Thickneſs, is a ſufficient Meaſure for all the Uſes of Life, whether in *Geometry*, properly ſo call'd, or in *Navigation*, *Fortifying*, or *Aſtronomy*; For, in all theſe Sciences, the Nature of the Meaſure is not Requir'd, but ſome fixt Standard or other of it; Thus in finding the Area or Contents of a Field, or of this Room, in which I write, I can Meaſure it by Walking over the Sides of it, that is, by certain Portions of Space, which I Mark out, or by a Foot of Wood, Braſs, or Iron, or any other Material Being; in which yet I conſider nothing, but the Abſtracted Length of the Meaſure, without Examining into the Nature of it, whether it be Real Space, Wood, Braſs, or Iron.

And this is the Caſe of *Mathematicks* in General, which Imagines nothing more than the Abſtracted Lengths, or Breadths, or Depths, of Things without Conſidering the Intrinſick Nature and Conſtitution of them, which, notwithſtanding, ought to be a Rule of the Mind's Abſtractions, and not it's own Will and Pleaſure; The *Mathematicks* therefore, which we propoſe of *Solids* and *Matter*, we cannot but think are Conformable to the Firſt, and what has been hitherto advanced in that Knwoledge nothing, more than the Reſult of the Latter; That is, We in our Mathematick Abſtractions take Nature for our Guide, whereas the *Mathematicians*, who have Wrote before Us, make their own Arbitrary Will and Pleaſure their Conductors; And upon this Iſſue we dare put our whole Reaſonings in that Science; For if our Abſtractions are ſuitable to Nature, as that will always Stand, the Wills and Arbitrary Fancies of Men muſt, ſome time or other, ſubmit to it, if they are not, we ſhall readily reſign our own Sentiments to thoſe who have Preceded us on theſe Subjects; Seeing, what we Intend in theſe Principles, is only to over-turn if Poſſible, and Demoliſh thoſe Structures, which ſome fond Perſons have Raiſed in their own Imaginations, either in Reſpect of *Mathematicks* or *Aſtronomy*, without any Warrant from the Reality of Beings, as they Exiſt; And altho' *Pythagoras* was the Firſt, whom we know of, who Introduc'd theſe Abſtractions amongſt the *Grecians*, which we find in the Elements of *Euclid*, for the *47th Prop. B. 1.* of which, it is ſaid, he Sacrific'd an Hecatomb, and altho' he Deriv'd theſe Abſtractions from the Famous *Zoroaſtres* of *Bactria*, whoſe Scholar he is thought to have been, and from the *Ægyptians*, whoſe Prince *Meres* the *Philoſopher*, the 28th in Deſcent from *Meſraim*, 941 Years after the Flood, and 675 before the Firſt *Olympiad*, is Affirm'd to have Invented the *Elements* of *Geometry*, yet neither His Authority nor Theirs in *Mathematicks*, nor *Ariſtotle*'s in *Phyſicks*, nor the preſent *Philoſopher*'s in Both, whom we are now Inquiring into, ought to Diſcourage or Hinder any Impartial Reſearches after Truth and Reaſon.

Dr. Prideaux's Connection, &c. Eratoſthenes's Table in Dr. Cumberland on Sanchoniatho's Phænician Hiſtory.

The Sixth Lemma Conſider'd. Sir Iſ. Newt. Lem. 6. Lib. 1. §. 1.

§. 14. The Sixth Lemma is, That if any Arc, which is Given in it's Poſition, as *AB*, be Subtended by the Chord *AB*, and in ſome Point *A*, in the Middle of it's Curvature, ſhou'd be Touch't by a Right Line *AD* Produced on both Hands, and afterwards the Points *A*, *B*, ſhou'd come together and Coaleſce, it is Affirm'd, That the Angle *BAD*, contain'd under the Chord and the Tangent, will be Diminiſh'd to Infinity and at Length Vaniſh.



For let *AB* be Produc'd to *b*, and *AD* to *d*, the Points *A*, *B*, therefore Coaleſcing, and conſequently *Bb* or *Ab* lying wholly out of the Curve, it is Manifeſt, that the Right Line *Bb* or *Ab* will either Coincide with the Tangent *Ad*, or will be Drawn betwixt the Tangent and the Curve; the Laſt is Impoſſible, becauſe it is Contrary to the Nature of the Curve, to have more than one Tangent to the ſame Point, from whence it follows, that the Firſt will obtain, that is, that *Ab* or *Bb* will Coincide with *Ad*, and that the Angle *DAB* will Vaniſh.

WHICH Demonſtration comes to this, that *AB* is FIRST ſuppos'd to be the Chord of an Arc, that is, a Right Line Subtended to it, and then by the Coalition of the Points *A* and *B* not to be a Right Line, but the Point of a Curve; Unleſs therefore the Infinite Diminution of a Right Line can be Prov'd to be the Point of a Curve, or, which is the ſame, Unleſs the Point of a Right Line can be Evidenc'd to be all one with the Point of a Curve Line, the Demonſtration is not Valid, which it is Impoſſible this Philoſophy ſhou'd do, ſince, as we have ſhewn, it is not Acquainted with what the Nature of a Curve or a Right Line is, whilſt it Conſiders Them without Diſtinction, tho' they are as Different Quantities, as a Line or a Surface, or that Compar'd with a Solid; All which it yet hopes to ſolve by Diminiſhing both to Infinity, when it Comprehends, what Infinity is, ſtill Leſs, and which by it's own Confeſſions is Various, ſo that what is Infinite in one Reſpect may be Nothing in another, which if True, tho' the Curve *AB* may be Infinitely Diminiſh'd at the ſame time that it's Subtenſe is, it is not Evident, that the Point of the Curve may not be Infinitely Greater than that of the Right Line.

Lemma 11. Schol. Lib. 1. §. 1.

SECOND, Moreover whatever has been ſaid, in Relation to Lemma 2 and 3, with the Corollaries, will in like Manner have a Place here.

THIRD, If *AB* the Chord is no Part of a Right Line, a Curve cannot be Conſider'd, as a Polygone of an Infinite Number of Sides, ſeeing it will be a Part of the Curve it ſelf, which Doctrine of Polygones this Philoſophy yet Eſpouſes; if it is the Part of a Right Line and is Inſcrib'd in the Curve the Angle *DAB* will not Vaniſh even upon an Infinite Diminution; becauſe it is only the Angle of Contact, which is Equal to Nothing, or Leſs than any Right-Lin'd Angle Given.

Elem. 3. Prop. 16. Schol. Tacquet.

FOURTH, That *Bb* or *Ab* cannot be a Tangent to a Curve, where *Ad* is made to be one, nor Comprehend an Angle with it, is Demonſtrated in *Euclid* upon the Suppoſition, that a Curve Line cannot Coincide with a Right, excepting in one Point; To prove therefore, that *Bb* or *Ab* does not make an Angle with *Ad*, becauſe *Ad* can only Touch the Curve in the Point *A*, which is ſuppos'd, is no other than to Prove by ſuppoſing; for if that is not ſuppos'd, *Ab* or *Bb* by Coinciding with the Curve may Conſtitute an Angle *bAd*, nor will it be neceſſary, that *Ab* or *Bb* ſhou'd fall without the Curve; what I mean is, that a Curve may, for all that is yet Demonſtrated in *Euclid*, or any where elſe, Coincide with a Right Line, where the Receſſion of the Tangent from the Center, and the Concavity of the Curve to it, is ſo Small, that there ſhall not be the Leaſt Difference, as in the Infinitely Large Circles of *Saturn*'s or *Jupiter*'s Orbits.

ON the Contrary, if it is not ſuppos'd, but prov'd from the Nature of a Curve and Right Line, which conſtantly Recede from each other, it cannot yet be Evinc'd, that the Points *A*, in the Tangent and the Curve, Coaleſce or are the Same; ſince, if the Nature of the Curve is Different from that of the Right Line in every Part betwixt *Ad* and *Ab*, it is neither Rational nor Juſt to think, becauſe they Touch each other in the Point *A*, that from that Contact they become of the ſame Nature; the Curvity of the one and the Rectitude of the other will ſtill in all Reaſon and Senſe be Preſerv'd; Thus, if a Line Touches a Surface or a Solid, it do's not by that Contact change either into a Surface or a Solid; which if true, tho' the Angle *dAb* ſhou'd Vaniſh, it wou'd not be any Proof, that *A*, when it Reſpects the Right Line, and when it Reſpects the Curve, wou'd be the ſame.

BUT what is farther to be taken Notice of here is, that all Matters are Clear and Indiſputable concerning the Angle of Contact, according to this Lemma; For if the Angle *BAD* upon an Infinite Diminution Vaniſhes, or is Equal to Nothing, it is Evident, that the Angle of Contact is not Leſs than any Acute one Given, becauſe the Angle of Contact cannot be Leſs than Nothing, to which an Acute Angle Infinitely Diminiſh'd is by this Lemma Equal; And yet it is known, that this has been a Controverſy betwixt the moſt Celebrated Geometers, and not as yet finally Determin'd, tho' Dr. *Wallis* has ſpent no Leſs than near 60 Pages in Folio on that Subject; *Clavius* and his Advocates Affirming on the one Hand, that the Angle of Contact is an Angle Diviſible in *Infinitum* by the Peripheries of a Circle, that it is yet Leſs than any Acute Angle Given, and that the Angle of the Semicircle is Leſs than

Dr. Wallis Oper. Math. Vol. 2. Pag. 605. ad Pag. 664. De Angulo Contactus.

than a Right Angle, but Greater than any Acute Angle Given; whereas *Peletarius* on the other Side Contends, that it is no Angle or Quantity, that the Tangent Coincides with the Curve, and that the Angle of the Semicircle is the ſame with and Equal to a Right one; And notwithſtanding all, which Dr. *Wallis* has Argued at Length upon this Occaſion, I do not ſee any better Solution of thoſe Difficulties and Paradoxes, which are Commonly Propoſed on this Head, than what is Deriv'd from the Doctrine I before Alledg'd in Relation to Curves, namely, that They are of an Heterogeneous Nature to Right Lines, and conſequently, that Rectilineal Angles and Angles of Contact or Contingence can be no more Compar'd with Each other, than Lines with Surfaces, or Surfaces with Solids. And this perhaps will give an Account of that Diſpute, which we find betwixt the Lord *Brouncker* and Dr. *Wallis* concerning the Diameters of Certain *Curves* or *Paraboloeids* in the Mathematical Works of the Latter.

Dr. Wallis Oper. Math. Vol. 1. P. 231.

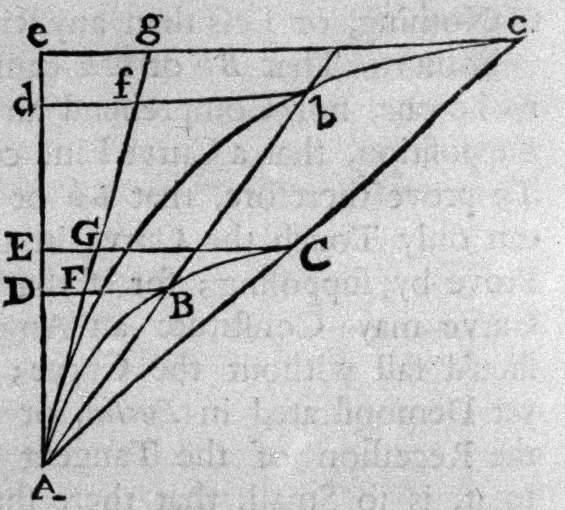
The Seventh and Eighth Lemmas Explain'd. Sir Iſ. Newt. Princip. Lem. 7, 8.

§. 15. THE Seventh Lemma is, That the ſame Things being ſuppoſed, which were in the Former Lemma, the Ultimate Reaſon of the Arc, Chord, and Tangent to one another, is the Reaſon of Equality; which, ſo Evidently depending upon the Former, there will be no Occaſion to conſider here.

THE Eighth is, That if the Right Lines Given, *AR*, *BR*, with the Arc *AB*, the Chord *AB*, and the Tangent *AD*, conſtitute Three Triangles *ARB*, *ARB*, *ARD*, and the Points, *A*, *B*, afterwards come together; the Ultimate Form of the Evaneſcent Triangles is that of their being Similar, and their Ultimate Reaſon, that of Equality; which likewiſe is Demonſtrated upon the Union of the Points, *A*, *B*, in the ſame manner, as has been already Examin'd.

The Ninth Lemma Conſider'd and Examin'd. Sir Iſ. Newt. Princip. Lem. 9. Lib. 1. §. 1.

§. 16. THE Ninth Lemma is, That if the Right Line *AE* and the Curve *AC*, Given in their Poſition, mutually Interſect each other in a Given Angle *A*, and *BD*, *EC*, which meet the Curve in *B* and *C*, are Apply'd as Ordinates to the Right Line *AE* in Another Given Angle, and the Points, *B*, *C*, ſhou'd afterwards Unite or Coaleſce in *A*; The Areas of the Triangles, *ADB*, *AEC*, will be at laſt to one another in a Duplicate Reaſon of their Sides; Which alſo is Deriv'd and Concluded from the ſame Kind of Reaſoning.



AND in all theſe Caſes it is ſuppos'd, that by an Infinite Diminution the Curve is Reſolv'd into a Right Line, and puts on the Properties of it, the Contrary of which we Affirm, and have Endeavour'd to Evince; If it be ſaid, that in Reality it is Confeſs'd, the Points of a Curve and of a Right Line are not the ſame, but yet to Appearance they may be Conſider'd as ſuch, becauſe our Senſes cannot Diſtinguiſh betwixt them; We Anſwer, FIRST, that, if theſe Principles are not Real, and are only Calculated upon falſe Appearances, it cannot well be Imagin'd, how they ſhou'd Account for Effects in Nature, which are Real and not to be Conſider'd as falſe Appearances.

SECOND, Suppoſing we ſee a Planet, as one of *Jupiter's* Satellits for Inſtance, Revolving in a Circle, and we Reduce that Curve to a Tangent and Centripetal Force, of which it ſhould prove the Curve does not Conſiſt, it will not be a juſt Explanation of the Real Revolution of that Planet, becauſe the Tangent and the Line, which Expreſſes the Centripetal Force, upon an Infinite Diminution, cannot by our Senſes be Diſtinguiſh'd from a Point in the Curve.

The Tenth Lemma Conſider'd and Examin'd. Sir Iſ. Newt. Lem. 10. Lib. 1. §. 1. Dr. Gregory's Aſtronom. Prop 23. §. 4. Lib. 1.

§. 17. THE Tenth Lemma is, That Spaces, which a Body Deſcribes by any Regular Force Urgeing it, are in the very Beginning of the Motion in a Duplicate Reaſon of the Times. Let the Times be Expreſs'd by the Lines, *AD*, *AE*, and the Velocities Produc'd by the Ordinates, *DB*, *EC*, and the Spaces Deſcrib'd by theſe Velocities will be, as the Areas, *ABD*, *ACE*, Deſcrib'd by theſe Ordinates, that is, by *Lem.* 9, in a Duplicate Reaſon of the Times, *AD*, *AE*, in the very Beginning of the Motion.

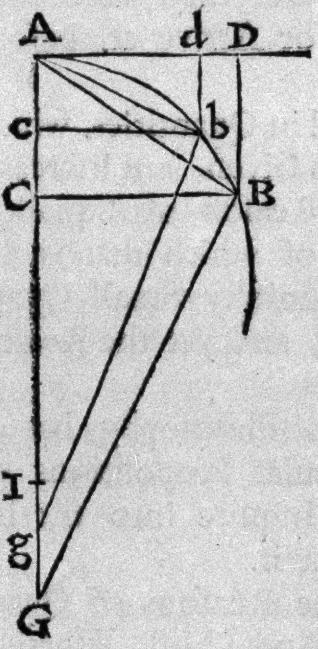
THIS Lemma, tho' it is Propos'd in the General of a Regular Force, and of that Force Acting upon or Urgeing Body Uniformly, is Primarily meant of Gravity, and that Explication, which by the Mechanick Philoſophers has been Given of it; In which Account, It is ſuppos'd, that all Matter is Similar and Alike, that it only conſiſts of Extenſion into Length, Breadth, and Depth, that it is Solid or Impenetrable, that it is Moveable and Diviſible, *&c.* all which are Properties, which make it Paſſive in it's Nature, and are Form'd in the Mind in Order to uſe it at Pleaſure; And the whole Maſs of Matter, as Philoſophy goes and is now underſtood, is nothing more than an Heap or Congeries of Abſtracted Notions Arbitrarily put together, as we have ſhewn in the Firſt Part of the *Principles* of *Natural Philoſophy*.

After this is done, it is in the next Place ſuppos'd, that Gravity or the Regular Force Acts upon this Abſtracted Matter by Continual Impulſes, ſo that in One Inſtant of Time it receives one Stroke from Gravity, in a Second Inſtant of Time Another, and in a Third, the Stroke or Impulſe from Gravity is ſtill Repeated, and ſo on; From whence the Times are Repreſented by one Side of the Triangle, which proceeds as 1, 2, 3, 4, *&c.* and the Impulſes or Strokes from Gravity, or the Velocities, which are the Effects of them, by the other, which has the ſame Arithmetical Progreſſion; and what ſeems to be Intended in this Lemma, by a Regular Force in General is only any Force, which Acts with the ſame Uniformity, be it Greater or Leſs than Gravity, with which Gravity is thought to do, that is, which Produces Effects Proportional to the Times, in which they are Produced.

Both theſe Doctrines we have therefore Conſider'd in the Firſt Part of our Said *Principles* of *Philoſophy*, namely, the Homogeneouſneſs of Matter, and the Uniform Impulſes of Gravity, and that which Remains therefore upon this Lemma, to Convince us of, is, that if we reduce, ADB, AEC, to Infinitely Small Parts towards A, they will be the ſame as Rectilineal Triangles in the Beginning of the Motion, that is, upon an Infinite Diminution of the Times and the Forces; which likewiſe has been already Conſider'd and Refuted in this *Doctrine* of *Naſcents* and *Evaneſcents*.

The Eleventh Lemma Explain'd. Sir Iſ. Newt. Princip. Lem. 11. Lib. 1. §. 1. Dr. Gregory's Aſtronom. Prop. 24. §. 4. Lib. 1.

§. 18. The Laſt Lemma is, That the Evaneſcent Subtenſe of the Angle of Contact is Ultimately in a Duplicate Reaſon of the Subtenſe of the Conterminate Arc; that is, AB Square is to Ab Square, as BD is to bd.



To which we need not Anſwer any more, than that in order to this Demonſtration, there is ſtill Requir'd a Coaleſcence of the Points, D, B, G, into thoſe of, d, b, g, and all the Conſequences of ſuch a Coalition are Affirm'd, which have been hitherto Diſprov'd.

Besides, if ſuch a Coalition is Allow'd, it is Plain, that $AB.Ab::AD.Ad$, but $AD.Ad::DBq.dbq$, becauſe AbB is a Curve Convex to AD the Abſciſs, that is, the Forces, according to §. 10, will be Greater in the Ordinates than in the Abſciſs, and therefore it will not be as $ABq.Abq::BD.bd$, but, if any Thing, on the Contrary, as $AB.Ab::BDq.bdq$.

The Scholium of the Eleventh Lemma Explain'd and Examin'd. Sir Iſ. Newt. Princip. Lem. 11. Schol. Lib. 1. §. 1.

§. 19. We ſhall therefore conclude this Inquiry into the *Doctrine* of *Evaneſcents*, with an Examination into the General Defenſe, which is made of it.

It is ſaid therefore, that theſe Lemmas are Premis'd, that the Perplexed Demonſtrations *ad Abſurdum* of the Antient Geometers might be Avoided; that by the *Method* of *Indiviſibles* they are indeed rendred more Contracted, but becauſe the *Hypotheſis* of *Indiviſibles* is Harſh and Leſs Geometrical, the Preſent *Method* of *Naſcent* and *Evaneſcent Reaſons*, which is ſaid to be Demonſtrated, is Subſtituted in it's Room; That is, the Ultimate Sums or Reaſons of the Evaneſcent Quantities, and the Firſt of the Naſcent, or the Limits of the Sums or Reaſons are conſider'd in this Method, and not the Quantities themſelves, as Diviſible to Infinity; And therefore that, when this Philoſophy ſpeaks of Quantities, as conſiſting of Particles, or makes uſe of ſmall Curve Lines for Right, it wou'd not be underſtood to mean Indiviſibles, but Evaneſcent Diviſibles, not the Sums or Reaſons of Determinate Parts, but the Limits or Determinations of thoſe Sums or Reaſons.

It yet Confeſſes, that there may be Two Objections made againſt this Method, Firſt, that of Evaneſcent Quantities there is no Ultimate Proportion, ſince, before they Vaniſh, it is not the Laſt, and after they Vaniſh, there is None.

In Anſwer to this we are told, that by the ſame Argument it may be Conteſted, whether there is any Ultimate Velocity of a Body proceeding to a Certain Fixed Point; for ſuch a Velocity, before it reaches the Point, will not be the Ultimate, and after it has reach'd it, will be no Velocity; The Anſwer to which is ſaid to be Eaſy, namely, that by the Ultimate Velocity is Meant or Intended, Neither that, with which a Body is carried, before it Arrives at the laſt Point, nor after it has Arriv'd to it, but that, which it Poſſeſſes at the very Point of it's Arrival, with which it Reaches the Laſt Point, and with which it's Motion Ceaſes; In like Manner, by the Ultimate Reaſon of Evaneſcent Quantities, is to be underſtood the Reaſon of Quantities, not before they Vaniſh nor after, but with which they Vaniſh. Thus alſo the Prime or Firſt Reaſon of Naſcent Quantities is the Reaſon with which they Riſe; and the Prime and Ultimate Sum is that, with which they Begin and Ceaſe to be, whether to be Increas'd or Diminiſh'd.

There is farther a Limit or Term, which Velocity in the End of it's Motion can Reach, but not go Beyond, this is the Ultimate Velocity; And the ſame Reaſoning obtains in Reſpect of the Limit or Term, which is Fixed to all Quantities and Proportions, that have a Beginning and an End; And ſince this Limit is Certain and Definite, it is a Problem truly and really Geometrical, to Determine it; But all Things, which are Geometrical, are juſtly allow'd and made uſe of in the Determining of what elſe is ſo.

A Second Objection, which is own'd may be made againſt this Doctrine, is, that, if the Ultimate Reaſons of the Evaneſcent Quantities are Given, there will alſo be Ultimate Magnitudes confeſs'd, and ſo every Quantity will conſiſt of *Indiviſibles* contrary to *Euclid*'s Doctrine of *Incommenſurables.*

This Objection however is ſaid to depend upon a falſe Hypotheſis, for the Ultimate Reaſons, with which Quantities Vaniſh, are not in Truth the Reaſons of Ultimate Quantities, but the Limits, to which the Reaſons of Quantities Decreaſing without End always Approach, and to which they can come nearer than according to any Given Difference, but can never go Beyond, nor Touch or Arrive at, before the Quantities are Diminiſh'd to Infinity.

This Matter, we are Inform'd, will be more clearly underſtood in Quantities, which are Infinitely Great; ſince if Two Quantities, whoſe Difference is Given, are Increas'd to Infinity, their Ultimate Reaſon will be Given, namely, a Reaſon of Equality, and yet the Ultimate or Greateſt Quantities will not be Given, of which that is the Reaſon, namely, of Equality; and the ſame may be ſaid of Infinitely Small Quantities, which are not Given, but are Diminiſh'd without Bounds, and yet the Reaſon of them may be Known and Determin'd.

An Examination into the Doctrine Aſſerted in the foregoing Scholium.

§. 20. This is a Perfect Account of the Notions, which this Philoſophy has of it's *Method* of *Naſcents* and *Evaneſcents* in General, the Particular Propoſitions of which we have before Conſider'd, and we ſhall now moreover Inquire into the Intimate Thoughts, which it has Communicated to us concerning it.

The whole of which is Reſolv'd into This; namely, Firſt, The Reaſons of Quantities; Second, The Quantities of which they are the Reaſons; And Third, Whether we can conſider the One without the Other; Theſe are the Three Things to be Examin'd into, in the Preſent Doctrine.

Before I do which, I muſt obſerve, that the *Doctrine* of *Indiviſibles* is own'd to be of the ſame Kind, as it Really is, with that of theſe *Naſcents* and *Evaneſcents*, the Former yet is Diſcarded, as being Contrary to *Euclid*'s *Doctrine* of *Incommenſurables*, and for the ſame Reaſon ought this to be, which is as Directly Oppoſite and Repugnant to it, in which Curves and Right Lines of the moſt Different Nature and Conſtitution are ſuppos'd to Coincide in a Point, and therefore to obtain one Common Meaſure; To omit this notwithſtanding,

First, This Philoſophy ſays, it only Conſiders the Reaſons of Quantities, and not the Quantities themſelves; whereas, in the Anſwer to the Firſt Objection, it ſuppoſes, that there is an End of the Velocity, which is made uſe of as a Parallel Inſtance to Quantity, and therefore, that there as an End of the Quantity it ſelf, whoſe Evaneſcent Reaſon is Conſider'd, and in the Second Objection from Incommenſurables and Quantities Infinitely Great, that there is no End of the Quantity, whoſe Reaſon is Conſider'd, which is a Contradiction.

SECOND, It is in the Next Place to be Obſerv'd, that even in the Reaſons of Quantities, if thoſe only were to be Regarded, it does not make any Juſt Concluſions, according to it's own Sentiments; for albeit Quantities are never ſo Different, if they are but ſuppos'd Infinitely Increas'd or Diminiſh'd, their Reaſon will be that of Equality according to this Philoſophy, notwithſtanding, that all the Naſcent and Evaneſcent Reaſons of Different Quantities are Imagin'd to be Different, which is again a Contradiction.

BUT if Quantities only, which are Equal, upon the Infinite Increaſe or Diminution of them, have their Reaſons of Equality, there is no great Myſtery in the Matter, becauſe, if Quantities are Equal, their Reaſons or Proportions will be ſo too.

THIRD, After what we have ſaid in Confutation of the Preſent Doctrine of Naſcents and Evaneſcents from it ſelf, we ſhall only Affirm, that Quantities and the Reaſons of Quantities are the ſame, and that we cannot conſider the One without the Other; that is, the Reaſons of Things are their Quantities, and *Vice Verſâ*, Quantities are nothing elſe but the Reaſons of Things.

FOR what are Quantities themſelves, unleſs the Reaſons or Relations of Things, when compar'd with one another? Thus a Right Line, or a Curve, or a Rectilineal, or Curve Figure, as a Square and a Circle, are Quantities only, as they are capable of Relation; for if they are barely Conſider'd, as one Uniform and Abſtracted Extenſion without any Properties of what they are in Reſpect to other Lengths or Figures, either contain'd in themſelves, as Portions of them, or Diſtinct from them, they cannot be in any Senſe Term'd Quantities; when therefore we Inquire into the Ultimate Reaſons of Reaſons', if that is a Proper Way of Speaking, which yet is the Real Meaning of this Philoſophy, we muſt know what thoſe Reaſons are, whoſe Ultimate Reaſons we wou'd Determine, or elſe it will be Impoſſible for us to Demonſtrate with any Degree of being Intelligible; that is, it will be neceſſary for us to Inform our Selves of the Nature of thoſe Lines or Figures, whoſe Ultimate Reaſons are Inveſtigated, before we can Fix or Aſcertain ſuch Ultimate Reaſons.

FOURTH, Moreover, according to the Doctrine of Velocities in the *Geometry* of *Solids*, one and the ſame Line to Appearance may be capable of a Thouſand Different Values, if the Velocities, by which it is Produc'd, are Various; whereas the Preſent *Method* of *Naſcents* and *Evaneſcents* does only ſuppoſe the ſame Abſtracted Length, as it Lies in the Figure, without Conſidering, with what Velocity it is cauſ'd or receives it's Dimenſion.

Geometria Solidorum ſive Materiæ.

FIFTH, Laſtly as Length or Breadth is Accounted one Similar and Invariable Extenſion in this *Method*, ſo is Infinity Equally Determin'd to be one Uniform Exhauſtion of Quantity; whereas, if Quantities are Different, one Uniform Infinity cannot Equally Exhauſt both, for Inſtance, if One Line in Reſpect of Another, as in the *Geometry* of *Solids*, may have the ſame Relation, as that of a Line and a Surface or a Solid, one Uniform Infinity can no more Exhauſt the Points of Each of them, than by Infinitely Diminiſhing a Line we can Exhauſt a Surface or a Solid; and the ſame may be ſaid of Two Different Lengths, for Example, let One be the Double of the Other, if Infinity is one Uniform Thing, they cannot be Equally Exhauſted by it, but when one is Exhauſted, there will be only Half of the other Deſtroy'd.

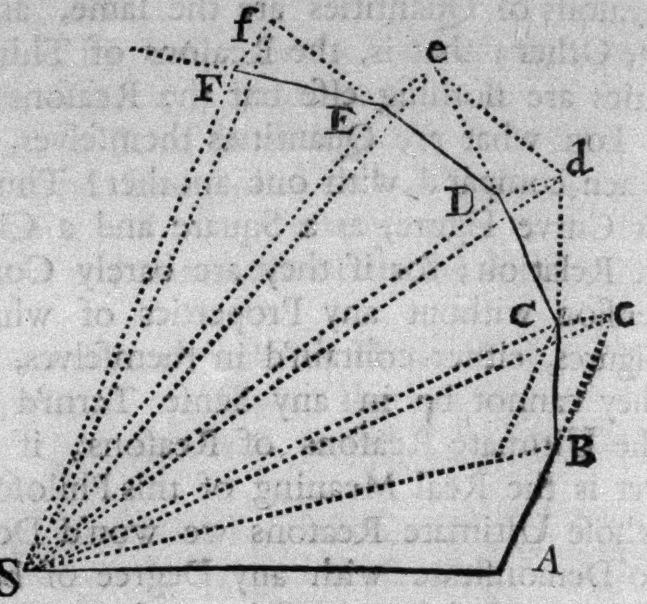
IF Infinity is not one Uniform Idea, but puts on Different Qualities and Shapes, it is in Short no more than this, that is, an Univerſal Deſtroyer of Unequal Quantities, which the Mind Employs to Devour and Annihilate all the Excreſcencies or Exorbitancies of them, which it has no Occaſion for in it's Abſtracted Reaſonings, and which, if not taken out of the Way, wou'd be Troubleſome to it; and ſomething like this is the Caſe of Infinity, when Apply'd to Quantities Infinitely Increas'd, which let them be what they will, and how Different ſoever Originally, ſhall from thence, it ſeems, become Equal; Infinity Deſtroys the Difference, and makes them all one, which is Impoſſible; for an Infinite Number of Inches cannot be the ſame, as an Infinite Number of Yards, or Miles, or Semidiameters of the *Moon*, of the *Earth*, or of the Orbits of *Jupiter* or *Saturn*; unleſs Infinity is ſomething not certain, excepting that it obtains one Quality, which is, of making all Things Equal, which, when Finite, were not ſo.

CHAP. IV.

Concerning the Fundamental Propoſitions of this Philoſophy, and the Application of them to the Solving the Revolutions of the Planets.

The Firſt Propoſition Conſider'd and Examin'd.
Sir Iſ. Newt. Princip. § 2. Prop. 1. Lib. 1.
Dr. Gregory's Aſtronom. Prop. 11. §. 2. Lib. 1.

§. 1. HAVING in the Former Chapter Conſider'd the *Doctrine* of *Naſcents* and *Evanaſcents*, we are now come to the Propoſitions and Theorems of this Philoſophy; The Firſt of which is, That if a Body is Mov'd according to the Direction from *A* to *B*, and at the ſame Time is Urg'd by a Centripetal Force from *A* to *S*, a Curve Line will be Deſcrib'd, and whilſt the Body Deſcribes that Curve Line, the Areas will be Proportional to the Times of the Body's Revolving in ſuch a Curve; or, which is the ſame, as it is Expreſs'd in this Philoſophy, The Areas, which Bodies Deſcribe, when carry'd about by Radius's drawn to an Immoveable Center of their Forces, are in Immoveable Planes, and are Proportional to the Times of their Revolving.



THUS, Let the Time be Divided into Equal Parts, and in the Firſt Portion of it, Let the Body Deſcribe by it's Innate Force the Right Line *AB*, the ſame Body, if nothing were to Hinder or Obſtruct it, wou'd proceed to *c*, and by Law the Firſt Deſcribe *Bc* = to *AB*, ſo that by Drawing the Lines, *AS*, *BS*, *cS* to the Center *S*, there wou'd be Equal Areas perform'd *ASB*, *BSc*, but when the Body comes to *B*, Let the Centripetal Force be ſuppos'd to Urge it by one Impulſe, and by that means make it Decline from *Bc*, and proceed forwards in the Line *BC*; Let *Cc* be drawn Parallel to *BS*, and meet the Line *BC* in *C*, If this is ſuppos'd, upon the Completion of the Second Portion of Time, by *Corol.* 1. of the Laws, the Body will be found in *C*, in the ſame Plane with the Triangle *ASB*; Join *S*, *C*, and the Triangle *SBC* will be Equal to the Triangle *SBc* = Triangle *SAB*, becauſe *SBC*, *SBc*, have the ſame Baſe *SB*, and lie within the ſame Parallels, *SB*, *Cc*; By a like Method of Arguing *SCd*, *SCD* will be Prov'd Equal, and ſo on; From whence it will follow, that, in Equal Times, Equal Areas are Deſcrib'd in a Plane, which is Unmov'd, and by Compounding any Sums of theſe Areas, *SADS*, *SAFS*, are to One Another, as the Times of their Deſcription.

LET now the Number of the Triangles be Increas'd, and the Latitude of them Diminiſh'd to Infinity, and their Ultimate Perimeter *ADF*, by the 4th *Coroll.* to *Lem.* 3. will be a Curve Line; and therefore the Centripetal Force, by which a Body is perpetually Drawn out of the Tangent of this Curve, will Act Continually and Inceſſantly upon it; And any Areas, *SADS*, *SAFS*, Deſcrib'd Proportional to the Times of Deſcription will be, in this Caſe, Proportional to the ſaid Times, *Q. E. D.*

IN which Demonſtration we ſhall Conſider; First, The Generation of this Figure upon the Suppoſition of the Preſent Abſtracted Forces; Second, What Alteration Different Magnitudes in the Bodies, which are Mov'd, wou'd Cauſe; Third, What wou'd be the Conſequence, if other Forces were Introduc'd, whether Projectile or Attractive in a Different Proportion to each other, from what is here Preſum'd; Fourth, What wou'd follow, if Forces perfectly Diſtinct from either of theſe ſhou'd be made uſe of.

The Forces by which this Figure is Generated, and the Way of Gene-

§. 2. FIRST, It is ſuppos'd, that *A*, by it's Innate Force, proceeds from *A* to *B* in one Portion of Time, and in an Equal Portion of it Deſcribes *Bc* Equal to *AB*, that is, a Perfect Inertia of Matter is Preſum'd, and that upon it's Motion it will have a Conſtant and Uniform Progreſs, unleſs it is Interrupted by ſome External

Agent, but to conſider Matter, as an Unactive Maſs, and only Capable in it ſelf of Receiving that Impulſe, which we Give it, is Firſt Making it a Creature of the Mind, and then Uſing it at our own Pleaſure; For this we have Prov'd to be the Senſe of a Similar, Paſſive, or Homogeneous Matter, And what we Mean and Intend in theſe Principles is only, with all Submiſſion, to Take it out of the Hands of the Philoſophers, and Reſign it to it's own Nature and the Great Author of it.

ration by Them liable to many Objections; And Firſt of the Projectile Force.

Let us yet ſuppoſe, that Matter or Body is Similar, Paſſive, or Homogeneous, and that it has no Action of it's own; we Ask, Does Almighty God Impart that Action to it, by which it proceeds Uniformly from *A* to *B*, or is it done by the Mind of the Philoſopher? If the Firſt, How is it known in what Proportion an Infinite Being Acts upon it? Why may not the Impreſſion, ſuitable to the Energy of the Almighty Agent, be ſo Forcible and Strong, that no Centripetal Action whatſoever ſhall be able to divert it's Progreſs? The Caſe is too Evident and Plain, that, if this Projectile Force was Deriv'd from God, Matter wou'd be as little in the Power of the Philoſophers to Manage and Direct it, as they pleas'd, as if they had never ſuppos'd it Paſſive; and therefore ſuch a Conceſſion wou'd deſtroy the very Intention of making Matter Paſſive and Homogeneous, namely, that of Contriving it into what Shapes, Figures, or Motions, ſhould be thought Fit or Convenient for a Preſent Solution.

If therefore the Progreſs of Matter or Body from *A* to *B* is under the Will and Direction of the Philoſopher, Let us now Inquire which way that Motion is Produced, when Matter is ſuppos'd to be perfectly Unactive in it ſelf; the Mind cannot by Thinking put it into Motion, nor can our own Imaginations, be they as Vigorous as they will, Actuate a Dead Lump, and Throw it into an Uniform Projection; Since then the Preſent Scheme does not Inform us, how in Unactive Matter a Projectile Force from *A* to *B* is Impreſs'd upon it, tho' all, which is here propoſed to be Demonſtrated, were true upon the Suppoſition of ſuch a Projectile Force, yet, when apply'd to Nature, the Body wou'd be Quieſcent in *A*, and there wou'd not be the Leaſt Advance made from *A* to *B*, and conſequently, the Revolution of the Planets cou'd not be Explain'd by ſuch a Scheme; If it is again ſaid, that Almighty God Impreſſes this Force upon Bodies, which, being Paſſive of themſelves, wou'd continue that Force *in Directum*, if not Interrupted by Gravity, which is ſuppos'd to be Deriv'd from him likewiſe, we ſhall ſeem, I'm afraid, to make the Sovereign Diſpoſer of the Univerſe a Capricious Kind of Agent, whilſt he Puſhes the Unactive Maſs One Way and Draws it Another, and Created it Unactive too, that he might be Employ'd in ſuch a Ludicrous Paſtime; It is told us, that Nature always takes the ſhorteſt Methods to Effect it's own Purpoſes, and it is a Fundamental Maxim of the Preſent Philoſophy; but ſurely then the wiſe Former and Maker of Nature it ſelf wou'd not Create a Maſs without Action, in order afterwards to Impreſs Two Different ones upon it, which might be Effected by one; Indeed this Notion has too great a Reſemblance of the Fancies and Artifices of Philoſophers to be Entitled to the being a Divine, which is plainly no more, as we ſhall ſhow, than a Human Contrivance.

After having ſaid what foregoes, we ſhall now Proceed to Inquire into the Generation of this Figure, ſuppoſing *A* to be a Body conſiſting of Homogeneous Matter, and that there is a Projectile Motion given to it, which carries it Uniformly from *A* to *B*, and from thence to *c*, and ſo on; Either then the Projectile Motion AB is Equal to $Bc = Cd = De = Ef$, or not, If Equal, when the Body Departs from *C* to *d*, it will have the ſame Projectile Force, as when it Proceeds from *B* to *c*, or from *A* to *B*, Notwithſtanding, that it is continually Urg'd by a Contrary Force, which Preſſes it towards *S*, which is Abſurd and Irrational; unleſs a Projectile Motion can riſe out of it's Oppreſſion from Gravity with as much Vigour at *C*, as before that Load was thrown upon it at *A* or *B*, which will ſhow a Vivacity in Matter ſomething Superior to that Tameneſs and Paſſiveneſs, which this Philoſophy preſcribes to it; Becauſe, if a Force was to Act Contrary to the Projectile, that is, from *B* to *A*, from *c* to *B*, or from *d* to *C*, the Projectile from *A* to *B*, from *B* to *c*, or from *C* to *d* wou'd be evidently Diminiſh'd, and if Equal to it, wou'd be Deſtroy'd, how therefore a Force Acting towards *S*, which is in ſome Meaſure Contrary to it, ſhou'd not in the leaſt Diminiſh the ſame Projectile Force, is Impoſſible to ſay.

If AB, Bc, Cd, De, Ef, are not Equal; In the Firſt Place, that will be Deny'd, which is here Affirm'd and Suppoſ'd, *viz.* that AB, Bc, Cd, &c. are Equal. Second, There is no more Reaſon to Suppoſe or Affirm, that Bc is Equal to AB, than that Cd is, for when the Body Moves from the Point A to B, ſuch a Projectile Force is not Clearer from an Attractive one, than Bc is; and therefore, if Cd is not Equal to Bc, for the ſame Reaſon Bc will not be Equal to AB, which is yet ſuppos'd, and the ſame may be ſaid of the Reſt. Third, If the Projectile Motion AB is not Equal to Bc Equal to Cd, what is it Equal to? If AB is Equal to Bc, and BC to Cd, CD to De, &c. which is the only Caſe Remaining, it will be Inconceivable, how the Motion of the Body in AB, BC, CD, DE, &c. which is Compounded of a Projectile and Gravitating Force, ſhould be Equal to the Motion of it in Bc, Cd, De, &c. where the Force is only ſuppos'd to be a Projectile one, Abſtracted from a Gravitating. Fourth, If BC, Cd, &c. are not Equal, the Reaſon of it will be, that BC is a Compound Force, and Cd only a Projectile, But how a Projectile ſhould riſe out of a Compound without being Affected with the Gravity, which was Impreſs'd from B to C, that is, how there ſhou'd ever be a ſingle Projectile Force after ſuch a Compound, is altogether Inexplicable.

Concerning the Attractive Force in this Figure, and what Relates to it.

§. 3. Having conſider'd the Projectile Force in the Preſent Scheme, let us now Examine into the Attractive, which may be Expreſs'd by cC, dD, eE, fF, ſince the Effect is neither Greater nor Leſs than the Cauſes, which produce it; The Effect is the Body's Revolving in the Diagonal Lines BC, CD, DE, whoſe Projectile Force is Bc, Cd, De, and conſequently, the Gravitating may be Deſcrib'd or Repreſented by Cc, Dd, Ee, according to this Philoſophy.

Either therefore theſe Attractive Forces, as the Projectile before, are Equal, or not; If they are Equal, ſince Bc or elſe BC muſt be Equal to Cd by what Precedes, and ſince the Attractive Quality cannot draw the Body down from one Projectile Force more than another, that is, ſince the Angles cBC, dCD will be Equal, it will follow, that thoſe Triangles are likewiſe Equal, and that therefore Bc is Equal to Cd, and BC to CD, or that BC is Equal to Cd, and Bc to CD, both which have been already Diſprov'd.

Moreover, if AB is the Projectile, where is the Gravitating Force, which belongs to it, that is Equal to Cc the Gravitating Force of the Projectile Bc? and why does not the Figure begin at B rather than at A, Unleſs it were for a private Reaſon, becauſe BC does not look ſo much like a Projectile, which is Compounded of that and Gravity, as AB does?

If Cc is not Equal to Dd, Equal to Ee, that is, if there is not the ſame Attractive Force, which Perverts the Body from the Tangent or Projectile, Matter, which is own'd to be perfectly Paſſive, will Aſſume of it ſelf Different Powers of Gravitation, and therefore are ſuch, as cannot be Deriv'd from any other Cauſe than that of the Pleaſure of the Philoſopher.

It may be farther ſaid, that the Propoſition, upon which the Preſent depends, namely, that the Force of the Diagonal may be Reſolv'd into the Sides of the Parallelogram, has been already Prov'd to be falſe in our Conſideration of the Laws of this Philoſophy, and the Corollaries of them, and therefore Bc and cC are not in Truth Expreſſive of the Force in BC, tho' we have hitherto Acknowledg'd it, and Argued againſt the Preſent Theorem upon that Suppoſition.

Lastly, We have ſhewn in the Examination into the Doctrine of Naſcent and Evaneſcent Reaſons, that if the Right Lines Bc, Cd, cC, dD, were Diminiſh'd to Infinity, it is not neceſſary they ſhou'd form a Curve, which this Propoſition Affirms; Since the Nature of One or Two Right Lines, as Bc and cC, tho' Join'd and United together, may be Infinitely Different from the Nature of a Curve, which cannot, in ſome Caſes, be Meaſur'd by any Right Lines, or any Conjunction of them whatſoever; It is alſo ſomething Extraordinary to make, in this Demonſtration, the Centripetal Forces not Tend to the Center, but Act in Lines Parallel to thoſe, which are Directed thither; But This we Omit.

Concerning the Magnitude of the Body, the Revolution of which is Inquir'd.

§. 4. Second, The next Thing to be conſider'd is the Magnitude of the Body, the Revolution of which is Inquir'd, if Matter or Body in General is nothing elſe than an Abſtracted Idea, it has no Qualities beſides thoſe, which the Mind of the Philoſopher gives it, which may Band about it's own Creature, as it pleaſes, and let the Bulk of ſuch Matter be Greater or Leſs, it will be the ſame; For if Matter conſiſts of Extenſion in General, Solidity in General, and the like Abſtracted Properties, a Greater or Leſs Magnitude of them will be Equally Obedient to any Impulſes, which ſhall be thought convenient to Impart to it, but then all this will only Reſt in the Fancy, and there Determine.

ON the other Hand, if Matter has any Real Properties and Affections belonging to it, as particularly that of Gravity, Impreſs'd upon it from an Elaſtick Medium, or Innate, which this Philoſophy owns, This Scheme, which lies before us, will be Incapable of Affording us the Leaſt Satisfaction or Demonſtration Concerning the Revolutions of the Heavenly Bodies, for which it is Deſign'd; Since, if *A* is Endued with an Actual Gravity, and ſuch Gravity is Proportional to the Quantity of Matter, which is contain'd in any Body, according to this Philoſophy, if the ſame Projectile Force *AB*, or *Bc*, &c. is Communicated to Different Portions or Bulks of Matter, it will have very Different Effects from what are Expreſs'd in this Figure, which ſuppoſes that Force to be Uniformly the ſame; Effects ſo Different, that if the Force of Gravity is Infinitely Increas'd by the Immenſe Magnitude of the Body, the Projectile may be Nothing in the Compariſon, by which the Body, inſtead of being Revolv'd, will Drop to the Center, and if the Force of Gravity is Infinitely Diminiſh'd by the Extreme Smallneſs of the Body, the Projectile may be Infinite in the Compariſon, and Carry it forward *in Directum* without any Revolution.

IF it is ſaid, that according to the Greater Quantity of Matter or the Leſs, the Projectile Force Imparted to it will be Greater or Leſs, but yet Uniform, ſo as to keep up the Body from Sinking to the Center, or going Forward *in Directum*; Who is the kind Perſon, that Moderates and Tempers this Projectile Force with ſo much Beneficence and Goodneſs, but the Same, Who had the Skill to Contrive and Invent it? I mean the Ingenious Author of this Philoſophy Sir *Iſ. Newton*, Who, by the ſame Method, may at his Pleaſure Communicate what Impreſſions he will to Matter, and no one ſhall Preſume to Contradict the Humour.

FOR the Expreſſions of Gravity *cC*, *dD*, *eE*, &c. will be Various, if the Bulk or Magnitude of the Body is ſo, and therefore cannot be Explain'd by a Uniform Gravity Repreſented by thoſe Lines, but whilſt the Projectile is *Bc*, the Gravity may be Ten, a Hundred, or a Thouſand Times Greater or Leſs than *Cc*, which, if Produc'd or Leſſen'd in that Proportion, wou'd cauſe an Unſpeakable Confuſion in the Preſent Figure, and wou'd in no manner be Capable of being Exacted to it; Unleſs it is ſaid, that the Accelerating Force to the Center continues the ſame, however Different the Magnitudes are, which are Accelerated, and therefore *Cc* will Expreſs ſuch Attractive or Accelerating Force without any Alteration; To which it may be Reply'd, that this would not be True, if it cou'd be even Prov'd, that Matter is Similar and Homogeneous, or that all Bodies Deſcend Equally, and in Equal Times, to their Center of Attraction, both which Notions we have yet Endeavour'd to Confute in the Firſt Part of the *Principles* of *Philoſophy*, which we have ſome Years ago Publiſh'd, and can be only True, if we conſider Matter as a mere Abſtracted Thing, which is the ſame, whether we take a Greater or Leſs Portion of it, and which is likewiſe a Perfect Fancy and Imagination.

What wou'd be the Conſequence, if other Forces were Introduc'd, whether Projectile or Attractive, in a Different Proportion to each other from what is here Preſum'd.

§. 5. THIRD, The Third Thing propos'd was to Examine, what wou'd be the Conſequence, if other Forces were Introduc'd, whether Projectile or Attractive, in a Different Proportion to Each other from what is here Preſum'd, for, in the Preſent Caſe, at the ſame Time that the Projectile Force wou'd carry the Body Revolv'd from *B* to *c*, the Attractive wou'd Draw it from *c* to *C*; If therefore the Projectile Force ſhou'd now be Infinitely Increas'd or Diminiſh'd, whilſt the Attractive Draws it from *c* to *C*, namely upon the Suppoſition of the Accelerating Force being the ſame, the Attractive Force wou'd be carry'd away *in Directum* by the Projectile in the Firſt Caſe, and in the Laſt Ruſh to the Center; or if the Projectile were to continue the ſame, and the Attractive Force was Infinitely Diminiſh'd or Increas'd in a Body of the ſame Magnitude, the like Effects wou'd Follow, and Nothing of this Figure wou'd Enſue; from whence it is plain in all Reſpects, that this is Nothing more than an Abſtracted and Mathematick Reaſoning, which conſiders an Uniform Projectile and Attractive Force without Determining what it is, which yet is Neceſſary in a true Reaſoning.

§. 6. FOURTH,

What the Conſequence, if Forces perfectly Diſtinct from either ſhou'd be Introduc'd.

§. 6. Fourth, And laſtly, if Forces perfectly Diſtinct from Either of theſe, as thoſe in Fire, Vegetation, *&c.* or the Expanſive in General, are Introduced into the Figure, it is Manifeſt, that the Projectile and Attractive will be ſo much Alter'd, that the Preſent Figure will be entirely Annihilated and Confounded.

The Second Propoſition Explain'd. Sir Iſ. Newt. Princ. Prop. 2. §. 2. Lib. 1. Dr. Gregory's Aſtronom. Prop. 12. §. 2. Lib. 1.

§. 7. The Second Propoſition is, That every Body, which, when it is Mov'd in any Curve Line, and by a Radius Drawn either to an Immoveable Point, or to a Point that proceeds Uniformly in a Rectilineal Motion, Deſcribes Areas about that Point, which are Proportional to the Times, is Urg'd by a Centripetal Force Tending to the ſame Point.

The Demonſtration of this Propoſition ſo evidently depends upon the Former, and the Firſt and Second Laws, which have been already Conſider'd, that there is no Occaſion particularly to Examine into it.

The Third Propoſition Explain'd. Sir Iſ. Newt. Prop. 3. §. 2. Lib. 1. & Schol.

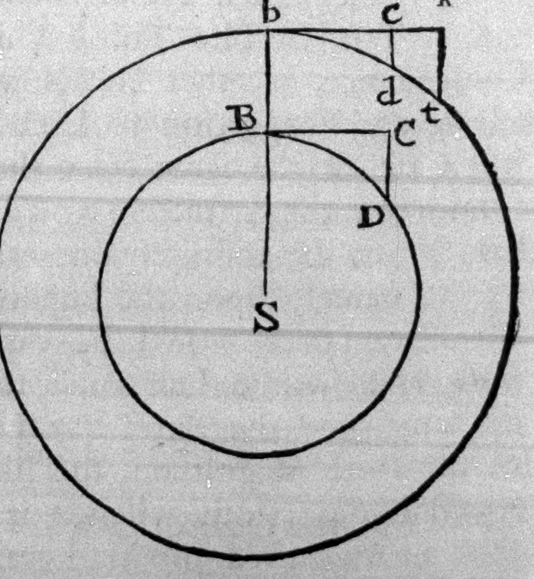
§. 8. The Third Propoſition is, That Every Body, which, by a Radius Drawn to the Center of Another Body, however Mov'd, Deſcribes Areas about that Center Proportional to the Times, is Urg'd by a Force Compounded of a Centripetal Tending to the other Body, and of the whole Accelerating one, by which the other Body is Urg'd.

The Demonſtration likewiſe of this Propoſition is concluded from the 6th Corollary of the Laws, which have been Examin'd, from the Firſt of thoſe Laws, and from the foregoing Propoſition the Second; and therefore it will not be here alſo neceſſary to make any particular Inquiry into the Preſent Reaſoning.

We ſhall notwithſtanding obſerve, that when the Firſt Propoſition had Inform'd us, that the Areas, which Bodies Deſcribe by Radius's Drawn to an Immoveable Center of their Forces, were Proportional to their Times, and on the Contrary, the Second had told us, that Bodies, which were ſo Circulated, were Urg'd by a Centripetal Force, both which Propoſitions Reſpect Curves in General, whoſe Radius's are not Determin'd whether Equal or not, we are yet in the Scholium of this Propoſition Perſuaded to Believe, that an Equable or Uniform Deſcription of Areas is an Index or Characteriſtick of a Center, whoſe Curve is Circular, and whoſe Radius's are Equal; whereas, if it were ſo, the Firſt Propoſition wou'd not be ſo General, as it is Exhibited to us, and if it is ſo General as it is Repreſented, an Equable Deſcription of the Areas can be no more Concluſive of a Circular than of any other Curve *in Spatiis Liberis*, ſince the Spaces are ſuppos'd to be ſo, in the Firſt Propoſition and the Demonſtration of it; that is, we cannot be willing to allow any more in a Philoſophy, which pretends to Mathematick Demonſtration, than what is Juſtly and Strictly concluded from ſuch Demonſtration.

The Fourth Propoſition with it's Corollaries and Scholium Explain'd. Sir Iſ. Newt. Prop. 4. §. 2. Lib. 1. Edit. 1. Dr. Gregory's Aſtronom. Prop. 25. §. 4. Lib. 1.

§. 9. The Fourth Propoſition is, That of Bodies, which Deſcribe Different Circles by an Equable Motion, the Centripetal Forces Tend to the Centers of the ſame Circles, and are to Each other, as the Squares of the Arcs, which are Deſcrib'd in the ſame Portion of Time, Divided by the Radius's of the Circles.



Let the Bodies, B, b, by Revolving in the Circumferences of the Circles, BD, bd, Deſcribe at once the Arcs BD, bd, ſince therefore by their ſole Innate or Projectile Force the Bodies, B, b, wou'd Deſcribe the Tangents BC, bc, it is Evident, that the Centripetal Forces are thoſe, which perpetually Draw the Bodies from the Tangents to the Circumferences of the Circles, and conſequently are to One Another in the Firſt Reaſon of the Naſcents, CD, cd, and Tend to the Centers of the Circles, by Prop. 2; becauſe the Areas Deſcrib'd by the Radius's are ſuppoſed Proportional to the Times; Let the Figure tkb be made Similar to the Figure DCB, and by *Lem.* 5, the Line CD will be to the Line kt, as the Arc BD to the Arc bt, And by *Lem.* 11. the Naſcent Line tk will be to the Naſcent Line dc, as bt *Square*, to bd *Square*, and *ex Æquo* the Naſcent Line

DC will be to the Naſcent Line dc, as $BD \times bt$ to bd *Square*, or, which is the ſame, as $\frac{BD \times bt}{Sb}$ to $\frac{bd \text{ Square}}{Sb}$, and therefore (upon the Account of the Equal Reaſons $\frac{bt}{Sb}$ and $\frac{BD}{SB}$) as $\frac{BD \text{ Square}}{SB}$ to $\frac{bd \text{ Square}}{Sb}$. *Q. E. D.*

FROM this Propoſition and the Demonſtration of it, It is Infer'd.

Corol. 1. THAT the Centripetal Forces are as the Squares of the Velocities Divided by the Radius's of the Circles; For DC is to dc, as $\frac{BD \text{ Square}}{SB}$ is to $\frac{bd \text{ Square}}{Sb}$ by the Propoſition, and BD and bd Repreſent the Velocities in Each, that is, the Arcs are as the Velocities. Or the Forces are in a Reaſon Compounded of a Duplicate Reaſon of the Velocities Directly, and of a Simple Reaſon of the Radius's Inverſely.

Corol. 2. That the Centripetal Forces are Reciprocally, as the Squares of the Periodical Times Divided by the Radius's; That is, theſe Forces are in a Reaſon Compounded of the Duplicate Reaſon of the Velocities Directly, and of a Simple Reaſon of the Radius's Inverſely; as alſo in a Reaſon Compounded of the Simple Reaſon of the Radius's Directly, and of a Duplicate Reaſon of the Periodical Times Inverſely. Which is eaſily ſhewn by the Arcs, BD, bd, Repreſenting their Circles.

Dr. Gregory's Aſtronom. Prop. 26. § 4. Lib. 1.

FOR ſuppoſing, V Expreſſes the Velocities, T the Periodical Times, q the Squares of Each, and c the Cubes; Since the Periodical Times are in a Reaſon Compounded of the Reaſon of the Radius's Directly, and of a Reaſon of the Velocities Inverſely, that is, ſince $BD,T . bd,T :: \frac{SB}{BD,V} . \frac{Sb}{bd,V}$, it will be, $BDq,T . bdq,T :: \frac{SBq}{BDq,V} . \frac{Sbq}{bdq,V}$, and $\frac{BDq,T}{SB} . \frac{bdq,T}{Sb} :: \frac{SB}{BDq,V} . \frac{Sb}{bdq,V}$, or $\frac{Sb}{bdq,T} . \frac{SB}{BDq,T} :: \frac{bdq,V}{Sb} . \frac{BDq,V}{SB}$; from whence it will follow by Inverſion, that $\frac{BDq,V}{SB} . \frac{bdq,V}{Sb} :: \frac{SB}{BDq,T} . \frac{Sb}{bdq,T}$, but by *Corol.* 1. $DC . dc :: \frac{BDq,V}{SB} . \frac{bdq,V}{Sb}$, and therefore $:: \frac{SB}{BDq,T} . \frac{Sb}{bdq,T}$, or $:: \frac{bdq,T}{Sb} . \frac{BDq,T}{SB}$.

Corol. 3. FROM whence it Enſues, if the Periodick Times are Equal, that both the Centripetal Forces, and the Velocities, will be as the Radius's, and *Vice Verſâ*; For $DC . dc :: \frac{SB}{BDq,T} . \frac{Sb}{bdq,T}$, or $:: SB . Sb$, if BDq,T is $= bdq,T$; Likewiſe $\frac{BDq,V}{SB} . \frac{bdq,V}{Sb} :: \frac{SB}{BDq,T} . \frac{Sb}{bdq,T}$, or $:: SB . Sb$, if BDq,T is $= bdq,T$; and therefore $BDq,V . bdq,V :: SBq . Sbq$, or $BD,V . bd,V :: SB . Sb$.

Corol. 4. IF the Squares of the Periodick Times are as the Radius's, the Centripetal Forces will be Equal, and the Velocities in a Dimidiate Reaſon of the Radius's, and *Vice Verſâ*; For ſince $DC . dc :: \frac{SB}{BDq,T} . \frac{Sb}{bdq,T}$, if DC is $= dc$, $\frac{SB}{BDq,T} = \frac{Sb}{bdq,T}$, that is, $SB . BDq,T :: Sb . bdq,T$, and by Alternation, $BDq,T . bdq,T :: SB . Sb$; Or, on the Contrary, if this Proportion obtains, DC is $= dc$; Or, if $BD,T . bd,T :: SB|^{\frac{1}{2}} . Sb|^{\frac{1}{2}}$. For This Reſults from the Former.

Sir Iſ. Newt. Princip. Edit. 2.

THE Velocities alſo will be in a Dimidiate Proportion of the Radius's, ſince $\frac{BDq,V}{SB} . \frac{bdq,V}{Sb} :: \frac{SB}{BDq,T} . \frac{Sb}{bdq,T}$ by *Corol.* 2, if therefore $SB . Sb :: BDq,T . bdq,T$, it will be as $\frac{BDq,V}{SB} . \frac{bdq,V}{Sb} :: \frac{SB}{SB} . \frac{Sb}{Sb}$, which is Evident from Proportionals being ſubſtituted in Diviſion inſtead of Proportionals, by which Means the Reaſons of the Quantities will be ſtill Preſerv'd the ſame; But, if $\frac{BDq,V}{SB} . \frac{bdq,V}{Sb} :: \frac{SB}{SB} . \frac{Sb}{Sb}$, it will be as $BDq,V . bdq,V :: SB . Sb$; And $BD,V . bd,V :: SB|^{\frac{1}{2}} . Sb|^{\frac{1}{2}}$, or $\sqrt{SB} . \sqrt{Sb}$.

Sir Iſ. Newt. Princip. Edit. 2.

Corol. 5. If the Squares of the Periodical Times are as the Squares of the Radius's, The Centripetal Forces are Reciprocally as the Radius's, and the Velocities Equal, and *Vice Verſâ.*

The First is thus Evinced, for $DC \,.\, dc :: \frac{SB}{BDq,T} \,.\, \frac{Sb}{bdq,T}$ by *Corol.* 2, Therefore $DC \times SB \,.\, dc \times Sb :: \frac{SBq}{BDq,T} \,.\, \frac{Sbq}{bdq,T}$; If Conſequently $\frac{SBq}{BDq,T}$ is $= \frac{Sbq}{bdq,T}$, that is, if $SBq \,.\, BDq,T :: Sbq \,.\, bdq,T$, or Alternately, if $BDq,T \,.\, bdq,T :: SBq \,.\, Sbq$, $DC \times SB$ will be Equal to $dc \times Sb$, or, which is the ſame, $DC \,.\, dc :: Sb \,.\, SB :: \frac{1}{SB} \,.\, \frac{1}{Sb}$. The ſame will obtain, if $BD,T \,.\, bd,T :: SB \,.\, Sb$.

Sir Iſ. Newt. Princip. Edit. 2.

Second, That the Velocities in ſuch a Caſe will be alſo Equal is Manifeſt, ſince $\frac{BDq,V}{SB} \,.\, \frac{bdq,V}{Sb} :: \frac{SB}{BDq,T} \,.\, \frac{Sb}{bdq,T}$ by *Corol.* 2, i. e. $BDq,V \,.\, bdq,V :: \frac{SBq}{BDq,T} \,.\, \frac{Sbq}{bdq,T}$, if therefore $\frac{SBq}{BDq,T}$ is $= \frac{Sbq}{bdq,T}$, that is, if $BDq,T \,.\, bdq,T :: SBq \,.\, Sbq$, BDq,V will be $=$ to bdq,V, or the Velocities will be Equal.

Corol. 6. If the Squares of the Periodical Times are as the Cubes of the Radius's, the Centripetal Forces will be Reciprocally, as the Squares of the Radius's, and the Velocities Reciprocally in a Dimidiate Reaſon of the ſame Radius's, and *Vice Verſâ.*

The First, becauſe $DC \,.\, dc :: \frac{SB}{BDq,T} \,.\, \frac{Sb}{bdq,T}$, as before; therefore $DC \times SBq \,.\, dc \times Sbq :: \frac{SB \times SBq}{BDq,T} \,.\, \frac{Sb \times Sbq}{bdq,T}$, and Conſequently, if $\frac{SBc}{BDq,T}$ is $= \frac{Sbc}{bdq,T}$, that is, if $BDq,T \,.\, bdq,T :: SBc \,.\, Sbc$, $DC \times SBq$ will be Equal to $dc \times Sbq$, or $DC \,.\, dc :: Sbq \,.\, SBq :: \frac{1}{SBq} \,.\, \frac{1}{Sbq}$. If $BD,T \,.\, bd,T :: SB|^{\frac{3}{2}} \,.\, Sb|^{\frac{3}{2}}$, or $\sqrt{SBc} \,.\, \sqrt{Sbc}$, it will be the ſame, As it neceſſarily follows from the Former.

Sir Iſ. Newt. Princip. Edit. 2.

The Second, becauſe $\frac{BDq,V}{SB} \,.\, \frac{bdq,V}{Sb} :: \frac{SB}{BDq,T} \,.\, \frac{Sb}{bdq,T}$, as before; Conſequently $\frac{BDq,V \times SBq}{SB} \,.\, \frac{bdq,V \times Sbq}{Sb} :: \frac{SB \times SBq}{BDq,T} \,.\, \frac{Sb \times Sbq}{bdq,T} :: BDq,V \times SB \,.\, bdq,V \times Sb$; if therefore $\frac{SBc}{BDq,T}$ is $= \frac{Sbc}{bdq,T}$, or, which is the ſame, if $BDq,T \,.\, bdq,T :: SBc \,.\, Sbc$, $BDq,V \times SB$ will be Equal to $bdq,V \times Sb$, that is, $BDq,V \,.\, bdq,V :: Sb \,.\, SB :: \frac{1}{SB} \,.\, \frac{1}{Sb}$, and therefore $BD,V \,.\, bd,V :: Sb|^{\frac{1}{2}} \,.\, SB|^{\frac{1}{2}} :: \frac{1}{SB|^{\frac{1}{2}}} \,.\, \frac{1}{Sb|^{\frac{1}{2}}}$. Or, $\sqrt{Sb} \,.\, \sqrt{SB}$, or $\frac{1}{\sqrt{SB}} \,.\, \frac{1}{\sqrt{Sb}}$.

Sir Iſ. Newt. Princip. Edit. 1.

Lastly, It is Infer'd, that all theſe Corollaries Follow in Reſpect of the Periodick Times, the Velocities, and the Forces, with which Bodies Deſcribe Similar Parts of any Similar Figures, and which have Centers in a Similar Poſition, if the Demonſtration of what foregoes is Apply'd to theſe Caſes, and the Application is made by Subſtituting an Equable Deſcription of Areas for an Equable Motion, and the Diſtances of Bodies from the Centers for Radius's.

Schol. Propoſ. 4.

We are farther in the Scholium of this Propoſition told, that the Caſe of the Sixth Corollary obtains in the Heavenly Bodies, according to Sir *Chriſtopher Wren*, Dr. *Hook*, and Dr. *Halley*.

Moreover that, by the Aſſiſtance of this Demonſtration, we may Collect the Proportion of the Centripetal Force to any Force, which is known, as that of Gravity. For if a Body is Revolv'd by the Force of it's Gravity in a Circle Concentrical to the Earth, this Gravity is it's Centripetal Force, but from the Deſcent of Heavy Bodies the Time of one Revolution is Given, and an Arc Deſcrib'd in any Given Time is had by an Inference, which is Deriv'd from this Propoſition, Namely, that the Arc, which a Body Deſcribes in any Given Time, whilſt it Revolves Uniformly in a Circle with a Centripetal Force Given, is a Middle Proportional betwixt the

Diameter of the Circle, and the Deſcent of the Body perform'd with the ſame Given Force, and in the ſame Time, that is, ſince $DC \,.\, dc :: \frac{BDq}{SB} \,.\, \frac{bdq}{Sb} :: \frac{BDq}{2SB} \,.\, \frac{bdq}{2Sb}$, therefore $2SB \,.\, BD :: BD \,.\, DC$, or $DC = \frac{BDq}{2SB}$, and Conſequently, DC is to the Force of Gravity as $\frac{BDq}{2SB}$ is to the Space, which an Heavy Body Deſcribes by it's Fall in the ſame Time.

We are alſo Inform'd from the Preſent Scholium, that this Propoſition may be Demonſtrated another Way; Let a Polygone of what Number of Sides ſoever be Deſcrib'd in a Circle, and if the Body, by Moving with a Given Velocity in the Sides of the Polygone, ſhou'd be Reflected from the Circle at each of the Angles; The Force, with which it Strikes upon the Circle in each Reflection, will be as it's Velocity, and therefore the Sum of the Forces in a Given Time will be as that Velocity, and the Number of Reflections conjunctly, that is, (if any Polygone is Particularly Given) as the Length Deſcrib'd in that Time, and the ſame Length Divided by the Radius of the Circle, or as the Square of that Length Divided by the Radius, and conſequently, if the Sides of the Polygone are Diminiſh'd to Infinity, ſo as to Coincide with the Circle, as the Square of the Arc Deſcrib'd in a Time Given Divided by the Radius; And this is ſaid to be the Force, with which a Body Urges or Preſſes upon the Circle, and to this is Equal the Contrary Force, by which a Circle conſtantly Repells the Body towards the Center. There is likewiſe another Demonſtration of this Propoſition in the Second Edition of theſe *Principles*, which is Collected and Form'd from *Prop.* 2, and *Corol.* 2. *Prop.* 1, as alſo from *Corol.* 4, of the ſame *Prop.* and from *Lem.* 7, all which have been ſeverally Conſider'd.

The Velocities are ſuppos'd to be as the Arcs they Deſcribe, which afterwards is Contrary to the Corollaries.

§. 10. This being the Chief Propoſition, on which the Revolutions of the Heavenly Bodies Depend, as Explain'd by this Philoſophy, we ſhall be the more Minute in the Examination of it; It is therefore Firſt ſuppos'd, that Two Bodies Move in Different Circles, ſo that the Arcs, *BD*, *bd*, ſhou'd be Perform'd together, and that thoſe Arcs are as the Velocities, with which they are Deſcrib'd, which we ſhall afterwards conſider, And ſhall only ſay at Preſent, that as the Arcs *BD*, *bd* are Perform'd at once, according to the Propoſition, their Circles, which they Repreſent, ſhould be ſo likewiſe, whereas all the foregoing Proportions in the Corollaries ſuppoſe the Periodical Times Different, Contrary to the Propoſition, which ſuppoſes Them the ſame.

Projectile Forces in Tangents are likewiſe here ſuppos'd, but not Prov'd.

§. 11. The Second Thing Preſum'd is, that, by a Sole *Innate* or Projectile Force, the Bodies, *B*, *b*, wou'd Deſcribe the Tangents *BC*, *bc*, and therefore the Centripetal Forces are thoſe, which perpetually Draw the Bodies from the Tangents. In Anſwer to which, it is Alledg'd, that we have already Inquir'd into the Projectile Force, which from whence it ſhou'd be deriv'd into a Paſſive Matter, as we ſaid, we could not Imagine, unleſs from Almighty God himſelf, or from the Will and Pleaſure of the Philoſopher, both which were Conſider'd, but now it ſeems by one word only Introduced of *Innate*, the whole Difficulty is ſolv'd, if that is meant by *Vis Inſita*, for there is no other Senſe Remaining of it; If ſo, how is it poſſible to Conceive, that Matter, which is abſolutely *Paſſive*, ſhould be *Naturally* Endued with a Projectile, or indeed with any Force?

Sir I. Newt. Princip. Def. 3.

'Tis true the *Vis Inſita* of Matter is Defin'd to be the Power of Reſiſtance, by which every Body, as much as in it lies, Perſiſts in it's State of Quieſcence, or Motion Uniformly *in Directum*; Therefore the *Vis Inſita* at the moſt is only the Perſeverance of a Body in that State, in which it is; But the Queſtion here is, how that State of Motion or Reſt is Produced in Unactive Matter, which is not Accounted for; And in General it may be ſaid, that the Perſeverance in a State cannot be *Innate* to Matter, unleſs upon a Suppoſition of Reſiſtance or Action in it, and which is Affirm'd in the Explanation of the Preſent Definition to be the Proper Inertia of it, by which it is ſaid, it comes to paſs, that every Body is difficulty Diſturb'd from the State in which it is; which is ſomething Singular indeed, that a bare Inertia of Matter, which ſuppoſes no Action at all, and that it is Intirely made up of Solidity, Extenſion, Diviſibility, and ſuch kind of Properties, which Include not the leaſt Activity in them, ſhou'd be Endu'd with a Poſitive Action, or a Poſitive Reſt, and in both Caſes with a Reſiſtance to the being Diſturb'd from the State, in which it is; Tho' the Perſeverance of each State does not here properly come into the Account, but how that State is Introduced upon it, which cannot be any Ways conceiv'd, how it ſhou'd be, if Matter is only Solid, Extended, Diviſible, Figur'd, and in General conſiſts of ſuch Qualities, which neither Have, nor can Infer, any Action in Them; Beſides which, if Perſeverance is a Reſiſtance to the being Diſturb'd, there is, as we ſaid, an Action in the Reſiſtance, that is, in the Inertia; or Matter is ſuppos'd to be Active and Unactive at the ſame Time.

Supposing yet this Projectile Force, it is again Imagin'd, that it proceeds in the Tangents, *BC*, *bc*, but it is not from thence Evident, that it will proceed in a Second Tangent Drawn from *D* and *d*; For if the Centripetal Force is Capable of Subdueing the Projectile, ſo as to Draw it to *D* and *d*, how the Projectile Force ſhou'd again Conquer and Subdue the Centripetal, ſo as to make the Projectile proceed in a Second Tangent, is Inconceivable; And the ſame may be ſaid of a Third, or a Fourth Tangent *in Infinitum*.

The Centripetal Forces, and the Proportions of them not Prov'd.

§. 12. The Third Thing Preſum'd is, that the Centripetal Forces are to one another in the Firſt Reaſon of the Naſcents, *CD*, *cd*, and that they Tend to the Centers of the Circles, by the Second Propoſition, becauſe the Areas Deſcrib'd by the Radius's are ſuppos'd Proportional to the Times; As to the Naſcent and Evaneſcent Reaſons, they have been already Conſidered and Refuted, and that *CD*, *cd* Tend to the Centers of the Circles is not Evident from the Second Propoſition, Firſt, becauſe that Propoſition is not Prov'd as we have before Obſerv'd, and in the next Place, becauſe the ſame Propoſition only Alledges, that Every Body, which, when it is Mov'd in any Curve Line and the Radius is Drawn to a Certain Point, Deſcribes Areas about that Point Proportional to the Times, is Urg'd by a Centripetal Force Tending to that Point; Whereas the Centripetal Forces *CD*, *cd* Tend to no ſuch Point, but Act in Directions Parallel to *BS*; 'Tis true *C*, when in *D*, Approaches nearer to *S* the Center, But it is not by a Force, which is Centripetal, for then it wou'd be Urg'd from *C* to *S*, but by an Imaginary one Aſſum'd by the Philoſopher, which Preſſes Side-ways and Parallel, becauſe it beſt ſuits the Preſent Scheme and Reaſoning.

The Similar Triangles aſſumed in this Demonſtration not Evinc'd.

§. 13. After this it is Deſired, that *tkb* ſhould be Similar to *DCB*, and then by Lemma the 5th and 11th the Preſent Propoſition is Concluded.

As to what Reſpects thoſe Lemmas, they have been before Conſider'd and Examin'd, in the *Method* of *Naſcent* and *Evaneſcent Reaſons*; and as to Similar Figures, where the Curves are Different, which enter into their Compoſition, it is a *Poſtulatum*, which it is Impoſſible to Grant, ſince, altho' the Angle *bkt* ſhould be Equal to the Angle *BCD*, yet *kbt* will not be Equal to *CBD*, unleſs we can ſuppoſe, that *BD*, Drawn by the Radius *SB* from the Point *b*, has the ſame Curvity and Direction with *bd*, Deſcrib'd by the Radius *Sb* from the ſame Point *b*, which is Contrary to Common Senſe and Experience.

On the other Hand, it is not Prov'd, that *kt*, falling upon any Part of the Circle *bt*, will make an Angle Equal to that, which is Comprehended betwixt *CD* and *BD*, for where the Curvities of Unequal Circles differ in every Point, it is not Imaginable, nor can it be Conceiv'd, how Lines Drawn Parallel to the Radius's of thoſe Circles ſhoud ever make Equal Angles. If yet it is ſaid, that *BD*, *bt*, are to be Conſider'd as Right Lines in the laſt Reſult, then *BD* and *bt* will Coincide with *BC* and *bk*, and Conſequently there will be no Triangles, and therefore no Similar ones.

The Firſt Corollary Examin'd, and the Proportion it aſſerts Refuted, and why the Moments, with which all Concentrical Circles or Similar Arcs are Deſcrib'd, are Equal, and Leſſer Circles are more Difficulty Deſcrib'd than Larger.

§. 14. We ſhall now Grant theſe ſeveral Preſumptions, and that $DC . dc :: \frac{BDq}{SB} . \frac{bdq}{Sb}$, The Corollaries yet, which are Drawn from hence, will deſerve to be Conſider'd.

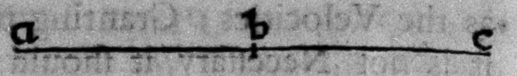
The Firſt of which is, that the Centripetal Forces are as the Squares of the Velocities, divided by the Radius's of the Circles, that is, $DC . dc :: \frac{BDq,V}{SB} . \frac{bdq,V}{Sb}$, In which Caſe, the Velocities are ſuppos'd to be as the Arcs, which are Deſcrib'd; And in General this Doctrine takes it for Granted, and makes it a Fundamental Maxim to it's Reaſonings, that Every Space, which is Deſcrib'd, is Compounded of the Time and the Velocity, which are us'd in it's Deſcription, and therefore, the Times in the Preſent Caſe being Equal, the Spaces Deſcrib'd, or the Arcs, will be as the Velocities; Granting therefore this to be ſo for the Preſent in Right Lines, it is not Neceſſary it ſhould be ſo in Curve ones, at Leaſt according to this Philoſophy; Since BD, bd, tho' Deſcrib'd in Equal Times, may not be as the Velocities, becauſe the Centripetal Forces CD, cd have their Effect, which will Interrupt, and Deſtroy that Proportion ſo far, that if the Times are the ſame, and the Spaces BD and bd Equal, yet the Velocities may not be; becauſe, where the Centripetal Force is Greater in CD than in cd, it may Require a Greater Velocity to Perform an Equal Arc in BD than in bd, tho' the Times are Equal; for it muſt want a Greater Force to turn a Point, into a Greater Curvity BD than into a Leſs bd, from it's Rectilineal Motion in BC, bc, becauſe D Declines more from C than d from c, or, which is the ſame, the Centripetal Force is Greater in D than in d, and therefore will demand Proportionably a Greater Degree of Velocity or Force to Actuate and Impell it in the Circle.

THIS is the more Probable, becauſe from hence the Moments, with which all Concentrical Circles are Deſcrib'd, will become Equal, as in all Reaſon and Senſe they ought to be; for how Abſurd is it to Think, that Motions, which begin and end at once, and are carry'd on in all the Intermediate Parts Uniformly, ſhould not be Generated by Forces, which are Æquipollent? Which if ſo, the Arcs will not be as the Velocities, but in a Compound Reaſon of the Velocities, and the Centripetal Forces, and ſuppoſing BC, bc, to Repreſent the Velocities, and CD, cd the Centripetal Forces, $BD \,.\, bd :: BC \times CD \,.\, bc \times cd$; and in Reſpect of the Velocities and Centripetal Forces in Similar Arcs, BD, bt, it will be $BC.bk::kt.CD$, ſuppoſing bk the Velocity, and kt the Centripetal Force of bt; So that if we take BD, bd, as Two Equal Arcs, both the Arcs themſelves, and every Point, which Forms them, will be of very Different Natures, and not Equal in any Particular, excepting that perhaps they are Meaſur'd by the ſame Aperture of the Compaſſes; ſince every Imaginable Equal Portion of the Arcs BD, bd, and the Arcs themſelves, conſiſt of Velocities and Centripetal Forces, which entirely Differ from each other, and we may as well ſay, that an Inch Line is Equal to an Inch Square or Cube, becauſe they ſeem to be ſo in Extent, or by the Meaſure of the Compaſſes, as that BD is Equal to bd for that Reaſon; For by what foregoes they are Different in their Nature and Conſtitution, and as much Slower, as the Velocity in BD is, than that in bd, ſo much is it the more Compact and Stronger, and as much Quicker, as the Velocity in bd is, than that in BD, ſo much is it the more Expanded and Dilated, or Weaken'd; that is, the Points Impell'd in BD are Heavier, and in bd Lighter, the Centripetal Force in the One being Greater than that in the Other: And this is ſaid in Relation to the Deſcription of Circles themſelves, Abſtracted from and without Conſidering Bodies Revolv'd in them, which may poſſibly be the Reaſon, why Leſſer Circles are Deſcrib'd with Greater Difficulty than Larger from the ſame Center.

A new kind of Force or Velocity Propos'd even in Right Lines, which may be Different in the ſame Spaces, the Time being the ſame, and the ſame in Different ones, and is Agreeable to and Confirm'd by Nature.

§. 15. AND This brings me to ſpeak of a Certain Kind of Velocity, even in Right Lines, which this Philoſophy has not ſo much as thought of, and which may be Equally Preſum'd, as well as the Preſent, which Depends upon it's being in a Reciprocal Proportion of the Times, in which it is Perform'd, ſuppoſing the Space the ſame, in which it Acts; that is, a Simple Velocity is Deſcrib'd in a Certain and Determin'd Time, a Double Velocity in Half that Time, a Triple in a Third Part of it, *&c.* On the Contrary, Half ſuch a Velocity will be in Double the Time, and a Third Part in a Triple of it; whereas from the *Geometry* of *Solids*, it is Evident, that the Proportions of Velocity or Force may be Infinite in the ſame Time, and the ſame Space, and we may add on the Contrary, that the Proportions of Time and Space may be Infinite, where there is the ſame Force or Velocity: That is, there may be the Same Line Deſcrib'd, which, tho' in Length it is the Same, ſhall be Capable of Different Values, Quantities, or Eſtimations, and Different Lines Deſcrib'd, which, tho' Different in Length, ſhall be Capable of the Same Values or Denominations, that is, ſuppoſing the Lengths of the Lines to Expreſs the Times, and the Values or Denominations of them the Velocities.

Thus it will not be deny'd, but that the Line *ab* may be twice Deſcrib'd in the Time Expreſs'd by *ab*, whilſt the ſame Line *ab* may be only once Deſcrib'd by a Different and a Slower Velocity; if therefore what is Deſcrib'd in Length is Confer'd upon the Force, that is, if there is a Double Force in every Part of *ab*, where it would have Deſcrib'd a Double Line for a Single one, in every Part of it, where it would only with a Single Force have Deſcrib'd a Single one, it will be the ſame, as the Repreſenting to us Different Actions or Forces in the ſame Points of Space, and in the ſame Time; the Eaſineſs of which Conception is Manifeſt, from Preſſing a Finger upon a Table with Different Degrees of Force, or from the Action of Fire, and of Electricity, or the Magnet, or any other Action, Perform'd in the ſame Time and Space, which yet are Different in their Natures and Activity; The ſame may be ſaid of Gravity, for if we take a Globe of Lead, and another of Cork, and ſet Them upon a Plane, Nothing can be more Evident, than that the Gravitating Action of the One, and of the Other, is Different in the ſame Time, and in the ſame Space.



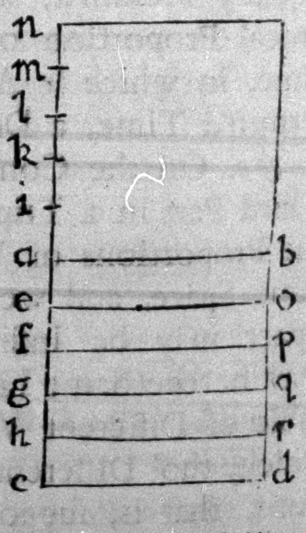
On the other Hand, as there may be Lines Deſcrib'd, which are in Length and Time the ſame, tho' Different in their Values or Denominations, ſo may there be Lines, which are Different in Length, Deſcrib'd in the ſame Time, and yet their Values the ſame; For if *ab* may be Deſcrib'd Twice in the ſame Time by a Double Velocity, whilſt the ſame Line is Deſcrib'd Once by a Single one, and the Double Velocity may be Confer'd upon a Double Force in each Point of *ab*; For the ſame Reaſon the Double Force Compreſs'd in *ab* will be Equal to the Double Velocity, which repeated *ab* Twice, and which is Equal to *ab*, *bc*; All the Difference, which will be, is, that the Action or Force is Compreſs'd in the one Caſe into *ab*, and Diffus'd or Dilated in the other, where it Scatters or Diſperſes it's ſelf, as far as *c*; Notwithſtanding which, *ab* in ſuch a Circumſtance will be Equal to *ac*; and thus it is in the Gravitating Force, where Gold is to Water, as 19 to 1, if therefore we take 19 Times as much Water as Gold, their Gravitating Forces will be Equal, tho' their Spaces are Different, and as 19 to 1.

And this is in a great Meaſure what we Intended in the *Geometry* of *Solids*, or of *Matter*, in which we Endeavour'd to Conform Mathematicks to Nature, and not to the Arbitrary Abſtractions of the Mind, Deriv'd originally from the Italick Sect of Philoſophers, who made all Things to conſiſt in Numbers and Abſtractions, and from the Notions in Logicks of Quantity and Extenſion, and from thence Spread into all the *Elements* of *Geometry*, as well as into all the *Syſtems* of *Philoſophy*, which have been Propos'd to us from the Times of *Pythagoras* the *Geometer*, and *Ariſtotle* the Great Inventor of *Logicks*, to the Preſent Age we Live in.

A new Generation of Mathematick Quantities Conformable to the above ſaid Notions, and Correſponding with Nature, as it Really Exiſts.

§. 16. And that the Generation of Mathematick Quantities, by Drawing a Cube into a Line, *&c.* and from thence Forming both Lines, Squares, and Cubes, Each of them of Different Dimenſions from what are vulgarly Receiv'd, is Agreeable to this Reaſoning we have already ſhewn in the ſame *Geometry* of *Solids*, or of *Matter*, and ſhall here farther Explain.

Let *ab* be one Conſtant and Determined Line, which is Produced by a Certain and Fixt Degree of Velocity; In the flowing of the Point from *a* to *b*, the only Reaſon of that Line's being Deſcrib'd is, that *a* paſſes thro' every Point of it to *b*, let the Velocity be what it will: From which Generation of *ab* it muſt be Acknowledged, that wherever a Point, as *a*, paſſes, it Produces Length, and for the ſame Reaſon, wherever a Line paſſes Parallel to it's ſelf, it Produces Breadth, and wherever a Surface or Breadth paſſes Parallel to it's ſelf, it Produces Space or Depth; The Queſtion is, whether a Cube carry'd Parallel to it's ſelf, according to the Direction of the ſame Line, may not Produce a Cube of a Different Value, Depth, or Space, which we Affirm it may.



Thus let *abcd* Repreſent the Side of a Cube, it is Evident, that if *ab* was Conſider'd as the Side of a Square, (which Square ſhou'd follow the Motion of it's Side *ab*, and be Perpendicular to the Plane *nb*) and ſhou'd paſs thro' the ſeveral Diſtinctions in an, from a to i, k, l, m, and n, ſuch a Motion of the Square of ab would Produce the Common Cube or Solid of the Mathematicians; As therefore ab before Repreſented the Square of it, Let eo, fp, gq, hr, cd Repreſent the Squares of which they are the Sides, it is Manifeſt, if $abcd$ is now Mov'd upwards from a to n, at the ſame Time that ab has paſſed thro' the ſeveral Points of an, and conſequently has Deſcrib'd a Cube, eo will have paſſed thro' all the ſame Points, excepting thoſe which lie betwixt m and n, and conſequently will have Deſcribed another Cube, excepting what Remains to be Accompliſhed betwixt m and n, if eo therefore Riſes to n, it will have Perform'd a Second Cube, as ab had done before, and by the ſame Argument fp, gq, hr, cd, will Perform or Deſcribe a Third, Fourth, Fifth, or Sixth Cube, when they arrive Reſpectively at n; From whence it will follow, that the Cube $abcd$ multiply'd into the Line an will be a Cube, whoſe Every Square after ſuch a Multiplication will be as much a Cube, as Every Parallel Superficies of a Common Cube is a Square, and by the ſame Methods of Generation: And the like Kind of Reaſoning obtains in the Multiplication of this Second Cube into the ſame Line an, *&c.*

Nor can it be oppos'd to this Generation of Mathematick Solids, to the *4th*, *5th*, *6th*, or *100th* Dimenſion, or to Infinity, that it is only Deſcribing the ſame Space over-again, without any conſequent Effect, ſince, if the Line ab in it's Motion, according to the Direction of an, Produces a real Square, which is Conſtant in that Space, in which it is Produced, for the ſame Reaſon the Lines eo, fp, gq, &c. will; that is, if the one has an Effect, which is Conſtant, the other will, or elſe they will all Equally have none, and the Line ab Multiply'd into an will not ſo much as Produce a Square.

We ſhall only therefore farther obſerve upon this kind of Geneſis, that the Reaſon of the foregoing Objection ſeems to lie in this, that Men have already form'd in their Minds their Idea of Length and Breadth, or of Space in General, as actually Exiſting in Nature, and which becomes Unvariable, whatever paſſes thro' it; Thus, Albeit Ten Thouſand Waves ſhould ſucceed each other thro' the ſame Space, the Space will ſtill remain Unalter'd, and conſequently it is thought, if a Million of Squares are carry'd Parallel to themſelves thro' the ſame Cube, or ſo many Lines thro' the ſame Square, both the Square and the Cube will be ſtill Conſtant and Unvary'd.

But if we Examine truly into this Sort of Arguing, the Caſe will appear to be Extreamly Different; for in the Firſt Inſtance, there is a ſtanding Meaſure of Space originally Fixed in the Mind, and which will be Immutable, whatever paſſes thro' it, but in the Second, namely, in the Generation of Mathematick Quantities, there is nothing ſuppoſed before the Production of thoſe Quantities, not ſo much as Length or Breadth; and conſequently, ſince Length is Produced, where there was no Length before, and Breadth, where there was no Breadth, *&c.* for the ſame Reaſon there may be more Dimenſions than Three Produced in the ſame Abſtracted Space, which were not there before, namely, by ſuch Actual Productions as are already Deſcrib'd; For Ten Dimenſions may as well Exiſt together as One, ſince they all of them Exiſt in nothing but Themſelves, that is, Five, Six, or Ten Dimenſions do not Exiſt in Three Dimenſions, no more than Three in Two, but they Exiſt in Themſelves, where Nothing of Them before Exiſted.

The Corollaries from this Doctrine.

§. 17. *Corol.* 1. From hence it follows, That as a Drawn into ac Produces a Line, ſo ca on the Contrary, Drawn into the Point a, ſo as Every Point of it ſhould paſs Succeſſively thro' a, will Produce in it a Quantity Equal to a Line.

Corol. 2. Likewise, if ca is Drawn Succeſſively thro' each Point in an, namely, a, i, k, l, m, n, and thoſe Points are Infinite, there will be a Line Produced in each Point, and conſequently, a Square Produced in the Line.

Corol. 3. Also, if ab is carry'd Parallel to it's ſelf, According to the Points a, i, k, l, m, n, it will Produce a Square, and if eo, fp, gq, hr, cd, are Succeſſively made to paſs thro' the ſame Points, and Parallel to Themſelves, they will Produce a Cube within the ſame Limits of the Square bn.

Corol. 4. Therefore every Point in ſuch a Cube will be Equal to a Line, every Line Equal to a Square, and every Square Equal to a Common Cube or Solid, and the Cube will be a Dimenſion of the Fourth Power.

Corol. 5. If, after this Cube is Produced, it is again multiply'd into the ſame Line an, ſince ab, which was a Line before, is now a Square, if it is Drawn into an, the

Square *bn* will be Equal to a Cube; And ſince *eo*, *fp*, *gq*, *hr*, *cd* are Succeſſively drawn into the ſame Line *an*, the Square *bn* will be Equal to the Dimenſion of the Fourth Power, and the Cube will be of the Fifth, in which Every Point will be Equal to a Square, Every Line Equal to a Common Mathematick Solid, and Every Square Equal to a Dimenſion of the Fourth Power, *&c.*

Corol. 6. FROM which it follows in General, according to what we have Expreſs'd in the *Geometry* of *Solids*, *Prop.* 1. *Corol.* 9. *Par.* 1, that m^3, m^4, m^5, m^6, m^7, &c. *in Infinitum* may be Each of them Compreſſed into a Cube, which is the ſame, as to Length, Breadth, and Depth, according to the Ordinary and Common Abſtractions.

WE have only however propos'd this, as a Way of Generating Quantities, which are Different in their Dimenſions, whilſt the Abſtracted Spaces continue the ſame; but that there are in Fact ſuch Quantities conſidered in Mathematicks, is Evident from the Lemma Prefixed to the *Geometry* of *Solids*, and from *Chap.* 4. *§.* 11. of the *Principles* of *Natural Philoſophy*, and Dr. *Wallis*'s *Arithmetick* of *Infinites* there Cited, which is the Foundation of the Preſent Celebrated *Doctrine* of the *Infinite Series*.

FROM whence, inſtead of the *Elements* of *Geometry*, which we poſſeſs, a Thouſand and a Million of *Euclids* might be Form'd, and the *Pomœria* of *Mathematicks* Enlarged without Bounds, and made as Univerſal and Comprehenſive as *Algebra* it ſelf. And that ſuch a Generation of Quantities is Agreeable to Nature is Plain, in which we find an Immenſe Number of Various and Different Bodies Exiſting in the ſame Space, as Gold, Lead, Marble, Wood, Water, Air, Fire, Light, and Æther, and the ſeveral other Infinite Species and Kinds of Bodies, which lie Intermediate to them, whoſe Different Conſtipations of Action, or whoſe Expanſive and Contractive Forces, may be Repreſented by theſe Different Extenſions, which can never be Reduced to an Unvary'd and Abſtracted one of Mere Length, Breadth, and Thickneſs: If therefore the Preſent Philoſophy is miſtaken in it's Notions of Velocities being always Reciprocal to the Times, and in it's Conception of Space, it's whole Reaſoning in This and the Subſequent Corollaries is Falſe, which ſuppoſes the Common and Vulgar Abſtractions of both.

The Second Corollary of the Preceding Propoſition Examin'd.

§. 18. THE Second Corollary from this Propoſition is, That the Centripetal Forces are Reciprocally as the Squares of the Periodical Times Divided by the Radius's; which Depends upon this Propoſition, that the Periodical Times are in a Reaſon Compounded of a Reaſon of the Radius's Directly, and of a Reaſon of the Velocities Inverſely, that is, $BD, T \,.\, bd, T :: \frac{SB}{BD, V} \,.\, \frac{Sb}{bd, V}$, and therefore, ſince it Aſſumes the Receiv'd Abſtractions of Velocity and Space for certain Maxims, is liable to the ſame Confutation.

The Third, Fourth, Fifth, and Sixth Corollaries.

§. 19. As the Third Corollary entirely Reſts upon what foregoes, ſo there is no Occaſion for any Conviction of the Untruth of it, and as the Fourth, Fifth, and Sixth Corollaries Inſiſt upon the Proportions of the Third, and are only Affirmations of the ſame Thing, by Multiplying, Dividing, Inverting, and Alternation, according to the Common and Fixed Rules in Mathematick Proportions, it will be unneceſſary to Enter into a more Critical Examination of them.

Objections againſt the Sixth Corollary obtaining in the Heavenly Bodies.

§. 20. AFTER a particular Inquiry into this Propoſition and the Corollaries of it, we ſhall conſider what is offered to us in the Scholium it ſubjoins, Namely, that the Sixth Corollary obtains in the Revolutions of the Heavenly Bodies, even according to what our own Country Men, as it is ſaid, have Collected a-part from each other, to wit, Sir *Chriſtopher Wren*, Dr. *Hook*, and Dr. *Halley*, In Anſwer to which we ſhall ſay;

FIRST, That the Proportion of this Corollary is not Prov'd, unleſs upon Abſtractions, which are Arbitrary, Uncertain, or Falſe.

SECOND, When this is Apply'd to the Planets, they are Conſider'd or Imagin'd to Conſiſt of a like Homogeneous Matter, whoſe Weight is Proportional to it's Quantity, which Notion has been already Examin'd into, whereas by the Nature of all the Matter, which Surrounds us, it is more Reaſonable to think it is Various and has Different Forces of Gravitation Peculiar to it, and conſequently, that Each Planet has, beſides what is Deriv'd from the Proportion of it's Greater or Leſs Diſtance from the Center.

THIRD, This Reaſoning is ſaid to be Confirm'd by the Obſervations made in the Heavens by Aſtronomers.

How it is Confirm'd by the Obſervations made in the Heavens by Aſtronomers.
Sir Iſ. Newt. Princ. Lib. 3. Phænom. 1, 2. Edit. 2.
Dr. Gregory's Aſtronom. Prop. 21. §. 4. Lib. 1.

§. 21. SINCE Firſt, the Motion of the Secondary Planets Revolving about any Primary one is ſo Adjuſted, that the Squares of the Periodick Times are in the ſame Reaſon with the Cubes of the Diſtances of the ſaid Planets from the Center of the Primary.

THUS in the Satellites of *Jupiter*, where the Periodick Times of the Innermoſt, the Second, the Third, and the Outermoſt, are Reſpectively, as $1\frac{3}{4}$, $3\frac{3}{5}$, $7\frac{1}{8}$, and $16\frac{3}{4}$ nearly, and their Diſtances, as $5\frac{2}{3}$, 9, $14\frac{1}{3}$, and 25; The Square of the Periodick Time of the Innermoſt, to wit 3, is to 13 the Square of the Periodick Time of the Second Satelles, as 170 the Cube of the Diſtance of the Innermoſt, to 736 the Cube of the Diſtance of the Second from the ſame Center.

IN like manner 3 is to 51 the Square of the Periodick Time of the Third Satelles, as 170 to 2890 the Cube of the Diſtance of this Third from the ſame Center of *Jupiter*.

ALSO 3 is to 280 the Square of the Periodick Time of the Outmoſt Satelles, as 170 to 15800 the Cube of the Diſtance of the ſame Outmoſt Satelles; and *ex Æquo* the ſame Reaſon obtains, if any Two other Satellites are Compared with One Another, that is, the Second with the Third, or with the Outermoſt, or the Third with the Outermoſt; And this Reaſon is ſtill more Exact, if the Diſtances and Periodick Times are taken more Accurately.

LASTLY, it is ſaid the ſame happens in the Satellites of *Saturn*, if thoſe Numbers are Aſſumed, which are Expreſs'd in the 15th *Prop.* of this Aſtronomy.

Anſwer to the foregoing Obſervations.

§. 22. FIRST, If therefore we Allow theſe Obſervations to be True, which yet may be Conteſted, as we have ſhewn in another Place, it will only follow, ſince the Principles before are Prov'd to be Falſe, that this Doctrine is no Other than an Ingenious Contrivance of the Mind to ſolve Phænomenons, which were already known, and that theſe Phænomenons are not deriv'd from a Strict, and Mathematick Reaſoning, but that an Hypotheſis is Invented and Pretended to be Mathematick by the Lines and Figures, which it draws, and then Adapted to the Solutions it Propoſes.

SECOND, In Reſpect of the Numbers, thoſe, which are Expreſs'd in the 15th Propoſition, do indeed Agree with thoſe, which are mentioned here excepting in the Diſtance of the Outmoſt Satelles, which is there ſaid to be as $25\frac{1}{3}$, whereas here it is only 25, which will cauſe ſome Difference at Leaſt in the Proportion Aſſign'd upon that Diſtance.

BUT that which is more is, it is not Evident, that the Numbers Alledg'd obſerve the Proportion Requir'd, for the Square of $1\frac{3}{4}$, the Periodical Time of the Inmoſt Satelles is $1+\frac{3}{4}\times 1+\frac{3}{4} =$ to $1+\frac{6}{4}+\frac{9}{16} =$ to $\frac{49}{16} = 3\frac{1}{16}$; Whereas in the Proportion, it is made to be only 3; Likewiſe the Periodick Time of the Second Satelles is $3\frac{3}{5} =$ to $\frac{18}{5}$, and the Square of it $= \frac{324}{25} =$ to $12\frac{24}{25}$, Whereas in the Proportion it is made to be 13; Thus alſo in the Diſtances, $5\frac{2}{3}$ is that of the Innermoſt Satelles, the Cube of which is $\frac{17\times 17\times 17}{3\times 3\times 3} =$ to $\frac{4913}{27} =$ to $181\frac{26}{27}$, which yet the Proportion makes to be 170; The Diſtance of the Second is ſaid to be 9, the Cube of which is $9\times 9\times 9 =$ to 729, which notwithſtanding is in the Proportion Expreſs'd by 736; what therefore can we conclude from ſuch a Reaſoning? Unleſs this, that either $3\frac{1}{16} . 12\frac{24}{25} :: 181\frac{26}{27} . 729$ is the ſame as $3 . 13 :: 170 . 736$, which is Impoſſible, or elſe that the Squares of the Periodick Times are not as the Cubes of the Diſtances, or which is the ſame, that the Periodick Times are not in a Seſquiplicate Reaſon of Them, according to *Corol. 6.*

WE muſt farther Add, that $3 . 13 :: 170 . 736$ is not a Juſt Proportion, unleſs 2208 is Equal to 2210, which are the Products from a Multiplication of 3 into 736, and of 13 into 170.

IT will be unneceſſary to Inquire into the other Proportions having conſidered theſe, ſince they admit of the ſame Reaſoning and Compariſon.

WE muſt likewiſe obſerve, that the Inmoſt Satelles of *Jupiter* is Diſtant from the Center of it's Planet, according to *Caſſini* in his Obſervations thro' a Teleſcope, 5 Semidiameters of *Jupiter* only, the Second Satelles 8, the Third 13, and the Outmoſt

moſt 23; *Townley* and *Flamſtead*, by their Obſervations thro' a Micrometer, do not make the Diſtances much more, but *Borellus* Enlarges them to $5\frac{2}{3}$, $8\frac{2}{3}$, 14, and $24\frac{2}{3}$, in which Obſervations there is the Difference of a whole Semidiameter, that is, according to ſome, of 40000, and according to others, of 25000 Miles at the Leaſt; which is a ſufficient Evidence of the Great Uncertainty of theſe Obſervations, when they neither Agree in the Number of theſe Semidiameters, nor in the Meaſure of the Semidiameters themſelves; However to Aſcertain this Matter, Recourſe is had to Calculations of their Diſtances, from obſerving the Eclipſes of theſe Satellites, and yet here they likewiſe differ, for *Flamſtead* makes them 5.578, 8.876, 14.159, 24.903, and then the Computation of them, from the Periodick Times, fixed them at 5.578, 8.878, 14.168, 24.968, but when this Philoſophy rather choſe to take *Caſſini*'s Obſervations, poſſibly out of ſome Diſguſt to the Former, and when he had Aſſign'd the Diſtances from the Eclipſes of the Satellites to be $5\frac{2}{3}$, 9, $14\frac{23}{60}$, $25\frac{3}{10}$, with Great Facility the Periodick Times were made to Juſtify theſe Obſervations, and accordingly the Diſtances Ariſing from them were found to be 5.667, 9.017, 14.384, 25.299; Let us now therefore take this Laſt Improvement, and tho' the Numbers come pretty near Each other, yet if we conſider, what the Difference of an Unit is, which at the moſt is above 40500, and at the leaſt above 26000 Miles, we ſhall not perhaps ſo readily ſee the Conformity of the Diſtances from the Periodick Times to thoſe which are found by Obſervation; for $5\frac{2}{3} = 5.666$ Subſtracted from 5.667 leaves a Difference of .001 = one Thouſandth Part of 40000 or 26000 Miles, Namely, a Difference of 40 or 26 Miles more according to the Periodick Times than is found by Obſervation, and 17 Times that Number of Miles, that is, 680 or 442 more in the one Caſe than the other in the Diſtance of the Second Satelles, and 40 or 26 Miles more in the Diſtance of the Third Satelles, and the ſame in the Fourth; And yet this is Argu'd upon the Utmoſt Conceſſions, which can be made or are deſir'd by this Philoſophy; For if we take other Obſervations the Difference will be much Greater.

Sir Iſ. Newt. Princip. Edit. 1.

Sir Iſ. Newt. Princip, Edit. 2.

THE Diſtances of *Saturn*'s Satellites, Computed by the Periodick Times, ſtill leſs Correſpond with thoſe, which are Affirm'd from Obſervation, which, meaſured by Semidiameters of his Ring, are $1\frac{19}{20}$, $2\frac{1}{2}$, $3\frac{1}{2}$, 8, 24; The Diameter of his Ring is the Double Seſquiquartan of the Diameter of *Saturn*, or $\frac{9}{4}$ or $2\frac{1}{4}$ of it, which, according to ſome, Contains 67870 Miles, or, according to others, only 43925; If the Firſt, $2\frac{1}{4}$ of that Number will Amount to 152707 Miles, which alſo is had another Way, namely, by adding to 33935 Miles the Semidiameter of *Saturn*, the Diſtance betwixt the Ring and his Body, and the Breadth of the Ring, which are Equal, but the Breadth of the Ring by Obſervation is 21209 Miles, and $2 \times 21209 = 42418$, and the Sum of the Two Numbers will be 76353, the Semidiameter of the Ring, which Doubled Produces the Diameter 152706, the ſame Number nearly as before; If we now take 43925 Miles for the Diameter of *Saturn* $2\frac{1}{4}$ of it is $= 98831$ Miles, the Diameter of the Ring, on the other Hand, add to the Semidiameter of *Saturn* Twice the Breadth of his Ring, and Double that Number, it ought to Produce 98831 the Diameter of the Ring as before, which it does not, but 128761; Let us however here make uſe of the leaſt Number, and the Semidiameter of the Ring will be 49415, that is near 50000 Miles, and in the Former Caſe 76353 Miles, which will be Equal to Unit in the Preſent Calculations.

Hugen's Coſmotheor. P. 102.
Dr. Gregory's Aſtronom. Prop. 15. Lib. 1.
Mr. Whiſton. Prælect. Aſtronom. Pag. 236.

THE Diſtance of the Firſt or Interior Satelles is not Different by the Periodick Times, from what is found by Obſervation 1.95 being = to $1\frac{19}{20}$, likewiſe in the Second Satelles 2.5 = to $2\frac{1}{2}$, but then in the Third Satelles, the Difference betwixt 3.52 and $3\frac{1}{2}$ is $\frac{2}{100}$, or Two Hundredth Parts of an Unit, which is 1526, or at the Loweſt 988 Miles; In the Fourth Satelles the Difference is ſtill more, for betwixt 8.09 and 8 there are Nine Hundredth Parts, that is, a Difference of 6867, or at leaſt of 4446 Miles.

AND thus far the Exceſs has been on the Side of the Periodick Times, that is, the Diſtances are Greater from Them, than what are Confirm'd from Obſervation, but ſo Uncertain and Fallible are theſe Calculations, that in the Fifth Satelles the Diſtance is leſs by Twenty Nine Hundredth Parts from the Periodick Times, than what is obſerv'd, that is, the Difference betwixt 23.71, and 24 is $\frac{29}{100}$, or 22127

Miles, which is more than the whole Breadth of *Saturn's* Ring, or by the Leaſt Computation 14326 Miles, which is near Twice the Earth's Diameter.

If we now Proceed to the Primary Planets, in the firſt Place the Obſervations of *Bullialdus* and *Kepler* Differ from one another, and thoſe of *Bullialdus*, which are the neareſt to the Computations from the Periodick Times, are yet not Correſpondent to Them.

Sir Iſ. Newt. Princip. Lib. 3. Phænom. 4. Edit. 2.

The Middle Diſtance of the *Earth* from the *Sun* is ſuppoſed to conſiſt of 100000 Parts, that is, according to ſome of 81000000, and according to others of 54000000 of Miles; the Reaſon of this Diſſenſion in Opinions Ariſes from the Difficulty of finding the *Sun's* Parallax, from whence Aſtronomers have Variouſly Reported the *Earth's* Diſtance from it, ſome making it no more than 749 Diameters of the *Earth*, whilſt others will have it to be 10000 or 12000, and the Neweſt and moſt Accurate Obſervations fix it at about 5000; however let us take 81 or 54 Millions of Miles, as before ſet down, 100000th Part of which will therefore be either 810 or 540 Miles = to the Unit in theſe Calculations; by *Kepler's* Obſervation, the Diſtance of *Saturn* from the *Sun* is 951000 of theſe Parts, by *Bullialdus's* 954198, ſo that the Latter has 3198 more than the Former, which Amount to 2590380 Miles in the higheſt Computation, and in the Loweſt to 1726920; and the Periodick Times have 392 Parts leſs than *Bullialdus's* Obſervation, which in the higheſt Computation is 317520 Miles, and in the Loweſt 211680: And in the ſame Way and Method may the Diſtances of *Jupiter*, *Mars*, *Venus*, and *Mercury* from the *Sun* be Examin'd and Compared with thoſe, which are Derived from the Periodick Times, wherein we ſhall find the Difference of ſeveral Thouſand Miles conſidered as Minutiæ, which are to be neglected and ſignify Nothing.

'Tis true, when we come to ſuch Large Units of Diſtance, as ſhall Comprehend ſeveral Hundred Miles in them, we may ſeem in our Arithmetical Figures not to vary much from the Obſervations of Aſtronomers, but if thoſe Units have their True Value Aſſign'd them, the Diſagreement will Appear, and we ſhall not be ſo forward to Think, that Thouſands and Millions of Miles are to be look'd upon as a Cypher, in the Real Diſtances of Bodies one from another, for we are not here ſpeaking of the Apparent Diſtances of Them.

Besides, the Diſtance of the *Earth* from the *Sun* is Unaſcertain'd, by the Parts of which the Diſtances of the other Planets are Computed; and in the Secondary ones, whoſe Diſtances are Meaſured by the Diameters of the Primary, thoſe Diameters are as little Determined.

Then, when we come to the Obſervations of theſe Diſtances, the Method of Taking Them, which we have conſidered in another Place, is Lubricous, Difficult, and Fallible, and the Reſults of Them in Fact, Vary, and Diſagree.

If, after This, we Recur to the Periodick Times, They make the Diſtances ſometimes Equal, at other times Greater, and then Leſs, than what is found by the moſt Accurate Obſervations, to which they ſhould always Correſpond, or elſe be always in the Exceſs, or always in the Defect, ſuppoſing thoſe Obſervations were Juſt and True; and if they are not True, it is not more Certain, that the Periodick Times are Right in fixing the Diſtances, when they Agree with Them, than when they Differ from Them.

So that upon the whole we cannot ſee any Demonſtrative Grounds for us to Acquieſce in this Doctrine; we rather think it from hence Appears, that as the Principles of this Philoſophy, which have been Prov'd to be Falſe, were only Contriv'd to ſolve theſe Phænomenons, ſo on the Contrary, that theſe Phænomenons are Fitted by certain Artificial Adjuſtments to Anſwer, as far as Poſſible, ſuch Looſe and Precarious Principles.

We have alſo ſhewn, that it is more Reaſonable to believe, that the Squares of the Periodick Times are not as the Cubes of the Diſtances, or the Periodick Times as the Roots of thoſe Cubes, but that they are as the Diſtances.

The other Demonſtration of this Propoſition not much Different from the Former.

§. 23. As for the other Demonſtration of this Propoſition from an Inſcrib'd Polygone, it does not ſeem to be much Different from the Former, excepting that what the Tangents Perform in the Firſt Caſe is done by Chords in the Laſt, and as there the Body is Drawn down from the Tangent to the Circumference by the Centripetal Force, and by that means is Continually made to Proceed in a

New Tangent, ſo here the Reflexive Force Directs it in a New Chord, and which Reflexive Force likewiſe Urges the Body towards the Center, and is alike Arbitrary and Unproved.

If the Principles Aſſumed by this Philoſophy are Falſe, tho' they ſhould Explain the Phænomenons of the Heavens, it will not excuſe them from being a Mere Contrivance, and without any Foundation in Nature.

§. 24. We might Proceed to the Remaining Propoſitions of this Second Section, and to thoſe of the Third, and ſhew in what Reſpect the Particular Principles, upon which they are Demonſtrated, are Falſe, or Uncertain, as well as how they are ſo in Depending upon others, which have been hitherto Examined, but that being wholly unneceſſary, which yet we could Eaſily do, we ſhall at preſent Wave that Confutation, and only Add, that if the Phænomenons or Theories in the Heavens are Anſwerable to what is Demonſtrated in theſe Sections, it is Evident, that theſe Propoſitions were Artfully Contrived to Give a Solution of Them, by Aſſuming what Sort of Abſtractions this Philoſophy thought were the beſt Adapted to that End; but if ſuch Abſtractions are Erroneous, and the Mind of the Philoſopher Takes and Rejects what he pleaſes in order to Form his Solutions, This, however Mathematical it may Appear, will be no better an Account of the Revolutions of the Planets than *Deſcartes*'s *Vortices*, or the *Striate Particles* in Reſpect of the Loadſtone; for where, as in the preſent Caſe, our Reaſonings are Adjuſted to ſerve a Purpoſe we have in View, let 'em be Mixt with Lines and Angles, or not, They will ſignify no more to the Explaining of Nature, if they are in Themſelves Arbitrary or Falſe, than if we ſhould Aſſume the Abſtracted Notions of Figure, Matter, and Motion, to do the ſame Feat for us, and ſhould Take upon our-ſelves to Shuffle and Combine them at our Pleaſure; The Abſurdity of which Way of Proceeding we have before ſhewn.

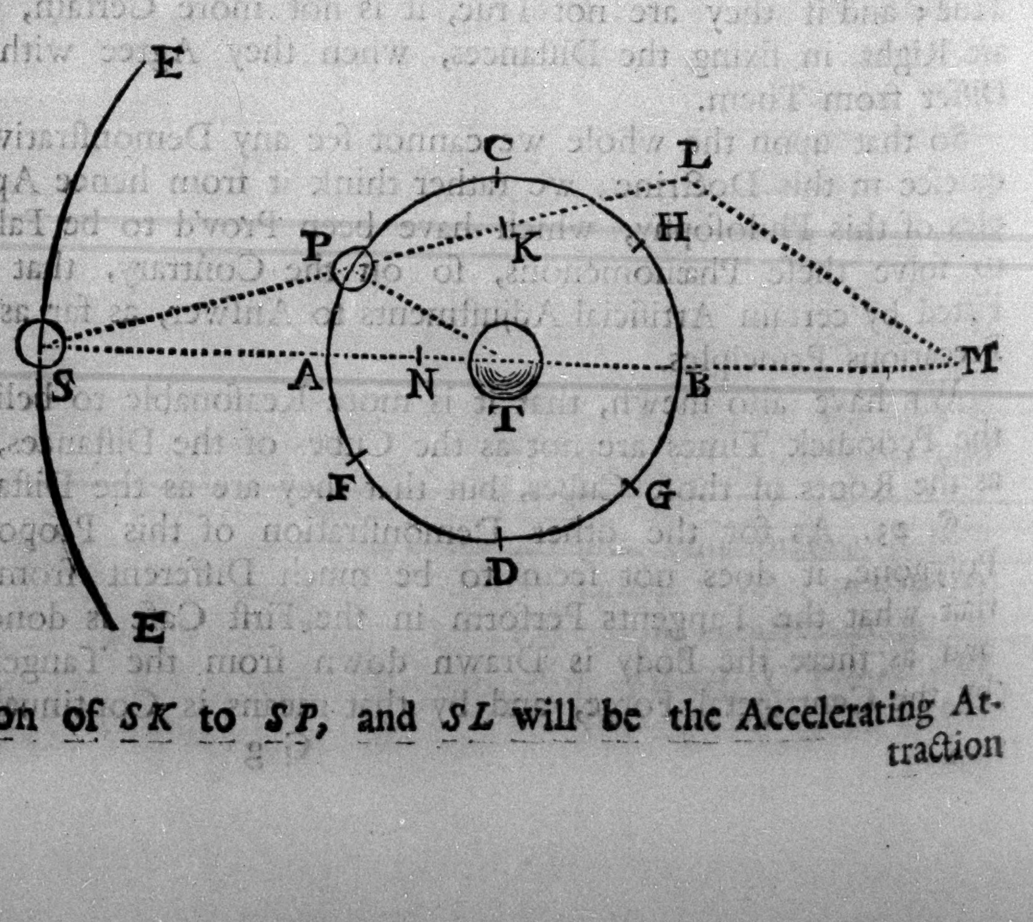
CHAP. V.

Concerning the Theory Propoſed by this Philoſophy for the Solution of the Errors and Irregularities of the Planets.

The Propoſition, upon which the Theory of the Errors of the Planets Depends, Explain'd.
Sir Iſ. Newt. Princip. Prop. 66. §. 11. Lib. 1. Edit. 2. Dr. Gregory's Aſtronom. Lib. 4. §. 1.

§. 1. AFTER having conſidered the General Theory of the Planets, it will be Requiſite to Examine into That, which is brought to Explain the Errors and Irregularities of Them; The Propoſition Alledg'd is, That if Three Bodies, whoſe Forces Decreaſe in a Duplicate Reaſon of their Diſtances, mutually Attract each other, and the Accelerating Attractions of any Two to a Third are to one Another Reciprocally, as the Squares of the Diſtances, and if alſo the Leſſer Bodies are Revolved about the Greateſt; upon theſe Suppoſitions it is Affirm'd and Pronounced, that the Interior Body Revolv'd about the Innermoſt and Greateſt, by Radius's being Drawn to it, will Deſcribe Areas more Proportional to the Times, as well as a Figure more nearly Approaching to the Form of an Ellipſis, which has it's Umbilicus in the Concourſe of the Radius's, if the Greateſt Body is Agitated and Affected by theſe Attractions, than if the ſame Greateſt Body ſhou'd either be Quieſcent, and not Attracted by the Two Leſſer, or being much Leſs or much More Attracted, ſhould be either much Leſs or much More Agitated or Affected.

Caſ. 1. Let the Leſſer Bodies *P* and *S* be Revolved in the ſame Plane about the Greateſt *T*; of which Leſſer Bodies, Let *P* Deſcribe the Interior Orbit *PAB*, and *S* the Exterior *SE*; Let *SK* be the Middle Diſtance of the Bodies *P* and *S*, and let the Accelerating Attraction of the Body *P* towards *S*, in that Middle Diſtance, be Expounded or Set forth by the ſame Line *SK*; Let *SL* be to *SK* in a Duplicate Reaſon of *SK* to *SP*, and *SL* will be the Accelerating Attraction



traction of the Body *P* towards *S* in any Diſtance *SP*; Join *PT*, and Draw the Parallel *LM*, which ſhall Meet *ST* in *M*, and the Attraction *SL* (by *Corol.* 2. of the Laws) will be Reſolv'd into the Attractions *SM*, *LM*, by which means the Body *P* will be Urg'd by a Threefold Accelerating Force.

The Firſt is That, which Tends to *T*, and Ariſes from the Mutual Attraction of the Bodies *T* and *P*; If this Force was only to be Conſidered, the Body *P* would Deſcribe round the Body *T* (whether Unmoved, or Agitated by this Attraction) by the Radius *PT* Areas Proportional to the Times, and an Ellipſis, whoſe Umbilicus is in the Center of the Body *T*, by *Prob.* 6. and *Corol.* 2, 3. *Theor.* 21.

The Second Force, by which the Body *P* is Urg'd, is that of the Attraction *LM*, which, as being Parallel, ſince it Tends from *P* to *T*, if it is Superadded to the Former, will Coincide with it, and by that means will Cauſe, that the Areas ſhould be ſtill Proportional to the Times, by *Corol.* 3. *Theor.* 21. But, becauſe it is not Reciprocally Proportional to the Square of the Diſtance *PT*, it will Compound, together with the Former Force, a Force which Declines from that Proportion, and the more, by how much Greater the Proportion of this Force is to the Former, if other Things are ſuppoſed Equal; and therefore ſince by *Prob.* 6. and *Corol.* 2. *Theor.* 21. the Force, by which an Ellipſis is Deſcribed about the Umbilicus *T*, ought to Tend to that Umbilicus, and to be Reciprocally Proportional to the Square of the Diſtance *PT*, that Compound Force of *PT* and *LM* by Receding from ſuch Proportion will Cauſe, that the Orbit *PAB* ſhould Depart from the Figure of an Ellipſis, which has it's Umbilicus in *T*, and the more, by how much Greater the Deviation is from that Proportion, and therefore, by how much Greater the Proportion of the Second Force *LM* is to the Firſt *PT*, if other Things are Equal.

The Third Force is that of *SM*, which Attracts the Body *P* according to a Line Parallel to *ST*, and will with the other Forces mentioned Compound one, which is no longer Directed from *P* to *T*, and which will ſo far Recede from that Determination, as the Proportion of this Third Force to the Former Forces is Greater, if other Things are Equal, and therefore will Cauſe, that the Body *P* will not by the Radius *TP* Deſcribe Areas Proportional to the Times, and the Deviation from this Proportion will be ſo much the more ſenſible, as the Proportion of this Third Force to the others is Greater; This Third Force alſo will Increaſe the Deviation of the Orbit *PAB* from the aforeſaid Elliptick Form, upon two Accounts, both becauſe it is not Directed from *P* to *T*, and becauſe it is not Proportional to the Square of the Diſtance *PT*; From whence it is Evident, that the Areas are then moſt of all Proportional to the Times, when the Third Force, the other Forces Remaining, is the Leaſt, and that the Orbit *PAB* then Approaches the neareſt to the aforeſaid Elliptick Form, when as well the Second as the Third Force, but eſpecially the Third, is the Leaſt, the Firſt Force Remaining.

Let the Accelerating Attraction of the Body *T* towards *S* be Expreſs'd by the Line *SN*, and if the Accelerating Attractions, *SM*, *SN*, were Equal, Theſe by Drawing or Attracting the Bodies *T* and *P* Equally, and according to Parallel Lines, would make no Change in their Situation to each other, The Motions of thoſe Bodies would be the ſame (by *Corol.* 6. of the Laws) as if theſe Attractions were Abſent or Taken away; And by a like Reaſon, if the Attraction *SN* is Leſs than the Attraction *SM*, it will Take away or Deſtroy the Part *SN* of the Attraction *SM*, and there will only Remain the Part *MN*, by which the Proportionality of the Times, and the Areas, and the Elliptick Form of the Orbit, will be Diſturbed.

Also, If the Attraction *SN* ſhould be Greater than the Attraction *SM*, the only Diſturbance to the Proportionality, and the Orbit, would Ariſe from the Difference *MN*, Thus by the Attraction *SN*, the Third Attraction *SM* is always Reduc'd to the Attraction *MN*, The Firſt and Second Attractions of *PT* and *LM* Remaining Unaltered; And therefore the Areas and the Times do then moſt of all Approach to a Proportionality, and the Orbit *PAB* to the aforeſaid Elliptick Form, when the Attraction *MN* is either None, or as Little as Poſſible, that is, when the Accelerating Attractions of the Bodies *P* and *T*, Made towards *S*, come as near as can be to an Equality, to wit, when the Attraction *SN* is not Nothing, nor yet Leſs than the Leaſt of all the Attractions *SM*, but a kind of Middle Attraction betwixt the

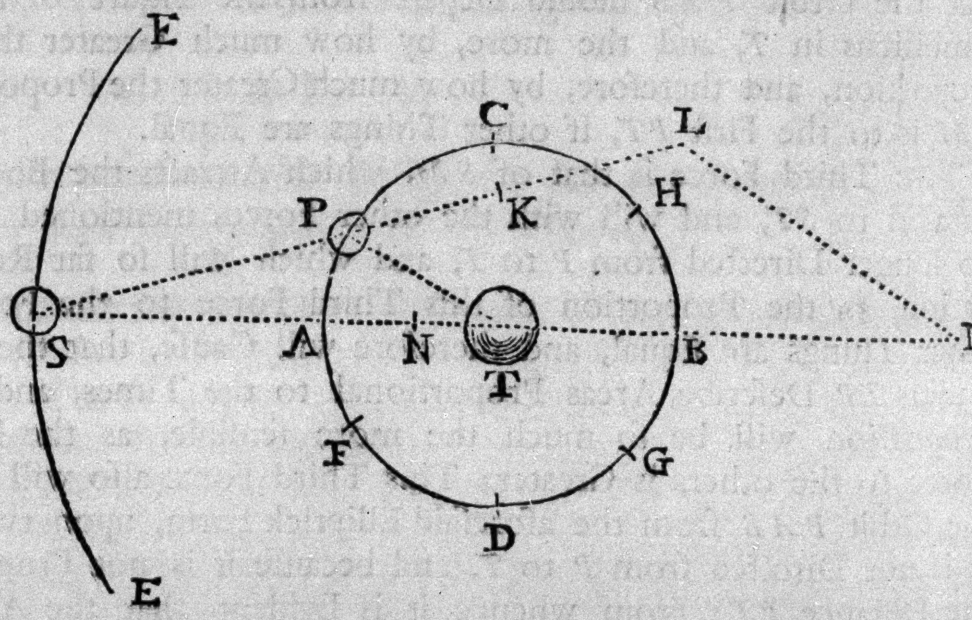
Greateſt and Leaſt of all the Attractions of *SM*, that is, not much Greater nor much Leſs than the Attraction *SK*. *Q. E. D.*

Caſ. 2. Let now the Leſſer Bodies *P*, *S* be Revolved about the Greateſt *T* in Different Planes, and the Force *LM* by Acting according to the Line *PT*, ſituate in the Plane of the Orbit *PAB*, will have the ſame Effect as before, nor will it Wreſt or Turn the Body *P* from the Plane of it's Orbit; but the other Force *NM*, by Acting according to the Line, which is Parallel to *ST* (and therefore, when the Body *S* is Converſant out of the Line of the Nodes, is Inclined to the Plane of the Orbit *PAB*,) beſides the Diſturbance of the Motion in Reſpect of Longitude, which has been already Explain'd, will produce a Diſturbance of Motion in Reſpect of Latitude, by Drawing the Body *P* from the Plane of it's Orbit; and this Diſturbance, in any Given Situation of the Bodies *P* and *T* to each other, will be as that Generating Force *MN*, and therefore will be the Leaſt, where *MN* is the Leaſt, that is, as before, where the Attraction *SN* is not much Greater nor much Leſs than the Attraction *SK*. *Q. E. D.*

The Firſt Suppoſition, that P *will Deſcribe Areas Proportional to the Times, not prov'd by what foregoes.*

§. 2. This Propoſition and the Demonſtration of it is not only the Foundation of the Theory for Explaining the Errors and Irregularities of the Planets, but of That, which is brought to give us an Account of the Flux and Reflux of the Sea, which therefore we ſhall Examine in the ſeveral Diſtinct Parts of it.

In this Demonſtration it is therefore Firſt ſuppos'd, that if *P* was only Attracted by *T*, it would Deſcribe Areas Proportional to the Times, which Doctrine has been before Conſidered, and does not want here a Second Refutation; And yet all which we have Argued upon that Head muſt be ſhewn to be Falſe, before the Preſent Propoſition can in any Senſe be Allow'd to be True.



The Reſolution of the Force SL *into* LM *and* SM *not Evinced.*

§, 3. The other Two Forces Alledg'd are *LM* and *SM*, into which *SL* is ſuppoſed to be Reſolved; what therefore has been in General ſaid againſt ſuch a Reſolution of Forces is likewiſe to be Anſwer'd, before this Reaſoning can obtain; Beſides which, *LM* is Preſum'd to be Drawn Parallel to *PT*, whereas according to that Reſolution of Forces it might be as well in any other Direction, by which means the Line *SM* would become either Longer or Shorter than what is here Expreſs'd, and conſequently, ſince the Attraction of *S* to *T* will be the ſame, to wit, *SN*, the Force *NM* may be Different from what is here ſuppoſed, and therefore will produce Different Effects from Thoſe, which are Concluded from it.

No Reaſon given, why SK *the Middle Diſtance ſhould be the Standard, to which all the other Diſtances ſhould be Exacted.*

§. 4. However, ſince this Philoſophy has Choſen the Parallel Direction to *PT*, becauſe it is the beſt ſuited to it's Deſigns, we ſhall now Conſider it, as Propoſed to us; in order to Produce which, *SL* to *SK* is Taken in a Duplicate Proportion of *SK*, the Middle Diſtance to *SP*, which is any Diſtance Aſſign'd of *P* from *S*; whereas it is not Evident, why the Middle Diſtance ſhould be Aſſum'd in Preference to any other, ſo as to be a Standing Rule, to which the Reſt ſhould be Refer'd or Compared; Since, when the Body is at the Greateſt or at the Leaſt Diſtance, there is Nothing in the Nature of the Thing, which ſhould Exclude it from the ſame Pretenſions of being the Standard of the others, Excepting that of Greater or Leſs, which Equally obtains againſt the Middle Diſtance, ſeeing it is Leſs than the Greateſt Diſtance, and Greater than the Leaſt.

Granting the Parallel Direction LM, *and the Middle Diſtance* SK, *this Reaſoning is liable to another Objection.*

§. 5. GRANTING yet this Hypotheſis of the Middle Diſtance being the Standard, to which all the Reſt ſhould be Exacted, which is no other than an Arbitrary one, and that *LM* is one of the Forces into which *SL* is Reſolved, it is far from being Apparent, that a Parallel Force, which is ſuppoſed to Act from *L* to *M*, ſhould Act from *P* to *T*, that is, that Two Different Points, as *M* and *T*, ſhould be the ſame; If *LM* is on the other Hand only an Expreſſion of the Force Acting from *P* to *T*, it is ſomething Extraordinary, that a Force, which Acts Directly from *P* to *S* according to this Hypotheſis, ſhould exert it ſelf in any Part of it from *P* to *T*, unleſs we can Imagine, which is at preſent Incomprehenſible, that a Force, which is deem'd to Act one Way, ſhall be Capable in any Degree of Acting a Contrary; For if *P* to *S* is a Determined Force, as it is ſuppoſed to be, if it can Act from *P* to *T*, it muſt be by Enlarging the Force from *P* to *S*, and Taking ſo much from it by a Contrary Force from *P* to *T*, which ſhall make it Equal to the Determined one from *P* to *S*, which is no more than making the Force what it was before, and Annexing an Arbitrary Addition and Subtraction of a Foreign one, which deſtroy each other. If, notwithſtanding, we could Allow of what has been hitherto Diſputed, we ſhould have this Advantage from it, of Confirming that, which we have ſo often Contended for in our *Geometry* of *Solids*, and in this *Philoſophy*, that there may be Various Forces or Velocities Conceiv'd in the ſame Inſtant of Time, and in the ſame Line, Surface, or Solid; for thus *P* to *T* is ſuppoſed to Act in the ſame Time and Manner, when that Force is only Conſidered, as when *LM* is Join'd and United with it, which we mention as an Inſtance of the Conceivableneſs of that Doctrine, which we have Advanced of Different Forces in the ſame Space, even according to the Notions of the Philoſophy, which now obtains; Since, if a Force may be Vary'd in one Determined Point and Inſtant, it may be ſo likewiſe in a Determined Line, Surface, or Solid, which are only a Continuation of Points, or a Multiplication of Them.

The Force LM *not Accounted for.*

§. 6. MOREOVER, if the Force Expreſs'd by *PT* is that, which is Capable of Retaining the Body *P* in it's Elliptical Orbit, if there is an Addition of an Undetermined Force *LM*, and the Law of it's Acting is ſo, It will be Impoſſible to Reduce the Body *P* to any Rule of it's Revolving; ſince ſuch a Force may be ſo Great, that the Projectile may not be Able to keep it from Carrying the Body *P* to the Center *T*, or on the Contrary ſo ſmall, that it will not be ſufficient to Retain it in any Orbit, beſides that which the Force *PT* Preſcribes, and therefore will be the ſame as if there was none; and in any Intermediate Degrees, the Projectile Force and the Gravitating *LM* being only General and Abſtracted Terms, there is nothing can be Concluded from them, ſince by putting any particular Caſes, whether of the Projectile or Gravitating Force, and their Definite Proportions to each other, ſeveral Different Concluſions would Ariſe, which would be little Agreeable to the preſent Abſtracted Philoſophy, becauſe it's Reaſoning can be only true upon one Particular Proportion betwixt the Projectile and Gravitating.

The Force SM *not Accounted for.*

§. 7. AFTER the Force *LM*, we are to Conſider that of *SM*, which is the other Force into which *SL* is Reſolved, whoſe Greater Efficacy is ſaid to Conſiſt (and which is no longer Directed from *P* to *T*) in it's being Greater in Proportion to the Two Former, namely, *PT* and *LM*, where other Things are Equal, that is, as the Force of *M* to *S* is more Prevalent, it will obtain a Proportional Effect by it's Action from *P* in a Parallel Line to *ST* in Drawing the Body *P* from it's Orbit *PAB*.

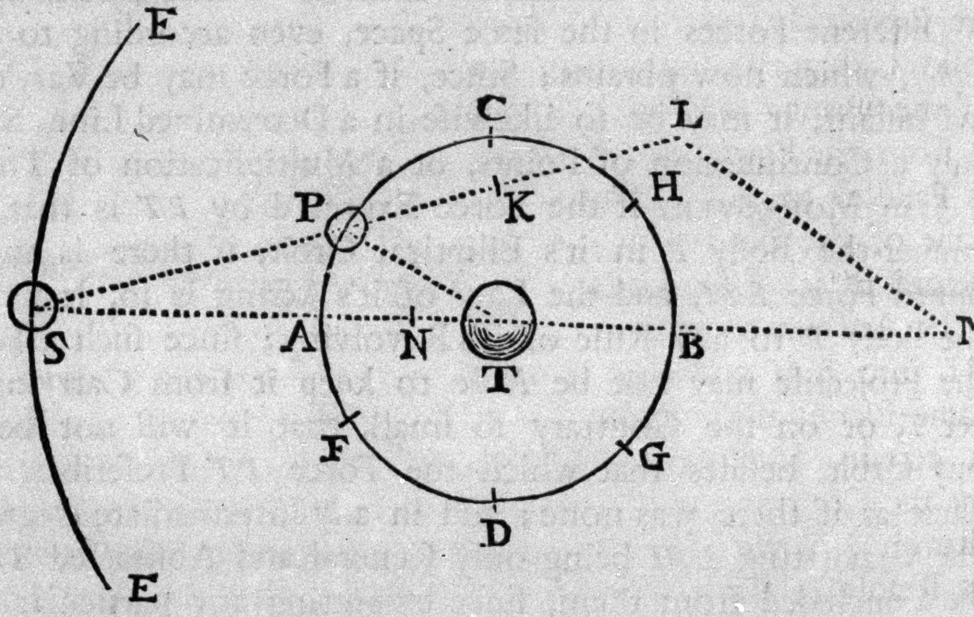
To which it may be Reply'd, FIRST, That if the Projectile Force is as Strong as that of *MS*, the Parallel Force from *P* to *TS* will be ſo far from Prevailing againſt the Forces *PT*, *LM*, that it may nearly leave the whole Province of Determining the Revolution of the Body to Them with little or no Alteration, and conſequently not Draw it from it's Orbit.

SECOND, If the Projectile Force is ſuppoſed to be Balanc'd with the Centripetal *PT*, ſo as to Deſcribe the Orbit *PAB*, It will be neceſſary to Determine their Proportions to each other, that we may be Inform'd what Degree of a Projectile Force in Conjunction with the Gravitating will Deſcribe ſuch an Orbit; For if the Projectile Force in *P* may be Different, where the Gravitating Force *PT* is the ſame, the Orbit *PAB* cannot be Deſcribed, unleſs Different Cauſes of the ſame Nature, as thoſe of the Projectile and Gravitating Force, can produce the ſame Effect; if the Projectile Force is the ſame, where the Gravitating Force is Different, as in the ſeveral Parts of an Elliptical Orbit they are ſuppoſed to be, It would not be Amiſs for this Philoſophy to ſhew us, what Proportion of the Projectile Force to the Gravitating would be ſufficient to Retain a Body from Flying off in a Tangent; for if there is not an Exact Balance betwixt the Two Forces, the one or the other will Prevail, and the Body will either ſink to the Center, or Proceed in a Right Line; if therefore an Equal Balance in the Two Forces is Neceſſary in whatever Point of the Elliptical Orbit that is ſuppoſed, in any other it will be Impoſſible to obtain; becauſe the Gravitating Force is Different, where the Projectile is the ſame.

THIRD, It is far from being Manifeſt (whatever Theories may be Contriv'd of the Reſolution of Forces, which has been ſhewn to be only a Contrivance), that a Body, which Tends from *P* to *S*, ſhould Act in any Reſpect in a Line Drawn from *P* Parallel to *TS*, unleſs a Body, which Actually Gravitates according to the Direction of *PS*, does Actually Gravitate in a Different Direction by the Power and Efficacy of the ſame Force, which Gravitates from *P* to *S*, which is Abſurd and a Contradiction, much Leſs can it Gravitate by the ſame Force from *P* to *T*, or *L* to *M*, as we have before Obſerv'd.

The Force SM *being Equal to* SN *not Accounted for.*

§. 8. ANOTHER Thing Aſſerted in this Reaſoning is, that if *SN* Repreſents the Accelerating Attraction of *T* to *S*, it will be either Equal to *SM*, or Greater, or Leſs; in the Firſt Caſe *P* and *T* will be Equally Attracted, and in the Second and Third the Difference of them will be Reſolv'd into *NM*.



As to the Firſt Suppoſition of *SN* and *SM* being Equal, ſince that will be at Equal Diſtances of *P* and *T* from *S*, it is certain, that the Attraction will not be Equal, altho' the Accelerating Forces are, but the Real Attraction will be Proportional to the Magnitude of the Bodies, where the Accelerating Forces are the ſame; and albeit this might perhaps make no Diſturbance in the Orbits, if the Revolution of each of the Bodies *P*, *T*, was only Conſider'd, becauſe the Different Sums of the Accelerating Forces might poſſibly occaſion no more Difference in the Progreſs of each Body, than Two Different Motions, whoſe Velocities were the Same, would; Yet, notwithſtanding this, where *P* and *T* are Refer'd in their Forces to a Third Body *S*, the Balances betwixt the Force of *P* and *S*, and *T* and *S*, will not be the ſame, unleſs the Two Different Sums of the Accelerating Forces are ſo, which is Impoſſible; if therefore the Sum of the Accelerating Forces in *T* is Greater than that in *P*, it will be either much more Difficult, or much more Eaſy, to Draw the One out of it's Orbit, than the Other, for the ſame Reaſon, that Two Bodies, which are Mov'd with the ſame Velocities, and whoſe Motions or Sums of Velocities are Different, will have a Different Effect, when Acted upon by the ſame Force.

NOR can there be any Thing Objected againſt what foregoes, unleſs it is Deny'd, that the Sum of the Accelerating Force in *T*, in Reſpect of *S*, is Greater than that of *P*, in Reſpect of the ſame Body, or which is all one, unleſs it is Affirm'd, that *S* has the ſame Influence upon *P* and *T*, which are ſuppos'd to be Endued with Two Different Sums of the ſame Accelerating Quality.

BESIDES which, that the Accelerating Forces in *P* and *T* are the ſame, can be no otherwiſe Prov'd or Imagin'd than upon the Suppoſition of an Homogeneous and Abſtracted Matter, the Abſurdity of which has been already ſhewn; For, if there ſhou'd be as much Difference betwixt the Matter, which Conſtitutes the Planets, as there is betwixt Earth, Water, Fire, and Æther; I believe no one will be apt to Grant their Accelerating Forces to the Center are the Same, tho' the Portions of Matter were Equal.

The Force MN *not Accounted for.*

§. 9. If therefore the Firſt Suppoſition of *SN* being Equal to *SM*, at Equal Diſtances of *P* and *T* from *S*, is not Certain, the Second of their being Unequal, and of their Difference being Reſolv'd into *NM*, will be likewiſe Unprov'd.

Let us yet Grant this, and that the Attraction *SN* is leſs than the Attraction *SM*, it is not Evident, which yet is Affirm'd, that *SN* would take off ſo much from the Attraction *SM*, as *SN* Repreſents, and conſequently, that there would only Remain the Difference of *NM*.

Because *SN*, when it Expreſſes the Force from *T* to *S*, is Different from *SN*, conſider'd as a Part of *SM*, which Explains a Force Acting from *P* Parallel to *SN*, and not Tending to the Point *S*, and therefore *SN* will not Equally Draw *P* and *T* to the Center *S*, nor will *NM* be the only Difference betwixt the Forces *SN* and *SM*, which is Aſſerted.

Again, If this were likewiſe Allow'd, namely, that *NM* was the Difference betwixt the Forces *SN* and *SM*, yet there will be a quite Contrary Reſult from *SN* being Greater than *SM*, or being Leſs than it; in the Firſt Caſe, *S* will be Drawn from it's Orbit, and in the Second, *P* will be Diſturb'd in it, whereas it is ſuppos'd, that, in both Caſes, *P* will be Affected only in it's Revolution: But this we ſhall Conſider more Particularly in the following Sections, after we have Examined into the Second Caſe of the Preſent Propoſition.

Concerning the Revolution of the Bodies in Different Planes, and the Force of MN *in that Caſe.*

§. 10. As to the Second Caſe it is now ſuppos'd, that the Leſſer Bodies *P* and *S* are Revolv'd about the Greateſt *T* in Different Planes, from whence it is ſaid to follow, that the Force *LM* by Acting according to the Line *PT*, ſituate in the Plane of the Orbit *PAB*, will have the ſame Effect, as before, nor will it Diſturb the Body *P* from the Plane of it's Orbit; whereas *MN*, by Acting according to a Line, which is Parallel to *TS*, Diſturbs the Body *P*, when that of *S* is Converſant out of the Line of the Nodes, and Induces a Variation in Reſpect of it's Latitude, as well as before in it's Longitude.

In Anſwer to which, it is Firſt to be obſerv'd, that the Reſolution of the Force *SL* into *LM* and *SM* is upon the Suppoſition of thoſe Lines being Drawn in the ſame Plane, if therefore Two Different Planes are Conſider'd, that is, which are of a certain Inclination to each other, as *PAB*, *ESE*, the Gravitation of *S* to *P*, or the Contrary, will be Different from what is Expreſs'd by *SL*, when *P* and *S* are Revolv'd in the ſame Plane, and conſequently, the Forces *LM*, *SM*, into which it is Reſolv'd; from whence it is Certain, that the Real Forces *LM*, *SM* are not the ſame in the one Caſe, as in the other, and therefore, that both *LM* and *SM* may Occaſion by their Real Forces Different Diſturbances of the Orbits, *PAB*, *ESE*, from what is here Confeſs'd.

Second, If *S* and *P* are Revolv'd in Different Planes, *MN* will either Act Parallel to *ST*, and *ST* being out of the Plane, in which *AT* lies, *TS* will be Inclin'd to that Plane, and with it Conſtitute the Angle *STA*, or elſe *MN* will Act Parallel to *AT*, in both which Caſes *MN* will fall out of the Plane, in which *STM* is in this Figure ſuppos'd to Lye, and conſequently, *LM* will in neither Caſes be in the ſame Direction with *PT*, which is Preſum'd; nor on the other Hand, will *MN* in the Firſt Caſe Attract *S* in a Parallel Line to *ST*, which is Affirm'd, and in the Second, *MN* is Imagin'd to Draw *S* Parallel to *ST*, whereas the Obvious and Real Attraction is in a Line Inclin'd to it, and Conſtituting an Angle, *STA*, with it; or, which is the ſame, *MN* is ſuppos'd to be a Right Line at the ſame Time, that *NT* or *AT* with *TM* form an Angle *NTM* or *ATM*, which is Impoſſible.

The Firſt Corollary of this Propoſition not Conſider'd, for the Reaſon Alledg'd.
Sir If. Newt. Princip. Præfat. Lib. 1.

§. 11. Having Conſider'd the Propoſition in General, we ſhall now Proceed to the Corollaries of it.

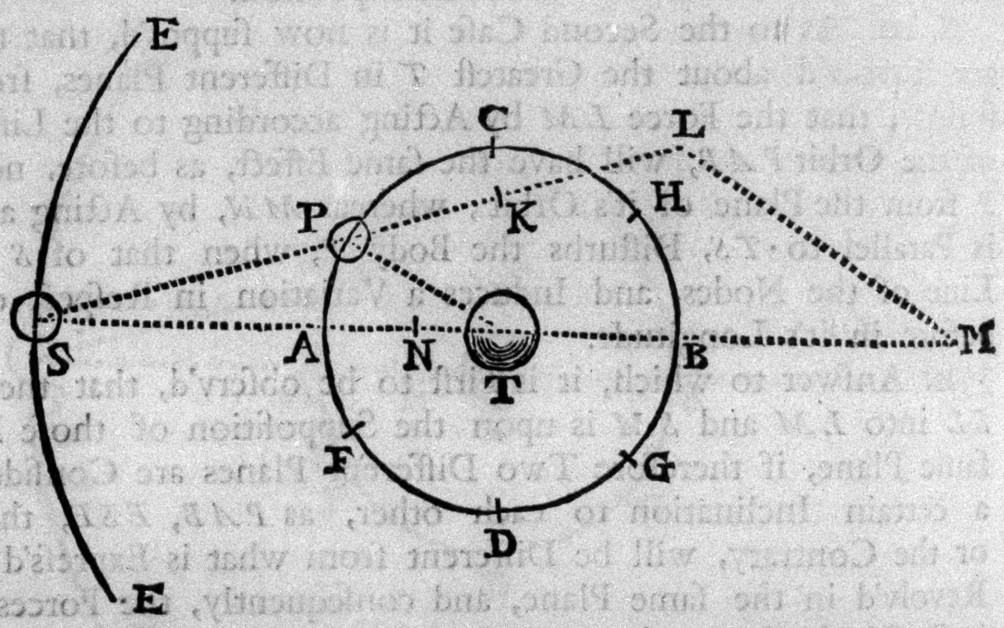
The Firſt we ſhall omit, as it Concerns more Bodies than Two Revolving about a Greater, ſince our Intention is to Confine this Inquiry to the Revolutions of the *Earth* and *Moon* about the *Sun*, in order to Explain which this Hypotheſis was principally Contriv'd, for if this is not True, neither in it's Principles nor it's Solutions, we have no Reaſon to Extend it any Farther.

The Second Corollary of this Propoſition Explain'd.
Sir Iſ. Newt. Princ. Prop. 66. Corol. 2. Lib. 1. Edit. 2. Dr. Gregory's Aſtronom. Prop. 1. §. 1. Lib. 4.

§. 12. The Second Corollary is, That in a Syſtem of Three Bodies *T*, *P*, *S*, if the Accelerating Attractions of any Two to a Third are to one another Reciprocally as the Squares of the Diſtances, the Body *P* with the Radius *PT* will Deſcribe an Area about the Body *T* with a Greater Celerity near the Conjunction *A* and the Oppoſition *B* or near the Syzygiæ, than near the Quadratures *C* and *D*; For all the Force, by which the Body *P* is Urg'd, and the Body *T* is not Urg'd, and which does not Act according to the Line *PT*, Accelerates or Retards the Deſcription of the Area, as it is Directed *in Antecedentia*, or *in Conſequentia*, and ſuch is the Force *MN*; Since This in the Paſſage of the Body *P* from *C* to *A* Tends *in Antecedentia*, and Accelerates the Motion; Then in it's Paſſage from *A* to *D* it is Directed *in Conſequentia*, and Retards it; From thence to *B* it Tends *in Antecedentia*, and again Accelerates the Motion; And laſtly, it Retards it in the Paſſage from *B* to *C in Conſequentia*.

Objections againſt the Truth of the Second Corollary.
Dr. Keill's Aſtronom. Lect. 5. Pag. 63. & 73. Edit. 1.

§. 13. As to what Reſpects the foregoing Corollary, before we Conſider it, we ſhall Examine, how this Scheme is Applicable to the Motion of the Secondary Planets about the Primary ones; ſince here Two Leſſer Bodies are Imagin'd to be Revolved about a Greater, that is, *S* and *P* about *T*, whereas *P* ſhould be Revolv'd about *S*, whilſt *S*, round which the Revolution is made, is Carry'd about *T*; But it is Prov'd in Aſtronomy, that it is the ſame Thing, whether the *Sun* and *Moon*, or *S* and *P*, are Revolv'd about the *Earth* or *T*; or whether the *Moon* is Revolv'd about the *Earth*, and both of them round the *Sun*, or *P* about *T*, and both about *S*.



Having Premis'd this, we ſhall now Inquire into the Reaſoning Alledg'd in §. 12, in which it is ſaid, that the Attraction of *MN* Accelerates *P* in it's Paſſage from *C* to *A*, that is, that the Difference betwixt the Direct Attraction of *P* to *S*, according to the Line *MS*, and the Direct Attraction of *T* to *S* Cauſes this Effect, which Difference muſt be Greater or Leſs than the Attraction of *T* to *S*, neither of which Suppoſitions will Juſtify the Preſent Reaſoning; for ſince the Accelerating Forces are ſuppos'd to be the ſame both in *P* and *T* at Equal Diſtances, or, which is the ſame, ſince the Bulk of each Body do's not come into the Computation of the Forces of them, and ſince the Diſtances from *S*, which only are Imagin'd to cauſe a Difference in their Forces, are variable from *C* to *A*, (for *P* when Situate in *C* will be at a Greater Diſtance from *S* than *T* is, and at a Leſs than it when Situate in *A*) it will follow, that, in the one Caſe or the other, *P* will not be Accelerated from *C* to *A*; Becauſe, if *C* or any Intermediate Point betwixt *C* and *A* is at a Greater Diſtance from *S* than *T* is, it is as Poſſible that *SL* ſhould fall as much below *SK* the Middle Diſtance, as it is here ſuppoſed to be Produced beyond it, and conſequently, *SM* would fall below *ST* or *SN*, which Repreſents the Attraction of *T* to *S*; which if ſo, *MN* would not Accelerate the Motion of *P* from *C* to *A*, ſince the Difference betwixt the Direct Attraction of *T* to *S*, and *P* to *S*, would be Leſs than Nothing.

On the other Hand, if *P* is Situate in *A*, or in any Intermediate Point betwixt *A* and *C*, which is at a Leſs Diſtance from *S* than *T* is, *SL* will be Extended beyond *SK* the Middle Diſtance, and *MN* will be a Difference, betwixt the Two Attractions of *P* and *T* to *S*, Greater than Nothing; But then the Queſtion will be, whether even ſuch a Difference will Accelerate the Motion of *P* from *C* to *A*; for if *MN* Acts by a Superior Force to all others, and according to the Line *MS* or a Line Parallel to it Drawn from *P*, it ſeems to be Evident, that it will be ſo far from Accelerating the Motion of *P*, that it will in Effect Retard it; Becauſe Nothing can Accelerate a Projectile or a Centripetal Motion, of which *CPA* is ſuppos'd to Conſiſt, unleſs ſuch a Force, as falls in with the one or the other, to both which, yet, *MN* is plainly Oppoſite, and neither Concurs in it's Efficacy with the Tangent of the Circle *CPA*, nor with the Radius of it; for *MN*, Acting from *P* in a Parallel Direction to *MS*, both Acts againſt *PT*, and a Tangent Drawn to *P*, that is, both againſt the Centripetal Force, and the Projectile; and in the Syzygiæ *A* and *B*, where the Acceleration ſhould be the Greateſt, *P* will be the moſt Retarded, becauſe *MN* will then Act in a Direct Oppoſition to the Centripetal *AT*, *BT*, and to the Tangents or the Projectile at *A* and *B*, where *MN* Hangs like a Weight upon the Projectile, and Cloggs and Obſtructs it's Progreſs, or elſe Propells it, in Lines Parallel to *BM*, from *D* to *B*, and by that means Preſſes upon *P*, and Endeavours to thruſt it from it's Orbit, eſpecially in the Parts towards *B*, which cannot but be an Impediment to it's Courſe thro' *DB*, ſo far from Advancing it; This is ſaid, ſuppoſing *MN* to be here Repulſive, which yet is not ſhewn, and may be juſtly Queſtion'd, and if it is not Repulſive, it has not the leaſt Pretence to Accelerate the Planet's Motion in that Portion of it's Orbit.

P is Imagin'd to be Accelerated from *C* to *A*, becauſe *P* is more Attracted to *S*, than *T* is, by the Exceſs of the Force *MN*, this Force yet is not ſuppos'd to Draw *P* farther from *T* towards *S*, ſince, if ſo, it might not ſpend it ſelf in the Acceleration of *P*, but in making it Deſcribe a Larger Orbit without any Acceleration at all; On the Contrary, it is ſuppos'd to Accelerate *P*, and to Contract it's Orbit by that Acceleration, ſo that the Syzygiæ *A* and *B* Approach nearer to *T* than the Quadratures *C* and *D*, all which is ſaid without any Proof or Demonſtration; for why *P* ſhould come nearer to *T* by Gravitating more towards *S* than it, or why *P* ſhould be Accelerated rather than Deſcribe a Larger Circuit about *T*, than it would have otherwiſe done, is Equally Inconceivable; but when we come to Prove the Repulſive Power of *MN*, by which *P* is Quickned in it's Progreſs from *D* to *B*, we are told, that *T*, Gravitating more to *S* than *P*, is Attracted more towards it by the Force *MN*, than *P* is, and conſequently, that *T* Receding from *P*, ſituate betwixt *D* and *B*, towards *S* is the ſame, as if *P* ſhould Recede from *T* with the ſame Velocity towards *M*, and therefore *MN* has here a Repulſive Force; *MN* conſequently is allow'd to have a Power of Making *T* Approach towards *S*, in order to Prove this Repulſive Force, which was not Granted before to it, in Reſpect of *P* in it's Paſſage from *C* to *A*, which it did not Draw towards *S* from it's Orbit, but only Accelerated in it, and by that means cauſed it to Recede from *S* and Approach towards *T*; and yet Preſuming all this to be True, which is no more than ſaying Things at Pleaſure, *MN*, by Driving and Impelling *T* towards *S*, would only Effect, that *P* ſhould Deſcribe *DB* at a Greater Diſtance from *T*, and in the Portion of a Larger Orbit about it, which would not Increaſe but Diminiſh it's Apparent Velocity without any Real Repulſive Force at all.

Let us now Conſider the Progreſs of *P* from *A* to *D*, which is ſaid to be Retarded, but why it ſhould be any more, than from *C* to *A* is not Evinced; For *MN*, which Acts from *P*, in every Point betwixt *A* and *D*, Parallel to *MS*, and againſt the Centripetal Force *PT* and the Projectile in every Individual Tangent to thoſe Points, will have the ſame Effect in the Progreſs of *P* from *A* to *D*, as it had from *C* to *A*; for it makes no Variation, whether *P* moves from *C* to *A*, or from *A* to *D*, whilſt the Centripetal Force *PT* is the ſame in both Caſes, and the Projectile Force in the Tangent is on both Hands Loaded with a Gravitating Force, as *MN*, which is Foreign to it, and Deſtructive of it; Thus, ſuppoſing Two Tangents, the one at *C* and the other at *D*, which is the moſt favourable Caſe, which can be put for the Preſent Philoſophy, and let the Projectile Force in the Tangent at *C* Act the ſame way with the Gravitating *MN*, and at *D* the Contrary, I ſay the Projectile Force will be Equally hindered from having it's Proper Effect in the one Caſe, as in the other; For if the Gravitating Force of *MN* in *D* is Equal to the Contrary Projectile in a Tangent Parallel to *NM*, and Acting from *N* towards *M*, the Projectile will be Deſtroy'd and not able to Proceed in a Second Tangent from *D* towards *B*; and the ſame may be ſaid of a Tangent in *C*, if the Gravitating Force *MN* is Equal to the Projectile in that Tangent, for tho' they Tend the ſame way of *M* to *N*, yet the Gravitating Force continuing ſtill the ſame, the Projectile will not be able to Proceed in a Second Tangent from *C* towards *A*, becauſe it was Clogg'd with a Weight before,

which

which Equally with the Projectile Urged it out of the Circle; and therefore it cannot Proceed in a New Tangent Inclin'd to the Former, and towards the Center *T*, and for the ſame Reaſon it will be Obſtructed in Proceeding in a Third, Fourth, or any other Tangent.

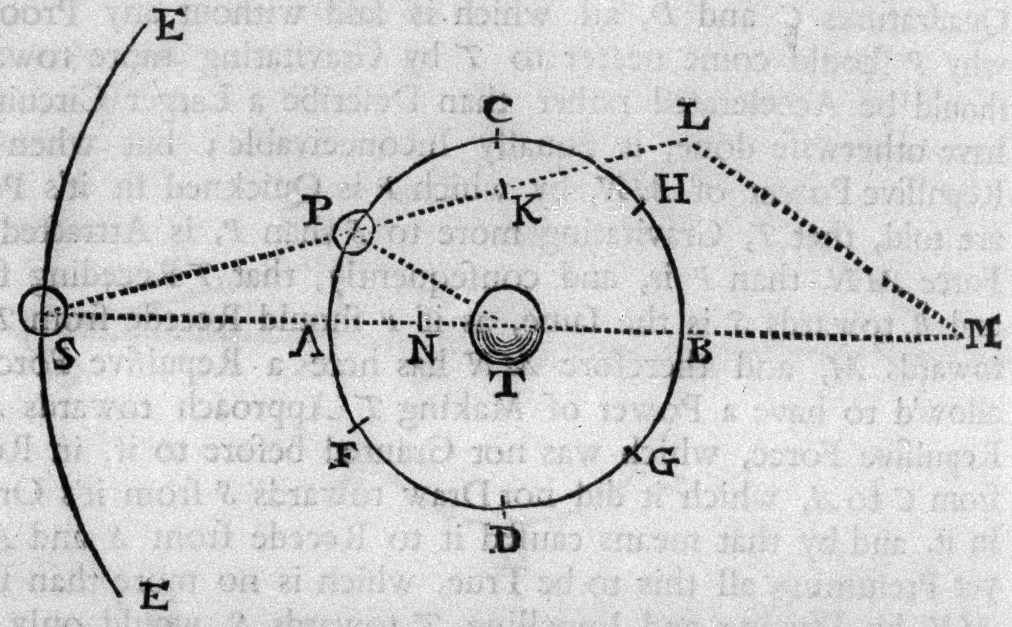
If *MN* the Gravitating Force in *C* is Greater than the Projectile in the Tangent, it will ſtill more Obſtruct the Projectile in it's Motion, and if *MN* is Leſs, it will Leſs hinder the Projectile, but not Accelerate it; And what has been ſaid in Reſpect of the Tangents and the Projectile Forces, at *C* and *D*, will obtain in Proportion in all the Points betwixt *C* and *A*, and betwixt *A* and *D*.

As to *P* being Accelerated betwixt *D* and *B*, and Retarded betwixt *B* and *C*, it is liable to all the Objections of it's Progreſs betwixt *C* and *A*, and *A* and *D*, and of *MN* being Conſidered, as a Repulſive Force in the Planet's Courſe thro' *BC*, as before thro' *DB*; All this is ſtill Argued upon the Suppoſition of a Gravitating and Projectile Force, which Forces have been already Conſidered and Shewn to be Arbitrary, as alſo upon the Suppoſition of Matter being Similar and Homogeneous in it's ſelf, which Notion likewiſe has been Refuted, and that the Diagonal Force is Reſolved into the Sides of the Parallelogram, which has not been Proved.

The Third Corollary Conſidered and Examined. Sir Iſ. Newt. Princip. Prop. 66. Coroll. 3. Lib. 1. Dr. Gregory's Aſtronom. Prop. 1. §. 1. Lib. 4.

§. 14. The next Corollary is, that by the ſame Reaſoning it is Evident, that the Body *P*, other Things being Equal, is Moved with a Greater Velocity in it's Conjunction and Oppoſition, than in it's Quadratures.

Which in the First Place is not True in this Reaſoning, however it is in Fact, unleſs the foregoing Corollary is ſo.



Second, Suppoſing the foregoing Corollary is True, it is not any more Evident, that *P* is Accelerated in it's Conjunction at *A*, from being Accelerated from *C* to *A*, than that it is Retarded there, from it's being Retarded in it's Progreſs from *A* to *D*, and the like may be ſaid concerning it's being Accelerated in it's Oppoſition at *B*; for why *CA*, in which *P* is Accelerated, rather than *AD*, in which it is Retarded, ſhould give the Denomination of *P* being Accelerated rather than Retarded in the Parts on each Side next adjoining to *A*, is as Incomprehenſible as it is Precariouſly and Arbitrarily Aſſerted in this Hypotheſis; or why *DB*, in which it is Accelerated, rather than *BC*, in which it is Retarded, ſhould give the Denomination of it's being Accelerated, rather than Retarded in the Parts adjoining to *B* in it's Oppoſition.

What farther has been Argued, in Reſpect of the Conjunction and Oppoſition of the Revolving Body, againſt it's being Accelerated, Equally obtains in it's Quadratures againſt it's being Retarded, That is, a Point, as *A* or *B* in the Syzygiæ, or *C* and *D* in the Quadratures, which lies Intermediate to an Acceleration and Retardation, can be no more ſaid to be Accelerated than Retarded, and *Vice Verſâ*.

The Fourth Corollary Explain'd and Conſidered. Sir Iſ. Newt. Princip. Prop. 66. Corol. 4. Lib. 1. Dr. Gregory's Aſtronom. Prop. 2. §. 1. Lib. 4.

§. 15. The Fourth Corollary is, That the Orbit of the Body *P*, other Things being Equal, has a Greater Curvity in the Quadratures, than in the Conjunction and Oppoſition: Becauſe Bodies, which obtain a Greater Velocity, are Leſs Deflected from a Rectilineal Progreſs, or, which is the ſame, Recede Leſs from a Right Line.

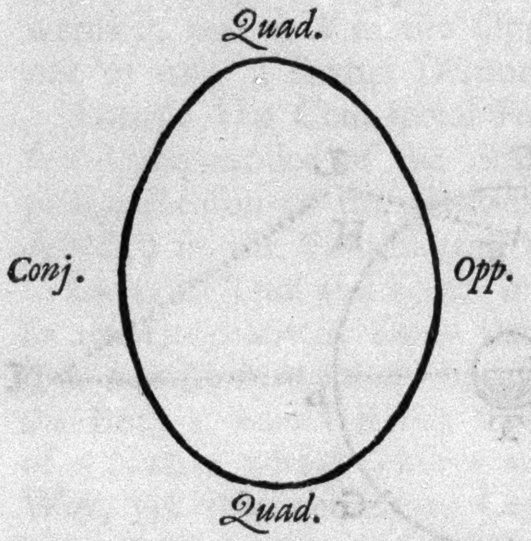
Besides, The Force *KL*, or *NM*, in the Conjunction and Oppoſition, is Contrary to the Force with which the Body *T* Attracts the Body *P*, and therefore Diminiſhes it; But the Body *P* will Recede Leſs from a Right Line, where it is Leſs Urged towards the Body *T*.

First, As to the Firſt Reaſon Alledged, it Evidently depends on the Truth of the Two preceding Corollaries, for if it cannot be Proved from them, that the Body *P* is Moved with a Greater Velocity in it's Conjunction and Oppoſition, than in it's Quadratures, it ſignifies Nothing to tell us, that Bodies, which obtain a Greater Velocity, are Leſs Deflected from a Rectilineal Progreſs.

Granting which Corollaries, it is not yet Plain, that Bodies, which obtain a Greater Velocity, are Leſs Deflected from a Rectilineal Progreſs, unleſs upon this Suppoſition, that the Greater the Force is, which Acts according to the Direction of the Tangent, the Leſs Power will the Centripetal have to Bend and Incline the Body to the Center, about which it is Revolved.

Whereas all that, which concerns a Projectile or a Tangent Force and a Centripetal, has been already Evidenced to be a Creature of the Mind, and that the Reſolution of Curves into ſuch Forces is either Diſputable or Groundleſs; Beſides which, *MN*, which is ſuppos'd to Increaſe the Velocity of the Revolving Body *P*, does not Act according to the Direction of the Tangent, and therefore cannot be Conſidered as a Force united with the Projectile, from which the Centripetal Draws and Impells the Body, which is Revolved.

Farther, If this were likewiſe Allow'd, it will follow, that, in the Conjunction and Oppoſition, where the Velocity is Swifter, the Body *P* will Approach nearer to the Center, and in it's Quadratures will Retire farther from it, ſo as to Deſcribe an Orbit Reſembling an Ellipſis, according to the Figure Annext; for in that Figure only an Orbit has it's Oppoſite Sides in one Caſe more Rectilineal, and in the other of a Greater Curvity; which is Impoſſible upon the Preſent Suppoſition, where the Projectile Force is continually Increaſing and Decreaſing, and the Centripetal Continues the ſame; ſince, if we Imagine any Determined Centripetal Force, by which the Body *P* is Attracted to the Center, and a Projectile, by whoſe Counter-Action to the Centripetal it is kept up in it's Orbit from falling to ſuch Center, if the Projectile is Increaſed, as in the Conjunction and Oppoſition, it will not make the Centripetal Force Greater, but take off and Diminiſh from it's Urgency to the Center, and conſequently, will not cauſe the Body in it's Revolution to Approach nearer to it; On the Contrary, if the Projectile Force in the Quadratures is Diminiſhed, it will not Diminiſh the Gravitating, but rather Increaſe it, by Leaving it more Entirely to it's own proper Tendency, from whence the Body will not Recede from, but Approach nearer to the Center of it's Revolution.

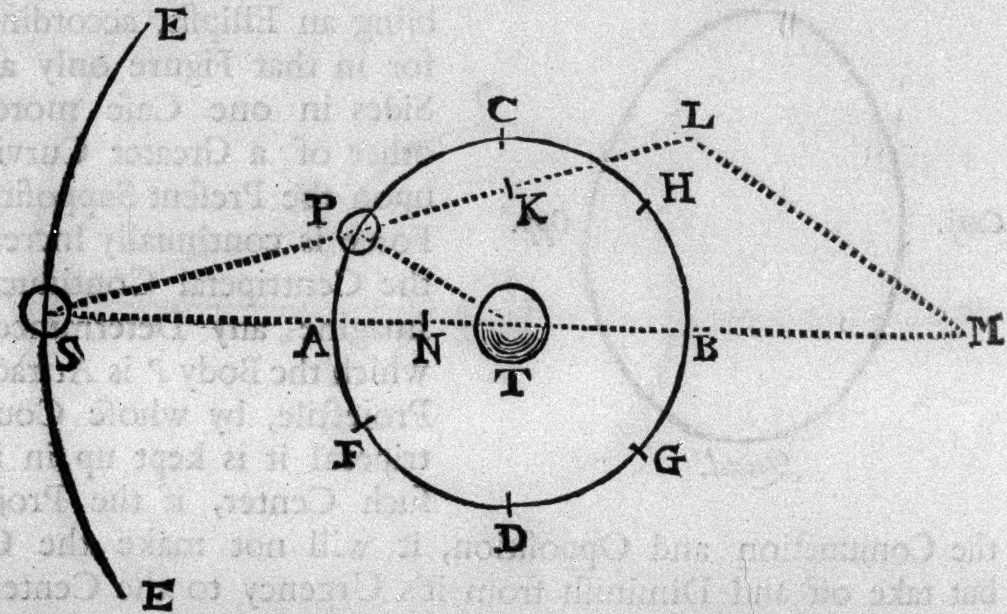


Second, As to the Second Reaſon Alledged for the Proof of this Corollary, namely, that the Force *MN*, in the Conjunction and Oppoſition, is Contrary to the Force, with which the Body *T* Attracts the Body *P*, and therefore Diminiſhes it, from whence the Body *P* will Recede Leſs from a Right Line, where it is leſs Urged towards the Body *T*: This Reaſon, we ſay, is Firſt in the General not true in an Orbit, which is ſuppoſed to be Deſcrib'd by a Gravitating, and a Tangent or Projectile Force; Becauſe where a Force, as *MN*, Acts directly Contrary to a Gravitating, yet ſince the Projectile is join'd with the Gravitating, and they are Both in an Æquilibrium, ſo that the Body *P* does neither Fall towards the Center, nor Proceed in a Tangent, it is Evident, that the Force *MN*, which Diſturbs it from that Æquilibrium, as much Obſtructs it from Going forward in a Tangent, as from Tending to the Center; and conſequently, the Body *P* will Equally Recede from a Right Line, as it does from the Center, and therefore not Leſs, where it is leſs Urged towards the Body *T*, ſince it would be Carry'd nearer to *S*, and Leave the Tangent in ſome Meaſure as well as the Center.

We add Secondly, Suppoſing the Force *MN* contrary to the Gravitating Force of *P*, if it had the Effect of making it Recede Leſs from a Right Line, that is, if it Reſign'd to the Projectile a Greater Force, it would at the ſame Time at leaſt make the Gravitating Leſs, and therefore, that Part of the Orbit, which Recedes Leſs from a Right Line, ſhould Recede more from the Center, whereas the Contrary is Affirm'd in this Hypotheſis of Forces, to wit, that in the Conjunction and Oppoſition *P* Approaches nearer to the Center, and in the Quadratures is farther Retired from it.

Third, If *MN* Acts directly Contrary in the Conjunction and Oppoſition to the Gravitating Force of *P* to *T*, That is either meant in the Points *A* and *B* only, or elſe in all the Intermediate Points betwixt *C* and *A* in Conjunction, and betwixt *D* and *B* in Oppoſition; if the Firſt, it will ſignify nothing to the Rectilineal Progreſs of the Body *P* betwixt *C* and *A*, or *D* and *B*, ſince this Reaſon obtains only in the Points *A* and *B*; if the Second, it will not be eaſily Conceived, how *MN* Acts Contrary to the Gravitating Force from *P* to *T*, unleſs in an Oblique Direction, which if Allow'd of, it does likewiſe to all the ſeveral Tangents, or the Projectile Forces of the Points in *CA*, or *DB*, and conſequently does not Accelerate the Velocity of *P*, but Retards it, as was before Obſerved.

Lastly, If all was Granted, which has been Diſputed, ſince the Accelerations are ſuppoſed betwixt *C* and *A*, *D* and *B*, and the Retardations betwixt *A* and *D*, *B* and *C*, it would follow, that the Greater Curvity of the Orbit would not be in the Quadratures, but in thoſe Parts of it, which lie betwixt the Conjunctions or Oppoſitions, and the Quadratures, that is, from *A* to *D*, and *B* to *C*; and the more Rectilineal Progreſſes of *P* would not be in the Conjunctions and Oppoſitions, but in the Parts lying betwixt the Quadratures and Them, that is, from *C* to *A*, and *D* to *B*.



The Fifth Corollary Explained. Sir Iſ. Newt. Princip. Corol. 5. Prop. 66. Dr. Gregory's Aſtronom. Prop. 2. Lib. 4.

§. 16. The Fifth Corollary is, that the Body *P*, other Things being Equal, will Recede farther from the Body *T* in the Quadratures, than in the Conjunction and Oppoſition, which has been already Conſider'd and Depends upon the former Corollaries.

The Sixth Corollary Explained and Examined. Sir Iſ. Newt. Princip. Corol. 6. Prop. 66. Dr. Gregory's Aſtronom. Prop. 3, 4, &c. Lib. 4. & Corol. Prop. 7. Lib. 4.

§. 17. The Sixth Corollary is, That ſince the Centripetal Force of the Central Body *T*, by which the Body *P* is Retained in it's Orbit, is Increaſed in the Quadratures by the Addition of the Force *LM*, and Diminiſhed in the Conjunction and Oppoſition, or in the Syzygiæ, by the Ablation of the Force *KL*, and by Reaſon of the Greatneſs of *KL* is more Diminiſhed than Increaſed, and ſince that Centripetal Force is (by *Corol.* 2. *Prop.* 4.) in a Reaſon Compounded of the Simple Reaſon of the Radius *TP* Directly, and of a Duplicate Reaſon of the Periodical Time Inverſely, it is Manifeſt, that this Compound Reaſon is Diminiſhed by the Action of the Force *KL*, and conſequently, the Periodical Time, if the Radius of the Orbit *TP* Continues the ſame, is Increas'd, and that in a Dimidiate Reaſon, in which that Centripetal Force is Diminiſhed, if the ſame Radius therefore is Increas'd or Diminiſh'd, it is likewiſe Manifeſt, that the Periodical Time is more Increaſed or leſs Diminiſhed than in the Seſquiplicate Reaſon of the ſaid Radius (by *Corol.* 6. *Prop.* 4.) If that Force of the Central Body ſhould gradually Languiſh and Decreaſe, the Body *P*, being leſs and leſs Attracted, would perpetually Recede farther from the Center *T*, and on the Contrary, if that Force ſhould be Increas'd, it would Approach nearer to it; Therefore, if the Action of the Remote Body *S*, by which that Force is Diminiſhed, be Increas'd and Diminiſhed by Turns, the Radius *TP* will be alſo Increaſed and Diminiſhed by Turns, and the Periodical Time will be Increaſed and Diminiſhed in a Reaſon Compounded of the Seſquiplicate Reaſon of the Radius and the Dimidiate Reaſon, in which that Centripetal Force of the Central Body *T* is Diminiſhed or Increaſed by the Increaſe or Diminution of the Action of the Remote Body *S*.

IN Anſwer to which, it is FIRST to be Conſidered, that the Centripetal Force of the Body *T* does not Retain the Body *P* in it's Orbit, becauſe there is a Projectile one Equally Requir'd by this Philoſophy, and becauſe it is not farther Prov'd, by what foregoes, that the Earth or Planets, for which the Preſent Hypotheſis is Contrived, are Retained in their Orbits by a Centripetal Force even in Conjunction with a Projectile, but poſſibly may be ſo by an Expanſive one of the Sun.

SECOND, Allowing this, the Centripetal Force of the Body *T* is not Increas'd by the Addition of *LM* in the Quadratures, and Diminiſh'd by the Ablation of *KL* in the Syzygiæ, unleſs upon the Suppoſition, that *SL* is Reſolved into the Forces, *LM*, *SM*, which has been before Conſider'd and Diſproved.

GRANTING which, as *SK* is ſuppoſed to Repreſent the Attraction of the Middle Diſtance, *KL*, when it falls in with the Line *SM*, will, if any Thing, in the Conjunction at *A* Diminiſh the Centripetal Force of the Body *T*, becauſe the Force Tends from *L* to *K*, and therefore Draws *A* from *T* towards *S*; and on the Contrary, in the Oppoſition at *B*, *LK* will Diminiſh the Centripetal Force of the Body *T*, by Impelling the Body *P* Leſs from *B* towards *T*, than *T* towards *S*, but then the Effects will be very Different, ſince the ſame Force *LK* Acts upon *P* in the Firſt Caſe and *T* in the Laſt, which are Bodies of a very Different Magnitude; Whereas it is Affirm'd in this Philoſophy, that the Effects are Equal and Alike by an Ablation of the Force *LK* in the Syzygiæ: Beſides, that *LK* in the Conjunction at *A* Acts by Attraction towards *S*, and at *B* in the Oppoſition by a Repulſion from it, is not ſo Evident as not to want a farther Demonſtration.

THIRD, The Centripetal Force is not Proved by *Corol.* 2. *Prop.* 4, to be in a Reaſon Compounded of the Simple Reaſon of the Radius *TP* Directly and of a Duplicate Reaſon of the Periodical Time Inverſely, as is Evident from what has been Alledg'd in the Examination of that Propoſition and the Corollaries of it.

LASTLY, Tho' the Force of the Central Body ſhould, upon it's Increaſe from *LM* in the Quadratures, Draw the Body *P* nearer to *T* by Acting more ſtrongly upon it, it is not Evident, that, upon a Diminution of that Force by *KL* in the Syzygiæ, the Body *P* would Retire farther from *T*, where the Body *P* is ſuppoſed to Conſiſt of a Nature wholly Paſſive and Unactive; for tho' an Action ſhould Impell it one Way, yet upon the entire Ceaſing of that Action, it would not be Impell'd another, but would remain Quieſcent in Reſpect of that Action, and much Leſs upon the Diminution of an Action, by which it is Impell'd one Way, would it be Impell'd, or Move a Contrary.

NOR conſequently, if the Action of the Remote Body *S*, by which the Central Force is Diminiſhed, ſhould be Increaſed and Diminiſhed by Turns, will the Radius *TP* be likewiſe Increaſed and Diminiſhed by Turns; for if by the Action of *S* the Action of *T* ſhould be ſo far Diminiſhed, as to be in an Equal Balance with it, that is, that by their Contranitent Forces they ſhould Deſtroy each others Effects towards their own Centers, the Body *P* would not lengthen it's Radius *TP* and Retire towards *S*, but would be perfectly Quieſcent in Reſpect of both theſe Forces; if therefore the Action of *T* is only ſo far Diminiſhed, as not to be Entirely Deſtroyed, it will ſtill be more Impoſſible for *TP* to lengthen it's Radius, or for *P* to be Impell'd a Contrary way to *T*, nor is it neceſſary it ſhould be ſhortned, only it would then obey the Projectile, and be carry'd Round *T* at the Diſtance *PT*; And if the Action from *P* to *T* is not only Deſtroy'd, but ſurmounted by the Superior one of *S*, *P* will Retire towards *S* and *TP* will be Produced, but then *T* will have loſt it's Influence, and the Body *P* will no longer be Revolved about the Center *T*, but Round that of *S*.

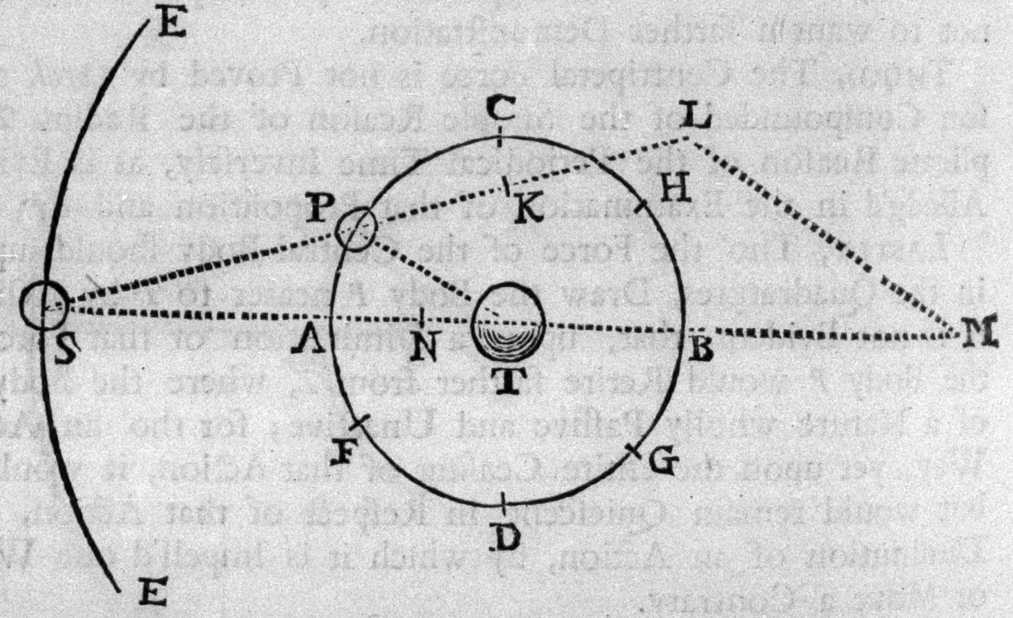
BUT that, which is ſtill more Unaccountable, is, that *LM* is ſuppoſed to fall in with *PT*, or the Centripetal Force, and to Increaſe it in the Quadratures, and *KL* to Act Counter to it, and therefore to Diminiſh it in the Syzygiæ, from whence, according to this Doctrine, it ſhould follow, that the Radius *PT* is Diminiſhed in the Firſt Caſe, and Increaſed in the Laſt; whereas we are before told, that the Radius *PT* is Longer in the Firſt Caſe, and Shorter in the Laſt, that is, Longer in the Quadratures, where there is ſaid to be a Greater Curvity of the Orbit, and Shorter in the Syzygiæ, where there is ſaid to be a Leſs.

WHAT therefore is Reaſoned concerning the Periodical Times being Increaſed or Diminiſhed upon theſe Preſumptions, which are Falſe and Groundleſs, muſt be Equally ſo.

The Seventh Corollary Explained and Examined.
Sir Iſ. Newt. Princip. Prop. 66. Corol. 7. Lib. 1.
Dr. Gregory's Aſtronom. Prop. 8, 9, 10. Lib. 4.

§. 18. THE Seventh Corollary is, That, from what foregoes, it is Conſequent, that the Axis or the Line of the Apſides of an Ellipſis, Deſcrib'd by the Body P, as to it's Angular Motion, makes it's Progreſs and Regreſs by Turns, but yet it's Progreſſes are Greater, and by that Exceſs of the Progreſſion, in each Revolution of the Body, it is Carry'd *in Conſequentia*.

FOR the Force, by which the Body P is Urged towards the Body T in the Quadratures, where the Force MN Vaniſhes, is Compounded of the Force LM, and of the Centripetal Force, by which the Body T Attracts the Body P; The Former Force LM, if the Diſtance PT be Increaſed, is Increas'd almoſt in the ſame Reaſon with this Diſtance, and the Latter Decreaſes in that Duplicate Reaſon, and conſequently, the Sum of theſe Forces Decreaſes in leſs than a Duplicate Reaſon of the Diſtance PT, and upon that Account, by *Corol.* 1. *Prop.* 45, Cauſes the Aux or the Upper Apſis to Return; On the Contrary, in the Conjunction and Oppoſition the Force, by which the Body P is Urg'd towards the Body T, is the Difference betwixt the Force, by which the Body T Attracts the Body P, and the Force KL, and that Difference, becauſe the Force KL is nearly Increas'd in a Reaſon of the Diſtance PT, Decreaſes in a Greater than a Duplicate Reaſon of that Diſtance, and therefore, by *Corol.* 1. *Prop.* 45, Cauſes the Aux or the Upper Apſis to Proceed; In the Places, which lie Intermediate to the Syzygiæ and Quadratures, the Motion of the Aux Depends jointly upon both theſe Cauſes, ſo that as the one or the other Prevails, it Proceeds or Returns; from whence it follows, that, ſince the Force of KL in the Syzygiæ is almoſt doubly Greater than the Force LM in the Quadratures, the Exceſs in the whole Revolution will be on the Side of KL, and will carry the Aux in each Revolution *in Conſequentia*.



To which it may be Firſt Reply'd, that in the Quadratures MN is ſuppoſed to Vaniſh, becauſe then the Body P is in a Middle Diſtance from S, which has been before Conſidered; Granting which, LM is Imagined to unite in it's Action with PT, upon the Reſolution of the Force SL into LM, MS, which likewiſe has been already Examin'd.

AND yet FIRST, Allowing both, it is not Proved, that LM, upon the Increaſe of PT, is Increaſed almoſt in the ſame Reaſon, becauſe, ſuppoſing PS and KS not much Different, which will be, when P is not far from the Tranſverſe or Conjugate Axis of it's Orbit, LM will be to PT, as LS to PS, and if PT is Lengthened or Increaſed, LM will be to PT in the ſame Proportion of LS to PS, which will be as the Square of the Middle Diſtance KS to the Square of the Diſtance PS, and therefore LM, will not be Increas'd almoſt in Proportion as PT is, but in Proportion as the Squares of KS and PS.

SECOND, It is not moreover Certain, if the Diſtance PT is Increas'd, that the Centripetal Force is Diminiſhed in a Duplicate Reaſon of ſuch Diſtance, according to what has been before Alledg'd, and therefore *Corol.* 1. *Prop.* 45, is Cited to no purpoſe, ſince by that the Aux or Uppermoſt Apſis is Proved to Return upon the Conceſſion, that the Sum of theſe Forces is Diminiſhed in Leſs than a Duplicate Proportion of the Diſtance PT, which is not Affirmed or Evinced, if the one Force is not Increas'd nearly as the Radius is, and the other Diminiſhed in a Duplicate Reaſon of ſuch Increaſe of the Radius. Not to add, that the Propoſition and the Corollary here Refer'd to are both of them Argued upon ſuch a Confuſ'd and Precarious Reaſoning, and upon ſo many Preſumptions of the Truth of Fluxions, and other Principles of this Philoſophy, that it would not be hard to Conteſt almoſt every Part of each; and which Propoſition we ſhall afterwards Examine.

It may be Secondly Reply'd on the other Hand, that, in the Conjunction and Oppoſition, the Force, by which the Body P is Urged towards the Body T, is not the Difference betwixt the Force, by which the Body T Attracts the Body P, and the Force KL; for KL in the Conjunction Acts in P from A to S, and therefore the Force, by which the Body P is Urged towards T, is ſuppoſed to be Diminiſh'd ſo far by that Contrary Attraction, whereas, according to what has been before Proved, if the Attraction from A to S, and A to T were Equal, A would not therefore Retire from T or to S, where the Difference betwixt ſuch Forces is Equal to Nothing, that is, if the Projectile is not Conſider'd, for that indeed in ſuch a Caſe would carry the Body in a Tangent from A, and there would be no Radius PT at all, and conſequently, if KL is Leſs than the Attractive Force of P to T, there may be an Actual Tendency of P to T at the Diſtance PT, without a Diminution of it, if KL is Greater, P will be Drawn from A towards S, and will have no Tendency to T, and therefore cannot be Retained in an Orbit by it; ſo that in the Firſt and Laſt Caſe there will be no Radius PT, and therefore no Revolution of the Aux or the Upper Apſis, and in the Second, ſince PT will continue the ſame, no Progreſſion or Regreſſion of it; If the Projectile Force is Conſidered, and is Equal to the Centripetal, in the Firſt Caſe, it will Direct the Body P in a Tangent, as was before ſaid; in the Second, it will ſurmount the Centripetal, and will likewiſe promote it in a Tangent, tho' with a Leſs Force, unleſs the Projectile is Diminiſhed in Proportion to the Centripetal, and then, as was mentioned before, PT will be the ſame in it's Revolution without any Progreſs or Regreſs of the Apſis; and in the Laſt Caſe, P will be Revolved round S, if KL is Greater than PT and the Difference Equal to the Projectile, if Leſs and the Projectile Greater, P will go off in a Tangent; And the ſame may be ſaid in the Oppoſition of P at B, with this Additional Objection, that it is not Certain, that KL Acts in a Contrary Direction to the Centripetal Force, when P is in it's Oppoſition at B, according to what has been before Conſidered, in Relation to the Action of MN.

Bbsides which, it is not Evident, that ſuch a Difference, if it were Granted, would Decreaſe in a Greater than a Duplicate Reaſon of the Diſtance PT, becauſe it is not Proved, that the Force KL is nearly Increas'd in a Reaſon of that Diſtance.

Lastly, If yet the Difference was Allowed to Decreaſe in a Greater than a Duplicate Reaſon of the Diſtance PT, it is not Certain, that the Propoſition and the Corollary Refer'd to are true, for the Reaſons above-mentioned.

The Eighth Corollary Explained and Examined.
Sir Iſ. Newt. Princip. Corol. 8. Prop. 66.
Dr. Gregory's Aſtronom. Prop. 11. Lib. 4.

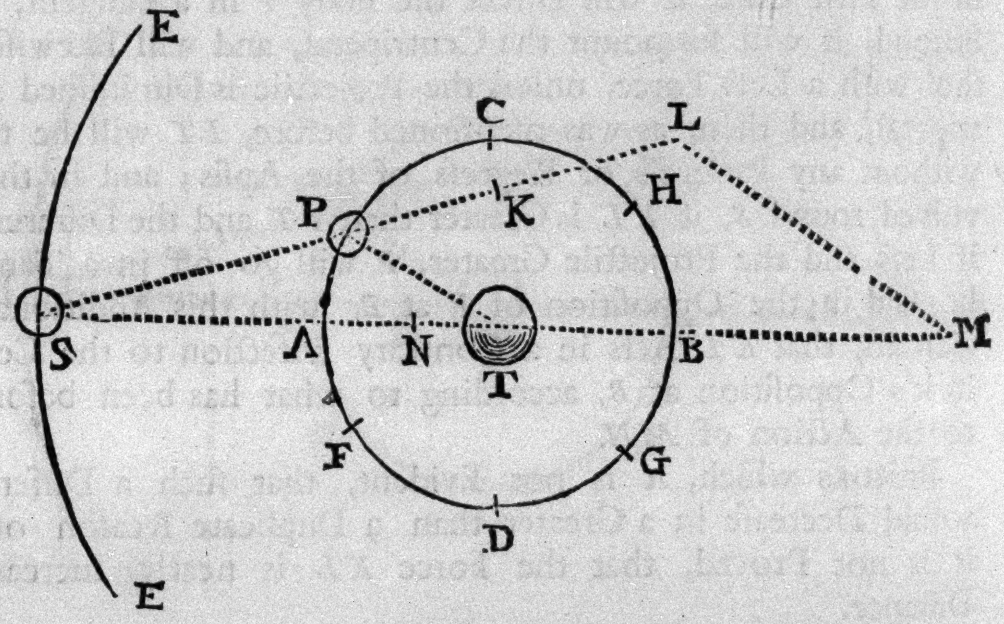
§. 19. The Eighth Corollary is, That ſince the Progreſs and Regreſs of the Apſides Depends upon the Decreaſe of the Centripetal Force, made in a Greater or Leſs than a Duplicate Reaſon of the Diſtance TP, in the Body's Paſſage from the Lower Apſis to the Upper, as alſo upon the like Increaſe in it's Return to the Lower, and conſequently, ſince this Progreſs and Regreſs is the Greateſt, where the Proportion of the Force in the Upper Apſis to the Force in the Lower Apſis Recedes the moſt from a Reciprocal Duplicate Reaſon of the Diſtances, It is manifeſt, that the Apſides in their Syzygiæ, by the Force KL or $NM-LM$ being Subſtracted, will Proceed Swifter, and in their Quadratures will Recede Slower, by the Addition of the Force LM.

But Firſt it muſt be Anſwer'd, that the Progreſs and Regreſs of the Apſides does not Depend upon the Decreaſe of the Centripetal Force, made in a Greater or Leſs than a Duplicate Reaſon of the Diſtance TP, in the Body's Paſſage from the Lower Apſis to the Upper, and upon a like Increaſe in it's Return to the Lower, Unleſs the Former Corollary is True, which we have ſhewn is not Certain, or rather that it is in Fact falſe; Beſides which, we ſhall Secondly add, that in General it is an Inconceivable Notion, that an Increaſe or Diminution of the Centripetal Force, according to a certain Proportion, ſhould be a Cauſe of the Progreſs or Regreſs of the Body P, or of it's Swifter or Slower Progreſs or Regreſs; ſince, if the Force from P to T is Diminiſhed, which before was Equal to the Projectile, ſo far as to Retain

Body *P* in it's Orbit, the Projectile will Direct it in a Tangent, when once it becomes Superior to the Centripetal, and on the Contrary, if the Force from *P* to *T* is Increas'd, it will be Superior to the Projectile, and conſequently will not be Retained in an Orbit, but will Tend to the Center; and therefore we may very juſtly Diſmiſs this Account of the Progreſſes and Regreſſes of the Apſides, as at the Beſt Precarious and Groundleſs, nevertheleſs at the End of theſe Corollaries we ſhall make a Particular Examination into the Fundamental Propoſitions of this Doctrine, That is, in our following Chapter.

The Ninth Corollary Explained and Examined. Sir Iſ. Newt. Princip. Prop. 66. Corol. 9. Dr. Gregory's Aſtronom. Prop. 12, 13. Lib. 4.

§. 20. THE Ninth Corollary is, That if any Body ſhould be Revolved about a Center in an Ellipſis by a Force, that was Reciprocally Proportional to the Square of it's Diſtance from ſuch a Center, and preſently after, in it's Deſcent from the Upper Apſis or Aux to the Lower, that Force ſhould by the perpetual Acceſs of a New one be Increas'd in more than a Duplicate Reaſon of the Diſtance Diminiſhed, it is manifeſt, that the Body by the Perpetual Acceſs of that New Force, being continually Impelled to the Center, would more Verge or Incline to that Center, than if it was Urged by the Force only Increaſing in a Duplicate Reaſon of the Diſtance Diminiſhed, and therefore would Deſcribe an Orbit Interior to the Elliptical, and in the Lower Apſis would Approach nearer to the Center than before, from whence the Orbit, by the Acceſs of this New Force, will become more Excentrical; If now the Force, in the Receſs of the Body from the Lower Apſis to the Upper, ſhould Decreaſe by the ſame Degrees as it Increaſed before, the Body would Return to the Former Diſtance, and therefore, if the Force ſhould Decreaſe in a Greater Reaſon, the Body being now Leſs Attracted will Aſcend to a Greater Diſtance, and ſo the Excentricity of the Orbit will be ſtill more Increas'd; Therefore, if the Reaſon of the Increment and Decrement of the Centripetal Force be Increas'd in each Revolution, The Excentricity will be Increaſed; and on the Contrary, the ſame Excentricity will be Diminiſh'd, if that Reaſon Decreaſes.



THUS in the Syſtem of the Bodies *T*, *P*, *S*, when the Apſides of the Orbit *PAB* are in the Quadratures, that Reaſon of the Increment and Decrement is the Leaſt, and the Greateſt, when they are in the Syzygiæ; If the Apſides are placed in the Quadratures, the Reaſon near the Apſides is Leſs and near the Syzygiæ Greater than a Duplicate one of the Diſtances, and from that Greater Reaſon Ariſes the Swifteſt Motion of the Aux; But if the Reaſon of the whole Increment and Decrement in the Progreſs betwixt the Apſides be Conſidered, it is Leſs than a Duplicate one of the Diſtances; The Force in the Lower Apſis is to the Force in the Upper Apſis in Leſs than a Duplicate Reaſon of the Diſtance of the Upper Apſis from the Umbilicus of the Ellipſis to the Diſtance of the Lower Apſis from the ſame Umbilicus; and on the Contrary, when the Apſides are placed in the Syzygiæ, The Force in the Lower Apſis is to the Force in the Upper in a Greater than a Duplicate Reaſon of the Diſtances; For the Forces *LM*, added in the Quadratures to the Forces of the Body *T*, Compound together Forces, which are in a Leſs Reaſon, and the Forces *KL* Subſtracted in the Syzygiæ from the Forces of the Body *T* leave Forces, which are in a Greater Reaſon; The Reaſon therefore of the whole Decrement and Increment in the Paſſage betwixt the Apſides is the Leaſt in the Quadratures, and the Greateſt in the Syzygiæ, and conſequently, in the Paſſage of the Apſides from the Quadratures to the Syzygiæ, it is continually Increaſed, and Increaſes the Excentricity of the Ellipſis, and in the Paſſage from the Syzygiæ to the Quadratures, it is continually Diminiſhed, and Diminiſhes the Excentricity.

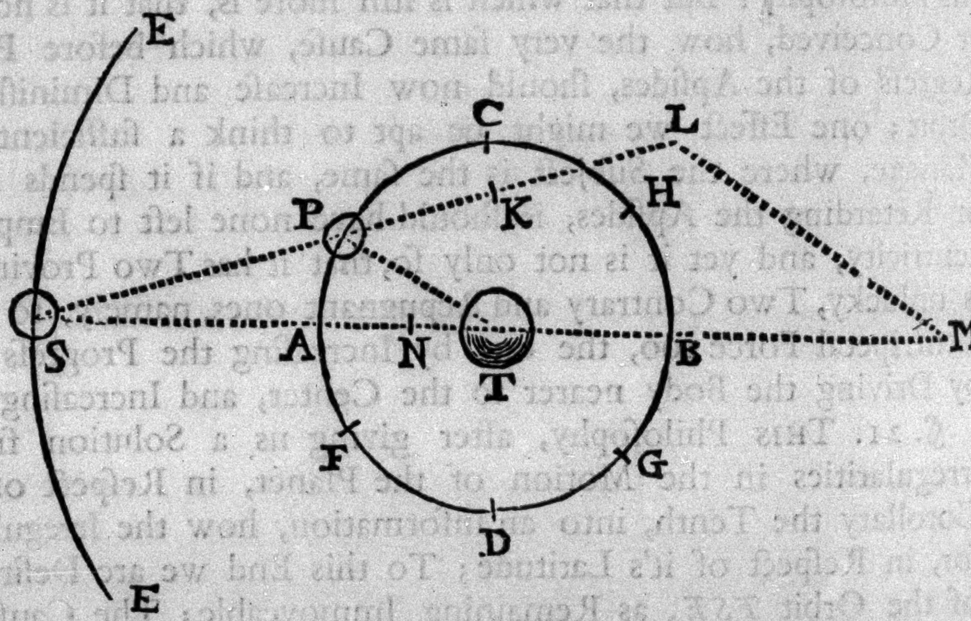
In Anſwer to which we ſay, that if the Body is more Attracted in it's Deſcent from the Upper Apſis to the Lower than in a Duplicate Reaſon of it's Diſtance Diminiſhed, by which in Conjunction with the Projectile it was before Retained in it's Orbit, the Centripetal being now Superior to the Projectile, and having overcome it ſo far, as by it's Exceſs to Draw the Body nearer to the Center in the Lower Apſis, it does not ſeem Reaſonable to allow the Projectile any farther Power to Raiſe the Body to a Greater Diſtance from the Center, for if one Force has once been actually Superior to another, how that ſhould Revive and be Reſtored to it again, which was Loſt, unleſs by the Will and Pleaſure of the Philoſopher, will not be ſo eaſily Imagin'd; for to ſay the Projectile Force continues to Puſh and Protrude the Body from the Center, notwithſtanding it is Surmounted by the Centripetal, is to affirm it to be an Innate, and therefore a Reviviſcent Force, when the Centripetal is taken off, which perfectly contradicts the Notion which this Philoſophy has of Matter, that it is perfectly Paſſive, and continues and perſeveres in a certain State of Motion or Reſt, till it is Diſturbed out of it; But here it ſeems, when a Body is Diſturbed out of it's Projectile Motion by a Superior Centripetal, when that Superiority of Centripetal Force is taken off, it can Re-aſſume it's Projectile without any new Impreſſion; which, what is it elſe but to Affirm, that the Projectile is an Innate Force in it, when yet all ſuch Innate Forces are actually Diſclaim'd by this Philoſophy? But that which is ſtill more is, that it is no ways to be Apprehended or Conceived, how the very ſame Cauſe, which before Produced the Progreſs and Regreſs of the Apſides, ſhould now Increaſe and Diminiſh the Excentricity of the Orbit; one Effect we might be apt to think a ſufficient Task for one Cauſe to Manage, where the Subject is the ſame, and if it ſpends it's Force in Accelerating or Retarding the Apſides, it ſhould have none left to Employ in Changeing the Excentricity, and yet it is not only ſo, that it has Two Provinces Aſſign'd it, but, which is unlucky, Two Contrary and Repugnant ones, namely, to Increaſe the Projectile and Centripetal Force too, the one by Increaſing the Progreſs of the Apſides, the other by Driving the Body nearer to the Center, and Increaſing the Excentricity.

The Tenth Corollary Explained and Examined. Sir Iſ. Newt. Princ. Prop. 66. Corol. 10. Dr. Gregory's Aſtronom. Prop. 15. Lib. 4.

§. 21. This Philoſophy, after giving us a Solution from it's Principles of the Irregularities in the Motion of the Planet, in Reſpect of it's Longitude, leads us, Corollary the Tenth, into an Information, how the Irregularities may be Accounted for, in Reſpect of it's Latitude; To this End we are Deſired to Conceive the Plane of the Orbit *TSE*, as Remaining Immoveable; The Cauſes of any Irregularities or Errors, as before Explained, are *NM*, *LM*; The Force *ML*, by Acting always according to the Plane of the Orbit *PAB*, will never Diſturb the Motion, in Reſpect of Latitude, nor yet the Force *NM*, where the Nodes are in the Syzygiæ, becauſe it Acts according to the Plane of the ſame Orbit, but when the Nodes are in the Quadratures, it chiefly Diſturbs them, and by continually Drawing *P* from the Plane of it's Orbit, it Diminiſhes the Inclination of the Plane in the Paſſage of the Body from the Quadratures to the Syzygiæ, and again Increaſes the ſame in the Paſſage from the Syzygiæ to the Quadratures, from whence it comes to paſs, that, whilſt the Body is in the Syzygiæ, the Inclination of the Plane is the Leaſt, and Returns to it's Former Magnitude nearly, when the Body Arrives at the next or ſucceeding Node; But, if the Nodes are placed in the Octants after the Quadratures, that is, betwixt *C* and *A*, and *D* and *B*, it will be found from what has been already Explained, that, in the Paſſage of the Body *P* from either of the Nodes to the 90th Degree, the Inclination of the Plane is perpetually Diminiſhed, afterwards in the Paſſage thro' the next 45 Degrees, ſo far as to the next Quadrature, the Inclination is Increas'd, and then thro' the next 45 Degrees to the ſucceeding Node it is Diminiſhed; The Inclination therefore is more Diminiſhed than Increaſed, and for that Reaſon is always Leſs in the Subſequent Node, than in the Preceding; And by the ſame Reaſoning, the Inclination is more Increas'd than Diminiſhed, where the Nodes are in the other Octants, betwixt *A* and *D*, *B* and *C*; Therefore the Inclination, where the Nodes are in the Syzygiæ, is the Greateſt of all: In the Paſſage of the Nodes from the Syzygiæ to the Quadratures, in each of the Appulſes or Arrivals of the Body to the Nodes, the Inclination is Diminiſhed, and becomes the Leaſt of all, when the Nodes are

are in the Quadratures, and the Body in the Syzygiæ; afterwards it Increaſes by the ſame Degrees, by which it before Decreas'd, and, when the Nodes Arrive at the next Syzygiæ, the Inclination Returns to it's firſt Magnitude.

In Anſwer to which, First, it is not Proved, That, when the Nodes are in the Syzygiæ, the Forces, *LM*, *MN*, do not Diſturb the Motion of the Body *P*, in Reſpect of Latitude, if they do, when the Nodes are placed in the Quadratures; that is, let *EST*, or *ETE*, be an Immoveable Plane, and let *AB* be the Line of the Syzygiæ, whilſt the Plane *ACB* is Elevated above the Plane *ETE*, and *ADB* Depreſſed below it, it is ſaid, that *LM*, *MN* cannot make any Diſturbance in the Orbit, altho' it is Acknowledged they may, when *CD* is the Line of the Nodes, and *CBD*, *CAD*, the Planes, are the one Depreſs'd below, and the other Elevated above the Immoveable Plane *ETE*, which ſeems to be Impoſſible; for *MN* muſt as much Draw *P* towards *S*, in ſome oblique Poſition or other, from *C* or *D*, which are not in the ſame Plane with *AB* the Line of the Nodes, as *MN* will Draw *P* from *A* or *B* towards *S* in ſome oblique Poſition or other, where the Line of the Nodes is *CD*, and conſequently, not in the ſame Plane with *A* or *B*; That is, ſuppoſing *EST* or *ETE* to be the Immoveable Plane, when the Nodes are in the Syzygiæ, *D* and *C* will be the one Depreſſed below, and the other Elevated above it, and when they are in the Quadratures, *B* and *A* will be ſo, and therefore in both Caſes, there will be a Diſturbance from the Force *MN*, or elſe in neither; for if *MN* Acts Parallel to *TS*, or the Plane *ETE*, which it is ſuppoſed to do, it will Attract *P* in *C*, as well as in *A*, when Raiſed above it, to a Point nearer to that Plane, than it would by it's own Force have otherwiſe Deſcribed.



Second, When the Nodes are in the Syzygiæ, that is, when *C* is Raiſed above the Plane *ETE*, without a Compariſon of them to what they are in the Quadratures, it is not Certain, that the Force *MN* has thoſe Effects, which are ſuppoſed, of Acting both by an Exceſs and a Defect upon the Body *P*, according to what has been before Argued, that is, by way of Attraction and Repulſion.

Third, The Reſolution of the Force *SL* into *LM*, *MS*, is likewiſe not Proved, as we have already ſhewn.

Fourth, The Accelerating of the Body *P*, by the Force *MN*, from *C* to *A* is Inconſiſtent with it's Inclining the Plane *BCA* towards *ETE*, ſince the one Draws it according to the Orbit, and the other Draws it down from it.

On the other Hand, when the Nodes are in the Quadratures *C*, *D*, there are the following Objections againſt the foregoing Account.

First, That *MN* cannot, by continually Drawing the Body *P* from the Plane of it's Orbit, Diminiſh the Inclination of the Plane in the Paſſage of the Body from the Quadratures to the Syzygiæ, and again Increaſe the ſame in the Paſſage of it from the Syzygiæ to the Quadratures; Becauſe, ſuppoſing *CAD*, where the Nodes are *C* and *D*, to be Raiſed above the Immoveable Plane *ETE*, or Depreſs'd below it, for the Caſe is the ſame; The firſt Queſtion will be, when the Body *P* is in *C* or *A*, if the Force *MN* has an Effect to Draw it towards the Plane *ETE*, how the Body *P* will ever Riſe from that Plane, in it's Paſſage from *A* to *D*; For if the Force *MN* is Increas'd in it's Progreſs from *C* to *A*, it is not ſhewn, why it ſhould not as gradually Draw the Body *P* to the Immoveable Plane, and conſequently keep it continually ſo Inclin'd, as it is at *A*; for, tho' *MN* ſhould be Gradually Diminiſh'd from *A* to *D*, it would not be Conſequent, that *P* ſhould Raiſe it ſelf from the Plane *ETE*, unleſs it was

Endued with ſome Innate Force, which ſhould thus Aſſiſt it's Elevation, which is not Acknowledged.

Second, If this could be done, it is not Evident, as was ſaid before, how *MN* ſhould Accelerate *P* in it's Paſſage from *C* to *A*, which is Affirmed, and yet Perform the other Province of Drawing it down from it's Orbit, which Attraction from it's Orbit muſt Retard it's Motion.

Third, It is not ſhewn, how *CAD*, conſidered as belonging to a Secondary Planet, is the Part of an Orbit Inclined to *ETE*, unleſs by ſuppoſing, that *P* Deſcribes an Orbit Inclined to it, which, in Reſpect of the Earth's Orbit about the Sun in Particular, the Moon for Inſtance moſt certainly does; But why it ſhould do it, is far from being Explain'd by this Philoſophy, which Preſumes and Suppoſes the Inclination of the Orbit, and then makes an Arbitrary uſe of the Force *MN* to Draw the Body *P* out of that Orbit; whereas, if the original Inclination of the Orbit of the Secondary Planet to the Primary was not Suppoſed, all the other Reaſonings upon it would be of no Importance; If it is Suppoſed and not Accounted for from any Principles in this Philoſophy, it's Deviations from it's own Orbit may be as well Suppoſed and as juſtly Accounted for from ſuch a Suppoſition, as it's Grand Deviation from the Orbit of it's Primary Planet is by a mere Suppoſition only.

Fourth, Since *MN* Acts according to the Plane *ETE*, or in a Direction Parallel to it, and not in any oblique one from *A* to *S*, when the Body *P* is in *A*, it cannot Diminiſh the Angle of Inclination, tho' it is there Imagined to have the Greateſt Effect, the Angle of Inclination being there the Leaſt; becauſe *P*, when in *A*, Gravitates to *S* in a Line Parallel to *TS*, and conſequently, the Force *MN* will only Draw *P* from it's Orbit at an Equal Diſtance from *TS*, and therefore will not more Incline *TA* or *TP* to it, than if there was no ſuch Attraction; for tho' it muſt be Confeſſed, that if *P* at *A* is Drawn out of it's Orbit, in a Parallel Direction to *TS*, to any Point next *A* towards *S*, if a Line is Drawn from *T* to ſuch a Point, it will have a Leſs Inclination to *TS*, than *TA* has, yet when *P* is in ſuch a Point, if it is Attracted to *T* in a Parallel Direction to *TA*, which there is as much Reaſon for, as that it ſhould be Attracted to *S* in a Parallel Direction to *TS*, the Angle of Inclination of *PT* to *TS*, will be the ſame in both Caſes; for beſides, if *P* in *A* is Drawn into another Plane, the Center ſhould be Removed into the ſame Plane, otherwiſe there could be no Revolution, according to *Prop.* 1. of this *Philoſophy*, which ſuppoſes an Immoveable Plane neceſſary to the Revolution, and therefore if *P* Shifts from one Plane to another, when the Center is the ſame, the Foundation of all the Preſent Reaſoning is Deſtroyed.

Fifth, The Reſolution of the Force *SL* into *LM*, *MS*, as before, is Unproved, and whatever has been alledged in Affirmation of theſe Principles.

Sixth, In the Paſſage of *P* from *A* to *D* all the Arguments are the ſame againſt the Plane of the Orbit *CAD* Increaſing in it's Inclination to *ETE*, which were us'd againſt it's being Diminiſh'd in the Paſſage of the Body from *C* to *A*.

As to the other Progreſſes of the Body from *D* to *B*, and *B* to *C*, it is not neceſſary to conſider them, unleſs upon this Account, that there is one Objection, which lies againſt them in Particular, namely, that it is not certain, that a Defect of the Force *MN* will have the ſame Effects, as the Exceſs of it.

There will be therefore nothing more to ſay on this Head, Excepting, that when the Nodes are Placed in the Octants, or in any other Poſition, The Reaſoning, with ſome Circumſtantial Allowances, will be the ſame.

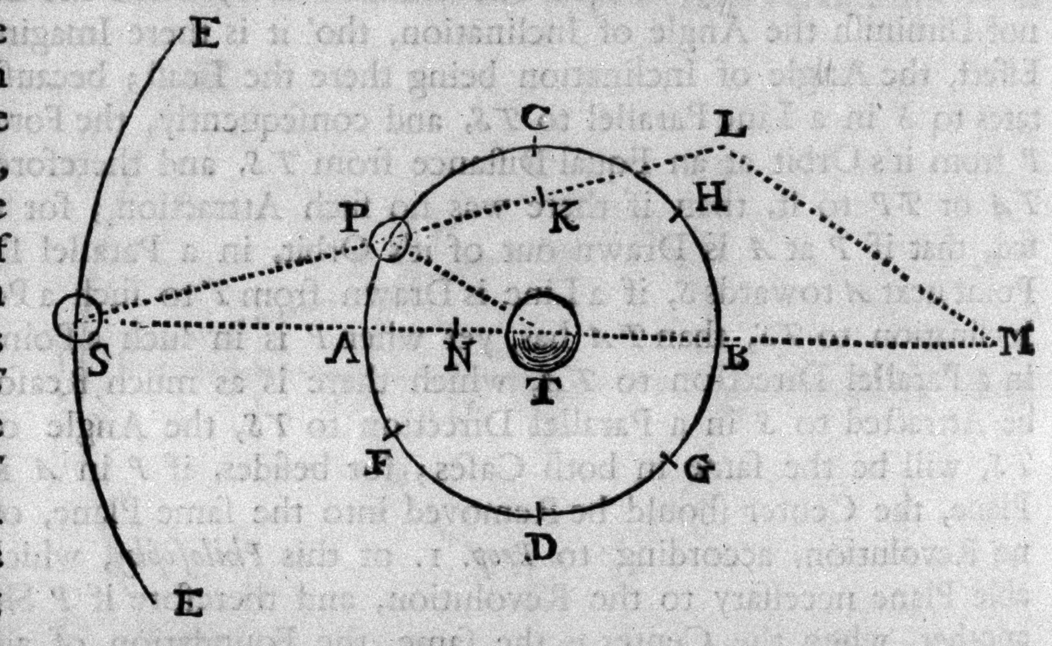
§. 22. The Eleventh Corollary is, That, ſince the Body *P*, when the Nodes are in the Quadratures, is continually Drawn from the Plane of it's Orbit, and that towards the Part *S* in it's Paſſage from the Node *C*, thro' the Conjunction *A*, to the Node *D*, and to the contrary Part, in it's Paſſage from the Node *D*, thro' the Oppoſition *B*, to the Node *C*, it is Evident, that, in it's Motion from the Node *C*, the Body Perpetually Recedes from the Plane of it's Orbit, till it comes to the Next Node, and conſequently, in this Laſt Node paſſes at a Greater Diſtance from the Firſt Plane *CD*, thro' the Immoveable one *ETE*, that is, not in the other Node *D* of that Plane, but in a Point, which Verges or Inclines from thence towards the Body *S*, and therefore is a New Place of the Node Tending Backwards, that is rather towards *A* than *B* in Reſpect of *D*; and by a like way of Reaſoning, the Nodes will

The Eleventh Corollary Explained and Examined.
Sir Iſ. Newt. Princip. Prop. 66. Corol. 11.
Dr. Gregory's Aſtronom. Prop. 14. Lib. 4.

Recede from the Places they obtained before, in the Paſſage of the Body from *D* to *C*; and therefore the Nodes, when Situate in the Quadratures, have a Continual Regreſs, but when in the Syzygiæ, where the Motion of *P* Receives no Diſturbance in Reſpect of it's Latitude, the Nodes are Quieſcent; and in the Intermediate Places, they Share in both theſe Influences, and by Conſequence, being either Actually carry'd Backwards, or being Actually Stationary in each Revolution, they find a Regreſſion, or are carry'd *in Antecedentia*.

But in the First Place, we muſt be Allow'd to obſerve, that this Corollary Depends upon the Former, ſince, if the Body *P* is not Drawn from it's Orbit in the ſame Method, which this Philoſophy Preſcribes, as, by what is before Proved, it is not, there can be no Argument from thence for the Regreſſion of the Nodes.

Second, Tho' that were Granted, namely, that *P* was Drawn from it's Orbit in it's Paſſage from *C* to *D*, it would not follow, that there ſhould be a Regreſſion of the Nodes; Becauſe *MN* would only produce this Effect at the moſt, to wit, that *P*, ſuppoſing the Nodes in the Quadratures, would be Attracted to a Plane below *CAD*, and which would have a Leſs Inclination to the Plane *ETE*, that is, *CAD* would Vibrate upon the Axis *CD*, and conſequently there would be no Regreſſion of the Nodes in the ſame Plane, but they would be Quieſcent, and only the Inclination of *CAD* to *ETE* would be Diminiſhed and Increaſed by Turns.



Third, This is the more Confirm'd, becauſe it is Inconceiveable, when the Nodes are in the Quadratures *C* and *D*, that they ſhould Recede from thoſe Points, if *CAD* is conſidered as a Plane Inclined to *ETE*; ſince *P*, when it is in *C* or in *D*, muſt have the ſame Attraction to *S* from the Force *MN*, and therefore, if the Node is in *C*, notwithſtanding the Attraction of *MN*, it muſt be likewiſe in *D*, notwithſtanding ſuch Attraction; If yet the Node *D* is Attracted by *MN* to a Point on this Side *D* towards *A* or *S*, in like Manner *C* ought to be, from whence the Line of the Nodes will not be carry'd in a Revolution *in Antecedentia*, but Parallel to *CD* towards *A* or *S*.

Fourth, If the Plane *CAD* Interſects the Plane *ETE* on this Side *D*, it's Inclination to *ETE* will be in a Greater Angle, and therefore Increas'd by the Force *MN*, whereas by the Preceding Corollary it was Diminiſhed by the ſame Force, which is a Contradiction.

Sir Iſ. Newt. Prop. 66. Corol. 12.

Fifth, All the Objections, which lie againſt the Preceding Corollaries and this Propoſition, are Equally certain here; As to the Twelfth Corollary, which is, that all the Errors, Deſcrib'd in theſe Corollaries, are ſomething Greater in the Conjunction of the Bodies *P* and *S*, than in their Oppoſition upon Account of their Greater Generating Forces *NM* and *LM*, we ſhall omit the Examination of it, as Depending on the Former, and Proceed to the Thirteenth.

The Thirteenth Corollary Explained and Examined. Sir Iſ. Newt. Princip. Prop. 66. Corol. 13.

§. 23. The Thirteenth Corollary is, That ſince the Reaſons of theſe Corollaries do not Depend upon the Magnitude of the Body *S*, all, which has been ſaid before, Obtains, where the Magnitude of the Body *S* is Determined or Suppoſed to be ſuch, that the Syſtem of the Two Bodies *T* and *P* may be Revolved round it; and from the Increaſe of the Body *S*, and conſequently of it's Centripetal Force, from which the Errors of the Body *P* do Ariſe, all thoſe Errors will become (in Equal Diſtances) Greater in this Caſe than in the other, where the Body *S* is Revolved about the Syſtem of the Two Bodies *P* and *T*.

In Reply to which it muſt be First ſaid, that nothing of what foregoes is True, and therefore, that which is here Alledged cannot be, ſo far as it is a Conſequence of it.

Second, This is only an Accommodation of the Preſent Scheme to the Revolution of the *Earth* and *Moon* about the *Sun*, and for that Reaſon does not concern the Argument we are now Conſidering, Namely, the Merits and Truth of the Preſent Hypotheſis in General; tho' it may be ſaid, that by the Increaſe of the Body *S*, and it's Centripetal Force, ſomething more than an Increaſe of the foregoing Errors may be Produced, whilſt the Projectile Forces Continue the ſame, namely, the Attraction or Detruſion and Compreſſion of the whole Syſtem of *P* and *T* to the Center *S*.

As to what Reſpects the 14th, 15th, 16th, and 17th Corollaries, it will not, after what has been ſaid, be neceſſary to Enter into a Diſquiſition of them, ſince they chiefly Relate to a Calculation of theſe Errors and Irregularities in the Primary and Secondary Planets, but if ſuch Irregularities are not juſtly Accounted for, it will be as Certain, that their Quantities and Proportions are not.

The Eighteenth and Nineteenth Corollaries Explained and Examined. Sir Iſ. Newt. Princip, Prop. 66. Corol. 18, 19.

§. 24. The Eighteenth Corollary is an Application of this Doctrine to the Flux and Reflux of the Sea, in order to ſolve which, it is ſuppoſed, that ſeveral Fluid Bodies are Moved round the Body *T*, at Equal Diſtances from it, with the ſame Laws and Conditions, with which the Body *P* is Revolved about the ſame Body *T*, Afterwards from theſe Fluid Bodies becoming Contiguous a Fluid Ring is Conceiv'd to be form'd Round and Concentrical to the Body *T*; If ſo, the ſeveral Parts of the Ring, by performing all their Motions according to the Laws of the Body *P*, will Approach nearer to the Body *T*, and will be moved Swifter in the Conjunction and Oppoſition of them and the Body *S*, than in the Quadratures; and the Nodes of this Ring, or it's Interſections with the Plane of the Orbit of the Body *S* or *T*, will be Quieſcent in the Syzygiæ, but out of the Syzygiæ they will be Moved *in Antecedentia*, or which is the ſame, will have a Regreſſion, and this Regreſſion will be the Swifteſt in the Quadratures, and Slower in the other Intermediate Places; The Inclination of the Ring will be alſo Vary'd, and it's Axis will Oſcillate in each of the Revolutions, and when the whole Revolution is Performed, it will Return to it's former Situation, unleſs ſo far as it is Carry'd Round by the Preceſſion of the Nodes.

After this, it is, in Corollary the Nineteenth, ſuppoſed, that the Globe of the Body *T*, which conſiſts of a Matter which is not Fluid, is Enlarged and Extended to this Ring, and in a Channel, which is Excavated thro' it's Circuit, contains the Water, and is uniformly Revolved with the ſame Periodical Motion about it's Axis; This Liquid being Accelerated and Retarded by Turns, as in the foregoing Lemma, will be Swifter in the Syzygiæ and Slower in the Quadratures than the Superficies of the Globe, and conſequently will have a Flux and Reflux in the Channel after the ſame Manner as the Sea has; The Water by Revolving about the Quieſcent Center of the Globe, if the Attraction *S* is taken away, will acquire no Motion of a Flux and Reflux, and the ſame Reaſoning Obtains in a Globe, which Proceeds uniformly *in Directum*, and in the mean Time Revolves about it's own Center, by the 5th *Corol.* of the Laws, as alſo in a Globe, which is uniformly Drawn from it's Rectilineal Courſe, by the 6th *Corol.* of the Laws, but let the Body *S* Approach, and the Water will be immediately Diſturbed by it's Unequal Attraction, for the Attraction of the Water which is Nearer will be Greater, and of that which is Remoter Leſs; But the Force *LM* will Draw the Water Down in the Quadratures, and will make it Deſcend to the Syzygiæ, and the Force *KL* will Draw it upwards in the Syzygiæ, and will ſtop the Deſcent of it, and make it to Aſcend as far as the Quadratures.

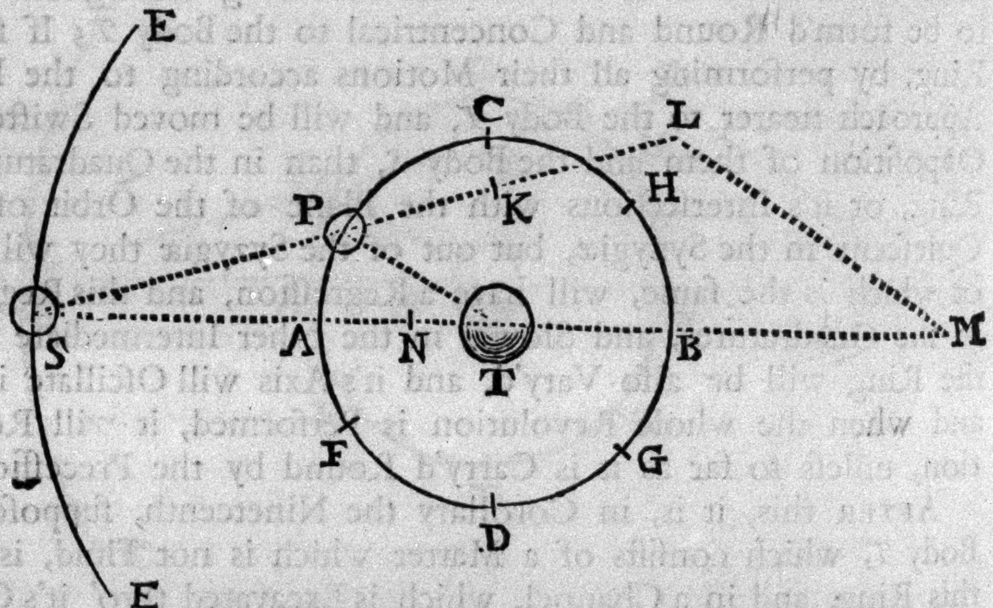
This Account of the Flux and Reflux of the Sea ſo evidently Depends on the Laws of Motion in the Revolution of a Secondary Planet about a Primary, that there will be no Occaſion very Particularly to conſider it, after a Confutation of thoſe Laws.

We ſhall only First ſay, That if all thoſe ſuppoſed Laws of Revolution were True, it is not very Conceiveable, how they are Applicable to the Ocean, unleſs it made ſuch a Ring in it's Channel about the *Earth*, and was Inclin'd in the ſame Manner to the Plane *ETE*, as the Orbit of the *Moon* is, neither of which Things are in Fact true, that we know of.

Sir Iſ. Newt. Princ. Lib. 3. Prop. 6. Corol. 1, 2.

Second, If this were ſo, ſince Water in this Philoſophy is ſuppoſed to be of the ſame Homogeneous Matter with any other Solid Matter, and only Differs in it's Quantity, which is Proportional to it's Weight, and not in it's Real Nature and Conſtitution, ſeeing, in it's Revolution about the Center *T*, one Kind of Matter, as Water, is Diſturb'd, there is no Reaſon Given, nor can be, why another ſhould not, as Earth, or even the whole Surface of the Globe it's ſelf, if it had the ſame Inclination with this Imaginary Ring of Water, unleſs that it in Fact is not.

Third, Let this be Granted, yet the Action of *S* would not obtain the Effects Propoſed, for it could only Diſturb every Parcel of the Fluid Ring, as it Paſſed, according to the Laws Obſerved in the Revolution of *P*; the Conſequence of which would be, that in the Paſſage of any Part of the Fluid Ring from the Quadratures to the Syzygiæ, there would be a Quicker Motion, which would again Flag and Languiſh in it's Paſſage from the Syzygiæ to the Quadratures, in Caſe each Fluid Part preſerved it's Situation, and what Acceleration any Part acquired in it's Paſſage from *C* to *A*, or Retardation from *A* to *D*, would be ſtill Increas'd by the Succeeding Revolutions; From whence would Reſult nothing of a Flux or Reflux, as is Pretended and Imagined, but a Puſhing or Protruding forwards of the ſame Fluid Ring, according to the Velocities Impreſs'd from *C* to *A*, and the Retardations of them from *A* to *D*; If the Parts of the Fluid Ring do not preſerve the ſame Situation in ſucceeding one another, but the preceding Parts, as betwixt *C* and *A*, which are mov'd Swifter, are heaped and thrown upon thoſe which are mov'd Slower, as betwixt *A* and *D*, All the Conſequence will be, that they will now move Slower from *A* to *D*, than before betwixt *C* and *A*; from whence it will follow, in either Caſes, that the Parts of the Fluid Ring will proceed Swifter betwixt *C* and *A*, and Slower betwixt *A* and *D* without any Imaginable Reflux, which we can at preſent Apprehend, that is, that the Waves, by the Accelerating and Retarding Action of the Moon, will ſtill ſucceed one another in a Swifter or a Slower Motion, without any Reflux or Regreſſion of Them.



Fourth, And as to the Waves of the Fluid Ring being Raiſed or Elevated in the Quadratures, and Depreſs'd in the Syzygiæ, it will be ſtill Leſs to the purpoſe of Accounting for a Flux and Reflux of them, in this way of Raiſing and Depreſſing them, tho' poſſibly Another, which we have already Explained, may be the only one to ſolve this Difficulty; However, ſince the Preſent Elevation and Depreſſion of the Waters is Aſcribed to the Celerity or Slowneſs of their Motion in the Syzygiæ and Quadratures, and ſince that can have no ſuch Effect, namely, to Elevate or Depreſs them, as has been Proved in the Preceding Article, and before in the Examination of that Corollary, which Reſpects the Enlargement of the Orbit of *P* in the one Caſe and the Depreſſion of it in the other, ſuch an Explication can be of no Force or Significance.

Book 1. Chap. 5.

Fifth, Since the Water is Swifter in the Syzygiæ and Slower in the Quadratures, it ſhould be more Depreſſed in the Syzygiæ, and more Prominent and Convex in the Quadratures, whereas the Contrary is here Affirmed, That it is more Elevated in the Syzygiæ, and more Depreſſed in the Quadratures; 'Tis true, the Forces *LM* in the Quadratures, and *KL* in the Syzygiæ are called in to Remedy this Defect; But is it not ſomething Strange, that the Two Acceſſory Forces, *LM* and *KL*, ſhould be able to Effect more than the Main and Principal Force *MN*, by which this whole Machine is Acted? And it is Evident, that *MN*, according to this Philoſophy, would Accelerate the Waters in the Syzygiæ, and Retard them in the Quadratures, and by Conſequence, Depreſs them in the one Caſe and Raiſe them in the other, in Contradiction to Fact and Experience.

SIXTH, 'Tis true LM in the Quadratures Drives the Waters towards the Center by Acting from P to T, and KL in the Syzygiæ Draws them from it, and both Aſſiſt to the Elevation of the Waters in the Syzygiæ, and the Depreſſion of them in the Quadratures, and if we ſuppoſe farther MN not to Accelerate the Waters in the Syzygiæ, but to Draw and Attract them from the Center, which Effect it is ſaid likewiſe to have, MN would alſo Contribute to the Production of the ſame Phænomenon; but here lies an Objection againſt this Account, that tho' MN, KL, may be eaſily Imagined to be Attractive of the Waters in the Conjunction A, and conſequently to Raiſe them, yet how the ſame Forces ſhould be Repulſive of them in the Oppoſition B, is not nor can be Explained, as we have before ſhewn in the Examination of that Corollary.

SEVENTH, The Fluid Ring it ſelf, as was before Intimated, is an Imaginary Buſineſs, and cannot be Juſtified from any Thing which we find in Nature; for how is the Ocean like ſuch a Fluid Ring, which Spreads it ſelf like one Vaſt Pond, Broken and Interrupted with Various Continents of Land, and has no more the Reſemblance of a Fluid Circulating in one continued Courſe about the Center of the Earth, than of any Thing elſe to which it is the moſt unlike: We ſhall not therefore conſider this Account of the Flux and Reflux of the Sea any farther, tho' there are ſeveral other Difficulties, which lie againſt it, nor yet the Doctrine of the Planets, as it is Compared with what is Obſerved in their Revolutions, albeit we might there likewiſe ſhew, that there are as many Adjuſtments of Calculation and Suppoſitions made to Effect, that the Appearances of Nature ſhould anſwer theſe Hypotheſes in Reaſoning, as we have ſhewn that there are Adjuſtments of Reaſoning, and Suppoſitions made to Anſwer theſe Appearances, and particularly in the Theory of the Moon, ſo much Celebrated in the preſent Philoſophy. But our Deſign being only to Inquire into the *Principles* of *Philoſophy*, we ſhall not at preſent trouble our ſelves to Examine, how they agree with what is propoſed to be ſolv'd by them, ſince, if the Principles are Falſe, it is Impoſſible the Solutions ſhould be True; Thus in the Magnet, if the *Striate Particles* of *Des-Cartes* are a Romance and a Ludicrous Fiction, as all will acknowledge them to be, who know any Thing in theſe Studies, tho' they ſhould be pretty well Adapted to the Solution of ſeveral Phænomenons in that Body, no one of Senſe would yet take them to be any thing Real or Exiſting in Nature.

Concluſion of this Chapter. Dr. Gregory's Aſtronom. Prop. 35. Lib. 1.

§. 25. FROM an Examination into the Principles, by which the Irregularities of the Planets are Explained, we might Proceed to the Doctrine of Comets, as to the Orbits of which it is not fully Determined, whether they are more truly Repreſented by a Parabola, an Hyperbola, or an Ellipſis, and all that is Proved, or Endeavour'd to be Proved, is, that a Comet is Moved in ſome Conick Section or another, in one of whoſe Umbilicus's is the Sun; The Parabola is rather choſen, but without any certain Proofs of it's being the Trajectory of the Comet. In Truth and Reality, if the Orbits of the Planets are not regularly and truly Adjuſted by this Philoſophy, the Extravagant Deviations of the Comets from ſuch Rules of Motion or Acting muſt be Leſs Accounted for. It will not therefore be neceſſary to Inquire into the Orbits, which the Comets Deſcribe, ſince the Cauſes Aſſign'd of ſuch Revolutions are Precarious or Falſe, all that will be Expedient to do, if any Thing, is to know upon what Reaſons the Caudæ or Tails of them are Explained, nor will this be neceſſary, ſince they do not Proceed upon any Mathematick Principles, but upon the Gueſſes and Conjectures of Men, which are Various according to Dr. *Gregory*, *Prop.* 4. *Lib.* 5, and which perhaps we may find ſome other Time to Examine. We ſhall therefore Conclude theſe Corollaries with an Admiration, that thoſe Philoſophers, who pretend that their Reaſon, not their Religion, is the only Diſtinction of Men from Brutes, ſhould fall into Notions, which have no Reaſon to ſupport them, that they ſhould Run themſelves into all the Follies and Abſurdities Imaginable, as is Evident, when they come to be Examined by Common Senſe, and at the ſame Time Think to ſecure themſelves in all theſe, by Drawing of Lines, and Circles, and Angles about them, as a Kind of Fortification and Defence for ſuch Errors.

CHAP. VI.

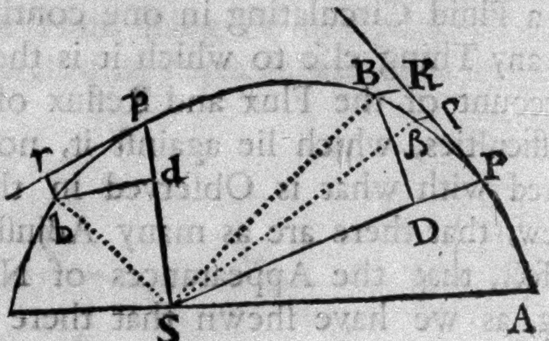
Concerning thoſe Propoſitions, which Relate to the particular Forces, by which the Planets are Retained in their Orbits.

The Remaining Propoſitions in the Phyſical Aſtronomy Explained and Examined.

§. 1. WE have in the Two foregoing Chapters Examined into ſeveral Propoſitions Relating to the Phyſical Aſtronomy, but in the Ninth and Tenth Corollaries of the Former Chapter, where the Apſides and the Excentricity of the Orbits were Conſidered, we were Obliged to take ſome Things for Granted, which were not there Proved, and which we Promiſed to ſubmit to a farther Inquiry, which when we have done, there will be ſcarcely any Thing Remaining, which will Require our Labour or Trouble in theſe Matters.

The Firſt Propoſition Explained. Sir Iſ. Newt. Princip. Prop. 6. §. 2. Lib. 1. Dr. Gregory's Aſtronom. Prop. 37. Lib. 1.

§. 2. THE Firſt Propoſition, which we ſhall take Notice of is, that if a Body is Projected according to the Direction of any Right Line, and at the ſame Time is Urged by a Centripetal Force Tending to the Center S, ſo that, by a Motion Compounded of theſe Two, it ſhould Deſcribe the Curve APp, to which in any Point P, let PR be a Tangent, and from the Point B, which is the next to P, let BD be Drawn Perpendicular to the Right Line SP, and BR Parallel to it, and let there be the like Conſtruction made at any other Point of the Curve, as p; It is Affirmed in this Propoſition, that the Centripetal Force in P is to that in p, as $\frac{Spq \times bdq}{br}$ to $\frac{SPq \times BDq}{BR}$, or, that the Centripetal Force in P is Reciprocally, as $\frac{SPq \times BDq}{BR}$, where $PRBD$ is Indefinitely Small.



FOR, Let the Force in P, which Tends towards the Center S, be called V, Let the Time in which the leaſt Arc PB is Performed, or in which the Body by it's *Vis Inſita* alone would Deſcribe the leaſt Tangent PR, be called T, Let the Centripetal Force, Acting from the Point p, be called v, and Let the Time, in which pb is Performed, or in which the Tangent pr would be Deſcribed, be called t; Let alſo the Arc $P\beta$ be Taken, which the Body Deſcribes in the ſame Time, in which it Deſcribes the Arc pb, and Let $\beta\rho$ be Drawn Parallel to PS; Theſe Things being ſuppoſed, the Reaſon of the Small Line BR to that of br is Compounded of the Reaſon of BR to $\beta\rho$, and of the Reaſon of $\beta\rho$ to br, or $\frac{BR}{br} = \frac{BR \times \beta\rho}{\beta\rho \times br}$, but, by Lemma 11th of the Method of Naſcents and Evaneſcents, BR is to $\beta\rho$ as PBq to $P\beta q$; and theſe Arcs PB, $P\beta$, ſince they are the Leaſt or the Evaneſcent ones, are as the Triangles of an Equal Height BSP, βSP, that is, by *Prop.* 1*ſt*, and 2*d*. of this Philoſophy, as the Times, in which they are Deſcribed by a Radius Drawn to S, or by the Preſent Conſtruction, as the Times, in which the Arcs PB, pb are Deſcribed; Again the Small Line $\beta\rho$ is to that of br as the Cauſes, which Produce them, that is, as the Centripetal Force in P to that in p, and therefore the Reaſon of the Small Line BR to that of br, is Compounded of the Duplicate Reaſon of the Times, in which PB, pb, are Deſcribed, and of the Reaſon of the Centripetal Force in P to that in p, that is, $\frac{BR}{br} = \frac{T^2}{t^2} \times \frac{V}{v}$, and therefore $\frac{V}{v} = \frac{BR \times t^2}{br \times T^2}$; But by *Prop.* 1, of this Philoſophy T is to t as the Area SBP is to the Area Sbp, or as the Double of the Former $SP \times BD$ is to the Double of the Latter $Sp \times bd$; If therefore in the Place of the Reaſon $\frac{t^2}{T^2}$ we ſubſtitute $\frac{Spq \times bdq}{SPq \times BDq}$, $\frac{V}{v}$ will be $= \frac{BR \times Spq \times bdq}{br \times SPq \times BDq}$, that is, by Dividing both by $BR\times br$, $V.v::\frac{Spq\times bdq}{br}.\frac{SPq\times BDq}{BR}$; and therefore the Centripetal Force in P to S is Reciprocally, as the Solid $\frac{SPq\times BDq}{BR}$. *Q.E.D.*

The Vis Inſita *or the Projectile Force and the Gravitating Precarious and Uncertain.*

§. 3. In Anſwer to which, we muſt obſerve, Firſt, That the Body is ſuppoſed to be Projected in the Tangents PR and pr, or to Proceed by it's *Vis Inſita* in thoſe Tangents, which Projection or *Vis Inſita* is wholly Undefined and Undetermined in this Figure, whether it is Conſtant or Mutable, and what Proportion it bears to the Centripetal Force, which Tends towards S; It is only ſaid, in General and at Large, to be ſuch, as, in Conjunction with that Force, will Deſcribe the Curve APp. We have given our Objections againſt the Projectile Force, or *Vis Inſita*, and the Gravitating Force or Centripetal already, which need not be here Repeated, upon the Truth of which, yet, all this Reaſoning ſtands; the Propoſitions here quoted for the Support of this Demonſtration have been alſo Examined and Refuted, and we ſhall only ſay farther, that, if the Planets are Revolved round the Sun by it's Expanſive Force Carry'd about it's Axis by a Perpetual Gyration, this whole Doctrine of Projectile and Centripetal Forces will fall to the Ground; and that they are thus Revolved, there are ſome Arguments to make us Believe it, and none, which can perhaps in Fact Diſprove it, and therefore this Imaginary Deſcription of Curves is Falſe or Uncertain, and far from being Mathematically Evinced or an Indiſputable Propoſition of Geometry.

The ſeveral other Suppoſitions in this Demonſtration Examined.

§. 4. Again, In the Second Place, if this *Vis Inſita* or Projectile Force is no more than ſuch, which, with the Centripetal Force here particulary Defined and Expreſſed, is capable of Deſcribing the Curve APp, this is only Telling us, that we may adjuſt Matters ſo betwixt the Projectile and Centripetal Force, that the Latter ſhall have the Proportion which we Aſſign it, tho' we do not Prove, that the Projectile will comply with thoſe Conditions, which we are pleaſed to Preſcribe to it; Thus $\frac{BR}{br}=\frac{T^2}{t^2}\times\frac{V}{v}$, from which Equation this Propoſition is Concluded.

That is, First, Suppoſing, that PR the Tangent is or would be Deſcribed by the *Vis Inſita* alone in the ſame Time T, as the Arc PB is by the Projectile and Centripetal Force together, that is, Suppoſing, that the Force PB conſiſts of the Forces PR and RB, and that the Diagonal Force is Reſolvible into the Lateral ones, a Doctrine, which has been before Conſidered and Objected to; Beſides which, the Diagonal here is a Curve, which was there ſuppoſed to be a Right Line, and we have ſhewn, that tho' a Curve is Infinitely Diminiſhed, it cannot put on that Form.

Second, By Aſſuming the Arc $P\beta$, which the Body is to Deſcribe in the ſame Time, as it Deſcribes the Arc pb, and by making the ſaid Aſſumed Arc Leſs than PB, it is ſuppoſed, that PB is not the Leaſt Arc, which yet was before ſuppoſed to be the Leaſt Arc, and therefore in Order to bring this Demonſtration to it's Concluſion, Two contrary Suppoſitions are Introduced, that PB is the Leaſt Arc, and that it is not the Leaſt Arc, which cannot but be a very Convincing Way of Geometrick Reaſoning.

Third, By Aſſuming the Arc $P\beta$ Leſs than the Arc PB, it is ſuppoſed, that the Time, in which the Arc $P\beta$ or pb is Deſcribed, is Leſs than the Time in which the Arc PB is Deſcribed, as Pp is Leſs than PR; for if the Time was Equal or Greater, Pp would be Equal to or Greater than PR, which would not ſuit with this Demonſtration; if therefore pb is an Arc Equal to PB, and is Deſcribed in a Leſs Time than PB is, it is Deſcribed with a Greater Velocity, and conſequently, if $P\beta$ is Deſcribed with the ſame Velocity as PB, the Time Pp will be Leſs to make it Anſwerable to the Arc pb, which it Repreſents; and from hence it is, in part, Infer'd, that $\frac{BR}{br}$ is $=\frac{T^2}{t^2}\times\frac{V}{v}$, that is, by, in part, ſuppoſing it. For it is not Proved, that T or the Time of PR is Greater than t or the Time of pr = to the Time of Pp, or that the Velocity in pb is Greater than the Velocity in PB, Imagining them to be the Leaſt Arcs and Equal, unleſs by ſuppoſing it.

If it is ſaid, that the Point p will Move Faſter in a Mathematical Figure, when it is nearer to S, and the Point P Slower, when it is farther from it, and therefore, that the Firſt will Deſcribe an Equal Arc in a Leſſer Time than the Laſt, That will not be

True, becauſe, in Concentrical Circles, the Velocity of the Arcs is Greater, the farther you Remove them from the Center, and the Leſs the nearer they Approach to it, in the ſame Time.

If it is farther Alledged, that, ſince it has been already Proved, that the Areas PSB, pSb are Proportional to the Times, therefore the Arc PB at a Greater Diſtance from the Center S muſt be Deſcribed in a Longer Time, and pb in a Shorter; That neither has been Proved, as we have before ſhewn, nor is it Agreeable with Concentrical Circles, nor is it, as yet, made uſe of in this Demonſtration, but is Introduced afterwards, when we are Preſcribed to ſubſtitute thoſe Areas inſtead of $\frac{t^2}{T^2}$.

Fourth, As the Times of the Deſcriptions of the Arcs are Arbitrarily Aſſumed, ſo are the Forces, with which they are Deſcribed; for if pb is an Arc, which is Deſcribed with a Greater Velocity, and $P\beta$ is one, which is Deſcribed with a Leſs, tho' they are both of them Deſcribed in the ſame Time, yet the one will not be a juſt Repreſentative of the other, becauſe the Component Forces of the Projectile and Centripetal in both will be Entirely Different, and therefore we might as well have made Choice of any other; for $P\rho$ and $\rho\beta$, of which $P\beta$ is Compounded, are very Different from pr and rb, which is the Compoſition of pb, as $P\beta$ is Different from pb.

Fifth, The *Vis Inſita* or Projectile Force in P is ſuppoſed Equal to that in p or not; If Equal, pr will be $= P\rho = PR$, and then there will be an End of this Demonſtration, becauſe then likewiſe rb will be $= \rho\beta = RB$, and there will be no Difference in the Centripetal Forces, whoſe particular Reaſons and Proportions are here Endeavoured to be Aſcertain'd; If the *Vis Inſita* or Projectile Force in P is not Equal to that in p, this Philoſophy would do well to Inform us, how a *Vis Inſita* can be Intended and Remitted, or how one Certain and Immutable Force, with which Bodies would be Carry'd *in Directum*, if not otherwiſe Diverted, can be Altered.

For it is not ſufficient to ſay, that the Centripetal Force being Different in Different Parts of the Orbit makes the Projectile Force Different, ſince the Difference of the Centripetal Forces is Expreſſed by Lines Drawn Parallel to the Radius of the Curve, and conſequently, the Projectile Forces, if Equal, ought to be Expreſſed in every Part of the Curve by Equal Tangents, or elſe ſome Portion of the Centripetal Force will Enter into the Compoſition of the Tangent, and BR and br will not be the Full and Entire Expreſſions of it; Which if ſo, it follows, that $P\rho$ is not a Juſt Expreſſion of the *Vis Inſita* or the Projectile Force in P or p, or that any other is, Excepting BR, becauſe the Projectile Force of every the Leaſt Arc, whether PB or pb, muſt be Equal.

It muſt be farther ſaid, that if the Projectile Force, or the *Vis Inſita*, is Equal in every Part of the Curve, and Expreſſible by Equal Tangents, it will be ſomething Difficult to Account for the Various Velocities and Accelerations in theſe Curves from the mere Principle of a Force Acting directly towards the Center, and yet there is no other to Account for them, eſpecially, how a Body ſhould Move faſter in p than in P, when it is Nearer to the Center than when it is farther from it by a Principle of Gravity only, when the Projectile Force is Acknowledged to be the ſame. On the Contrary, if it ſhould be again Affirmed, that the Projectile Force is Different in Different Points of the Curve, it will be neceſſary to ſhew the Proportions of theſe Projectile Forces to each other, before we can pretend to Determine thoſe of the Centripetal, and we ought as well to Inquire, what Proportion PR, $P\rho$ and pr obtain amongſt themſelves, as what BR, $\beta\rho$, and br do.

Sixth, After the foregoing Suppoſitions, this Demonſtration Proceeds to tell us, that the Small Line BR is to the Small Line br in a Compound Reaſon of BR to $\beta\rho$, and of $\beta\rho$ to br, that is, $\frac{BR}{br} = \frac{BR \times \beta\rho}{\beta\rho \times br}$, and therefore by the Lemma before Cited from the *Method* of *Naſcents* and *Evaneſcents*, $BR . \beta\rho :: PBq . P\beta q :: T^2 . t^2$, ſince PB, $P\beta$ are the Leaſt Arcs, and are as the Triangles of the ſame Height BSP, βSP, that is, by the 1*ſt* and 2*d* Propoſitions of this Philoſophy, as the Times T, t, in which they are Deſcrib'd, or by the Conſtruction, as the ſame Times T, t, in which the Arcs PB, pb are Deſcribed; Again, the Small Line $\beta\rho$ is to the Small Line br, as the Cauſes, that is, as the Centripetal Forces in P and p, which Produce them, or as V to v, and therefore $\frac{BR}{br} = \frac{BR \times \beta\rho}{\beta\rho \times br} = \frac{T^2}{t^2} \times \frac{V}{v}$. But,

1st. As to the Compoſition of Reaſons, BR is to br in a Compound Reaſon of BR to PR, and of PR to br, that is, $\frac{BR}{br} = \frac{BR \times PR}{PR \times br}$, as alſo of BR to $P\rho$ and $P\rho$ to br, that is, $\frac{BR}{br} = \frac{BR \times P\rho}{P\rho \times br}$, as alſo of BR to pr and of pr to br, that is, $\frac{BR}{br} = \frac{BR \times pr}{pr \times br}$, &c. In the Firſt Caſe, ſince the Projectile Force is as the Time, $\frac{BR \times PR}{PR \times br} = \frac{V \times T}{T \times v} = \frac{BR}{br}$. In the Second, $\frac{BR \times P\rho}{P\rho \times br} = \frac{V \times t}{t \times v} = \frac{BR}{br}$. In the Third Caſe, $\frac{BR \times pr}{pr \times br} = \frac{V \times t}{t \times v} = \frac{BR}{br}$. In all which Caſes it is Evident, that nothing more is Proved, than that BR is to br, as V to v, or as the Centripetal Force in P to that in p, nor can this Demonſtration, if rightly Conſidered, Prove any more or any farther.

2d. SUPPOSING this Compoſition of Reaſons, which is here Aſſumed, that is, that $\frac{BR}{br}$ is $= \frac{BR \times \beta\rho}{\beta\rho \times br}$, it is in the Firſt Place not Evinced, that $BR.\beta\rho :: PBq.P\beta q$, becauſe the Lemma Alledged is not Prov'd; And in the next Place, becauſe $BR.\beta\rho :: PR.P\rho$ upon Account of Similar Triangles, but PR to $P\rho$ is as one Projectile Force to another, or as one Line to another, and not as the Squares of thoſe Lines.

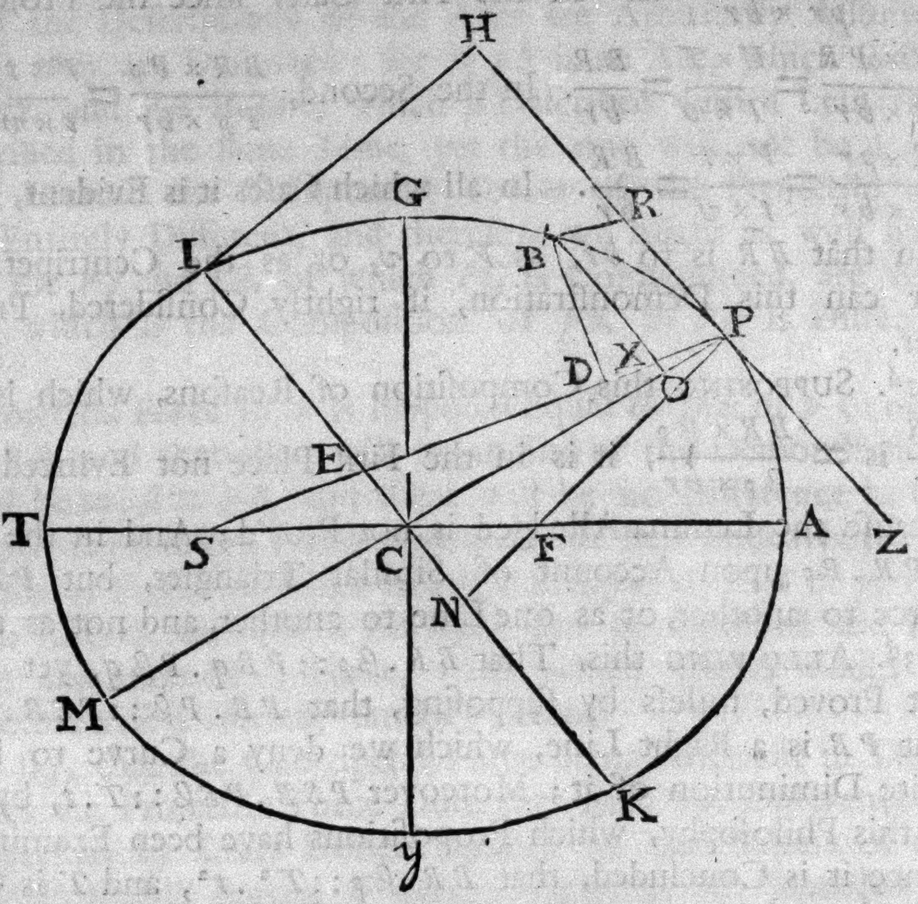
3d. ALLOWING this, That $BR.\beta\rho :: PBq.P\beta q$, yet that $PBq.P\beta q :: T^2.t^2$ is not Proved, unleſs by ſuppoſing, that $PB.P\beta :: PSB.PS\beta$, or which is the ſame, that PB is a Right Line, which we deny a Curve to be Reducible to, by any Infinite Diminution of it; Moreover $PSB.PS\beta :: T.t$, by Propoſition the *1ſt* and *2d* of this Philoſophy, which Propoſitions have been Examined and Refuted; And from hence it is Concluded, that $BR.\beta\rho :: T^2.t^2$, and T is Greater than t only, becauſe it is ſuppoſed originally to be ſo, that is, the Arc pb is ſuppoſed to be Deſcribed in a Leſs Time than the Arc PB.

4th. LASTLY it is ſaid, that $\beta\rho.br :: V.v$, but ſince PB is the Leaſt Arc, why BR ſhould not be as the Centripetal Force in P is not ſhewn, and conſequently, why BR is not to br as $V.v$, and therefore $BR.\beta\rho$ in a Proportion of Equality, which better Agrees with PB, $P\beta$, being the Leaſt Arcs, that is Equal, from whence the whole Conſtruction at P will Vaniſh, and we ſhall be only Informed, what it is to make a Diſtinction without a Difference; For it ſeems to be Impoſſible, that $\frac{BR}{br}$ ſhould be $= \frac{\beta\rho}{br} = \frac{V}{v} = \frac{T^2}{t^2} \times \frac{V}{v}$. And thus by theſe Demonſtrative ſteps we are Led to the Sum of all, that $\frac{BR}{br}$ is $= \frac{T^2 \times V}{t^2 \times v}$, when not one Particular of the whole is Proved.

SEVENTH, If $\frac{BR}{br}$ is $= \frac{T^2 \times V}{t^2 \times v}$ it muſt be owned, that $\frac{V}{v}$ is $= \frac{BR \times t^2}{br \times T^2}$ by Multiplying the Equation by t^2, and Dividing it by T^2. But when it is ſaid, That $T.t :: SBP.Sbp$ or as the Double of That, to the Double of This, or $:: SP \times BD.Sp \times bd$, here the Demonſtration again Fails; Becauſe the Firſt Proportion Depends upon what is not yet Proved, that is, the Firſt Propoſition of this Philoſophy, and the Second upon $SP \times BD$, and $Sp \times bd$ being Double of the Areas SBP and Sbp, which is not True, unleſs PB and pb, which are the Arcs of a Curve, are the ſame as Right Lines. And it is ſomething Strange and Peculiar in this New Method of Reaſoning, that there ſhould be a Difference made in the Leaſt Arcs PB, $P\beta$, or in the very Point P, for there thoſe Arcs are Naſcent or Evaneſcent, and yet that there ſhould be None made betwixt a Right Line and a Curve, or betwixt the Tangent PR and the Arc PB, as to the Triangle PBS, and on the other Hand, that there ſhould be a wide Difference made betwixt them, when we come to Conſider the Centripetal Forces BR and $\beta\rho$; This is Saying and Unſaying, making a Diſtinction, and then Rejecting it, and then Aſſuming it again, for which there can be no Account given, Excepting that of the Will and Pleaſure of the Philoſopher, who was Reſolved to Conclude in his own Way, whatever became of the Premiſes; for from hence we are Carry'd to the End of this Demonſtration in a Succeſſion of Algebraick Equations, which are Acknowledged and cannot be Conteſted.

The Law of the Centripetal Force in the Ellipſis Explained.
Sir Iſ. Newt. Princip. Lib. 1. §. 3. Prop. 11.
Dr. Gregory's Aſtronom. Lib. 1. Prop. 39. §. 5.

§. 5. The next Propoſition, which we ſhall Conſider in this Doctrine, is the Law of the Centripetal Force Tending to the Focus of an Ellipſis, in which a Body is ſuppoſed to be Revolved; Let APM be an Ellipſis, and S the Focus or Umbilicus of it, to which the Force Tends, by which a Body is Drawn from a Rectilineal Motion, which it would Deſcribe by it's *Vis Inſita* or Projectile Force, and is made to Circulate in an Elliptical Orbit, This Centripetal Force, and the Law of it, is here the Inquiry; Draw the Conjugate Axes TA, GY, which will Croſs each other in the Center C, and thro' any Point P in the Curve of the Ellipſis Draw the Tangent PZ, and the Diameter PM together with it's Conjugate ICK Parallel to the ſaid Tangent, and to ICK let fall the Perpendicular PN; Let the Parallelogram $PCIH$ be Compleated, and the Right Line IH will be a Tangent to the Ellipſis, by *Prop.* 17. *Lib.* 1. of *Conick Sections*; Join SP, which will Interſect IK in E; Thro' the Point B, which is ſuppoſed to be the next and Immediately Contiguous to P, Draw BR Parallel to PS, and BX to PZ, which will Meet PS in X and MP in O, The Centripetal Force Tending to S is by the foregoing Propoſition Reciprocally, as the Solid $\frac{SPq \times BDq}{BR}$, which, what it is in the Ellipſis, muſt be Computed and Determined from the Nature of it.



In Order to which, let the Principal *Latus Rectum* of the Ellipſis, or that, which belongs to the Greater Axis TA, be called L, the Reaſon of $L \times BR$ to BDq is Compounded of the Reaſons of $L \times BR$ to $L \times PO$, and of $L \times PO$ to $MO \times PO$, as alſo of $MO \times PO$ to BOq, of BOq to BXq, and of BXq to BDq; that is, $\frac{L \times BR}{BDq}$ is $= \frac{L \times BR \times L \times PO \times MO \times PO \times BOq \times BXq}{L \times PO \times MO \times PO \times BOq \times BXq \times BDq}$. But, 1ſt, $\frac{L \times BR}{L \times PO} = \frac{BR}{PO} = \frac{PX}{PO} = \frac{PE}{PC}$, Becauſe XO is Parallel to EC, $= \frac{AC}{PC}$, For PE is $= AC$ by a foregoing Lemma, which there is no occaſion particularly to Conſider here, and which is at Preſent Granted. 2*d.* $\frac{L \times PO}{MO \times PO}$ is $= \frac{L}{MO}$. 3*d.* $\frac{MO \times PO}{BOq}$, by *Prop.* 21. *Lib.* 1. of *Conick Sections* is $= \frac{MCP}{ICq} = \frac{CPq}{ICq}$. 4*th.* $\frac{BOq}{BXq} = \frac{1}{1}$, or the One is Equal to the Other, becauſe the Points B and P Coincide or come together, by which means BO is $=$ to BX. 5*th.* $\frac{BXq}{BDq} = \frac{PEq}{PNq}$, becauſe the Triangles BDX, PNE are Equiangular, upon the Account of the Angles D and N being Right ones, and BXD, PEN Alternate ones from the Parallels BX, EN, but $\frac{PEq}{PNq} = \frac{CAq}{PNq}$, Becauſe $PE = CA$ by the aforeſaid Lemma; Farther, by *Prop.* 72. *Lib.* 4. of *Gregory à Sancto Vincentio* $GC \times CA = PCIH = IC \times PN$, that is, $CA . PN :: IC . GC$, and therefore $CAq . PNq :: ICq . GCq$, and conſequently, $\frac{CAq}{PNq} = \frac{ICq}{GCq} = \frac{BXq}{BDq}$.

From whence, namely from the ſeveral Steps, thro' which we have Gone, ſince $\frac{L \times BR}{BDq}$ is $= \frac{L \times BR \times L \times PO \times MO \times PO \times BOq \times BXq}{L \times PO \times MO \times PO \times BOq \times BXq \times BDq}$, and ſince $\frac{L \times BR}{L \times PO}$ is $= \frac{AC}{PC}$, $\frac{L \times PO}{MO \times PO} = \frac{L}{MO}$, $\frac{MO \times PO}{BOq} = \frac{CPq}{ICq}$, $\frac{BOq}{BXq} = \frac{1}{1}$, and $\frac{BXq}{BDq} = \frac{ICq}{GCq}$, It follows, that $\frac{L \times BR}{BDq}$ is $= \frac{AC \times L \times CPq \times 1 \times ICq}{PC \times MO \times ICq \times 1 \times GCq} = \frac{AC \times L \times PCq}{PC \times MO \times GCq}$, Alſo by *Prop.* 13. *Lib.* 1. of *Conick Sections*, $AC \times L = 2GCq$, and therefore $\frac{L \times BR}{BDq}$ is $= \frac{2GCq \times PCq}{PC \times MO \times GCq} = \frac{2PC}{MO}$;

But ſince the Point B, in the Preſent Caſe, is Next and Immediately Contiguous to the Point P, O is very little or nothing Diſtant from it, that is, MO is $= 2PC$, and therefore $L \times BR = BDq$; Conſequently, ſince the Centripetal Force is, as has been in the Preceding Propoſition Proved, Reciprocally as the Solid $\frac{SPq \times BDq}{BR}$, and ſince in the Ellipſis BDq is $= L \times BR$, the Centripetal Force in the Ellipſis will be Reciprocally, as $\frac{SPq \times L \times BR}{BR} = SPq \times L$, and ſince L is a Conſtant and Invariable Quantity, the Centripetal Force in the Ellipſis will be Reciprocally as SPq, that is, Reciprocally as the Square of the Diſtance of the Body Revolved from the Focus or Umbilicus, to which the Centripetal Force Tends. *Q. E. D.*

The ſeveral Objections which lie againſt the foregoing Demonſtrations.

§. 6. Wonderful ſure muſt be the Name or Power of Demonſtration, which can Force the Aſſent without Convincing the Underſtanding, and yet this ſeems to be the Preſent Caſe; For,

First, That the Centripetal Force Tending to S is Reciprocally, as the Solid $\frac{SPq \times BDq}{BR}$ is far from being ſhewn, we have on the Contrary Evinced, that the Reaſoning, by which that Proportion is Concluded, is Sophiſtical, Deſultory, and Falſe upon ſeveral Accounts, notwithſtanding which, the whole Strength of the Preſent Demonſtration Entirely Reſts and Depends upon the Truth and Certainty of that Proportion.

Second, The next Thing neceſſary to this Demonſtration is to make $L \times BR = BDq$, ſo that SPq may ſtand by it ſelf; and this is done in ſo Looſe and Arbitrary a Manner, by Begging ſo many Uncertainties for Real Truths, and Subſtituting in the Room of others ſo many Quantities, which are better Fitted for the Purpoſe of Reducing BDq to $L \times BR$, and which yet is not Effected, that it may be called any Thing rather than a Mathematick Calculation of the Law of the Centripetal Force of a Body Revolving Round the Focus of an Ellipſis.

Thus, First, $\frac{L \times BR}{BDq}$ is ſaid to be $= \frac{L \times BR \times L \times PO \times MO \times PO \times BOq \times BXq}{L \times PO \times MO \times PO \times BOq \times BXq \times BDq}$, and ſo it is, becauſe $\frac{L \times PO \times MO \times PO \times BOq \times BXq}{L \times PO \times MO \times PO \times BOq \times BXq}$ is $= \frac{1}{1}$, which multiplyed into $\frac{L \times BR}{BDq}$ is certainly $=$ to $\frac{L \times BR}{BDq}$, But then we may as well ſay, that $\frac{L \times BR}{BDq}$ is $= \frac{L \times BR \times L \times PX \times MP \times PX \times BXq}{L \times PX \times MP \times PX \times BXq \times BDq}$, or $\frac{L \times BR}{BDq} = \frac{L \times BR \times L \times PS \times MP \times PE \times ICq \times IEq}{L \times PS \times MP \times PE \times ICq \times IEq \times BDq}$.

Second, Granting yet this Compoſition of Reaſons, $\frac{L \times BR}{L \times PO}$ is ſaid to be Equal to $\frac{BR}{PO} = \frac{PX}{PO} = \frac{PE}{PC}$, Becauſe XO is Parallel to EC, $= \frac{AC}{PC}$, Becauſe PE is $= AC$ by the

Lemma Cited. But ſince, by the Fourth Article of the Compoſition of Reaſons in the Preceding Demonſtration BO is $= BX$, why $\frac{PX}{PO}$ ſhould be Equal to $\frac{PE}{PC}$ or $\frac{AC}{PC}$ rather than $PO =$ to PX cannot be ſhewn, and conſequently, why $\frac{PX}{PO}$ ſhould not be $= \frac{1}{1}$; Or on the Contrary, if $\frac{PX}{PO}$ is not in a Reaſon of Equality, then $\frac{BOq}{BXq}$ in the ſaid Fourth Article is not.

Third, $\frac{BXq}{BDq} = \frac{PEq}{PNq}$, becauſe upon Account of the Similar Triangles BDX, PNE, $BX.BD::PE.PN$, but if we may Take BO inſtead of BX, it will be as $BO = BX.BD::PC = PE.PN$, from whence $PC = AC$, and therefore again, as before, $\frac{PX}{PO} = \frac{AC}{PC} = \frac{1}{1}$, and AC and PC Two very Different Semiaxes of the ſame Ellipſis, or even AC and GC the Semiaxis Principal and Conjugate or Tranſverſe will be Equal, and the Ellipſis changed into a Circle, for the Point P may be taken, as this Demonſtration is General, in any Part of the Elliptical Curve, or elſe it is not General, and therefore of no Significance or Importance at all.

Fourth, Conſequently, tho' PE is $= AC$ by the Lemma Cited, PC is not, and yet according to what foregoes PC may be likewiſe $=$ to AC; Again $\frac{CAq}{PNq}$ is $= \frac{ICq}{GCq}$, becauſe PE is $= CA$, but if we take PC in the Room of PE, will $\frac{PCq}{PNq}$ be $=$ to $\frac{ICq}{GCq}$? And yet this may be Affirmed, if BX is $= BO$.

From all which it is very Apparent, that this Philoſophy uſes theſe Naſcent or Evaneſcent Quantities juſt as it Pleaſes, ſometimes there ſhall be no Difference betwixt them, and at other Times a very Senſible one, then it ſhall again Vaniſh, and afterwards Riſe into a Reaſon and Proportion, as Regularly and Mathematically, as if there was ſomething more than Notion in it; And upon the Whole, which is a little Admirable, all this Pompous Compoſition of Forces is Reduced to a Proportion of Equality, and $\frac{L \times BR}{BDq}$ is $= \frac{1}{1}$.

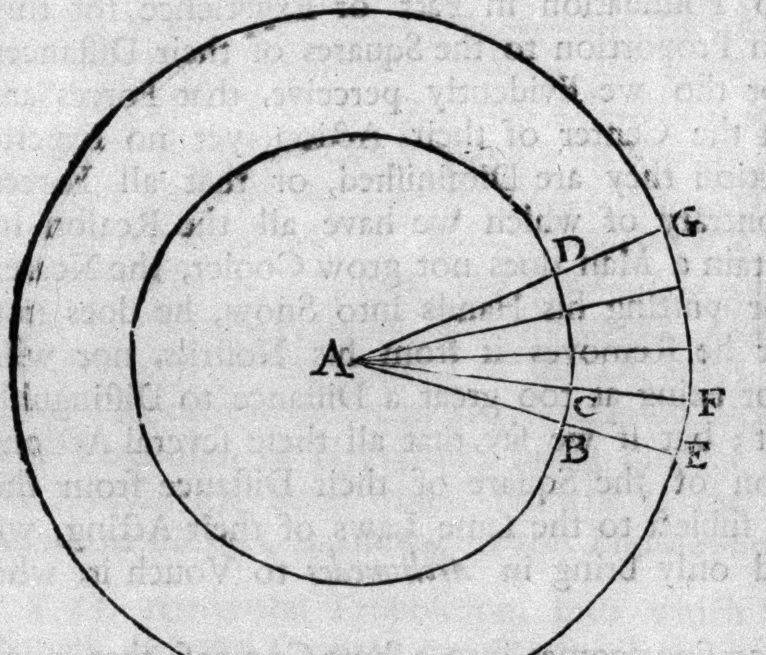
Fifth, After the ſeveral Reaſons, which are Subſtituted in the ſtead of the Former, $\frac{L \times BR}{BDq}$ is ſaid to be $= \frac{AC \times L \times CPq \times BOq \times ICq}{PC \times MO \times ICq \times BXq \times GCq}$, and ſince $\frac{BOq}{BXq}$ is $= \frac{1}{1}$, or $BO = BX$ and $\frac{ICq}{ICq} = \frac{1}{1}$, $\frac{L \times BR}{BDq}$ will be $= \frac{AC \times L \times PCq}{PC \times MO \times GCq}$. But in the 1ſt Place, if $\frac{BOq}{BXq}$ is $= \frac{1}{1}$, we have ſhewn that $\frac{AC}{PC}$ may be $= \frac{1}{1}$, and $\frac{PCq}{GCq} = \frac{ACq}{GCq}$, and conſequently, $\frac{L \times BR}{BDq} = \frac{L \times ACq}{MO \times GCq}$, from whence there could be no farther Progreſs to a Concluſion of what this Demonſtration pretends to Affirm. In the 2*d* Place, if $\frac{BOq}{BXq}$ is not $= \frac{1}{1}$, $\frac{L \times BR}{BDq}$ will not be $= \frac{AC \times L \times PCq}{PC \times MO \times GCq}$, but $= \frac{AC \times L \times BOq \times PCq}{PC \times MO \times BXq \times GCq}$, which will not again ſuit with the following Concluſion. In the 3*d* Place we muſt likewiſe ſay, that ſince, becauſe $BOq = BXq$ and $ICq =$ to it ſelf, they are Removed out of the Compoſition of Reaſons, it is not Evident, why Originally $\frac{L \times PO \times MO \times PO \times BOq \times BXq}{L \times PO \times MO \times PO \times BOq \times BXq}$ ſhould not, and then $\frac{L \times BR}{BDq}$ would be $= \frac{L \times BR}{BDq}$, which would not ſignify much to the Preſent Demonſtration.

Sixth, At Length after a great Deal of Artifice in Shuffling and Changeing of Reaſons, it is found, that $\frac{L \times BR}{BDq}$ is $= \frac{2PC}{MO}$, but then a new Conceſſion is wanted, and therefore it is Affirmed, that $2PC$ is $= MO$, or, that PO is $= 0$; and ſince $BX = BO$, $PX = BR$ is $= 0$, from whence $\frac{L \times BR}{BDq}$ is $= 0$, and conſequently, this whole Demonſtration in the laſt Reſult will in Reality and in Truth amount to Nothing.

Seventh, Since $L \times BR = BDq$ is $= 0$, $\frac{SPq \times L \times BR}{BR}$ in the Ellipſis will be $=$ $SPq \times 0 = 0$, and ſo this Demonſtration Ends with telling us, that it has Demonſtrated Nothing at all. *Q. E. D.*

The Demonſtration, as Extended to all Actions Propagated from the ſame Center, Conſidered.
Dr. Halley's Miſcellanea Curioſa.
Dr. Keill's Intr. ad Ver. Phyſ. Lect. 1.
Dr. Gregory's Aſtronom. Prop. 48. Lib. 1.

§. 7. And notwithſtanding this Appears to be, as I have Repreſented it, ſo fond of late have Philoſophers been of the Cogency and Strength of this Reaſoning, that they have Extended it to all Actions of Light, Heat, Cold, Smells, &c. which are Propagated from a Center, as well as of Attraction or Gravitation, to which it is here only Confined, but not being willing to make uſe of a Projectile and a Centripetal Force upon which this Demonſtration Depends, becauſe they knew of no ſuch Forces in Light, Heat, Cold, or Smells, they have Contrived another Way to let us into this Secret, which is, by ſhewing that the Rays AG, AF, AE, in a Sphere, are more Cloſe and Compact nearer the Center, and more Diffuſed and Diſperſed in a farther Removal from it, in that Proportion, which the Spherical Surface GE has to that of DB; and ſince, by *Archimedes de Sphæra & Cylindro*, it is Proved, that Spherical Surfaces are to each other, as the Circles Deſcrib'd by the ſame Radius's, and ſince thoſe Circles are in a Duplicate Reaſon of the ſaid Radius's, it follows, that the Compaction, Denſity, or Spiſſitude of the Rays in DB, is to the Denſity or Spiſſitude of them in GE, as the Square of AG or AE to the Square of AD or AB, and ſince the Intenſion or Force of any Action is in Proportion to the Compaction or Spiſſitude of the Rays, ſuch Force or Intenſion will be Reciprocally, as the Squares of the Diſtances.



But First, How far this Demonſtration is Applicable to Gravity, we have already Conſidered in the Firſt Volume of our *Principles* of *Natural Philoſophy*, and as there, ſo here likewiſe we cannot but Affirm, that it is an Abſtracted Reaſoning, and ſo far True, as it is *Archimedes*'s, that Spherical Surfaces are in a Duplicate Reaſon of their Semidiameters. If yet we are willing to Extend it any farther, what we ſay is Precarious and without a Foundation in Reaſon or in Fact.

For Second, It is not Evinc'd, that the Spiſſitude of the Rays in DB and GE is Reciprocally, as the Spaces Poſſeſſed by them, and that therefore in DB they are more Compact and Denſer, and in GE more Dilated and Diffuſed, becauſe if ſo, it would not only Arrive to Rays of Action, but to thoſe of Space, and the Space DB would be Thicker and Denſer than GE, which is not Acknowledged, tho' it may Perhaps be True.

Besides, the Spiſſitude of the Rays is Reciprocal to the Spaces, which they Poſſeſs upon the Suppoſition, that there is the ſame Number of Rays in DB and GE, as likewiſe in a Similar Arc of the Smalleſt Circle Drawn round A, and conſequently, ſuppoſing DB to be Entirely full of Rays, the ſame Rays can Poſſeſs no more Space in GE than in DB, if therefore a Space is taken out from EG Equal to DB, all the Reſt muſt be Vacuity, to which theſe Rays do not Reach; from whence it will follow, that if *GE* Repreſents the Earth's Orbit, and the Spherical Surface produced by it, and *DB* ſuch a Surface, the neareſt we can Imagine to the Sun in *A*, as the Spherical Surface of the one is ſeveral Millions of Times Greater than the other, ſo there will be only in the Spherical Surface of the Earth's Orbit a Space Illuminated Equal to the next Spherical Surface to the Sun, and all the ſeveral Millions and Millions of Space beſides will have no Rays at all to Pervade them: And the ſame may be ſaid of other Actions, that if this Reaſoning were True, they would not be Diſperſed thro' all Space, which they are ſuppoſed to be, but at a Conſiderable Diſtance from their Center they would be Perceived at Certain Intervals, and by Interruptions, as at *E*, *F*, and *G*, without any Uniformity of their Diffuſion.

FARTHER, Not only the Number of Rays is ſuppoſed to be the ſame in *GE* and *DB* to make the Spiſſitude of them Reciprocal to the Spaces, which they Poſſeſs, but the Force and Action of them is ſuppoſed to be as the Spiſſitude of the Rays, that is, the Action of the Ray at *G* is Equal to the Action of it at *D*, and it is not Diminiſhed by it's Diſtance, for if it were, the Diminution of it's Force need not be Conferred upon the Rarity or Dilatation of the Rays, ſince every Ray would be Diminiſhed in it's Force ſeparately, and by it ſelf, the farther it was Carry'd from the Center of it's Action; Beſides, if every Ray was Diminiſhed in it's Force, the farther it Receded from the Center of it's Action, the Diminution of Force would not be as the Spiſſitude of the Rays was Diminiſhed, which this Demonſtration Affirms, but in a Compound Reaſon of the Diminution of the Force of the Rays, and of the Diminution of the Spiſſitude of them; On the Contrary, if every Single Ray, conſidered by it ſelf, is the ſame in it's Force at a Greater or Leſs Diſtance from the Center of it's Action, which I ſay is here ſuppoſed, That of all Things ſhould have been Proved, becauſe it is a Paradox, and will want very Fine and Subtile Arguments to ſupport it.

WE have alſo ſaid, that there is no Foundation in Fact or Experience for this Aſſertion, That Forces are Diminiſh'd in Proportion to the Squares of their Diſtances from the Center of their Action; For tho' we Evidently perceive, that Forces are Diminiſhed the farther they are from the Center of their Action, yet no Experiments yet have ſhewn in what Proportion they are Diminiſhed, or that all Forces are ſubject to the ſame Laws; The Contrary of which we have all the Reaſon in the World to believe is True; 'Tis Certain a Man does not grow Cooler, the Nearer he comes to the Fire, nor Warmer for putting his Hands into Snow, he does not Smell *Aſſa Fœtida* Stronger, the farther he Removes it from his Noſtrils, nor will he Hear a Conſort of Muſick better for being at too great a Diſtance to Diſtinguiſh the Variety of Notes, which Compoſe it; but if we ſay, that all theſe ſeveral Actions are Diminiſhed in the ſame Proportion of the Square of their Diſtance from the Center of them, and that they are all ſubject to the ſame Laws of their Acting, we ſhall Affirm a very great Falſhood, and only bring in *Archimedes* to Vouch it, who never ſaid any ſuch Thing.

ALL, which can be ſaid upon this Occaſion, comes into a ſhort Compaſs, that every Action is the Strongeſt at the Center of it, that it is afterwards Weaker by Receding from thence, and by being Diffuſed into a Larger Space, each part of which it Repleniſhes and Fills, that this is not done by Rectilineal Rays, but by a Diffuſion of the Force, that this Diminution of Action upon both theſe Accounts is Various according to the Nature, Temper, and Genius of the Agent and Patient, which Nature and Temper of the Agent and Patient is to be Determined from the Degree of the Expanſive and Contractive Force, which belongs to them.

The Corollaries from the foregoing Propoſition need not to be Conſider'd. Dr. Gregory's Aſtronom. Prop. 39. Coroll. 1, 2, 3, 4, 5, and 6. Lib. 1.

§. 8. BUT to Return to the Phyſical Aſtronomy, from which we have thus far Digreſſed; As to the Corollaries, which are Derived from the Preceding Propoſition, in Reſpect of the Ellipſis being Changed into a Circle by the Focus's Approaching infinitely near to each other, or into a Right Line by their Approaching infinitely near to the Vertices, as alſo into a Parabola, if one of the Focus's Remains Fix'd, and the other Removes to an Infinite Diſtance from it, or into an Hyperbola, ſuppoſing the one is Fixt, and the other is Carry'd to more than an Infinite Diſtance from it, ſo that now the Vertices *A* and *T* ſhould lie betwixt the Foci *S* and *F* with their Curves bent Contrary ways; It is Affirmed, that the ſame Law obtains in all theſe Caſes, which has been before Demonſtrated; That is, in this Doctrine a Right Line, a Circle, a Parabola, and Hyperbola are all to be Reduced to a kind of Ellipſes, tho' their Properties are never ſo Different and Various, and all this is Done in Order to bring them under the ſame Law of the Centripetal Force, which is ſaid to be Obſerved in the Ellipſis; againſt which there might be ſeveral Objections Alledged, into which Diſpute there is yet no Occaſion to Enter, ſince the aforementioned Law of the Centripetal Force is not proved to Obtain in Ellipſes themſelves.

§. 9. The next Propoſition, which we ſhall mention is, that if ſeveral Bodies Revolve about a Common Center, and the Centripetal Force is Reciprocally as the Square of the Diſtance from the Center, the Squares, it is ſaid, of the Periodical Times in Ellipſes are as the Cubes of the Tranſverſe or Greater Axes, or, which is the ſame, the Periodical Times are in a Seſquiplicate Proportion of the ſaid Axes; which Propoſition there is no Occaſion particularly to Conſider, unleſs the Centripetal Force is Reciprocally as the Square of the Diſtance from the Center, which is not hitherto Evinced; Beſides which, there are other Things, which may juſtly be Objected againſt this Demonſtration, which we ſhall now Paſs over, till the Way is Clear'd and made Plain to the Preſent Concluſion, and for the ſame Reaſon the Corollaries of this Propoſition concerning the Circle or any other Orbits are Omitted, for if the Grand Law of the Centripetal Force is not True, it will be in Vain to Expect any Thing of Conviction and Certainty from Reaſonings, which Depend upon a Wrong or a Precarious Foundation.

That the Squares of the Periodical Times in Ellipſes are as the Cubes of the Tranſverſe or Greater Axes, not Proved.
Sir Iſ. Newt. Princip. Prop. 15. Lib. 1.
Dr. Gregory's Aſtronom. Prop. 40. Lib. 1.

§. 10. Upon the ſame Account, it will not be neceſſary to Examine into the following Propoſition, that, ſuppoſing the ſame Law of the Centripetal Force, as before, the Velocities of Bodies, Revolving about a Common Center, in any Points Given, are in a Reaſon Compounded of a Subduplicate Reaſon of the Principal *Latus Rectum*'s, belonging to the Lines, which they Deſcribe, and of an Inverſe Reaſon of the Perpendiculars, Drawn from the Common Center to the Tangents of the Points, where the Bodies are; Becauſe that Law of the Centripetal Force is Deny'd, and all the Concluſions, which are made from it; Moreover, we Affirm, that here likewiſe lie ſome Particular Objections againſt this Demonſtration upon the Account of the Velocities, which are Introduced in it, and of which we have Diſcourſed in other Parts of the Preſent Philoſophy. We muſt farther add in Reſpect of the Corollaries here, and of thoſe, which belong to the Former Propoſitions, that ſeveral of them will be the ſame, tho' the Law of the Centripetal Force was Different from what is Pretended to be Demonſtrated; Thus, tho' the Centripetal Force was not Reciprocally, as the Squares of the Diſtances, but as the Cubes or Biſquares, or any other Proportion of them, the Reaſoning would be much the ſame, as to the Circles, Parabolas, or Hyperbolas being Ellipſes, for if any Different Law obtain'd in the Ellipſis, the ſame would likewiſe in the Circle, Parabola, or Hyperbola, conſider'd as Ellipſes, according to this Philoſophy.

That the Velocities are in any Points given in a Reaſon Compounded of a Subduplicate Reaſon of the Principal Latus Rectum's *and of the Inverſe Reaſon of the Perpendiculars let down to the Tangents of the Points where the Bodies are, not Proved.*
Sir Iſ. Newt. Princip. Prop. 16. Lib. 1.
Dr. Gregory's Aſtronom. Prop. 41. Lib. 1.

§. 11. Another Propoſition, into which we ſhall Inquire, is, that the Primary Planets and the Comets are Drawn from a Rectilineal or Projectile Motion, and made to Revolve in Elliptical Orbits by Forces, which Tend to the Sun, and are Reciprocally, as the Squares of their Diſtances from it's Center.

The Law by which the Primary Planets are kept in their Orbits, not Certain.
Sir Iſ. Newt. Princip. Prop. 2. Lib. 3.
Dr. Gregory's Aſtronom. Prop. 42. Lib. 1.

As to the Planets, the Former Part of the Propoſition is ſaid to be Proved from *Phænomenon* the 5th, *Lib.* 3. which Informs us, that the Primary Planets Deſcribe Areas Proportional to the Times by Radius's Drawn not to the Earth, but to the Sun, becauſe, in Reſpect of the Earth, they are ſometimes Progreſſive, then Stationary, and at other Times Retrograde, whereas, in Reſpect of the Sun, they are always Progreſſive, ſo as in Equal Times to Deſcribe Equal Areas; and from *Prop.* 2. *Lib.* 1, which tells us, that every Body, which is Moved in any Curve Line Deſcrib'd in a Plane, and by a Radius Drawn to an Immoveable Point, or a Point, which has an Uniform Progreſſion, Performs Areas about that Point Proportional to the Times, is Urged and Impelled by a Centripetal Force Tending to ſuch Point.

The Second Part of this Propoſition is Affirmed alſo to be Proved from *Phænomenon* the 4th, *Lib.* 3, which Relates, that the Periodical Times of the Primary Planets, and of the Earth Round the Sun, are in a Seſquiplicate Reaſon of their Middle Diſtances from it, or which is the ſame, that the Squares of the Periodick Times are as the Cubes of the Diſtances, and from *Prop.* 4. *Lib.* 1, which alledges, *Corol.* 6, that if the Periodick Times are in a Seſquiplicate Reaſon of the Radius's, the Centripetal Forces will be Reciprocally, as the Squares of the Radius's.

However, This Part of the Propoſition is reckoned likewiſe to be ſtrictly Demonſtrated from the Quieſcence of the Nodes and the Apſides, Becauſe the Leaſt Deviation from the Duplicate Reaſon, by *Prop.* 45. *Lib.* 1. *Corol.* 1, would produce a very ſenſible Motion of the Apſides in each Revolution, and in ſeveral a very Enormous one.

This Part alſo of the Propoſition is ſaid to be Proved from it's being Acknowledged, that the Planets are Revolved in Ellipſes, and from being before ſhewn, that the Law of Bodies, Revolving in Ellipſes, is, that their Centripetal Forces ſhould be Reciprocally, as the Squares of the Diſtances. And as to Comets, the ſame is Affirmed of them, becauſe they are ſuppoſed to Deſcribe Ellipſes, which by the Infinite Diſtance of the Focus's are Changed into Parabolas.

The Arguments, which may be brought againſt this Account of the Revolution of the Primary Planets.

§. 12. First, The *Pythagorick* or *Copernican Syſtem* is here Suppoſed and not Proved, and the Diſpute betwixt that and the *Ptolemaick* and *Tycho Brahe's* is left where it was, and nothing Propoſed in the *Phænomenon*, but that the Planets are Progreſſive, Stationary and Retrograde to us on the Earth, which would not appear ſo, if we were in the Sun, which the Aſſertors of all the Syſtems no doubt Equally knew, and which yet did not hinder, but was indeed the Occaſion of their ſeveral Hypotheſes; and as to the Equable Deſcription of the Areas about the Sun, it is no more an Argument for the Truth of the *Copernican Theory*, than the Unequal Deſcription of the Orbits or Curves ſhould be an Argument againſt it, the Equality of the Deſcription of Areas being of no Advantage, that I know of, to an Hypotheſis, unleſs we could ſhow a Reaſon, why it ſhould be ſo, in Preference to an Equal Deſcription of the Curve.

Nay, Imagining it to be Fact, that the Areas were Deſcrib'd from the Aphelium to the Perihelium of the Planet in a Double or Triple, a Duplicate or a Triplicate, or in any other Arithmetical or Geometrical Proportion of the Times, it would be to the full as Beautiful to an Hypotheſis, as if they were Deſcrib'd in a Proportion of Equality, tho' I am apt to ſuſpect, that this Phænomenon of Equal Areas being Deſcribed in Equal Times is not exactly True, and that it is only a Fancy or Notion Invented to ſerve ſome Explanations in Aſtronomy, which *Kepler* began, and whom the Reſt have Followed.

'Tis true by this Equable Deſcription of the Areas, the Velocity of the Planet in the Perihelium, and it's ſlow Progreſs in the Aphelium is Immediately Inferred, but if thoſe and other Phænomenons may be Accounted for upon Different Suppoſitions, it is not ſo Evident, why the Equable Deſcription of Areas about the Sun ſhould Determine us to Prefer the *Copernican* to any other Hypotheſis; I do not ſay this, that I would be thought rather to Incline to the *Ptolemaick* or *Tycho's* than *Copernicus's*, but only to ſhew, that the Preſent *Phyſical Aſtronomy* leaves that Controverſy very much, where it found it, and raiſes it's own Structure upon the Conceſſion, that the *Copernican Syſtem* is True, without Confirming it by any Invincible Demonſtrations, unleſs what may Reſult from it's future Reaſonings.

Second, After the *Phænomenon*, we are Referr'd to *Prop.* 2. *Lib.* 1, for a Proof of the Firſt Part of this Propoſition, which has been already Conſidered and Examined, and has been ſhewn to fall ſhort of, what is Pretended, a Demonſtration; Since it cannot be Proved, that a Curve is Deſcribed by thoſe Two Forces, a Projectile and a Centripetal; To which we ſhall add, that it may as well be Performed by an Expanſive Force from the Sun in the Preſent Caſe, as a Centripetal one to it, of which we ſhall afterwards Diſcourſe.

Third, The Second Part of this Propoſition is as illy Proved, as the Former, becauſe the Phænomenon, that the Squares of the Periodick Times are as the Cubes of the Diſtances, is altogether Uncertain, for tho' the Periodick Times may be well enough Agreed on amongſt Aſtronomers, yet the Diſtances of the ſeveral Planets from the Sun are not ſo, and even that of the Earth from it admits of Variations from the beſt Obſervers; Becauſe we are not capable by any Inſtruments Preciſely and Exactly to Determine them: But the Diſtances of the Planets from the Sun we have more particulary Examined into in another Place.

GRANTING all which, we have ſhewn, that *Prop.* 4. *Lib.* 1. and *Corol.* 6, which ſays; that if the Periodick Times are in a Seſquiplicate Reaſon of the Radius's, the Centripetal Forces will be Reciprocally, as the Squares of the Radius's, as well as all the other Corollaries belonging to the ſame Propoſition, want that Evidence, which might be Expected from a Demonſtration of them, and which therefore we need not here Repeat. But ſtill in Juſtification of this Reaſoning it is ſaid, that the Quieſcence of the Nodes and the Apſides is a full Conviction of the Truth of it, ſince the leaſt Deviation from a Duplicate Proportion, by *Prop.* 45. *Lib.* 1. *Corol.* 1, would produce an Evident Motion in them, even in each Single Revolution.

THIS may ſeem to be a very forcible Argument for the Validity of the Preſent Doctrine, but if we Conſider, that the Propoſition here Cited is ſtrictly Connected with, and Depends upon that, which we have Explained in §. 5. of this Chapter, and which Affirms, that the Law, by which a Body Revolves in the Ellipſis, is, that it's Centripetal Force is Reciprocally, as the Square of it's Diſtance, we ſhall eaſily ſee, that it ſignifies no more, than to ſay, that if an Immoveable Ellipſis is Deſcribed by the Duplicate Proportion, a Moveable Ellipſis will not be; but this is no Proof, that an Immoveable Ellipſis is Deſcribed by that Duplicate Proportion; a Deviation from a Duplicate Proportion makes the Ellipſis Moveable, upon the Suppoſition, that a Duplicate Proportion belongs to an Immoveable one, but no one will therefore from ſuch a Deviation ever ſurely Prove, that a Duplicate Proportion belongs to an Immoveable one, and yet this is the Strength and Solidity of the preſent Argument.

LASTLY, As to the Law of the Centripetal Force, by which an Ellipſis is ſaid to be Deſcribed, and which is thought to be Reciprocally, as the Square of the Diſtance, we have fully Examined that Propoſition in §. 6. of this Chapter. So that all the Demonſtration, which we have of this Grand and Magnificent Propoſition in the Phyſical Aſtronomy concerning the Revolution of the Primary Planets, Reſts either upon Phænomenons, which cannot be fully Acknowledged, or upon Propoſitions which are not Evinced, or on Deductions from them, which are not Rational and Juſt, and for which we may deſervedly ſuſpend our Aſſent to it, at leaſt till theſe Objections are Anſwer'd and Cleared.

An Account of the Method of Proceeding in this Philoſophy, and the Reaſons againſt it in General.

§. 13. I ſhall only here add, that it has been for ſome Time a Myſtery to me in this Philoſophy, what ſhould be the Occaſion of Introducing ſo New, and ſo Looſe, and Latitudinarian a Way of Reaſoning in it, and ſo Apparently Forced and Strained, and what ſhould be the Spring and Riſe of the Whole being Caſt into the Preſent Model; Which upon a farther Conſideration and Inquiry ſeems Obvious and Plain.

Sir Iſ. Newt. Pref. Lib. 3. De Mundi Syſtemate.

FOR, This Philoſophy deſigning Chiefly and Principally to give an Account in a Mechanical Way of the Revolutions of the Heavenly Bodies, it obſerved, what were acknowledged to be the General Phænomenons or Appearances of them; Namely,

Sir Iſ. Newt. Princip. Phænom. Lib. 3.

1st. THAT the Primary Planets Deſcrib'd Ellipſes about the Sun, which was Placed in one of the Focus's of ſuch Ellipſes.

2d. THAT the Secondary Planets Deſcrib'd Ellipſes about the Primary ones, which were likewiſe Placed in one of the Focus's of ſuch Ellipſes.

Phænom. 1, 2. Lib. 3.

3d. THAT the Secondary Planets about *Jupiter* and *Saturn* Performed Areas Proportional to the Times of their Deſcription, and that the Squares of their Periodical Times were as the Cubes of their Diſtances, or which is the ſame, that their Periodick Times were in a Seſquiplicate Proportion of the ſaid Diſtances.

Phænom. 4, 5. Lib. 3.

4th. THAT the Primary Planets and the Earth about the Sun Perform'd Areas Proportional to the Times of their Deſcription, and that the Squares of the Periodical Times were as the Cubes of their Middle Diſtances from the Sun, or, which is the ſame, that their Periodical Times were in a Seſquiplicate Proportion of the ſaid Middle Diſtances.

Phænom. 6. Lib. 3.

5th. THAT the Moon about the Earth Performs Areas Proportional to the Times of their Deſcription.

FIRST, As to theſe Phænomenons themſelves, tho' Propounded by *Kepler*, and ſince Aſſented to by moſt Aſtronomers, We ſay, the Uncertainty of Aſtronomical Obſervations in General is ſo Great, that methinks it is but a very fallible Foundation to Build a whole Syſtem of Philoſophy upon; For *1ſt*, As we before Mentioned, however known the Periodical Times of the Primary and Secondary Planets are, the Diſtances of the Firſt from the Sun, and the Laſt from the Primary Planets are Diſputable and Unaffirmed; Beſides, 2*d*, there is not ſuch Evidence for the Areas being Proportional to the Times of their Deſcription in the Revolution of the Primary or Secondary Planets, as makes it Inconteſtable and beyond Contradiction; Nor farther, 3*d*, Does the Proportion of the Periodick Times being in a Seſquiplicate one of the Diſtances ſo manifeſtly Appear, that it ſhould be Aſſumed as a Demonſtrative Truth, for Reaſons before Alledged; And it is ſomething Strange, and looks like a Colluſion in the Beginning of this Philoſophy, that when it Confeſſes this to be the Proportion in the Secondary Planets, becauſe the Orbits are nearly Circular, in the Primary ones it is obliged to take the Middle Diſtances only, becauſe the Orbits are more Elliptical, and this Proportion could not Anſwer all the Diſtances of the Planets Revolving in ſuch Orbits.

And yet theſe are the Grand Truths, upon which the Whole of this Philoſophy ſtands, and which ſeem to be no more at the Beſt than Aſſertions of *Kepler*, to which other Aſtronomers have thought fit to Agree, becauſe Two or Three plain Phænomenons, as they were Imagin'd, would better be Explained, than a Thouſand Various and Different ones; But ſurely we ſhould not take our Notions of all Nature from fallible Obſervations in Aſtronomy, and Obſervations Adjuſted and Reduced by Artifice and Contrivance to what are Termed Phænomenons in the Heavens. Nay, ſuppoſing theſe Phænomenons in the Heavenly Bodies to be Accurate and True, and not Framed and Modelled by the Aſtronomer to his own Purpoſes, is it Juſt and Reaſonable to Preſcribe Laws to Nature, and all the Beings in it from them, or from what we continually Know and Experience to Dictate to their Actions? However it is Plain, that upon theſe ſuppoſed Truths or Phænomenons the Preſent Philoſophy is built, and it may be properly ſaid to be an Account or Explanation of them.

Sir Iſ. Newt. Princip. Prop. 1. Lib. 1.

Def. 3. & 5.

Sir Iſ. Newt. Princip. Lex. 1.

Sir Iſ. Newt. Princip. Leg. Corol. 1.

Sir Iſ. Newt. Lem. 3. Corol. 4. De Methodo Rationum Primarum & Ultimarum.

Second, The next Thing therefore, which was to be Atchieved, was to ſhew, what Forces would be Requiſite to Deſcribe a Figure, whoſe Areas ſhould be Proportional to the Times of their Deſcription, and this Introduced *Prop.* 1. of this Philoſophy; and the Deſcription of theſe Areas being to be made about an Immoveable Center, from hence Aroſe the Centripetal Force and the *Vis Inſita*, and conſequently, Definitions the 3*d* and 5*th* were Eſtabliſhed; but the *Vis Inſita* being only Defined to be a Power in Matter of Reſiſting, by which every Body, as far as in it was, Perſevered in it's State of Reſt or of Motion *in Directum* or Strait forward, ſuch a Definition could ſignify nothing to the Reality of the Thing, unleſs this Definition was afterwards Advanced into a Law of Nature, which is done Law the Firſt, which ſays, that every Body perſiſts in it's State of Reſt, or of Motion *in Directum*, unleſs it is forced from that State by an External Impreſſion; To Prove this Propoſition likewiſe it was neceſſary to Introduce *Corol.* 1. of the Laws, that a Body Deſcribes with it's Joint Forces the Diagonal of a Parallelogram in the ſame Time, in which the Sides of it are Deſcribed by the Separate Forces; Nor yet was this ſufficient to Evince the Preſent Propoſition, excepting Corollary the 4*th* Lemma 3*d* of the *Method* of *Naſcent* and *Evaneſcent Reaſons* was Allowed, which Lemma we have already Conſidered.

From this one ſuppoſed Phænomenon therefore of the Planets Deſcribing Areas Proportional to the Times of their Deſcription, which is *Kepler's* Aſſertion, is Derived this whole Philoſophy; and this Firſt Propoſition is Proved by ſo many Aſſumed Definitions, Laws, and Lemmas, none of which, as we have ſhewn, are Evinced, that it would give any Rational Man a Diſtaſte and a Thorow Averſion to it. But, which is more, the Definitions, the Laws, and Lemmas of this Philoſophy are Originally made and Contrived from this Propoſition, and then are uſed to Prove it.

Sir Iſ. Newt. Princ. Prop. 4. Lib. 1. Corol. 6. & Schol.

The Definitions of Names being thus Eſtabliſhed into Things and Realities, the Laws Allowed, and the Lemmas in a Certain way Demonſtrated, all which were neceſſary to Confirm the Preſent Propoſition, it was eaſy from thence to Proceed to the 4*th* Propoſition, and the 6*th* Corollary of it; but, how that is made to Appear, I have already Declared, and which is nothing more than a Reaſoning Accommodated to what Sir *Chriſtopher Wren*, Dr. *Hooke*, and Dr. *Halley* had Embraced, which is Equally Wrong and Proceeds upon the ſame Unjuſtifiable Abſtractions; which ſhews the Artifice of this Philoſophy, that it does not ſo much Proceed upon the Reaſon of Things, as caſts about and Conſiders, how it may ſuit it ſelf to the Acknowledgments and Confeſſions of the Greateſt and moſt Celebrated Men in theſe Studies.

For we have Evidently Evinced, that this Sixth Corollary and the Propoſition, from which it is Derived, is Falſe in Reaſon, and therefore cannot be Applicable to any Motions in Nature. The Corollary is, That, if the Periodical Times are in a Seſquiplicate Reaſon of the Diſtances, the Centripetal Forces will be Reciprocally, as the Squares of thoſe Diſtances, which this Philoſophy finding to Agree with what *Hugens* had pretended to Demonſtrate in Reſpect of the Centrifugal Forces, and with what Sir *Chriſtopher Wren*, Dr. *Hooke*, and Dr. *Halley* had done, all was Certain and Sure, as to that Point, and it had nothing more to Endeavour than the Third Thing, which was to ſhow, that this Reaſoning of the Mechanical Philoſophers exactly Correſponded with the Revolutions of the Planets in Ellipſes, by which it would at once oblige Aſtronomy, as it was Propoſed by *Kepler* and his Followers, and the Mechanical Doctrine, as it was Propoſed by *Hugens*.

Dr. Keill's Intr. ad Ver. Phyſicam. De vi Centrifugâ & Motu Circulari.

This being the main Intention and Deſign to Reconcile the Mechanical Philoſophy, with the Aſtronomy and Obſervations of *Kepler*, the next Step, which was made, was to Prove, ſince *Kepler* Affirms that the Planets are Revolved in Ellipſes, and the Mechanical Philoſophers Affirm, that if the Periodical Times are in a Seſquiplicate Reaſon of the Diſtances, the Centripetal Forces are Reciprocally, as the Squares of ſuch Diſtances, that Bodies Revolved in Ellipſes obſerve that Reciprocal Duplicate Proportion.

Third, And from hence this Philoſophy is Carry'd to Propoſition the 11th, §. 3. *Lib.* 1. that is, to Inquire into the Law of the Centripetal Force, by which a Body Revolves in an Ellipſis round the Focus or Umbilicus of it, which, by Lemmas or Aſſumptions not ſufficiently Proved, and by Compounding of Reaſons at Pleaſure, and other Arbitrary Methods, at length Concludes, what was before-hand Deſigned, that the Centripetal Force is Reciprocally, as the Square of the Diſtance.

Sir Iſ. Newt. Princip. Prop. 11. Lib. 1.

And this is the Occaſion of *Prop.* 6th, 7th, 8th, 9th, and 10th, being Premiſed to it, whereas this Propoſition was the Firſt in View, and when there were ſeveral Things neceſſary to Clear it, thoſe Things were built into Independent Propoſitions of themſelves, as if they had no Relation to any Thing; Albeit they were Originally Contrived and Invented, as the Laws and Definitions before, to ſerve one End and Purpoſe, which in the Preſent Caſe was that of making the Centripetal Force in the Ellipſis Reciprocally, as the Squares of the Diſtances; and with this Intent the 6th Propoſition in Particular was Apparently Introduced, and which, as well as the 11th, we have at Large Conſidered and Examined, and could eaſily do the other Intermediate ones, if it was worth the Time or the Trouble; but it would be an unneceſſary Expence of both after what we have ſaid; and we ſhall only obſerve, that, in General, in this Philoſophy the Firſt Things in it were the Laſt Thought of, and the Laſt Firſt, as may be Evidenced in every Part of this Syſtem.

Sir Iſ. Newt. Princip. Prop. 6, 7, 8, 9, & 10. Lib. 1.

Fourth, From theſe Thoughts and Notions, and when the Mind was Warmed with them, it was Natural to Proceed to an Inquiry into the Law of the Centripetal Force Tending to the Umbilicus of an Hyperbola, ſuppoſing a Body Moved in it, or what the Law of the ſaid Centripetal Force Tending to the Umbilicus of a Parabola would be, ſuppoſing a Body was Moved in that Figure. This brought the Preſent Philoſophy into a Train of Ideas and Reaſoning concerning the *Conick Sections* for many Propoſitions together to §. 7. *Lib.* 1. which ſhews the Great and Wonderful Art and Skill of the Geometer in Manageing of Proportions and Quantities, but ſeems to be of little Uſe or Advantage to the Knowledge of Nature.

Sir Iſ. Newt. Princip. Prop. 12. Lem. 13, 14. Prop. 13. to §. 7. Lib. 1.

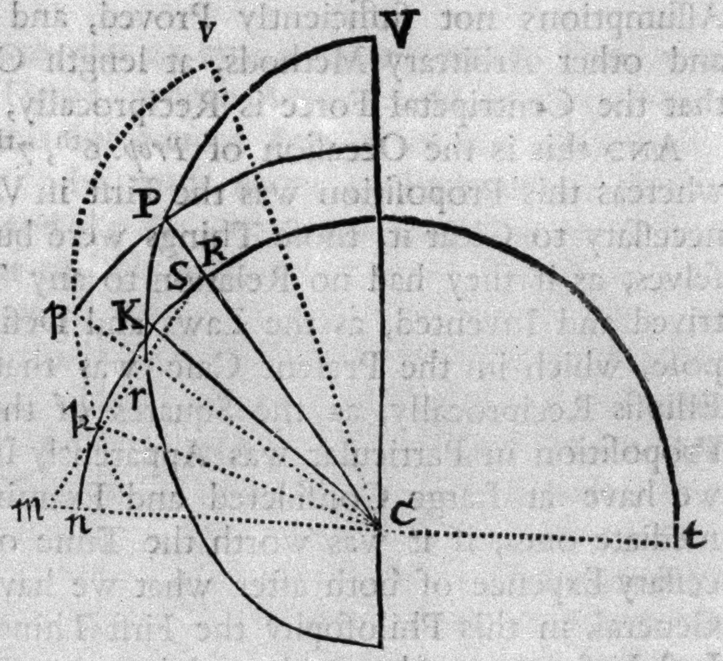
Fifth, Sect. 7th. is concerning the Rectilineal Aſcent and Deſcent of Bodies, and ſuppoſing the Centripetal Force is Reciprocally Proportional to the Square of the Diſtance of the Places from the Center, to Determine the Spaces, which a Body Deſcribes in any Given Times by a Perpendicular Deſcent. The 8th is Concerning the Invention of Orbits, in which Bodies are Revolved, when Acted by any Centripetal Forces whatſoever, till at Length we come to §. 9. which gives an Account of the Motion of the Apſides, which we ſhall now Conſider.

Sir Iſ. Newt. Princip. §. 7, & 8. Lib. 1.

THIS is a ſhort Explanation of the Riſe, Beginnings, and Structure of the Preſent Phyſical Aſtronomy, in which there will upon a ſtrict Examination be found to be more Artifice and Contrivance than Truth, more Quickneſs and Skill in Geometry than Exactneſs in Reaſoning, and more Poſtulatums and Aſſumptions and Arbitrary Abſtractions, than Real Laws of Nature, and Theorems of Philoſophy; It muſt be Confeſs'd, that this Reaſoning has one Advantage on it's Side, ſince it is about Beings ſo Remote from us, that, if an Hypotheſis is pretty well laid together, it will be the Harder to Confute it, becauſe we know not, what the Circumſtances and Conditions of thoſe Beings are, and therefore muſt take them, as they are Explained, unleſs we can ſhew a Falſity in the Hypotheſis it ſelf; in which it Agrees with the Corpuſcular Philoſophy, which by it's Infinitely ſmall Particles puts it out of our Power of Demonſtrating the Abſurdity of it by our Senſes; For thus the Phyſical Aſtronomy Relates to Bodies, which are ſo Infinitely Removed from our Inſpection, that we muſt Deſpair of Confuting it's Arguments by Pointing to and Shewing the Bodies themſelves; But as from Reaſon we have Evinced the Extreme Folly of ſuch Infinitely ſmall Particles Exiſting in Nature, as the Conſtituents of Bodies, in like Manner we may by Inquiring into the *Rationale* of this Aſtronomy, make it Evident, that all this *Apparatus* of Lines and Angles, of the *Vis Inſita*, and the Centripetal Force, is nothing, but mere Pageantry, and has no Subſiſtence or Being in Reality.

Concerning the Motion of a Body in a Moving or a Quieſcent Trajectory. Sir If. Newt. Princip. Prop. 43. §. 9. Lib. 1. Dr. Gregory's Aſtronom. Prop. 3. §. 2. Lib. 4.

§. 14. THE next Propoſition therefore which we ſhall Examine is that, which is to Inform us, how a Body may be alike Moved in any Trajectory Revolving about a Center of Forces, as another Body in a Trajectory, which is Quieſcent. In the Orbit VPK, which is Given in it's Poſition, Let a Body P be Revolved from V towards K, from the Center C Let Cp be always Drawn Equal to CP, and form an Angle VCp Proportional to the Angle VCP; The Area, which the Line Cp Deſcribes, will be to VCP the Area, which the Line CP Deſcribes at the ſame Time, as the Velocity of the Deſcribing Line Cp to the Velocity of the Deſcribing Line CP, that is, as the Angle VCp to the Angle VCP, namely, in a Given Reaſon, and therefore Proportional to the Time; Since then the Area, which the Line Cp Deſcribes in an Immoveable Plane is Proportional to the Time, it is Evident, that the Body, by ſome Determinate Centripetal Force Urging it, may be Revolved together with the Point p in that Curve Line, which the ſame Point p in the Reaſon Explained Deſcribes in an Immoveable Plane; Let the Angle VCv be made Equal to the Angle PCp, and the Line Cv to the Line CV, and the Figure vCp to the Figure VCP, and the Body, being always in p, will be Moved in the Perimeter of the Revolving Figure vCp, and will Deſcribe an Arc of it vp in the ſame Time, in which another Body P Deſcribes an Arc VP Similar and Equal to it in the Quieſcent Figure VPK: Let therefore the Centripetal Force be Inquired by *Corol.* 5. *Prop.* 6, by which a Body may be Revolved in that Curve Line, which the Point p Deſcribes in an Immoveable Plane, and the Problem is Solved. *Q. E. F.*



BUT let theſe Philoſophers give me leave to Anſwer, FIRST, That a Body Deſcribing an Area Proportional to the Time is Demonſtrated upon the Suppoſition of an Immoveable Plane, of a *Vis Inſita* or a Projectile Force Proceeding in a Tangent, and of a Centripetal one, by *Prop.* 1, of this Philoſophy; and conſequently, if Cp in the Moveable Plane pCv Deſcribes Areas Proportional to the Times, it will be neceſſary to ſhow us, how a Moveable Plane is the ſame as an Immoveable one, and where and what is the Projectile Force in this Caſe, which Acts in a Right Line or a Tangent, and then we ſhall acknowledge or not, that there is a Centripetal

Force too of one Kind or other, by which the Figure Deſcribed by p in the Moveable Plane vCp is Performed.

SECOND, Suppoſing Cp to be Drawn always Equal to CP, and p to be in the Moving Orbit at the ſame Diſtance from v, as P from V in the Quieſcent one, that is, that p in the one Caſe Moves exactly as P in the other, excepting that vCp is Revolved about C, whilſt VCP is an Immoveable Plane, by which the Difference of the Motions p and P will be Conferred upon VCv, or which is the ſame, upon vC Departing from VC; It is not yet Evident, how VCp will be an Angle Proportional to VCP; Becauſe, tho' Cp and CP from V and v will Deſcribe Areas Proportional to the Times, ſuppoſing VCP and vCp to be Quieſcent Planes, according to this Philoſophy, and tho' VC in it's Progreſs to vC will, according to the ſame Philoſophy, Deſcribe Areas Proportional to the Times, ſuppoſing the ſaid Planes to be Moveable ones, yet it will not follow, that Cp Deſcribes Areas Proportional to the Times, when the Plane vCp is Moveable, ſince the Body p will not have the ſame Effect, either in it's Projectile or Gravitating Force, when the Plane vCp is Revolved round C, as the Body P has, when VCP is Quieſcent, and therefore will not Deſcribe Areas Proportional to the Times in the Moveable Plane, if P does in the Quieſcent one; for we are not to Conſider p Revolving in vp in the ſame Manner, as P in VP, and make all the Difference of their Revolutions to lie in Cv being Carry'd in an Angular Motion from CV; which, notwithſtanding, is the Senſe of this Propoſition, and which therefore takes for Granted, what is to be Proved, that p Deſcribes Areas Proportional to the Times in a Moving Plane, becauſe P does in a Quieſcent one.

THIRD, Suppoſing p did Deſcribe Areas Proportional to the Times, yet ſince it does it under Different Circumſtances, than what were before Alledged in Propoſition the 1*ſt* of this Philoſophy, when Figures were made by a Projectile and a Gravitating Force, it would not Enſue, that p likewiſe was Revolved by ſuch Forces, ſeeing the Projectile Force it ſelf is here a Circular one, which before was ſuppoſed always to Act in a Tangent.

FROM whence we may Conclude, that here is a new Aſſumption made in the Preſent Philoſophy, and which is no otherwiſe Proved than by the Neceſſity of it to it's Future Explications.

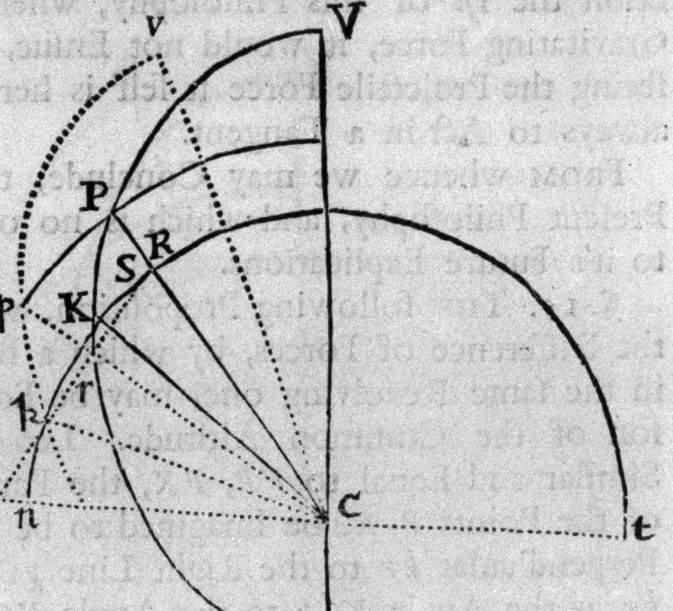
Concerning the Difference of the Forces, by which a Body is Moved in a Quieſcent or Revolving Trajectory.
Sir Iſ. Newt. Princip. §. 9. Prop. 44. Lib. 1.
Dr. Gregory's Aſtronom. §. 2. Prop. 4. Lib. 4.

§. 15. THE following Propoſition, which we ſhall likewiſe Examine into, is, that the Difference of Forces, by which a Body in a Quieſcent Orbit, and another Body in the ſame Revolving one, may be Equally Moved, is Inverſely in a Triplicate Reaſon of the Common Altitude. Let vp, pk the Parts of a Revolving Orbit be Similar and Equal to VP, PK, the Parts of a Quieſcent one, and let the Diſtance of the Points P, K, be Imagined to be Infinitely ſmall; From the Point k Draw the Perpendicular kr to the Right Line pC, and Produce it to m, ſo that mr may be to kr as the Angle VCp to the Angle VCP; and ſince the Altitudes of the Bodies PC, and pC, KC and kC are always Equal, it is Evident, that the Increments or Decrements of the Lines PC and pC are always Equal; and therefore, if each of the Motions of the Bodies, when they are in P and p, are Diſtinguiſhed into Two, by *Corol.* 2, of the Laws, the one of which is Determined towards the Center, or according to the Direction of the Lines PC and pC, and the other is Tranſverſe to the Former, and according to Lines Perpendicular to PC and pC, the Motions towards the Center will be Equal, and the Tranſverſe Motion of the Body p will be to the Tranſverſe Motion of the Body P, as the Angular Motion of the Line pC to the Angular Motion of the Line PC, that is, as the Angle VCp to the Angle VCP; Conſequently in the ſame Time, in which the Body P with both it's Motions Arrives at the Point K, the Body p with an Equal Motion towards the Center will be Equally Moved from p towards C, and upon the Determination and Concluſion of that Time will be found ſomewhere in the Line mkr, which Drawn thro' the Point k is Perpendicular to pC, and will by it's Tranſverſe Motion acquire a Diſtance from the Line pC, which will be to the Diſtance, which the other Body P acquires from PC, as the Tranſverſe Motion of the Body p to the Tranſverſe Motion of the Body P, that is, as the Angle vCP to the Angle VCP, or as mr to kr, and therefore the Body p at the End of ſuch Time will be found in m; Theſe Things will be ſo, when the Bodies p and P are Moved Equally according to the Lines pC, and PC, and conſequently are Urged by Equal Forces according to thoſe Lines; Let now the Angle pCn be to the Angle pCk, as the Angle VCp to the Angle VCP, and let nC be Equal to kC, and the Body p at the End of the ſame Time will be found in n, and ſo is Urged by a Greater Force than the Body P, if the Angle mCp is Greater than the Angle kCp, that is, if the Trajectory or Orbit vpk is Moved *in Conſequentia*, or if it is Moved *in Antecedentia* with a Greater Celerity, than the Double of that, with which the Line CP is carry'd *in Conſequentia*; and by a Leſs Force, if the Orbit is Moved Slower *in Antecedentia*; and the Difference of the Forces will be, as the Interval of the Places mn thro' which the Body p is Transferred by it's Action, in the aforeſaid Time. From the Center C with the Radius Cn or Ck, Let a Circle be Deſcribed, ſo as to Interſect the Lines mr, mn Produced, in s and t, and the Rectangle $mn \times mt$ will be $= mk \times ms$, by the *Elements* of *Euclid*, and $mn = \frac{mk \times ms}{mt}$; and ſince the Triangles pCk, pCn are given in their Magnitude, kr and mr, and their Difference mk, and their Sum ms are Reciprocally, as the Altitude pC; and therefore the Rectangle $mk \times ms$ is Reciprocally, as the Square of the Altitude pC; Alſo mt is Directly as $\frac{1}{2}\,mt$, that is, as the Altitude pC, and conſequently the Naſcent Line $mn = \frac{mk \times ms}{mt}$ is Reciprocally, as the Cube of the Altitude pC. *Q. E. D.*

Dr. Gregory's Aſtronom. Prop. 4. Corol. 1. Lib. 4.

The Objections which lie againſt the Foregoing Demonſtration.

§. 16. But notwithſtanding the Authority of the Propoſer, and the Capital Letters Affixt to Show us, or to Perſwade us, that the Propoſition is ſtrictly Proved, we muſt beg the Liberty to Inquire, whether it is ſo or not.



And First, PK and pk being ſuppoſed Infinitely Small, all which has been Objected againſt the Method of Naſcent and Evaneſcent Reaſons will have a Place here.

Second, Since $mr \,.\, kr :: VCp \,.\, VCP$, upon which Proportion this whole Reaſoning Proceeds, that is likewiſe taken for Granted, which we have Diſproved in the foregoing Propoſition, namely, that the Centripetal Force of the Body p in the Revolving Trajectory is the ſame, as that of P in the Quieſcent one, ſince the Tranſverſe or Projectile Force will make an Alteration in the one, which is not from the Quieſcence of the Orbit Induced upon the other; and therefore, tho' pC and PC, kC and KC are Equal, the Increments and Decrements of thoſe Lines will not be ſo.

Third, The Diſtinguiſhing the Motion of each Body P and p into Two by *Corol.* 2 of the Laws has been Examined, and the Reaſoning Refuted.

Fourth, Granting which, the Forces, into which the Motion of each Body P and p is Reſolved, are not Juſtly and Truly Aſcertained, ſince in the Quieſcent Trajectory, the Forces, with which P Revolves, are Confeſſed to be a Centripetal to C, and a Projectile one in a Tangent according to the foregoing Propoſitions in this Philoſophy, whereas here, in order to make them Correſpond with the Compound Forces of p, thoſe of P are Affirm'd to Conſiſt of a Centripetal to C, and a Tranſverſe, Perpendicular to PC, and which is Meaſured by the Angle VCP, Deſcribed by the Angular Motion of the Line PC, which is very Different from the *Vis Inſita* Acting in a Tangent, becauſe this Tranſverſe Motion it ſelf, being an Angular one, is made up of a Centripetal and a Projectile by this Philoſophy, and therefore cannot be ſubſtituted in the Room of a Simple Projectile; Nor is it ſufficient to ſay, tho' here ſeems to lie the Fallacy, that the Tranſverſe Motion is ſuppoſed to be Perpendicular to PC, as the Projectile is, and therefore the one may be Aſſumed for the other, this is not ſufficient; Becauſe a *Vis Inſita*, Proceeding Equably in a Tangent, can never be Reduced to, or Expreſſed by an Angular Motion, which Conſiſts of that, and a Centripetal one; Beſides, ſince the Areas VCP are Proportional to the Times of their Deſcription, the Angular Motion of CP from VC will be very Unequal, and therefore an Improper Repreſentative of the *Vis Inſita*, which goes on uniformly.

FARTHER, Which ſhews this Philoſophy do's Juſt what it Pleaſes, Allowing what has been before Diſputed, the Right Line, which is Perpendicular to PC, does not ſo much as Exhibit this Tranſverſe or Angular Motion, no more than the Sine of any Angle is the Expreſſion of the Angle, that is, the Perpendiculars to PC are not Proportional to the Angles VCP, or to the Elongations of PC from VC. In the Moving Trajectory all theſe Things are ſtill more Embarraſſed and Confuſed, for in this Caſe the Tranſverſe Motion is ſaid to be as the Angular of pC, or as the Angle VCp, againſt which not only all, which has been before Alledged, is Valid, but more, It is altogether Unaccountable, how a Perpendicular to pC, which would Repreſent the Tranſverſe Motion according to this Propoſition, if the Trajectory was Quieſcent, ſhould be made uſe of to Explain the Tranſverſe Motion, when the Trajectory is ſuppoſed to be a Revolving one, and how mk, which is in the ſame Direction with kr ſhould Exhibit or Expreſs the Tranſverſe Motion of VCv, that is, of a Line, which is Parallel to mC, and Inclined in an Angle to mk; or it is Inconceivable, as well as not Proved, that $mr.kr::VCp.VCP$, or as the Tranſverſe Motion of the Body p, to the Tranſverſe Motion of the Body P, ſince the Tranſverſe Motion of the Body p cannot be Reſembled to the Angular one of pC from VC, which Conſiſts of Cp from Cv, and of Cv from CV at the ſame Time, but the Angular Motion vCp is Compounded of a Centripetal and Projectile Force, which therefore cannot be Expreſſed by a Projectile, or a Line Perpendicular to pC, and the Angular Motion VCv is Compounded likewiſe of a Centripetal and a Projectile, which has a Different Direction from the Former, being Parallel to Cm, and therefore cannot alſo be Expreſſed by the ſame Perpendicular to pC, and conſequently it is not Evinced, that the Body p will be found in m.

MOREOVER, if $mr.kr::VCp.VCP$, mr muſt be Evidently Greater than kr, and therefore the Tranſverſe Force mr will Carry p out of it's Elliptical Orbit from k to m, the Centripetal Forces being Equal; if ſo, and the Tranſverſe Force is Superior to the Centripetal, it cannot be ſhewn, why the ſame Tranſverſe Force ſhould not Carry the Body forward in the ſame Right Line, without permitting it to Revolve in an Ellipſis, or any other Curve whatſoever; unleſs it can be ſuppoſed, that the Centripetal Force has a Greater Power to Recollect it ſelf in m towards C, than it had in k, when it was nearer to it; If yet this was not Effected by the Superiority of the Tranſverſe Force, the Ellipſis or Trajectory notwithſtanding would be Spoiled, whereas this Propoſition is Contrived to ſhew us, how and with what Forces a Body may be Moved in a Revolving Ellipſis or Trajectory.

FIFTH, One would think here were Poſtulatums and Aſſumptions ſufficient already to give an Inquirer enough of the Preſent Demonſtration, and yet it has not done Asking; the Body p is brought one way or other to the Point m, but this would ſignify nothing, if there was no Difference in the Centripetal Forces, which have hitherto been Imagined to be the ſame in p and P; in order therefore to make this Difference, it is Preſcribed, that the Angle pCn ſhould be made to pCk, as the Angle VCp to the Angle VCP, and that the Line nC ſhould be Equal to kC, and ſo the Body p will be found in n, and therefore is Urged with a Greater Centripetal Force than the Body P, if the Angle mCp is Greater than the Angle kCp, that is, if the Orbit vpk is Moved *in Conſequentia*, and the Difference of the Forces will be mn.

BUT here alſo it is to be obſerved, that VCP, VCp are ſtill the Meaſure and Standard of the Tranſverſe Forces, which has been already Objected againſt, Granting which, it is now Affirmed, that $pCn.pCk::VCp.VCP$, and before $::mr.kr$, ſince therefore nkr may be as juſtly the Meaſure of pCn or pCm, as mkr, mk will be Equal to nk, and m and n will Coincide, and we are got no farther than we were at our Firſt ſetting out in this Demonſtration. Beſides which, nC is made Equal to kC, that the Body p may be at the ſame Diſtance from C in n, as in k, and yet without any Apparent Cauſe, the Centripetal Force in n is made Greater than in k, and no wonder, becauſe the Body p is as Arbitrarily Removed from m to n, as that Greater Centripetal Force is Arbitrarily Introduced; For if the Diſtances from C in k and n are Equal, what ſhould make the Body p Gravitate more in n than in k? If it is ſaid, That is Neceſſary to bring it from m (whither the Tranſverſe Force would have carry'd it) to n, the Neceſſity is Acknowledged, but from whence p ſhould have a Greater Centripetal Force at m, ſo as to Determine it to n, is the Inquiry, eſpecially when the Tranſverſe Force has already Carry'd p out of it's Orbit to m.

Farther, Suppoſing the Body p in n, it is yet out of the Moving Ellipſis or Trajectory, whereas this Propoſition was to give an Account of the Forces neceſſary to keep a Body within ſuch a Moving Ellipſis or Trajectory, and is accordingly apply'd in all the Subſequent Philoſophy.

Sixth, Having at Length with a great Deal of Labour, and with no Small Satiſfaction to it ſelf, tho' with little to any Impartial Conſiderer, Settled the Difference of Forces, which is Propoſed to be mn, this Demonſtration Proceeds by the *Elements* of *Geometry* to Aſcertain, what this Force muſt be; A Circle therefore is Drawn by the Radius nC, or kC, the Secants of which are mt, ms, and the Lines Intercepted betwixt the Point m and the Circle, are mn, mk, and therefore, by the ſaid *Elements*, $mn \times mt$ is Equal to $mk \times ms$, and $mn = \frac{mk \times ms}{mt}$, which, by what I can Perceive, is the only certain Thing in the whole Proceſs of the Preſent Reaſoning. But then, when it goes on again upon it's own Principles and Aſſumptions, we are Loſt and Bewilder'd as before, and find Nothing, which is Clear or Concluſive.

Thus the Triangles pCk, pCn are ſaid to be Given in their Magnitude, but wherefore? Becauſe the Angles pCk, pCn are as VCP to VCp, which has been already Conſidered; But ſuppoſing they are ſo, it does not follow, that kr and mr are Reciprocally, as the Altitude pC, ſince in the Triangle pCn, mr has nothing to do, unleſs mr and nr are the ſame, and then mn or the Difference of the Forces is Nothing; If therefore kr and mr are not Reciprocally, as the Altitude pC, their Difference mk, and their Sum ms, are not in that Reciprocal Proportion of the Altitude pC, nor the Rectangle $mk \times ms$ Reciprocally, as the Square of ſuch Altitude.

To make mn this Imaginary Difference of Equal Forces to be Reciprocally, as the Cube of the Altitude pC, it is farther Aſſumed, that mt or $\frac{1}{2}mt$ is in a Direct Proportion, as the Altitude pC, which is Impoſſible, ſince, if $pCk . pCn :: kr . mr$, m will be Determined by that Proportion, and which Determination of m will be ſo Uncertain, that neither mC, nor mt, nor $\frac{1}{2}mt$ will be Directly as pC, and conſequently, $\frac{mk \times ms}{mt}$ will not be Reciprocally, as the Cube of the Altitude pC, and yet this was the Propoſition to be Demonſtrated.

The Corollaries not neceſſary to be Conſider'd, nor the following Propoſitions of this Section.
Sir Iſ. Newt. Princip. Prop. 44. & Corol. & Prop. 45. Corol. Lib. 1. Dr. Gregory's Aſtronom. Prop. 4. Lib. 4. Corol Prop. 5, 6, 7, & Corol.

§. 17. After this Point was Confirmed, this Philoſophy had Room to Play and Divert it ſelf with Quantities and Proportions from *Corol.* the *1ſt* to the *6th*, which Concludes this Propoſition, and from thence Proceeds to *Prop.* the *45th*, and it's Two Corollaries; None of which there is any Occaſion to Examine, unleſs becauſe it Introduces it's Fluxions here, which we ſhall afterwards Conſider, and thus Ends Section the Ninth.

The Doctrine of the Revolution of the Apſides farther Explained.

§. 18. And now we are able to give a more Particular Account of the Doctrine of the Revolution of the Apſides in this Philoſophy, which we before Promiſed, and could not ſo well Explain, till we had made an Examination into the foregoing Propoſitions; For the Direct Conſequence, which follows from the Preceding Demonſtration, and upon which the Doctrine of the Revolution of the Apſides Depends, is, That if from a Centripetal Force, which is Reciprocally, as the Square of the Altitude, a Force be Subſtracted, which is Directly as the Altitude, a Force Remains, which in it's Receſs from the Center Decreaſes more, or is Leſs than according to a Duplicate Reaſon of the Altitude Increaſed, or which in it's Acceſs to the Center Increaſes more, or is Greater than according to a Duplicate Reaſon of the Altitude Diminiſhed; in both which Caſes, the Apſides will be Progreſſive, or be Carry'd *in Conſequentia*; On the Contrary, if to a Centripetal Force, which is Reciprocally, as the Square of the Altitude, be added a Force, which is Directly as the Altitude, a

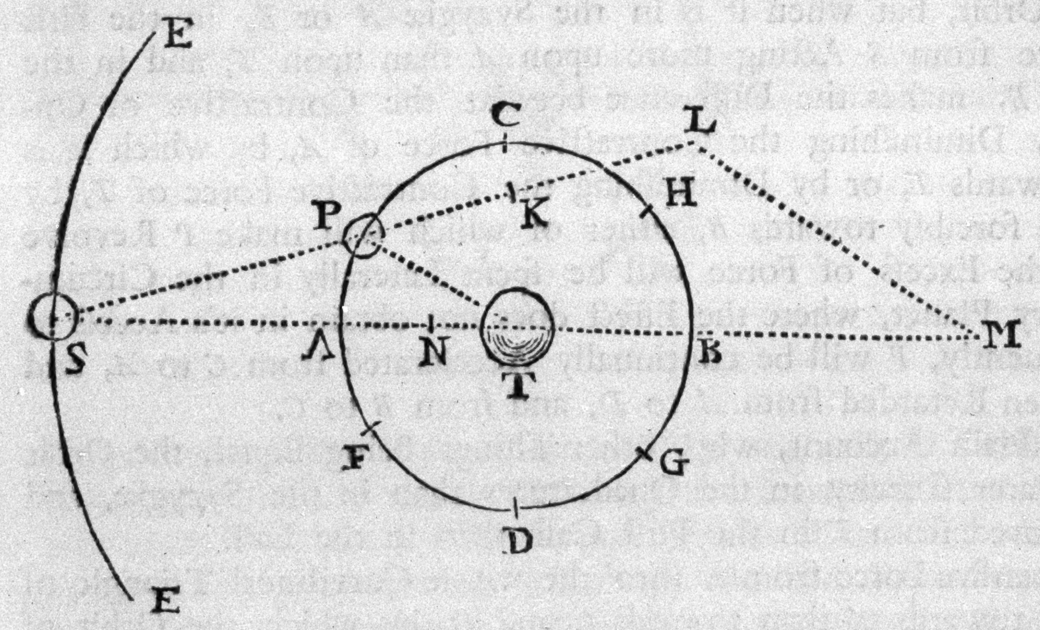
Force is Compounded, which in it's Receſs from the Center Decreaſes Leſs, or is Greater than according to a Duplicate Reaſon of the Altitude Increaſed, or which in it's Acceſs to the Center Increaſes Leſs, or is Leſs than according to the Duplicate Reaſon of the Altitude Diminiſhed; in both which Caſes, the Apſides will be Regreſſive or be Revolved *in Antecedentia*; If now this be Apply'd, which is Deduced from the Demonſtration Premiſed, and particularly from that, which is Alledged in the Middle of it, Concerning the Orbit being Carry'd *in Conſequentia* or *in Antecedentia*, as the Centripetal Force is Greater or Leſs than in a Duplicate Proportion, If this be Apply'd to what we have before Produced concerning the Apſides in the Corollaries to the *66th* Propoſition of this Philoſophy, which we have already Examined, the Whole of this Doctrine will be fully Explained; and as we have ſhewn this Demonſtration to be Precarious and Inconcluſive upon ſeveral Accounts, it follows, that this Doctrine of the Revolution of the Apſides ſubſiſts in the Fancy and Imagination only, and has Nothing to ſupport it, but Lines and Figures, Arbitrary Aſſumptions, and Poſtulatums, which cannot be Granted, and therefore is deſervedly Rejected by us.

An Explanation of the Phænomenons of the Primary and Secondary Planets from the Expanſive and Contractive Forces.

§. 19. WE ſhall now therefore Endeavour to give an Explanation of theſe Phænomenons of the Primary and Secondary Planets from the Expanſive and Contractive Forces, and ſhall take them in the ſame Order, as they are Propoſed to us *Prop. 66. Lib.* 1. of this Philoſophy.

Sir Iſ. Newt. Princip. Prop. 66. Corol. 2, 3. Lib. 1. Dr. Gregory's Aſtronom. Prop. 1. Lib. 4.

1*ſt*. IN a Syſtem of Three Bodies *T*, *P*, and *S*, the Laſt of which has an Expanſive Force, as the Sun, by which it Repells all Bodies from it, and the other Two a Contractive, by which they Draw all Bodies towards them, as the Earth and Moon, or a Primary and Secondary Planet, or Two Primary, or Two Secondary ones, for the Caſe is much the ſame, when we Conſider them as Endued with the like Contractive Forces, Let us therefore take the Firſt Inſtance of a Primary and Secondary Planet, namely of the Earth and the Moon, ſince the Reſt may be Reduced to it with Certain Allowances, There ſeem to be Two Reaſons, wherefore the Secondary Planet *P* in it's Orbit round the Primary *T*, is Accelerated from *C* to *A*, Retarded from *A* to *D*, and again Accelerated from *D* to *B*, and again Retarded from *B* to *C*, and conſequently, why the Secondary Planet *P* Moves with a Greater Velocity in the Syzygiæ *A* and *B*, than in the Quadratures *C* and *D*, other Things being Equal.



FIRST, Becauſe, the Expanſive or Repulſive Force of the Sun Spreading and Diffuſing it ſelf from *S* thro' the whole Triangle *CST*, *P*, when in *C*, has all that Force to Surmount in it's Deſcent towards *A*, and therefore Moves with a Leſs Velocity in *C*, where the Expanſive Force of the Sun Acts very near Equally upon the Primary Planet in *T*, and on the Secondary in *C*, but when the Secondary Planet Arrives at *P*, it has a Leſs Expanſive or Repulſive Force Acting in the Triangle *PSA* to Subdue, by which means it Moves with a Greater Velocity continually in it's Deſcent from *C* to *P*, and from thence to *A*.

IN it's Aſcent from *A*, it has all the Expanſive Force *TSD*, which Acts from *S*, Tending the ſame way with it's own Motion, and therefore is Accelerated in *A*, but Leſs in *F*, becauſe the Triangle *FSD* of the Expanſive or Repulſive Force is Leſs, from whence it is ſtill Leſs Accelerated in it's Paſſage from *F* to *D*.

LIKEWISE from *D* to *B* it is again Accelerated, yet Leſs in *D*, becauſe in the Triangle *TSD* the Expanſive Forces from *S* are nearly Equal in *T* and *D*, but More in *G*, becauſe the Expanſive Forces *GSD*, which Tend the ſame way, are Greater, and the

Moſt in *B*, becauſe the Expanſive Forces *BSD*, Tending the ſame way, are then the Greateſt.

AGAIN in it's Paſſage from *B* to *C* it is Retarded by the Expanſive Force Acting Contrary to it, in *B* Leſs, in *H* More, becauſe *HSB* is Greater than *SB*, and in *C* the Moſt, becauſe the Expanſive Forces in the Triangle *CSB* Acting Contrary ways are then the Greateſt; All this is ſaid upon the Suppoſition, that, of it ſelf, *P* Revolves about *T* uniformly by Certain Forces Adjuſted betwixt them, which are not here Conſidered.

SECOND, Another Reaſon of this Acceleration and Retardation in the Syzygiæ and Quadratures is, That in the Firſt Caſe the Contractive Forces are Diminiſhed, in Reſpect of each to the other, and in the Laſt not ſo. To Explain this more Particularly; Thus *P* Revolves about *T* by that Exceſs of the Contractive Force, which the one has above the other, whether of *T* above *P*, or of *P* above *T*, for ſince that Exceſs of Force, by which they Draw againſt, and in Oppoſition to each other, cannot bring them Together, it muſt of Neceſſity be ſpent Sideways, and by that means occaſion a Revolution; the Greater ſuch Exceſs therefore is, the Quicker will the Revolution be, and the Leſs ſuch Exceſs is, the Slower; For by this very Method it is, that a Stone Revolves in a Sling, the Hand Attracting the String one way, and the Stone another, the Exceſs of which Two Forces is conferred upon the Circular Motion Produced from the Conteſt between them; This does not obtain in Two Forces Drawing againſt each other, and Gravitating to a Third as the Earth, becauſe ſuch Gravitation hinders the Revolution, which, as it is an Argument, that the Earth and Moon and the other Planets do not Gravitate to the Sun, ſo it is a Reaſon, why any Sublunary Forces cannot be Objected againſt this Doctrine.

FROM whence it comes to paſs, that, when the Secondary Planet is in the Quadratures *C* or *D*, the Expanſive Force from *S* Acting nearly alike upon both, it Proceeds uniformly in it's Orbit, but when it is in the Syzygiæ *A* or *B*, in the Firſt Caſe the Expanſive Force from *S* Acting more upon *A* than upon *T*, and in the Laſt upon *T* than upon *B*, makes the Difference betwixt the Contractive or Oppoſite Forces Greater, by Diminiſhing the Contractive Force of *A*, by which it is Drawn more forcibly towards *T*, or by Diminiſhing the Contractive Force of *T*, by which it is Drawn more forcibly towards *B*, either of which will make *P* Revolve Faſter about *T*, becauſe the Exceſs of Force will be ſpent Laterally in the Circumvolution of the Secondary Planet, where the Effect does not obtain in it's Acceſs to the Primary, and conſequently, *P* will be continually Accelerated from *C* to *A*, and from *D* to *B*, and as often Retarded from *A* to *D*, and from *B* to *C*.

Sir Iſ. Newt. Princip. Prop. 66. Corol. 4, 5. Lib. 1. Dr. Gregory's Aſtronom. Prop. 2. Lib. 4.

2*d*. FROM hence is a Plain Account, why, other Things being Equal, the Orbit of the Body *P* has a Greater Curvity in the Quadratures than in the Syzygiæ, and therefore is farther Removed from *T* in the Firſt Caſe than in the Laſt.

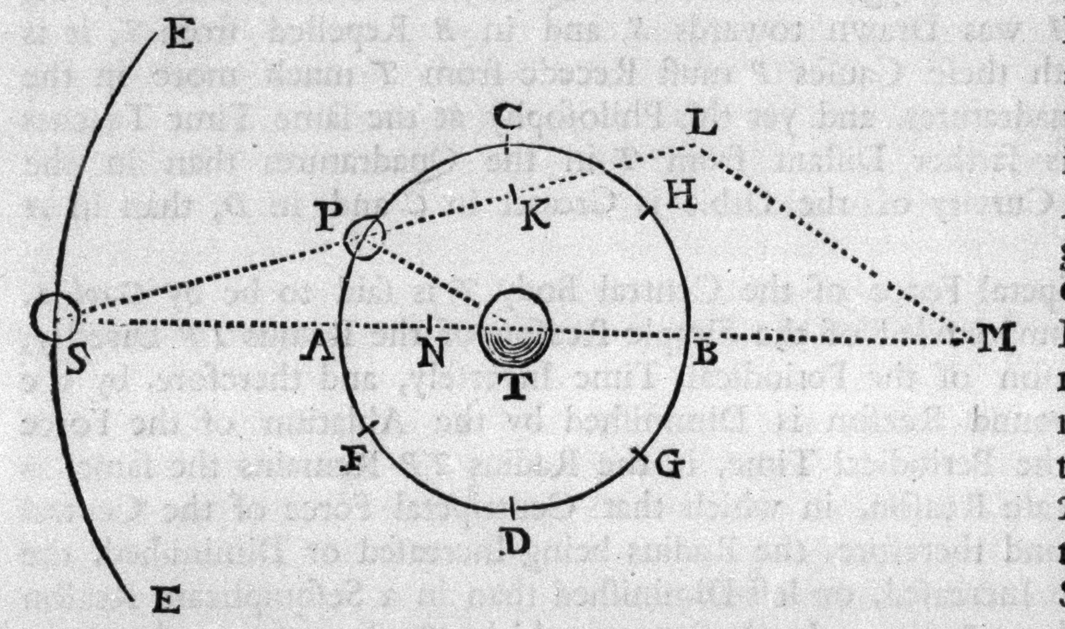
FIRST, Becauſe the Expanſive Force from *S*, thro' the whole Curvilineal Triangle of it *CSDAC*, is Stronger towards *A* than towards *C* and *D*, by which the Orbit of *P* is Flattened and Depreſſed in *PAF*, and made more Prominent in the Quadratures; ſo likewiſe in the Curvilineal Triangle *CSDBC*, *CBD* is Depreſſed towards *T*, becauſe the Expanſive Force from *S* Acts Stronger in *C* and *D*, and the Parts Adjoining, than in *HBG*, which therefore Sinks nearer to *T*, or the Secondary Planet *P*, which Deſcribes the Orbit, Approaches to *T*, which is the ſame.

SECOND, Becauſe the Expanſive Force from *S* Deſtroys ſome of the Contractive Force of *P* in *A*, by which it leſs Counter-Acts to *T*, and therefore is Drawn nearer to it, as alſo when *P* is in *B*, the ſame Expanſive Force from *S* Deſtroys ſome of the Contractive Force in *T*, by which it leſs Counter-Acts to the Force of *P* in *B*, and therefore Approaches nearer to it, which is not done in *C* and *D*, where the Expanſive Force Acts nearly alike upon both, and conſequently thoſe Points, and ſuch, which are Adjacent to them, are farther Removed from the Body *T*; but how theſe Phænomenons are Accounted for by a Gravitating Force, which Draws *A* towards *S*, and Repells *B* from *T* is not, as we have Shewn, nor can be Explained; This is ſaid Excluſive of the Excentricity of the Orbit, and the Motion of the Apſides, for by that the Body *P* may Recede farther from *T* in the Syzygiæ than in the Quadratures, as is Affirmed in this Philoſophy.

3*d*. FROM the ſame Expanſive and Contractive Forces, it is alſo Evident, that the Aux or the Summa Apſis *B* will be Regreſſive or Move towards *G*, when the Body *P* is in *C*, or in it's Quadrature; Becauſe, being there Retarded in it's Motion, if it were to Proceed round it's Orbit with the ſame Diminiſhed Celerity, it would be neceſſary for *B*, or the Summa Apſis, to be Carry'd towards *G*, that it may Arrive at the Point of the Orbit, or the Summa Apſis, from whence it ſet out, in the ſame Periodical Time; On the Contrary, when the Body *P* is in the Syzygia *A*, and is Moved with a Greater Velocity, the Aux or the Summa Apſis *B* will be Progreſſive towards *H*; Becauſe, the Motion of the Body being now Accelerated, if it were Carry'd with the ſame Acceleration thro' the Reſt of the Orbit, it would come too ſoon to *B*, and which therefore is Removed towards *H*, that it may ſtill obſerve the ſame Periodical Time in it's Revolution; Alſo for the ſame Reaſon, when the Body *P* is in *D*, it's other Quadrature, the Summa Apſis is Carry'd *in Antecedentia* from *B* towards *G*, and when in the Syzygia *B*, *in Conſequentia*; And ſince the Acceleration is Greater than the Retardation, the Summa Apſis, in each Revolution of the Body *P*, is Carry'd *in Conſequentia*.

Sir Iſ. Newt. Princip. Prop. 66. Corol. 6, 7, 8. Lib. 1. Dr. Gregory's Aſtronom. Prop. 8, 9, 10, 11. Lib. 4.

AND from hence it will likewiſe follow, that, when the Apſides are in the Syzygiæ, they will have a Swifter Progreſſion, and, when in the Quadratures, a Slower; Becauſe the Body *P* is more Accelerated in the Syzygiæ, when the Apſides are in the Syzygiæ, than when they are in the Quadratures, ſince the Triangle *CST* of the Expanſive Force, (by which *P* is Retarded, when in *C*, and is Leſs ſo in *P*, where the Triangle *PST* is Leſs) is Leſs, when the Apſides are in the Syzygiæ, than when in the Quadratures; as alſo the Expanſive Force from *S* makes a Greater Difference betwixt the Contractive Forces of *A* and *T*, and *T* and *B*, when the Apſides are in the Syzygiæ, than when they are in the Quadratures, becauſe in the Firſt Caſe there will be originally a Greater Difference betwixt the Contractive Forces, which Difference the Expanſive from *S* will Increaſe, and in the Laſt there will be a Leſs Difference betwixt them, and which therefore will not be ſo much Increaſed by the Action of the ſaid Expanſive, but it has been before ſhewn, that the Greater the Difference is betwixt the Contractive Forces, the more Force will be ſpent Sideways, and the Body *P* thereby Accelerated in it's Revolution.



THERE are therefore in this Solution only Two Difficulties Remaining; 1*ſt*, How the Removal of the Apſis backward or forward to make the ſame Periodick Time of the Body *P* in it's Orbit is to be Explained; 2*d*, How it can be Proved, that the Acceleration of *P* is Greater, in the Whole, than it's Retardation, ſo as to Cauſe a Progreſſion of the Apſides.

As to the Firſt it may be ſaid, that at the ſame Time, that the Expanſive Force from *S* Retards the Body *P* in *C*, or the Quadrature, there is likewiſe a Reflection of it from *P* in *C* to *B*, by which the Aux or the Summa Apſis is Protruded from *B* towards *G*, and by which it becomes Regreſſive.

ON the other Hand, when *P* is in *P*, *A*, and *F*, the Expanſive is Diverted towards *G*, by which it is Repell'd back towards *B* and *H*; when again *P* is in the Quadrature *D*, the Expanſive is Reflected towards the Parts *B* and *H*, and hinders the Progreſſion of *G* or *B* towards *H*, or rather makes it Regreſſive; and laſtly, when *P* is in *G*, *B*, or *H*, the Expanſive Force Carrys *B* forwards by a Reflection of it from *G* to *B*, and *B* to *H*.

As to the Second, that the Acceleration is Greater than the Retardation in the whole Orbit, is manifeſt, becauſe the Retardation of *P* Proceeds from the Expanſive Force, as before Explained, and in which the Difference of the Contractive Forces obtains little or no Effect, but the Acceleration of it Ariſes from a Double Cauſe, from the Expanſive Force leſs Obſtructing it's Motion, and from it's Increaſing the Difference of the Contractive Forces betwixt *T* and *P*.

Whatever Objections may be made againſt this Solution, they perhaps would Vaniſh, if we knew more Perfectly the Laws of the Expanſive and Contractive Forces, but whatever they are, as I am ſufficiently aware, what they are or can be, yet as this Solution Depends upon Principles, which are the ſame, as it Connects all theſe Phænomenons together, and makes them to have a Dependance upon each other, ſo it is Preferrable at leaſt to the Preſent Account, which Introduces New, and Diſtinct, and Independent Propoſitions to ſolve the Motion of the Apſides, and perfectly Different from what are uſed to Explain the Acceleration and Retardation of the Body in it's Orbit.

Sir Iſ. Newt. Corol. 6. Prop. 66. Lib. 1.

Thus First, It is ſaid, that the Centripetal Force of the Central Body *T*, by which the Body *P* is Retained in it's Orbit, is Increaſed in the Quadratures by the Addition of the Force *LM*, and is Diminiſhed in the Syzygiæ by the Ablation of the Force *KL*, and by Reaſon of the Greatneſs of the Force *KL* is more Diminiſhed than Increaſed; conſequently, if the Centripetal Force of *T*, by which *P* is Retained in it's Orbit, is Leſs in the Syzygiæ than in the Quadratures, ſince before by the Action of *MN*, *P* in *A* was Drawn towards *S*, and in *B* Repelled from *T*, it is Manifeſt, that from both theſe Cauſes *P* muſt Recede from *T* much more in the Syzygiæ than in the Quadratures, and yet this Philoſophy at the ſame Time Teaches us to Believe, that *P* is farther Diſtant from *T* in the Quadratures than in the Syzygiæ, and that the Curvity of the Orbit is Greater in *C* and in *D*, than in *A* and *B*.

Second, This Centripetal Force of the Central Body *T* is ſaid to be by *Corol.* 2. *Prop.* 4. in a Reaſon Compounded of the Simple Reaſon of the Radius *TP* Directly, and of a Duplicate Reaſon of the Periodical Time Inverſely, and therefore, by the firſt Article, this Compound Reaſon is Diminiſhed by the Ablation of the Force *KL*, and conſequently the Periodical Time, if the Radius *TP* Remains the ſame, is Increaſed in a Subduplicate Reaſon, in which that Centripetal Force of the Central Body *T* is Diminiſhed, and therefore, the Radius being Increaſed or Diminiſhed, the Periodical Time is more Increaſed, or leſs Diminiſhed than in a Seſquiplicate Reaſon of this Radius, by *Corol.* 6. *Prop.* 4. In Anſwer to which, *Corol.* 2. *Prop.* 4. has been already Examined and Refuted, as likewiſe *Corol.* 6. *Prop.* 4.

Granting which, if the Periodical Time is Increaſed by the Ablation of *KL*, which therefore muſt be in the Syzygiæ, the Velocity is Diminiſhed in the Syzygiæ, as the Periodical Time is Increaſed, whereas, before, we were Informed, that the Body *P* was Accelerated in the Syzygiæ, and Retarded in the Quadratures, by the Action of *MN*; and yet now by the Ablation of *KL*, it ſeems, it is Retarded in the Syzygiæ, and I ſuppoſe by the Addition of *LM* for the ſame Reaſon Accelerated in the Quadratures, tho' theſe Forces Act at the ſame Time, and obtain their Effects together, that is, *MN* which Accelerates, and *KL* which Retards in the Syzygiæ, as alſo, *MN* which Retards, and *LM* which Accelerates in the Quadratures.

Sir Iſ. Newt. Princip. Prop. 66. Corol. 7. Dr. Gregory's Aſtronom. Prop. 8. 11. Lib. 4.

Third, It is Affirmed, that in the Quadratures, where *MN* Vaniſhes, the Force, with which the Body *P* is Urged towards the Body *T*, is Compounded of the Force *LM*, and of the Centripetal Force, with which the Body *T* Attracts the Body *P*; the Force *LM*, if the Diſtance *PT* is Increaſed, is likewiſe Increaſed nearly in that Proportion, and the Latter Force Decreaſes in a Duplicate Reaſon of the ſame Diſtance, and therefore the Sum of theſe Forces together Decreaſes in a Leſs than a Duplicate Reaſon of the ſaid Diſtance *PT*, and conſequently, by *Corol.* 1. *Prop.* 45. it Cauſes, that the Aux or the Summa Apſis is Regreſſive, or Moves *in Antecedentia*.

On the Contrary, In the Syzygiæ, the Force, by which the Body *P* is Urged towards the Body *T*, is the Difference betwixt the Force, with which the Body *T* Attracts *P*, and the Force *KL*, and becauſe the Force *KL* is Increaſed almoſt in Proportion to the Diſtance *PT*, That Difference Decreaſes in a Greater than a Duplicate Reaſon of the ſaid Diſtance; and therefore, by *Corol.* 1, *Prop.* 45. it Cauſes, that the

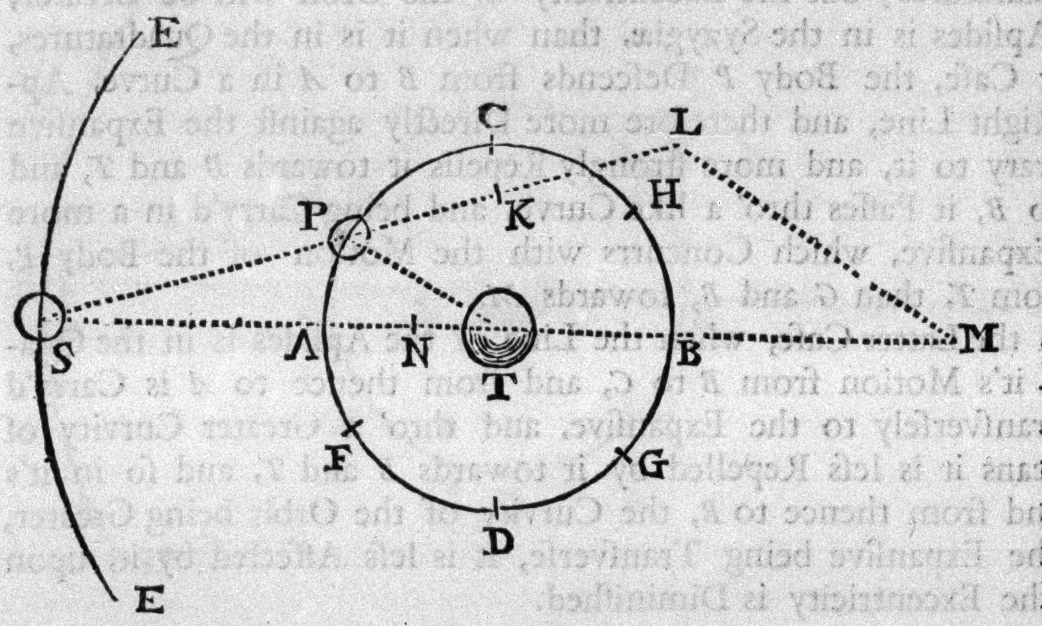
Aux or the Summa Apſis is Progreſſive, or Moves *in Conſequentia*, and ſince *KL* is Twice as big in the Syzygiæ, as *LM* in the Quadratures, the Exceſs in the whole Revolution will fall on the Side of *KL*, which will Carry the Aux *in Conſequentia*.

In Reply to which we have ſhewn, that this 45*th* Propoſition, and it's 1*ſt* Corollary is not Proved, that there is no Account given of the Revolution of the Ellipſis, and conſequently of the Apſides, that this Departure from the Duplicate Proportion being the Reaſon of the Progreſs or Regreſs of them is a Chimæra, and that we may as well take any Thing elſe to ſolve ſuch Progreſſion or Regreſſion, as the aforeſaid Deviation from the Duplicate Proportion. Beſides which, Granting the moſt, tho' there ſhould be a Receſs from the Duplicate Proportion, ſuppoſing the Apſides or the Ellipſis were Moved, it is not ſhown, that ſuch a Receſs from the Duplicate Proportion is a Cauſe of their Motion, which is Preſumed and Unaccounted for.

And Laſtly, The Revolving of Ellipſes or Orbits at once is a Mathematical Scheme and Projection, but to Imagine there is ſuch a Thing in the Heavens, is bringing us back to the Old and Exploded Epicycles, and Allowing that to the Modern Ellipſes, which we would not Grant to the Circles of the Antients.

Yet all theſe Various and Independent Matters are to be Confeſſed and Acknowledged, and Twenty more, in order to Prove this Progreſſion and Regreſſion of the Apſides, which at Length is not Obſerved in any of the Planets, in Reſpect of which they are Eſteemed Quieſcent, and all Calculations made concerning them, are upon the Suppoſition of their being ſo ; Notwithſtanding which, this Doctrine is laid ſo General, as to make us Think and Believe, that even their Apſides are Progreſſive and Regreſſive likewiſe, for what is here Demonſtrated ſeems to Relate not only to the Secondary Planets, but the Primary.

Sir Iſ. Newt. Princip. Prop. 14. Lib. 3. Dr. Gregory's Aſtronom. Prop. 43. Lib. 1.



To blind this Buſineſs of the Apſides ſtill farther, we have a Great ado made about the Seſquiplicate and Duplicate Proportions of the Diſtance *PT*, whereas we have before Obſerved, that whatever Solutions we may give of the Heavenly Bodies, as to their General Appearances, it is nothing Leſs than a Piece of Oſtentation and Vanity to pretend to Aſcertain the Preciſe Diſtances of the Planets from the Sun, or from each other, by any Inſtruments Aſtronomers can make uſe of; And this I am the more Confirmed in, when I have known good Mathematicians, in the Meaſuring the Height of a Turret only, of no more than Fifty Yards, to be Miſtaken in their Calculations, and to Differ from each other Two or Three Yards, as ſome of this College, where I now Write, may well Remember.

4*th*. From the ſame Expanſive and Contractive Forces we may Explain the Increaſe and Decreaſe of the Excentricity of the Orbit *BCAD*, for if the Body *P* is ſuppoſed to Deſcend from the Upper to it's Lower Apſis, namely, from *B* to *A*, where the Apſides are in the Syzygiæ, it is manifeſt, that the Expanſive Force from *S* will Cauſe *P* to Deſcribe an Orbit Interior to the Elliptick *BCA*, towards the Parts *CA*, ſo as to make *A* Approach nearer to *T*, by which the Excentricity will be Increaſed.

Sir Iſ. Newt. Princip. Prop. 66. Corol. 9. Lib. 1. Dr. Gregory's Aſtronom. Prop. 12, 13. Lib. 4.

First, Becauſe the Expanſive Force, Acting againſt *P* in it's Deſcent from *B* to *C*, and from thence to *A*, Repells it in every Step towards *B*, and conſequently, *BHC* from *T*, and *CPA* towards it.

Second, Becauſe the Expanſive Force from *S* Acting more ſtrongly upon *A* than upon *B*, the Contractive Force of *A* is thereby Diminiſhed, and conſequently does

not

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not Attract *T* ſo forcibly towards it, from whence the Contractive Force of *T* has a Greater Influence to Draw *P* in *A* towards it ſelf, and *P* in *A* by that means comes nearer to *T*.

If therefore *P* was to Aſcend from *A* to *D* and *B* with the ſame Conditions, as it Deſcended from *B* to *C* and *A*, it would Deſcribe *ADB* juſt the ſame kind of Orbit, and Equally Excentrical with *BCA*; But as the Expanſive Force from *S* Acts now the ſame Way with the Motion of the Body *P* in it's Aſcent from *A* to *D* and *B*, it Impells the Body *P* nearer to *T*, in it's Paſſage from *A* to *D* than in it's Paſſage from *C* to *A*, and Repells it farther from *T* in it's Paſſage from *D* to *B*, than in that from *B* to *C*, upon which Account the Excentricity is again Increaſed, and there is one Part of the Orbit Deſcribed Interior to *AD*, and another Exterior to *DB*.

From whence it follows, that the Excentricity of the Body *P*, Revolving in an Excentrical Orbit, is twice Changed in every Revolution, and that it is the Greateſt, when *P* is in the Syzygiæ, and the Leaſt, when in the Quadratures, and that in it's Paſſage from the Quadratures to the Syzygiæ, it is continually Increaſed, and in it's Paſſage from the Syzygiæ to the Quadratures, it is continually Diminiſhed.

Suppoſing now the Line of the Apſides to be placed in the Line of the Quadratures *CD*, ſo that the Aux or Summa Apſis ſhould be in *C*, and the Ima Apſis in *D*, it is Plain for the ſame Reaſons, as before, that the Greateſt Excentricity will be in *A* and *B*, and the Leaſt in *C* and *D*, that is, the Greateſt will be in the Syzygiæ, and the Leaſt in the Quadratures; but the Excentricity of the Orbit will be Greater, when the Line of the Apſides is in the Syzygiæ, than when it is in the Quadratures, Becauſe, in the Former Caſe, the Body *P* Deſcends from *B* to *A* in a Curve, Approaching nearer to a Right Line, and therefore more Directly againſt the Expanſive Force, which Acts contrary to it, and more ſtrongly Repells it towards *B* and *T*, and in it's Aſcent from *A* to *B*, it Paſſes thro' a like Curve, and being Carry'd in a more Direct Stream of the Expanſive, which Concurrs with the Motion of the Body *P*, it is Repulſed farther from *T*, than *G* and *B*, towards *M*.

On the Contrary, in the Latter Caſe, when the Line of the Apſides is in the Quadratures, the Body *P* in it's Motion from *B* to *C*, and from thence to *A* is Carry'd more Obliquely and Tranſverſely to the Expanſive, and thro' a Greater Curvity of the Orbit, by which means it is leſs Repelled by it towards *B* and *T*, and ſo in it's Motion from *A* to *D*, and from thence to *B*, the Curvity of the Orbit being Greater, and it's Paſſage thro' the Expanſive being Tranſverſe, it is leſs Affected by it, upon both which Accounts the Excentricity is Diminiſhed.

From whence it is conſequent, that, in ſeveral Revolutions of the Body *P* compared with each other, the Excentricity of it's Orbit is the Greateſt, when the Apſides are in the Syzygiæ, that it is from thence Perpetually Diminiſhed in the Paſſage of the Apſides from the Syzygiæ to the Quadratures, where the Excentricity is the Leaſt, and in the Paſſage of the Apſides from the Quadratures to the Syzygiæ, that it is again Increaſed.

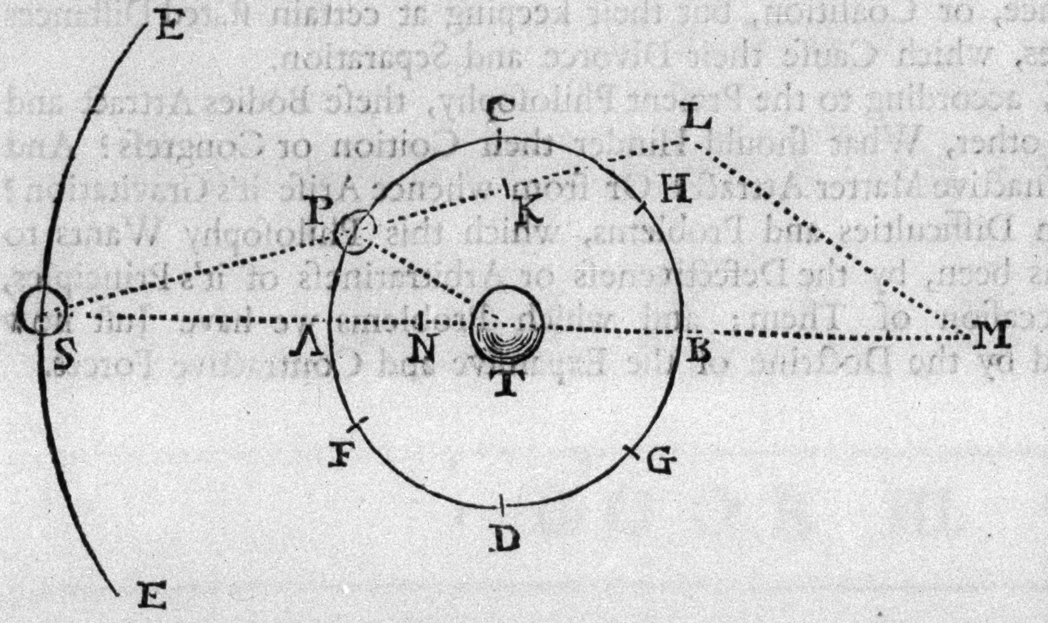
Having given our own Explanation of the Varying Excentricity of the Orbit *BCAD*, and having before Examined into that of the Preſent Philoſophy, it will be ſeen by a Compariſon, which Account is the more Probable.

Sir Iſ. Newt. Princip. Prop. 66. Cor. 10, 11, 12. Lib. 1. Dr. Gregory's Aſtronom. Prop. 14. 15, 16. Lib. 4.

5th. We ſhall therefore Proceed to the Errors in Latitude, which likewiſe are Explicable from theſe Expanſive and Contractive Forces; Let *EST* be an Immoveable Plane, and *BCA* Raiſed above, as *BDA* is Depreſſed beneath it, and conſequently in this Poſition, the Line of the Nodes *BA*, in which the Plane *BCADB* Interſects the Immoveable one *EST*, is in the Syzygiæ, when the Body *P* therefore is in *B* or in *A*, the Expanſive Force from *S* Acting according to the Direction *SA* or *SB* can make no Alteration in the Inclination of the Orbit *BCA* to *EST*, but when the Line of the Nodes is in the Quadratures *CD*, and the Body *P* is in *A* Elevated above the Immoveable Plane, it is manifeſt, that *P* will be Depreſſed towards the ſaid Plane by the Action of the Expanſive Force, for the ſame Reaſon, that *P* is Accelerated to *A* in it's Deſcent from the Quadrature to the Syzygia; becauſe there is a Leſs Expanſive Force betwixt *P* in *A*, and the Immoveable Plane, than there is above it, and when *P* is in *B* Depreſſed below the Immoveable Plane, it is by the ſame Exceſs of the Expanſive Force beyond *P* made to Approach to the ſaid Plane, for all the Expanſive Force beyond the Immoveable Plane in either Caſes is to be neglected, as Acting another way, and not Contrary to, or againſt the Inclined one *CAD* or *CBD*.

From hence it therefore follows, that the Inclination of the Orbit of *P* to the Immoveable Plane is the Greateſt, when the Nodes are in the Syzygiæ, and the Leaſt when in the Quadratures, all other Things being Equal, and that the ſaid Inclination is Diminiſhed in the Paſſage of *P* from the Quadratures to the Syzygiæ, and is again Increaſed in it's Paſſage from the Syzygiæ to the Quadratures, and conſequently, when *P* is in the Syzygiæ, the Inclination of the Planes is the Leaſt, which Returns to it's former Magnitude, when *P* Arrives at the next Node; Alſo in the Paſſage of the Nodes from the Syzygiæ to the Quadratures, the Inclination of the Planes is Diminiſhed and is the Leaſt, other Things being Equal, where the Nodes are in the Quadratures, then it Increaſes by the ſame Degrees by which it before Decreaſed, and upon the Return of the Nodes to the Syzygiæ, the Inclination of the Planes obtains it's firſt Magnitude.

The next Thing to be Accounted for is the Regreſſion of the Nodes, which Intirely Depends upon the Explication, which is Given of the Inclination of the Orbit's being Increaſed and Diminiſhed by Turns, and therefore, what the Action of *NM* is ſuppoſed to do in the Former Caſe, and which is Apply'd to the Latter, the Expanſive Force from *S* will afford us a ſufficient Solution of; namely, why the Planes are Differently Inclined in the Syzygiæ and the Quadratures, and conſequently, why the Nodes when Placed in the Quadratures are Regreſſive, and when in the Syzygiæ Quieſcent, and in the Intermediate Places, as they partake of the Conditions of the Syzygiæ and the Quadratures, are more ſlowly Regreſſive, and therefore, why the Nodes, being either Retrograde or Stationary, will be Carry'd *in Antecedentia*.



§. 20. **Having** thus Endeavoured to Explain the Errors and Irregularities of the Body *P* in it's Orbit by the Expanſive Forces of the Sun and the Contractive of the Planets, and ſhewn what Objections lie againſt the Received Expoſition of theſe Matters, we ſhall only farther in General add, that it ſeems more Agreeable to Nature, Philoſophy, and a juſt Reaſoning, to ſuppoſe, that the Orbit of *P* is Affected by ſome Force, which is Univerſal, and which is Spread and Diffuſed round it, by which thoſe Changes and Alterations are Produced in it, than to Imagine they are Derived from the Action of Two Simple Lines *MN* and *KL*, which at the Beſt owe their Origin to a Chimerical Reſolution of Forces, which we have before Conſidered and Examined.

Some General Reaſons, why the Doctrine of the Expanſive and Contractive Forces is Preferrable to that of Attraction or Gravitation.

If it be Alledged, that it is Attributing too Great an Effect to the Expanſive Force of the Sun, to make it Capable of Repelling a whole Planet, when we do not find the Rays of it ſufficient to Remove the Smalleſt Pebble.

We Anſwer, **First**, That the very ſame Argument may be Urged againſt a Planet's being Attracted by the Sun, ſo as to make it Change it's Situation, ſince, if it could Attract a whole Planet, it might, one ſhould think, eaſily Attract any Small Part or Portion of it, which lies Looſe and Unconnected with it, and therefore Draw it from it's Surface, which we do not yet Experience in the Planet we Inhabit.

Second, The Reaſon, why either the Attractive or the Expanſive Force of the Sun cannot Draw or Repell any looſe Body upon the Surface of the Planet in Diſtinction from the Whole, is Plain, becauſe by the ſame Force, with which any Body is Drawn or Repelled, every other Body of the ſame Magnitude may be Drawn or Repelled, and conſequently the whole Planet, which Conſiſts of ſeveral Bodies of the ſame Magnitude or Nearly, ſuppoſe that Magnitude, what in Reaſon we will.

Third, But yet we find very ſenſible Effects of the Sun's Expanſive Force, in it's Diſſolution of Bodies, which are Inflammable, and even of Mettals by a Burning Concave, and in it's Diſſipation of Waters and Vapours; On the Contrary, that the Sun has an Attractive Power, or that any Parcels of Matter on our Earth Gravitate towards it, we have not the leaſt Intimation from any Thing we Obſerve.

Fourth, And as to the Contractive Forces, we find the Earth is Endued with ſuch a Force, by which it's Parts are Conſtipated, and Condens'd together, and Cemented with each other, and by which all Groſs Bodies Deſcend and Gravitate to it, in order to ſuch Cement and Union, in like Manner, as the Expanſive Force Diſſipates or Repells Bodies, which come within it's Sphere of Activity; That therefore other Bodies, as the Moon and Planets, which alſo are Opake and Denſe, ſhould be Endued with the ſame Contractive Force is not Improbable, or without a Foundation in Experience and Reaſon, by which their Parts Tend to each other, and by which all External Bodies, that are Groſs and Denſe, are Drawn and Incited to a Conjunction with Them; If therefore ſeveral Bodies, as the Earth, Moon, and Planets Draw Different Ways by ſuch Contractive Forces, and the Sun Repells Them, the Effect will not be their Concurrence, or Coalition, but their keeping at certain ſtated Diſtances Agreeable to the Forces, which Cauſe their Divorce and Separation.

On the Contrary, If, according to the Preſent Philoſophy, theſe Bodies Attract and Actually Tend to each other, What ſhould Hinder their Coition or Congreſs? And how can a Paſſive and Unactive Matter Attract? Or from whence Ariſe it's Gravitation? Which are all of Them Difficulties and Problems, which this Philoſophy Wants to have Solved, after it has been, by the Defectiveneſs or Arbitrarineſs of it's Principles, the ſole Cauſe and Occaſion of Them; and which Problems we have Juſt now ſhewn may be Explained by the Doctrine of the Expanſive and Contractive Forces.

THE

THE PRINCIPLES OF THE PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK III.

CONCERNING THE CHIEF PROPERTIES OF MATTER, AS ALSO, CONCERNING THE PRINCIPLES OF CHYMISTRY, ANATOMY, PNEUMATICKS, AND HYDROSTATICKS.

THE PRINCIPLES of the PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK III.

Concerning the Chief PROPERTIES of MATTER, as alſo, Concerning the PRINCIPLES of CHYMISTRY, ANATOMY, PNEUMATICKS, and HYDROSTATICKS.

CHAP. I.

Concerning the Chief Properties of Material Beings.

§. 1. DESIGNING in this Chapter to Treat of the ſeveral Properties we find in Material Beings, I ſhall obſerve in the Firſt Place, that as they are Diverſify'd amongſt one another into Taſts, Smells, Colours, Sounds, Gravity, Extenſion, *&c.* ſo there are Various and Numberleſs Degrees in the ſame kind of Property; In the Four Firſt this is Acknowledged, but it is not ſo fully Apprehended to obtain in Gravity and Extenſion.

The Properties of Material Beings Various in Degrees as well in Gravity and Extenſion, as in Taſts, Smells, Colours, and Sounds.

THUS in Taſts we eaſily Allow of Acid, Sweet, Bitter, *&c.* and that Each of theſe have a Thouſand Varieties in them; in Smells, of Fragrant and Fœtid, with ſeveral other Intermediate ones, which are likewiſe each of them Diſtinguiſh'd into a Multitude of others, which Differ in their Impreſſions, but are the ſame in Kind and Species; Colours Appear under many Particular Forms, and each Form is Divided into New Shades and Complexions; and Sounds Conſiſt of Diverſe Notes, which are likewiſe Capable of Innumerable Modulations; And if it were not for the Prejudices we had Received from ſome Syſtems in Philoſophy, we ſhould conclude the ſame of Gravity and Extenſion; In Gravity we Experience an Infinite Number of Gradations from Gold, which is the Heavieſt Subſtance we know, to Fire, which is the Lighteſt, and has a quite Contrary Force to that of Gravitation; And in Extenſion the Eye is Filled with a Greater or Leſs Conſtipation of Parts, with which the ſame Dimenſional Space appears to be Differently Replete, and the Hand is Informed of Various Degrees of Roughneſs or Smoothneſs, of Rare and Denſe, of Solid and Fluid, to an Infinite Diverſity; but, which is the Misfortune, we have Learned to ſpeak of Gravity and Extenſion in an Abſtracted Manner, and from thence have Imagined the one, and the other, to be the Same and Invariable, tho' our Senſes Inform us, and Nature Aſſures us, that each of them Admits of as Great a Variety, as the other aforementioned Properties of Material Beings, ſo eaſy it is for our Senſes, our Reaſon and Judgment, to be Corrupted by our Abſtractions and our Fancies.

Eſſential and Accidental Properties of Matter Diſowned and Refuted.
Princip. of Natur. Philoſ. Chap. 6, 7, &c. Part 1.

§. 2. ANOTHER Thing we muſt obſerve in Reſpect to the Properties of Material Beings is, that they are Generally Diſtinguiſh'd into Eſſential and Accidental; Solidity and Extenſion are in the Number of the Firſt, and Taſt, Smell, Colour, and Sound Appertain to the Laſt, becauſe the Firſt are found in all Kinds of Matter, whereas the other are not; On the Contrary, Matter, when Defined in General, we have formerly ſhewn to be a Creature of the Mind, as well as it's Two Eſſential Properties, Solidity and Extenſion, and when we come to any Particular Portions of Matter, as they Exiſt in Nature, thoſe, which are Termed Accidental Properties, will be as Eſſential to thoſe Portions of Matter, in which they Inhere, as any other the moſt Eſſential Properties of it; Beſides the Dividing the Properties of Matter in this way is making our Senſations the Standard of the Real Qualities of it; Thus if we Exclude Smell from being Eſſential to Matter, becauſe we have no Senſations of it in ſeveral Portions we are Converſant with, in many of thoſe Portions even Dogs and ſeveral other Brutes will Convince us of our Miſtake, and in all, it is not certain, but ſome Animals or Inſects may Receive thoſe Senſations from it, which we find Impoſſible in our ſelves; What therefore we are to do, is to Conſider the Properties of Matter, as they Exiſt in it, without any ſuch Frivolous and Idle Diſtinctions, Formed by our Fancies, and Contrived to ſerve our Particular Hypotheſes, and to Try, whether we can give the Subſtratum or Eſſence of it, and from thence we ſhall know, what Properties are Eſſential to and Immediately Derived from it.

What the Eſſence or Subſtratum of Matter is, Proved from the ſeveral Properties of it.

§. 3. THE Subſtratum or Eſſence of Matter therefore I Take, and Think I may Affirm, to be Action or Force, and if I can Produce thoſe Actions and Forces, which will not only ſolve all the Phænomenons of Matter, which we are Acquainted with by our Senſations from it, but even thoſe, which may poſſibly Arrive to other Animals of a Diſtinct Nature and Species from us, I hope I ſhall have Given a full Account not only of the Eſſence of Matter, but of it's Real and Eſſential Properties, for I know of no other that belongs to it, the Accidental ones being only, as we before ſaid, either Calculated according to our own Senſations in Nature, Abſtracted from thoſe of other Animals, or upon our own Hypotheſes in the Corpuſcular Scheme.

THAT Action or Force in General is the Eſſence or Subſtratum of Matter, is Evident from this Plain Axiom, that it would be Impoſſible for us to have any Senſations from Matter, unleſs by ſome Kind of Action or other Impreſs'd upon our Minds from it, we ſhall Inſtance in thoſe already Mentioned.

Princip. of Natur. Philoſ. Chap. 10. Part 1.

TAST is Produced by an Immediate Application of the Matter to the Organ of Senſe, either therefore this Senſation muſt be Cauſed by the Action of the Mind without any Influence from Matter, and the Mind would perceive Taſt without ſuch Application of Matter to the Organ, which is Impoſſible, or elſe by the Figure of the Corpuſcles, which Compoſe this Matter, which has been Confuted in our *Principles* of *Natural Philoſophy* in the Chapter of Taſts, or elſe it muſt be by the Agitation and Motion of the Parts of this Matter (which we here ſuppoſe not to be a Fluid but a Solid and Quieſcent Body) by the Saliva without any Particular Force in the Matter, and then all Taſts of Solid and Quieſcent Bodies would be the ſame, or laſtly, by ſome Peculiar Action or Force in the Matter it ſelf, which is what we Aſſert and Defend.

Princip. of Natur. Philoſ. Chap. 10. Part 1.

SMELL is Cauſed either by Effluviums from the Odorous Body, if the Corpuſcular Account Prevails, or by Small and Minute Percuſſions, and Infinitely ſhort Vibrations, or Momentany Impreſſions of it, on the Ambient Plenum, Air or Æther, which we Maintain; if the Laſt, the Caſe is Evident, that there muſt be ſome Force or Action in Matter, from whence theſe Gentle Strokes are Promoted thro' the Æthereal Fluid to our Organs of Senſe, if the Firſt, the Arguments, which we have Alledged againſt ſuch Effluviums in the ſame Chapter, and againſt the whole Atomick Syſtem are to be Anſwered, and therefore want no Repetition here.

COLOUR Proceeds from the Reflection of the Rays of Light, which is either Heterogeneous and Diſſimilar in it ſelf, and Conſiſts of Various Colours, which are

Connate to it, and Originally in it, which the Preſent Philoſophy Affirms, or it is Homogeneous and Alike, and Receives it's New Forms from the Impreſſions and Minute Percuſſions of Natural Bodies, which we Propoſe; if the Former, our Chapters Writ againſt the Modern Explanations of Light and Colours are to be Review'd and Refuted, if the Latter, it is again manifeſt, there muſt be ſome Power, Force, or Action in Bodies Peculiar to them, which occaſions thoſe Changes and Alterations in the Luminous Subſtance.

Princip. of Natur. Philoſ. Chap. 2, 3, 4, &c. Part 2.

SOUND is Confeſſed on all Hands, for here the Corpuſcles of Matter and the Atomical Philoſophy are entirely Excluded, to be the Effect of Vibration, Percuſſion, or Elaſticity; and ſince Sounds are Different, where the External Percuſſion is the ſame, the Internal Percuſſions, Vibrations, or Elaſticities of the Sonorous Bodies muſt be Different, and therefore owing to Peculiar Forces, Actions, or Minute Vibrations in the Bodies themſelves.

GRAVITY, If Different in the ſame Quantity and Portion of Matter, which is what we Aſſert, muſt Ariſe from a Different Innate Force or Action in it, and the like may be ſaid of Solidity and Extenſion, which we ſhall more Particularly Conſider.

SOLIDITY is only Felt by us, but Extenſion is both Felt and Seen, if therefore we ſhew, that theſe Senſations could not be had, unleſs from ſome Certain Action in Matter, we ſhall Prove, that thoſe Properties of Matter, which are Eſteemed to be the moſt Eſſential, Derive their being ſo from the Action of it.

SOLIDITY is to be Conſidered Two ways, either as it is Defined in General to be an Excluſion of all other Bodies out of the Space or Dimenſions, which any Certain Body Poſſeſſes, or as it is an Actual Senſation; In the Firſt Senſe, it is only an Abſtracted Notion, and a Fancy and Creature of the Mind, thus Water, or Air, or Æther, is as Solid as Gold, as there is no Difference in the Impenetrability of them; But does any one Really Feel or Perceive, that there is no Diſtinction betwixt theſe Subſtances? Or that the one Excludes the other from being in the ſame Place? No, we Reaſon and Argue our Selves into ſuch an Opinion, that it is a Thing Impoſſible, becauſe Water or Air cannot be Æther, nor Æther Gold, which would be, if the one could Poſſeſs the very ſame Dimenſions, which the other does, that is, was Exactly of the ſame Nature; but that Water, or Air, or Æther, may not be Compreſs'd within the ſame Dimenſional Space, in which Gold Exiſts, can be no more Proved even from Reaſon, than that Water, if there were Forces in Nature to Compreſs it to a Nineteen Times Narrower Space than it Poſſeſſes, would not be Gold, or that Gold is not the Reſult of ſuch a Compreſſion by Real Forces Actually Exiſting in Nature; This Solidity therefore we have nothing to do with at Preſent.

IN the Second Senſe, Solidity is an Actual Senſation, by which we Perceive, that Gold Reſiſts more to a Penetration into it by our Hand or a Finger than Water does, Water than Air, and Air than Æther; but unleſs there was ſome Force or Action in Body, there could be no Reſiſtance to our Feeling of it, nor could we Feel it at all, and there could be no Difference of Reſiſtance to our Feeling, unleſs there were Diſtinct and Peculiar Actions or Forces, which made that Reſiſtance; This Appears to be ſo Plain and Evident, that there can be no Grounds to Conteſt it, and therefore all that we can ſay of Matter, as to the Solidity of it, from our Senſations is only, that it is it ſelf Real Force or Action, and Wholly and Entirely Conſiſts in it.

EXTENSION is to be Conſidered Two ways, as Solidity was, either as one Uniform, Invariable Thing, and the ſame in all Bodies, and then it is an Abſtraction and Creature of the Mind, or as an Actual Senſation; The Former will ſerve well enough to Diſcourſe of in Logicks, or in Geometrick Demonſtrations, but can ſignify nothing to Philoſophy, which Inquires into the Real Properties, which Exiſt in Nature, and therefore we muſt take the Latter; Which, as it is a True and Explicit Senſation, may be Felt and Seen, but how we ſhould Feel Extenſion, any more than Solidity, without an Impreſſion upon our Organs of Senſe, or how that Impreſſion ſhould be made without Action or Force, is Impoſſible to ſay or Imagine; nor is it Probable, that we could See Extenſion, unleſs the Body, which was Extended, was Endued with ſome Force or Action, for if it was Entirely Paſſive, there is no more Reaſon, why the Rays of Light ſhould not Flow along it's Surface, than be Reflected from it, but if there is an Action or Force in Body, which Repells the Rays, there is a juſt Argument for it's being ſeen.

THE Different Conſtipation of Parts, which we obſerve in the Extenſion of Wood, Stone, Mettals, *&c.* is a Conviction likewiſe, that this Force or Action is Different in Bodies; and the Various Colours of them Confirm the ſame Opinion, ſince we have ſhewn in our *Principles* of *Natural Philoſophy*, and in the Chapters Relating to Colours, that the Accounts, which have been Given of them, by *Deſcartes* or Sir *Iſ. Newton*, upon the Hypotheſis of a Similar and Paſſive Matter, are liable to Many and Invincible Objections; And till we find Equal Difficulties Chargeable upon an Active Matter, we muſt beg Leave of the Philoſophers to Depart from them into thoſe Sentiments and Perſwaſions.

What the Actions of Matter are, and that they Conſiſt of Various Degrees of the Expanſive and Contractive Forces Exerting themſelves Differently in the ſame Individual Points of Space.

§. 4. SINCE therefore all Matter Depends upon, and Conſiſts in ſome Force or Action, the next Inquiry will be, what that Force or Action is, which we Diſtinguiſh into the Expanſive and Contractive, and which has hitherto led us thro' all the Mazes and Labyrinths of Nature; It has been Generally thought, of which Opinion the Preſent Philoſophy Appears to be, that a Double Force or Velocity will Carry a Body in the ſame Time to a Double Length, and a Triple Force or Velocity to a Triple one, likewiſe on the Contrary, that a Body Carry'd with the ſame Velocity in Double the Time will Deſcribe a Double Length, and in Triple the Time a Triple one; but it ſeems to have no Notion of a Double or a Triple, or any other Multiple Force or Velocity Exiſting in the ſame Space, and in the ſame Time, which the Expanſive and Contractive Forces we Defend, will Explain, and than which Nothing is more Certain and Evident.

FOR if Two Equal Balls of the ſame Matter and Subſtance are ſtruck with Two Different Forces in the ſame Inſtant of Time, the Forces Communicated muſt be Different in the very Firſt Point of their Velocity or Motion, if not, they are Equal, and Different Forces will afterwards Ariſe from an Equality of them, which is Impoſſible; if therefore they are Different in the Firſt Point of Space, thro' which they Paſs, for the ſame Reaſon they will be in a Second, Third, a Hundredth, Two Hundredth, a Thouſandth, or a Millionth, *&c.* 'Tis true a Body will be Carry'd to a Greater Length by a Greater Force, as for Inſtance, to Double the Length by Double the Force, and for that Reaſon, that there muſt be Double the Force to carry it to Twice the Length, it muſt have Double the Force to carry it the ſame Length, the Double Proportion of Effect continuing undiminiſh'd, for otherwiſe it would ſink and dye at the ſame Diſtance from the Beginning of it's Velocity and Motion, at which the other does, that is, when both the Balls have Deſcribed the ſame Length they will be Quieſcent; whereas, ſince the Ball, which has the Double Force, will be Carry'd as far again, after the other has Ceaſt it's Motion, it is plain it came with a Double Force thro' every Point of their Moving together.

I mean, if it had not an Activity and an Energy in the ſame Time, and in the ſame Space, Double to that of the Other, it could not Produce an Effect of Motion, or of Deſcribing a Space Double to that, which the other does; for nothing is a Plainer Axiom than this, than that the Time continuing the ſame and the Activity too, the Balls, which are ſtruck, muſt ceaſe their Motion at the ſame Diſtance from their Firſt Moving, and nothing can carry the one to a Double Diſtance of that of the other, excepting we ſuppoſe their Forces Double to each other in every Point of Space, in which they Move in Conjunction.

THIS Notion of Different Velocities in a Point will be ſtill more manifeſt in Forces Communicated to Bodies, which are not Actually moved by them, as in Preſſure; Thus with the Force of Screws we may Impart a Different Velocity in a Point, and in the ſame Inſtant of Time, for a Different Velocity in a Point, I call a Different Action produced in that Point, from a Various Force apply'd to the Production of it; And the ſame may be ſaid of the Preſſure of our Hands, or our Fingers, upon a Body, which is Unmoved, and of the Variety of Impulſes from Percuſſion upon Bodies, which are Solid and Reſiſting; In all which Caſes, there are Different Actions and Forces produced in the ſame Point of Space, and in the ſame Inſtant of Time, unleſs it can be ſaid or affirmed, that there is no Effect Evident, when yet the Cauſe is Apparent and Unconteſted.

The like Inſtances to theſe are ſuch, which Relate to the Revolutions of Bodies round a Center, which according to the Different Force, with which they are Agitated and Retained in their Circles, do acquire a Different Velocity; It is generally Acknowledged, that a Circular or Periodical Motion is Compounded of a Centripetal in the Curve, and a Centrifugal in the Tangent or Projectile, it muſt be alſo Confeſs'd, that the Centrifugal is Greater, as the Velocity is Greater, with which the Body is Revolved, and conſequently in every Point of the ſame Circle, there may be Different Forces or Velocities produced, which will have the Effect upon the Æther, which Surrounds it, of Different Kinds of Preſſure.

From hence it is therefore, that we perceive thoſe Various Senſations from Matter, which we do; All Bodies Solid and Fluid are Compounded of a Centripetal, a Gravitating, or Contractive Force, by which they not only Tend to the Center, but by which they are United and Cemented together, and this Force of Gravitation or Contraction is in the ſame Point of Space, and the ſame Inſtant of Time, Various and Changeable in Different Sorts and Portions of Matter; as alſo of a Centrifugal, Elaſtick, or Expanſive, which likewiſe in Different Kinds of Matter is Changeable and Various in it's own Nature; Theſe Forces therefore, together with the Infinite Mixtures and Combinations of them, are the Real Subſtratums of all Bodies, and upon which all the Properties of them do not only Depend, but from which they are Derived.

How the ſeveral Properties of a Solid Body, as Gold, are ſolved from the Expanſive and Contractive Forces.

§. 5. Thus where a Body is Solid and Heavy, Ductile, Fuſible, and Yellow, as Gold for Inſtance.

It's Solidity, by which we cannot Penetrate into it, is owing to it's Particular Degree of Contraction, by which the Parts of it Tend to one another, and to the Center, as alſo to it's Expanſive Force, by which it Repells any Force, which Endeavours to enter into the Place, which it Poſſeſſes.

It's Gravity is owing to it's Principle of Contraction towards the Earth, which is ſomething Diminiſh'd by that of it's Expanſion from it, tho' in a Small Degree.

It's Ductile and Fuſible Nature is Derived from it's Expanſive, being Aſſiſted by the Expanſive Force of Percuſſion, or that of Fire, by which it in ſome Meaſure Surmounts it's Force of Contraction.

It's Colour, which is near as Ruddy as Fire, is Derived from it's Expanſive Force, which may be probably as Great as that is, but which only Acts with that Force within it's own Dimenſions, and is only capable of making it's Impreſſions upon the Æther, which immediately Surrounds it, by Reaſon of it's being held back and detained by the much Superior Proportion of the Contractive Force, with which it is Mixt and Blended.

And from whence Appears the Neceſſity of a Plenum, ſince, if there were not ſuch an Æther, it would be Impoſſible for us to have any Senſation of this Force, or indeed of any Colour, or any Viſion in General, namely, unleſs by the Mediation and Interpoſition of it.

How the ſeveral Properties of Fire are ſolved from the ſaid Forces.

§. 6. On the Contrary, let us take Fire or Flame, that it has a Certain Degree of Gravitation or Contraction, is not only Evident from Mr. *Boyle's* Experiments, which Evince that Fire or Flame is Ponderable, but is likewiſe Manifeſt from this Plain Reaſon, that if it were not ſo, it would be Diſſipated at once, no Fire or Flame would continue together, none would ſubſiſt near this Earth, but if it had an entirely Centrifugal or Expanſive Force, it would be loſt and would Vaniſh in a Moment; Beſides Fire or Flame cannot ſubſiſt without ſome kind of Fuel, or other, to ſupport it, which always has a Contractive Force belonging to it, and with which therefore it is as conſtantly United.

But as Gold has the Greateſt Contractive Force, and perhaps the Greateſt Expanſive joined together, by which it obtains the Properties above-mentioned, ſo Fire or Flame ſeems to have the Greateſt Expanſive Force, and the Leaſt Contractive, of any other Material Being.

To it's Contractive Force it is owing, that it is not Diſſipated and Diſperſed, that it Exiſts in a Lamp or a Cinder, and Precipitates to the Earth with it,

To it's Expanſive Force it is owing, that it Dilates and Extends it ſelf to every Part, where there is a Matter, whoſe Contractive Force is not too Great to be overcome, and to be Changed and Transformed into an Expanſive by it.

By this alſo it Strikes the Æther with that Force and Velocity, which Excites either an Heat, or Burning in it, to our Senſations of Feeling, and Imparts to our Sight, by it's Impreſſions upon the Organs of that Senſe, the Strong and Vivid Red, and the peculiar Brightneſs and Luſtre, we Receive from it.

How the ſeveral Properties of Fluids are ſolv'd from the ſame Principles, and the Various Kinds of Fluids Determin'd.

§. 7. As to Fluids, what is Common to them with other Material Subſtances is derived from the aforementioned Contractive and Expanſive Forces, and what is Peculiar to them, namely, their Fluidity, ariſes from an Equal Balance betwixt thoſe Forces, by which means, tho' the Parts of a Fluid have a Certain Degree of Tendency to each other, yet being endued with an Equal Degree of Force of Retiring from each other, they accordingly ſeparate upon the leaſt External Application to Divide them, and this ſeems to be the Nature of a Perfect Fluid; But by how much more Fluids Reſiſt to a Separation of their Parts, by ſo much is the Contractive Force more Prevalent than the Expanſive, or by how much more they are Inclined or Diſpoſed to ſuch a Separation without an External Force, by ſo much is the Expanſive more Prevalent than the Contractive.

An Inſtance of the Firſt Sort of Fluids ſeems to be That of Clear and Limpid Water, which by it's Expanſive Force is capable of Reducing ſeveral Kinds of Matter, as Salts, and Earths, and Minerals, to it's own Expanſive Force, in like manner, as Fire does Combuſtible Subſtances to it's Expanſive; from whence poſſibly proceed Perennial Fountains, with their Various Mineral Taſts, which ſeem to be Produced by the Expanſive Force of Water Acting upon theſe Waterable Subſtances, if we may ſo call them, and Converting them into it ſelf, juſt as Flame Acts with a Greater Degree of the Expanſive Force upon Subſtances, which are Inflammable, from whence Ariſe the Vulcanos, as thoſe of *Ætna* and *Veſuvius*, which have continued Burning for ſo many Ages; Water has an Expanſive Force in Reſpect of Wood, which it Swells and Dilates, and which Froſt Contracts, as is Evident in Doors and the Frames of Saſhes.

Of the Second, The more Viſcous and Tenacious ones, as Oils and Glutinous Subſtances, the Latter of which, when Cooled and laid upon Wood, have ſo Great a Contractive Force, as to be Equal, if not Superior, to that of the Wood it's ſelf; and it is not certain, that when Oils are infuſed betwixt Poliſh'd Plates or Marbles, but they may have the ſame Contractive Force with Glews, tho' it is Generally ſaid, that they only keep the Air from Entering betwixt the Plates, and the Reaſon of their Adhering ſo firmly together is aſcribed to the Incumbent Preſſure of the Atmoſphere; To this kind may be added Metals in their Fuſion, and Mercury.

Of the Third, The more Spirituous ones, as all Kinds of Wine and Strong Waters, Oil of Vitriol, Spirit of Turpentine, *&c.* Alcohol or Rectified Spirit of Wine, which will of it ſelf, if a Small Quantity of it is poured into a Spoon, immediately Evaporate and become Inviſible; Alſo Air, Æther, Fire, and Flame, ſeem to have likewiſe a Greater Degree of the Expanſive Force than of the Contractive, and of the Four laſt mentioned, Æther, Fire, and Flame, appear to have ſo little of it, that, inſtead of Giving a Reſiſtance to Material Beings in their Actions, they Accelerate and Contribute to them.

All the Phænomenons of Nature ſolv'd from theſe Principles.

§. 8. All the Properties of Matter mentioned in Philoſophy are Derived from hence, whether thoſe which are Termed Accidental, or thoſe which are Called Eſſential; All the Principles in Chymiſtry, and the Operations Obſervable in it, as Fermentation, Precipitation, Coagulation, Chryſtallization, *&c.* and all the Phænomenons, which Preſent themſelves to us from the Animal, the Vegetable, or the Mineral Kingdom, all the Principles of Anatomy, and the Motion of the Lungs, of the Blood, and of the Muſcles, of the Animal Spirits, or the Nervous Juice, and the Union of the Human Syſtem with our Minds, and the Sympathy, which the one has with the other, are likewiſe Deducible from theſe Forces of Expanſion and Contraction.

The Eſſential or Philoſophick Properties of Matter, when Conſidered as Real, Accounted for from Expanſion and Contraction.

§. 9. The Eſſential Properties of Matter, according to the Corpuſcular Philoſophy, are thoſe of Extenſion, Solidity, Figure, Diviſibility, Mobility, and being, in it's own Nature, Unactive, to which we may add, with Sir *Iſ. Newton*, Attraction or Gravitation; when theſe Properties, which I call the Philoſophick ones, are only Conſidered in the Abſtract, they are, as I have already ſaid in the Firſt Part of the *Principles* of *Natural Philoſophy, Chap. 6. Book* 1. Nothing more than Creatures of the

Mind, and of thoſe there can be no other Account given, than that the Philoſopher was pleaſed, for his own Purpoſes, which he had in View, and for the Convenience of his Solutions, to make thoſe Abſtractions.

Extenſion Accounted for from hence.

BUT let us take theſe Properties, as they really Exiſt in Nature, and we ſhall eaſily find, in the Firſt Place, that Extenſion is Various and Different in Matter, both to our Sight and to our Feeling; That we See or Feel the Extenſion of Matter, I ſay is owing to it's Expanſive Force, by which it Acts, in Feeling, immediately upon the Organs of our Senſe, and, in Seeing, upon Æther, and, by it's Intermediation, upon the Eye, the Humors, and the Retina of it; We likewiſe hold, that this Action of Extenſion is by a Pulſe or a Velocity Exerting it ſelf in a Point or a Surface, without any Evident Change of it's Space or Situation; that farther, Extenſion ſeems Various to us, that is, Rarer or Denſer to our Sight, according as this Expanſive Force does Act upon more or fewer Points of the Æther, and ſo upon the Eye, and to our Feeling, as it more or leſs Affects the Organs, which Tranſmit thoſe Senſations to us.

Solidity Accounted for.

As to Solidity, by which our Finger, or any other Body, is Prohibited from Entering into the Space or Dimenſions, which another Body Poſſeſſes, ſo far as it is an Abſtracted Idea, it is a Creature of the Mind, and is of no Significance or Importance to Nature; but if we conſider Solidity, as an Actual Reſiſtance, by which one Body obtains a Force of Excluding others from Penetrating or Piercing into it's Space, unleſs by a Superior one; ſuch a Property is entirely Accounted for, both from the Force of Contraction, which keeps Matter United and Cemented together, and from the Force of Expanſion, which Repells or Subdues the Force, which attacks it; and the Various kinds of Solidity are derived from the Various Forces of Expanſion and Contraction.

FROM which it is manifeſt, that altho' there cannot be a Penetration of Dimenſions, where Solidity is conſidered and look'd upon as an Abſtracted Idea, yet there may be one in Reality, according to what has been Affirmed in the Firſt Part of the *Principles* of *Philoſophy*, that is, there may be Different Forces either of Contraction or Expanſion, or Different Degrees of Action, in the ſame Dimenſional Space; and if Gold, for Inſtance, is 19 Times Heavier than Water, ſetting aſide both their Expanſive Forces, 19 Parts of Water will be Compreſs'd into the ſame Space with one of Gold, to make that Subſtance; ſuppoſing likewiſe, that the Contractive Force is Equal to the Gravitating, which may not in fact be; ſince the Parts or Portions of Matter may Tend to each other with a Different Force and Inclination from that, with which they do to their Center, unleſs all Matter was Similar and Homogeneous, which it is not; for in that Caſe, it is reaſonable to think, that the Forces of Contraction would be exactly Equal and Correſpondent to thoſe of Gravitation.

Princip. of Natur. Philoſ. Chap. 4. §. 11. Part 1.

Figure Accounted for.

FIGURE is another Eſſential Property in the Corpuſcular Philoſophy, for which it does not Account, which is Defined and Deſcribed to our Sight or our Feeling by Diverſe Portions of Matter Exerting their Different Forces of Expanſion within certain Limits and Boundaries, in Diſtinction from the Expanſive Force of the Æther, which ſurrounds it.

THUS, becauſe a Globe or a Cube of Gold, of Glaſs, or of Marble, or Wood, or of any other Body Solid, or Fluid, Diſplays it ſelf in a Different Expanſive Force upon the Æther, or our Organs of Senſe, from the Expanſive Force of Æther it ſelf, within ſuch ſtated Terms and Limits, therefore it is, that we ſee Bodies, or feel them, to be Endued with ſuch Figures Agreeable to the Extent of their Peculiar Actions.

Diviſibility Accounted for.

ANOTHER Eſſential Property of Matter is Diviſibility, which the Corpuſcular Philoſophy knows the Uſe, but not the Cauſe of, and which Ariſes from the Expanſive Force in Matter, for if there was a Contractive Force in it, which was Infinite, and not Moderated and Allay'd by an Expanſive one, it would be Impoſſible ever to Separate one Part of Matter from another; but the Contractive Force of Matter being Finite, or being Tempered with an Expanſive, upon both theſe Accounts it is capable of a Separation; either by the Expanſive Force of Diſſolvents, as Gold by the Spirit of Nitre mixt with Sal Armoniac, and Silver and other Metals by the Spirit of Nitre alone; or elſe, by Attrition, Percuſſion, and other External Violences, which are all ſo many Expreſſions or Conveyances of the Expanſive Force of the Mind Exerted upon Material Beings.

Mobility Accounted for.

A Fifth Eſſential Property of Matter in the Corpuſcular Philoſophy is that of Mobility, which is likewiſe Explained from theſe Forces, For if any Portion of Matter, as before in Reſpect of Diviſibility, had an Infinite Force of Contraction, and the Force of Contraction was Correſpondent to that of Gravitation, or of it's Contraction to the Earth, it would be Impoſſible for any Body to be Moved; but as every Body has only a Finite Tendency of it's Parts to each other, and of all of them to the Center, and as it's Parts and it's Whole are Endued with an Expanſive Force; Upon both theſe Accounts, as before Bodies were Diviſible, ſo are They Moveable, when there is a Superior Expanſive Force, either by Springs, as in Automatons, or by Percuſſion, or by Levers, which are Derived from a Superior Expanſive Force; or by Weights, which are Derived from a Superior Contractive, and which, in ſuch a Caſe, have the Nature of an Expanſive.

The Inertia of Matter what in Reality and how the Immenſe Bodies of the Sun and Stars may be ſuſpended in Æther, as alſo the Planets, if they were not Agitated by their Actions amongſt one another, and by the Expanſive of the Sun.

THE Sixth Property of Matter, which is that of it's Inertia or being perfectly Unactive, cannot indeed be Accounted for from Forces, which are Active in that Senſe, in which we have Explained them, nor is there ſuch an Inertia of Matter in Nature; For there is no Material Being, which has not ſome Action or other belonging to it, by which we are made ſenſible of it's Exiſtence, and without which we could not be furniſh'd with any Senſation at all of it.

ALL the Real Inertia of Matter is, that it is Capable of being Moved, or in General of yielding to a Superior Force, which ſhall Act upon it, whether the Expanſive or Contractive, not that it will continue in a State of Motion or Reſt after it is once put into either; ſince it has an Innate Force in it ſelf Derived from God, which will Prevail, when the Force, which is Impreſſed upon it, Ceaſes, which it will do, unleſs that Force is Infinite, which is not the Caſe of Percuſſion, or Projection in this Philoſophy.

THUS in Fire, when Water, which in Reſpect of Fire has a Contractive Force, is apply'd to it, if in a Great Degree, the Contractive Force extinguiſhes the Expanſive, but if in a Leſs, tho' the Fire ſubmits for a Time to the Contractive of the Water, yet, when that Force is ſpent, it riſes Stronger and Livelier than before.

THUS alſo in Reſpect of the Contractive Force, or of any Solid and Heavy Body, if it is ſtruck with a Racket, when that Force is ſpent, which is Derived from Percuſſion, it Re-aſſumes it's own Contractive Force and Tends to the Center, for the Contractive Force does not only Act, ſo as to Unite the Parts of Matter amongſt themſelves in any Portion of it, but ſo as to Unite all the Portions of the Univerſe together, and the Expanſive Force Acts the Contrary way to Diſpell and Diſunite them; It muſt be farther here obſerved, that it is ſomething Wonderful and Strange, that the Great Prince and Coryphæus of the Corpuſcular Philoſophy Sir *Iſ. Newton*, whoſe Name ought not yet to be mentioned without the Greateſt Regard, ſhould aſſert this Inertia of Matter, againſt which we Contend, and from that Aſſertion, which is the Foundation of all his Philoſophy, ſhould endeavour to Prove, that Gravitation or a Centripetal Force is a Quality or Property, which belongs to all Matter in General, which is neither more nor leſs than a Contradiction.

IF it be Enquired, what would become of a Contractive or Solid Body upon Percuſſion or Projection, ſuppoſing it was at an Infinite Diſtance from all Matter, ſo as not to Gravitate to any Portion of it, we Anſwer, after that External Force, which is Communicated to it, is Loſt, or Surmounted by the Contractive of the Body, which would be at ſome Finite Diſtance from the Beginning of it's Motion, becauſe the Force, which is Impreſſed upon it, is Finite, It would Ceaſe to Move, and by it's own Contractive Force would remain Quieſcent in Æther, there being no External Force to determine it one way more than another, and it's Contractive Force having no other Effect than to make the Parts Tend Inwards, and to reſiſt Percuſſion, or any foreign Violence; The like may be ſaid of an Expanſive Body, and it's Parts Exerting themſelves Outwards; And this Probably may be the Reaſon, why that Stupendous Maſs of Matter the Sun, as alſo, the fixt Stars continue ſuſpended in Æther, and why this Earth and the Planets would do the ſame, were they not Agitated by their Actions amongſt one another, and by the Expanſive of the Sun.

Gravitation Accounted for.

THE laſt Philoſophick Property of Matter is Gravitation, which is the ſame kind of Contractive Force in Reſpect of Different Bodies, which is Exerted in the Uniteing and Cementing of the Parts of the ſame Body; but as the Contractive Force, by which the Parts of one Body Tend to each other, may differ in the Degree of it's Velocity in a Point from the Contractive Force of the Parts of a Second, a Third, or a Fourth Body, *&c.* ſo alſo the Force of Gravitation, by which Bodies Tend to others, may be likewiſe Different, albeit we may not obſerve that Difference in their Tendency to the Earth, by Reaſon of the Vaſt and Prodigious Contraction it Exerts, in Reſpect of all Bodies in General; tho' we cannot but think, notwithſtanding all the Experiments made uſe of to Prove, that Bodies Deſcend with an Equal Force to the Earth, that a Rock falling from the Top of the Alpes would be Carry'd down to it with a more Aſtoniſhing Violence, than a Pebble dropt from that Height; and in Fact, if Hiſtorians are True, The Former once fell with ſo Great a Force, as to over-whelm a Whole Village, which lay beneath, and ſunk to ſuch a Depth into the Ground, as to Produce a conſiderable Lake in the Place, where it made it's Impreſſion.

Dr. Heylin's Geography.

§. 10. HAVING thus far Gone thro' the Eſſential Properties of Matter Alledged by Philoſophers, and Endeavour'd to give an Explanation of them, we ſhall now Proceed to thoſe, which are Term'd Accidental ones, ſuch are Hardneſs, Softneſs, and Fluidity, Rarity and Denſity, Reſiliency and it's Contrary Quality, Heat and Cold, Smoothneſs and Roughneſs, Brittleneſs and Limberneſs, Taſts and Smells, Sounds, Light and Colours, Opakeneſs and Tranſparency.

The Properties of Matter Termed Accidental ſolved from the foregoing Principles, in Particular Hardneſs, Softneſs, and Fluidity, Rarity and Denſity, Reſiliency and it's Contrary Quality, Heat and Cold, Smoothneſs and Roughneſs, Brittleneſs and Limberneſs.

As to Hardneſs, it is evidently Explained by a Contractive Force, which the Greater it is, by which any Portion of Matter is Cemented together, the more Difficult muſt it be to make a Separation in it; Softneſs ariſes from a leſs Degree of ſuch a Contractive Force, and Fluidity from a nearly Equal Mixture of the Expanſive and Contractive, according to what has been already mentioned.

RARITY and Denſity, as was likewiſe ſaid before, Conſiſts in a Leſſer or a Greater Number of Parts, Exerting their Expanſive and Contractive Force, in Conjunction, in the ſame Space.

RESILIENCY is, where the Expanſive Force is Vigorous and Strong, but yet is ſo far Ty'd down by the Contractive, as not to Move conſiderably out of the Space, which the Body Poſſeſſes; and the Contrary Quality is, where the Contractive entirely Prevails over the Expanſive, which therefore does not obtain in Nature, ſince there is no Body, which has not Sound or Colour, both which are Indications of ſome Degree or other of the Expanſive reſiding in it.

HEAT is manifeſtly Derived from a Great Degree of the Expanſive, and Cold from a Great Degree of the Contractive.

SMOOTHNESS and Roughneſs many Times Ariſe from the Parts of Bodies, being leſs or more Prominent from their Surfaces, which is of no Conſideration in Philoſophy, excepting in the Corpuſcular; But we may likewiſe farther obſerve, that Bodies, which are Equally Poliſh'd, and whoſe Surfaces to all Appearance are a-like Levigated and Plane, do ſome of them feel Harſher, and others more Oily and Smooth, as in Steel and Ivory or Water, the Former of which ſeems to be Derived from it's Contractive Force being Greater than that of Ivory or Water, and it's Expanſive Force Joined with that Contractive being in a Greater Proportion to the Contractive in Steel than in the others, upon both which Accounts, there will be more Reſiſtance to the Hand, carry'd along the Surface, in the Former Caſe than in the Latter; From hence Planes Rubbed, by which the Expanſive is Excited, Feel Smooth, that is, the Hand Slips off, or is Repelled from them.

BRITTLENESS appears to Proceed from the Expanſive Force, Acquired from the Fire, by which an eaſier Separation of Matter is made, as in Steel, Glaſs, Brick; and Limberneſs from the Contractive, by which a Divulſion is more Hardly, and with Greater Difficulty performed, as in Iron, Clay, Leather and Paper, which have but a Small Degree of the Elaſtick or Expanſive.

§. 11. As to Taſts, they are Derived from the Expanſive Force, from whence Solid and Conſiſtent Bodies, which have but a Small Portion of the Expanſive, and which is kept and detained from Exerting it ſelf out of it's own Space by the much Superior Contractive, have little or no Taſt at all.

Taſts ſolved from hence, and their Various Diſtinctions Aſcertained and Accounted for.

AND Spirituous Bodies, or Fermenting ones, as alſo Vegetables, *&c.* which have the Greateſt Degree of the Expanſive Force, are Endu'd with a very Lively and Vigorous one, and on the Contrary Water, in which the Contractive and Expanſive ſeem to be in an Æquilibrium, the Taſt is Flat, Languid, and Inſipid.

ACCORDING to the Various Degrees and Proportions of this Expanſive Force, which is Superior to the Contractive, but yet is Joined with it, are our Taſts Diverſify'd, which may be Diſtinguiſh'd in General into Sweet, Bitter, Sowre, Harſh, and Smooth, and the ſeveral Degrees, Compoſitions, and Combinations of them.

From whence Sweet and Sowre Ariſes.

SWEET Ariſes from a Greater Degree of the Expanſive Force, and Sowre from a Greater Degree of the Contractive, and ſeem to have the ſame Relation to our Taſt, as Heat and Cold have to our Feeling; For as what is Sweet and Warm gives a Life and Vigour to the Mind, ſo what is Sowre and Cold makes it Shudder and Tremble: The Reaſon of which can be no other, than that, as there is an Expanſive Force in the Mind Superior to the Contractive in the Airy, ſo accordingly it is Affected with Pleaſure or Pain, as thoſe Forces Agree, and are Coincident with, or are Oppoſite and Contrary to it's own; On the other Hand it muſt be likewiſe Confeſs'd, that in the Auſtere there is a Contractive Force in the Mind Superior to the Expanſive, by which it's Thoughts are kept and retained together, and which makes it more delighted with Sowre Taſts than with Sweet, with Cold than with Heat, and with Grave and Serious Things more than with what is Light and Trivial; From too Great a Degree of the Expanſive Ariſes Madneſs and a Diſſipation of Thought, and from too Great a Degree of the Contractive Ariſes Melancholy; Wit, Jovialneſs, and Debonairneſs, are owing to the Former, and Thought, Study, and Contemplation, to the Latter; The Medium betwixt the Airy and the Auſtere is the Juſteſt.

Concerning the Expanſive and Contractive Forces of the Mind.

WHEN I affirm Expanſive and Contractive Forces to belong to the Mind, as well as to Matter, I do not affirm, that the Mind is Matter, no more than, when I aſſert, that Fire and Gold have both theſe Forces, I aſſert that Fire and Gold are the ſame; So Extenſion is Neceſſary and Eſſential to Body and to Abſtracted Space, which yet are Eſſentially Different. Beſides which, I have given my Reaſons, wherefore I am aſſur'd and perſwaded, that the Mind is a Being entirely Spiritual, as it has Thought, Perception, Memory, and Judgment, nothing of which Matter has, and as it has an Innate Force Diſtinct and Separate from it; All which I affirm is, that it has theſe Contractive and Expanſive Forces, and upon which it's Union with Matter does, in Conjunction with what has been before alledg'd, Depend; For altho' their Subſtances Differ, yet ſince they agree in their Forces of Expanſion and Contraction, by thoſe it comes to paſs, that they have a Sympathy or Coalition with each other, and are in that Senſe alſo, as in Another, which was there Mentioned, United and Cemented together.

Chap. 6. §. 28, 29, 30. Book I.

THAT the Mind has an Expanſive Force, and in Different Degrees, is Evident and Manifeſt, not only from what has been before ſaid, but from the Different Strength, which it Exerts in Percuſſion, in Walking, Running, and in all the Motions and Actions of the Body; That it has likewiſe a Contractive Force is Plain from it's Power of Reſtraining, Curbing, or Contracting thoſe Motions.

Concerning Fevers, Dropſies, and Lethargies.

See Chap. 6. §. 29. Book I.

FROM hence it is, that in Fevers, or in Drinking Spirituous Liquors to Exceſs, in which there is the Greateſt Expanſive Force of the Blood, Madneſs or a Diſſipation of Thought Enſues; And in a Lethargy or Dropſy, or in the Stone in the Kidneys, when the Serum is ſtopt and circulates back with the Blood, that is, when there is the Greateſt Contractive Force in it, and it Approaches neareſt to a Stagnancy, a Flatneſs and Languor of Thought is conſequent, and the Expanſive Force of the Mind being Diminiſh'd, it Tends to Sleep and to Slumbers; And nothing is found in Experience to Contribute more to the Real and Conſtant Vigour of the Mind, and of the Body, than a Temperate Diet both in Meats and in Drinks, that is, ſuch which have not too Great an Expanſive, nor too Great a Contractive Force in them, or in other Terms, which are not too Solid and Heavy, or too Light and Spirituous.

THE Queſtion with Phyſicians is, in the Two Extremes afore-mentioned, what is the Proper Cure for them; But, without Entering into their Arguments about Acid and Alkali, In both theſe Caſes of a Fever or Madneſs, or of a Dropſy and Lethargy, (for as to the Stone, unleſs the Obſtruction to the Paſſage of the Urine thro the Ureters can be Removed, it is Incurable, of the Fatality of which Diſtemper, I have Formerly had the moſt Melancholy Inſtance in the Death of my Deareſt Father, one of the moſt Excellent and Pious of Men) I ſay, in both theſe Caſes,

THE Firſt Method, according to this Doctrine of Expanſive and Contractive Forces, to be uſed, is the utmoſt Temperance and Regularity in Meats and Drinks, to ſee if Nature will Return to it ſelf, and it's own Proper and Peculiar Forces of Expanſion and Contraction will Prevail, which in all Likelihood they will, if theſe Diſtempers are Induced by any Exorbitancy in the uſe of either the Expanſive or Contractive Forces in them; If this does not prove Effectual,

FIRST, In Madneſs or in Fevers, it ſeems to be Reaſonable to think, that the too Great Expanſive Force in the Blood is to be Diminiſh'd and Abated; Not by Taking Blood away, becauſe, that which Remains will have the ſame Temper and Craſis, and there will be a Want of it to ſupply a Nouriſhment to the Body, and which can very hardly be made by any New Digeſtions, when the Stomach, the Entrails, the Lacteals, the Veins and Arteries, the Lungs and the Heart, are in the utmoſt Diſorder, But by Altering the Expanſive Force of it, in which Conſiſts the main Difficulty; for if we Throw Water upon Fire in a ſmall Quantity, it cauſes it to Burn the Fiercer, if in a Large, it Extinguiſhes it, but Air keeps it in a Conſtant and Equable Tenour.

WHAT therefore ſeems to be done in this Caſe is, in the Higheſt and Greateſt Fever, Firſt, to Take the Air, ſo as the Motion, by Rideing in a Chair or Litter, ſhould not Excite or Exaſperate the Expanſive Force of the Blood, but keep it in the ſame Temper, in which it is.

THE next Thing is to be Regular in Diet, and to uſe Nothing, which has an Apparent Exceſs of the Expanſive Force in it.

AND the Third to take Gradually, and by Times, thoſe Things which have the Greateſt Contractive Force in them, as Water, Cooling Draughts, and the Juice of Lemons with the Rind.

SECOND, On the Contrary in a Lethargy or Dropſy, the Contractive Force of the Blood is to be Subdued, and to be Raiſed and Excited by Expanſive Forces into a Quicker Circulation.

WHICH is to be done, FIRST, By a Temperate and Regular Diet, which has not too Great an Expanſive or Contractive Force in it.

SECOND, By Taking the Air, and by the Briskeſt and moſt Lively Exerciſes, either by Rideing, or Walking, but eſpecially by the Former, and if poſſible, in the Midſt of Froſt and Snow.

THIRD, By Making uſe Gradually, and by Times, of Spirituous and Expanſive Liquors, to Envigorate and Enrich the Blood.

THAT this Method in Dropſies is a Juſt one, I have had Experience of, from the Effect it had upon a Fellow of this College, my Good Friend, who dy'd of one, who yet, when he was given over by his Phyſicians, was wonderfully and to every Body's Admiration Reſtored by theſe Preſcriptions, ſo far as to Survive his Imagin'd Death Three Years, and which, if they had been Purſued, would have Prevented a Relapſe of his Diſtemper, which at Length was too Hard and Difficult for him to Surmount. And I do not Queſtion, if the Method, which I have mention'd in Fevers, were Followed, but Phyſicians would Find a much Greater Succeſs in their Practice upon that Malady; This, which I wrote many Years ago, I Perceive is now Confirmed by Dr. *Hancock's Febrifugum Magnum*, which is nothing elſe but Water. All Diſtempers or moſt of them lie betwixt theſe Two Extremes, a Fever or Phrenzy on the one Hand, and a Dropſy or Lethargy, and Melancholy on the other.

A farther Proſecution of the Theory of Taſts

BUT to Return to Taſts, Sweet we have Affirmed to be owing to the Expanſive Force of Bodies, and Sowre to the Contractive; And it is Evident, that the Sweetneſs of Sack has a more Expanſive Force upon the Mind, that is, Intoxicates and Diſſipates it more than the Juice of an Apple or a Lemon, as the Heat of Fire Creates a Greater Diſorder in it than the Severity of a Froſt, the Firſt of which is an Expanſive Force and the Laſt a Contractive. And from hence, as was ſaid before, it is, that Phlegmatick Perſons, or thoſe, who have a Greater Degree of the Contractive Force in their Conſtitutions, are more Pleaſed with Cold Weather, and thoſe who are more Sanguine, with Heat and with Fire; which will alſo give a Reaſon, why the World will be always Divided, ſo long as there are theſe Two Principles in our Natures; The Witty, the Chearful, and Lively, under what Denominations of Religion or Learning ſoever, will be Averſe to the Serious, the Grave, and the Contemplative, and theſe to the Others, From the Difference of the Expanſive and Contractive Forces in Them.

From whence Bitter Ariſes, and how the Bark of Peru *Operates in Agues and Intermitting Fevers.*

If therefore Sweet is Derived from the Expanſive, and Sowre from the Contractive, Bitter, which is a Mixture of Both, will Proceed from a Mixture of thoſe Forces; Which is the Reaſon, that Bitters in General are Found and Experimented to be Wholeſome and Salutary to us, eſpecially, that of the Bark of *Peru*, in Agues and Intermitting Fevers, where the Expanſive and Contractive Forces are in a Perfect Struggle, and have in a Manner an Equal Contention with each other, which is manifeſt from the Hot and Cold Fits, the Conſtant Attendants of that Diſtemper.

See Chap. 6. §. 31. Book 1.

The Way of Operation therefore of this Grand Specifick in Medicine ſeems to be by it's Applying both to the Expanſive and Contractive Forces in the Blood, and Subduing the Exceſſes of both of them by thoſe Innate Forces of Expanſion and Contraction, of which it is Compounded, the Expanſive Force of the Blood, by the Contractive of the Bitter, and the Contractive Force of the Blood, by the Expanſive of it, But being more Expanſive than Contractive, it's Greateſt Influence is upon the Contractive of the Blood, from whence theſe Diſtempers ſeem to Ariſe.

From whence a Smooth Taſt and an Harſh one.

A Smooth Taſt, as that of Oil, and a Rigid or an Harſh one, as that of a Sloe, Ariſe from the ſame kind of Forces, which belong to Bodies which Feel ſo, in like Manner, as Sweet and Sowre are to Taſt, what Warm and Cold are to our other Senſations; and conſequently, the Harſhneſs of Taſt appears to be Derived from a Greater Contractive Force in the Sloe, Compared with that of Oil, and a Greater Expanſive Force in the Sloe, Compared with the Contractive of it, than in the Oil Compared with it's Contractive; the Greater Expanſive will make it ſtrike with more Force upon the Palate, and the Greater Contractive will make that Force of a leſs Sweet or Fiery Nature, and bring it nearer to a Sowre or a Cold; and that this is the Caſe of Oil and the Juice of Sloes is pretty Evident, ſince the Firſt is Inflammable, and the Laſt the Contrary.

Smells and the ſeveral Kinds of them ſolved from the above-ſaid Principles.
Princip. of Natur. Philoſ. Chap. 10. Part 1.

§. 12. The next Accidental Property of Matter is that of Smells, which likewiſe may be much Better Accounted for, from theſe Contractive and Expanſive Forces, than from any Imaginary Effluviums whatſoever, the Abſurdity of which in the Firſt Part of the *Principles* of *Philoſophy*, I have Endeavoured to ſhew; For if there is an Expanſive Force in Bodies, and that in Various Degrees, where that Expanſive Force is kept down to the Surface or Space, in which it Exiſts, by a Superior Contractive, there will no Smell Enſue, and this is the Caſe of Metals and Minerals. But where the Expanſive Force is Superior to the Contractive, as in Spirituous Bodies or Fermenting ones, and in Vegetables, Smells are Produced; for by their Brisk and Lively Action upon the Æther, from their Expanſive Force in Conjunction with their Contractive, they ſpread their Impreſſion round them, without any Diminution of their Parts by Effluviums, and are always prepared to Excite the ſame Senſations in us. Thus alſo, where the Contractive is Superior to the Expanſive, if yet the Expanſive is Vigorous and Quick in it's Action, Smells may be Excited in us, as in Odorous Woods, and other Solid Bodies, which ſend a Perfume from them.

Why Water and Fire have no Smell, and yet why both of Them have, if Joined, the Firſt with an Expanſive Force, and the Laſt with a Contractive.

But in Water, where the Contractive and Expanſive ſeem to be in an Exact Æquilibrium, and in Fire, where the Expanſive has little and an almoſt Imperceptible Contractive Join'd with it, as well as before in Solids, which have little and an almoſt Imperceptible Expanſive, we do not Experience any Smells. And yet, if by Fermentation an Expanſive Force is added to Water, or to Metals, or to Minerals, or if to Fire a Contractive, Smells are Produced; Thus Stagnant Waters by being Agitated by the Rays of the Sun, and Metals and Minerals, by their Diſſolution in Spirit of Nitre, become Fœtid, and any Thing burnt in the Focus of a Lens does alſo Emitt a Smell Agreeable to the Subſtance, which is Inflamed. From all which it is manifeſt, that there muſt be a Great Degree of the Expanſive Force, and that it muſt be Join'd with a Good Degree of the Contractive, to produce the Senſation of Smell in us.

The Reaſon of which is Evident, becauſe an Entirely Contractive, it is Plain, cannot do it; nor an Entirely Expanſive, without a ſufficient Force of the Contractive, to Moderate and Allay the Violence of it; becauſe otherwiſe it would not Affect our Organs of Smelling in a Certain Degree Requiſite for that Senſation, but would Confound and Diſorder them by it's Exceeding Expanſiveneſs and Volatility; And an Expanſive and Contractive in an Æquilibrium, as in Water, cannot Exert it ſelf enough outwards.

Smells either Fragrant, or Fœtid, or Intermediate to both.

Smells may be Diſtinguiſh'd into Fragrant and into Fœtid, or Pleaſing and Offenſive, with the ſeveral Intermediate Degrees of them.

From whence Fragrancy.

Fragrancy or Perfume is owing to a Greater Degree of the Expanſive Force, in Conjunction with a Leſs Degree of the Contractive, by which it Acts upon the Æther, and by it upon our Noſtrils; And from hence it is, that moſt Flowers have an Agreeable Smell to us, and if Bruſh'd, by which their Expanſive Force is Gently Rais'd and Excited, that Fragrancy becomes Stronger and more Vivid; Thus Steel or the String of an Harp, when ſtruck, is thrown into a Multitude of Vibrations.

From hence likewiſe in the Spring, when the Graſs and Flowers are Young and Vigorous, and their Expanſive Force Active and Lively, and when the Air is in it's Freſheſt Temper, and it's Elaſticity Strong, by a mutual Acting upon one another, there is a peculiar Fragrancy Raiſed, and the Whole Air ſeems to be Perfumed by it.

From the ſame Cauſe are produced all thoſe Delightful Smells, which we meet with in the Fields, in the Summer Time, or in the Harveſt, namely, from the Expanſive Force of the Bloſſoms, of Corn and Fruits, with which we are every where Entertained, and from the Expanſive Force of the Æther and the Sun's Rays, eſpecially, when Attended with ſome Moiſture or a Contractive Force, according to what has been already Mentioned.

Thus alſo in Woods and Spices, which grow in Hot Countries, ſuch as Cedar, Cinnamon, Cloves, Mace, and in Fruits, ſuch as Lemons, Oranges, Citrons, and the like, there is a Peculiar Fragrancy, upon Account of the Expanſive Force Impreſs'd upon them from the Extraordinary Heat of the Sun in thoſe Climates, with which they continually Act againſt the Contractive Forces in their Sowre Juices, by which they are detained, and upon the leaſt Friction or Rubbing, the Expanſive Force Raiſes and Exerts it ſelf into Action, and thereby Excites the Pleaſing Senſations we Receive from thoſe Bodies.

Rohault. Phyſic. Chap. 25. §. 12. Lib. 1.

Lastly, A Purulent Matter, produced from the Beaver, at Firſt has a Fœtid and Diſagreeable Smell, but being for ſome Time Expoſed in the Heat of the Sun's Rays, it at length Changes to a Tolerable one, and afterwards is Endued with a moſt Extraordinary Fragrancy, which cannot be owing to any Thing, excepting the Expanſive Force Impreſs'd upon it from the Sun's Action, and this is what is called Musk, and is of much the ſame kind with Civet. Amber-Greaſe is ſuppoſed to be Formed from the Excrement of the Sea, which it throws off and ſeparates from it ſelf by it's Exceeding Working and Fermentation, and probably has therefore at Firſt a Stinking and Loathſome Smell, as other Water has upon a like Emergence of it's more Corrupted Parts, but afterwards, by being Envigorated from the Heat of the Sun's Rays, it Acquires in all likelihood an Expanſive Force, from whence it's Fragrancy is Derived.

From whence a Fœtor.

And, as we have made a Fragrancy of Smell to Proceed from a Greater Degree of the Expanſive Force, Acting with a Leſs Degree of the Contractive, ſo it is Reaſonable to Think, that a Fœtor, or Loathſomeneſs of it, muſt be owing to a Greater Degree of the Contractive Actuated by a Leſs Degree of the Expanſive.

Thus in the ſeveral Caſes now Recited, if Flowers, or Bloſſoms, or Fruits, which before were Fragrant, or Spices and Odorous Woods, are Bruiſed or Beaten, and Pounded to a Good Degree, by which their Spring and Elaſticity, or Expanſive Force is Diminiſh'd and Broken, and their Contractive more obtains, they loſe their Delicate Perfume, and ſometimes change into a Smell, which is Diſagreeable and Offenſive; And in Musk, whilſt the Contractive Prevails in it's Purulency, and is Excited by a Small Expanſive, the Smell is Stinking and Nauſeous, as is even that of Flowers, as Poppies, when they are of a Soporifick, Cold, and Contractive Nature.

'Tis alſo Manifeſt, that Spirit and Oil of Vitriol, which Conſiſt of a Strong Contractive, Excited by an Expanſive, produce a Fœtid Smell, as do likewiſe Metals, Lead, eſpecially, when in Fuſion, the like do Salt-Petre, and Gun-powder, which is made of it, when Agitated by the Expanſive, and Glue when Heated, all which in their own Nature have a ſufficient Degree of the Contractive Force belonging to them, and ſend forth a moſt Fœtid Smell; And Moſt Solid and Combuſtible Subſtances, the more they have of the Contractive, are the more Offenſive in the Smell, which they Emit.

THE Contractive Force of Hair, or of Wool, which are made from thoſe Juices of Animals, which have the utmoſt Phlegm and Tenacity in them, when Excited by the Expanſive Force of Fire, as a Tough Phlegm in General Raiſed into Action the ſame way, will create a moſt Nauſeous Smell from the Expanſive Force Communicated to them; Pit-Coal, which has a leſs Contractive Force, will create a leſs Nauſeous one, as having more of the Oil, Sulphur, and Inflammable, or Expanſive, and leſs of the Phlegm and the Contractive; Wood, as having ſomething of the Expanſive Force of the Vegetable, will create a ſtill leſs Nauſeous Smell to us upon it's being Inflam'd; and Straw or Stubble, which has ſtill leſs of the Contractive, the leaſt of all.

THE Smells, which I have hitherto taken Notice of, are ſuch, which all or the Generality Agree to be Fragrant, or Fœtid, and which ſeem, the one to Pleaſe and Rejoyce the Mind, and the other to be Nauſeous and Offenſive to it; Becauſe the Expanſive and Contractive Forces of the Mind have ſuch a Certain Degree, to which the Firſt are more Suited and Adapted, and the Laſt of too Phlegmatick and Contractive a Temper for it to Reliſh, or Like it's Impreſſions.

As to other Smells, which Ariſe from Intermediate Forces of Expanſion and Contraction, they are Approved or Rejected by the Mind, according to the Various Proportions of the Expanſive and Contractive, which belong to it.

THUS, as in Taſts, ſo likewiſe in Smells, and even in Perfumes, thoſe which are Pleaſing to Minds, which have a certain kind of Delicacy from the Expanſive Force of them, ſhall be hateful to others, which have a more Robuſt Expanſive, that is, which Conſiſts of a Greater Degree of the Contractive join'd with a Strong Expanſive, to which more Rancid Smells ſhall be the Better Perfume; which ſeem to be the Two Caſes of an Elegant and a Polite Perſon, and whoſe Parts are Fine and Expanſive, and of a Plain and Hardy Soldier, whoſe Genius is Great, Commanding and Diffuſive, from the Expanſive Force of his Mind, and Solid, and Grave, and Serious from the Contraction of it; The Firſt of which Tempers cannot be better Repreſented than by the Characters we have of the Poliſh'd Age of *Auguſtus*, and the Laſt than by thoſe of the Firſt and Antient *Romans*, who would Prefer a Leek or an Onion to all the Refined Taſts or Smells in the World.

Sounds and the Variety of them Derived from the ſame Principles.

§. 13. FROM Taſts and Smells we ſhall Proceed to Sounds, which likewiſe may be Explained from theſe Forces of Expanſion and Contraction, and the Various Degrees and Combinations of them: And in this Caſe, the Quickeſt and Livelieſt Sounds are Produced from the Greateſt Expanſive or Elaſtick Force, join'd with a Leſs Contractive, and the Dulleſt and Heavieſt from the Greateſt Contractive Force, join'd with a Leſs Expanſive.

Not from Vibration, which is only a Conſequence of the Expanſive and Contractive Forces.

LET us Firſt take thoſe Sounds, which are Termed Muſical, and it is very Evident, the Quicker the Vibration of any String in Muſick is, that is, the Stronger the Expanſive Force, and the Leſs the Contractive, the Acuter and Sharper the Sound is; and on the Contrary, the Heavier and Duller the Vibration is, the Sound is in like manner Graver; The Nature of which Sounds are not to be Aſcertain'd from the Vibrations of the Strings or the Bodies, which are ſuppos'd to Produce them, but from the Innate Force and Velocity, which each would have in one Point only; The Quicker or Slower Vibrations are only Conſequences of ſuch Innate Forces, and each of them will occaſion Like and Similar Sounds from the Beginning to the End of their Motion, as Proceeding from the ſame Cauſes, namely, their Peculiar and Diſtinct Forces; As to the Vibrations themſelves, they do no more than continue the Ringing or Repeating the ſame Sound in our Ears, and therefore Sound does not Conſiſt in them, tho' the Continuance of the ſame Sound does; 'tis nothing more than if a Sharp Percuſſion or an Heavier was made upon the Æther in a Point, a

Line, or a Surface, for the Producing of the Senſation of Differing Sounds in us, and the Tremulous Motion has no other Effect, than that of Repeating the ſame kind of Senſation to our Minds, which will be the ſame, tho' this Force were to Act in a Point only, or in a Longer or Shorter Vibration, and which will fully Explain that Grand Difficulty in the Theory of Sounds, how it comes to Paſs, that the Sound of the ſame Muſical String continues the ſame thro' all it's Vibrations, be they Shorter or Longer.

The Great Difficulties in the Theory of Sounds Explained.

We may here add the Reaſon, which has not been given in any Philoſophy, why an Elaſtick Force in Bodies is Attended with a Vibration upon Percuſſion, namely, becauſe ſuch a Body is Compounded of an Expanſive and Contractive Force; The Expanſive therefore being Raiſed by the Expanſive of Percuſſion Reſults to the ſtroke, and is Drawn back by the Contractive, which not being able to Retain it, it again Recoils by the Force, which the Percuſſion Excited in it, and is again Drawn back by the Contractive, the Elaſtick Medium aſſiſting in this Vibration, till the whole Force of the Original Percuſſion is ſpent, and the Expanſive and Contractive remain at Reſt in their Firſt Point or Surface.

How the Voice is Formed, and the Different Modulations of it.

From the Muſical Notes upon Strings, if we Proceed to thoſe of Voices, the ſame Expanſive and Contractive Forces will prevail in the Solution of theſe Phænomenons; For the Quicker or Brisker the Air or Æther is ſtruck, not by the Epiglottis, which continues always open, for our Conſtant Breathing, excepting when any Thing is Conveigh'd over it into the Stomach, in which Caſe it Cloſes, but by the Muſcular and Animal, and Elaſtick Motion of the Lungs, and the *Aſperia Arteria*, Derived from the Expanſive of the Mind, the Shriller and Sharper the Voice is.

So we find it, when there is the Extremeſt Emotion in the Mind from any Fright or Ailment, which is brought upon it, and when it's Expanſive Force is Excited in the Greateſt Degree to Repell the Grievance, that it Exerts it ſelf upon the Muſcles, and the Rough Artery, and thoſe by the Elaſtick Force, which is Communicated to them from the Mind, Act upon the Air in the Cavity of the *Aſperia Arteria*, which occaſions the Shrieks and Screams, and the Sharpeſt Noiſes, which we hear upon ſuch Emotions.

This Exceeding Expanſive Force of the Mind joined with the Contractive Force of the Muſcles, which it Raiſes and Excites, and from them Strikes upon the Contiguous Air in the Rough Artery, is the Cauſe of the utmoſt Shrillneſs in Sounds, which we Experience: On the Contrary, where the Mind is but dully Affected, and where the Spirits are Sunk and Low, where the Activity of it is not ſo Great, and the Muſcles are not Moved by it with that Quickneſs and Vivacity, the Air, Included in the Lungs and the *Aſpera Arteria*, finds a more Languid Percuſſion, and conſequently gives a Graver and Deader Sound to the Ear, which proceeds from the Greater Power of the Contractive.

And this is the Reaſon, why Men can Fall and Raiſe their Voices at Pleaſure, and Change them into a Thouſand Different Notes, according as the Mind thinks fit to Exert more or leſs of it's Expanſive and Contractive Forces, which belong to it, and by them Act upon the Muſcles for their Expanſion or Contraction, from whence Different Velocities are Communicated to the Air, and from it to the Tympanum, by which that Variety of Sounds, which we Perceive, is produced in us.

Hence it is, that Children and Young Perſons, whoſe Spirits are Brisker and Livelier, and whoſe Expanſive Force of Mind is Gayer and Quicker, as well as that of the Muſcles, and therefore is in a Greater Proportion to the Contractive of them, in their Voices have a Shriller and Sharper Note.

But in Perſons, who are Grown up, and are more Advanced in Years, and in whom a Gravity of Thought, and a Greater Contractive Force obtains both in the Mind, and in the Muſcles, in Proportion to the Expanſive, by which Means the Air is ſtruck by the Organs of the Voice with a more Solid, but a leſs Elaſtick Force, a Graver Tone prevails; Another and a Concurrent Cauſe of the Difference in Voices is the Narrowneſs or Wideneſs, that is, the Contraction or Expanſion of the Larynx by the Muſcles of it. For which Reaſons alſo Women have Voices, which are Generally Shriller than Men's are.

Why ſome Sounds are Muſical and others not.

From the ſame Principles may the Cauſe be Derived, wherefore ſome Sounds are Harſh and Unpleaſing to us, and others Harmonious and Muſical; For if the Expanſive or Contractive Force, whether in Vocal or Inſtrumental Sounds, is Strained to an unnatural Degree it Produces Sounds, which are Diſagreeable, but if it is Exerted within certain Limits, ſo as not to Proceed from a Violence upon our Natures, nor ſo as to offer a Violence to them, ſuch an Expanſive or Contractive Force occaſions Sounds, which are Harmony and Muſick to us.

THUS a Shriek or a Scream, which Proceeds from too Great and Violent an Agitation from the Expanſive, and an Hoarſeneſs, which Proceeds from too Great a Degree of the Contractive, as in Colds, which are owing to an Exceſs of the Contractive, Produce Sounds, which are not Pleaſing to us; And the ſame may be ſaid of the Strings in Muſick, where the one has a ſqueaking Sound from too Great an Expanſive Excited in it, and the other an Hoarſe Baſs from too Great a Contractive.

FROM the like Methods of Reaſoning we alſo know, that where Perſons have a Greater Contractive Force in their Minds, and in their Muſcles, the Inſtruments of their Motion, The Baſs is more Agreeable to them; and thus we find in Experience, that thoſe, who are more Grave and Tragical in their Thoughts, are more Pleaſed and Delighted with ſuch Muſick; And on the Contrary, thoſe, who are more Briſk and Airy, and have more of the Expanſive in their Minds, like the Quicker and Briſker Sounds much Better. We muſt farther Obſerve, that thoſe Sounds are likewiſe Eſteem'd Muſical by us, which Approach the neareſt to Human Voices, or the Modulations of them, as being Naturally Delighted moſt with what Agrees with, or Comes neareſt to, our own Species, and for the ſame Reaſon, Vocal Muſick has Generally the Preference with us to Inſtrumental; Thus, the Barking of a Dog, or the Roaring of a Lion, was never thought to be very Harmonious, but the Notes of Birds Extremely ſo.

Why there is a Sympathy in Uniſons.

From hence alſo it is Plain, why there is a Sympathy in Uniſons, ſo that upon the Striking of the one, the other Trembles; For upon the Percuſſion of the Uniſon, It, by it's Natural and by it's Innate Force of Expanſion joined with the Contractive, which it Excites, Reſults to that Percuſſion, by which means there is an Impreſſion made upon the Air or Æther, which likewiſe, having a Certain Degree of the Expanſive and Contractive Force, Reſults or Recoils upon the Uniſon, and this Reciprocal Percuſſion of the Uniſon upon the Air or Æther, and of the Air or Æther upon the Uniſon, is the Cauſe of that Particular Vibration we Perceive, and which Vibration will continue, till the Original Force Impreſs'd upon the Uniſon, from the Expanſive Force of the Mind, and of the Muſcles, which it makes uſe of in ſuch a Percuſſion, is ſpent: For, if we conſider exactly, how can it be Imagin'd, that any other Effect ſhould proceed from the Percuſſion of the Uniſon, unleſs the Reſult of it peculiar to that Innate Degree of Expanſion or Elaſticity, which it has, againſt which the Contractive Force of the String, and of the Centers of the String's Tenſion Re-acts? This Contention betwixt the Expanſive and Contractive Force of the Uniſon Raiſes a like Conteſt in the Expanſive and Contractive Forces of the Air or Æther about it, ſince the Air or Æther is ſo Formed and Diſpoſed, as to be ſuſceptible of any Impreſſions, from whence it becomes Expanſive by Heat and Contractive by Cold, and according to the Various Actions it Receives and Conveys, Odorous, Coloured, and Sonorous.

THIS therefore being Allowed, as it ſeems to be Certain, it is Evident, that when the Air or Æther around has the ſame Percuſſion given to it by the Uniſon, which is plain from our Hearing the Sound of it every Way, the like Effect will be Produced by the Air's Acting upon another Uniſon, which has the ſame Degree of the Expanſive and Contractive Force with that, which it Receives from the Former. But this Reciprocal Percuſſion of the Uniſon and Air or Æther, being of a Peculiar Force, Compounded of one Particular Degree of the Expanſive and Contractive; From hence it is, that it Affects by it's Motion no other String, but what has the ſame Expanſive and Contractive Force with it.

Why Sound is Stronger when Impreſs'd upon a Thicker Medium than upon a Thinner.

WE may likewiſe from theſe Principles give an Account, why in General Sound is Stronger, when Impreſs'd upon a Thicker Medium of Air or Æther, and Diminiſh'd, when Impreſs'd upon a Thinner; Becauſe the Expanſive and Contractive Force of the Air or Æther, is in the Former Caſe thereby Increas'd, and in the Latter Impaired and Leſſen'd; for in the Former Caſe, the Expanſive Force is Increas'd, by the Increaſe of it's Elaſticity, and the Contractive, by it's Compreſſion and Denſity, but where the ſame Percuſſion is made upon a Greater Expanſive and Contractive Force, the Sound will be the Greater; Thus, if we ſtrike upon a Solid Body with the ſame Force, as we do upon Wool or a Feather, the Sound will be much Greater in the Firſt Inſtance, than in the Laſt.

Why Increas'd in the Speaking Trumpet.

From hence farther is Manifeſt, the Reaſon why the Sound is Increas'd in the Speaking Trumpet, as it is in any Inſtrument of Vocal Muſick whatſoever; becauſe the Percuſſion of the Voice is made, not upon mere Air or Æther, but upon Bodies, which have a Greater Degree of the Expanſive and Contractive, by which means, the Sound becomes more Solid and Strong; and the more Parts are Affected, and Struck, which will be the Larger the Cavity is, the Greater will be the Sound, that is, if the Cavity is not ſo Large, as that the Force of the Voice cannot reach it with it's full Strength and Vigour.

Whence the Ringing Noiſe in Choirs of old Cathedrals.

From this Percuſſion of the Voice upon Elaſtick Bodies, Proceeds the Ringing and Ecchoing Noiſes, we always find in Choirs, eſpecially in Cathedrals, which are Antient, and whoſe very Walls and Roofs have Acquired from the Air, by a Long Tract of Time, a more Peculiar Rigidneſs and Elaſticity, for in New Buildings, we do not Experience the ſame Sonorouſneſs.

Farther, The Propagation of Sound is likewiſe Explained from this Doctrine, and the Immediate Ceaſing of it, at a Nearer or Greater Diſtance from the Sonorous Body, upon the Vibration of it Ceaſing.

How Sound is Propagated.

The Firſt is Explicated from this Plain Axiom, that where there is a Percuſſion upon Air or Æther every way, as there is from the Sonorous Body, which Trembles and Vibrates, not from a Line, or an Axis, but from a Center every way, that Air or Æther will ſtrike that which is Contiguous to it, which likewiſe will Vibrate every way, and Communicate it's Impreſſions all around, to a certain Diſtance, where the Force is Spent and Loſt; which Diſtance is More or Leſs, according to the Force and Strength of the Percuſſion, which Depends upon the Proportion, which there is of the Expanſive and Contractive in the Sonorous Body; thus the Sound of a Bell will be heard Farther than an Harp or a Violin, becauſe there is a Greater Degree of the Expanſive and Contractive, in the Former Caſe, than in the Latter.

From hence alſo Sound may be Propagated thro' a Solid Body, becauſe Percuſſion may be, and the ſtriking of one Part upon another, in the ſame Way and Manner, as in Air or Æther.

The Laſt is alſo Evident, becauſe where Percuſſion, which is the Cauſe of Sound Ceaſes, the Effect, which is Sound, will Ceaſe at the ſame Time.

Concerning Sounds, which are not Muſical.

Thus far we have conſidered Sounds, which are Term'd Muſical, and have Endeavoured to ſhew how they are Derived from the Expanſive and Contractive Forces of Matter; We ſhall now Proceed to other Sounds, and from them Demonſtrate ſtill farther the Truth of this Doctrine.

Why Fire or Flame does not Produce Sound, nor the Percuſſion of Solid Bodies upon Æther, nor the Motion of the Hand in Air.

And Firſt, where there is a Great Expanſive Force, and a Small Degree of the Contractive, there is no Sound Produced, as in the Rays of the Sun, the Flame of Fire, a Lamp, or a Candle, the Reaſon of which is, becauſe there is no Reſiſtance made to ſuch Percuſſions, the Air or Æther on the Contrary Joins with, and Aſſiſts and Promotes theſe Actions.

On the other Hand, where there is a Great Degree of the Contractive, as in Solid Bodies no Sound is Conſequent, becauſe the Air or Æther has a Preſſure, as well as an Elaſticity, by which it Concurrs with the Contractive Force of them.

And why Sound is Produced by Water or Fuel thrown upon Fire.

But if a Contractive Force, as Water, or a Solid Body, as Fuel, is thrown upon Fire, a Sound is Produc'd, becauſe their Actions do not Concurr, but there is a Reciprocal Percuſſion upon each other, and a mutual Reſiſtance.

From hence it is, that if we Move our Hand in Air no Sound Ariſes, becauſe there is no ſenſible Reſiſtance to the Motion of it, but if we Strike the Air or Æther with an Expanſive Force, Joined with a Contractive, to which it Reſiſts, as in the Caſe of a Switch or a Wand, a Sound Enſues.

From all which it is Plain, as we have before Aſſerted, that there muſt be a certain Degree of the Expanſive and Contractive, Acting upon Air or Æther, to Produce the Senſation of Sound in us.

Or by the Exploſion of Gun-Powder.

WHICH is farther Confirmed from what is Experienc'd in the Exploſion of Gun-Powder, which Striking upon the Air or Æther, with the Expanſive Force of Fire Joined with the Contractive of Nitre, is Reſiſted by it, and by that Percuſſion, which always ſuppoſes a Reſiſtance or Re-action to the Percuſſion, the Mind comes to be Informed of the Contraſt there is betwixt them, by the Sound Produced from ſuch Action or Re-action, but the Sound is Small, as the Reſiſtance is ſo, when the Exploſion of Gun-Powder is made in Open Air.

ON the Contrary, when the ſame Expanſive and Contractive Force is Included in a Cannon, or a Mortar, or in a Rock, the Prodigious Expanſive Force of the Gun-Powder, Joined with the Small Contractive of it, Striking againſt the Vaſt Contractive and the Small Expanſive Force of the Canon or Mortar, or the Rock, there is the Greateſt Oppoſition, which is Poſſible, betwixt theſe Forces, and conſequently the Greateſt Sound is Produced from them, becauſe the Force of Percuſſion is the Greateſt.

MOREOVER the Small and Gentle, tho' Quick and Lively Percuſſions of the Strings of an Harp, would not Produce Sound, unleſs they ſtruck upon the Air or Æther in Various Degrees, and were variouſly Reſiſted by it; for if the Air or Æther was entirely Obedient to the Impreſſions of theſe Elaſtick Strings, and did not Recoil and Reſult upon them, there could be no more Sound from them, than there is from the Flame of a Lamp or a Candle.

Or by Percuſſion upon a Solid Body.

THUS alſo in Hard and Conſiſtent Bodies, what is the Reaſon that we hear a Sound upon the Percuſſion of them? Unleſs that the Expanſive Force of the Percuſſion, Derived from the Expanſive Force of the Mind, Raiſes the Expanſive Force of the Solid Body, which before lay Dormant by the Superior Contractive, which Oppreſs'd it, and thereby Exerts it ſelf more Strongly and Powerfully upon the Air or Æther, which again Recoils upon it, and at the ſame time Strikes the Tympanum of the Ear by it's Elaſticity, and Informs the Mind from thence of the Struggle or Competition, which it has with the Percuſſion Derived from it; for if the Expanſive Force of the Body only ſtruck the Air or Æther, as it does in Colour, without a Reſiſtance from it, the Cauſe of Sound and Colour would be much the ſame, which it is plain they are not, ſince there may be a very Lively Colour, as in Flame, where yet there is no Sound which accompanys it; And that the Expanſive of the Air or Æther, upon the Percuſſion of the Body, muſt likewiſe Act by it's Elaſtick Force both ways is Evident, becauſe otherwiſe no Impreſſion could be Communicated to the Ear; And this Doctrine is Confirmed in Fact, ſince we find that any Elaſtick Force, as that of the String of an Harp, or the Tone of a Bell, which does Communicate Sound, does Vibrate and Tremble, and Imparts it's Percuſſions all the ways, which are Imaginable.

FROM hence it is, that if we Strike a more Elaſtick Body, as a Stone, or a Marble, it will Reſult upon the Air or Æther more ſtrongly from that Percuſſion, and conſequently cauſe a Brisker or Livelier Sound to us, than if the ſame Force of Percuſſion is given to a Leſs Elaſtick Body, as Lead, which Produces to us a Flatter and a Deader; Steel upon Percuſſion Recoils more ſtrongly than Iron, Silver than Gold, Metals than Stone, and Stone than Clay or Earth, for the ſame Reaſon; and conſequently, the one Produce Sounds more Quick and Alert than thoſe of the other; And even in the Animal World, Noiſes are Produced from Mutual Quarrels and Conteſts, and Silence from Acquieſcence.

THUS alſo, if Percuſſions are made with Different Forces upon the ſame Body, the Greater the Expanſive Force of Percuſſion is, the Greater will be the Expanſive of the Body upon Air or Æther, it will be ſo much the more Excited and Raiſed into Action, and by Conſequence will occaſion a ſtronger Sound to our Senſation; For, if we Strike a Stone or a Marble with a Leſs Force of Percuſſion, it will Produce a Leſs Sound, and if with a Greater, a more Intenſe one, and the ſame obtains in Lead, or in any other Bodies.

Light and Colours ſolved from the ſame Principles; The Firſt of which is by a Percuſſion of

§. 14. FROM Sounds if we Proceed to Light and Colours, and the Refractions and Reflexions of them, we ſhall find, that the Expanſive and Contractive do here likewiſe Prevail, and that they are Entirely derived from theſe Forces.

LIGHT, as that of the Sun, of Fire, of the Flame of a Lamp or a Candle, is manifeſtly owing to the Expanſive Force of thoſe Luminous Bodies, which, Striking upon Air or Æther, Produce the Senſations of a Vivid Yellow, or Orange in us; By Air or Æther, I here, and all along, mean a certain ſubtile Medium, which is ſuſceptible of theſe Various Percuſſions from Bodies, and of which all the World is Full, which is Endued with a Degree of Action and Elaſticity, by which it Communicates the Impreſſions, made upon it, to our Senſations, from the moſt Diſtant Parts of the Univerſe.

the Expanſive Force of the Luminous Body upon the Æthereal Medium.

THO' this Medium is Alter'd and Changed in it's Force by other Forces, which Mix with it, eſpecially near the Sun, or the fixt Stars, the Earth, or the Planets, and the Various Comets, which belong to our or any other Solar Syſtem; which Forces, which mix with it, are Different, according to the Different Expanſive and Contractive Forces, of which the Sun, or the Star, or the Planet, or the Comet, is Compounded; From which Diverſe and Sundry Mixtures their ſeveral Atmoſpheres are Formed.

THUS the Atmoſphere about the Sun muſt be of an Exceeding Expanſive Nature, and that about our Earth, in Compariſon with it, of a more Sullen, Heavy, and Contractive, and as to the Moon, it ſeems to have None at all; That Body being in all Probability nothing more than a Vaſt and Stupendous Rock, Mountainous and Craggy, Placed near the Earth, by the Great and All-wiſe Creator, for the Reflexion of Light, the Raiſing of Tides, and it's Drawing forth of Plants and Vegetables, by it's Contractive Force in Conjunction with the Expanſive of the Sun, and for a Source of Froſt and Cold, or the Contractive, as the Sun is of Heat and the Expanſive; and which ſeems to be the Deſign of the ſame Infinitely Intelligent Being, in Reſpect of *Saturn*'s Ring, and His, and *Jupiter*'s, *Satellites*, and all thoſe, which are called Secondary Planets, with Regard to their Primary.

See Book 1. Chap. 5. §. 21.

BUT allowing for the Variety of Atmoſpheres, Ariſing from the Different Natures of the Bodies, which they Surround, it is Reaſonable to think, that the whole Univerſe is filled with one Uniform Active Medium, Ordained and Produced by the Expanſive Force of the Infinite Mind, which Medium is capable of Tranſmitting the Actions of one Body to another, tho' at an Immenſe Diſtance.

FOR how can any one Conceive, that Bodies ſhould Act upon one another by Percuſſion, which is the Preſent Caſe, without any thing to Conduct or Convey the Action? We may as well Affirm, that we can See, or Hear, or Taſt, or Smell, without any Impreſſion made upon our Organs of Senſation, as that any Percuſſion ſhould be Communicated from one Body to another, without a Medium to Tranſmit the Force which is Impreſs'd.

IN Anſwer to this it is Affirmed, that Light is a Body, and that it is Emitted from the Sun, and Arrives to us in 7 or 8 Minutes of Time, from Obſervations made upon the Eclipſes of *Jupiter*'s *Satellites*, and that it always is Propagated in a Direct Line, all which are ſuppoſed to be Arguments, that Light does not Proceed from a Percuſſion upon a Medium, but from an Emanation of it from the Luminous Subſtance; Becauſe, if it was by a Percuſſion upon the Medium, Light, conſidered in it ſelf, would not be a Body, nor would it be Emitted from the Sun, nor, as it is Apprehended, would it come to us in a certain Proportion of Time, but in an Inſtant, nor would it be Propagated in a Direct Line, but would be Diffuſed into all the Parts of the Medium, upon which the Impreſſion is made; In Refutation of which we ſay,

Sir Iſ. Newt. Opticks.
Molyneux's Dioptricks.

FIRST, That Light may have ſeveral of the Properties, which belong to Body, if it is Produc'd by a Percuſſion of the Expanſive Force of the Sun upon the Medium, which is Spread and Extended thro' the whole Univerſe, is Confeſſed; Becauſe the Medium, which is Excited by ſuch an Expanſive Force, is Body, and is Endued with the Various Properties and Affections of it, and Light in this Caſe is the Medium thus Excited.

Light not Proved to be an Emanation of Particles from the Luminous Body.

SECOND, Light is alſo Emitted from the Sun, by it's being Derived from and Owing to the Percuſſion of it's Expanſive Force upon ſuch a Medium, by which it is Propagated to all the Parts of it's Syſtem; But to ſuppoſe Light to be an Emanation of the Particles of the Sun, thro' a Void Space, and that thoſe Particles are carry'd with that Exceeding Velocity, as to come from thence to us in 7 or 8 Minutes of Time, is ſomething Incredible, and has many Difficulties Attending it.

For Firſt, in ſeveral Thouſand Years, by ſuch a Vaſt and Continual Efflux of Parts in every Minute, it would be Impoſſible, that the whole Body of the Sun ſhould not be entirely Waſted and Conſumed, and if all the Comets in the World were to Fall into it, as no Body knows, that ever any one did, it would not be a ſufficient Aliment for ſuch a Prodigious Expence of it's Particles, as ſhould every Minute and every Inſtant ſucceſſively Fill the whole Orb of the Solar Syſtem, and perhaps ſeveral Millions of Miles beyond it.

Besides, What becomes of theſe Particles? Here is in ſome Thouſands of Years, in all Probability as many Particles ſent out from the Sun, as would Conſtitute a Maſs a Million or Ten Hundred Million Times Bigger than the whole Body of it, and yet no one can tell, where they are Lodged; not in the Air or Æther, for there is no ſenſible Increaſe of it's Denſity, nor in the Earth, the Moon, or the Planets, or fixt Stars, which we do not find Receive any Material Augmentation to their Bulk; So that in this Way of Philoſophy, there are Millions and Millions of Particles of Matter, and even of whole Suns, are Spent and Exhauſted by Light, and no Mortal can ſay, where they are, or give any more Account of them, than if they had Really and Entirely Vaniſh'd.

Moreover, For any Particles of Matter to be carry'd, ſeveral Millions of Miles, by a Local Motion, in 7 or 8 Minutes of Time, is ſo Contrary to all the Phænomenons of Nature, that to mention it only is to Explode it; And ſuch an Aſſertion only Proceeds from this Philoſophy's maintaining a Vacuum, which could never Inform us of ſuch a Being, as the Sun, unleſs it ſent it's Emiſſaries ſo many Millions of Miles in every Inſtant of Time to Tell us of it; For we do not Experience or Know of any ſuch or the like Velocities in Being. And here I muſt ſay, that the Greateſt Effects in Nature are Produced by Silent and Unheeded Cauſes, and we may, I believe, Affirm, that, in General, the Reaſon, why we Underſtand ſo little of Nature is, becauſe we are apt to Attribute every Effect to ſome Extraordinary Violence, made uſe of to Produce it, whereas Nature Acts by Gentle Methods, and almoſt, many Times, Undiſcernible and Imperceptible ones to us, and that muſt be the trueſt Philoſophy, which Explains it in that Way.

Farther, ſuch a Number of Particles, be they as Small, as this Philoſophy can Imagine them, if yet they are Particles of Matter, Actuated with the ſuppoſed Velocity, would, like a Rapid and Violent Stream, bear down all before them, and we ſhould not be Heated, but Knockt down by the Rays of the Sun.

Thus even in the Infinitely ſmaller Velocity of a Canon Ball, it is ſaid, if it comes near to us, we are Affected very ſenſibly by the Motion, which it Impreſſes upon the Air, or Æther, or Medium, which Surrounds us, and it is very Evident, that we are ſcarcely able to ſtand againſt Violent and Tempeſtuous Winds, As to whoſe Particles, tho' perhaps they may be Groſſer, according to the Corpuſcular Philoſophy, yet their Velocity and Force is not a Millionth-Millionth Part of what is Attributed to the Particles of the Sun by the ſame Philoſophy.

Third, But if we only allow, that there is a Percuſſion of the Medium by the Expanſive Force of the Sun, ſuch a Percuſſion may be Communicated by it in 7 or 8 Minutes of Time; Becauſe we in Fact find, that Sound, which is own'd to be from Vibration, and therefore from Percuſſion, is Propagated with a Great Velocity without any Rapid Motion of Parts, as is Evident in Eccho's, in which we hear the Return of the Sound, almoſt as ſoon as it is Produced by Percuſſion upon the Sonorous Body, without any Violent Agitation of Parts in Air or in Æther; but the Expanſive Force of Fire or Flame being ſtill Quicker, and to which there is little or no Reſiſtance given from Air or Æther, or from the ſame Medium, thro' which Sound is Promoted, to which there is a Reſiſtance given, the Percuſſion of it is Communicated thro' it with an Exceeding Greater Velocity, ſo far, that we are ſuppoſed to ſee Light, as ſoon as it is Produced in Lightening or in the Flaſh of Gun-Powder, tho' we hear the Sound of Thunder, or the Report of a Cannon, Later.

Light may be variouſly Propagated, where the Expanſive Force of the Luminous Body

But even this kind of Percuſſion, namely, that of Fire or Flame, may be Different, as in it's Force, ſo in the Propagation of it thro' the ſame Medium, or where the Percuſſion has the ſame Force, as that of the Sun, and the Medium is Different, the Propagation of it may upon that Account be likewiſe Different; and therefore it is no Unimaginable Thing, that ſuch a Percuſſion, in ſome Caſes, may be Communicated

municated in an Inſtant, where the Expanſive Force of the Luminous Body is extremely Great, the Medium of the ſame Nature, and the Diſtance Small, thro' which it is Derived; and in other Caſes, that the ſame Percuſſion may be Communicated in Various and Different Portions of Time, as in 7 or 8 Minutes, where the Percuſſion is made from an Exceeding Expanſive Force, and is Communicated thro' Different Mediums, and where the Diſtance of the Luminous Object, as that of the Sun, is Large and Great; for if there is any Contractive Medium, which Intervenes, as that of the Earth's Atmoſphere, or any other, it will give a Check and Interruption to the Propagation of ſuch a Percuſſion, and the Largeneſs of the Diſtance, thro' which the Action is Communicated, will alſo in ſome Proportion or other Diminiſh the Force of the Action.

is the ſame, and the Mediums are Different, or where the Expanſive Force is Different and the Medium the ſame.

On the Contrary, where the Percuſſion is Different, and the Medium and Diſtance of the Luminous Object are the ſame, and much more, where the Diſtance is Different, it is Reaſonable to ſuppoſe, that Light will be Propagated to us in Different Portions and Spaces of Time; It would perhaps be hard to Evince this by Experiments, but this is Evident, that the Flame of a Lamp, or a Candle, will not Communicate it's Light to ſo Great a Diſtance, as a ſtrong Blaze of Fire, nor will a Meteor be ſeen ſo far as the Sun, the Reaſon of which can be only, that the Expanſive Force, and the Percuſſion conſequent upon it, is not ſo Vigorous in the one as in the other, and by the ſame Argument it's Propagation ſhould not.

Fourth, As to the Propagation of Light in a Direct Line, this may be acknowledged, tho' the Propagation of it was by the Percuſſion of the Sun upon the Elaſtick Medium, and not from an Emanation of any Solar Particles; becauſe a Force will be Directed and Communicated that Way, in which the Impreſſion is made.

But then on the Contrary, if Light was Produced from an Emanation of the Sun's Particles, it would be Derived in Radius's Drawn from the Sun's Body, by which means there would be a Million of Streams of Light Iſſueing in a Streight Courſe from the Sun, with as many Intermediate Spaces, to which Light would not Arrive, and by Conſequence, the Open Day would be Checquer'd with Alternate Portions of Light and Darkneſs, which is what we do not Experience.

The Rectilineal Propagation of Light, by an Emanation of Particles from the Sun, not Adequate to the Phanomenons of it, whereas a Percuſſion of the Expanſive Force upon a Medium is.

Besides which, upon the Interpoſition of a Cloud, by which theſe Streams of Light would be Intercepted, or in a Dark Chamber, where it's Ingreſs is only Permitted at a Foramen in the Window-Shut, a Perfect Darkneſs would Enſue, which again is Contrary to what every one's Obſervation will Inform him of; In all which Caſes it is Evident, that Light does Diffuſe it ſelf out of any Streight Lines Drawn from the Sun; and therefore, if an Emanation of the Sun's Particles can give no Account of this Diffuſion of Light, and Percuſſion from the Expanſive Force of it can do it, for that Reaſon alſo the one is Preferrable to the other in an Explanation of Nature, and which may be eaſily done.

For the Sun or any other Luminous Body ſtriking by it's Expanſive Force upon the Elaſtick Medium in Direct Lines, and the Medium not Reſiſting, as in Sound, but Communicating and Promoting the Force without Oppoſition, Light is by that means Propagated *in Directum*, but the Medium being Elaſtick, the Percuſſion will Excite a Vibration Side-ways, ſince Elaſticity or the Expanſive Force always Acts from a Center, as the Contractive does towards one, by which means there will be a Diffuſion of Light to every Part, and it will Spread and Enlarge it ſelf out of the Lines, thro' which the Original Communication is made.

And from hence it is, that Light is more Vivid and Strong, and has a Yellower Complexion in the Direct Rays of the Sun, and is Weaker and has a Paler Colour out of them, or in a Shade, becauſe the Firſt Proceeds from the Direct Percuſſion of the Expanſive Force of the Sun, and the Laſt from the Lateral Percuſſion of the Direct; the Latter of which muſt certainly be more Languid than the Former, and is perhaps upon an Equal Balance nearly with the Contractive of the Medium.

Why Light is Different.

There is alſo another Reaſon, why the Light is not ſo Strong in the Shade, whether of a Cloud, or of Trees, or of a Tower, or any other Body, becauſe the Light, which is Reflected from them, is Alter'd and Chang'd by the Peculiar Expanſive and Contractive Forces of thoſe Bodies; Thus the Shade of a Stone Edifice is Lighter than that of a Grove, and the Shade of a Cloud is Lighter or Darker, according to the

Why Shades of Light are Different.

the Various Compoſition of the Contractive and Expanſive Forces of the Cloud, which Cauſes it.

What Darkneſs is.

THIS therefore being the Nature of Light, we may readily Conclude, what Darkneſs is, that it is Produced, not by a bare Privation of Rays, or of the Expanſive Force, but where there is the Leaſt Expanſive Force Communicated to our Organs of Seeing, and conſequently, where the Contractive moſt Prevails; which Contractive we could not be ſenſible of, unleſs Excited by ſome Small Degree at leaſt of the Expanſive.

FOR it is Impoſſible for us to have any Poſitive Idea from a Privative Cauſe, and therefore it is, that we perceive ſeveral Degrees of Darkneſs, which could not be from one Abſolute Privation of Light, but muſt Proceed from a Greater Degree of the Contractive in the Medium, and a Leſſer of the Expanſive in ſeveral Proportions; Thus the Night is not ſo Dark, when the Moon or the Stars appear, as otherwiſe.

What Opacity and Tranſparency is.

FOR which Reaſon Bodies, which have the Greateſt Proportion of the Contractive, and the Leaſt of the Expanſive, are Opake, as Earths, Stones, Metals, and Minerals; and thoſe, which have the Greateſt Proportion of the Expanſive, and the Leaſt of the Contractive, or where the Contractive and Expanſive are almoſt Equal, are Tranſparent, as Pure Flame, and Spirituous Liquors and Fluids in General, alſo Water, Air, and Æther, where the Contractive and Expanſive are perhaps nearly Equal; Thus likewiſe Glaſs, which acquires an Exceeding Expanſive Force from the Fire, is Tranſparent; Metals, Diſſolved by it, will be Changed into Glaſs, and by Spirit of Nitre into a Tranſparent Fluid, and afterward into Chryſtals; all which Phænomenons are Evidently derived from the Expanſive Force prevailing over the Contractive, or being Equal to it nearly; for by that means, by theſe ſeveral Bodies, Acting from a Center, and not in ſo Great a Degree, or in an Equal nearly, towards one, there is a Paſſage permitted to the Rays of other Bodies thro' them; but on the Contrary, where the Contractive Force is Great, that is, where the Bodies Act towards a Center, or, which is the ſame, where the Forces of them Preſs together with more Vehemence, ſuch a Paſſage of the Rays is Interrupted, and the Body thereby becomes Opake.

THIS Doctrine is Confirmed farther from our Fingers, which are, if look'd upon in the Common Light of the Sun, Opake from the Contractive Force of our Fleſh and Skin, but if they are oppoſed to the Strong and Vivid Rays of it, they become in ſome ſort Tranſparent; becauſe the Expanſive Force of the Sun, Joining with the Expanſive Force of the Blood, Surmounts the Contractive of the Fleſh and Skin to a certain Degree, and induces that Tranſparency, which we in ſuch a Caſe Experience.

Why Melancholick Perſons are more Pleaſed with a Darker than a more Enlightened Medium.

FROM this Account of Light and Darkneſs, we may know the Reaſon, why Melancholick Perſons are more pleaſed with a Dark, a Gloomy, or a Shaded Medium, than with a more Light and Enliven'd one, and with Graver and Darker Colours, than with the more Flaſhy and Airy, becauſe the Contractive Forces of their Minds are Superior to the Expanſive of them.

Whence the Variety of Colours.

As to Colours, we have Endeavour'd to ſhew in the firſt Part of our Principles, that they are Various Degrees of the Expanſive Force, occaſioned and produced by the Tranſmiſſion of it thro' the Edges of the Priſm, that is, by the Various Contractive Forces, which it meets with, and by which it is Reſiſted and Oppoſed in ſuch a Tranſmiſſion.

THUS the Strongeſt and Livelieſt Colour, and which comes neareſt to the Complexion of the Sun's Rays, but is ſomething Darker, is a Fiery Red, and Proceeds from the Expanſive Force of the Sun being the leaſt Diminiſh'd by the Contractive of the Glaſs, as Paſſing thro' the Thinneſt Part of it, that is, thro' the Extremity of it's Edges; the Priſmatick Orange is a Weaker Red, and the Priſmatick Yellow a Weaker Orange, Green a Weaker Yellow, and Blue a Weaker Green, and Violet a Weaker Blue, and Deep Violet a Weaker Violet; all which are produced from the Expanſive Force of the Sun's Rays being variouſly Enervated by Paſſing thro' Greater Thickneſſes of the Priſm, and therefore meeting with Different Degrees of the Contractive Force of it, which in like Proportions Weakens and Impairs the Expanſive Force of them. This alſo gives an Account of that Agreeable Diverſity of Colours, which we many Times obſerve in the Clouds upon the Riſing and Setting of the

Sun, which is owing to the Different Oppoſition made to the Expanſive Force of the Solar Beams by the Different Contractive of the Clouds, and the Different Obliquity of the Rays, which Impinge upon them in a Thouſand Various Inclinations, both which Contribute to the Changeing the Expanſive Force into thoſe ſeveral Actions and Enervations of Action, from which we ſuppoſe Colours to Ariſe.

From this Doctrine it alſo follows, which we find in Fact, that the Fiery Red Rays have the Leaſt Degree of Refrangibility, that is, the Greateſt Expanſive Force, or the Greateſt Degree of Activity, Orange are the Next Leaſt Refrangible Rays, Yellow after Them, Then Green, Blue the Next, and Violet and Deep Violet are the moſt Refrangible, that is, have the Leaſt Proportion of the Expanſive Force, or the Leaſt Degree of Activity, and conſequently, are the moſt Removed from the Complexion of the Sun's Rays, and Approach the neareſt to Darkneſs.

Sir Iſ. Newt. Opticks.

Mr. Hauksbee's Exper. Pag. 29.

And upon theſe Principles, all the other Phænomenons of the Priſmatick Colours may be Explained, in which there are Various Mixtures and Combinations of them, and in which, as the Expanſive or Contractive Force more Prevails, the Colours are Brighter, or Heavier and Duller; Mr. *Hauskbee*'s Experiments do alſo abundantly Confirm this Doctrine, From which he aſſerts, That Different Sorts of Bodies afford us remarkably Different Lights, Different in Colour, and Different in Force and Vigour; which can Proceed from nothing elſe, unleſs their Various and Innate Proportions of the Expanſive and Contractive.

What Whiteneſs is. Sir Iſ. Newt. Opticks.

Hence likewiſe, when all of them are mixt together in a due Proportion, a White, not the Solar Light, which is Yellowiſh, is Produced; becauſe, there being an Equal Balance of the Contractive and Expanſive Forces, a Colour, as the Whiteneſs of Light, is Cauſed, which holds it ſelf Indifferently to all Colours; ſo that the Whiteneſs of Light is the ſame to Colour, as Tranſparency is to the Colours, with which Tranſparent Bodies are Tinged, they both Proceeding from nearly an Equal Balance of the Contractive and Expanſive Forces.

The Difference betwixt Tranſparent Coloured Bodies, and Opake ones.

From hence farther it is, that Tranſparent and Colour'd Bodies are Fluid, or of a Glaſſy Subſtance; becauſe in both thoſe Bodies either the Expanſive Prevails, or is nearly Equal with the Contractive; but where Bodies are Colour'd and Opake, the Contractive Prevails, tho' not to the Suppreſſion of the Expanſive, but to the Detaining it's Action within a Point or a Surface; whereas Spirit of Nitre, which has a Strong Expanſive Force Joined with ſome Degree of the Contractive from it's Salts, Exerts it ſelf out of a Point or a Surface, by Corroding and Diſſolving Metals by it's Expanſion, as Fire or Flame does Wood or Fuel, and Converting them into a Fluidity.

What Red, Orange, and Yellow in Fluid Bodies, Proceeds from. Sir Iſ. Newt. Opticks.

The Colour therefore of Fluid and Tranſparent Bodies Proceeds, not from the Tranſmiſſion of the Colour'd Rays, as Sir *Iſ. Newton* Affirms, but from that Particular Degree of the Expanſive Force, which is join'd with a Leſſer Degree of the Contractive, in ſuch Bodies, which Acting upon the Elaſtick Medium Produces that Senſation in us; and according as the Colour Approaches nearer to that of the Sun, which has the Greateſt Expanſive Force of any Being we know in Nature, the Greater is the Expanſive Force of ſuch a Fluid, if the Contractive Forces are Equal, which is the Caſe of Fiery Red, Orange, and Yellow.

Spirit of Nitre, Oil of Vitriol, and White or Yellow Wines, are more Expanſive than Red or Purple Wines and Fluids.

Thus we find, that Spirit of Nitre and Oil of Vitriol are of a Yellowiſh Colour, which have the Greateſt Expanſive Force of any Fluids, as is Plain from their Corroſive or Burning Quality; and thoſe, which are called White Wines, and which are of a Yellowiſh Colour, are more Expanſive, that is, are Stronger and more Intoxicating than thoſe, which are of a Deep Red or Purple, if other Things are Equal; Alſo, Sacks, Ales, and Brandies kept, are Expanſive and Yellow; On the Contrary, Rheniſh, which comes the neareſt to a White, has the leaſt Expanſive or Intoxicating Force of all, as Water likewiſe has.

It is alſo obſerved, that Red or Purple Wines are generally more liable to Produce the Stone in us, and the Yellowiſh Wines more apt to Diſſolve or to Prevent it; that the Firſt are more Aſtringent, and the Laſt in fact Looſening and Diuretick; all which are manifeſt Arguments, that the Firſt kind of Fluids have more of the Contractive, and leſs of the Expanſive Force in them, and the Laſt more of the Expanſive and leſs of the Contractive: Suitable to which, Chymiſts aſſure us, that the Red Abound in Tartar, or a Contractive Subſtance, but the White not.

'Tis alſo Evident, that the One are Sweeter and Smoother, and the other more Rigid and Harſher, which is likewiſe another Inſtance, according to what we have ſaid concerning Taſts, that the one are more Expanſive, and the other leſs ſo.

What Red, Orange, and Yellow in Solid Bodies Proceeds from.

Red, Orange, or Yellow, on the other Hand, in Solid Bodies, if of the ſame Colour with the Red, Orange, or Yellow, in Fluids, has the ſame Expanſive Force with the other, but is kept down and detained by the Superior Contractive to a Point or a Surface, in which it Acts upon the Elaſtick Medium with an Equal Force of Velocity, tho' Exerted in a Point or a Surface, with which Red, Orange, or Yellow, in a more Spirituous Subſtance, does.

Thus Clay Burnt acquires a Redneſs, and Lead Calcin'd produces Minium, which have a Great Degree of the Contractive Force, and are Endued from the Fire with a Great Proportion of the Expanſive, which yet cannot Exert it ſelf any farther than a Point or a Surface, by Reaſon of the Vaſt Contractive, which obſtructs it, ſo that from thence we only perceive the Redneſs of each Body, without any other conſiderable Effect from it.

Whence the Yellowneſs of Gold.

And the ſame may be ſaid of Gold, which by it's Yellowneſs or Orange, has the Greateſt Expanſive Force, but as from it's Weight it has the Greateſt Contractive, by that means the Expanſive is Loaded to that Degree, that it cannot Exert it ſelf, without a foreign Force to Excite it, beyond a Point or a Surface, and therefore it only Acts by it's Expanſive upon the Elaſtick Medium, and thereby Produces the Senſation of Orange or Yellow in us; Sound it cannot Impreſs upon us by ſuch an Expanſion, becauſe there is no Reſiſtance made to it, nor Smell, nor Taſt, becauſe it does not Act out of a Point or a Surface, that is, the Contractive is Superior to the Expanſive, whereas, in Taſt and Smell, the Expanſive is Superior to the Contractive.

What the Nature of Poiſons, as Spirit of Nitre, or Oil of Vitriol, which are Yellow, and White Sublimate and Arſenick.

And that Gold has a Great Proportion of the Expanſive, is Apparent, not only from it's Colour, but from what is Affirm'd, that the Leaf of it, or any Diſſolution of it, is the beſt Cordial, that is, that it has the Greateſt Expanſive Force Join'd with a Contractive, which can be agreeable to our Natures; for there may be an Expanſive Force Join'd with a Contractive ſo Extraordinary, as Spirit of Nitre, or Oil of Vitriol, which ſhall Immediately Diſſolve them, and this is the Caſe of Poiſons, which have too Great an Expanſive Force for the Contractive of our Conſtitutions; as there are others, to wit, White Sublimate, Arſenick, *&c.* which have too Great a Contractive for the Expanſive of them; What has been here ſaid of Red, Orange, and Yellow, is alſo Applicable to the Various Colours of other Bodies, making juſt Allowances for their Different Expanſive and Contractive Forces.

The Reflections and Refractions of Light, ſolved from theſe Principles.

§. 15. From the Colours of Light we ſhall now paſs on to the Refractions and Reflections of it.

How the Reflection of Light is Performed.

The Reflection of Light therefore Proceeds from the Expanſive Force of it, by which it Diſperſes it ſelf from a Lamp or the Sun, but Meeting with the Contractive Force of a Solid Body, which is Superior to it, it is hinder'd in it's Progreſs, and would there Ceaſe and Vaniſh, or elſe be Diffuſed upon the Surface of it, like Water or any other Fluid Pour'd upon the ſame Surface; and conſequently, there are Two Reaſons, which cauſe the Return of it's Rays: Firſt, It's own Native and Expanſive Energy from the Center of any Body, by which it Recoils from the Contractive, which it cannot ſubdue; Second, The Expanſive Force, which is mixt with the Contractive in every Solid Body, in a certain Proportion and Degree, and which forwards and aſſiſts the Reverberation; And as this Expanſive Force is Greater in Solid Bodies, the Rays of Light are Reflected and Repuls'd more ſtrongly. Thus Wood or Stone are not ſo Vivid in their Reflections, as Burniſh'd Metals, nor Iron or Lead ſo Active, as Steel, Becauſe they are not ſo Expanſive.

Light not always, and in General, Reflected in the ſame Angle, in which it is Incident.

It cannot therefore be affirmed in General, that Light is Reflected in the ſame Angle, in which it is Incident, ſince in Different Circumſtances of the Repelling or Luminous Body, the Reflections of Light from it may be in Different Angles; It is notorious in Fact, that we ſee the Colours of Bodies in all Angles of Reflection, tho' the Sun or Candle, which Illuminates them, is in a Perpendicular Situation to them; The Light of the Sun or a Lamp by it's Force of Expanſion Diffuſes it ſelf every way, and by the Joint Contractive and Expanſive Force of the Body is Repell'd every way, and at all Angles. From hence it is, that Light is weaken'd by Reflection, becauſe it meets with the Contrary and Duller Force of Expanſion and Contraction in Bodies, which by their Impreſſion Change and Diminiſh the Force of it.

FROM hence likewiſe it is, that Concave and Burniſh'd Metals, or Poliſh'd Glaſſes, Reflect Light to a Focus or Point, and Convex ones from one, by which means in the Firſt Caſe it is Stronger and more Collected, and in the Laſt Weaken'd and Diſpers'd.

Why Concave Metals, or Glaſſes Reflect Light to a Focus, and Convex ones to ſeveral.

BECAUSE the Contractive and Expanſive Force of the Concave Body, being Superior to the Expanſive of the Light, Repells it; and as it's Parts are in a Situation, by which it's Actions are made to Tend the ſame Way or to a Point, they accordingly Determine the Reflection of Light to one; on the Contrary in Convex Reflecting Bodies, the Expanſive and Contractive Forces in them are Superior to the Expanſive of Light, which is Repell'd by them, but as their Parts are in a Situation of Convexity, by which they Act Contrary ways, Light is alſo Repell'd to Different Points, and is thereby Diſſipated and Weaken'd. Or elſe, This may be perhaps owing to a Superiority of the Light's Expanſive Acting againſt and Reſulting from the Contractive, there of the Concave, and here of the Convex.

THE Conſequence of which is, that Objects, which are Repreſented to us in a Thicker and a more Conſtipated Light, appear Larger, and in a Thinner and more Diluted, Smaller; that the Former becomes Inflammant, and the Latter is leſs than the Natural Heat of the Sun's Rays.

Why Objects appear Larger in the Firſt Caſe and Diminiſh'd in the Laſt, and why Light is Inflammant in the one Circumſtance, and the Reverſe in the other.

AND in Plane Bodies, a middle Effect is Produc'd betwixt Both, and Light is Reflected much the ſame, as what it was when Incident; Objects are not much Increas'd or Diminiſh'd, nor the Heat of the Sun's Light, and there is no Difference betwixt the Incident and Reflected Light, excepting what Ariſes from the Expanſive and Contractive Force of the Repelling Body, by which it is in ſome Meaſure Weakened.

The Caſe of Plane Glaſſes.

ON the other Hand, the Refraction of Light Proceeds from the Superiority of the Expanſive Force of Light, above that of the Expanſive and Contractive of the Bodies, thro' which it is Refracted, by which it Forces it's Paſſage thro', inſtead of being Repell'd by them; And as in Convex ones, the Parts Act towards Different Points, the Expanſive of Light Acts in Oppoſition to them, and Collects it ſelf to one by Surmounting their Force of Diſperſing it; and in Concave Bodies, where the Parts Act towards the ſame Point, the Expanſive of Light Acts with a Contrary Direction, and Surmounting that Force of Contraction, Tends and Diſperſes it ſelf towards the Sides of the Concave, and thereby becomes Weaker; or elſe perhaps this may be owing to the Greater Force of the Glaſs, which draws the Light to a Focus in the Convex, and from one in the Concave; and from hence, as before, Light will be more Denſe and Conſtipated in the Firſt Caſe, and thereby become Inflammant, and Objects will appear Greater in it, and in the Laſt Caſe, it will be Thinner and more Diluted, and Objects appear Diminiſh'd; And in Plane Glaſſes, the Refracted Light obtains a Middle Effect betwixt both theſe.

How the Refraction of Light is Performed in General, and in Particular thro' Concave and Convex Glaſſes.

See Book 1. Chap. 5. §. 43.

HAVING gone thro' the ſeveral Eſſential and Accidental Properties of Matter, as they are Termed, we ſhall now go on to give an Explanation of the ſeveral Operations and Effects in Chymiſtry from theſe Expanſive and Contractive Forces, after we have taken Notice, that if we Admit of theſe Forces in Bodies, there will be no Occaſion for any Imaginary ſubtile Spirit, which is ſuppoſed to Pervade all Groſs Bodies, and which is Latent in them, by which their Parts attract each other, and is the Cauſe of the Coheſion of them, by which alſo Electrical Bodies are ſaid to Act, and to have a Power of Attracting and Repelling Bodies at Greater Diſtances, by which Light is Emitted, Reflected, and Refracted, and all Senſation is Excited, and Animal Motion Performed according to the Direction of the Will, namely, by the Vibrations of this ſubtile Spirit; ſince all theſe are Accounted for by theſe Real Expanſive and Contractive Forces, which are the Eſſence and Subſtratum of Matter, and from which, as we have ſhewn, all it's Properties are Derived.

Sir Iſ. Newt. Princip. Math. Lib. 3. Pag. 484. Edit. 1.

§. 16. BUT before we Proceed to Chymiſtry, we ſhall from theſe Principles give a Solution of thoſe Strange and Wonderful Phænomenons, which Ariſe from the Friction of Glaſs.

The Strange and Wonderful Phænomenons Ariſing from the Friction of Glaſs, ſolved from the ſame Principles.

LET a Glaſs Sphere be Exhauſted of it's Air, and Revolved round upon an Axis by the Motion of a Wheel, and let it be Rubbed whilſt Revolving, till it has acquired a ſufficient Heat by it's Friction; and from a Semicircular Arc, large enough to Encompaſs the Sphere, Let ſeveral Threads of Yarn hang Perpendicularly over it;

Mr. Hauksbee's Experiments, Plate 7. Pag. 191. Mr. Whiſton. & Mr. Hauksbee, Plate 6. Pag. 20,

after the Glaſs Sphere has acquired a certain Degree of Heat, the Threads will riſe from their Perpendicular Situation, and Point like ſo many Radius's to the Center of the Sphere; if the Semicircular Arc is placed under the Sphere, or if it is fixt in an Horizontal Poſition, and in both Caſes with the Concave Part towards it, the ſame Effect will be Conſequent, that is, the Threads will Riſe and Direct themſelves, like ſo many Radius's, to the Center of the Sphere.

And on the Contrary, if the Inward Cavity of the Sphere is now ſuppoſed to Communicate with the Air, and there is an Axis Inſerted into it, with a Small Circle affixt thereto, and Threads of Yarn are faſtened to the Circle; upon the Circumvolution and Friction of the Glaſs Sphere, the Threads will Riſe and Extend themſelves from the Circle, and it's Axis in ſo many Radius's Pointing towards the Spherical Surface; and whilſt the Threads are under the Influence of the Glaſs's Electricity, they may be Moved by the Finger without Touching them, and even by the Finger and Breath, thro' the Glaſs it ſelf.

Of the Friction of the Glaſs-Sphere; and the Pointing of the Threads towards it, how ſolved.

In order to ſolve which Phænomenons, it muſt be obſerved, that Glaſs has a Great Contractive Force from the Flints and Sands, of which it is Compoſed, which, tho' not the Heavieſt, are ſome of the Hardeſt and moſt Compact Subſtances; that, by their Fuſion in the Fire, a Great Degree of the Expanſive is acquired to them; that, by the Former of theſe, Glaſs is of a Cloſe and Solid Contexture, and, by the Latter, becomes Fluid in Heat, and Suſceptible of any Impreſſion, and when Cold is Brittle, it's Parts eaſily Separated, and is to a Great Degree Elaſtick and Expanſive under the Reſtriction and Conſtraint of it's Contractive; when therefore the Expanſive is Excited by Friction, it Exerts it's ſelf in Radius's from the Center, as the Actions of all Spherical and Convex Bodies do, thro' the Ambient Medium; the Threads of Yarn, which by their hanging Perpendicularly in the ſame Medium, have a Contractive or Gravitating Force, which is contrary to the Expanſive, ſuitably upon this Action of the Expanſive thro' the Medium, which was before in Æquilibrio, Exerts it's Contractive in an Oppoſite Direction to the ſame Center, from which the Expanſive Acted, to preſerve the Equal Poiſe, which the Expanſive had Diſturb'd and over Balanc'd, and which, by ſuch Tendency of a Contrary Force, is again Reduc'd to an Æquilibrium, or as near as theſe Two Acting Forces can do it.

On the other Hand, if a Circle and the Threads of Yarn affixt to it are placed within the Glaſs Sphere, and the Air Communicates with the Cavity of it, the Threads will Riſe from the Center and Extend themſelves to the Concave of the Glaſs, becauſe the Expanſive Force of it, Excited by Friction, does now Exert it ſelf Inwards, and towards the Center, as a Spherical and Concave Body does, where it's Action is Determined to the Medium, which lies within it, and by conſequence, Diſturbs the Æquilibrium of ſuch a Medium; the Contractive Force therefore of the Yarn, having a Contrary one, Diſperſes and Dilates it ſelf from a Center in the Medium ſo Affected, and obtains the Power of an Expanſive; This kind of Action of the Expanſive and Contractive, in Oppoſition to each other, has been Exemplified in the Revolutions of the Heavenly Bodies, in the Actions betwixt the Moon and the Earth, and in the Reflections and Refractions of Light, and is farther juſtified by theſe Experiments, which have been hitherto deſervedly Accounted very Surprizing. If what we have ſaid is not ſufficiently Convincing, perhaps the Suppoſition of an Expanſive Force in Yarn, which it Evidently has from Balls, which are made of it, will give us a Solution; if we will but Allow, that as the Expanſive and Contractive Act in Oppoſition to each other, ſo the Expanſive Forces Unite with and Tend to each other, and therefore the Threads of Yarn will Tend to the Expanſive of the Glaſs Excited by Friction.

From hence it is Manifeſt, that an Æquilibrium of the Air is neceſſary to theſe Phænomenons, for if the Expanſive Force of the Glaſs, and the Contractive of the Yarn were to Exert their Actions in Vacuo, where there is little or no Æquilibrium of Force to be Preſerved, they would each of them ſpend their Forces of Expanſion and Contraction by themſelves, without any Effect produced in Relation to each other.

We may likewiſe Conclude, that there is a Certain Force and Action Diſpers'd thro' the whole Univerſe, ſince otherwiſe the Expanſive Force of the Sun, and the Contractive of the Earth, Moon, and Planets, would Exert themſelves without any

Reſpect to, or Influence upon each other, unleſs there was ſuch a Medium, deſtined to Communicate their Actions, and very much in an Æquilibrium with it ſelf.

That this Refined Æquilibrium may be eaſily Diſturbed by a Finger, or a Breath, is no Wonder, or even thro' the Glaſs it ſelf, which in this Caſe acquires an Expanſive Force, and is as Permeable as the Air, which Surrounds it, tho' not to the Touch, yet as to any Forces, which may Diſturb the Æquilibrium, of which the Glaſs makes up only one Part of the Moment.

That theſe Effluvia muſt be very ſubtile is very Plain, becauſe they are none at all, all theſe Phænomena only proceeding from the Innate Forces of Bodies compared with one another, without the leaſt Diminution of their Matter or their Particles.

What I have here in General ſaid might perhaps ſatisfy the Excellent Mr. *Hauksbee*, if he were now alive, in Reſpect to ſeveral Queries, which he makes, and whoſe Obſervations and Experiments, and his Judicious Remarks upon them, deſerve to be in the Hands of all Lovers of Philoſophick Truth, who will not Reſign their Notions of Nature to the Preſent Mathematick and Abſtracted Commenters upon it.

How the Purple Light is Produced.

The Electricity of Glaſs, or the Tendency of the Threads to the Center, or to the Concave Surface of it's Sphere, requires the Air in the one Caſe or the other, but to Produce the Phænomenon of Purple Light a quite Contrary Circumſtance is Expected, that is, a Vacuum or rather a Leſſer Force in the Medium, than in Common Air; The Reaſon of which is Plain, becauſe the Expanſive Force of the Glaſs, when Excited by Friction, Acting upon a Leſs Force than that of Common Air, Raiſes it into a kind of Flame, which when a Greater Force is Introduc'd, as that of Common Air, it's Effect is Diminiſhed, and then Vaniſhes, or is Extinguiſh'd by it, as Common Flame by Water; That this Flame is a Purple one, Proceeds from the Weakneſs of the Expanſive Force of Glaſs, tho' Excited to the utmoſt Degree, and from the Contractive Force of it, which hinders it's Expanſion, according to what we have already Alledged concerning that Colour; This Purple Flame darts Inwards, as the Air is admitted, by Endeavouring with an Impetus to recede from it, as being of a Contractive Force in Reſpect of it, and for the ſame Reaſon, that Flame ſtrikes Upward.

How the Blue Sparkles, and the Crackling Noiſe are Cauſed.

The Blue Sparkles, which it Emits, ariſe from the Friction, by which the Expanſive Force of the Glaſs is ſo far raiſed, as to be Superior in ſome Minute Particles to the Contractive of it, from whence they fly off, and by the Expanſive being Increas'd the Purple is heightened into a Blue; and the Crackling Noiſe is cauſed by the Contranitency, which there is betwixt the Expanſive and Contractive, as in the Expanſive of Flame, and the Contractive of Wood, or the Expanſive of Percuſſion, and the Contractive of Glaſs, Stone, or an Anvil, or any other Contranitent Body to the Expanſive.

Why a Glaſs Cylinder becomes Electrical.

A Glaſs Cylinder, either Solid or Hollow, by Friction becomes Electrical, This is ſolved by what foregoes, and there is no Occaſion to exhauſt the Air in the laſt Caſe, becauſe it's Action is Cloſe and Compact by the Sides of the Figure lying ſo near each other, and therefore not much Interrupted by the Intervening Air, which rather complys with it, and makes it the ſame, as if a Solid one was uſed.

Why no Heat, but that of Friction will Produce this Electricity.

No other Heat, but that of Friction, will Produce this Electricity in Glaſs, becauſe by Friction it's own Proper Expanſive Force is Raiſed and Excited, whereas any other Heat Proceeds from an Expanſive Force, which is Different from it's own, and Acts not in Concurrence with, but in Oppoſition to it, and therefore rather Weakens and Impairs it.

Why this Electricity Exerts it ſelf thro' Glaſs, leſs thro' Muſlin, and not at all thro' Wood.

It will Exert it's Expanſive or Electrical Force thro' Glaſs, becauſe it has the ſame kind of Expanſive Force with it, tho' not Excited, and leſs thro' Muſlin, becauſe it is of a more Contractive and Glutinous Nature and obſtructs it's Action; and for the ſame Reaſon, if the Glaſs Cylinder lies betwixt Two Pieces of Wood, it's Electricity fails, becauſe of their Clammy, Glutinous, and Contractive Forces in Reſpect of the Quick and Elaſtick Force of the Glaſs, which is therefore Oppoſed, Vanquiſh'd, and Dampt by them.

Why it Attracts and Repells Leaf Gold, and other Small and Light Bodies, and why ſuch Bodies, when fully Repell'd, cannot be made to Touch the Glaſs, till they have been Reflected from ſome other Body.

LASTLY, It Attracts and Repells Leaf-Gold, and the like Small and Light Bodies by Turns, and when once they have been fully Repell'd, they cannot be made to Touch it, till they have been Reflected from ſome other Body.

THE Reaſon of which is, that the Expanſive Force of the Glaſs, being Raiſed and Excited by Friction, is in a kind of Conteſt with it's Contractive; when therefore the Expanſive Prevails, the Contractive of the Gold is carry'd in Oppoſition to it, as the Threads of Yarn were before, to preſerve the Equal Balance of the Medium, when the Contractive prevails the Leaf-Gold Reſults from it to the Side of the Expanſive, and will not be made to Return upon the Expanſive's Prevailing, that is, by the utmoſt Force of Electricity, becauſe it has now acquired an Expanſive Force, and therefore cannot Act in Oppoſition to it, as before, unleſs it obtains a new Contractive Force, by Touching ſome other Body, which is endued with it, after which it will again, upon the Expanſive Force Prevailing, Act with the Contractive and in Oppoſition to the Expanſive, and ſo *in Infinitum*.

FROM which it is Manifeſt, that there is not only a Contagion in our Minds, from the Company we Converſe with, of Good or Evil, of Mirth or Severity, but that it is a General Law in Nature, and that even Material Subſtances are Infected with the Expanſive or Contractive, as they Act in Concert and in Conjunction with thoſe Forces.

IT would be eaſy, from theſe Principles of the Expanſive and Contractive Forces, to ſolve, I believe, every one of the Strange and Surprizing Phænomenons Exhibited to us, from the very Ingenious Experiments of Mr. *Hauksbee*, I have only given an Inſtance of ſome, and ſhall now Proceed to the Equally Wonderful ones in Chymiſtry.

CHAP. II.

Concerning the Principal Phænomenons Obſervable in Chymiſtry.

The Preſent Method of Chymiſtry not ſuited to a Philoſophick Inquiry. Lemery, Mr. Vagani, and others.

§. 1. IT has been hitherto the Method of Authors and Teachers in this Science to Treat of Chymiſtry, as if they were Giving a Collection of Receipts, for the Compoſition of Drugs and Medicines, for the Trade of Apothecaries, and for the Uſe of the Phyſician, rather than Forming a Syſtem of Experiments for the Satiſfaction of the Philoſopher, and the Inquirer into Nature and it's Operations; and from hence it is, we find them Diſtributing their Analyſis of Bodies into that of the Mineral, the Vegetable, and the Animal Kingdom, and under thoſe ſeveral Heads to furniſh us with the Chymical Reſolution of Gold, Silver, Copper; of Guiacum, the Bark of *Peru*, Opium; or of Vipers, Urine, Honey, Hartſhorn, and the Like.

The Method propoſed more adapted to ſuch an Inquiſition.

§. 2. WHEREAS the Reverſe of ſuch a Procedure is what becomes a Philoſopher, who is not ſo much concerned to know the Particular and Artificial Manner of Analyzing theſe Bodies, as the General Effects, which Reſult from ſuch an Analyſis; the Firſt being the Buſineſs of the Operator and the Mechanick, and the Laſt of the Studious and the Inquiſitive; Conſidering this many Years ago, I had Drawn up ſeveral Papers in order to put this Science into a Different Method, from what had been Practiced by the Profeſſors of it; Namely, by Ranking and Diſpoſing their Proceſſes under thoſe General Phænomenons, in which they Terminated, as Spirits, Oils or Sulphurs, Water or Phlegm, Salts and Earth, Calcination, Chryſtallization, Precipitation, Coagulation, Condenſation, Diſſolution, Rarefaction, Pulverization, Inflammation, Diſtillation, Sublimation, Digeſtion, Extraction, and Colour; and tho' I have been Prevented in Finiſhing theſe Papers by many other Studies, which have Employed my Inquiries, it is a Satisfaction to Me to find the ſame Method Approved of, by the Learned and Ingenious Dr. *Freind*, in his *Prælectiones Chymicæ*.

The Preſent Principles of Chymiſtry Precarious and not Proved. Lemery Engliſh Edit. 3. Pag. 9.

§. 3. I Shall therefore go on in the way, which I Originally propoſed to my ſelf in the Purſuit of Nature, and Firſt conſider the Principles of Chymiſtry, which by moſt are confeſs'd to be thoſe of Spirit, Oil, Salt, Sulphur, and Earth; which yet, it is Plain, are very Imaginary ones, becauſe all Bodies cannot be Reſolved into theſe Principles; All of them are found in Animals and Vegetables, but not ſo eaſily in Minerals, and in Gold and Silver, not ſo much as one of them, of which there can be no Separation made at all.

AND on the Contrary, tho' Glaſs is made of Sand, and Aſhes or Salt, there can be no Salt Extracted from it, from whence it is Inferred, that Bodies may be Compounded of Principles, into which they cannot by any Art be Reſolved; which ſeems altogether Impoſſible; for if there were Spirit, Oil, Sulphur, and Salt of Glaſs, no doubt they would be as much found in it by the Operations of Chymiſtry, as thoſe Principles are Experienc'd to be in Animals and Vegetables; but as we never heard of any of them, ſo it muſt be Confeſs'd, that all of them are Equally wanting in the one, as they abound in the other, and that Glaſs in that Reſpect is of a like Nature with Gold or Silver; Spirit, Oil, and Sulphur, are not of ſo ſullen a Diſpoſition, as not to obey the Influences of the Quick and Active Motion of the Fire, if any ſuch were Latent in the Subſtances we have Named.

§. 4. IF yet we allow the Expanſive and Contractive Forces in Bodies, which I have been hitherto Defending, the Caſe is Plain, that as all Bodies conſiſt of theſe Two Forces in Various Degrees of them, ſo where the Expanſive Force is moſt Prevalent, as it certainly is in Animals and Vegetables if Compared with Metals, Spirit, Oil, and Sulphur, which are of an Expanſive Nature, may be Produced and Separated by the Expanſive Force of Fire, and want only the Aſſiſtance of that Force for their Formation and Exertion.

Why Glaſs and ſome Metals have none of the Chymical Principles, and Animals and Vegetables, all of them.

ON the other Hand, when, as in Glaſs, the utmoſt Expanſive Force of Fire has been uſed in it's Production, no Chymical or Analyzing Force of the Fire can alter it's Conſtitution and Diſpoſition; the Expanſive and Contractive are ſo ſtrongly united, that it will be Impoſſible to ſeparate them into Diſtinct Principles, and they will either unite with one another, as far as Poſſible in Fuſion, notwithſtanding a new Expanſive Force endeavours to diſturb them, or by Friction, when the Greateſt Conteſt is Excited betwixt them, as has been before Explained, there will be only Small Glaſſy Particles Emitted from them.

AND ſince Gold and Silver obtain the ſame Obſtinacy, we may conclude they are only more Conſtipated Glaſſes, whoſe Expanſive and Contractive Forces are Greater, and more Cloſely and Firmly united to each other, and that in all Probability they are made and formed in the Earth, by the Exceeding Heat of the Sun, or by Subterraneous Fires Mixing with, and Fuſeing of Certain Earths or Contractive Forces, which are proper for ſuch Productions, as Sand or Aſhes are Converted by a like Expanſive Force into Glaſs; Minerals are not ſo Contractive as Metals, nor ſo Expanſive as Vegetables or Animals, and therefore the Principles of Spirit, Oil, and Sulphur may be Extracted from them, more eaſily than from Metals, and more hardly than from Animals and Vegetables.

§. 5. WE have ſhewn in the foregoing Section, why ſome Bodies, which have a Greater Contractive Force, and where the Expanſive is firmly united with it, have none of the Chymical Principles, and where the Expanſive Force more Prevails, that they have all of them; we ſhall now conſider the Principles themſelves, which are Evidently Reſolved into theſe Expanſive and Contractive Forces; Spirit, Phlegm, and Oil, proceed from the Prevalence of the Expanſive Force, and Salts and Earth from the Contractive, and therefore the one may be called the Active, and the other the Paſſive; and tho' this Diſtinction is Different from what is maintained by Chymiſts, who make Spirit, Oil, and Salt to be the Active Principles, and Phlegm and Earth to be the Paſſive ones, yet the Juſtneſs of it will appear from hence, that even Spirits are acknowledged to be nothing elſe than Salts Diſſolved in Phlegm, and certainly the Diſſolvent muſt therefore be the Active Principle, and that which is Diſſolved the Paſſive; Phlegm is Fluid, and Salts not ſo, but when Diſſolved in one; And laſtly, Phlegm in Diſtillation comes before the Spirits, when they are Fixt, and after them, when they are Volatile, which ſhews that it is even more Active than ſome Spirits, tho' leſs Active than others, and that it may be deſervedly Rank't amongſt the Active Principles, and therefore falls under the Expanſive; whereas Salt remains Diſguiſed in the Earth, after the other Principles are Extracted, and conſequently belongs to the Contractive.

The Real Principles of Chymiſtry are the Expanſive and Contractive Forces in Various Mixtures and Combinations of them.

Lemery, Pag. 10.
Dr. Freind, Pag. 7.

Lemery, Pag. 5. and Pag. 4.

§. 6. As to Spirits, they are Subtile, Piercing, and Light Subſtances, and more in Motion than any other of the Principles, and are of Three Sorts, the Spirit of Animals, the Burning Spirit of Vegetables, and the Acid Spirit; the Firſt, as of

The Different Sorts of Spirits in Chymiſtry Accounted for by theſe Forces.

Lemery, Pag. 3. 9, 10.

Hartſhorn, is ſaid to be nothing but a Volatile Salt Diſſolved by Phlegm, the Second, as Alcohol, or Spirit of Wine, Spirit of Juniper and Roſemary, an Exalted Oil; and the Third, as the Spirit of Vinegar, Tartar, and Vitriol, an Acid Eſſential Salt put into Fuſion, and made Fluid by Fire.

The Spirit of Animals therefore Proceeds from a ſmall Expanſive Force into a ſmall Contractive one, as Phlegm has but a ſmall Degree of Expanſion, being only Fluid and not Fiery, as rather Raiſed by Fire, than Formed by it; and the Volatile Salt has but a ſmall Degree of Contraction, being made in ſome Reſpect Expanſive by the Expanſive Force in the Circulations of the Humours in the Animal, from which it is Extracted.

The Burning Spirit of Vegetables is therefore Compounded of a Greater Degree of the Expanſive, than the Former Spirit, and of almoſt an Equal one of the Contractive, by the Firſt of which it becomes Inflammable, and by the Laſt Oleous and Clammy, and the Parts more ſtrongly Cohere and are more Firmly united to one another, than in Spirit of Animals; and this is Evident from the Different Conſtitution of Animals and Vegetables, the Former being ſupported by the Proper and Weaker Expanſion of the Circulation of it's own Humours, which are of a leſs Contractive Nature, and the Latter by the vaſt Expanſive Force of the Sun, Acting upon a Groſſer, more Contractive, and Earthy Matter.

The Acid Spirit therefore is Formed from a Great Expanſive Force of Fire into a Great Contractive of Tartar or Vitriol, and is the Reverſe of thoſe Forces, which Enter into the Compoſition of the Spirit of Animals, by the Former of which it becomes a Spirit and Fluid, and by the Latter Acid and Fixt, as to it's Salts, and it is not Inflammable, becauſe the Expanſive is ſufficient to raiſe the Contractive into a Spirit, but not to diſpoſe it to a farther Activity.

From all which it is Apparent, that the Expanſive Force of the Fire has a Great Influence in Forming and Compounding theſe Spirits, and that they are not properly in Bodies themſelves, but as ſuch Bodies contain thoſe Principles and Diſpoſitions of Force, which render them capable of being Raiſed and Animated into Spirits by the Application of the Expanſive of Fire.

Concerning the Spirit of Saturn *and* Venus. Lemery, Chap. 5, 6. Part 1.

§. 7. From hence it is, that Spirits may be Drawn from Lead and Copper, becauſe their Natures are more Expanſive than thoſe of other Metals, as is Evident from the eaſy Fuſion, the Pliantneſs and Softneſs of the Firſt, by which it Approaches towards a Fluid, and from the Quick and Ready Diſſolution of the Parts of the Latter, even by Warm Water only, and it's Changeing into Ruſt and Verdegreeſe, by a Drop of Cold; From this Innate Expanſive Force it is, that they abound with Sulphurs, but to Exalt theſe Metals into a Spirit, Three Things are Required, all which Contribute to the raiſing their Innate Expanſive; Firſt, that Lead ſhould be Calcin'd into a Ceruſſe, or Copper Reduc'd to a Ruſt or a Verdegreeſe; Second, that this Ceruſſe or Verdegreeſe ſhould be mixt with a Fluid, as Vinegar, by which it Incorporates into a Salt or a Chryſtal; And Third, that it ſhould be Excited into the Higheſt Expanſive, of which it is Capable, by Fire and Diſtillation.

Concerning Spirit of Salt and Nitre. Lemery, Chap. 15, 16, Part 1.

§. 8. Spirit of Salt is made by all the Ways, which can Promote and Advance the Innate Expanſive Force, which belongs to it; It has a Great Degree of the Contractive, by it's Parts being ſo cloſely United, that the utmoſt Force of Fire is not ſufficient to diſengage them, unleſs Potter's Earth is mixt with it, or ſome other the like Subſtance; it is yet not ſo Contractive as Metals, becauſe it is Endued with a Quick and Pungent Taſt, which can only Proceed from it's Expanſive, whereas Metals in General have little or none belonging to them, and it's Parts are not ſo Solid nor ſo Heavy; it is alſo formed from a Fluid, as Sea-Water or Urine, *&c.* and conſequently is leſs Contractive and more Expanſive in it's Nature than Metals are.

The Firſt Step, taken to Raiſe it's Natural Expanſive Force into an Artificial Spirit, is, to Dry it in the Beams of the Sun, or over the Fire; Second, to Pound and Grind it to a ſmall Powder; Third, to mix Potter's Earth with it, which has an Expanſive by it's having a Deterſive or Corroſive Force; Fourth, to Temper it with Water, which, by it's being capable of Diſſolving it, has likewiſe an Expanſive Force; Fifth, to Dry this Compoſition in the Sun; And Laſtly, to Diſtill it in a Reverberatory Furnace, Firſt, by a Gentle Fire to ſeparate the Phlegm from it, and then, by Increaſing it to the utmoſt Degree, when the Spirits will come out in White Clouds, and continuing it in this Condition for 12 or 15 Hours; From all which Procedure it is Manifeſt, that every Gradation Tends to the Exalting the Expanſive Force of the Salt, till it is Formed and Excited into a Spirit, which yet, after all theſe Operations, retains a Contractive Force, which is Manifeſt from it's Acidity.

Spirit of Nitre is Drawn much the ſame Way, namely, by Reducing the Salt to a Powder, then mixing it with Potter's Earth, and afterwards Diſtilling it in a Large Retort, and in a Reverberatory Furnace, by a ſmall Fire till the Phlegm is ſeparated, and from thence Increaſing the Fire by little and little to the Second Degree, when the Spirits will come forth in White Clouds, and fill the whole Receiver; and from ſtill Increaſing the Fire to the Laſt Degree, the Vapours will come Red, and the Operation will end in about 14 Hours.

The Difference therefore betwixt theſe Two Spirits can only Proceed from the Difference of the Expanſive Forces of the Salts, from which they are Drawn, and from that of Nitre being Greater, and having a leſs Contractive Force than that of Common Salt.

That Nitre has a Greater Expanſive Force than Common Salt is from hence Evident, becauſe it is Confeſs'd to be more Volatile; it is made from a Fluid, and by the Airs Acting upon that Fluid, both which have an Expanſive Force, and by the Contractive Force of the Cold and Solid Stones, to which it Adheres, as in Cellars and in old Ruined Buildings; whereas Common Salt is Formed by the Evaporation of the Fluid from the Expanſive Force of the Sun's Rays, and leaving a Matter behind, which cannot be Evaporated by it, and which therefore cannot be Separated by the utmoſt Force of Fire; Nitre is alſo found in the Hotteſt Countries, as in *Ægypt*, where the Expanſive Force of the Sun is the ſtrongeſt.

From whence a Solution follows of the foregoing Experiment, that, with the utmoſt Force of Fire, the Spirituous Vapours of Common Salt will be only White, whereas thoſe of Nitre will be Red; which can only Proceed from the Greater Natural Expanſive Force of the one than of the other; for the Senſation of Orange or Red is Produced in us by a Greater Action upon our Organs, and White with a Leſſer, or by an Æquilibrium of the Expanſive and Contractive Blended together.

Why the Spirituous Vapours of Common Salt are White, and thoſe of Nitre Red.

From hence alſo it follows, that what Spirit of Nitre Diſſolves, the Spirit of Salt will Precipitate, becauſe the whole Moment of Spirit of Nitre has an Expanſive Force, in Reſpect of the whole Moment of the Spirit of Common Salt, which has a Contractive, and therefore the Latter, having a Greater Contractive than the Former an Expanſive, what this Diſſipates and Diſſolves, the other Precipitates and Drives to the Center by a Contrary Action.

Why Spirit of Salt Precipitates, what the Spirit of Nitre Diſſolves.

§. 9, Spirit of Sal Armoniack is made from a Salt, which has ſtill a more Volatile and Expanſive Force than Nitre.

Concerning Spirit of Sal Armoniack. Lemery, Chap. 15. Part 1.

The Natural Sal Armoniack is found in the Hotteſt Countries, as in the Parts of *Africk*, which are the neareſt to the Torrid Zone, upon Earth, which has Imbibed the Urine of Animals, and which is Sublimed into a Volatile Salt, by the Action or Expanſive Force of the Sun; Artificial Sal Armoniack is made at *Venice* with Five Parts of Urine, one of Sea Salt, and half a Part of Chimney Soot Boiled together, and Sublimed into a Volatile Salt, in which the Sea Salt cloſely Unites with the Volatile; the Reaſon of which is, becauſe the Contractive Force of the Sea Salt Acting towards a Center, whilſt the Expanſive of the Volatile Acts from one, they Cling and Faſten to each other, and are thereby ſtrongly Cemented together.

In order to Exalt this Salt into a Spirit, it is mixt with Quick-lime, after they have been both Powder'd apart; Water is then Poured upon it; and the Reſt is Performed by Diſtillation, and Increaſing the Fire to the Second Degree only, by Reaſon of it's Exceeding Volatility; all which Methods contribute to the Raiſing and Exciting the Expanſive Force into a Spirituous Subſtance.

Aqua Regalis is a Solution of Sal Armoniack in Spirit of Nitre; or it may be made with Equal Portions of Salt-Petre and Sal Gemme, with thrice as much Potter's Earth, Powder'd and Diſtill'd in the ſame Manner, as Spirit of Nitre; or it may be made with Common Sea Salt.

What Aqua Regalis is, which Diſſolves Gold.

And from hence is Manifeſt, the Reaſon of the following Phænomenons;

1ſt. Why Spirit of Nitre, or Aqua Fortis, which is only an Addition of the Spirit of Vitriol to it by a joint Diſtillation of both, will Diſſolve Silver and other Metals, but not Gold, and conſequently, why Spirit of Nitre will Raiſe a Precipitation into a Tranſparent Fluid.

2d. Why Aqua Regalis will Diſſolve Gold, but not Silver, and Spirit of Salt Diſſolve Leaf of Gold, which Aqua Fortis is not able to do.

3d. Why Spirit of Sal Armoniack, or Oil of Tartar *per Deliquium*, will Precipitate Gold Diſſolved by Aqua Regalis, and other Metals by Spirit of Nitre, as alſo, why Spirit of Salt will Precipitate what Aqua Fortis Diſſolves.

4th. Why in Spirit of Nitre Dulcify'd, or Mixt with Alcohol, or Spirit of Wine Dephlegm'd, tho' the one is an Acid, and the other a Sulphur, and not an Alkali, a Great Heat, and afterwards a ſtrong Ebullition, Ariſes, the ſame which is obſerved in the mixing of Oil of Vitriol, which is an Acid, and Oil of Turpentine, which is an Æthereal one.

5th. Why a Mixture of Equal Quantities of Spirit of Sal Armoniack and Spirit of Wine will cauſe a Coagulum.

6th. Why Oil of Vitriol, which is an Acid, mixt with the Acid Spirit of it or with Water, Produces many Times a Conſiderable Ebullition.

Why Spirit of Nitre or Aqua Fortis Diſſolves Silver and other Metals.

§. 10. As to the Firſt, that Spirit of Nitre or Aqua Fortis Diſſolves Silver, and other Metals, Proceeds from hence, namely, from a Great Contractive Force, as that of Salt being Excited into a very Great Expanſive one, as has been obſerved in the Diſtilling and Forming this Spirit; on the other Hand, Metals ſeem to be a Solid, Denſe, and Compacted Salt, into which they are capable of being Reſolved, that is, the Baſis of them is, as before in Spirit of Nitre, a Great Contractive Force, Compreſs'd into a Greater, with a Various Mixture of the Expanſive in a Small Degree, from whence ariſes the Difference of their Colours; or, which comes to the ſame, the Baſis of them is a Various Degree of the Expanſive Condens'd and Compreſs'd into a very Great Contractive; when therefore theſe Forces are Placed together, the Expanſive Force of the Spirit being Superior to the Contractive of the Metals, it Diſſipates and Diſſolves them and Reduces them to a Fluid, that is, to as Great an Expanſive as their Contractive Forces will admit of.

Why Aqua Fortis is a leſs Powerful Diſſolvent than Spirit of Nitre.

And hence it ſeems to be, that Aqua Fortis is a leſs Powerful Diſſolvent than Good Spirit of Nitre; becauſe Vitriol is leſs Volatile than Nitre is, and the Expanſive Force is thereby weaken'd, as Spirit of Vitriol is not ſo ſtrong as Spirit of Nitre; for the Moments of their Forces are Compounded of a Contractive and Expanſive, and if the Contractive of both were Equal, yet if the Expanſive of Nitre is Greater than that of Vitriol, it's Moment will be Proportionably Greater; and which is the Reaſon, that Oil of Vitriol, which is longer in Drawing, and Requires a more Intenſe Fire to do it, by which it's Expanſive is Increaſed, is Stronger and has a Greater Moment than the Spirit of it; From whence it is found Requiſite in making Salt or Vitriol of Mars, to moderate the Force of Oil of Vitriol by a Mixture of Spirit of Wine, which yet is Performed by Spirit of Vitriol without any ſuch Mixture.

Why Spirit of Nitre will Raiſe a Precipitation into a Tranſparent Fluid.

And for the ſame Reaſon Spirit of Nitre will raiſe a Precipitation, which has a Contractive Force, into a Tranſparent Fluid, in which the Expanſive is Superior to the Contractive, and which is the Occaſion of that Quick and Ready Tranſmiſſion of the Rays of Light thro' it, which have likewiſe an Expanſive one.

Why Spirit of Aqua Regalis Diſſolves Gold, when Spirit of Nitre cannot.

§. 11. On the Contrary, Spirit of Nitre will not Diſſolve Gold, which is a Metal of a Greater Contractive, as well as a Greater Expanſive than the Reſt, the Firſt is Evident by it's Weight and it's Compactneſs, and the Laſt, by it's Colour and it's Softneſs and Ductility; there muſt therefore be an Addition to the Moment of the Former Diſſolvent, by Diminiſhing the Expanſive, as little as can be, and Increaſing the Contractive, ſo as the Expanſive may Act with a Greater Weight and not a Leſs Energy, which is Performed by a Solution of Sal Armoniack in it, which, tho' Exceeding Volatile, is yet below the Expanſive Force of Spirit of Nitre, and by the Contractive Force of Salt, a Weight is added to the Contractive of it, or elſe by Diſtilling Sal Gemme or Common Salt together with Nitre and Potter's Earth, which will have the ſame Effect, ſince there will be the Contractive Force of Salt, as before, and the Volatility of it's Spirit by Diſtillation will be a little Inferior to that of the Spirit of Nitre; For in all theſe Caſes, the Contractive Force is in ſome Meaſure Repreſented by Weight in Bodies, and the Expanſive by the Velocity of them, the Moment of which is Greater, as the Velocity and the Weight is Greater.

AND therefore Aqua Regalis will not Diſſolve Silver, becauſe the Contractive Force being Greater and the Expanſive Leſs, than in Spirit of Nitre or Aqua Fortis, it's Action is too Heavy and Dull, that is, is not Quick and Expanſive enough to cauſe ſuch a Diſſolution.

THUS a Great Weight of Lead may be Moved by an Equal Weight of Lead, with a certain Degree of Velocity or the Expanſive Force annext to it, which the Greateſt Percuſſion of the Rays of the Sun, of Fire, of a Wand, or any other Expanſive Force, where the Contractive is Small, ſhall not be able to Stir, and this ſeems to be the Caſe of Aqua Regalis Diſſolving Gold, and the Spirit of Nitre not doing it.

ON the Contrary, a Feather, or any other Light, or Expanſive Subſtance, is more eaſily Moved with a Breath or by an Expanſive Force, which has the leaſt Degree of the Contractive, than by the Motion and Action of Lead, which has a Greater Contractive and Leſs Expanſive, and which inſtead of Moving the Feather, Oppreſſes it's Action by it's Superior Force of the Contractive, and this ſeems to be the Caſe of Spirit of Nitre or Aqua Fortis Diſſolving Silver, and Aqua Regalis not doing it.

FROM hence it is likewiſe Manifeſt, why Spirit of Salt Diſſolves Leaf of Gold, which Aqua Fortis or Spirit of Nitre is not able to do, becauſe Gold, as before, Requires a Greater Contractive and a Leſs Expanſive to Diſſolve it, and Spirit of Salt has a Greater Contractive Force than Spirit of Nitre, according to what has been already ſaid in Reſpect of each of them; and why Spirit of Salt can Diſſolve Leaf of Gold, when it cannot a Maſs of it, and which only Aqua Regalis can do, the Reaſon ſeems to be, becauſe there is not ſo Great an Expanſive Force required to Diſſolve the one as the other, as Spirit of Salt has a Leſs Degree of the Expanſive, than Spirit of Nitre mixt with Sal Armoniack has.

Why Spirit of Salt Diſſolves Leaf of Gold, which Spirit of Nitre cannot.

LASTLY, From the ſame Principles, Vinegar Diſſolves Lead, which Aqua Fortis cannot, and Aqua Fortis Quick-ſilver, which Vinegar cannot, becauſe, Lead requires a Leſſer Contractive and a Leſs Expanſive to Move it, which is found in Vinegar, and Quick-ſilver a Greater Expanſive and a Greater Contractive, which is found in Aqua Fortis; the Firſt Metal having a Leſs Expanſive from it's Softneſs, Ductility, and Dull Colour, and a Leſs Contractive from it's Weight; and the Laſt a Greater Expanſive from it's Volatility and Bright Colour, and a Greater Contractive from it's Weight.

Why Vinegar Diſſolves Lead, which Aqua Fortis cannot. Lemery's Principles of Chymiſtry, Pag. 25. Edit. 3.

§. 12. SPIRIT of Sal Armoniack Precipitates Gold, which Aqua Regalis, or Silver, *&c.* which Spirit of Nitre or Aqua Fortis Diſſolves; becauſe Spirit of Sal Armoniack has a Greater Expanſive Force and a Leſs Contractive than Aqua Regalis or Spirit of Nitre, and therefore Weakens and Enervates the Contractive of the Diſſolvents, which was neceſſary to the making an Impreſſion upon thoſe Metals, by which their Moments become too Light and Spirituous to ſupport their Diſſolutions; On the Contrary, Oil of Tartar per Deliquium, which is a Salt, that by Calcination and Fire firſt, and by the Air afterwards acquires a Dull and Languid Degree of the Expanſive, and retains a Conſiderable one of the Contractive, will cauſe the ſame Precipitations; becauſe here the Contractive is too Powerful for, and Oppreſſes, the Expanſive of the Diſſolvents, and by a contrary Force Drives to the Bottom, what the Former had Raiſed.

Why Spirit of Sal Armoniack Precipitates Gold, which Aqua Regalis, or Silver, &c. *which Spirit of Nitre or Aqua Fortis Diſſolves.*

THUS likewiſe Spirit of Salt, which obtains an Expanſive Force from it's Spirit, and a Good Degree of the Contractive from the fixtneſs of it's Salt, Precipitates what Spirit of Nitre Diſſolves, by Acting contrary to, and Loading the Expanſive Force of it, from whence the Metal ſubſides; for Spirit of Salt has a Leſs Degree of the Expanſive and a Greater of the Contractive than Spirit of Nitre, as is Evident before from the Formation and Compoſition of them.

§. 13. IN Spirit of Nitre Dulcify'd, that is, where Spirit of Nitre is mixt with Alcohol, or Spirit of Wine Dephlegm'd, tho' the one is Termed an Acid, and the other is not, what is called an Alkali, but is eſteemed a Sulphur, there is a Great Efferveſcence, and afterwards a Strong Ebullition ariſes.

Concerning Spirit of Wine and Alcohol, and why Spirit of Nitre mixt with Spirit of Wine

Dephlegmed, or Alcohol, Cauſes an Ebullition. Lemery, Chap. 19. Part 2.

But before we Endeavour at a Solution of this Phænomenon, it will be neceſſary to give an Account, by what Methods and from what Principles the Spirit of Wine is made, as we have already done in Spirit of Nitre; Wine is the Muſte or Juice of Grapes, which by Fermentation is Raiſed into a Spirituous Subſtance, and is more or leſs Groſs, according as it abounds more or leſs with Tartar, which conſiſts of Earthy and Saline Parts, which are Drawn from the Earth into the Grape; Muſte is Sweet and not in the leaſt Intoxicating, and Changes into Wine by the Expanſive Force of the Air, which Ferments it, ſo that the Contractive Force, which is that of Tartar and Salt, is Firſt Rais'd and Excited by Vegetation, by which it acquires a Degree of the Expanſive, and is the Cauſe of the Agreeable Taſt in Grapes Compounded of the Expanſive and Contractive, by the Firſt of which they are Sweet, and by the Laſt have a little Acidity and Sharpneſs; upon the Bruiſing and Preſſing of the Grapes, the Expanſive is ſomething Increas'd, and the Contractive ſtill Diminiſh'd; and laſtly, by Fermentation, in which the Air Performs the Office of Fire in the Diſtillations of more Contractive Subſtances, the Expanſive is ſtill raiſed to an Higher Degree, by which means it becomes Intoxicating and Spirituous, and the Contractive is farther Diminiſh'd by a Separation of the Tartar from it to the Sides of the Veſſel, or Precipitating to the Lees; ſo that, when it is Wine, we may conſider it, as Conſiſting of a Great Degree of the Expanſive, and of a very Small one of the Contractive.

This Expanſive Force is ſtill farther raiſed by Diſtillation and a Gentle Fire into Brandy or Aqua Vitæ, when that which is Drawn off into the Receiver becomes Inflammable, and Greater Plenty of Brandy is Extracted from Wines, which are Lighter and more Spirituous, that is, which have a Greater Degree of the Expanſive, and a Leſs of the Contractive, than from ſtronger Wines, which have a Greater Contractive Force in them; Spirit of Wine is the Diſtillation of Brandy in a Vaporous Bath, which ſhews the Exceeding Expanſiveneſs of it; and when it is Dephlegm'd, that is, cleared of all it's Contractive Force, as much as Poſſible, it becomes, what is, by the Chymiſts, called Alcohol; which therefore is a Liquor raiſed from a Tartar and Salt in the Earth, which has a Moderate Contractive Force to an Expanſive in Alcohol, which has little or none at all, for take a Spoonful of it, and throw it in the Air, and it will be immediately Diſſipated, without Falling, or Contracting into Drops, as all other Liquors do.

This therefore being the Nature of Spirit of Wine Dephlegm'd, or of Alcohol, namely, that it obtains the utmoſt Degree of the Expanſive, and the moſt Minute, which can be Imagined of the Contractive, it will not have any Moment to Diſſolve Metals, but Spirit of Nitre, which has a Great Degree of the Expanſive, and a Good Degree of the Contractive, will do it, and in the Operation cauſes an Efferveſcence.

On the Contrary, the ſame Alcohol, or Dephlegm'd Spirit of Wine, which has a Great Expanſive Force, and a Small Contractive, will have that Effect upon Spirit of Nitre, which has a Leſs Expanſive Force, and a Greater Contractive, by cauſing an Efferveſcence and Ebullition, and by Diſſolving it's Contractive Force, and Dulcifying it's Acidity, which Spirit of Nitre, which has a Greater Expanſive Force, and a Leſs Contractive than Metals, has upon them, by Producing alſo an Efferveſcence and Diſſolving their Contractive.

From whence Efferveſcence and Ebullition is cauſed in General, and what are Acid and Alkali.

From whence it follows, that Efferveſcence and Ebullition ariſes from the Conteſt, which there is betwixt Two Contrary Forces, the Expanſive and Contractive; and it is neceſſary it ſhould be ſo; for in any Fluid, where the Forces are the ſame, there is a perfect Quieſcence, as in Water, in Spirit of Nitre, Oil of Vitriol, Spirit of Sal Armoniack, *&c.* which mixt with others would occaſion the utmoſt Tumult and Diſorder; which can therefore only Proceed from the Difference and Diſparity of ſuch Forces, which Contend with each other, till the one or the other Prevails, which is the Reaſon of their Ebullition, and, whilſt ſuch a Conteſt laſts, the Medium about them is ſuitably Affected, is raiſed into a Ferment, and ſtrikes a Senſation of Heat upon the Organs, which are apply'd to it.

And Bodies do not cauſe this Efferveſcence and Ebullition, as they are Acid or Alkali, but as they have Expanſive and Contractive Forces, Acting Contrary and in Oppoſition to each other; Thus an Acid Spirit has a Great Expanſive Force, and a

Small Contractive, as Spirit of Nitre, Oil of Vitriol, *&c.* in Reſpect of Alkalis or Salts Calcined, which have a Great Contractive from the Salt, and a Small Expanſive from the Fire; And from hence it is, that what is an Acid in one Caſe, may be an Alkali in another, for what has an Expanſive Force in Reſpect of Metals, may have a Contractive in Reſpect of Liquors, which are more Spirituous than it; Thus Spirit of Nitre is an Acid, or has an Expanſive Force in Reſpect of Metals, which have a Contractive or Alkalic, but in Reſpect of Alcohol, Spirit of Nitre has a Contractive, and Alcohol an Expanſive; Spirit of Salt, in Reſpect of Leaf Gold, is Expanſive, in Reſpect of Aqua Fortis, is Contractive.

Lemery, Chap. 27. Part 2. Edit. 3.

For the ſame Reaſon it is, that Oil of Vitriol, which has a Great Contractive Force, as is Evident from it's Acidity, and from the Nature of Vitriol, which is a ſtrong Compacted Salt, and Oil of Turpentine, which is a Vegetable and Æthereal Oil, and has therefore a Great Degree of the Expanſive, both from the Nature of it, which is a Searching and Penetrating Liquor, that Diſtills in hot Countries, as in *Italy*, *Spain*, *Cyprus*, the Iſle of *Chios*, *Provence* and *Dauphine*, from ſeveral Kinds of Trees by the Inciſion of them, and from the Diſtillation and Increaſe of the Fire to Produce it; the Heat of the Sun, the Vegetation, and Fire in Diſtilling it, all, Contributing to Raiſe and Exalt the Expanſive Force of it; For the ſame Reaſon it is, that theſe Two Oils, by the Oppoſition of their Contractive and Expanſive Forces, occaſion an Efferveſcence and Ebullition; And the ſtrong Ebullition, which ariſes from Pouring Aqua Fortis or Oil of Vitriol upon an Alkali Salt, Proceeds from no other Cauſe, the Oil of Vitriol being the ſame or a like Force to the Alkali Salt, as the Oil of Turpentine is to the Oil of Vitriol; the Former, namely, Oil of Vitriol and Oil of Turpentine, in their ſeveral Reſpects to the Latter, that is, Alkali and Oil of Vitriol, having a Greater Expanſive Force and a Leſs Contractive, and the Latter a Greater Contractive and Leſs Expanſive.

Lemery, Chap. 18. Part 1. Pag. 419, 421.

Thus alſo Spirit of Vitriol mixt with Oil of Vitriol, or Oil of Vitriol with Water, will cauſe a conſiderable Ebullition, for Oil of Vitriol has a Greater Contractive and a Leſs Expanſive than the Spirit, it being Confeſs'd by Chymiſts, that the one is the more fixt Part of the other; Spirit of Vitriol, therefore in this Caſe, will Perform what Oil of Turpentine did in the Former, and having a Greater Expanſive and a Leſs Contractive, than the Oil, will raiſe it into an Ebullition, as Fire does Water.

On the Contrary, Oil of Vitriol, albeit it has a good Degree of the Contractive, yet having a Vaſt Expanſive Force likewiſe, tho' Inferior to that of it's Spirit, Expands it ſelf in Water, which has a Small Expanſive Force, and a Greater Contractive, and therefore Oil of Vitriol is to Water, as Spirit of Vitriol to the Oil; Juſt, as before, Oil of Vitriol was to the Alkali Salt, as Spirit of Turpentine to Oil of Vitriol.

Oil of Vitriol is a Fluid Fire or a Burning Fluid, becauſe it's Expanſive Force is exceeding Great, and it's Contractive very Conſiderable, by which there is a Weight and Moment added to the Expanſive, which makes it approach to the Nature of Fire: By the ſame way, all other Effervеſcences and Ebullitions may be Expounded, we ſhall therefore now only add ſomething concerning Fire and Flame.

Concerning Fire and Flame Producible in Chymiſtry. Sir If. Newt. Optic. Quer. 31.

§. 14. If Two Parts of a Spirit, Drawn and Compounded from Oil of Vitriol and Nitre, are Poured upon one Part of Oil of Cloves, or Caraway Seeds, or any Ponderous Oil of Vegetable or Animal Subſtances, or Oil of Turpentine thickened with a little Balſam of Sulphur, there will immediately be Produced a Burning Flame, the Reaſon of which is the ſame, as that of the foregoing Phænomenons,

The Expanſive Force of Oil of Vitriol is Loaded with too Great a Degree of the Contractive, which Force is therefore corrected by Nitre, which is more Volatile than Vitriol, and made ſtill more Expanſive by it's Diſtillation with the Oil of it.

On the Contrary, Spirit of Nitre would have too Great an Expanſive Force, by which it is capable in Conjunction with it's Contractive to Diſſolve Metals, a Middle Proportion of both theſe Forces therefore muſt be taken to Diſſolve the Oils above-mentioned.

The Oils muſt be Ponderous, and Oil of Turpentine Thicken'd with Balſam of Sulphur, that they may have a certain Degree of the Contractive to be ſurmounted by that of the Vitriol Actuated by the Expanſive, they muſt be the Oils of Vegetable or Animal Subſtances, that they may have a certain Degree of the Expanſive, by which they are diſpoſed to riſe into Flame by the Action and Force of the Compound Spirit of Nitre.

Lemery, Chap. 3. Part 1. Pag. 110.

For the ſame Reaſon it is, that Tin Powder'd, by the Extreme Force of Fire in Calcining it, acquires a Degree of the Expanſive, which becomes Inflammable by Mixing a Diſſolution of Silver, Precipitate with Salt Water, and Dry'd; the Spirit of Nitre, made leſs Expanſive by the Diſſolution, obtaining the Effect of the Compound Spirit before, and the Powder'd Tin, that of the Ponderous and Vegetable Oil.

Thus alſo Common Fire and Flame Proceeds from the Expanſive Force of Fire and Air, which has a Degree of the Contractive in Reſpect of the Former, Acting upon the Sulphureous or Expanſive Nature of the Fuel, and it's Contractive Force, by which ſuch Bodies become Solid and Heavy; Hence all Vegetables, Wood and other the like Materials Dry'd, Coals and Sulphureous Bodies, are Inflammable in Various Degrees, from their Expanſive Force, but Stone, Clay, Earth, and Metals, not ſo, by their being too Contractive; Hence likewiſe, Oils from their Expanſive Force are Inflammant, but Salts from their Contractive do not Admit of that Property.

Why a Veſſel of Boiling Water ſhall be Cool at the Bottom and Hot on the Sides.

It has been Eſteemed a Problem in Nature hard to be Solved, tho' what is Juſtified by Common Experience, why in a Veſſel full of Water, ſet over a Fire, the Water ſhould Boil, when the Bottom of the Veſſel is, in Compariſon, Cool, and yet the Sides of it ſhall be Extremely Hot; The Reaſon of which is, becauſe the Expanſive Force of the Fire, not being able to ſubdue the Contractive of the Metal, Paſſes to the Contractive of the Water, betwixt which and the Fire the Conteſt therefore lies, and is raiſed into an Ebullition by it; but as the Expanſive Force Aſcends by the Sides of the Veſſel, the Conteſt will be betwixt the Fire and the Contractive Force of the Metal, from whence the Intenſe Heat of it is cauſed; when the Veſſel is taken off the Fire, the Bottom of it grows Hot by the Expanſive Force, which now Prevails in the Water, Returning upon the Contractive of the Metal, betwixt which Forces now the Conteſt lies.

Air, which in Reſpect of Fire, has a Contractive Force, and which being the Greateſt in Cold and Froſty Weather, makes Fire burn more Fiercely, in Reſpect of Water, has an Expanſive, by which it Ferments and Actuates it, and in certain Caſes Raiſes it into Vegetation.

Concerning Air or Æther, and the Cauſe of it's Elaſticity and it's Effects in Vegetation.

§. 15. Air or Æther is a Medium known to be Elaſtick, but whence that Elaſticity is Produced, has been little Explained, which Probably Ariſes from it's Expanſive Force, Mixt in a due Proportion with it's Contractive; the Firſt being chiefly Impreſs'd upon it by the Sun's Expanſive Force, and the Moon's Contractive, which Acts in Conjunction with it, and in Oppoſition to the Earth, as has been ſhewn in Tides; and the Laſt from the Earth's Contractive, which Acts contrary to the other; by which means the Air is a Medium Stretch'd and Pulled betwixt Two Oppoſite and Contranitent Forces, from whence it becomes Elaſtick, like a String or Wire under Tenſion from Forces Acting a Different way at the Two Extremities of it.

Whence the Colours of Natural Bodies, and how Drops of Water becomes Spherical.

This Elaſtickneſs of Air or Æther is the Cauſe of it's being ſuſceptible of the leaſt Percuſſions from the Expanſive Force of Bodies, by which we ſee and are made ſenſible of the Various Colours of them; From the ſame Elaſtick and Expanſive Force, Exerting it ſelf Equably on Water dropt in Air, and from the Contractive Force of the Water, by which it's Parts are Equably Drawn to each other, it is, that the Figures of theſe Drops are Spherical.

Whence Plants and Trees have their Cylindrical Form, and Circular or Oval Foliage.

Plants and Trees, likewiſe in General, have a Cylindrical Form upon the like Account, the proper Expanſive Force in Vegetation, diſplaying it ſelf Round to a certain Degree, is bounded by the Contrary Expanſive or Contractive of the Air, which Acting Equably on all Hands, keeps the Vegetating Force within the Circular Limits it Preſcribes, excepting where it Shoots upwards in the Stock, or downwards from the Root by a Superiority of Force, by which means the Cylinder is Framed; when the main Expanſive Force is ſpent, and the Trunk can riſe no Higher, it then Shoots out into Lateral Directions and Branches, which for the Reaſon above are ſtill Cylindrical, but when theſe are Formed, and the Expanſive is Enervated and Weaken'd, it ſinks into Flat Leaves, which yet are generally Circular, or Approaching

ing to that Figure by the ſame proper Expanſive Force of Vegetation, and the contrary Expanſive of the Air Acting Equably upon it.

See alſo Chap. 5. §. 20. Book 1.

The laſt Effort in Vegetation is the Expanſive Force ſtriking out into ſharp Points, which is the Cauſe, that moſt Leaves are ſurrounded with little Indentations, and which Proceed from the final Exertion of the Expanſive, in the Plant or the Tree, and the Renitency of the Air's Contractive, or Contrary Expanſive; I have mentioned thus much of Air here, as Eſteeming it amongſt the Spirits, of which I am now Treating, tho' the moſt Moderate and Gentle, and the beſt Adapted to all the uſes of Animal Life and Vegetation, of any other Spirit whatſoever.

Why Equal Quantities of Spirit of Sal Armoniack and Spirit of Wine will Cauſe a Coagulum.

§. 16. **Equal** Quantities of Spirit of Sal Armoniack and Spirit of Wine will cauſe a Coagulum, becauſe the Moments of their Forces are Equally Balanced and Contrary to each other; From whence they will Cling together, and mutually Cohere, and Produce a Poſitive or Active Reſt, or Union of their Forces; Spirit of Wine mixt with Spirit of Nitre Cauſes an Ebullition and Diſſolves it's Acidity, as has been before ſhewn, that is, the Expanſive Force of the Spirit of Wine is Superior to the Contractive of Spirit of Nitre; but Spirit of Sal Armoniack is more Volatile and Expanſive than Spirit of Nitre, and therefore comes nearer to the Expanſive Force of Spirit of Wine, and their Forces are Contrary, becauſe Spirit of Wine is an Oleous one, that is, almoſt entirely Conſiſts of an Expanſive, but Spirit of Sal Armoniack has from it's Salts a Contractive; And for the ſame Reaſon, Oil mixt with a Saline Liquor in a Mortar will form an Unguent called Nutritum, which can Proceed from nothing but the Expanſive of the Oil, which is Inflammable, Acting contrary to the Contractive of the Salt, which Decrepitates in Fire, with nearly Equal Forces, by which they are United and Cemented together in a cloſer Coheſion; And this may give an Account of the Conſiſtency of all Bodies, which ſeem to be little more than ſtronger Coagulums, ariſing from the Mutual and perhaps nearly Equal Actions of the Expanſive and Contractive, whoſe Forces are yet much Greater than thoſe here alledged of Wine and Sal Armoniack.

Lemery, Chap. 17. Pag. 399.

Lemery, Chap. 18. Part 1. Pag. 421.

From hence alſo it is Evident, why Spirit of Vitriol, remaining ſome Time upon Iron, reincorporates into Vitriol, and Spirit of Nitre poured upon Salt of Tartar returns into Nitre; Green Engliſh Vitriol, out of which the Spirit is formed, is ſaid to partake of Iron, and Mars or Iron may be Reduced to a Green Vitriol, by Diſſolving it in the Spirit, Filtrating and Evaporating the Solution to a Skin, and ſetting it to Cool; As therefore the Contractive Force of Iron may be Exalted into a Green Tranſparent Chryſtal by the Expanſive Force of the Spirit, when made to Evaporate and Set to Cool, ſo it is no wonder, that the ſame Expanſive Force of the Spirit, remaining for ſome Time upon the Iron, by which means it is Evaporated and Weakened, ſhould be changed into Vitriol by the Contractive of the Iron, and ſo likewiſe, that Spirit of Nitre poured upon Salt of Tartar ſhould by the Contractive Force of the Latter, and the Evaporation of the Expanſive of the Former, Coagulate into Nitre; ſince theſe, as thoſe others before of Spirit of Wine and Spirit of Sal Armoniack, ſeem to Act with nearly Equal and Contrary Forces.

Concerning the Depart, *by the Diſſolvent's quitting one Metal, which Precipitates, and betaking it ſelf to another, with the Reaſon of it.*

§. 17. **There** is one Phænomenon more, which I ſhall take Notice of in Reſpect of the Spirits in Chymiſtry, which is that of the *Depart*, which is the Parting of Metals, by the Diſſolvent's quitting one Metal, which Precipitates to the Bottom, and betaking it ſelf to another. Thus when Aqua Fortis has Diſſolved Silver, and Copper is put into the Diſſolution, the Silver Precipitates, and the Copper is Diſſolved; Iron Precipitates Copper, Lapis Calaminaris Iron, and the Liquor of fixt Nitre Precipitates Lapis Calaminaris.

Lemery, Chap. 2. Pag. 92, 93. Part 1.

The Reaſon of which ſeems to be this, that Silver requires all the Expanſive and Contractive Force of Aqua Fortis or Spirit of Nitre to Diſſolve it; Copper, by an Addition of it's Contractive Force, Acts againſt the Expanſive of the Spirit of Nitre, by which it is ſo far weakened as to be no longer able to ſupport the Silver, which therefore ſubſides, but the Expanſive thus weakened and retaining ſomething of the Silver, which it has Diſſolved, is ſufficient to ſubdue the Leſs Contractive Force of the Copper; and thus we find even Diſtill'd Vinegar, which has a much leſs Expanſive and Contractive Force than Spirit of Nitre, able to Diſſolve Verdegreeſe, which is only a Copper opened and reduced to a Ruſt by the Fermenting Spirits of Tartar; what therefore Diſtilled Vinegar can Perform upon Copper opened, it is reaſonable to think, that Spirit of Nitre weakened in its Expanſive Force, but increaſed in it's Moment of the Contractive by the Silver, which it ſupports, may Perform in Reſpect of Copper, which is crude and unopened, and not ſo fully prepared for a Solution.

For the ſame Reaſon, Iron, whoſe Specifick Gravity or Contractive Force is to Copper, as ſomething more than 7 to 9, as Copper is to Silver as about 9 to 11, will Precipitate Copper, whilſt the Spirit of Nitre will ſtill obtain a ſufficient Expanſive Force to Diſſolve the Former; and the ſame may be ſaid of Lapis Calaminaris, and the Liquor of fixt Nitre, which Decreaſe in their Contractive Forces from the foregoing, as the Expanſive Force of the Spirit of Nitre or Aqua Fortis is Impaired by the Solutions.

Concerning the Phlegm, and the Oil of Chymiſts.

§. 18. Having therefore gone thro' what we intended in Relation to the Spirits in Chymiſtry, we ſhall now Proceed to the next Active Principles in it, which are the Phlegm and the Oil, and which are only ſtill Leſſer Degrees of the Expanſive Formed and Raiſed by the Fire, and are more or leſs Expanſive, according to the Greater or Leſs Expanſive Nature of the Subſtances, from which they are Extracted; Phlegm, as being little more than a Fluid, has but a Small Degree of the Expanſive, and Oil, where it is Inflammable, has a Greater; and where it is not, has a Greater Degree of the Contractive, as in Oil of Vitriol, which is not Raiſed but by a very Violent Force of the Expanſive of the Fire, concerning which we have already ſpoken.

The Principal Difference betwixt Spirit, Phlegm, and Oil.

The Principal Difference betwixt the Spirits, Phlegm, and Oil in Chymiſtry, Abſtracting from the Natures, from which they are Drawn, which would diverſify them to Infinity, ſeems to be in General this, that Spirit acquires the Greateſt Degree of the Expanſive by the Operation of the Fire, and which it is capable of Exerting upon other Bodies, to which it is Applied; Oil has a Leſs Degree of the Expanſive, and which is always joined with a Greater Degree of the Contractive, from whence it becomes a Thicker and more Glutinous Subſtance than the Spirit, and therefore is not ſo capable of Exerting it's Action upon other Bodies, but yet is Prepared, by this Mixture of the Expanſive with the Contractive, to obey the Expanſive Force of Fire, and to riſe into a Flame.

Phlegm acquires but a Small Portion of the Expanſive, and has but little of the Contractive, being nothing elſe than the Moiſture of Bodies, which ſits looſe in them without partaking of the Real Qualities of them, and therefore eaſily comes forth in Vapours, and differs not much from Common Water, Excepting that it receives ſome Small Impreſſions from the Active Principles, and has thereby a more Deterſive, that is, Expanſive Vertue than Common Water; which is alſo the Caſe of Rain and Dew, being Impregnated with, and made more Expanſive by, the Spirit of the Air; we ſhall therefore Say nothing farther of the Phlegm propoſed to us by Chymiſts, but Proceed more particularly to Specify their Oils.

Lemery, Pag. 163.

The ſeveral Oils of the Chymiſts Conſidered.

Lemery, Pag. 4. 7. 11. 19. Edit. 3.

§. 19. Oil or Sulphur is ſaid to be a Sweet, Subtile, and Unctuous Subſtance, which riſes after the Spirit, and is Inflammable; which Character exactly Agrees with the Forces we before Aſſigned it, that it has a Leſs Expanſive and a Greater Contractive, than that which belongs to Spirit, tho', like Spirits, it muſt be Different, according as the Nature, from which it is Extracted, is more or leſs Expanſive or Contractive.

Why Oil of Roſemary or Lavender will Swim upon Water, whilſt Oil of Box, Guaiacum, or Cloves, Sinks to the Bottom.

Thus Oil of Roſemary or Lavender will Swim upon Water, whilſt Oil of Box, Guaiacum, or Cloves, Precipitates to the Bottom; which is not a Proof, as Chymiſts Imagine, that theſe Oils are drawn Impure, and that the Former are mixt with Spirits, and that the Latter are filled with Salts; but that Roſemary and Lavender have a more Expanſive and Volatile Nature, which is Evident from the Briskneſs, and Liveline ſs of their Smell, and that Box, Guaiacum, and Cloves, have a more Contractive, which likewiſe may be eaſily acknowledged from their Fainter and Duller ones; and therefore the Oils of the one will be Lighter and more Expanſive, and thoſe of the other more Contractive and Heavier.

Why ſome Oils are Drawn by Inciſion, others

Hence it is alſo, that ſome Oils may be drawn by Inciſion only into the Root or Trunk of Trees, as that of Turpentine; others by Beating and Preſſing the Fruits and Seeds, which are Contained in Them, as thoſe of Olives, Almonds, and Nuts;

whilſt others are not Extracted without the Greateſt Force of Fire, as Oil of Vitriol; the Firſt ſeems to have the Greateſt Degree of the Expanſive, the Second a ſtill Leſs, and the Laſt, tho' it has a Great Degree of the Expanſive, yet is joined with a Great Degree of the Contractive.

by Expreſſion, and a Third by Fire.

That Oil is Inflammable not from Salts, but from the Expanſive Force, which thoſe Salts acquire.

§. 20. It is Affirmed by Chymiſts, that nothing but Oil can properly be ſaid to be Inflammable, and that the Oil is ſo much the more ſo, as the Salts, with which it is cloſely United, have been more or leſs Spiritualized, as if the Salts contributed to their Inflammation.

Whereas, as we Eſteem Salt to be of a Contractive Nature, and Flame to be of an Expanſive one, it will be hard to conceive, how the one ſhould be Aſſiſtant to the other; eſpecially, when it is acknowledged, that the more Volatile and Expanſive the Salts are, which are mixt with the Oil, the readier it is to be Imflamed; and on the Contrary, that fixt Salt entirely Hinders and Obſtructs Inflammation, as is Evident from Sea Salt thrown into the Fire, which ſerves not to Raiſe and Excite, but to Extinguiſh it.

It muſt therefore be ſaid, that Spirit of Wine, which is reckoned to be an Oleous one, and which muſt conſequently be the moſt Perfect Oil, which is Poſſible, is only Salt, or the Contractive Force, Exalted to the Higheſt Degree of the Expanſive; for that all Bodies have Salt, or a Contractive Force, has been ſhewn, and that this Contractive Force is raiſed into the utmoſt Expanſion in Alcohol or Spirit of Wine Dephlegm'd; any Oil therefore is nothing more than Salt Volatilized, or the Contractive Force Actuated with a Great Degree of the Expanſive; From whence it Enſues, that it is not the Mixture of Oil with Nitre or any other Volatile Salt, which is the Occaſion of Inflammability, but the Raiſing a Salt, or a Contractive Force, into a ſufficient Degree of Expanſiveneſs to be Acted upon by Fire and thrown into a Flame.

If therefore Salt Petre mixt with any Oily Subſtance makes ſuch Oil more eaſily Inflamed, whereas Nitre is not it ſelf Inflammable, it Proceeds from hence, that the Contractive of the Oily Subſtance, or the Salt of it, is more fixt than that of Nitre, and which, when mixt with it, abates that Contractive Force, and aſſiſts the Expanſive of the Oil; ſo that if it were Poſſible to draw the more Spirituous Part from ſuch Oil, the Salt or Contractive Force remaining would be leſs Inflammable than Nitre it ſelf, which is in ſome meaſure Evident from the Dull Salts or Earths, which are left after ſuch Oily Extractions are made. Or elſe this muſt be from the Contractive of the Salt, Exciting the Expanſive of Flame by Oppoſition, as Water or Air does Fire.

Oils of whatever Sort, only Spirits with a more Contractive Force belonging to them, and the Nature of them Explained. Lemery, Pag. 19, 12.

§. 21. It is ſaid, that the Oil of Vegetables is the Fat and Unctuous Part of Earth mixing and inſinuating it ſelf with Salt, and that many Things are improperly called Oils, as Oil of Tartar per Deliquium, Oil of Vitriol, and that of Antimony.

As to the Firſt, it may be more truly ſaid, that Vegetable Oil is a more Phlegmatick, that is, a more Contractive Burning Spirit of Vegetable, which has a Leſs Degree of the Expanſive than the Former, but is much of the ſame Nature; And as to the Second, that Oil of Vitriol, and ſo of the Reſt, may be as juſtly Termed an Oil, as the Spirit of it may be acknowledged to be ſuch, ſince it is a more Contractive Spirit only of the ſame Mineral; that is, we cannot but think, that as Spirits are either thoſe of Animals, Vegetables, or Minerals, and accordingly are either Volatile, Burning, or Acid, or have a Middle Expanſive and Contractive Force belonging to them, as in the Firſt Caſe, or a Greater Expanſive, with a Little Contractive, as in the Second, or a Leſs Expanſive and a Greater Contractive, as in the Third, ſo it is ſuitably in the Oils of them, tho' the Expanſive Force in their Oils is Leſs, and the Contractive Greater, than in their Spirits; and conſequently theſe Oils are ſtill Salts, or the Contractive Force, not raiſed to ſo high a Degree in Them, as in the Spirits; and there may be Volatile, Burning, and Acid Oils, as well as Spirits.

The Oils of each ſort alledged in Chymiſtry.

To Confirm what I ſay, I ſhall only Inſtance in ſome of the Oils of each Sort alledged in Chymiſtry; in Reſpect of the Oil of Minerals, I ſhall take thoſe of *Mars*, *Saturn*, *Venus*, Antimony, and Vitriol; of Vegetables, thoſe of Tartar, Cloves, Turpentine, and Cinnamon; of Animals, thoſe of Vipers and Hartſhorn; in the Firſt of which we ſhall find ſome Marks, or other, of a Greater Contractive Force, than what is Experienc'd in the Oils of Vegetables or of Animals, by their Vertue

Oil of Mars. Lemery, Pag. 174. *Oil of* Saturn. Pag. 136. *Oil of* Venus. Pag. 147. *Oil of Vitriol.* Pag. 425.

of Removing Obſtructions, which is the Caſe of the Oil of *Mars*, which Fire and Expanſive Forces cannot do; or by Cleanſing and Cicatrizing of Ulcers, and Reſiſting Putrefaction or Expanſion, which is the Caſe of Balſam or Oil of *Saturn*, and of the Liquor or Oil of *Venus*, of the Cauſtick Oil of Antimony, or that Prepared with the Acid Spirit, and Oil of Sugar, tho' it's Effect is in a leſs Degree than the Former; or by their Harſhneſs, Acidity, and Weight, and their Styptick Quality, which is the Caſe of the Oil of Vitriol, and of all the others foregoing in Different Degrees, according to the Different Nature of the Bodies, from which they are Drawn, and the Methods of Drawing them; theſe therefore may be properly enough Termed Oils, which have a more Contractive Nature, and are not Raiſed into ſo High a Degree of the Expanſive Force, as the Oils of Vegetables or of Animals, as the Spirits of Minerals are not ſo Expanſive, as thoſe of Vegetables or of Animals: The Oils of Animals and Vegetables are leſs Contractive, and more Expanſive, than thoſe of Minerals, Evident Marks and Characters of which we may obſerve from their Smells, which are Greater and Stronger than thoſe of Minerals, and which ſhews they have a Leſs Contractive and a Greater Expanſive Force, from their Vertues, that is, their being Good to Smell to in Hyſterical Vapours, which Proceed from too Great a Contractive Force or Melancholy in the Mind, and from the ſame Exceſs of the Contractive Force in the Humours and Animal Spirits of the Body, as alſo their being Good to rub Paralytick Parts with, and for Cold Pains, both which Diſtempers Proceed from too Great a Contractive Force in the Animal Syſtem; And this is the Caſe of Fœtid Oil of Tartar by Diſtillation, the Spirit alſo of which is approved in the Palſie, Aſthma, and Scurvy, is likewiſe uſed in Hyſterical Maladies, and for the Epilepſie, which are all of them Diſorders, that Ariſe from an Exceſs of the Contractive Force, and it is ſaid to work by Urine and by Sweat, which are manifeſt Tokens of the Remedy having the Nature of the Expanſive, and that the Oil is of the ſame kind of Force with the Spirit, tho' not ſo Expanſive.

Lemery, Pag. 592. Edit. 3.

Oil of Tartar, per Deliquium. Lemery, Pag. 594.

THE Oil of Tartar per Deliquium, by it's Compoſition and Formation, being only the fixt Salt of Tartar Calcined and made more Expanſive by Fire, and then Diluted by the Air, muſt be leſs Expanſive and more Contractive than the Fœtid Oil by Diſtillation; and therefore, tho' the Vertue of this Oil is Aperitive, and conſequently Expanſive, yet it is given for Obſtructions, as the Oil of *Mars* is, becauſe of the Good Degree of the Contractive, which is joined with the Small Expanſive Force of it.

FROM this Account of the Oil of Tartar per Deliquium is Confirmed, what we before mentioned concerning it, which was, that it acquired a Dull and Languid Degree of the Expanſive, from it's Calcination, and retained a Conſiderable one of the Contractive, from it's Salt.

Oil of Cloves. Lemery, Pag. 495.

THE Oil of Cloves is ſtill of a more Expanſive and a leſs Contractive Nature than that of Tartar, which is Evident from the Firſt being Odoriferous, and the Laſt Fœtid; ſo Spirit of Wine is more Expanſive than Spirit of Nitre and Vitriol, the Former of which is Agreeable both to the Taſt and Smell, the Latter very Abominable and Offenſive; It is alſo too Expanſive and too little Contractive to remove Groſſer or more Heavy Obſtructions, which Oil of Tartar can do, but is ſufficient to Diſſipate the Cold Humours in the Tooth-Ach, and is ſaid to be Good in Malignant Fevers and the Plague, which therefore perhaps originally Proceed from a Contractive Force prevailing in the Blood, as That is the Reaſon, in Intermitting ones, of the Cold Paroxyſms obſervable in them; This Oil is alſo Drawn with a ſmall Degree of Fire, which ſhews the Expanſiveneſs of it, and is Inflammable by a Compound Spirit of Nitre.

Oil of Turpentine. Lemery, Pag. 640.

OIL of Turpentine Diſtills from an Inciſion only into ſeveral ſorts of Trees, which grow in hot Countries, and is of the Conſiſtency of a Balſam, in Winter, and of an Oil in Summer, it is Diuretick, Good for the Stone, the Nephritick Cholick, the Retention of Urine, *&c.* has a Biteing Taſt, and is Inflammable by a Compound Spirit of Nitre; all which ſhew, that it has a Greater Expanſive and a Leſs Contractive than Oil of Tartar, and, if not a Leſs Expanſive, a Greater Contractive than Oil of Cloves, it being more Diſagreeable to the Taſt, and more Capable of Removing Greater Obſtructions, and more Thick, and Conſiſtent, and Ponderous.

Oil of Cinnamon. Lemery, Pag. 484.

Oil of Cinnamon is more Expanſive and leſs Contractive than Oil of Turpentine or Tartar, it is more Odoriferous, and is of no uſe in Obſtructions, only ſtrengthens the Stomach, and aſſiſts Nature in her Evacuations, which ſhews it has not ſo Great a Moment as the others; the ſame may be ſaid of the Oils of Lavender, Roſes, and Orange Flowers, of Box, Juniper, and Anis-ſeed, All which, with that of Cloves before, either Swim upon the Water or Precipitate to the Bottom, as they have a Leſs or Greater Contractive Force belonging to them.

Oil of Vipers. Lemery, Pag. 673.

Oil of Vipers is almoſt of the ſame Nature by it's Vertues with the Fœtid Oil of Tartar, it is ſaid to be Good in the Epilepſie, Palſie, and Hyſterical Caſes, and it's Smell is ſo Offenſive as ſcarcely to be Endured, and it ſo far, in ſome Meaſure, Agrees with the Contractive Force of the Oil of Tartar; but then it is more Expanſive by the Volatile Salts, which Vipers contain, none of which are to be found in Tartar, and therefore it is alſo uſed in Malignant Fevers and Agues, and the Bitings of Venemous Creatures, which Proceed from too Great a Contractive Force infuſed into the Blood, and mixing with it, and which Oil of Tartar has too Little an Expanſive Force to Diſſolve, tho' it is capable of Diſſolving a Greater, as in the Epilepſie and Palſie; becauſe it's Expanſive Force joined with the Contractive, which is Greater in Tartar than in Vipers, has a Greater and Stronger Moment than the Expanſive of Vipers joined with it's Contractive; and it is very Reaſonable to think this is ſo, ſince the Contractive Force in Animals, and eſpecially in ſuch a Fiery and Spirituous one as the Viper is, muſt be thought to be much Leſs from the continual Circulations of the Humours in it, than that of a fixt Salt agitated only by a Temporary Fire in Calcineing and Diſtillation.

Oil of Hartſhorn. Lemery, Pag. 735.

Oil of Hartſhorn, as well as the Volatile Salt and Spirit of it, is made in the ſame Manner as that of Vipers, it is more Expanſive and leſs Contractive, and therefore is not ſo Powerful and Momentous a Remedy as the other: From all which has been ſaid concerning Oils, it ſeems to be Evident, that they are of the ſame Nature with the Spirits of the Bodies, from which they are Extracted, only are more Contractive and leſs Expanſive than they.

Concerning the Salts of Chymiſts, with the Concluſion.

§. 22. The Firſt Contractive Principle in Chymiſtry, and the only one, I take to be Salt; and as Spirit Principally conſiſts of the Expanſive, and as Oil is a Duller Spirit, or Expanſive Force, ſo Earth is nothing, but a Duller Salt or Contractive, whoſe Force is Weakened and Impaired by the Expanſive of the Fire, as Common Earth is a Salt, whoſe Contractive Force is Diminiſhed by the continual Action of the Rays of the Sun; Salt in General is Defined by Chymiſts to be a Principle, which remains Diſguiſed in the Earth after the other Principles are Extracted, that it is a Fixt and Incombuſtible Subſtance, which gives Bodies their Conſiſtence, and preſerves them from Corruption; all which are Evident Tokens of a Force Acting contrary to an Expanſive one, which I all along call a Contractive.

Lemery, Pag. 4. Edit. 3.

That it cauſes a Diverſity of Taſts, according as it is Differently mixt, is not ſo Evident, but on the Contrary, that all Taſts Proceed from the Expanſive Force, and that the Taſts of Salts themſelves are Derived from their having a certain Degree of the Expanſive, tho' extremely Small, in Reſpect of their Contractive; for Metals, which have ſtill a Greater Contractive Force than Salts, have little or no Taſt at all, and if the Contractive Force of Salts is raiſed into a Higher Expanſive by Fire and Diſtillation, their Taſt is ſtill Stronger; thus alſo, Muſte has a weaker Taſt than Wine, and that, than Brandy, which are only ſeveral Degrees of Exalting Salt or Earth, into the Expanſive, by Vegetation, Fermentation, and Diſtilling with Fire.

Salts are affirmed to be of Three Sorts, the Fixt, the Volatile, and Eſſential; The Fixt are Evidently thoſe of a Contractive Force almoſt Entirely; Volatile are the Fixt raiſed into a Degree of the Expanſive by Fire; and Eſſential are ſuch likewiſe raiſed into a certain Degree of the Expanſive by Vegetation; It was our Intention to have Examined particularly into each of theſe, in the ſame Manner, as we have done into the Spirits and Oils of the Chymiſts; as alſo into Calcination, Chryſtallization, Precipitation, Coagulation, Condenſation, Diſſolution, Rarefaction, Pulverization, Inflammation, Diſtillation, Sublimation, Digeſtion, Extraction, and Colour; and we have ſome Papers written by us Concerning ſeveral of them; we would alſo have Inquired into the Particular Conſtitution of Gold, Silver, Tin, Copper, Lead, Iron, and Mercury, into Minerals, Vegetables, and Animals, according to the Chymical way of Analyzing them; but as the Sheets would Riſe to an Expence too Great, and the Encouragement which is Given to my Proceeding in theſe Matters is too Small, and as I have Propoſed my Notions in General in this Science, which may be eaſily Purſued thro' all the Particulars Above-mentioned by a Diligent and Laborious Hand, I ſhall Excuſe my ſelf any farther Trouble on this Head, and Leaving what Remains to thoſe, who are Willing to take the Pains I have done, Go on to another Science we Propoſed to Treat of, which was Anatomy.

CHAP. III.

Concerning the Phænomenons in Anatomy, and Firſt of the Common and Proper Containing Parts of the Abdomen, *the* Cuticula, *the* Reticulum Mucoſum, *the* Cutis, *the* Membrana Adipoſa *and* Carnoſa, *the* Communis Membrana Muſculorum, *the* Peritonæum *and* Omentum.

Anatomy Diſtinguiſh'd into Enterology, Myology, and Oſteology.
Mr. Rolfe's Syllabus Enterologiæ & Muſculorum.
Dr. Haver's Oſteologia Nova.
The Common Containing Parts of the Abdomen.
Mr. Rolfe's Syllabus Enterologiæ, Prælect. 1.
Dr. Gibſon's Anatomy, Chap. 3. Book 1.
Concerning the Cuticula *with a full Account of it from the Anatomiſts.*
Mr. Rolfe's Syllabus Enterologiæ, Prælect. 1.
Dr. Gibſon's Anatomy, Chap. 3. Book 1.
Dr. Keill's Chap. 1. §. 3.
Dr. Drake's Anthropologia Nova, Chap. 2. Book 1.
Blancard's Anatomia Reformata, Cap. 34. Pag. 742.

§. 1. ANATOMY may be Diſtinguiſhed into Three Parts, Enterology, Myology, and Oſteology, to each of which we ſhall Aſſign ſome Chapters, and in our Treating of Them, we ſhall Purſue the ſeveral Parts of an Human Body, according to the Order of Diſſection, and Endeavour to Explain them by our Doctrine of the Expanſive and Contractive Forces.

§. 2. THE *Abdomen* is Firſt Diſſected, as containing the Entrails, which are the moſt liable to Putrefaction and Corruption, the Integuments or Coverings of which, as alſo of the whole Body, or moſt of the Parts of it, are the Ἐπιδέρμις, the *Cuticula*, or Scarf-Skin, the *Parenchyma*, or *Reticulum Mucoſum*, the Δέρμα, or *Cutis*, the *Membrana Adipoſa* and *Carnoſa*, and the Common Membrane of the Muſcles.

§. 3. THE *Epidermis* or *Cuticula* Inveſts the whole Body, and is ſaid to be Raiſed by an hot Iron from the Heel, where it is the Thickeſt, and to be made by the *Reticulum Mucoſum*, or by a Viſcous and Oleous Vapour of the Blood, Condenſed by the External Air or Cold, or by a Tranſparent Juice Iſſuing from the Capillary Nerves, and Coagulating at the Extremities of them; Others will have it Proceed from a Seminal Principle, becauſe it is found on Infants New-Born, which cannot be owing to a Condenſation from Cold; It is more Compact than the Skin, becauſe the Serous Humour, which Paſſes thro' the Laſt is ſtopt by the Firſt, as in Puſtules and Bliſters; It is yet full of Pores for Sweat and Perſpiration, wants Nerves and Blood Veſſels, and therefore is void of Senſe, can hardly be Separated from the Skin by a Knife, but may eaſily by a Veſicatory or Cautery; It Peels off in Scarlet or Burning Fevers, and the Small Pox, but a new one Preſently ſucceeds it; It appears by Microſcopes to be made up of ſeveral Lays of Exceeding Small Scales, which Cover one another, more or leſs, according to the Different Thickneſs of the *Cuticula* in the ſeveral Parts of the Body, and in the Lips, where the Scales appear Plaineſt, becauſe the Skin is Thinneſt, they only in a Manner touch one another; Theſe Scales are ſuppoſed to be either the Excretory Ducts of the Glands of the Skin, as is thought to be Evident in Fiſhes, or elſe theſe Glands have their Pipes opening between the Scales; *Lewenhoeck* reckons, that in one Cuticular Scale, there may be Five Hundred Excretory Channels, and that a Grain of Sand will Cover Two Hundred and Fifty Scales, ſo that one Grain of Sand will Cover One Hundred Twenty Five Thouſand Orifices, thro' which we daily Perſpire; He thinks the *Cuticula* to be an Expanſion of the ſaid Excretory Ducts of the Glands of the *Cutis*, or Skin, and, as is ſaid, with Great Probability; The Humour, which is Separated by thoſe Glands, being Pent in between the Scales, cauſes frequent Itchings, to the Removing of which, Nature Directs us to Rubbing, Waſhing, or Bathing; It ſticks cloſe to the Surface of the Skin, to which it is alſo tied by the Veſſels which Nouriſh it, tho' they are ſo Small, as not to be ſeen.

IF inſtead of Raiſing it by a Cautery on a Dead Body, we Examine that, which is Raiſed by Cantharides, or any other Veſicatory, we ſhall find it to Conſiſt of ſeveral Membranes exceedingly Thin and Fine, and not deſtitute of Bodies Reſembling

Glands, which in that State are Viſible to the Naked Eye, and therefore, it is Imagin'd, muſt have Veſſels belonging to it, tho' not Diſcoverable by Microſcopes.

The Solution of the ſeveral Phænomenons of it, from the Doctrine of the Expanſive and Contractive Forces.

§. 4. THIS is a full and ſhort Account, which the Anatomiſts give of the *Cuticula*, which ſeems to Ariſe from the Expanſive Force of the Blood and Spirits on the one Hand, and the Contrary Expanſive of the Air on the other, which in Reſpect of the Former is Contractive; for from hence it follows, that a Denſe, Continued, and Membranous Subſtance will be Produced, as the *Epidermis* or *Cuticula*, namely, from Two Oppoſite Forces, which are nearly Equally Balanced, as the Expanſive and Contractive in this Caſe are, and in the *Fœtus* in the Womb, the Liquor of the *Amnion*, in which it Swims, ſupplies the Place of the Contractive; For this Reaſon it is, that ſome have Thought the *Cuticula* to be made by a Viſcous and Oleous Vapour of the Blood, others by a Tranſparent Juice Iſſuing from the Capillary Nerves, and both of them, that it is Condenſed by Cold, becauſe it is Actually formed by the Expanſive Force of the Blood and Spirits, and the Contractive of the Air Reſiſting and Repelling their Action; If the Expanſive Force on one Side is Increaſed by a Cautery or Veſicatory, or by an Inflammation, or if the Contractive of the Air is on the other Diminiſh'd, as in Cupping Glaſſes, the *Cuticula* is Raiſed and Expanded from the *Cutis*, and Bliſters and Puſtules are Cauſed; This likewiſe gave an Handle to ſome to Imagine it was Derived from a Seminal Principle, becauſe the Blood and Spirits are ſuch, and are Different and Peculiar in Men, and by Conſequence, the *Cuticula*, from what they are in Brutes; *Lewenhoeck*'s Notion alſo of it's being an Expanſion of the Excretory Ducts of the Glands of the Skin Receives from hence a Solution; It Peels off in Burning Fevers, and the Small Pox, by an Exceſs of the Expanſive above the Contractive, and is again Renewed by an Equal Poiſe of thoſe Two Forces; It is not Neceſſary, that it ſhould be full of Pores for Sweat and Perſpiration, but that it ſhould be Separable only by an Increaſe of the Expanſive Force, when the Blood and Spirits are Excited by Heat or Exerciſe; It is void of Senſe, when Divided from the Expanſive Force of the Spirits, as any other Part of the Body would be, but whilſt United with it, as in it's Natural State, it is the Inſtrument of Feeling; It's Surface conſiſts of Innumerable ſmall Aſperities, or little Scales, becauſe the Expanſive Force of the Blood and Spirits and the Contractive of the Air do not Act againſt each other by one Uniform Diffuſion, but in Punctual and Unequal Exertions, as Common Fire and Flame does, by which Liquors, in Boiling or Growing Hot, Contract a like Uneveneſs in their Surface.

THERE is no occaſion for Veſſels to nouriſh it, or Nerves to make it Senſible, all that is Required for both is it's Adherence to the Syſtem of the Expanſive Forces of the Blood and Spirits; It may be Divided into ſeveral Membranes, and may ſeem to have Glands from the Application of Different kinds of Expanſive Forces to it, as Water is Raiſed by Heat into Cuticles and Bubbles; And the *Cuticula* appears to be nothing elſe than an Animal Fluid Agitated by the Expanſive Force of the Blood and Spirits, and Condenſed and Congealed by the Contrary Expanſive or Contractive Force of the Air, and in the Womb by the Liquor of the *Amnion*; which ſhews the wonderful Providence in Nature, in Preparing ſuch a Liquor, ſince, without it, the Syſtem of the Expanſive Forces in a *Fœtus* would be Diſſipated and Diſſolved, having nothing to Counter-Act to it, from whence no Production would Enſue, but an Entire Abortion and a Corruption of the whole *Fœtus*.

How Hair is Produced and the Reaſon of it's Different Colours, Curlings, and Figures.

WHERE the Expanſive Force of the Blood or Spirits is the Greateſt, by which it over-balances the Contractive of the Air, there is a Protruſion of the *Cuticula* into Hair, as near the Brain, the *Medulla Spinalis*, and the Heart, and in the Armpits, where the Contractive Force of the Air is Diminiſhed by the Continual Adheſion of the Arms to the Body, and where it is the Leaſt, as in the Hands and Feet, there is no ſuch Protruſion, or a very Small one; According to the Different Nature and Degree of theſe Expanſive Forces of the Blood and Spirits, the Colour of theſe Protruſions alter; Red ariſes from an hot Conſtitution, or from the Greateſt Expanſive Force of the Blood and Spirits; White from a Cold one, or from the Leaſt Expanſive Force of it, from whence old Men grow Grey, and younger Perſons upon a ſudden Paſſion of Fear, or by Immenſe Care or Study, by which the Expanſive Force of the Blood and Spirits is Weakened and Enervated; and Black or the Deepeſt Red from a Great Degree of the Expanſive, mixt with a Great Degree of the Contractive of the Blood and Spirits; the Firſt are conſequently, Cholerick, Briſk, and Lively in their Parts; the Second, Calm, Sedate, and Cool; and the Laſt, Melancholick, Quick, and Vigorous, but Solid, Judicious, and Grave: It is generally Streight in Cold Countries, and Curled and Contorted in Hot, becauſe the Contractive Force of the Air is more Certain, Uniform, and Determined in it's Reſiſtance in the Firſt Caſe, and more Deſultory, Various, and Unequal in the Laſt; and in the ſame Climate, where the Expanſive Force of the Blood and Spirits is leſs Regular and Uniform, the Hair from thence becomes likewiſe Curled and Contorted, and where it is Streight, it is a Sign of a Greater Certainty, Unalterableneſs, and Immutability in their Action; It is Triangular, or Four-ſquare, or Round in it's Figure, as the Expanſive Force of the Blood and Spirits Exceeds more or leſs, or is Equal to the Contractive of the Air, which ſurrounds it; if more, it Exerts it ſelf in Three Angles, if leſs, in Four, and approaches nearer to a Circle, if Equal, it is Circular.

Why Plants have Different but Regular Forms in their Leaves and Stalks.

Which alſo will give an Account of the Different Forms of Plants, and the Stalks of Them, which come nearer to a Circle, or depart farther from it, as the Proper Expanſive Forces of them, Derived from their Seminal Principle, are nearer to an Equilibrium with the Contractive of the Air, or are more Removed from it; and the Figures will be always or moſtly Regular, becauſe the Superiority of Action in the Expanſive above the Contractive of the Air muſt Exert it ſelf alike, and therefore in Three, Four, Five, or an Hundred Equidiſtant Angles from each other.

Why Men are of Different Statures from one another, and in the Various Periods of their Lives, as alſo why Animals are of Different Bigneſſes from Men.

From this Doctrine likewiſe is Determined the Different Stature and Bigneſs of Men in the ſeveral Periods of their Lives, and Compared with one another, as alſo the Various Proportions of Animals and Inſects; Thus the Expanſive Force of the Blood and Spirits in Inſects being Small, the Syſtem is Extended but a ſhort way before the Contractive of the Air becomes an Equal Balance to it; in Birds and the Smaller kind of Animals the Expanſive Force is Increaſed, by which means the Syſtem is Extended farther, before it is weak enough to be Reſiſted and Equi-Poiſed by the Contractive of the Air; and in larger Animals, as the Lion, the Horſe, the Ox, and the Elephant, the Expanſive Force of the Blood and Spirits is ſtill Augmented, from whence the Syſtem of them is more Dilated before it's Force is Diminiſhed to an Equilibrium with the Contractive; and thus it is in Men, ſo that the Greater Stature any one Acquires, it muſt be from thence Inferred, that the Expanſive Force of his Blood and Spirits is Greater, and that in young Men it is Greater than in Infants and Children, till it obtains it's utmoſt Force in Perſons of Maturity, when there is no farther Growth or Expanſion of the Syſtem, the Extreme Parts of it being in an Equal Balance with the Contractive of the Air, with which it is Encompaſs'd, or in Fiſhes with the Contractive of Water; and the ſame kind of Reaſoning obtains in Plants and Trees, if for that of the Blood and Spirits in Animals, we ſubſtitute the Expanſive Force of the Nutritious Juices and the Vegetating Quality of Them.

Why it is not Impoſſible but Men might have Gigantick Forms before the Deluge, and Plants and Animals might be Greater than we now find them.

See alſo Chap. 5. §. 46. Book 1.

Lastly, If the Expanſive Force therefore of the Blood and Spirits in Men and in Animals, and of the Nutritious Juices and their Vegetating Quality in Plants and Trees, in the Antediluvian World, was Greater than in the Preſent, and the Contractive Force of the Air Leſs, which in a Different Craſis and Temper of Things is not Impoſſible, and particularly, ſuppoſing the Surface of the Earth was much Higher and more Removed from the Center of it, when all Beings would have a Leſs Force of Gravitation and Contraction, and a Greater of Expanſion, than in this lower and more Contracted Form, it is not Inconceivable, that Men and Animals, and Plants and Trees, might be much Larger, and more Longæval and Durable, than we now find and Experience them to be, and that for ſome Ages, after the Diſſolution of the Earth's Original Frame by the Flood, ſuch Gigantick Proportions might be in ſome Degree Continued, till they ſunk in Proceſs of Time to the ſettled Standard of Magnitude and Duration, which is agreeable to the Modern Conſtitution of the Earth we Inhabit. This we ſay in Anſwer to the Cavills of Irreligious Men, who Think ſuch Matters Incredible, which do not agree with the Modern Frame and Structure of Beings.

Thus far we have conſidered the *Cuticula* of the Anatomiſts, which is the utmoſt Term and Boundary of the Animal Syſtem, and how it comes to be Defined by certain Limits from the Nature and Degree of the Expanſive Force of the Blood and Spirits, and the Contractive of the Air, which hinder it's Dilatation *in Infinitum*.

§. 5.

§. 5. THE next Subſtance, which lies Immediately under the *Cuticula*, and above the *Cutis*, or Skin, is the *Reticulum Mucoſum*, or *Parenchyma*, which in the *Europeans* is a White Slime, and in the *Æthiopians* a Black, ſcraped off from the Inner Side of the *Cuticula* Raiſed, and which ſerves to fill up the Interſtices made by the *Nervous* or *Pyramidal Papillæ*, Jetting out from the Surface of the *Cutis* or Skin; Of which there is little more to be ſaid, excepting, that it ſeems to be a Matter Prepared for a ſupply of the *Cuticula*, upon any Accidental Defect of it, that it is thrown out from the Expanſive Syſtem of the Blood and Spirits, as an Excrement of it, and more particularly from the *Nervous* Juice, which is of a Contractive Nature, and therefore more eaſily ſubmits to the Contractive Force of the Air, when it is Expoſed to it, which Blood, or Fat, or Spirits, would not do, and conſequently is Immediately Condenſed by it.

Concerning the Reticulum Mucoſum, *or* Parenchyma.
Syllabus Enterologiæ.
Dr. Gibſon's Anatomy, Chap. 3. Book 1.
Dr. Keill's §. 4. Chap. 1.
Dr. Drake's Anthropologia Nova, Chap. 3. Book 1.
Blancard's Anatomia Reformata, Cap. 34. Pag. 742.

§. 6. THE *Cutis* or Skin, beſides it's own Proper Subſtance, which is made up of *Nervous Fibres* very cloſely Interwoven one with another, and of a *Parenchyma*, which fills up the Inequalities and Interſtices thereof, has it's *Nervous* or *Pyramidal Papillæ*, which are ſmall Strings ſeen to break off by pulling the *Cuticle* from the *Cutis*, and are thought by *Malphigius* to be the Ends of the Nerves Perforating the Skin, and not Raiſed from the Skin's proper Subſtance, but theſe *Pyramidal Papillæ* cannot be ſeen without Optick Inſtruments, and can hardly be ſhewn by the Common way of Diſſection; It has alſo, lying betwixt the ſaid *Papillæ*, the *Glandulæ Miliares*, or *Sudoriferæ*, which Convey the Sweat thro' the Skin, and Separate it for it's Excretion; It has likewiſe certain Bulbs or Protuberances, in which the Roots of the Hair are fixt, and it's Veſſels are *Sanguineous*, *Nervous*, and *Secretory*; It is Thin in the Forehead and Sides, and under the Heel it is Thickeſt; It is Thinner in Children and Women than in Men, and in thoſe that Live in Hot Countries, than thoſe that Live in Cold; From whence, according to *Spigelius*, thoſe who are born in Cold Countries, when they come under the Equinoctial Line, are often taken with Fevers, becauſe the Great Heat there Excited in the Body by the External Air cannot Exhale thro' the too Thick Skin; It is Imagined to be full of Pores, and that thoſe, who Deny them, oppoſe not only Reaſon but ocular Demonſtration; the Paſſing of Quick-ſilver thro' a Sheep-skin, we are Told, Evidences this, and that Mr. *Boyle* has Tried the ſame in a Piece of the Skin of a Man's Arm, and this is ſtill farther Proved, it is Thought, by the Iſſueing out of Freſh Sweat from a Man's Fingers, when the Former is wiped off; It is ſaid to be of a moſt Exact Temperature, neither too Hot, nor too Cold, that it might judge more Accurately of the Temperature of Tangible Things, It's Nervous Part, which is naturally Cold, being ſomething Warmed by the Arterial Blood, for it has many both Capillary Arteries and Veins Diſtributed thro' it, as alſo abundance of Nervous Twigs, which Endue it with a moſt Exquiſite Senſe; Each of the Miliary Glands Receives a Nerve and an Artery, and ſends out a Vein or Excretory Veſſel, which laſt Paſſes to the *Cuticula* for the Diſchargeing the Matter of Sweat and Inſenſible Perſpiration, and for the Moiſtening the *Cuticula*, and the *Pyramidal Papillæ*, that they may not be Dry, which would very much Hurt the Senſe of Touching; The Skin is Six Times Thicker than the Scarf-Skin, and ſerves for an Emunctory of the whole Body, and it is ſaid, that the Sum of the Particles, ſtrained thro' the Cuticular Glands, is by *Sanctorius* reckoned to amount to about 50 Ounces a Day, ſo that, ſuppoſe a Man's Body to weigh 160 Pounds, in 51 Days we Perſpire a Quantity Equal to the Weight of the whole Body, but in what Time, or if ever, the whole Subſtance of the Body is Changed, is no Eaſy Matter to Determine. And thus far the Anatomiſts.

Concerning the Cutis, *or Skin, with a particular Account of it, Extracted from the Anatomiſts.*
Syllabus Enterologiæ.
Dr. Gibſon's Anatomy, Chap. 3. Book 1.
Dr. Keill's §. 4. Chap. 1.
Dr. Drake's Anthropologia Nova. Chap. 3. Book 1.
Blancard's Anatomia Reformata, Cap. 34. P[illegible] 742.

§. 7. THE Expanſive Syſtem of the Blood and Spirits, being now Freed from the Impreſſions and Contractive Force of the External Air, is more at Liberty to Exert it's own Actions.

A Solution of the Preceding Phænomena of the Cutis *from the Doctrine of the Expanſive and Contractive Forces.*

AND Firſt, the Syſtem of the Spirits or the Nervous Fluid Expanding it ſelf, as far as it can, is ſtopt at the *Cuticula* by the ſaid Contractive Force of the Air, and from thence Divaricates into a Thouſand Streams Croſſing and Interſecting each other, which Conſtitutes the Subſtance of the Skin, and Forms thoſe Nervous Fibres, of which it Conſiſts, and which are ſo cloſely Interwoven one with another; Hence alſo come the *Pyramidal Papillæ*, and the ſmall Nervous Strings, which break off by Pulling the *Cuticula* from the *Cutis*; On the other Hand, the Syſtem of the Blood, *How the Nervous Fibres, the Arteries, and Veins of it are Formed, as to the Various Rivulets of the Fluids.*

Expanding it's ſelf likewiſe, is ſtopt at the *Cuticula* and the Contractive Force of the External Air, and made to Diffuſe it's ſelf into Innumerable Rivulets, which Compoſe the Blood Veſſels, and which are Diſperſed by that means thro' the whole Skin, and thus all the Arteries of it ſeem to be Formed, in which the Blood Retires from the Heart; This Expanſive Force of the Syſtem of the Blood continuing, and the Blood not being able to Proceed any farther, is by it's ſaid Expanſive Force Returned back and Repelled from the *Cuticula*, and the Contractive Force of the Air, towards the Heart, by which the Veins are Produced; But the Nervous Fluid, being of a Cold and Contractive Nature, Terminates in the Skin, without any Circulation, or Recourſe, or Repulſe from it.

Whence Sweat ariſes, and how made from the Contractive Force of the Nervous Fluid being Weakened.

INTERMEDIATE to the *Pyramidal Papillæ* Lie the Miliary or Sudoriferous Glands, and the Mouths of the Ducts, which open into them, and which are made by the Sweat; which ariſes from a Conteſt betwixt the Contractive Force of the Spirits, or the Nervous Syſtem, and the Expanſive of the Blood; for tho' they are both of them taken together of an Expanſive Nature, in Reſpect of the External Air, yet in Compariſon with each other, the Nervous Syſtem is of a Cold or a Contractive Temperature, and the Blood of an Hot or an Expanſive; and therefore it is, that all Heat Diſſolves and Enfeebles the Nerves and Spirits, and all Cold Contracts and Envigorates them; And on the Contrary, all Heat Excites and Inflames the Expanſive Force of the Blood, and all Cold Weakens and Impairs it: When theſe Two Syſtems are Equally Balanced, the whole Human Frame or Compoſition is in Exact Order; but if the Expanſive Force of the Blood is too much Increas'd by Exerciſe or Heat, or the Contractive Force of the Spirits is too much Diminiſh'd by Labour or the Depreſſion of them, as in Faintings, or in Sleep, Sweat Enſues; which is neither a Nervous nor a Sanguineous Fluid, but a Nervous Weakened and Diſſolved, by which it becomes more Liquid than the Nervous, but Retains a Degree of it's Contractive by it's Saltneſs, as all Salts are of a Cold and Contractive Temper.

How it Paſſes thro' the Cutis.

AND from hence it is, that the Sweat can Force it's way thro' the Subſtance of the *Cuticula*, without the Suppoſition of any Pores in it, when the Nervous Fluid and the Blood cannot; becauſe the Nervous Fluid is of too Contractive, Conſiſtent, and Cold a Nature to do it, and the Blood's Expanſive is Reſiſted and Repelled by the Contractive of the Air; whereas the Sweat, being a Contractive Excited by the Expanſive of the Blood, is Leſs oppoſed by the Contractive of the *Cuticula* and the Air, and, Approaching more to it's own Nature, is more eaſily Admitted and Received by it.

THUS far we have Conſidered the Fluids, which Enter into the Skin, of which the Firſt or the Nervous is ſenſible, and ſeems to be of the like Subſtance with the Brain, and to be ſeveral Proceſſes from it, but the Blood and Sweat do not appear to be Endued with that Power.

How the Coats of the Veſſels, or the Solids, in the Cutis *are Produced.*

THE next Thing to be Examined and Accounted for is the Solids, which are made up of the Veſſels, which Contain and Circulate, or Secrete the Fluids, and are Derived from the Contractive Force of the Nervous Fluid, and the Expanſive of the Sanguineous; for as the *Cuticula* is Formed from the Expanſive Force of the Animal Syſtem and the Contractive of the External Air, which Acts againſt it, ſo by the Expanſive Force of the Blood being Acted againſt by the Contractive of the Nervous Fluid, the Coats of the Blood Veſſels are Produced, and thoſe of the Nerves by the Expanſive Force of the Blood, and the Heat ariſing from it, Acting againſt the Contractive of the Nervous Fluid; from which Contractive Force it is, that they and all the Membranes of the Body are ſaid to Deduce their Original from the *Meninges*, or the *Pia* and *Dura Mater* of the Brain, and from whence they come to be Furniſhed with ſuch an Exquiſite Senſe; Thus alſo we ſee, by the Expanſive Force of Fire and the Contractive of Water Acting againſt each other, a Film or ſmall Membrane is Conſtituted and Framed.

How the Circulations of the Fluids in the Cutis *and the Animal Syſtem are Performed by the ſame*

To theſe Two Oppoſite and Contending Forces is likewiſe owing the Circulation of the Fluids in the Skin, and thro' the whole Body; and the Revolutions of the Humours in the Animal World are Accounted for in the ſame Manner, and upon the ſame Principles of the Expanſive and Contractive, as thoſe of the Heavenly Bodies in the Material; For the Contractive Force in the Nervous Syſtem, to which is annext the Expanſive Force of Senſe or Intelligence, Acting continually againſt the Expanſive in the Sanguineous, which in Reſpect of the Mind is Contractive, is the ſame, as the Almighty Intelligence and the Expanſive of the Sun Acting againſt the Contractive of a Planet; for the Action of the Mind or of Senſe being Superior in the Nervous Syſtem, ſince the Counter-Action, or Contractive Force, in the Sanguineous cannot prevail againſt it, it muſt Circulate about it, for the ſame Reaſon, as a Planet about the Sun; Thus, The Blood Acts by it's Force againſt the Nervous Coats of the Veſſels, which Contain it, and the Nervous Coats Contract themſelves againſt that Force, which, being too Weak to Reſiſt the Contraction, Proceeds forwards in the Arteries, from whence ariſes the Progreſs of the Fluid in it's Proper Channel, and the Pulſation, by which it is Promoted; From hence alſo it is, that every Artery has a Nerve, which Attends it, to fortify and ſtrengthen the Contractive Force of the Nervous Syſtem, in order to Propagate the Motion of the Sanguineous, and the Blood is carry'd back without the Aſſiſtance of ſuch Contraction, by Protruſion only thro' the Veins, and by Valves, which hinder it's Regreſs, and therefore has no occaſion of ſuch Nerves to accompany them, nor is there found any Pulſation in Them.

Laws, as thoſe in the Heavens, namely, of the Expanſive and Contractive Forces.

THUS Rivers alſo are Probably made to Circulate about the Earth by an Oppoſition betwixt the Expanſive and Contractive Forces; for Water having an Expanſive Force in Reſpect of Earth, which is Evident from it's being eaſily Raiſed into Exhalations and Vapours, would Exert it's Activity that way, if it were not ſurmounted by the Contractive of the Earth, which therefore ſpends it ſelf Side-ways by a Revolution round it; ſince, if there was nothing, but the Gravitation of it, it ſhould Tend to the Center and be bound down towards it in a Perpetual Stagnation, unleſs we ſuppoſe a Declivity in the Channel, which is not Evident in a Sphere, as the Earth.

That Rivers Circulate about the Earth the ſame Way.

THE Thickneſs or Thinneſs of the Skin depends upon Two Cauſes, the External Air, and the Internal Syſtem of the Blood and Spirits; The Greater the Contractive Force of the Internal Syſtem is in certain Parts, the Thicker will the Skin be, and the Greater the Expanſive, the Thinner; 'tis Thickeſt therefore in the Head and Back, becauſe it lies next to the Brain, and the *Medulla Spinalis* or *Oblongata*, which are Cold and Contractive, and under the Heel, becauſe the Expanſive Force of the Blood is Leſs in thoſe Parts; In the Fore-head, Palm of the Hand, and Lips, it is Thinneſt, becauſe the Contractive Force is Leaſt there from the Brain and Nerves, and the Expanſive the Greateſt from the Blood, as is Evident from the Redneſs and Floridneſs we obſerve in Them; And thus it is Thinner in Children and Women than in Men, becauſe the Expanſive Force of the Blood Prevails moſt in the Firſt, and the Contractive of the Nervous Fluid in the Laſt, as is Manifeſt from the Weakneſs, Beauty, and Sanguine Colour of the one, and the Strength, Robuſtneſs, and the Paler or Duller Complexion of the other; But there is likewiſe another Cauſe of this, which we before-mentioned, namely, the External Air, for the more Cold and Contractive the Temper of that is, the Skin will be thereby made Thicker, and the more Warm and Expanſive, the Thinner; Men therefore being more Expoſed to the Cold and the Contractive Force of the Air than Women or Children, their Skin is upon that Account more Denſe and Thicker, and for the ſame Reaſon Their's who Live in Colder Climates, than Their's who Live in Hot ones; And, that an Expanſive Force of Blood or Heat ſhould Diſſolve Continuity, and a Contractive one of the Nervous Juice or Fluid, or of Cold in the External Air, ſhould Contribute to it and Increaſe it, is not any ways hard to Imagine.

The Cauſe of the Different Thickneſs and Thinneſs of the Cutis *Derived from the ſame Principles.*

As to the Pores of the Skin there is no occaſion for them, as we have ſhewn there is not in the *Cuticula*, ſince the Conſiſtency of it is ſuch, as to Permit a Paſſage to any Forces, which it is convenient to Nature ſhould be Tranſmitted thro' it, and it is Confeſſed, that the Famous Dr. *Gliſſon* denied any ſuch Pores to belong to it; Thus a Pin or a Needle can make it's way thro' a Solid or a Fluid, when Acted with a ſufficient Force, and there is no Greater Difficulty in Apprehending, how any Fluid of the Body ſhould obtain the ſame Effect in Reſpect of the Skin, where it is Expelled by the Expanſive Force of the Animal Syſtem and Protruded thro' it, which is the Caſe of Sweat, and therefore can be no Proof of ſuch Pores; and as for Quick-ſilver Paſſing thro' a Sheep-Skin, or the Skin of a Man's Arm, it is only an Evidence of what I have been Affirming; Beſides upon ſome Extraordinary Strainings even Blood has Iſſued thro' the Skin, for which yet it never was ſuppoſed, that any Pores were Formed, and conſequently, it's very Subſtance muſt be capable of Yielding

Not neceſſary that there ſhould be any Pores in the Cutis, *the Tranſmiſſion of Sweat being otherways Accounted for.*

ing to and Allowing the Tranſmiſſion of a Force, which is Superior to it, that is, of an Expanſive Superior to it's Contractive.

Concerning the Fat, the Membrana Adipoſa, and Muſculoſa, with a Summary Account of Them.

Syllabus Enterologiæ. Dr. Gibſon's Anatomy, Chap. 3. Book 1. Dr. Keill's Chap. 1. §. 6. and 7. Dr. Drake's Anthropologia Nova, Chap. 4. Book 1. Blancard's Anatomia Reformata, Cap. 18. Pag. 392.

§. 8. UNDER the Skin lies the *Membrana Adipoſa*, in the Cells of which the Fat is Contained, and which is allowed to be of an Oily Subſtance Condenſed by Cold, with which the Membranous Cells are filled, as the Cells in the Pulp of an Orange are filled with it's Juice; and Dr. *Gliſſon* ſays, that the *Membrana Adipoſa* is a Part of the *Carnoſa*, the outer Part of which is full of ſuch Membranous Cells; *Malpighius* Informs us, that the Blood Veſſels are Expanded like the Boughs of Trees every where thro' the Membrane, that lies under the Fat, and the Membranous Cells, full of Fatty Globules, Veſicles, or Bladders, hang upon their Ends, like Leaves upon the Boughs; The Cells are of almoſt an Oval Figure, and are like the Lobules of which the Lungs are Compoſed, or rather like a Conglomerate Gland; They are Knit to one another by the Membrane, out of which they are Formed; and thro' the ſame Membrane and Cells, not only Capillary Veins and Arteries, but *Ductus Adipoſi* alſo Run, which Swell with Fat, eſpecially in an Animal newly Killed; This Fatty Membrane Extends it ſelf all over the Body, except in the Lips, the upper Part of the Ear, the Eye-lids, and Fore-head, and ſome few other Places, and the Fat is ſuppoſed to be Bred of an Oily or Sulphureous Portion of the Blood, Sweating, like a Dew, out of the Veins, and afterwards Concreting; It is Different from the Suet or Tallow of the *Omentum* or Caul, in this, that Fat is eaſily Melted, but not ſo eaſily Congealed, whereas Suet is not eaſily Melted, but is eaſily Congealed, Fat is not Brittle, but Suet is; ſome Reckon the Marrow of the Bones to be a Third ſort of Fat. It Defends the Body from the External Air, Preſerves the Natural Heat, Hinders the too Great Diſſipation of Spirits, Fills up the Interſtices of the Muſcles and Wrinkles of the Skin, and thereby Contributes to the eaſy Motion of the Former, and to the Beauty of the Latter, and keeps all Parts from Dryneſs and Breaking; It Beſmears the Extremities of the Cartilages, the Jointings of the Greater Bones, and the Veſſels, that they may Paſs ſafely; *Malpighius* thinks the Principal uſe of it, whilſt it is mixt with the Blood, is to be a continual *Pabulum* or Food of the Natural Heat, whereby the Vital Flame is Perpetuated, Alſo to mitigate the Acrimony of the Blood, and unite the Saline Particles of it, ſo as they may become Aſſimilable to the Solid Parts for their Nutriment, ſerving to thoſe Particles, as Oil to Colours in Painting, or Lime to Stones in Building, and that in Caſe of Famine it Paſſes into the Nouriſhment of the Body, ſo that ſome Creatures Live during a whole Winter without any other Food, and an Atrophy never becomes Mortal, till all the Fat is Waſted; In this Membrane are alſo a Number of Little Glands, which are accompany'd with Lymphatick Veſſels, which carry back any Seroſity, that is, Superfluous; In ſome the above-mentioned Veſicles are ſo full, that the Fat is an Inch, or an Inch and half Thick, and in others they are almoſt Flat.

The ſeveral Kinds of Fat, and it's Uſes, according to Anatomiſts.

What a Membrane is, and the ſeveral Sorts of Them.

MEMBRANES are Webs of ſeveral Sorts of Fibres Interwoven for the Covering and Wrapping up of ſome Parts; Their Membranous Fibres give them an Elaſticity, whereby they can Contract and cloſely Graſp the Parts they Contain, and their Nervous Fibres give them an Exquiſite Senſe, which is the Cauſe of their Contraction, therefore they can ſcarcely ſuffer the Sharpneſs of Medicines, and they are Difficultly united, where there is a Solution of Continuity, or Loſs of their Subſtance; In their Texture there are a Number of ſmall Glands, which ſeparate an Humour fit for Moiſtening the Parts, which they Contain; Thoſe that Cover the Solid Parts are properly called Membranes, as the *Peritonæum*, which Wraps up all that is Contained in the *Abdomen*, the *Pleura*, that, which is in the *Thorax*, the *Perioſteum* the Bones, and the *Pericardium* the Heart; Thoſe, which Form the Coats of Veſſels, and which Contain the Humours, as thoſe of the Veins, Arteries, Stomach, Bladder, Inteſtines, *&c.* are called Tunicles or Coats, and thoſe, which Cover and Embrace the Brain, as the *Dura* and the *Pia Mater*, are called *Meninges*; of all theſe Kinds of Membranes ſome are Thin and ſome are Thick, and the ſame Membrane is Thick in ſome Places and Thin in other, as the *Membrana Adipoſa*, which is Thicker in the Neck than in any other Part of the Body; The uſe in General of Membranes is to Cover and Wrap up the Parts, to Strengthen them, to Save them from External Injuries, to Preſerve the Natural Heat, to Join one Part to another, to Suſtain ſmall Veſſels and the Nerves which Run thro' their Duplicatures, to ſtop the Returning of the Humours in the Veſſels, as the Valves ſtop the Returning of the Blood in the Veins and Heart, of the Chyle in the Lacteals and Thoracick Duct, and of the *Lympha* in the Lymphatick Veſſels.

By the *Membrana Adipoſa* is moſt commonly underſtood that Part of it only, which lies next the Fleſh, and which contains but little Fat in it's Cells, and therefore Appearing more Membranous than the Reſt is ſaid to be the Baſis of the *Cellulæ Adipoſæ*, and even ſome Part of this hath been taken by Anatomiſts for the *Membrana Carnoſa* upon the Account of it's Redneſs, for here the Blood Veſſels lie very Thick, the Veſicles not being Diſtended with Fat.

The Fat, which is a Groſs Whitiſh Oil, is nothing but the Oily Part of the Aliment ſeparated from the Arterial Blood by the Adipoſe Glands, and carry'd by Peculiar Ducts to the Membranous Cells, from whence it is Tranſmitted to the Blood again by the Veins; In Corpulent Perſons, who have their Blood too much Charged and all their Organs too much Clog'd with it, Senſation is Dampt, and they are apt to fall into a Drowſy Lethargick Condition, which ſometimes Ends in ſudden Death, and the Abundance of it ſeldom fails of being Attended with Inactivity and Somnolency.

Under the *Membrana Adipoſa* lies the *Membrana Communis Muſculorum* or *Muſculoſa*, and makes a Duplicature with it; It is Spread all over the Body, except the Skull according to ſome; It is Knit by Fibres to the Adipoſe Membrane that lies above it, and to the Proper Membrane of each Muſcle that lies under it; 'Tis Thin, but Strong, 'tis not Yellow like the Adipoſe, but Whiter and more Tranſparent, and is ſaid by Dr. *Havers* in his *Oſteologia Nova* to be every where Glandulous, by which a Mucilaginous Liquor is Diſtilled upon the Muſcles to Lubricate them; Theſe Membranes receive Veins, Arteries, and Nerves from the Parts which are neareſt to them; As to the *Membrana Carnoſa*, which lies in Brutes betwixt the *Adipoſa* and *Muſculoſa*, It ſeems to be the Cauſe, by it's Expanſive Force, of that General Protruſion of Hair thro' the whole Body, according to the Explanation given §. 4.

How the Adeps or Fat is formed from the foregoing Principles.

§. 9. This being a full Account from the Anatomiſts of the Fat and the Membranes, which lie under it, we ſhall now Conſider them one after another; As to the *Adeps* or Fat, it ſeems to be Formed from the Expanſive Force of the Blood, brought by the Arteries to the Adipoſe Membrane, according to *Malpighius*, and there Meeting with the Nerves, which contain a Contractive Subſtance, there being a Duplicature of Membranes alſo, the upper Red, where the Blood Veſſels Run, and the lower White, where the Nervous ones are Spread and Diffuſed, from their Mutual Action upon each other an Intermediate Liquor is Produced, not ſo Red as Blood, nor ſo White as the Nervous Fluid, not ſo Hot and Warm as the Former, nor ſo Cold and Phlegmatick as the Latter, from whence a Pale Yellowiſh Oil ſucceeds, and which we have before ſaid appears to be the Nature of Sweat, which is an Exuding of this Oil.

How the Veſicles, in which the Fat is Contain'd, are Produced.

The Bubbles and Veſicles are made by the Two Contending Forces, as is done by Heat and Water mixing together, or in the Pulp of an Orange by the Sun's Expanſive Mingleing with the Cold and Contractive Juice of the Fruit and Tree; The Expanſive alſo Exerts it ſelf ſtill in ſome Meaſure in the Oil, and Forms the Contractive into the Bladder or Film, which ſurrounds it, to which the Contractive Force of the External Air, which Penetrates by it's Contrary Expanſive, we may eaſily Imagine, thus far, is Aſſiſtant, and Helps to Conſtitute theſe Veſicles, as it manifeſtly does in the Formation of other Bubbles; and they are like the Lobules, which Compoſe the Lungs, or like a Conglomerate Gland for the ſame Reaſon.

Why no Fat in the Lips, the upper Part of the Ear, &c.

There is no Fat in the Lips, the upper Part of the Ear, the Eye-lids, and Forehead, becauſe in theſe Parts the Contractive Force of the Air, or the Expanſive Force of the Blood is too Great, or the Contractive Force of the Nervous Fluid too Small to Produce a due Mixture for Fat, or a Conteſt or Ebullition betwixt thoſe Forces, which is Requiſite to the Attemperature of the Oil, and the Raiſing of the Globules and Veſicles; hence alſo there is a Sulphureous Quality in the Fat from the Expanſive Force of the Blood, and a Cold or Condenſing one from the Contractive of the Nervous Fluid and of the Air.

How this Account agrees with what we gave before of Chymical Oils.

This likewiſe Agrees with the Account, which we before gave of Chymical Oils, which follow the Spirits, which are of an Expanſive Nature, and riſe before the Phlegm, and the other Principles, which are of a Contractive; ſo that it is a Force

Intermediate to and Compounded of the Expanſive and Contractive, according to what is here Affirmed and Maintained.

Confirmed by Dr. Grew's *Experiment.*

BESIDES which, Dr. *Grew*'s Experiment, quoted by Dr. *Havers* in his *Oſteologia Nova* and his Diſcourſe therein of the Marrow, Admits from what foregoes of a Solution; which Experiment is to ſhew us, how we may Imitate the Fat of an Animal Body: Take Oil Olive, and pour it upon High Spirit of Nitre, then Digeſt them for ſome Days, by Degrees the Oil becomes of the Colour of Marrow, and at laſt is Congealed or Hardened into a White Fat or Butter, which Diſſolveth only by the Fire, as that of Animals, and it hardens moſt upon the Exhalation of ſome of the more Sulphureous Parts of the Spirit of Nitre; from whence the true Congealing Principle is a Spirit of Nitre, and the Making of Fat is but the durable Congelation of Oil, and Animal Fat, but the Curdling of the Oily Parts of the Blood, either by ſome of it's own Saline Parts, or by the Nitrous Parts of the Air mingled therewith.

THUS far Dr. *Grew*, in which Experiment we have all the Ingredients, which we ſuppoſe to be in Animal Fat; for Oil Olive, being a Vegetable Oil, and Convertible into Flame, has Evidently a Sulphureous and Expanſive Nature, which likewiſe is in a Different Degree in the Blood, Spirit of Nitre from it's Salt and Cold has a Contractive Nature, tho' Actuated and Excited by Fire into a Spirit, Anſwerable to the Contractive Temper of the Nervous Fluid, which is alſo of a Cold Complexion, but Raiſed and Animated by the Mind, and to which is joined the Elaſtick and Contractive or Contrary Expanſive of the Air; It Diſſolves by the Fire, becauſe the Expanſive by that means prevails againſt the Contractive, and hardens by an Exhalation of the more Sulphureous or Expanſive Parts of the Spirit of Nitre, becauſe the Contractive then Prevails againſt the Expanſive.

And by the Difference between Common Fat, and the Sevum *of the* Omentum *or Caul.*

FRON whence we may Conclude, that the Common Fat of the Body has more of the Expanſive Force of the Blood in it, and leſs of the Contractive of the Nervous Fluid, and that the *Sevum* of the *Omentum* or Caul, has more of the Contractive Force of the Nervous Fluid in it, and leſs of the Expanſive of the Blood, becauſe the Firſt is hardly Congealed, and the Laſt as difficultly Melted; The Firſt is not Brittle, but Fluid, from the Expanſive Force, the Laſt is, from the Contractive; The Artificial Oil or Fat is Whiter than the Animal, as Oil Olive is much Whiter than Blood, which is a Deep Red, and the Nervous and Nitrous Fluids not extremely Different, or not to ſuch a Degree; Here is therefore no Occaſion to ſuppoſe any Oily Parts of the Blood, nor any Saline Parts of it, but to take the Expanſive or Sulphureous Force of the Blood it ſelf, and the Saline or Cold and Contractive of the Nervous Fluid to form this Production.

The Effects and Uſes of the Fat Conſonant to the Account we have given of it.

HITHERTO we have Conſidered the Nature of the Animal Fat, and are now to Examine the Uſes and the Effects of it; It Defends the Body from External Air, for Penetrating thro' the *Cuticula* and Skin, it is met and oppoſed by the Blood and Spirits, from which very Conteſt or Mixture the Fat and it's Veſicles are Produced, as we before obſerved, and therefore it is not reaſonable to Think it can go much farther, beſides the very Nature of Oil is in ſome Meaſure Reluctant to the Impreſſions of Air, the Firſt being Expanſive, and the Laſt Contractive; by being of a Middle Temper betwixt Blood and the Nervous Fluid, that is, by being neither too Hot nor too Cold, it is well Fitted to Lubricate, Supple, and Moiſten the ſeveral Parts; but as to it's being a *Pabulum* or Food for the Blood, before it's Secretion out of the Maſs of it, is not ſo Evident, becauſe it is not Manifeſt, that it is not made, as above-mentioned, by a Mixture of the Blood and the Nervous Fluid Principally; After which, as in Famine or an Atrophy, it is not unlikely, but it may be Converted to ſuch a *Pabulum*, it being eaſy to Imagine, that the Oil by Increaſing the Expanſive in Circulation may be Changed into ſuch a Subſtance; That Fat ſhould Incline Men to be Lethargick, and to a Somnolency, is no Wonder, according to the Account we have Given of it, ſince it Ariſes from the Contractive Force of the Nervous Fluid, being Diminiſhed by the Expanſive of the Blood, but by how much more the Force of the Spirits is Depreſs'd, by ſo much Greater muſt the Propenſity be to Drowſineſs and Sleep, thus the Expanſive Force of Fire, Wines, and Strong Liquors Acting upon the Contractive Force of the Nervous Fluid, Produces the ſame Effect, and all Narcoticks, which are of an Hot and Expanſive Nature; and therefore it is, that when we are Warm in our Beds, that is, when the Nervous Fluid and it's Contractive Force is in ſome Meaſure Diſſolved by the Expanſive of Heat, we ſink into Slumbers, which are not ſo readily Induced upon us, when we are Cold in Them; And on the Contrary, Froſt and Cold Agreeing with and Exciteing the Contractive Force of the Nervous Fluid, the Mind is Exhilarated and Enlivened by it; By the Firſt the Nerves Tremble and are Enfeebled, and by the Laſt Envigorated and Strengthened.

Why Fat Perſons are leſs Active and Lively in their Minds.

HENCE alſo it is, that where the Veſicles are Flaccid, and where Men are of a Thin and Lean Complexion, the Contractive Force of the Nervous Fluid is too Strong and Powerful for the Expanſive of the Blood, which therefore is not capable of Diminiſhing it's Force, ſo as to Produce an Oil, or a Fat, or very Little, which is the Reaſon, that Perſons of ſuch a Craſis and Temper are generally more Quick and Lively in their Senſe, than others, who are more Fat and Corpulent.

How the Membranes of the Body are Formed.

§. 10. AFTER the Fat, we are to Examine into the Adipoſe Membrane, and the other Membranes of the Body, which are formed chiefly by the Cold and Contractive Force of the Nervous Syſtem, from whence is Derived their Solidity and Continuity; This is Evident from the Web of Nervous Fibres, which Conſtitute them and from their Elaſtick and Contractive Force, ſuch as we Experience in an Elaſtick, Froſty, and Cold Air, as alſo from their Exquiſite Senſe; And as the Expanſive Force contributes to Fluidity, and to a Diſſolution of Parts from each other, when joined together by the Contractive Force, ſo the Contractive Force Cements, Unites, and Continues Them, when Diſſolved and made Lax by the Expanſive.

Why the Parts of a Membrane upon a Separation of them are difficultly United, and in General why Hard Bodies are, but Soft or Fluid ones not.

THIS Contractive Force, by which every Part Contracts towards it ſelf, and it's own Center, makes them upon Separation to be difficultly United, whereas the Expanſive Force Tending from it ſelf, and it's own Center, to that of another, Renders the Cement and Union more eaſy, becauſe there is an Inclination to it; Hence a Fluid, as Water, readily Unites, but Ice, which has a Contractive Force, do's not; The Contractive Force, which is Endued with an Elaſtick, as Glaſs, or Ice, or the String of an Harp or a Violin, Reſults to any Impreſſion from the Expanſive it has Joined to it, by which it Cloſes with any Expanſive Force or Percuſſion, which is Impreſſed, But if the Contractive Force is Broken or Interrupted, it is hard to Unite it upon the Account of it's Contractive Quality; And thus it is in the Human Syſtem, a Membrane or a Fibre, which has a Contractive Force is not eaſy to be Cemented, but a Fleſhy Subſtance, which conſiſts of an Expanſive Force of the Blood, and approaches towards a Fluidity, endeavours at a Re-union; and Bodies, which have a Contractive Force, as Glaſs, Ice, *&c.* are Cemented only by Introducing an Expanſive, and Melting them again into a Fuſion, but then after this Re-union, it is the Cold or Contractive Force, which gives them their Solidity and Tenacity, by which they become leſs Penetrable to any Expanſive Force or Impreſſion made upon them; From whence it follows, that there muſt be a Warmth or Expanſive Force from the Sanguineous Syſtem, to diſpoſe the Parts of a Membrane to a Union with each other, and a Cold and Contractive Force from the Nervous Syſtem, to make them Solid, Senſible, and Tenacious, and which in Fact and Experience is the Frame and Compoſition of a Membrane, which has it's Nerves, it's Fibres, and it's Blood Veſſels, and a Cold and Contractive Force attending the one, and an Expanſive and Warm one, the other.

THIS gives a Solution, as before, why Anatomiſts have Thought, that all the Membranes and Coats, diſperſed thro' the Body, owe their Origine to the *Meninges*, or the *Pia* and *Dura Mater*, ſince it is manifeſt, that the Brain, and the Nervous Syſtem Derived from it, has a Peculiar Efficiency in Forming of Them.

The Grand Cauſe of all the Motions in the Animal Frame is the Contractive Force of the Nervous Syſtem, and the Expanſive of the Sanguineous.

WE are likewiſe by this Theory Informed of the Grand Cauſe of all the Motions in an Animal Body, namely, the Contractive Force of the Nervous Syſtem, and the Expanſive of the Sanguineous; for we have before *Chap. 6. §. 20. Book 1.* ſhewn it to be an Univerſal Law of Nature, that where Two oppoſite Forces Subſiſt and Act againſt each other, the Conteſt or Motion will be continued, till the one or the other Prevails, and if the Forces are in an Equal Balance, the Motion will become Perpetual, till that Æquilibrium is Deſtroyed; Thus the Heart and the Brain are the Two Chief Seats of the Expanſive and Contractive Forces, and are joined to each other, and to all the Parts of the Body by Innumerable Arteries ſent out from the Firſt, and by Nerves from the Laſt; Theſe Act againſt one another by their Expanſive and Contractive Forces, and Produce the Circulations obſervable in the Human or Animal Frame; and if the Expanſive Force of the Blood is too much Increaſed by Heat or Violent Exerciſe, or High Meats, or Spirituous Liquors, the Contractive Force of the Brain, or the Nervous Syſtem, Languiſhes, and if the Contractive Force of the Brain and the Nervous Syſtem is too much Promoted by Cold Applications of Air or Water, and by a Thin and Meagre Diet, Rheums, Catarrhs, Defluctions, and Agues are Cauſed, by which the Expanſive Force of the Blood is Diminiſhed; and nothing Tends more to Health, than the Preſerving both theſe Syſtems in a due and regular Poiſe with each other, and in which Conſiſts the whole Art and Science of Phyſick, after we have Conſidered the Nature of the Mind.

Not Improbable that the Semen *of the Male Parent is the Source of the Contractive and the* Ovum *of the Female, of the Expanſive.*

IT is not farther Improbable, that the Principles of the Brain in the *Fœtus* are Derived from the *Semen* of the Male Parent, which ſeems to be a Matter Endued with Senſe, as the Brain and the Nervous Fluid is, and has been Obſerved by *Lewenhoeck* to be made up of Millions of Animalcules; This therefore Impregnating the *Ovum* of the Female Parent, which we may juſtly ſuppoſe is of a Warm and Expanſive Nature, Firſt cauſes that Beating Red Speck, which Anatomiſts have Imagined to be the Heart; from which Two Subſtances, mixt in the *Ovarium*, the Contractive and Expanſive Forces of the Brain and the Heart appear to take their Origine, tho' the *Fœtus* afterwards receives in the Womb, by the means of the *Placenta*, an Acceſſion to the Latter of theſe Forces, from the Blood Veſſels of the Mother, albeit, others will not allow ſo much as that.

Why the Fœtus *comes into the World with it's Head foremoſt.*

THIS may however give an Account, why a *Fœtus* comes into the World with it's Head foremoſt, ſince the Brain being of a Contractive Nature Tends downwards, and the Expanſive Syſtem of the Blood upwards, when looſ'd from the Womb, ſuitable to all other Contractive and Expanſive Forces.

How the Mind is Proved to be Diſtinct from the Nervous and Sanguineous Syſtems, tho' United and Connected with them.

WE have hitherto only conſidered the Nervous and Sanguineous Syſtems in the Compages and Structure of an Human Body, in which we have made the Firſt to be of a Contractive Force, and the Laſt to be of an Expanſive; but as it is Evident, that the Nervous Syſtem has a Prodigious Force of Action and Expanſion Exerted by the Muſcles in a Quick, Lively, and Nimble Motion and Agitation of all the Limbs, we muſt ſuppoſe a Power Independent of, and Superior to, the Contractive Force of the foregoing Syſtem, which we call the Mind, and is the Spirit, which Actuates and Informs it.

Becauſe thoſe Syſtems, tho' they may Account for the Circulation of the Fluids, and the Conſiſtency of the Solids, they cannot Account for our Voluntary Actions.

THE Circulations of the Humours may be Performed in an Animal Syſtem from the Expanſive and Contractive Forces in them, but thoſe Expanſive and Contractive Forces cannot Account for the Free and Arbitrary Motions and Exertions of the whole Frame to one Part of Space or another, without any Previous Determination, unleſs from a Voluntary and Intelligent Agency, annext to the Syſtem it ſelf; But if we join a Spirituous or Expanſive Force from the Mind to the Contractive Force of the Nervous Syſtem, it may then have an Expanſion Superior even to that of the Blood; for thus Nitre, as all Salts, is of a Contractive Nature, and Wine of an Expanſive, and yet when Nitre is Exalted and Volatilized into a Spirit by Fire, and the Operations of Chymiſtry, it's Force is Mightier and Stronger than Alcohol, or the Fineſt rectify'd Spirit of Wine, which is Weak and Languid in the Compariſon.

Alſo becauſe if it Reſided as an Active Property in the Brain and Nerves, it would Exert it ſelf every where alike.

THAT the Mind, which Actuates the Nervous Syſtem, is Diſtinct from, tho' Joined and Connected with it, is manifeſt from hence, that if it Reſided, as an Active Property in the Brain and Nerves, it would Exert it ſelf always, and every where, in the Nervous Syſtem alike, and the Nervous Subſtance would be in it's own Nature Expanſive and not Contractive, as Spirit of Nitre is, and every Portion of that Subſtance would be Endued with the ſame Degree of Thought, Senſe, Perception, and a Freedom and Liberty of Action; On the Contrary, the Brain, and the Nervous Subſtance Derived from it, is in its own Nature Contractive, and is Expanſive only as the Mind Inſpirits and Envigorates it; nor does the Active Quality, Communicated to it by the Mind, Exert it ſelf alike in the Brain and Nerves, but Arbitrarily and at Pleaſure, ſometimes one Part or Member of the Body being moved by the Expanſive Force of them, and ſometimes another; neither yet is there the ſame Degree of Senſe, Thought, Perception, and a Freedom of Acting, in every Portion of the

Nervous Subſtance; For tho' we Feel and Perceive at our Fingers Ends, we do not Think there, and tho' all the Nervous Matter or Subſtance is furniſhed with Senſe, by which there is a Communication of it with the Mind, yet there is not the Mind, there is not the Reaſon, the Will, and Underſtanding in every Part of ſuch Nervous Subſtance, which reſides in the Brain only; which ſhews, that the Mind is not a Property of the Nervous Matter, but a diſtinct Being and Subſtance annext to it.

AGAIN, The Mind has both an Expanſive and a Contractive Force in the Properties and Affections of it, for it has Qualities belonging to it, as we have before ſhewn, which are directly Oppoſite and Contrary to each other; It has the Vertues of Sobriety, Humility, and Goodneſs, of Application and Study, Frugality and Prudence, and Wiſdom, which are Impreſſions upon it of a Cooler Temperament and Nature, and fall in with the Contractive Force of the Nervous Matter, or Subſtance; whilſt it is not Incapable of the Warmer and more Vicious Paſſions of Intemperance, Pride, Cruelty, Licentiouſneſs of Thought, Extravagance, Indiſcretion and Folly, which Ariſe from the Expanſive Force of the Sanguineous; Since therefore the Nervous and Sanguineous Syſtems are perfectly Repugnant and Oppoſite to each other, and cannot Conſiſt together in one, the Mind, which is capable of Receiving and Admitting the Impreſſions of Each, muſt be Different and Diſtinct from Both.

Likewiſe becauſe the Mind has an Expanſive and Contractive Force, which are Single and Divided in the aforeſaid Syſtems.

MOREOVER it is known, That Diſtempers, of a Contractive and of an Expanſive Nature, may both of them begin in the Mind by Paſſions raiſed there; Thus Grief, or Immenſe Study, or a Sudden and Panick Fear, may cauſe Melancholy and an Entire Depreſſion of the Contractive Force of the Nervous Syſtem; and Pride, an Intemperance of Paſſion, and a Vehement Emotion of the Mind, Madneſs and a Violent Fever in the Sanguineous; On the Contrary, if theſe Diſtempers have their Origin from the Animal Syſtem, the Firſt begins in the Nervous, and the Laſt in the Sanguineous; which ſeems to be a Demonſtration, that the Mind is a Subſtance Different from Both, and Adapted to Either, by it's Union and Conjunction with Them.

It admits of Diſtempers, which are Single and Divided in thoſe Syſtems.

LASTLY, The Mind, tho' Diſorder'd, continues Firm and Strong in it's Perceptions in both theſe Caſes, whilſt the ſeveral Syſtems of the Body are Impair'd and Weaken'd, and ſeems then only to Affect a Diſſolution from them, when it finds it's ſelf out of a Capacity, and unable any farther, to Govern the Irregular and Tumultuous State Introduced into Them.

The Mind is Strong, when thoſe Syſtems are Weakened.

THUS far we have ſpoke of the Membranes in General, and how they are Formed; And as to the uſes of them, there is nothing Alledged on that Head, which is not agreeable to the Account we have Given: We ſhall only therefore farther obſerve, that thoſe Membranes are the Thinneſt, which Encompaſs and Lie neareſt to the Contractive Forces in the Body, as the *Meninges*, or the *Pia* and *Dura Mater*, which Intercede betwixt the Scull and the Brain, the Coats of the Nerves, and the *Perioſteum* which Inveſts the Bones; and thoſe are Thickeſt, which Encompaſs or Lie neareſt to the Expanſive Forces in the Body, as the *Peritonæum*, the *Pleura*, the *Pericardium*, and the Coats of the Blood Veſſels; Becauſe the Contractive Force Tends Inwards, and therefore Diminiſhes the Thickneſs of the Former, but the Expanſive Force Dilates and Extends it ſelf Outwards, and by that means Swells and Increaſes the Thickneſs of the Latter; as alſo becauſe the Expanſive Force requires a Greater Contractive to Balance it, and a Contractive a Leſs.

Why the Membranes are the Thinneſt, which encloſe the Contractive, and the Thickeſt, which encloſe the Expanſive.

ALSO the Adipoſe Membrane is Thicker in the Neck, than in any other Part; ſince we have ſhewn, that the Air is Aſſiſting to the Forming of Fat in Conjunction with the Blood and the Nervous Subſtance, of all which there are Great Quantities; for there is not only the External Air, but that, which Paſſes thro' the *Pharynx* and *Larynx* into the Stomach and Lungs; there is likewiſe the *Medulla Spinalis*, and the ſeveral Cervical Nerves, which are Derived from thence in the Number of the Contractive Forces; and the Variety of Blood Veſſels Propagated from the Heart to the Head, which belong to the Expanſive; from which Quantities of the Contending Forces it is Rational to Think, there will be a Greater Portion of Fat Elaborated and Made.

Why the Adipoſe Membrane is Thickeſt in the Neck.

OF the Membranes, there is nothing therefore now Remaining for us to ſpeak to, excepting the *Membrana Carnoſa*, which lies above the *Membrana Communis Muſculorum*, or the *Muſculoſa*, and beneath the *Adipoſa*, and which is not to be found in Men,

Why the Membrana Carnoſa *is not in Men, but in Brutes.*

Men, and is obſervable in Brutes only; the Reaſon of which ſeems to be, that the Nervous Subſtance or the Contractive Force is more Powerful and Predominant in Men, and the Sanguineous or Expanſive in Brutes, by which, in the Former, the *Carnoſe Membrane* is Thinner and more Contracted, and in the Latter, more Dilated and Thicker; and upon the ſame Account, as the *Meninges* and *Perioſteum* are Thinner than the Coats of the Blood Veſſels, or the *Peritonæum*, *Pleura*, or *Pericardium*: From this Membrane and the Exceſs of the Expanſive Force ſeems to Ariſe that Great Protruſion of Hair obſervable in Brutes.

Why the Structure of the Animal Compages conſiſts of Fluids, and of Solids, which are Vaſcular.

LASTLY, This Theory of the Expanſive and Contractive Forces exactly Correſponds with what the Beſt and Lateſt Anatomiſts have Affirmed concerning the Compages and Structure of an Human Body, that it conſiſts Entirely of Fluids and Solids, the Laſt of which are ſaid to be Vaſcular; That is, however Solid any Part may Appear, it is yet Aſſerted, that it is nothing elſe than a Congeries of Hollow Tubes and Veſſels, containing ſome Liquors or other, which are Proper and Peculiar to them; For the Coats and Veſſels we have ſhewn are owing to the Contractive Force of the Nervous Syſtem, and poſſibly to the Internal Air of the Body, and the Humours of which they are the Conduits, to the Expanſive of the Blood and the Contractive of the Nervous Fluid, or to a Various Mixture of Them.

Concerning the Peritonæum *and* Omentum. Syllabus Enterologiæ. Dr. Gibſon's Anatomy, Chap. 4. & 5. Book 1. Dr. Keill's Chap. 2. §. 3. & 4. Dr. Drake's Anthropologia Nova, Chap. 6. & 8. Book 1. Blancard's Anatomia Reformata, Cap. 33. & 18.

§. 11. IN Diſſection, next under the *Membrana Adipoſa* and the *Membrana Communis Muſculorum* lie the Muſcles, of which there are Five Pair; the Outermoſt are the Obliquely Deſcending, the Next are the Obliquely Aſcending, the Third the *Recti*, the Fourth the Pyramidal, and the Inmoſt, the Tranſverſe; which being Removed, the *Peritonæum*, or the Inmoſt Coat of the Belly, called the Rim, Appears; It adheres above to the Midriff, below to the Share- and Flank- or Hip-Bones, in the Fore-Part firmly to the Tranſverſe Muſcles, but chiefly to their Tendons about the *Linea Alba*, behind to the Fleſhy Heads of theſe Muſcles looſely; It's Figure is Oval, and it's Subſtance Membranous, and it's Origin, according to *Fallopius* and Dr. *Gliſſon*, from the Superior and Inferior *Plexus* of Nerves of the *Abdomen*, for from them it cannot be Separated without Tearing; others Derive it from the Ligaments, by which the *Vertebræ* of the Loins and of the *Os Sacrum* are Knit one to another; It is Double every where, but appears to be ſo chiefly about the *Vertebræ* of the Loins, and in the *Hypogaſtrium*; For in the Former Place, between it's Membranes, lie the *Vena Cava*, the *Aorta*, the Receptacle of the Chyle and the Kidneys, and in the Latter the Bladder, and in Women the Womb.

IT has Three Perforations Above, where it adheres to the Diaphragm or Midriff; the Firſt on the Right-Side, for the Aſcending Trunk of the *Vena Cava*; the Second on the Left, for the Gullet to Deſcend by, with the Nerves of the Eighth Pair inſerted into the upper Orifice of the Stomach; and the Third, for the Deſcent of the *Aorta* and the Intercoſtal Nerve, and for the Aſcent of the *Ductus Chyliferus*; Below, It has Paſſages for the *Rectum*, for the Neck of the Bladder, and in Women for the *Vagina* of the Womb, Alſo for the Veins, Arteries, and Nerves, which Proceed down to the Thighs; Before, in a *Fœtus*, for the Umbilical Veſſels; It has likewiſe Two Proceſſes in Men, and Two in Women, which there is no occaſion here particularly to Deſcribe; If it breaks, it Cauſes a Rupture, either in the Groin or Navel: It has Arteries and Veins from the Mammary and Epigaſtrick, and from thoſe of the Midriff, and from the Phrenick, and has ſlender Threads of Nerves from the Pairs of the *Vertebræ* of the Loins, and it's uſe is to aſſiſt the Contraction of the Belly for the Expulſion of the Excrements, and to give an outer Coat to the Inteſtines.

UNDER the *Peritonæum*, in Diſſection, lies the *Epiploon*, *Omentum*, or Caul; It is compoſed of Two Membranes, betwixt which the Veſſels run, and the Fat and Glands are placed; They are very Thin, and, where the Fat hinders not, Tranſparent; It is compared by Dr. *Gliſſon* to a Woman's Apron, when the lower Part of it is Turned up, and Tuck'd under the Girdle; For it conſiſts of Two Leaves, whoſe upper Edges are Knit to Different Parts, but it's Bottom is Continued, and Reſembles a Pouch or a Satchel; The Fore-Leaf, beginning at the little Lobe of the Liver, which it Involves, Adheres to the Under or Hollow Side of the ſaid Liver, to the *Duodenum* or *Pylorus*, ſomewhat towards their Fore-Side, to the Bottom of the Stomach, and to the Suture of the Spleen; The Hinder or Inner Leaf is Continued from the Former, and, Paſſing from the Spleen backwards, Returns towards the Right-Side again, by the way

Adhereing to the *Pancreas*, to that Part of the *Colon*, which runs under the Stomach, to the Back, to the *Duodenum*, ſomewhat towards it's Back-Side, and at laſt is Joined or Continued to the Fore-Leaf at the aforeſaid little Lobe of the Liver; as for it's Bottom, it is knit to no Part, but hangs Looſe upon the Inteſtines, Reaching commonly a little below the Navel, and ſometimes to the very Bottom of the Belly; It alſo Cleaves to the *Peritonæum*, it aboundeth with Veſſels of ſeveral Sorts, which do not Terminate in it, but are carried thro' it to other Parts, namely, Arteries, Veins, and Nerves, which it is not Requiſite here particularly to Deſcribe; *Malpighius* thinks he has Diſcovered a fourth Sort, which he calls *Ductus Adipoſi*, but this is very uncertain, and there is nothing but Conjecture and Diſputed Obſervations for them; According to Dr. *Wharton*, in his *Adenographia*, there are Lacteal Veſſels paſs thro' it from the Stomach to the Common Receptacle of the Chyle; It has likewiſe Glands, which are ſaid to contain much Fat, and into one of which, that is joined to the *Pylorus*, the Lacteals are Inſerted; The Fat is unequally Spread over it, and is gathered chiefly there, where the Veſſels run, the Spaces betwixt being only Membranous; It is Collected in little Membranous Cells, Included betwixt the Two Inveſting Membranes, and it's Matter is Derived from the Blood Veſſels, according to the Common Opinion, but according to *Malpighius* from the *Ductus Adipoſi*; the Fore-Leaf has more Fat than the Hinder, and the upper Part of the Fore-Leaf, that is knit to the Stomach, is the Fatteſt of it: In very Fat Perſons, this Part grows to a Great Bulk, but in Conſumptive, there is little left beſides the Membranes and Veſſels; And *Peyerus* ſays, that in all Hydropick Perſons, that he ever Diſſected, the Caul was always Extenuated and Putrid.

The Uſes of it are, To Cheriſh the Heat of the lower Part of the Stomach, and the Inteſtines; To Miniſter Nouriſhment to the Parts, in a long Want of Food, it's Fat being received into the Veins, and mixt with the Blood; To convey Veſſels ſafely to other Parts; To keep the Inteſtines Moiſt and Glib; And to knit looſely together, the Stomach, Liver, Spleen, *Pancreas*, *Colon*, and *Duodenum*.

Thus alſo the *Syllabus* divides the *Peritonæum* into the *Lamina Exterior* and *Interior*, the *Proceſſus*, from whence is derived the *Tunica Elythroides*, or the *Vaginalis Teſtium*, and the Umbilical Veſſels, that is, the Veins, Arteries, and *Urachus*, And the *Omentum* into the *Ala Superior* and *Inferior*, the *Burſa*, the *Lamellæ*, the *Adeps in Cellulis*, and the Veſſels, namely, the Arteries, Veins, and Nerves.

§. 12. This is the Account, which the Anatomiſts give of the *Peritonæum* and *Omentum*; We have therefore, in Philoſophy, ſcarcely any Reaſon to add more, than what has been ſaid concerning the Fat and the Membranes; however we ſhall go on to ſhew, how theſe Phænomenons of an Human Body Conſiſt and Agree with our Doctrine of the Expanſive and Contractive Forces.

The ſeveral Phænomenons ſolved Relating to Them.

The Muſcles lie under the Adipoſe Membrane, and are entirely Formed, as to the Subſtance of Them, of Nervous Fibres and an Heap of Capillary and Infinitely ſmall Blood Veſſels; For the External Air, being now hinder'd by the Fat, and it's Membrane, from Proceeding any farther into the Human Frame, the Syſtems of the Expanſive and Contractive Forces of the Sanguineous and Nervous Matter are at full Liberty to Exert themſelves againſt each other; and by that Means are Spread and Diffuſed into a Thouſand Streams and Rivulets of Blood on the one Hand, whilſt the Nervous and Contractive Matter on the other Surrounds and Inveſts them with Coats, againſt which the Expanſive Force of the Blood Acting ſhapes them into Round and Hollow Canals, and the Contractive Force of the Nervous Matter Repelling ſuch Expanſion, the Blood becomes Progreſſive, and the Canals Tubular; whilſt in the mean Time the Nervous Matter is likewiſe carried in Oppoſition to the Sanguineous into Innumerable Filaments and Threads, which have alſo a Force and Power of Contraction.

How the Muſcles, lying over the Peritonæum, *are Formed.*

How the Blood Veſſels are Produced and why Tubular.

And from hence is derived the True and Natural Action of a Muſcle, which is the ſame, as that of the Heart, or any Artery, namely, that of a Syſtole and a Diaſtole, or of a Contraction and Dilatation; the Firſt Ariſing from the Cold or Contractive Force of the Nervous Matter, and the Laſt from the Hot or Expanſive of the Sanguineous; and it is a wrong Opinion of Anatomiſts, to Imagine, that the Proper Action of a Muſcle is Contraction only, ſince Dilatation is as Peculiar and Eſſential to it;

The Action of the Muſcles, whence derived, and what.

the One Proceeding from the Real Contractive Force of the Fibres of the Muſcle and the Coats of the little Blood-Veſſels, that is, from the Contractive of the Solids, and the Other from the Expanſive of the Blood, and the Fluids.

Why we are more Lively and Brisk in Cold Weather, than in Hot.

THIS Theory of the Muſcles again, ſuitable to what we have ſaid before, gives us a Reaſon, why Cold and Froſty Weather makes us more Active and Lively, as alſo Cold Bathing; when Hot and Sultry Weather, or the Warmth of a Fire, or the uſe of Spirituous Liquors, Renders us more Faint, Languid, and Unactive; Becauſe the Firſt Strengthens and Corroborates the Contractive Force of the Nervous Syſtem, and the Laſt Weakens and Diſſolves it.

How the Peritonæum *is Cauſed.*

UNDER the Muſcles, when Removed, the *Peritonæum* is Diſcovered, which, being a Membrane, does chiefly conſiſt of a Contractive Force, and is Evident from the Origin, which Anatomiſts aſcribe to it, that is, the Superior and Inferior *Plexus* of Nerves of the *Abdomen*, and the Ligaments, by which the *Vertebræ* of the Loins, and of the *Os Sacrum* are Knit one to another; by it's Adhereing to the Midriff, and to the Share- and Hip-Bones, and to the Tendons of the Tranſverſe Muſcles, which are all of a Cold, Solid, and a Contractive Nature; and by it's Looſe Adheſion to the Fleſhy Heads of the ſame Muſcles, which are of a more Sanguineous and Expanſive one; The Action alſo is from hence Contractive, by which it is aſſiſting to the Periſtaltick Motion of the Inteſtines, and to the Expulſion of the Excrements from Them: The Contractive Force of the Internal Air, it is Probable, obtains ſome Effect in the Conſtitution and Conſolidation of this Membrane, but being leſs Contractive than the External Air, and the Expanſive Force of Heat from the Blood being Greater, and the Contractive Force of the Nervous Syſtem being ſpread thro' it, It is of a Tenderer Nature than the *Cuticula*, or outward Skin, as well as of a more Senſible.

Why it is Double about the Vertebræ *of the Loins more eſpecially.*

IT is Double, eſpecially about the *Vertebræ* of the Loins, and the *Hypogaſtrium*, from a neceſſary Diviſion of it by Superior Forces; in the Former Place, by the Expanſive Force of the Blood in the *Aorta*, and the *Vena Cava*, and of the Chyle, in the Receptacle of it, by the Blood alſo in the Emulgent Arteries, and the Expulſion of the Serum by the Kidneys; and in the Latter, by the Deſcent of the Serum into the Bladder, and ſo to it's Excretion from the Human Syſtem.

The Reaſon of it's Perforations.

IT's Perforations are owing to the ſame Cauſe of Superior Forces, the Expanſive Force of the Blood in the *Aorta* and *Vena Cava*, of the Protruſion of Meat thro' the Gullet, and of the Chyle thro' the *Ductus Chyliferus*, of the Expulſion of Excrement thro' the *Rectum*, of the Blood which paſſes down thro' the Arteries to the Thighs, and up thro' the Veins from thence, the Nerves by it's Oppoſite and Contractive Force continually Attending them, and laſtly, of the Blood in the Umbilical Veſſels; And by a Superior Force, Ruptures are Cauſed, or an Enlargement of certain Perforations: It gives an outer Coat to the Inteſtines from a Diffuſion of it's Contractive Force, by which thoſe Coats likewiſe obtain their Periſtaltick Motion, as will be ſhewn afterwards.

The Phænomenons of the Omentum *Explained.*

THE *Epiploon, Omentum*, or Caul, lies next under the *Peritonæum*, and immediately covers the Inteſtines, which conſiſts of Fat and a Membrane, the Laſt of which is derived from a Diffuſion or Extenſion of the Coats of the Numerous Veſſels, which Paſs thro' it, which makes it ſo Exceeding Thin, and from the Coats of the Entrails to which it Adheres; The Firſt ſeems to be made, as before, from a Mixture of the Expanſive Force of the Blood, and of the Contractive of the Nervous Matter, and of the Internal Air, which is the Reaſon, that it is chiefly gathered there, where the Veſſels Run, and is found in the Greateſt Plenty in the upper Part of the Fore-Leaf, where it is Joined to the Stomach, which Abounds with Veſſels of it's own, and is Repleniſh'd with Air, which Enters with the Aliment; There is little of it in Conſumptive Perſons, becauſe the Expanſive Force of the Blood, and the Contractive of the Nervous Matter, is too much Weakened and Impair'd to be capable of Producing it; and it is Groſſer, and leſs Oily and Fluid, than the Fat of the Adipoſe Membrane, upon the Account of the Internal Air being more flat and leſs Spirituous and Springy than the External, and of the Contractive Force of the Nervous Matter being Greater; from which firſt Cauſe it is, that there are fewer Veſicles or Bladders in This, than in That.

CHAP. IV.

Concerning the Oeſophagus *or Gullet, the* Ventricle *or Stomach, the* Inteſtines *or Guts; and of the Mouth and Palate.*

§. 1. AFTER the *Abdomen* is laid open, and the *Omentum* is taken off, the Inteſtines appear, which being continued to the Stomach, and That to the Gullet, it will be neceſſary to Diſcourſe of Them together.

An Account of the Gullet, Stomach, Guts, and of the Mouth and Palate, by Anatomiſts. Syllabus Enterologiæ. Prælect. 2. Dr. Gibſon's Anatomy, Chap. 6, 7, 8. Book 1. Dr. Keill's Anatomy, § 5, 6, 7. Chap. 2. Dr. Drake's Anthropologia Nova, Chap. 9, 10, 11. Book 1. Blancard's Anatomia Reformata, Cap. 19, 20, 21. Tauvry's Rational Anatomy, Chap. 1, 2, 3. Part 1.

The Gullet.

THE *Oeſophagus, Stomachus, Gula,* or Gullet, begins at the Root of the Tongue behind the *Larynx,* which Part of it is called the *Pharynx,* and Deſcends from thence between the Wind-pipe and the *Vertebræ* of the Neck, and the Four Firſt *Vertebræ* of the *Thorax,* in a Direct Line; at the Fifth *Vertebra* it bends a little to the Right-ſide, by giving way to the Deſcending Trunk of the Great Artery, which it Accompanies to the Ninth *Vertebra,* and then Turns a little to the Left and Climbs upon the Artery, and about the Eleventh *Vertebra* Paſſes thro' the Midriff a little on the Left-ſide of it's Nervous Centre, at a Hole Diſtinct from the Great Artery, and is Inſerted or Continued unto the Left Orifice of the *Ventricle* or Stomach, which is Situate immediately under the Midriff; in the Fore-part on the Right-ſide it is Covered with the Hollow-ſide of the Liver, on the Left-ſide it is Touched by the Spleen; it's Back-ſide upon full Feeding bears againſt the *Vena Cava* and *Spine,* and to it's Bottom Length-ways is the Caul knit; the Back-ſide of it's Bottom reſteth on the *Pancreas,* and the Fore-ſide on the *Colon,* all which promote it's Heat; It is Longiſh and Pretty Round, very much Reſembling a Bag-pipe in Shape; It hangs Croſs the Body, and it's Two Orifices, by which the Meat is Received in from the Gullet, and let out into the Guts, ſtand higher up than it's Middle, ſo that it's upper Part makes, as it were, the Concave of an Half-Moon. The *Cardia,* or Left Orifice, hath Orbicular Fibres, that it might be exactly ſhut leſt Fumes, and the Heat, ſhould break out to the Hindrance of Concoction, and Annoyance of the Head. The *Pylorus* has it's Inmoſt and Nervous Coat full of Wrinkles, and it's Middle or Carnous has Two Ranks of Fibres, Annular or Tranſverſe, to ſtreighten or ſhut up this Paſſage upon Occaſion, and ſuch as run Length-ways, which draw the reſt of the Stomach towards this Door, for the Diſtribution of the Chyle into the *Duodenum;* but if they draw the Contrary way, Vomiting is Produced, and the Choler and Pancreatick Juice ſometimes pumpt into the Stomach.

The Stomach.

THE *Ventricle,* or Stomach, is more Capacious on the Left-ſide than on the Right; It's Left or Upper Orifice is Continued from the Gullet, by means whereof it is knit to the Midriff; and it's Right or Lower Orifice is Continued to the *Duodenum;* It's Bottom, in the whole Length of it, has the upper Edge of the Fore-Leaf of the Caul adhereing to it, by whoſe Mediation it is joined to the Liver, Back, Spleen, *Colon* and *Pancreas;* the Upper Orifice is called *Cardia,* and the Lower is termed *Pylorus.*

The Guts.

THE *Inteſtines* or Guts are Continued from the *Pylorus* to the *Anus,* and are knit together by the *Meſentery,* by which, and by the Intervention of the *Omentum,* part of which Adheres to the Beginning of the *Duodenum* and the Middle Part of the *Colon,* they are tied to the Back, and fill the Greater Part of the Cavity of the *Abdomen;* and the Length of them is about Six Times the Perſon's Length, whoſe they are; They have a Periſtaltick, Vermicular, or Worm-like Motion, which begins at the Left Orifice of the Stomach, and is Promoted quite thro' them.

THEY are Divided into Two Sorts, the Thin or the Small, and the Thick or the Large; the Firſt are placed Above, are Long, Equal, Round, Rolled about in Wreaths, Full of Wrinkles, Interwoven with Innumerable Veins, and Chyle is found in them; On the Contrary, the Thick are placed Below, are Short, Anfractuous, without Twiſting Wreaths, Endued with Fewer Veins, and the Wrinkles or Folds in the Guts are Expanded, and there Reſult from them certain Receſſes or Cells, and the *Fæces* are found in them; The Thin poſſeſs the Umbilical *Region* and *Hypogaſtrium,* and are diſtinguiſhed into the *Duodenum, Jejunum,* and *Ileon;* The Thick are the *Cæcum, Colon,* and *Rectum,* at the Extremity of which is placed the *Sphincter.*

The Duodenum. THE *Duodenum* is ſo called, from being ſuppoſed by the Antients to be about Twelve Inches Long, which in Sheep is ſo, but in Men not above Four Fingers; It reaches to the *Pancreas*, and is not knit to the *Meſentery* but the *Omentum*; It is Thicker in it's Membranes, than the *Jejunum* and *Ileum*, but it's Paſſage ſtraiter; Towards it's Lower End it has commonly Two Ducts Leading into it, the *Ductus Choledochus Communis*, by which the Bile enters this Gut from the Liver, and a little below the *Ductus Pancreaticus*, by which the Pancreatick Juice is convey'd into it from the *Pancreas* or Sweet-Bread; tho' ſometimes theſe Two Ducts are joined into one, and open by one Mouth into this Inteſtine.

The Jejunum. THE *Jejunum* is ſo called, becauſe it is for the moſt Part empty, both upon the Account of the Multitude of Lacteals, which are Inſerted into it, by which the Chyle is carried off from it, and of the Fermentation of the Acrimonious Choler with the Pancreatick Juice, which are brought into the Inteſtines juſt before it's Beginning; It is about Eight Feet in Length, and begins on the Right-ſide, under the *Colon*, where the *Duodenum* Ends, and the Guts begin to be Wreathed, and is Continued to the *Ileum*; From which it is Diſtinguiſhed by it's Emptineſs, and by it's Greater Number of Veins and Arteries, from which it looks Reddiſh; as alſo by the nearneſs of the Folds or Wrinkles of it's Inmoſt Coat one to another, which are but about half an Inch Diſtant, whereas in the *Ileum* they are a whole Inch, or more; and laſtly, by it's Coats, which are Thicker than thoſe of the *Ileum*.

The Ileum. THE *Ileum*, ἀπὸ τȣ̃ εἰλεῖσθαι, is ſo called from it's many Turnings and Windings, is in Length about Twenty Feet, and is the Narroweſt of all, being about a Finger's Breadth in Diameter; It is Diſtinguiſhed from the *Cæcum* or *Colon*, which make one continued or ſtrait Canal, the Lower-ſide of which the *Ileon* aſcending Pierces, and Enters Tranſverſely to it, and into the Latter whereof it's Inner Coat hangs looſely the Length of half an Inch, Forming the Valve it ſelf of the *Colon*, and is the Limit, that divides the *Cæcum* from it: When this Inteſtine falls into the *Scrotum* it Cauſes a Rupture, and when there is a Contortion of it, or an Obſtruction in it, or an Intruſion of one Part into another, the Iliack Paſſion, in which the Excrement is voided by the Mouth.

The Valvulæ Conniventes. THE Wrinkles in the Inſide of theſe Inteſtines are, what the Modern Anatomiſts Term the *Valvulæ Conniventes*, which are made from the Inmoſt Coats being longer than the Two other, and which therefore ſtreighten the Paſſage, and detain the Chyle, that the Lacteals may have more Time to Imbibe it; Theſe Wrinkles are neceſſary, that the Inmoſt Coat may comply with the Outward and Muſcular, upon it's Diſtention and Dilatation, which, when Contracted, Determines it into theſe *Plicæ* or Folds, the Inner Coat being, like the Outward Skin or *Cuticula*, without any Senſe or Motion in it ſelf, as having neither Nervous nor Fleſhy Fibres.

The Cæcum. OF the Thick Guts the *Cæcum* is the Firſt, being only a Continuation of the *Colon*, and is called the *Blind*, becauſe one End of it is Cloſed, and the other is Joined with the Cavity of the *Colon* by a large Mouth, which opens into it, tho' in Men it is only about as Thick, and half as Long, as a Larger Earth-Worm ſtretched out at Length; It is Bigger in Infants than in Men, and in a *Fœtus* is a Repoſitory for the *Fæces*, and in Sound Perſons is generally Empty; In Four-footed Beaſts it is always full of Excrements, in Apes it is Larger than in Men, in Dogs than in Apes, and in Conies, Squirels, and Rats, Largeſt of all, in Proportion to their Bodies.

The Colon. THE *Colon*, ſo called from κῶλον or κοῖλον, becauſe it is the Holloweſt and Wideſt, or ἀπὸ τȣ̃ κωλύειν, from Detaining the Excrements, begins at the *Os Ilium* on the Right-ſide, and Aſcending by it's *Spine*, Arrives at the Right Kidney, to which Parts it is joined by a Membranous Connexion; From thence bending Left-ways it Creeps under the Liver, by the Gall-Bladder, to the Bottom of the Stomach, to the whole Length of which it is tied by the Mediation of the Fore-Leaf of the Caul, as it is to the *Pancreas* and Loins by the Mediation of the Hinder-Leaf; Then it comes to the Lower Part of the Spleen, to which it is likewiſe Joined, and afterwards Touching and firmly Adhereing to the Left Kidney by Fibres, it comes to the Left *Os Ilium*, from which Deſcending by the Left Groin to the *Pelvis*, it embraceth the Bottom of the Bladder behind on each Side; Then it Aſcends upwards by the Right Groin near the Place, from whence it Firſt took it's Riſe, and thence Marching back again towards the Left-ſide, and Running it ſelf in betwixt the *Ileum* and the Back-Bone, it Reaches to the Top of the *Os Sacrum*, and there Unloads it ſelf into the *Rectum*; It is about Seven Feet Long, and goes almoſt quite Round the *Abdomen*, next to the Muſcles, that it may better be Compreſſed by Them for Avoidance of the Excrements; It is Furrowed on the Outſide, and Ridged in the Inner, by which it becomes Divided into Cells, which Proceeds from the Inmoſt Coat Exceeding the Outward in Length, but chiefly from the Ligament, which is half an Inch Broad, that Runneth on the Upper and Middle Part of this Gut, and is much ſhorter than the Membranes of it; Theſe Cells hinder the Excrements from flowing into one Place, which would Compreſs the Parts Adjacent, and keep the *Fæces* from Deſcending too haſtily, that we may not have a continual Need of going to Stool; On it's Outſide, eſpecially from it's Paſſing by the Spleen to it's Joining to the *Rectum*, it has a Great many Fatty Knots, which ſerve to Moiſten and Lubricate it, that the *Fæces* may Paſs more eaſily; The *Rectum* has the like for the ſame Reaſon: The *Valve*, which, as before Expreſſed, is the Inmoſt Coat of the *Ileum* Propending or Hanging out Flaggy into the *Colon*, Cauſes that Flatuoſities cannot Aſcend from this Latter into the Former, much leſs Excrements Regurgitate; but if the Periſtaltick Motion be Inverted, as in the Iliack Paſſion, ſo that the Inmoſt Coat of the *Ileum* be drawn back from it's Depending into the *Colon*, the Excrements of the *Colon*, and Clyſters themſelves, may Aſcend up thro' the Small Guts into the Stomach, and be thrown up by Vomit.

The Rectum. THE *Inteſtinum Rectum*, or Strait Gut, begins at the Firſt *Vertebra* of the *Os Sacrum*, where the *Colon* Ends, and Paſſes directly Downwards to the Extremity of the *Coccyx*; 'Tis faſt tied, on it's Back-ſide, to both theſe Bones, by Mediation of the *Peritonæum*, to keep it from Falling out; and on it's Fore-ſide in Men it grows to the Neck of the Bladder, from whence, in the Pain of the Stone there, a *Teneſmus* or a Continual Inclination to go to Stool happens, but there is a Muſculous Subſtance comes between; It is a Foot in Length, not ſo Wide as the *Colon*, but it's Membranes Thicker.

The Sphincter. THE *Sphincter* Muſcle is placed at the very Extremity of the *Rectum*, by which it is cloſe Purſed up to hinder the Continual Exit of the *Fæces*, and is Forced open by a Voluntary Relaxation, and the Compreſſion of the Guts, and their Contents, by the Contraction of the *Abdominal* Muſcles; by too much Straining, or by Aloetick Purges, the *Rectum* ſometimes is ſo far Protruded, as to require an Artificial putting up again.

Three Tunicks. THIS is a ſhort Account from Anatomiſts of the Inteſtinal Paſſage from the *Pharynx* to the *Anus*, as to the Windings, Turnings, and the Various Situation of it; The next Thing to be Deſcribed is the Subſtance and Contexture of it; And in this all the ſeveral Parts of it Agree, *viz.* in being Compoſed of Three Tunicks or Coats, the Common, the Muſcular, and the Nervous or Tendinous, which laſt conſiſts of Blood-Veſſels, Little-Glands, and a Cruſt or a kind of *Cuticula*, with a ſort of Down or Velvet upon it; beſides which, the Stomach and Inteſtines, in General, have their Blood-Veſſels, their Nerves, and their Secretory Ducts.

The Common. THE outmoſt Tunick, or the Common, is very Thin, being only Endued with Membranous Fibres, and thoſe very Slender, and in the Gullet is ſuppoſed to be Derived from the Midriff, or the *Pleura*, or the *Peritonæum*, or from the Ligaments of the *Vertebræ* of the Neck and Breaſt, upon which it Reſteth, or from all of them: In the Stomach, it's Fibres, being Nervous, are Strait, Running from one Orifice to the other, and Encompaſſing both it's Bottom and Sides in their whole Longitude, and they Croſs, at Right Angles, the Carnous ones lying next under: In the Inteſtines it is of the ſame Fibrous and Membranous Texture, and is Derived mediately from the *Peritonæum*, but immediately from the Membranes of the *Omentum*, and of the *Meſentery*.

The Muſcular. THE Second Tunick, which lies under the Former, is the Carnous or Muſcular, which, In the Gullet, conſiſts of Two Ranks of Fleſhy Fibres, which Aſcend and Deſcend obliquely, Spiral-wiſe, and do mutually Intercuſſate, or Croſs, one another, ſo as that the Fibre, which, before it met with another to Interſect, did lie underneath another, Rides upon that which it Interſects, and ſo continues uppermoſt, till it comes to a Second, under which it Paſſes again, and ſo on by Turns: In the Stomach,

mach, it alſo Conſiſts of Two Ranks of Fleſhy Fibres; the Outer of which are the more Numerous, and are Tranſverſe, Running Croſs, or Round, the Stomach; the Inner go ſtrait from the Left Orifice to the Right on the Top of the Stomach, and the Remainder of Them tend along each Side of it, obliquely forwards toward the Bottom, and meet one another there: In the Inteſtines, it likewiſe conſiſts of Two Ranks of Fleſhy and Moving Fibres, the Outer is of ſtrait ones, which reach along the whole Length of the Inteſtines, and cut the Circular ones under them at Right Angles, and their Tendon ſeems to be the outmoſt Coat, which being wholly Nervous, or as it were Tendinous, is wrapt about the whole Rank of theſe Fibres; the Inner is Annular or Tranſverſe, or, as others, Spiral, which, Encompaſſing the whole Cavity of all the Inteſtines in very cloſe Order, is Inſerted into the Hem or Selvedge of the *Meſentery*, as into it's Tendon.

The Nervous. THE Third Tunick or Inmoſt is Nervous, Lined with a Velvet or Downy Subſtance, and, In the Gullet, the Fibres are ſtrait, according to the Ancients, but according to Dr. *Willis*, they are variouſly Interwoven one with another, it is continued to the Membrane, which covers the Mouth, Jaws, and Lips, and Deſcends Three Fingers Breadth below the Mouth of the Stomach, from whence Proceeds that Great Conſent among theſe Parts in Vomiting: In the Stomach, it is the ſame, Nervous, Lined with a Downy Cruſt, and it is Endued with all Sorts of Fibres, but the ſtrait are the moſt Numerous; It is beſet with abundance of Small Ring-like Glands, which appear, when the Down is taken off, which is called the Glandulous Coat, and which when Removed, the Nervous one is Diſcovered, along which Abundance of Arteries and Blood-Veſſels Run, that Terminate in it's Inner Superficies, which Adheres to the Glandulous Coat; There is alſo a *Parenchyma*, which ſome think wholly Glandulous, which is Diſperſed, eſpecially in it's Middle or Muſcular Coat, and is not Sanguineous, but of a Slimy Nature; it fills the Inequalities, which ariſe from the Texture of the Fibres, and is ſcraped off from the ſmall Guts in Great Plenty, by thoſe, who make Strings for Muſical Inſtruments; and it is Evident, the Subſtance of the Stomach and Guts is the ſame: In the Inteſtines, the Downy Coat ſerves as a Filtre for the Chyle, before it Enters into the *Vena Lactea*, and hinders Excoriation from ſharp Humours.

Blood-Veſſels, and Nerves. ALL theſe, The Gullet, Stomach, and Inteſtines, have Arteries, Veins, and Nerves, Communicated to them from the Adjacent Parts, and which are Spread and Diſperſed thro' their whole Subſtance.

Glands. THERE are alſo the Glandules, called the *Tonſillæ* or Almonds, which lie in the Throat, at the Root of the Tongue, next to the Gullet, and Two more on the Backſide of it, which have their Arteries, Veins, and Nerves, likewiſe belonging to them; The *Tonſillæ* are ſaid to be of a Yellowiſh Colour, and their Subſtance to be like Concreted Honey, only of a firmer Conſiſtence; They have a Large Duct or *Sinus*, which opens into the Mouth, and Diſcharges into it the Liquor Separated in Them.

IN the Inmoſt Coat of the Inteſtines, Anatomiſts, and *Peyerus* in Particular, have Obſerved abundance of Glands, beſides thoſe, which Form the Glandulous Coat of Dr. *Willis*; They appear like little Teats with a ſmall Pore at the Top towards the Cavity of the Gut, and their Baſes, or Back-ſides, have Blood-Veſſels and Nerves Inſerted into them, and there Iſſues out of the Pore a Clammiſh Humour; There are few of them in the *Duodenum* or *Jejunum*, and they are the moſt Numerous in the *Ileum*.

Actions. THE Two Chief Things obſervable in theſe Parts, are the Chylification of the Stomach, and the Periſtaltick Motion of the Inteſtines. This is a ſuccinct Account of the Inteſtinal Paſſage, as to the Situation, Subſtance, and Action of it; but ſince we are to Diſcourſe of Digeſtion, and the Chyle, before we leave this Subject, it will be Requiſite to ſubjoin here a Deſcription of the Palate and Mouth, to which the ſaid Inteſtinal Paſſage is continued, and which conſiſts of ſeveral Parts.

Concerning the Mouth and it's Parts. THE Teeth and Gums we omit, as commonly known.

The Palate. THE Palate, or Roof, of the Mouth is Concave like a Vault, and Formed in the *Os Sphenoides*, which is covered with a peculiar Glandulous Fleſh, and a Thick Tunicle full of little Holes for the *Saliva*, which is ſeparated by the Glands, to Paſs thro' into the Mouth; There are ſaid to Spring out of it Innumerable ſlender Ducts, which Perforating the Membrane make it like a Sieve.

The Uvula.

THE *Uvula* is a Red, Spongy, and Longiſh *Caruncle*, which, being ſomewhat Broad at it's Baſis, hangs down from the Middle of the Palate, where the Noſtrils open into the Mouth, with a Small but Bluntiſh End.

The Tongue.

THE Tongue has Two Membranes, The Outer, which covers the upper Part only, and is pretty Smooth in Men, but Rough in Brutes with Cartilaginous Prickles riſing out of it, and ſtanding, like the Teeth of Wool-Cards, towards the Root of the Tongue; The Inner covers the whole Tongue, is Thin and Soft, and has many *Papillæ* Protuberating out of it, which in the upper Part Intrude themſelves into the Pores of the Outer; this Inner *Malpighius* calls a Nervous and Papillar Body: The Subſtance of the Tongue is ſaid by ſome to be That of a Gland, by Others to be Peculiar to it ſelf, and by Dr. *Wharton*, *Spigelius*, and *Malpighius*, to be That of a True and Real Muſcle; the Laſt of whom Informs us, that immediately under the aforeſaid Membranes, there lie ſtrait Fleſhy Muſcular Fibres, whereby the Tongue is drawn Inwards, and Shortened, but that the Center of the Tongue conſiſts of a manifold Kind of Fibres, Long, Tranſverſe, and Oblique, which, Riding one upon another, are Interwoven like a Mat; and yet Dr. *Gibſon* thinks it cannot be Properly called a Muſcle, becauſe no Muſcle ſerves to Move it ſelf, but ſome other Part, and becauſe one Muſcle is not Moved by another, as the Tongue is by ſeveral Pair; It is Connected to the *Os Hyoides*, *Larynx*, and *Fauces*, and by a Membranous Ligament to the Parts under it; It has it's Veins, Arteries, and Nerves, derived into it from the Adjacent Places.

Glands, The Parotides.

BESIDES the Glands above-mentioned, there are the *Parotides*, the Conglobate, and Conglomerate, which are both Seated in the Hollow under the Ear, and were formerly Reputed Emunctories of the Brain; but now the Conglobate are ſuppoſed to ſeparate the *Lympha* from the Arterial Blood, and convey it by Lympheducts into the Jugular Veins; the Conglomerate have not only Inſerted into them Veins, Arteries, and Nerves, but there alſo Springs out of each a Salival Duct by many ſmall Roots, which Running along opens into the Cavity of the Mouth, into which it Diſcharges the *Saliva*, which it Receives from the *Parotis* of it's own Side, and is called *Ductus Salivalis Stenonianus* from it's Inventor *Steno*.

Nuckianæ.

THE Second Glands, called *Nuckianæ* from Dr. *Anthony Nuck* a Dutch-Man, are ſeated in the Orbit of the Skull, wherein the Eye is placed, their Figure is Various in Different Perſons, Oblong, Oval, or Triangular, they have Arteries, Veins, and Nerves Inſerted into them; Each has a Salival Duct, which Deſcending downwards comes to the upper Part of the Gum in the upper Jaw, near to the Second Double Tooth, Reckoning from behind forwards, where it Empties it ſelf very near that of *Steno*'s.

The Maxillar.

THE Third Glands are the *Maxillar*, which are External or Internal; The External are ſmall and of little Moment, they are ſeated outwardly about the Middle of the Lower Jaw, where an Artery, a Vein, and a Nerve, Aſcend into the Muſcles of the Face, they ſeparate a *Lympha*, and Convey it into the Neighbouring Jugular Veins; The Internal are ſeated immediately within the Lower Jaw, their Hinder-ſide is next to the *Parotides* and Jugular Glands, and is Thicker, Rounder, and Redder; They grow Thinner by Degrees, and are Extended betwixt the Jaw and the Muſcles of the Tongue, as far forward as to the Chin; They are Conglomerate, and have each a proper Salival Duct belonging to them, which Spring by many Roots out of the Thicker and Hinder Part of the Glands, and run ſtrait forwards to the Middle of the Chin, where they End juſt within the Gums, and have a certain *Papilla* affixt to their Orifice, whereby they can eaſily Diſcharge themſelves, and yet nothing Return out of the Mouth into them.

The Sublingual.

THE Laſt Glands are the *Sublingual*; They lie underneath the Tongue on each Side, and each of them ſends forth a Salival Duct, which, Running Parallel with thoſe of the Internal Maxillar Glands, open in the ſame *Papillæ*, but have a peculiar Orifice of their own, ſtraiter than that of the other.

THE uſe of all theſe Glands is to ſeparate the *Saliva*, and to convey it into the Mouth by the Salival Ducts; the Origin of the *Saliva* is ſaid to be moſt Probably derived from the Arterial Blood, the Serous Part of which, the Gland beſtowing a ſubacid Quality upon it, is made into Spittle, to the Compoſition whereof ſome think a Nervous Juice is Contributed, becauſe Larger and more Numerous Twiggs of Nerves are Communicated to the Glands, than to moſt other Parts, which yet have a more Exquiſite Senſe than theſe; againſt which Dr. *Nuck* alledges, that if the Nerve, that Runs to any Gland be either hard tied or cut in Sunder, the Secretion of the *Saliva* will not thereupon Ceaſe, but only will proceed more Slowly; The *Saliva* is eſteemed to be a Ferment, and by ſome to be an Univerſal one for Meats and Drinks, and it is Thought to have Acids, Volatile Salts, and Oleous Particles, in it.

How the Fœtus *is Formed in the Womb, by the Intervention of the Expanſive and Contractive Forces.*

How the Mouth is Formed.

§. 2. To Account in Philoſophy and Nature for the Cavity of the Mouth and the Inteſtinal Paſſage, we muſt ſuppoſe, that in a *Fœtus* in the Womb, which has a Brain, or a Nervous Subſtance chiefly from the *Semen*, or a Nervous Juice, Imparted by the Male Parent, and a Heart or a Red Beating Speck from the *Ovum* of the Female, as before Explained, which Acting againſt each other by their Contractive and Expanſive Forces, and Exciteing the Animal Motion in the Syſtem, that is a-Forming, Produces by Degrees that Syſtem, we muſt I ſay neceſſarily ſuppoſe, that there muſt be an Aliment to ſupport, and Mature the growing *Embryo*, which making it's way by it's Expanſive Force, opens a Paſſage to it ſelf, and is the occaſion of that *Foramen*, we call the Mouth; For there being now Two oppoſite Forces contending in the *Ovarium* the Contractive and Expanſive, and the Contractive being the ſtrongeſt, by which the *Ovum* is Impregnated and Actuated, and is brought down thro' the *Tubæ Fallopianæ* into the Womb, and the Human Syſtem endeavouring to Preſerve it's own Æquilibrium, the Expanſive Force of the Nutritious Juices in the Female Parent is Directed towards the Contractive, which is Derived into it from the Male, in order to Preſerve an Equal Balance betwixt them.

Why the Head and Brain in the Embryo, *is Larger in Proportion to the Reſt of the Parts of the Human Syſtem, and how the* Fœtus *is Expelled from the Womb.*

For which Reaſon the Head, and the Brain, in the *Embryo* is Exceeding Large in Proportion to the other Parts of the Body, that is, the Contractive Force is much Greater than the Expanſive; but the Expanſive Force of the Nutritious Juices of the Female Parent, being ſtill ſent in to Counterpoiſe the Contractive, the Body and it's ſeveral Members Increaſe, till the Contractive and Expanſive are *in Æquilibrio*, when the Infant is come to Maturity, and preſently after is by the Superior Expanſive Force of the ſaid Parent Expelled from the Womb.

Why the Mouth is in the Head, and at that Diſtance from the Fore head rather than in it.

From hence it is Manifeſt, why the Mouth is in the Head, and at that Diſtance from the Fore-head, rather than in it, becauſe the Contractive Force in the Brain is too Great to admit the Expanſive Force of the Nutritious Juices from the Mother, but yet the Expanſive Forces of thoſe Juices exert themſelves, as near to the Contractive, as Poſſible, in order to Balance and Oppoſe it.

How the Cavity of the Palate is Formed, and the Tongue.

The Nutritious Juices, Entering here, lie for ſome Time, and Expand themſelves againſt the Contractive of the Nervous, by which means the Cavity of the Mouth and Palate is Formed; and the *Os Sphenoides*, being the Boundary of theſe Forces, is Acted upon by both, and is beaten and hardened into a Cartilage, and from thence into a Bone: A way being now made for the Expanſive Force of the Nutritious Juice into the New Syſtem, there is a continual Acceſſion of it, by which the Former is protruded Downwards, and Forms the Tongue, which Acting againſt the Contractive, Adheres to the *Os Hyoides*, and afterwards by Reverberating againſt the Contractive Force of the Brain, and the Contractive Acting againſt it, gives it that Facility of Motion we Experience in it, and which is alſo the Reaſon, why the Under Jaw is Expanſive and Moveable, whilſt the Upper is Contractive and Quieſcent; The Nutritious Juices ſtill Continuing their Influx to Balance the Contractive Force of the Brain, Form the Fleſh of the Gums and Palate, which are ſtill oppoſed by the Contractive Forces of the Brain, from whence are Produced the Cartilages or Bones under them.

How the Nerves and Blood Veſſels betwixt the Heart and Brain are Formed.

And all theſe Nutritious Juices are ſtill Aſſiſtants, and come in, as Auxiliaries, to the Expanſive Force of the *Ovum*, and the Red-Beating Speck, the Heart, which is the Seat of the Expanſive, which in the mean Time Exerts it ſelf in Streams or Radius's againſt the Contractive of the Brain, whilſt that Oppoſes it, by ſending out Nerves and a Nervous Subſtance againſt it, by which the Nerves, and the Nervous Coats of the Blood-Veſſels betwixt the Heart and Brain are Formed, and the Blood is Incloſed and Reſtrained within them.

How the Inmoſt Coat of the Gullet, Stomach, and Inteſtines is Formed.

THE Aliment and the Expanſive Force of it Preſſing on, and not being able to Proceed upwards, by Reaſon of the Repulſion of the Contractive Force of the Brain, nor forwards, by Reaſon of the *Medulla Oblongata* and *Spinal* Marrow, which is a Dilatation and Extenſion of the Brain in order to Reſiſt the Expanſive Force from the Maternal Juices; and the Aliment alſo coming to the Aſſiſtance of the Heart, and it's Blood, and the Expanſive Force of them, Tends downwards, and Proceeds towards the Heart; The Contractive Force of the Brain and Nervous Subſtance at the ſame Time is not Idle, but Exerts it ſelf from the Brain, the above-mentioned *Medulla Oblongata*, and *Spinal* Marrow, againſt the ſaid Expanſive of the Aliment and Nutritious Juices, and by that Contraction Inveſts and Surrounds them with the Nervous Coat before Deſcribed, or the Inmoſt Coat of the Gullet, Stomach, and Inteſtines.

Concerning the Infant, when Born, and how the Down or Velvet is Induced upon the Innermoſt Coat of the Inteſtines.

§. 3. AFTERWARDS, when the Infant is Born, and Produced into the open Air, this Coat, being extremely Senſible and Tender, is the occaſion of thoſe frequent Pukeings and little Vomitings of Children, even upon Receiving only the Milk of the Mother into the Stomach, and of the Convulſions and Inquietudes ſo Common to them; but by Degrees from the Contractive Force of the Air, which Enters every Day with the Aliment, the *Cruſta* is Induced upon it, and a kind of Down or Velvet, from the Shootings out of the Nervous Subſtance of the Coat, which are hardened by the ſaid Contractive Force of the Air into Small Iceicles, which, together with the *Cruſta* thus Congealed and Condenſed by the Air, Form a kind of *Cuticula* to the Gullet, Stomach, and Inteſtines, which Defends the Nervous Coat from any Internal Injuries, as the Real *Cuticula* does the *Papillæ* of the Nerves of the *Cutis* from External ones; That the Air has ſuch a Contractive, and Congealing or Condenſing, Force is Evident, not only from it's Effects upon the *Cuticula*, but upon Bread, Clay, Mortar, Stone, and a Thouſand other Bodies, which it Reduces from Soft and Pulpous Subſtances into Hard and Conſiſtent ones, and even Fluid ones into Ice, where, as in Winter, the Contractive Force is Increas'd by an Abſence of the Sun's Expanſive, and the Greater Influence of the Moon's Contractive.

That the Moon has a Cold or Contractive Force, and is of a Globular Figure, Demonſtrated from a Total Eclipſe of the Sun.

WE ſuppoſe here the Moon to have a Cold and Contractive Force, not only for the Reaſons Alledged, *Book* 1. *Chap.* 5. §. 21. but becauſe in a Total Eclipſe of the Sun, there is an Apparent Coldneſs and Chillneſs Introduced into the Air, which cannot be ſo well Accounted for by a bare Interception of the Sun's Rays, which we Experience in the Warmeſt and Darkeſt Nights, as by an Actual Impreſſion of Cold from the very Body and Subſtance of the Moon, which was very Senſible in that Famous Eclipſe, which was Total at *Cambridge*, where I now Write, and which happened on the 22*d* of *April* 1715. betwixt 9 and 10 of the Clock in the Morning; And when, by Looking upon it thro' a Small Teleſcope, without any Smoak't Glaſs, My ſelf, as well as ſeveral Others, in the Garden of this College, Diſcerned the Body of the Moon, as it came under the Sun, to be not of a Flat, but, Manifeſtly and Apparently, of a Globular Figure, which perhaps will be ſcarcely Credited by our Preſent Philoſophers; Tho' there are Arguments enough to Think it ſhould be ſo; Since, when the Side of the Moon, which is Obverted towards us, is ſtrongly Irradiated, there is a Wide Diſtance of it's Extremeſt Parts Obſervable by us, as in the Full Moon; And why it ſhould not be, in the other Two Sides, where the Rays of the Sun Paſs by them to the Eye, and therefore Receive Impreſſions from them, cannot be eaſily Imagined, or why we ſhould not ſee the Moon Globular, as well as any other Nearer and Leſſer Globe would appear ſo, if Interpoſed betwixt us, and a Lamp or a Candle. There was nothing Remarkable here in the Laſt Eclipſe, *May* the 11*th* 1724. betwixt 5 and 6 in the Afternoon, by Reaſon of the Clouds, which Intervened, unleſs a Gloomy Darkneſs, like that before Snow, and ſomething of Cold, which ſtill Confirms the Cold and Contractive Force of the Moon. But to Return to Anatomy.

THAT this is ſo, is farther Evident, becauſe, if this Contractive Force in the *Cruſta*, and the Downy Coat, is Diſſolved by the Expanſive one of ſtrong Liquors, Vomitings in the Stomach, and Diſorders in the Inteſtines, and a Dejection of Appetite Enſue, which cannot be owing to any Thing elſe, excepting to the Inmoſt and Nervous Coat, being from thence made bare, and thereby rendered too Tender to admit

mit the Impreſſions of any Expanſive Force, even from the Common Aliments only, which we take in for our Suſtenance.

Concerning the Conteſt betwixt the Expanſive and Contractive Forces in Forming the Coats of the Inteſtines.

§. 4. THE next Tunick, or Coat, which lies Immediately above the Nervous, or Inmoſt, is the Carnous or Muſcular, which conſiſts chiefly of a Fleſhy, Sanguineous, and Expanſive Subſtance, and runs over the Nervous Coat, which is of a Contractive one in order to be a Balance and Poiſe to it, after which comes the outer Coat, which is of a Nervous and Tendinous, and therefore of a Contractive Nature, and there the Oppoſition Ends; in which the Contractive has the Prevalence, or Superiority, which is Derived from the Brain and from the Male Principle, as the *Ovum*, the Aliment, the Blood, and the Nutritious Juices, are the Female Principle of our Animal Conſtitutions: And this ſeems to be the Reaſon of this Alternation of Forces; Firſt, there is the Brain from the *Semen*, and the Heart or Red-Beating Speck from the *Ovum*, the one Contractive, the other Expanſive, which begin the Competition of the Two Forces, by which the Animal Syſtem is Formed; After which there is the Aliment or Nutritious Juice ſupplied from the Mother, which is Expanſive, to Balance which the Brain or Contractive Force Extends it ſelf into the Nervous and Inmoſt Coat; which is again Counter-Acted by the Expanſive Force of the Carnous and Muſcular Coat; to Balance which the Nervous and Tendinous, and Outer, is Formed, and then the Æquilibrium is Perfected; in which, as the Contractive Force began, ſo it Ends the Conteſt; when the Expanſive has ſufficiently ſupported it ſelf againſt it, by a continual Infuſion of New Aliment and Nutritious Juices, which are all of an Expanſive Force: The Outer, Nervous, or Tendinous, Coat is very Thin, becauſe the Contractive Force in this Ferment or Contraſt is now Spent and Exhauſted, and therefore is ſparingly Spread upon the Expanſive of the Carnous or Muſcular.

How Dr. Gliſſon's, *and* Steno's *Notions of Fibres are Accounted for by this Doctrine, and the Reaſon of the Different Direction of Them in the ſaid Coats.*

§. 5. The Fleſhy or Muſcular Fibres, which are obſerved in the Second or Carnous Coat, and the Different Direction of them, are alſo Derived from the Contractive Force of the Nervous Subſtance; upon which Account Dr. *Gliſſon* Defines them in a Muſcle, to be in Figure like a Thread, Slender, Tenacious, Tenſile, and Irritable, and made of Spermatical Matter, which Deſcriptions agree with the Contractive Force of the Nervous Subſtance; and that which is properly Fleſh, is either a Sanguineous *Parenchyma*, Flowing thro' the Intervals of the Fibres, and Coagulated by their Cold or Contractive Force; or elſe, according to *Steno*, the Fibres are Vaſcular, knit cloſely together in the Tendon, and hanging more looſely in the Fleſhy Parts, and filled with a Fluid, I ſuppoſe a Sanguineous one; Be it, which it will, here are Evidently the Foot-ſteps of the Two Contractive and Expanſive Forces Contending with each other, and the Contractive or Nervous is Confeſs'd to be of a Spermatick Nature, as the Expanſive, of a Sanguineous, in Fleſhy or Muſcular Fibres: As to the Directions of theſe Fibres, the Outer Rank, in the Inteſtines, is of Strait ones, and the Inner of Circular or Spiral, and, in the Gullet and Stomach, they are nearly Analogous to the above-ſaid Directions, either by Spirals Interſecting each other, as in the Gullet, or by Strait or Oblique ones Interſecting the Circular, as in the Stomach; The Reaſon of which ſeems to be, that as the Aliment and Nutritious Juice or Blood is Progreſſive Downwards, upon the Account mentioned above, and at the ſame Time, by it's Expanſive Force, Exerts it ſelf into a Circumference from a Center, ſo, accordingly, the Nervous Subſtance by it's Contractive Force, Attending and Oppoſing the Former, is carry'd down in Strait Fibres, and caſts it ſelf likewiſe into Circular or Spiral ones to Balance and Poiſe thoſe Strait and Circular, or Spiral, Actions in the Expanſive.

The Reaſon of the Increaſe of the Nervous Subſtance, ſo as to Form the Brain, the Medulla Oblongata, *&c.*

§. 6. As to the Increaſe of the Contractive Force of the Spermatick or Nervous Subſtance, by which it is Enlarged into the Brain, the *Medulla Oblongata*, the *Spinal* Marrow, the Nerves and Nervous Coats, that ſeems to be owing to the *Albugineous* Liquor, that Sweats out of the Inmoſt Membrane of the *Uterus*, which is a Nervous one, and is Imbibed by the *Chorion*, which is Soft and Spongy, and from thence Convey'd into the *Amnion*; as alſo by the prevailing Force of the Contractive, by which it Converts ſome of the Nutritious Juices into it's own Nature, as the Contractive Force of Cold, or the Expanſive of Heat can do other Subſtances, which is Evident in Froſt and Fire, by which thoſe Forces are Exemplified.

§. 7. AFTER

The foregoing Reaſonings Confirmed by Dr. Harvey's *Experiments.* Dr. Gibſon's Anatomy. Chap. 33. Book 1.

§. 7. AFTER I had gone thus far in Explaining theſe Matters upon my own Principles, and from what Appeared to Me to be Rational, I was ſurprized to ſee my Reaſonings ſo exactly Confirmed from the Obſervations of the Great and Celebrated Dr. *Harvey*, the Firſt Aſſertor of the Circulation of the Blood, who, Diſcourſing concerning the Order of the Generation of the Parts in a Chicken, Says; There appears at the very Firſt, a Red Leaping *Punctum* or Speck, a Beating Bladder, and Fibres Drawn from thence containing Blood in them, and, as much as one can Diſcern by Accurate Inſpection, Blood is made before the Leaping Speck is Formed, and the ſame is endued with Vital Heat, before it is ſtirred by the Pulſe; and becauſe the Beating Bladder and the Sanguineous Fibres, that are Produced from it, appear Firſt of all, it is Conſentaneous to Reaſon, that the Blood ſhould be before it's Receptacles; Namely, the Content before its Container, and that this is made for the ſake of the other; He confeſſes it to be a Paradox, that Blood ſhould be made and moved, and endued with Vital Spirit, before any Sanguifying or Motive Organs are in Being, and that the Body ſhould be Increaſed before the Organs appointed for Concoction; namely, before the Stomach and Bowels are Formed; but neither of theſe are Greater Paradoxes, than that there ſhould be Senſe and Motion in the *Fœtus*, before the Brain is Compoſed, and yet he ſays the *Fœtus* Moves, Contracts, and Stretches out it ſelf, when there is nothing Conſpicuous for a Brain, but clear Water; 'Tis alſo owned, that the Mouth, Gullet, Stomach, *&c.* are gradually Formed in a *Fœtus*, that the Nutritious Juice enters by the Mouth, that it Receives no Blood from the Mother, nor is nouriſh'd by it; all which agrees Perfectly with what has been before Argued from the Contractive and Expanſive Forces, and which Paradoxes I have Endeavoured to Explain.

The Reaſon of the Different Cavities of the Gullet and Stomach.

§. 8. THE next Thing to be Conſidered in the Mouth, Gullet, Stomach, and Inteſtines is their Various Cavities, that of the Palate and Mouth we have already Accounted for; The Aliments or Nutritious Juices, being Repuls'd from the Contractive Force of the Brain, and the *Medulla Oblongata*, and *Spinal* Marrow, Deſcend down towards the Heart, or the Red Beating Speck; and by the Contractive Force Inveſting them, the Coats of the Gullet are Formed, with a Cavity not Conſiderable, by Reaſon of the Perpendicular Deſcent of the Aliment; but when in it's Progreſs it comes near the Heart, it is Detained by the Expanſive Force in ſome Meaſure, by it's Inclination and Propenſity towards it, being of a like Nature with it, yet being Repelled by the Contractive, which Attends it, and which ſtill Inveſts it with Coats and Membranes, it in part Circulates about it, and makes the Figure of the Stomach Lunar from the *Cardia* to the *Pylorus*, with the Concave bending towards the Heart; This Detention of the Aliment in it's Paſſage muſt neceſſarily Amaſs it together in Greater Quantities, and conſequently muſt Enlarge the Cavity of the Stomach; and for the ſame Reaſon it is more Capacious on the Left ſide, which is neareſt the Heart, than on the Right: And that Forces of the ſame Nature, whether Expanſive or Contractive, do Propend and Incline to each other, is Plain from the Magnet, Electrical Subſtances, and Glaſs, Attracting, and being Attracted by, Bodies of the like Kind; From Fire Drawing Flame to it, and the Eaſy Propagation of it thro' Hot Bodies, and the more Difficult thro' Cold; as alſo from the Mutual Gravitation of Heavy Bodies to each other, which have a Contractive Force, and from the Eaſy Propagation of Froſt and Cold thro' Phlegmatick and Groſſer Subſtances, and the more Difficult thro' Hot and Fiery ones, which have an Expanſive.

The Reaſon of the Small Guts, Venæ Lacteæ, *and* Meſentery.

§. 9. THE Contractive Force now Prevailing, after the Expanſive of the Aliment is driven from the Heart, thro' the Stomach, to the *Pylorus*, it Streightens it's Coats, Thickens it's Folds and Wrinkles, and Preſſes the Chyle, the more Expanſive Force of the Aliments, thro' it's Sides, into new Derivations, which being ſtill ſurrounded by the Contractive, the *Venæ Lacteæ* are Produced, and the Guts become Small in the *Duodenum*, the *Jejunum*, and *Ileum*; And to Fortify and Strengthen the Contractive of the Guts, Freſh Auxiliaries are ſent out from the *Spinal* Marrow, which Joining to the Contractive Force of the Guts, which is of the ſame Nature with it, the *Meſentery* is Formed and Connected with the Inteſtines; On the Contrary, to Balance in ſome Meaſure theſe Contractive Forces, the Bile, an Hot and Expanſive Liquor, is conveyed into the *Duodenum* from the Liver, together with the Pancreatick Juice from the Sweet-Bread, which, as Sweet, is of an Expanſive Nature, and Innumerable

Veins and Arteries from the Heart are Interſpers'd in the *Jejunum* and *Ileum*, which ſerve to the ſame Purpoſe, all which, in Conjunction with the Aliments, keep the Contractive Force from cloſeing the Guts, and ſhutting up the Inteſtinal Paſſage, by their Oppoſite and Expanſive Forces, which was almoſt done at the End of the *Ileum*, which hangs Lank and Flaccid into the *Colon*, as before Deſcribed.

The Reaſon of the Large Guts, of the Expulſion of the Fæces, *and why the Male Principle Prevails againſt the Female.*

§. 10. The Aliments now coming into the *Colon*, after the former Conteſt is Ended, being Deprived of their Principal Expanſive Force, the Chyle, are Changed into *Fæces*, which being of a more Contractive Nature, the Contractive Force of the Coats Acts leſs againſt them, becomes more Compliant, Widens their Paſſage, and Expands their Wrinkles and Folds; and there are Fewer Blood-Veſſels, becauſe there is leſs Occaſion to ſupport, from the Heart, the Expanſive of the Aliment, which is now Fæculent and Groſs, neither fit for the Sanguineous, nor the Nervous Subſtance, and therefore Ready and Prepared for Expulſion by them both.

But it muſt be Obſerved, that the Contractive Force, has from the Firſt to the Laſt, from the Entering of the Aliment into the Mouth, to the Excretion of the *Fæces* from the *Rectum*, by it's Inveſting and Incloſeing it, by Repelling it from the Heart, by Contracting it in the Small Guts, and giving a Greater Space and Room to it in the Large, ſtill Prevailed; That is, the Male Principle above the Female; and which is Agreeable to what is certain Matter of Fact, that the Solids, which are of a Contractive Force, are Greater, in Proportion, than the Fluids in the Animal Syſtem, and that Children, generally ſpeaking, more Reſemble the Father in their Features and their Diſpoſitions than their Mother; tho' the *Albugineous* Matter from the *Uterus*, may, when very Contractive, and the *Ovum* from the Mother, when very Expanſive, have a Different Effect, and make them carry ſtronger Impreſſions of the Maternal Qualities, where the *Seminal* or Male Principle is Weaker.

The Reaſon of the Circumvolution of the Small Guts, and the Circuit of the Colon *thro' the* Abdomen.

§. 11. As for the Circumvolution of the Small Guts in the *Umbilical Region* and *Hypogaſtrium*, it is agreeable to the Expanſive Forces ſtill Exerting themſelves, and the Contractive Forces Acting againſt them, the Firſt of which, when Reſiſted by the Laſt, muſt of Courſe find out new Turns and Ambages to avoid the Inſults of the Former.

The *Colon*, which is the Beginning of the Large ones, takes the whole Circuit of the *Abdomen*; for the Remaining Parts of the Aliment having ſtill an Expanſive Force, and the Contractive Force being Diminiſh'd, Tend again towards the Heart, Aſcending up on the Right ſide of the *Abdomen*, then Bending to the Left, and Running along the Bottom of the Stomach, and being Repelled once more from the Heart, by the Contractive Force of the Coats, they now Deſcend down the Left ſide of the *Abdomen*, Paſs from thence to the Right, and preſently after are carry'd into the *Rectum*, where they are at Length finally Excluded from the Human Syſtem by the abovesaid Contractive Force.

The Reaſon of the Blood-Veſſels and Nerves, and the Nature of the Glands.

§. 12. And ſuitable to the foregoing Account of the Inteſtinal Paſſage, the Brain ſends it's Nerves to ſupport the Contractive Force, and the Heart it's Arteries and Veins to ſtrengthen the Expanſive, thro' the whole Canal. As to the Glands already Deſcribed, the *Tonſillæ* or Almonds, the *Parotides*, the *Maxillar*, and *Sublingual*, it is Plain they are chiefly of a Contractive Nature, both by their Conglobations and Conglomerations, by their being ſuppoſed to ſeparate the ſerous Matter of the Arterial Blood, and to beſtow an Acid Quality upon it; and by the Large and Numerous Twigs of Nerves, which enter into their Compoſition; The Glands therefore are ſtrong Efforts of the Contractive Force, Derived from the Brain and Nervous Syſtem, and the Arteries found in them are Efforts of the Heart, and the Sanguineous, which are but Small in the Compariſon; and the Liquor, or Fluid, ſeems on that Account to be of a Cold and Contractive Temper, which is Derived from them, and not Separated from the Blood, which only Enters them to Oppoſe and Qualify their Contractive Force, but Made and Diſtilled from the Brain, and Convey'd into the Mouth to oppoſe the Expanſive of the Aliment there; and from whence Ariſes the Fermentation ſo much Celebrated, and almoſt univerſally Affirmed by Anatomiſts, and is the Beginning of Chylification, which is afterwards Perfected in the Stomach, and the *Duodenum*; To give an Account of which we ſhall now Proceed.

How Chylification is Performed.

§. 13. IT muſt therefore be Firſt obſerved, that the Aliments, which we take in at our Mouths, are endued with a certain Degree of Force, in Order to become ſuch; If the Expanſive is too Great, as in Fire, or the Contractive, as in Metals or Stones, it is Evident, that Subſtances of this Nature are no ways Adapted to give us any Nouriſhment, ſince we can neither Eat Fire, nor Digeſt Gold or Marble.

Subſtances of too Expanſive or too Contractive a Nature not fit for Chylification, but thoſe of a Middle Temper betwixt both.

FARTHER, any Hot or Spirituous Liquors on the one Hand, or any Cold and Phlegmatick Juices, as thoſe of Cucumbers, ſome kinds of Melons, *&c.* on the Other, are alſo Prejudicial to the Animal Frame; From whence it is Plain, that Liquids and Solids, of a Middle Temper betwixt both, and which have a due Proportion of each, are the moſt Agreeable to our Natures and Conſtitutions; And ſo they ought to be, If what we have before alledg'd is true concerning them, which, we have ſaid, Conſiſt of an Equal Balance of the Expanſive and Contractive Forces.

Fruits better than Meats, and Meats Boiled or Roaſted than Raw.

THUS the Fruits of Trees, in our own Climate, are Generally eaſily Digeſted, becauſe, as they Receive a Contractive Force from the Earth, out of which they Spring, ſo do they likewiſe an Expanſive one from the Sun, by which they are Raiſed to a certain Elevation above the ſaid Earth; and if ſome of them are of too Contractive and Cold a Nature, they are found to be more Wholeſome, by an Application of the Expanſive Force of Fire to them; Thus alſo Grain, which has a leſs Expanſive Force, and Riſes to a leſs Height from the Earth, wants the Grinding and the Baking, which adds to that Expanſive, before it is thought to be a proper Nutriment for the Body; 'Tis the ſame in Salads, which are Eſteemed to be of too Cold and Contractive a Temper, unleſs ſomething Warm, as Oil and Muſtard, which are of an Expanſive Force, are Joined with them. And in Meats or Fleſh, it is Evident by what foregoes, that they Conſiſt of a Nervous and Sanguineous Subſtance, that is of a Contractive and Expanſive Force, and the Reaſon, why Fire is apply'd to them, in the Boiling or the Roaſting of them, in order to make them more Conſonant and Agreeable to the Animal Oeconomy, is to Exalt the Expanſive Force of them, which are too Contractive from their Nervous Fibres; Roaſted are more Spirituous, Expanſive, or ſooner Digeſted than Boiled, becauſe the Expanſive of Fire is Greater than that of Hot Water; Hence alſo it is, that Fiſh, which have a Cold and Contractive Nature, Require the Expanſive of Wine.

Small Beer or Small Ale Preferrable to Strong Ales or Wines, or to Water.

THUS alſo in Liquids, Water is Eſteemed of too Cold and Contractive a Temper, Strong Ales, Wines, Spirits and Brandies, of too Hot and Expanſive a one, and both of them Prejudicial to our Conſtitutions; whereas Small Beers, and Small Ales, and Cyders, and other Liquors, which are Gently Fermented, that is, which Conſiſt of a Small Contractive Force, as Water, and which are Exalted into a Moderate Degree of the Expanſive, by the Infuſion and Fermentation of Grain, or Fruits, with it, which from Vegetation and the Sun's Influence are Endued with an Expanſive, are Allowed to be the moſt Wholeſome, and the moſt Friendly to our Natures.

Wines Preferrable to Brandies, and Red Wines more Eligible than White.

AND for the ſame Reaſon, Wines are Preferrable to Spirits or Brandies, and the Red Wines more Eligible than the White; Becauſe the Former, we ſpeak of Red Wines and White, have a Greater Degree of the Contractive Force mixt with their Expanſive, that is, are Cooler, and Leſs Hot and Expanſive, than the Latter; which may not be only Collected from the Red, having more Tartar than the White, but from the Taſt, which is more Rough and Subacid, and not ſo Sweet, Hot, or Spirituous; from the Colour, which is Deep and Purpleiſh, and is an Indication, all other Things being alike, that the Force is leſs Violent and Active, the White or Yellow, on the other Hand, Explains the Force to be more Fiery and Expanſive; as alſo from the Effects, the Firſt is Coſtive, the Laſt, Diuretick and Laxative, the Firſt drank Soberly occaſions no Diſorders, but comforts the Head and Bowels, the Laſt Heats the Blood, makes ſtronger and more laſting Impreſſions on the Brain, and deſtroys the Tone of the Stomach; and both Spirituous Liquors and Wines are always better, when Mingled, and Qualified, with the more Contractive Force of Water: This therefore ſeems to be a ſettled and fixt Rule for the Aliment of the Animal Syſtem, that it ſhould be Compounded of a Moderate Contractive Force joined with a Moderate Expanſive; ſo that in every Thing, which we take in for our Suſtenance, there are the Foundations and Principles of Fermentation and Diſſolution, and conſequently, in ſome Meaſure, of Chylification.

How the Aliment acquires a Greater Degree of the Expanſive, and is Oppoſed by the Contractive, from whence a Fermentation Ariſes and the Saliva is Produced.

Afterwards, when the Aliment is Received into the Mouth, if Solid, it is Chewed, Comminuted, and Macerated in ſome Meaſure by the Teeth, and in the Grinding acquires a certain Degree of the Expanſive Force, which Promotes and Aſſiſts that, which is Innate to it; In Fluids, where the Expanſive Force is Greater, there is no occaſion for ſuch Maceration or Comminution; the Expanſive Force in both is likewiſe advanc'd by the Heat and Expanſive Force in the Mouth, ariſing from the Fleſhy Muſcles of the Tongue, and the Carnous and Sanguineous Subſtance, with which the whole Cavity of the Mouth is Lined, which is all of an Hot and Expanſive Nature; On the other Hand, from the Brain and the Glands before Deſcribed, there is a Juice Imparted thro' the Salival Ducts, and thro' the little Glands of the Palate, which is of a Cold and Contractive Temper, which being Heated and Warmed by the Expanſive above-mentioned cauſes a Gentle Ebullition and Fermentation, as Water Boils with a Fire, or an Acid Ferments with an *Alkali*, and Raiſes the *Saliva* into little Bubbles, and a Froth, which mixing with the Aliment, a new Diſſolution enſues from the Intimate Fermentation Produced betwixt theſe Two oppoſite Forces, and with which it Proceeds thro' the Gullet into the Stomach.

That the Juices, which Diſtill from the Brain and Glands, which we have ſhewn to have a Contractive Force, are likewiſe of a Contractive one, is manifeſt from the Saltneſs and Coldneſs of them, and that Conſiſtency, which is Obſervable in the Phlegmatick or Purulent Matter, which is Derived from them, and is Excreted by our Noſtrils, or our Mouths, and which is a certain Proof of what Kind and Temper thoſe Juices are, of which this is the Excrement.

How the Aliment is ſo ſoon Diſſolved by the Mixture of theſe Expanſive and Contractive Forces.

That theſe Juices, mixt with the Expanſive Force of the Aliments, have ſo quick and ſudden an Effect in the Diſſolution of them, Proceeds from their being, tho' of a Contractive Nature, yet Exalted into a Degree of the Expanſive by the Spirituous Force of the Brain, from whence they Flow, and from whence they become leſs Contractive than Common Water, and put on a more Forcible Quality in Diſſolving the Aliment, which is already Expanſive, than Spirit of Nitre has in the Solution of Metals, which are vaſtly Contractive; On the Contrary, the Expanſive Force of Heat from the Blood Operates more Effectually upon the Aliment to the Fuſeing of it, than Fire can do in the Boiling of Meat in Water, becauſe it's Expanſive Force Ariſing from Blood, is more Fitted and Adapted to Diſſolve Blood, or Fleſh, or any other Aliment of the like kind of Expanſive Force, than Fire is, which is too Strong, and too Diſproportioned to it.

Why Spirituous Liquors or Hot Herbs are Prejudicial to Chylification.

From hence we may be able to give a Reaſon, why Spirituous Liquors, and high Seaſoned Meats, and Smoaking or Chewing of hot Herbs, are Detrimental to Digeſtion, becauſe they Spoil and Deſtroy the Nervous, Spirituous, and Contractive Juices, which Flow into the Mouth from the Brain, and Raiſe the Expanſive of the Heat and Blood to too Exorbitant a Degree for an Equal and Juſt Fermentation with the Contractive.

Why the Salival Juices are not an Univerſal Menſtruum.

Hence alſo we may be Informed, that theſe Juices are not an Univerſal *Menſtruum*, or Diſſolvent, but Adapted only to Aliments, which have certain Degrees and Proportions of the Expanſive and Contractive Forces; Likewiſe, that theſe Juices, being Formed from the Brain, and being of a Gentle, Spirituous, and Contractive Force, and meeting with the Expanſive, and from thence Producing a Fermentation and Diſſolution of the Aliment, are not to be conſidered, as a Spirit of Nitre, ſo that any Injury ſhould Ariſe from them to the Coats of the Stomach; Becauſe their Spirituous and Contractive Force cannot be Greater than the Nervous Syſtem, from which they are Produced, and from whence the Coats of the Stomach are alſo made; whereas Spirit of Nitre is a vaſt Saline and Contractive Force, Exalted into a Prodigious Degree of the Expanſive, and thereby capable of Diſſolving Metals, which theſe Juices cannot Stir or Move; Beſides which, the Diſſolution of the Aliment is not Performed barely by the Diſſolving Forces of theſe Juices, as a *Menſtruum*, but by their Fermenting with the Expanſive Force of the Aliment, and the Heat of the Sanguineous Syſtem.

How Digeſtion is Performed in the Stomach.

The Stomach is now Cloſed up, and the Aliments, which conſiſt of Meats Boiled and Roaſted, that is, of a Moderate Expanſive and Contractive Force, Moiſtened with the Fermented *Saliva*, and Mixt with the Small Beer taken in at our Meals, which has alſo a Gentle Fermentaceous Quality, are left to Digeſt in the Cavity of it; The

Heat of the Heart above, and of the Inteſtines below, and particularly of the *Colon*, which is filled with a warm Excrement, and which runs along the Bottom of the Stomach, very much Contribute likewiſe to the Maceration and Diſſolution of the Aliments into a Pulp, and ſo into a Chyle, by their Expanſive Forces; That is, Firſt of all, there is a Principle of Diſſolution in the Solid Aliments themſelves, both from the Contractive Force being Small, and from the Expanſive Force, which is Contained in them; Second, being Received into the Mouth, they are Broken and Comminuted by the Teeth, by which the *Saliva* Enters into their Subſtance, and again Rends and Tears it by it's Contractive Salts, Actuated by it's Spirit, Moiſture, and Fluidity, which are Expanſive, and thereby Repreſents a Milder Spirit of Nitre, which is alſo a ſtronger Salt, or a Contractive, Raiſed into a ſtronger Spirit and Expanſive by Fire; Third, the Expanſive of the *Saliva* is Increas'd by Fermenting with the Heat of the Mouth and Palate, and the Diſſolution of the Solid Aliments is farther thereby Promoted; Fourth, the Liquids Poured into our Stomachs at our Meals, by their Fermentaceous and Expanſive Force, and by their Fluidity, forward the ſaid Diſſolution, and the Heat of the Stomach, Heart, and Bowels, by their Expanſive, Compleats it, and Turns the Whole into a White Chyle, like Barley Broth, or a Milky Liquor; For all Expanſive Forces Act towards a Diſſolution, as all Contractive ones towards a Coagulation; Attrition therefore has nothing to do in this whole Proceſs, which is only a Creature of that Abſurd Philoſophy, the Atomical or Corpuſcular.

Why the Chyle is White, and how it is Gradually Changed into an Higher Expanſive in it's Paſſage from the Stomach, and Jejunum, to the Subclavian *Vein.*

THE Chyle is White and Milky from an Equal Balance betwixt the Expanſive Forces and the Contractive, that is, betwixt the Oily or Expanſive Forces of the Aliments, and the Saline and Contractive of the Nervous Juices, from whence no Particular Colour enſues, but a White, which is Indifferent to all, as we have ſhewn in our Philoſophy of Light and Colours; Thus an High Spirit of Nitre, mixt with Oil-Olive, and digeſted in a Gentle Heat for ſome Days, becomes of the Colour of Marrow, and afterwards Changes into a White Fat or Butter, according to Dr. *Grew*'s Experiment before Recited: The Chyle, being thus Formed in the Stomach, Paſſes thro' the *Pylorus* into the *Duodenum*, at the Bottom of which, and before it's Entrance into the *Jejunum* and the Lacteal Veſſels, the Bile from the Liver, and the Pancreatick Juice from the *Pancreas*, or Sweet-Bread, mixes with it, which are both of an Expanſive Nature, and the Former of them of an Hot and Fiery Temper; the Expanſive Force of the Chyle is therefore ſtill Augmented, with which it Tends to the Common Receptacle of it, and from thence it is carry'd thro' the *Ductus Thoracicus* to the Subclavian Vein, and ſo thro' the *Vena Cava* to the Heart, where it becomes of the Higheſt Expanſive Nature, and is Transformed into Arterial Blood to be diſpens'd thro' the whole Body.

How it is from thence Exalted into Blood, and Proceeds to the Heart.

FROM which it is manifeſt, that as the Contractive Force of the Aliment is Diminiſh'd, and the Expanſive Increas'd, it gradually Tends to the Common Receptacle, and the Thoracick Duct, and from thence to the Venal Blood, which Raiſing again it's Expanſive Force in the Subclavian, it urges it's way thro' the *Cava* to the Heart, the Seat of the Expanſive; and which is a Confirmation of what we before Alledged, that the Expanſive Force of the Aliment in the Stomach Tended towards the Heart, but was hindered in it's Progreſs to it from the Contractive and Nervous Syſtem, which Formed the Coats of it; The more Expanſive Forces of the Aliment being Directed this way, the more Contractive conſtitute the *Fæces*, which remain in the *Colon*, and are Expelled by the Periſtaltick or Vermicular Motion of the Guts, which we ſhall now Conſider.

The Periſtaltick Motion of the Inteſtines Accounted for from this Doctrine.

§. 14. IT has been ſhewn, that the Coats of the Inteſtines are Alternately Muſcular and Fleſhy, or Nervous and Fibrous, that the Fleſhy are Interwoven with Nervous Fibres, and the Nervous with Fleſhy ones, that they are likewiſe Endued beſides with Nerves, and Blood-Veſſels, both Veins and Arteries; from which Temperature and Complexion of them, it is manifeſt, that they conſiſt of a Sanguineous or an Expanſive Force, and of a Nervous or Contractive, which being nearly in an Equal Balance, and Acting againſt each other from the neceſſity of their own Natures, the Firſt from the Center or towards the Surface of the Body, and the Laſt from the Surface of the Body or towards the Center, cauſe a Reciprocation of Motion, by which the Cavity of the Inteſtines is Enlarged and Contracted by Turns; from whence in the Firſt Caſe, the Contents are Permitted to Deſcend, or Urge forwards, being Protruded by an Acceſſion of new Aliment behind, and in the Laſt they are Squeezed and Compreſs'd, ſo that one Part of them Proceeds, and the other Retires back, which upon a Second Enlargement and Dilatation or Expanſion of the Inteſtines, is again Urged forwards, and ſo thro' the whole Inteſtinal Pipe or Canal; which gives alſo an Account of what *Peyerus* has obſerved in all his Live Diſſections, that there is a Reciprocation of Motion in the Guts, Upwards and Downwards, and of the acknowledged Undulation of them; The Aliment alſo, and the Fermentations it undergo's, as before Deſcribed, makes this Periſtaltick or Vermicular Motion ſucceſſive from one Part to another, as it is Promoted forwards by it's Expanſive and Fermentaceous Force, and the Protruſion of New.

This Motion Agreeable to that of the Arteries, Muſcles, &c.

SUCH a kind of Motion, as this, is likewiſe Agreeable to what will be afterwards ſhewn to be that of the Arteries and Muſcles, and the other Parts of the Body; being a Gentle Pulſation, Ariſing from the Action of leſs Expanſive and Contractive Forces, as the other is a more Strong and Violent one, Derived from Greater.

Why the Fibres of the Carnous Coat are Circular or Spiral, and Direct or Linear.

THE Fibres of the Carnous Coat are Circular or Spiral, and Direct or Linear; Becauſe the Motion of the Guts, being Expanſive and Contractive in Circles, and ſucceſſive, Forms thoſe Intermediate Circles or Spirals ſucceſſively, and being Progreſſive or Regreſſive the Direct or Linear ones; For the Contractive or Quieſcent Force, of which Nature the Fibres are, muſt poſſeſs the Intermediate Spaces of Action, as in a Ferment, the Parts, which are the moſt Groſs and Contractive, are Separated from the Fermentation; or in General, as a Leſs Force ſubmits to a Greater; and it is Evident in the Carnous Coat, where the Sanguineous or Expanſive Force Prevails, the Nervous or Contractive muſt ſubmit in thoſe Directions of Circular or Spiral, or ſtrait Lines, which are Preſcribed to it; And in the ſame Manner, the very Creſts or Lines in our Hands are made by the Contraction and Expanſion of them. From this Doctrine it follows, that the Inteſtinal Paſſage in Various Animals may be Infinitely Vary'd, and that in a Worm or Inſect it may be Strait, without the Stomach, or the Circumvolutions of the Inteſtines, there being nothing more Required than ſuch a Paſſage for the Aliment, or the Expanſive Force, to be Diſtributed to the Syſtem.

CHAP. V.

Concerning the Liver, the Gall-Bladder, the Spleen, and the Pancreas.

An Account of the Liver, the Gall-Bladder, the Spleen, and the Pancreas, *by Anatomiſts.*
Syllabus Enterologiæ. Prælect. 2.
Dr. Gibſon's Anatomy, Chap. 11, 12, 13, 14, 15, 16. Book 1.
Dr. Keill's Anatomy, §. 11, 12, 13. Chap. 2.
Dr. Drake's Anthropologia Nova. Chap. 15, 16, 17. Book 1.
Blancard's Anatomia Reformata, Cap. 23, 24, 25.
Tauvry's Rational Anatomy, Chap. 8, 9.

§. 1. THE Liver is Seated in the upper Part of the *Abdomen*, about a Finger's Breadth Diſtance from under the Diaphragm, or Midriff, which Parts the *Abdomen* from the *Thorax*, in the Right *Hypochondre* under the ſhort Ribs, which, being of a Great Bulk, it in a Manner fills, and Reaches from thence towards the Left ſide, a little beyond the *Cartilago Enſiformis*, or Pit of the Stomach; It's Upper Side is Convex or Round, and Smooth, the Under is Hollow lying on the Right ſide of the Stomach and *Pylorus*, it's lower Edge reaches below the ſhort Ribs on the Right ſide, and very near as Low as the Navel before. In Dogs, and many other Brutes, it is Divided into Diverſe Lobes, but in Men it is Undivided and Continuous.

IT has Three Ligaments; Firſt, The *Suſpenſorium*, which Suſpends or Ties it up to the Diaphragm, and is Broad, Membranous, and Strong, Ariſing from the *Peritonæum*, and forming the outer Membrane of the Liver, into which it Deſcends, and is ſtrongly Faſtened to the Common Sheath, or *Involucrum*, of the *Vena Cava*, there where the Umbilical Vein is continued to it; Second, The *Umbilicale*, or Umbilical Vein, which after the Birth Cloſes up, and Hardens into a Ligament; It is Directly oppoſite to the Former, Paſſes out of the Hollow ſide of the Liver by the *Porta*, and Terminates in the Navel; by this the Liver is kept from Aſcending, upon the Motion of the Diaphragm upwards in Reſpiration; Third, The *Latum*, by which the Liver adheres to the *Cartilago Enſiformis*, it is Thin, and Flaccid, but yet Strong, Broad, and Doubled, Ariſing from the Membrane, wherewith the Liver is Encompaſs'd, and is a Duplicature of it; This hinders the Liver from Fluctuating to the one or the other Side, or towards the Back.

The Situation, Ligaments, Coats, and Subſtance of the Liver.

Besides theſe Three Ligaments, it has ſeveral other Connexions to the Neighbouring Parts by the Veſſels, that come into it or go out of it; It's Common Membrane, which is very Thin, Springs, as has been ſaid, from the *Suſpenſorium*, and Cleaves firmly to the Subſtance of the Liver, it is ſometimes Separated from it by a Wateriſh Humour, Iſſuing out of the Capillary Veins, or Lymphaticks, whereby Wateriſh Puſtules are Ingendred, and if theſe Break, and the Water Falls into the Cavity of the Belly, it is in ſome Part the occaſion of the Dropſy, called *Aſcites*; It's Proper Tunick or Membrane is that, which Inveſts each Lobe and Gland, which belongs to it; It's Subſtance, according to *Malpighius*'s Obſervations by the Microſcope, to which the preſent Anatomiſts Agree, Conſiſts of Innumerable Lobules, which commonly have each of them Six Sides like a Die, and are Formed of ſeveral little Hexangular Glands like the Stones of Grapes, and are each Cloathed with a Proper Circumambient Membrane, and are Joined to one another by Membranes continued from the Circumambient, and Running Tranſverſely, yet ſo, as that certain *Rimulæ*, or little Chinks, Reſult from the joining of the Sides of the Lobules together.

Of the Blood-Veſſels and Bilary or Excretory ones, and that they have no Anaſtomoſes, as alſo of the Glands.

The Whole Liver is made up of theſe Little Grape-ſtone-like Glands, and of diverſe Sorts of Veſſels, and therefore Reaſon concludes it neceſſary, that there ſhould be a Commerce betwixt theſe Glands and Veſſels, tho' Senſe cannot Diſcern the very ſlender Extream Twigs of the Veſſels, that are Inſerted into the Glands; It is farther Obſerved by the ſame Perſon, that the little Branches of the *Porta*, *Cava*, and *Porus Bilarius*, do run thro' all, even the leaſt Lobules in an Equal Number, that the *Porta* does the Office of an Artery, and has ſo Great Society with the *Porus Bilarius*, that both their Twigs are ſtraitly tied together in the ſame Cover; Alſo, that the Shoots of the ſaid Veſſels are not joined by *Anaſtomoſes*, by which the one opens into the other, but that the Grape-ſtone-like Glands are a Medium between the Importing and Exporting Veſſels, ſo that by the Interpoſition of theſe, the Importers Transfuſe their Liquor into the Exporters; From whence he Concludes the Liver to be a Conglomerate Gland Separating the Bile, and becauſe it is uſual for the Conglomerate Glands to have, beſides Arteries, Veins, and Nerves, a proper Excretory Veſſel, as in the *Pancreas*, *Parotides*, &c. Diſperſed thro' their Subſtance, Receiving and Carrying away the Humour ſeparated in them, this kind of Veſſel in the Liver is the *Porus Bilarius* with the Gall-Bladder; and tho' ſeveral Lymphatick Veſſels have been ſaid to Ariſe from the Liver, leſt it ſhould have a Double Excretory Veſſel, he thinks, that ſeeing in other Places the Lympheducts have not their Origin from Conglomerate, but Conglobate Glands, therefore they do not Spring from the Liver it ſelf, but from thoſe Conſpicuous Conglobate Glands, that are in the Hollow of the Liver under the *Involucrum* or *Capſula*, where the Trunks of the *Porta* and *Porus Bilarius* Enter into it; for as *Steno* obſerves, the Lympheducts of the *Parotides* do not Spring from them, but from a Conglobate Gland, which is Contiguous to them.

The Vena Cava, *and the* Vena Portæ.

It hath Two Sorts of Veins, in the upper Part the *Vena Cava* Entereth into it, and Spreads it ſelf all thro' it in the Lower, as well as Upper Part; into the Lower Side the *Vena Porta* is Inſerted, whoſe Branches likewiſe Run thro' it's whole *Parenchyma*, and ſerves for an Artery in bringing Blood to it, it's Proper Arteries being very Small and Few.

Of the Vena Portæ *or* Porta, *and the* Vagina Portæ, *or* Capſula Communis.

As to the *Vena Portæ*, or the *Porta*, from the πύλαι, where it Enters the Liver, and the Veins in General, the *Galeniſts* have Affirm'd them, and they ſeem, to have their Root in the Liver, tho' it is more Properly in thoſe Parts, where they Receive their Blood from the Arteries, than in the Liver or the Heart, whither they Convey it; but in Reſpect of the *Porta*, ſome Think, that the *Vena Umbilicalis* ought to be Accounted it's Root or Original, becauſe it is Firſt Formed in the *Fœtus*, and Inſerted into the *Porta*, but the Umbilical Vein, after the Birth, Degenerating into a Ligament, cannot from that Time be Eſteemed the Root or Origin of the *Porta*; Others therefore think, that, becauſe it's Branches, every where Inſerted into the Inteſtines, bring Blood from thence to the Liver, and not *Vice-Verſâ*, thoſe ought to be Accounted it's Roots, and it's Diviſions within the Liver, it's Branches, and ſo, ſtrictly, they ought to be Accounted; As it Enters the Liver, it is Inveſted with another Coat, which is called the *Vagina Portæ*, or Sheath, the *Capſula* or *Involucrum*, or it's Caſe or Cover, and the *Capſula Communis*, becauſe the *Porus Bilarius* is Involved in it, as well as the *Porta*.

A farther Deſcription of the Porta, *and an Account of it's Branches.*

THIS outer Coat it has from the Membrane, that Cloaths the Liver, tho' it is more Denſe and Carnous, with which all it's Ramifications are Inveſted, upon which Account, of it's having a Double Coat, upon it's Bringing Blood to the Liver for it's Nouriſhment, and upon it's being Endued with an obſcure Pulſation, according to Dr. *Gliſſon*, from the *Arteria Hepatica*'s Inſertion into the *Capſula*, it has the Nature of an Artery; A little within the Liver it Forms a *Sinus* or Heart, by it's Smaller Syſtole and Diaſtole belonging to it, and without which, it is hardly thought, the Blood would paſs out of the Larger Branches of the *Porta* into the Narrower, and ſo on into the Roots of the *Cava*; This *Sinus* is more Conſpicuous in an *Embryo*, becauſe the Great Influx of the Nutritious Juice, by the Umbilical Veins, Enlarges it, as is alſo the *Tubulus* or *Canalis Venoſus*, which paſſes Directly out of this *Sinus* into the *Cava*, almoſt oppoſite to the Mouth of the Umbilical Vein, that opens into the *Sinus*; This Canal or Pipe is of the ſame Subſtance and Texture with a Vein, and Enters into the *Cava* juſt as it Penetrates the Diaphragm, where Two other Great Branches out of the Liver are Inſerted into the *Cava*, and in the ſame Place this Pipe is alſo knit to the Suſpenſory Ligament, and after the Birth grows it ſelf into a Ligament; There are no Anaſtomoſes of the Capillaries of the *Porta* into thoſe of the *Cava*, according to Dr. *Gliſſon*, in his Book *de Hepate*, but the Blood Ouzes thro' the Glandulous *Parenchyma* of the Liver out of the Capillaries of the Former into thoſe of the Latter; After the Trunk has paſſed the Liver, it is Divided into ſeveral Twiggs and Branches; Namely, the *Cyſticæ Gemellæ*, the *Pylorica* or *Gaſtrica Dextra*, the *Splenica*, *Meſenterica*, *Ventricularis*, *&* *Coronaria*, the *Epiplois*, or *Omentalis*, *Dextra & Poſtica*, *Gaſtro-Epiplois Siniſtra*, *Hæmorrhoidalis Interna*, *Inteſtinalis* or *Duodena*, &c. ſo named from the Parts, thro' which they are Carried; Their uſe is to bring back to the Liver, from the Stomach and Guts, that Blood which Remains after their Nutrition, and which was brought to them by the Meſaraick Arteries, to be Circulated back by the *Cava* to the Heart.

A Deſcription of the Cava, *and of it's Aſcending and Deſcending Trunks.*

THE *Vena Cava*, from it's Large Cavity is that, into which, as into a River or Common Channel, all the other Veins like Rivulets, excepting the *Pulmonaria*, Empty themſelves; It's Root may be ſaid to be in the Liver, for by it's Capillaries it Receives the Blood, that is Tranſcolated thro' the Glandulous *Parenchyma* of the Liver, from the Capillaries of the *Porta*, and by it's Aſcending Trunk, which forthwith Enters the Diaphragm and Marches up the *Thorax*, Conveys it to the Right Ventricle of the Heart.

THE Capillary Branches of the *Cava*, as thoſe of the *Porta*, are ſpread thro' the whole Subſtance of the Liver; and the farther the Capillaries have their Origin in the Liver from the *Cava*, the Larger their Channel comes to be at their Arrival at it; but they do not all meet together in one Common Trunk within the Liver, as thoſe of the *Porta* do, but Empty themſelves apart into the *Cava* without the Liver.

THE Deſcending Trunk of the *Cava* is ſomething Narrower than the Aſcending, and Paſſing down along with the Great Artery, or the *Aorta*, continues undivided till the Fourth *Vertebra* of the Loins, before which there are ſeveral Branches from it; as, the *Venæ Adipoſæ*, for the Coat and Fat of the Kidneys, the *Emulgents*, which bring back to the *Cava* that Blood which the *Emulgent* Arteries carry'd to the Kidneys with the *Serum*, the *Spermaticks* or *Vaſa Præparantia*, and the *Lumbares*, for the Membranes of the Spinal Marrow; After which it is Divided into Two Equal Branches, called *Iliaci*, becauſe they paſs over the *Os Ileon*, as they go down to the Thighs; juſt about the Diviſion, there Spring Two Veins, the *Muſcula Superior* for the *Peritonæum*, the Muſcles of the Loins, and the *Abdomen*, and *Sacra* for the Marrow of the *Os Sacrum* and the Membranes that Cloath it; Afterwards the *Iliacal* Branches are again Divided into the *Interior*, from whence ariſe the *Muſcula Media* for the Muſcles of the Hip and Buttocks, and the *Hypogaſtrica* for the *Hypogaſtrium*, and into the *Exterior*, from whence the *Epigaſtrica* for the *Peritonæum* and Muſcles of the *Abdomen*, the *Pudenda* for thoſe Parts, and the *Muſcula Inferior* for the Buttocks; and then the Branches of the *Cava*, being paſſed out of the *Abdomen* into the Thighs, begin to be called *Crural*.

Nerves of the Liver.

BESIDES theſe Veins, the *Porta* and *Cava*, the Liver has Nerves alſo, but they are Extended only to the Membrane and Veſſels of it, as the Arteries were, ſo that the *Parenchyma* has but a very Dull Senſe.

Lymphedučts and Valves.

The Lymphedučts have not been yet obſerved to be Diſperſed in the ſaid *Parenchyma*, and *Malpighius* thinks they are not Derived from the Liver at all, but from the afore-named Conglobate Glands; They carry a moſt Limpid and Pellucid Juice from the Liver along the *Meſentery*, and open themſelves into the *Receptacle* of the *Chyle*; for in *Viviſection* make a Ligature about any one or more of them, and we may preſently ſee them Swell betwixt the Ligature and the Liver, but be Empty on the Side towards the Receptacle; Likewiſe there are Valves in them, which open towards the Receptacle and cloſe towards the Liver, and which hinder any Thing from Returning to it, and therefore are not Lačteals Conveying Chyle to the Liver in order to be Sanguify'd, as the Ancients Imagined: Theſe Lymphedučts are alſo obſerved to Cover the External Surface of the Stomach, and are Diſcharged into the Receptacle of the Chyle.

It's Office or Ačtion.

Hippocrates ſays, the Fountain of Blood is the Heart, the Place of Choler is in the Liver, which is thought to come very near the Truth; But the *Greeks* and *Arabs* generally held, that the Meſaraick Veins Received the Chyle from the Guts, and brought it to the Liver, by which it was Turned into Blood, which was carry'd from it into all the Parts of the Body by the Veins, but it is Plain, that none of the *Venæ Lačteæ* are Inſerted into the Liver, and conſequently, no Chyle is Imported into it; It's true Ačtion therefore is to Separate the Bile from the Blood, which is brought to it by the *Porta*, whether by Colature or Fermentation is Undetermined; *Ariſtotle* and the Ancients thought this Bile to be a mere Excrement, and of no uſe but, by it's Acrimony, to Promote the Excretion of the Guts; *Diemerbroeck* and the Moderns, on the other Hand, Eſteem it to be more than a mere Excrement, and therefore believe it to be a Ferment for the Blood and Chyle, made from the Blood Flowing into the Liver by the *Porta*, and Mixing with an Acrimonious, Saltiſh, and Subacid Juice, Formed in the Spleen, of the Arterial Blood Conveyed thither by the Arteries, and of the Animal Spirits by the Nerves, which is brought into the *Porta* by the *Ramus Splenicus*; by the means of which Acid Juice, and the Specifick Vertue or Cočtion of the Liver, the Spirituous Particles, both Sulphureous and Salt, lying hid in the Venous Blood, are Diſſolved, Attenuated, and become alſo a little Acrimonious and Fermenting; the Thinneſt Part whereof, like Clear Water, being Separated by the Conglobate Glands in the Hollow-ſide of the Liver, is carry'd off in the above-mentioned Lymphedučts: But the Fermentaceous Spirits of Greater Acrimony, mixt with the Thicker and more Viſcid Sulphureous Juices, and more ſtrongly Boiling, when they cannot Enter the Conglobated Glands, and yet thro' their Ebullition are Separated from the Blood, as Yeſt from Beer, become Bitter and are called Bile; which, being Tranſcolated thro' the Grape-ſtone-like Glands into the Roots of the *Porus Bilarius*, and Gall-Bladder, paſſes thro' them by the *Dučtus Communis* into the *Duodenum* or *Jejunum*; where mixing with the Pancreatick Juice, and the Alimentary Maſs Diſcharged from the Stomach, a Fermentation is Cauſed, by which the Chyle is Forced towards the Receptacle of it by Innumerable Lačteals in the above-mentioned Inteſtines; but the ſaid Fermentaceous Juice, being palled by Degrees in the following Guts, Chyle is more ſlowly Separated, and the Lačteals are therefore Fewer, and what Remains of this Fermenting Matter mixes with the *Fæces* in the Thick Guts, and by it's Acrimony Irritates them to Excretion; This farther is certain, that tho' the Liver does not Sanguify, yet it cannot be Eſteemed to ſeparate Bile as an Excrement only, becauſe it is a very noble Part, ſeeing the Diſeaſes of it are generally ſo Dangerous, and the Wounds in it are ſo Commonly Mortal; and by conſequence, that Liquor, which it Separates muſt have ſome noble Uſe, and ſuch as is very Neceſſary to Life.

It's ſeveral Bilary Veſſels.

The Bilary Veſſels, belonging to the Liver, are the Κύϛις Χολήδοχος, *Veſica Fellea* or *Bilaria*, or the Gall-Bladder, the *Porus Bilarius* or *Meatus Hepaticus*, the *Dučtus Cyſt-Hepaticus*, and the *Dučtus Choledochus* or *Communis*.

The Veſica Fellea *or* Gall-Bladder.

The *Veſica Fellea* or Gall-Bladder is about Two Inches in Length, and One in Breadth, by it's upper Part it Adheres to the Liver, and the Lower Reſts on the Right ſide of the Stomach and the *Colon*; It has Three Membranes or Coats, the outmoſt is Common with the Liver; the Middle is Thick and Strong, and conſiſts of Three Ranks of Fibres, the Outermoſt are Tranſverſe, the Middlemoſt Oblique, and the Innermoſt

Streight; The Inmoſt Coat is Nervous or Tendinous, Lined with a Glandulous one, and a Cruſt or Velvet upon it, as in the Stomach and Inteſtines; And the Glands ſeparate a Mucous Humour to Defend the *Veſica* from the Acrimony of the Bile: It has a Neck and a Bottom; The laſt is the Largeſt, and Contains the Bile or Choler, and is of the ſame Complexion with it, commonly Yellow, but ſometimes Greeniſh, Blackiſh, *&c.* The Neck, or *Meatus Cyſticus*, is the Narroweſt, being about as Wide, as a Gooſe Quill, and about Two Inches Long; betwixt this, and the *Veſica*, there is a Fibrous Ring, which ſtreightens the Paſſage, and hinders the too haſty Depletion of the *Veſica*; the other End of the Neck is Joined to the *Porus Bilarius*, and they both make the *Ductus Communis*, which is Inſerted into the Beginning of the *Jejunum*, or the End of the *Duodenum*: 'Tis a Controverſy with Anatomiſts, which way the Bile is Convey'd into the *Veſica*; Dr. *Cole*, with the Ancients, thinks it to be by the Neck from the *Porus Bilarius*, and to obviate the Objection of a Reciprocation of Humours in the ſame Veſſel, which is thought to be Abſurd, he ſuppoſes, that the Gall Paſſes out of the *Veſica* only upon the Diſtribution of the Chyle, but at other Times it is Received into it from the *Porus*, and Stored up there againſt the next Occaſion; Dr. *Gliſſon* on the other Hand, and the Moderns, believe that the Bladder is filled by it's Fibrous Roots, that are Diſperſed thro' the Liver, and that the whole Trunk of theſe Roots Enters that Part of the Bladder, where it is ſtreightened by the Fibrous Ring, tho' it is hardly to be Found or Diſcerned, which is not Strange, ſince the Inſertion of the *Ureters* into the Urine-Bladder are very difficultly Diſcovered: The Bile in the Gall-Bladder is Thinner, more Acrimonious and Fermentative, that in the *Porus Bilarius*, Thicker and Milder: The *Ductus Cyſt-Hepaticus* was found out by *Perrault*, whoſe Numerous Twigs are Diſperſed thro' the *Parenchyma* of the Liver, from thence they Aſſemble into Three Roots, and thoſe Unite into one Trunk, which has a Double Implantation, one into the *Porus Bilarius*, and the other into the Middle of the *Veſica*.

The Ductus Cyſt-Hepaticus.

Alphonſus Borellus de Motu Animalium, from the Continual and Speedy Efflux of the Bile by the *Ductus Communis* into the *Duodenum*, believes there is a Circulation of it; becauſe in a Day's Time, from a Perſon Faſting, there paſs Thirty Four Pounds of *Bilious* Juice into the *Duodenum* by the Common Duct, whereas the whole Maſs of Gall Amounts not to above Two Pounds; ſuch a Quantity he thinks cannot be Produced by way of Fermentation in the Liver, but that it is Separated Mechanically by Cribration from the Minute Veſſels of the *Porta*, thro' the Pores of the Glandules of the Liver, as the Urine in the Kidneys, and that this Circulation is Performed thro' the *Abdomen* by the *Vena Meſaraica* into the Trunk of the *Porta*, thence to the Liver, thence thro' the *Bilious* Veſſels into the *Duodenum*, to Return again by the Meſaraick Veins.

Veins, Arteries, and Nerves of the Veſica.

THE *Veſica Fellea* hath Two Veins, the *Cyſticæ Gemellæ*, which Spring from the *Porta*, it hath Twigs of Arteries from the Right Branch of the *Cœliaca*, and a Small Thread-like Sprig of a Nerve from the Meſenterical Branch of the Intercoſtal: Many Times Stones are Found in this Bladder, which are Lighter and more Spongy than thoſe of the Urinary, and will Swim above Water, which thoſe will not do.

The Porus Bilarius.

THE *Porus Bilarius*, or *Meatus Hepaticus*, which carries the Thicker Sort of Choler, paſſes Directly from the Liver to the *Ductus Communis*; within the Liver it's Trunk and Branches are Inveſted with a Double Coat, it's Proper one, which it Retains without the Liver alſo, and it's Common, Derived from the Membrane of the Liver, which wraps up the *Porus* and *Porta* together, and is called the *Capſula Communis*; It's Roots within the Liver are Equally Divided with thoſe of the *Porta* every where, excepting where the Roots of the *Veſica* are Spread, they are more Large and Numerous than thoſe of the *Veſica*, Drawing Choler from all the Parts of the Liver, ſaving whither the Roots of the Bladder Reach, tho' more Thick and Viſcous, and leſs Acrimonious; The *Porus* is Found in all Creatures, but the *Veſica* is Wanting in ſeveral, as Harts, Fallow-Deer, the Sea-Calf, and thoſe which have a whole Hoof, as an Horſe.

The Ductus Communis Choledochus.

THE *Porus* and *Meatus Cyſticus* Joining Form the *Ductus Communis Choledochus*, which, Piercing the outer Coat of the Inteſtine, paſſes betwixt that and the Middlemoſt, about the Twelfth Part of an Inch, and then Piercing that marches down farther betwixt it and the Innermoſt Coat about half an Inch, and at laſt opens with a Round Mouth into the Inteſtine; which oblique Inſertion ſerves Inſtead of a Valve to hinder any Thing Regurgitating out of the Gut into the Duct: There are no Anaſtomoſes or Inoſculations of the Roots of the Bilary Veſſels with thoſe of the *Porta*, they Terminate in the *Parenchyma* of the Liver, and Imbibe the Choler there Separated from the Blood.

THE *Ductus Communis* is ſometimes Inſerted at the Bottom of the Stomach, and then the Party Vomits Choler, and ſometimes at the lower End of the *Jejunum*, from whence follow *Bilious* Dejections; The Bile is generally Yellow, but in a Morbid State Degenerates into Pale-Colour'd, Eruginous, Porraceous, Vitelline, Reddiſh and Blackiſh, which is the Cauſe of very Violent and Acute Diſeaſes, as the *Cholera Morbus*, Dyſentery, Cholick, *&c.*

HAVING gone thus far in Recounting from Anatomiſts their Notions and Opinions of the Liver, the *Porus Bilarius*, and the Gall-Bladder; I ſhall Excuſe my ſelf the Trouble of Entering at preſent into the Deſcription and Uſe of the Spleen, and the *Pancreas*, which may be ſeen in the Authors, and the Places of thoſe Authors, whom I have Quoted, and if we can give ſome Solution of theſe Parts, from our Principles of the Expanſive and Contractive Forces, the ſame may be Extended to an Explanation of the Others.

How the Placenta, *or Womb-Liver is Produced.*

§. 2. To Account for theſe, or any other Parts of the Human Syſtem, it will be Neceſſary for us to look back to the *Ovarium* and the Womb, and the Formation of the *Fœtus* there; 'Tis therefore known, that after the Impregnation of the *Ovum* by the *Semen*, the Firſt of which we make to be of an Expanſive Force, and the Laſt of a Contractive, the *Ovum* is Inveſted with a White and *Albugineous* Liquor, and Two Membranes which Incloſe it, the Inner, which is the *Amnion*, and the Outer, which is the *Chorion*; the Liquor we ſuppoſe to be of a Contractive Nature, and is Aſſembled by the *Semen*, to Balance the Expanſive of the *Ovum*, and the Films or Membranes, which Surround them, are Condenſed into thoſe Coats or Tunicks by the Air, which is Included, and is of a Contractive Nature, having a like Effect upon the *Albugineous* Fluid, as a Colder or a more Contractive has upon Water, when it hardens it into Ice, or as Common Air has upon a *Lixivium*, when it Induces a Film or Cruſt upon it, or as the ſame Air has upon a Tough Phlegm, which it preſently Stiffens into a Skin. The Expanſive Force alſo in the Animal Syſtem Contributes to the ſame Effect, which makes the Contractive Force, if it cannot Diſſipate and Surmount it, to Coaleſce and Unite Cloſer together; That the *Albugineous* Liquor is likewiſe Contractive, ſeems Evident from the White of an Egg, which Coagulates upon the Application of the Expanſive of Fire or Hot Water to it, an Effect, which does not ſo ſoon obtain in the Yelk.

IN the mean Time, by the Glowing Warmth of the Womb, and of the Blood-Veſſels diſperſed in it, a ferment of this *Albugineous* Liquor is Cauſed, that is, by the Expanſive of the one Acting upon the Contractive of the other, from whence a Glandulous Subſtance Ariſes, or a Congeries of Veſicles or Bladders, as in all other Fermentations, and which by Degrees grow into a *Parenchyma*, which is called the *Placenta* or Womb-Liver; and as the Expanſive Force Prevails in this Fermentation, it is Attracted by, and Joins it ſelf to, the Womb and the Blood Veſſels of it; as all Forces of the ſame kind, whether Expanſive or Contractive, Unite with each other.

How the Liver is Formed in the Fœtus, *and it's Hexangular Veſicles or Glands, the* Porus Bilarius *and* Ductus Communis.

§. 3. WHEN the *Placenta* is thus Connected with the Womb, the Blood Contained in the Umbilical Veſſels Iſſues with an Expanſive Force, which is not to be Reſiſted, and is carried thro' the *Chorion* and *Amnion*, and the *Albugineous* Liquor Included in them, to the Expanſive Force of the *Fœtus*, which is afterwards the Liver, and is Removed at ſome Diſtance from the Contractive of the Brain; In this Paſſage the Expanſive Force of the Maternal Blood is Excited by an Oppoſition of the Contractive Force of the *Albugineous* Liquor, and becomes more Fiery and Expanſive from ſuch an Oppoſition, as Fire by an Affuſion of Water, or the Ventilation of Air; and with this Temper it Enters the *Fœtus*, and the Blood is Turned into an Hot Bile or Choler, and from thence Acquires it's Acrimony, and Yellow Colour, which are both Indications of it's Expanſive and Fiery Nature; Such a new Expanſive Force therefore Incurring into the Animal Syſtem, in order to Preſerve the *Æqui-*

librium of it, the Contractive Force is Increaſed by the *Albugineous* Fluid from the *Amnion*, which is conveyed into the Syſtem, and by the Aſſemblage of the Watry or Contractive Humours already in it, which the Brain had Collected for it's Support and Maintenance againſt the Expanſive of the *Ovum* and the Heart, theſe Contrary Forces Meeting, where the Umbilical Vein Enters the *Fœtus*, cauſe a Ferment there, and a Stoppage of both Liquors in the Conteſt, till the Liver is Formed, which from this Mutual Oppoſition Riſes into the Grape-like or Hexangular Glands or Veſicles before-mentioned, as Bubbles in a Fermentation, which, by the Air Included in the Syſtem, and by the Methods of the Expanſive and Contractive Forces before Deſcribed are hardened into Veſſels, which Surround and Contain by their Contractive Force the Expanſive of the Bile, which ſtill pouring in by the Umbilical Vein Proceeds towards the Heart, the Seat of the Expanſive; and being ſtill Reſiſted and Oppoſed by the Contractive Force, it is Inveſted with Coats, by which the *Porus Bilarius*, and it's Branches in the Liver, and the *Ductus Choledochus Communis* are Formed; which yield a Paſſage to the Expanſive Force of the Bile forwards, but Compreſs and Reſtrain it into Small Currents and Channels by their Force of Contraction; and which Coats are made from the Contractive, *Albugineous*, or Watry Humour, Condenſed by the Contractive Force of the Air Included in the Syſtem, and by that of the Nerves, which are Diffuſed in them, as alſo by the Contractive Cloſeing upon the Inſults and Action of the Expanſive.

Why there are Lympheducts found in the Liver, Gall-Bladder, Spleen, Pancreas, *&c. but not in the Guts or Inteſtines, the Reins, or in the Nervous Syſtem.*

§. 4. HENCE likewiſe it is, that the Liver, the Gall-Bladder, the Spleen, and the *Pancreas*, meet with Lympheducts in Great Plenty, being no where found in the Guts or Inteſtines; The Reaſon of which is, becauſe the Bile or Choler, as we before Obſerved, is of a Fiery and Expanſive Nature, and the Pancreatick Juice is Eſteemed by Anatomiſts to Aſſiſt to the ſame End and Purpoſe with the Bile in the *Duodenum* or *Jejunum*; It was therefore neceſſary to Preſerve the *Æquilibrium* of the Human Syſtem, that a *Lympha*, or Water, ſhould be Collected to thoſe Parts, which is of a Contractive and a Cooler Temper, in order to Rebate the Violence and Fury of the Expanſive in them; which Equal Balance in the Syſtem is as naturally Maintained, without ſuppoſing the Forces to Act by Choice or Inſtinct, as one Column of Water, which is Higher than another, Deſcends to a Lower, in the Caſe of Hydroſtaticks; becauſe the Expanſive and Contractive Forces muſt be Equally Poiſed in the Animal Oeconomy, or there will be a Diſſolution of it, and conſequently, if the Expanſive Prevails, as in the Bile, the Contractive, as in the *Lympha*, muſt of Neceſſity flow to it, to ſupport the *Æquilibrium*; upon the ſame Account we find a *Lympha* in the *Pericardium*, which Encompaſſes the Heart, and Lymphaticks Attending the *Arteria* and *Vena Pulmonaris* in the Lungs, but none in the Reins or Urinary Parts, nor in the Nervous Syſtem.

Why the Liver is ſo neceſſary a Part in the Human Syſtem, and why the Wounds in it are ſo Dangerous and often Mortal; and of the Nature of the Bile.

§. 5. As the Choler or Bile paſſes from the Liver into the *Jejunum* or *Duodenum*, and mixes with the Chyle, and is carry'd thro' the Lacteals of the *Meſentery* to the Receptacle of it, ſo the *Lympha* likewiſe is conveyed by it's Ducts from the ſame Bowel, thro' the ſame *Meſentery*, and to the ſame Receptacle, to Qualify and Moderate the Heat and Effervеſcence of the Bile, but ſtill to keep up a Ferment, by which the Chyle is Prepared, to be Exalted into Blood upon it's Mixture with the Venal Maſs in the *Subclavian*, and it's Entrance afterwards into the Heart; which ſhews how neceſſary the Liver is to the Human Syſtem, and Explains the Reaſon, why Wounds in it are ſo Dangerous and often Mortal; for tho' it do's not Sanguify, as the Ancients thought, yet it aſſiſts towards Sanguification, by Produceing the Bile, which is as Requiſite to the Forming of the Expanſive of the Blood, as the Chyle is; This Expanſive Force of the Bile alſo is that, which Precipitates, and Gives an Exertion of, the Chyle into the Lacteals with ſo much Activity and Vigour, as to leave the *Jejunum* commonly Empty, and by which the Aliment, and the Chyle, Receives a new Impreſſion of Volatility, and becomes ſtill more Fluid and Refined; Which, on the other Hand, Mixing with the *Fæces*, Diſſolves, Stimulates, and Urges them to an Expulſion, and if in too Great a Quantity, cauſes the afore-mentioned Bilious Dejections.

Why the Liver is of an Hot Temperature, and it's Colour

§. 6. THE Liver being thus Framed in the *Fœtus*, it is manifeſt it muſt be of an Hot Temperature and Conſtitution, and therefore we find it of a Yellower Colour than any other Entrail, which Approaching neareſt to the Colour of Fire, which is

the

Yellowiſh, and why the Umbilical Vein is Changed into a Ligament.

the Standard of the utmoſt Expanſive Force, is a certain Mark of ſuch a Force Reſiding in it; When therefore the *Fœtus* is ſeparated from the Womb, and the Maternal Blood ceaſes to flow thro' the Umbilical Vein, it is Contracted into the Umbilical Ligament; Becauſe, as we have ſhewn, the Coats of the Vein, or any Blood-Veſſel, have a Contractive Force, by which they Preſs Inwards, and there being no Expanſive Force of the Blood in the Cavity of it to Reſiſt that Contraction, it by Conſequence cloſes; On the Contrary, the Liver being of an Expanſive Force, and Surrounding it by Compreſſing of it, Aſſiſts to it's Contraction, and it is not Evident, that the *Suſpenſorium* and *Latum* might not be Originally Conduits, or, in ſome way or other, upon the Firſt Forming of the *Fœtus*, Receptacles of Blood, and Cloſed and Contracted in the ſame Manner; ſince the *Tubulus* or *Canalis Venoſus*, which Paſſes from the *Sinus* into the *Cava*, and is of the ſame Subſtance and Texture with a Vein, after the Birth is likewiſe Contracted and Cloſed into a Ligament.

The Reaſon, why the Blood Flows from the Inteſtines to the Liver after the Separation of the Fœtus *from the Womb, and how the Bile is ſtill Formed by the ſame Method of Nature.*

§. 7. However this ſeems to be Manifeſt, that, as this Bowel was Firſt made by the Influx of the Venal Blood from the Womb, ſo upon the Stoppage of that Influx, and the Separation of the *Fœtus* from it, it was Neceſſary it ſhould be ſupplied with ſomething Analogous to that Influx, for the Preſervation of the Animal *Æquilibrium*; which is the Reaſon, that the Blood Flows from the Inteſtines in ſmall Canals, and at length Meeting and Conſtituting the Trunk of the *Porta* Enters the Liver, ſince the Bilary Veſſels and Lymphaticks carry every Thing from it, and is a Solution, why the *Porta* does, in this Caſe, tho' a Vein, Perform the Office of an Artery in Conveying Blood to the Liver, tho' Veins in all others Conduct it back from the Parts, in which they are Implanted, to the Heart: And that Bile will ſtill be Produced is plain, ſeeing there is now an Aſſemblage of *Lympha* to this Part, and Lymphaticks Formed, which Contain it, there is likewiſe an Influx of Venal Blood, and moreover there is the Liver, which has an Hot and Expanſive Force belonging to it; and conſequently, the ſame Effect will be Obtained, as before from the Maternal Blood Paſſing thro' the *Lympha* or Liquor of the *Chorion* and *Amnion*, at leaſt by the Help and Aſſiſtance of the Natural Expanſive Force and Heat of this Bowel: Alſo the Blood Flows from the Inteſtines upon this Occaſion, where there is the leaſt need of it; the Conteſt there lying chiefly betwixt the Expanſive Force of the Aliment, and the Contractive of the Coats of thoſe Inteſtines.

Why the Liver is Round and Convex on the Upper-ſide and Concave on the Lower, and as to the Subſtance of it, it was before Explained.

§. 8. The Situation of the Liver, we have hitherto Explained, and ſhewn, that it muſt be Gathered and Contracted about the Umbilical Vein; The ſame Doctrine will alſo give us an Account of it's Figure, which on it's upper Side is Smooth, Round, and Convex, and on it's Lower is Hollow and falls into Edges, ſuitable to ſuch Concretions; for whilſt the Liver is Forming from the Coagulation of the Expanſive and Contractive Forces, and is kept and ſuſtained in the ſame Place, the Parts will Deſcend from above Downwards, and leave the upper Surface Convex and Smooth, and ſhape the Edges and Concavity below by their Natural Gravitation; For thus any Soft or Conſiſtent Subſtance, as Clay or Dough, if Round, and ſupported by the Finger, will fall into the ſame Figure; and that, if it were not for this Gravitation, and it's Support from the *Suſpenſorium*, the Liver would be Round, or Globular, may be Concluded from an Equal Balance of Forces nearly, from whence the Coagulation Enſues.

The Subſtance of the Liver, we have likewiſe ſhewn to be Glandulous and Vaſcular, and to be Repleniſh'd with an Hot Bile or Choler, made by the Paſſage of the Venal Blood thro' the Contractive Liquor of the *Chorion* and *Amnion*, and afterwards by the Venal Blood, and the *Lympha*, and the Liver it ſelf, which has now Contracted a Bilious, Hot, or Expanſive Force; And as to the Glands being Hexangular, that Proceeds from the Nature and Degree of the Contractive Force, which has been upon another occaſion Explained.

Malpighius's *Commerce betwixt the Glands and the Veſſels of the Liver not certain, nor*

§. 9. As to what is ſaid by *Malpighius*, that Reaſon concludes it neceſſary, that there ſhould be a Commerce betwixt the Glands and Veſſels, tho' Senſe cannot diſcern it, that does not appear ſo very certain; becauſe the Glands may be only ſmall Bubbles or Veſicles, Raiſed by the Fermentation, which Conſtitutes the Liver, and the Bile may Run betwixt them, or if it is Contained in them, it may by it's Ex-

that there are any Anaſtomoſes in it.

panſive Force break thro' the Sides of them, (which may again cloſe by their Contractive Force,) and enter into the Veſſels, which convey it to the *Jejunum*, and ſo to the Lacteals, and from thence to the Heart; It is not therefore Requiſite, that the Shoots of the *Porta* and *Porus Bilarius* ſhould be joined by Anaſtamoſes, nor that the Glands ſhould be the Medium betwixt the Importing and Exporting Veſſels, ſince nothing of this is Diſcovered by Obſervation, but it is Rational to Think, that the Bile is Spread thro' the Liver, and betwixt the Veſicles and Glands, as well as Contained in them, and that by it's Expanſive Force it from thence Enters into the Branches of the *Porus Bilarius*, and ſo is carry'd to the Lacteals and the Heart, to whoſe Expanſive Force it Preſſes.

In Relation to the little Branches of the *Porta*, *Cava*, and *Porus Bilarius*, Running in an Equal Number in the Liver, it Agrees with what we have Mentioned; the *Cava* being the Vein, and the *Porta* the Artery, and the *Porus Bilarius* the Conduit of the Bile, it has been alſo Explained, how the *Porta* comes to Perform the Office of an Artery.

Not Evident that the Liver is a Conglomerate Gland.

§. 10. But that the Liver is a Conglomerate Gland, is not Evident or Proved any farther than ſuch, as we have Repreſented it to be; Namely, that it Conſiſts of Glands or Veſicles, and of Innumerable ſmall Veſſels of the *Porta*, the *Cava*, and the *Bilarius* Diſperſed in it; and it is not Improbable but there may be Lymphaticks, tho' ſo Small as not to be Diſcovered by Obſervation, ſince a ſmall Affuſion of *Lympha* will Excite the Blood into Choler, and therefore is neceſſary to the Produceing it; and that this is ſo ſeems Manifeſt, ſince in Livers, which are Diſeaſed, there is not only a *Lympha* Collected on the outward Surface of it, but there are Bladders, filled with it, ſcattered about in it's Subſtance, which Proceeds from a Watry or Dropſical Conſtitution, when the *Lympha* is Aſſembled in too Great a Quantity to the Liver, and the Contractive is Prevalent over the Expanſive.

How the Lympha is ſupply'd to the Liver, and the Lympheducts Formed.

§. 11. We alſo conceive, that the *Lympha* is not chiefly carry'd to the Liver by the *Porta*, and mixt with the Blood, and then ſeparated from it to be convey'd off by the Lymphaticks, but that it is Originally drawn thither from the Liquor of the *Chorion* and *Amnion*, and afterwards ſupplied from the watry Steams of the *Abdomen*, which Ouze thro' and Moiſten the Subſtance of the Liver, and Increaſe the Heat and Ferment of it, and what Remains unuſeful to that Part is ſent in Veſſels to the Receptacle of the Chyle; and conſequently, when, as in an *Aſcites*, the *Abdomen* is full of Water, the Liver is Corrupted with too Great an Affluence of it, and not the *Abdomen* fill'd with a Diſtillation of the *Lympha* from the Liver, as is Taught by Anatomiſts; It is plain there is a *Lympha* in the *Abdomen* from the Diſtemper we call the *Aſcites*, and it is very Rational to believe, that in an Healthful State of the Body, there is ſome convey'd into it, which by the Heat of the Liver is Raiſed and Exhaled towards it, as Vapours by the Sun, that is, the Expanſive Force of the one is Ris' againſt and Oppoſed by the Contractive of the other, which coming to the Liver heightens it's Heat and Ferment, but when the Expanſive Prevails, theſe Steams are ſent off in Rivulets of *Lympha*, as Vapours fall down in Showers by the Prevalence of the Sun; And from hence it is, that theſe Streams of *Lympha* Ariſe from thoſe Conſpicuous Conglobate Glands, that are in the Hollow or Under-ſide of the Liver, where the Trunks of the *Porta* and *Porus Bilarius* Enter into it; for the Expanſive Force of the Liver Prevailing, Repells the Aſcent of the *Lympha*, and Determines it to a lower Situation at the Bottom of it's Subſtance, where it is carry'd off in Ducts made by it's obeying the Expanſive Force of the Bile and Liver, and the Contractive Force of the *Lympha*, and Included Air, and Nervous Syſtem.

Why the Parenchyma *of the Liver has but a Dull Senſe, and how the* Porta *is Inveſted with a Second Coat.*

§. 12. The *Parenchyma* has but a dull Senſe, there being few Nerves but what are Spread upon the Surface and Veſſels; which is Agreeable to the Account we have Given of it's Production from a Coagulum of Bile and *Lympha* after a mutual Conteſt betwixt the Expanſive of the Venal Blood and the Contractive of the *Lympha*, which are Liquors, that have no Senſe belonging to them; The Nerves by their Contractive Force aſſiſt to the Formation of the Membrane of the Liver, and the Coats of it's Veſſels, as well as to the Imparting a Senſe to them.

The *Porta*, when it Enters the Liver, is Inveſted with a Second Coat from the Membrane, with which the Liver is Encloſed, by an Adherence of the Contractive

Force of the Membrane to the Firſt Coat of the *Porta*, as Forces of the ſame Nature always Unite with and Propagate each other; and for the ſame Reaſon, the Portal or Arterial Blood, and the Bile, which are of an Expanſive Nature, join in Separate Channels under one Common *Capſula* or *Involucrum* in Oppoſition to the *Lympha*, and the Venal Blood of the *Cava*, which are of a more Contractive Temper.

Why the Porta *has an Obſcure Pulſation.*

§. 13. WHAT Dr. *Gliſſon* Obſerves of the Obſcure Pulſation of the *Porta* is eaſily Accounted for from the Principles we have Alledged for the Solution of the Periſtaltick or Vermicular Motion of the Inteſtines; ſince the *Porta* is Endued with a Double Coat, a Membranous and a Carnous, the Former of which has a Contractive Force, and the Latter an Expanſive, by which they Mutually and Reciprocally are Contracted and Expanded, which is the very Caſe of Pulſation; 'tis yet an Obſcurer one, than in the Arteries, becauſe the Blood, which is Aſſiſting to the Expanſive Force of the Carnous Coat, being Venal, has a leſs Degree of Expanſion than the Arterial, as will be afterwards ſhewn, and the Membranous Coat having but a ſmall Number of Nerves, has alſo a leſs Degree of Contraction, by which the Pulſation becomes Leſs Vehement, and Strong; On the Contrary, that this Obſcure Pulſation is owing to the Inſertion of the Hepatick Artery into the *Capſula*, which Dr. *Gliſſon* Thinks, cannot be Imagined, unleſs we can ſuppoſe, that little Twigs of Arteries are capable of Shakeing a whole Trunk, and Producing a Greater Motion in Larger Veſſels, than what is Experienced in Themſelves, which are Smaller.

Why there are no Anaſtomoſes of the Capillaries of the Porta *into thoſe of the* Cava.

§. 14. THERE are no Anaſtomoſes of the Capillaries of the *Porta* into thoſe of the *Cava* according to Dr. *Gliſſon*, nor ſhould there be according to the Principles above; ſince the Blood is Diſcharged from the *Porta* into the Liver to be Turned into Bile, which could not be, if there was a Continuation of the Capillaries of the *Porta* to thoſe of the *Cava*, and the Blood was carry'd on in one Channel; On the Contrary, when there is a ſufficient Quantity of the Influent Blood from the *Porta*, made into Bile, that is, when the *Lympha* and the Bilious Quality of the Liver, which it has Contracted by the Methods before Deſcribed, have Converted as much Blood into Bile, as their Forces are capable of Doing, the Blood, ſtill Flowing in by the *Porta*, is Returned by the *Cava* and it's Branches, whoſe Mouths lie open in the Liver ready to Receive it, as thoſe Mouths and the Cavity of the Branches were made and formed by the Expanſive Force of the Superfluous Blood, Preſſing towards the Heart, and the Coats by the Contractive of their Nerves, and of the Included Air; and perhaps the *Lympha*, being of a Contractive Force likewiſe, may be Aſſiſting to the ſame Effect.

The Aſcending Trunk of the Vena Cava, *and it's Root Accounted for, and why it is Wider than the Deſcending.*

§. 15. THE Root of the *Vena Cava*, as to it's Aſcending Trunk, it is very Probable, was originally in the *Fœtus* from the Umbilical Vein, which is Continued to it, aud after the Birth is Contracted into a Ligament, from thence the Capillaries of the *Vena Cava*, which Import the Superfluous Blood from the *Porta* into it, may be Accounted the Root of it; The Umbilical Blood went in a Current directly to the *Cava*, and by the Contractive Force of the Nerves had by Degrees a Coat, which Surrounded and Formed that Vein, but afterwards, when the *Porta* ſupplied the Blood to the Liver from the Inteſtines, and was ſpread in ſeveral Currents thro' the *Parenchyma* of it, the Superfluous Blood by it's Expanſive Force Tended back to the Heart in Various Channels, and conſequently Entered the Aſcending Trunk of the *Cava* in Different ones, to which Courſes of the Blood the Contractive Force of the Nerves and the Included Air, and poſſibly of the *Lympha*, gave their Tunicks, and Shaped them into Veins and Conduits for it's Conveyance; The Aſcending Trunk of the *Cava* is alſo Wider than the Deſcending, becauſe the Firſt was originally Formed by the Umbilical Vein, and by one Current and Influx of Blood made thro' the one into the other, whereas the Deſcending Ariſes from ſeveral and various Rivulets Emptying themſelves into it, according to the Diſtinctions of Branches before ſet Down and Deſcribed; which there is no Occaſion here any farther to Conſider, till we come to the Arteries, from whoſe Superfluous Blood theſe Branches of the Deſcending *Cava* are Conſtituted and Formed; in like Manner, as the Branches of the Aſcending are from the Superfluous Blood of the *Porta*, and in all Probability without any Imaginary Anaſtomoſes, which we ſhall then more fully Examine into.

Our Account of the Bile, and it's Sanguifying Quality Confirmed by Diemerbroeck.

§. 16. As to *Diemerbroeck*'s Account of the Bile, it very much Confirms what we have Propoſed concerning it, that it is made from the Blood of the *Porta*; He owns an Acrimonious, Saltiſh, and Subacid Juice Derived from the Spleen, and Formed by the Arterial Blood and Animal Spirits, which is Anſwered by our Contractive Force of the Air, and *Lympha*, Fermenting with the Expanſive of the Blood; He Confeſſes a Specifick Vertue and Coction of the Liver, we a Natural Heat and Expanſive Force of it, as before Explained: As to the Sulphureous and Salt Particles lying hid in the Venal Blood, we Acknowledge no ſuch Latent Qualities, but Think they are Produced, and that the Blood becomes Acrimonious and Fermenting, That is, that the Expanſive Force of it is Exalted, and Render'd more Fiery by the Methods above Specified: We likewiſe Imagine, that the *Lympha* is not only Separated from the *Porta*, but that it is Collected from the Cavity of the *Abdomen*, by which the Heat of the Liver is Increas'd above the Natural Temper or Expanſive of the Blood; that the Conglobate Glands do not Separate the *Lympha* from the Blood by way of Colature, but that the *Lympha*, Paſſing that way and being Excluded and Repelled from the Liver by the Expanſive Force of it, does by it's Contractive Force harden it into a Schirrous Fleſh or a Conglobate Gland, which Fleſh would otherwiſe have had a more Expanſive Force, that is, a Sanguineous one: How the Chyle is Urged towards the Receptacle of it by the Expanſive Force of the Bile, and ſo to the Heart, we have Explained before, which *Diemerbroeck* does not, who only ſpeaks of an Acrimonious and Fermentaceous Quality, in General, in the Bile, but does not Inform us, how that ſhould Protrude and Thruſt the Chyle towards the Receptacle of it: As to the Expanſive Force of the Bile, we Agree with him, that it is Diminiſh'd, the farther it goes from the Liver, and therefore has a leſs Effect in the Thick Guts, than it has in the Small ones, and by conſequence, that there are fewer Lacteals in the Latter than in the Former, and that it aſſiſts by it's Expanſive Force to the Excretion of the *Fæces*, we alſo acknowledge with him that the Bile is not an Excrement; On the Contrary, we ſay it Sanguifies, tho' not in the Liver, yet in Exalting the Expanſive Force of the Chyle upon it's Mixture with it, by which, and it's Entrance into the *Subclavian*, it is Transformed into Venal Blood, and by the Contractive Force of the Air in the Lungs, and the Pulſation of the Heart, into Arterial, as will be afterwards more fully Deſcribed.

The Reaſon of the Veſica Fellea, *or Gall-Bladder.*

§. 17. As to the *Veſica Fellea*, or Gall-Bladder, it ſeems to be Evidently ſupplied by it's Fibrous Roots from the Liver, not only from the Abſurdity of the contrary Opinion, but from this, that the Roots of the *Porus Bilarius* draw Choler from all Parts of the Liver, Excepting, whither the Roots of the Bladder Reach, which is a plain Argument, that the Choler there is Convey'd by thoſe Roots to the *Veſica*; Beſides there is a Reaſon for this *Veſica*, namely, the Conglobate Gland, and the *Lympha* Paſſing that way in a Greater Quantity, which being of a Contractive Force, is Equally Balanc'd by the Expanſive Force of the Bile Paſſing towards it in thoſe Small Roots, and which Collecting to an Head is Inveſted by the Contractive Force of the Air, and *Lympha*, and the Nerves, with it's Mucous and Tendinous Coat, and afterwards with it's Fibrous, to which the Liver adds it's Membranous; This alſo gives an Account why the moſt Active Part, and therefore the moſt Subtile and Acrimonious, of the Bile is Engaged in this Encounter, and Lodges in the *Veſica*, whilſt the more Viſcid, and Groſſer, is carry'd from the Liver by the *Porus Bilarius* to mix with the Chyle.

An Account of Borellus*'s Circulation of the Bile.*

Borellus De Motu Animalium, Prop. 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153. Cap. 10. Par. 3.

§. 18. I Think there is little more neceſſary to be ſaid in Reſpect of the Liver, the Bile, and it's Veſſels, as to Philoſophy, unleſs we conſider *Alphonſus Borellus*'s Circulation of it, which we ſhall now Inquire into: His Notion is, that the Glands are certain *Faſciculi* of Veſſels of ſeveral Kinds, of Arteries, Veins, Excretory Ducts, and Nerves; that there are no Anaſtomoſes of Arteries into Veins, becauſe there could be nothing then Separated from the Blood, which would Run in one continued Canal, nor yet of Arteries into the Excretory Ducts, becauſe then the Superfluous Blood could not be Received into the Veins; and that therefore it is neceſſary there ſhould be ſome Repoſitory of the Blood and Juices Different from the Veſſels, which Import or Export them; that the Imported Juices are mixt in theſe Repoſitories, and afterwards the Veins and Excretory Ducts Receive and Convey off thoſe, which are

Suited and Adapted to them; The Queſtion is, whether theſe Repoſitories of the Juices are mere Receptacles of them, or whether they obtain a Fermentative Quality, by which they Change and Alter the Blood Imported to them: Againſt their having a Fermentative Quality it is Objected, that Fermentation requires ſome Time to perform it in, and cannot poſſibly be Raiſed and Effected in the Momentany Paſſage of the Blood or Juices thro' them; that, by the Continual Flux of the Blood or Juices into the Glands, ſuch a Fermentative Quality would by Degrees be Impaired and Weaken'd, and at laſt be wholly Deſtroy'd, as a Vinegar Veſſel by a Continual Affuſion of Water looſes it's Acrimony; On the Contrary it is thought, that a Separation is made of Heterogeneous Fluids in the Glands Mechanically, thro' the Small *Foramina* of the *Faſciculi* of Veſſels, by way of Incuneation or Colature, where the Orifices of the Veſſels are Excavated Conformably to the Figure of the Blood or Juices, which they Receive.

As to the Bile, that it is made by the Liver is Confeſſed by all the Modern Anatomiſts, but whether by Fermentation, or Colature and Incuneation, is Diſputed; For the Firſt Opinion there are Two Reaſons Aſſigned: Firſt, Becauſe the Bilious Particles are very cloſely Embraced by the Blood of the *Vena Portæ*, and are not united by ſimple Contact; For in *Oximel*, the Vinegar, which is mixt with the Honey, by the ſimple Contact of Parts is ſenſible to the Taſt, whereas the Bitter and Bilious Juice, which is mixt with the Blood in the *Abdomen*, is not, it's *Aculei* being Entangled in the Glutinous Parts of the ſaid Blood, as Diſtilled Vinegar is Dulcified by Lead, or Powder of Coral; Second, Becauſe the Blood is not carry'd into the Liver with an Impetuous Motion, as it is in the Reins from the Emulgents, but Proceeds by a Gentle Diſtillation from the *Porta* into it, and conſequently, ſince a firm Connexion of Parts requires Time and Fermentation to ſeparate them, it is not Unlikely, but Nature may have taken that Method in Separating the Bile.

THE Laſt Opinion, that it is done by Incuneation or Colature is thus Defended; ſince, if it was Effected by Fermentation, there ſhould be a Fermentaceous Juice in the Liver, as Bitter as the Bile it ſelf is, or ſuch a Juice ſhould be Convey'd to it from other Parts; On the Contrary, the Liver is of a Sweet Taſt, and there are no ſuch *Vaſa Deferentia* in Experience or Obſervation; Beſides it ſeems very hard to Imagine, that a Fermentation ſhould be Raiſed in the ſmall Veſſels of the *Porta*, and in the ſtreight Pores of the Hepatick Glands.

BUT for the Mechanical Secretion of the Bile by Incuneation or Colature many Things may be ſaid; in the Firſt Place there is a *Vis Motiva*, namely, the Impulſe of the Blood from the Compreſſion of the *Abdomen* and the Air drawn in by our Breath; there is alſo a Continued Motion; there are the Small Canals of the *Porta*, and the Pores of the Glands; and there are Two Orders and Kinds of Veſſels for the Reception of the Different Fluids, the Venous, and the Bilary.

THIS being Premiſed, in order to Prove the Circulation of the Bile, he goes on to ſhew: FIRST, That only one Part of Blood, which is Leſs than the Twenty-Fifth Part of the whole Sanguineous Maſs, Paſſes thro' the Liver in one Circulation; Becauſe there is no other Blood enters into the *Vena Portæ*, and goes out of it, in the Time that one Circulation of the Blood is performed thro' the Heart, beſides that which is ſent into it by the Meſenterick and Cæliack Arteries; but the Diameters of thoſe Arteries are but $\frac{1}{5}$*th* of the Diameter of the Trunk of the *Aorta*, as it comes out of the Heart, and ſince the Circular Orifices are in a Duplicate Proportion of the Diameters, the Orifices of the Meſenterick Arteries will be $\frac{1}{25}$*th* of the Trunk of the *Aorta* Proceeding from the Heart; and the Bulk or Quantity of the Fluids, Circulating with the ſame Velocity, and in the ſame Time, have the ſame Proportion with the Orifices, thro' which they Paſs, that is, the Blood Flows with an uniform Velocity thro' all the Arteries.

SECOND, That the Bile, which is Collected by the Bilary Veſſels, does not ſeem to be Separated from the Sanguineous Maſs, out of, or which is Exterior to, the *Abdomen*; For if the whole Sanguineous Maſs, which does not uſually Exceed in Weight 20 *l.* was uniformly Tinged by the Bilious Juice, which does not commonly Exceed 2 *l.* That Part of the Sanguineous Maſs, which Paſſes thro' the Liver in one Circulation, can only be Purged of it's Bile, and therefore, by what foregoes, there can be only $\frac{1}{25}$*th* Part, or one Ounce of the Bile Separated from the Blood in the

Firſt Circulation, as there is only $\frac{1}{25}$*th* Part of the whole Maſs of Blood Paſſes thro' the Liver in that Circulation; and ſince that Part of the Blood, which is Purged of it's Bile, mixes afterwards with the whole Maſs, which is ſtill Tinged with it, in the Subſequent Circulations, there will be ſucceſſively $\frac{1}{25}$*th* Part of the Remaining Bile Purged or Separated from it.

If therefore a Calculation is made in the ſame Manner, as is done in the Excretion of the Urine, it will Appear, that after Six Hours, in the 100*th* Circulation of the whole Blood, $\frac{1}{250}$*th* Part of the whole Bile will be Separated by the Liver, that is, Leſs than $\frac{1}{10}$*th* Part of an Ounce; and after Twelve Hours, in the 200*th* Circulation, there is not Separated by the Liver more than 1000*th* Part of the whole Bile, or the 80*th* Part of an Ounce; and ſince the Bile is not Repaired, unleſs by taking in new Aliments, and what is Separated in the Liver is continually carry'd off by the *Porus Bilarius* and the *Ductus Cyſticus*, it would follow, that, after a Day's Faſting, thoſe Bilary Veſſels would be entirely Empty, but this is Falſe in Experience; For after ſeveral Days Faſting, nay ſome Months, the *Porus Bilarius* and Gall-Bladder in Serpents are Found as Full and Turgid, as Before; And therefore the Bile is not Separated from the Sanguineous Maſs, which is Circulated thro' the Arteries and the *Vena Cava*, that is out of the *Abdomen*.

Third, That the Bile, which is Separated by the Liver, upon Faſting, in the Space of one Day, is Sixteen Times More than that, which is in the Sanguineous Maſs: For the Bilary Veſſels are not like the Arteries or Meſaraick Veins, which End in Capillaries, but rather like the Branches of the *Vena Cava*, which, as Rivers, Form larger Canals from a Collection of ſeveral little Rivulets Falling into them; Thus the Bilary Veſſels, which carry the Bile from the Liver, have their Riſe from Capillaries, and End in Large Orifices in the *Duodenum*, and are always full of Bilious Juice; They are alſo like the *Vena Cava* in their Courſe, and in the Velocity with which the Fluids are Moved thro' both, the Impelling Cauſe of them being the ſame, namely, the Common Compreſſion of the *Abdomen*, and the Conſtriction of the ſame Circular Fibres; And ſince the Quantities, or the Bulks of the Fluids, which Paſs thro' Unequal Canals with the ſame Velocity, and in the ſame Time, obtain the ſame Proportion to each other, which the Orifices of the Veſſels have, the Bile, which is carry'd thro' the Bilary Veſſels to the Blood, which is carry'd thro' the *Cava* will have that Proportion; But the Diameter of the *Porus Bilarius* is not Greater than $\frac{1}{30}$*th* Part of the Diameter of the Greateſt Trunk of the *Vena Cava*, and the Diameter of the *Ductus Cyſticus* is of the ſame Meaſure, and therefore the Diameter of the One, will be to the Sum of the Diameters of the other Two, as 30 to 2, and the Circular Orifices, as the Squares of thoſe Numbers, or as 900 to 4, that is, the Circular Orifices of the Bilary Veſſels will be $\frac{1}{225}$*th* Part of the Circular Orifice of the *Vena Cava*, and the Fluids, which Paſs thro' the ſame Orifices in the ſame Time, will be in a like Reaſon; And becauſe, in every Circulation of the Blood, 20 Pounds of it are Tranſmitted thro' the *Vena Cava*, or 240 Ounces, therefore in the ſame Time, that one Circulation of the whole Sanguineous Maſs is Performed, there will Flow out of the Bilary Ducts $\frac{1}{225}$*th* Part of it, that is, ſomething More than one Ounce of Bile; Moreover, ſince in one Hour there are 16 Circulations of the Blood, in 24 Hours there will be Tranſmitted thro' the *Vena Cava* 7680 Pounds of it, and at the ſame Time $\frac{1}{225}$*th* Part, or 34 Pounds of Bilious Juice thro' the Bilary Veſſels, whereas the whole Bile, which is Contained in the Sanguineous Maſs, does not Exceed 2 Pounds, and therefore the whole Bile, which is Separated by the Liver upon Faſting, is 17 Times More than that, which is Contained in the Sanguineous Maſs, as 34 is 17 Times Greater than 2.

Fourth, That ſo Great a Quantity of Bile, as 34 Pounds, cannot be Generated in an Human Body upon Faſting by Fermentation; For he Thinks it cannot be Imagined, that the Bile ſhould be made by a Bilious Quality in the Liver, Changeing and Transforming the Blood into it, as Wine Turns to Vinegar, if put into a Veſſel, Infected and Tinged with that Liquor; Since it would be very Fooliſh and Abſurd in Nature to Corrupt that Blood into a Noxious Excrement, fit to be Separated and Expelled from the Animal Syſtem, which yet it had before taken ſo much Pains to Elaborate and Increaſe, and therefore ſuch a Procedure is Unworthy of it, which Acts all Things Wiſely and with Deſign.

From all which it is laſtly Concluded, That the ſame Maſs of Billious Juice, which is Diſcharged from the Liver is brought back, by a Circulation of it, to the Place from whence it began it's Motion, and it is Affirmed, that by the ſame Arguments, by which the Circulation of the Blood was Demonſtrated and Proved by our Celebrated *Harvey*, the Circulation of the Bile may be Evinced; For, as That is Inferred from the Heart's continually Pouring Blood into the Arteries, and from the whole Sanguineous Maſs, which in Men does not Exceed 20 Pounds, being Tranſmitted thro' the Heart in 3 or 4 Minutes of an Hour; ſo This is Aſſerted and Argued, from the whole Bilious Maſs not being Increaſed, where there is no new Acceſs of Aliment, from it's not Exceeding 2 Pounds, from it's conſtantly Flowing out of the Bile-Veſſels, and into the *Duodenum*, as alſo from ſomething Leſs than an Ounce of Bile Paſſing thro' the ſaid Ducts in the Time of one Circulation of the Blood; Since conſequently in the Time, in which 40 Circulations of the Blood are Performed, that is, in 2 Hours and an half, the whole Maſs of Bile, which does not Exceed 2 Pounds, is carry'd from the Liver into the *Duodenum*, that the ſame Effect may be continued from the Liver, it is neceſſary there ſhould be a Circulation of the Bilious Maſs.

After this we have Three Propoſitions; Firſt, That it does not ſeem Poſſible, that the Circulation of the Bile ſhould be Performed thro' the Heart by a Mixture of it with the Arterial Blood, but thro' the *Abdomen* only; Second, That there are Ducts, which may be Aſſigned, by which the Circulation of the Bile may be Performed thro' the *Abdomen*, that is, by the Meſaraick Veins; Third, That there is a Neceſſity and Uſe of the foregoing Circulation of the Bile, in Reſpect of which Propoſitions, I refer my Self to the moſt Incomparable Author.

The Circulation of the Bile Agreeable to and Confirmed by our Doctrine.

§. 19. As to the Circulation of the Bile, it is not only Rendered Probable and almoſt Certain from the Arguments above, but alſo from an Original Circulation of Fluids in thoſe Parts in the very *Fœtus*, Diſtinct from the Circulation of the Sanguineous Maſs, and Peculiar to them; Thus in the *Funiculus* or *Inteſtinulum*, beſides the *Urachus*, are Comprehended Two Arteries and the Umbilical Vein, by the Latter of which a Nutritious Juice, and ſome Blood, is convey'd from the *Placenta Uterina* to the *Vena Porta*, and ſo to the Liver, and by the Former, the Groſſer Part of the Nutritious Juice and Blood is carry'd back, and ſome of it Depoſited in the *Amnion*, to be Suck't in by the Mouth of the *Fœtus* for it's Nouriſhment that way, and the reſt is Returned to the *Placenta*; So that in the Firſt Formation of the Human Syſtem, here is a Circulation of Fluids from and towards the Liver, and therefore upon the Birth, when the Navel String is Cut, and that Circulation is Stopt, it is Reaſonable to Think, that Nature finds out a New way to continue the ſame Motions, or what are nearly Analogous to them, by conveying Blood to the *Vena Porta* from the Meſaraick Veins, and Returning it from thence, and the Liver, by the Bilary Veſſels.

This quick Circulation of the Blood and Nutritious Juices thro' the Liver in the *Fœtus*, will likewiſe Contribute to the Heat and Expanſive Force of it; and conſequently Confirms and Eſtabliſhes our Doctrine concerning it, and if the Bile afterwards is frequently Tranſmitted thro' the ſame Bowel, it Exalts that Expanſive Force and makes it capable of Converting Blood, which has a Leſs Degree of the Expanſive, into it's own Nature and Action.

That the Bile is not Separated from the Blood by a Colature, but made by the Expanſive Force of the Liver.

§. 20. Thus far therefore we Agree with this Celebrated Writer, but when he ſuppoſes, that the Bile is Separated from the Blood by a Colature in the Liver, he ſeems to be far from Truth, and his Arguments are as Weak, by which he Defends his Opinion: He ſays, that Fermentation Requires ſome Time to Perform it in, and cannot be Raiſed and Effected in the Momentany Paſſage of the Blood thro' the Liver.

To which we ſhall Firſt Anſwer from himſelf, that the Blood is not carry'd into the Liver with an Impetuous Motion, as it is in the Reins from the Emulgents, but Proceeds by a Gentle Diſtillation from the *Porta* into it, and conſequently, ſince a firm Connexion of Parts Requires Time and Fermentation to ſeparate Them, it is not certain, but Nature may have taken that Method in Separating the Bile; Second, We Anſwer, that to Convert Blood into Bile, it is not neceſſary to Imagine there is any Fermentation, but that the Action or Expanſive Force of the Liver is immediately Impreſs'd upon and Communicated to the Blood, as Fire, or the Expanſive Force of it, to Flax or Gun-Powder, without any ſuch Fermentation: He farther ſays, that by the Continual Flux of the Blood or Juices into the Liver, ſuch a Fermentative Quality would by Degrees be Impaired and Weakened, and at laſt be wholly Deſtroy'd, as a Vinegar Veſſel by a Continual Affuſion of Water looſes it's Acrimony; but if we conſider the Liver, as a Gentle Animal Flame, and the Blood, like a kind of Fuel, Convertible into it's Nature, this Objection will be of no Weight or Importance.

As to what is Alledged, that the Bile is Bitter, and therefore the Liver ought to be ſo, if it Made and Fermented, and did not Percolate the Bile, whereas the Liver is Sweet; It may be Anſwered, that Sweet and Bitter are only Various Degrees of the Expanſive Force, Acting upon the Palate and the Tongue; and conſequently, the Expanſive Force of the Liver, Exerting it ſelf upon that of the Influent Blood, may produce a Bitter, tho' the Liver is Sweet; as Fire can render Sweet Things Bitter, by Parching of them, and Impreſſing too Great a Degree of the Expanſive upon them, ſo as to make them Offenſive to the Taſt by their too Violent Action.

It is likewiſe thought hard to Imagine, that a Fermentation ſhould be Raiſed in the ſmall Veſſels of the *Porta*, and in the ſtreight Pores of the Hepatick Glands; but this Difficulty will Vaniſh with thoſe, who do not ſuppoſe the Bile to be Produced by Fermentation, but by an immediate Impreſſion of the Expanſive Force of the Liver upon the Blood Imported into it, by which it Changes it's Sanguineous Temperament into a Bilious one.

On the other Hand, there is Little urged on the Side of Colature or Incuneation: Excepting, that there is a *Vis Motiva* from the Compreſſion of the *Abdomen*, and the Air drawn in by our Breath; that there is a continued Motion; that there are ſmall Canals of the *Porta* and Pores of the Glands, and Two Orders of Veſſels for the Reception of the Different Fluids, the Venous and the Bilary; all which will be as neceſſary for the Circulation of the Bile, and carrying off the Superfluous Blood, which is not Converted into it, as for the Separating of it by Colature or Incuneation.

A Solution of the Pancreas, *and it's Juice.*

§. 21. As to the *Pancreas*, which Forms a Lymphatick Juice, which we reckon to be of a Contractive Nature, ſince it Contains Abundance of Volatile Salts in it, we may juſtly Eſteem it deſigned for an Equal Balance to the Liver in the Human Syſtem, in Preparing a Liquor of a Contractive Force, which ſhould be a Poiſe to the Expanſive of the Bile; and which it therefore Meets in the *Duodenum* or *Jejunum*, and Attends it in ſeveral Ducts to the Thoracick, and to the Receptacle of the Chyle, and from a Mixture of which the Blood may poſſibly owe in a Great Meaſure it's Red Colour, or a Deep Yellow, it's Flame and Heat, and it's Saltneſs and Volatility.

And it is not Impoſſible, but the Formation of it may be from another Gentler and Milder Fermentation within the Body of the Included Air, the Nervous Juice, and the Blood, From whence it's *Lobuli* and *Acini Glanduloſi* Ariſe; but as the Expanſive Force Prevailed in the Liver by the Great Influx of Blood Heated, ſo in the *Pancreas* the Contractive by the Nervous Juice and the Aſſemblage of *Lympha*; by which means it became capable afterwards of Converting the Fluids Imported into *Lympha*, as the Liver obtained the Quality of Changeing them into Bile.

The Nature of the Spleen, and it's uſe Explained.

§. 22. We ſhall only here add in Reſpect of the Spleen, *Lien*, or Milt, that it ſeems to be a Bowel made by Nature for the Excrement, or Effœte and Languid Part of the Bile, when it's Expanſive Force is Worn off and Spent, tho' Anatomiſts have hitherto been at a Loſs to aſſign a Uſe for it, whoſe ſeveral Opinions it will be too Tedious here to Recount, being either Contradicted in Fact or not Supported by Reaſon, as may be ſeen at Large in Dr. *Gibſon*; What therefore we are apt to believe is the Truth, is, that the Meſaraick Veins convey ſome Part of the Excrementitious or Languid Bile to the Spleen, where, when there is too Great a Quantity of it, it Lodges and Grows ſtill more Acid and Vapid; that from thence it is carry'd into the *Ramus Splenicus*, and ſo to the Liver, where it is again Exalted into Bile; And to Enliven the Bilious Excrement and Prepare it for New Bile, that the Influx of the Nervous Juice, and of the Blood, in ſome Meaſure Contributes;

This Account of the Spleen Agrees with *Borellus*'s Circulation of this Fluid, and Avoids all the Abſurdities, which are Chargeable, either upon the Hypotheſes of the Ancients, or thoſe of Dr. *Gliſſon*, *Velthuſius*, Dr. *Mayow*, *Blancard*, or Dr. *Havers* in his *Oſteologia Nova*.

AND from this Notion of the Spleen ſeveral Phænomenons may be Explained, which otherwiſe could not be; FIRST, That tho' there are Two or Three Spleens, out of each of them a Veſſel Paſſes into the *Ramus Splenicus*, that is, to convey the Acid or Vapid Bile to the Liver to be again Recocted into New Bile.

SECOND, That in Infants New-Born it is of a Red Colour, in thoſe of a Ripe Age ſomewhat Blackiſh, and in Old Men of a Leaden or Livid Complexion; Becauſe the Excrementitious or Languid Part of the Bile in Infants has the Greateſt Expanſive Force, in thoſe of a Ripe Age a Leſſer, or a Deeper Red, and in Old Men the Leaſt, or a Leaden and Livid, or Violet; For thus the Bile it ſelf has theſe Different Forces, being in Young Children more Hot and Expanſive, in Grown Perſons leſs ſo, and in Old Men leaſt of all, other Juices and Temperaments of the Animal Oeconomy being Equal; And in Conſequence of this, we find Children Hot and Froward, Men Sedate, and Old-Age Dull.

THIRD, From hence it obtains that Affinity with the Liver, which made *Hippocrates* and *Ariſtotle* Term it the Left, or the Baſtard one.

FOURTH, This alſo gives a Reaſon, why Wounds in it are commonly Mortal, as in the Liver, becauſe they are both Subſervient to the Forming of the Bile; and Obſtructions in it occaſion Grievous Pains, as it contains an Acid Liquor, which wants to be Raiſed into a Bilious one by Circulating thro' the Liver; the Stoppage of which Circulation, and the Contractiveneſs or Acidity of the Fluid Increaſing by it, muſt neceſſarily afflict the Patient in a ſenſible Manner; And yet if the Spleen is taken wholly away, there may no Great Inconvenience Enſue, becauſe then the Circulation will be Performed by the Meſaraick Veins, and there will be no Stagnancy of an Acid Fluid in the Cells and Glandules, which Compoſe it.

FIFTH, And therefore likewiſe it is, that the Fluids take ſo Tedious a March from the Spleen to the Liver by the *Ramus Splenicus*, whereas they might have been readily conveyed into the Trunk of the *Cava*, which lies near it, if the Juices, Elaborated or Lodged in it, had not been for the Service of the Liver.

LASTLY, Hence it is, that when the Quantity of Choler or Bile is too much Increaſed by an High or Luxurious Diet, or by Wines and Hot or Expanſive Liquors, Eager and Violent Paſſions are many Times Produced in the Mind; which are often Attended with a Sinking and Depreſſion of it, from a too Great Quantity of the Languid, Vapid, and Excrementitious Bile Stagnating in the Spleen, or at leaſt not Circulating ſo Expeditiouſly, as when there is a Leſs Load or Maſs to be Diſcharged from it thro' the *Ramus Splenicus* to the Liver; and which therefore cauſes Pains in ſome Meaſure Approaching to thoſe, which Reſult from Obſtructions in the ſame Bowel; which likewiſe, as the Liver and *Pancreas*, is Formed by the Fermentation or Conteſt betwixt the Expanſive and Contractive Forces, the Vaſcular Parts being made by the Contractive, and the Fluids contained, by the Expanſive. To Explain this a little farther; The Expanſive Force Acting from a Center, or Centers, Dilates it ſelf, by which a Fluidity or a Separation of Parts Enſues, the Contractive Force Tends towards a Center, or Centers, from whence a Solidity or Connexion of Parts Ariſes; And when theſe Two Forces meet together, it is neceſſary, in certain Circumſtances, that the Expanſive ſhould Exert it ſelf Outwards in the Contained, and the Contractive Exert it ſelf Inwards in the Container; That is, that the one ſhould be the Fluid, and the other the Veſſel, that the one ſhould Endeavour to Dilate and Expand it ſelf, and the other to Reſtrain and Encloſe ſuch Dilatation or Expanſion; And theſe Forces Acting againſt each other, the Contractive will ſurround the Expanſive, as being Superior in the Animal Syſtem, by what we have before ſhewn.

CHAP. VI.

Concerning the Meſentery, the Receptacle of the Chyle, the Thoracick Duct, and the Via Lactea, *or the Paſſage of the Chyle into the Blood.*

An Account of the Meſentery, the Receptacle of the Chyle, the Thoracick Duct, and the Via Lactea, *or the Paſſage of the Chyle into the Blood, by Anatomiſts.*
Syllabus Enterologiæ. Prælect. 3.
Dr. Gibſon's Anatomy, Chap. 9, 10. Book 1.
Dr. Keill's Anatomy, §. 7, 8. Chap. 2.
Dr. Drake's Anthropologia Nova. Chap. 12. Book 1.
Blancard's Anatomia Reformata, Cap. 22.
Tauvry's Rational Anatomy, Chap. 3. Part 1.

§. 1. THE next Thing to be Conſidered in the Animal Frame is the Meſentery, and the Paſſage of the Chyle into the Blood: The Account of which Former is, That it is ſo called from it's being placed in the Midſt of the Inteſtines, that it is a Membranous Part ſituated in the Middle of the Lower Belly, and ſerves both for Conveying Veſſels to and from the Guts, and Tying them together ſo, as they ſhould not be Entangled aud Confounded; It is not above a Span diſtant from the Center to it's Circumference, and yet in that it Comprehends all the Guts, excepting the *Duodenum* and Part of the *Rectum*, which are about Nine or Ten Yards in Length; It is almoſt of a Circular Figure, which is the moſt Capacious, and tho' it be Narrow and Plain at it's Riſe, it's Circumference is Wrinkled and Enlarged into ſo many Folds, as to be Three Ells in Extent; It has a Double Origin, the Higher is at the Firſt, and the Lower at the Third *Vertebra* of the Loins; It is Compoſed of Two Common Membranes, which are Propagated or Continued from a Duplicature of the *Peritonæum*, and betwixt theſe Two it has a Third Membrane, which is Proper, and is Thicker than either of the other, in which the Glands are Seated, and by which the Veſſels are Conducted; It is Stuffed with Fat betwixt it's Membranes, tho' in Dogs, Cats, and ſuch like Animals, this Part is very Thin and Tranſparent, in Human *Embryos* there is None, and in Lean Perſons but Little; It's Common Parts are Veins, Arteries, Nerves, and Lympheducts, and it's Proper are Glands and the *Venæ Lacteæ*.

The Veins and Nerves of the Meſentery.

THE Veins are called *Meſaraicæ*, and are Branches of the *Vena Portæ*, and the Arteries are Branches of the *Arteria Meſenterica*, and Paſs as the Veins do.

THE Nerves are thus Accurately deſcribed by Dr. *Willis*, in his Book *De Cerebro*, *Cap.* 25. As ſoon as the Intercoſtal Pair is Deſcended as Low, as over againſt the Bottom of the Stomach, it ſends forth on each Side a large Meſenterick Branch, each of which is again Divided, and makes Two *Plexus*'s in each Side; in the Middle of theſe is the Greateſt *Plexus* of all, which, as he ſays, is like the Sun amongſt the Planets, from which Twigs and Numerous Fibres are Diſperſed into all the Parts of the Meſentery, which accompanying the Sanguiferous Veſſels in their whole Proceſs, do Climb upon, and Twiſt about them; It has likewiſe, according to *Spigelius*, other Nerves, which Spring from the *Spinalis Medulla* between the Firſt, Second, Third, and Fourth *Vertebræ* of the Loins.

The Lympheducts of it.

THE Lymphaticks or Lympheducts, according to *Tho. Bartholin*, a Learned *Dane*, who Firſt found them out, and that not many Years ago, are of Figure Long and Hollow like a Vein, but very Small and Knotty, having many Valves, which Permit the *Lympha* or Water to Paſs to the Chyliferous Veſſels, and to ſome Veins, but hinder it's Return; They are of a Pellucid and Chryſtalline Colour, conſiſting of a Tranſparent and moſt Thin Skin, which being broken, and the *Lympha* Flowing out, entirely Vaniſhes; They are almoſt Innumerable, and according to *Malpighius*, always have their Riſe and Proceed from Glands, but according to *Steno*, they either Riſe from or are Inſerted in them; He farther ſays, that there are Three Sorts of Them; Firſt, Such, which from their Origins, but what thoſe are he does not Determine, are Diſperſed over the Surface of the Firſt Glands they Approach unto; Second, Such, which Run from the Hollow Side of one Gland to the Gibbous of another; And Third, Such, which Run from the Hollow Side of the ſaid Glands to the Veins, into which they Diſcharge their *Lympha*; As to their Inſertion, it is Agreed by Anatomiſts, that Thoſe under the Midriff Diſcharge their Liquor into the Receptacle of the Chyle, Thoſe in the *Thorax* into the Thoracick Duct, and Thoſe of the Neck, Arms, *&c.* into the Jugular Vein; It is alſo Affirmed by *Diemerbroeck*, that they open into many other Veins, and for which he Cites *Steno*; *Frederick Ruyſch* likewiſe, his Country Man, Aſſerts, that by Ligature and Structure of the Valves, he has plainly ſeen, that all the Lympheducts in the Lungs do carry their *Lympha* into the Subclavian, Axillar, and Jugular Veins.

What the Lympha is and it's uſe, according to Anatomiſts.

WHAT this *Lympha* is, is much Diſputed, *Bartholin* thinks it to be mere Limpid Water, with which is mixed a Thinner Part of the Chyle fit for Nutrition, and that the Former is thrown off by Sweat, and the Latter carry'd on by Circulation; *Gliſſon*, that it is a Liquor Condenſed from the *Halitus* of the Blood, like Dew, Driven into theſe Veſſels, and Flowing back with the Vehicle of the Aliment brought by the Nerves; *Segerus* and *Sylvius*, that it is the Animal Spirits, or is made of Them, which being Diſtributed into all Parts by the Nerves, are partly there Diſſipated and Conſumed, and partly Condenſed into this Water; *Diemerbroeck*, that it is a Fermentaceous Liquor, Separated from the Serous Part of the Blood in the Conglobate Glands, yet not Simple, but Impregnated with much Fuſed and Volatile Salt, and alſo with ſome Sulphureous Particles, which, when it is convey'd to the *Vaſa Chylifera*, makes the Chyle Thinner, and apt to Dilate eaſily in the Heart, and when to the Veins, Prepares likewiſe the Venous Blood, now grown Thick, for ſuch Dilatation; This is certain, that it Differs from the *Serum*, for if a Little of it is gathered in a Spoon, and ſuffered to Stand, without ſetting it over a Fire, it will Turn to a Jelly, which the *Serum* will not do; Some of the Lympheducts of the Meſentery paſs only thro' it from other Parts, as from the Liver, *&c.* but Many have their Riſe in it.

The Glands of the Meſentery.

IT has many little Softiſh Glands, fixed in it's Proper Membrane Covered on each Side by the Two Common ones, and beſet with Fat, out of which, when Cut, may be Squeezed a Whitiſh Liquor; They are Whiter in Young People than in Old; and in Men Fewer and Smaller than in other Creatures; The Biggeſt is at the Riſe or Center of the Meſentery, called by *Aſellius*, tho' Improperly, the *Pancreas*, into which all the *Venæ Lacteæ* are Inſerted; When theſe Glands grow Schirrous and Hard, or are any ways Obſtructed, ſo that the Chyle cannot be Strained thro' them, there follows a *Fluxus Cœliacus*, or *Chyloſus*, upon a Continuance of which, an Atrophy enſues, and the Party dies Tabid; and perhaps, it is ſaid, from the ſame Obſtruction, by which the Courſe of the *Lympha* is ſtopt, and the Lympheducts by an over Extention burſt, an *Aſcites* is partly cauſed.

It's Venæ Lacteæ, *and their ſeveral Kinds.*

THE *Venæ Lacteæ*, or Milky Veins, ſo called from the White Colour of the Chyle, which they carry, were firſt Diſcovered by *Caſpar Aſellius* in the Year 1622, upon Diſſecting a Live Dog; They are ſlender Pellucid Veſſels, having but a Single Coat, Diſperſed in Great Numbers thro' the Meſentery; Their Riſe is from the Inmoſt Membrane of the Inteſtines, where their Mouths are hid under a kind of a Spongy Cruſt, or *Mucus*, thro' which, by the Preſſion of the Guts, the Chyle is Strained and Received by the Mouths of theſe Veſſels; From whence they Proceed the Readieſt way to ſuch Glandules of the Meſentery, as are neareſt to Them, but in their Paſſage many Small ones, uniteing to one another, do commonly grow into one Large Trunk, namely a Pretty way, before they Inſinuate themſelves into the Gland, but in their very Entrance into it, or a Little before, this Trunk Separates again into New Branches, More and Smaller than the other, and thus far they are called *Radicales*, or *Primi Generis*; Out of the Gland there Spring again New Capillary ones, which by and by Meeting together make one Trunk as before, which, keeping it's Courſe towards the Center of the Meſentery, Enters as many Glands as lie in it's way, being Divided into New Branches, juſt before it's Entrance into each Gland; but whilſt all the Trunks Bend one way, they alſo, Meeting with one another, do in their Proceſs, ſeveral of them, Grow into one, and at Length all the Trunks Arrive at the Great or Middle Gland of the Meſentery, call'd Improperly the *Pancreas* by *Aſellius*, which moſt of them Enter into, but ſome of them paſs over it's Surface, and preſently after, They all Empty themſelves into the Great or Common Receptacle of the Chyle, that lies behind the ſaid Gland; Thoſe that were Inſerted into it Riſing out of it in like Manner, as they did before out of the Leſſer Glands; As they Run from one Gland to another, they are called *Secundi Generis*; and from their having paſſed all the Glands to their opening into the Common Receptacle, they are called *Tertii Generis*.

The Common Receptacle of the Chyle.

THE Common Receptacle and the *Ductus Thoracicus* were Firſt found out in Horſes, by *Bartholomæus Euſtachius*, as Appears by a Book he Writ *Anno* 1564, but the true Uſes of them were Firſt aſſigned by *Pecquet Anno* 1651; It is called the Common

Receptacle, becauſe it Receives both the Chyle and *Lympha* Promiſcuouſly; the Latter of which continues to Glide into it, after the Former is all Diſtributed, and to Aſcend by the *Ductus Chyliferus Thoracicus*, which therefore might as juſtly be Termed *Lymphaticus*; It is ſeated under the Celiack Artery and Emulgent Veins, about the Middle Diſtance between the Kidneys, and the *Capſulæ Atrabilariæ*, upon the *Vertebræ* of the Loins, for the moſt Part rather towards the Left Side; *Pecquet* and *Caſpar Bartholin* Affirm, that 'tis alſo Situate betwixt the Tendons or Appendices of the Diaphragm, by the Motion whereof it is Preſs'd, and the Contents of it Propelled; It is of a Membranous but Thicker Subſtance in Men than in Brutes, but not ſo Capacious, ſeldom being ſo Large, as to admit one's Finger's End.

The Ductus Thoracicus.

Out of it there Springs a Duct, that preſently Aſcends up into the *Thorax*, behind the Deſcending Trunk of the *Arteria Magna*, where it begins to be called *Ductus Thoracicus*, which, according to *Sylvius*, might be more fitly called *Spinalis*, as it runs along the Inſide of the *Spina Dorſi*; This Duct, having paſs'd the Midriff, Marches upward under the Great Artery, to about the Fifth or Sixth *Vertebra* of the *Thorax*, where it Turns a little Aſide from under it to the Left Hand, and ſo underneath the Intercoſtal Arteries and Veins, and the Gland *Thymus*, it Aſcends to the Left Subclavian Vein, into whoſe Lower Side it opens, juſt there, where the Left Jugular Vein enters into it on the Upper Side, ſo that their Mouths face one another; but it opens not into this Vein with any Large Orifice, but by Six or Seven little Ones, which are Covered in the Cavity of the Subclavian with one Broad Valve, looking towards the *Cava* from the Shoulder, whereby there is Granted to the Chyle and *Lympha* a free Paſſage out of the *Ductus Chyliferus* into the Subclavian, but their Return out of the Vein into the Duct is Prevented, as well as that of the Blood Contained in the Subclavian; The Duct ending thus in the Subclavian, the Chyle, which it Conveys, Paſſes with the Blood, Returning by the *Cava*, into the Right Ventricle of the Heart; This Duct has many *Semilunar Valves*, that hinder the Aſcending Chyle and *Lympha* from Deſcending back again, which *Valves* are thus Diſcovered, becauſe the Chyle contained in the Duct may eaſily be Preſſed upwards, but by no means downwards.

The Difference betwixt the Venæ Lacteæ *and the Meſaraick Veins.*

The *Venæ Lacteæ* differ from the ordinary Meſaraick Veins; First, In Bigneſs, for the Meſaraicks are Bigger, but the *Lacteæ* are more in Number; which is thought to be neceſſary, becauſe there muſt be more Chyle Conveyed thro' the Latter for the Nouriſhment of the whole Body, than Blood, thro' the Former, Remaining from the Nouriſhment of the Inteſtines only.

Second, In Colour, the Lacteals being White and Clear, and the Meſaraicks of a Dusky and Blackiſh Complexion.

Third, In their Inſertion, the one Terminating in the Common Receptacle, and the other in the Liver.

The Difference betwixt the Lacteals and the Lympheducts.

Betwixt the Lacteals and Lympheducts there is not the ſame Diſtinction, and ſome Queſtion, whether the ſame Veſſels be not in the Meſentery Common Conduits for both Liquors, the Chyle and the *Lympha*; This however is obſerved, that during the Time of Digeſtion, there are no Lymphatick Veſſels to be ſeen in the Meſentery, but it Exhibits an infinite Number of Milky ones; and on the Contrary, after Digeſtion, all the Milky Veins Diſappear, and a prodigious Number of Lymphaticks preſent themſelves to our View. The Lacteals have a pretty many Valves, but not ſo many as the *Ductus Thoracicus*, and which may be Diſcovered the ſame way, as thoſe of the Latter.

Why the Chyle Tends towards the Heart, according to the preſent Anatomiſts.

The Way, which the Chyle takes in it's Paſſage to the Heart, has been hitherto Deſcribed, but what ſhould be the Reaſon or Cauſe of this Motion is to be Inquired; It is thought not to Proceed from the Attraction of the *Lacteæ*, as if they Sucked up the Chyle out of the Guts, for ſuch Elective Attraction has been a long Time Exploded, but the true Reaſon is Imagined to be the Preſſure of the Guts, whereby the Chyle is Squeezed thro' their Spongy Inner Cruſt or Coat into the Mouths of the *Lacteæ*; Which Preſſure Proceeds partly from that undulating Contraction of the Guts, that is Performed by their own Fibres, which one may plainly obſerve in Conies opened Alive, and partly from their being heaved to and again by the Muſcles of the *Abdomen*, and the Diaphragm in Reſpiration; After which the whole Progreſs of the Chyle is ſuppoſed to be made by a Continual Protruſion of it to the Heart; and the Aſcent of it is reckoned to be aſſiſted by a Mixture of the

Lympha with it in the Receptacle and the Duct; not only becauſe the Chyle is thereby Dilated, but more Eſpecially, becauſe a Motion is Impreſſed upon the *Lympha* from the Pulſation of the Heart, whereby it is made to Circulate by the Lymphe-ducts, as well as the Blood by the Veins: This is the Account, which the Anatomiſts give of the Chyle, and the *Lympha*, and the Paſſage of them to the Heart, and of the Ducts, by which they are Conveyed thither; which we ſhall now moreover Explain by our Doctrine of the Expanſive and Contractive Forces.

Why there is a Nervous Duct of the Spinal Marrow Anſwerable to the Alimentary one, and the Aorta, *and how the Meſentery, and it's Veſſels are Formed, as well as the Ribs and other Bones in the Animal Syſtem.*

§. 2. And Firſt it is to be Obſerved, that as there is an Alimentary Duct from the *Oeſophagus* down thro' the whole Canal of the Inteſtines to the *Rectum*, by which an Expanſive Force is conveyed thro' the Body, of which Force our Aliments principally Conſiſt, as likewiſe the *Aorta*, ſo to Balance theſe there is likewiſe a Contractive Force of the Spinal Marrow Propagated from the *Cerebellum*, and the *Medulla Oblongata*, thro' the *Vertebræ* to the *Os Sacrum*; And as the Chyle Paſſes from the Alimentary Duct towards the Center of the Meſentery, ſo from thence a Contractive Force is Diffuſed, namely, from the *Vertebræ* of the Loins, to Meet and Oppoſe, and Counterpoiſe the Expanſive of the Former; the Contractive Force from the Brain, and the Medullary Subſtance Derived from it, Inveſting the Expanſive of the Chyle, and Blood, and *Lympha* with Coats, and Cementing thoſe Coats together with Membranes, which Form the Meſentery; And it is not Improbable, that the Ribs are only Protruſions of the ſame Contractive Force from the Spinal Marrow; a Membrane, a Cartilage, and a Bone differing chiefly in their Degrees of Hardneſs, which ſeems to be Induced upon them from their Lying betwixt the Two Contrary Actions of the Expanſive and Contractive Forces of the Nervous and Sanguineous Syſtems, by which they are Conſtipated and Condenſed in the ſame Manner, as Steel by the Expanſive of the Fire and of the Hammer, and the Contractive of Water and of the Anvil; and the like may be ſaid of the other Bones.

Why the Meſentery is Simple in it's Origin, and Circular in it's Figure, and the Reaſon of of it's Fat, and it's Nerves, and the Plexus's *of Them.*

§. 3. The Meſentery is Plain and Simple in it's Origin at the *Vertebræ*, as the Contractive Force is ſo, but is afterwards Diſplayed in ſeveral Wreaths and Folds, and Enlarged into a Circumference, by it's Exerting it ſelf againſt the Expanſive from the Alimentary Duct, and the Chyle, and Blood, and *Lympha* Diſperſed in the ſaid Meſentery.

It is not above a Span from it's Center, becauſe the Forces on each Hand have that Degree of their Exertion in their mutual Conteſt with each other; and it is nearly Circular for the ſame Reaſon, becauſe Equal Forces, Acting againſt each other, muſt Produce Equal Lines, and being Propagated from the Riſe of the Meſentery, thoſe Lines muſt be Radius's of a Circle Drawn from a Center.

It is Compoſed of Two Common Membranes, which are Continued from a Duplicature of the *Peritonæum*, which has alſo in Part it's Origin from the *Vertebræ* of the Loins; Betwixt theſe Two it has a Proper Membrane, which is Thicker than either of the other, becauſe the Blood-Veſſels, the Chyliferous, and Lymphatick, have their Courſe thro' it, on which Account the Contractive Force is Gathered in Greater Quantity about the Expanſive in thoſe Veſſels, and the Membrane thereby becomes Thicker; And from hence it obtains the Middle Situation; For the Contractive Force Acting againſt the Expanſive, firſt Inveſts the Fluids with Coats, then Strengthens and Unites them by Membranes and Glands, which Intercede the Veſſels; and laſtly, Encompaſſes the Proper Membrane with one on each Side, that it may ſufficiently Encloſe the Expanſive Force, againſt which it Oppoſes it ſelf, by which the Expanſive is Obliged and Determined to a Circulation.

It is ſtuffed with Fat betwixt it's Membranes, which Ariſes, as has been ſaid, from a Mixture of the Expanſive and Contractive in their Mutual Conteſt, but in Lean Perſons, in Human *Embryos*, and in Dogs and Cats, and ſuch like Animals, there is little of it, becauſe the Contractive is ſo far Prevalent over the Expanſive, that there is not a due Mixture of Them; Which is Evident from the Account we gave before of Lean Perſons and of a *Fœtus*; and Dogs and Cats have from their Fierceneſs or the Prevalence of the Nervous Syſtem, and the Defect of the Muſcular and Sanguineous an Exceſs of the Contractive.

The Nerves, as Containing a Contractive Force, diſperſe Themſelves in vaſt Numbers thro' the Meſentery, in order to aſſiſt to this Effect of Forming theſe Membranes by their Fibres every where Detach'd to Encounter the Expanſive of the Blood and Alimentary Fluids, as well as that alſo of the Bile, which here Mixes with them; Hence thoſe Two Notable *Plexus*'s are made, one of which is Compared to the Sun amongſt the Planets, and hence they accompany the Sanguiferous Veſſels in their whole Proceſs, and Climb upon, and Twiſt about Them by their Oppoſition againſt and Counter-Action to Them; And hence laſtly it is not Improbable, that there are other Nerves, according to *Spigelius*, beſides the Meſenterick, which Spring from the *Medulla Spinalis*, between the Firſt, Second, Third, and Fourth *Vertebræ* of the Loins, which Iſſue from thence to Curb and Reſiſt the Expanſive in Conjunction with the Foregoing.

What it's Lympheducts are, and the Lympha *with which they are Repleniſhed.*

§. 4. THE Lympheducts ſeem to carry a Water, which bears the ſame Proportion of Force to the Animal Syſtem, as Common Water does to the Mundane, that is, it is in a kind of Middle Proportion betwixt the Expanſive and Contractive; Common Water is not ſo Contractive, as Salts, Froſt, or Metals, nor ſo Expanſive, as Spirits, Heat, or Fire, but is capable of being eaſily Cemented into Ice, or Reſolved into Vapours, by the Application of one or the other; and conſequently may be Converted in Cold Places, as in Vaults, into a Nutrition for Stones, and in Hotter into one for Plants and Vegetables; Thus likewiſe the Animal *Lympha* appears to be an Intermediate Fluid betwixt the Nervous and Sanguineous Forces, that is, the Contractive and Expanſive ones, and therefore is Expanſive in Compariſon with the Contractive Force, and Contractive, if Compared with the Expanſive; But it is more Spirituous, and has a Greater Expanſive Force than Common Water upon Two Accounts.

FIRST, Becauſe all the Fluid and Solid Aliments we take in, Excepting Water, are furniſh'd with a Moiſture or Watry Humour, but Raiſed into a Degree of the Expanſive by Heat or Fermentation, even before they are Received into the Mouth or Stomach, after which that Spirituous or Expanſive Force is again in ſome Meaſure Increaſed.

SECOND, Becauſe the Contractive Forces of the Body themſelves, as the Nervous Juice, Membranes, Cartilages, and even Bones, are Temper'd with a Greater Degree of the Expanſive, by the Continual Circulation of Warm and Expanſive Fluids amongſt them, than the Contractive Forces in the Mundane Syſtem; and therefore the *Lympha* of the one will be more Spirituous and Expanſive than the mere Water of the other, and this Notion of it will Account for all the Phænomenons of theſe Lymphaticks.

Why the Lymphaticks are Inveſted with a Thinner Coat than the Veins, and the Arteries with a Double, and the Reaſon of their Knots, and their Valves.

§. 5. THEY are Inveſted with a very Thin Membrane, or Coat, whereas the Veins are with a much Thicker, and the Arteries with a Double one; Becauſe the *Lympha* has a much leſs Expanſive Force than the Venal Blood, and ſtill Leſs than the Arterial, and therefore the Contractive Force from the Nervous Fluid is leſs Aſſembled about the Former, than the Two Latter, and a leſs Quantity ſerves to Incloſe, in order to Balance and Poiſe it: They are Long and Hollow like a Vein for the ſame Reaſon, which was given for the Veins being ſo; Becauſe the Expanſive Force being hindred from Dilateing it ſelf by the Contractive, which Inveſts it, muſt Exert it ſelf forwards and Form the Cavity of a Canal: They are Small; Becauſe the Expanſive Force of the *Lympha* is very Minute in Compariſon with the Blood, and therefore Enlarges it ſelf to a ſmaller Compaſs in it's Ducts, than the other; and are Knotty, becauſe the Contractive Force frequently Prevails againſt the Weakneſs of the Expanſive; to which Cauſe likewiſe it's Valves may be Attributed, which are only an Intruſion of the Contractive Force into the very Courſe and Conduit of the Expanſive, and which muſt be owing to the Prevalence of the Former, and to the Inability of the Laſt to Reſiſt it, that is to the Weakneſs of it: Theſe Valves open towards the Common Receptacle, and ſhut the Contrary way; Becauſe tho' the Contractive Force is ſtrong enough to Interrupt the Current of the *Lympha* at certain Intervals, yet the Expanſive Force of the *Lympha* is not ſo Languid, as to be Entirely Obſtructed by it, but by Preſſing forwards ſtill Removes and Opens the Obſtruction; But if it is Endeavoured to be forc'd back again, the Valves cloſe; becauſe ſuch an Impreſſion is Contrary to the Expanſive Force of the *Lympha* Forwards, which therefore becomes Contractive, and joins with the Contractive of the Valves, by which they both unite againſt the Violence Induced upon them; or, which is the ſame in Effect, the Expanſive Force of the *Lympha*, which Exerted it ſelf Forwards, having now a Contrary Determination Impreſt, it's Force is Loſt and Deſtroy'd, and therefore incapable of opening the Valve Backwards, or Surmounting the Contractive Force, which before it was ſufficient to do.

Why the Lymphaticks of the Meſentery have their Origin from the Canal of the Inteſtines.

§. 6. They, according to *Malpighius*, Riſe from Glands, and, according to *Steno*, Riſe from them or are Inſerted in them, but neither of them Determine, what thoſe Glands are, from whence they have their Origin; tho' it ſeems to be Manifeſt, that they Begin, as the Lacteals do, from the Alimentary Duct, or the Canal of the Inteſtines, and which is expreſly Affirmed by other Anatomiſts, which is the Reaſon, that ſometimes we find the very Lacteals filled with *Lympha*, which could not be, if it were not Derived into them from the ſaid Alimentary Duct; Beſides it is Natural for us to Think, that the Primary *Lympha*, before it Arrives at any Glands, is made from the Firſt Concoction and Leaſt Alteration of our Food, and that it is the Moiſture of it Separated from the Chyle, and Elevated into a Small Degree of the Expanſive by it's Paſſage thro' the Stomach to the *Jejunum*.

Why, during Digeſtion, the Lacteals appear in the Meſentery, and afterwards the Lymphaticks.

§. 7. And hence that Remarkable Phænomenon of *Tauvry* Admits of a Solution, that, during Digeſtion, the Lacteals appear in the Meſentery in Great Numbers, after which they Vaniſh, and the Lymphaticks Fill and Riſe in their Stead; Which muſt be owing to the Greater Expanſive Force of the Chyle, and a Leſſer Degree of it in the *Lympha*, by which the one Repells the other and Enters Firſt the Meſaraick Veſſels; and when it has thus opened a Paſſage, to it ſelf, to the Common Receptacle, the *Lympha* is left to Succeed it by a Slower and Heavier Expanſive; And for the ſame Reaſon it continues to Glide into the Common Receptacle, and to Aſcend in the *Ductus Thoracicus*, after the Chyle has Relinquiſh'd it.

Why theſe Lymphaticks Tend from their Firſt Origin to other Glands, and why the Greateſt Gland is in the Center of the Meſentery, and the ſmalleſt in the Coats of the Inteſtines.

§. 8. These Lymphaticks Tend from their Firſt Origin in the Glandules, or Small Knots, of the Inteſtines, which it is Probable are Formed by a Contractive Force to Curb and Reſiſt the Greater Expanſive of the Chyle and the Leſſer of the *Lympha*, to other Glands, which likewiſe are ſeveral Congeries and Aſſemblages of the Contractive Force to oppoſe the Expanſive of the ſame Chyle and *Lympha*, and which are Greater, as we Approach nearer to the Center of the Contractive; and therefore the Greateſt Gland is in the Center of the Meſentery, and Leſs, as we Come nearer to the Center of the Expanſive, and conſequently, the ſmalleſt Glands are in the Coats of the Inteſtines; which alſo furniſhes us with a Reaſon, why the Meſentery is Enlarged from a Center to a Circumference, becauſe at the one the Contractive Force moſt Prevails, and at the other the Expanſive.

Why theſe Lymphaticks are from their Origin diſperſed upon the Surface of the Subſequent Glands, and do not paſs thro' them.

§. 9. Hence alſo it is, that the Chyle and the *Lympha* paſs thro' the ſmall Glands of the Inteſtines, notwithſtanding their Obſtruction, becauſe their Contractive Force is Inconſiderable in Compariſon with the Expanſive of the other; but when the Lymphaticks in their Progreſs towards the Common Receptacle Arrive at any Subſequent Glands, they do not Paſs thro' them, but are Diſperſed on their Surface, becauſe the Contractive Force of the Latter is too Great to be Surmounted by the Former; On the Contrary, the Lacteals paſs thro' the ſame Glands, the Expanſive Force of the Chyle, which they Contain, being Greater than that of the *Lympha*, but yet they are always Diſperſed into ſeveral Small Channels from a Larger Duct, before they enter the Glands, by Reaſon of the ſame Oppoſition from the Contractive Force of them, and from thoſe Small Channels unite again into one Duct, after their Paſſage thro' the Glands, by the Property in Nature before Explained of Similar Forces Aſſembling to each other, by which alſo ſeveral Branches unite into one Trunk before their Entrance into the Glands; Laſtly for the ſame Reaſon, when all the Trunks of the Lacteals Approach to the *Pancreas Aſellii*, or Middle Gland of the Meſentery, moſt of them Enter into it by the ſame Superiority of the Expanſive Force, but ſome are Repelled by the Contractive Force of the Gland, which Force is here the Greateſt, and are made to Climb upon it's Surface; And all of them preſently after unite in the Common Receptacle by the ſame Property, Quality, or Affection in Nature, of Similar Forces Attracting each Other, by which They Aſſemble together, and Coaleſce into one.

How the Chyle and Lympha *are Altered in their Paſſage from the Inteſtines to the*

§. 10. It muſt be farther Obſerved, that in the Paſſage of the Chyle and *Lympha* from the Inteſtines to the Common Receptacle, their Forces are Altered by this Repulſion from the Glands, and their Re-union afterwards, for tho' there do's not ſeem to be any Percolation of the Chyle or *Lympha* thro' them, yet their Expanſive Forces

are

Common Receptacle, not by a Percolation of the Glands, but by a Change of their Forces.

are Changed by the Contractive of the Glands, and very Probably Diminiſhed by them, ſo that Creeping up by the Thoracick Duct to the Subclavian, they do not Enter that Vein in one Large Orifice, but in Six or Seven Little ones, the Expanſive of the Venal Blood in ſome Meaſure Repelling the Chyle and *Lympha*, which is now almoſt of a Contractive Nature, as when it was of a more Expanſive, the Glands, which are of a Contractive, had done; For this Reaſon alſo the *Ductus Thoracicus* is carry'd up on the Side of the Great Artery, the Expanſive of it's Fluids being Weak, and wanting to be Animated and Supported by the Cheriſhing Force of ſo Powerful a Neighbour: And hence likewiſe it was neceſſary, that the Bile, which is Hot and Expanſive, ſhould be conveyed to theſe Parts, to Act againſt and Poiſe the ſtrong Contractive, which was Propagated from the *Vertebræ* of the Loins, and the Center of the Meſentery; with theſe Forces, namely, the Languid Expanſive of the Chyle, the *Lympha*, and the Venal Blood, and the more Exalted of the Bile, the Entrance is made into the Right Ventricle of the Heart, which therefore Mixing with the Blood there, which is ſtill of a more Expanſive Force, muſt Cauſe an High Degree of Accenſion, and Contribute to the Vaſt Pulſation in it, which we ſhall afterwards conſider.

Why the Lacteals and Lymphaticks are either Inſerted into the Common Receptacle, or the Thoracick Duct, or into Veins.

§. 11. THIS alſo gives an Account, why the Lacteals and Lymphaticks are either Inſerted into the Common Receptacle, or the Thoracick Duct, or into Veins; For the Expanſive Force of the Blood in the Arteries is too Great to admit of a Force in the Lacteals and Lymphaticks, which, by the Smallneſs of it's Expanſive, has in Compariſon almoſt the Nature of a Contractive, and the Contractive Force in the Nerves is Repulſive of the Expanſive of the ſame Lymphaticks and Lacteals; On the Contrary, that Lacteals may convey their Fluids into Lacteals, or Lymphaticks into Lymphaticks, is Evident, by that Law of Nature, we before Mentioned, by which Similar Forces Aſſemble to each other; Thus alſo Lymphaticks may Transfer their Juices into Lacteals, and Both their ſeveral Liquids into Veins, becauſe they have all a low Degree of the Expanſive, and nearly Approach to one another in their Diſtinct and Particular Forces.

The Account of the Lympha *before Given, Agrees with the Notions of Anatomiſts, and ſhews why it Turns to a Jelly without Heating it, which Serum will not.*

§. 12. WHAT this *Lympha* is we have already Explained, that it ſeems to be Water Exalted into a ſmall Degree of the Expanſive in the Stomach and Inteſtines, and which well Enough ſuits with the Opinions of *Bartholin*, *Gliſſon*, *Diemerbroeck*, and *Segerus* Concerning it, which were before Recounted; And that by Standing, without Heating it, it Turns to a Jelly, which *Serum* will not do, is owing to the Low Expanſive of the *Lympha*, being ſubdued by the Small Contractive of the Air, from whence it Grows into a Conſiſtence; *Serum* on the Contrary is of too Heavy and Phlegmatick a Nature, as Common Water likewiſe is, to be thus Affected, Conſtipated, or Coagulated by the Gentle Force of the ſame Contractive, which yet ſubmits to a more Violent one, as that of Froſt and Extreme Cold; That is, Common Air has the ſame Influence to Reduce the Animal *Lympha* to a Contractive, which a Frozen one has to Impreſs it on Urine and Common Water; which ſhews the one is a more Fine and Delicate Fluid, and the other a more Lumpiſh and Groſs: As for the Lympheducts, which Paſs from the Liver thro' the Meſentery, 'tis not Impoſſible, but ſome of the Meſaraick Veins may Convey a *Lympha* in them to it, and by an Aſſemblage of *Lympha* from other Parts it may be Remanded back to the Meſentery to Attend and Balance the Expanſive Force in the Bilious Veſſels.

Why the Glands in the Meſentery are Whiter in Young Perſons than in old, and Fewer and Smaller in Men than in Brutes.

§. 13. THE Reaſon of the ſeveral Glands in the Meſentery, and of that in the Center of it, called *Pancreas Aſellii*, we have before Aſſigned: Why they are Whiter in Young Perſons than in Old, Proceeds from the Chyle being ſo, with which they are Tinged, and that, from the Expanſive Force of the Chyle being Greater in the one than the other, as all the Forces of the Animal Syſtem are; Theſe Glands are likewiſe Fewer and Smaller in Men than in Brutes, becauſe upon the Account of the Continual Aliment taken in by the Latter, which is of an Expanſive Force, and a Continual Digeſtion of it in the Stomach and the firſt Inteſtines, the Expanſive Force from the Alimentary Canal is Leſs in Men than in Brutes; and conſequently the Contractive Force from the *Vertebræ* of the Loins, and the Center of the Meſentery, muſt be Greater to Oppoſe and Balance the Expanſive, and therefore the Glands in the Meſentery muſt be more Numerous and Greater in the Latter than in the Former, in Brutes than in Men.

Why all the Venæ Lacteæ *are Inſerted into the* Pancreas Aſellii.

§. 14. All the *Venæ Lacteæ* are Inſerted into the *Pancreas Aſellii*; Becauſe the whole Expanſive Force of the one is Directed againſt and Unites to Oppoſe and Surmount the Contractive of the other, which is there the Greateſt: And when theſe Glands grow Schirrous and Hard, or are Obſtructed, That is, when the Contractive of them is too Great to be overcome by the Expanſive of the Chyle, it cannot be Conveyed into the Blood for a ſupply of it; the Blood cannot be Furniſhed to the ſeveral Parts of the Animal Syſtem for their Nouriſhment, and by conſequence, an Atrophy muſt Enſue, and the Perſon die Tabid; and by a like Obſtruction of the *Lympha*, an *Aſcites* is probably Cauſed.

Why the Common Receptacle is Membranous, and Thicker in Men than in Brutes.

§. 15. The Common Receptacle is Membranous, that is, of a Contractive Force; becauſe the Chyle and *Lympha* are of an Expanſive, which therefore it Surrounds and Incloſes; It is Thicker in Men than in Brutes, but not ſo Capacious, becauſe the Contractive and the Nervous Force is Greater in the Former, and the Alimentary and Expanſive in the Latter.

Why the Meſaraick Veins are Bigger, and the Lacteals more Numerous and Smaller.

§. 16. The Meſaraick Veins are Bigger, and the Lacteals more Numerous and Smaller, not for the Reaſon above Aſſigned, §. 1. ſince the Largeneſs of the one may be Compenſated by the Number of the other, but becauſe the Expanſive Force of the Venal Blood, tho' Leſs than that of the Arterial, is Greater than that of the Chyle; from whence the Cavity of the Meſaraick Veins is Dilated to a Larger Circumference than that of the Lacteals, and the Coats or the Contractive Forces, which Encompaſs them, are Greater and Stronger; The Veins are Fewer, becauſe the Expanſive Force, which they carry, is Stronger and therefore more United, that of the Lacteals Weaker, and therefore more Diſperſed by the Contractive.

Why the Chyle is White and the Blood Red.

§. 17. The Chyle alſo is of a White Colour, and the Blood of a Red, becauſe the Expanſive Force of the Firſt is Leſs, than that of the Laſt; It being known, that Rays of a Red Colour are the Leaſt Refrangible, that is have the Greateſt Expanſive Force, and thoſe of a White one Proceed from a Mixture of all the Different Degrees of the Expanſive Forces, as the Chyle may be juſtly Imagined to do.

That the Motion of the Chyle and Lympha *is not by the Preſſure of the Guts and a Protruſion, but by the Expanſive Force of the Chyle and* Lympha, *Tending towards the Expanſive of the Blood in the Heart.*

§. 18. Lastly, As to the Motion of the Chyle and *Lympha*, it is ſuppoſed to be by a Preſſure of the Guts, and afterwards by a Continual Protruſion of thoſe Fluids to the Heart; Againſt which there is this Objection, that the Contraction or Preſſure of the Guts cannot be ſo Great, as to bring the Sides of them together, without which, it is not to be Conceived, how ſo Thin a Fluid, as the Chyle and *Lympha*, ſhould be Forced into the Small Mouths of the Lacteals of the Meſentery, by a mere Periſtaltick or Vermicular Motion of the Guts, which is all the Contraction, they own or acknowledge: Beſides, afterwards, in the Lymphaticks and Lacteals, theſe Fluids cannot be Promoted or Protruded to the Heart by an Impreſt Force from the Inteſtines, for the Reaſon we have given before, becauſe by a Protruſion only they cannot be Forced back again, which the Tender Valves, in ſuch ſmall Veſſels, would certainly admit of, if there were not ſomething in the Real Forces of the Liquids and Solids, which Contradicted and Oppoſed it.

Farther, Tho' this might be Allowed, and that the Valves opened towards the Common Receptacle, and ſhut the Contrary way, yet it cannot eaſily be Imagined, how the Chyle and *Lympha* ſhould be Thruſt upwards thro' the Thoracick Duct by a Protruſive Force only; Since, according to this Account, the Gravitation of the Fluids backwards might be Superior, or Equal, to any Propulſion of them forwards, from whence there would be an entire Obſtruction to the Circulation of them.

On the Contrary, we ſay, againſt which there are none of theſe Difficulties, that the Expanſive Force of the Chyle and *Lympha* Tends towards the Expanſive Force of the Blood in the Heart, by that Law of Nature, which we have often Mentioned, of Similar Forces Aſſembling to each other; but the Contractive Force from the *Vertebræ* of the Loins coming in to Balance this Expanſive, theſe Fluids do not Proceed immediately to the Heart, but are carry'd thro' the Meſentery, the Receptacle, and Thoracick Duct, as before Explained.

Why the Chyle and *Lympha* do not Tend towards the Liver, which we have ſaid has a Great Degree of the Expanſive, but towards the Heart, is for Two Reaſons.

First, Becauſe the Courſe of both of them is Determined towards the Heart, or the Red Beating Speck in the *Embryo*, before the Liver is Formed, which is not done, till after the Joining of the *Placenta*, or Womb-Liver, to the *Uterus*.

SECOND, Becauſe the Expanſive Force of the Bile is ſo Great, that it Repells the Smaller one of the Chyle and *Lymphæ*, and Urges it towards the Receptacle and Thoracick Duct, and from thence to the Subclavian and the Heart: And this is the Reaſon, wherefore the Chyle cannot be Forced into the Lacteals by a mere Protruſion, according to Dr. *Drake*; Becauſe in a Dead Subject the Expanſive Force of it is Loſt, and the Tendency of it to a Similar Force in the Venal Blood Ceaſes, and cannot be Repaired by any Protruſion.

CHAP. VII.

Concerning the Thorax, *Diaphragm*, Pleura, *and* Mediaſtinum.

An Account of the Thorax, *Diaphragm*, Pleura, *and* Mediaſtinum, *and of the Heart and Lungs by Anatomiſts, and Firſt of the* Thorax *or Breaſt in this Section.* Syllabus Enterologiæ. Prælect. 3. Dr. Gibſon's Anatomy, Chap. 3, 4, 5, &c. to the 14. Book 2. Dr. Keill's Anatomy, Chap. 3. §. 1, 2, 3, &c. to the 6*th*. Dr. Drake's Anthropologia Nova. Chap. 1, 2, 3, &c. Book 2. to Book 3. Blancard's Anatomia Reformata, Cap. 2. to the 10*th*. Tauvry's Rational Anatomy, Chap. 4, 5, &c. to the 8*th*. Alphonſus Borellus de Motu Animalium, Cap. 4, 5, &c. to the 9*th*.

§. 1. THE next Part to the *Abdomen*, which is Examined in an Human Diſſection, is the *Thorax* or Breaſt; Which is Bounded above by the *Claviculæ* or Channel Bones, below by the Diaphragm or Midriff, which ſeparates it from the *Abdomen*, in the Fore-part by the Breaſt-bone and Cartilages, in the Sides by the Ribs, and behind by the *Vertebræ* of the Back; It's Figure is in a Manner Oval, ſomewhat Flat before and behind, whereas in Beaſts it is ſomewhat Sharp; It's proper Containing Parts are either External, as the Breaſts, the Muſcles, and the Bones, or Internal, as the *Pleura*, the *Mediaſtinum*, and Diaphragm.

As to the Breaſts, they have Arteries, Veins, Nerves, and Lymphaticks, in Common with other Parts of the Body, which it is not neceſſary here Particularly to Deſcribe and Diſtinguiſh; only that the Arteries and Veins are Branches of the Subclavian and Intercoſtal, and that the Nerves are from the Vertebral Pairs, and from the Sixth Pair of the Brain; And from the Quantity of Fat in them *Malpighius* Collects there are *Ductus Adipoſi* Diſperſed in their Subſtance; This may be ſaid, that they conſiſt of many Glandulous Bodies of a Different Bigneſs, of which there is one Bigger than the Reſt, juſt under the *Papilla*, or Nipple, which is of an Exquiſite Senſe, Spongy, Covered with a Thin Skin, and full of Little Holes; It is made of Two Orders of Fibres, the Smalleſt make a Fine Net-work within the Larger Spaces of the Net-work of the Larger Fibres; The Spaces betwixt the Glands are filled up with Fat, and there are Abundance of Veſſels that go from one to another, and are all Incloſed by the *Membrana Carnoſa*; Theſe Glands ſeparate the Milk or Chyle from the Blood, Iſſueing out of the Left Ventricle of the Heart, to which the Chyle is not yet Aſſimilated, and which is driven hither along the Thoracick Arteries, unleſs we allow the *Venæ Lacteæ* to come hither, which Opinion is not thought Probable; And in the *Papilla* all the *Tubi Lactiferi* End; The *Areola* is a Small Circle, which Surrounds the *Papilla*, which in Virgins is Pale and Knotty, in thoſe that are with Child, or give Suck, Brown, and in Old Women Black, as the *Papilla* is Red in Virgins, Livid in thoſe that give Suck, and in Old Women Blackiſh; The *Tubuli Lactiferi* are Seven or Eight, or more Small Pipes, which have ſeveral Croſs Canals, by which they Communicate with one another, and they grow Smaller, and Smaller, to their Extremities in the *Papilla*; The Milk is conveyed to them from the Arteries, which Terminate in the Glands, that ſeparate the Chyle from the Blood brought thither, as was before ſaid; which Chyle Runs into the *Tubuli Lactiferi*, upon the Suction of the Child: As to the other External Containing Parts of the Breaſts, namely, the Bones and Muſcles, we ſhall Leave them, with the Anatomiſts, to be Diſcourſed of Diſtinctly and Apart from the Preſent Argument.

That the Milk in the Breaſt is not the Chyle Separated by the Glands of the Mammæ *from the Arterial Blood, but that it is Made and Digeſted in the* Mammæ *by a*

§. 2. THAT which is here to be Conſidered is, what Account can be given of theſe Parts, and the Milk, which they contain, from the Doctrine of the Expanſive and Contractive Forces; Some have Thought, that the Chyle may paſs Directly to the Breaſt by Lacteals from the Stomach, or the Common Receptacle, or the *Ductus Thoracicus*, or from the Womb; but *Diemerbroeck* and Others, who are of this Opinion, are Confuted by *Steno*, Dr. *Wharton*, and Dr. *Needham*; and *Tauvry* has ſhewn, that all the Chyle is Tranſmitted thro' the Subclavian Vein, both by Experiments from Dr. *Lower*, and by his Own, which Confirm them: Anatomiſts therefore have Generally gone into the Sentiments before Explained, that the Chyle is Separated by the Glands, in the *Mamma* or Breaſts, from the Blood conveyed to them by the Mammary or Thoracick Arteries; But how this ſhould be is as little Intelligible as the other; Since the Chyle is ſuppoſed to be Transformed into Blood in the Heart, and how it ſhould be Separated from it by the Glands, when it is no longer in it, but is Changed into another Subſtance, is not to be Conceived; To Say, that the Chyle is not perfectly Aſſimilated into Blood, as Dr. *Gibſon*, and Dr. *Keill*, or, To Say, as *Tauvry*, that the Blood is Compounded of all Particles, of Milky, of Bilious, Urinous, *&c.* is To Say indeed, but not To Prove or Evince the Aſſertion.

Mixture with the Fat and the Nervous Juice.

It is more Probable therefore, that as the Chyle is made in the Stomach, and the Firſt Inteſtines, the Bile in the Liver, the Blood in the Heart, and the Nervous Fluid in the Brain, ſo that the Milk is in the Breaſts, and that it Differs from the Chyle, tho' not much in Colour, yet in Quality and Subſtance; And therefore Dr. *Wharton*'s Argument is of no Force againſt Milk being made from the Blood; Since according to him, if Chyle is Firſt Excocted into Blood, and then made into Chyle or Milk, Nature would Act in Vain and Fruſtrate her own Work: Whereas we Affirm, that Chyle is Effected and Produced from the Digeſtion of the Aliment, but Milk from a Digeſtion of the Blood in the Glands, and the Fat of the Breaſts, and that as the Cauſes, ſo the Fluids Formed and Conſtituted from them, are Different.

Milk and Chyle Different in their Natures.

Thus, tho' our Aliment is Red, or Green, or Yellow, or of any other Colour, it is Reduced to a White Chyle, and is Reſembled in it's Conſiſtence and Colour to Barley-Broth; And by the ſame or the like Operations in the Animal Syſtem, it is not Inconceivable, that Blood may be Reduc'd to a Milk; which is very Different from the Former Fluid, ſince no one ever Imagined, that Water-Gruel could by any Art be Transformed into thoſe Coagulums, of which Milk is known to be Capable by every Days Experience; Beſides that the Conſiſtence of the one is much Greater than that of the other.

This Account of the Milk in ſome Meaſure Confirmed by Malpighius, *and Dr.* Drake.

What Confirms this is, that *Malpighius* likewiſe is of Opinion, that the Buttery Part of the Milk Iſſues from the Fat of the Breaſts, and Dr. *Drake* Aſſerts, that in all Milk the Butter and Oil of it is a Great Part, and that it ſeems to conſiſt of Nothing but Water and Oil united by the Artifice of Nature, perhaps by the Intervention of Salts, with which Milk however Sweet abounds, and he tells us, that by the Mediation of Salts, Water and Oil may be ſo Mixed, as very much to Reſemble Milk, tho' we cannot come up to the Mechaniſm of it, and Make it: In Anſwer to *Malpighius*, Dr. *Gibſon* Objects, in the ſame way as Dr. *Wharton* before, that it is Improbable, that Nature ſhould Separate the Oily or Fatty Particles of the Chyle, to the End they may be Mixt with the ſame again; Which Objection ſuppoſes Two Things, which we Deny; Firſt, That Chyle and Milk are the ſame; And Second, That Fat is made from Chyle; Since we have given an Account how Fat is Formed before, and that it has as much it's Peculiar Production from the Expanſive and Contractive Forces, as Chyle it ſelf has; And we may as juſtly Affirm, that Chyle is made from Fat, as this from the other; Beſides which, there is no one ever Dream'd, that Barley-Broth or Water-Gruel, to which Chyle is Compared, Contained much Oil or Butter in it.

How Milk is made in the Mammæ.

We cannot therefore but Think, that Milk is made by the Expanſive Force of the Blood Mixing with that of the Fat, which is of a Lower Degree, and with the Nervous Juice, which is of a Contractive one; The Contractive Force gives it it's Salts, and the Expanſive it's Sweetneſs and Oil, and there is Apparently a *Lympha* or Water, in which the Red *Globuli* of the Blood Swim, beſides that which is Imported by the Lympheducts obſerved by Dr. *Wharton*; From whence there will be all the Ingredients, which Dr. *Drake* requires to make a Milk of, and theſe are all worked up together by the Animal Force, which he calls, the Artifice of Nature, that is, by theſe Expanſive and Contractive Forces Acting amongſt each other in the Animal Syſtem, which cannot be by any Methods Imitated exactly out of it; As the Animal Oil or *Lympha* cannot be Repreſented by a Vegetative one and common Water, the Two Former having a Peculiar and a Greater Degree of the Expanſive Force than the Latter, and ſuch a Degree, as cannot be Induced upon them, but by the Digeſtions and Circulations, and other Motions, in the Animal Frame.

Dr. Gibſon's *Objection againſt this Explanation of no Force.*

As to the Nervous Juice or Fluid, which we take to be, not any Imaginary *Succus*, but of the ſame Kind and Subſtance with the Brain, Dr. *Wharton* concurrs with us in Attributeing a Share to it in the Formation of Milk, and the Objection which is made againſt it by Dr. *Gibſon*, we ſhall here conſider, namely, the Weakneſs, which it muſt needs Induce upon the Mother, to have ſo much of it Drained away by Suction, which is alſo a Reaſon alledged againſt the Blood's being Exhauſted in the ſame Manner; But is not the Anſwer eaſy, that the ſame Objection would lie againſt the Chyle? Since the Blood and the Nervons Fluid would be Defrauded of ſo much Chyle, as would be Requiſite to ſupply their Syſtems; And it is not Material, whether we take ſuch a certain Quantity of Blood and Spirits away, which will be Replac'd by the ſucceeding Chyle, or whether we take ſuch a certain Quantity of Chyle away, which was by Nature deſtined to Replace the Expence of Them.

Another Objection, which may be Raiſed from the Adipoſe Membrane *and the* Omentum, *Conſidered.*

If to this Doctrine it is Oppoſed, that therefore, if This were True, Milk ſhould be Produced by the *Adipoſe Membrane*, and by the *Omentum*; As to the Firſt, it is a Confirmation of what we have ſaid, that a Kind of Milk is Produced by it; ſince the *Reticulum Mucoſum* or *Parenchyma*, which lies betwixt the *Cutis* and *Cuticula*, is a White Milk or Slime, which is Scraped off from the Inner Side of the *Cuticula*, when Raiſed from the *Cutis*; And as for the Laſt, it has been ſhewn, that the Fat of the *Omentum* is not of an Oily and Expanſive, but in ſome Degree of a Contractive Nature, by it's Brittleneſs, and it's Proneſs to be Congealed; There is yet a Greater Quantity of Milk in the Breaſts than under the *Cuticula*, becauſe there is a Greater Influx of Blood, and a Greater Proportion of Fat in Them.

Why the Breaſts Swell in the Time of Puberty and the Eruption of the Menſes, *and why there are no ſuch Phænomenons in the Male Syſtem, with the Reaſon of Procreation.*
Alphonſus Borellus De Motu Animalium, Cap. 12. Par. 2.

§. 3. Hence alſo the Breaſts Swell about the Time of Puberty, and the Eruption of the *Menſes*; Becauſe we have ſaid the Expanſive and Sanguineous Syſtem Prevails moſt in the Female Sex, and the Nervous and Contractive in Men; and from the *Fœtus* to Puberty there being a Conteſt betwixt the Expanſive and Contractive Forces in Women, the Expanſive now becomes Predominant, and the Blood Diſcharges it ſelf by the Hypogaſtrick Arteries in the Monthly *Profluvia*, and by the Thoracick or Mammary into the Breaſts, where by a Mixture with the Nervous and Contractive Juice it is Elaborated into Fat, as before Explained, in order to Form Milk upon Attracting ſtill a Greater Quantity of Blood that way by Suction; On the Contrary this Phænomenon is not obſervable in Men, there is no Swelling of the Breaſts, nor no Efflux of the *Menſes*, becauſe the Contractive Force is moſt Prevailing in Them; of which Kind is the *Semen*, and which is Acknowledged by *Borellus* to be of the Nature and Subſtance of the Brain, and is Agreeable to what we before Alledged concerning it; and the Diſcharge of it is as much an Indication of the Contractive Force being Superior in Men, as that of the *Menſes* is of a Superior Expanſive in Women: And as we have all along ſhewn, that to Preſerve the Animal Syſtem there muſt be a Kind of *Æquilibrium* betwixt thoſe Two Forces, ſo from hence we are farther Confirmed in that Opinion, becauſe it is neceſſary even to the Procreation of it, that is, a due Mixture of the Expanſive with the Contractive is ſo.

Why the Milk Flows into the Breaſts at certain Times only.

§. 4. But to go on with the Phænomenons of the *Mammæ* or Breaſts; This gives a Solution of the Difficulty, which has been thought Impoſſible to be Explained, namely, why the Milk flows into the Breaſts at certain Times only: Becauſe upon Puberty, and by an Exceſs of the Expanſive Force at that Age, the Blood is Derived into thoſe Parts, where there is the leaſt Obſtruction to it, and conſequently, where there is a Paſſage open to it out of the Body, as by the *Hypogaſtricks* and the *Papillæ*; by the Former of which it is Diſcharged, but meeting with a Nervous or Contractive Force in the Latter, and in the *Mammæ*, together with the Lympheducts, it is ſoftened into Fat and Oil and there Remains; Afterwards upon Suction, there being more Blood drawn that way from the Mammary or Thoracick Arteries, it is Transformed into Milk by a Mixture with the Oil, *Lympha*, and the Nervous Fluid, as we have already ſhewn and Evinced.

Why it is Wanting in Old Perſons, and what Office the Glands Perform in the Mammæ.

§. 5. In Old Women, as the Expanſive Force Abates, and as the *Mammæ* by the Contractive Force, upon a Deficiency of the Expanſive, Harden into Glands and Veſſels, that is, as the Blood, and Oil or Fat, Declines, the Milk is no farther Produced; And if *Diemerbroeck*'s Inſtances are True, which may be much Queſtioned, and may be only the Prate and Tattle of mere Old Women, that Milk has been ſupplied

from them, it muſt be, upon the Account, that their *Mammæ* have ſtill Retained their Fat or Oil, and that there has been a plentiful Influx of Blood into them, which will be found to be as Rare, as Men preſerving their Senſes and Reaſon in their full Force and Vigour to Fourſcore, or an Hundred; Certainly it cannot Proceed from Imagination, becauſe almoſt every Old Man or Old Woman might, upon ſuch a Suppoſition, be Capable of giving Suck to Infants, whereas we have only Two Examples mentioned by him, the one of the Former, and the other of the Latter.

As to the Glands therefore, they are not, as Anatomiſts Imagine, to ſtrain the Chyle from the Blood, but having a Contractive Force, which we Explained before in the Glands of the Meſentery, they Repreſs and Reſtrain the Expanſive of the Blood, and Bring it to a Temper nearer to Milk, which is Perfected, by the Oil and *Lympha* in the Breaſts, into that Balſamick Fluid, which is ſo neceſſary to the Conſervation of the Infant; and which by the Compoſition, we have Named, muſt be very well and rightly Adapted to that Purpoſe; that is, to the Maintaining the Riſing *Æquilibrium* of Forces in the Child, without any Detriment or Prejudice to Them.

Why the Largeſt Gland is ſituate under the Papilla, *and how the* Papilla, *the* Tubuli Lactiferi, *and the* Areola *are Formed.*

§. 6. THIS Account of the Glands gives us alſo a Reaſon, why the Largeſt is ſituate under the *Papilla*, which the Preſent Anatomy cannot do; For if the Glands are to be Conſidered only as Strainers of the Chyle from the Blood, the Largeſt would not be Placed there, where all the Milk in the Breaſts is already made, and where there is no farther occaſion for a Percolation of it; On the other Hand, as we ſhewed in the Meſentery, the Glands muſt be the Greateſt, where the Contractive Force was the Greateſt from the *Vertebra* of the Loins, and where the Expanſive was the Leaſt from the Alimentary Duct, or Canal, ſo here the Gland under the *Papilla* muſt be the Greateſt, becauſe the Contractive Force of the Nervous Juice, and *Lympha*, is the Greateſt by the Extreme Senſibleneſs of thoſe Parts, and the Expanſive Force of the Blood is the Leaſt, as Mixing with the Fat and Oil of the Breaſts, and being oppoſed thro' the Whole of them by thoſe Contractive Forces; and the *Tubuli Lactiferi* grow Smaller and Smaller to their Extremities in the *Papilla* upon the Increaſe of the ſaid Contractive, and Diminution of the ſaid Expanſive Forces; Laſtly, As there is ſome Blood Wreſtles thro' all this Oppoſition, it's Expanſive Force is thereby Augmented and Heightened, as we before ſaid of the Blood, which paſſed thro' the Liquor of the *Amnion* to the Liver of the *Fœtus*; This Expanſive Force therefore Exerts and Diſplays it ſelf beyond the Breaſt, by which the *Papilla* is Formed, and from the Vivacity and Livelineſs of it, the Red Colour Enſues, which upon a Decay of the Expanſive, Firſt Turns to a Livid, and Then to a Blackiſh; The *Areola* ſeems only to be ſo many ſmall Efforts of the Expanſive towards the Raiſing ſeveral other *Papillæ* round the Large one, but the Expanſive being too Weak there is nothing more Produced than certain little White Knots or Protuberances in the Skin, and the Expanſive Force of the Blood, which Thruſt them out, Languiſhing by Age, they become ſomething like the *Papilla*, Firſt Brown and Then Black, as before Recounted.

The Account of the Diaphragm, Pleura, Mediaſtinum, *and* Thymus, *by Anatomiſts.*

See the Places Cited above, §. 1.

The Diaphragm, it's Situation and Subſtance.

§. 7. HAVING laid the Cavity of the *Thorax* Open, the Diaphragm, *Pleura*, *Mediaſtinum*, Heart, and Lungs, Appear.

THE Diaphragm or Midriff Divides the Trunk of the Body into Two Ventricles, the *Abdomen* and *Thorax*; It is Exceeding Nervous, and, thro' the Great Conſent it has with the Brain, when Enflamed or Diſtempered Affects the Mind; It is almoſt Round, Excepting it's Two Appendages, whereby it is Faſtened to the *Vertebræ* of the Back and Loins; It is Seated Tranſverſely or Acroſs the Body, only Slopeing a little Backwards, and is as Broad as the Width of the *Thorax*, for it's Edges are Faſtened to the Lower Part of the *Sternum*, to the Ends of the Loweſt Ribs, and to the Loweſt *Vertebra* of the *Thorax*; It's Subſtance is Muſcular, Conſiſting of Carnous and Tendinous Fibres, like other Muſcles; It has formerly been thought to be only one Continued Muſcle, but *Caſpar Bartholin* Divides it into Two, a Fore and an Hinder, or an Upper and a Lower, as the Diaphragm is ſomething ſlanting Backwards; The Upper Muſcle by one End, *viz.* it's Head, Adheres Circular-wiſe to the Ribs and to the Appendix of the *Sternum*, the Other paſſeth into the Tendon, which makes the Nervous Center of the Diaphragm, and is Spent on or Continued unto the Fleſh of the Lower Muſcle, and ſo the whole Midriff becomes like one Digaſtrick or Double-Belly'd Muſcle; the ſame Author has obſerved, that the Fore-part of the

Upper Muſcle Shoots forth a Tendon to every Baſtard-Rib on each Side the *Sternum*, to which Ribs the Upper Part of the Tranſverſe Muſcles of the Lower Belly alſo Adheres: The Lower Muſcle Springs from the *Vertebræ* of the Loins and Back, and Riſes from thence by Two Appendices, the Right and Left, the Former of which Exceeds the Latter in it's Length, and in the Number of it's Carnous and Tendinous Fibres; The other Extremity of this Muſcle is joined in an Oval to the Center, or Tendinous Part of the Diaphragm, and the Fibres of both the Upper and Lower Muſcles Tend in a Kind of Radius's from their Centers to their Circumferences; The Upper Muſcle is Thin and Broad, the Lower is Thick and has a very Fleſhy Belly.

It's Coats and Perforations. Dr. Gibſon Fig. 2. & 3. Tab. 13. Book 2. Chap. 12.

The Upper Side of the Midriff is Cloathed with the *Pleura*, and it's Lower with the *Peritonæum*, to the Upper Membrane the *Mediaſtinum* and *Pericardium* are Knit, and ſometimes the Loweſt Tips of the Lobes of the Lungs, but this is Preternatural; It is Perforated in it's Tendinous Part by the *Vena Cava* Aſcending from the Liver, and in the Fleſhy Part of it's Lower Muſcle by the Gullet, and Two Stomachick Nerves Springing from the *Par Vagum*, where the Fibres do not Run ſtreight from the Appendices, as before, but bend towards and Encompaſs the Orifice, by the Conſtriction of which the Gullet is exactly Cloſed, ſo that the Continual Motion of the Diaphragm cauſes no Regurgitation of the Contents of the Stomach, nor is the Aſcent of any Vapour Permitted thro' it; The Laſt Perforation is made behind the Appendices by the Deſcent of the *Aorta* along the *Vertebræ* of the Loins and Back, the Thoracick Duct, a Branch of the *Vena Azygos*, and the Intercoſtal Nerve.

It's Arteries, Veins, and Nerves; And the Reaſon of Laughing.

It has Two Arteries called *Phrenicæ* from the Deſcending *Aorta*, and as many Veins from the Aſcending *Cava* in their Paſſage thro' it, with ſome other Peculiar ones, from the *Vena Adipoſa* and *Arteriæ Lumbares*; It's Nerve, which is called the Nerve of the Diaphragm, Ariſes from the Vertebral and Communicates with the Intercoſtal Pair, according to Dr. *Willis*, from whence he Aſſigns a Reaſon of the Great Conſent of the Midriff with the Heart, Brain, and Face, when a Man Laughs; For when the Imagination is Affected with Pleaſure, the Heart would fain Triumph by Throwing off it's Burthen, wherefore, that the Blood may be the Quicklier Emptied out of it's Right Ventricle into the Lungs, and conſequently out of the Left into the *Aorta*, the Diaphragm, being Inſtigated by the Nerves from the Intercoſtal, is drawn upwards with a more Rapid Syſtole, and often Repeating it's Jumps, bears up the Lungs and cauſes them with more Quickneſs and Frequency to Diſcharge the Air and Blood, but the ſame Intercoſtal Nerve, which Communicates below with the Nerve of the Diaphragm, is Continued above with the Maxillar Nerves, and therefore, when a Cackling is begun in the Breaſt, the Geſtures of the Mouth and Face anſwer thereto; and when the Diaphragm is wounded in it's Nervous Part, the Muſcles of the Face ſuffer Convulſions and an Involuntary Laughter, or the *Riſus Sardonius*, is Cauſed.

It's Uſe.

The Uſe of the Diaphragm is, Firſt, To Divide the *Thorax* from the *Abdomen*, that Noiſome and Impure Vapours may not Aſcend from the more Ignoble Parts, as the Guts, to offend the more Noble, as the Heart; Second, To help the Muſcles of the *Abdomen* in Excluding the Excrements and, in Women, the *Fœtus*, and to Preſs the Stomach and Inteſtines for the Diſtribution of the Chyle; Third, To aſſiſt Reſpiration, which is ſuppoſed to be Performed in the following Manner: The Diaphragm from the *Vertebræ* to the Circumference, according to *Steno* and *Bartholin*, is Convex towards the *Thorax*, and Concave to the *Abdomen*, whoſe Firſt Motion is downwards, ſo that from a Convex Laxity it is Depreſs'd to a Plane Surface, as *Bartholin* Teaches, without being Extended, as *Steno* Suggeſts, or Thruſt into the *Abdomen*, as *Diemerbroeck*; This Depreſſion of the Diaphragm to a Plane Surface, by which it's Circumference is Extended, muſt of Courſe Elevate the Sides of the *Thorax*, which *Diemerbroeck's* Protruſion of it into the *Abdomen* could not do, and yet this is what we Experience. The Cavity of the *Thorax* being thus Enlarged, the Air by it's Preſſure or Elaſticity Iſſues in at the *Trachea*, *Aſpera Arteria*, or Wind-Pipe, and the *Bronchus*, which is the Lower Part of it, and immediately Poſſeſſes all the *Bronchia*, or Branches of the *Bronchus*, and *Veſiculæ* of the Lungs, and Puffs them up and Inflates them, by which means Inſpiration is Effected; On the Contrary, the Diaphragm now Returning to it's Former Convex Situation, and Moving upwards into the *Thorax*, it's Sides fall towards each other, whereby it's Cavity is Diminiſhed, and by this Compreſſion of the *Thorax*, and the Contraction of the Muſcular Fibres of the *Veſiculæ*, and of the Inner Coat of the *Trachea* and it's *Bronchia*, the Air is Expelled from the Lungs by the ſame way, by which it Entered, which is the Account given of Expiration; and thus, Inſpiration and Expiration ſucceeding each other Alternately, Reſpiration is Cauſed.

THE Motion of the Midriff is thought to be Mixed, but rather Animal and Voluntary, than Natural and Neceſſary, becauſe, tho' it Moves in our Sleep, yet, when we are Awake, we can Stop, Slacken, or Haſten it's Motion.

The Pleura, *it's Membranes, Perforations, Arteries, Veins, and Nerves.*

THE *Pleura* is a Double Membrane, which Covers all the Cavity of the *Thorax*, and is White, Thin, and Hard, Reſembling the *Peritonæum*, the Inner Membrane, or that next the *Thorax*, is the Thickeſt; 'Tis ſmooth on it's Inſide, and bedew'd with a Wateriſh Humour, Proceeding probably from Vapours Raiſed from the Blood, and Condenſed by the Comparative Coldneſs of this Membrane, but the Outſide, which is fixed to the *Perioſteum* of the Ribs, is Thinner and Rough; It ſends out from it's Inſide certain Nervous Fibres even in Healthful Perſons, which are Inſerted into the Inveſting Membrane of the Lungs, and Hinder the Aſcent and Deſcent of them in Reſpiration, without any Great Injury to their Breathing, tho' ſometimes, when they tie the Lungs too Cloſe, they cauſe a Continual and Incurable *Dyſpnœa*; It's Perforations below have been already mentioned, thoſe above are for the *Vena Cava* Deſcending, and the *Aorta* Aſcending, the *Gula*, the Rough Artery, Lacteals, Lympheducts, and Nerves; It Ariſes from the Membranes, which Inveſt the *Medulla Spinalis*, and from the *Vertebræ* of the Back, from whence it comes forward on each Hand by the Sides to the *Sternum*, or Breaſt-Bone, under which the Membranes of each Side are joined together, and ſo becoming Double, it goes back again ſtreight from the Middle of the Breaſt to the Back, Dividing the Cavity of the *Thorax* into Two Parts, and one Lobe of Lungs from the other, and is called the *Mediaſtinum*; It has Veins and Arteries from the Superior Intercoſtal Branches, and they are ſaid to Run between the Two Membranes of the *Pleura*, betwixt which, when any Matter Impoſthumates, from whence an Inflammation is Cauſed, a Pleuriſy is ſuppoſed to be Produced; It hath Nerves from Twelve Vertebral Pair, namely, from all the Pairs of the *Thorax*, for from betwixt each of the Twelve *Vertebræ* of the Back there Springs a Pair of Nerves.

The Mediaſtinum, *it's Subſtance, Cavities, and Uſes.*

THE *Mediaſtinum* is Formed by a Continuation of the *Pleura*, as Above, and conſequently conſiſts of Four Membranes, being a Duplicature of the *Pleura*, which conſiſts of Two; As it Tends from the *Sternum*, where they are ſtrictly United, the Duplicated Membranes are Sever'd, ſo as to Contain the Heart with it's *Pericardium* in the Cavity, which Intervenes their Separation; After which they again Unite as Cloſe, as before, but preſently Part, to Form as Long, tho' a Narrower, Cavity for the *Oeſophagus*, or Gullet, the *Vena Cava*, and Stomachick Nerves to Deſcend by; Where it is Parted, it is Thinner and Softer than the *Pleura*; The Outſide towards the Lungs is Smooth, but the Inner is Rough, by Reaſon of the Fibres, whereby in ſome Places it adheres to the *Pericardium*, and there is Fat diſperſed in it, eſpecially about it's Veſſels; It has Veins, Arteries, and Nerves, and ſome Lympheducts, as other Parts, and it's Chief Uſes are, To divide the Lungs, ſo that if one Part is Wounded, the other may be Secure, To keep the Heart from Falling upon the Back, or Breaſt-Bone, or the Ribs upon our Bending our ſelves Backwards, Forwards or on one Side, To give a Paſſage to Veſſels, and ſupport the Diaphragm againſt the Weight of the Stomach and Liver, which hang by it.

The Thymus, *it's Subſtance and ſuppoſed Uſe.*

THE *Thymus* is a Glandulous Body, ſituate in the Upper Part of the *Mediaſtinum* under the *Claviculæ*, it is Whitiſh, Soft, and Spongy, and Larger in Children and Women, than in Men; In Infants it conſiſts of Three Glands, and is ſomething like the Sweet-Bread, but in Adult Perſons it Drys and Contracts up into one Subſtance; It has no Proper Duct, whereby any Thing is conveyed into any Peculiar Cavity, and therefore is Eſteemed in the Number of Conglobate Glands, tho' Dr. *Drake* makes it a Conglomerate one; The Jugular Veins and Arteries paſs thro' it, as they go to the Neck; In Children it has Liquor like Chyle in it, and it's Nerves and Lympheducts are Acknowledged; The Uſe of it is ſcarcely Known, and only Gueſs'd at by Anatomiſts, the Later think it is a kind of *Diverticulum* to the Chyle in the Thoracick Duct, when over charged in Children, and that it conveys a *Lympha* from the Adjacent Parts to the ſaid Duct; It having no Valves, for if Wax or a Liquid is Injected into the *Ductus Thoracicus*, it will paſs into the Lympheducts of the *Thymus*: We ſhall now, as Uſual, Examine theſe Phænomenons of the Animal Syſtem by the Doctrine of the Expanſive and Contractive Forces.

How the Diaphragm Separates the Thorax *from the* Abdomen; *how it is Produced, and why it is Slanting, and when Inflamed ſtrongly affects the Mind.*

§. 8. AND Firſt, the Diaphragm Separates the *Thorax* from the *Abdomen* by the Exertion of the Expanſive and Contractive Forces athwart the Body, which could not be done in any other Place of the *Thorax*, or the *Abdomen*, they being both already poſſeſſed by the Lungs and Heart, and by the Stomach, Liver, and Inteſtines, which as we have ſhewn are Primarily Formed, Excepting the Lungs, which will afterwards be Proved; In this vacant Space therefore the Contractive Force of the Nervous Fluid from the Spine Advances, which is Confirmed by Anatomiſts, who acknowledge it's Riſe to be in Two Nervous Appendices Produced from the *Vertebræ* of the Back and Loins, that it is Derived from the Membrane, which Inveſts the *Medulla Spinalis*, and the like; And hence it is, that the Tendinous Fibres, of which it is Compoſed, are Propagated, as a kind of Radius's from the ſaid *Vertebræ*, as a Center; But whilſt this is doing, the Expanſive Force of the Blood from the Deſcending *Aorta* Exerts it ſelf in the Phrenick Arteries, to whoſe Aſſiſtance likewiſe the Lumbar Arteries Arrive, in order to Balance the Contractive Force of the Nervous Fluid from the Spine, by which it's Carnous or Fleſhy Fibres are made, being Conſtipated by the ſaid Contractive Force from Blood into Fleſh; To which the Nerve of the Diaphragm alſo Contributes: The Diaphragm is alſo Slanting, being Lower at it's Riſe in the *Vertebræ*, or in it's Nervous and Contractive Part, and Higher at it's Ending in the *Sternum* and Ribs, or in it's Carnous and Expanſive, becauſe Naturally the Contractive, as it Tends to a Center in it's Force, Deſcends likewiſe towards the Earth, but the Expanſive, Dilateing it ſelf from a Center, Riſes and is Elevated from it: Nor is it any wonder, that ſo Nervous a Membrane, which is immediately Derived from the Spinal Marrow ſhould, when Enflamed or Diſtempered, ſo ſtrongly Affect the Mind, whither all Nervous Senſations are Ultimately Referred.

Why the Diaphragm is a Digaſtrick Muſcle by the Intervention of the Nervous Center, and why it is Convex to the Thorax, *and Concave to the* Abdomen, *as alſo, why the Lower Muſcle is Thicker and more Fleſhy than the Upper,* &c.

§. 9. THE Reaſon, why this Muſcle of the Diaphragm, which otherwiſe would be but one, is Divided into Two by the Intervention of the Nervous Center, ſeems to be the Paſſage of the *Vena Cava* Aſcending from the Liver, which, Containing the Expanſive Force of the Blood, Heats the Parts about it, to Balance which therefore the Contractive Force of the Nervous Fluid Aſſembles to it, and Conſtitutes that Part, which is called the Tendinous, juſt as about the *Aorta* the ſame is done by the Appendices from the *Vertebræ*; On the other Hand, the Expanſive Force of the Aliment, which is carry'd thro' the *Oeſophagus*, is Small and Gentle in Compariſon of that of the Blood, and therefore is not Encloſed with ſuch a Degree of the Contractive Force, but yet the Tendinous Fibres Bend towards it, and more nearly Encompaſs it, and Leave their Rectilineal Progreſs in order to Counter-Poiſe it: And for the ſame Reaſon the Diaphragm is Convex towards the *Thorax* and the Heart, and Concave to the Guts and *Abdomen*, the Contractive bending towards the Expanſive in this Caſe likewiſe: The Lower Muſcle of the Diaphragm is alſo more Thick and Fleſhy, and the Upper Thinner, becauſe the Contractive Force at the *Vertebræ* or Origin of the Former is Greater than that of the Tendinous Part of the Latter; and conſequently, the Expanſive Force of the Fleſhy Fibres muſt be Greater and Stronger to Balance the one than the other: The Midriff Adheres Cloſe to the *Sternum* and Ribs on each Side, not by the Expanſive Force of the Fleſhy Fibres, but by the Contractive of the Nervous and Tendinous ones; for, having now made it's way thro' the Expanſive, it more ſtrongly Cements and Unites to the *Perioſteum*, which is of the ſame Nervous and Contractive Force with it; And this is the Reaſon that *Bartholin* Affirms, that the Upper Muſcle Shoots forth a Tendon to every Baſtard-Rib on each Side the *Sternum*; The Upper Side of the Diaphragm is Lined by the *Pleura*, and the Lower by the *Peritonæum*, which are more Fleſhy Membranes, that is, have a Greater Expanſive Force, and are made by it to Encompaſs and Confine the Contractive of the other, that it ſhould not be too Prevailing.

The Phænomenon of Laughing not Accounted for by Anatomiſts, and which Receives a Solution from the foregoing Principles.

§. 10. As to Laughing, there is an eaſy Account given of it from theſe Principles, without ſuppoſing that Communication of the Intercoſtal Nerve with the Nerve of the Diaphragm below and the Maxillar above, and the Heart's Endeavouring to throw off it's Load, according to Dr. *Willis*; For what? Theſe Nerves do not only Communicate with each other, but there is a Connexion almoſt betwixt all the Nerves of the Body, as may be ſeen in the Syſtem of them, Expreſs'd Tab. 16. Appendix to Vol. 1. of Dr. *Drake*'s *Anthropologia*, and in Dr. *Willis*'s *Anatomy of the Brain*, Tabb. 9, & 11; And it is not Probable, that the Communication here mentioned ſhould Produce ſuch Particular and Singular Effects in the Parts ſpecified in Diſtinction from the Reſt; And why, when the Imagination is Pleaſed, the Heart ſhould Endeavour to throw off it's Burden, is Unexplained, nor is there any Natural Dependency ſhewn of the one upon the other; Beſides the *Cachinnus*, or Noiſe, occaſioned by Laughing, cannot be owing to the Quick Expulſion of the Air from the Lungs barely, for a Quicker or Shorter Inſpiration or Expiration can no more Produce ſuch a kind of a Sound, than our Quick or Short Breathing can produce a Note, or a Modulation of the Voice; On the other Hand, Joy, Pleaſure, Delight, and Satisfaction, are Affections in the Mind of an Expanſive Force, and are Raiſed in it by ſome Expanſive Force in it ſelf, or which is External to it; Thus Wines and Spirituous Liquors, and Wit, which conſiſts in a Velocity, and a Diſſolution or Diſſipation, or Airyneſs, of our Ideas, are Evidently of an Expanſive Nature, and occaſion Mirth, and Laughter, and Jollity, and as Similar Forces produce Similar, Mirth and Laughter likewiſe muſt be of an Expanſive Nature; And from hence it comes to paſs, that as ſoon as the Mind is thus Affected, it Communicates it's Expanſive Force to the Nervous Syſtem, which is of a Contractive, and which Reſults and Rebounds upon the Impreſſion, and Exerts it ſelf in Leaping, Jumping, Danceing, Laughing, and the Like; in the ſame Manner, as the String of an Harp, which is of a Contractive Nature, is Raiſed into Vibrations by Percuſſion, which is of an Expanſive, and in the ſame Manner, as we have ſhewn all Motions are Produced and Continued by an Oppoſition of the Expanſive and Contractive; which Solution, if True, is a plain Conviction of the Diſtinction of the Mind from the Body, and that it is not the Intercoſtal or Diaphragmatick Nerve, which is the main Inſtrument of Laughing, but the whole Nervous Syſtem; and which is the more Confirmed from hence, that Men do not in that Action ſhake their Maxillar Muſcles and their Diaphragm only, but their Sides and their whole Body.

The Cachinnus *from hence Explained.*

§. 11. As to the *Cachinnus* in Particular, or the Noiſe made in Laughing, it is a Quick Return of the Voice from the Quick Vibrations of the Nervous Syſtem, Part of which Acts upon the Lungs, but the Voice it ſelf is formed, as all other Modulations of it, by the Intervention of the *Aſpera Arteria*, and the Palate; an Explanation of which comes under the Doctrine of Sounds, which, as we have before ſaid, always Ariſe from the Reſiſtance of the Contractive to the Expanſive, which is here Communicated from the Mind to the Nervous Syſtem, and from thence to the Lungs and Air Included, and is Reſiſted by the Contractive of the Rough Artery, Palate, and Elaſticity of the External Air.

How Tears are Cauſed, Agreeable to the ſame Principles.

§. 12. This Account is ſtill farther Evinced from Conſidering, what Grief and Sorrow is, and how Tears are cauſed; for ſince theſe Affections are the Reverſe of the Former, if it is True, that they Proceeded from an Expanſive Force in the Mind, Theſe muſt from a Contractive, which it is Plain they do; For Afflictions have a Natural Tendency to make Men Wiſe and Judicious, and Judgment conſiſts in the Gravity, Cloſeneſs, and Connexion of our Ideas, which are manifeſtly of a Contractive Nature; When therefore the Mind is in a State of Grief, which always makes us Conſiderate and Thoughtful, it has a Contractive Force Impreſs'd upon it; Thus alſo, if there is a Melancholy, or Languor in the Blood, which muſt be owing to a Contractive Power, the ſame State is Induced upon the Mind, and it is Inclined to Grieve and to be Sad; This Contractive Force in the Mind, Imparting it ſelf to the Nervous Syſtem, has alſo quite Contrary Effects from what were before Produced, as it ought to have, for now the Nervous Syſtem does not Reſiſt and Rebound againſt the Incurring Expanſive of the Mind, but Submits to and Complys with the Contractive of it, the Forces being now of the ſame Nature; Hence it Contracts it ſelf, Draws it ſelf Inwards, and Retires within the Sphere of it's Proper Activity; Conſequently, we find the Body in a Reſerved and Sedentary Poſture, the Head Drooping, the Legs Acroſs, and the Arms Claſped within each other, which are all ſo many Indications of a Contractive Force Prevailing, and Evident Marks and Proofs of it, as they are Contrary to Leaping, Jumping, Dancing, *&c.* The Tears alſo fall on ſuch Occaſions, becauſe the Nervous Syſtem, and with it the Brain, being Contracted, the *Lympha*, which always ſtands in the Ventricles of it, is Preſs'd and Squeezed from thence, and Carry'd thro' the *Foramen Lachrymale*, ſituate at the Inner *Canthus*, or Corner of the Eye; to which Opinion, tho' he Rejects many others, *Diemerbroeck* alſo is willing to Subſcribe.

The Account above not Contradicted, by the Riſus Sardonius.

THE *Riſus Sardonius* does not Contradict the Solution before Given, ſince where ſo Nervous a Part as the Diaphragm is Wounded, it muſt neceſſarily Affect in a ſenſible Manner the whole Nervous Syſtem, and therefore the Muſcles of the Face, and the Maxillar Nerves Inſerted into Them.

The Diaphragm of little or no Uſe to Reſpiration.

§. 13. AMONGST the uſes of the Diaphragm the Principal is ſaid to be, that of Aſſiſting Reſpiration, whereas we have ſhewn how it comes to be Formed, and that in the Formation there does not ſeem to be any ſuch Deſign or Intention; Nor does the Account Propoſed to us of Reſpiration appear to be True; In order to which it is Preſumed, that the Firſt Motion of the Diaphragm is Downwards, by which the Cavity of the *Thorax* is Enlarged, and the Air thereby Ruſhes into the Lungs in Inſpiration; But which way is this Motion downwards Performed? Not by the Incurſion of the Air, for the Cavity is originally Enlarged before ſuch Incurſion; Not by the Contraction of the Diaphragm, as a Muſcle, becauſe that would draw the Sides of the *Thorax* together, and diminiſh it's Cavity, whereas in Inſpiration they are Raiſed and Elevated, and the Cavity is Widened and Increas'd; Not by the Diſtenſion or Dilatation of it, becauſe that is not the Proper Action of a Muſcle, and only is ſubſequent upon the Contraction of it, and therefore it's Firſt Motion cannot be Downwards; But if the Firſt Motion is Upwards, there being no Air yet Inſpired, the Diaphragm will only Preſs out what Air Reſides in the Lungs before Inſpiration, and upon it's Moving Downwards, ſuch Air will be Replaced, which does not come up to the Phænomenon, as it is Propoſed, of Inſpiration and Expiration, which is here Endeavoured to be Explained; Beſides it is manifeſt, that it is but little Air, that we Inſpire and Expire at once, ſo that there does not ſeem to be any occaſion for the Enlargement of the Cavity of the *Thorax* by the Methods Alledged, that is, by the Deſcent of the Diaphragm, and Leaving a Room or Space for the Lungs to Swell in from the Incurring Air; Nor, which is Material in the Preſent Account, is it really Proved, or can it be, that the Diaphragm does Aſcend and Deſcend; Since another Motion may be Aſſigned to it, which is more Probable, and will Equally Anſwer the Particulars we obſerve of it.

How Reſpiration is Performed, and what is the Real Action of the Diaphragm as well as all other Muſcles, and how Accounted for.

§. 14. CONSIDERING therefore what foregoes, it is more Conſiſtent with Reaſon to Think, that Reſpiration is Performed in another, that is, in a ſimpler Manner, namely, not by the Local Motion of the Diaphragm, the other Muſcles of the *Thorax* we at preſent omit, which yet have their Share in this Action, but by the bare Diſtenſion of it from the Expanſive Force of the Fleſhy Fibres, and the Conſtriction of it from the Contractive of the Tendinous; The Contractive Force of the Latter is allowed by all Anatomiſts, and we only beg the Favour to Introduce the Expanſive of the Former, as Equally Natural and Proper to the Action of a Muſcle, with it's Contraction; And from whence Muſcular Motion in General may be Explained.

Why the Action of the Thorax *and Diaphragm is to and from the* Vertebræ, *and not upwards and downwards.*

WE have already ſhewn, that the Nervous or Tendinous Fibres of the Diaphragm Proceed from the *Vertebræ* towards the *Sternum*, and the Ribs on each ſide it; and therefore the Contractive Force of theſe Fibres will draw the Ribs and *Sternum* towards the *Vertebræ*, and the Expanſive Force of the Carnous, Acting againſt and in Oppoſition to the Former, will Protrude the ſaid Ribs and *Sternum* from the ſame *Vertebræ*; which, if any one will Reflect and Look upon himſelf, when he Breaths, he will find to be the Real Action of the *Thorax*; This likewiſe Explains, why the Ribs on one Side do not in Reſpiration Approach to, or Recede from thoſe on the other, but are carry'd Jointly to and from the *Vertebræ*.

On the other Hand, if we ſuppoſe the Diaphragm to be Moved upwards and downwards in the *Thorax*, in the Firſt Caſe, the Ribs ought to come Nearer to each other, as well as to the *Vertebræ*, and in the Laſt, to Retire from each other; and the Conſtriction and Dilatation of the *Thorax* would be Circular, and not from the *Sternum* to the *Vertebræ*, contrary to Fact and Experience; This alſo Informs us, how Reſpiration may be Perfect, tho' the loweſt Tips of the Lobes of the Lungs ſhould Adhere to the Midriff, becauſe they may eaſily follow the Motion of the Diaphragm backwards and forwards from the *Vertebræ* to the *Sternum*, but not ſo well it's Motion downwards without ſome remarkable Impediment to it.

For which Cauſe the Nervous Fibres, which are ſent out from the *Pleura*, and are Inſerted into the Inveſting Membrane of the Lungs, ſometimes cauſe an Incurable *Dyſpnœa*, which is not ſo readily Conceived, if the Diaphragm is Moved upwards and downwards, as if the Tips of the Lobes Adhere to it, whilſt it is Moved backwards and forwards, for in ſuch a Circumſtance, the Nervous Fibres would very much obſtruct the Motion; Laſtly, This Gentle Conſtriction and Expanſion of the Diaphragm much better Agrees with the ſmall Influx and Reflux of Air, which we find in Breathing, than the Turbulent Ruſhing in and Expulſion of it, which the other Doctrine ſeems to Patronize and Eſpouſe by Enlargeing and Contracting the Cavity of the *Thorax*.

The Queſtion Determined, whether the Motion of the Diaphragm is Natural and Neceſſary, or Animal and Voluntary.

§. 15. The laſt Queſtion concerning the Diaphragm, is, whether the Motion, which belongs to it, is Natural and Neceſſary, or Animal and Voluntary, or whether it is a Mixture of both; To which we Anſwer, that in the Human Frame, which we now Conſider, there is no Diſtinction betwixt Natural and Animal, or betwixt Neceſſary and Voluntary, and therefore the Queſtion ceaſes; For the Natural Motion in it is the Animal, and the Neceſſary is the Voluntary, ſince there can be no Motion, which is not Animated, and no Animation, which is not with the Concurrence of the Mind, and no Concurrence of the Mind without the Conſent of the Will, that is, every Action, as it is an Animal one, is ultimately a Voluntary one, as Volition is Inſeparable from Thinking.

Thus Sitting ſtill is as Voluntary an Action as Walking, Sleeping as Wakeing, and the Motion of the Diaphragm as the Reſt of it, for ſince we can Stop, Slacken, or Accelerate it's Motion by the Will, it muſt be by the Will that it is Perpetuated and Continued; Muſcular Motion is confeſs'd to be Performed by the Will, which alſo is an Animal one, and therefore the Neceſſary Motion of a Muſcle is a Voluntary one; The Circulation of the Blood, which, if any other, is a Neceſſary Motion, is yet a Voluntary one, both as it is a Muſcular, and as the Heart it ſelf is a Muſcle; and as we can obſtruct it by Coughing or Straining, and by a Voluntary Expulſion of the Air from the Lungs, or permit it to Flow thro' them at our Pleaſure: Nay, Theſe very Expanſive and Contractive Forces, which we have been Explaining, and which ſeem to be as Neceſſary in their Laws and Actions, as any Thing in our Syſtem, could not obtain their Effect without an Animation, that is, without a Mind Endued with, and Exerting, it's Faculties, One of which is Volition; of which Opinion, I alſo, ſince my Writing of This, find *Borellus* to have been.

Borellus De Motu Animalium, Prop. 80. Cap. 6. Par. 2.

How the Pleura, Mediaſtinum, *and* Thymus, *are Formed from the ſame Principles.*

§. 16. The *Pleura* likewiſe, as well as the Diaphragm, is formed by a Contractive Force from the *Medulla Spinalis* and the *Vertebræ* of the Back, with which the Expanſive of the Blood Mixing from Veins and Arteries of the Superior Intercoſtal Branches, the Membrane is made; The Contractive Force is farther Strengthened by Twelve Vertebral Pair of Nerves, which will require an Addition of the Expanſive to Balance it; The Expanſive and Contractive being thus Increas'd, as alſo the Latter being Stronger from being nearer the Brain the Source and Fountain of it, which again is to be Balanced by an Expanſive, when they arrive at the *Sternum*, their Forces are ſtill Vigorous and Active, and ſince they Meet each other there in oppoſite Directions, and cannot Proceed forwards, 'tis neceſſary they ſhould Cling faſt together, and at the ſame Time Preſs jointly backwards in a ſtrait Line from the *Sternum* to the *Vertebræ*, by which the *Mediaſtinum* is Conſtituted and Framed: Which Forces muſt yet give way to the Expanſive of the Heart, and therefore Separate upon their Approach to it, but Unite again afterwards, till they Separate a Second Time for the Expanſive in the *Oeſophagus* and *Vena Cava*; And upon theſe Separations, the *Mediaſtinum* is Softer and Thinner than the *Pleura*, becauſe the Expanſive Forces, which Divide it, Diminiſh the Contractive; The Contractive alſo takes it's Courſe from it's Riſe in the *Vertebra* along the Ribs, which are of a Contractive Nature, becauſe Similar Forces, according to the General Law we have Mentioned, Tend to each other, and the Expanſive follows the ſame Track in order to Counter-poiſe it; The *Pleura* is Whitiſh from the Contractive Force.

THE *Thymus* is Formed from the Contractive Force of it's Nerves and Lympheducts, and from the Expanſive of the Jugular Veins and Arteries; It ſeems to be of little uſe, and only the Natural Product of the Concourſe of thoſe Forces; It is Larger in Children and Women than in Men, becauſe the Expanſive Force is Greater in Them, which Diſtends and Swells it, and for the ſame Reaſon it is Divided into Three, ſince it belongs to the Expanſive Force to Separate and Divide as well as Diſtend; But in Men whoſe Contractive Force is Greater, it is Leſs, and Compinged into One; In Children, Chyle may ſupply the Place of the Venal Blood, and therefore is many Times found in it.

CHAP. VIII.

Concerning the Heart, and it's Pericardium, *the* Aorta *and* Cava, *the* Arteria *and* Vena Pulmonaris, *and Circulation of the Blood.*

An Account of the Heart and it's Pericardium *with the* Aorta *and* Cava, *the* Arteria *and* Vena Pulmonaris, *and Circulation of the Blood, by Anatomiſts.*
See the Places Cited Above, Chap. 7. §. 1.
The Heart, it's Situation, Subſtance, Fibres, Nerves, Arteries and Veins.

§. 1. THE Next Parts, which we are to conſider in the Human Syſtem, are the Heart and it's *Pericardium*, with the *Aorta* and Circulation of the Blood: The Heart is the Source of Vital Heat, and the Principal Bowel of the Body, which no Perfect Animal wants, nor can long ſurvive it's Wounds; It is ſituate in the Middle of the Breaſt, Encompaſs'd with the *Pericardium* and *Mediaſtinum*, and it's Lower Tip or *Mucro* Inclines a little to the Left Side; It hangs Looſe in it's *Capſula* or Caſe, only ſuſpended by the Veſſels, that go in and out of it's upper Part or Baſis, to which the *Pericardium* Adheres; In Beaſts, that Feed upon Graſs, it is near the Middle of the whole Body, from the Head to the Tail, but in Men, and moſt Carnivorous Animals, whoſe Necks are Shorter, it is nearer to the Head; It has a Firm, Thick, and Denſe Subſtance, Thinner, and Softer in the Right Side, Thicker, and more Denſe in the Left, but moſt Compact and Hard at it's Tip, only on the Left Side of the Tip it is Thin, as Conſiſting moſtly of the Concourſe of the Inner and Outer Membrane; It's *Parenchyma* is made up chiefly of Muſcular Fibres, ſo that the Heart it ſelf may be Eſteemed a Muſcle.

THE Fibres are of Three Kinds; FIRST, The outermoſt, which Run in ſtrait Lines from the Apex to the Baſis, or which Run, tho' not quite from the Apex, yet Parallel to Them which do.

SECOND, Thoſe which lie next under the Former, and Aſcending obliquely from the Left Side towards the Right Terminate in the ſame Baſis, Imitating a Snail-Shell, or Screw, by their Spiral Circuit.

THIRD, The Inmoſt, which are alſo oblique, but Run a contrary Courſe to the Former, Ariſing from the Right Side of the Heart, and from thence Aſcending Spiral-Wiſe towards the Left, when, having Embraced each Ventricle of the Heart, they Terminate in the Baſis on the Left Side.

THE Fibres of the Left Ventricle differ not from thoſe of the Right as to Kind, only they are conſiderably Stronger, and all of them are Inſerted into a Tendon, that is ſpread over it's Baſis under the Auricles, Part of which Tendon at the Egreſs of the *Aorta* in ſome Creatures, as in a Stag, becomes Bony; It's Shape is that of a Cone with it's Baſis upwards, and the Lower Part is called it's *Apex* or *Mucro*; It is Bigger in Men than in other Creatures, conſidering the Proportion of their Bodies, and Leſſer and more Denſe in Hot and Bold Men, than in the Cold and Cowardly; In Adult Perſons it is commonly Six Fingers Breadth Long, and Four Broad at the Baſis; Outwardly it is Covered with a proper Coat, which is Thin, but Strong and Denſe, and very hard to Separate from it, and is the ſame with the outer Coat of the Great Artery, as that, which Cloaths the Ventricles on the Inſide, is Continued unto and Common with that Thin Skin, that Covers the Inſide of the Arteries, like a *Cuticula*; and hence it is thought Probable by *Diemerbroeck*, that the Arteries borrow theſe Coats of the Heart, as the Nerves borrow their Two Tunicles from the *Pia* and *Dura Mater* of the Brain; Upon this Membrane, which Inveſts the Heart, there grows ſome hard Fat about the Baſis; It is not nouriſhed by the Blood or Chyle Received into it's Ventricles, but by the Veſſels of it's *Parenchyma*; It has Two Arteries, which Spring from the *Aorta* before it's Paſſage out of the *Pericardium*, and are called *Coronariæ*, becauſe they Encompaſs the Baſis of the Heart, like a Diadem, by fetching a Circuit on it's Surface, from whence they are branched ſeveral ways to the *Apex* of the Cone, where they Inoſculate; It has alſo as many Veins, which go by the ſame Name; and it's Nerves are Derived from the *Plexus Cardiacus* and *Par Vagum*; It is Diſtinguiſhed, in it's Parts, into it's Two *Auriculæ*, it's Two Ventricles, and it's *Septum*.

It's Auriculæ, *and their Syſtole and Diaſtole.*

THE *Auriculæ*, or Ears, are ſo called from ſome Similitude they have with thoſe of the Head, for they Riſe from a Long Baſis upon the Baſis of the Heart, and End in an Obtuſe Point, making an Obtuſe Triangle; They are Two Appendages of it, and are Seated over it's Ventricles, and are of the ſame Fabrick and Uſe, being both Muſcles, and made up of a like Order of Fibres, which are carry'd into oppoſite Tendons, whereof that at the Baſis of the Heart is Common to it, and theſe *Auriculæ*, and the other Runs along their upper Part; The Right is Larger and Softer, the Left is Leſs, but more Firm; Their Superficies is Smooth, when they are Filled, but Wrinkled, when Empty, and the Left more than the Right; When They are Cut open, there Appear in their Cavity many Fleſhy Columns Running from the Upper to the Lower Tendon, and betwixt Them there are pretty Deep Ditches, or Long Cavities, but Fewer in the Right, than in the Left; They are Dilated and Contracted, as the Heart is, but at Different Times; For the Syſtole of the Ventricles is at the ſame Time with the Diaſtole of the *Auriculæ*, and the Syſtole of the *Auriculæ* with the Diaſtole of the Ventricles; ſo that the *Auriculæ* Receive their Blood from the Veins, while the Ventricles Expell theirs into the Arteries, and when the Ventricles are Relaxed and Empty in their Diaſtole, the Auricles Force their Blood into Them by their Syſtole: They have Coronary Arteries and Veins, in like manner as the Heart, from the Former of which their Subſtance is nouriſh'd, and by the Latter the Superfluous Blood is carry'd Back to the Heart to be again Circulated thro' the Arteries; They ſerve to Receive the Venal Blood immediately out of the *Vena Cava* and *Pulmonaris*, and to Meaſure it into the Ventricles, into which, that they may Expell it with the Greater Force, Their Internal Fibres or Columns Ariſing from their Root, where they are Joined to the Baſis of the Heart, Reach directly outward towards the *Vena Cava* and *Pulmonaris*, and in the Syſtole of the *Auriculæ* Graſp the Blood Contained in their Cavity, like ſo many Fingers, and Squeeze it into the Ventricles, whilſt they are Relaxed in their Diaſtole.

It's Ventricles with their Conſtriction and Dilatation.

THE Ventricles Receive the Blood from their Auricles, and Expell it into the *Arteria Pulmonaris* and *Aorta*; The Right is the Wideſt, and almoſt Semicircular, but does not Reach down to the *Apex* or *Mucro* of the Heart, whereas the Left does, and is almoſt Round; Their Sides are Hollowed into Diverſe Interſtices or Furrows, and Interwoven with Carnous Fibres Reaching this Way and That; Which are more Numerous in Men's Hearts than in Thoſe of any other Animal, tho' ſuch as are Big, as Horſes, *&c.* have them Larger; Theſe Fibres or Fleſhy Columns ſerve to Streighten and Conſtringe the Ventricles, and the Furrows betwixt them help their Sides to Cloſe more exactly in their Syſtole; The Fibres are More and Stronger, and the Furrows Deeper in the Left Ventricle, than in the Right, and are alſo in the Side of the *Septum*, tho' that Side of it, which looks to the Right, is nearly Smooth; for there was occaſion for a Greater and Stronger Conſtriction in the Left, than in the Right, becauſe the Firſt Expells the Blood to the Extremeſt Parts of the Body by the *Aorta*, but the Laſt by the *Arteria Pulmonaris*, thro' the Lungs only.

It's Septum.

THE *Septum* divides the Ventricles from one another, and is a Carnous and Denſe Partition that ſtands like a Wall betwixt them; It is Hollow towards the Left Ventricle and has it's Fibres and Clefts, as was juſt now ſaid, but Convex and Smooth towards the Right; It has been a Conteſt with ſome, whether there is not a Paſſage thro' this *Septum* out of one Ventricle into the other, but none ſuch is found, nor is it likely there ſhould be any; becauſe, if there were, there would be no need of the *Foramen Ovale* and the *Canalis Arterioſus* in the *Fœtus*, by the Former of which the Blood paſſes out of the *Cava* into the *Vena Pulmonaris*, as it is Entering the Left Ventricle, and by the Latter out of the *Arteria Pulmonaris* into the *Aorta*.

It's Pericardium, *and the Blood and Water Contained in it.*

THE Heart is Incloſed by it's *Pericardium*, *Capſula*, or *Involucrum*, which is Membranous and of the ſame Figure; It Springs at the Baſis of it from the outer Common Coats of thoſe Veſſels, that Enter into the Heart, and which are borrowed from the *Pleura*; It's Outſide Adheres to the *Mediaſtinum* by many Fibres, and is Continued to it at the Baſis of the Heart, where the Veſſels Perforate it; It's Lower End is knit firmly to the Center or Nervous Part of the Diaphragm, which is ſaid by *Bartholin* to be Peculiar to Men, and that in all other Creatures it hangs Looſe; It has Nerves, Arteries, Veins, and Lympheducts, as other Parts, and any Tremors or Inordinate Vibrations are thought to Proceed from the Succuſſion of this Membrane, and to Differ from the Pulſation of the Heart; It contains in it a Serous Liquor, which in healthful Bodies is Reddiſh, like Water wherein Fleſh has been Waſhed, and is ſuppoſed to be Bred of Vapours Exhaleing from the Heart, which are Stopt and Condens'd by this Membrane, or elſe to Drop out of the Glands ſeated at the Baſis of the Heart, and what is Superfluous is Imagined to be carry'd off by it's Lymphaticks to the *Ductus Thoracicus*; This Liquor is not Naturally above Two Spoonfuls, tho' in Diſeaſed Perſons it ſometimes Increaſes to half a Pound, or a Pound; It ſerves for Moiſtening the Heart and making the Superficies Slippery, that it may Move the Better; Worms have been obſerved to Breed in the *Pericardium*, which Subjects the Perſons, who have them, to Palpitations of the Heart and Swoonings.

The Five Perforations of the Pericardium.

IT has Five Perforations: Firſt, One made by the Aſcending Trunk of the *Cava*; Second, Another by the Deſcending, both which Enter the Right Ventricle of the Heart; from whence there goes out the *Vena Arterioſa*, or *Arteria Pulmonaris*, into the Lungs, which makes a Third Perforation; A Fourth is made by the *Arteria Venoſa*, or *Vena Pulmonaris*, which Enters the Left Ventricle of the Heart; And a Fifth by the *Aorta* or Great Artery going out of the ſame.

The Aſcending Trunk of the Vena Cava, *which makes the Firſt Perforation.*

As to the Branches of the Deſcending *Cava*, they have been already Deſcribed, when we Treated of the Liver, and the Aſcending Trunk was Traced as far as the Midriff from the ſame Bowel, where we ſhall now take it up and purſue it; Upon the Paſſage thro' which it ſends forth into it's Subſtance the *Vena Phrenicæ*, which alſo Run thro' the *Mediaſtinum* and *Pericardium*, and are ſuppoſed by ſome to Abſorb the *Pus* Collected in the *Thorax*, from whence it is Circulated thro' the Heart, and at Length Diſcharged by Urine from the Emulgent Arteries.

The Foramen Ovale *and* Canalis Arterioſus.

FROM the Diaphragm the *Vena Cava* Aſcends Undivided to the Right Auricle of the Heart, but before it Enters it, having Pierced the *Pericardium*, it Emits the *Venæ Coronariæ* before-mentioned, which carry back the Superfluous Blood to the *Cava*; and where they open into it, there is a Valve fixed, which Permits the Blood to Return by them into the *Cava*, but Hinders any from Paſſing out of the *Cava* into Them; Before this Aſcending Trunk of the *Cava* opens into the Auricle, it is Joined by the other, which Deſcends from the *Claviculæ*, a certain Tubercle or Protuberance Intervening betwixt them upon their Junction, which Hinders the one from opening into the other in a Direct Line, and makes them go obliquely to the Left Hand, as they Enter the Auricle, otherwiſe the Deſcending Blood would have met the Aſcending to the Stoppage or Retardation of both; Immediately beyond this Protuberance out of the United Trunk, there goes a Paſſage called *Foramen Ovale* along the Baſis of the Heart to the *Vena Pulmonaris* in a *Fœtus*, juſt as it Enters into the Left Auricle, which Cloſes up and is Obliterated as ſoon as the *Fœtus* is Born; as alſo, from the *Arteria Pulmonaris* to the *Aorta* there is a Pipe called *Canalis Arterioſus*, which Conveys the Blood from the one to the other, and which upon the Birth Degenerates into a Ligament; Theſe Ducts are Formed, becauſe there is little or no Motion of the Lungs in the Womb, and no Paſſage of the Blood thro' them, Excepting what is Required for their Nutriment and Growth.

Concerning the United Trunk of the Cava *opening into the Right Ventricle of the Heart.*

THE United Trunk of the *Cava* opens by one Large Orifice into the Right Ventricle of the Heart, (but *Steno* affirms that it goes no farther than the Auricle,) into which is Poured all the Blood, that Returns in Circulation from all the Parts of the

Body, except the Lungs; At the Orifice of the *Cava* there Grows a Membranous Circle, which is Cleſt into Three Membranous Valves, looking Inwards and called *Tricuſpides*, which Permit the Blood to come in, but not to go out, and whoſe Action Dr. *Lower* thus Explains; Out of the Sides of the Right Ventricle there grow certain *Papillæ*, or Round and Long Caruncles or Fleſhy Columns, from whoſe Top Proceed ſome Tendinous Fibres, which are knit to theſe Membranous Valves, and Encompaſs the Orifice of the *Cava* ſo, that when the *Mucro* of the Heart is drawn up towards the Baſis in it's Syſtole, the Fibres, and conſequently, the Membranes or Valves are Slackened, and Hanging looſe are Driven upwards by the Blood that is Squeezed up in the Syſtole, by which means this Inlet is exactly Cloſed, ſo that no Blood can flow Back into the *Cava*, but is Expelled into the *Arteria Pulmonaris*, whoſe Orifice is now Open; On the Contrary, in the Diaſtole the *Mucro* Receding from the Baſis, the *Papillæ* and Fibres are Drawn down, and the Membranes and Valves with Them, by which the Orifice is Uncloſed, and a Paſſage Opened for more Blood to Flow in, the Former being Expelled by the Laſt Syſtole.

The Deſcending Trunk of the Cava *from the* Claviculæ, *which makes the Second Perforation.*

The Trunk of the *Cava*, which Deſcends from the *Claviculæ* has many Branches, which Empty themſelves into it, before it Diſcharges it ſelf into the Heart, which we ſhall only juſt Mention, Referring to Anatomiſts for a more Particular Deſcription of Them; As Firſt the *Azygos* or *Vena Sine Pari*, which Enters the *Cava* juſt before it's Tranſmiſſion thro' the *Pericardium*, and Receives into it the Inferior Intercoſtals from the Interſtices of the Eight Loweſt Ribs, on Each Side; from whence they Aſcend from the Eighth or Ninth *Vertebra* of the Breaſt to the Fourth or Fifth, and ſo up thro' the Right Side of the Cavity of the *Thorax*: The next are the Superior Intercoſtals, which are Derived from the Interſtices or Intervals of the Three or Four Uppermoſt Ribs, but ſometimes they Diſcharge themſelves into the *Azygos*, and then they are Wanting to the *Cava*; but it is obſervable, that where the *Azygos*, and the Superior Intercoſtals Enter the *Cava*, there are Valves placed, which Admit the Influx of Blood into it, and Prohibit the Reflux from it; After theſe, upwards, under the *Claviculæ*, the Subclavian Veins, one from the Right and the other from the Left, Empty themſelves into the *Cava*, where they Unite into one Deſcending Trunk, which Subclavian Veins, Continued out of the *Thorax*, are called the Axillar.

The Subclavian Receives ſeveral Branches Upwards and Downwards.

In it's Lower Side; First, The Mammary, which are Internal and External; The Former from the Ends of the Ribs, the Intercoſtal Spaces, and the Glands of the *Mammæ*; The Latter from the Navel and near the Epigaſtrick Veins, from whence they Aſcend to the ſaid Subclavian: But ſo Deſultory is the Courſe of the Veins, that ſometimes theſe Mammary Enter into the *Cava* and not into the Subclavian.

Second, The *Mediaſtina*, which is Derived from the *Mediaſtinum*, and ſometimes Diſcharges it ſelf into the *Cava*.

Third, The *Cervicalis* or *Vertebralis*, which is Derived from the *Vertebræ* of the Neck, and the Membranes, which Inveſt the Marrow contained in them.

Fourth, The *Muſcula Inferior*, which comes from the Lower Muſcles of the Neck and the Upper of the *Thorax*, and ſometimes Empties it ſelf into the Jugular Vein.

In it's Upper Side, First, The *Muſcula Superior* from the Upper Muſcles of the Neck.

Second, The External and Internal Jugulars, which have Valves to hinder the Return of the Blood out of the Subclavians; The External from the Membranes, which Cover the Brain, from the Fore-head, Temples, Head, Face, Chaps, and Neck. They Run juſt under the Skin, and are thoſe which are opened for Diſtempers of the Head, or a Quinzy, *&c.*

The Internal from the *Dura Mater*, and the Rough Artery or Wind-Pipe.

Into the Subclavian is alſo Inſerted, as before-mentioned, the *Ductus Thoracicus*, and the *Ramus Lymphaticus*, which brings the *Lympha* from the Arms, Neck, *&c.* but this ſometimes Enters the Jugular.

The Vena Arterioſa, *or* Arteria Pulmonaris, *which makes the Third Perforation.*

The *Vena Arterioſa*, or *Arteria Pulmonaris*, Obtains the Nature of an Artery, both in carrying Blood out of the Right Ventricle of the Heart to the Lungs, and in it's being Inveſted with a Double Coat, like other Arteries; At it's Riſe out of the Right

Ventricle, it's Orifice is Guarded with Three Membranous Valves, looking outwards, called the Semilunar, or Sigmoides, becauſe they Repreſent the Old Greek Sigma, or the Engliſh C; They are made, according to *Steno*, of the Tendons of the Fibres of the Heart, which go into the Subſtance of the Artery it ſelf; In the Syſtole they Open and Permit the Blood to Paſs out of the Ventricle into this Artery, but in the Diaſtole they Shut, ſo as none can Return; A little beyond theſe Valves, in a *Fœtus*, the *Canalis Arterioſus* Springs, which Runs croſs the Breaſt to the *Aorta*, and Conveys the Blood out of this Artery thither, without Paſſing thro' the Left Lobe of the Lungs, or thro' the Left Ventricle of the Heart, but, as was before ſaid, upon the Birth turns Ligamentous; After it's Paſſage out of the *Pericardium*, it Bends towards and Aſſociates it ſelf with the *Aſpera Arteria*, or Wind-pipe, and Attends it in all it's Ramifications thro' the Lungs; and where the Branches of the Wind-pipe End in the Little Cells or Bladders after Deſcribed, the Twigs of this Artery, being Interwoven with thoſe of the Vein, Embrace them like a Net, in order, as it is Imagin'd, that the Air may more Intimately mix with the Blood.

The Arteria Venoſa, *or* Vena Pulmonaris, *which makes the Fourth Perforation.*

THE *Arteria Venoſa*, or *Vena Pulmonaris*, Obtains on the other Hand the Nature of a Real Vein, in carrying Blood to the Heart, and in being Inveſted with a Single Coat only, as other Veins; It accompanies the Rough Artery, and the *Arteria Pulmonaris*, in all their Branchings in the Lungs, and conveys the Blood from the ſmall Twigs of the ſaid Pulmonary Artery by it's own Capillaries into Two Trunks, which afterwards Unite into one, which opens into the Left Ventricle of the Heart; At it's Orifice there are placed Two Membranous Valves, beſides which it has no other, called *Mitrales*, becauſe together they Reſemble a Biſhop's Mitre; They are of a ſtronger Contexture than the *Tricuſpides* in the Right Ventricle, and ſo are the Fibres, which Aſcend to them from the *Papillæ* or Fleſhy Columns, for the Reaſon above given; Theſe Valves ſhut in the Syſtole of the Heart, and the Orifice of the *Aorta* opens, by which means the Blood is Hindred from Returning into the Pulmonary Vein, and Directed to Proceed in the ſaid Great Artery; Juſt as the Pulmonary Vein Enters the Left Auricle, the *Foramen Ovale*, a Pipe or Canal, opens into it coming from the *Cava*, as was mentioned before, and at it's Orifice into this Vein there is a Valve placed, that hinders any Blood from Returning into the *Foramen*.

THERE is one Thing Remarkable in this Pulmonary Artery and Vein, That, whereas in all the other Arteries, the Blood is of a Bright Scarlet Colour and in the Veins of a Black Purple, on the Contrary, in the Pulmonary Artery it is of a Black Purple and in the Vein of a Bright Scarlet, which can be owing to nothing but to the Mixture of Air with it in the Lungs.

The Aorta, *or Great Artery, which makes the Fifth and Laſt Perforation.*

THE *Aorta*, or Great Artery, which is the Laſt Veſſel, that Perforates the *Pericardium*, is That, from which all the Arteries, Excepting the Pulmonary, are Derived; At it's Riſe from the Left Ventricle of the Heart, it hath Three Valves looking outwards, called *Semilunares*, being altogether like thoſe at the Orifice of the *Arteria Pulmonaris* in the Right Ventricle, and both They and the *Aorta*, according to *Steno*, are Conſtituted of the Tendons of the Fibres of the Heart; Theſe hinder the Blood from Returning out of the Great Artery into the Heart again; The Orifice of the *Aorta*, or the Tendon of the Heart, which is Continuous to it, in ſome Creatures, eſpecially in Stags, does often, as was ſaid before, and ſometimes in Men, become Bony; It Aſcends out of the Left Ventricle, and is immediately Turned into an Arch, and from thence Deſcends downwards, which is called the Deſcending Trunk, and that which is called the Aſcending, according to Dr. *Ruyſch*, is only Branches ſent upwards from the Bending Arch of the Main Trunk of the *Aorta*; Before it comes out of the *Pericardium* it conveys the Coronaries to the Heart, it's Auricles, and to it's own Coats; but when it has Pierced it, it is Divided into what is uſually Termed the Aſcending and Deſcending Trunk.

It's Aſcending Trunk, and it's Branches.

THE Aſcending Runs up under the *Vena Cava*, Lies upon the Wind-pipe, and preſently Sends forth Two Large Branches, the one to the Right, the other to the Left Arm, called the Subclavian, as Marching under the *Clavicula*, and after they are out of the Breaſt, the Axillar: They Send out ſeveral Branches, both from their Lower and Upper Side.

FROM the Lower, The Superior Intercoſtal to the Interſtices or Intervals of the Four Uppermoſt Ribs, to the Neighbouring Muſcles and Spinal Marrow; Theſe are ſometimes Propagated from the Cervical Arteries, coming out thro' the Holes of the *Vertebræ*.

FROM the Upper, FIRST, The Mammary to the Breaſts, and down the *Abdomen* to the Epigaſtrick Arteries Aſcending, with which they are ſuppoſed by ſome to Inoſculate, contrary to the Circulation of the Blood; ſince no Blood can Aſcend by the Mammary, nor Deſcend by the Aſcending Twigs of the Epigaſtrick.

SECOND, The Cervical or Vertebral to the *Vertebræ* and Muſcles of the Neck, the Spinal Marrow and it's Membrane, which Inveſts it, to the *Pia Mater*, on which a wonderful Net-like *Plexus* of them is Formed, to the Subſtance of the *Cerebellum*, and to the Carotid Arteries, with which uniteing, Part of the *Rete Mirabile* is made.

THIRD, The Muſcular to the Muſcles of the Neck and Arms.

AT the ſame Place or very Near, where the Aſcending Trunk ſends out the *Subclavian* Side-ways, The Remainder is Divided into Two, called *Carotides*, which Aſcend directly Upwards, tho' the Right ſometimes Ariſes from the Right *Subclavian*; They are ſuſtained at their Riſe by the *Thymus*, beſtow certain Twigs on the *Larynx*, Tongue, the Muſcles of the *Os Hyoïdes*, and the Neighbouring Glands, paſs up on each Side by the Wind-pipe to the Jaws, where they are each of Them Subdivided into the External and Internal Branches.

THE External and Smaller is Diſperſed into the Muſcles of the Cheeks, Fore-head, Temples, Lips, and all the outer Parts of the Head and Face, and into the Roots of the Teeth of the Lower Jaw.

THE Internal and Larger into the *Larynx*, Tongue, *&c.* the Glands behind the Ears, and the Spongy Parts of the Palate and Noſe, and the Roots of the Teeth of the Upper Jaw; The Remainder Climbs upon the Scull, and at it's Baſis is Divided into Two Branches.

THE Leſs and Hindermoſt is ſpent upon the Inner Muſcles of the Neck, the Membrane of the Spinal Marrow, and the *Dura Mater*, at whoſe *Sinus* it Ends.

THE Larger, likewiſe, is ſpent in ſeveral Twigs on the *Dura Mater*, and being Interwoven with many ſmall Slips of the Cervical Artery, Produces the *Rete Mirabile*, which is more Conſpicuous in Beaſts than Men; It afterwards Pierces thro' the *Dura Mater*, and Enters the *Pia*, Mixes with the Twigs of the Cervical Artery, with which it Accompanys the Spinal Marrow to the Loins; It ſends alſo one ſmall Branch with the Optick Nerve to the Eye, and ſupplying ſtill more Twigs to the Subſtance of the Brain and *Pia Mater*, which Unite with thoſe of the Cervical Artery, it makes the *Plexus Choroides*.

It's Deſcending Trunk and it's Branches.

THE Deſcending Trunk of the *Aorta* goes down by, and cleaves to, the Gullet, and is the Reaſon, why Perſons, when Hot, are ſo much Cooled by a Draught of Water; It's Branches are as follow,

FIRST, The Inferior Intercoſtals to the Interſtices of Eight or Nine of the Lower Ribs, the Marrow of the Back, the Muſcles of the *Vertebræ* and *Thorax*, and, according to *Ruyſch*, to the *Bronchia* of the Wind-pipe. When it comes to the Midriff,

SECOND, The Phrenical to the Diaphragm, *Mediaſtinum*, and *Pericardium*. When it comes down by the Spine to the Fifth *Vertebra* of the Loins,

THIRD, The Cœliacal, from the Fore-part of the Trunk, to the Stomach, the Hollow of the Liver, and the Trunk of the *Vena Portæ*; where it is Divided into Two Branches, the Right and Left.

THE Right, which is the Smaller, Aſcending from it's Upper Part, ſends out the *Gaſtrica Dextra* to the *Pylorus*, and the *Cyſticæ Gemellæ* to the Gall-Bladder.

FROM the Lower Side. 1. The *Epiplois Dextra* to the Hinder and Inner Leaf of the Caul, and to the *Colon* Annexed to it.

2. The *Inteſtinalis* to the *Duodenum*, and the Beginning of the *Jejunum*.

3. THE *Gaſtroëpiploïs Dextra* to the Bottom of the Stomach, and the Caul, which Adheres to it.

4. THE *Hepaticæ* to the Inveſting Membrane of the Liver, the *Capſula Communis*, the Gall-Bladder, and *Porus Bilarius*.

THE Remainder of this Right Branch Enters the Meſentery in ſeveral Twigs.

THE Left Branch of the *Cœliaca*, which is called the Splenick, and ſometimes Springs Immediately from the *Aorta*, is Larger than the Right, and ſends forth,

FROM it's Upper Side, The *Gaſtrica Major* to the Stomach, and then is Divided into the *Coronaria Stomachica*, which Encompaſſes the Upper Orifice of it like a Garland, and is Spread in Twigs upon it's Ventricle; and the *Gaſtrica Siniſtra*, which is Carry'd to the Upper Part of the Stomach and to the *Pylorus*.

FROM it's Lower Side, The *Epiplois Poſtica* to the Hinder Leaf of the *Omentum*, and the *Colon* Adjoining, and the *Epiplois Siniſtra* to the Lower and Left Side of the *Omentum*.

JUST as this Splenick Branch Enters the Spleen, there Ariſes out of it's Upper Part *Vas Breve Arterioſum*, which goes ſtrait to the Left Part of the Bottom of the Stomach, and the *Gaſtroëpiploïs Siniſtra*, which, being ſuſtained by the Upper or Fore Leaf of the *Omentum*, ſends ſome Twigs to the Stomach, and to the ſaid *Omentum*; After which the Splenick Branch Enters into the Spleen.

FOURTH, The Superior Meſenterick from the Fore-part of the Trunk to all the Upper Part of the Meſentery, where it is Accompanied by the Meſaraick Vein of the *Porta*, to the *Jejunum*, *Ileum*, and Part of the *Colon*.

FIFTH, About the Second *Vertebra* of the Loins from each Side of the Trunk an Emulgent to the Kidneys, where it is Separated into a Thouſand Twigs, which are Inveſted, with the Veins, in one Common *Capſula* Borrowed from the *Pelvis*, and End with their Capillaries in the Glands; where the *Serum* is Secreted, and carry'd from them by the Urinary *Siphons* into the *Pelvis*, and from thence thro' the *Ureters* into the *Veſica*, in order to be Expelled the Animal Syſtem.

SIXTH, The Spermatick, or *Arteriæ Præparantes*, from the Fore-part of the Trunk, ſeldom from the Emulgents, to the *Epididymides* and *Teſtes*, and in Women, to the *Ovaria*, and Bottom of the Womb.

SEVENTH, The Inferior Meſenterick, a little before the *Aorta* is Divided into the *Rami Iliaci*, to the Lower Region of the Meſentery, to the *Colon*, *Rectum*, and *Anus*, where it makes the Internal Hæmorrhoidal Artery.

EIGHTH, The *Lumbares*, out of the Back-ſide of the *Aorta*, to the Neighbouring Muſcles of the Loins, the *Peritonæum*, and, thro' the Holes of the *Vertebræ*, to the Membrane of the Spinal Marrow, and to the Marrow it ſelf.

WHEN the *Aorta* is Deſcended to the Fifth or Laſt *Vertebra* of the Loins, and to the Top of the *Os Sacrum*, it begins to Climb upon the *Vena Cava*, under or behind which it Paſſed thus far, and is from thence Divided into the *Rami Iliaci*, and immediately upon the Diviſion there Springs out the *Arteria Sacra*, whoſe Twigs Penetrate into the Marrow of the *Os Sacrum*.

THE *Rami Iliaci* are Subdivided into the Interior, and Exterior Branches.

FROM the Interior Proceed. 1. The Inferior Muſcular to the *Glutæi*, or Muſcles of the Buttocks, to the Iliack Muſcle, and the *Pſoas*.

2. THE Hypogaſtrick, to the Bladder, *&c.* and in Women to the Bottom of the Womb and it's Neck, from whence Iſſue the *Menſes*, in their Monthly Purgation, and to the *Anus*, where it makes the External Hæmorrhoidal Artery.

3. THE Umbilical, by the Sides of the Bladder, betwixt the Membranes of the *Peritonæum*, to the Navel, and Paſſes, in a *Fœtus*, into the *Placenta Uterina*, but after the Birth, Turns to a Ligament.

FROM the Exterior. 1. The Epigaſtrick, betwixt the *Peritonæum* and *Muſculi Recti* of the *Abdomen*, to the Mammary Artery, at the Navel; tho' it does not Inoſculate with it, as is ſuppoſed.

2. THE *Pudenda*, to the Secret Parts.

THE *Rami Iliaci* after This go out of the *Abdomen* into the Thighs, where they are called *Crurales*.

The Chief Controverſies of Anatomiſts Relateing to the Heart, and it's Veſſels.

UPON the Heart and the Veſſels, which Appertain to it, There are Three Grand Controverſies, or Queſtions.

FIRST, What is the Cauſe of the Heart's Motion?

SECOND, How the Pulſation of the Heart, and Circulation of the Blood is Performed?

THIRD, How Blood is made of Chyle, from whence it's Heat and it's Colour, and whether the Body is Nouriſhed by it, and in what Manner?

§. 2.

§. 2. We ſhall now, as uſual, Inquire what Solution our Doctrine of Expanſive and Contractive Forces can give of theſe Phænomenons.

It is Agreed on all Hands, that a Red-Beating Speck Appears in the *Embryo* before the Heart is Formed, and that this Ariſes upon the Impregnation of the *Ovum*; That the *Ovum* has Originally an Expanſive Force belonging to it, is manifeſt in Fowls, both from it's Yellow Colour, which is ſome Indication of it, as it Approaches to that of the Sun and of Fire, the Great Sources in Nature of the Expanſive, and from it's Strength, as an Aliment, which chiefly conſiſts in that Force, as we have before ſhewn; On the other Hand, the *Semen*, which Impregnates it, has a Contractive Force, and accordingly was never deemed fit for Nouriſhment, and it's Colour is near to a White, as Froſt, and Snow, and Salt, the Greateſt Inſtances we know of the Contractive, are; ſo that the One may be Eſteemed the Animal Fire or Warmth, and the Other the Animal Froſt or Cold, or Salt, of the Growing Syſtem: Whilſt therefore the *Ovum* Continues Unimpregnated, the Expanſive Force of it Remains Undiſturbed and Quieſcent in it ſelf; but as ſoon as the Contractive Force, which is Oppoſite to it, Supervenes, a Motion and Ferment is Cauſed, and the Expanſive of the *Ovum* is Excited, and the Red-Beating Speck is Produced, which by it's Pulſe declares it's Force to be, what we have Mentioned; And now the Animal Flame begins, which Burns to the laſt Period of our Lives.

How the Heart or Red Beating Speck is Produced in an Embryo, *and by what Forces the Impregnation of the* Ovum *is Perform'd, and the Animal Syſtem begins to Riſe.*

§. 3. Thus in Mechanical Forces, where a Contractive and Expanſive Act oppoſite to each other, a Motion is continued ſo long, as thoſe Forces ſubſiſt; Thus it is in the Revolutions of the Heavenly Bodies; and Thus in the Mixture of Chymical Liquors of a Contractive and Expanſive Nature, all which we have already Explained: Life alſo has been Compared, and not Unjuſtly, to a Lamp and a Taper, as it depends upon the like Forces; For here there is the Expanſive of Fire or Flame, to which the Oily or Expanſive Subſtance continually Runs and Aſſimilates it ſelf, and the Contractive of Air, and if either of Theſe are not Regularly ſupplied, it is Extinguiſhed and Dies: Laſtly, that this Animal Flame Commences in the Red-Beating Speck, and from the Forces alledged, is Proved even in our own Syſtem, from the Common Inflammations and Redneſſes Produced in it, which are always in a Wound by the Expanſive Force of the Blood, and the Contractive of Air or Cold Mixing with it; For if the Air is kept off, the Expanſive of the Blood Preſerves it's own Craſis and Temper; And Salts, which are of a Contractive Nature, applied to a Freſh and Large Wound cauſe it to Redden and Feſter, but Balſams and Salves, which by their Inflammableneſs are of a Gentle Expanſive, Cheriſh and Aſſimilate with the Blood, and gradually Cloſe and Heal it; Colds likewiſe produce Fevers, and a Contractive Force, Mingling with the Expanſive of the Blood, Exalts it into the Greateſt Heat and Ferment; And the Venal Blood, which is of a Deep Purple, by it's Paſſage thro' the Lungs and Pulmonary Artery, that is, by Meeting with the Contractive of the Air there, is Raiſed into a Bright Scarlet, which is the Colour of the Arterial Blood, or which is the ſame, into an Higher Fever or Flame, or into a more Elevated Degree of the Expanſive, and which is immediately Perceived by the Pulſation, which Enſues, and the Vivid Colour it puts on, which are both Irrefragable Arguments of ſuch a Force and the Increaſe of it. And the ſame Blood Expoſed to the open Air in Venæſection Changes into a like Scarlet, as is known to every Obſerver. Hence alſo it is, that *de Graef* in the Diſſection of Rabbits, half an Hour, Six, Twenty Four, Twenty Seven, and Forty Eight Hours, after the *Coitus* or Injection of the *Semen*, or Contractive Force, found the *Cornua* of the *Uterus* to Redden, and the Limpid Liquor of the *Ovum* gradually to change into that Colour.

The foregoing Account Confirmed from the Mechanical Forces, from the Revolutions of the Heavenly Bodies, from the Mixture of Chymical Liquors, from a Lamp or a Taper, and from the Common Inflammations Produced in our own Syſtems, from the Arterial Blood, and de Graef's *Experiments in Diſſection.*

§. 4. The Animal Flame being Kindled in the Manner Deſcribed, that is, the Red-Beating Speck being Produced as above, it Scatters, like Fire, and Diſperſes it's Influence, and Converts what is Alimentary to it, that is, the Lower Degree of the Expanſive in the *Ovum*, into it's own Force, from whence Streaks of Blood are preſently after Obſervable in it, the Contractive Force, like the Air, ſtill Fanning and Exciteing the Vital Flame; And as the Aliment of the *Embryo* or *Fœtus* is of an Expanſive Nature, as what we afterwards Receive in for our Nouriſhment is, it becomes the *Pabulum* or Fuel of the Blood or Animal Fire, by which there is a Continual Increaſe of it; And the Contractive Force is on the other Hand ſupplied by the

How the Animal Flame is Preſerved and Continued by a New ſupply of the Expanſive and Contractive Forces. And what are the Rudiments of the Heart and Brain.

the *Albugineous* Liquor ſurrounding the *Ovum*, and by the Nerves of the Womb, and of the *Plexus Meſenterii Maximus*, which Run into the *Ovarium*, by the Aſſiſtance of Which, the Seminal Principle Subdues other Forces to the Contractive, as the Sanguineous does to the Expanſive, from which Conjunction of the Contractive Forces are Derived the Rudiments of the Brain, which at Firſt is little more than a Watry and Thin Fluid.

The Reaſon of the Streams of Blood before the Heart is Produced, and of the Diſtance of the Heart from the Brain.

The Red-Beating Speck, which is afterwards the Heart, therefore being the Seat and Center of the Expanſive, and the Brain that of the Contractive, and being Forces oppoſite to each other, as the Red-Beating Speck ſends out Rays and little Streams of Blood towards the Latter, ſo the Latter emits Nerves and Saline or Contractive Juices towards the Former, in order to Preſerve the *Æquilibrium* betwixt Them: And the Diſtance betwixt the Heart and the Brain is Determined, when that *Æquilibrium* is Fixt and Aſcertained in the *Fœtus*, and is ſtill in Proportion Enlarged in Men, as the Expanſive and Contractive Forces Increaſe by new and continual Additions of Nouriſhment to both, till the Animal Syſtem is Arrived to it's utmoſt Maturity.

How therefore the Blood is Formed before the Veſſels, which contain it, which is Acknowledged by Dr. Harvey, *and is a Matter of Aſtoniſhment to him; and the Right Ventricle of the Heart is made.*

§. 5. From this Account it is manifeſt, that the Blood is Formed before the Veſſels, which contain it, according to what is Acknowledged and Wondered at by the Eminent Dr. *Harvey* and the Anatomiſts, who have Followed him; But the Red-Beating Speck being, as was ſaid, the Center and Seat of the Expanſive, Freſh Blood is continually Amaſs'd to it, by it's Power of Converting Alimentary and Expanſive Juices into it's own Nature, as Fire does the Fuel, which is Miniſtred to it, and by the Law of Nature, which Aſſembles Forces of the like Kind together.

On the Contrary, the Nerves from the Brain Endeavour to ſurround it with their Contractive Force to Balance the Expanſive, from whence the Right Ventricle of the Heart is Formed; for in a *Fœtus*, as there is no uſe of the Left, by Reaſon of the *Canalis Arterioſus* Conveying the Blood from the *Arteria Pulmonaris* immediately to the *Aorta*, ſo we have no occaſion at Preſent to conſider it, but to ſuppoſe the Heart to Conſiſt only of One.

How the Heart is the Source of Vital Heat, why the Wounds in it are Mortal, and why it hangs looſe in it's Capſula.

§. 6. We ſhall now therefore from this Doctrine go Regularly on, and Aſſign a Reaſon for all the Phænomenons of the Heart and it's Veſſels in Order, as they are before Recounted from Anatomiſts.

It is the Source of Vital Heat, that is, the Blood, which is Contained in it, the Heart it ſelf being nothing more than a Thick Muſcular Coat, which Inveſts it; The Heat of the Blood Ariſes from the Expanſive Force Acting in Oppoſition to the Contractive, as has been ſhewn before in the Red-Beating Speck, and is the Caſe of an Efferveſcence Produced in Chymical Liquors of Contrary Forces; And the Blood in the Heart is the Source of this Vital Heat, becauſe the Blood here is Originally made from the ſaid Red-Beating Speck, and afterwards Converts other Alimentary Juices into the ſame Subſtance, till Growing into a Maſs, it is ſurrounded by the Contractive with Muſcular Coats, and Included in the Ventricle; The Wounds in it are Mortal, for the ſame Reaſon, as the Cutting of any conſiderable Artery is ſo, the Blood being thereby Let out into the Body, the Circulation Diverted, and the whole Animal *Æquilibrium* entirely Deſtroyed; It hangs looſe in it's *Capſula*, and is only ſuſpended in the Breaſt by it's Veſſels; For the Red-Beating Speck was ſolitary and alone, and by Degrees Collected a Maſs of Blood or the Expanſive Force to it, which afterwards was Cloathed with the Fibres of the Contractive.

Why the Figure of it is Conical, and why the Mucro *Inclines to the Left.*

§. 7. It's Figure is Conical with the Baſis upward; becauſe the Principal Conteſt betwixt the Expanſive and Contractive Forces is on that Surface, which lies Intermediate to Them, that is, to the Heart and Brain, and therefore the Expanſive will there make the Greateſt Reſiſtance to the Contractive and will Diſplay it ſelf the Moſt, after which the Expanſive will be Continually and Gradually Diminiſh'd, and the Contractive cloſe in upon it accordingly, from whence it will Surround it in ſtill Leſſer Circles, till it Ends in the *Mucro* and *Apex* of a Cone; Which *Mucro* or Tip Inclines a little to the Left Side, becauſe the Chief Weight of the Action betwixt the Expanſive and Contractive Forces lies on the Right, that is, the Right Ventricle is Loaded with Blood and Preſſed by the Contractive, whereas the Left, if Formed, is as yet Empty, which therefore muſt Elevate the *Mucro* to the Left, and ſink the Loaded Ventricle to the Right.

Why it is ſituate Nearer to the Brain in Men than in Brutes, Why it Adheres to the Pericardium *at the Baſis, and why it's Subſtance Differs in Thickneſs on the Right Side, the Left, and at the* Apex *or Tip.*

§. 8. It is ſituate Nearer to the Brain in Men than in Brutes; becauſe the Contractive Force is Greater, as we have before Evinced, in the one than the other, by which the Heart is drawn Nearer to it in the Firſt, than in the Laſt Caſe.

It Adheres to the *Pericardium* at the Baſis, but not at the Tip; becauſe the Contractive uſes all it's Efforts there, where there is the Greateſt Expanſive Exerted againſt it, and therefore Unites the *Pericardium* to the Muſcular Fibres of the Heart; On the Contrary, where the Expanſive Force is Weak, and the Contractive ſufficiently Prevalent, as at the *Mucro*, the *Pericardium* do's not Preſs to it's Aſſiſtance.

It has a Firm, Thick, and Denſe Subſtance; becauſe the Expanſive Force of the Blood Amaſſed together being Great Required a ſtrong Contractive to Balance and Encloſe it; and therefore the Arteries, which contain a Leſſer Expanſive Force of the Blood, have Thinner Coats, and the Veins, whoſe Expanſive Force of Blood is ſtill more Diminiſhed, Thinneſt of all: For the ſame Reaſon, the Subſtance of the Heart is Thinner and Softer on the Right Side, thro' which the Venal Blood Flows, which has a Leſs Expanſive, and Thicker and more Denſe on the Left, thro' which the Arterial is carry'd, and which has a Greater Expanſive; but the Tip is the moſt Hard and Compact, from the Expanſive, which is here ſubdued, giving little Diſturbance to the Contractive, and from the Contractive Forces Uniteing more firmly together after their Conteſt with, and Superiority over, the Expanſive by the Law of Nature, by which Similar Forces Aſſemble and Cement together, and are Excited, if not Vanquiſhed, by a Contrary Action.

The Reaſon of the Courſe of the Fibres of the Heart, and why the Fibres of the Left Ventricle are Stronger than thoſe of the Right.

§. 9. Farther, in Reſpect of the Fibres, which Compoſe theſe Coats, the Innermoſt Run Spiral-Wiſe from the Baſis to the Tip, Deſcending from the Left to the Right; becauſe the Expanſive Force of the Blood lying in the Right Ventricle, the Contractive Force is carry'd in it's Action againſt it from the Left to the Right; the Contractive alſo Deſcends, as it is Derived from the Brain, and Spirally, becauſe the Expanſive Force Exerting it ſelf in Oppoſition to it, it cannot Deſcend in a Strait Line from Left to Right, and therefore muſt do it in a Curve; That is, every Step, which the Contractive takes, is ſomething Elevated by the Expanſive above a Rectilinear Deſcent, and conſequently is made to Wind about the Heart in a Spiral: The Second Courſe of Fibres Deſcend from the Right to the Left, For the Contractive Meeting with an Oppoſition from the Expanſive in it's Former Courſe, the ſame Contractive, Deſcending ſtill from the Brain, Attempts a Contrary, which by the Expanſive is notwithſtanding again Diverted into a Spiral, which Forms the Middle Coat: Laſtly, the Contractive Force from the Brain ſending out Freſh ſupplies againſt the Expanſive of the Blood, it at Length proceeds in ſtrait Lines from the Baſis to the Tip, or in Lines Parallel to Them; becauſe the Balance is now Compleated betwixt the Two Forces, and the Contractive Left to go in it's own Direction, and ſo the Heart with it's outmoſt Coat is Finiſhed: The Fibres of the Left Ventricle are ſtronger than thoſe of the Right; becauſe the Expanſive Force of the Arterial Blood is Greater than that of the Venal, and therefore the Contractive Force in the one Caſe muſt be Superior to that in the other, to make an *Æquilibrium* of the Expanſive with the Contractive.

Why a Tendon is ſpread over the Baſis of the Heart, and why the Heart is Bigger in Men than other Creatures, and yet leſs in Bold Men, than in the Cowardly.

§. 10. There is a Tendon ſpread over the Baſis of the Heart; becauſe the Expanſive Force there Exerts it's whole Effort againſt the Contractive, and conſequently the Contractive from the Brain will be the moſt Conſtipated and Confirmed at that Surface in Oppoſition to it: For which Reaſon alſo at the Egreſs of the *Aorta*, where the Expanſive of the Blood is ſtill Increaſt, that Part of the Tendon in ſome Creatures becomes Bony, that is, is Guarded with a Greater Contractive.

Hence the Heart is Bigger in Men than in other Creatures, becauſe the Brain is ſo, being as Big again as an Oxe's, for to Preſerve an *Æquilibrium* betwixt the Two Forces, if the Source of the one is Enlarged, that of the other muſt be in Proportion; but it is Leſſer and more Denſe in Bold and Couragious Men than in the Cowardly, becauſe the Spirituous, Nervous, or Contractive Force is Greater in the Firſt, and the Expanſive and Sanguineous in the Laſt; which is Certain in Experience, as the Meagre and the Enthuſiaſtick are more Nervous and Bold than the Fat and the Sanguinary.

How the Fat about it's Baſis is Produced, and the Coronariæ *Formed, and*

§. 11. The Fat about it's Baſis is Produced from a Mixture of the Blood, and Nervous Juice, as other Fat; and it is not nouriſh'd from the Blood Contained in it, but by the Veſſels Diſperſed in it's *Parenchyma*: For as ſoon as the Blood is Pulſed

out

it's Fleſh and Subſtance Concreted and Made.

out of the Left Ventricle of the Heart thro' the *Aorta*, Part of it Runs immediately to the Heart by the Appetite of Nature, which carries it towards a Similar Force, the Expanſive within the Heart, from whence it was Expelled by the Contractive, and forms the Innumerable Branches of the *Coronariæ* upon the Baſis of it, where the Expanſive is the moſt Senſible and Strong; from thence it Deſcends to the Tip, and Penetrates into and Diffuſes it ſelf in Various Rivulets and Meanders thro' the whole Subſtance; On the other Hand, the Contractive Force in like Manner, and by a like Appetite of Nature Aſſembles to the Contractive of the Heart, and Attends this Expanſive from the *Plexus Cardiacus* and *Par Vagum* of the Nerves, and Congeals and Hardens it into Fibres and Fleſh, by which means the *Æquilibrium* is ſtill Preſerved betwixt the Two Forces, and the Parts Equally Nouriſhed.

The Cauſe of the Syſtole and Diaſtole of the Heart.

§. 12. The Right Ventricle of the Heart being made, as above, with it's Fibres and Coats, an eaſy Account is given of the Reaſon of it's Motion and the Pulſation of it; For the Blood by it's Expanſive Force Swells and Dilates it, both that within it's Cavity, and that which is Spread and Scattered in it's Subſtance, that is, the Fleſhy Part of it, which is the Cauſe of it's Diaſtole; and the Nervous Fibres and Coats by their Contractive Force Streighten and Compreſs it, from whence it's Syſtole Ariſes; And theſe Forces being in an Equal Balance, or nearly ſo, mutually ſucceed each other by Turns, neither being able to Overcome, Surmount, or Deſtroy and Annul the other; Like the Weight of a Clock and it's Pendulum, in which the Former is the Contractive and the Latter the Expanſive derived from the Expanſive of the Hand, which puts it into Motion; and which therefore Conſiſts of Two Oppoſite Forces Balanceing each other, from whence it's Motion Commences, and which, if either of them is Stopt or Hindered from it's Exertion, Ceaſes.

How the Beginning of the Arteria Pulmonaris, *the* Canalis Arterioſus, *and the* Aorta *with it's Branches, are Formed.*

§. 13. The Blood, being by the Syſtole or Conſtriction of the Heart Driven out of it in a *Fœtus*, by it's Appetite to the Expanſive hovers upon the Baſis, whither the Contractive Force Purſues and Surrounds it, and as the Blood is ſtill Pulſed forwards and Expands it ſelf, and the Nervous or Brainy Subſtance by it's Contractive Incloſes and Inveſts it, The Beginning of the *Arteria Pulmonaris* and the *Canalis Arterioſus* is Formed, and after that, the *Aorta* with it's ſeveral Aſcending and Deſcending Branches; For a Progreſſive Expanſive Incloſed by a Progreſſive Contractive muſt of Neceſſity produce hollow Tubes and Canals, ſuch as the Arteries are.

And from hence it is, that the Inner and Outer Coats of the Arteries are the ſame with thoſe of the Heart, being Conſtituted in the ſame Manner: And as to the Aſcending and Deſcending Branches of the *Aorta*, why the Blood ſhould Deſcend is not hard to Conceive, ſince it is Endued with it's own Proper Gravity, but why it ſhould Aſcend, or Receive a Lateral Direction, is ſomething Difficult to Explain, unleſs upon this Doctrine of the Expanſive and Contractive Forces.

How the Branches of the Aorta *Aſcend upwards, and in General, How the Blood Veſſels and Nerves are capable of Receiving all Manner of Directions.*

§. 14. The Reaſon of which therefore is, becauſe, as the Contractive Force of the Nervous Subſtance is carry'd from the Brain towards the Heart, ſo the Expanſive of the Sanguineous is Reciprocally carry'd from the Heart towards the Brain to Preſerve the *Æquilibrium* of the Animal Syſtem, by their Forces Acting in Oppoſition and in Counterpoiſe to each other; And this is ſo True, that the Carotid and Cervical Arteries, thro' which the Blood Aſcends to the Brain, are ſo Large and Numerous, that a Third Part at leaſt of the whole Maſs is Conveyed thither by Them, which, Paſſing thro' the Capillary Arteries in ſmall Quantities, is Congealed and Transformed into the Contractive Force of the Brain, in order to ſupply the *Medulla Oblongata*, which is the Trunk of the Nerves, with new Matter for their Conſumption upon the ſeveral Parts, to which they are Branched and Propagated; And this made *Malpighius* think it Probable, that the Coagulative, or Concreſcible, *Serum* was Filtred in the *Cortex* or Glandulous Part of the Brain from the Arterial Blood, and from thence Transferred by the Fibres of the *Corpus Calloſum* to the ſaid *Medullæ*; Upon the ſame Account, the Expanſive Force of the Blood is carry'd in a Lateral Direction by it's Acting Contrary and in Oppoſition to the Contractive from the *Vertebræ* of the *Thorax*, Back and Loins, and the ſeveral Nerves and Fibres ſent from thence; and which will alſo ſolve all other the Innumerable Variety of Directions, Upwards, Downwards, and Tranſverſe, or Oblique, in which the Sanguineous and Nervous Matter is Diſperſed thro' the whole Body.

Why the Proceſſes of the Heart and

§. 15. We are likewiſe hereby Informed, why the Proceſſes of the Heart and Brain open towards each other, whereas the Parts, which are Averſe, are Incloſed; For the

Blood and the Brain Exerting their Actions continually againſt each other, that is, their Expanſive and Contractive Forces, there muſt of Courſe be a continual Outlet to them that way, but the Expanſive in the Heart being Leſs at the Tip, as was before Explained, the Contractive Prevails and Inveſts it; On the Contrary, if the Brain is not Sollicited by any Action from the Expanſive, it Cloſes and Contracts of it ſelf, and therefore, under the *Os Frontis*, the *Synciput*, and *Occiput*, is Continuous without any ſuch Proceſſes.

Brain open towards each other.

§. 16. Thus far we have gone, ſuppoſing the *Fœtus* to be ſtill in the Womb, whilſt the Lungs are of no Uſe, and the *Canalis Arterioſus* and *Foramen Ovale* obtain their Proper Offices: But after the Birth, Things are otherwiſe, for then the Air Ruſhing into the Lungs, gives a new Turn to the Circulation of the Blood, and it now Paſſes no Longer thro' the *Canalis Arterioſus* from the Pulmonary Artery into the *Aorta*, nor from the *Vena Cava* into the Pulmonary Vein thro' the *Foramen Ovale*; which not being kept open by the Expanſive Force of the Blood Acting againſt their Sides, their Contractive Force Prevails, by which they Cloſe, and the Canals are thereby Converted into Ligaments; On the other Hand, the Blood from the Right Ventricle of the Heart Expands it ſelf againſt the Incurring Force of the Contractive of the Air, in order to Oppoſe and Balance it, from whence it is Diſperſed into the Innumerable Branches of the Pulmonary Artery, whoſe Coats are ſupplied to it by the Method above Deſcribed; But being able to Expand it ſelf no farther by the Oppoſition, which is made to it from the Contractive Force of the Air, it Returns in Streams towards the Heart the Fountain of the Expanſive, from whence the Branches of the Pulmonary Veins are Formed; and the Right Ventricle being already Poſſeſſed, it Forces it ſelf into the Left Side, and Makes or Compleats the Left Ventricle; and being Excited and Exaſperated into an Higher Degree of the Expanſive by the Oppoſition it has met with, it Penetrates to the very Tip, and Dilates it ſelf ſo, that the Cavity of the Left Ventricle becomes Prominent into the Right, and the *Septum* is Concave to the Former, and Convex to the Latter; but here being Reſiſted by the Coats of the Heart, and Propelled by their Contraction, it is Driven into the *Aorta*, which is already made to Receive it.

How after the Birth, the Canalis Arterioſus *and* Foramen Ovale *are Changed into Ligaments, and the Branches of the Pulmonary Artery and Vein are Formed, as alſo how the Left Ventricle of the Heart is made.*

§. 17. From hence it is Plain, that if there was any Communication betwixt one Ventricle and another by a *Foramen* thro' the *Septum*, this whole Contrivance of Nature would be Deſtroyed; And Either, when the Blood in the Right Ventricle was Expanded into the Lungs, the Blood in the Left would Follow thro' ſuch *Foramen* and not Paſs into the *Aorta*, from whence the *Aorta*, and all the Arteries Derived from it, would Change into Ligaments, and the Body Wither into a Skeleton; Or Elſe it would Paſs into the *Aorta* and not into the *Foramen*, in which Caſe the *Foramen* would Change into a Ligament, and then it would have been in Vain for Nature to have made it, that is, only to ſtop it up again; Or laſtly, the Blood would Flow out of the Right Ventricle into the Left, without Paſſing thro' the Lungs, from whence it would want it's Accenſion, from the Contractive Force of the Air, into Arterial Blood, which is ſo neceſſary to it's Pulſation and the Circulation of it, and the Pulmonary Artery and Vein would be Contracted into Ligaments: From all which it is Evident, that as no *Foramen* was ever obſerved in the *Septum*, ſo that there Really is none; Namely, both from the Diſtinct Formation of the Ventricles, and from the neceſſary Conſequences, which would Enſue in the Human Syſtem, which are not Experienced in it.

That there is no Communication betwixt the Left Ventricle and the Right by a Foramen *thro' the* Septum, *and the Reaſon of it.*

§. 18. This Account alſo Furniſhes us with a Reaſon, why all the Blood Flows to the Right Ventricle, and is Diſcharged and Diſperſed from the Left; The Firſt, becauſe, before the Left Ventricle was Formed, or Compleated, the Blood Aſſembled to the Heart by that Law of Nature, which makes Similar Forces to Cement and Unite together; The Laſt, becauſe, as the Influx of the Blood continually Increaſes it's Quantity, there muſt be ſome way for it's Diſtribution and Efflux, which therefore is by the Pulmonary Artery and Vein into the Left Ventricle, from whence by the Contractive Force of it's Coats it is Expelled into the *Aorta*, as already Deſcribed.

Why all the Blood Flows to the Right Ventricle, and is Diſcharged and Diſperſed from the Left.

§. 19. The Auricles are of the ſame Nature with the Ventricles, and ſerve for the ſame Uſe, and are only the Firſt Efforts of the Contractive Force to Encloſe the Expanſive of the Blood, which is Finiſhed and Effected in the Ventricles; and therefore the Fibres in the Auricles, as before in the Ventricles, are Stronger in the

What the Deſign of the Auricles is, and how the Pulſation of them is Accounted for.

Left

as well as that of the Ventricles, and the **Aorta.**

Left than in the Right, that is, the Contractive Force is ſo, of which they Conſiſt; becauſe the Expanſive of the Arterial Blood is Greater and Stronger than the Venal, to Balance which a ſtronger Contractive is Required.

The Pulſation of the Auricles Proceeds likewiſe from the ſame Cauſe with that of the Ventricles, Namely, from the Expanſive Force of the Blood, and chiefly from it's Accenſion in the Lungs by the Contractive Force of the Air, and from the Contractive Force of their Coats; So that it is very Probable, that the Principal Pulſation Ariſes from the Left Auricles and Ventricles of the Heart, when the Blood has been Inflamed, and the Expanſive Force of it Excited in the Lungs; However it is Plain, that the Syſtole of the Ventricles muſt be at the ſame Time with the Diaſtole of the Auricles, and the Syſtole of the Auricles with the Diaſtole of the Ventricles; And the ſame Law ſeems to be Obſerved thro' the whole Arteries, that the Syſtole of the Superior Parts ſhould be at the ſame Time with the Diaſtole of the Inferior, and the Syſtole of the Inferior, with the Diaſtole of the Superior, or *Vice Verſâ*; Since This Appears to be Requiſite in a Courſe of the Expanſive, as of the Blood, and of the Contractive, as of the Coats, which Inveſt it; For ſuppoſing the Blood to be Entering the Auricle, by it's Expanſive Force concurring with the Sanguineous Fibres, It immediately Dilates and Diſtends it, till the Contractive Force of the Nervous Fibres and Coats being Excited, It Repreſſes on the Blood and Fleſhy Fibres, and Conſtringes the Auricle, by which the Blood is Driven into the Ventricle, which is Dilated and Diſtended by it, whilſt the Auricle ſubſides and is Contracted; On the other Hand, the Contractive Force of the Ventricle is now Exerted, and whilſt the Blood is Expelled from it by it's Conſtriction, New Blood Flows into the Auricle, which again by it's Expanſive Force Dilates and Diſtends it.

In like Manner, in any Two Contiguous Parts of the *Aorta*, or it's Branches, in the Deſcending Trunk, when the Blood Enters the Superior Part, a Dilatation is Cauſed by it's Expanſive Force, and that of the Sanguineous or Fleſhy Fibres, which is ſucceeded by a Conſtriction from the Contractive of the Coats and the Nervous, and which Propells the Blood to the Inferior Part, which now Swells and is Dilated, whilſt the Superior Sinks and is Contracted, and whilſt the Inferior is Contracted after it's Dilatation, the Superior is Dilated by a Freſh Influx of Blood, and a Renewal of the Expanſive; And what is ſaid here of a Superior and Inferior Part of the Deſcending Trunk of the *Aorta*, obtains of an Inferior and Superior in the Aſcending: And by theſe Methods a Progreſſive Pulſation appears to be Propagated thro' the whole Sanguineous Syſtem, not much Unlike to, or Differing from, the Vermicular or Periſtaltick Motion of the Inteſtines; So that Nature in the Animal Frame, as well as in the Mundane, Acts Agreeably and Conſiſtently with it ſelf, ſince it's Motions are Uniform, and are Derived from the ſame Principles; and which is a Forcible, if not an Invincible, Argument for the Truth of Thoſe, which we have Alledged.

That the Pulſe of the Arteries in the ſeveral Parts of them is Diſtinct and Independent of the other Parts, and of the Heart, and the Reaſon.

§. 20. From hence it is Manifeſt, that the Pulſe of the Arteries does not Depend upon, nor is Communicated to Them from the Pulſation of the Heart, but that every Pulſe is Diſtinct and Independent, and Ariſes from the Influx of Blood into the ſeveral Parts, and the Reciprocal Contraction and Dilatation of thoſe Parts from the Expanſive of the Blood and of the Sanguineous or Fleſhy Fibres, and the Contractive of the Nervous; And therefore it is, that in Experience we find the Pulſe of the Wriſt keeps no Time with that of the Temples, or of the Upper Part of the Noſe, nor Theſe with the Grand Pulſe of the Heart, which is Eaſy for any one himſelf to Obſerve, and is an Indication of Their being Separate and Independent Motions, and ſuch, as are Agreeable to the Account, we have given of Them.

How the Heart comes to Beat after it's Separation from the Animal Syſtem.

§. 21. This likewiſe Informs us, how the Heart comes to Beat, and to Retain a Pulſation even after it's Separation from the other Parts of the Body; becauſe, tho' the Blood no longer Flows into it, yet the Fleſhy or Sanguineous Fibres Preſerving their Expanſive Force, ſo long as the Heart continues Warm, Act againſt the Contractive of the Nervous; From whence there will be a Reciprocation of Motion, till the Expanſive is quite ſubdued, and the Heart becomes Cold and Contractive: And this is farther Confirmed from the Pulſe being Revived, after the Heart has Ceaſed Beating, by an Affuſion of Warm Blood, or even Warm Water upon it; For thereby the Expanſive of the Sanguineous, or Fleſhy, Fibres is again Excited, and Enabled to Prolong it's Conteſt with the Contractive, till it again Languiſhes and Expires, and the Contractive finally Triumphs and Prevails.

How the Circulation of the Blood is Performed.

§. 22. So that from this Doctrine the Circulation of the Blood, it's Colour and Nutrition, and the Motion of the Heart, are Neceſſary, tho' they are Difficulties, which have hitherto Confounded and Puzzled the Greateſt and moſt Celebrated Anatomiſts; For the Blood Aſſembles to the Heart by that Law of Nature, by which Similar Forces are Collected to each other, and it is Expelled from thence by the Contractive Force of it's Coats and Nervous Fibres, which cannot fail of Produceing the Circulation Mentioned; More particularly, according to the foregoing Account, the Heart being Formed, the Blood is Pulſed out of the Right Ventricle into the Left, and ſo into the *Aorta*, for it is Impoſſible for it to Proceed thro' the *Mucro* or Tip; The Cauſes, Springs, and Directions of which Motions have been already Explained; From the *Aorta* it is carry'd thro' the Branches of it to all Sides, by the Contractive Force of the Coats and Nervous Fibres, which Protrude and Propell it from the Heart towards the Extreme Parts, where it is Made or Congealed into Fleſh by the Contractive Force of the ſaid Nervous Fibres, and becomes the Nouriſhment of the Carnous Syſtem; But, the Blood ſtill Flowing into thoſe Parts, by it's Expanſive Force being now Superior to the Contractive, it Aſcends, and Tends towards the Heart by the afore-mentioned Law, from whence it was Driven, but, being ſtill Followed and Balanced by the Contractive, it is Inveſted with Nervous Coats, from whence the Veins are Formed, and the Circulation Perfected.

How the Parts of the Body are Continually Renewed.

§. 23. ALSO, we may from hence give an Account, how the Parts of the Body are continually Renewed; for the Blood being Condenſed into Fleſh by the Contractive Force of the Nervous Fibres, as it Rots and Putrifies it Grows Expanſive, and is therefore carry'd off by the Expanſive Force of the Superfluous Blood in the ſeveral Parts, and is ſupplied by the Condenſation of New Blood into Fleſh, from whence Ariſes the Perpetual Renovation of Them; But when the Expanſive Force of the Blood grows ſo Weak and Languid, that it cannot Elevate and Raiſe the Corrupted Parts, an Univerſal *Tabes* Enſues, and a Diſſolution of the whole Animal Oeconomy.

The Colour of the Blood, and the Motion of the Heart Explained.

§. 24. As to the Colour of the Blood we have ſhewn, that it Ariſes from an Increaſe of the Expanſive in the *Ovum* by the Oppoſition of the Contractive of the Seminal Principle, and that the Chyle is Raiſed into that Colour by Mixing with the Blood, as Fuel is Converted into Fire; And as to the Motion of the Heart, we have Explained it from the Expanſive of the Blood, and the Carnous Fibres on the one Hand, and the Contractive of the Animal or Brainy Subſtance, and the Nervous Fibres on the other, Acting againſt each other, ſo as neither of the Forces can Prevail, from whence Proceeds a Reciprocation of Them.

The Opinions of the Anatomiſts Vulgarly Received, concerning the Heart and the Blood, Confuted from what foregoes.

§. 25. THERE ſeems nothing therefore Remaining in Relation to the Heart, Excepting to Examine the Famous *Alphonſus Borellus*'s Opinions Concerning it, ſince what we have ſaid will ſufficiently Confute the Fancies of Anatomiſts, which have been vulgarly Received about it; As, That it's Motion Proceeds from an Influx of Animal Spirits conveyed by the Nerves, altho' the Heart Moves in the *Embryo*, before either Brain or Nerves are Formed, and in Living *Fœtus*'s when cut out from the Body and Entirely Separated from all the Parts of it; Or elſe from the Rarefaction and Accenſion of the Blood, tho', according to Dr. *Lower*'s Experiment and his Friend's, the Heart continues it's Pulſation, when no Blood Flows into it, and only Beer and Wine, or Broth, are Derived into it's Ventricles;

THAT Chyle is Changed into Blood by the Vital Spirit, and Active Principle of the Latter Looſening, by Fermentation, the Salt, Sulphur, and Spirit of the Former, and Aſſimilateing and Aſſociateing Parts to it, which are of a like and ſuitable Nature;

THAT Blood becomes Hot by a Vital Flame, which is Lodged in the Heart, and which Heats the Blood, as it Paſſes thro' it, tho' it is Clear, the Heart is nothing but a mere Muſcle; or by a Saline Subſtance Mixing with a Sulphureous, or an Acid with an *Alkali*, tho' no ſuch Subſtances are Obſerved in the Blood; or by an Accenſion of it, which is Unexplained;

And laſtly, That the Colour of the Blood is owing to the Heart, or to the Mixture of Salt and Subacid Juices with Sulphureous, as in the Diſtillation of Sal Nitre, or in Pouring Oil of Vitriol on Conſerve of Roſes, by which a Pale Red is Changed into a Deep one: Theſe we ſhall omit particularly to Conſider, becauſe they are Refuted by what foregoes, or, ſo far as they ſtand Good, are Explicable by the Precedeing Doctrine; The Vital Spirit and Sulphur, being Different Expreſſions of our Expanſive Force, and the Acid and Saline Subſtances, of the Contractive.

Borellus*'s Account of the Heart and it's Action.*
De Motu Animalium, Cap. 5. Par. 2. Prop. 37.

§. 26. As to *Borellus* in order to Explain the Mechanical Motion of the Heart, he Firſt gives the Structure of it from *Malpighius*, *Lower*, and *Bellinus*, and makes it not to be a *Parenchyma*, but a Muſcle, conſiſting of ſtrong Carnous Fibres, of the ſame Priſmatick Figure, of the ſame Colour, Conſiſtence, and Tenacity, with the Fibres of other Muſcles, which cannot be Drawn out into a Greater Length, as they are Renitent to a Diſtraction, but are Contracted of their own Accord, have a Tenſion, are Inflated, and acquire an Hardneſs, when they Act in the Heart's Pulſation, and in Boiling become Groſs and Turgid, and are eaſily Torn in Pieces and Broken; But this Fibrous Conſtitution of the Heart Differs from that of other Muſcles, it's Fleſh being Firm, Hard, Uniform, and Dyed with a Red Colour, not Lax and Soft, nor are it's Priſmatick and Carnous Columns Separated from it's Membranes, and it's Innumerable Tendinous Fibres, as the Carnous Fibres of the other Muſcles are; Thus alſo it's Carnous Fibres are not in Strait and Parallel Lines, but Spiral, and Variouſly, and with a wonderful Artifice, Interwoven with each other, ſomething Agreeable to what has been before Deſcribed.

Besides which, there are Ten Millions of Millions of the Ramifications of Nerves, Derived from the *Par Vagum*, Scattered and Diſperſed thro' the Carnous Fibres of the whole Heart; After which he Proceeds to Inform us of the Two Coronary Arteries from *Bartholin*, and of the Auricles, and the Trunk of the *Vena Cava*, all which were before Mentioned.

De Motu Animalium, Cap. 5. Par. 2. Prop. 38.

In a following Propoſition he farther ſhews the Action of the Heart, as before he had given the Structure of it; That in Pulſation, it's whole Carnous Subſtance, as is Evident from Senſe, is, like that of other Muſcles, Inflated upon the Tenſion, and Hardened with the utmoſt Violence, but that the Swelling of the Fibres is not Deſigned by Nature, as in other Muſcles, to ſhorten the Diſtance betwixt the Extremities, and by that means to Draw them together, and the Joints, in which they are Inſerted; On the Contrary, in the Heart the Fibres are Shortened, that they may be Inflated and Swell, and thereby Straiten and Contract the Concave Surface of it, in order to Preſs out and Expell the Blood, which it Contains; which is Apparent from hence; That in every Pulſation or Tenſion of the Heart, the Contents of it are in Fact Driven into the Arteries, as by a Syringe, and if we Thruſt a Finger thro' a Wound made into a Live Heart, which *Bellinus* and *Lower* Obſerved, it will, at every Pulſation or Tenſion of it, be Caught and Encloſed as in a *Forceps*, and the Fleſh, upon it's Inflation and Hardening, Surrounds and Embraces it with Force, and Buries it in it's Subſtance, whilſt the External Surface of the Heart is not Increaſed or Altered by ſuch a Turgeſcence or Inflation.

De Motu Animalium, Cap. 5. Par. 2. Prop. 43, 44, 45, 46, 47, & 48.

What is afterwards Alledged to ſolve this Repletion of the Cavity of the Heart, from an Hollow *Glomus*, Compoſed of Threads Equally Thick, and of a like Conſiſtence, Wound Spirally Round a Bladder, blown up to Form an Intermediate Cavity, that if the Cavity of the *Glomus* is filled up by a Moiſtening of the Threads, the Internal Spires of the *Glomus* muſt be Wrinkled and Pleated, whilſt the External Remain in their full Tenſion, from whence the Mechanical Operation, by which the Cavity of the Moiſtened *Glomus* is filled up, is Explained, is ſo Abſurd and Ridiculous an Account of the Repletion of the Cavity of the Heart in it's Pulſation or Tenſion, that, however Mathematical it may Appear by the Figures it makes uſe of, it ſcarcely Merits an Anſwer.

The Repletion of the Cavity of the Heart not ſolved by the Madefaction of the Glomus.

§. 27. For, tho' it is certain, that if the Cavity of the *Glomus* is filled up by a Moiſtening of the Threads, thoſe Threads, be they Circular or Spiral, muſt be bent Inwards in *Plicæ* or Wrinkles in order to Penetrate into and Poſſeſs ſuch Cavity, yet it is not Evident, that the Cavity of the *Glomus* will be filled up by Moiſtening the Threads, or that they will Deſcend into ſuch Cavity by their Madefaction; On the

Contrary, Threads, which are Lax and Flaccid, by Moiſture become more Stretched and Shorter, whereas this Doctrine ſuppoſes, that Threads, which are Stretch'd, by Moiſture are Rendered more Lax, Flaccid, and Longer.

BESIDES, This Doctrine Informs us, that the Cruſt of the *Glomus* and the External Threads, of which it is Compoſed, do not Change their Figure by Madefaction, and there is no Reaſon Aſſigned, why the next Interior Coat of Threads ſhould do it; 'tis True, we are Told, that there is an Inflation of the Internal Threads, which Augments their Superficies, and ſince the External Coat is Immoveable, ſuch Augmentation of the Superficies muſt be ſpent Inwards towards the Center of the *Glomus*, by which it will become Wrinkled; but an Inflation of the outward Coat is likewiſe Acknowledged, and yet no Augmentation of the Superficies Owned; If therefore the Superficies of the outward Coat is not Augmented by Madefaction, why ſhould the Inner? If it is, why ſhould the Inner Tend in Wrinkles to the Center of the *Glomus?* Moreover, This is contrary to the Experiment in the Heart, in which the outward Surface is not Augmented, to Explain which the *Glomus* is Introduced.

FARTHER, Suppoſing the Madefaction of the Threads of the Interior Coats was capable of Protruding them in Wrinkles towards the Center or Cavity of the *Glomus*; Here would be a Languid Motion of the Internal Parts of the *Glomus* towards it's Cavity or Center, but how will this ſolve that Force and Vehemence, with which the Heart Acts in Pulſation? Here would be a Soft, Looſe, and Pulpy, Subſtance from the Relaxation of theſe Spirals into the Cavity of the *Glomus*, but where ſhall we find from thence the Hardneſs and Induration Acquired upon the Muſcular Tenſion? On the Contrary, the Principles, we have Advanced of the Expanſive and Contractive Forces, furniſh us with a Complete Solution of the Phænomenons Propoſed.

Borellus*'s Phænomena of the Heart ſolved by the Doctrine of the Expanſive and Contractive Forces.*

§. 28. THE Carnous Fibres cannot be drawn out into a Greater Length, but are Renitent to a Diſtraction, whilſt they are Contracted of their own Accord; Becauſe they conſiſt of the Expanſive Force of the Blood ſubdued by the Contractive Force of the Nervous Fluid, by which they are Rendered Tenacious and Solid, the Contractive Force therefore Prevailing, the Fibres are Diſtracted Hardly, but Contracted Eaſily: They are of a Priſmatick Figure; Becauſe the Expanſive Force, Exerting it ſelf in their Formation, and being Oppoſed by the Contractive, Shoots into Angles, as is done in all other Chryſtallizations, and the Number of the Angles are Different, according to the Different Proportion of Force betwixt the Expanſive and Contractive, as we have before Explained: They become Turgid in Boiling, and are eaſily Torn in Pieces and Broken; Becauſe, by the Application of the Expanſive Force of Heat and Fire, the Contractive is Diminiſhed, by which their Parts in a Turgeſcence are Separated farther from each other, and in Tearing and Breaking of them are entirely Diſunited: They have a Tenſion, are Inflated, and Acquire an Hardneſs, in the Pulſation of the Heart; Becauſe, when the Blood Flows into the Ventricles of it, the Expanſive Force is exceedingly Increaſed, and Raiſes and Excites the Remaining Expanſive in the Fibres, which therefore Swell and are Dilated; This likewiſe cauſes the Contractive Force to Exert it ſelf with Greater Vigour in the Fibres, in Oppoſition to the Expanſive, to which the Nerves from the *Par Vagum* Joining their Contractive, an Hardneſs is Produced: And from this vaſt Contention of Forces the Cavities of the Heart are Filled, and, the Contractive Prevailing, the Expanſive Force of the Blood is Expelled into the Arteries, and the Heart is Cloſed, till it is again opened by a new Influx of the Expanſive Force of Blood, which is in like Manner Driven out, as before.

See Chap. 3. §. 4. of this Book.

THE outward Surface of the Heart in the mean Time is little or nothing concerned in this Action; Becauſe, as we have already ſhewn, the Contractive Force is there ſo fully Eſtabliſhed, that it is not Affected with the Expanſive of the Influent Blood.

See §. 9. of this Chapter.

THE Fleſh and Carnous Fibres of the Heart are Firmer and Stronger than thoſe of other Muſcles; Becauſe the Conteſt of the Two Forces is here Greater, and the Contractive Prevailing muſt be likewiſe here the Greateſt in Reſpect of any other Carnous Fibres.

AND for the ſame Reaſon, the Carnous Fibres in the Heart are more ſtrictly Joined and Cemented with it's Membranes and Tendinous Fibres, which are of a Nervous and Contractive Nature, than in other Muſcles; Becauſe the Expanſive Force of the Blood being Greater, the Contractive Forces are more united together in order to Oppoſe and Balance it.

FROM whence Appears alſo the Neceſſity of ſuch an Infinite Number of Ramifications of Nerves from the *Par Vagum*, which have a Contractive Force, to ſupport the *Æquilibrium* againſt the Expanſive of the Incurring Blood.

See §. 9. of this Chapter.

AND why theſe Fibres of the Heart are Spiral, we have before Explained, which will give a Solution, why thoſe of other Muſcles are Direct and Parallel.

THIS Doctrine therefore Declares to us plainly, how the Cavities of the Heart are filled by a Tenſion and Inflation of the Fibres of it, how they become Hard, Conſtipated and Cloſe, and how this Action is Performed with the utmoſt Vehemence and Energy, which is not done by the *Rugæ* or *Plicæ* of the Threads of the *Glomus*, and the Madefaction of Them.

De Motu Animalium, Cap. 5. Par. 2. Prop. 53. 54.

FROM hence alſo an eaſy Account may be given, why the *Mucro* of the Heart in it's Syſtole ſtrikes againſt the Left Part of the Breaſt; Becauſe after the Blood has Paſſed thro' the *Vena Pulmonaris*, it becomes more Accended, and by Conſequence more Expanſive, the Contractive Force will therefore be Greater on that Side, and the Conteſt betwixt both the Forces Irritated and Increas'd; from whence, as we have ſhewn, the Pulſation, Tenſion, or Erection, of the Heart Ariſes, and therefore the Action of the Heart will Preponderate to the Left; And as to it's Auricles, we have already Conſidered, and Given an Explanation of, the Syſtole and Diaſtole of Them.

Borellus's *Explanation of the Force and Power of the Motive Faculty of the Heart, and of the Efficient Cauſes of it's Motion.* De Motu Animalium, Cap. 5. Par. 2. Prop. 56. to 76. & Cap. 6. to Cap. 7.

§. 29. WHAT Remains of *Borellus* upon the Heart is to Inquire into the Force and Power of it's Motive Faculty, and into the Immediate Efficient Cauſes of it's Motion.

As to the Firſt, he tells us after a Series of many Mathematical Propoſitions. *1ſt.* That the Motive Force of the Fibres of the Heart, conſidered by it ſelf, is able to ſuſtain a Greater Weight than of 3000 Pounds. *2d.* That all the Arteries of an Animal, after Pulſation, could not Subſiſt, if they were wholly Exhauſted of their Blood, and therefore always Retain ſome in Them after ſuch Pulſation. *3d.* That the Motion of the Blood in the Arteries has Thrice the Velocity of that, with which the Heart is Moved and Impells the ſame Blood. *4th.* That the Power of the Heart, which Conſtringes it, is Leſs than the Reſiſtence, which the Blood Contained in it's Ventricles Exerts, in a Subſeſquialterate Proportion. *5th.* That the Force, which the Blood Contained in the Ventricles ſuffers from the Compreſſion of the Heart, is Equal to the Force, with which the ſame Blood, like the *Embolus* of a Syringe, Impells the Blood into the Arteries, to whoſe Reſiſtence it has the ſame Proportion, as 1 to 40 nearly. *6th.* That the Abſolute Motive Force, by which the Heart is Conſtringed, is to the whole Force, with which the Blood in the Arteries is Reluctant to it's Expulſion, as 1 to 60. *7th.* That the Motive Force of the Heart Exceeds in it's Moment the Reſiſtence of the whole Blood of the Arteries, and the *Faſciæ*, which Hinder their Dilatation, which Reſiſtence is Greater than the Force, which Reſults from a Weight of 180000 Pounds. *8th.* That in every Pulſation of the Heart, either a whole Maſs of 3 Ounces of Blood, or ſome Portion of it, is Expelled from the Heart, not into, but on the Outſide of, the Arteries by a Motive Force Diſtinct and Different from the *Embolus* of the Heart, Conſidered as a Syringe. *9th.* That the Cauſe, which Expells the Blood on the Outſide of the Arteries thro' the Orifices of them, is the Compreſſion and Reſtriction of the Arteries by the Mechanical Force of the Human Syſtem, by which the Fibres of the Veſſels and Bowels are Contracted. *10th.* That the Power of the Heart in one Pulſation only ſurmounts a Reſiſtence, which is Greater than a Weight of 135000 Pounds, and ſince there are at Leaſt 1000 Pulſes in an Hour, it will in that Time have ſurmounted a Reſiſtence, which is Greater than a Weight of 135000000 Pounds, and in a Day, a Reſiſtence Greater than a Weight of 3140000000 Pounds, an Amazeing and Incredible Force, if it were not ſo, that the Force of Percuſſion in it's own Nature is ſo Vaſt, that it is Able to Surmount any Finite Reſiſtence.

As to the Second, or the Efficient Cauſes of the Heart's Motion, he Aſſerts contrary to the Ancients. 1*ſt*. That the Immediate Cauſe of the Motion of the Heart, which is a Muſcle, does not Differ from that which makes the Muſcles of the Joints to Move. 2*d*. That the Firſt and Mediate Cauſe of the Motion of the Heart does Differ from that, which Actuates the other Muſcles. 3*d*. That the Motion of the Heart is Performed by certain Stated and Neceſſary Laws of Mechaniſm, as well as that of other Automatons. 4*th*. That it is Poſſible, the Motion of the Heart may be Effected by the ſame Animal and Cognoſcitive Faculty, by which that of the Muſcles of the Joints is Produced, tho' we do not Obſerve it, by Reaſon of it's being ſo Habitual to us.

THESE are the Chief Propoſitions of *Borellus* in Relation to the Heart, beſides thoſe, which we have already Conſidered, and which we ſhall now Examine.

Borellus's *Proof of the Motive Force of the Fibres of the Heart Inſufficient and Weak upon ſeveral Accounts.*

§. 30. AND FIRST, as to the Motive Force of the Fibres of the Heart being able to ſuſtain a Greater Weight than of 3000 Pounds, it is Proved upon a Suppoſition, that all the leaſt Fibres of Different Muſcles in the ſame Animal are Equally Strong, and Exert an Equal Motive Force in the ſame Time; and that therefore, ſince the Carnous Fibres of the Heart are nearly Equal in Bulk to one of the Temporal Muſcles, and one of the Maſſeters, and their Motive Force is known, that of the Heart may be likewiſe Determined: But it is a large Conceſſion to allow, that the leaſt Fibres of Different Muſcles in the ſame Animal have an Equal Force; and we might as well Preſume, that the Blood obtains as Great an Expanſive Force, and the Nervous Fluid a Contractive in the Extreme Parts, as in the Heart and the Brain, as that the Fibres, which are Compounded and Formed of thoſe Forces, are every where the ſame; The Contrary rather ſeems to be Evident, that the Fibres Vary and Differ in their Forces, according to the ſeveral Degrees of the Expanſive Force of the Blood, and of the Contractive of the Nervous Fluid, of which they are Conſtituted and Made.

IN order likewiſe to Prove this Motive Force of the Fibres of the Heart, the Cortex of it is Imagined to Repreſent a Gun, it's Internal Fibres, upon their Turgeſcence and Inflation, ſerve for the Powder upon it's being Fired, and the Blood, Expelled from it, is the Bullet Diſcharged; From whence it is Reaſoned, that as every Grain of Powder in it's Exploſion Impells a Correſpondent Particle of the Bullet, and all the Grains together Impell ſo many Particles of the Bullet, as there are Grains, ſo in like Manner each of the leaſt Fibres of the Heart Acting together Impell Equal Particles of the Sanguineous Bullet; and it's whole Reſiſtence is Compounded of ſo many Particles, every one of which is Equal to the Power of one of the leaſt Fibres, as there are leaſt Fibres in the whole Heart: But is it not ſomething Extraordinary, that here the Heart is made to Reſemble a Gun, and it's Fibres the Exploſion of Gun-Powder, when before it was a *Glomus*, and it's Fibres Expreſſed by the Madefaction of Threads, And after a Syringe, and it's Fibres the *Embolus?* Is not this to Reconcile Fire and Water? And to Play the *Proteus* inſtead of the Philoſopher? Beſides, the Exploſion of Gun-Powder, which is ſo neceſſary to this Doctrine, is not Truly and Juſtly Explained, whilſt the Particles of it, and the Number of them, are only Conſidered, as if all Matter were Similar, and not their Real Force, which, in Diſſimilar, may be much Greater in a Particle of One Kind, than a Bare and Common Impulſe in a Thouſand of Another.

What the Truth is in Reſpect of the Motive Force of the Heart.

§. 31. WHAT therefore we ſhall Conclude with upon this Propoſition is only, that tho' we may give an Account of the Motion, or Motive Force, of the Heart, yet it is not likely, that we can Aſcertain the preciſe Degree of it's Force, excepting in General, that it is much Leſs than what is here Aſſigned, and that it is Little more than what Exceeds an *Æquilibrium* of Two oppoſite Forces; That the Heart does not Surmount the whole Maſs of Blood by it's Pulſations and Ejecting it out of it's Cavities, but that in Reſpect of it ſelf the Blood is in an Equal Balance of it's Parts nearly, it's Expanſive Force being throughout the Human Compages not Extremely Different, and the Exceſs of the Contractive Force above it in the Nervous Syſtem ſeems to be the Cauſe of it's Expulſion from the Heart, it's Promotion thro' the Arteries and Veins, and conſequently of it's Circulation, as we have before Evinced.

Second, That therefore the Arteries are never Empty, but always Retain ſome Blood, and are even Generally almoſt Full, is Aſſented to and Granted; For otherwiſe, neither the *Æquilibrium* of the Expanſive in the Blood could be Maintained, nor the Exceſs of the Contractive in the Nerves Exerted, nor conſequently the Pulſation of the Heart and Arteries Continued, which Ariſes from the Conteſt of thoſe Two Forces; This therefore we readily Admit, and it is farther juſtified by Fact and Experience.

Third, That the Motion of the Blood in the Arteries has thrice the Velocity of That, with which the Heart is Moved and Impells the ſame Blood, is Proved upon this Suppoſition; That the Inflation of the Heart, the Reſtriction of it's Cavities, the Expulſion of the Blood from the Ventricles, the Motion of it within the Arteries, and the Protruſion of the *Cruor*, or of the Blood which was Pre-exiſtent in Them, are all Performed in the ſame Time; Whereas there muſt be an Inflation of the Heart before the Cavities are Filled, and the Cavities muſt be filled before the Blood is Expelled; and the Motion of the Blood within the Arteries is as much owing to the peculiar Contractive Force of Them, as to the Expulſion from the Heart; and the Promotion of the *Cruor* belongs alſo to the ſame Cauſe, according to what has been before Explained.

Fourth, That the Power of the Heart, which Conſtringes it, ſhould be Leſs than the Reſiſtence, which the Blood, Contained in it's Ventricles, Exerts, in a Subſeſquialterate, *viz.* as 2 to 3, or any other Submultiple Proportion, is Inconceivable; Since, if the Reſiſtence of the Blood within the Heart was Greater than the Compreſſive or Contractive Force of it, which way would it be Poſſible for the Blood to be Expelled out of it? On the Contrary, we ſuppoſe the Expanſive Force of the Blood to be Inferior to the Contractive of the Muſcular Fibres; Beſides, the Preſent Propoſition Proceeds upon a Reaſoning, which Preſumes, that the External Fibres Act as a Cord Revolved Round a Globe or a Cylinder, and that therefore the Power, which Contracts the External Fibres of the Heart, is to the Reſiſtence of the Blood Compreſſed, as the Semidiameter of a Circle to it's Circumference, or Leſs than as 1 to 6, and that the Internal Fibres Act as a *Cuneus*, or an *Embolus*, of a Syringe, and Impell the Blood from the Periphery of the Heart to it's Center: To which there needs be no other Anſwer given, than that this Philoſophy, if it Pleaſed, might make the Heart a Milſtone, which was able to Grind all our Aliment into Blood, and there would be an End of the Matter; for how will a Cord and a Cylinder, or an *Embolus* or a Wedge, give us any Account of the Erection and Inflation of the Heart, and of it's Exceeding Life and Energy? which are not to be Repreſented by ſuch Dead and Unactive Powers, and can be only ſolved by the Expanſive and Contractive Forces we Defend.

Fifth, And for the ſame Reaſon we have no occaſion to Examine into the Fifth, Sixth, Seventh, Eighth, Ninth, and Tenth Propoſitions before Mentioned, which all Depend upon the like Precarious Fancies.

What are the Efficient Cauſes of the Heart's Motion.

§. 32. As to the Second Series of Propoſitions concerning the Efficient Cauſes of the Heart's Motion, there is no neceſſity of Inquiring into them particularly, we ſhall only ſay, that we have already Explained what they are; and perhaps there will be found to be no Great Diſagreement in this Celebrated Author from what we have Aſſerted, Excepting that, as he in General Affirms in *Prop. 3d*, That the Motion of the Heart is Performed by certain Stated and Neceſſary Laws of Mechaniſm, as well as That of other Automatons, and *Prop. 4th.* That the Motion of the Heart may be Effected by the ſame Animal and Cognoſcitive Faculty, by which that of the Muſcles of the Joints is Produced, without giving any Account how this may be done, ſo we have on the Contrary ſhewn the Way and Manner of it from the Laws of the Expanſive and Contractive Forces, tho' ſuch a Mechaniſm is Impoſſible to be ſolved by the Common Laws of the *Cuneus* or *Embolus*, or a Cord Revolved about a Globe or a Cylinder, which this Philoſophy Eſpouſes.

Nor can we allow what Dr. *Drake* Propoſes for a Solution of the Diaſtole of the Heart, which he Confeſſes is Unexplained by Dr. *Lower*, *Borellus*, and the beſt and moſt Accurate Anatomiſts, Namely, that it is owing to the Air Incumbent upon the Lungs, and which Acts as an Antagoniſt Muſcle to the Heart; ſince the Pulſe of the Heart does not keep Time with the Action of the Lungs, For we can Breath Faſter or Slower, whilſt the Pulſe Continues Uniform; nor can it be owing to the Moment and Pondus of the Influent Blood, ſince Fluids, which Paſs in a continued Canal, as the Blood in the Heart and Arteries, are in an *Æquilibrium*, and have not any peculiar Weight or Preponderancy at all; And as to the Syſtole, which he thinks ſufficiently Explained by thoſe Learned Men, we muſt Alledge, that however they may Agree in making Contraction the Proper and Natural State of the Heart, or any other Muſcle, they have not ſhewn, how it comes to be ſo, which we have before Accounted for from the Contractive Force of the Nervous Syſtem.

My Intention to have Proceeded to Inquire particularly into the Lungs, Brain, Eyes, Muſcles, Bones, and Marrow.

§. 33. I Intended from hence to Proceed to the Lungs, to the Brain, the Eyes, the Muſcles, Bones, and Marrow, and to Inquire into Them in the ſame particular Method, which I have hitherto Preſcribed to my Self, and into all the Remaining Phænomenons of the Animal Syſtem.

How the Lungs and Noſtrils in General are Formed, and why the Latter are betwixt the Fore-head and Mouth, and the Former Terminate in the Thorax.

In Reſpect of the Lungs, I would have ſhewn from the ſame Doctrine of the Expanſive and Contractive Forces; That the *Bronchia* of them and their Veſicles or Bladders are Formed from the Contractive of the Nervous Syſtem, and the Expanſive of the Air; That the Expanſive of the Air obtains a Middle Proportion betwixt the Contractive of the Brain, and the Expanſive of the Blood and the Aliment; That therefore it's Entrance into the Animal Frame, in the *Fœtus*, is betwixt the Fore-head, where the Contractive of the Brain Prevails, and the Mouth, which is made by the Expanſive of the ſaid Aliment; That the Noſtrils are Flatter at Firſt, and afterwards more Prominent by the above-mentioned Expanſive of the Air; and That the Air, which is of a Middle Force betwixt the Contractive of the Brain, and the Expanſive of the Blood and Aliment, Terminates it's Progreſs in the Lungs by the Superiority of thoſe Forces, which have their Canals and their Conveyances in Pipes and Tubes Diſperſed thro' the whole Syſtem.

What in General the Brain is, and it's Frame and Conſtitution.

In Reſpect of the Brain, I would have likewiſe Evinced; That it Conſiſted of a Cold and Contractive Force; That the Blood-Veſſels in it, which are of an Expanſive, are Small and Few upon the Account of the Prevalence of the Contractive; That yet the Degrees of the Contractive are Different in it, the one Part being more Conſtipated and Denſe, and the other more Watry and Fluid, or Expanſive; That from the Different Natures of theſe Forces, we are to Judge of the Conſtitution of the Brain, and it's Influences and Effects upon the Mind, or the Contrary, and not from the Texture of it's Parts, or the Diſtinctions made of Them, by the Nomenclators in Anatomy.

How the Eyes are Formed, what their Nature, and whence their Senſations are the moſt Delicate.

In Reſpect of the Eyes, I would have Proved; That they are Formed by the Watry Part of the Brain, and are the Contractive of it, Raiſed into the utmoſt Degree of the Expanſive; From whence they may be ſaid to be the Life and Spirit of it; and therefore that there is that particular Connexion betwixt the Temper and Conſtitution of the Brain and Eye, the Latter of which is ſaid to be the Index of the Former, and of the Mind and Underſtanding, which Reſides in it; Hence it is, that having ſo near an Alliance with the Brain, and conſequently with the Mind, it Communicates all the moſt Delicate Senſations to us; Whereas thoſe Perceptions, which we have from our other Senſes, whoſe Organs are leſs Nervous, and are Mixed with the Sanguineous Force, are of a Groſſer and Duller Information.

Whence their Life and Spirit and why They are Double as ſeveral other Parts of the Human Syſtem.

The Contractive of the Brain is Raiſed into this Spirit, or Expanſive, of the Eye by it's Conteſt with the Sanguineous Force (For it is owned by Anatomiſts to be ſurrounded with Muſcles, which have the Sanguineous Force in them, with which the Brain is not) and ſurmounting it, Starts out and Riſes into thoſe Fiery Balls, which give that Life and Beauty to the Animal Frame, and which Acquire, by that means, as Great a Degree of the Expanſive, as the Contractive of the Brain is capable of Receiving; Why they are Two, or the Noſtrils, or the Hands, or Feet, or why the Fingers are Five, and the Heart, Brain, and Mouth but One, Is to be Determined from the Courſe of the Nervous and Sanguineous Fluids in the Animal Syſtem, which, by what we have Recounted of Them, would not be Impoſſible to do.

What the Muſcles are, and whence the Difference of their Belly and Tendons.

In Reſpect of the Muſcles; The Belly of them Conſiſts of Nervous, and Sanguineous or Carnous, Fibres, by the Former of which they are Contracted, and Expanded by the Laſt, upon the Action of them from the Impulſe of the Mind on the Nervous Syſtem; and their Tendons are more Contractive by their Adheſion to the Bones, which are of a Contractive Nature, and to which they therefore Aſſimilate and Unite; but are kept Pliant by the Motion of the Muſcles, and their Vicinity to the Expanſive of the Sanguineous Part of Them; So that lying betwixt the Bone and the Belly of the Muſcle, they partake of a Middle Force betwixt both, being neither ſo Contractive as the one, nor ſo Expanſive as the other.

The Nature of Bones, and how they are Formed and Continued thro' the whole Syſtem.

The Bones are of a Contractive Force, and Ariſe from the Salts, or the moſt Contractive Parts, which are Separated from the Animal Compages, and are to the Marrow, and Aſſemble about it in order to Encloſe it, as the Nervous Subſtance is to the Expanſive of the Blood, which it Embraces; All the Difference is, that the Expanſive of the Blood is Quick, Agile, and Lively, whereas that of the Marrow, whoſe Subſtance will be afterwards ſhewn from it's Compoſition, is Flat, Heavy, and Languid: And therefore the Equal Balance is Preſerved in the Animal Syſtem by the Duller and Heavier Contractive in the Bones, and the Salts which Compoſe them, Acting againſt the Duller and Heavier Expanſive of the Marrow, which they Encloſe; In like Manner, as we have before Explained the *Æquilibrium*, in the ſame Syſtem, to be Maintained by the Contractive of the Quick and Nervous Fluid of the Brain, and the Expanſive of the Quick and Sanguineous Fluid of the Blood. Theſe Bones and this Marrow go thro' the whole Animal Frame upon the ſame Account, and for the ſame Reaſons, as the Nervous and Sanguineous Fluids do, which have been before Explained.

What the Marrow is.

The Marrow ſeems to be a Mixture, as Fat is, of the Contractive of the Nervous Force by the Nerves, which are ſent into it, and of the Expanſive of the Sanguineous by the Blood-Veſſels, which are Diſperſed in it; by which Mixture it is plain, that the Expanſive Force is much Diminiſhed by the Contractive of the Nerves, and that it is leſs Inflammable than Fat or Oil, by the Salts, and the Greateſt Contractive Force of the Bones, which Surrounds it.

I would have Enlarged more particularly upon theſe Subjects, but the Philoſophers and Anatomiſts being in a Different Way of Thinking, I ſhall Satisfy my ſelf with what I have Propoſed to them, and only take Notice of Two Things more, Wakeing and Sleeping, and the Paſſions of the Mind.

Of Wakeing and Sleeping. Dr. Cheyne's Eſſay on Health and Long Life.

As to Wakeing and Sleeping; Wakeing appears to be Derived from an Exceſs in the Contractive Force of the Nervous Syſtem, and the Expanſive of the Mind Acting upon it, the Latter of which Excites the Former into an Elaſticity or Springineſs, and is the Cauſe and Occaſion of all the Actions of Life, and the many Motions and the Varieties of them obſervable in it; Hence Perſons, who are Studious, and whoſe Mind and Nerves are Employed, are the moſt Wakeful; As alſo from a Defect in the Expanſive of the Sanguineous, from whence Perſons, who are the Leaſt Repleniſhed with Blood, and who want the Neceſſaries of Life, are the Leaſt Diſpoſed to Doſeing and Slumbers: On the other Hand, Sleep is owing to an Exceſs in the Expanſive Force of the Sanguineous Syſtem, when the Mind and Nerves are Paſſive; or to a Defect in the Contractive of the Nervous; Thus the Voluptuous and the Ideot are alike Inclined to Sleeping.

Why we are Taller when we Riſe, than when we go to Bed. Mr. Waſſe of Ayno, and Mr. Profeſſor Smith of Cambridge.

And from this Doctrine it will follow, what has been a late Obſervation of Philoſophers, that we are Taller by ſome Parts of an Inch, when we Riſe, than when we go to Bed, and I would Venture to ſay, tho' not Obſerved, Thicker and Plumper; Becauſe, as we have before Aſſerted, In Sleep the Expanſive Force moſt Prevails, which Dilates and Diſtends all the Dimenſions of the Body; But when we Wake, the Contractive, which Compreſſes and Conſtringes them. And thus by Sleep Men Grow Fat and Diſtended, and Lean and Contracted by Watching.

Why there is a Viciſſitude of Sleeping and Wakeing in the Animal Frame.

It muſt be alſo ſaid, That this Viciſſitude of Sleeping and Wakeing is a certain Sign and Indication of theſe Two Oppoſite Forces Exiſting in our Natures, and that there is a continual Conteſt betwixt them, by which ſometimes the one and ſometimes the other Prevails, till the Contractive at length ſubdues the Expanſive, the Animal Flame is Extinguiſhed, the Soul Flies off, and Death Enſues.

Why the Paſſions are of Two Oppoſite Kinds, and are to be wholly Suppreſſed, or Indulged in their Turns.

As to the Paſſions, They are all Reſolvible into thoſe, which are of an Expanſive and Contractive Nature, and, to Preſerve the Animal *Æquilibrium*, are to be Entirely Suppreſſed as much as Poſſible, or to be Indulged in their Turns in a Moderate Degree; If a Man were always to Grieve, or always to be Joyous, he would make his Frame Sink by the Contractive Force of the one Paſſion, or Diſſipate and Diſſolve it by the Expanſive of the other, and the Like may be ſaid of the Reſt.

CHAP.

CHAP. IX.

Concerning the Expanſive and Contractive Forces.

§. 1. WE have hitherto made all Nature to Conſiſt in Thought and Intelligence, or Action, for all that we Experience or find in it, is either Thought and Intelligence, or Matter, which we Affirm to be nothing elſe than Action; This therefore we ſhall ſtill farther Explain in this Chapter.

That Matter Conſiſts of Nothing elſe but Action.

THAT the Parts of Matter are Actuated by Various Motions is owned by the Preſent Philoſophy, but it is fancied, that the Parts of Matter themſelves are Solid, Maſſy, and Impenetrable, that they are Quieſcent, and only the Seats of Action, which may be Imparted and Communicated to them, whilſt they have an Abſolute Inertia in their own Nature; On the Contrary we cannot but be Perſuaded, that this Opinion of Matter is a Creature and an Abſtraction of the Mind, whereas in Truth, and in Fact, It appears to be nothing but Action it ſelf: Thus in Fire, for Inſtance, or in the Sun, our Perceptions from Either are thoſe of Light, Heat, and Colour, which are Evident Proofs of the Action of Both; and if we go farther than our Perceptions, by which we can only Judge of the Nature of Things, and will Aſſert, that this Action Proceeds from Solid Parts, which are Agitated, we mix Preſumptions and Imaginations of our own with our real Senſations and our undoubted Experiences; For from them, All that we can juſtly Conclude concerning thoſe Subſtances, is the Action of Them.

An Inſtance in Fire, and the Sun, or the moſt Fluid Bodies.

LET us now take a Solid, Heavy, and Reſiſting Body, as Gold, we Perceive it oppoſes our Hand Endeavouring to Penetrate into it; It Gravitates upon it, when Suſtained by it; and the Eye is ſenſible of it's Brightneſs and Yellow, which are likewiſe Inſtances and plain Indications of it's Action; and what we Experience from a whole Globe of it, we are convinced belongs to every the Minuteſt Particle, which can poſſibly fall under the Cognizance of our Senſes; If yet we are Reſolved to make this Subſtance conſiſt of ſmall Solid and unactive Particles, we depart from Nature to our own Fancies, and from Experience to Imagination, and Introduce into Matter and Bodies, what we never Actually found in Them; but if we will Adhere to Fact, and Regulate our Notions by our Informations from Material Beings, we ſhall then Confeſs, that they furniſh us with no other, than what manifeſtly Evince nothing more than the Action of Them.

Another Inſtance in Gold, or the moſt Solid and Compact ones.

IF any one ſhall ſay, that mere Action is Inconceivable without ſome Solid Subſtance to ſupport ſuch Action, and in which it ſhould Inhere; That only Proceeds from our being Uſed to conſider Things in the Corpuſcular Way, which has continually Inculcated to us it's Atoms or Particles, and the Moving of Them, and to our Obſervations of Groſſer Bodies Moving in Nature, which Bodies we have Conceived to be Solid and Unactive in Themſelves, without Conſidering, that even They Conſiſt of nothing elſe but Action.

Why we are Apt to Imagine, that Action in Matter muſt have ſome Solid, Unactive Subſtance to ſupport it.

§. 2. THIS Action we have alſo ſaid is Diſtinguiſh'd into the Expanſive and Contractive Forces, which, and the Different Combinations of them, are the occaſion of thoſe Diverſities of Matter, which we Feel and See to Exiſt in Like and Equal Portions of Space; Thus an Infiniteſima of Fire, which has no Pores or Vacuities in it, ſhall be Different from an Infiniteſima of Gold or Lead, and an Infiniteſima of Gold or Lead ſhall be Different from that of Water, or Wood, or Stone, which has likewiſe no Pores or Vacuities in it; And any Equal Globes Compoſed of ſuch Infiniteſima's ſhall be alſo Different from the Various Mixtures of theſe Forces.

The Various Kinds of Matter Ariſe from Different Degrees of the Expanſive and Contractive Forces.

§. 3. WHAT we mean by an Expanſive Force is an Action, which is Similar and Homogeneous in any Body, either in the Whole or in it's Parts, and which Exerts it ſelf Equably from a Center every Way; For Example, let us take a Globe of Fire, or that Immenſe one of the Sun, we ſuppoſe the Exertion of ſuch a Body to be outwards, not by an Actual Motion, but by a Force, which Endeavours to Dilate and Expand it ſelf every way from the Space, in which it Reſides, and that every Point in the ſame Globe has a like Property and Affection of Exerting it ſelf from thence in

The Expanſive Force Explained.

the ſame Manner, ſo that the Action of the Whole, and of every Part, is Entirely Similar and Homogeneous.

THAT the Sun, or Fire, has this Expanſive Force of Exerting it ſelf from a Center, or ſeveral ones, that is outwards, is Evident from our Feeling, and it's Burning Quality, by which it Diſſolves the Nerves, the Inſtruments of our Senſations, Cauſes Bliſters, Diſſipates Clouds and Vapours, Raiſes and Elevates Water into the Air, and Rends and Tears in Pieces Wood, and other Inflammable Subſtances, and even Metals by the Fuſion of them; and that every Particle of Fire has, in Proportion to it's Quantity, the ſame Force, we know by the leaſt Spark of it Burning and Diſſipateing the ſame Way.

The Contractive Force Explained.

§. 4. ON the other Hand, what we mean by a Contractive Force is an Action, which is Similar and Homogeneous in any Body, either in the Whole, or in it's Parts, and which Contracts it ſelf Equably to a Center, or Centers, from all Sides; For Example, the moſt Solid and Compact, and the Coldeſt Body we can mention or ſuppoſe, as the Moon, or the moſt Metallick Ice we are capable of Imagining, or this Earth we Inhabit.

THAT the Moon has this Contractive Force is plain from the Tides it occaſions, by which the Waters Tend towards it; That the Earth likewiſe has, is as manifeſt from Heavy Bodies Gravitating to it; and That Solid and Compact Subſtances are Endued with the ſame Power, is Notorious from the Difficulty we find in Separating or Dividing of them; That they feel Cold to us, Proceeds from the ſame Cauſe; For as Heat Diſſolves, Cold Contracts our Nerves, the Inſtruments of our Senſations; As Heat Diſſipates and Diſunites Material Beings, Cold Hardens and Cements them; Which are all Inſtances of a Force Acting Inwards, and towards a Center, or Centers; And as every Particle of theſe Bodies has, in Proportion to the Quantity of it, the ſame Influence and Effect, it is Reaſonable to Conclude, it has the ſame kind of Force with the Whole, that is, ſuch a Contractive one, as we have been Deſcribing.

How the Expanſive and Contractive Forces may be Communicated to Bodies, which ſurround them.

§. 5. How the Expanſive Force Exerts it ſelf Outwards, and makes it's Impreſſions upon all Bodies, which come within the Sphere of it's Activity, is Conceivable enough, that is, how a Fire, or the Sun, ſhould by it's Exertion thro' the Plenum communicate it's Force to it, and the Beings which Exiſt in it, and by that means either Heat or Burn them; But how the Contractive Force ſhould obtain any Effect External to it, whilſt it's Action is ſuppoſed to be Inwards, and towards a Center, or Centers, may be difficult to Apprehend; Thus, how the Earth, which has a Contractive Force, ſhould cauſe Bodies to Deſcend to it, and the Moon, the Waters to be Raiſed and Elevated from it; But this will Admit of an Eaſy Solution, if we Conſider, that a Contractive Force has the Nature of an Expanſive, only Acting from a Different Center, and in a contrary Direction; What therefore any Expanſive Force can Produce or Effect, in a Plenum of Matter, by a Progreſſion into it or a Repulſion of it, a like Influence will a Contractive Force have by a Regreſſion from it, or an Attraction of it.

The Reaſon of the Preſent Conſtitution of the Univerſe, and why the Sun and Stars, and Planets are Placed at that Immenſe Diſtance from each other.

HENCE it is, that there is ſuch an Immenſe Diſtance betwixt the Sun and the Planets; becauſe the Sun by it's Expanſive Force Reduces all Material Beings to it's own Action, or elſe Repells them from it, the Intermediate Æther being ſubject to it's Impreſſions of Light and Heat; Whilſt the Planets on the Contrary, Preſerving their Contractive Force, Recede from the ſame Medium Inwards, into which the Sun Expands it ſelf, and Attract and Draw the Plenum, and all other Bodies, towards them.

The Difference betwixt Air and Æther, and why the Former is Elaſtick.

How the Forces of Nature, the Expanſive and Contractive, are Analogous to the Poſitive

ANOTHER Reaſon of this Conſtitution of the Univerſe is, that Forces of the ſame Nature Tend to each other, from whence, all, which is of an Expanſive Force, as Fire, Unites with the Sun and Stars, and what is of a Contractive one, with the Earth and the Planets of each Syſtem; By which likewiſe that Immenſe Intervall of Æther, which is of the ſubtileſt Matter, is cauſed, having neither the Expanſive of the Sun, nor the Contractive of the Planets, but being Obedient to either, as their ſeveral and particular Forces Prevail; Air on the other Hand has an Elaſtick Force, from the Expanſive of the Sun Mixing with the Contractive of the Earth, by which upon Compreſſion it is Dilated, and Vibrates upon Percuſſion; And from a Conteſt betwixt the Expanſive of the Sun and the Contractive of the Planets, as we have before ſaid, the Revolutions of the Latter about the Former are Produced and Made. And this Account of theſe Two Forces is Juſtified by all the Quantities, which we know, or have been hitherto Propoſed in Nature or in the Mathematicks, which are either

Negative or Poſitive, that is, Regreſſive or Progreſſive, Expreſſed by $-$ or $+$, and which have Equally their Abſolute Effects in the Ratio, or Reaſon, of Things, which is Evident in all Algebraick Demonſtrations, as we have ſhewn they have in Nature.

and Privative Quantities of Mathematicks and Algebra.

§. 6. In Reſpect of theſe Expanſive and Contractive Forces, it may be farther Inquired, how they are Communicated, and which way, and by what means they Prevail; To which, it is Anſwered, that the Expanſive Force, for Inſtance, of Fire Prevails by Turning and Converting every Thing into it's own Expanſive Force or Action; Thus a Spark is Kindled into a Flame, and one Portion of Fire or Flame Unites to Another.

The Mechanical way by which the Expanſive and Contractive Forces are Propagated and Increaſed.

THE Firſt may be eaſily Conceived; For a Spark Exerting it ſelf every way from a Center, if it meets with a Contractive Force, as of Stone, which it cannot Subdue, it is Extinguiſh'd and Dies, but if with a Light and Expanſive Subſtance, upon which it can make it's Impreſſions, as Tinder, or Linnen Burnt, it Communicates it's Force, in the ſame Manner, as Motion is Produced by Impulſe, and the Effect, in both Caſes, Partakes of the Nature of the Cauſe; And by the ſame Method this Aſſemblage of the Expanſive Force will Impart it's ſelf to what elſe is Inflammable, by Impreſſing it's own Action upon it, and Converting it into it, till a Flame is Raiſed ſufficient to Promote the like Force to what Degree, or as far as, we Pleaſe.

THE Second, how one Expanſive Force Unites with another, is alſo Evident from the above-mentioned Explanation, if thoſe Forces are Contiguous; for if the Expanſive Force Reduces, as far as Poſſible, what is Contrary to it, to it's own Action, it cannot but Cement, and Concur, with it's own, for that very Reaſon; if they are not Contiguous, they muſt be either Hindered from Uniteing by Contrary Forces, which they cannot Overcome, or they will Overcome thoſe Forces by Impreſſing their own Action upon them, till they do Unite.

FROM hence it is, That if we take Two Lamps, or Candles, and ſet them a-Burning at ſome Diſtance from each other, they will Riſe in Diſtinct Flames in a Conical or Pyramidal Form; Becauſe they Exert themſelves in an Oppoſite Direction by their Expanſive Force to the Contractive of the Earth, and the Medium of Air, or the Atmoſphere, which Tends towards it; and the Flame grows Pointed at Top, becauſe the Expanſive Force is there Leſs, and the Contractive of the Air or Atmoſphere, in Reſpect of it, Greater; But if we place theſe Flames in a nearer Situation to each other, they will on the Sides of them manifeſtly come together, and endeavour at an Union. And thus in Fire, the Fuel at firſt Separates it into Different Flames, till at length the whole Receives the Action of the Expanſive Impreſſed upon it, and the ſeveral Expanſive Forces are United into one.

Why the Flames of Two Lamps or Candles Riſe in a Pyramidal Form, and when Set together Unite on their Sides.

THAT therefore Forces of the ſame kind Tend to one another, It is Plain is not by an occult Quality, but by the Method, by which all Motions are Communicated to Material Beings; and there is no more difficulty in Conceiving, how an Expanſive Force ſhould Produce an Expanſive, than how an Impulſe from the Hand ſhould Produce Action or Motion in Body, the Effect is Anſwerable to the Cauſe, and Follows, and Participates of, it's Nature; And we have ſhewn, Granting this Principle only, that it is eaſy to Account for a Tendency to an Union of the ſame Forces.

The Uniteing of Similar Forces is not by an occult Quality, but by the known and confeſſed Laws of Mechaniſm.

WHAT we have ſaid of the Expanſive Force is Equally Applicable to the Contractive, ſince they Differ, not in Force, but in the Direction of it, and therefore a Contractive Force will Impreſs the ſame Force upon Bodies, which are not, as Fire, too Expanſive to be overcome or ſubdued by it; Thus the Contractive Force of the Earth and the Moon, where the Expanſive of the Sun is not too Predominant, will Turn Water into Ice, and Earth into Metals, Minerals, and Stones, which are all therefore Formed in the Inward and Subterraneous Receſſes of our Planet; and it is not Certain, ſince we can only Penetrate a few Yards of Four Thouſand Miles, of which the Semidiameter of the Earth nearly Conſiſts, but there may be Subſtances of a much Greater Contractive Force than any we meet with juſt under the Surface of it.

How Water is Turned into Ice, and Earth into Metals, Minerals, and Stones; and that there may be more Compact and Solid Bodies near the Center of the Earth, than any we find near the Surface of it.

AND for the ſame Reaſon Heavy and Compact Bodies Deſcend to the Earth, which have a Contractive Force, becauſe they will be either Hindered by ſome Oppoſite Expanſive Force, which they cannot Surmount, or elſe they will Reduce that Force by their Contractive, till they are United and Cemented with that of the Earth, which is of the ſame Kind and Species of Action with Them.

Why ſome Bodies Deſcend to the Earth, and others Aſcend from it.

Thus Heavy Bodies are Actually Hindered from Deſcending by the Expanſive of Fire, but otherwiſe Unite with and Precipitate to the Earth, As in Smoke and Vapours.

That there is no Determined Center of theſe Expanſive and Contractive Forces, and therefore none of the Earth or Sun.

§. 7. Since therefore theſe Expanſive and Contractive Forces conſiſt in a Similarity or Likeneſs of Action, tho' the one Exerts it ſelf from a Center, and the other towards one, yet it cannot be ſaid, that ſuch Centers are any fixed or determined Points, becauſe the Whole and Every Part of theſe Expanſive and Contractive Forces are Homogeneous; and conſequently, when we ſay, that Bodies, as the Planets, are Repelled from the Sun by this Expanſive Force, we do not mean, that they Recede from the Exact Center of it, or when we Affirm, that Bodies Deſcend to the Earth, do we Intend to ſay, that they do not Deſcend to the whole Maſs of it's Contractive Force, without any Diſtinction of a particular Center belonging to it; On the Contrary it ſeems Plain, that the Sun is to be Conſidered as an Aſſemblage of the Expanſive Force, without any particular Center of it's Action, and that the Earth is to be looked upon, as a Congeries of the Contractive, without any ſuch certain Point to which the Contraction is made; For no One ever Dreamed of ſuch a Center in a Fire, or a Flame, or of ſuch a Point in Amber, or the Load-ſtone.

As to the Firſt, all the Reaſon, which can be given for it, is only, that the Planets Revolve round it, as if there were ſome Fixed and Determined Center of it; But it is known, there is ſuch an Immenſe Diſtance betwixt It and Them, that the whole Body of the Sun bears no Great Proportion to ſuch a Diſtance; Beſides, the moſt Accurate Account of the Revolutions of the Planets ſuppoſes, there is no ſuch fixed Center in the Sun, the Common Center of Gravity of all the Planets, tho' Quieſcent, yet by the Agitation of the Sun, being ſometimes in it's Surface, and at other Times, a little Above or Below it; which may be owing to what we Affirm, that the Sun really has no Center, and that the whole Body of It is to be Eſteemed, as one Maſs of the ſame Similar and Homogeneous Action.

Sir Iſ. Newt. Princip. Math. Philoſ. Natur. Prop. 12. Lib. 3.

As to the Second, the Spherical Surface of the Waters in the Ocean is Imagined to be an Argument of it, which may be as Rationally Explained by ſuppoſing the Earth is nearly Globular, which is Confeſſed, and that the Contractive Force of it being of that Figure Preſcribes the ſame to the Waters, which manifeſtly obey the Impreſſions it gives, as we have ſhewn they do in the Tides, in the Regulating of which the Earth has it's Influence, as well as the Sun and Moon.

See Chap. 5. Book 1. §. 22, 23, &c.

What is the Reſult of Two Equal or Unequal Expanſive or Contractive Forces mixed together, and how from thence the Phænomenons of Fluids or Solids upon their Mixture are Solved.

§. 8. From this Explication of the Expanſive and Contractive Forces it follows,

First, That each Expanſive Force, or each Contractive Force, Conſidered by it ſelf, has it's Center of Action every where, and Acts Uniformly from, or towards, it's ſeveral Centers; Thus Fire Acts outward in the whole Maſs of it, and in every Part of that Maſs; Heavy and Compact Bodies Act Inwards, in the Whole, and in every Part of the Whole.

Second, That Bodies, or the Parts of them, which have the ſame Expanſive or Contractive Force, are in an Equal Poiſe and Balance, and therefore at Reſt amongſt themſelves; Thus Solids, and the moſt Fiery and Spirituous Fluids, are alike Quieſcent.

Third, That if Two Expanſive or Two Contractive Forces, that is, which Act to or from the ſame Center or Centers, are Equal, and there is an Addition of the one to the other, namely, of the Expanſive to the Expanſive, or the Contractive to the Contractive, there will be no Alteration Produced in the Degrees of the ſeveral Forces; becauſe the Actions will be Similar and Homogeneous, and therefore in an Equal Poiſe or Balance, by Article the Second; And hence it is, that Fiery and Spirituous Fluids mixed with Fiery and Spirituous ones of the ſame Nature, or Solids with Solids, as Gold with Gold by Fuſion, continue the ſame.

Fourth, That if Two Expanſive Forces are Unequal, and the one is Blended or Mixed with the other, the Greater will Increaſe the Leſs, but not a Leſs, a Greater; For Inſtance, if an Expanſive Force Acts from a Center with a Force of Two, and another from the ſame Center with a Force of Five, the Force of Two on both Hands will be Equal, and ſo far Similar and Homogeneous, and therefore, by Article the Third, there will be no Alteration Produced; but the Force of Five Exerting it ſelf ſtill with an Exceſs of Three, ſuch Point or Center will be Acted by that Impreſſion, and what is ſaid of one Point or Center, holds Good in all of the ſame Bodies, which have the ſame Nature and Similarity of Action.

FROM hence it is, that the Expanſive Force of Fire or Flame, Mixing with a Greater or Leſs Expanſive Force of Oil, or Fuel, and the Different kinds of it, becomes more or leſs Vigorous in it's Action or Force; And from the ſame Cauſe it is, that if we mix a more Fiery or Spirituous Fluid with a Leſs, as Spirit of Wine with Water, the Expanſive Force of the Water is Increaſed, and that of the Spirit of Wine is Diminiſhed.

LIKEWISE, If Two Contractive Forces, which Tend towards the ſame Point or Center of Action, are Unequal and Compounded together, the Caſe will be the ſame, as before in the Expanſive, that is, the Forces will be in an Æquilibrium, ſo far as they are Equal, and the Exceſs of the Greater Contractive Force above the Leſs will only cauſe an Alteration.

FROM hence it is, That Gold mixed with Silver or Copper in Fuſion loſes of it's Weight, and Silver or Copper Receives an Addition to it, and ſo in other Metals, where an Heavier and a Lighter are Fuſed together.

Why ſome Fluids are more Eaſily, and ſome Solids more Difficultly, Separated than others; and why they have Inclinations and Tendencies to and from the Earth Different from thoſe of their Parts to each other.

§. 9. AND from this Doctrine of the Expanſive and Contractive Forces we are alſo Informed, why the Parts of ſome Fluids are more eaſily Separated, and the Parts of ſome Solids more Difficultly, than thoſe of others; Becauſe, in the Former Caſe, the more Expanſive any Fluid is, the more Eaſy is the Separation of it, and in the Latter, the more Contractive any Solid is, the more Difficult is ſuch Separation.

IT muſt be here Obſerved, in Reſpect of the Expanſive and Contractive Forces of Bodies, that they are not always Equal to their Expanſive from the Earth, or their Contractive to it; Thus the Expanſive Force of Spirit of Nitre, and Oil of Vitriol, is very Conſiderable in it's Diſſolving of Metals, which Fire or Flame cannot do, and yet Fire or Flame Recedes from the Earth, whilſt the Spirit and Oil Gravitates towards it; On the other Hand, Gold Gravitates more, or has a Greater Contractive Force towards the Earth, than a Ruby or a Diamond, and yet the Parts of the Latter have a Greater Contractive Force to each other, than the Former; The Reaſon of which is, becauſe Fire and Flame are Entirely of an Expanſive Nature, whereas Nitre and Vitriol are originally of a Contractive one, and are made Expanſive by the Force of Fire, and therefore Retaining ſtill a Degree of their original Contractive, they Gravitate to the Earth, tho' Exalted into Spirit and Oil; Thus alſo Gold Gravitates more, or has a Greater Contractive Force towards the Earth, than a Ruby or a Diamond; Becauſe the Firſt is Formed in the Bowels of it, and in the Midſt of the Contractive Forces, as all other Metals are, and in Colder Countries and Climates, whereas the Latter are Produced nearer to the Surface of the Earth, where the Contractive does not ſo much Prevail, and in Hotter and Warmer Climates; But Gold is more eaſily Separated, is more Fuſible and Ductile, than the others; Becauſe the Expanſive Force has Exerted it ſelf to the utmoſt in the Formation of the Latter, and yet their Contractive Force continues, whereas there has been but a ſmall Expanſive Force, in Compariſon, Applied to the Production of the Former, and therefore, when a Greater Supervenes, it is Diſſolved or Separated in the Parts of it; For ſo Earth and Clay, tho' they are eaſily Separated, yet have an Intenſe Force of Gravitation, whereas Glaſs and Chryſtal, which are made by Fire and of Flint, are with Difficulty, tho' they have a Leſs Force of Contraction or Gravitation to the Earth; Another Reaſon may be, that Gold and Metals are poſſibly Formed from Quick-ſilver, which is ſaid to be *Mater Metallorum*, the Volatility of which will make the Metals, which are Fixed and Produc'd from it, more eaſily Separated, tho' their Contractive Force to the Earth may not be Diminiſh'd, but Increaſed.

THIS therefore will lead us to Enquire, what Effects will Ariſe from a Mixture of the Expanſive and Contractive Forces.

An Inertia Produced from Equal Degrees of the Expanſive and Contractive Forces, and what Æther, Air, and Water are.

§. 10. IF Two Forces are Equal, the one an Expanſive, and the other a Contractive, They muſt upon their Application to each other, like + and − in Algebraick Quantities, and in the Motions of Bodies, Deſtroy each others Effects; For if one Force Exerts it ſelf from a Point, and another towards the ſame Point, the Conſequence will be a Quieſcence or Inaction, or Nothing in Reſpect of an Expanſive or Contractive Force; and a Syſtem of Matter, which is Compounded of ſuch Forces, muſt be Endued with an entire Inertia, and be Paſſive to any External Impreſſions; There does not ſeem to be any Material Being, which comes ſo near to this Compoſition, as the Corporeal Space, or the Æthereal Plenum, which Surrounds us; It is Colourleſs and Tranſparent, and has no Heat in it ſelf, and therefore has little or no Expanſive Force to Produce the Senſation of Colour or Heat in us; It has an Inſenſible Gravitation, and is eaſily Separated in it's Parts by our Moving in it, and Dividing it by the leaſt Percuſſion, and therefore is void of any Apparent Contractive Force, and conſequently is capable, by this Inertia, of Receiving and Obeying any Foreign Impulſe; It is alſo Conſtituted and Formed by the Expanſive Force of the Sun, and the Contractive of the Earth, which we may reaſonably Imagine are here nearly in an Equal Balance, ſince otherwiſe the Expanſive Force of the Sun, if Superior, would Repell the Earth farther from it, or, if Inferior, the Contractive of the Earth, would Approach nearer to it, till it had Strength and Force enough to Repell it.

Air and Water are the next Material Beings, which come the neareſt to this Conſtitution of Forces, by their want of Colour, Heat, and Activity, and their Defect in Gravitation and Contraction, by which almoſt all Bodies Sink and Gravitate in them, and by which they eaſily yield to any Impreſſion, or become Fluid; but theſe yet have certain Forces of Expanſion and Contraction belonging to them, by which they ſhew their Forces are not in an Exact Æquilibrium, as is Evident from their Elaſticity, Colour, and Power of Diſſolving Bodies on the one Hand, and their Re-union of Parts, upon their being Separated, and their Gravitation, on the other.

Other Fluids alſo by their Colour, and their Spirituous Qualities, and their Power of Diſſolving Bodies, manifeſtly declare their Expanſive Force, and by the Aſſemblage of their Parts to each other, and by their Gravitation, their Contractive, and that they are not in an Equal Balance; Solids, in like manner, Evince, that by their Colour they have a plain Expanſive, and by the cloſe Cement of their Parts, a Contractive one, and that they are not in an Æquilibrium.

An Inertia Various in Different Bodies, and the occaſion of Fluidity.

§. 11. But it deſerves to be Enquired, whether theſe Equal and Expanſive Forces, however Different, Acting againſt each other, will Produce the ſame Quieſcence or Inertia, which it does not Appear, that they will, ſince, where the Cauſes are Different, the Effects muſt be ſo too; Thus let us take a Leſſer Expanſive Force of the Sun's Rays, and a Leſſer Contractive of Earthy Matter, and the Fluidity of Æther, or Air, or Water, is Produced; if we take a Greater Expanſive Force of the Sun's Rays, as in a Burning Convex or Concave, and a Greater Contractive, as of Gold, the Fluidity or Fuſion of it is cauſed; and yet the Fluidity, or Inertia, or Quieſcence of the one, is not Equal to that of the other; Thus alſo, if Two Bodies meet with Equal Forces, which are Leſſer, and other Two meet with thoſe which are Greater, tho' there is an Equal Deſtruction of their oppoſite Forces, yet the Effect or the Reſult on both Hands is not Equal, for it will be eaſier to Separate them in the Firſt Caſe, than in the Laſt; And it is the ſame even in Mathematical Quantities, for tho' $a-x$ is $=0$, and $b-z=0$, it will not be immediately Conſequent, that $a-x$ is $=b-z$, no more than a is $=b$; for there may be as many various Noughts, as there may be Infinites, and which are Different according to the Different Quantities, from which the Relations of Infinite and Nought are Derived, as a Nought or an Infiniteſima of a Cube or a Surſolid is not Equal to that of a Line or a Square; but if we mean by $+$ and $-$ an Affirmation and Negation of the ſame Thing, it is an Abſtracted Idea, and do's not deſerve to be conſidered in Nature or in Real Quantities. For, as in the Inſtance of Two Concurring Bodies before, if I preſs my Two Hands together with an Equal and a Contrary Force, and then with an Equal and Contrary Force, yet Double or Triple of the Former, it will not, I hope, be ſaid that the Effect in both Caſes is Equal, tho' there is a like Deſtruction of the Contranitent Forces.

Whence Volatility and Coheſion, Mobility and Diviſibility.

§. 12. If now Two Forces are Unequal and Act contrary to each other from a Point, (and that which is the Caſe of one Point, will be the Caſe of an Hundred or a Million, which are ſuppoſed to have the ſame Conditions in a Syſtem of them, that is, in any Material Being) what will be the Effect? Namely, That ſo far as they are in an Equal Balance, a Different Fluidity would be Produced, that is, a Leſs or Greater, as the Equal Forces are Greater or Leſs, according to what we have before Explained; But ſo far as the Expanſive is Superior, that Fluidity will be changed into Fire and Volatility, and ſo far as the Contractive is, into Solidity and Coheſion.

Let us in the Firſt Place Examine into Fluids, Æther we have already Mentioned and Endeavoured to Account for; Wine and other Spirituous Fluids, are Fluid by an Equal Balance of their Expanſive and Contractive, but are Spirituous by their Expanſive; And this Spirit is Lighter or Heavier, as the Expanſive is Greater, which Actuates the Contractive, and the Contractive is Leſs, or as the Contractive is Greater, which is Actuated, and the Expanſive is Leſs; ſo that every Spirituous Fluid is a Contractive Force Acted by a Superior Expanſive, which Contractive, as it is Different gives likewiſe a Different Moment and Impreſſion to the Expanſive; and from hence ariſes the Different Spirituouſneſs of Water, Wine, Brandy, Alcohol, *&c.*

On the Contrary, Solids proceed from a Prevalence of the Contractive over the Expanſive, by which it Curbs and Reſtrains it from Exerting it's full Force and Energy, and preſcribes it's own Bounds and Limits to it's Action, but tho' the Contractive Prevails the Expanſive is mixed with it, and gives a Different Impreſſion and Moment to the Contractive, as the Contractive did before to the Expanſive; And from hence it is, that the Expanſive of Solid Bodies Exerts it ſelf only on and within the Surface of the Solid, and by it's Impulſes upon the Æther occaſions the Various Senſations of Colour, which we Receive from them, when the Æther is Excited by the Sun's Expanſive, and rendered ſuſceptible of ſuch Minute Agitations of it; Sometimes alſo the Expanſive, notwithſtanding the Contractive, continues ſo Vigorous and Strong, as to Raiſe the Senſations of Smell in us, as in Odorous Bodies; And in General, the Capacity of Bodies being moved by the Expanſive Force of Percuſſion, Ariſes from this Mixture of the Expanſive in all Bodies, as likewiſe their Capacity of being Separated and Divided; for if the Contractive Force was Infinite in the one Caſe or the other, it would be Impoſſible, that Bodies ſhould be either Moveable or Diviſible.

§. 13. The Source and Origine of theſe Forces in all Material Beings, which we know, is the Sun, or Fire, or Heat, namely, of the Expanſive, and the Earth, the Moon, and the Planets, or Cold and Salts, of the Contractive; ſo that we may truly Affirm, that all the Various Portions of Matter are Compounded of Infinitely Various Degrees, and Various Combinations, of Heat and Cold, Mixed and Tempered together.

The Source and Origine of the Expanſive and Contractive Forces, and the Method of their Acting upon each other.

And the Method of Acting of theſe Forces one upon another ſeems to be, that the Expanſive Aſſiſts and Promotes the Expanſive, and the Contractive the Contractive; Thus any Inflammable Subſtances, which have a good Proportion of the Expanſive Force in them, by the Application of the Expanſive of Fire or Flame are eaſily Reduc'd to an Expanſive, and Cold and Contractive Bodies, and which are not Inflammable, as Water and Earth, by the Application of a Contractive, to Froſt and Ice.

On the other Hand, the Expanſive Acting upon a Contractive, if not Superior to it, Increaſes the Contractive, and a Contractive Acting upon an Expanſive, if not Superior, Increaſes the Expanſive; Thus Fire Increaſes the Weight of Lead in Calcining, and that of other Contractive Bodies, Water grows Colder for being Boiled, and thus we may Imagine that Mercury by a Heat, which would not Diſſipate it, that is, by a Gentle one, would grow Fixed and Heavier, that is, that Gold might be from thence Produced; On the Contrary, Water, thrown upon the Fire, if not in too great a Quantity, or a Froſty Air, makes it burn the Fiercer, and Water poured upon Quick-Lime cauſes an Intenſe Heat, and an Acid upon an Alkali an Efferveſcence: The Reaſon of which appears to be Plain, that as one Force will Act in Conjunction with another Force of the ſame kind, ſo Two contrary Forces will Act, as much as Poſſible, in Oppoſition to each other.

How Gold may poſſibly be Produced. See alſo §. 32. Chap. 3. Book 1.

§. 14. We ſhall not therefore now Enlarge any farther upon theſe Forces, to ſhew the Great Advantage there is from them in Explaining all the Effects we obſerve in Being, only Conclude by Saying, That they will much better ſerve that Purpoſe than any Principles, which have been hitherto Propoſed by the Defenders of *Epicurus*'s Syſtem of a Similar and Homogeneous Matter, it's Atoms and the Gravity of them; and, That there is an Eaſy Solution given of the Coheſion of Matter, and how there may be Different Conſtipations of it in the ſame Space, namely, from the Various Combinations of Action in it; Neither of which Problems are in any Meaſure Accounted for by the *Epicurean* Scheme, and the Atoms with their Gravity, which it Aſſerts, and which It Thinks to juſtify by Intricate Propoſitions, and Demonſtrations,

Theſe Principles more Univerſal and Extenſive than the Corpuſcular.

in the Mathematicks, which have no Relation to the Point in Queſtion, and which ſhew more the Sagacity and Penetration of the Geometer, than the Solid Reaſoning or Senſe of the Philoſopher, and of a Strict and Impartial Inquirer into Nature; The Circulation of which not only in the Heavens, which we have before Explained, but in our Sublunary World, is Owing to theſe Expanſive and Contractive Forces: For the Reaſon of Water and Earth Riſing into Plants, Trees, and Vapours, is Derived from the Expanſive of the Sun, and their falling Back again in Dew, Rain, and Leaves, is from the Contractive of the Earth; which Expanſive and Contractive Forces of the Sun and Earth, being nearly in an Æquilibrium on the Surface of the Latter, as above-mentioned, is the occaſion of thoſe continual Revolutions of Material Beings, which we Behold.

From theſe Principles the Exiſtence of an Infinite and All-wiſe Agent may be Immediately Concluded.

§. 15. WHAT we have more to do is to Evince, from theſe Principles, the Neceſſity of an Infinite and All-wiſe Agent, that, as we originally Propoſed, we may make the Demonſtration of a God the Reſult and Concluſion of all our Reſearches and Inquiries into Philoſophy; And what can be more Evident than that there is ſuch a Being, from the Principles, which we have laid Down?

IF there were an Infinite and All-wiſe Agent, of Conſequence we muſt ſuppoſe there would be an Infinite Action Produced from him, and that this Infinite Action would be ſo Adjuſted in the ſeveral Parts of it, as to Cauſe the moſt Regular Order and Diſpoſition of Things; On the Contrary, if we ſhew that this Infinite Action is ſo Adjuſted, we muſt be obliged to Acknowledge, that there is ſuch an Infinite and All-wiſe Agent.

AND Nothing can be more Evident than this; Since, notwithſtanding all the Infinite Variety of Forces, which are in their own Nature Contrary to each other, yet they are ſo Diſpoſed by the Infinite Wiſdom and Intelligence of the Almighty and Great Creator, as to Produce an Æquilibrium in the Whole, and an Harmony in all the Beings of the Univerſal Syſtem.

I Have not any Thing more at Preſent to Add in Reſpect of theſe Forces, Unleſs to Inform the World; That this Doctrine of the Expanſive and Contractive Forces was what I originally had a Notion of, before my Reading the Leaſt of what Sir *Iſ. Newton* has Advanced in his Mathematick Principles of Philoſophy; That I have Mentioned it, many Years ago, in my *Geometria Solidorum ſive Materiæ*, which was in Part Written by me, when I was Under-Graduate in this Univerſity; That I was not a little Pleaſed to find ſo Great a *Genius*, as Sir *Iſaac*, to Confirm my Sentiments in Reſpect of a Gravitating Force in Matter, which is a Species and Kind, or rather a Conſequence, of the Contractive, which I Maintain; and That what Firſt led me into theſe Opinions, was, that I found Different Appearances of Material Beings in the ſame Space, which could not be Accounted for by conſidering Matter, as conſiſting only of a Trine Dimenſion; I therefore Examined, as carefully as I could, what might be the Cauſe of ſuch Different Appearances, and could not Perceive, after the ſtricteſt Scrutiny, that it could be Attributed to any Thing elſe, Excepting to a Difference of Action; This Incited me to Inquire, what that Difference of Action was, which I could not but Acknowledge muſt have a contrary Direction, and could be Reſolved into nothing more Rational than an Expanſive and Contractive one: And, if to all this it ſhould be Objected, that I make Two contrary Actions to Exiſt in the ſame Space, I ſhall only Anſwer, that this we may Experience, if we Pleaſe, every Minute in our Lives, if we will but Preſs one Force againſt another in Oppoſite and Contrary Tendencies.

Geometria Solidorum ſive Materiæ. Pag. 15.

CHAP. X.

Concerning Pneumaticks and Hydroſtaticks, and a Real Space, or Extenſion, or of Air and Æther, and the ſeveral Kinds of Experiments made in Them.

The Difference betwixt Real and Abſtracted Extenſion.

§. 1. THAT Real Extenſion, or the Space, which Exiſts in Nature, has all the Properties of that Subſtance, which is called Matter, and beſides thoſe, ſome others peculiar to it ſelf, we ſhall in this Chapter ſhew; The Properties, which are Aſcribed to Matter in General, are theſe, *viz.* Extenſion, Impenetrability or a Non-Penetration of Dimenſions, Solidity or Reſiſtence or Firmneſs, Gravity, Mobility, and, as the Corpuſcular Philoſophy ſays, Figure and Diviſibility.

FIRST, As for the Extenſion of Space no one doubts of it, ſince Thoſe, who Defend a *Vacuum*, make it to conſiſt in nothing elſe; and Thoſe, who Maintain a *Plenum*, make it of the very Eſſence of Body; But in Both it is an Abſtracted Idea, as generally uſed by Them, and Similar according to that Idea; Whereas, as it Exiſts in Air, Æther, and ſeveral Parcels of Matter, that is, in Nature, it is very Different; One is Cloſer and more Compact, Another Thinner and Rarer, One Stronger and Firmer, Another Weaker and more Diluted, According to the Different Proportions of the Expanſive and Contractive Forces, Acting in the ſame Abſtracted or Similar Space.

Impenetrability of Two Kinds, the Firſt Abſtracted, the Second Real, and may be Admitted in Nature.

§. 2. SECOND, Impenetrability, or a Non-Penetration of Dimenſions, is likewiſe Acknowledged by both theſe Sects of Philoſophers in one Senſe, that Space cannot Penetrate Space, or that one Part of Space cannot Poſſeſs another at the ſame Time, that the other Poſſeſſes it; but on the Contrary it is Acknowledged, that tho' this cannot be, yet Body, which is Extended, may Poſſeſs Space, which is likewiſe confeſſed to be Extended, which Two Opinions we ſhall Conſider.

Sir If. Newt. Princip. Definit.

1*ſt.* THE *Primarius Locus*, as it is Termed, or Place, in it's Proper Signification, is thought to be Immoveable, and therefore cannot Enter into another Place; or if it was not Immoveable, if one Place could Penetrate another, Place would Exiſt in Place, and conſequently one of them only would be Primary, which is no more than to ſay, our Idea of one Part of Extenſion is not our Idea of another; For in Truth, Place, or the *Primarius Locus* is only a ſtanding Meaſure of Extenſion delineated in the Mind; It is an Abſtracted Idea, and nothing in Nature and Reallity; It is a determinate Image of Space made and ſhaped by the Mind, ſo as to Agree to any Body we are Willing to Conſider; The Reaſon of which Procedure of the Mind ſeems to be, that, not obſerving any Thing in Space, but Length, Breadth, and Depth, and finding no Reſiſtence in it to the Motion, or Dimenſions, of Body, and at the ſame Time, that all Space is in a perfect Repoſe, it from thence Concludes, that Body does Move in Space, and conſequently Poſſeſſes a Portion of it, agreeable to it's own Bulk; Another Reaſon is, our Perceiving Extenſion every where, without Conſidering the Real Properties and Affections, which belong to it, which leads us to Form our Notion of it's being Abſtracted and Similar; but what has Contributed the moſt to this are the Elements of Geometry, and the Abſtractions of Length, Breadth, and Depth Inculcated in them, we are therefore to Examine,

2*d.* WHETHER Body does Move in, or Poſſeſs Real Space, or, which is all one, Æther; We ſay not, no more than a Plumbet Moves in the very Dimenſions of Water.

1*ſt.* BECAUSE the Mind falſely Concludes, that there is neither Motion nor Reſiſtence in the Æther, or Real Space, for the Reaſon only that it Perceives none; Since the Senſes are very unequal Judges concerning a Matter ſo Refined and Subtile, that it muſt needs Eſcape them; Thus there is a Gravitation of one Body to another, which yet is not Perceptible, becauſe it is nothing in Compariſon with their Gravitation to the Earth; our Feeling more eſpecially is ſo Groſs, that we cannot diſtinguiſh betwixt the Reſiſtence of one Body, and another, even where it is extremely Different; and therefore there is no wonder, if it is not capable of Informing us in the Reſiſtence of the Æther; tho' our Feeling our ſelves to Exiſt, as

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to the External Parts of our Hands and Faces, may poſſibly proceed from the Gentle Impulſe of the Surrounding Space, which we do not take Notice of however, becauſe the Perception is ſo Conſtant, as a Man ſcarcely feels the Cloathes he Wears, unleſs when he ſets his Mind to reflect upon thoſe Perceptions; it is yet very Evident we can feel Cold and Heat, which are nothing more than Different Modifications of the Fluid we are Defending, as they are of Water; And indeed, when thoſe Qualities of Affecting us are Abſent, I do not ſee, that we ſhould have any Manner of Senſe without us; That is, if there were no Action at all of Space or Æther upon our Organs, we could neither See nor Feel, nor have any other Senſation.

2*d.* If Body Exiſted in Abſtracted Space, that Abſtracted Space, or Extenſion, will be Commenſurate to Body or not, for Inſtance, a Globe of ſuch Space to a Globe of Gold of Equal Length, Breadth, and Depth; If Commenſurate, it will be Diſperſed thro' every Part of the Gold, that is, it will Enter into each Point of it, and conſequently, ſince the Extenſion of Gold does not only conſiſt in Length, Breadth, and Depth, but in Thickneſs too, the Globe of Space or Æther will be as Thick, ſeeing it is Commenſurate to it, as the Gold it ſelf, which will not only Cauſe a Penetration of Dimenſions, which this Philoſophy denies, but is contrary to it's own Notion of Space or Æther; If it is not Commenſurate to it, then there will be ſome Parts of Gold, which do not Exiſt in Space, which is again Contradicted by the ſame Philoſophy: But if Body does not, nor cannot Poſſeſs Space, or Enter into it's Dimenſions, that is, ſuppoſing them both to Retain their own Natures, *viz.* Gold and Æther, Then is Real Space or Extenſion Equally Impenetrable with Body.

I ſay ſuppoſing them both to Retain their own Natures, for if they do not, there may be a Penetration of Dimenſions, in one Senſe, ſince a Penetration of Dimenſions may be ſaid to be either,

1*ſt.* When the Dimenſions of one Body Penetrate into thoſe of another, and at once that other Preſerves it's own Dimenſions; This Penetration is Impoſſible in all Matter, and Space or Real Extenſion, Equally; for it is no more than to ſay, the one Body cannot be it ſelf, and another, at the ſame Time, that is, It Excludes all Things from being, what It is.

2*d.* A Penetration of Dimenſions may be ſaid to be, when the Dimenſions or Bulk of one Body not Porous are contrived into the Bulk or Dimenſions of another, which is likewiſe not Porous, without any Increaſe of the Length, Breadth, and Depth of ſuch Bodies; That is, when a Globe of Gold not Porous, for Example, is Compreſſed into another of Equal Bigneſs not Porous without any Alteration of the Magnitude of the Globe, and with only a Change of the Denſity of it; This Penetration of Dimenſions we ſay is poſſible, and that Matter is not Impenetrable in this Senſe, and therefore not Real Extenſion; Becauſe we take all Matter and Space, as they are Real in Nature, to be nothing elſe than Different Combinations, Compoſitions, or Congeries of Actions, which Actions are Reſolved into the Various Degrees and Aſſemblages or Compoſitions of the Expanſive and Contractive Forces, which, however Diverſified, may and do ſubſiſt in the ſame Abſtracted and Similar Space.

Solidity and Reſiſtence is likewiſe Abſtracted, or Real, the Former is a Creature of the Mind, the Latter, what we find in Nature.

§. 3. Third, Solidity, or Reſiſtence, or Firmneſs, as Extenſion, is commonly no more than an Abſtracted Idea Formed by the Mind, and is generally Reputed to be the ſame in all Bodies, the Dullneſs of our Perceptions not allowing us to diſtinguiſh it; whereas, in Truth, there is as much Difference, as in Colours themſelves, or any other the moſt Variable Property in Matter; Solidity, as it is an Excluſion of all Matter from that Place, or Space, which it takes up, is one Thing in all Sorts of Matter, and admits of no Change, This we conſidered under the Head of Impenetrability; But that, which we ſpeak of now, is of a Different Nature; Namely, 'tis that Degree of Force, Reſiſtence, or Action, which any Body obtains againſt the Actions or Violences of other Material Beings; Nor does this Solidity only reſpect the Reſiſtence of a Body to any other Matter Penetrating into the Space it is forced to Leave, but Penetrating into the ſame Space, which it Poſſeſſes, I mean into the ſame Length, Breadth, and Depth, *i. e.* not only it's Oppoſition to Removeing out of it's

Place, but it's Oppoſition to the being Contracted or Condenſed into a Leſs, tho' it was ſuppoſed to have no Pores or Vacuities before ſuch a Condenſation; For if Water could be Compreſſed into a Nineteenth Part of the Space, which it Poſſeſſes, it is not certain, but it would be as Solid as Gold; and tho' we have no Methods, by which we can Effect ſuch a Compreſſion, or indeed any Compreſſion at all, it is not Evident, whether Nature by it's Two Great Inſtruments of the Expanſive and Contractive Forces, or by Heat and Cold, may not do it.

§. 4. THAT Real Space or Extenſion has a Reſiſtence may be Proved by Experiments from *Galileus's Syſtema Coſmicum*, which are Known and Common; For a Stone may be thrown farther than Cotton; An Arrow projected Lengthways will go farther, than one Tranſverſely; On the Contrary, the Wind blowing carries away the Tranſverſe one, and leaves the other; Agitate the Air, as much as poſſible, with a Fan or Napkin in a Narrow Room or any other, bring in immediately the ſmalleſt Candle imaginable Lighted, or the fineſt Leaf-Gold, and there will be no Motion Apparent, the Air in that Inſtant Returning to it's Reſt, which is an Argument of it's Reſiſtence; The Clouds, which hang in the Air, when they come Lower, fall into Rain, which may be, becauſe, tho' there is a Reſiſtence, yet there is not enough to ſupport them; In Rideing, the Air Acts with Violence upon our Faces; Air is apt to carry the lighteſt Things with it, but not the heavieſt in it's Motion, and Atoms Swim every where in it's Subſtance, when a Feather Sinks; If the Earth Moves round it's Axis, it muſt carry the Air and all Projectiles with it, which Demonſtrates a great Force in the Air, for, if it did not thus carry Them, a Bullet ſhot Upwards would not fall in the ſame Place, nor a Stone thrown Downwards deſcend in a Perpendicular; nor would a Bullet Shot to the Weſt with the ſame Force, as another to the Eaſt, deſcribe Equal Lines, but one as much ſhorter than the other, as the Earth deſcribes a Space from Weſt to Eaſt; If Air therefore has a Force or Action, it has, *à fortiori*, a Reſiſtence; A Globe of Lead will alſo make 600 Returns in a Pendulum, whilſt one of Cotton or Wood will not make above 3 or 4; *Rohault's Vitrea Lacryma*, Broken and Shatter'd to Pieces by the Intruſion of Air, likewiſe ſhews this Action or Reſiſtence; And all theſe Demonſtrations of Fact Equally Extend to one Point of Real Space or Extenſion with another, and as much Evidence a Continuity of Reſiſtence or Action, as the Phænomenons we obſerve in Water of Bodies Sinking and Swiming in it, which yet is Acknowledged to have no Pores or Vacuities, ſo far as the beſt Microſcopes could ever Diſcover.

That Real Space or Extenſion, tho' not indeed an Imaginary or Abſtracted one, has a Solidity or Reſiſtence. Galileus's Syſtema Coſmicum.

§. 5. DR. *Wallis* Endeavours to compute the Air's Reſiſtence to all Projectiles by this Lemma, as the moſt Rational that occurrs; That the Reſiſtence of the Air is Proportional to the Celerity, for in a double Celerity, there is to be Removed in the ſame Time twice as much Air, which is a double Impediment, in a Treble thrice as much, *&c.* Therefore, according to this Computation, there is Reſiſtence in every Point; And upon this Computation, the *Linea Projectorum* Reſembles a *Parabola Deformed*, which Agrees with Sir *Iſ. Newton*, who makes it a Curve Approaching nearer to an *Hyperbola* than a *Parabola*, upon the Suppoſition of ſuch a Perpetual Reſiſtence; He alſo tells us, that a Viſcous Medium ſhall more Reſiſt than one more Fluid, tho' of like Intenſive or Specifick Gravity; That a Sharp-Headed-Arrow finds an Eaſier Paſſage, than an Obtuſe or a Blunt one, thro' the Medium, and the ſame Pyramid with the Point, than with the Baſe forward; That Different Mediums equally Liquid, and other Circumſtances alike, Reſiſt in Proportion to their Specifick Gravity, becauſe there is a Heavier Medium to be Removed by the ſame Force; That the Heavier Projectile being once in Motion, and being Equally Swift, and all other Circumſtances the ſame, Moves thro' the ſame Medium more ſtrongly than a Lighter, in Proportion to it's Intenſive Gravity, the Force being Greater in Proportion for the Removal of the ſame Reſiſtence; All which can be no otherwaies Accounted for than by ſuppoſing, that there is a Continued Action or Reſiſtence in Real Space or Extenſion, and which we alſo Defend, that there is an Innate Contractive Force in Heavy Bodies, which makes them Act with a Greater Moment againſt the Expanſive of Air or Æther, than Lighter or more Expanſive ones.

The Continued Reſiſtence or Action of Real Space or Extenſion Acknowledged by Dr. Wallis *and Sir* Iſ. Newton *in their Notion of Projectiles.* Lowthorp's Abridgement of Philoſoph. Tranſact. Vol. 1. Pag. 572. Sir Iſ. Newt. Princip. Prop. 10. Schol. Lib. 2.

Dr. Wallis Philoſ. Tranſ. Pag. 545. Vol. 1. Lowthorp's Abridgm.

Dr. *Wallis* farther tells us. 1. *Si vis ut* V *moveat Pondus* P, *vis* mV *movebit* mP, *cæteris paribus, puta per eandem Longitudinem, eodem Tempore, h. e. eadem Celeritate:* Therefore the Reſiſtence is Equable, if that Computation proceeds juſtly.

2. *Si vis* V *movere poteſt Pondus* P *Celeritate* C, *vis* mV *movebit vel* P *Celeritate* mC, *vel Celeritate* C *Pondus* mP, *vel quodvis* P *ea Celeritate, ut factum ex Pondere & Celeritate ſit* mPC, *atque hinc dependet omnium Machinarum pro facilitandis Motibus conſtruendarum ratio,* viz. *ut quâ ratione augetur* P, *eâdem minuatur* C, *quo fiat, ut*

$$V . PC :: V . mP \times \frac{1}{m} C = PC.$$

3. *Celeritatis gradus ſunt Longitudinibus eodem Tempore tranſactis proportionales, vel Temporibus in eâdem Longitudine tranſactis reciprocè proportionales*; Either therefore there is no Reſiſtence at all, or an Equable one.

4. *Impingentia corpora reſilient ab invicem, ſi Impetus ex vi Reſtitutivâ vel Elaſticâ ſit progreſſivo major.*

5. *In motibus acceleratis & retardatis Impetus pro ſingulis momentis is reputandus eſt, qui gradui Celeritatis tum acquiſito convenit; ubi per curvam fit motus, directio motûs ea eſt, quæ Tangentis; ſi acceleratus Motus ſit & per curvam, Impetus in ſingulis punctis erit ut acceleratio & obliquitas Tangentis; hic motus Pendulis competit.*

6. *Impetus obliquè Impingentis eſt ad ejuſdem Impetum, qui eſſet, ſi directè Impingeret, cæteris paribus, ut Radius ad Secantem Anguli Obliquitatis; Et eadem eſt ratio Gravitationis Gravium obliquè Deſcendentium ad eorundem perpendiculariter Deſcendentium Gravitationem*; Therefore, as in oblique Deſcents on the Surfaces of Water, ſo in Air Motion is more Retarded; and in General, it is Plain, that a Continued Reſiſtence in Æther is ſuppoſed by theſe Philoſophers.

The Continued Reſiſtence or Action of Real Space or Extenſion Acknowledged by Sir Chriſtopher Wren, Hugens, Bernoulli, *and* Leibnitz, *in their Notion of the Velocities of Bodies in their Mutual Occurſe and Impulſe, or the Motion of them in Curves, or in their Elaſticity.* Philoſoph. Tranſact. Pag. 547, 548. Vol. 1. Lowthorp's Abridgm. as alſo, Pag. 550, 551, 553, 554.

§. 6. Sir *Chriſtopher Wren* ſays. 1. *Velocitates corporum propriæ & maximè naturales ſunt ad corpora reciprocè proportionales*; Therefore there muſt be a Reſiſtence, beſides what Ariſes from their own Gravity; for the *Vires acceleratrices* to the Center are owned to be alike in both the Bodies, the ſame or an Equal Force therefore will change both the Bodies from that Determination, and ſince they take another, which is Reciprocally Proportional to their Bulks, there muſt be as much Reſiſtence, as if they were Reſiſted in every Part; and conſequently is to be Attributed to the Medium, thro' which they Move; For the Acceleration of the Parts in either Bodies is owing to one and the ſame Force, and therefore it is one and the ſame Force, which will alter that Acceleration.

2. *Corpora habentia proprias Velocitates etiam poſt Impulſum retinent proprias.*

3. *In Colliſione Corporum, ut in Librâ ſuper Bina Fulcra vel Centra,* 2 *Corp.* 1 *Corp.* :: 1 *Veloc.* 2 *Veloc.* therefore, if they meet *permutatis velocitatibus*, they are the ſame ſtill, if both Acted with Equal Force.

4. *Corpora Improprias Velocitates habentia ex Impulſu Reſtituuntur ad Æquilibrium.*

Hugens tells us in his *Regulæ de Motu Corporum ex Mutuo Impulſu.* 1. *Si Corpori quieſcenti duro aliud æquale corpus durum occurrat, poſt contactum, hoc quidem quieſcet, quieſcenti verò acquiretur eadem, quæ fuit in Impellente, Celeritas.*

2. *At ſi alterum illud Corpus æquale etiam moveatur, feraturque in eâdem Lineâ rectâ, poſt contactum Permutatis invicem Celeritatibus ferentur.*

3. *Corpus quâmlibet magnum à Corpore quâmlibet exiguo & qualicunque celeritate Impacto movetur. Quæ eſt Regula Generalis Determinandi Motum.*

4. *Quantitas motûs duorum Corporum augeri minuivè poteſt per eorum Occurſum, at ſemper ibi remanet eadem quantitas verſus eandem partem, ablatâ inde quantitate motûs contrarii.*

5. *Summa productorum, factorum à Mole cujuſlibet corporis duri ductâ in quadratum ſuæ Celeritatis, eadem ſemper eſt ante & poſt Occurſum eorum.*

6. *Corpus durum quieſcens accipiet plus motûs ab alio corpore duro ſe majori minorivè per alicujus tertii, quod mediæ fuerit quantitatis, interpoſitionem, quàm ſi percuſſum ab eo fuiſſet immediatè, & ſi corpus illud interpoſitum fuerit medium proportionale inter duo reliqua, fortiſſimè omnium aget in quieſcens.*

7. *Conſiderat Hugenius Corpora ejuſdem materiæ, ſive cujus Moles Æſtimatur ex Pondere.*

8. *In Corporibus Sphæricis demonſtrare ſe poſſe ait, (& videtur ipſi eandem eſſe Naturæ Legem reliquis omnibus, ſive Duris, ſive Mollibus, ſive directè, ſive obliquè Occurrentibus) Centrum commune Gravitatis duorum vel quotlibet Corporum æqualiter ſemper promoveri verſùs eandem partem in Lineâ rectâ ante & poſt Occurſum.*

All which Laws or Rules do either ſuppoſe no Action or Reſiſtence in Space at all, or an Equable and a Continued one; If the Firſt, they are mere Abſtracted Notions, and do not Explain, what the Real Motion of Bodies is from Mutual Impulſe; If the Laſt, That Reſiſtence and Action in all the Parts of Space is Confeſſed, which we Contend for.

It is farther ſaid by Others. 1. *In Cycloide Grave quodlibet cadens ex quovis ejùs puncto attingit fundum eodem tempore, quo eum attingeret, ſi caderet ex quovis ejuſdem puncto alio.*

2. *Globulus ſuſpenſus è funiculo juſtæ longitudinis, & intra duas Cycloides vibrans, movetur in Cycloide, ergo vibrationes ejuſmodi ſunt Synchronæ*; This is Demonſtrated by a Perſon of Quality; And the ſame by *Hugens* in his *Horolog. Oſcillator.*

3. That an Heavy Body Deſcends Quicker in a Cycloid Drawn, than in a Right Line Drawn, betwixt Two Points, not in the ſame Vertical Line, but Unequally diſtant from the Horizon; This is Proved from *Bernoulli's Probl. pag.* 551. and another *Probl. pag.* 553. and is ſhewn by Dr. *Keill* in his *Introd. ad Ver. Phyſic. Lect.* 15. *Pag.* 221. *Lemm. Edit.* 3.

4. *Leibnitz, Pag.* 554. *Philoſ. Tranſact. Abridged, Vol.* 1. ſays, That a Spring being Bent to the ſame Degree, will always Unbend it ſelf in the ſame Time, Provided it find the ſame Freedom of Unbending it ſelf ſuddenly, which Proves there is a Reſiſtence or Action in Space: And all the foregoing Propoſitions Evince it, or elſe, that they are merely Abſtracted and Mathematick Reaſonings without giving us any Explication of the Actions and Forces in Nature.

§. 7. That *Des-Cartes* makes the Particles of his Celeſtial Matter, by being Reflected from the Surface of the Earth, to drive down into their Places thoſe Terreſtrial Bodies, which they find above them, §. 20, 21, 22, 23. of his laſt Book of his *Principia Philoſophiæ*, is known.

A Continued Action or Reſiſtence in Space Acknowledged by Philoſophers in their Accounting for the Deſcent of Heavy Bodies. Philoſ. Tranſ. Pag. 560. Vol. 1. Lowthorp's Abridgm. & Pag. 561.

Dr. *Halley* Anſwers: 1*ſt*. That his Obſcurity in §. 23. is an Argument of his Confuſed Idea of Gravity. 2*d*. That it is unlikely, that a Body moving Downwards ſhould be ſo moved by an Impulſe Upwards. We do not here Defend *Des-Cartes's* Account of Gravity, only ſhew, that Philoſophers have Thought a Continued Action in Space no Inconceivable Thing.

Dr. *Voſſius* Aſcribes it to the Diurnal Rotation of the Earth.

Dr. *Halley* Anſwers: 1*ſt*. That there is Demonſtration, that Bodies, circularly Moved, Recede from the Center of their Motion, and if not kept faſt by ſuch a Principle, as Gravity, would be caſt into the Air in a Tangent. 2*d*. That the Effect of this Principle of Gravity is nearly Equal every where, or rather Leſs towards the Equinoctial than towards the Poles, as very certain Experiments Evince, whereas, where the Motion was Swifteſt, the Effect would be the moſt Conſiderable; which is a Proof, either that the Weight of the Air does contribute to Gravity, or that Bodies being Moved in a Circular Motion with the Air, have a leſs Circular Impreſſion to hinder their Deſcent near the Poles than near the Equinoctial: So that let us take Dr. *Voſſius*, or Dr. *Halley*, there is an Action Acknowledged in Space, and a Continued one.

Others Aſſign the Preſſure of the Atmoſphere.

Dr. *Halley* Anſwers. 1*ſt*. That the Atmoſphere's Preſſure is it ſelf from it's own Gravity. 2*d*. That the Preſſure of the Upper Parts of the Atmoſphere, by bending the Springs of the Lower, give the Lower a Force of Reſiſtence Equal to the Bearing or Compreſſive Force of the Upper, having of it ſelf no Force at all. 3*d*. That 'tis unlikely there ſhould be any Deſcent of a Body by the Intervention of the Atmoſphere, becauſe there is an Equal Preſſure of it Upwards and Downwards. 4*th*. By a plain Experiment a Feather in *Vacuo* Deſcends with near the ſame Velocity as a Stone, from which it is Evident, that the Atmoſphere is ſo far from being the Cauſe of Gravity, that it hinders it's Effect; becauſe the Atmoſphere is nearly of the ſame Specifick Gravity with a Feather, but the *Vacuum Boylianum* is not, which ſhows, that in Air there is Gravity and Reſiſtence, or that there is an Action every where, and Diſperſed thro' every Point of Space.

Again, Others make the Earth Attract as a Loadſtone.

To which He Anſwers, That the Loadſtone Attracts only in and about it's Poles, but the Earth near Equally in all it's Parts.

Lastly, Others Aſſign a Sympathetical Attraction betwixt the Earth and Bodies, whereby they have, as it were, a Deſire to be United to it.

He Anſwers *1ſt.* This is to ſay, that Bodies Deſcend, becauſe they Deſcend. *2d.* Beſides, a Feather would then Deſcend with the ſame Appetite, or Deſire, as a Stone. He afterwards tells us, what the Properties of Gravity are, according to the New Syſtem, which we have before Inquired into in our Account of the Phyſical Aſtronomy. He ſays;

First, By this Principle the Earth and all the Heavenly Bodies are kept from Diſſolution, ſo that any Part of them cannot Recede far from the Reſpective Centers, and tho' violently thrown from them they ſtill Tend to be Re-united. But this Doctrine of the Projectile and Gravitating Force has been already Examined, and if it had not, what is This, unleſs the Sympathetical Attraction he before Condemns?

Second, That all Bodies Deſcend towards a Point, which either is, or is very near the Center of the Earth, conſidered as a Globe, to which it Approaches, being Really a Spheroid. Which Center we have before Denied and Confuted, as well as the Earth's being a Spheroid.

Third, That the Center is fixt, becauſe, if it Changed it's Place, the Low-Lands of that Side of the Globe, to which it Approached, would be over-flown, and the oppoſite Side exhibit New Iſlands; but the Iſlands of the Mediterranean, near which the oldeſt Writers Lived, are much what they were in their Times, and therefore no Alteration of the Center; That the Univerſal Deluge is beſt Accounted for by an Approach of the Center to the Parts of the then Habitable World, which, if it were only a 2000*th* Part of the Radius, would be ſufficient to Drown the Higheſt Hills. All which is Calculated and Affirmed upon the Suppoſition of the Truth of the Phyſical Aſtronomy, which is Diſputed.

Fourth, That in all Parts of the Surface of the Earth, as well as in all Points Equidiſtant from the Center, the Force of Gravity is nearly Equal; That the French Obſervers near the Equinoctial, and Himſelf at St. *Helen*'s 16 Deg. South Lat. have found however, that it is there neceſſary conſiderably to ſhorten the Pendulum to make it vibrate Seconds; He thinks this Proceeded from the Height of the Place above the Sea, where the Gravity being Diminiſhed, it was Requiſite to ſhorten the Pendulum in Proportion; but this we have Conſidered and Refuted in the Firſt Book of the Preſent Philoſophy.

Fifth, That the Impediment being Removed, as in the Experiment of the Feather in *Vacuo*, the moſt Compact and moſt Looſe, the Greateſt and Smalleſt Bodies, Deſcend the ſame Spaces in Equal Times; Which, together with the former Property, ſhews the Difference betwixt Gravity and Magnetiſm, this Affecting only Iron, and towards it's Poles; Hence 'tis ſaid to be Evident, that there is no poſitive Levity, that Cork, which Swims on Water, or a Feather in the Air, can no more be ſaid to be Light than Iron, which Swims on Mercury, they are only Bulk for Bulk, not ſo Heavy as the Fluid they Swim in; This is Contradicted by our Doctrine of the Expanſive and Contractive Forces; However Air is here Confeſſed to be as much a Fluid, as Water, and as Water has no Vacuities, ſo neither is it neceſſary, that Air or Real and Material Space ſhould have.

Sixth, That this Power Increaſes as we Deſcend, and Decreaſes as we Aſcend, in Proportion to the Squares of the Diſtances from the Center Reciprocally, ſo as at a Double Diſtance to have but a Quarter of the Gravity; *Vid. Miſcell. Curioſ. Vol.* 1. *Pag.* 14. Upon this Principle, Sir *Iſ. Newton* is ſaid to have made out all the Phænomenons of the Celeſtial Motions; And it is thought to be very Rational, that the Attractive or Gravitating Force ſhould Exert it ſelf more in a Small Sphere, and be weaker in a Larger, but the Surfaces of Spheres are as their largeſt Circles, *i. e.* as the Squares of their Radii, therefore this Force will be Diffuſed in Larger Spheres, and Contracted in Smaller; That is, in Proportion to the Squares of the Radii, and will be always Equal upon the Surface of the ſame Sphere; We have in another

Place Examined into the Truth of this Reaſoning, and ſhall only Obſerve, that it is therefore here again ſuggeſted, that the Action or Force is not Inherent in Bodies, but in the Spaces they Paſs thro', which have Different Actions, as they are at Different Diſtances from the Earth; and according to the Different Diffuſions of Space, that the Action is variouſly Diffuſed thro' it; which is a plain Aſſertion of an Action or Force Continued thro' every Point or Portion of it.

Seventh, That Bodies, tho' they really Tend to the Center, Deſcend in Parallel Lines, the Difference being ſo Small, as by no means to be Diſcovered; And therefore a Reſiſtence or Force in Æther may really be, where it is not Diſcernible; nay it is Evident by this Philoſophy; for Gravity, which is a Reſiſtence or Force, tho' it Increaſes and Decreaſes, as in Property the *6th*, yet is not ſenſible in Projects or Deſcending Bodies, becauſe they fall from a ſmall Height; and all Bodies upon the Surface of the Earth Gravitate to each other, which no one could ever yet Perceive.

A Continued Force, Reſiſtence, or Action in Space Confeſſed by Philoſophers in their Accounts of the Velocities of Falling Bodies. Philoſ. Tranſ. Pag. 563. Vol. 1. Lowthorp's Abridgment.

§. 8. This Action, or Force, in all Parts of Space is likewiſe Confirmed from Thoſe Accounts, which Philoſophers give us of the Velocities of Deſcending Bodies.

First, The Velocities of falling Bodies are ſaid to be Proportionate to the Times from the Beginning of their Falls; Suppoſe a Body has the Velocity of 32 Feet at the End of 1″ of Time, it will have Double of 32 Feet at the End of 2″, Triple at the End of 3″, becauſe, each 1″ of Time, a new Velocity is added Equal to the Former from the ſame Power of Gravity continually Acting thereon; ſuch Force therefore cannot be Inherent in Bodies, but in the Space they Paſs thro', or in the Earth, for Body could not Increaſe it's own Force or Velocity, but, in every Point of Extenſion being ſtill thruſt Downwards with an Equal Action, it muſt continually Acquire a new Velocity Proportional to it's Progreſs; If then this Power or Force Exiſts in Space, we have what we Contend for; If in the Earth, It is neceſſary, that it ſhould be Diffuſed from thence thro' all Space, wherever Gravity Prevails, otherwiſe Bodies could not be Affected by it; And take which we will, a Continued Action or Force is owned on both Hands to Reſide in the Univerſal Expanſe, if there is an Univerſal Gravitation thro' it.

Second, The Spaces Deſcribed by the Fall of a Body are as the Squares of the Times from the Beginning of the Fall; For they are as the Areas of Triangles, which are as the Squares of their Homologous Sides, therefore as 1. 4. 9. 16. &c. and the Differences, according to *Galileus*, as 1. 3. 5. 7. &c.

Third, The Velocity, which a falling Body Acquires in any Space of Time, is Double to that, wherewith it would have Moved thro' the Space Deſcended by an Equable Motion in the ſame Time, *i. e.* the Velocity of falling Bodies is Double to that, which in the ſame Time would have Deſcribed the Space faln thro' by an Equable Motion.

These Three foregoing Propoſitions depend upon this, that in every Moment of Time there is a Triangle Produced by this Velocity, becauſe what Law obtains in Larger Spaces, the ſame does in Smaller and even in the Minuteſt, and which have been before Conſidered in Book the Firſt of this Philoſophy.

Fourth, All Bodies, on or near the Surface of the Earth, in their Fall Deſcend ſo, as at the End of the firſt *2d* of Time they have Deſcribed 16 Feet 1 Inch *London* Meaſure, and Acquired the Velocity of 32 Feet 2 Inches in a *2d* of Time; Becauſe it is Demonſtrated by *Hugens*, *25th Prop. Part 2d. Horol. Oſcillat.* That 16 Feet 1 Inch is the Deſcent of a Body in a *2d* of Time, and by *Prop. 3d.* the Velocity will be Double thereto; and near to this it hath been found by ſeveral Experiments; near, becauſe the Swiftneſs of the Fall hinders us from Determining exactly it's Quantity, and which Difference perhaps may be owing to the Reſiſtence or Action of the Medium.

A Continued Action or Reſiſtence in Space Acknowledged by Dr. Halley, Sir Rob. Moray *&c. in their Doctrine of Projects.*

§. 9. From theſe Four Propoſitions, all Queſtions concerning the Perpendicular Fall of Bodies are ſaid to be eaſily Solved, and the Doctrine of Projects likewiſe Deducible, If we Aſſume the Two following Axioms.

1ſt. That a Body, ſet a-Moving, will Move on Continually in a Right Line with an Equable Motion, unleſs ſome other Force or Impediment Intervene, whereby it is Accelerated, or Retarded, or Deflected, which depends upon this, that Matter is perfectly Paſſive; But this Axiom we are not willing to Allow; becauſe we form our

Philoſ. Tranſ. Pag. 563. 565. 571. 580. 583. Lowthorp's Abridgm. Dr. Halley, Pag. 563.

our Notions of Matter from what we Generally Perceive, and the Generality of Matter being of a Sluggiſh and Groſs Nature, we from thence Conclude that all is ſo; whereas there are a Thouſand Inſtances of Fire, Spirituous Liquors, Vegetables, *&c.* which are not ſo, and even the groſſeſt Parts of Matter have Properties, according to this Philoſophy, more Active, than what our Senſes Inform us of, as, the Gravitation of one Portion of Matter to another, and Others, which are diſcernible by our very Senſes, which no ways Agree with a Paſſive Nature, as Elaſticity or Springineſs, Vegetation, *&c.*

2d. That a Body, being Agitated by Two Motions at a Time, does by their Compounded Forces paſs thro' the ſame Points, as it would do, were the Two Motions Divided and Acted ſucceſſively, or, that the Diagonal is Compounded of the Sides of a Parallelogram; This Axiom has alſo been Examined into, Book the Second, and the Objections againſt it need not be Repeated here. However it is Affirmed;

Pag. 565.

First, That the Motion of all Projects is in the Curve of a Parabola, becauſe the Project is Compounded of Two Forces, the Projectile, which, if there is no Impediment, moves Equal Spaces in Equal Times, and is as 1.2,3.4, the other Deſcending, and is as the Squares of the Times, or as 1.4.9.16, which by Axiom the *2d.* meet in their ſeveral Reſpective Points; the Firſt are Parallel to the Ordinates of a Parabola, and the Second to it's Abſciſſes, and the Property of a Parabola is, that the Abſciſſes are in a Duplicate Reaſon of the Ordinates. Since therefore a Parabola ſuppoſes no Reſiſtence, that Deſcents are in Parallel Lines, which may as well be denied as Reſiſtence, and that Bodies without an Impediment move Equal Spaces in Equal Times, whereas the Air Reſiſts, as well as Gravity, it is Plain, that this is nothing elſe than a Mathematick and Abſtracted Reaſoning.

Pag. 571.

Second, That the Force of a Shot is always the Leaſt at the Vertex of the Parabola, and at Equal Diſtances from it, it's Force is always Equal; From whence, according to this Reaſoning, a Bomb ſhall have as much Force, as it ſeems, when it is wholly Spent, as when it Firſt comes out of the Mortar; ſo that, notwithſtanding, This, in an Abſtracted Reaſoning, may be true, when we only conſider the Projectile and Gravitating Force, yet, ſince it is not in fact True, it is Manifeſt, that the Reſiſtence or Action of Space is very Material in the Doctrine of Projectiles, which occaſions ſo Great a Difference from what is Concluded by a Geometrick Deduction.

Third, That in the Elevation of a Mortar; *1ſt.* If very Great, the Bombs fall too Perpendicularly and bury themſelves in the Ground without any Effect, and that therefore the Beſieged unpave their Streets; *2d.* That, if they come more obliquely, they many Times break upon, or near, the Surface of the Earth; From whence it is plain, that there is a Reſiſtence or Action of the Air or Æther, or Real Space, as in other Fluids, in which Heavy Bodies are leſs Obſtructed in their Force, when Projected Perpendicularly, than when Obliquely; *3d.* That by Diminiſhing or Increaſing the Quantity of Gun-Powder, the ſame Effects may be Produced as by a Greater or Leſs Elevation of the Mortar, and to more Advantage; becauſe you know the Force of Gun-Powder, whereas the Elevation is uncertain in Ships, where the Waves are ſo; Which is an Evident Eviction, that the Reſiſtence or Action of the Medium, or Space, has a conſiderable Influence in Produceing theſe Phænomenons, ſince it may be Conquered by a determinate Force of Powder, as well as by the particular Elevation of the Mortar; *4th.* That a Mortar 45° Elevated is little Different, in it's Projection, from one Elevated ſome Degrees above, or Depreſſed below, that Poſition; Which may be Poſſibly, becauſe ſuch an Elevation is in the middle Situation betwixt the greateſt and the leaſt Reſiſtence or Action of the Air or Æther, or Real Space; for in a Perpendicular Elevation probably there is the Leaſt, and in an Horizontal the Greateſt, the Reaſon of which may be, that in an Horizontal all the Parts of the Æther, or Real Space, have an Equal and the Greateſt Gravity, and therefore the Greateſt Action or Reſiſtence; but in a Perpendicular Elevation the Gravity of the Æther, or Real Space, is Diminiſhed upwards, and conſequently the Action or Reſiſtence; Nor are theſe our Concluſions altogether Unjuſtified from Dr. *Halley*, who ſays, That the foregoing Rules would be Rigidly true, were it not for the Oppoſition of the Medium, whereby the Projectile Motion is Retarded, as well as the Deſcent, being not exactly as the Squares of the Times: 'Tis certain Air is to Water in Weight, as 1 to 800 nearly; However in large Shot of Metal, whoſe Weight ſurpaſſes many Thouſand Times that of Air, and whoſe Force is very great in Proportion to the Surface, wherewith they, namely Shot, preſs thereon, this Oppoſition is ſcarcely Diſcernible; For by certain Experiments, in a ſolid Braſs Shot of 4½ Inches Diameter and 14 *l.* Weight, the Ranges above and below 45° were nearly Equal, tho' the under Ranges went rather the fartheſt; In a ſmaller Braſs Shot of about 1½ Inch Diameter, caſt by a Croſs-Bow to 400 Feet, the Force being much more Equal than in the Mortar Piece, the Under Ranges evidently out-went the Upper; So that in the Smaller Shot, tho' the Air's Reſiſtence is to be Accounted for, yet in Larger it may be Neglected, and his Concluſion is; If ſo, it will be Objected, that the Air's Reſiſtence is Leſs towards the Horizon than the Perpendicular; To which he Anſwers, That the Air's Reſiſtence is really Greater, but the Nearer we come to a Perpendicular, the more Direct is the Projectile Motion contrary to the Deſcent, which muſt therefore diminiſh the Force of the Project; To which we Anſwer, That, according to the Mathematick Doctrine, the Projectile Motion muſt be the ſame, any way, as alſo the Gravitating. But this we are Agreed in, that there is a Force, Action, or Reſiſtence in Real Space or Extenſion. We ſhall now Proceed to Sir *Rob. Moray*'s Obſervations.

Sir Robert Moray, Pag. 580.

FIRST, Take leſs Powder than a full Charge, and ſtill Diminiſh it every Charge, and it will throw a Bullet ſtill a ſhorter and ſhorter Way, then Increaſe it, and it will thro' it farther and farther, till it comes to a juſt Charge, when, if you Increaſe it never ſo much, it will be of no Advantage; We ſay, becauſe it's Force is not Infinite to Vanquiſh the Reſiſtence, or Action, of the Air, or Real Space.

SECOND, The Powder and Bullet is to be Rammed home Equally at every Shot, he means in Trials made, to know the juſt Charge of a Piece, tho' the looſer the Powder lies, it Fires the better; We ſay, becauſe the Contractive Force in the Ramming ought to be Equal, to know what the Expanſive Force of the Fire and Powder is able to Effect againſt the Contractive of the Bullet and the Medium, or the Action and Reſiſtence of Lead or Braſs and of Real Space; and the Looſer the Powder is, the Better it takes Fire; Becauſe the Expanſive Force of the Powder is Greater, the Leſs it is Rammed and Contracted together, and therefore more Readily Joins and Unites with the Expanſive of Fire.

THIRD, Powder finding more Reſiſtence from a Bullet Drives it out with Greater Violence, as in Screw Barrels, or where Wedges or wooden Tampions are made Uſe of, (the Deſcription of which he gives *Pag.* 582.) which may Increaſe the Force of the ſame Charge of Powder, till the Piece burſts; The ſame Quality is alſo Experienced in Air, that the more it's Action is Compreſſed or Reſiſted, the Stronger it is, which ſhews, that Real Extenſion has a certain Action or Force with Powder it ſelf; and the Phænomenons on both Hands may be Explained by the Expanſive, Acting againſt the Contractive.

FOURTH, Guns fixt from Recoiling by a certain Weight, laid upon their Carriages or Barrels, will require leſs Powder to Shoot a Bullet the ſame utmoſt Length, than when they Recoil freely; Becauſe none of the Motion or Force is ſpent contrary to the Project, but is wholly Reflected upon it; Or rather, as We ſay, becauſe the Contractive Force is Greater, as in the former Phænomenons, and therefore the Expanſive of the Powder is Increaſed.

FIFTH, If there be the ſame Weight in the Carriage, there will be the ſame Recoil; as if you cut off the Muzle and place it upon the Carriage, the ſame Recoil will be, as when Continued in one Barrel, becauſe there is the ſame Reſiſtence to the Violence of the Powder; Or, according to our Doctrine, becauſe the Action of the Contractive againſt the Expanſive is in both Caſes Equal.

Mr. Greaves, Pag. 583.

MR. *Greaves* Informs us, That a Cylinder-Bore carries with much the ſame Force, other Circumſtances being Equal, as Bullet, Length and Weight of the Piece, with 8, 9, or 10 *l.* of Powder.

THAT a Taper Bore, the Weight an 100 *l.* more, 4 Inches longer, and the ſame Bullet, carries with the ſame Force as the Firſt, with 7 *l.* of Powder.

THAT a Braſs Piece carries thro' all the 3 Butts with 9 *l.* of Powder, and 18 *l.* Bullet of Iron, whereas an Iron one, with 4 *l.* of Powder and 9 *l.* Bullet of Iron only Entered the ſecond Butt, and a Braſs one with 4 *l.* of Powder and 9 *l.* Iron Bullet carried thro' 2 Butts, and with 3 *l.* of Powder almoſt thro' 2 Butts.

FROM whence it ſeems Evident, that the Expanſive Force of the Metal, or the Elaſticity or Springineſs of it, Adds a Force to the Action or Expanſive Force of the Powder; and from the Firſt Experiment, that the Force of Powder does not lie in the Quantity, but in the Specifick Action, or Expanſive Force, of it, as the Gravity or Acceleration of Bodies to the Center likewiſe does, and perhaps it is a General Law of Action; But then it will be Objected, that 3 or 4 Grains of Powder would be as Effectual as 3 or 4 *l.* to which we Anſwer, ſo it will, *Cæteris paribus*, that is, if the Reſiſtence or Contractive Force as Gravity, or any other Reſiſtence or Contractive Force, as the Straitneſs of the Bore, has no greater a Proportion to 3 or 4 Grains, than it has to ſo many Pounds; *viz.* the proper Actions of Bodies are to be only conſider'd in this Caſe, and how the one differs from the other in Quality, Abſtracting from the Quantity, *&c.* Thus for Inſtance, a ſmall Particle or Point of Iron may be probably carry'd as far by 3 Grains, as 9 *l.* by 3 *l.* of Powder, every Thing elſe I ſay being conſider'd as equal, excepting the Specifick Action or Expanſive Force of the Powder, compar'd with the Specifick Reaction, or Contractive, of the Iron. Thus the Specifick Action of a Drop of Water or of an Ocean of it is the ſame.

THESE Experiments however prove the Reſiſtence, or Action, of the Air, or Real Extenſion, ſince there are different Forces of Powder requir'd to carry the ſame Weight a ſhorter Space, or a longer. And ſeeing all the Experiments of this and the foregoing Sections, as well as of the following, Extend to every Point or Portion of Space without Diſtinction, We from thence Affirm, that ſuch Reſiſtence, Force, or Action, is Continued, Uniform, and Uninterrupted.

A Continued Reſiſtence, Force, or Action, in real Space or Extenſion, Acknowledged by Philoſophers in their Notions of Wind-Guns, Jects d' Eau, *Velocity of Air, ruſhing into an Exhauſted Receiver, and of Wind produced by the Fall of Water,* &c. Philoſoph. Tranſact. Vol. I. Pag. 584, 585, 586, 587, 593. Lowthorp's Abridgm. Dr. Papin, Pag. 584.

§. 10. A Ball of Lead may be Caſt to a very great Diſtance by Compreſſion or Rarefaction; *Rohault* deſcribes an Inſtrument for the Firſt, and Dr. *Papin* for the Laſt, and which amounts to this, that if, by certain Methods preſcrib'd, you Compreſs the Air in a Cylinder, it's Action being much greater than that of the External, it will violently Ruſh out, and carry along with it the Lead, which is Included; On the Contrary, if you Extract the Air out of a Cylinder by a Pneumatick Engine, the External Air will be too Powerfull for the Internal, and conſequently drive the Lead thro' it to a great Diſtance; The Effects of Rarefaction are little leſs than thoſe of Compreſſion, or which is all one, the Force of Compreſſed Air, in Reſpect of Common Air, is not much more than the Force of common Air, in Reſpect of the Rarefacted; However from both theſe Experiments it is Manifeſt, that there is a vaſt Reſiſtence, Force, or Action in the Air, or in real Extenſion, and that ſuch Force, or Action in General, is Uninterrupted and Continued in it, ſince theſe Experiments Obtain in any Parts or Parcels of Space whatſoever; Theſe likewiſe are Inſtances of the Expanſive and Contractive Forces in Nature, for Common Air, Contracted becomes more Expanſive, and in its natural State is Expanſive, which Power we may ſuppoſe it Derives from the Expanſive of the Sun, as, what it has of the Contractive, is from the Earth, the Moon, and the Planets.

Dr. Papin, Pag. 585.

FIRST, In Order to Compute the Velocity of Air Ruſhing into an Exhauſted Receiver, or Wind-Gun, we are told, that *Jects d'Eau* riſe not ſo high, as their Source, by Reaſon of the Reſiſtence of the Medium, tho' they have originally a Force or Swiftneſs, which would carry them to that Height, being an Hydroſtatical Principle; Which may be Explain'd from the Expanſive and Contractive Forces; For if Water, for Inſtance, has a Comparative Expanſive Force in Reſpect of the Earth, which has a Contractive, the Parts of it, if permitted, will Riſe to the ſame Height from it, whilſt the Reſt are equally Balanced with one another.

SECOND, From which Hydroſtatical Principle this Propoſition is Deriv'd, That of Fluids equally Preſs'd the Lighter riſes higher than an Heavier, in the ſame Proportion as one is Heavier than the other, that is, the Height is in a Reaſon Reciprocal to the Weight; Which may be likewiſe Explained from the Expanſive and Contractive Forces; For the Lighter any Body is, the more Expanſive is it's Force, and the farther it will Recede from the Contractive of the Earth; Thus Spirituous Liquors, and Fire, will riſe higher than in a Reciprocal Proportion to their Weight, by their exceeding Expanſive Force and their Volatility: From this Expanſive Force, is Solved that Phænomenon alſo, which the common Hydroſtaticks cannot do; Namely, Why, upon mixing a Spirituous Fluid with Water, it Diffuſes and Diſperſes it's ſelf thro' it in a Thouſand Undulations, whereas if Lighter, it ſhould Swim at the Top, or, if Heavier, Subſide to the Bottom; The Reaſon of which can be only, that the Expanſive Force Exerts it's ſelf in all Directions, wherever it is oppoſed by the Contractive of the Water, as the Expanſive of Fire or Flame, in the Contractive of the Fuel, which it Surmounts and Vanquiſhes.

THIRD, We are farther told, that of Fluids equally Preſs'd the Lighter Acquires a Greater Velocity, the Heavier a Leſs, and are to one another Reciprocally, as the Square Roots of their Specifick Gravities, *i. e.* As the Square Roots of their Heights, Propoſition the *1ſt*. That is, the Squares of their Velocities are Reciprocally as their Heights, or the Spaces deſcrib'd by the fall of Bodies, as the Squares of the Times from the beginning of the Fall; But the Velocities are Proportionate to the Times of Deſcent, therefore the Squares of the Velocities may be ſubſtituted for the Squares of the Times; And in the ſame Proportion as Bodies Preſs Downwards, the Fluids muſt Riſe Upwards.

FOURTH, To know then the Velocity of the Air, driven by any Degree of Preſſure, we are, it is ſaid, to Enquire, what the Velocity of Water is under the ſame Preſſure, but as much as the Square Root of the Gravity of Water exceeds the Square Root of that of Air, ſo much the Velocity of Air will the Velocity of Water.

FIFTH, The Preſſure of Air is = to the Preſſure of 32 Feet of Water, which would make the Water Aſcend with a Velocity ſufficient to carry it 32 Feet, and therefore the Velocity is = to 45 in 1″ of Time, tho' it is 64 according to Dr. *Halley*, or rather 16 Feet is only deſcended in a ſecond, and the Velocity = to 32 Feet, therefore 32 Feet is deſcended in 2 Seconds and the Velocity is 64.

SIXTH, The Gravity of Air to Water, tho' it is known to be Variable by the Height, Heat, or Moiſture of the Atmoſphere, in General is ſaid to be as 1 to 840, *Franceſco Lana*, as 1 to 640, their Square Roots therefore will be as 29. 1, and the Velocity of Air 29 Times greater than that of Water, or 29 Times 45 = to 1305 Feet will be perform'd by it in a Second of Time.

FROM All which it appears, that Dr. *Papin* has the ſame Notion of Air, as of a thinner Fluid, and conſequently Endu'd with Action, Force, or Reſiſtence, and it is plain by it's Ruſhing with all that Velocity into the exhauſted Gun or Receiver. From hence we are carry'd to Dr. *Pope*'s Obſervations of Wind, being produced by the Fall of Water in the Braſs Works at *Tivoli*.

A Small Pipe paſſes betwixt a Tube and the Hearth, which has a Foramen in the middle to the Air; There is a conſtant Fall of Water into the Tube, and if the Foramen is ſtopt, there is as conſtant a Courſe of Wind thro' the Pipe to the Fire, which ſerves inſtead of a pair of Bellows, if that End of the Pipe towards the Fire is ſtopt and the Foramen open'd, there will be a Wind ſtrong enough to make a Ball play. So *Rohault*'s or *Des-Cartes*'s Braſs Ball fill'd with Water produces Wind, if ſet to the Fire, as likewiſe Mr. *Boyles*'s Æolipile.

Dr. Walter Pope, Pag. 586.

THERE are two Queſtions, which We make upon this. *1ſt*. Whether the Wind is Water Rarefy'd, but Water ſtill, or whether it is another Fluid, Thin'd in it's Dimenſions and Increaſed in it's Activity by the mixture of Fire or the Expanſive. *2d*. Whether the Stream of Wind to the Fire proceeds from the Attraction of that Element, or from the diminiſhed Reſiſtence of the Air in the Pipe, as Air Ruſhes into an Exhauſted Receiver. *1ſt*. It ſeems not to be Water, but a Fluid of a different Extenſion and Activity, and tho' it may be poſſibly condenſed into it, as Clouds and Winds may, yet whilſt they are Clouds or Wind, they are another Fluid. *2d*. It is Evident, that Fire has a Power of Attracting Liquids, which have the ſame Expanſive Force with it, as Oil in Lamps; And the Fumes of Water and other Liquors Boiling, it is Probable, Proceed from the Expanſive Force of the Fire, and the Contractive Force of the Fluid and Veſſel; which may be the Cauſe, why the Fumes do not Aſcend directly, but Hover upon the Veſſel, being Drawn down by the Veſſel and Fluid, and Repell'd by the Fire, and which at length by the Superior Force of the Expanſive of the Fire are carry'd upwards; The Reaſon therefore of the Stream of Wind to the Fire ſeems to be from the Conteſt there is betwixt the Expanſive Force of the Fire and the Contractive of the Water, which, when too Weak for the Fire, is Attracted by it, when too Strong and too Diſagreeable to it's Nature, is Repelled by it.

Dr. Hooke,
Pag. 587.
Fig. 180.

From Dr. *Hooke's* Horizontal Sails for a Wind-Mill, nothing more, or leſs, can be Collected, but that the Air Reſiſts more to the Vane, when ſtanding a-croſs to the Wind, than when ſet ſharp againſt it, and is ſtill a Confirmation of the Force, or Reſiſtence, or Action, of the Plenum we Defend.

Franceſco Lana and Dr. Hooke.

Franceſco Lana tells us in his Account of a Flying Chariot, ſomething like which we find in *Sturmius's Collegium Curioſum*, that if a Piece of Braſs, for Example, could be drawn out to that Thinneſs in forming a round Globe, as to make the Braſs Weigh leſs than the Air included in the Globe, if we were to draw out the Air, the Globe would Aſcend, and the ſaid Globe, being made extremely Thin and Large, it might at length be able, not only to Aſcend it ſelf, but to carry up 3 or 4 Men in it: To which Dr. *Hooke* Anſwers, that tho' the Preſſure of the Air is Equal on Equal Surfaces, yet in Larger the Preſſure is proportionably Increaſed, ſo that the Braſs Globe muſt be made likewiſe proportionably Stronger, and therefore it will not be Lighter, as it is made Larger.

From Which it is Evident, that the Preſſure is equally diffus'd thro' all the Air, or nearly, and therefore the Reſiſtence, Force, or Action, Univerſal; becauſe the Preſſure Encreaſes, as the Surface does, according to the Acknowledgement of the laſt Philoſopher, who ſeems to be the Wiſer of the Two, and leſs Addicted to ſuch Strange and Unaccountable Whimſies, as thoſe are, of Flying in the Air; To which Dr. *Wilkins* gave Riſe in his Natural Magick, and who was as Weakly follow'd by his Admirers, who Thought in Time, that Voyages to the Moon would be as Frequent, as thoſe to the *Levant*, or the *Caribbees*.

Dr. Papin,
Pag. 593.
Fig. 184.

The Contrivance of a Perpetual Motion by a Pair of Bellows and Mercury is thus Confuted by Dr. *Papin*; Becauſe the outward Preſſure of the Air would be Superior to the inward Preſſure of the Quickſilver, and hinder the Bellows from opening, the firſt being Equal every where to 27 Inches of Quickſilver, and the laſt only to 20, ſince the greateſt Preſſure is as 40, and the leaſt as 0, and the reſt in an Arithmetical Progreſſion betwixt 0 and 40, therefore $\frac{1}{2}$ the Number of the greateſt Term, *viz.* 20, into the whole Number of the Preſſures 40 is = to the whole Arithmetical Progreſſion; From which likewiſe a Reſiſtence, Force, or Action in every Part of the Medium is own'd and confeſſed by this Celebrated Philoſopher or *Wag*, who was Able to Confute *Another*, who was a Member of the College, where I Write, upon the ſame Topick. Let us now take Mr. *Conyers's*, and Sir *Samuel Moreland's* Speaking Trumpets.

A Continued Reſiſtence, Force, or Action in Real Space or Extenſion, owned by Philoſophers in their Opinions of their Doctrine and Propagation of Sound. Philoſoph. Tranſact. Vol. 1. From Pag. 593. to 596, *&c. Mr.* Conyers and Sir Sam. Moreland. Mr. Walker, Pag. 594, &c.

§. 11. From Conſidering of which it ſeems Evident, that the Æther has the Nature of other Fluids, becauſe by the Angularly Arch'd ones the Voice is ſaid to be Stopt, that is, the Progreſs of the Æther is hinder'd by the Sides of the Angles Opposing themſelves to it's Waves, and ſo driving them back upon the Succeeding; which if ſo, it is Manifeſt, that there is a Reſiſtence, Force, or Action diffuſed thro' the Medium, in which Sound is Promoted.

Mr. *Walker*, in his Account of the Swiftneſs of Sounds, and their Reflections, or their Ecchoes, Informs us,

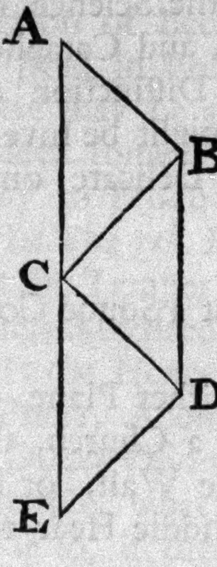
First, That by Obſerving Ecchoes in a Calm Morning a Sound moves 1526 Feet or 1378, at other Times ſomething above or under 1200 in a Second of Time, *Merſennus* makes it 1474, Mr. *Boyle* above 1200 Feet; The Academy *Del Cimento* by the Exploſion of Harquebuſſes, computed from the Flaſh to the Sound, 5739 Feet in 5 Seconds, which is leſs than any of the reſt by 1261 Feet in 5″, or 252, at leaſt, in 1″, which therefore was Deſcrib'd before the Flaſh came to the Eye, and Conſequently the Progreſs of the Flaſh to the Eye was in the ſame Time, as 252 Feet were Deſcrib'd by the Progreſs of the Sound; *viz.* The Flaſh Deſcrib'd 5739 Feet in the fifth Part of a Second; Granting therefore the Time, in which the Light of the Sun comes to us, as is prov'd to be $\frac{1}{6}$ of an Hour, or 10′, or 600″, or 3000 fifth Parts of a Second, the Action of the Sun may be known in Reſpect to that of Fire, ſuppoſing the Diſtance, or ſuppoſing the Action the ſame, the Diſtance may be known.

Second, That at Mid-way the Motion of the Sound from the Harquebuſſes was in half the Time exactly, ſo that the Space it Deſcribes is Proportional to the Time, which ſhows, that the Reſiſtence or Action of the Medium is every where Uniform, and A-like.

THIRD, He thinks by ſome Experiments, which He try'd, that the Sound Mov'd Quicker, when it was Calm, than in an Equable Wind, when it mov'd half way with it; Becauſe the Reſiſtence would be greater to it's Return than it's Aſſiſtance was to it's being carry'd forward.

FOURTH, That Sound moves Swifter at firſt than afterwards; The Reaſon muſt be, becauſe of the Reſiſtence or Action of the Æther.

FIFTH, In St. *John's* Cloyſter in *Oxford*, which is 104 Feet 7 Inches Long, the Sound was Reflected 11 Times in two Seconds; In New College Cloyſter, which is 160 Feet 8 Inches, 7¼ Ecchoes were Perform'd in 2 Seconds; The Firſt Repetition was Slower than the Second, or Third; The Reſt Slower in Order, the Third than the Second, the Fourth than the Third, *&c.* That is, the Subſequent were Slower than the Precedent.



BECAUSE The firſt was ſtruck farther off upon the Wall, the reſt in the Manner following, *A* is the Place, where the Perſon ſtands, *B*, the Firſt Percuſſion of the Voice, *C*, the Second, *&c.* But *AB* is longer than *AC*, and *AD* longer than *AC*, *AE* longer than *AD*, *&c.* So that the Eccho is always Heard from the Place, againſt which the Sound ſtrikes; and if the Wall is Smart and Hard, an Eccho will be produced, becauſe Sound is always from Reſiſtence, Reaction, Elaſticity, or Repulſion, which happens in Old Buildings, where the Walls are fixt by Nitre; but if they are Soft, and have little Reſiſtence, Elaſticity, or Repulſion, the Eccho will be Flat or none at all, as in New Buildings, and in Brick ones, rather than Stone, and in Clay Walls, rather than Brick; and from hence in Groves, where no Building is near, an Eccho may be Heard either from the Reſiſtence of the Nitrous, Stagnating, or fixt Air, or from the Old Trunks, which are fixt to that Degree of Reſiſtence; From Hence it appears likewiſe, that a Perpendicular oppoſite Wall is not Requiſite, becauſe you may Hear an Eccho in any Direction, if the Reſiſtence is Sufficient to Rebound it; and it is exactly the ſame, as if any one were to ſtrike an Hammer upon a Wall, which would be Heard, tho' we ſtood in the ſame Line with the Wall; This is prov'd,

1*ſt*. FROM *Fig.* 189. Where the Sound is not Toſſed Perpendicularly to the Wall. 2*d*, From the Ecchoes following one another Quicker, where the Parallel Walls are nearer. 3*d*, From a Door Reflecting a Baſer or Duller Sound than a Wall, which gives a Brisker. 4*th*, From Diſtinguiſhing the Eccho from the Clap, till you come within 7 or 8 Yards of the Wall, the Diſtance being then ſo ſmall that it Deſcribes the 16 Yards in too ſhort a Time; From whence we may alſo gather, what Time is too Minute for the Succeſſion of our Ideas to Diſtinguiſh, or in what Briskneſs our Thoughts can Act; For if a Sound Deſcribes 400 Yards in a Second, it will Deſcribe 16 in about 63 Thirds, which therefore is the utmoſt Quickneſs of our Thoughts. 5*th*, From ſtanding in the midſt between Two Parallel Walls and hearing the Sound twice as Thick, as ſtanding near one Wall, becauſe the Perpendicular betwixt the Parallels and the Oblique Lines cannot be Diſtinguiſht by the 4*th* Article.

SIXTH, The Reaſon why *Fig.* 190 the Sound is not Toſſed ſo long betwixt the Walls, when you ſtand at the End of one or the other, as when you ſtand in the middle betwixt them both, is, becauſe the Sound is Toſſed more Perpendicularly, and therefore Reflects oftner; whereas, when it is more Oblique, it is ſooner Loſt; But why don't we Hear Two Ecchoes at a Time? It is Anſwered, becauſe the Direction is one Way; tho' that may ſometimes be, that we may Hear Two Ecchoes at a Time.

SEVENTH, The Reaſon of an Eccho in a Vault is ſomething like a Speaking Trumpet, where, all the Parts of the Vault being in a manner ſtruck upon by the Fluid Sound Succeſſively, it gives a continu'd Noiſe, juſt as if we ſhould Conceive a moſt thin Subſtance ſcatter'd to every Part of a Vault or Dome, and thence Rebounded to us; Mr. *Walker* tells us, that it Proceeds from the Toſſing of the Sound from one Side to another. And from all this it is abundantly Manifeſt, that the Æther is a Fluid and has a Reſiſtence, or Action, in every Part of it, which Diſperſes in a Stream thro' a whole Dome, meets with a Reſiſtence, Reaction, or Repulſion in every Part, and Cauſes a ſort of diffus'd Murmur or Ringing, which we find in Choirs; And the ſame Account may be given of Violins, not that the Sound is Toſs'd about in them, but 'tis done by one Current of Æther, Impinging upon the Concave and Sonorous Bodies of them; And this likewiſe may be the Reaſon, why the Strings and the Bow are fixt with a Reſinous Subſtance, *viz.* That they may be Capable of a ſmarter Reſiſtence, or Repulſion. It may be alſo Enquired, whether Fluids in the Air are not of different Kinds, ſome of a Nitrous, others of an Oily, a Third of this Reſinous Nature, the laſt of which, if it falls upon Walls, will impart an Eccho, or perhaps the firſt, but where the Second Prevails, it Damps it; Moreover from this Doctrine of Sounds, ſeveral Phænomenons are propoſed to us, which very much Confirm our Philoſophy of the Expanſive, and Contractive Forces; For Reaction, Elaſticity, or Repulſion, is the Expanſive Force we Contend for in Bodies, and where there is a Flatneſs and Deadneſs of Sound Produced, it is an Eviction of our Contractive.

Biſhop of *Ferns* and *Leighlin*, Pag. 596.

THE Biſhop of *Ferns* and *Leighlin*, in his Account of Sounds, makes them Direct, Refracted, or Reflexed; From whence, He Thinks, might be Derived the Science of Acouſticks, Diacouſticks, and Catacouſticks, or Phonicks, Diaphonicks, and Cataphonicks; They likewiſe, He Imagines, may be conſider'd under another Diſtinction of being Produced or Propagated; As to the Firſt, He Believes, that Ways might be Invented to Imitate the Sounds of Birds, Beaſts, *&c.* And Perhaps more Delicate ones Contriv'd, than Any we yet Know.

As To the Second, or the Propagation of them, He tells us;

1ſt. THE Medium is to be Quieſcent and Thin, ſo in a Calm Night a Sound Goes Fartheſt.

2d. THE Sonorous Body is to be placed near a ſmooth Wall, either Plane, or Arch'd Cycloidically, or Elliptically, rather than Circularly; Thus in a Church, the Preacher is better Heard ſo, Eſpecially by thoſe, who Stand near the Wall, or by thoſe, who lay their Ears to the oppoſite Wall, and thoſe in the Middle Hear the Worſt.

WE Say, becauſe as a Shot Goes Farther and Brisker by laſhing down the Piece and keeping it from Recoiling, ſo the Steadineſs of the Wall behind Rebounds the Voice upon the Audience with greater Force, and the Arch Collects it into that Force, which the Cloſer it is, the Better, and therefore the Elliptical Figure is Preferrable to the Circular; The oppoſite Wall being better for Hearing than the Middle, is an Argument, that a Wall by Reverberating gives a freſh Force to the Sound, which therefore is not wholly Paſſive, but Renitent and Springy, and is a Proof of the Repulſive, or Expanſive, Force in Bodies, which we Affirm.

3d. ON Water, Sound is Propagated to a greater Diſtance, and is Softer; On Land, Harſher and more Periſhing; Which is Plain from Bells on the Water, which are Softer, and from a great Gun at Sea, which is farther Heard, and ſo in a Plane, to a far greater Diſtance, than on Uneven Ground; The Laſt of which is, becauſe there is nothing to Break or Interrupt the Courſe of the Æther in the one Caſe, whereas there is in the other; The Firſt, becauſe the Water is leſs Nitrous or Springy, and therefore the Air about it, with which it is Mixt, which may Contribute likewiſe to it's farther Progreſs, becauſe the Springineſs or Expanſiveneſs of the Air makes ſome ſort of Reſiſtence to Sound, which Spring, and Conſequently Reſiſtence, is weaken'd by the Moiſture of the Water. For,

4th. AIR Paſſing, tho' over the even Superficies of an hard and reſiſting Body, is Retarded in it's Undulations or Revolutions.

5th. THE Biſhop's Account therefore of the Reſurges of the Water, Promoting by a kind of Elaſticity the Motion of the Air, and yeilding to it's Impreſſions, ſeems not Valid; becauſe Sound is Wafted too Swift thro' the Air, to Owe any of it's Celerity to the Phlegmatick Motions of Water.

6th. SOUNDS are Propagated beſt thro' an Arch, near a Wall, the Surface of Water, or of Earth, to which if we lay our Ears, we may hear Sounds at a Great Diſtance; Thus the Trampling of a Troop of Horſe, in a ſtill Night, on a large Plane, the Ear being cloſe to the Ground, may be Heard at an Incredible Diſtance.

7th. HENCE Narrow Arches will Propagate a Whiſper from one End to the other, which is therefore Thought to be the beſt Place for a Conſort; becauſe the Sounds would Conſolidate and Embody into one, which becomes a true Compoſition of Sounds, in which is the Life and Soul of Conſort. But perhaps it may be Objected, that it will be too Loud and Harſh, ſince a Whiſper can be ſo eaſily Diſtinguiſh'd.

In The *6th.* Experiment, how the Æther or Medium in a large Plane ſhould Tranſmit Sound ſo well, is not Evident, becauſe the Vegetables, which are ſcatter'd upon it, muſt needs Interrupt the Courſe of the Æther, therefore Sound is not Tranſmitted that way, but by the Trembling or Expanſive Force or Springineſs of the Ground, and perhaps the ſame may be ſaid of Water, which has alſo a great Degree of Elaſticity, when it meets with a Force which is likewiſe Expanſive; On the Contrary in Clay Ground, which has a Contractive Force, we ſhall never Hear the Sound at that Diſtance, becauſe it Damps and Stifles it; The firſt has the Nature of a Wall, as in the preſent Experiment, the other of a Door, as in the Third of Mr. *Walker*, or of Silver in the firſt Caſe, and Lead in the laſt; But if the Æther tranſmitted the Sound, in the preſent Experiment, it would do it, as well on Clay, as on a Large and Bounding Plane; So that here we have another Reaſon of Ecchoes, *viz.* The Expanſive Force or Springineſs of ſome ſort of Ground or Soil, in Preference to another; That is, where there is neither Grove nor Wall, there may be an Eccho from the Ground, as well as a Sound upon Trampling; And indeed ſome Airs, which have an Elaſtick or Expanſive Force, may be diſpoſed Poſſibly to Produce the ſame Effect, as in an hard Froſt.

In The *7th.* Experiment it is not unlikely, but it may not only depend upon the Arch, but the Temper of the Stone, as being Tremulous, Expanſive, and Active, from a Springineſs Acquired from the Air by a Long Tract of Time, not Flat, Contractive and Heavy.

From Hence the Biſhop Proceeds to Compare;

First, Acouſticks to Opticks, or Direct Viſion, and Runs the Parallel betwixt them; In Direct Viſion The Looking thro' a Tube cauſes a different Sight of a Picture; For Example, what is only in Projection in the Picture, ſhall ſeem Real thro' a Tube; Be it ſo, if therefore there were nothing but an Intermediate Space or Vacuum, That could be no ways alter'd, for Things, if at all Viſible, would appear the ſame with or without the Tube; Nor is it enough to ſay, that the Rays of Light are Material but Space not, ſince the Light, by which we ſee Objects, is not the direct Rays of the Sun, or Emanations from it, but the Æther actuated by the Impreſſion of that Luminous Body, no more than the Light of the Candle is the Emanations or Effluviums of the Candle, and not Impreſſions made by it upon the Æther; For if they were really Rays or Parcels of the Suns Matter, or of the Candle, the Light of a Cloudy Day would be the ſame as Sun-Shine, and the Flame of a Candle would be ſpent in an Inſtant, before it was Poſſible to Convert any farther Matter into it's own Nouriſhment; From hence therefore, we have alſo an Argument for the Diffuſive Action and Force of Æther. The Reaſon, why a Tube makes that Alteration, may be, becauſe the Projection is caſt Flat upon the Eye, whereas a Perpendicular being rais'd to the Sight, it Poſſibly makes the Object appear ſo too, as all Things appear according to the Circumſtances of the Medium, thro' which they are Preſented to us. The Parallel in Sounds is that of thoſe, which paſs thro' a Tube or a Trumpet, by which they Flow like a Current, and are altered, Namely, by an Increaſe of them from the Expanſive Force of the Concave Metal, againſt which they Impinge.

Second, He Compares Refracted Sounds or Diacouſticks, to Viſion or Dioptricks, in the Purſuit of which Compariſon he tells us, That,

1ſt. A Man, or any Object, appears larger thro' a Thicker than a Thinner Medium; We ſay, becauſe the Dimenſions in the ſame Space are Larger in a Thicker than a Thinner Medium; And therefore when Impreſs'd upon the Æther, will act in a greater Circuit than ſo much Æther, and is a Proof of it's Continued Force or Action.

2d. A Fine Glaſs Bubble fill'd with clear Water, and plac'd before a burning Lamp or Candle, helps it to dart forth it's Rays to a prodigious Length and Brightneſs; We add, Probably for the ſame Reaſon as before, becauſe the Water is a thicker Medium, and therefore Magnifies the Flame ſo far, that the Bubble appears to be nothing elſe, and the Flame being Increaſed the Length muſt be; This likewiſe is an Argument for a Continued Force or Action in Æther.

3d. The Manifeſt Refraction of Æther thro' Convex Glaſſes is an Evident Inſtance of it's Force or Action, and Conſequently of it's Reſiſtence, which are only Relative Terms of the ſame Thing, for what is Action one way is Reſiſtence another.

4th. As There are *1ſt.* Spectacles, *2d.* Perſpectives, *3d.* Microſcopes, *4th.* Polyſcopes; So the Biſhop thinks there might be contriv'd, *1ſt.* Otacouſticks for old Men to Hear with, *2d.* Others to Hear Things, otherwiſe Inaudible, at great Diſtances, *3d.* Others, *viz.* Microphones, to Hear the Voices of Inſects, *4th.* Others, *viz.* Polyphones, to make a Violin a Conſort.

But For my own Part I cannot believe, that there is ſuch an Analogy betwixt Hearing and Viſion, that what may be Effected to Aſſiſt the one, may likewiſe be contriv'd for the other; becauſe, *1ſt.* The Organ of the Ear is not ſo Curious and Senſible, as that of the Eye, and therefore if the Nature of Sounds were never ſo well Underſtood, it is not likely, that the Æther, however directed, ſhould find the ſame Admittance to the Mind by one Paſſage, as another. *2d.* It is not Probable, Sound conſiſting in Elaſticity, that there are any Bodies Elaſtick enough to Reverberate, for Example, the Voice of a Mite or an Inſect. The Biſhop goes on,

Third, To Reflected Viſion and Hearing, or Catoptricks and Catacouſticks.

And, Here it is likewiſe again Evident, that there is Action, Force, or Reſiſtence, in the thinneſt Æther; ſince we ſee the ſame Images of Things Reflected from the ſame Looking-Glaſs, remove it into what Part of the Æther we will, which Reflection could not be made without a Continued Force, or Action, or Expanſiveneſs, in the Æther; For if there were the ſmalleſt Part of Space, which was not Expanſive, by which Expanſiveneſs the Reflex Image is Communicated to the Eye, in ſome Poſition or other it would be Diſcernible, which would Diſcover, that what was Repreſented, as Part of the Image in one Place, would be a void Space in another.

As To Reflex't Audition or Ecchoes He Tells us, That.

1ſt. A Sound falling Directly or Obliquely upon any Denſe Body, and of a ſmooth Superficies, there muſt be Ecchoes more or leſs, tho' they are not always Heard; If it is Parallel to the Surface, no Eccho is Produced. We may Add, that it ſeems to be a General Law of Nature, that the Greater the Reſiſtence or Oppoſition is, in the Actions of Bodies amongſt Each other, the Greater is the Noiſe or Sound which Reſults from ſuch a Conteſt. And the ſame may be ſaid, in ſome Reſpect, in that of Minds.

2d. If the Surface is Rough and Uneven, the Sound will be thereby Broken and Extinguiſh'd.

In Relation to theſe two Aſſertions, we ſay the Eccho may not be Heard, not only upon the Account of the Unevenneſs of the Surface, or of the Sound being Parallel to it, but becauſe the Surface, however Smooth, is of an Unelaſtick, and Contractive Force, which Damps the Sound, without a Capacity of Returning it.

3d. A Plane Surface Reflects the Sound in a due Tone and Loudneſs, Allowing a Decreaſe Proportionable to it's Diſtance. It muſt be Added, Suppoſing ſuch Surface is Elaſtick and Expanſive.

4th. A Convex Reflects a Smaller and Weaker, tho' Quicker, and a Concave a Bigger and Stronger, tho' Slower, and alſo Inverted, but never according to the Order of the Words.

Of Which we Propoſe the following Solution; The Concave muſt Reflect a Bigger Sound, becauſe it Collects the Sound, or the Motion of the Fluid Æther, Stronger upon the ſame Account, but Slower, becauſe a Contracted Medium, or Condenſed, muſt be Slower than one at Liberty from ſuch Reſtraints; The Convex muſt Reflect a ſmaller and Weaker, yet Quicker, for the contrary Reaſon; The Concave Inverts, becauſe ſome Sounds ſtrike the Sides ſooner, whilſt others, which were Spoke firſt, fall on the Bottom of the Concave, and therefore the Inverſion is never according to the Order of the Words; The Convex on the Contrary does not Reflect the Sounds, which Strike on the Sides, which are Diverted another Way, and only thoſe, which fall directly, or nearly, upon the Apex of the Convex; which may be a Second Reaſon, why thoſe from the Convex are the Quicker, and thoſe from the Concave, Slower, Thoſe falling on a Nearer Point, and Theſe on a Remoter.

5th. It Is ſaid to be Impoſſible to Contrive a ſingle Eccho, which ſhall Reflect all the Words Inverted, becauſe the Words laſt Spoken muſt be firſt Repell'd, which cannot be; tho' in Compound Ecchoes, where many receive from one another, this Inverſion is thought to be not Impoſſible. It does not, we ſay, ſeem, that the one is more Impoſſible than the other, for if in a ſingle Eccho one reflecting Superficies was ſet ſo far off, that it ſhould not Reverberate the firſt Sound, till a nearer had Reverberated the laſt, a ſecond Superficies the Sound next to the laſt, *&c.* The Words would be Inverted; Let us, for Inſtance, conceive five Wells of very different Depths, ſo near one another, as to be within the Compaſs of one, we might Hear, what we Spoke firſt, in the Deepeſt, laſt, and what we Spoke laſt, in the Shalloweſt, firſt, ſuppoſing they were filled to different Heights of Water.

6th. SOME Bodies will Eccho back but one Determinate Note, it is ſaid, and in one certain Place and no other; The Biſhop concludes this to be from the Concavity, or Archedneſs, of theſe Bodies, and their undue Poſition to any other Place.

ON the Contrary we may ſuppoſe this to be rather from the Degree of Elaſticity, which perhaps may not be Capable of Reflecting a Baſs-Note, tho' it may a Treble, or *Vice Verſâ*; for by the Temper of the Wall, whether Nitrous, Reſinous, or Oleaginous, the Elaſticity muſt be alter'd, and no more Capable of Reflecting ſome Notes upon the Percuſſion of Æther, than the Strings of a Baſs-Viol are Capable of Reflecting thoſe of a Violin upon the Application of the Hair, which Produces the Sound in both; that is, any Æther, Striking upon ſuch a Degree of Elaſticity, will ſtill Occaſion the ſame Note; And as to Hearing it in one certain Place, and no more, that ſeems not to Proceed from the Poſition of the Body, which Reflects, but the Direction of the Elaſticity one particular Way, and from it's Expanſion to ſuch a certain Degree.

7th. THE Ecchoing Body Reflects more of the Sound, when farther off, than when nearer, and therefore ſome Ecchoes Repeat but one Syllable, ſome one Word, and ſome many.

8th. A Multiple Eccho may be produced from one Sound, and will be either Confuſed, or Succeſſive, as a Laugh from an Ha—, ſeveral Claps from one, and from one Word many of the ſame Tone and Accent; The Biſhop Thinks this is done by ſo placeing the Ecchoing Bodies at unequal Diſtances, that they Reflect all one Way, and not one on the other.

9th. ONE Sound, by a diverſe Poſition of the Reflecting Body, ſhall produce many Ecchoes different, both as to their Tone, and Intenſion; So that one Inſtrument might ſeem not only many of the ſame Sort and Size, but even a Conſort of different ones.

To Theſe Obſervations the Biſhop ſubjoins the three following Problems.

Probl. 1ſt. To make the leaſt Sound equal to the greateſt, or a Whiſper as Loud as the Shot of a Cannon, by which the Minuteſt Sounds in Nature might be Heard.

Probl. 2d. To Propagate the leaſt Sound to the greateſt Diſtance, by which the Whiſtling of a Weather-Cock would be Heard at Sea many Leagues, and the ſtrength of Winds meaſur'd, allowing for their Moiſture; for the Drier a Wind, the Louder it Whiſtles.

Probl. 3d. To Conveigh a Sound from one Extreme to the other without being Heard in the Middle, that a Man might Converſe with his Friend at a Diſtance, and not be Heard.

FIRST, The Reaſon of Experiment, the *7th.* Which we have not before taken Notice of, ſeems to be, becauſe the Elaſticity or Expanſive Force of the Ecchoing Body, when we ſtand too near, is overpower'd by too great a Violence of the Expanſive of the Voice or Sound, ſo as not to be able to Exert it ſelf, but at a greater Diſtance it Impinges with a leſs Force, by which Means the Ecchoing Body, according to the Degree of it's Force, Reflects a Syllable, a Word, or two, *&c.* Becauſe, when you ſtand farther off, it has Liberty to Play and Rebound, as it is Succeſſively touch'd by the Impulſe of the Æther, flowing from the Voice or the Sonorous Body.

SECOND, The Reaſon of Experiment the *8th.* is, becauſe of the Extreme Elaſticity or Expanſive Force of the Ecchoing Body, whereby, being once touch'd by the Impinging Æther, it gives Two, Three, Four, or Five Rebounds with the ſame Force and Determination; So that Poſition ſeems to have nothing to do in this Phænomenon.

THIRD, By being Reflected upon Walls of different Elaſticity or Expanſiveneſs, the Tone will be vary'd from Treble to Baſs, Tenor, *&c.*

So That the Biſhop, if this is True, is in an Error, not only, when he Affirms, that theſe Variations of the Eccho proceed from the different Poſitions of the Ecchoing Bodies, but when he likewiſe thinks, that a true Underſtanding of the Progreſſion of the Æther would make Acouſticks, as Capable of being Improv'd, as Opticks; for it is plain in Opticks, as they are now Received, and as He Underſtands them, that the Improvements in that Science are not owing to the Knowledge of the Medium, but of the Figures and Bodies, by which the Medium is Refracted or Reflected; and ſo likewiſe, to uſe his own Parallel from Opticks, Acouſticks are not to be Improv'd from a Knowledge of the Medium, but of the Nature and Degree of Elaſticity, or Expanſion, in the Repelling Body; as the Sound, which a Hammer would Produce upon a Wall, a Door, a Bell, or an Anvil, is not to be known by the Poſitions of thoſe Bodies, or the Motion of the Hammer, but from the Force of Recoiling upon the Stroke; So that his three Problems in Acouſticks will appear to be Chimerical. From all which, for this is the Concluſion we would make, it is Evident, that there is as much real Action, Force, or Reſiſtence in the Æther, as there is in any one Body ſtriking upon another, or as there is in a Stone Refracted in, or Reflected from Water.

As For Problem the Firſt, ſuppoſing it Feaſible, we ſhould not Hear the Minuteſt Sounds, but different ones produced, from, indeed, a Minute Cauſe; In Opticks 'tis otherwiſe, becauſe we are Aſſiſted in Diſcovering the real Figures of ſmall Bodies, their Situation and Poſition, which Figure, Situation, or Poſition are the ſame in great Ones as they are in Minuter; But Sound Conſiſts in it's very Magnitude, and therefore, if Increaſt by any Methods, is different from the Real; unleſs we Conſider the Tone only, that is Baſs or Treble, *&c.* Which might Poſſibly be diſcover'd; But ſince the Reflection of Sound chiefly Conſiſts in Elaſticity, or Expanſion, where ſhall we find Bodies Elaſtick or Expanſive enough to Reflect the ſtill Note of a Mite or a Leech-Worm, if they have any Organs to make one?

An Account of Stone Henge, *and the Reaſon of that Fabrick.*

I Have in the foregoing Account mention'd Walls acquiring a Force by Proceſs of Time, which did not Originally belong to them; Which brings to my Mind, what I have many Years ago obſerved of the Famous Building of *Stone-Henge* in *Wiltſhire*, upon my Travells into thoſe Parts; It is generally Eſteemed to be Compoſed of a Factitious Stone, or of Natural ones, which were Originally of the ſame Prodigious Magnitude and Size, in which we now behold them; Whereas, not to Inſiſt upon the ſeveral Objections, which lye againſt thoſe two Accounts, nothing ſeems to be more Evident to me, than that it was Raiſed, as all other Buildings, by a Cement of Mortar; For in many of the Tranſverſe Stones, which are laid upon the Perpendicular ones, there is Evidently a Disjointing of them into ſeveral ſmall and ſquare Stones, whoſe Mortar or Cement is Periſhed; I was ſtill more Confirmed in this Opinion, when I ſaw a Wall, Adjoining to the Cathedral Church at *Wincheſter*, Hardened into Stone; I likewiſe found, that the Stones call'd, by the Adjacent Villagers, the *King's-Stone* and *his Men*, nothing elſe, but the Remains of an Old Building; Theſe lye betwixt *Long-Compton* and *Endſtone* in *Oxfordſhire*; The ſame Sort of Stones I obſerved betwixt *Abingdon* and *Milton*, in the ſame County, in ſome of which there was a Plain Cement Diſcernible, and Harden'd into Stone; and *Hemlock-Stone* betwixt *Stapleford* and *Starly* in *Derbyſhire*, I could plainly Perceive had the ſame Marks of Mortar and Cement in ſeveral Parts of it, which were yet Harden'd into the ſame Subſtance; But we need go no farther, than to a Building of Stone next to the College where I Write, one of the moſt Famous in the World, King *Henry* the *6th's* Chapel in *Cambridge*, the outward Surface of which has acquired a Particular Hardneſs, whilſt the inward Parts of the ſame Stone are Soft and Moldering: From all which I conclude, that the Elaſtick and Expanſive Force of the Air, and Sun, has a Power of producing, in a long Courſe of Time, an Hardneſs and Elaſticity in Mortar, Equal to that, which we find in Stone it ſelf, and that the ſame Expanſive Force can make a ſofter Stone more Hard and Elaſtick; And thus all Stones and Flints upon the Earth are Formed; In the ſame Manner, as Fire, or an Expanſive Force, Hardens Clay, and Imparts an Elaſtick or Expanſive Force to it, which before had a Dull and a Contractive.

As to the Deſign of this Building of *Stone-Henge*, it ſeems to have been to Perpetuate the Memory of the Entire Victory of the *Saxons* over the *Britons*, under their great Leader *Hengiſt*, from whom it Derives it's Name; and therefore we find the *Britiſh* Prince *Ambroſe*, who was the laſt Champion in that Cauſe, Bury'd in an Obſcure Village not far from thence, which alſo has it's Name from him, and is now commonly called *Amesbury*, and in their Writings *Ambroſe-Bury*; There are other Accounts of this Matter, but which, upon

Examination, will not be found ſo Conſiſtent as this; The long Reign of the *Saxons* in this Iſland will give a Reaſon, why that Monument of their Triumphs ſhould ſtand undiſturbed ſo many Years, till it was beyond the Will, or the Power, of any to Ruin it, by it's becoming Venerable to a *Saxon* Poſterity, and Impoſſible to be Deſtroy'd by a *Britiſh*.

A Continued Reſiſtence, Force or Action, in Real Space or Extenſion, Confeſſed by Philoſophers in their Notions and Sentiments of the Barometer, and in their Theories and Conclusſions Concerning the Air or Atmoſphere formed upon it. Philoſoph. Tranſact. Vol. 2. Pag. 3, 4, 5, 6, 7, &c. Mr. Lowthorp's Abridg. Dr. J. Beal. Pag. 3, 4, 5.

§. 12. WE Shall now Proceed to the Doctrine of Barometers, and Firſt, as it is Propoſed by Dr. *Beal*, who Tells us, that Mr. *Boyle* was the firſt who made them Publick with Obſervations upon them in the Years 1659 and 1660. And he Thinks it to be one of the moſt Wonderful Diſcoveries, that was ever made in the World, to Weigh Clouds and Weather, and to prove the Sereneſt Air to be the moſt Heavy, and the Thickeſt Air, when Clouds are ready to Diſſolve, or Dropping, to be the Lighteſt.

WHICH Indeed ſeems to be both very Extraordinary, and very Incredible; On the Contrary we ſay, that the Reaſon, why the Quick-Silver is higher in Serene Weather, than in Wet and Cloudy, is not from the Weight of the Air, but from it's Spring, Elaſticity, or Expanſive Force, or from it's Contractive, which in Moiſt Weather is Diminiſh'd, and more ready to Yeild to the Gravity or Action of the Quick-Silver, than in Dry; So that the Weight of the Air may be greater, when the Action or Expanſion, or Contraction of it, is ſmaller; For Inſtance, Water is Heavier than Ice, and yet any Body, as Quick-Silver ſhall Deſcend in Water, where it cannot in Ice; And by a Parity of Reaſon, the Mercury may Deſcend in a Moiſt Atmoſphere, when it will not in a Frozen, or Contractive, or in a Serene, Elaſtick, or Expanſive one.

THE Barometer, he made Uſe of, was a Cane or Tube, 35 Inches long, of a ſlender Cavity, and a thick Glaſs, and the Height of the Mercury was never but very little above 30 or 30½ Inches.

HIS Experiments are, as Follow, of which I ſhall Endeavour to give the Solutions, according to our Doctrine of the Expanſive and Contractive Forces.

1ſt. Decemb. 13th, 1664, The Mercury ſtood at 30¼ Inches, when it was fickle Fair in the Evening.

2d. June 22d. 1664. It ſtood at 30½ Inches, if no Miſtake, at 5 in the Morning, when it was ſettled Fair; This is however certain, that it continued High in ſettled Fair, and Warm Weather, and as High in the Hotteſt Summer, as the Coldeſt Winter.

3d. IT Yet was Higher in Winter and Summer, in cold Mornings and Evenings, than in the Warmer Mid-Day, and deſcended lower upon Rain, or great Winds; Yet was ſometimes High in Stormy Winds of 3 or 4 Days Continuance; It was alſo Lower after Rain than before it.

The Variations of the Height of the Mercury Solved from the Doctrine of the Expanſive and Contractive Forces.

THE Reaſon of the foregoing Experiments is plain; That the Mercury is higher, whether in Winter or Summer, when the Weather is Fair or Cold, is, becauſe the Contractive Force of the Air is Increaſed, in Cold Weather, and in Fair, the Expanſive, both which will Act againſt the Mercury, and Hinder it from Deſcending, the One by Compreſſing the Parts of Air together, ſo as to thruſt up the Mercury, or by Contracting the Cavity of the Tube; The Other by giving them an Elaſticity and Expanſion, by which they Repel the Mercury, or Mixing with it, Dilate it; And in ſome Stormy Winds it is Higher, where they are Dry and Cold, upon the Account of their Contractive: On the Contrary it Sinks in Mid-Day, or upon Warmth, becauſe That Weakens the Contraction or Reſiſtence of the Air; Alſo upon Rain, becauſe Moiſture has the ſame Effect, and on Stormy Winds, where they are Warm or Moiſt: In ſhort, where Cold and the Contractive Force, or Dryneſs and the Expanſive, Prevails, the Reſiſtence of Air to the Deſcent of the Mercury is ſtrong, where Heat and Moiſture, Weak and Feeble.

4th. STRONG Fumes, or Odours, and thick Smoaks, let into a Cloſet, do not much alter the Height of the Mercury; The Reaſon of which ſeems to be, becauſe they are not of a Moiſt Nature but Dry, and do not therefore Weaken the Expanſive or Elaſtick Force of the Air, which Enervation of it ſinks the Mercury; But the abundant Fumes of Water Boyling, which are Warm and Moiſt, Poſſibly would do it.

5th. THE Mercury is Higher in an Eaſt or North Wind, than in a South and Weſt, which is for the ſame Reaſon; becauſe the two Firſt are Cold and Dry, and the two Laſt Warm and Moiſt.

6th. THE Greateſt Alterations in the Height of the Mercury are not above 2¾ or 2⅞ or near 3 Inches at the moſt; Becauſe the Expanſive or Contractive Force of the

Air cannot be very conſiderably Diminiſhed or Increaſed by the Variations of Heat and Moiſture.

7th. In Days, which were Clear, and Dewy Nights, when the Moiſture Deſcended in great Quantities, the Baroſcope held the ſame; becauſe the Dryneſs or Expanſive Force of the Air, and the Coldneſs or Contractive Force of the Dews, produce much the ſame Effect, as in Experiment the *3d.*

8th. In a Bright and Clear Day, and in the Next, which was Darken'd by ſome Rain and Snow falling, the Mercury was the ſame, ſo on High Winds and in Calms it was the ſame; This is already Accounted for, ſince Bright and Clear, or the Expanſive Force, Produces the like Effects with Darkneſs, cauſ'd by Cold Rain, and Snow, or the Contractive, and High Winds, that are Cold, the like with Calms, that are Dry.

9th. December 17th. The Wind, Froſt, and Quick-Silver, were the ſame, the Day before there was a large Black Cloud and the Sky was Darken'd, yet Froſt and Wind the ſame. *December 18th.* The Quick-Silver fell $\frac{1}{4}$ of an Inch, tho' the Sky and Air were Clear, Bright, and Cold, as before; This ſeem'd a Myſtery to him, till he ſent his Servant Abroad Caſually, who diſcover'd the remote Hills about 20 Miles off Cover'd with Snow, by which he Thinks the Air was Lighten'd and the Mercury fell: This looks to be a very fancifull Account of the Matter; On the Contrary, we cannot but ſtill Affirm, that the Alteration of the Height of the Mercury Proceeds from the greater or leſs Reſiſtence of the Air, *i. e.* From the greater or leſs Expanſive or Contractive Force of it, tho' the Height, under which it never falls, may Proceed from another Cauſe, which we ſhall afterwards Explain; Since I ſay, we ſtill Affirm it, there might be ſome Unheeded Circumſtance, which might Occaſion an Alteration of the Air's Spring, ſo far as to Sink the Mercury $\frac{1}{4}$ of an Inch.

10th. The Change of the Height of the Mercury is ſeldom great at a Time, tho' once it fell $\frac{1}{4}$ of an Inch in a Day; We ſay, becauſe the Alteration of the Air's Spring is Gradual and Small.

11th. The Mercury he found to be high in Thick and Miſty Weather, tho' it is uſually lower then; it is acknowledg'd, that it Continu'd above a Week, and at laſt, that the Weather was Senſibly more Cold, and that there was an Eaſt, and North-Eaſt Wind; Which Agrees with what we have Advanc'd, that the Mercury may Riſe or Fall in Miſty and Cloudy Fogs, or in a clearer Air, according as thoſe Fogs or Air are Cold and Contractive or not; here they were Cold, being the Effects poſſibly of a North-Eaſt, or Eaſt Wind, and therefore the Mercury Roſe.

The Variations of the Height of the Mercury Explained from the Doctrine of the Expanſive and Contractive Force.

12th. Upon Dark thick Miſts riſing, and when there was ſome ſmall Rain, the Mercury held at a great Height; Becauſe the Darkneſs, Miſts, and Rain, were of a Cold and Contractive Complexion, and the Craſis or Temper of the Air thereby Altered.

13th. If The Mercury Riſes after a Fall of Rain, a ſettled Serenity Enſues, if it Subſides, the Weather is broken and Showry; Becauſe in the firſt Caſe, the Air recovers it's Spring or Expanſive, or rather it is a Sign, that by it's Spring and it's Expanſive, it forced the Rain down, and preſerv'd it ſelf from the Impreſſions of it's Moiſture; In the ſecond it is Plain, the Spring or Expanſive of the Air is quite loſt and Diſſolv'd, and has little or no Force remaining, it's Craſis is Flaccid, and it is able to Support, neither any Clouds, nor any Quick-Silver, above it's neceſſary Standard.

14th. The Weather and our Bodies are more Chill, Cold, and Drooping, when the Mercury is the Loweſt, and the Air Lighteſt; Which is Agreeable to what we Advance, that the Chiller and more Drooping we are (which is one Sort of Cold Perceivable by us, tho' a Moiſt, not a quick one, as that of Froſt) the Mercury is the Lower; becauſe the Craſis of the Air is Flat and Diſſolute, and Strikes not ſo briskly upon our Senſes, but Unbraces the whole Animal Oeconomy.

15th. The Loweſt Deſcent of the Mercury is near 27$\frac{1}{2}$ Inches; It was 28$\frac{1}{2}$ in great Storms, and much Rain, *Octob. 25th*, in the Morning; Alſo 28, when the Winds were quiet, and there were Thick and Dark Clouds on the *26th*, in the Morning; But 27$\frac{1}{2}$, that Day and ſome Days following, when there was frequent Rain, and by Rain and the South Wind the Mercury ſunk $\frac{1}{2}$ an Inch.

All which Phænomenons are Accounted for, in the Manner Explained, by our Doctrine of the Expanſive and Contractive Forces.

16th. THE Aſcent of the Mercury was above $\frac{1}{2}$ an Inch more than 30 Inches, ſometimes higher to an *8th*, or a *10th*, Part of an Inch; The Air was then Bright and Clear, and there was a gentle Froſt, yet Melting by the Sun's Heat; The Air was Silent, and there was no Wind ſtirring, excepting what was Remarkable by a Curious Vane, which ſhow'd the Wind to be in the Eaſt directly; then it went to the North-Weſt, and haſted again to the Eaſt, during which Time the Mercury Deſcended a little: We ſay upon the Account of the Weſt being a Warm and a Moiſt Quarter, but upon returning to the Eaſt, the Mercury Aſcended from the Expanſive; All which is Agreeable to what we have ſaid.

AND We ſhall only Add, that from all, which foregoes, it is Manifeſt, that Dr. *Beal* did Imagine and Believe, that there was a Real and Continued Force, Reſiſtence, or Action in Space, tho' he was Wrong in ſuppoſing, that ſuch a Force Conſiſted in Gravitation.

Dr. Wallis. Philoſophical Tranſactions, Vol. 2. Pag. 6. Mr. Lowthorp's Abridg.

WE Shall therefore now Proceed to the Obſervations, which Dr. *Wallis* has made upon the ſame Subject.

HE Tells us, that the Mercury is not Higher than 30, nor Lower than 28 Inches, or not $\frac{1}{16}$ of an Inch Lower or Higher; The Mercury was well cleans'd from Air, tho' Mr. *Boyle*'s was much better cleans'd (whoſe Barometer he compar'd with his own, being both in *Oxford*), for the Mercury ſtood always Higher, in Mr. *Boyle*'s, near $\frac{1}{4}$ of an Inch; The Reaſon was Thought to be, becauſe the Doctor's Mercury was Heavier than Mr. *Boyle*'s, or becauſe Mr. *Boyle*'s was better cleans'd from Air; But how ſhould a light Body Expell'd make an Heavier Riſe Higher? Or becauſe his Bore was larger than Mr. *Boyle*'s; But if a Weight of a Pillar of Air is equal to a certain Pillar of Mercury in Height, the ſame will hold in a leſſer or a greater Column of it in Diameter; As for the Glaſs, That ſeems to be ſo far, by any Aptitude of it's Parts, from Aſſiſting Quick-Silver's Riſing in the Tube, that, it is Plain, it Rejects it by the Mercury's Convexity at the Top of it.

Concerning the Suſpenſion of the Mercury in the Tube.

As Therefore the Variation of it's Height has been Endeavour'd to be Accounted for, by the Change of the Air's or Æther's Action, Spring, or Expanſive and Contractive Force, ſo now we are to Account for it's Suſpenſion in the Tube; which is not to be done by the Gravity of the Air or Atmoſphere, as has been Generally Apprehended, but by the Mutual Action, and by the Expanſive and Contractive Forces of the Glaſs and Quick-Silver, as alſo of the Air, or Æther, or Atmoſphere Surrounding it.

The Reaſons, why the Mercury is not Suſpended in the Tube by the Weight of the Air, or Atmoſphere.

THOSE, Who will have the Weight of the Air the Cauſe of the Suſpenſion, Run into Difficulties, which are not to be Anſwer'd, and Depend upon Reaſons, which may be; For,

1ſt. IF it was the Weight of the Air, which really ſuſtain'd the Mercury, why does it not do it, when the Tube is Inverted in Air or Water, as well as Quick-Silver? Since the Weight of the Air, if any, will be in both Caſes the ſame; but if we Aſſign any other Cauſe, it is Plain, there is ſomething more in the Buſineſs, than the Weight of the Atmoſphere.

2d. IT is not Imaginable, how ſo great a Weight ſhould be Senſible in it's Preſſure upon the Quick-Silver, and not upon our Hands, or Faces; Eſpecially, when by the Laws of Fluids, tho' there is an Abſolute Gravitation in the Air, yet there is not a Comparative one, but all the Parts of it are in Æquilibrio; And therefore any Thing, which is in Air, does not Perceive the leaſt of ſuch Gravitation or Preſſure.

3d. IF there were really an Incumbent Weight upon the Mercury in the Veſſel able to Support the Mercury in the Tube, the ſame Force would be able to Raiſe it into it, but That we do not find; Thus we Perceive by the Preſſure of Water in one Tube of a Recurve Siphon, the Water is Rais'd in the other, as well as Supported; Beſides, the Aſſertors of Gravity own, that the Mercury Riſes, and Subſides, in the Tube, as the Atmoſphere is Heavier, or Lighter, and therefore ſhould be Rais'd into it by the ſame Weight.

4th. THE Experiment of Monſieur *Hugens* is an Invincible Argument againſt it's being Upheld by the Weight of the Atmoſphere, in which the Mercury is kept up to 75 Inches, which is above double the common Height; 'Tis true the Preſſure of a Subtiler Fluid, beſides that of the Air, is here Introduced, becauſe Marbles and other Smooth Surfaces will Adhere together even in Vacuo, and the Uppermoſt will draw the Undermoſt after it, tho' ſome Pounds of Lead hang upon it; But *Hugens* did not Conſider,

ſider, that, in the commoneſt Elevations of Mercury, That Preſſure of a Subtiler Fluid, if any ſuch there is, could not be wanting, unleſs it was Created juſt to ſerve that Turn, and was afterwards no where to be found in Nature; Nor does Dr. *Wallis's* Elaſticity Solve the Matter much better, which is Increaſed Proportionably to the Preſſure; Becauſe, if that Rule held, the largeſt Tubes or Cylinders of Quick-Silver might be Suſtain'd, and not only to 30 Inches, but 30 Yards, ſince the Elaſticity of the Air, by being Preſſed, Repreſſes in Proportion.

5th. Which gives us a Fifth Argument againſt this Account of the Suſpenſion of Mercury by the Gravity of the Atmoſphere; For if a Cylinder of Air of $\frac{1}{2}$ an Inch Diameter, for Example, can Support by it's Gravity a Cylinder of Mercury, 30 Inches High and $\frac{1}{2}$ an Inch Diameter, a Cylinder of Air of 20 Inches Diameter will as well Support a Cylinder of Mercury 30 Inches High and 20 Diameter; Which does not Happen.

6th. If the Weight of a Cylinder of Air was Equal to a Cylinder of Mercury of 30 Inches, how comes it to Paſs, that the Surface of Mercury in the Veſſel, which is round the Tube and is only Preſſed by the Air, which Surface is greater than the Baſis of the Cylinder of the Mercury within the Tube, does not Raiſe or Suſtain a Pillar of Mercury, in Proportion Higher, as the Pillar of Air is Bigger?

The Reaſons Alledged, why the Mercury is Suſpended by the Gravity of the Air or Atmoſphere, Inconcluſive.

As to the Reaſons for this Doctrine, They are only,

1ſt. That the Mercury Riſes and Subſides according to the Gravity, or Lightneſs of the Atmoſphere; Which we have already Confuted; Becauſe the Alterations of the Height of the Mercury, as we have Proved, Proceed from different Cauſes. *2d.* That it is Lower in the Higher Parts of the Atmoſphere, and Higher in the Lower; Which does not Ariſe from the Leſs or Greater Gravity of the Incumbent Air, but from it's Reſiſtence or Spring, or it's Contractive or Expanſive Force, being Changed in thoſe different Situations.

Thus the Mercury may fall Lower at the Top of High Mountains upon the Account of Moiſt Clouds and Vapours, which would Break and Diſſolve the Elaſtick or Expanſive Force of the Air; or the Medium may be Thinner, and not ſo Contractive, by which it may be leſs able to Reſiſt the Deſcent of the Mercury; The Contrary of which may happen at the Bottom of ſuch Mountains.

To Return to Dr. *Wallis's* Experiments.

1ſt. In Thick Foggy Weather the Quick-Silver Riſes, in Sun-Shiny likewiſe, and the Clearer the more, for if Clouds Intervene, it Falls, and upon Breaking out of the Sun, Riſes; it's Riſing in Thick Foggy Weather, He Attributes to the Weight of the Air, which might be as well Aſcribed to it's Reſiſtence, being more Conſtipated and Condenſed, that is, to the Increaſe of it's Contractive; As to it's Riſing in Sun-Shine, if ſo, it muſt be owing to the Increaſe of the Expanſive; Tho' by after Experiments He found the Contrary, that it rather Subſided: But when we ſay the Fogs are Denſe and Contractive, That ſtill muſt be judg'd by their Nature, for there may be ſome Moiſt Fogs and Warm, which will make the Mercury rather Fall than Riſe, by Diſſolving the Elaſtick and Expanſive Force of the Air.

2d. In Rain it Falls, which was likewiſe Experimented by Dr. *Beal*, and was before Accounted for.

3d. In Snowy Weather, and ſometimes in Hoar-Froſt, it Falls, but not ſo much as in Rain; becauſe theſe are only Colder Rains, in which the Expanſive and Contractive Force of the Air is Enervated and Weakned in ſome Meaſure, as it is in Rains.

4th. In Windy Weather Univerſally it Falls, more than in Rain; He ſays, becauſe it Hinders the Gravity of the Air from Preſſing Downwards; on the Contrary, we rather Think, becauſe Winds Take off from the Air's Expanſive or Contractive Force, and Hinder it from Exerting it's Action; tho' it is own'd, where the Winds are Cold, as Eaſt, or North-Eaſt, the Mercury Riſes; and therefore Perhaps this Windy Weather was from a different Quarter, which was Warm and Moiſt, and which certainly would make the Mercury Subſide.

5th. The Mercury has fallen without any Viſible Cauſe at Home, but looking Abroad, Rains have fallen at diſtant Parts, which He thinks Lightened the Air; But this ſeems to be unlikely, that the Mercury ſhould be Affected by ſuch Remote Cauſes, and it is more Probable, that the Cauſe was to be Sought at Home from the different Temperament of the Air, from what it uſed to be, which was Unheeded; His Ar-

Argument for his Opinion is this; That, the Air being Lighten'd on a different Side, and become more Thin and Rare by the Fall of the Rain, the Groſſer Air, which lyes towards the Barometer, Diſcharges it's ſelf towards the Rarer Part of the Atmoſphere; Which might Poſſibly be true, if Rains falling from Particular Clouds, and not Settled and General ones, Weaken'd the Reſiſtence or Spring of the Air, but by Dr. *Beal* it is Plain, that Partial and Accidental Rains do not Weaken it's Reſiſtence, ſince after ſuch Rains the Mercury Keeps it's Station, or Riſes Higher; Whereas, in General and ſettled Rains, the whole Craſis of the Air is Diſſolv'd, and is not able to Support the Mercury, as it otherwiſe would do; and therefore Dr. *Wallis*'s Rains falling are no Argument, why the Air, which lyes towards the Barometer ſhould Preſs that Way, where the Rains fell, ſince after ſuch Rains the Reſiſtence will be as great or greater than before; Beſides, if the Air ſhould Preſs from the Barometer, it would be Succeeded by the Air on the other Side of it, which would Supply the Place of that, which was carry'd from thence.

6th. HE Retracts his Experiment about the Sun-Shine making the Mercury Riſe Higher, or rather Endeavours to Reconcile his laſt Obſervation of it's Falling at ſuch Times, with his firſt of it's Riſing, but at the ſame Time Owns, that in Froſty Weather it Riſes, tho' before He had Affirm'd, that in a Hoar-Froſt it Falls; He Endeavours however to Reconcile his Elevation and Depreſſion of the Mercury upon Sun-Shine thus, that at firſt it was not Clear'd enough from the Air, and Conſequently, the Heat Increaſing, it's Elaſticity Rais'd it, but afterwards the Air Diſentangling it's ſelf, and Getting to the Top of the Mercury, it Depreſs'd it; But the more Obvious Account ſeems to be, that in the firſt Caſe the Rays of the Sun fell upon a Medium, which was Dry and Elaſtick, and by that Means Increaſed the Expanſive Force of it, by which the Mercury was Raiſed; and in the laſt upon a Moiſt, by which the Expanſive Force of the Rays was Diminiſhed, and the Medium became Diſſolute and Flaccid, from whence the Mercury Subſided.

7th. THIS Experiment Informs us, that the Quick-Silver, Striking againſt the Ice in the upper Part of the Tube, gave a Smart Noiſe, not a Dead one, which is uſual to Quick-Silver; which does not ſeem to Proceed from a void Space, but from the Nitrous, and Springy, or Expanſive Nature of Ice, which would therefore, by it's Brisk Reſiſtence to the Quick-Silver, Produce that Tone; as we have ſaid in Ecchoes, the Tone very much Depends upon the Elaſticity, or Degree of Reſiſtence, in the Bodies, which Strike one againſt another.

8th. THE Higheſt Standard of the Mercury was 30 $\frac{1}{8}$ Inches in Froſt, the Loweſt 27 $\frac{7}{8}$, the Uſual Height 29 Inches, or a little more; The Reaſon, why Dr. *Wallis*'s Mercury Roſe not ſo High, as Mr. *Boyle*'s, or Dr. *Beal*'s, ſeems to be the Largeneſs of the Bore, which, He ſays, was greater than Mr. *Boyle*'s, and Dr. *Beal* Acknowledges his to be very Small; Becauſe the Action betwixt the Mercury and Glaſs would be Weakened from the Gravity of the former, and the Mercury would Subſide.

Why the Mercury is Suſpended in the Tube.

BUT The Principal Phænomenon Remains ſtill to be Accounted for, Namely, why the Mercury is Suſpended to the Uſual Height; Which, we ſay, chiefly Depends upon the Mutual Action, or the Expanſive Force, of the Glaſs and Quick-Silver, tho' the Expanſive Force of the Air, or Æther, is Aſſiſting to the ſame Effect; It is Manifeſt, that Glaſs has Acquired a Conſiderable Expanſive Force from the Fire, in the forming of it from Sands and Flints, Mercury has likewiſe a very Evident one from it's Exceeding Volatility; When theſe Actions, or Forces, therefore are at Liberty to Exert themſelves upon each other, they muſt Unite, as being both of an Expanſive Nature, and as they Preſs towards each other by that Expanſion; The Contractive Force, on the Contrary, of the Mercury, from whence it's Gravity is Derived, being much Superior to that of Glaſs, would draw theſe Expanſive Forces from each other, were there not an Expanſive Force in the Air, or Æther, which Acted in Oppoſition to the Contractive of the Mercury; By both which Concurrent Cauſes, it is at length Supported, and Suſtained, in the Tube; And from hence it is,

FIRST, That Water, if Immitted into a ſmall Tube, Adheres ſo cloſely to the Glaſs, as not to be Expelled from thence by it's own Gravity, or by the Strongeſt Breath, which can be Apply'd to it;

SECOND, That Dr. *Wallis* Imagined, there was an Aptitude of Parts betwixt the Glaſs and Quick-Silver, and that Dr. *Hooke* Affirmed, there was a Coheſion of them, as Sir *Iſ. Newton*'s Followers, an Attraction of the one to the other;

THIRD, That the Bore, or the Cavity, of the Tube muſt be Small; Becauſe the Action lyes betwixt the Mercury and the Glaſs, and not betwixt one Portion of Mercury and another, which is in Æquilibrio, and which would Subſide by it's Gravity, if not Supported by this Mutual Action;

FOURTH, That the Mercury at the Top of the Tube has a Convex Surface; Becauſe the Expanſive Force of the Glaſs is Greater than that of Quick-Silver, which is Loaded with a great Degree of the Contractive, and therefore by that Superiority of the Expanſive, Repells it; And which may be one Reaſon, why the Mercury Riſes in Dry Sun-Shiny, or Froſty Weather, when it is Expanſive, as it Contributes to the Expanſive Force of the Glaſs, and Cooperates, and Concurs, with it.

THE Next Experiments, we have, are thoſe of the Eminent Mr. *Boyle*, Who Tells us;

Mr. Boyle, Pag. 8. Vol. 2.

1*ſt*. THAT Obſervations upon the Barometer Diſagree, when the State of the Air is the ſame, very much for want of taking Notice of the Height of the Country, or it's Low Situation; He Inſtances in two Barometers, which He had, the one at *Oxford*, which was Low, by the *Thames*, the other at *Stanton* St. *John's*, on a Moderate Hill; the laſt was always between $\frac{2\ \&\ 3}{8}$ of an Inch lower than the other, in which Proportion He found them Riſe and Fall by Conſent; This has been Accounted for without the Gravity of the Atmoſphere; Namely, by the Different Temper and Expanſive Force of it, which might therefore make a leſs Reſiſtence, when Higher, than when Lower; Theſe Obſervations, He ſays, may Teach us, that the Subterraneous Steams Riſing into the Air do more Uniformly Affect it in Lower and Higher Places of the Atmoſphere, than He durſt at firſt Conclude; To which it is Anſwer'd, that however that may be, it Teaches us at the ſame Time, that the Height of the Quickſilver, or Lowneſs of it, does not Depend upon the Gravity of the Air, but upon the Atmoſphere's being leſs Expanſive or leſs Contractive when Higher, than when Lower, by the Diminution of either of which Forces the Mercury will Fall; that is, that there is a different Force from that of Gravity, which Cauſes the Alteration.

2*d*. HE Informs us, that once theſe two Barometers vary'd from the Proportion before Mention'd, *i. e.* the Barometer at *Stanton*, which uſ'd to be Lower than that of *Oxford* $\frac{2\ \text{or}\ 3}{8}$ of an Inch, was $\frac{1}{4}$ of an Inch different from it's uſual Proportion, without any Manifeſt Change of Weather, either in one Place or the other; This therefore is a Proof, that there may be ſome Domeſtick Cauſes, as we ſaid before, or ſome Particular Craſis or Temper of the Air in that Place, which Occaſion'd the Alteration; Which is eaſily Solv'd by Suppoſing, that the Air's Expanſive or Contractive Force might be Weaken'd by ſuch Private Cauſes, but cannot be Solv'd by the General Gravity of the Air, which muſt certainly be much the ſame at all Times, and under all Circumſtances of Heat or Cold, of Moiſture or Dryneſs.

3*d*. THE Quick-Silver was Higher than, He Thinks, has been Obſerv'd in *England*, *viz.* $\frac{5}{16}$ above 30 Inches, tho' Dr. *Beal* has Obſerv'd it 30 Inches and $\frac{1}{2}$ and more; Upon Conſiderable Showers, after a long Drought, it fell at *Stanton* to $\frac{3}{8}$ Beneath 29 Inches, which He Foretold, not in Quantity indeed, but that the Mercury would Fall; We ſay, becauſe the Expanſive Force of the Air was Diſſolv'd by the Moiſture of it, Agreeably to what has been before Affirmed.

4*th*. HE Says it is Difficult to ſettle a General Rule for the Riſe and Fall of Mercury; the moſt General is, that when High Winds Blow, the Mercury is the Lower; We have already given a Reaſon of this; Namely, becauſe the Expanſive or Contractive Force of the Air is Diminiſh'd, and it's proper Force is taken off in General, by ſuch an Emotion in it; But if this is the moſt General Rule, it is Evident, that the Weight of it has not much to do in this Matter, ſince that would be much the ſame, notwithſtanding the Caſual and Accidental Perturbation of it; Thus in Water, Move it as we Pleaſe, it will ſtill nearly Retain it's own Natural Gravity.

Sir William Beeſton, Pag. 9. Vol. 2.

SIR *William Beeſton* Obſerved in *Jamaica*,

THAT the Mercury was one Degree below Changeable in the Morning, and as the Heat of the Sun Increas'd, that it Fell within one Degree of Rain; that, leaving open his Window, the Sun Shone upon the Tube, and in $\frac{1}{2}$ an Hour it Sunk 3 Degrees; He Shut his Window, and within an Hour it Roſe to within one Degree of Changeable; Which fully Confirms, what we Say, that the Alteration of the Standard of the Mercury Proceeds from the Change of the Expanſive or Contractive Force of the Air.

THE Biſhop of *Cloyne* Says,

THAT, in very much Thunder, the Mercury was Conſiderably Lower than ever He Obſerv'd it, He does not give the Reaſon; Namely, becauſe the Thunder, which is always Accompany'd with Lightening, breaks the Expanſive or Elaſtick Force of the Air, which is Evident by Showers Generally Falling upon it, and ſhows the Diſſolv'd and Languid Craſis of the Medium upon ſuch Accidents.

Biſhop of Cloyne, Pag. 9. Vol. 2.

MR. *Townley* goes on to Aſſure us by his Experiments,

Mr. Townley. Pag. 9. Vol. 2.

1*ſt.* THAT Mr. *Flamſteed's* Barometer at *London*, was more than $\frac{3}{10}$ of an Inch Higher than his at *Townley*, which is Seated in a Valley, yet is Higher than the Low Lands near the Sea, where the Standard Differs little from that of *London*; That is, the Barometer Stood Higher in the Atmoſphere at *Townley*, than at *London*, and therefore the Mercury was Lower; But this is no Proof of the Air's Gravity, as we have ſhewn.

2*d.* THAT *November* 18*th*, 1674. The Mercury Fell very Faſt at 2 in the Afternoon, and Roſe at 4; Becauſe perhaps the Air grew Sharper and Colder at 4, and at 2, was Poſſibly Moiſter, being in *November*.

WE Proceed to Dr. *Hooke's* Baroſcope, which Conſiſts of 2 Canes, and 3 Cylinders, the Canes are fill'd with Mercury, Spirit of Wine Dy'd with Cochineal, and Oil of Turpentine; The two laſt are made Uſe of, becauſe they are very nearly of the ſame Specifick Gravity, tho' the Oil Swims at Top; But if the Spirit be Highly Rectify'd, it will be Lighter than the Oil, which yet, by Mixing with it a Drop or two of Water, will Preponderate and Sink below it.

Dr. Hook, Pag. 11, 12. Vol. 2.

IN this Baroſcope it is Obſerved,

1*ſt.* THAT each 13 Foot of Oil and Spirit will Raiſe the Mercury 10 Inches.

2*d.* THAT, when the Atmoſphere is Heavy, the Mercury will Riſe into the firſt Cylinder, and leave the ſecond, which the Spirit of Wine takes Poſſeſſion of; The Spirit of Wine leaves the Cane, which the Oil Seizes, and Quits the third Cylinder; On the Contrary, when the Air is Light, the Mercury Sinks from the firſt Cylinder, and Enters the ſecond, the Spirit of Wine Riſes from the ſecond Cylinder and Enters the Cane, and the Oil of Turpentine leaves the Cane, and Replaces it's ſelf in the third Cylinder; ſo that the Difference of the Mercury at the Higheſt, and Loweſt, will be Inlarg'd into the Length of the whole ſecond Cane.

3*d.* THAT the Dilatation and Contraction of the Spirit of Wine, which may happen in this Baroſcope, is to be Accounted and Allowed for, by the like Dilatation and Contraction of a Thermometer of equal Size.

WE ſhall here therefore take Notice upon the foregoing Obſervations;

FIRST, That from the Deſcription of this Baroſcope, 'tis Plain, that Fluids Mixing together, do not Preponderate, or Aſcend by the Reſpective Gravity, or by the Lubricity or Oilineſs of their Parts, but by their Natural Inactivity and Sluggiſhneſs, or their Contractive Force on the one Hand, or by their Fire, and their Spirituouſneſs, or their Expanſive, on the other;

SECOND, That from the 1*ſt.* and 2*d.* Obſervations, 'tis true Mercury does Aſcend by Preſſure in this Experiment, that is, of the Oil and Spirit, but whether this Preſſure Proceeds from their Comparative Gravities, or from the Exertion of the Expanſive of the Oil and Spirit, againſt the Contractive of the Mercury, is not Determined; we Inſiſt upon, and Defend, the Latter, in Conjunction with the Expanſive and Contractive of the Glaſs and Air;

THIRD, That it is not unlikely but Mercury may be Dilated or Expanded by Heat, or the Expanſive Force, and be Contracted by Cold, or the Contractive, as well as Spirit of Wine in the Thermometer, tho' not in ſo great a Degree, upon the Account of the Weight and Contractive Force of the Mercury, in Compariſon with that of the Spirit of Wine; And this is not only not unlikely, but pretty Manifeſt and Certain.

THE next Obſerver upon the Barometer, whom we ſhall Conſider, is Dr. *Halley*, Who Informs us;

FIRST, that *May* the 26*th*, 1697. between one and two in the Afternoon on the Top of *Snowdon*-Hill in *Carnarvonſhire*, which was Meaſur'd by Mr. *Caſwell* to be 1240 Yards High, the Mercury ſtood at 26, 1 Inches, and when He came to *Llanberris*, at the

Dr. Halley, Pag. 13. Vol. 2.

the Foot of the Hill, about 6 in the Evening He found it 29,4 Inches, and this Experiment He thrice Repeated; He alſo as often Experimented very near the Surface of the Sea, the next Night at 8 in the Evening, that it ſtood at, 29,9 Inches, and the Air continued both before and after in the ſame State; Therefore He Concludes, that 1240 Yards of Air is = to 3 Inches $\frac{8}{10}$ of Mercury, or thereabouts, and that $\frac{1}{10}$ of an Inch of Mercury is = to 99 Foot of Air very nearly; Mr. *Derham*, *September* 1696. finding his Experiments on the Top of the Monument in *London*, ſomething Different from Dr. *Halley*'s, Namely, that $\frac{1}{10}$ of an Inch of Mercury was = to 80 Foot of Air, and $\frac{2}{10}$ = to 160, Tryed the ſame Experiment with more Niceity, and Exactneſs, *November* 1697, when $\frac{1}{10}$ of an Inch was = to 82 Foot, and $\frac{2}{10}$ = to 164, *i. e.* the Mercury Fell in that Proportion; But He found by ſtaying above, ſomewhat long, the Preſſure of the Atmoſphere was alter'd, and therefore, going up and down Quicker than before, He found the ſame Proportion as at firſt; He thinks however, that if the Mercury ſinks $\frac{1}{10}$ of an Inch at 82 Foot Aſcent, when the Barometer is at 30 Inches, it may not yet ſink ſo much, when at 29.

Mr. Derham, Pag. 14. Vol. 2.

This ſeems to be the Irrefragable Argument for the Gravity of the Air, being the Cauſe of the Aſcent of the Mercury, and Conſequently of it's Suſpenſion; Which Gravity is far from being prov'd by theſe Experiments; Becauſe the Air's Expanſive and Contractive Forces, as we Aſcend, may be Proportionably Diminiſh'd, and it is, what Philoſophers themſelves own, that the Medium is ſtill Rarer, as we go Higher into it, and therefore the Expanſive and Contractive Forces muſt by Conſequence be Leſſened, upon which Forces we have made both the Suſpenſion of the Mercury in the Tube, and the Variation of it's Standard to Depend.

We ſay, that Gravity cannot Produce theſe Effects, becauſe no Gravitating Preſſure is felt from the Atmoſphere by our ſelves, or any other Bodies, upon the Account of the Æquilibrium in it; Nor is it enough to Anſwer, that in the Æquilibrium of a Fluid, tho' the Parts do not Preponderate, and ſo their Gravity is not felt, yet they may Preponderate, where there is nothing to React againſt them, in the Tube above the Quick-Silver; ſince the Parts of the Atmoſphere, as to any Actual Gravitation, are Perfectly in Æquilibrio, and if the Quick-Silver were away, and that Space were Abſolutely Void, would Aſcend no more than as that Void Space had leſs Action or Force than the Expanſive and Contractive of the Atmoſphere, and not according to the Force of the whole Incumbent Column, which is ſuppoſed to Gravitate; For ſuch a Force would be ſo much Superior, as to Break in Pieces the Glaſs, which does not happen even in an Exhauſted Receiver.

'Tis True, where there is no Reſiſtence or Action to the Contrary Side, the Air has a Prodigious Force, which is Manifeſt in it's Ruſhing into Spaces Exhauſted of it, as in the Wind-Gun, Mention'd before, of Dr. *Papin* by Rarefaction; No One yet ever Pretended to Explain this Effect by the Gravitation of the Air, but by it's Elaſticity; 'Tis this Expanſive Force joyned with it's Contractive, which has ſo vaſt a Power in the Barometer, which, tho' Flat and Languid in it's Natural State, Exerts it's ſelf to ſo great a Degree, where there is none to Oppoſe it; The Mercury therefore in the Barometer, when it has Conquer'd all the Force of Air in the Tube, or left none there, finds the ſameForce from the External Air, as if the Mercury were Shot into a Wind-Gun; For it is the ſame, whether it hinders ſuch a Weight from Deſcending in the Tube, or whether it Shot ſuch a Weight into it, which was of equal Force. We do not however here Affirm, that this Expanſive and Contractive Force of the Air Supports the whole Standard of Quick-Silver in the Tube, ſince we have ſaid the Mutual Action or Expanſive Force of the Glaſs and Quick-Silver has an Important Effect in the Production of this Phænomenon.

What we have Alledged concerning Air and Mercury holds in ſome Proportion in Air and Water, which, as *Rohault* Informs us, Riſes to $1\frac{1}{2}$ Foot above 30, in the Siphon, which ſhows the Force of the Air's Action to no Action, or a very Inconſiderable one, to be much the ſame; For Suppoſing we take 30 Foot of Water, 80 Foot of Air, according to Mr. *Derham*, is = to $\frac{1}{10}$ of an Inch of Mercury, *i. e.* 960 Inches of Air = to $\frac{1}{10}$ of an Inch of Mercury, or 9600 Inches of Air = to 1 Inch of Mercury; On the other Hand 30 Foot of Water are = to 30 Inches of Quick-Silver, becauſe the Air has that Reſiſtence or Reaction to both, *i. e.* Mercury is to Water as 12 to 1, but Water is to Air, as 800 to 1; therefore by this Computation, likewiſe, Mercury is to

Air, as $12 \times 800 = 9600$ to 1; which is not the Gravity of the one to the other, but there is that Force of Action in the Air, Exceeding the Action of what is call'd a Vacuum, into which it would Ruſh, if not hinder'd by an equal Force; and upon theſe Principles, all the Variations of the Mercury are eaſily Accounted for, and not by Gravity; For as this Force in Air is Increas'd, as was ſaid before, by Cold, or Dryneſs, or Elaſticity, or by Heat ſometimes, or Diminiſh'd by Warmth or Moiſture, or by Winds, ſo it has a greater or leſs Power, to Act upon a Non-Reſiſting or Inactive Medium; And as the Air grows Thinner, and more Dilated, by it's Height, it is Plain, there muſt be a leſs Difference betwixt a leſs Active, or more Dilated, and a Non-Reſiſting or Inactive Medium; This likewiſe gives an Account why, if we Incline the Barometer or the Tube of Water, the Mercury or Water does not Vary, which it ſhould do, if it's Height Depended upon a Perpendicular Gravity of the Atmoſphere.

FROM Hence it is likewiſe Apparent, that there is not a Void Space in the upper Part of the Barometer, ſince there is a Proportion given betwixt the Reſiſtence of the External Air and that, *viz.* as 9600 to 1.

LASTLY, here is an Eaſie Account given of Mr. *Hugens*'s Experiment, which has ſo much Perplexed the Philoſophick World, which could never be done by the Gravity of the Atmoſphere, tho' the Lord *Brouncker* tells us, that there is Really a greater Gravity in it, than what will Support 29 or 30 Inches of Mercury; on the Contrary, Gravity is a Firm and conſtant Principle, and therefore, notwithſtanding any Concuſſion of the Tube, would Support the Mercury at 75 Inches, according to Mr. *Hugens*'s Experiment, whereas, upon Shakeing it, it Falls to it's uſual Standard; Dr. *Wallis* Aſcribes it to the Elaſticity of the Air, which would lie under the ſame Inconveniency, as the former; for Elaſticity, according to his Explanation of it, would not be the leſs, but the greater, by an Incumbent Load, and therefore upon Concuſſion and the Mercury endeavouring to Deſcend, ſuch Elaſticity would Proportionably Increaſe and hinder it's Deſcent; Mr. *Hugens* would have it, that there is a Preſſure beſides of a Subtile Matter, which is a *Carteſian* Romance, and entirely Precarious and Groundleſs; It can therefore be Reſolv'd into nothing but this, that the External Air endeavouring to follow the Mercury, which Proceeds into the Vacuum of the Tube, which it would Poſſeſs, if the Mercury was away, fixes it's ſelf cloſe to the Mercury in the Ciſtern, and there Suſtains it; becauſe if the Mercury ſhould Deſcend without any Succuſſion, it would by it's Gravity, which is none in the Tube, where it is in an Æquilibrium, be able to Preponderate the Force of Reſiſtence in the Air, which has none at the Top of the Tube to Oppoſe it, and whoſe Force in Reſpect of that would therefore be Infinite.

THIS at leaſt may be one Reaſon of the Suſpenſion of the Mercury to ſo great and unuſual an Height; but if we will Allow the Doctrine of the Expanſive and Contractivce Forces, this Matter I believe will ſtill be Clearer; for the Expanſive Force of the Mercury, Acting againſt the Expanſive of the Glaſs, Cements them to each other, and the Contractive of the Mercury or it's Gravitating Force, being Balanced in it's ſelf in ſome Meaſure, and what remains unbalanced being Supported by the Expanſive or Contractive of the Ambient Air or Plenum, Nothing can make the Mercury Deſcend, but Diſturbing this Æquilibrium by the Expanſive of Percuſſion.

HOWEVER, upon the Reſult of the whole, we may draw this Concluſion, which is what we Originally Intended, that Philoſophers, in their Diſputes about theſe Matters, do own a Continued Action of Gravity, or Elaſticity, Diffuſed and Diſperſed through every Point of Space, to which, it is Evident, theſe Experiments Reach or Extend.

SECOND, Dr. *Halley*, in Order to Aſcertain from this Doctrine the Height of the Atmoſphere, makes the Weight of Water to that of Air, as 840, 852, or 860 to 1, by ſeveral Experiments of Water upon the Barometer, which ſtood in each of them at 29 Inches $\frac{3}{4}$ in Summer, when the Air was Rarefy'd, and therefore the Thinner; ſo that 800 to 1, He takes to be a Moderate Proportion of Water to the Air in a more Natural State.

Dr. Halley, Pag. 14. Vol. 2.

BUT the Weight of Mercury to Water is as $13\frac{1}{2}$ to 1, and therefore the Weight of Mercury to Air, as 10800 to 1, or 900 Foot of Air = to 1 Inch of Mercury, and the Atmoſphere will be 5,1 Miles High, if the Air were equal in all Parts, but being Expanded in the ſame Proportion, as the Incumbent Weight of the Atmoſphere Decreaſes, each Space Anſwering to an Inch of Quick-Silver, grows Greater and Greater, as we go Higher ; and Conſequently the Expanſion of the Air being Reciprocally, as the Heights of the Mercury, the Atmoſphere will reach at the Higheſt, to 45 Miles, See the Computation, *Pag.* 16. *Vol.* 2. and at 41 Miles the Air is ſo Rarefied, that it takes up 3000 times the Space it Occupies here, and at 53, 30000 times the Space, and therefore it is unlikely that the Air's Spring ſhould Exert it's ſelf to that Degree ; and Conſequently 45 Miles is the Higheſt.

In Anſwer to this we ſay, that the Height of the Atmoſphere, or Gravity of it, cannot be Deduced from Experiments upon the Barometer ; For, beſides the Arguments we have brought againſt it, and the Explanation we have given, it is very Unfair in Reaſoning to Aſcribe the ſame Effect to two different Powers of Producing it ; the Mercury Sinks from 30 Inches, or more, to 27 in the ſame Height of the Atmoſphere, the Reaſon of which is the Alteration of the Temperature of the Air, as theſe Philoſophers Acknowledge, tho' they Aſcribe that Effect to the Lighter Temperature, we to the leſs Active or leſs Reſiſting ; It Sinks in the ſame Proportion, if we carry it to an higher Part of the Atmoſphere, what's the Reaſon? Why now Truly, not becauſe the Temper of the Air is alter'd, but becauſe there is ſo much direct Gravity cut off from the Incumbent Column ; Whereas, we Say ſtill, that it Proceeds from the ſame Cauſe, as the other, an Alteration in the Temper of the Air, or it's Force of Action or Reſiſtence ; For here it is own'd by thoſe who Oppoſe us, that the Air's Expanſion is Increas'd, or, which is all one, it's Force Diminiſh'd, as the Mercury Falls ; Moreover, it is not Improbable, that the Air's Reſiſtence, Action, or Force, may not only be Diminiſh'd 3000, or 30000 times, but 30 Millions, or to Infinity ; as Sir *Iſ. Newton* ſeems likewiſe to Apprehend ; and therefore Dr. *Halley* cannot Conclude from the Impoſſibility of ſuch an Expanſion, that the Height of the Atmoſphere is only 45 Miles at the moſt ; Becauſe, according to the Doctrine of the Expanſive and Contractive Forces, there may be ten Millions of Degrees of both, without Suppoſing Elaſticity or Expanſion to bear any Proportion to the Preſſure of the Incumbent Atmoſphere.

However Dr. *Halley* ſays this Computation, which is Affirmed upon the Suppoſition of the Atmoſphere's Gravitating upon the Mercury, ſeems to be Aſcertained from a Computation made from the Crepuſculum's continueing till the Sun's Falling 18 Degrees below the Horizon ; which yet is the ſame Kind of Demonſtration, which *Varenius* Confutes, and by which *Nonnius* and *Clavius* made the Height Indeed of the Atmoſphere about 44 Miles, Dr. *Halley* 44, 4. See *Pag.* 17. *Vol.* 2. but *Varenius* 246, and another, whom *Varenius* Mentions, made it 52, which is within one Mile of that Number, which Dr. *Halley* reckons Impoſſible, *viz.* 53 ; ſo that from ſo Uncertain a Computation nothing Certain can be Inferred, Eſpecially to Confirm Another, which is equally Uncertain, and as we ſay, Falſe.

Third, by this Computation Dr. *Halley* ſays, the Air is Expanded to 3000 times the Space it Occupies here, and by Experience it may be Condenſed into a *60th*, ſo that it is Capable of being Compreſſed into the 180000*th* Part, *viz.* 60 × 3000*th*, of the Space it would take up, if free from Preſſure ; which great Contraction and Expanſion, he thinks, is not to be Accounted for by the Compariſon of Wool or Cotton, as Mr. *Boyle* would have it, or other Springy Bodies ; for what Texture or Compoſition of Parts is Capable of ſuch Contraction or Dilatation? a Queſtion, He ſays, which ſeems very hard to be Reſolved ; Farther, that the Rarefaction of the Air cannot Proceed in Infinitum, as is Proved the Condenſation cannot, He ſays is Plain, becauſe there muſt be a Determinate Magnitude of the Natural State of each Particle, as in Wool, and the like, which have a Determinate Bulk, which they cannot Exceed, when freed from all manner of Preſſure.

This Expanſion of the Air we ſhall therefore Conſider, and, as it ſeems to me, there is no more Difficulty in Conceiving Air to be Expanded to 3000 times it's Space, when Compreſſed, or to 180000, when Rarefy'd to the utmoſt, which He allows, than to 30000, or 1800000, which He will not ; Nor is there any Abſurdity to Imagine it to be Expanded to Infinity, as Sir *Iſ. Newton* would have it ; 'Tis true it cannot be Condenſed by Experiments to Infinity, nor can it be Rarefyed by them to Infinity ; But all our Force in Experiments is only Finite, which we can Uſe, whether to

Rarefy or Condenſe it; It is alſo True, that every Particle has it's Determined Magnitude; But the Particles of the Corpuſcular Philoſophy I take to be here no ways Concerned; And as to the Particles of Wool, they continue the ſame, whether Condenſed or Dilated, and their Action only is Different, and therefore is an Improper Inſtance: In ſhort, a Similar Matter can be no other than it's ſelf, and cannot be Expanded but to a certain Degree; But the Queſtion is not, whether the ſame Kind of Similar and Homogeneous Matter can be Infinite in it's Expanſion, but whether Matter may not put on Various Forms, and Equally fill Space; The Air, Conſidering it to be Compoſed of Determined Parts, or Atomes, according to the Corpuſcular Scheme, can reach no farther, than thoſe Parts will Permitt it; But if we Conſider it as a Body, Capable of Changing it's Dimenſions, and being Thicker or Thinner, that is, having a more Compreſſed or a more Diluted Action, and yet Continued thro' Space, it may be Expanded to Infinity; that is, being a Body eaſily wrought into any Form by it's very ſmall Reſiſtence, it may be Conſtipated, as far as the Finite Force of Engines will do it, and may be Dilated to Infinity by the Expanſive Force of the Sun; As for Example, tho' by the *Florentine* Experiments of the Academy *Del Cimento*, Water cannot be Compreſſed, yet it may be Dilated and made Lighter, by being Changed into Air, by the Expanſive Force of the ſaid Body, and Air by being Changed into Æther, and yet all three fill Space; On the Contrary, Matter may be Contracted into Gold, or an Adamant, which Force will be Infinite in Reſpect of the Contractive of Æther; that is, the Expanſive Force of Matter, or the Contractive, may be Increaſed to Infinity in Nature, tho' not by our Experiments, or by Finite Agents.

Fourth, He tells us, that the whole Weight of the Atmoſphere is Various, being Counterpoiſed by 28 Inches and $\frac{1}{2}$ ſometimes, and at others, by 30 $\frac{1}{2}$, which, if 2 Inches of Mercury are Equal to the 15*th* Part of the Gravity of the Air, will make the Gravity of the Air ſometimes Lighter by a 15*th* Part, than at other times.

This ſeems to be a Sufficient Argument, why the Gravity of the Air is not here Concerned; For by the Alterations of the Atmoſphere, from any Cauſes whatſoever, can we Poſſibly Suppoſe, that there ſhould be ſuch a Difference in the ſame Place, and many times without any great Alteration of the Air, as that it ſhould be Diminiſhed a 15*th* Part of it's Weight? Whereas, Allowing the Force, or Action, of the Air, to be different, the Solution is Eaſy.

Fifth, we are Told, that Heat and Cold do Conſiderably Dilate and Contract the Air, and Conſequently Alter it's Gravity; On the Contrary, we do not Think they can to any great Degree, at leaſt not to Increaſe, or Diminiſh it, to a 15*th* Part of the Whole; for whether Water be Boiled or Frozen, which is more than happens to Air from any Natural Cauſes, they are much of the ſame Specifick Gravity, tho' Ice is ſomething Lighter; which is a Plain Argument, that the Variations of the Mercury Depend upon the Expanſive and Contractive Force of the Air, ſince in Froſt it Riſes Higher, and yet, according to an Analogy with Water, the Air ſhould be Lighter; However Ice will Reſiſt to the Deſcent of any Body, more than Water, and therefore Air, when Frozen, may, tho' it is Lighter.

Sixth, He ſays, if we add to this the Mixture of Effluvia or Steams, which Aſſimilating into the Form of Air, are kept Suſpended therein, as Salts Diſſolved in Liquors, or Metals in Corroding Menſtrua, as in Spirit of Nitre, which are Heavier by much than Air, it's Gravity will be Increaſed.

On the other Hand we Alledge, if theſe Metals, or Salts, Aſſimilate into Air, and are Suſpended there, it is a Proof they are then in Æquilibrio with it, and therefore do not Gravitate amongſt one another, nor upon any Body, as Mercury, which is Incloſed in them; but they may Increaſe the Contractive Force of the Air, *i. e.* not only to keep the Mercury from Deſcending Lower, but to make it Riſe Higher, by adding a Moment to the Expanſive, becauſe there will be then a greater Force, in Proportion to the Boylean Vacuum, in the Top of the Tube, and Conſequently a greater Force of Ruſhing into it, if the Quick-Silver did not Intercept it.

Seventh, We are Informed, that Condenſations of Air are Confined to a certain Degree, for if the Air is Compreſſed to an 800*th* Part of the Space it takes up here, it's Conſiſtence would be equally Denſe with that of Water, which Yeilds not to any Force by the Experiments of *Florence*, or the Academy *Del Cimento*.

IF this is a Surmiſe, becauſe He has Thought it is Prov'd from Barometers, that Air is to Water, as 1 to 800, the Experiment is wanting; if it is Matter of Fact, that Air can be Compreſſed into an *800th* Part of it's Space, it is Contrary to his other Experiment of it's being Condenſed into a *60th* Part only, ſince He takes no greater a Proportion to Prove the utmoſt Contraction and Dilatation of the Air, which would be *180000th* Part, when Contracted, of what it would be without any Preſſure; but Suppoſing Common Air could be Condenſed to that Degree of 800, and no Force could afterwards Condenſe it, no more than Water, it only Proves, that the Reſiſtence or the Expanſive or Contractive Force of ſuch Water, or ſuch Air, Compreſſed, is Equal to any Force we can bring againſt it, and not that it is Infinite; or if Water or Air Condenſed is Infinite in it's Expanſive or Contractive Forces or Reſiſtence, it does not Prove againſt what We have before in General laid Down, that there are Degrees of the Expanſive or Contractive Force or Reſiſtence; for in thoſe Experiments, there is an equal Preſſure upon the Water on all Hands or nearly, but the Weakeſt Reſiſtence, or Expanſive or Contractive Force, will be as Firm as the Strongeſt, where the Preſſure upon it on all Hands is Equal.

THAT there are different Degrees of the Expanſive, or the Contractive Force, is Evident from Quick-Silver and Water, for if we Separate the Parts of Quick-Silver and let them lie near together, they will tend to each other with a wonderfull Celerity, and ſeem to Embrace ſo cloſely, that one would think there was ſome more than common Sympathy betwixt them; whereas the Parts of Water do not ſhow the ſame eager Propenſity to each other.

EIGHTH, Dr. *Halley* tells us, that the Variations of the Air Compenſate one another, and ſo leave the Gravity of the Air to obtain it's Effect, for when it is Hot, Vapours are Raiſed, which Load the Air as much, as the Heat Lightens it; But if ſo, how comes the Mercury to Riſe, when it is Suppoſed the Air is full of Vapours by the Heat, if thoſe Vapours Compenſate the Heat only?

THEREFORE it does not ſeem ſo well Atteſted, that the Mercury does Riſe in Hot Weather, tho' it is Generally Receiv'd, and the Reaſon Commonly given is, becauſe the Air is Crowded with Vapours; 'tis certain Dr. *Halley* here Reckons the one does but Balance the other; and afterwards, when he Enumerates the Principal Obſervations upon the Barometer, He does not Mention that of it's Riſing in Hot Weather; And by an Experiment already Recited the Sun being let in upon it, the Mercury Sunk, and when the Window was Shut, it Roſe.

See Chap. 1. Book. 1. of this Philoſophy.

AND as for the Air being charg'd with Groſs Vapours, tho' in the cleareſt Day in Hot Weather, I cannot believe there is any ſuch Thing, tho' Dr. *Halley* here Affirms it, but that what Moiſture is Raiſed by the Expanſive Force of the Sun's Rays is Changed into a different Complexion, Nature, and Temper, than that of Water, *viz.* into a fine Fluid Subſtance; and as for Rain after exceſſive Heat, *1ſt.* that does not always follow, ſince there have been very Hot and Dry Summers, when the Winters have not been Rainy for Years together, *2d.* when Rain, or Dew, does Succeed, that Proceeds not from the Latent Groſs Parts of the Air Congealing, but from the ſame Alteration in the Craſis of the Air to Produce Moiſtneſs from a different Cauſe of Cold and the Contractive Force, as Dryneſs Proceeded from the Expanſive Force of the Sun; So that the Air is not the Heavier, but the Lighter, for ſuch Imaginary Vapours, which are Chang'd by the Sun into the Expanſive.

NINTH, Dr. *Halley* farther Aſſures us, that Mr. *Caſwell*'s Experiment, on *Snowdon-Hill* in *Carnarvon-Shire*, Agrees very nearly with his Calculation; the Height of the Hill is 1240 Yards, and the Mercury ſtood at $25\frac{6}{10}$, 4 Inches below it's uſual Standard; the Space in Dr. *Halley*'s Calculation Anſwering to 4 Inches near the Surface is 1288 Yards, which is not more than 48 Yards above Mr. *Caſwell*'s Obſervation; which according to Mr. *Derham*, is about equal to half $\frac{1}{10}$ of an Inch of Mercury; and Dr. *Halley* found the Difference to be 3 Inches $\frac{8}{10}$, ſo that it is $\frac{4}{10}$ leſs than Mr. *Caſwell*'s, and therefore ſhould be, according to Mr. *Derham*, 4 × 82 Yards lower, *viz.* 328; and in the firſt, the Mercury ſhould be half $\frac{1}{10}$ of an Inch Higher than Mr. *Caſwell* Obſerv'd it, and in the ſecond, the Space or Height of the Hill 400 Yards lower than Mr. *Halley*'s Calculation has ſett it, which therefore is no Argument of the Truth of ſuch Calculation. Nor is it Impoſſible but this Difference of the Standard of Mercury at the Top, and at the Bottom of the Hill, may Proceed, not only from what we before Mentioned of the different Expanſive and Contractive Forces of the Air, when we gave an Account of this Phænomenon, but from the Expanſive Force of the Mercury towards the Expanſive of the Glaſs, which may Riſe Higher by a greater Oppoſition of the Contractive of the Earth, and Fall lower by a Leſs, for Contraction, or Gravitation, is Leſs at greater Diſtances, and greater at leſs Diſtances, from it; thus the Expanſive of Fire, and Flame, Riſes Higher, by an Oppoſition from the Contractive of Water, and one Force is often in Nature Excited and Exaſperated by it's Contrary. Or it may be, as the Mercury is Heavier, it's Expanſive Force has a greater Moment, and Adheres, and Preſſes, more Strongly to the Glaſs Tube, and as it is Lighter, more Weakly, and therefore ſtands Higher in the Tube, at the Bottom of the Hill, than at the Top; And Perhaps Succuſſion in the Aſcent up the Hill, may have ſome Influence in this Phænomenon.

TENTH, in General, Dr. *Halley* Aſcribes all the Variations of the Mercury to two Cauſes. Dr. Halley, Vol. 2. Pag. 20.

1*ſt.* THE Principal is the Variable Winds in the Temperate Zones, which are not ſo in the Torrid, and therefore, the Mercury at St. *Helena*, and *Barbadoes*, and ever betwixt the Tropicks, ſtands much at the ſame Height; The Winds, if they have any Effect, muſt Produce it, by heaping up the Air together and Diſperſing it, and Carrying to the Mercury, or from it, different Exhalations, that is, by Increaſing or Diminiſhing the Gravity of the Air in both Caſes; As for the firſt, the Winds Blowing from the ſame, or contrary Quarters, no more add to the Weight of the Air, than Streams or Currents of Water, meeting or taking the ſame Courſe, Increaſe the Specifick Gravity of Rivers; and for heaping Air together by the Winds, it ſeems to be only a mere Chimæra; if it be ſaid, that Clouds Fly about, in Abundance, in Stormy Weather, 'tis Anſwered, and for that Reaſon they are Lighter, and it is Obſerv'd, that even Rain is kept up by the Force of ſuch Storms, and Conſequently Winds will rather Diſperſe than Heap the Air together; as to the Second, or Exhalations, that is Referred to his ſecond Cauſe.

2*d.* THE other General Cauſe is the uncertain Exhalation, and Precipitation, of Vapours lodged in the Air, which in great Meaſure depends upon the former, *viz.* the Winds; To which likewiſe we in General Anſwer, that Vapours lodg'd in the Air are Really no more than Parts of Water, or other Terreſtrial Fluids, Aſſimilated to Air; They become nearly of the ſame Specifick Gravity, and therefore do not Increaſe it's Weight to any tolerable Degree, ſo as to produce the Effect, which we want; and when they Precipitate, it is, becauſe they loſe their Expanſive Force, and put on a Contractive, and therefore Preponderate; ſo that, upon their Deſcending, the Air ſhould be Heavier, and upon their Aſcending, Lighter; The Contrary to which is commonly Affirmed by Philoſophers.

THESE Eight following are the chief Obſervations upon the Barometer, according to Dr. *Halley*.

FIRST, the Mercury Falls in Calm Weather, when the Air is Inclin'd to Rain; which He ſays is, becauſe the Air being Light, the Vapours are no longer Supported thereby, as they are become Specifically Heavier; To which we Anſwer; If the Air is Lighter, and the Vapours Heavier, than their ſeveral Intenſive Gravities, the Weight of the whole Air may be ſtill the ſame, and ſo the Mercury may not Deſcend, but that I ſuppoſe Dr. *Halley* does not Mean; For by what Methods ſhould the Intrinſick Gravity of the Air be Alter'd, or that of Vapours, and the one continue Air, and the other Vapours? tho' this may be by the Power of the Expanſive and Contractive Forces, but theſe He did not Conſider; What therefore He ſeems here to Think is, that the Vapours would Preſerve their own Nature, and Gravity, but being Collected together, and the Air keeping it's Natural State, the Vapours may Preponderate; If ſo, in ſuch a Column of Air, where it Rains, I Ask, whether, whilſt the Air Preſerves it's Natural Gravity, and the Vapours are Amaſſed together, ſo as to be Heavier than before, the whole Column is not Heavier, and Conſequently, whether the Mercury ſhould not Riſe, rather than Fall, upon the Suppoſition, that the Cauſe of it's Riſe is the Gravitation of the Atmoſphere? On the Contrary, we Affirm, as before, when the Weather is Calm, and there is a Diſpoſition to Rain, whether the Atmoſphere be Heavier or Lighter, it matters not, that the Craſis of the Air is Diſſolved, that it's Expanſive and Contractive Force is Leſſened, and therefore the Mercury muſt of Neceſſity Fall.

This however is only Dr. *Halley's* Account of it's being Lower in Rain; But why it is Lower in General at one Time than another, He ſays is, becauſe the Air of that Place is carry'd both Ways from it by Winds, Blowing two different Ways from the ſame Place, from whence, the Weight of the Air being Diminiſhed, the Vapours Fall; This is a Wonderfull Solution indeed, to bring Winds to what Place we Pleaſe, and then make them ſet out Fairly together, to Sweep all the Air away before them, in Order to make the Mercury Deſcend; but Suppoſing from this ſame Barometer, that two Winds ſhould make all the Room, that is Poſſible, for Deſcending Vapours; Would theſe Heavy Vapours have no Preſſure upon the Column, in which they Deſcend? As great, it is not Improbable, as the Preſſure is Leſs, which is taken off by the Winds, Suppoſing they take it off; Eſpecially, ſince it is before Obſerved by Dr. *Wallis* or Dr. *Beal*, that as the Air is Lighter, there is always a Preſſure to thoſe Parts, and very Juſtly, tho' Perhaps not at the Diſtance, as was there Imagined, and to Solve that Phænomenon.

Second, in good Serene ſettled Weather the Mercury is generally High; This Dr. *Halley* Attributes to Winds Blowing from different Quarters to the ſame Place, where the Mercury ſtands in the Barometer, by which the Air is Accumulated and made Heavier, and the Atmoſphere Higher, and the Rain is hindered from Falling, which He ſays is the Reaſon of the Serene good Weather, which Attends the greater Heights of the Mercury: Therefore, as before the Winds ſett out from one Point, ſo they are now to begin their Courſe from contrary Points, and to meet in the ſame; and by the like way of Reaſoning, if it would have Solved the Phænomenon better, He might have made Choice of all the Winds, and all the Quarters, in the Univerſe, and Diſpos'd and Shuffled them together, as beſt ſuited his preſent Purpoſe; This is not however Solving of Difficulties, but Blowing them away; That Windy and Bluſtering Weather yet, ſhould be called Serene, is a little Strange; If He means Serene afterwards, and that then the Mercury Riſes, He takes away the Support, which He gives, Namely, the Winds meeting in the ſame Point, or the Storms He had Rais'd to Suſtain it: And here I cannot but Obſerve, that according to Dr. *Halley*, the Mercury Falls, whilſt the Condenſed Vapours are Actually in the Air, which ſhould make it Riſe, and Riſes after the Winds are Over, which are the Reaſon He Alledges, that it does not Fall; Unleſs, I ſay, He makes good Serene Weather the ſame as Windy: The Solution therefore, as before, I take to be this, that in good Serene Weather, when there are neither Rains, which Diſſolve the Craſis of the Air, nor Winds, which Take off from it's Innate Force, the Mercury is generally High, whilſt the Air obtains it's Proper Force of Expanſion or Contraction, and of Endeavouring to Ruſh into the Tube, where there would be leſs Reſiſtence, if the Weight of the Mercury did not Counterbalance it's Preſſure.

Third, upon very great Winds, tho' not accompany'd with Rains, the Mercury Sinks Loweſt of all; 'tis ſaid *1ſt.* Becauſe theſe Winds do not Blow every where, and therefore the Stagnant Air, where they do not, cannot come in faſt enough to Supply the Deficiency of Air, which is carry'd off by them; *2d.* Becauſe they may Poſſibly, by their Horizontal Rapidity, Intercept the Perpendicular Preſſure, or Gravitation, of the Air: But *1ſt.* It is not certain, that the Winds carry off any Air, but it is rather certain, that theſe Winds are the extraordinary Motion of the Air it's ſelf, and as a Rapid Stream does not carry off the Water, ſo as to leave none to Succeed it, it may very Probably be ſo in the Air; that is, ſuppoſing here is a violent Current in one Place, and there is Air or Water Stagnates behind, yet as the one goes off the other comes on; And therefore we cannot but believe, that the Alteration made in the Air is not a Diminiſhing of it's Gravity, but of it's Force or Preſſure, as Quieſcent or Stagnating Fluids have a greater Preſſure than thoſe, which have an Activity, induced upon them, to take off that Preſſure; *2d.* As to the Horizontal Rapidity Diminiſhing the Perpendicular, *i. e.* the Gravitating, Force of the Air, it is true; But then it is likewiſe true, that it Diminiſhes the Preſſure to other Parts, as well as to the Center, that is, the Expanſive and Contractive Force of the Air, on which we have ſaid the Variations of the Mercury Depend.

We are here farther told, that Rain is the Conſequence of the Air's Rarefaction, becauſe being Heavier, it then Precipitates: On the Contrary, it is not Improbable, but that the Languid and Contractive Force, not only of the Air, but of the Vapours, is the Cauſe of Rain; The Air and Vapours grow leſs Expanſive, their Elaſtick Tone is Perfectly Chang'd, and ſo they Diſſolve and Fall; For if it Proceeded from Rarefaction, it ſhould Rain moſt in Sun-Shine, and Extreme Hot Weather, which it ſeldom does; and ſince in Sun-Shine the Mercury Falls, as has been Proved, if the Deſcent of Mercury, and Rain, Ariſes from Gravity, why ſhould not Rain Fall then two, that is, in Sun-Shine, when the Mercury is ſaid to Fall, and the Air is Rareſyed?

Fourth, *Cæteris paribus*, the Mercury is Higheſt on Eaſterly and North-Eaſterly Winds; Becauſe, it is ſaid, the Trade Winds conſtantly Blow upon the *Atlantick* Ocean, which are Weſt and South-Weſt, and therefore ſtop the Current of the Eaſterly and North-Eaſterly Winds, by which the Air is Accumulated, but if they Blow exceeding Strong, they overcome the Oppoſition from the Trade Winds, and ſo the Mercury Falls, which, on ſuch Occaſions, the ſame Perſon has Obſerv'd to ſtand as Low, as 29 Inches: To which we Reply, that the Accumulation of the Air by ſuch Means has been already Anſwer'd; and the Reaſon of this Phænomenon ſeems to be, becauſe thoſe Winds are Impregnated with a Cold, Nitrous, and Contractive, Force from the North, or with an Expanſive from the Eaſt and the Sun, both which Forces, we have ſhewn, may Contribute to the Riſe of the Quick-Silver.

Fifth, It is ſaid, in Calm, Froſty Weather Mercury is Generally High; Becauſe the North Winds poſſibly Blowing near, and being met at the Ocean by contrary Winds, may heap the Air towards the Barometer, and then bringing from the North a Denſe Frozen Air, the lower Parts of the Atmoſphere, being Contracted into a leſs Room, the upper Parts Deſcend upon it to make an Æquilibrium, and Conſequently Increaſes the Gravity of the Air on that Account: To which we Anſwer, that it is not Agreeable to Reaſon to Think, that in Calm Weather, the Winds ſhould be any ways Concern'd, Eſpecially where the Effect may be Accounted for without ſuch Foreign Cauſes; the Cauſe is Intimate and to be found in the Air it's ſelf, that is, in it's Coldneſs or Contractive Force, which being Increaſed muſt Neceſſarily Raiſe the Mercury by the Contraction of the Glaſs or the Quick-Silver, or both, into a ſmaller Cylinder, and muſt therefore Increaſe it's Length, or it's Height.

Sixth, after great Storms of Wind, when the Mercury has been Low, it generally Riſes again very Faſt; after a continu'd Storm of a South-Weſt Wind, it Riſes 1 Inch and $\frac{1}{2}$ in leſs than 6 Hours; Dr. *Halley* Affirms, becauſe, the Air being very much Rarefy'd by the great Evacuations, the Neighbouring Air runs in more Swiftly to bring it to an Æquilibrium, as Water runs faſter for having a greater Declivity; To which we Reply, that it is not Evident, that there is any Evacuation of the Air or any Chaſm made in the Atmoſphere by Winds; which ſeem to be only an Extraordinary Ferment or Agitation of the ſame Fluid, ariſing from a Conteſt in it betwixt the Expanſive Force of Heat and the Contractive of Cold, as there is betwixt the Acid and Alkali of the Chymiſts, and as this Agitation Languiſhes by Degrees, the Air is Reſtored to it's Natural Preſſure and Impetus upon the Mercury, which therefore Riſes, in a little Time after, from the Reſiſtence or Impulſe of the Air; I ſometimes call This, Reſiſtence, Impulſe, Elaſticity, or Impetus, becauſe what is Reſiſtence in it's own Fluid of equal Reſiſtence, or to a Body of Superior Reſiſtence or Impulſe, is Impulſe upon Vacuum, or a Fluid of a much leſs Reſiſtence; That is, we are to Conſider nothing in Nature, but the Relative Forces of it, there being none, which are Abſolute or Poſitive, without a Relation to others, and by which Relation the ſame Force is Infinitely Differenced and Varied.

Seventh, the Variations are greater in the more Northerly Parts, than the Southerly, at *Stockholm* than at *Paris*, as compared by Mr. *Paſcall*; It is ſaid, becauſe the Storms of Wind being Greater there, they make the Mercury Sink Lower than in other Places, and the Northerly Winds, bringing Ponderous Air from the Poles, and being Stopt by the Winds of the Ocean, make it Riſe Higher in the other Extreme, than we find in the more Southerly Parts: On the other Hand we Alledge, that the moſt Probable Account ſeems to be, what has hitherto Anſwer'd all the Phænomena of the Barometer, the Exceſſive hard Froſts, which are there, and which muſt needs Increaſe the Air's Contractive Force, and therefore, the Height of the Mercury, more than in any other Place, and ſince this Froſt is not conſtant, whenever it Changes, that very Froſt Contributes to making the Air the Moiſter; And Conſequently upon an Alteration of Weather, the Craſis and State of the Air is more Diſſolute, and Languid, by the Froſt's Changing into Moiſture, than in other Places, from whence the Mercury Sinks, as we find even here upon Thawes.

Eighth, Within and Near the Tropicks, or, as is afterwards ſaid, Near the Æquinoctial, there is little or no Variation, as Dr. *Halley* himſelf has Obſerv'd at St. *Helena*, and has found by Comparing it with the Accounts of Others; He tells us, becauſe the Winds there, Blow generally from the ſame Points, E. N. E. at *Barbados*, and E. S. E. at St. *Helena*; So that there being no contrary Currents of Air to Exhauſt or Accumulate it, the Atmoſphere continues much in the ſame State, tho' upon Hurricans, the moſt Violent of Storms, the Mercury has been Obſerv'd very Low; This however is but once in 2 or 3 Years, and it ſoon Recovers it's ſettled State of 29 Inches and $\frac{1}{2}$: Dr. *Halley* takes this to be a full Confirmation of his Doctrine, whereas it is rather the Contrary; For it is 1*ſt.* Own'd they have Winds, which Blow from the ſame Point, which therefore ſhould make the Mercury Sink by carrying off the Air; 2*d.* There is no Reaſon to Suppoſe there are not contrary Winds at a Diſtance, which Stop this conſtant Courſe of the Air, and we may as well Introduce the *European* Winds to Stop the *Tropick* ones, as He the *Atlantick*, to Check our *Engliſh*, and therefore the Mercury would ſometimes Riſe: And to this whole Doctrine, we ſay, where a Philoſopher will give himſelf the Liberty to Range through the Univerſe for Foreign Cauſes of a Domeſtick Effect, it will be hard if He cannot Solve all Phænomenons; ſince, by Rambling about, He may Pick up Cauſes for his Purpoſe, tho' He is Senſible at the ſame Time, He Overlooks ſeveral, which are not, and yet may bid as Fair to be Intereſted in the Production of ſuch Effects; For when at ſome times, one Wind ſhall be only Concern'd, and at others, Winds ſhall be Rally'd from diſtant Corners of the Globe, it is Plain, we do it, to Patch up an Hypotheſis, where we find it Deficient, and not to give a True and Adequate Solution of Nature: Our Account on the other Hand is Conſtant to it's ſelf, and Conſiſtent with what fore-goes, and Alledges no other Reaſon of this Phænomenon, but what is drawn from the Circumſtance of the Air, which hangs about the very Barometer, that is, the Temper and Conſtitution of it; For the Expanſive and Contractive Forces, we have Affirm'd, Produce the ſame or a like Effect; Therefore there being no great Variation of the Expanſive betwixt the Tropicks, as not lying, as the Temperate Zones do, betwixt the two contrary and contending Forces, or Principles, of Heat and Cold, or the Expanſive and Contractive, the Mercury keeps at the ſame Height; 'Tis True, there are Winds, which conſtantly Blow there, but thoſe cannot Influence the Mercury, unleſs they be of ſuch Force, as to Intercept the Preſſure of the Air to the Tube, which common Winds, or Breezes, cannot do, becauſe there is a greater Force of Direction towards the Tube, than there is a Force of Wind to Change that Direction, which only happens in Hurricans; Or when there is an Enervation of the Air, by a Particular Moiſture or Languor of it, the Mercury Sinks likewiſe.

Concerning the Cauſe of the different Weather and Winds within the Tropicks and the Temperate Zones, and of the Mercury's Aſcent and Deſcent from thence.

§ 29. I Shall here Add, ſince we are Entered into this Argument, what Appears to be the Cauſe of the different Weather and Winds within the Tropicks, and the Temperate Zones, and of the Mercury's Aſcent and Deſcent from thence.

First, Therefore we ſay, where Heat, or the Expanſive Force, Prevails, the Weather muſt be much the ſame, as to the Temperature of it, or where Cold, or the Contractive, does, and Conſequently the Mercury in all Likelihood would find very few if any Variations under the Poles, as it does not betwixt the Tropicks; Becauſe that Region is ſo far remov'd from the Sun's Heat, that it is wholly Obedient to the other Principle of Cold, or the Contractive Force, and Therefore the Craſis and State of the Air muſt be nearly the ſame; For that which makes it Alter, is ſometimes the one being Predominant, and ſometimes the other; Which happens in thoſe Countries, which are Situate betwixt the Articks and the Tropicks, where Heat and Cold, or the Expanſive and Contractive are in a kind of Struggle with each other, by which Means Winds and Storms are Produced; That is, when a Nitrous Medium and a Fiery meet, and Neither of them is able to Subdue the other; And this is the Reaſon, that our Storms are Generally betwixt Summer and Winter, or the Contrary, and ſeldome or never in ſettled Froſt or Heat; So that there are two Cauſes of Winds, our being Situate betwixt the Regions of Heat and Cold, or of the Expanſive and Contractive Forces, that is, in the Temperate Zones; and the Seaſons, which lie betwixt the Hot and Cold, or the Expanſive and Contractive ones of the Year, *viz.* Spring and Autumn; However, we may perhaps give a fuller Account of the Nature of Winds, when we come Particularly to Treat of them; We only Conſider them here, as they Relate to the Barometer.

Second, But as Heat and Cold, or the Expanſive and Contractive Force, are the firſt to be Conſider'd, as the Cauſes of the Alterations in the Air, ſo is the Medium, on which they make their Impreſſions; ſometimes it is perfectly Dry, and ſometimes Moiſt; If Dry, whether Heat or Cold, or the Expanſive or Contractive Force Intervene, the Mercury will Riſe, that is, where the Heat or Expanſive Force is ſuch, that it does not Weaken the Air's Spring or Moiſten it by Vapours, becauſe the Air then has it's full Force of Expanſion or Elaſticity; But if Moiſt, the Air grows Flat and Enervated, and Heat or Cold, or the Expanſive or Contractive Force, will do little to Reſtore it to it's Natural Force, and Conſequently Rain or Snow will make it Sink; And that there is an Enervation of the Air's Spring in Rain and Snow is Manifeſt, from the Dead Sound of Bells, which we Experience at ſuch Times; Therefore, according as theſe Principles Prevail, Heat and Cold, or the Expanſive and Contractive Forces, or the Enervation of them in the Languor and Moiſture of the Air, ſo is the Riſe or Fall of the Mercury; And when it Falls upon very great Winds, it ſeems to be owing to the Moiſtneſs of the Air, ariſing from the Conteſt of theſe two Forces, which Weaken and Impair each other, and therefore our Weſterly Winds, which come from the Ocean, being Loaded with Water, Act with greater Force to the Depreſſion of the Mercury, than Eaſterly and Dry ones, which come from the *German* Continent; The Former make the Mercury Sink, becauſe Moiſt, and therefore is a Proof, that it does not Riſe from the Weight, ſince Watry Winds muſt be Heavier than Airy; The Latter make it Riſe, becauſe Dry and Expanſive, and therefore the Natural Force of the Air is not Impair'd or Diminiſh'd; From all which it Appears, that neither the Weight of the Air, according to Mr. *Boyle*, nor the Winds, according to Dr. *Halley*, are the Cauſes of the Variation of the Heights of the Mercury, but the Dryneſs or Moiſtneſs of the Air, that is, the greater or leſs Expanſive or Contractive Force of it; For beſides, where a Barometer is kept in a cloſe Room, and it's Ciſtern likewiſe encompaſſed with a Box, what Effect can we ſuppoſe the Winds will Produce, where we Feel not the leaſt of them in an open Room? Yet 'tis true by Diſperſing their Wett Parts in the Air, and by Degrees Weakening it's Expanſive or Contractive Force that Way, 'tis Conceivable enough, how they ſhould be the Occaſion of ſo Senſible a Depreſſion of the Mercury.

The Obſervations upon the Barometer, by Dr. Liſter *Conſidered.* Philoſoph. Tranſact. Pag. 18. Vol. 2. Lowthorp's Abridgement.

§ 30. The next Obſerver upon the Barometer, whom we ſhall take Notice of, is Dr. *Liſter*, who Informs us,

First, That let the Weather be Cloudy, Rainy, Windy, or Serene, the Mercury is not Affected in St. *Helena* and *Barbados*, according to Dr. *Halley*; Becauſe, we ſay, if it is ſo, in all theſe Caſes the Temperature of the Air is much the ſame, being under the Influence of one, and the ſame, conſtant Principle of Heat, or the Expanſive Force, which keeps the Air, nearly, in the ſame Spring and Temper, notwithſtanding the Accidental Fluids, which may Swim in it.

Second, That in *England*, in a violent Storm, or when the Quick-Silver is at the very loweſt, it then Viſibly Breaks and Emits ſmall Particles; which He looks upon as a kind of Fretting, and Conſequently, at all times of it's Deſcent, that it is more or leſs upon the Frett.

1ſt. From Hence He Concludes, that it's Depreſſion is from a Contraction of the Quick-Silver, and Purging it's ſelf from Air, by which the Parts fall cloſer to one another, and the Air, which is Expreſſed, Riſing to the Top of the Tube, Increaſes it's Elaſticity, and ſo Depreſſes it from above.

2d. That the Quick-Silver Riſes in Hot and Froſty Weather, becauſe it then obtains it's Natural Expanſion, which it therefore always does between the Tropicks, which makes it never Fall. To which we Anſwer,

First, As to the Quick-Silver Emitting Particles in a Storm; That in all Likelihood Proceeds from a Moiſture Impregnated with Warmth; For if we put Water to Quick-Silver, it keeps it's Parts together, whereas if we ſet it over a Fire, or mix Unſlackt Lime with it, it preſently Separates into ſmall Parts; And therefore, as the ſame Moiſture, Impregnated with Fiery Particles, will Sink the Mercury by Weakening the

Air's Force, ſo will it make it Sparkle by Separating it's Parts into ſmall Globules; Which Agrees with what we have before ſaid, that Mercury Sinks in a Storm, becauſe the Expanſive and Contractive Forces are in a Conteſt with, and Weaken each other, that is, there is a Moiſture Impregnated with Warmth, or a Contractive Mixed and Blended with an Expanſive; And Conſequently, if the Mercury is not, at leaſt the Air and Medium, which Surrounds it, is upon the Frett, and may Cauſe it in the other.

SECOND, As for the Mercury's Contraction, the Parts are ſo cloſe, and even upon the Interpoſition of Groſſer Bodies, than that of Air, are ſo fully bent upon Uniting, that it may be very much Queſtioned, whether there is any ſuch Contraction; But it is common to Purge it from Air, it is ſaid, by Applying a Hot Iron, and therefore it has Interſtices; Granting it, yet it can never be Suppoſed, there are ſuch vaſt Chaſms in the Mercury, that it ſhould Sink two or three Inches upon that Account; For as to the Depreſſion of it from the Top of the Tube by the Air, which is Purged from it, it muſt be very Inſignificant, becauſe the Elaſticity of the Air, which is his ſecond Reaſon, is not much help'd by it; Since Suppoſing a Boylean Vacuum at the Top, and ſome Parts of Air Sprinkled in it, what can this Signify to the Encreaſing it's Force of Elaſticity, which muſt be very Small, that Ariſes from a Medium, which before had none at all, and now cannot be much Encreaſed by a few Particles of Air Diſpers'd in it?

THIRD, As to it's being in it's Natural Expanſion in Heat and Froſt, as it is in *Barbados* and St. *Helens*; if it is ſo, how comes it then in this Natural Expanſion to be only $29\frac{1}{2}$ Inches, as Dr. *Halley* has Experienc'd, and here to be 30, or ſometimes $30\frac{1}{2}$? Beſides Mercury, as Conſider'd of it's ſelf, without the Air being Intermixt with it, ſeems to be no more Capable of Expanſion or Contraction than Gold or Water, or any other Bodies, which really are not, by the Experiments *Del Cimento*; However that Heat and Froſt, or the Expanſive and Contractive Force of the Air, may have a like Effect upon Mercury, in Raiſing or Keeping it to a higher Standard, than otherwiſe, we have Shewn; The Firſt by Dilateing and Lengthening the Cylinder of Mercury by it's Expanding it, and the Laſt by Contracting the Cylinder into a Shorter Diameter, by which it's length is likewiſe Encreaſed; Alſo the Expanſive Force Acts upon the Mercury in the Baſon, by which it Repells it into the Tube, and the Contractive Compreſſes the Mercury in the ſaid Baſon, by which it again Riſes.

FOURTH, He ſays, 1*ſt*. that Heat and Froſt may Produce the ſame Dilatation of Mercury, or which is all one, according to his Notions, may Reſtore it to it's own Expanſion; Becauſe Salts Liquefy'd will Chryſtallize either by Heat or by Cold, that is, Return to their own proper Natures, by ſetting them in a Cold Cellar, or as *Zwelfer*, by keeping them conſtantly in Balneo; 2*d*. Water is Naturally Ice, as *Bornichius* a Learned *Dane* thinks, it is Imagined in Complement to his Climate, and Dr. *Liſter* himſelf Thinks ſo, becauſe Salt is naturally Rock, or Foſſile, which is the moſt like Ice, for it's Tranſparency, and eaſy Liquefaction, and Yeilding to the Impreſſions of Air, ſo that it is Difficult to keep it in it's Natural State of Chryſtallization; And all Salts in General ſeem to Propagate themſelves, and Vegetate in a Dry Form; The like is to be Obſerv'd in Quick-Silver, of it's being a hard Rock, and from it's Readineſs to Amalgamate with Metals, or Embrace a more fixt State upon all Occaſions.

To which it is Anſwered, as to his 1*ſt*. Argument from Salts, it is Deſtroy'd by his ſecond from Ice; Since, if Heat and Cold will make Salts Chryſtallize, they will not make Ice; 2*d*. if the Nature of Mercury is to be Fixt, it is not Reſtor'd to it's Nature by being ſtill Fluid and leſs Contracted; 3*d*. We ſay it Preſerves it's own Nature, whether Depreſſed or Elevated, and it is ſtill the ſame Mercury; We ſhall therefore, being entered into this Argument, Conſider the Reaſon of his Experiments, Abſtracted from their Solving the Variations of the Mercury, which they do not.

FIRST, Salts Liquefy'd Chryſtallize by Heat or Cold, or by the Expanſive and Contractive Force; Becauſe in the firſt Caſe, the Moiſture is Evaporated and Leaves the Salts to their own Natural Contractive Force or Quality of Fixation, which are ſaid to Vegetate in a Dry Form; And in the ſecond, the Cold or Contractive Force, being a Salt and of a Fixing Nature, it's ſelf Contributes to the producing the ſame Effect of a Contractive Force or of Fixation.

SECOND, Water is ſaid to be Naturally Ice, becauſe Salt is, which very much Reſembles it; This ſeems to be a very Triffling Diſpute Concerning the Word, Natural;

Since Bodies are Capable of being either Fixt, or Fluid ; And that Capacity is their Nature ; Nor is it more Natural for Water to be Fixt, or to be Ice, where there is a Power, as that of Cold, or the Contractive Force, to Produce ſuch an Effect in it, than it is for it to be Fluid, where there is another Powerfull Principle, or the Expanſive Force, which is the Cauſe of ſuch Fluidity ; The Nature of Bodies is not an Abſolute, but a Relative Thing, and Varies and Differs according to the Relations and Circumſtances, under which it Lies to other Beings, which are Applyed to it, Thus Ice is Water if Applyed to Fire, or the Expanſive Force, and Water is Ice, if Applyed to Salt, or the Contractive.

DR. *Liſter* adds by way of Corollary ſomething concerning Healthfull and Sickly Seaſons, that ſince, in very Cold and very Hot Seaſons, the Mercury and other Liquids have the leaſt Violence done them, the Humours of our Bodies in all Probability muſt be in ſome Meaſure accordingly Affected ; And that therefore Cold is Healthfull ; Which He 1*ſt*. Argues from the vaſt Number of Old Men and Women, which are to be found upon the Mountains of *England*, Comparatively to what are found elſe where.

2*d*. FROM Cold being more Agreeable to our Natures than Heat is, becauſe tho' the Blood of Quadrupeds, Fowls, and Men, is Hot, yet that of Inſects and Fiſhes is Cold, and He has found, that there are above a Hundred Species of the latter to one of the former.

3*d*. As a farther Argument of the little Injury, which Cold does to the Nature of Animals, He ſays He has ſeen Hexapode Worms, and Flies of divers Sorts, Frozen in the Winter, He has taken them up from the Snow, has caſt them againſt the Glaſs, and they would Endanger the breaking of it, and make it Ring, like ſo much hard Ice ; Yet when He has put the Inſects under the Glaſs, and ſet them before the Fire, they would, after a ſhort Time, nimbly Creep about, and be gone, if the Glaſs, which He Whelm'd upon them, had not ſecur'd them.

4*th*. AN Eminent Philoſopher Contradicts this Notion of Cold being Healthfull, and that other common Proverb, that a Green *Chriſtmas* makes a Fat Church-Yard, becauſe the Plague broke out at *London*, after a long and Severe Winter, 1665 ; To which He Anſwers, that a Plague is a Foreign Diſeaſe, and Proper to *Aſia*, and comes to us only by Trade and Infection from Commerce with thoſe Parts ; To which ſeveral Particulars we Alledge.

FIRST, That Cold, or the Contractive Force, cannot be Concluded to be Healthfull, from the Riſe of Mercury in the Tube, unleſs Heat, or the Expanſive Force, may be ſo too, by which it alſo Riſes ; However this we may ſay, that the Force of the Air, when Hot, or Cold, and Dry, is then the greateſt, it's Craſis ſtrongeſt, and therefore ſtrikes the moſt Briſkly, at ſuch Times, upon the Nervous or Sanguineous Syſtems, which makes us more Chearfull, and the Nerves more Active, and the Blood Circulate much better ; And in theſe Caſes the Mercury Riſes ; But if it is Cold, or Hot, and Moiſt, the Mercury Sinks, and the Air Droops, which not Affecting our Bodies with ſo lively a Motion, is therefore not ſo Healthfull ; So that the Healthfullneſs of Weather ſeems rather to Conſiſt in the Dryneſs, or in the Force and Springineſs of the Air, when the Expanſive or Contractive Forces are in the greateſt Vigour, and the Sicklineſs in it's Moiſture, or in the Flaccidneſs and Languor of it, when the Expanſive or Contractive Forces are broken and Diſſolved, that is, Moiſture ſeems to Proceed from the Expanſive of Heat, and the Contractive of Cold, Weakening and Impairing each other ; Thus the Expanſive of Fire, or the Sun, Acting againſt the Contractive of Ice, or Froſt, or Snow, Produces Water, a Thaw, or Rain ; From whence the Old Proverb may Poſſibly be true, that a Green *Chriſtmas*, or a Moiſt, makes a Fat Church-Yard.

SECOND, Cold can never yet be Inferred to be Agreeable to our Natures, from the Cold Vitals of other Inſects ; Since Our's are both Cold and Hot, that is, Conſiſt of the Nervous, or Cold, and of the Sanguinenous or Hot, or, which is the ſame, of the Contractive and Expanſive Forces ; and Thus Cold by it's Contractive Force Preſſes upon the Blood, and gives freſh Life and Motion to it ; For, as the Lympha of the Blood becomes a Jelly, when ſet in a Warm or Cold Place, I ſuppoſe by it's own Proper Inclination to Coagulate, ſo the Blood on the Contrary becomes more Fluid, either from the Expanſive of Heat, or the Contractive of Cold.

Third, As for the Inſects Reviving by the Fire, when Frozen in Snow; ſince their Vitals are Naturally Cold, it cannot be Injurious to them, and the extraordinary Heat of the Fire will equally put a Cold Fluid into Motion, when Frozen, as the gentle Heat of the Sun will, when it is not; And it is very likely, their Vitals are much of the ſame Nature with Water, and tho' Frozen into Ice may, becauſe of it's Simple Compoſition, eaſily be made to Circulate by Heat, whereas the Blood of other Animals, being more Compounded, when once firmly Congeal'd, 'tis Impoſſible by any Heat almoſt to Diſſolve it; Again, the Vitals of Inſects lying in a ſmall Compaſs, and being cover'd with a tender Skin, are more readily put into Motion by the Fire, than thoſe of other Animals, which lie Deep in the Fleſh, and are not eaſily Approach'd by any External Impreſſions.

Fourth, As for the Plague after a hard Winter, that Poſſibly might not Ariſe from any Foreign Infection, nor yet from Cold, and ſo will not Contradict the Old Proverb of a Green *Chriſtmas*, *&c.* But from an extreme Diſſolution of the Craſis of the Air, after it had been long held and bound up by a hard Froſt, as the worſt Thaws ſometimes ſucceed the ſevereſt Winters,

From Dr. *Liſter*, we ſhall Proceed to Mr. *Hugens's* Celebrated Experiment upon the Barometer, which is.

Monſieur Hugens's Experiment upon the Barometer Conſidered, and the Solution of it Examined. Philoſoph. Tranſact. Vol. 2. Pag. 23. Mr. Lowthorp's Abridgment.

§ 31. That a Tube, fill'd with Mercury Perfectly Purg'd of Air, and Inverted, will remain top full to the Height of 75 Inches, equal in Weight to 89 Foot of Water, and it is not known, whether it will not be Suſpended ſtill Higher; If the Mercury is not Purg'd of Air, or if a ſmall Bubble of Air enters into the Tube, or if the Tube be ſhaken or ſtruck, it Immediately Falls to it's Standard of about 29 Inches, and after ſome Ebbs and Flows, or Reciprocations, at length Settles at that Standard.

The Solution of which Phænomenon, by the ſaid Mr. *Hugens*, is as Follows,

That There is a Subtile Matter, which Penetrates Glaſs, Mercury, *&c.* which has a Preſſure Peculiar to it's ſelf, and by being added to that of the Groſſer Air, Supports the Mercury at the fore-mention'd Height; That this Subtile Matter, tho' it can Penetrate Glaſs, does not Preſs upon the Mercury in the Top of the Tube, when full, ſo as to make it Deſcend, for two Reaſons,

First, Becauſe the Parts of the Glaſs, which are Impervious, take off a great deal of the Preſſure, ſo that That, which Paſſes thro' the Pores, is very ſmall in Reſpect of the whole Surface of the Mercury at the Top of the Tube, whereas the Mercury in the Ciſtern is Preſſed by this Subtile Matter all over it's Plane; Second, as the Parts of the Mercury above, which lie under the impenetrable Parts of the Glaſs, are freed from the Preſſure of the ſubtile Fluid, ſo are they cloſely United to the Glaſs and to one another by a ſort of Coheſion, whereby the other, which lie under the Pores of the Glaſs, and are Expos'd to the Preſſure of the ſubtile Matter, are likewiſe kept Suſpended; Laſtly, if by any Violence there is the leaſt Room made for the ſubtile Matter to enter, it Ruſhes in and Purſues the Mercury, till the upper Preſſure of the ſubtile Matter is in an equal Balance with the lower, that is, when the Mercury is fallen to it's uſual Height.

There are five Things therefore to be prov'd in this Account; Firſt, that there is a Preſſure of ſuch ſubtile Matter, different from that of the groſſer Air, and able to Suſtain the Mercury's Height; Second, that there are Pores in Glaſs, which this ſubtile Matter Penetrates; Third, that there is a Coheſion of the Mercury with the Parts of Glaſs, and with it's own; Fourth, that this ſubtile Matter has no Effect, till there is Room made for it in the Top of the Tube, and then that, Deſcending till the Mercury is only 29 Inches high, it is in an Æquilibrium with the Preſſure of the ſame Subtile Matter below; Fifth, that the ſhaking or ſtriking of the Tube, or the Admittance of Air, or the Mixture of it with the Mercury, makes Room for this Subtile Matter.

Dr. *Wallis*, therefore, does not ſeem fully to Underſtand Monſieur *Hugens's* Hypotheſis, when He thinks, that the Glaſs at the Top of the Tube is every where Pervious according to this Account; And therefore his Firſt Objection is not of Force againſt it, that, ſince there is a Preſſure above thro' the Glaſs, he does not ſee, why that ſhould not be equal to the Preſſure below, and Conſequently, why, that Subtile Matter being in Æquilibrium, the Mercury ſhould not Deſcend by it's own Weight;

Whereas Monſieur *Hugens* denies there is a Preſſure thro' all Parts of the Glaſs, and therefore Affirms, that it is leſs above, than that, which lies upon the Ciſtern: Second, Dr. *Wallis* ſays, that granting the Preſſure is not ſo great thro' the Glaſs, as it is on the Mercury below, the ſame Obſtacle of Glaſs would remain, when the Tube is in Part empty'd, or the Quick-Silver Unpurg'd, the Pores of the Glaſs not being made more Open or Pervious by either; It is Anſwer'd, that nothing is more Notorious, than if a Veſſel was full of Quick-Silver, and cover'd with a Plate, in which ſome few ſmall Holes were made, and Water Stagnating upon it, that the Water would not Deſcend, till the Quick-Silver gave Place, and that a great Part of the Preſſure would be Intercepted by the Solid Part of the Plate; But if the Mercury ſhould be made to Deſcend, that then the Water would immediately Tranſmit it's ſelf thro' the Foramina, and lie upon the Mercury with it's whole Weight; Taking the Water then for the Subtile Matter, and the Cover for the Top of the Tube, and it will be Monſieur *Hugens's* Caſe, in which there is nothing Monſtrous, or Unaccountable: Third, Dr. *Wallis* Objects, that if it freely Penetrated Glaſs, the Preſſure of the Subtile Matter would be much the ſame, notwithſtanding the Glaſs, as without it, and therefore the Mercury would Deſcend at the very firſt, but this is Anſwer'd in the Firſt, and Second Articles; If it Penetrated with ſome Difficulty, and was, as it were, ſtrained thro' it, then the Mercury would not Deſcend at once as it does, but by Degrees; On the other Hand we ſay, that as ſoon as the Quick-Silver Unſtops the Foramina of the Glaſs, the Subtile Matter flows in like a Torrent, and with the greateſt Eaſe and Freedom Imaginable, and therefore the Mercury muſt Deſcend in an Inſtant, as is Experienced: Fourth, Dr. *Wallis* ſays, the Weight or Preſſure of this Subtile Matter cannot be ſo Conſiderable, as that, when let in at the Top of the Tube, it ſhould drive down the Mercury with that Vehemence; And I add, nor is it Conceivable, how it ſhould keep it up to that Height, for, according to Monſieur *Hugens*, it's Preſſure is much Greater than that of the Groſſer Air; Fifth, Dr. *Wallis* does not Underſtand, why the Mercury Unpurg'd from Air, ſhould Deſcend ſooner, than when it is Cleanſed from it, nor I ſuppoſe, Monſieur *Hugens*; Sixth, Dr. *Wallis* Thinks, the Coheſion of the Parts of Mercury is Inſufficient to Solve this Effect, and more Eſpecially it's Adheſion to the Glaſs, the Contact of which it rather Declines, tho' Dr. *Hooke* is ſaid to have found it ſtick to the Sides; as was Mentioned before.

MONSIEUR *Hugens*, to make good his Hypotheſis, of all the five Particulars, to be prov'd, only Endeavours to Evince, by two Experiments, one Part of the firſt, *viz.* That there is a Subtile Preſſing Matter.

FOR Firſt, He ſays, two Plates of Metal or Marble, whoſe Surfaces are Perfectly Plane, ſtick ſo cloſe together, that, the uppermoſt being lifted up, the undermoſt follows without quitting it; He took two of an Inch Square, the lowermoſt held up three Pounds of Lead, and would keep thus together, as long as you Pleaſed; He put them into an Exhauſted Receiver, in which the Preſſure of the Air did not Suſtain an Inch Height of Water, they yet did not Disjoin; He put Spirit of Wine betwixt the two Plates, they ſtill obtain'd the ſame Effect, both in a full and an Exhauſted Receiver; from whence He Thinks there is no more Reaſon to Doubt of the Preſſure of a Subtile Matter, than of the Air it's ſelf, to which this Effect is Univerſally Aſcrib'd.

To Which we Anſwer FIRST, that from Hence it is Plain, that the Adheſion of the two Plates does not Proceed from the Preſſure of the common Air; If it did, the Preſſure of the common Air muſt Contribute to this Effect, whereas by the foregoing Experiment the Subtile Matter does as much without it; That is, with the common Air the undermoſt Plate does not Suſtain a greater Weight of Lead, and without it not a leſs; So that it is hard to Imagine, how that ſhould be a Cauſe, which is no ways Aſſiſting in Producing the Effect; And Monſieur *Hugens* by this Experiment Deſtroys the Argument He makes uſe of, which is, that He may as juſtly Affirm, that the Coheſion of theſe Plates in an Exhauſted Receiver Proceeds from a Preſſure of Subtile Matter, as Others, that their Coheſion out of a Receiver Ariſes from a Preſſure of common Air, which all are Agreed in, but by this Experiment it is Evident that their Coheſion out of a Receiver, or in the open Air, is not owing to ſuch a Preſſure; and therefore Mr. *Hugens* can no more Juſtly Affirm, that it is owing to the Preſſure of a more Subtile Matter in the Receiver, than others have done, that it is owing to a Groſſer out of it; This way of Arguing ſeems to be True and Genuine enough, but yet

Concludes quite Contrary to what Mr. *Hugens* Deſign'd; Beſides, how can the Coheſion of theſe Plates be owing to the Preſſure of the common Air, when the Height of Water, which is Deriv'd from the ſame Preſſure, is Diminiſh'd in Vacuo, and this Coheſion is not?

SECOND, It will not be eaſily Underſtood, how in a Fluid, as the Air, in which an equal Balance is Maintain'd, there ſhould be ſuch a Preſſure; ſince we find in Water, whoſe Weight is much greater, a Freedom to Bodies of moving according to any Direction, and ſo likewiſe in Air, to theſe Plates, Singly or Jointly, tho' not from one another, which ſhows the Cauſe is to be ſought in the Plates, and not in the Air; If it be ſaid the Air has not Time to Succeed into the Vacuum betwixt the Plates, it ſeems to be a mere Fancy to Affirm it of ſo Subtile a Body, as that of Air, and ſuppoſing it had not Time, there would no Inconvenience Ariſe from thence; ſince granting the Space betwixt the Plates as Void, as the Medium in an Exhauſted Receiver, and nothing Succeeded into it, yet the Air is not ſo Thick and Conſtipated, that it would be Impoſſible to Compreſs it by the Disjoining of the Plates, tho' none of that Compreſſed Air ſhould Succeed into the Space betwixt the Plates.

THIRD, If therefore there is no Reaſon to Attribute this Effect to the Preſſure of the Groſſer Air, there is certainly leſs Reaſon to believe it Produc'd by that of a Lighter and more Subtile; there are at leaſt the ſame Objections againſt the Preſſure of this more Subtile Matter, which lie againſt the other; Namely, *1ſt.* It's not Raiſing the Water above an Inch, for if the Preſſure of this Subtile Matter has the ſame Effects with the Imaginary one of common Air to keep the Plates together, it will likewiſe have the ſame Effects with it to Raiſe the Water Higher; Eſpecially ſince this Preſſure is thought to be ſtronger than the Preſſure of common Air; *2d.* It's being in an Æquilibrium, for be the Medium Thinner or Groſſer, it will Obtain that Property, if it's Parts are Fluid and Homogeneous; If it is not Fluid, there can be no Motion Perform'd in it, and if it is not Homogeneous, nothing certain and Conſtant can be Affirm'd of it: It might farther be eaſily Demonſtrated, whether this Phænomenon was owing to the Preſſure of this Subtile Matter, by Examining the Weight of the Mercury in a Tube, which, if three Pounds, 'tis the ſame Weight with what the undermoſt Plate will bear by the Aſſiſtance of the ſame Preſſure, if not, this Subtile Matter is a mere Chimera.

FOURTH, What therefore has been generally Aſſign'd to the Preſſure of the Atmoſphere, it is Plain, requires another Cauſe, which we ſhall Venture to give; Namely, the Strong and Eager Attraction of theſe two Plates, Derived from the Expanſive Forces.

THE Queſtion is, FIRST, why that Attraction is ſometimes Senſible, and at other times not.

SECOND, How it is Perform'd, or the Reaſon and Manner of it.

FIRST, If two Bodies do not Touch one another, unleſs the Expanſive Force is very Strong, as in Electrical or Magnetick Bodies, and in Fire and Flame, It is not Senſible; Becauſe the Contractive Force to the Earth is Superior, and Takes off from that Inclination, which One Body has to Unite with another, by that greater Propenſity it gives them to the Earth, which has an Infinitely greater Contractive Force, than Bodies Generally are found to have of the Expanſive amongſt themſelves; So if we place one Drop of Quick-Silver near another on the ſame Table, they will not Unite till they either Touch, or very nearly Touch, each other; Becauſe they both have a ſtronger Contractive Force towards the Earth, than an Expanſive towards one another; But if they Touch in one Part only, they Immediately cloſe into one round Globule; Becauſe as the Contractive Force is ſtrong towards the Earth, ſo is that of the Expanſive by which they Unite; Water will have the ſame Effect, if two Drops of it are placed near each other, but the Union of them is not ſo Quick and Expedite, becauſe their Expanſive Forces are not ſo Strong, as is Evident from the Exceeding Volatility of Mercury in Reſpect of that of Water; On the other hand, if we ſet one Globe of Gold to another, they will not join by ſuch Contact, becauſe their Contractive Force is greater to the Earth than their Expanſive to one another; But if two Plates of Gold touch one another in more Parts than they do not touch, that is, if their Surfaces are Plane and Even, they will Unite by that Mutual Contact; Becauſe each Part Senſibly Expanding it's ſelf to each Part and the Contractive Force to the

Earth in both, being now as of one Body, ſuch Expanſive Force is Superior to any ſmall Contractive Force, which can be brought to Separate them, as of three Pounds of Lead; 'Tis True, they may be more eaſily Separated Sideways; Becauſe by that Means we Conquer the Expanſive Force, by which they Unite, of each Part by Degrees, that is, the Expanſive Force of a Line at once, whereas if we Endeavour to Draw them Perpendicularly from one another, we are to overcome the Expanſive Force, by which they Unite, of a whole Surface.

SECOND, It is not farther Improbable, but ſomething elſe is likewiſe Requiſite, to the Uniting of the Plates, *viz.* The ſame kind of Expanſive Force; So if we put a Plate of Metal upon a Surface of Poliſh'd Stone, ſince their Forces are Diſſimilar, they will not Unite, there will be no Adheſion of them, and they will no ſooner Cement together, than Oil and Water will Mix by a mutual Contact.

THIRD, And this may give ſome Account of the Coheſion of Matter, that is, this Expanſive Force of one Part to another is Probably the Cauſe of it, or which is all one, a natural Tendency of the Parts in Matter, which is Solid, tho' not many times Perceptible to us, by which they Unite with ſeveral Degrees of Tenacity or Force of Attraction.

See Pag. 455.

THE Second Experiment is of a Siphon of unequal Legs, by which Water is drawn out of a Veſſel, the Effect of which is Aſcrib'd to the Preſſure of the Air upon the Surface of the Water, which makes it Riſe in the ſhorter Leg, after we have Extracted the Air by Suction, and in the longer Leg it Deſcends by it's own Gravity; Monſieur *Hugens* has Obſerv'd the ſame Effect to Obtain in an Exhauſted Receiver, where the Water is Purg'd of Air; And therefore *1ſt.* He Concludes there muſt be a Preſſure of a more Subtile Air than that of the Atmoſphere; *2d.* And from hence He alſo thinks, by knowing to what Degrees of Force this Preſſure Reaches, that is, by Experiments made with longer Baroſcopes than Mr. *Boyle*'s, it will perhaps be found, that it is ſufficient to Cauſe the Union or Coheſion of Glaſs, and other Bodies, which, He is of Opinion, require a more Poſitive Cauſe of their Cementing, than bare Contiguity or Reſt, as *Deſcartes* Teaches. To which we Anſwer;

IF This Effect of the Water Riſing in the Siphon is certain, as well in an Exhauſted Receiver, as out of it, for the Reaſons Alledg'd it cannot be Deriv'd from the Air's Preſſure, and if not, as was likewiſe ſaid before, it is more Unreaſonable to Suppoſe it Proceeds from the Incumbent Weight of a ſtill thinner Fluid; But it is a wonderful Thing, that ſeveral of this Kind of Phænomenons ſhould be Attributed to Gravity, whereas we Perceive nothing at all of it our ſelves, and the holding out our Hands in the Air, or in Water, and the feeling no Preſſure from thence, is a Sufficient Confutation of ſuch Solutions; Or Suppoſing that the Inferior Column of a Fluid to the Hand Æquiponderates the Superior, and that therefore we Feel no Preſſure, how comes it yet to Paſs, that we are not Senſible of any, when Perhaps a Floor, or a Table, Supports one Side of our Hand, and Leaves the other to be Preſſed by the whole Weight of the Incumbent Fluid? 'Tis therefore Probable,

What the Nature of a Perfect Fluid is.

FIRST, That there is an equal Balance in Perfect Fluids of the Expanſive and Contractive Force nearly, as in Air and Water, ſo that tho' the whole may Gravitate, as Water, or not Gravitate, as Poſſibly Air does not, but is rather Expanſive, yet ſuch a Contractive or Expanſive Force is not Felt by any Animate, or it's Effects Perceived by any Inanimate Beings; *viz.* In the Preſſure, or the Expanſive Forces, one way, or the Contractive another, of the Parts of the Fluid; there is no manner of Force Senſible, the Forces being equally Poiſed on all Hands.

Why Bodies Deſcend.

SECOND, That where the Contractive Force of any Body, as Gold or Marble, is Superior to that of the Fluid, as Water, which Encompaſſes it, the Preſſure of the Fluid towards the Earth is ſo much leſs, as the other is greater, and therefore the Deſcent of Bodies is not more owing to the Contractive Force of the Solid, than to the Expanſive of the Fluid, for in this Reſpect, and in this Compariſon, the Water as much Aſcends in Reſpect, and Relatively, to the Gold or Marble, as the Gold or Marble Deſcends in Reſpect, and Relatively, to the Water.

Why Bodies Aſcend.

THIRD, That where the Preſſure of the Fluid is greater, the Preſſure of Real Space or Body, to the Earth is ſo much leſs, and therefore the Aſcent of Cork or Wood in Water, is not more owing to the Expanſive of the Solid, than the Contractive of the Fluid; For in this Reſpect, and in this Compariſon, the Water as much Deſcends in Reſpect, and Relatively, to the Cork or Wood, as the Cork or Wood Aſcends in Reſpect, and Relatively, to Water; What therefore we Imagine, and which we Believe to be the Truth in Nature, is, that Bodies Aſcend or Deſcend by their proper Expanſive and Contractive Forces in Relation to each other, ſo that what may be a Contractive Force in one Compariſon, may be an Expanſive in another.

The Reaſon, why Water Riſes in a Siphon.

The Reaſon, then which we give of the Water's Riſing in the Siphon, is, not the Weight or Preſſure of the Air, becauſe that cannot be the Cauſe of it, ſince the ſame Effect happens, according to *Hugens*, in an Exhauſted Receiver, nor can it Riſe without Suction; The Reaſon Conſequently, I ſay, muſt be this, that Suction haveing the Nature of an Expanſive Force, as it Acts againſt Gravity and the Contractive, becomes in ſmall Tubes, in which Suction can Prevail, Superior to the Contractive of the Water, and by that means Elevates it to a certain Height, after which it will Deſcend by it's own Contractive Force, and ſink in the Inferior Contractive, or the Comparative Expanſive of the Air, which lies under; or which is all one, if we Fill the Siphon with Water, and then put the ſhorter Tube into the Water, and leave the Orifice of the longer open, the Effect will be the ſame; Becauſe, then the Water will be Raiſed without Suction in the ſhorter Tube, and again Deſcend, as before, in the longer; But if the Tubes are equal, then there will be no greater Force in one Tube than another, and Conſequently, the Water, contained in the one, will Obey the Force of it's Contractive, and will Fall into the Air, and the other will not Fall into the Water, if leſs than 32 Foot High, as is Experienced.

On the other Hand if one Tube is longer than another, and both are Fill'd with Water, one Aſcending, which is the ſhorteſt, and the longeſt Deſcending, if it be askt, how the Water in the ſhorteſt comes to Aſcend Contrary to it's proper Gravity, we Anſwer, becauſe Fluids do not Act according to their Gravity, but their Preſſure or Expanſive and Contractive Forces, in Fluids of an equal Balance, and therefore the Water in the Baſon being Stagnant and in an equal Balance, that has no Influence upon the ſhorter Tube of the Siphon, but the Water in the longer Tube of the Siphon giving way, *i. e.* Weakening the Balance of the Air below, the Water in the ſhorter Tube Preſſes to that Part, where it finds the leaſt Reſiſtence, like Air into an Exhauſted Receiver, or which is the ſame, the Water Deſcending in the longer Tube has the Force of an Expanſive to that in the ſhorter, by which it Draws and Attracts it after it by it's Superior Force, and has thereby the Effect of Suction or an Expanſive.

So Again, if we take this Siphon and Invert it with it's Orifices upwards, and fill it with Water, the ſhorter Tube will Spout out Water, as the longer did in a contrary Poſition, becauſe the Contractive Force of the Water in the longer Tube now Prevails againſt the Contractive in the ſhorter, and therefore has the Force of an Expanſive, by which it is Raiſed into the Air in Spouts or Jetteaus, *V. Sturmius's Colleg. Curioſum.*

But Suppoſing Monſieur *Hugens* had Prov'd a Preſſure of this Subtile Matter from theſe two Experiments, He is yet far from Solving the preſent Phænomenon, becauſe He does not Pretend to give a Reaſon, why the Force of a Subtile Matter ſhould be more Weighty, and Raiſe more Mercury in the Tube than a Groſſer; That is, He has too much Senſe to go about to Prove, that a Lighter Body has more Weight than an Heavier.

This However was only one Article to be Prov'd,

The Second is, that there are Pores in Glaſs, which this Subtile Matter Penetrates at Pleaſure; which is no ways Evident, becauſe one would be apt to Suſpect there are no ſuch Pores, which are generally made by the Corpuſcular Philoſophy, to ſuit ſuch certain Parts as are cut out and ſhap'd for them; However we may Reaſonably Doubt of them, till they are Demonſtrated; Eſpecially, if the Appearances or Effects in Nature may be Accounted for another away, than by ſuch Imaginary Suppoſitions, which are Contriv'd only to ſerve the Purpoſe of Solving, what is thought Impoſſible to be done without them.

The Third is, that there is a Coheſion of the Mercury with the Parts of Glaſs, which Dr. *Wallis* Diſowns, and Dr. *Hooke* Affirms, and both may Poſſibly be in the Right, in different Circumſtances of it; for at the Top, the Mercury certainly Declines the Contact, and ſtands in a Convex, as all other Fluids do, where they are of a vaſtly different Specifick Gravity or Preſſure from thoſe Bodies, which Surround them; for in ſuch a Caſe the Actions or Forces are ſo different, that they cannot Unite, but the Quick-Silver is left to follow it's own Contractive Force to it's ſelf, rather than it's Expanſive towards the Glaſs; On the other Hand, if there is a Preſſure of Quick-Silver from the Parts above upon thoſe below, the Caſe is Alter'd, and ſince it is Preſſed, it Tends towards the Weakeſt Part, and therefore Urges, with it's Expanſive in Conjunction with the Expanſive of Preſſure, the Sides of the Glaſs, by which it Adheres to them, which Adheſion is Probably very Significant in Producing the Effect we are now Conſidering; And for that Reaſon a Tube of a large Bore would not Exhibit to us the ſame Phænomenon, as my Lord *Brouncker* very Juſtly Thinks and Owns,

THE Fourth is, that this Subtile Matter has no Effect, till the Pores of the Glaſs are unſtop'd by the Mercury's Subſideing; This therefore Depends upon the Glaſs haveing ſuch Pores, which are not Proved, nor are we willing to Acknowledge.

FIFTH, It will be Enquired, what the Reaſon is, why the Mercury Deſcends upon the Air's being Admitted, or upon ſhakeing of the Tube; Which Monſieur *Hugens* does not likewiſe Account for.

Lord Brouncker's *Explanation of the Foregoing Phænomenon.* Philoſoph. Tranſact. Vol. 2. Pag. 24. &c. Mr. Lowthorp's Abridgment.

§ 32. MY Lord *Brouncker*'s Notion of this Experiment is, as Follows,

1ſt. THAT There may be in the Air, where Mercury is only to be Supported, as in a full Tube, yet a greater Weight, than what will Suſtain only 29 Inches; *2d.* That this Purer or more Subtile Part of the Air is of a like Nature with the Groſſer, and therefore Springy, and Conſequently Acts, in that Reſpect, as much when a ſmall Portion of it is Admitted into the Top of the Tube, as when the whole Column is Incumbent. To which we Anſwer,

FIRST, That if there is a greater Weight in the Air, than what will Support 29 Inches of Mercury, it would always Exert it's Force, or elſe there ſhould be a Reaſon given, why it did not.

SECOND, That it is altogether Impoſſible, that the Weight, or Spring, of the Air ſhould ſometimes only Suſtain 29 Inches, and at others above Double, and almoſt Treble, that Number, without any ſenſible Alteration of the Air, and certainly without any Imaginable Reaſon, as well as without any Aſſign'd.

As for the Springineſs of the Air, we ſhall Examine it in Dr. *Wallis*'s Account of this Phænomenon, to which we now Proceed.

Dr. Wallis's *Opinion and Solution of the Abovementioned Phænomenon.* Philoſoph. Tranſact. Vol. 2. Pag. 25, &c. Mr. Lowthorp's Abridgment.

§ 33. DR. *Wallis*'s Account of this Matter is therefore this.

1ſt. THAT The Spring of the Air is Neceſſary to put Heavy Bodies into Motion, which are not Impell'd by any other Force; *2d*, That therefore the Elaſticity of the Air and the Non-Elaſticity of the Quick-Silver, is the Cauſe of this Effect; Since Water by ſeveral Experiments has been Prov'd to have none, and Conſequently Mercury in all Probability has not; *3d.* That Bodies at Reſt will not Move, without ſome External Force, as the Elaſticity or Pulſion of the Air, Admitted into the Tube, or Reſideing there, or elſe without Percuſſion, or Shakeing the Tube, or Strikeing upon it; *4th.* Gravity is a Conatus to a Motion Downwards, and the Queſtion is, why that Conatus is not Senſible in the preſent Experiment; *5th.* That Gravity may Poſſibly be only the Effect of Percuſſion, *i.e.* of Shakeing or Strikeing the Tube, or of Pulſion, *i. e.* of the Elaſticity of the Air, *viz.* That Gravity is no Innate Principle in Matter, but Proceeds from ſome External Cauſe, as Pulſion or Percuſſion, and therefore Bodies, Accounted Heavy, will not begin to Fall, till either Pulſion or Percuſſion Intervene, and Cauſe their Deſcent. To which we Anſwer;

THAT There may be ſeveral Reaſons given, why Dr. *Wallis*'s Hypotheſis is not Valid,

FIRST, As to the Springineſs of the Air, it ſeems to be no more than the Expanſive Force we have been Speaking of, *viz.* That, where the Force of the Air is Superior, and meets with a leſs Reſiſting Medium, it Expands it's ſelf into that Medium, and Water, or Quick-Silver, Probably would do the ſame, were they not hindered by their own Proper Contractive Force; That is, Air, in Reſpect to the Matter, which Compoſes the Earth, has but a ſmall Contractive Force towards it, whereas Water and Quick-Silver have a Senſible Propenſity to it, and therefore are Obliged to Obey the ſtrongeſt Direction; On the Contrary, where the Contractive Force is Superior to the Air's Expanſive, it is Compreſſed, which does not happen to Wa-

Water or Mercury, becauſe they have a Force of their own, Capable of Surmounting any we can bring againſt them; What I Mean is, that Poſſibly the Spring of the Air, which has been the Theme of ſo many Pens, is not, what is Pretended, an Elaſtick or Reſtituent Force in the Air, by which it Returns to it's Primitive State, after it's Preſſure is off, but rather a Property of Preſſing and Exerting it's ſelf that way, where is the leaſt Oppoſition, which is the Cauſe of it's Expanſion or Dilatation, and it's Yeilding to a Superior Force or Preſſure is the Cauſe of it's Contraction; But whatever is the Reaſon of the Air's Spring, or whatever it's ſelf is,

Second, There ſeems to be no Foundation for Aſſerting the Elaſticity of the Air to be Neceſſary to put Heavy Bodies into Motion, which are Impell'd by no other Force, and that therefore the Air Admitted into the Tube puts, by it's Elaſticity, the Quieſcent Parts of the Mercury into Motion; For the Air Admitted into the Quieſcent Parts of the Mercury ſtanding at it's uſual Height, does not Precipitate it into the Baſon, which it ſhould do, According to this Doctrine, and Air Admitted into the Mercury of the Baſon, ſhould Agitate and put it's Parts into Motion, which yet does not happen.

Third, It is ſaid, that Bodies at Reſt, will not Move without ſome External Force, as Percuſſion, *&c.* But the Queſtion is, why the Mercury is at Reſt, and does not Obey the Laws of it's own Gravity, as He owns in the Fourth Article we Mention'd.

From The Fifth, it is Told us, that Gravity may Poſſibly be only the Effect of Percuſſion, *&c.* This is Sufficiently Confuted by it's being Prov'd to be an Innate Force in Matter,

A Solution Propoſed of the preſent Phænomenon from the Doctrine of the Expanſive and Contractive Forces.

§ 34. Mr. *Hugens*'s Account of this Experiment, my Lord *Brouncker*'s, and Dr. *Wallis*'s, being Inſufficient, we ſhall Propoſe another, which is this.

First, That, the Tube being fill'd to 29 Inches, the Mercury, as was ſaid before, is there Suſpended by the Air's Expanſive and Contractive Force, and not by it's Gravity, in Conjunction with the Expanſive Force of the Mercury towards the Glaſs, by which it Unites and Cements with it; and by this Means there is an equal Balance of all the Forces.

Second, That where two Fluids are once in an Æquilibrium, an Addition of the ſame Fluid, on either Hand, does not Deſtroy the equal Balance; So if the Experiment be made in a cloſe Room, in Reſpect of Air, or in a Veſſel, where it is Impoſſible for the External Air to Enter, whether it be made in a larger Room or Veſſel, or in a ſmaller, the Effect will be the ſame, that is, a greater or leſſer Quantity of Air will not Deſtroy the equal Balance; On the Contrary, if a Tube be filled with Quick-Silver to twice, or Poſſibly to any Height, in Reſpect of the common Standard, ſo as there is no Air above, it will not Preponderate, for this Reaſon; Becauſe all the Parts of Quick-Silver are at Reſt, or in an Æquilibrium amongſt themſelves, as well as thoſe of Air, that is, the Expanſive Force of the Mercury towards the Glaſs, be the Glaſs Tube longer or ſhorter, will be the ſame, by which Expanſive Force it is United to it; The Expanſive Force of the Air, by which it Exerts it's ſelf againſt the Contractive or Gravitating Force of the Quick-Silver, is likewiſe the ſame, ſo as to hinder it's Deſcent, ſince the Parts of the Quick-Silver are equally Balanced amongſt one another; But if the Diameter of the Tube or Cylinder is too Large, the Expanſive Force of the Mercury towards the Glaſs in the Parts, which lie towards the Axis of the Cylinder will not be much, and will therefore Deſcend towards the Earth by their Contractive or Gravitating Force to it, by which Means all the Quick-Silver will Subſide; And thus it is not unlikely, but in a Siphon of unequal Tubes in length, Inverted upwards, if the Foramen of the longer is ſtopt, that the Water in the ſhorter Tube will no more Flow out, than if Inverted downwards, it will in the longer, if the Foramen of the ſhorter is likewiſe ſtopt, or in the ſhorter, if the Foramen of the Longer is, but if both are unſtopt, the Water will Aſcend thro' the ſhorter, and Deſcend thro' the longer; Since this was Writ, I find the Conjecture was not Wrong, being Experimented by *Sturmius* in his *Collegium Curioſum, Tentam* 5. *Pag.* 29. Which ſhews the equal Balance of the Parts of Water amongſt each other, in ſuch unequal Tubes.

Third, As to the Mercury's Deſcending in a Tube by Percuſſion, or when any ways ſhaken, it is Evident, that ſuch an Additional Force Deſtroys the Æquilibrium, above Deſcribed, of the Expanſive and Contractive Forces.

Fourth, If Air, or even the leaſt Bubble of it, enters, that alſo Deſtroys the ſaid Balance, ſince we make the Natural Expanſive or Contractive Force of the ſmalleſt Portion of Air to be equal to the largeſt.

Fifth, If there is a Vacuum, or thin Æther, at Top, that will have the ſame Effect, ſince it is not of the ſame Specifick Force in it's Expanſion with Mercury; And this alſo obtains in the Siphon, where the Foramina are upwards, for if we unſtop the Foramen of the longer Tube, the Water in the longer will Preſs upon that in the ſhorter, till it is Reduced to an Æquilibrium with it's ſelf, that is, Water does not Act upon Water, ſo as to Produce Motion, nor Air upon Air, nor Mercury upon Mercury, tho' they have all an Innate Force of Expanſion, which they Exert, when they meet with Bodies of a different Nature; Thus Air will Expand it's ſelf by it's proper and Innate Force into a Vacuum, or a thinner Medium, or into Water, tho' a thicker, Water into Salts and Earths, Spirit of Nitre into Silver, and *Aqua Regalis*, or Mercury, into Gold; Namely, that which has a greater Expanſive, Volatile, or Spirituous, Force will Exert and Expand it's ſelf into that, which has a Leſs, if the Contractive, which belongs to the leſſer Expanſive, is not too great to be Surmounted by ſuch greater Expanſive; For Air, or Water, cannot Penetrate into Iron or Marble, nor Spirit of Nitre, or Mercury, into a Diamond; And this Abundantly Proves the Difference we Maintain betwixt the Expanſive Force, and Gravity, ſince by the firſt, the preſent Phænomenon of Mr. *Hugens's*, as well as ſeveral others, may be eaſily Solv'd, but cannot by the laſt; and therefore what Philoſophers offer concerning ſuch Gravity of Air, or the Subtile Matter, is of no Significance to the Solution of this Important Problem.

Why Air Penetrates into Water, or Vacuum, Water into Salts, Spirit of Nitre into Silver, Aqua Regalis, *or Mercury, into Gold.*

The Laſt Explication, which Dr. *Wallis* gives, is, that there may be ſome Springy Parts of Air, Intermixt with the Mercury, which ſhall be able to Support it to a greater Height than uſual; Becauſe each Springy Part of Air has the ſame Force as a larger Quantity; But this, as He owns, Concludes, that Mercury Unpurg'd from Air will ſtand Higher, than that which is Purg'd, Contrary to the Experiment, which Preſcribes, that the Mercury ſhould be well Purg'd from Air, and Affirms, that if the leaſt Air is admitted, the Mercury Falls, which, according to Dr. *Wallis*, ſhould Riſe the Higher for ſuch an Acceſſion.

§ 35. We Shall now therefore go on to Mr. *Boyle's* Statical Baroſcope;

It Conſiſts of a Glaſs Bubble, about the Bigneſs of an Orange, Blown Extremely Thin, and Counterpois'd in a fine Balance, that would turn with the 30*th* Part of a Grain, by one Drachm and 10 Grains in Braſs or Lead, or which is beſt, Gold, as bearing the leaſt Proportion to the Magnitude of the Bubble; There may likewiſe be fitted to the Anſa, or Cheek, of the Balance an Arc of a Circle, Divided into 15 or 20 Degrees, that the Tongue of it, reſting over againſt theſe Diviſions, may ſhew the Quantity of the Angle, in which the Scale is Depreſſed on one Hand or another.

An Account of the Statical Baroſcope by Mr. Boyle. Philoſoph. Tranſact. Vol. 2. Pag. 28. Lowthorp's Abridgment.

This Statical Baroſcope ſhow'd the Variations of the Air, when no greater, than what alter'd the Height of the Mercury half a Quarter of an Inch, *viz.*

1*ſt.* Sometimes The Bubble was in an Æquilibrium with the Counterpoiſe.

2*d.* When The Atmoſphere was High, *i. e.* when the Air was Light, upon gently ſtirring the Balance, the Tongue would Play on the Side of the Bubble, which would Manifeſtly Preponderate.

3*d.* When The Air was Heavier, the Counterpoiſe would Preponderate, and would continue ſo for ſeveral Days, if the Air held it's Gravity.

4*th.* Upon Other Changes the Bubble would regain an Æquilibrium, ſo that He could Foretell the Riſe or Fall of the Mercury by looking upon the Bubble, which would Riſe and Fall Suitably to the other.

The Reaſon of which He Conceives to be, becauſe the Bubble is perhaps a 100 or 200 times bigger than the Metalline Counterpoiſe, and according to the Hydroſtatical Laws, 1*ſt.* If two Bodies of equal Gravity, but unequal Bulk, are Weigh'd in another Medium, they will be no longer Æquiponderant, but in a Heavier Medium, the Greater in Bulk, and Lighter in Specie, will loſe more of it's Weight than the Leſſer and more Compact, and in a Lighter Medium, leſs than the other in Proportion to the Diſparity of Bulk; 2*d.* And Conſequently, this Law will hold in the ſame Medium, as of Air, where it's Gravity is Alter'd.

5th. One Morning, when there was a Miſt, He Thought the Weight of the Air would Buoy up the Bubble, but it continu'd in an Æquilibrium.

6th. The Greater Length of the Beam would make the Balance more Exact; The Reaſon of which is, according to what we have Mentioned in Book the Firſt, becauſe the farther any Point is from the Center of Detention, or Contraction, the Velocity, or the Expanſive Force, will be the greater, and therefore the Balance will more eaſily turn.

7th. This Shows, that the varying Weight of the Atmoſphere is the Cauſe of the Riſe and Fall of the Mercury, ſince here can be no Pretence of a *Fuga Vacui* or a *Funiculus.*

8th. It Shows, that there is a greater Weight in the Air, than ſome Learned Men will allow him, tho' they allow him to have Prov'd ſome, ſince the Weight of the Quantity of an Orange is Senſible in no very Nice Balance.

9th. This Baroſcope is in one Reſpect better than the Mercurial, becauſe in that there is Skill and Trouble requir'd to free the deſerted Part of the Tube from Air.

10th. The Mercurial is uncertain, becauſe it is ſometimes more or leſs Cleans'd from Air, which He has Diſcover'd by the Statical; *viz.* that many times it has not been well Cleans'd from Air, which is done by makeing Uſe of an heated Iron.

11th. By This may be Computed the Weight of Air, which Anſwers to an Inch, or other Diviſion, of Mercury in the Tube; for when it is High, or Low, or Middle, ſet the Scale to an exact Æquilibrium by a Counterpoiſe to the Bubble, when therefore the Mercury is Riſen an Inch, *&c.* or Fallen, Reduce it again to an Æquilibrium, and the Weight put in, which muſt be Divided into Minute Parts of a Grain, will give the Weight Requir'd.

12th. This Will ſhow the Difference of the Weight of Airs in ſeveral Places, which Supports the Mercury in the Tubes at thoſe Places; for by the *11th.* we know the Weight of Air anſwering an Inch of Mercury, or other Diviſion, in one Place, and by the ſame we may know it in another, which Compar'd will give the Difference; But we muſt take our Obſervations in the ſame Height of the Air, and not one in a Vale, and the other on an Hill, becauſe then the Lower will certainly be Heavier.

13th. If The Bubble is Pois'd at the Foot of a Steeple, or Mountain, and at the Top, it will give the Difference of the Weight of the Air in one Place and the other.

To Which I Anſwer, that the whole of all this lies upon the Bubble's Preponderating in Lighter Air, and Aſcending in an Heavier; Of which there may be an eaſy Account given by it's Expanſive and Contractive Forces, and no one can be by it's Gravity.

First. We ſay, that none can be given by Gravity, from the *5th.* Article; ſince there was a Miſt, and no Alteration; which could only Proceed from this, that the Miſt Weaken'd ſo far the Air's Spring or Expanſive Force, that what was Increaſed in the Contractive, or the Weight, was Diminiſh'd in the Spring or the Expanſive, *i. e.* that the Expanſive being in a certain Proportion, the Weight, or Contractive, Imported, and the Spring, or Expanſive, Weaken'd, Deſtroyed one another, and therefore the equal Balance Continu'd.

Second, From ſeveral Inſtances, it is Thought, we may Compute the Weight of the Air Anſwering to a certain Portion, as an Inch, *&c.* of Quick-Silver; If therefore all this Depends upon Gravity, how comes it to Paſs, that the ſmall Diviſions of a Grain ſhall be equal to a certain Weight of Air, and that Weight of Air equal to Support ſuch an Inch, or whatever other Portion it is, of Quick-Silver? It is a known Axiom, if two Things are equal to a Third, they are equal to one another, and Conſequently if ſo much Air is equal to Perhaps an $\frac{1}{100}$*th* Part of a Grain, and that Air is likewiſe equal to $\frac{1}{8}$*th* of an Inch of Quick-Silver, or whatever it is; in this Caſe, ſince we take the leaſt, $\frac{1}{8}$*th* of an Inch of Quick-Silver is equal to an $\frac{1}{100}$*th* Part of a Grain, whereas it is much Superior to a whole one; So that all this Doctrine ſeems to fall, upon the Suppoſition of Gravity.

Third, What therefore we Affirm is, that ſuch ſmall Variations of the Statical Baroſcope ſeem more juſtly to be Attribnted to the Air's Spring or Expanſive Force, or elſe it's Contractive, being Increaſed or Diminiſh'd, than to any Imaginary Gravity, which is ſuppoſed to be in it; And nothing appears to be Conſiderable in Philoſophy, as to Material Properties and Beings, unleſs theſe Expanſive and Contractive Forces.

Dr. Gregory's Aſtronom. Lib. 5. Prop. 3. Pag. 401. Philoſoph. Tranſact. Menſ. Julii Ann. 1671 N° 73.

To Confirm this Expanſive Force of the Air, and it's Contractive, we ſhall only mention a Propoſition, the Demonſtration of which Philoſophers at preſent Acquieſce in, and which I have not now Time Particularly to Conſider; Namely, that if the Air Expands it's ſelf, according to this Law, which is ſaid to be Confirmed by Experiments, to Wit, that the Spaces, into which a given Quantity of it is Compreſſed, are Reciprocally Proportional to the Weights, which Compreſs it, a Globe of our Air, which is only one Inch Diameter, if it was of the ſame Rarity and Fineneſs with that, which is at the Height of one Semidiameter of the Earth from us, would fill all the Regions of the Planets, as far, as the Sphere of Saturn, and much beyond it; An Expanſion, which can be only Accounted for by our Philoſophy of Forces, which may be more Diluted and Diffuſed, or more Conſtipated and Denſe, to Infinity, and Space may be Filled and Repleniſhed with ſuch different Forces; But if we Reſolve all into a Similar Matter, and the Parts of it, if Air is Rareſy'd to that Degree, there muſt be ten Millions of Vacuities for one Part of Solid, Similar, and Inflexible Matter, and Conſequently Space cannot be Filled by it; I ſhould now have Proceeded to the Theory of the Winds, but haveing already Diſcourſed ſomething concerning them, as alſo concerning Hydroſtaticks, which can be only Accounted for by theſe Forces, I ſhall not Trouble my ſelf, or Others, with any farther Reaſonings, or Arguments, upon thoſe Subjects.

All That I am therefore to Conclude from this Chapter is, in General, that Philoſophers Diſpute about Real Space, as of a Space, which is full of ſome Properties, and Affections, beſides Length, Breadth, and Depth, which is our, and their Notion of a Vacuum; and that therefore they Tacitly Conſent to and Affirm a Plenum, tho' it is perfectly Inconſiſtent with their Fundamental Opinion of a Similar and Homogeneous Matter; For all their Arguments about Air, and the Gravity of it, do equally Extend to one Portion of Space, as well as another.

How the World is full of Matter, What it is, How Produced by Almighty God, and how it may be Annihilated by him in a Moment.

They Alſo Reckon this Force of Gravity to be Diſperſed thro' all Parts of Space, and therefore do Acknowledge, that there is a Diffuſion of Force every where; ſo that, albeit they do not Declare it, becauſe they in Terms Aſſert a Vacuum, yet they in Fact, and in their Reaſonings, Concurr with, and Aſſent to, what I have Defended, that all Space is Full, and that it is Full of ſome Force or other; They Call it Gravity, I Name it the Expanſive and Contractive Force, and that Expanſive and Contractive Force, or Action in General, I Maintain, to be the Eſſence of Matter; and therefore, that all Space is full of Matter, and that it is Immediately Produced, or Created, from and by the Power, Will, and Pleaſure, of the Infinite, Eternal, and Self-Exiſtent Being, by an Immenſe, and Unlimited Exertion of his Agency thro' the whole Univerſal Expanſe of Nature; And therefore may be Annihilated by him in an Inſtant, upon his withdrawing and Repreſſing ſuch Agency; All which Phænomenons the Corpuſcular, or Atomical, that is, the Preſent Philoſophy cannot Account for, nor does Underſtand.

THE PRINCIPLES OF THE PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK IV.

CONCERNING OPTICKS, DIOPTRICKS, AND CATOPTRICKS.

THE PRINCIPLES of the PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK IV.

CHAP. I.

Concerning the Doctrine of Reflection, as Propoſed by Deſcartes *from Impulſe and Reſiſtence.*

The Doctrine of Reflection and Refraction, of Opticks, Dioptricks, and Catoptricks, next to be Conſidered.

§. 1. HAVING In the former Books of theſe Principles Conſider'd the Notions and Apprehenſions, which the Corpuſcular Philoſophy gives us of External Being, and afterwards having Examined, whether the Mechanical could afford us a better Solution of thoſe Phænomenons of it, which it has Endeavour'd to Explain; We ſhall now Proceed to an Enquiry into the Doctrine of Reflection, and Refraction, and of Opticks, Dioptricks, and Catoptricks.

That the Corpuſcular, Mechanical, and Optical Philoſophy ought to be Deriv'd from the ſame Principles of Nature.

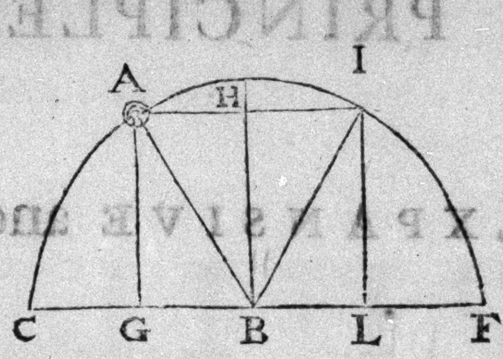
§. 2. BEFORE We do which, we ſhall only at Preſent ſay in General, that in a True and Adequate Account of Nature, the Principles, upon which we Propoſe to Explicate the ſeveral Appearances of it, ought to be Conſiſtent with themſelves; and Conſequently, if the Mechanical Philoſophy ſhould be granted Capable of Solving thoſe Effects, which the Corpuſcular cou'd not, as it is a Demonſtration againſt the Truth of the Corpuſcular Syſtem, ſo on the Contrary, if the Corpuſcular Philoſophy Accounts for the various Properties of Matter, which the Mechanical cannot, it is equally a Confirmation of the Incompetency and Defects of the Mechanical; and the ſame may be ſaid of the Mechanical Philoſophy and the Optical; Since if we would Diſcourſe with any Reaſon or Congruity upon the Nature and Eſſence of Things, there muſt be ſome General Principles laid down, from whence both the one and the Other, both the Properties of Matter, and the Laws of Motion, or of Opticks, may be Deriv'd, ſo as to Unite the Optical to the Mechanical, and that to the Corpuſcular Philoſophy, under one Natural Cauſe of their Exiſting, according to the Method Alledg'd in the Preface to the two firſt Parts of the preſent Principles Publiſhed many Years Ago.

After Light and Colours, the Laws of Reflection and Refraction, and the Doctrine of Opticks are deſervedly to be Examined into.

§. 3. But That we may Proceed in a certain Order and Method, and with that Cautiouſneſs, which is Requiſite in the Confutation of Principles, which have been hitherto Receiv'd with a General Applauſe; As we have Deſcended to each Particular in the Mechanical Philoſophy, and Concluded, our laſt Part of thoſe Principles Formerly Publiſhed, with an Examination into the Celebrated Theory of Light and Colours, and that of the Rain-bow, which we alſo Proſecuted in the Introduction to the preſent Volume of our Philoſophy; So we ſhall now Enter into a Particular Conſideration of the Laws of Reflection, and Refraction, and the Doctrine of Opticks, Dioptricks, and Catoptricks. And firſt we ſhall Examine into the Laws of Reflection, as Propoſed by *Deſcartes*.

An Account of the Doctrine of Reflection Propoſed by Deſcartes, Dioptric. Cap. 2. §. 1. 2. 3. Rohault. Phyſ. Cap. 15. Part 1. §. 2, 3.

§. 4. As To what Concerns Reflection, there are two Principal and Eminent Laws Aſſerted of it, 1*ſt*. That the Angle of Incidence is Equal to the Angle of Reflection, 2*d*. That Each of them are in the ſame Perpendicular Plane to the Reflecting Surface; Both which are Endeavour'd to be Prov'd by the following Demonſtration. *A* is Suppoſed to be a Body perfectly Hard, and to Fall upon the Surface of Another, *C F*, which is equally ſo; The Body *A*, which is thrown upon the Surface, *C F*, is likewiſe to be Conſider'd, as having neither Gravity, nor Figure, nor Craſſitude, and that there is no Reſiſtence to it's Motion from the Air; Granting this, it is ſaid, it will Neceſſarily follow, that the Motion will be Uniformly continued from A to *B*, and will be Reflected with the ſame Motion, likewiſe, but to what Part, or which way, is to be Inquired; It Deſcends from the Point, *A*, in the Circumference *C A I F*, to *B* it's Center; and ſince the ſame Motion is continued, with which it Fell, it muſt Riſe to ſome Point in the ſame Circumference, ſo as to make the Line of it's Reflection Equal to that of it's Incidence *A B*; But *A B* is Reſolv'd into two Motions, of which it is Compounded, *A G*, and *G B*, a Perpendicular and a Lateral, and as the Perpendicular is only Obſtructed, and is ſuppos'd to be chang'd into the Reflected *I L*, there only remains the Lateral to be Anſwer'd, which muſt be Equal to $BL = GB$; and Conſequently IL is = to GA, or the Sine of the Angle of Reflection is Equal to that of Incidence, and therefore the Angles are ſo; And if this Demonſtration is certain, it will follow, that the Angles of Incidence and Reflection are in the ſame Plane.



Argument the firſt, againſt this Doctrine, becauſe it ſuppoſes a Similar Matter, and that Matter to be Hard, and without Gravity, or any other Property.

§. 5. In Reply to which, there are ſeveral Things to be Objected againſt, in a Real, and not an Abſtracted, Philoſophy, which this Demonſtration ſeems only to Favour, that is, a Philoſophy Created by the Mind, without being any ways Agreeable or Adequate to Nature, as it in Truth Exiſts; For what can be more Abſurd, than from a Suppoſition of Bodies perfectly Hard, to Raiſe an Univerſal Theorem, which ſhall Extend to Bodies which are Perfectly Soft, to Bodies, which for all that we yet know, may have Properties Diametrically Oppoſite, to thoſe which this Demonſtration Aſſumes, as Light, Fire, Air, Water, *&c.* which cannot any other ways be Imagin'd to be Hard, unleſs upon the Hypotheſis of their being Reſolv'd into little Hard Particles of Matter, according to the Corpuſcular Syſtem, which has been already Sufficiently Confuted; And therefore we ſhall not now Inſiſt upon the Unreaſonableneſs of the preſent Suppoſition of Parts without Gravity, Figure, or Craſſitude, and which ſhall yet have Hardneſs; ſeeing it is Abundantly Evident from what we have before ſaid, as well as from this very Inſtance, that where Philoſophers Diſcourſe, or Demonſtrate in that Way, they only Prove, as Phænomenons Riſe, and according to the preſent Emergence.

Argument the ſecond, becauſe Matter in the preſent Corpuſcular Philoſophy, is allowed to be Proportional to it's Weight, which would Accelerate the Deſcending Body, and Retard it in it's Riſing.

§. 6. What We take to be our Immediate Province is to ſhew, that, allowing theſe Corpuſcular Notions, however Irrational, the Reaſoning of this Philoſophy is not Valid in Reſpect of the Laws of Action it Affirms; And in the firſt Place, let us make our Notion of the Reflected Body, as Abſtracted as we Pleaſe, in Relation to it's Figure or Groſſneſs, tho' both of them are to be Conſider'd in the Laws of a Real Reflection, we cannot yet take Gravity from Matter, according to the Modern Philoſophy, unleſs we will Annihilate Matter it's ſelf, to which Gravity is ſaid to be Proportional; If therefore a Gravity is allow'd to the Body *A*, tho' the

Motion from Impulſe was Uniformly continued to *B* and *I*, yet the Force of Gravitation wou'd make a conſiderable Difference betwixt the Deſcending Line *A G*, and the Aſcending *L I*, ſince in the firſt, it wou'd Accelerate the Impulſe, and in the laſt, Retard it.

§. 7. HOWEVER, Granting this Demonſtration to Proceed upon a mere Speculation of Impulſe, in the Abſtract, without Conſidering the Properties of the Body Impell'd, we ſtill Deny the Force and Evidence it Pretends to; Since if *A* has no Properties belonging it, nor *C F*, unleſs thoſe of Hardneſs, that is, of one not Yielding to the other, there can be no Reaſon Aſſign'd, why the Impulſe Communicated from *A* to *B*, in it's Incident Line, ſhould be Equal, in it's Reflected one, ſo as it ſhould come to ſome Point, or other, of the ſame Circumference, from which it was Originally Propagated; Becauſe, by the ſame Reaſon that *A*, when Impell'd in it's Perpendicular Incidence to *G*, (for the Lateral Directions may at preſent as well be ſuppos'd Equal, as not) looſes nothing of it's Impulſe at *G*, or B, but is Reflected from thence, by the ſame Argument, when it is Reflected from *B* or *L* to *I* in it's Perpendicular Reſult, it ſhou'd not ſtop there, but proceed in Infinitum; by which Means the Angle of Reflection wou'd be Infinitely different from the Angle of Incidence, for why a Bare and Abſtracted Impulſe ſhould Looſe it's Force at *I*, where there is no Obſtruction ſuppos'd, rather than at *G* or *B*, where there is one Acknowledg'd, will be Impoſſible to Determine. Nor will the Hypotheſis of an equal Velocity Solve this Difficulty, ſince *A B* is to be Prov'd = to *B I*, not by ſuppoſing it, but by the Forces, of which they are Compounded, which, tho' *G B* and *B L* may be equal, in Reſpect of *A G*, and *L I*, may be vaſtly Different; Becauſe they are not to be Meaſur'd by their Force of Velocity, but by the actual Motion, which is Deſcrib'd in Length. According to what is above Argued. Some have Endeavour'd to Refine upon this Demonſtration, by ſaying, that Nature always takes the ſhorteſt Courſe to ſatisfy two Determinations; But even this Suppoſes the Forces, or Determinations, Equal, which we have ſaid may be juſtly Conteſted.

Argument the third, becauſe the Perpendicular Impulſe in it's Reſult from L *to* I*, may for ought which this Philoſophy Alledges, be Continu'd in Infinitum.*

§. 8. BESIDES in Matter of Fact it is Evident, that Marble, or any other Elaſtick Body, as an Hard Ball of Wool, may be made to Rebound in a Perpendicular Direction much Higher than the Place, from whence the Impulſe was given; and therefore, ſince according to this Doctrine the Oblique Force *A B* is Compounded of a Lateral and Perpendicular, it is likewiſe certain in Fact from the foregoing Section, that the ſame Body may be made to Rebound with an Angle of Reflection, very different from the Angle of Incidence.

Argument the fourth, Evident in Fact, that Elaſtick Bodies may be made to Riſe Higher than the Place, from which they Fall.

§. 9. FOR Moreover, ſuppoſing that an Impulſe ſhould be given to *A*, which wou'd carry it 5 or 10 times as far, as from *A* to *B*, either ſome of that Force muſt Neceſſarily be loſt at *B*, or elſe the Reflected Line muſt be much greater than *A B*, either of which Conceſſions wou'd Abſolutely Deſtroy the preſent Demonſtration; For if there is not the ſame Motion after Reflection, as before it, *B I* will not be equal to *A B*, and if the Point at *B* can divide the Impulſe, as it pleaſes, it will be no more Neceſſary to think, that it will give an Equal, than an Unequal Portion, to the Lines of Incidence and Reflection, ſince it will have a Diſcretional Liberty of Acting, without Conſulting the Pleaſure of ſuch Abſtracted Reaſonings, or the Patrons of them.

Argument the fifth, becauſe the Line BI *may be much greater than* AB, *in a Real and not an Hypothetick Impulſe.*

§. 10. MORE Than this, it is not undoubted, and wants ſome Force of Conviction to ſhew, that an Impulſe Communicated from *A* to *B* will, from the Principles made uſe of, Neceſſarily Riſe after it's Obſtruction at *B*, and not Proceed in a Lateral Direction to *F*, even ſuppoſing the Line of Incidence is Equal to that, which it's Motion or Action Deſcribes after it's Incidence upon *B*, and that there is no other Affection Conſider'd than bare Impulſe and Hardneſs; Seeing there is nothing in Hardneſs which can give it a particular Determination, that is, the mere Reſiſtence of one Body to another, which hinders it's Progreſs one Way, cannot give it a Direction another Way; And there is nothing in Impulſe, but what wou'd give it a Different or Contrary to that, which is Experienc'd, or Fix it in the Point *B*,

Argument the ſixth, becauſe it is not Proved that the Impulſe may not be continued after it's Incidence, on the Hard Body, in a Lateral, as well as any other Direction.

§. 11. IF all this were True, this Reaſoning cou'd not Demonſtrate to us, how the Angle of Incidence and Reflection ſhould be in the ſame Plane; becauſe altho' the Line of Incidence and Reflection were Equal, that is, the Impulſe in both Caſes ſhould be Alike, yet it is not Evident, why they may not be in the ſame Sphere, as well as in the Circle only, which is here Aſſum'd, ſince the whole Argument Depends upon that

Argument the ſeventh, becauſe it is not Prov'd that the Line cf Incidence and Reflection, may not be in the ſame Sphere as well as in the ſame Circle.

that Equality, which may as well be Preſer v'd in the Radius of a Sphere, as in that of a Circle.

Argument the eighth, becauſe the whole depends upon the Incident and Reflected Line being equal, which is no other ways Prov'd, than by ſuppoſing it.

§. 12. We have ſaid in the foregoing Sections, that this Demonſtration Depended wholly upon the Suppoſition, that the Reflected Line was Equal to the Incident, for if *B I* is not Equal to *B A*, that is, if they are not Radius's of the ſame Circle, neither *G B* and *B L*, nor *G A* and *L I*, the Forces, of which the Radius's are Compounded, will be Equal, that is, the Sine of Incidence will be different from that of Reflection; If *B I* is = to *A B*, and this is by Suppoſition, and not Prov'd to be ſo, as it is Plain, it is not, from §, 7, 8, 9, ſince there is no Diſtinction betwixt *A B*, and *A G*, *G B*, and betwixt *B I*, and *B L*, *L I*, Excepting in the Compoſition and Reſolution of the ſame Force, to Conclude from thence, that the Sine of Incidence is Equal to the Sine of Reflection, is no more than to Prove it, by Suppoſing it; that is, *A B* is ſuppoſ'd = to *B I*, but *A G* and *G B* are the ſame in Effect as *A B*, and *B L* and *L I*, as *B I*, and therefore, by Subſtituting one Inſtead of the other, *A G* and *B G* are ſuppoſ'd = to *B L* and *L I*, or the Sine of Incidence = to that of Reflection, or which is all one, it is ſo, becauſe it is ſuppoſ'd to be ſo.

Argument the ninth, becauſe this Abſtracted Notion of Reflection is not Juſtifyed, by any Appearances in Nature.

§. 13. But Beſides the extreme Precariouſneſs of this Way of Reaſoning, it is likewiſe unhappy to it, that, when it is apply'd to Philoſophy, there is not one Phænomenon in ſeveral, which might be Named, to which it Agrees, or which it Gives any Explanation of; Thus Dirt, Clay, Salt, Sand, Glaſs, Stone, Iron, Lead, Gold, and all Sorts of Fluids, have little or no Reflection, that is, the Reflected Line, if any, is much ſhorter than the Incident; On the other Hand, all Elaſtick Bodies, as Wool, made into an hard and conſiſtent Ball, Marble, &c. if ſtruck with any Conſiderable Force upon a Reſiſting Body, Deſcribe a Reflected Line much longer than the Incident, if with a more Faint and Weaker, the Reflected Line will be ſhorter; Add to this, that in the ſame Bodies, whether Elaſtick or not, their Reflected Force will be Greater, as their Bulks are Diminiſh'd and their Impulſe Increaſed, and a Small Piece of Marble or Lead, which is ſtruck with a Greater, will be more Reflected than a Larger, which is ſtruck with a Leſs; in all which Caſes, there is not one Inſtance, in which the Sine of Incidence is Equal to that of Reflection, ſince the Lines are not.

The Preceding Arguments not Invalidated by pretending the Real Difference in Reflection Proceeds from the Qualities of Matter, which are Abſtracted from it in the preſent Rule; Argument the firſt, becauſe the Laws of Reflection in the Abſtract, which are falſe, are Endeavoured to be Juſtified by an Ignorance of the Cauſes of that Difference which is Obſerv'd in a Real one.

§. 14. If it be here Alledg'd, that theſe Inſtances are not any ways Concluſive againſt the foregoing Demonſtration, which Abſtracts from all other Qualities of Body, as Gravity, Figure, Elaſticity, &c. and only Retains that of a Mutual Reſiſtence upon Impulſe, and therefore, allowing for, what Effects ſuch Properties may have, that it is not certain, but this Theorem of the Angle of Incidence being Equal to the Angle of Reflection may Univerſally obtain; We Anſwer, that we have already ſhewn, that, bare Impulſe and Reſiſtence only Conſider'd, the Demonſtration is both Falſe in it's Argument, and at the beſt Ends in a Suppoſition of what was to be Proved; and we now ſay, that to Confirm ſuch a Demonſtration, when there is no matter of Fact to Support it, by telling us of the Poſſibility, that other Properperties in Nature, which were not Conſider'd, may Cauſe a Deviation from this general Rule, is no other than to Confeſs, that we do not know, what the Force and Impreſſions of thoſe Properties may be, nor which way they are Reconcileable with the pretended Laws of Reflection; That is, a Demonſtration of the Laws of Reflection, in the Abſtract which is Falſe, is Endeavour'd to be Juſtify'd by an Ignorance Acknowledg'd of the Cauſes of that difference of Reflection, which is Obſerv'd in Exiſtence and Reallity.

Argument the ſecond, becauſe Gravity in ſome Caſes does not Affect the Reflexive Force.

§. 15. But to Evidence farther the Vanity of ſuch a Refuge, we ſhall Inquire, what Effects Gravity, or Elaſticity, may Probably have, to Diſturb the Reflection of Bodies ſo far, as the Reflected Line ſhould not be Equal to the Incident, if it would be ſo from mere Impulſe and Reſiſtence. It is therefore in the firſt Place certain, that Gravity cannot make in ſeveral Caſes any Alteration, in the Real Reflection of Bodies, ſince Marble, which is Heavier than Water, is Reflected, whereas Water, which, in the Corpuſcular Philoſophy, is ſuppoſed to Conſiſt in the laſt Reſult of Hard and Solid Parts, is not to that Degree, altho' according to §. 13. the leſs the Parts of a Solid and Hard Body are, other Things being Equal, the more they are Reflected; It is alſo Plain in Experience, that if Fluids are not Admitted in the preſent Compariſon, that Dirt, Salt, Sand, and Glaſs, which are alſo Lighter than

Marble, have not ſo great a Reflection; If therefore theſe Bodies, Abſtractedly Conſider'd, would from Impulſe have Deſcrib'd Lines of Reflection Equal to the Incident, Gravity would not be any Obſtruction to ſuch a Deſcription.

§. 16. On the Contrary, Elaſticity does not Cauſe ſuch an Alteration in other Bodies, as ſhould hinder the Line of Incidence, from being Equal to that of Reflection, if it were Originally ſo, ſince Lead, or Gold, *&c.* which has little or no Elaſticity, and therefore cannot be any ways Interrupted in it's Reflection by it, does not Obſerve this Law.

Argument the third, becauſe Elaſticity in others does not.

§. 17. Lastly, if it is ſaid, that Elaſticity in the former Inſtances, and Gravity in the latter, is the Cauſe, that this Law of Reflection does not obtain, which it wou'd have from Impulſe and Reſiſtence; This will be Confuted from the ſame Heavy or Elaſtick Body Riſing to different Heights, according to the Velocity of the Impulſe, as is Evident in §. 13; For if *A*, having the ſame Elaſtick or Gravitating Force, will Riſe Higher from *B* upon one Impulſe, than it will upon another, ſuppoſing both thoſe Forces are taken away, *A*, if we Conſider only bare Impulſe, will Riſe Higher from one Impulſe than it will from another, and Conſequently will Fall under the Confutation of §. 7, 8, 9, and 10.

Argument the fourth, becauſe neither Elaſticity in the Inſtances of §. 16. *nor Gravity in thoſe of* §. 15. *will Affect the Reflexive Force ſo as to hinder the Prevalence of this Law, if it is True.*

§. 18. That we may not yet Reſign this Objection of Foreign Forces changing the Laws of Reflection without a particular Examination, we Perhaps may afterwards Conſider, 1*ſt*. what Effect Gravity wou'd have to Produce an Alteration in Reflection, 2*d*. what cou'd be ſuppoſed to Proceed from Elaſticity; And ſhall at preſent Obſerve, that if two new Forces, as Gravity, and Elaſticity, are made Uſe of to Explain, a Real Reflection, the Hypotheſis of Impulſe and Reſiſtence cannot do it; and that, if the Doctrine of the Sine of the Angle of Incidence being = to the Sine of the Angle of Reflection were True in the Theory, which we have ſhewn is not, yet it is far from being True in Nature, which in a Real Philoſophy we are only to Conſider, albeit it is made Uſe of in all Demonſtrations of it, as an Undoubted Axiom.

Argument the fifth, becauſe if Gravity and Elaſticity enter into the Nature of Real Reflexion, Impulſe and Reſiſtence will not be a ſufficient Cauſe to Explain it.

§. 19. That However, which has led Philoſophers into this way of Thinking, is the Advantage, they Apprehended, ſuch a Propoſition would be to them in Conſidering the Action of Light, and Laying the Foundation of a new Science in Opticks; They thought Light might be Abſtracted from Gravity and Figure, and the other common Affections of Bodies, and therefore, whatever was Demonſtrated upon ſuch Suppoſitions, would be true of it in General; We ſhall next then Examine into Matter of Fact, in Reſpect of the Reflection of Light, and See, whether it Juſtifies this Abſtracted Reaſoning of Reflection, which in it's own Principles we have Evinced to be, at the worſt, Falſe, and at the beſt, Precarious and Inconcluſive, as they are only an Affirmation of the very Thing, which is to be Prov'd, as is Explain'd in §. 12.

The Reaſon of this Doctrine is Derived from the Uſe it was thought it would be of in the Explication of Opticks, Cartes, Rohault, Hugens Molyneux, Sir Iſ. Newt.

§. 20. According to §. 12. if the Sines of the Angles of Incidence are Equal to the Sines of the Angles of Reflection, the Incident and Reflected Lines, or Forces, muſt be ſo likewiſe, and Reciprocally, neither of which, we ſhall Prove, Obtains in the Reflections of Light.

In Light, if the Rays of Incidence and Reflection are equal, the Sines will be ſo likewiſe.

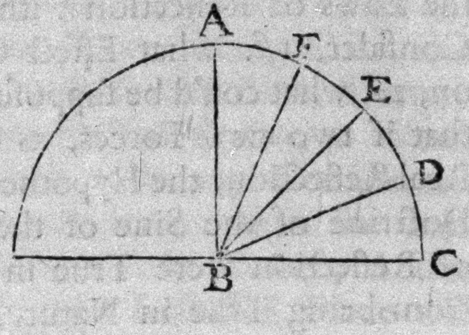
§. 21. That the Latter does not is Plain, ſince the Incident Line from the Sun to the Reflecting Body is Infinitely greater than the Reflected Line, and even in Objects, which are near us, the Images of them are ſometimes preſented to us at a much leſs Diſtance by Reflection, than that of thoſe Objects from the Reflecting Surface, upon which the Rays are Incident; Either therefore in both Caſes the Rays of Light are Equal in their Incident, and Reflected Force, or not; If they are, as the Light of the Sun is Deriv'd from it ſeveral Millions of Miles to the Reflecting Surface, it ought to be Reflected as many Millions of Miles from it, and the Brightneſs of Burniſht Gold would be as Viſible at that Interval, as at a nearer, which is not Experienced; Likewiſe, as the Satellites of *Jupiter*, and the Phaſes of *Venus*, are Diſcern'd from a Catoptrick Teleſcope at a ſmall Diſtance from it, they ought rather to be ſeen as far from ſuch a Teleſcope, as the Satellites of *Jupiter*, or the Phaſes of *Venus*, are Remov'd from the Reflecting Poliſh, and not at ſo ſhort a Diſtance, ſince the Rays falling in a manner Parallel, and Perpendicular, ſhould be Reflected with the ſame Direction, with which they were Incident, and Conſequently would either Aſſemble into no Focus at all, or at an Infinite Remove from their being Reflected; And laſtly, as com-

The Rays of Incidence and Reflection not equal in the Experiments of Opticks.

common Objects are Perceiv'd by Reflection from a Glaſs, the Reflected Rays ought to be as Strong and Lively at the Diſtance of the Object, which Cauſes the Reflection, as when we are nearer the Reflecting Surface; whereas in the firſt Caſe the Light is exceedingly Diminiſht, and the Object grows Obſcure, and ſcarcely to be Diſcern'd, altho' in the laſt, it is ſufficiently Diſtinct and Vivid; On the other Hand, If the Forces of the Incident and Reflected Lines are not Equal, the Sines or Angles of Incidence and Reflection will not be Equal, according to §. 12, 20.

The Angles and Sines of Incidence and Reflection, not equal, Prov'd from Experiments in Opticks.

§. 22. Nor does the former, according to the Diſtinction we made in §. 20. Univerſally obtain in Light, by any Thing we can Experience; ſince it is Manifeſt, that we ſee Colours Produced by the Rays of the Sun, thro' a Priſm, at all Angles of Reflection, tho' it is Impoſſible, in the ſame Poſition of the Priſm and the Wall, on which the Colours are Projected, that they ſhould Fall in all Angles of Incidence; And the ſame may be Affirm'd of Colours in General, if we allow they Proceed from the Appulſe of the Direct Rays of the Sun to the Colour'd Body, and from thence are Reflected to the Eye, which ſeems to be the Opinion and Sentiment of the preſent Philoſophy; For in either of theſe Caſes we may as well Aſſert that the Incidence *A B* ſhall Cauſe a Reflection to *C, D, E, F,* and at the ſame time Produce no Variation in the Angle of Reflection, as Imagine that Colours can be Determin'd to every Part from a like Incidence, if the Angle of Reflection is always Equal to the Angle of Incidence, as this Doctrine ſuppoſes, ſeeing the Coloured Image, or Body, as *B*, may be equally ſeen at *C, D, E* and *F*.



Prov'd from Colour'd, and Opake Bodies not Reflecting any Images to the Eye.

§. 23. Besides which even in the Reflections of Catoptrick Glaſſes, or Poliſht Marbles, *&c.* if the Images, preſented to us from them, are an Argument, that Light is Rebounded in the ſame Angle, as it Falls, in Opake and Colour'd and White Bodies, where no ſuch Images are offer'd to us, and Light is Acknowledg'd to be Reflected, it will be a Proof, that Light is not Return'd in the ſame Angle, in which it Falls; For if it be ſaid, that all the Rays are Reflected in one Caſe, and in the other ſeveral are Loſt and Abſorpt by certain Imaginary Pores, ſuch an Objection depends upon the Corpuſcular Philoſophy, which we have already Conſider'd and Refuted, *Part 2d. Chapter* of *Light* and *Colours*. Or if it was allow'd, an Interruption of the Homogeneous Matter, by the Intervening Pores, could only Cauſe an Interruption, or ſeveral Breaks, in the Features and Lineaments of the Image, if there was nothing more to be Conſider'd, than the Reflection, or the Rebound, of Light in Angles Equal to its Incidence, and the Capacity of Reflecting that Light in the Oppoſite Body, to Determine, what Subſtances are Reflexive of Images, and what are Opake, and Colour'd; From whence it is Evident, that there is ſomething more, which this Philoſophy does not Explain, and which is Requiſite to the Underſtanding the Various Reflections of Light, and the Laws of them.

Reflection of Light, and the Laws of it different, according to the difference of the Bodies on which it is Incident.

§. 24. However, if we Grant, according to the Suppoſition of §. 23. that Light is Incident in the ſame Angle, with which it is Reflected in Bodies Reflexive of Images, which yet is not True from §. 21. and will likewiſe Allow, according to §. 22. that Light, which is Incident on Colour'd Bodies is not, it will no ways Appear, that the Angle of Incidence and Reflection being Equal, is a general Property of Light, but on the Contrary, that it Varies according to the Bodies, on which it Impinges; The Cauſes of which no Philoſophy has hitherto Aſſign'd: From all which it will follow, that the Angle of Incidence being Equal to the Angle of Reflection, is no more a Property in Nature, than it's being Ten times Leſs or Bigger, and therefore is deſervedly to be Rejected from a true Philoſophy, as Groundleſs and not Anſwerable to the full Extent of Being, or Scarcely to any Part of it.

On the other Hand, if we Conſider all Bodies, as differing Faſciculi, Aſſemblages, or Congeries of Actions, or of the Expanſive and Contractive Forces, we may eaſily Conceive, how Light may be Reflected from them in a Thouſand Varieties, according to the Various Forces, on which the Light is Incident, and it's Various Repulſion from them.

CHAP.

CHAP. II.

Concerning Sir Iſaac Newton'*s Hypotheſis of Reflection from Impulſe and Attraction.*

The Deſign of this Chapter.

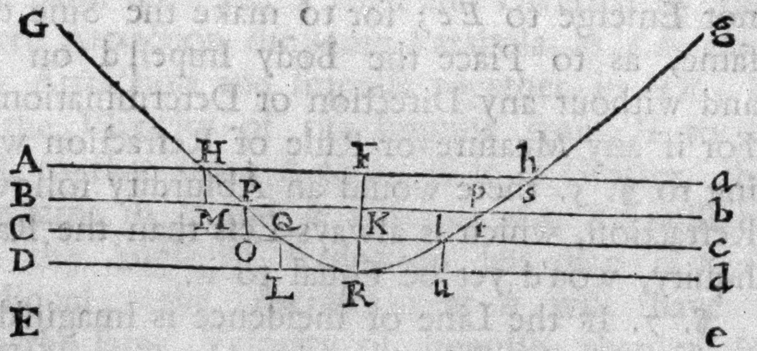
§. 1. WE Have in the Preceding Chapter endeavour'd to ſhew, that the Angle of Incidence being Equal to the Angle of Reflection, or on the Contrary, is not an Eſtabliſht Law in any part of Nature, nor even in the Rays of Light, and that if we Conſider it only, as an Abſtracted Reaſoning, it is Deficient, and Wants a farther Explanation; Since this Opinion notwithſtanding has ſo far Prevail'd, as to Draw into it's Patronage another Way and Method of Arguing, we ſhall farther Conſider it likewiſe, ſo as nothing may be Omitted, which ſhall ſeem Material in the preſent Argument.

If the New Hypotheſis of Reflection is True, the former muſt be Falſe, if not True, the former may be much leſs ſuppoſed to be ſo.

§. 2. THE New and very Ingenious Hypotheſis, for Solving the ſuppos'd Equality of the Angle of Incidence and Reflection in Light, is Founded upon a Conjunction of two Forces, Attraction and Impulſe, whereas the former Hypotheſis had only Conſider'd the laſt of them, and a bare Reſiſtence to it; If therefore this, which we are at Preſent to Examine into, is True, the other cannot but fall ſhort of that, which it pretends to Demonſtrate, becauſe two Forces will be Requir'd, whenas That had only Aſſumed one; If it is not True, ſince it muſt be yet own'd to be Introduc'd upon a more Rational and Accurate Inquiry than the other was, it will follow, that if this Hypotheſis is not Juſt, the Preceding, which beſides has been already Confuted, is much leſs ſo.

The New Hypotheſis of Reflection Explain'd.
Sir Iſ. Newt. Princip. §. 14. Lib. 1. Prop. 96. Pag. 230.

§. 3. THE Hypotheſis is, that if a Uniform Perpendicular Attraction from *D d* towards *A a*, that is, from a Rarer towards a Denſer Medium, is ſuppos'd, of any Body paſſing thro' the Space Intercepted by the Planes *A a*, and *D d*, and it is at the ſame time Impell'd in the Line *G H*, Granting likewiſe, that the Motion before Incidence is ſwifter than it is after, The Body, by the Line of Incidence being Inclin'd, will be at length Reflected, and the Angle of Reflection will become Equal to that of Incidence; For by the Doctrine of Refraction, which this Philoſophy had before Explain'd, and we ſhall at preſent ſuppoſe True, *H R* is a Parabolick Curve made by the Perpendicular Attraction from the Plane *D d* towards the Plane *A a*, and the Incident Force *G H*. Let therefore now the Obliquity of the Line of Incidence *G H* to the firſt Plane *A a* be ſuch, that the Sine of Incidence may be to the Radius of the Circle, of which it is the Sine, in the ſame Proportion, as it is to the Sine of Emergence out of the Plane *D d*, into the Space *D d*, *E e*, which Sine of Emergence will Conſequently be equal to the Radius of the Circle and the Line of Emergence Coincide with the Plane *D d*; but the Body or Light Impell'd cannot Proceed according to the Direction of that Plane, becauſe it is Attracted in a Perpendicular to the Medium of it's Primary Incidence, that is, to the Plane *A a*; nor is it Poſſible for it to Emerge into the Space or Medium *E e*, becauſe the Sine of Emergence is in the Line or Plane *D d*; It muſt therefore be Reflected back to *l p* and *h*, and Deſcribe in it's Return the ſame Portion of a Parabola, as it did before in it's Deſcent, ſeeing the Laws and Conditions of Motion are the ſame in the one Caſe, as in the other; And as this Demonſtration ſuppoſes the Attraction, and Impulſe, to Act Uniformly, according to a certain and ſtated Rule, ſo if that is Vary'd, and the Planes *A a*, *B b*, &c. Approach Infinitely near to each other, the ſame Proof Extends to all Caſes, that can be Aſſign'd of different Impulſes and Attractions in the ſeveral Spaces contain'd within the foremention'd Planes, whilſt the Infinite Diminution of the Intervals of the Planes, and the Infinite Increaſe of the Number of them, renders the Action of the Impulſe and Attraction Imperceptibly different, and Reduces it to one and the ſame Law, and Conſequently the Angle of Emergence *g h a* will be Equal to the Angle of Incidence *G H A*.



Tho' the Advantage of this Hypotheſis is ſaid to be, that it Accounts for Reflection and Refraction upon the ſame Principles, yet very Different ones are Aſſumed from what are Originally ſuppos'd.

§. 4. THIS Hypotheſis of Reflection is the more Eſteem'd and Applauded, as it is Thought to Solve Refraction upon the ſame Principles; but when we Conſider the whole, and Obſerve, that tho' Impulſe and Attraction are the Terms made Uſe of, yet that in Reality there are ſeveral other Foreign Suppoſitions Introduced to make this Doctrine Conſiſtent, it will not perhaps appear, that Refraction and Reflection are Accounted for from the ſame, or like, Cauſes by this Hypotheſis.

This Hypotheſis is not True, becauſe a given Proportion betwixt the Sine of Incidence and Refraction, is no ways Applicable to the Doctrine of Reflection.

§. 5. FOR at preſent Allowing the Refraction to be in a Parabola, which we ſhall afterwards Conſider, ſince the Sine of the Angle of Incidence is to the Sine of the Angle of Emergence in a given Reaſon, for Inſtance, as a to b, Either in the ſuppos'd Inclination and Obliquity of GH to the Plane Aa, in which the Sine of the Angle of Incidence is to have the ſame Proportion to that of Emergence, as it has to the Radius of the Circle, of which it is the Sine, the Reaſon of a to b is Preſerv'd, or not; If it is not, there is a new Suppoſition, and the ſame Hypotheſis does not Solve Reflection and Refraction, but another Arbitrary one is Introduced, which does not depend, nor has any Connection with the Laws or Doctrine of Refraction, excepting that Impulſe and Attraction are made Uſe of, in both Caſes, but with different Rules and Appointments of their Actions; If the Reaſon given is Preſerv'd in this Suppoſition, then the Sine of the Angle of Incidence will be either Leſs, or Greater, or Equal, to the Sine of Emergence, which is ſuppos'd the ſame with the Radius; If it is Equal, that is, if a and b are ſo, there can be no Refraction to or from the Perpendicular, which is the Problem to be Solv'd in Refraction; If it is Greater, the Sine of the Angle of Incidence will be Greater than the Radius, of which it is the Sine, which is Impoſſible; If it is Leſs than the Sine of Emergence, it will only explain the Proportion of Refractions from the Perpendicular, and not thoſe, which are made to it, whereas it ought to Account for the one, as well as the other, for the ſame Reaſon, as it Endeavours to Unite Reflection under the ſame Cauſe with that of Refraction.

This Hypotheſis is not Juſt, becauſe by makeing the Sine of Emergence equal to the Radius, that in Part is Aſſum'd, which is to be Prov'd.

§. 6. BUT what is ſtill farther Conſiderable in this Hypotheſis is, that it ſuppoſes the Sine of Emergence=to the Radius, and then Concludes, that the Body Impell'd will not Emerge to Ee; for to make the Sine of Emergence Equal to the Radius, is the ſame, as to Place the Body Impell'd on the Plane Dd by an Arbitrary Hand, and without any Direction or Determination from the Nature or Laws of Refraction; For if any Meaſure or Rule of Refraction was Obſerv'd in the preſent Caſe, according to §. 5. there would an Abſurdity follow, which is, in General, that a Sine of Refraction, which is always Leſs than the Radius in any Incidence or Angle of Obliquity, wou'd yet be Equal to it.

Not Juſt, becauſe even the Suppoſition of an Infinitely Oblique Incidence, would ſignify nothing to the Explaining of Reflection.

§. 7. IF the Line of Incidence is Imagin'd to be Infinitely Oblique to the Plane Dd, or it's Parallel Aa, when the Sine of Emergence Coincides with it, this will be nothing more than conſidering a Body Impell'd, according to the Preſcription of one Uniform and ſtrait Line, which what it will Signify to the Explaining Reflection upon the ſame Hypotheſis with that of Refraction, will be hard to Comprehend; If there is ſomething more Intended, than what we have Mention'd, it is this, that tho' we ſuppoſe an Infinite Obliquity of Incidence, the Sine of it will yet Preſerve the ſame Reaſon and Proportion to the Sine of Emergence as before; Becauſe the Sines of Incidence are to thoſe of Emergence in a given Ratio; If ſo, the Sine of an Infinite Obliquity will be either = to the Sine of the Emergence, or greater, or leſs, and will fall under the Confutation of the ſeveral Caſes Alledg'd in §. 5. We may alſo further ſay, that, in an Infinite Obliquity, the Radius is always ſuppos'd to be the Sine of the Angle of Incidence, and therefore a will be to b in a Reaſon of Equality, tho' on the Contrary, in Refraction, it is ſaid to be in a Reaſon of Inequality.

Such a Suppoſition not Agreeable to the Body's being Reflected in a Parabola.

§. 8. HOWEVER, that this Infinite Obliquity can be of no Service in the preſent Caſe is Manifeſt, ſince if we allow, by ſuch a Suppoſition, that the Body Impell'd is Arriv'd at R, and cannot Proceed any farther than Dd, to Effect which this Hypotheſis is Introduced, yet ſeeing, according to the preſent way of Reaſoning, it is to Riſe, by means of the Perpendicular Attraction, in a Parabola as $Rlph$, it muſt likewiſe Deſcend in one, or otherwiſe, the Angle of Incidence will not be Equal to that of Reflection, which is the very Propoſition intended to be Prov'd; But if the Parabola of Deſcent has the ſame Inclination to the Plane of Dd with the other, we

have

Nam'd, either *GHPQR* will be Infinitely Inclin'd to the Plane *D d*, and Conſequently *Rlph*, that is, the Force of Attraction, which is ſuppos'd to Raiſe the Body Impell'd from the Plane *Dd*, will not be able to do it, or if it is, and the Body Impell'd is Attracted to *h* for Example, the Deſcending Parabola *GHPQR* will be likewiſe in an equal Obliquity to the aforeſaid Plane, contrary to the Suppoſition of it's being Infinitely Inclin'd to it.

§. 9. WE cannot alſo but take Notice of another Suppoſition, which is directly Contrary to what is made Uſe of for the Solving of Refraction; Namely, whereas the Attraction draws the Body Impell'd upwards, which is moving downwards by Suppoſition in Accounting for Refraction, in Giving a Solution of Reflection, it is Conceiv'd to Act the ſame way likewiſe, but then the Body is ſuppos'd to be Aſcending; For if it is not ſuppos'd to Aſcend, it ſeems Impoſſible for the Attractive Force to Draw the Impellent from *R* upwards, which could not hinder it from Arriving downwards to it; Upon which we ſhall only Obſerve, that, in an Abſtracted Hypotheſis, as this of Reflection and Refraction, if Philoſophy has a Liberty to make as many Arbitrary Forces, Acting in as many Different Determinations, as the Effects, it wou'd Solve, Require, it is no more, than Explaining thoſe Effects by themſelves; Thus, if, when we wou'd Explain, what Reflection is, we take a Force, which ſhall Impel a Body downwards, and then Another, which ſhall Strike it upwards in an Abſtracted Hypotheſis, What is it more than to Adapt and Contrive our Suppoſitions at Pleaſure, according as we ſhall find there is Occaſion for them, from the Effects we Conſider, without Giving us any Truer a Knowledge of Nature, than we had before? Since by the ſame Methods we may Explain any Thing, that is, by ſuppoſing the very Thing to be Explain'd.

The Suppoſition uſed for the Solving Reflection is, that the Body is Reflected, which is Abſurd.

§. 10. AND This, beſides what has been ſaid, is Evidently done in the preſent Hypotheſis, for ſuppoſing *GH* by Impulſe to fall upon *Aa*, and from thence to Deſcend in a Parabola to *R*, there muſt either be a New Impulſe at *R* towards *Fh*, and Conſequently, a freſh Suppoſition Introduced contrary to the Former, or elſe it muſt be a Continuation of the ſame Impulſe from *GH*, and Turn'd and Diverted by the Action in *Aa*, towards *Fh*, for by mere Attraction the ſame Parabola, as *Rh*, cannot be Deſcrib'd, which was before by Attraction and Impulſe together, in *HR*; If it is a Continuation of the Impulſe, the Velocity of the Impulſe from *H* to *R*, will be either Greater, or at leaſt Equal, to that from *R* to *h*; If it is Greater, the Attraction *RF* being ſuppos'd to be the ſame in *H* and *h* towards *Aa*, *Rh* will be leſs than *RH*, and Conſequently *FRh* the Angle of Reflection leſs than *HRF*, the Angle of Incidence; If it is Equal, the Attraction at *h* will have a greater Force, where it Conſpires with the ſame Velocity of Impulſe, than at *H*, where it Oppoſes it, and *ul* will be Greater than *QL*, which again will make *FRh* the Angle of Reflection leſs than *HRF* the Angle of Incidence.

If this Hypotheſis of Impulſe and Attraction was True, the Angle of Reflection would be leſs than that of Incidence.

§. 11. WHEN farther, in this Hypotheſis, the Rays are Tranſmitted thro' the Glaſs, and afterwards Attracted back to it, in order to Produce Reflection, it will be Difficult to ſay, how there ſhould ever be any Refraction Produced at all; but this it ſeems is done by the Vibration of the Medium, which, when it Concurrs with the Incident Rays, Refraction is Produced by carrying them off from the Influence of the Glaſs, when it Meets or Oppoſes the Progreſs of the Rays in it's Vibration, Reflection is; But in the Firſt Place, this Vibration of the Medium is not True, unleſs a Stagnant Fluid, as the Air or Æther, is Naturally a Pendulous Subſtance, which no Philoſophy, but the preſent Celebrated one of Pendulums, has Aſſerted; Second, it is not Conceivable, how the Pulſe and Repulſe of the Air or Æther, allowing of it, ſhould have a Force to Rebound the Rays of Light, if Glaſs it's ſelf, by it's Solidity and Attraction, cannot do it.

The Vibration of the Medium to Produce Reflection and Refraction Precarious and Groundleſs.

Sir Iſ. Newt. Opticks. Concerning Fits of eaſily Reflection and eaſy Tranſmiſſion.
Prop. 12. Part. 3. Book. 2.

§. 12. ANOTHER Suppoſition in this Reaſoning is, that in the Formation of the Semi-Parabolas *HPQR*, and *Rlph*, the contrary Directions of the Impulſe will Produce the ſame Figure, which it ſeems Impoſſible, that they ſhould; For if *HM*, *PO*, *QL*, are ſeverally Expreſſed in the Abſciſs of the Parabola *FR*, they will be as 5. 3. 1. according to the Laws of Attraction, or Gravitation, that is, as the Differences of the Squares of a Series of Numbers in a Simple Arithmetical Progreſſion, ſuppoſing *R* to be the Vertex of the Semi-Parabola *HRF*, and therefore the Attraction

Different Forces are Imagined in this Hypotheſis Capable of Deſcribing the ſame Parabola.

in the Return of the Body Impell'd, Obſerving the ſame Law, as before, but being Aſſiſted by Impulſe, whereas before it was Retarded by it, *s h*, *t p*, *u l*, will be alter'd in their Proportion to the former; from whence it will follow, that if the Vertex of the Semi-Parabola *F R H* is Fixt and Determin'd in *R*, in Reflection it will ceaſe to be a Parabola, as it would Produce a Curve nearer to *F R* than *R h* the ſuppos'd Parabola, which would be far from proveing the Angle of Reflection Equal to that of Incidence.

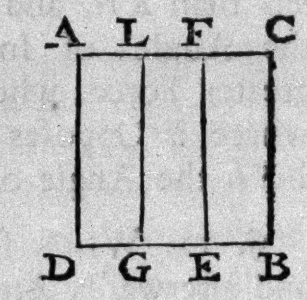
Another Preſumption in this Hypotheſis is, that the Body is Impelled beyond that which Reflects it, and that Refraction is Neceſſary to Reflection.

§. 13. Lastly, it is ſuppos'd, that to Produce Reflection, there muſt be a Progreſs of the Body Impell'd beyond the Medium, which Reflects, and that the Refraction of it is Neceſſary and Previous to it's Reflection; which is ſo Extraordinary, not to ſay, ſo Extravagant an Opinion, that nothing but the Authority of the Philoſophy, which Propoſes it, cou'd Support it; for if Refraction is Neceſſary to Reflection, ſince the Law of Refraction is, that the Sines of Incidence, and it, ſhould be as *a* to *b*, and thoſe of Incidence and Reflection, as *a* to *a*, it will be Requiſite, that *a* ſhould have it's Definite and Certain Reaſon of Inequality to *b*, that *a* may have it's own or a Proportion of Equality to *a*, than which nothing can be more Contradictory; Beſides that the Progreſs of the Body Impell'd is beyond that which Reflects it, there are not any Inſtances in Nature to Juſtify it, or to Shew, that if Light, for Example, Falls upon a Denſer Medium, it ought to Paſs thro' it, and be Reflected from a Rarer.

The ſeveral foregoing Supboſitions are made to be only one Hypotheſis, all which, if True in Abſtraction, yet are not ſo in Nature. Not an Infinite Obliquity of the Incident Rays. Not the Attraction of Light by the Reflecting Body.

§. 14. But granting all theſe Suppoſitions, which are yet call'd by the Name of one ſingle Hypotheſis, there is not any of them, as they are Inſufficient in an Abſtracted Reaſoning, but what are equally Defective, when Apply'd to External Being; It is Evident in the firſt Suppoſition, where the Line of Incidence is ſuppos'd to be Infinitely Oblique to the Reflecting Surface, ſince not only in common Bodies, but in Light alſo, the Rebound is made in all Angles of Incidence, excepting where the Incidence is Infinitely Oblique, which in ſuch a Caſe is Propagated in a direct Line, and therefore all Catoptrick Reflections, as well as all Reflections of other Bodies, are a Confutation of the Hypotheſis of an Infinite Obliquity.

§. 15. It is alſo Manifeſt in the ſecond Suppoſition, that ſuch an Hypotheſis cannot be Agreeable to the real Attractions of Bodies, becauſe if Glaſs, for Inſtance, was the Medium Attractive of the Rays of Light, let *A B* Repreſent a Cube, of which it is the middle Plane, ſo that half the Side of *A C*, Rais'd Perpendicularly from the Surface of the Paper I now Write on, ſhould be one half of the Depth of it, if ſuch a Cube Attracted the Rays of Light Perpendicularly from *B* to *C*, or *E* to *F*, or *G* to *L*, *&c.* when they Emerge out of the Side *D B*, according to the preſent Doctrine of Reflection, the ſame Light, the Conditions of the Body being the ſame, ſhould be Equally Attracted from *C* to *B*, *F* to *E*, *L* to *G*, *&c.* upon it's Emergence out of the Side *A C*; from whence would Reſult this Affection of the Glaſs Cube, either that the Forces of Attraction towards *D B* and Retraction towards *A C*, would be Equally Pois'd, and there would be no Attraction or Retraction of Light at all, which would wholly Deſtroy the preſent Hypotheſis, Since there would be no more Force to Raiſe the Light in Reflection, than what would be Equal to Depreſs it; or elſe that there would be a Parabola Deſcribed upon the Side *A C*, as well as upon that of *B D*, which would be alike Inconvenient to this Hypotheſis; Not to mention, that Light takes no ſuch Courſe in Fact, and Experience, as to be Trajected thro' the Cube, and from *D B* to Form a Parabola, then again to be Trajected and to Form another upon *A C*; Nor yet farther to mention, that all this Doctrine ſeems to be form'd upon that of Projectiles, which by Impulſe and by their Proper Gravitation to the Earth are ſaid to Deſcribe a Parabola, to which the Rays of Light by their Projectile Force and their Attraction to the Glaſs, Appear to be made Analogous; whereas we ſhall Prove in Examining into the Hypotheſis of Refraction, that there is no ſuch Analogy.



Not the Penetration of Light into a Denſer Medium, and the Reflection of it from a Rarer.

§. 16. It is farther as certain, that the third Hypotheſis is not Equal to the Nature of Things; Namely, that there is a Penetration of the Rays of Light in Particular into a Denſer Medium, as Glaſs or Water, and a Proceeding from thence into a Rarer, before they are Reflected; If there was, there could be no Reflection from the Surfaces

faces of theſe Bodies, which it is Plain there is, ſince if we make Plates of Glaſs as Thin, as we can Poſſibly Imagine, and hinder the Rays of Light from being Tranſmitted, by Quick-Silver laid behind it, or by any other Method, there will be a Reflection, where it is Impoſſible there ſhould be a Parabola; And on the other Hand, where the Plates of Glaſs are Tranſparent, ſo that the Parabola may be Form'd, there is a much leſs Senſible Reflection.

§. 17. But what is ſtill more to be Objected againſt the Compliance of this laſt Hypotheſis with Nature, is, that it can no ways Account for the Reflections of Light from common Bodies, as Marble or Steel, in which it is Apparent, that the Light does not Enter into the Bodies, from which it Rebounds; Beſides which nothing can be more certain than that a Ball of Wool, or any other Elaſtick Body, does not Penetrate into the Pavement, from which it is Reflected, whereas if this was a true Hypotheſis for the Solution of Reflection, it ſhould give us a juſt Reaſon of theſe Phænomenons, as well as thoſe of Light; Becauſe there is no more Pretence to Exact the Cauſe of Reflection to what we Experience in Light, than to what we Experience in Groſſer Bodies, nor to make the Reflections of the one or the other to Proceed from a different Force, than Reflections and Refractions in General, which are Endeavour'd, by this Hypotheſis, to be Brought and United under one Reaſon of their Appearance, or of their being Phænomenons in Nature.

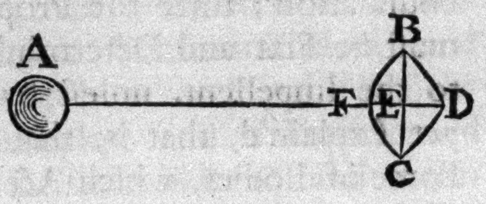
This laſt Hypotheſis not Agreeable to moſt of the Reflections Obſervable in Nature.

Chap. III.

Concerning Dr. Wallis's *Hypotheſis of Reflection from Impulſe and Elaſticity.*

§. 1. THERE is another Hypotheſis Remaining, which Appears to be the moſt Rational of any we have yet met with, for the Solution of Bodies being in General Reflected with the ſame Angle, with which they Fell, which is divided into two Propoſitions; The firſt of them is, that if a Heavy Body Impinges in a ſtrait Line upon one that is Fixt and Unmov'd, and either or both are Elaſtick, the Body, which Impinges, will Recoil with the ſame Celerity and Direction, with which it Impinged.

The Third Hypotheſis of Reflection from Elaſticity the moſt Rational.



§. 2. To Prove this Propoſition, one Caſe is firſt Conſidered, that is, where the Elaſtick Force is ſuppos'd to be in the Body, which Repells; Thus let A be the Body, which is carry'd with the Celerity rC againſt an Oppoſite and Elaſtick Obſtacle BEC, and let the Weight of it be mP, therefore the whole Moment will be it's Celerity Multiply'd by it's Weight, or $mPrC$, with which it Falls upon E; where it meets with the Reſiſtence BEC, which by the Force of the Impellent is Preſs'd Inwards to BDC, till the Reaction in BDC becomes Equal to the whole Compreſſion from the Incurring Body; but the Reſiſtence in BEC is Equal to the Moment in A, or $mPrC$, therefore the Reaction in BDC in it's Counterpoiſe to the Preſſure, which lies againſt it, is Equal to $2mPrC$; farther, ſince the whole Moment of A is loſt after the Compreſſion of BEC to BDC, BDC, as it is Thought, muſt neceſſarily Recoil with the Force of $2mPrC$, one half of which it ſpends in Reſtoring it's ſelf to BEC, and with the other it Reſults upon the Impellent, and Strikes it back to A with the ſame Celerity, with which it was Incident; And if the ſame Conditions are Obſerv'd in A, as are here ſuppos'd to be in BEC, or if they are Preſum'd in both, the Demonſtration will be the ſame.

The Hypotheſis Explain'd. Dr. Wallis's Opera, Math. Vol. 1. Prop. 1. Pag, 1018. De Elatere & Reſilitione.

§. 3. In Anſwer to this Proof we ſhall firſt ſay, that as the former Hypotheſis, which we have Conſidered, ſeems to be Calculated for the Doctrine of Opticks, and does not any ways Agree with the Phænomenons of other Bodies, ſo this on the Contrary is drawn from a Conſideration of Hard and Reſilient Bodies, and cannot be well Apply'd to the Reflections of Light, unleſs upon the Suppoſition, that Light is at leaſt a Solid and Hard Subſtance, if not Elaſtick, which as few will be willing to Allow, ſo it cannot be Prov'd, but upon the Preſumption of the Truth of the Corpuſcular Philoſophy, againſt which ſeveral Arguments have been already Produced.

This Hypotheſis depends upon the Properties of Hard, and Elaſtick, Bodies being applicable to Light or thoſe of Light to them, which is not Prov'd.

§. 4. And

This Hypotheſis Preſumes Matter to be Proportional to it's Weight, which has been before Refuted.

§. 4. And for the ſame Reaſon it is, we cannot Acquieſce in this Demonſtration, where the Weight of the Body mP Enters into it; ſince the Moment of the whole $mPrC$ is Computed from Multiplying the Celerity into the Weight, and Conſequently making Matter Proportional to it; a Poſition, which we have in the firſt Part of theſe preſent Principles Examin'd into, and given thoſe Reaſons, which then Occurr'd, why it could not in General be Affirm'd.

This Hypotheſis is not Evident, becauſe it is not Certain that there is any Change of Figure in the Body, which Compreſſes, or is Compreſſed, much leſs in the Rays of Light.

§. 5. Notwithstanding, Granting the Hypotheſis of the Corpuſcular Philoſophy, and the Equal Proportion of Weight to Matter, we are not ready to Acknowledge, till it is Evidenced, that in all Reflections, which are made, there is a Change of Figure, either in the Body, which Compreſſes, or which is Compreſſed; for if, to make this Demonſtration certain, it is Neceſſary, that BEC ſhould Retire to BDC, it will Exceed an Ordinary Comprehenſion, how the Small and Tender Rays of Light ſhould cauſe any ſuch Impreſſion on the Rigid and Obſtinate Surfaces of Glaſs or Steel; or, on the other Hand, how thoſe Rays ſhould be alter'd in the Figure of their Parts, which will have enough to do, to make us Believe, they have any Determin'd or Figur'd ones.

Such a Change of Figure not Prov'd even in Hard and Elaſtick Bodies, which by Experience Rebound outwards.

§. 6. Besides even in the common Reflections of Bodies, it is not to be Imagin'd, that the change of Figure Inwards any ways Contributes to their Reflection; ſince any Inſtances, which may be made uſe of to prove it, from the Senſible Jarring of Metals or Glaſs, do Evince the Contrary; Namely, that the Elaſtick Force Exerts it's ſelf outwards, and inſtead of being Compreſſed to D, as this Doctrine ſuppoſes, and then Recoiling to it's former Situation in E, Rebounds from E to F, and from thence Retires to E, from whence it Rebounded.

The Force of Preſſure and Reſiſtence will be in an Equal Poiſe, and therefore no Reflection, or if not, no Reflection Equal to the Incidence.

§. 7. And, that this is more Probable, is Evident, if we Conſider, that this Preſſure from E to D is Impoſſible, according to the preſent Account of it; for the Reſiſtence in the Obſtacle BC is ſuppos'd Equal to the Force of Impulſe in A, that is, to $mPrC$; but if $mPrC$ is Reſiſted by $mPrC$, what can we Conclude, unleſs that their Forces will be in an Equal Balance, and that A will be Quieſcent in the Point E, and not Proceed to D as is Preſum'd? If the Reſiſtence in the Obſtacle is Leſs than A, or $mPrC$, the Action of A, or $mPrC$, together with that Action, which Accrues to A from the leſs Reſiſtence of the Obſtacle will not Produce a Force of Elaſticity Equal to $2mPrC$, which on the other hand is likewiſe ſuppos'd, Excepting $mPrC+x$ an unknown Quantity is Equal to $2mPrC$.

The Reſiſtence of the Body Impell'd, not always Equal to the Action of the Impellent.

§. 8. Farther, the Suppoſition of Reſiſtence in the Obſtacle BEC being Equal to the whole Moment of A, the Incurring Body, ſeems to be without any Foundation; ſince the Proper and Innate Force of Reſiſting, which any Body has, muſt be Fixt and Determin'd, and cannot Change or Alter it's Degree of Recourſe to the Impellent, unleſs by Adding a new Property to Matter, which has not been yet Explain'd, that is, that it is more or leſs Active according to the greater or leſs Force of Bodies, which Act upon it; But, if in the Notions of the preſent Philoſophy Matter is Immutable and Homogeneous, and has one certain Power of Reſiſting, ſince the Impulſe upon it may be Infinitely Vary'd, Reſiſtence cannot in all Caſes, and only in one, be Equal and Commenſurate to them.

The Reaction of the Body Impell'd not always Equal to the Action of the Impellent, if yet it was, this Demonſtration would be Falſe.

§. 9. But if we ſhould Allow, what we have hitherto Endeavour'd to Diſprove, that BEC by the Force of the Impelling Body may be Preſs'd Inwards to BDC, that the Reſiſtence is Suitable to the Impulſe, and that the Force at D is Conſequently Equal to $2mPrC$, it will not follow from thence, that the Elaſtick Force is then in an Equal Poiſe with the Compreſſion; unleſs, whatever the Moment is of any Body, half the Elaſtick Force, or $mPrC$, is always Equal to it, which, as before in Reſiſence, ſo in Elaſticity likewiſe, is to make a Certain and Determin'd Degree of Elaſticity vary in Proportion to the Impulſe or Preſſure upon it; which yet, if True, any Body which is Elaſtick, the more it is Preſſed, the more would be it's Renitency, and the leſs it is Preſſed, the Repreſſure would be ſtill Proportionably ſo; from whence it would follow, that either there would be an Equal Poiſe betwixt them at firſt, or not all; if at firſt, there is no Occaſion, that E ſhould Retire to D, till there is an Equal Balance, which this Demonſtration ſuppoſes, ſince there would be that in E; if there is no Equal Balance at all, then BDC in it's Elaſtick Force will not be Equal to $2mPrC$, which is alſo Neceſſary in the preſent way of Demonſtrating.

§. 10. The Truth of the whole is this, that Elaſticity in any Body is a Certain and Definite Force, which may Exceed or be Exceeded by that of the Impellent Body, and which therefore cannot be always Anſwerable or Proportional to the Moment $mPrC$, which is Impreſs'd; for if it could, the Repreſſure of Wool, for Inſtance, or of any other Body, which is Elaſtick, would Increaſe it's Force of Elaſticity, according to the Weight which Oppreſſes it, when yet we find the Contrary in all Fact and Experience.

The Force of Elaſticity Peculiar to each Elaſtick Body Independent of any Foreign Actions from Impulſe.

§. 11. What yet is ſtill more, if the Force of Elaſticity in BDC was Equal to $2mPrC$, Equal or Proportional to the Preſſure, it would be Impoſſible to Reſtore it's ſelf to E from D, excepting upon the Suppoſition, that A had loſt it's whole Moment by Impinging upon the Reſiſtence BEC, which is Equal to it; if it had loſt it's whole Force at E, BEC could not be Preſſed to D, if it had not, till it came to D, the Reſiſtence at E was not Equal to the Moment in A, becauſe the Moment in A would be Superior to it.

According to this Doctrine it would be Impoſſible for the Body Impell'd to be Reſtored to it's firſt Situation, or elſe to be Diſturb'd from it.

§. 12. Moreover, how both Reſiſtence and Elaſticity ſhould be Equal to Preſſure or Impulſe, which this Demonſtration Suppoſes, excepting Reſiſtence and Elaſticity are the ſame Affections of Matter, which the preſent Philoſophy does not Acknowledge, will be Difficult to Apprehend, and much more to Demonſtrate, the firſt being only a Flat and Dead Force, and the other a Reactive.

It likewiſe Suppoſes Reſiſtence and Elaſticity in Effect to be the ſame Property of Matter which is not True.

§. 13. Should we farther Suppoſe, that A had loſt it's whole Moment at D, that the Reſiſtence at E is Equal to that Moment, and that the Elaſticity in BDC is Equal to $2mPrC$, what would Neceſſarily follow from theſe ſeveral Suppoſitions is only this, that $mPrC$, or half of $2mPrC$, would be Communicated to A, which had loſt it's Moment at D, and that the other Part would be Communicated to the Reſiſtence of BEC; which likewiſe would either be loſt or not; if the Reſiſtence is ſuppos'd to be loſt at D, as that of the Impellent Body is, it ought to be Reflected with the ſame Degree of Force as A, and by Conſequence the Obſtacle BEC will fly off from D with the ſame Celerity, and will reach the Point A at the ſame time with the Impellent; if the Force of Reſiſtence is not loſt, then having the Reſiſtence or Force of $mPrC$ towards A, and being Aſſiſted or United with an Equal one from the Elaſticity of BDC, it will have a double Force, and either Deſcribe in it's Return double the Line of DA, or at leaſt, if there is any other Imaginary Obſtacle hinders at B and C, Proceed to F, which is Contrary to what is at preſent Affirm'd.

Granting all it's Suppoſitions, Reflection would be Different from what this Philoſophy Aſſerts.

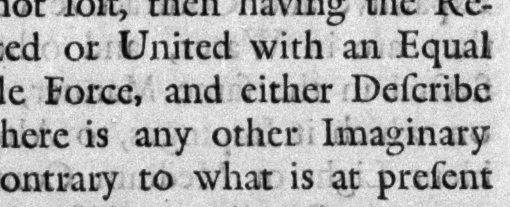
§. 14. Lastly, as the Force of the Impellent A Conſiſts of it's Pondus and Celerity, tho' $mPrC$ ſhould be Communicated from the Elaſtick Body to it at it's Return, either the whole Moment of A is loſt before ſuch a Communication, and Conſequently it's Weight, or not; If it's Weight, or mP, is as well as it's Celerity, it is contrary to what is both Suppos'd in this Philoſophy, and is Experienced in Nature; If it is not, the Force upon Reflection will be $mP \times mPrC$, and therefore the Celerity will be leſs in Reflection, than it was in the Incidence, which this Propoſition endeavours to Prove Equal.

The Weight not being loſt, tho' the Celerity is, will make the Impelling Body leſs Swift in it's Reflection than in it's Incidence.

§. 15. To Conclude upon this Theory of Reflection, we do not deny, but Bodies may Rebound, or be Reflected, in the ſame Angle, with which they are Incident; we yet deny, that in Philoſophy, and in a Real Science of Nature, it is any more a Property of Bodies, or of Light, than that the Angle of Incidence is 10 times Bigger or Leſs than that of Reflection, and that the Abſtracted Reaſonings to Prove the former are Valid, or can any ways Aſſiſt us in Accounting for the Phænomenons or Appearances of External Being.

It is not deny'd that the Angle of Incidence is Equal to that of Reflection, but it is deny'd that ſuch an Affection is proper to Light or to Bodies in General.

§. 16. As to what Concerns the Second Propoſition in this Hypotheſis of an Oblique Incidence (a Direct being before only Conſidered) Producing an Equal Angle of Reflection, ſince the Incident and Reflected Lines AB, and BC, are Reſolv'd into the two Components, AD and DB, on the one Hand, and BE and EC on the other, and the Conditions of the Direct Lines AD and EC are the ſame with thoſe we have already Examin'd, there is not any Thing more, which is Requiſite to be Conſidered in the preſent Argument, unleſs the Com-



The ſecond Propoſition in this Hypotheſis not Neceſſary to be Enquired into.

Dr. Wallis's Opera Math. Vol. 1. Prop. 2. De Elatere & Reſilitione.

Compoſition of Forces, which has been done under another Head of Philoſophy.

We ſhall only therefore farther Obſerve, that if there are any Demonſtrable Proofs, either in Mechanicks or Opticks, for the Equality of the Angle of Incidence and Reflection, we are not at preſent Concern'd to Anſwer them, ſince we now Inquire into the Principles of thoſe Sciences, which if they are Falſe or Precarious, the Deductions from them are ſo; and if any Plain Phænomenons are Urg'd againſt us, We Alledge they muſt be Accounted for another way, if the Fundamental Propoſitions, upon which they are Affirm'd, are not True, that is, by our Philoſophy of the Expanſive and Contractive Forces.

What the Real Nature of Reflection is, and why Tranſparent and Poliſhed Bodies Reflect Images to us, whereas Opake and Coloured ones do not.

§. 17. The Philoſophy of the Expanſive and Contractive Forces will therefore Inform us, that all Bodies Conſiſt, and are made up, as was ſaid before, of Various Aſſemblages and Congeries of Actions, which Actions are Ultimately Reſolv'd into the Forces now Mention'd; If, in Reſpect of Colours, a Black, Blue, or Purple, or any other of a Dark and Melancholy Aſpect are Conveigh'd to the Eye, it is a Sign, that the Light or Æther is ſo far from being Reflected, that it is Contracted in Various Degrees from the Eye towards the Body; If a Red, Orange, or Yellow, or any other of a more Lively and Vivid Complexion, it is a Proof, that the Light or Æther is Reflected with Various Degrees of Force from the Different Degrees of the Expanſive of the Bodies, which Reflect it, and that this is done in all Angles of Reflection; Or rather the Light or Æther is not Incident in ſtrait Lines or Rays upon the Bodies, from whence it is ſuppos'd to be Reflected, but Surrounds and Encompaſſes them, as Water, or any other Fluid, the Bodies, which are Immerſed in it; And as Fire Communicates it's Impreſſions to the Æther about it, without any one's ever Imagining or ſuppoſing ſuch an Incidence or Reflection, ſo the ſeveral Bodies, which Exiſt in the Æther, and are ſo many Various Kinds of Fire, Flame, or the Expanſive Force, differently Mixed and Loaded with the Contractive, Impart their Sundry Influences to the ſame Ambient Medium; On the other Hand, in Poliſhed, Tranſparent, and Uncolour'd Bodies, the Expanſive and Contractive Forces are nearly in an Æquilibrium, as we have ſaid they are in Water, and other Fluids; And therefore the Light or Æther is Reflected in the ſame Manner, as it Fell, with little or no Impreſſion from the Bodies, on which it Impinges, to Alter it's Quality or Force; or, which is the ſame, the Ambient Light or Medium Continues and Preſerves it's own Force without Receiving any very Peculiar or Diſtinguiſhing Impreſſions from the Bodies it Surrounds; Which ſeems to be the True Meaning of the Angle of Reflection being Equal to the Angle of Incidence; That is, the Æther or the Light is Rebounded the ſame as it Fell; Which gives alſo an Account, why Images are ſeen from ſuch Reflections, but not from others; Becauſe in the firſt Caſe the Incident Light or Æther is Unalter'd in the Reflected, but in the laſt the Reflected is Changed by the Forces of the Bodies, on which it Falls, into a different from the Incident.

Chap. IV.

Concerning the ſeveral Hypotheſes of Refraction.

The ſeveral Hypotheſis of Refraction Conſidered.

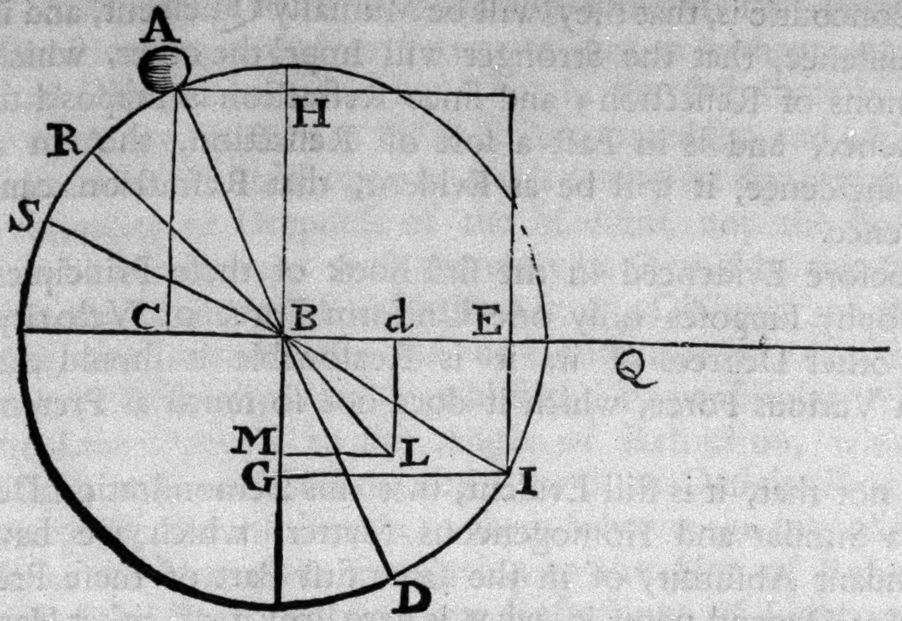
§. 1. After Conſidering the Doctrine of Reflection, as it has been hitherto Propos'd to us, we are next to Inquire into that of Refraction, which we ſhall do in the ſame Order of the Hypotheſes, which we before Obſerv'd in the ſeveral ones of Reflection.

The Laws of Refraction Explain'd according to the Hypotheſis of Impulſe and Reſiſtence. Deſcartes, Dioptric. Cap. 2. Rohanlt. Cap. 15. Par. 1. and Dr. Clark's Annotat.

§. 2. The firſt Demonſtration made Uſe of, to Prove the Laws of Refraction, and that the Sine of the Angle of Refraction in Particular is in a given Proportion to the Sine of the Angle of Incidence, is, that Suppoſing CBE to be a Superficies, which Reſiſts the Velocity of a Body in A Deſcending to B, ſo as that A ſhall looſe half it's Velocity in Surmounting the Reſiſtence at B, A will be Refracted, and, Inſtead of Proceeding in a Line continued from A thro' B to D, will be Diverted to I; Becauſe AB is Compounded of the Forces AC and CB, and ſince A has loſt half it's Velocity at B, it will be double the Time in Deſcribing the Radius of the ſame Circle; But the Perpendicular Motion AC is only Obſtructed, and therefore the

Lateral *CB* will in it's Velocity Deſcribe double the Line *CB*, that is, *BE*, whilſt *A* is Falling from *B* to the Circumference, and Conſequently, the Determination of *A* will be to I, ſo as *GI* may be double of *CB*, and *A* at the ſame time may Arrive at ſome Point in the Ambitus of the Circle, which by a plain Neceſſity, upon theſe Suppoſitions, muſt be in *I*.

Hugens's Dioptric. de Refractione Radiorum, Pag. 1.



§. 3. In Reply to which we are Oblig'd to take Notice; Firſt, that if with the ſame or any other Incidence, which we can Poſſibly Imagine of *A* to *B*, the Medium at *B* ſhould Reſiſt 3 or 4 times more than is here Suppos'd, this Demonſtration will not be Valid; ſince *BE* in Reſpect of *CB* muſt be in that Proportion, either 3 or 4 times longer than *CB*; in which Caſe *BE* will be Produced beyond the Circumference towards *Q*, and therefore there can be no Line drawn from *B*, which will Determine it's ſelf in the Circumference, which is Neceſſary in this Doctrine, and yet Take in and Comprehend another Line, which goes beyond it, *viz.* towards *Q*; And thus, if Quick-Silver, which is to Water, as about 12 or 13 to 1, is Capable of Refracting, and not of Reflecting only, an Incident Body, as will be Acknowledg'd, it muſt be done upon other Principles, than what are here Affirm'd; ſince if Water Diminiſhes the Velocity of the Incident Body in any Degree, Quick-Silver will do it, if Reſiſtence is Equal to Denſity, 12 or 13 times as much without any Reflection; or if we take common Mud or Earth, their Reſiſtences are much greater than that of Water, and if this Theory can Account for the latter, which it will ſcarcely do, it cannot Conſequently for the former, ſuppoſing their Reſiſtences only to be to that of Water, as 2 or 3 to 1, which yet it is Probable are in a greater Proportion.

There is no Reaſon, why the Reſiſting Body ſhould Diminiſh the Velocity in one certain Proportion, more than in others; which might be Aſſign'd.

§. 4. And this ſeems to be an Evident Proof, that, as before in Reflection, the Compounded and Diagonal Force ought to have been Demonſtrated from the Compounding, Namely, the Lateral and Perpendicular, ſo here likewiſe, that *A* will not be Refracted from *B* to *I* or any other Part of the Circumference, unleſs by ſuppoſing it, that is, by Circumſcribing and Aſcertaining the Diagonal Force, Antecedently to thoſe, into which it is Reſolv'd; Thus, for Example, in the preſent Caſe, *A* in it's Lateral Direction may Proceed from *B* to *Q*, as well as to *E*, but it is Originally ſuppos'd, that it ſhall not go beyond the Circumference, becauſe the Compound Refracted Force *BI* is not to Proceed any further than the Incident *AB*, which is Taking for Granted, what is Really to be Proved.

The Refracted Line is ſuppos'd Equal to the Incident, which is Preſuming the very Thing, which ſhould be Prov'd.

§. 5. However, leſt we ſhould be thought to Affirm only, without giving a Reaſon for the Aſſertion, tho' what we have ſaid is in it's ſelf Manifeſt, let us Conſider the Original Suppoſition, which is made to Prove this Doctrine of Refraction; Namely, that upon the Diminution of the Velocity of *A* at *B*, *A* ſhall Deſcribe after it's Reſiſtence at *B* the ſame Line, which it did, before there was any Reſiſtence to it's Motion; that is, tho' the Velocity of *A* is but half, in it's Deſcent from *B* to *I*, of what it was in it's Deſcent from *A* to *B*, the Line of it's Motion from *B* ſhall yet be Equal to the Line of Incidence to *B*; But to Compenſate for ſo Arbitrary an Hypotheſis, it is Contriv'd, that *A* ſhall be double the time in Deſcribing it, albeit there is no more Reaſon, why upon any Impediment given to Velocity, it's Decreaſe ſhould not be Meaſur'd by the different Lines it Deſcribes in the ſame Time, than by the ſame Lines in different Times, if nothing elſe is Conſider'd, but bare Reſiſtence; If therefore we take the former Meaſure, *A* after the Reſiſtence at *B*, will only Proceed to half the Line it had before paſs'd thro', and Conſequently, this whole Demonſtration Falls; Or if we take that other Meaſure, which is here ſuppos'd, ſo as the Velocity of *A* ſhould be Diminiſhed in it's Degree, and *A* only Proceed in a ſlower Motion,

If upon Refraction the Body is Diminiſht in it's Velocity, no Reaſon given, either why it ſhould not Deſcribe a leſs Line, or why in Reflection it ſhould Deſcribe an Equal.

it will be altogether Unaccountable in this Philoſophy, how A, when it is wholly Reflected from B, and meets with an entire Reſiſtence, ſhould Preſerve it's Velocity wholly Unimpair'd and Undiminiſhed.

Refraction not Capable of being Solv'd from Reſiſtence, ſince Reflection is not.

§. 6. FARTHER, as this Opinion of Refraction Depends in a great Meaſure upon Reſiſtence, which yet is not thought by a more Rational Account of Things at Preſent to Solve Reflection, but that a certain Elaſticity or Attraction is Requiſite, it is equally Impoſſible it ſhould Solve Refraction; For if two equally Reſiſting Bodies meet, the Confeſſed Reſult of ſuch a Concourſe is, that they will be Mutually Quieſcent, and if they are not Equal in their Reſiſtence, that the Stronger will Impel the other, which can never Solve the Phænomenons of Reflection; and ſince Refraction is ſuppos'd to Proceed from a kind of Reſiſtence, and is in Fact a ſort of Reflection, tho' in a different Angle from that of Incidence, it will be as Evident, that Refraction cannot be Solv'd from bare Reſiſtence.

If there is a Various Force or Velocity, ſuch a Difference of Forces ſhould be Explained, which is not done.

§. 7. BESIDES, as we have before Evidenced in the firſt Book of theſe Principles, *Chap.* 10. §. 8. 9. this Philoſophy ſuppoſes only one Uniform Force or Velocity, and therefore, if it Introduces other Degrees of it, it is Reaſonable it ſhould Account for the Cauſe of ſuch a Various Force, which it does not ſo much as Pretend to.

This Hypotheſis of Refraction depends upon Matter's being Similar and Homogeneous, which has been already Refuted.

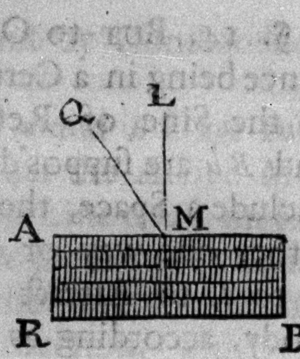
§. 8. MOREOVER, if it did not that, it is ſtill Evident, that this Demonſtration Depends upon a Suppoſition of a Similar and Homogeneous Matter, which we have Endeavour'd to ſhew the Abundant Abſurdity of in the ſame firſt Part of theſe Preſent Principles; For if it did not Depend upon it, what is here Prov'd of either Hard or Reſiſting Bodies in General, would Signify nothing to the Properties of thoſe Beings, which may Poſſibly have no ſuch Qualities; for it is not certain, that Fire, or Oil of Vitriol, have not Properties Contrary to thoſe of Plain Reſiſtence, by being Intenſely Active.

The Principles made Uſe of in this Philoſophy, if Truely Argu'd upon, would be ſo far from Proving the Sine of the Angle of Refraction to be in a certain Reaſon to the Sine of the Angle of Incidence that they could not Evidence, that there was any certain Sine of Refraction at all, but that the Angles of Refraction were Infinitely Variable.

§. 9. IF all this was yet Acknowledg'd, and the Degree of the Velocity was Chang'd upon the Reſiſtence at B, it would not follow, that the Refraction ſhould be made to I; ſince CB, one of the Reſolute Forces of AB the Reſolvend, is Unalter'd after the Reſiſtence at B, as before in Reflection; which if granted, as it was Prov'd, that the Angle of Incidence could not be Equal in all Caſes to the Angle of Reflection, ſo here we ſhall Demonſtrate, that the Angle of Incidence cannot be in a Certain and Definite Proportion to the Angle of Refraction upon theſe Principles; For of the Compound Force of AB, CB Continuing the ſame, and HB meeting with a Reſiſtence in B, the whole Alteration of the Velocity of A muſt be Conferr'd on the Perpendicular Direction, and Conſequently in the ſame time, that A after Reſiſtence comes to d, ſo as Bd ſhould be Equal to CB, it will Proceed from B to M, and Deſcribe a Line BM Equal to half BH, by which Means the Perpendicular Velocity will be as Slow again, as it was before the Impinging of A upon B; from whence the Refraction will be Determin'd to L, and not ſo much as Form any Sine of the Circle AD; If now the Velocity of A is Increaſed or Diminiſhed, the Reſiſtence at B will be Reciprocally Diminiſhed or Increaſed, and BM accordingly will be longer or ſhorter, whilſt Bd, Equal to BC, is ſtill the ſame; ſo that ML will be always in a Proportion of Equality to the Sine of Incidence AH, but the Angle of Refraction MBL will be Greater or Leſs, as ML Approaches nearer to Bd, or Retires farther from it, that is, as the Velocity of Impulſe is leſs, or more Intenſe, or, which is the ſame, as the Reſiſtence at B is leſs, or more.

If the Incidence is Chang'd, the Line Oppos'd to the Angle of Refraction may be Alter'd, which ſtill will not Agree with the Propoſitions of this Philoſophy.

§. 10. LASTLY, ſuppoſing the Incidence Chang'd, and A to Fall from R or S to B, the Reſiſtence at B will either be the ſame or not; If it is not the ſame, it muſt either Proceed from it's Encountering a greater Thickneſs of the Medium in an Oblique Deſcent, than in a Direct, or from a different Poſition of the Parts of it to the Incident Body; As to the firſt, it is Manifeſt, that if the Medium is two or twenty Inches Deep, the Reſiſtence is Preſum'd to be the ſame in theſe Principles; Beſides BI, the Continuation of the Incident Line SB, is Equal to BD, the Continuation of the Incident Line AB thro' the Refracting Medium; And, as to the Second, there is no Reaſon to Affirm, that the Situation of the Parts Obſtructs the Progreſs of A in one Caſe more than another, in the former Figure; ſince, altho' in the preſent, if the Body was to Fall from *L* to *M*, in the Poſition of the Parts *A B* it would find leſs Reſiſtence, than if it was to Fall Obliquely from *Q* to *M*, yet this is not the Caſe of any Medium, which is Homogeneous, where the Parts are in a like Poſition every Way, ſince if *R M* ſhould be drawn ſo, as *Q* might Fall in a Perpendicular to it, the Deſcending Body wou'd find as eaſy an Ingreſs, as in it's Incidence from *L* to *M*; 'tis True it is generally thought, that Perpendicular Directions to a Medium Penetrate into it with greater Facility, than Oblique ones do; but That in the firſt Place is not certain, and in the next if it were, is a Problem as yet to be Accounted for, and which at preſent we are not to Conſider, what we Inſiſt upon is, that it cannot be done by theſe Principles, ſeeing neither the Profundity or Deepneſs of the Medium, nor the Poſition of it's Parts are ſufficient for the Solution; it muſt therefore be Own'd by this Philoſophy, if it Reaſons Juſtly, that the Reſiſtence of a Medium in any Obliquity of the Incident Body is the ſame; If it is, the Lateral Force *B d* and Conſequently, *M L*, in the Figure, foregoing the laſt, may be Infinitely Vary'd, whilſt the Perpendicular *B M* Continues Unchang'd, and the Line Oppos'd to the Angle of Refraction, which will not be the Sine of a Circle, will not Obſerve any Certain or Definite Proportion to the Sine of Incidence, excepting that of Equality; from whence we may Affirm, that if the Sine of Incidence is in a given Proportion to the Sine of Refraction, by what, as is ſaid has been fully Experienced, in the Refractions of Light, we may yet from hence Affirm, that the preſent Doctrine does not Evince it.



§. 11. As to what concerns Experience, it is not any ways Reaſonable, allowing the Experiments, to make a general Law of Refraction, from the Refraction of Light, which may be as different in it's Action upon Water, or Glaſs, from that of other Material Beings, as the Heat of the Fire is from the Effects of a Froſt upon the ſame Bodies; Beſides that by Experiments it is Prov'd, that Light Refracts from the ſame Incidence into different Sines, which, whatever Evaſions are made Uſe of, can never be Reconcil'd with the Hypotheſis, as it is here Affirm'd, of the Sines of Incidence being in a given Proportion to thoſe of Refraction; ſince it is Impoſſible, where Light Falls with the ſame Velocity upon a Medium, and is Refracted in various Angles, that the different Sines of thoſe Angles ſhould be in the ſame Proportion to one Certain and Determin'd Sine of Incidence, and we have before Prov'd, that Light is in it's ſelf Homogeneous, and does not Originally Conſiſt of Diſſimilar Rays, and therefore is Incident with the ſame Velocity upon the Refracting Medium, or if we had not, according to §. 3. and 9, different Reſiſtences in the Refracting, or different Velocities in the Refracted Body, would Deſtroy the Abovemention'd Proportion.

A general Law of Refraction not to be made from the Refractions of Light, or if it were, is not ſo much as Agreeable to them.
Sir Iſ. New. Opticks: Prop. 6. Part. 1.
Principles of Philoſophy. Chap. 3. Part. 2.

§. 12. The next Hypotheſis we ſhall Examine into, after this of Reſiſtence, ſhall be another more Modern of Attraction, which is brought to Prove, that the Sine of Incidence is in a given Proportion to that of Refraction, or Emergence.

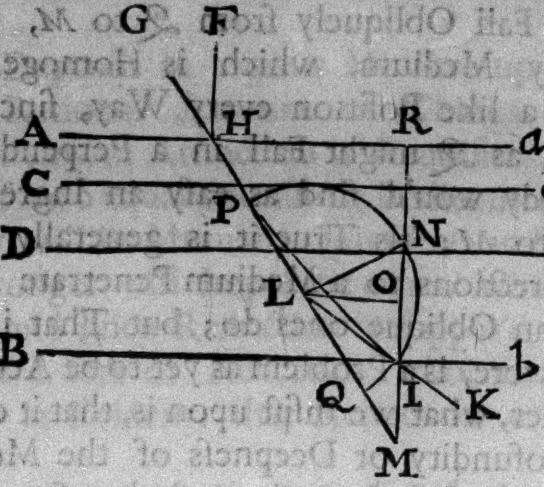
The next Hypotheſis is that of Impulſe and Attraction.

§. 13. The Title of the Section, in which this Hypotheſis is Propos'd, is, Concerning the Motion of the leaſt Bodies, which are Agitated by Centripetal Forces, that Tend to each of the Parts of any large Body; From whence it is Neceſſary for us to Obſerve, that this Reaſoning ſtill depends upon the Truth of the Corpuſcular Philoſophy and an Homogeneous Matter, and Reducing the Actions of the Subtileſt and moſt Active to the ſame Laws, which are found in the Groſſeſt or the Heavieſt; for otherwiſe, tho' the Bodies, which are Conſider'd in Mechanicks, may have a Force of Attraction amongſt one another, unleſs all Matter is Homogeneous, it will not be Neceſſary, that Light, or Subſtances, which are the moſt Refin'd, ſhould have, in their Compariſons with others, which are of a Thicker and more Unwieldy Nature; and Conſequently, what was before Alledg'd Concerning Reflection, upon this Suppoſition, will be Liable to the ſame Objection; That is, this Reaſoning ſuppoſes, that the Expanſive Force of Light is of the ſame Nature and Temper with the Contractive of other Bodies, or that the Expanſive and Contractive Forces are the ſame.

An Original Objection againſt this Hypotheſis, from it's making the Properties of Light, the moſt Active Body, of the ſame Kind with thoſe of the Groſſeſt and Heavieſt.
Sir Iſ. Newt. Princip. §. 14. Lib. 1.

An Explanation of the Preſent Hypotheſis. Sir Iſ. Newt. Princip. §. 14. Prop. 94. Lib. 1.

§. 14. But to Omit this; The Demonſtration of the Sine of the Angle of Incidence being in a Certain and Definite Proportion to the Sine of Refraction is as Follows; Aa and Bb are ſuppos'd to be Parallel Planes, which Include a Space, the Property of which ſhall be, either one way or other, from I to R, or from R to I, to Attract Perpendicularly an Incident Body, according as it Falls upon the Planes, Aa or Bb, thro' Mediums which are Similar, and are here ſuppos'd to be Denſer; Let it therefore, for Inſtance, Fall on the Plane Aa, and paſſing thro' the Intermediate Space, let it be Attracted in the Direction from I to R; on the Contrary, if the Incidence had been upon Bb, the Perpendicular Attraction would have been from R to I; and in either of theſe Caſes, the Attraction is ſuppos'd to be Equal in the ſame Diſtances from Aa or Bb, that is, in any Point of Cc or Dd Reſpectively, the Attractions are alike. Let now the preſent Figure be ſuppos'd to be Drawn, in which GH is the Incident Force, HI the Refracted, and IK the Emergent, IL is the Continuation of IK, the Emergent Force, till it meets with HM, which is the Production of the Incident GH, and RM is Perpendicular thro' the Point I to the Plane Bb, ſo as to Reach the Oppoſite Plane Aa, and Interſect the Continuation of GH in M; PNQ farther is a Circle Deſcrib'd from L with the Radius $LI = LN$, and LO is a Perpendicular from the Center to the Chord NI, which is therefore Biſected in O.



The Demonſtration, upon which this Doctrine is Founded, Explain'd. Sir Iſ. Newt. Princip. Ibid. Caſ. 1.

§. 15. Having given an Explanation of the Suppoſitions, and the Figure, Neceſſary to the Enſuing Demonſtration, we ſhall now Proceed to the Reaſoning upon both. It is therefore in the firſt Place Affirm'd, that if the Attraction, or Impulſe, is Uniform, the Refracted Force HI will Deſcribe ſome Portion of a Parabola, the Property of which will be, that the Rectangle, Comprehended under the Latus Rectum given, and IM, is $=$ to the Square of the Tangent HM, and HM will be Biſected in L; from whence it will follow, that RO and OM are Equal, ſince LO is Parallel to HR, and Conſequently MO and OR, ML and LH, Segments or Sides of Similar Triangles, therefore $IO = ON$, added to each, will make $MN = IR$; But IR is given, being the Interval betwixt the two Planes Aa and Bb, and for the ſame Reaſon MN will be given, which is $=$ to IR; NM therefore, which is given, into IM will be to the Latus Rectum, which is given, into the ſame IM in a given Proportion; But IM into the Latus Rectum is $= HMq$, and Conſequently NM into IM will be in a Given, or a Known, and Determin'd Proportion to HMq, or the Square of the Tangent of the Parabola; Farther, the Rectangle NM into IM is $=$ to the Rectangle PM into QM by a Propoſition in the Elements of Euclid, which Affirms, that from the ſame Point M, all the Secants of a Circle Drawn into the Line Intercepted betwixt M and the Circumference of the Circle, make Equal Rectangles; But PM into QM is $=$ to the Difference betwixt the Square of ML, and the Square of PL, or LQ, the firſt being $LQq + 2LQM + QMq$ and the laſt LQq, and the difference, $2LQM + QMq = \overline{PL + LQ + QM} \times QM$, $= PM \times QM = NM \times IM$, which has been Prov'd to have a given Proportion to HMq, and therefore betwixt $MLq - LIq$, and HMq, there will likewiſe be a Determin'd or given Proportion; Again HMq has a given Proportion to the fourth Part of it's Square $= LMq$, and therefore the Difference of MLq and LIq, or $MLq - LIq$, will have a given Proportion to MLq, the fourth Part of HMq. Laſtly, if $MLq - LIq$ has a given Proportion to MLq, $MLq - LIq - MLq$ will have a given Proportion to MLq, or LIq to LMq; And if we Extract the Roots of the Squares, which are in a certain Proportion, LI will have a given Proportion to ML; LI therefore being Prov'd to have a Known and Definite Reaſon to ML, ſince the firſt is Oppos'd to the Angle $LMI =$ to the Angle GHF, the Angle of Incidence, and the laſt to the Angle $LIM =$ to the Angle RIK the Complement of the Angle of Emergence $LIR = MIK$, the Sines of the Angles being Proportional to the Sides, which are Oppos'd to thoſe Angles, the Sine of the Angle of Incidence will be in a Given

Euclid. Prop. 36. Lib. 3. Tacquet. Corol. 1.

a Given Proportion to the Sine of the Angle of Emergence, and by Conſequence to the Sine of Refraction.

§. 16. THE firſt Suppoſition in this Demonſtration is, that *A a* and *B b* are two Active Planes, which Include a Medium entirely Unactive, and that the Active Medium *A a* or *B b* will Reſpectively Attract towards each, the Incident Line *GHI* on the one Hand, or *KIH* on the other, in their Progreſs thro' the Unactive Medium, in Equal Diſtances from *A a* or *B b*; If therefore we take *GHI* and *A a*, the one for the Incident Line, and the other for the Boundary betwixt the Active Medium, which is above *A a*, and the Unactive, which is below it, *Cc* and *Dd* will have their Attractions Peculiar to each, and Equal in the ſeveral Points of them in their Tendency to *A a*.

The firſt Suppoſition, that an Unactive Medium ſhould be Included betwixt two Attractive ones.

§. 17. IN which Hypotheſis, either the Plane *A a*, which is the Diſtinction betwixt the Mediums, is ſufficient to this Demonſtration, or not; If it is not, the Medium *B b*, Similar to that of *A a*, is alſo Requir'd, that is, an Unactive Medium betwixt two that are Active, which if Neceſſary, it will not Solve one Phænomenon of Refraction; ſince that is always made by Mediums, which lie Contiguous to each other, and have each of them a different Force or Action, and it was never Experienced, that two Unactive Mediums, and one Active, or two Active Mediums and one Unactive, were Requir'd in Order to produce Refraction; For Light in Particular is Refracted without any other Requiſite, than the difference of two Mediums only, a Rarer and a Denſer; Moreover, if two Similar and Active Mediums were Neceſſary to Refraction above the Plane *A a*, and below the Plane *B b*, the laſt Medium would Attract as much as the former, in equal Diſtances, and would Conſequently Diſturb the Perpendicular Attraction to *A a*, ſuppoſing the Incident Line to be *GH*, and therefore the Impulſe and Attraction together will not Deſcribe the Line *H I*, which is ſuppoſ'd.

Argument againſt ſuch a Suppoſition, that no ſuch Condition is found in Nature, or if it were, that it would not Effect what is Propos'd from it.

§. 18. FARTHER, if the Plane *A a* Attracted Light, for Example, in it's Paſſage from *H* to *I*, *B b* in Equal Diſtances would give a contrary Impreſſion to it, and Conſequently, when Light by Impulſe came to *Cc*, *Dd*, the Mutual Attractions of *A a* and *B b*, which draw contrary Ways, would Cauſe that *A a*, in the whole, would Attract towards it's Plane, as much as *B b* would do towards it's Plane, from whence the Attractions of both the Planes would be Equal, but Reciprocal, in their Force; Which would produce Effects very different and unlike from what are here Imagin'd.

If the Planes A a *and* B b *Attracted the Attractions would be Equal, but Reciprocal.*

§. 19. IF the Plane *A a* is ſufficient for the preſent Demonſtration without a Similar Medium, and an Active, at *B b*, the Angle of Emergence *MIK* will Vaniſh, ſince there will be no Emergence at all, or which is the ſame, *HI* will be Continued on in one Similar and Unactive Medium, without being Limited or Bounded in it's Progreſs, unleſs by the Reſtraint, which is put upon it from the Attractive Medium *A a*; from whence it will follow, that the Angle *KIM*, as it is not an Angle of Emergence, ſo will not be Fixed in any Particular Place, but may be Drawn from any Point betwixt *I* and *H*, as well as from *I*, there being no Circumſtance or Condition to Determine it to one Point more than to Another; But ſuch Angles, made by Lines Parallel to *IM*, and Interſecting the Curve *HI* in the ſeveral Points of it, would be Extremely different from the Angle *M I K*, and therefore it cannot be the ſame, whether we take the one or the other, and yet there is no Reaſon, why one may not Repreſent an Angle of Emergence, as well as another.

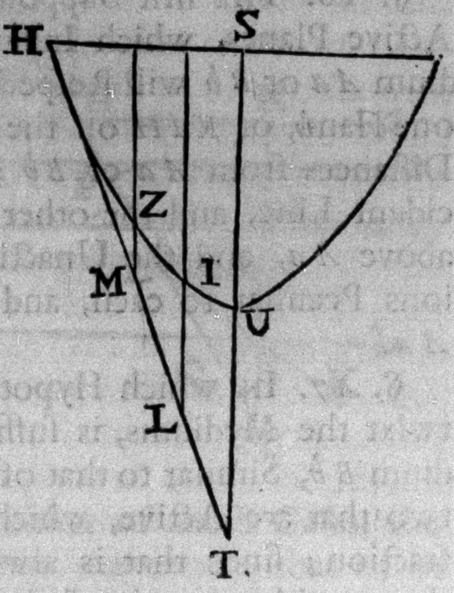
If only an Attractive Medium, and an Unactive, are Requiſite to Refraction, there would be no Angle of Emergence.

§. 20. AGAIN, for the ſame Reaſon that *HI* is Attracted by the Active Medium *A a* after it's Ingreſs at *H*, it ought to be Attracted before it's Entrance into it, and *GH* will Fall in a Curve, as well as be Refracted in one; But this is far from being Prov'd by Experience, and if it were True, would Deſtroy the Fundamental Suppoſition of the preſent Demonſtration, which is, that *GH* is a Projectile Force, and not Diſturb'd by any Foreign Impulſe; Nor would *LMI* be = to the Angle *GHF*, nor *LIM* to *RIK* nor *LIK* to *MIK*. But what is not to be Surmounted in this Hypotheſis is, that the Medium above *A a*, and below *B b*, is Preſumed to be Denſer than the Intermediate one betwixt them, that is, that Air is Denſer than Glaſs or Water, and therefore Light is Refracted to the Perpendicular thro' Water or Glaſs, and from the Perpendicular in Air; Whereas, as we have ſhewn §. 32, the Refraction to the Perpendicular Proceeds, as in Reaſon it ſhould, from a Greater Denſity or Contractive Force in the Glaſs or Water, than in Air, and the Refraction from the Perpendicular Proceeds from a Greater Expanſive Force in Air, than in the Other.

If there was only one Medium A a *which Attracted, the Incident Line would be a Curve, as well as the Refracted.*

A Second Suppoſition in this Way of Reaſoning Explain'd.

§. 21. ANOTHER Suppoſition in this Way or Method of Reaſoning is, that the Projectile Force from H to T is, as the Lines HM, HL, HT, that is, as the Times of Progreſſion to M, L and T, and the Gravitating or Attracting, MZ, LI, TU, as the Squares of the Times, in which HM, HL, and HT, are Deſcrib'd; upon which Conceſſion the Compounded Forces will Form the Parabola, $HZIU$, and the Tangent Drawn from one Point I, (*Fig.* §. 14.) ſo as to meet the Tangent HM, Drawn from another Point, as H, will Biſect it.



By the foregoing Suppoſition, the Laws of Refraction are made Analogous with thoſe of Projectiles.

§. 22. FROM whence it is Evident, that the Laws of Refraction are made Analogous, and Alike, with thoſe of Projectiles, Conſidered as they Gravitate to the Earth, and the Rays of Light are Imagin'd to Deſcend and to be Attracted towards the Active Medium HS, after the ſame Manner, as a Stone, thrown out of the Hand, is Drawn and Impell'd to the Surface of the Earth, from which it is Projected.

If this Analogy was Juſt, Refraction and Gravitation ought to be Reciprocal Terms.

§. 23. BUT, if ſuch an Analogy is allow'd of, the Terms of Gravitation and Refraction ought to be Reciprocal, and as we may Affirm a Ray to Gravitate to the Medium, when it is Refracted to it, ſo on the other Hand, we may Affirm a Stone to be Refracted by the Earth, when it Gravitates to it, which as it is very Diſagreeable to our common Senſe of Things, ſo at leaſt ſhews the Extravagant Reaſoning of this Philoſophy.

Inconceivable, that the Quick Beams of Light ſhould have a Property of Gravitation common with Lead or any other Projectile.

§. 24. BESIDES, that MZ, LI, TU, ſhould Gravitate to HS in the Rays of Light, according to the Proportion of what a Heavy Body does to the Earth, is very Inconceivable, unleſs we can Imagine, that the Active and Vigorous Rays of the Sun have the ſame Propenſity to Gravitation, as a Stone, or the Dead Weight of Lead, or any other Projectile.

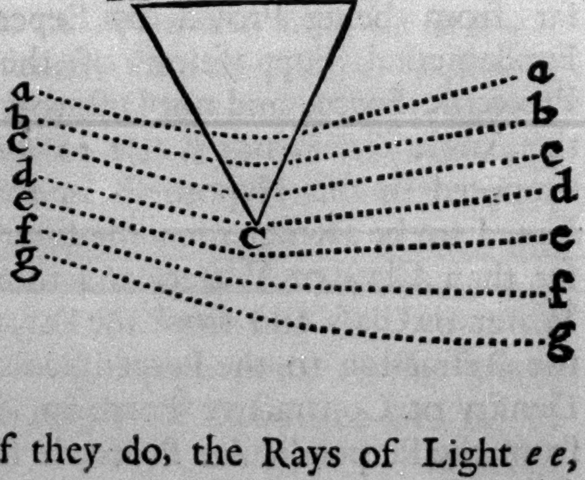
Light does not Gravitate to the Refracting Medium, as a Stone does to the Earth, by the Confeſſions of this Philoſophy.

§. 25. WHICH is ſo far from being True of Light, in Reſpect of Glaſs, or any other Body, as Poliſhed Steel, that this Philoſophy gives us an Inſtance of the Contrary, in the Examples, which it Produces to Prove ſuch a Gravitation or Attraction; for to make good the foregoing Reaſoning, or to give ſome kind of Approbation to it, There were two Things to be Conſidered; 1*ſt*. Whether in Reality the Sine of the Angle of Incidence and Refraction were in a certain Proportion to each other; 2*d*. Whether Light did Gravitate to a Refracting Medium, as other Bodies did to the Earth; If neither of theſe were True, this Philoſophy very Juſtly Apprehended, that it had only Propos'd an Abſtracted or a Chimerical kind of Reaſoning.

An Inſtance, that it does not from the Incurvation of it's Rays, when they Approach an Attractive Body.

Sir Iſ. Newt. Princip: Prop. 96. Lib. 1. Schol.

§. 26. IN the Scholium therefore to the 96*th Prop.* both theſe are Endeavour'd to be Prov'd, 1*ſt*. by ſaying, that *Snellius* found out, that the Secants of the Angle of Incidence and Refraction were in a given Proportion, and that *Des-Cartes* by Conſequence might eaſily Reduce it to the Sines of thoſe Angles; On the Contrary, we have Shewn, that *Des-Cartes*, and therefore *Snellius*, has not Prov'd this Proportion; the 2*d*. is Shewn from Matter of Fact by *Grimaldus*, that the Rays of Light are Attracted by Glaſs, Steel, Gold, Silver, Braſs, &c. ſo as to make an Incurvation of the Rays, aa, bb, cc, &c. towards ACB; But how this Incurvation of Rays to ACB is any ways Anſwerable to the Gravitation of a Stone to the Earth, is no more to be Comprehended, than how dd, which is Suppos'd to be only bended towards ACB, ſhould Actually Gravitate to C; For the Queſtion here is not, whether Glaſs or Steel, or any other Body, has the Power of Attracting, in ſome Meaſure, the Rays of Light, which may be Proved by our Principles, that they have, but whether they Attract them, as the Earth does Lead, a Stone, or any Heavy Body; If they do not, the Doctrine of Refraction and Projectiles cannot be the ſame, which yet is here ſuppos'd; If they do, the Rays of Light ee, dd,



dd, cc, &c. ought to Deſcribe either Parabolas, if they Touch the Body, or Ellipſes, if they do not, to the Attracting Medium, which it is not Preſum'd or Apprehended, that they do.

§. 27. Farther, the Matter of a Stone, or Lead, and that of Light, in their Compariſons with the Earth and Glaſs, *&c.* muſt be ſuppos'd to be the ſame, otherwiſe Light, in Relation to it's Action upon Glaſs, may have a very different Property from what a Stone or Lead has in Reſpect of it's Action to the Earth, and therefore from Compareing the Actions of the laſt, we cannot Infer a Certainty of the Actions or the Reſpects, which each has to the other in the former, unleſs the Matter, both in Light and in Lead, is the ſame, and their Properties ſo too, which is Impoſſible.

If the Preceding Analogy is True, it muſt be Preſum'd, that Matter is the ſame, both in Light and in Lead, and that the Properties in both are alike, which is Impoſſible.

§. 28. What we have therefore now farther to Examine into, after the Suppoſitions, is the Reaſoning upon them, which is us'd to Prove, that the Sine of the Angle of Incidence and Refraction is in a given Ratio; We at Preſent Own, what is ſaid to be Prov'd by *Galileus*, Concerning the Parabola form'd by Projectiles, but cannot Think it is Applicable to the Laws of Refraction; We likewiſe for the Preſent Acknowledge, that in (*Fig.* §. 14.) HM is Biſected in L, and that the whole Series of Argument from thence to $MLq - LIq$ having a given Proportion to MLq, is True and Unconteſted.

That the Sine of Incidence and Refraction are not in a given Ratio according to this Demonſtration.

§. 29. If therefore $MLq - LIq$ is to MLq, as a to b, for Inſtance, by Diviſion $MLq - LIq - MLq . MLq :: a - b . b$; from whence it will follow, that if b is greater than a, or LIq than MLq, the given Proportion will be, as $-c$ to b, and Conſequently the Extractions of the Roots will be Impoſſible, ſo as to give any Definite Proportion.

The Proportions betwixt them Indeterminable according to this Demonſtration.

§. 30. More than this, what is it, which is Meant by a given Proportion? If it is ſaid to be a certain Proportion, as a to b, it is not True, ſince this Demonſtration takes the Liberty to Change it in every Step of it's Reaſoning; If a to b may be Chang'd to any other known Proportion, yet that of $-c$ to b cannot be known; Or if it could, it would be eaſy to Prove any Quantities to be in a Known and Definite Proportion, if we ſuppoſe 'em ſo at firſt, and at the beginning of the Demonſtration, which is the preſent Caſe; Or if the Sine of the Angle of Incidence and Refraction are in a given Ratio of a to b, or any other Proportion, it cannot be Prov'd that they are ſo from their being in the Proportion of c to d, or g to l, or any other Reaſon we can Name, which is the Reſult of the preſent Demonſtration; Namely, that the Sine of the Angle of Incidence and Refraction, ſhall be in a certain Reaſon of a to b from very different and various Proportions, which cannot be Reſolv'd into it.

That this Demonſtration Proves the Sine of the Angle of Incidence and Refraction, to be in a given Ratio from Proportions in Quantities, which are Extremely different, and by which way of Reaſoning any thing might be Prov'd any Thing.

§. 31. As in Reflection there were three Hypotheſes of it, ſo we might Expect the ſame Number to Account for Refraction, but Elaſticity being one, and the beſt Hypotheſis for Reflection, the Reaſon of which we have already given, and there being no Proſpect of Solving Refraction by the ſame Principles of Elaſticity; upon that Account we have only two Solutions of the preſent Phænomenon, and muſt therefore, if they are any ways Satisfactory, Acquieſce in them, altho', as was before Obſerv'd, both Reflection and Refraction ſhould Rely upon the ſame Individual Hypotheſis for their Solution; Which we ſhall therefore now give from the Doctrine of the Expanſive and Contractive Forces.

The Reaſon why there are three Hypotheſes of Reflection and two only of Refraction.

§. 32. Refraction therefore, as Reflection, is different and Various, according to the difference and Variety of the Expanſive and Contractive Forces of the Incident Body, and of the Mediums of Incidence and Refraction, thro' which it is Carried; Thus Light, being of an Expanſive Force, and falling upon Water or Glaſs, which, in Reſpect of it, is of a Contractive, is Refracted to the Perpendicular, that is, it's whole Subſtance, by ſuch an Incidence and upon it's Ingreſs, without Conſidering it, as Conſiſting of Particular Rays, is Contracted, and Preſſed cloſer together, which before was more Dilated, Expanſive, and Diffuſed; Upon the ſame Account, Light Trajected thro' a more Spirituous or Expanſive Fluid than Water, will be leſs Refracted to the Perpendicular, that is, will Dilate and Diffuſe it's ſelf more in it; On the other Hand, if a Stone, which is of a Contractive Force, is Incident upon Water, which is of an Expanſive in Reſpect of it, it is Refracted from the Perpendicular, that is, it's Action is more Dilated and Diffuſed from ſuch an Expanſive

What the Nature of Refraction is, and how Accounted for.

Force in the Water; But Lead will be leſs Refracted from the Perpendicular, becauſe it's Contractive Force is greater than that of a Stone, and therefore the Expanſive of Water will make a leſs Impreſſion upon it to Divert it from the Perpendicular; Thus alſo, I ſuppoſe, a Groſſer Medium of Air will be leſs Diffuſed in Water, than a more Expanſive and a Lighter; And Probably a Virgula or Rod of Wood, which is of a Lighter and more Expanſive Nature, will Appear more Crooked and Diffuſed in it from the Perpendicular, than a Spatula, or a Rod, of Iron or Braſs, which is of a more Contractive; The latter of which, ſince my Writing of this, I have in Fact, in ſome Meaſure Experienced; For take a Baſon of Water, and thruſt a ſmall Virgula of Wood into it, and then one of Braſs, or Iron, the former will appear Longer and more Dilated in the Water, and the latter, Shorter and more Contracted; Hence alſo an Object Appears greater from a Denſer Medium thro' a Rarer, and leſs from a Rarer Medium thro' a Denſer; Becauſe, in the firſt Caſe, the Subſtance of Light is Expanded and Diffuſed, and in the laſt Compreſſed and Contracted: And in all theſe Inſtances, the ſame Proportions of the Expanſive or Contractive Forces are Preſerved betwixt the Mediums of Incidence and Refraction; which Probably gave Riſe to the Doctrine, which has been ſo much Laboured by the Mathematick Philoſophers, that the Sine of Incidence is in a certain Proportion to the Sine of Refraction.

CHAP. V.

Concerning the Fundamental Principles of Opticks.

The Refraction Alledg'd in Opticks to be next Conſidered.

§. 1. HAVING Already Conſider'd the two Hypotheſes Produced to Explain Refraction, that of Reſiſtence, and the other of Attraction, and ſinec we find in Opticks, that the former is Defended a different Way than we have yet Explain'd, we ſhall now Proceed to an Examination of thoſe New Hypotheſes, and which Particularly Reſpect the Refractions of Light.

Refraction in Opticks, as Explain'd by Leibnitz. Molyneux. Dioptr. Chap. 1. Part. 2.

Act. Erudit. Lips. Ann. 1682. Pag. 185.

§. 2. THE firſt is, that Light Proceeds, from the Radiating Point to that which is to be Enlightend, that way, which is of all the moſt Eaſy, and that two Mediums have their Determinate Degrees of Reſiſtence to the Progreſs of the Ray, from whence it follows, that the Sines of the Angles of Incidence and Refraction are to each other Reciprocally, as the Reſiſtences of the Mediums, that is, as the Reſiſtence of the Refracting Medium to that of the Medium of Incidence, and therefore will be in a given Proportion; 'Tis therefore Inquir'd by what Way, or Path, C ſhall Radiate to G, (*Fig.* §. 4.) ſuppoſing IE to be Air, and EK Water, or Glaſs, Separated from each other by the Surface AB, or which is the ſame, the Point E is ſought, which is to Refract the Incident Ray CE to G, that way, which of all is the Eaſieſt; But the Difficulty, which a Ray is to Surmount in it's Paſſage thro' any Medium, Conſiſts of the length of the Way it is to go, and the Reſiſtence of the Medium to it's Progreſs; Let m be the Reſiſtence of Air, and n of Water or Glaſs, the Difficulty of the Way from C to E will be, as the Rectangle $m \times CE$, and from E to G, as $n \times EG$; If therefore any Point, as F, is taken in the ſame Surface, $CE \times m + EG \times n$, being the leaſt of all Poſſible Ways, ought to be leſs than $CF \times m + FG \times n$; The Point E is now Requir'd with theſe Conditions; C, G, and AB, are given by Poſition, and Conſequently the Perpendiculars CH, GL, and the Line HL; Let CH be c, GL, g, and HL, h, and EH the Line ſought, y; EL therefore is $= h - y$, and $CE =$ to $\sqrt{c^2 + y^2} = p$, $EG = \sqrt{g^2 + y^2 - 2hy + h^2} = q$; wherefore $mp + nq$ ought to be the leaſt of all thoſe Quantities, which can be ſo Expreſſed, and y is to be Determin'd accordingly; But by the Doctrine *De Maximis & Minimis*, $myq = np \times h - y$, or $np.mq :: y, h - y$, and CE being put $= EG$, or p to q, $n.m :: y.h - y$, or the Reſiſtence of Glaſs or Water is to that of Air, as HE the Sine of Incidence to KG the Sine of Refraction, which was the Aſſertion to be Prov'd.

Act. Erudit. Lips. Ann. 1684. Pag. 467. 472.

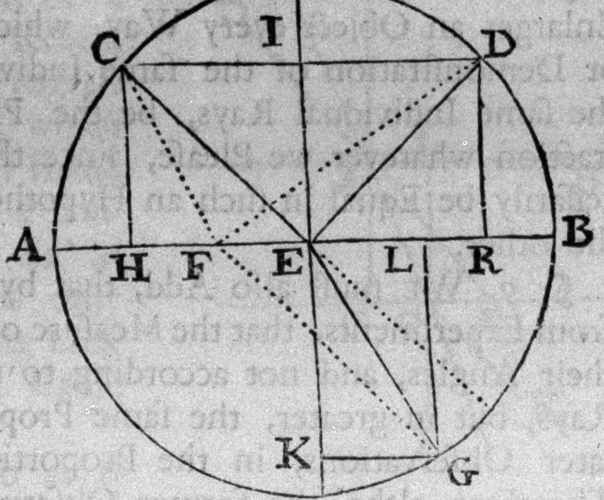
Argument the Firſt, againſt the forgoing Demonſtration, that CE *and* EG *are not Prov'd to be Equal.*

§. 3. BUT to this Demonſtration, Firſt it may be Reply'd, as before, when we Conſider'd the preſent Hypotheſis of Reſiſtence, that CE and EG are ſuppos'd to be Equal in their Incidence and Refraction, which is not Evident, that they are. However this may be ſaid in Favour of this Demonſtration, that it makes the Reſiſtence of Glaſs or Water to be Greater than that of Air, according to what we have Affirmed, §. 32. and Contrary to what Sir *Iſ. Newt.* Aſſerts, §. 14. and 20. of the foregoing Chapter.

§. 4.

Argument the Second, that CF, FG, *are the ſhorteſt Way, if that is not Preſum'd, which is to be Prov'd.*

§. 4. **Second**, it muſt be Own'd, that, as in Reflection *CE, ED*, are ſhorter than *CF, FD*, ſo in Refraction *CE, EG*, are ſhorter than *CF, FG*; but it has been Obſerv'd in the firſt Caſe, what we do now in the laſt, that the Compound Determinations are Originally Preſum'd, which ought to be Prov'd; Thus *CE, ED*, in Reflection are only the ſhorteſt way of Satisfying *CI, CH*, on the one Hand, or *ID, DR*, on the other; For if *CH, HF*, and *DR, FR*, were the Determinations, *CF, FD*, would be the ſhorteſt way of Satisfying them; From whence it is Evident, that *CI* and *CH* are Originally ſuppos'd, or, which is the ſame, that *CE* is = *ED*; And the Caſe is in ſome Meaſure Parallel in Refraction, where *CE* is ſuppos'd = to *EG*; for *HE, HC*, are Preſum'd to be the Determinations of Incidence, and *EL, EK*, of Refraction, upon which Hypotheſis *CE, EG*, will be the ſhorteſt Way; but ſuch an Hypotheſis is the Propoſition to be Demonſtrated; for if *CH, HF*, are the Determinations of Incidence, and *FL, LG*, of Refraction, that is, if *CE* is not = to *EG*, then *CF, FG*, will be the ſhorteſt Way, Contrary to what is ſuppos'd in the preſent Demonſtration; But that, which Overturns this whole Reaſoning is, that *CI* is ſuppoſed to be to *KG*, the Sine of Incidence to that of Refraction, Reciprocally, as the Reſiſtence of the Refracting Medium to that of the Medium of Incidence, and therefore Glaſs and Water are ſuppos'd to Reſiſt more than Air, which if ſo, the Refraction, it is Thought ſhould be from the Perpendicular and not towards it, the Contrary of which is Experienced, and which Difficulty we have Accounted for before from the Doctrine of the Expanſive and Contractive Forces, §. 32. *Chap.* 4.



Argument the Third, that Reaſonings from final Cauſes are Inconcluſive, unleſs the Laws and Properties of Nature are before known and Determin'd by Fact, and Experiment.

§. 5. **Therefore** all the Advantage, that I know of, in this Method of Reaſoning, is only to add a new Hypotheſis to the former; Namely, that Nature takes the eaſieſt Way to Perform her own Operations, and to this End Light is Reflected and Refracted according to the Laws before Mention'd; And from Hence, if the Cauſe or Reaſon of Refraction or Reflection was Inquir'd, it is Told us we might Anſwer, that the Reaſon of both was, that the Ray might Proceed the Eaſieſt Way Poſſible; In Reply to which we Say, however Pious Final Cauſes may be Imagin'd to be and tho' ſuch Laws of Reflection or Refraction were Obſerv'd, if at all, in Obedience to ſuch an Intention or Deſign in Nature, it yet can never be Evidenced, that there was ſuch a Deſign in Nature, till thoſe Laws are Prov'd, which according to §. 3. and 4. are not; that is, if ſuch a certain Law of Reflection or Refraction was Eſtabliſhed with an Intent and Purpoſe, that Things might be Atcheiv'd in the Plaineſt and Eaſieſt Way, that Certain Law in both ought firſt to be Demonſtrated, and if it is, the Final Cauſe cannot be an Additional Evidence to the Demonſtration, but a Conſequence and Deduction from it; if it is not Demonſtrated, then the End or Deſign of any Thing, which is not known, cannot be known neither; And in Truth, the End and Intention of the Almighty Diſpoſer of all Things can be only Collected from Properties in Nature, as they Really Exiſt; Since, if thoſe Properties are not Firſt and Originally Determin'd to Exiſt, it will be Impoſſible to Affirm the Deſign of what we are not certain does Exiſt.

Argument the Fourth, that Nature always takes the ſhorteſt Way, in it's Operations, not certain.

§. 6. **Farther** the Hypotheſis it ſelf is not Evident, that Nature takes always the ſhorteſt way in it's Operations in the Senſe, which this Demonſtration ſuppoſes, that is, that *CE* ſhould be a ſtrait Line, ſince it is Prov'd by Experiments, that there is an Incurvation of the Ray in it's Incidence thro' Air to Glaſs, or any other Refracting Medium.

Argument the Fifth, that the Sines of Incidence and Refraction are not Reciprocally as the Reſiſtences of the Mediums.

§. 7. **Moreover**, ſeeing Reſiſtence is in ſome Proportion, or other, to Denſity, and, as Sir *Iſ. Newton* ſays, *Cæteris Paribus*, in a Proportion of Equality, if certain Lighter and more Rarefy'd Fluids Refract more than Heavier and Denſer, the very Concluſion of this Demonſtration will be Falſe, that the Sines of the Angles of Incidence and Refraction are to each other Reciprocally, as the Reſiſtences of the Mediums, when yet the Fineſt Fluids, as Spirit of Wine, and Oil of Turpentine, and, as *Des Cartes* Informs us,

Dr. Hook's Micrographia Preface.

Sir Iſ. Newt. Princip. Prop. 96. Schol. Sect. 14. Lib. 1.

Air Refract Light more than Subſtances, which are of a Groſſer and Denſer Nature, and Conſequently more Reſiſting.

Argument the Sixth, that Incident Rays Refracted into the ſame Individual Rays cannot Solve the Enlargement of Objects every Way by Refraction. Dr. Wallis Philoſoph. Tranſact. N°. 187.

§. 8. Some likewiſe have Obſerv'd, that the Refraction by Glaſſes Apparently Enlarges an Object every Way, which cannot be Accounted for by any Suppoſition or Demonſtration of the ſame Individual and Incident Rays being Refracted into the ſame Individual Rays, be the Proportions of the Sines of Incidence and Refraction whatever we Pleaſe, ſince the Refracted Image and the Incident muſt Neceſſarily be Equal in ſuch an Hypotheſis, there being no more Rays in the one than the other.

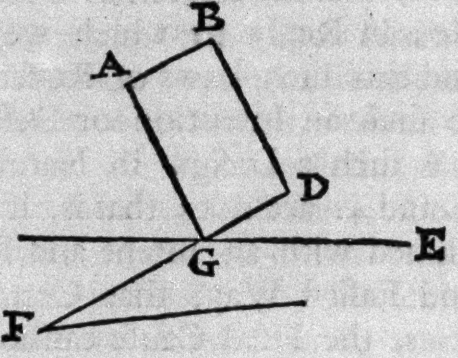
Argument the Seventh, that there are Experiments which Equally Prove, that Refractions are to be Meaſured by their Angles as well as by their Sines. Hugens's, Dioptr. De Refractione Radior. Pag. 2.

§. 9. We muſt alſo Add, that by *Vitellio* and *Alhaſenus* an *Arabian*, it was found from Experiments, that the Meaſure of Refractions was according to the Proportion of their Angles, and not according to that of their Sines, in leſſer Inclinations of the Rays, but in greater, the ſame Proportion did not Prevail, being according to the later Obſervations, in the Proportion of their Sines and not of their Angles; Therefore, altho' the former Obſervations cannot Determine, that all Refractions of a greater Inclination are made according to the Meaſure of their Angles, on the other Hand, it will not be certain, that the later Obſervations, that the Refractions are to be Meaſured by their Sines, where there is a greater Inclination, ſhould Aſſert the Properties of thoſe of a leſs to the ſame Meaſure, and it would be more Rational to give a Solution of both theſe Phænomenons, as they are Diſtinguiſh'd by Experiments, than to Reduce the one to the other.

That, if in Fact and Experience, the Sine of the Angle of Incidence was Proportional to that of Refraction, there is no Hypotheſis hitherto Alledg'd, which gives a Solution of ſuch a Phænomenon, and that Dr. Barrow's *does not.* Molyneux Dioptricks Chap. 1. §. 4. Part. 2. Dr. Barrow.

§. 10. Lastly, as in Section the *7th*, it was ſaid to be Experience, that ſeveral Lighter Fluids Refracted the Rays more than Heavier ones did, and that therefore the Reaſoning from Reſiſtence, or even from Attraction, could not be Valid, in Order to Solve theſe Refractions, ſo by the ſame Experiments it is found, that the Sine of the Angle of Incidence is in a certain Proportion to that of Refraction, at leaſt, as in the foregoing Section is Obſerv'd, in any greater Inclinations; If this yet were True, we Contend, that none of the Preceding Hypotheſes or Demonſtrations, do give us any Satisfactory Account of this Appearance.

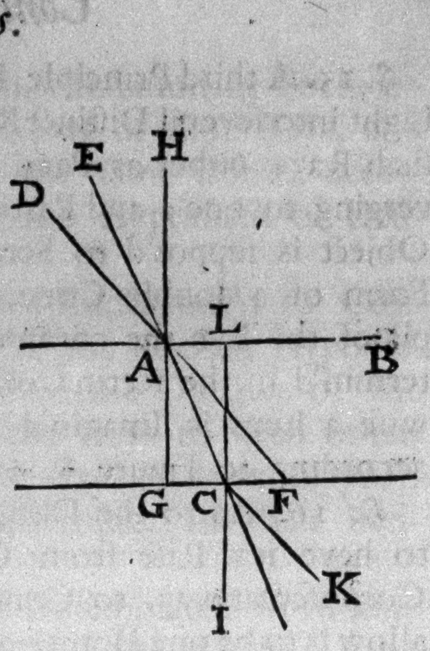
And after this it will not be Neceſſary to Conſider the Hypotheſis of Rays Falling upon the Refracting Medium *GE*, in a Parallelogram *AGDB*, and turning round on a Center *F*, ſo that the Ray *AG*, which Falls ſooneſt, ſhall be Diminiſhed in it's Motion, whilſt the other *BD*, which is farther remov'd from the Refracting Medium, is Accelerated, till they both enter the Medium, and are from thence carry'd on in an Equable Motion thro' it; We Say there is no Occaſion to Conſider this Hypotheſis, ſince, why *D* ſhould Move faſter, till it Enters *GE*, than it does afterwards in Reſpect of *G*, cannot be Determin'd; That is, why *G* ſhould ſtop, till *D* Arrives at *EG*, which is Impoſſible, whilſt *G* and *D* are ſuppos'd to be Points in the ſame Radius; Not to Mention the Chimerical Suppoſition of the Center *F*.



However this at leaſt we may Infer from the Reaſoning Alledg'd by Dr. *Barrow*, to Solve Refraction, that He was ſo far Penetrating and Sagacious as to ſee, that it could not be done by any of the Preſent Hypotheſes. And which I have once more Attempted, *Chap.* 4. §. 32. from our Doctrine of the Expanſive and Contractive Forces.

The ſecond Fundamental Principle in Opticks Conſider'd, viz. that the Progreſs of Light is Reciprocal thro' Different Mediums. Molyneux's Dioptricks Experiment 8. Part 1. Hugens's Dioptricks Prop. 1.

§. 11. Next to the Determin'd and Certain Proportion betwixt the Sine of Incidence and Refraction, upon which the whole Doctrine of Opticks Depends, is that other Fundamental Propoſition, that the Progreſs of Light thro' different Mediums is Reciprocal; Namely, if a Ray, falling from *D*, is Refracted to *C*, where the Diaphanous Medium, as Glaſs, is Placed on the Side of *AB* towards *C*, if the ſame Diaphanous Medium is Placed on the contrary Side towards *E*, the Refraction of *FA* will be to *E*, ſuppoſing *DA*, *AC*, make the ſame Angles of Incidence and Refraction, as *FA*, *AE*, becauſe all the Circumſtances are the ſame in both Caſes, ſince *FAG*, *HAE*, in the latter, are = to *HAD*, *CAG*, in the former. This therefore is rather an Axiom, or an Affirmation of the ſame Thing, than a Propoſition, or a Demonſtration; But, as it ſeems, this does not Reach, what is here Intended in Opticks by a Reciprocal Progreſs of Light thro' different Mediums, as Air and Glaſs, which is, that as *DA* is Refracted to *C*, ſo on the other Hand, *CA* is Refracted to *D*, ſuppoſing Light to take a contrary Courſe in the laſt Inſtance, from what it did in the firſt; for if it takes the ſame Courſe, it is only Conſidering one Individual Refraction by Traceing back the Effect *CA* to the Cauſe *AD*, as before the Cauſe *DA* was Affirm'd to Produce the Effect *AC*.



If Light Proceeds in Oppoſite Courſes in Refraction, the Reciprocal Progreſs of it not certain.

§. 12. If Light takes an Oppoſite Courſe, ſo as *D* ſhall be the Point of Radiation in one Example, and *C* in another, it is not altogether ſo Evident, what is here Endeavour'd to be Demonſtrated, or at leaſt the Demonſtrations made uſe of by the Writers of Opticks are no ways Concluſive of the Propoſition; ſince if the Luminous Point *C* Radiates thro' *CA*, with the ſame Force as *D* thro' *DA*, *CA* will Proceed to *E* in a Direct Line without any Refraction, if it Radiates with the ſame Force as *A* to *C*, that is with a Force that is Weaken'd and Impair'd, in it's Egreſs from *A* to *D* it will be Impoſſible to Recover that Vivid and Lively one, which is Expreſſed by *AD*, and Conſequently cannot be Return'd in that Direction; Nor is this Reciprocal Refraction Prov'd by the common Experiment of our ſeeing Objects Situate in *C*, at *F*, ſince that would Equally happen from a bare Inflection of the Viſual Rays from *DA* to *C*, without Preſuming any Retrograde Refraction of *CA* to *D*; becauſe the Eye at *D* may as well Purſue the Stroke of the Ray *AD*, back to *F*, whether *AD* is Refracted from *C*, or any Intermediate Points from *F* to *G*; however this is, what we have now to Conſider is only the Reaſoning made Uſe of to Prove, that Light Acting two contrary Ways thro' different Mediums, Produces in the whole the ſame Effect, that is, Deſcribes the Lines *CA*, *AD* backwards, as it does *DA*, *AC* forwards, which is not Manifeſt from the Arguments Alledg'd to Confirm it, and which likewiſe ſeems Averſe to common Reaſon.

If the Progreſs of Light was Reciprocal, Objects would appear, when Refracted, in a different Situation from what they do.

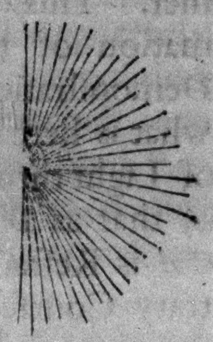
§. 13. For moreover, if this were True, as *DA* is Refracted to *C*, ſo likewiſe *AC* would be Refracted to *K*, and the Angle *KCI* would be = to *HAD*, ſuppoſing a different Medium *ABFG* to ſeparate two Similar ones lying on each ſide it, and in this Poſition of the Figure, Objects at *D* to the Eye at *K* would appear on the left Hand of their Real Situation, which is not found to be True in Experience; on the contrary Philoſophers Endeavour to Explain the Reaſon of our ſeeing Objects thro' the Glaſs of a Window in their Natural Poſture from this very kind of Refraction, becauſe the Refraction from *C* to *K* Undoes, and, as it were, Unravells the Effect Produced from the Refraction of *DA* to *C*, which how Juſtly, it is Eaſy to Determine.

If there was a Reciprocal Progreſs of Light, a Faint and Languid Ray of Light would be Refracted as ſtrongly, as a more Vivacious and Forcible.

§. 14. Besides, if Light is Refracted thro' Glaſs or Water, every one knows, the Rays come out of them with a Faintneſs and Languor, which is not in the leaſt Equal to the Vivacity and Force, with which they Enter; if therefore we Purſue back the Rays from *K*, it will be altogether Inconceivable, how the Weak and Feeble Light at *K* ſhould be Refracted in the ſame Line *CA*, and in the ſame Angle *ACL*, with the more Vigorous and Strong from *D* in the Line *AC*, and in the Angle *CAG*, which yet muſt be Acknowledg'd, if this Doctrine of Reciprocal Refractions is True; What therefore we cannot but Conclude is, that, as in other Parts of Philoſophy, ſo in this, an Abſtracted Reaſoning is Preferr'd to a Real Knowledge of Things, and that, by theſe Demonſtrations, there is nothing more Prov'd than a Simple and Plain Refraction of Light, Conſider'd as one ſettled Effect in Nature, which ſuppoſes a Refracted Ray *AC* from a Refracting one *DA* or *KC*, or Reciprocally, a Refracting from a Refracted, which is all the Reciprocal Refraction we can at Preſent Comprehend to be Explain'd, or in any manner Deducible from this way of Arguing. On the Contrary, all theſe Difficulties Vaniſh in our Doctrine of the Expanſive and Contractive Forces.

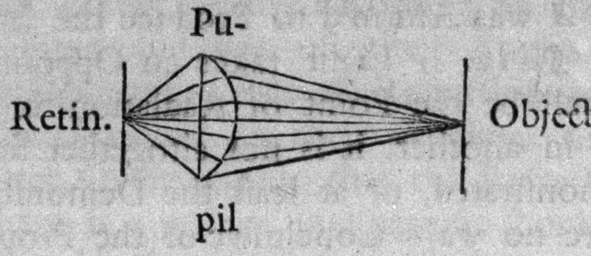
The third Principle of Opticks, which ſuppoſes the Diviſion of Light into Diſtinct Rays, Conſidered.
Molyneux's Dioptricks, Suppoſition 1. Part 1. Tab 2. Fig. 2. Defin. 13. 14. 15. 16. Tab. 2. Fig. 1. Defin. 18. Tab. 2. Fig. 3. Hugens's Dioptricks Paſſim.

§. 15. A third Principle, Preſum'd in the preſent Account of Opticks, is the Diviſion of Light into ſeveral Diſtinct Rays, as in the Preſent Figure, and the Conſidering ſuch Rays, either as Parallel, or Diverging from a certain Point, or Converging to one; and Particularly, every the Minuteſt Portion of a Viſible Object is ſuppos'd to Send out and Emit a Pencil of theſe Rays in the Form of a double Cone, whoſe one Vertex is the Viſible Point, the Pupil of the Eye the common Baſis of both, and the other Vertex is Determin'd in the Retina of it, Expreſſed in Figure §. 16; Thus likewiſe a Lens is Imagin'd to Collect ſeveral Parallel Rays to a Focus, according to Figure §. 17.



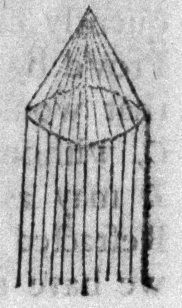
The Diſtinction of Light into little Rays or Filaments a Creature of the Mind.

§. 16. As to the Diſtinction of Light into Individual Rays, ſuch a Doctrine ſeems to have it's Riſe from Conſidering it, in the Corpuſcular way, to Conſiſt of Parts, but if we allow it to be one Homogeneous, Undiſtinguiſhed Subſtance, and of one Entire Nature, the Splitting and Separating it into little Filaments, or Rays, appears to be nothing elſe than a Creature of the Fancy, in Order to Form a Mathematical Romance in Opticks, as before has been Prov'd to be the Deſign of the Corpuſcular Syſtem, in Reſpect of Philoſophy.



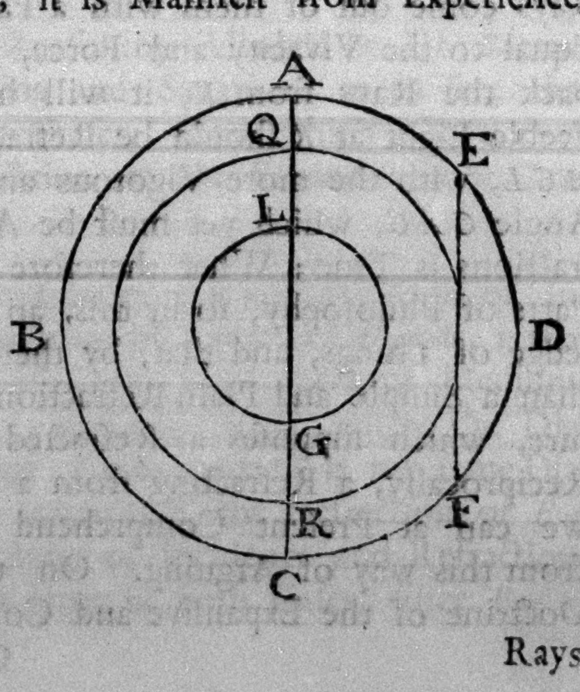
The Diſtinction of Light into ſmall Rays and Filaments Diſprov'd, from the Number of the Rays upon the Lens and in the Focus being the ſame.

§. 17. For if the Beams of the Sun, which, as coming in direct Lines from it, bid the faireſt for the Proof of ſuch an Hypotheſis, Conſiſted of little Rays, or ſmall Threads, or Filaments, deriv'd from thence, there could be no Collection of them by a Lens to a Focus, as in this Figure; Since the Surface of the Lens, which is Oppos'd to the full Light of the Sun, is Perfectly Illuminated without any Vacant Interſtices or Interruptions of Shadow, or not; If it is, ſince theſe Imaginary Rays are as cloſe, as it is Poſſible for them to be, they could not be Collected into a narrower Compaſs in the Focus; If the Surface of the Lens, which is Oppos'd to the Sun, is not Entirely Illuminated, ſince the Diameter of the Focus may not be a *20th* or *30th* part of the Diameter of the Lens, and therefore the Area of the one not a *400th* or *900th* Part of that of the other, and ſince the Number of Rays are the ſame before and after their Tranſmiſſion, it will follow, that the Illuminated and Shaded Parts of the Lens will be as the Dimenſions of the Focus to thoſe of the Lens, Subſtracting Unit, *i. e.* if there are a Million of Rays Incident on the Surface of the Lens, which are Collected together in the Focus, there will be 399, or 899, Shadow'd Spaces in the Lens for one that is Illuminated, which, if True, would be Abundantly Viſible, nothing of which has yet Appear'd from any Obſervations; But on the Contrary, the Surface of the Lens has not the leaſt Viſible Space, which is not Illuminated, as well as the Focus, and all the Difference is, not in the Extent, ſo far as each of them Reach, but in the Degree and Force of their Illuminations; and this is in a manner Acknowledg'd, in §. 19. where the Rays of the Sun are ſuppos'd to be Parallel, becauſe their Inclination to each other is Infinitely ſmall, that is, the whole Lens is Illuminated by them.



Diſprov'd from much the ſame Focus being Produced, when Various Parts of the Lens are Covered.

§. 18. But to put this farther beyond Diſpute, it is Manifeſt from Experience, that if, in a Lens, *A B C D* ſhould be the Surface Expos'd to the Sun, and *L G* or *Q R* the Circles with their Areas, or *A B C D Q R*, or *A B C A*, or *E A B C F E*, are cover'd by a Paper, the Rays, in all theſe Caſes, will be Collected to much the ſame Focus, and Burn with a like Fierceneſs; and therefore the Hypotheſis, in *Fig.* §. 17. of Parallel Rays Falling, and being Refracted with the Conditions there Specify'd and Expreſſed, cannot be Equal to ſuch a Solution, which muſt Depend upon ſome other unknown Property, betwixt the Light and a Lens, than what has been hitherto Preſum'd or Imagin'd; for how a Determin'd Number of



Rays, upon their Collection, ſhould Conſtitute a Certain and Inflammant Focus according to (*Fig.* §. 17.) and yet 100*th* or 1000*th* or a Millionth part of thoſe Rays, and in different Incidencies and Poſitions, ſhould nearly Produce the ſame Effect, according to the Preſent Figure, will not be Readily Solv'd by the Hypotheſis of Definite and Diſtinct Rays, or Filaments, in the Sun's Beams.

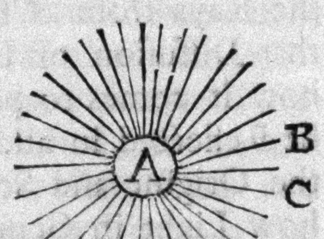
If Diſtinct Rays were allow'd of, the Paralleliſm of them is next to be Conſider'd.

§. 19. If this Notwithſtanding was True, which we have Endeavour'd to Confute, the Suppoſition of Parallel Rays is another Difficulty in the preſent Hypotheſis, for thoſe Rays are made to be Parallel, which by the Immenſe Diſtance of the Object, as the Sun, Cauſes an Infinitely near Approach of the Rays to each other, or elſe, which, Conſidering the Smallneſs of the Pupil of the Eye, may be ſuppos'd as Parallel, there being in ſo ſhort a Circuit little or no Difference in their Incidence.

Molyneuxs's Dioptricks Chap. 1. Defin. 7. Part. 1. Deſcartes Dioptricks Cap. 8. Hugens's Dioptricks Paſſim.

The Paralleliſm of the Rays not prov'd from the Infinite Diſtance of the Sun.

§. 20. As to the Rays of the Sun being Propagated and Diſſeminated from one Center; If there is a certain Number of Diſtinct and Individual Rays, Deriv'd from thence, which this Hypotheſis muſt ſuppoſe, they will be ſo far from Approaching to each other by Increaſing the Diſtance from the Luminous Body, that they will continually Enlarge their Divarication; Thus in the preſent Figure, if *A* is ſuppos'd to be the Sun, and to Propagate Rays from it's ſelf, the farther the Diſtance is, the greater will be the Diſperſion of the Rays *AB*, *AC*, from each other, and Conſequently the leſs Inclin'd to a Paralleliſm; If there is not a certain Number of Rays emitted from the Sun, but they grow Thicker and Cloſer, as they Retire from it, that new Property of a Circle is firſt to be Demonſtrated, which is Impoſſible; And therefore it will be Reaſonable in the next Place to Conſider, whether we are not to Inquire into the Nature of that Æther or Medium, which Surrounds us, and the Action of the Sun upon it, in Order to Solve the Refractions of it thro' a Lens, rather than Form our Notions of Philoſophy from Pictures, which have been made us of the Sun's Radiation, in Lines and Filaments, which ſeems to be the Real Foundation of ſo Groſs and Childiſh an Error, as that of ſuppoſing there are any ſuch Radiations.



§. 21. For it is not Sufficient to ſay, that the farther we are Remov'd from any Object, the Viſual Rays Approach the nearer, ſince that is a Problem in Viſion, which remains to be Solv'd, and will be afterwards Conſider'd, the Queſtion only here is, whether the Sun, as a Luminous Body, Emits a certain number of theſe Imaginary Rays, or not; If it does, it is very Evident and Manifeſt, that they muſt Divaricate from their Center, the farther they are Produced from it; If it does not, there will be no Occaſion to Conſider the Sun's Diſtance, ſince the Paralleliſm of the Rays muſt be Aſſerted from a nearer Cauſe, which has not been as yet Aſſign'd, and ſeems to be the Action of the Sun upon a Univerſal Æther or Plenum, which, when a Glaſs is Interpos'd, is Refracted.

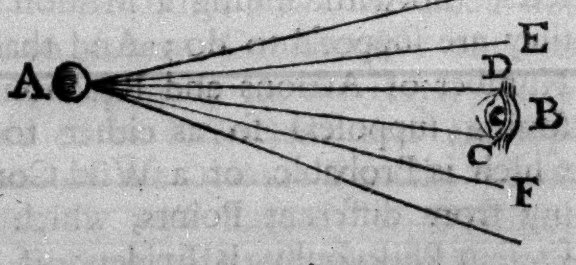
Not Prov'd from the Viſual Rays Approaching nearer, as the Object is farther Remov'd.

The Doctrine of the Paralleliſm of Rays ſuppoſes likewiſe that thoſe which fall from a near Object upon the Lens of the Eye are Parallel.

§. 22. But that, which makes this Doctrine ſtill more Inconceivable, is, that a Paralleliſm of the Rays is requir'd even in the Viewing of Objects, which are near us, in which Caſe the Pupil of the Eye is ſaid to be ſo ſmall, that the Rays, which Fall upon it, find little or no Diſtinction in their Incidence.

Not Prov'd, that the Rays from a near Object are Parallel.

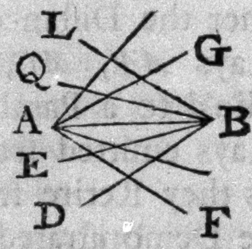
§. 23. Let us therefore Imagine the Object to be *A*, and the Eye *B*, and *DAC* the Inclination of the Rays, which Fall upon the Pupil of it; in this Caſe, ſuppoſing it is True, where ſo ſmall a Portion of the Radiating Sphere is taken in, that the Rays will not be eaſily Diſtinguiſhed from Parallel ones, as *AC* and *AD*, it is certain notwithſtanding, if the Eye be carried nearer to *A*, that it may be made to Intercept all the Rays, which are Comprehended betwixt *AF* and *AE*, and which will therefore have a very Senſible and Notorious Obliquity; and on the Contrary, if it is Placed at a greater Diſtance from *A* than in it's Poſition at *B*, *AC* and *AD* may Diverge to that Degree, as not to be Incident upon the Eye at all; ſo that, according to this Doctrine, ſince a Paralleliſm of the Rays is Required, which cannot be always



Preſerved, the Eye can only fully Diſcern an Object in one certain State and Poſture, or in that, which is not very Different from it; Whereas in Experience we Plainly and Clearly Diſcern Things, both at a very near, and at a very great Diſtance, and in all the Intermediate Degrees of Interval from the Object; we cannot likewiſe Omit to take Notice, that it is very Extraordinary, that Parallel Rays from a Remote Object, and Divergeing ones from a Near, ſhould be the ſame, in Effect, upon a Lens, or the Pupil of the Eye; But this Philoſophy can Reconcile Contradictions.

Another Principle Alledg'd in §. 15. is the Radiation of the Viſible Point; that ſuch a Radiation cannot be Prov'd from a Suppoſition of Light being Deriv'd in Direct Rays from the Sun.

§. 24. We are alſo Confirm'd in the Opinion of the Inſufficiency of the preſent Doctrine, from it's giving us no Account, how the ſeveral Points of an Object are Capable of this Wonderfull Radiation, for in this Philoſophy, which Conſiders Matter as Paſſive and Homogeneous, it muſt be from a bare Reflection of the Rays, which are Incident upon it, which Rays are either the Direct ones of the Sun, or elſe thoſe of Light, which are every where Diffus'd around the Object it Illuminates; If the firſt, upon the Suppoſition of the Angle of Incidence being Equal to the Angle of Reflection, which is Adopted into the preſent Science, as a fixt Axiom, it will be Impoſſible, that ſtrait Lines drawn from the ſame Point, as the Sun, ſhould be Reflected from another in the Object, on which they Fall, in the Circumference of a Circle, or a Sphere, or which is the ſame, that the Rays from *A* ſhould Produce thoſe from *B*; If the laſt, what that Diffuſion of Light is, will want to be Explain'd, that is, how it comes to paſs, that Light Acts in all Directions, ſo as to be Reflected in all, ſince the Sun, which is Acknowledg'd to be the Origine and Source of it, does not; or, which is Equivalent, how Light in General ſhould Determine it's ſelf from *E* and *D* to *B*, ſo as to be Reflected to *Q* and *L*, when the Suns Action is from *A* to *F*, *B*, and *G*; or, if it did not Contradict that Action, how the ſame Light Conſider'd, as Diſtinct from the Sun, ſhould take ſuch Arbitrary and Different Paths to the ſame Point, without any Known or Confeſſed Cauſe of it's Various Determinations.

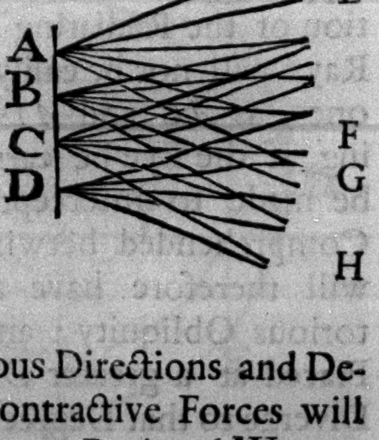


Nor yet from a Suppoſition of an Equal Diffuſion of it.

§. 25. If this were notwithſtanding Acknowledg'd, ſince Light is now ſuppos'd to be every where Promiſcuouſly Diffus'd, there would be no more Reaſon, why *E* and *D*, ſhould be Reflected from *B* to *Q* and *L*, than why the latter ſhould be to the former, and Agreeably in all other Caſes; from whence it would follow, that from ſuch a Univerſal Diffuſion of Light, Action would be Equal to Reaction, and there would be no Reflection at all, but a Perfect Æquilibrium in the Actions of Light, by which they would be a Mutual Curb and Reſtraint upon each other, ſo as Effectually to hinder any Returns of the Rays from the Bodies, on which they Impinged.

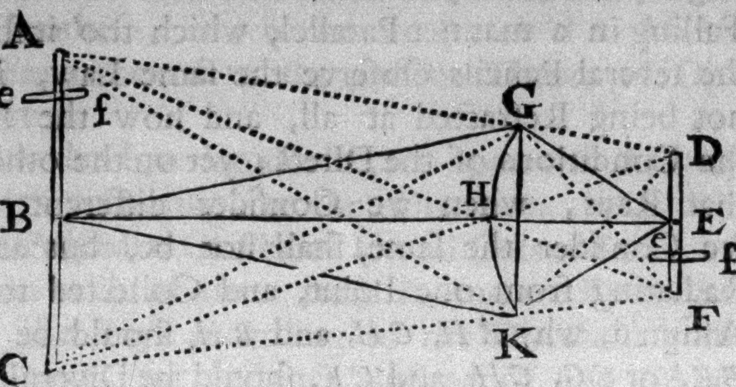
Such a Radiation would Produce the greateſt Confuſion Imaginable in the Forces of Light, ſo that either no, or a Wild, Effect would Enſue from it.

§. 26. If it be ſaid it is Matter of Fact, that Rays are Reflected in all Angles, from our ſeeing Objects in any Poſition of the Eye to them, we Anſwer, the Phænomenon is Acknowledg'd, but the Solution of it deny'd from the Hypotheſis of Diverging Rays; for, beſides what has been Alledg'd, it is farther to be Remarked, that on ſuch a Conceſſion, and granting all that can be Deſir'd, there would thoſe Difficulties ariſe, which would be altogether Inextricable; For it ſeems to be Impoſſible for the Points *A*, *B*, *C*, *D*, to ſend out their ſeveral Pencils of Rays *A E*, *B F*, *C G*, *D H*, without Occaſioning the utmoſt Confuſion upon their firſt ſetting out, from whence their Progreſs would be Entangled and Embaraſſed to that Degree, that it would be altogether Inconceivable, how they ſhould Produce the Effects, (notwithſtanding a Million of Interruptions,) which they are ſuppos'd to do; And that there muſt be an Infinite Number of Actions and Reactions, (which the preſent Reaſoning ſuppoſes) ſo as either to Cauſe an Equal Balance which is Probable, or a Wild Confuſion of the Forces Acting from different Points, which is the Plain Reſult of the Preſent Philoſophy, is Evident, if there are any Forces Confeſſed at all, which are Propagated from Various Points in Various Directions and Determinations. The Doctrine, Again, of the Expanſive and Contractive Forces will Extricate us from theſe Objections and Explain Nature in a much more Rational Way.



§. 27.

§. 27. A fourth Principle Preſum'd in this Philoſophy is, that the Rays, which Diverge from any Point in a Viſible Object, are Collected by a Lens to the ſame Point after their Tranſmiſſion thro' it, and that the Points, from which the Rays are Scatter'd, are to the Points, to which they are Refracted, in an Inverted Order, and that the Eye Purſues the Rays, and Returns it's Action back upon them in the ſame Strait Lines, by which the Impreſſion was made; Thus in the Figure here Inſerted, The Rays, which Fall from *A*, are Collected in *F*, thoſe from *B*, in *E*, and thoſe from *C*, in *D*, and ſo of any Intermediate Points betwixt *A* and *B*, or *B* and *C*; The Axes of which ſeveral Pencils of Rays are ſuppos'd to be in ſtrait Lines, as *A F*, *B E*, *C D*, and to Croſs each other in the middle Point *H*, betwixt *G* and *K*; It is alſo farther to be Obſerv'd, that the Object *A C* is preſum'd to be directly Oppos'd to the Refracting Medium *G K*.



The Fourth Principle Preſum'd in Opticks, and Explain'd in it's ſeveral Particulars.

Molyneux. Dioptr. Chap. 1. Defin. 13. Tab. 2. Fig. 1. Deſcartes's, and Hugens's Dioptr.

§. 28. First, as to the Axes of the ſeveral Pencils being in Direct Lines, it is Impoſſible they ſhould be, unleſs in this certain Poſition Obſerv'd in the preſent Figure, or unleſs, as the Object *A B C* Approaches nearer to *G H K*, or is Remov'd farther from it, the Image *D E F* is Proportionably Increaſed or Diminiſhed; for as the Lines *A H*, and *C H*, by the Acceſs or Receſs of the Object *A B C*, Divaricate and Widen, or Come cloſer together, ſo will *H D* and *H F*, being Oppoſite Lines, which are Continued to the former, and Act upon the ſame Center of their Motion *H*; From whence it will follow, either in the firſt Caſe, that the Repreſentation *D E F* can only be made in this Fixed and Unalterable Poſition of the Object to the Lens, which is Contrary to Fact and Experience, or elſe in the ſecond, that within the ſhort Space or Diſtance of double the Line *B H*, ſuppoſing *D F* to be half as Large as *A C*, the Object may Appear twice as great near *H*, as it does at *B*, and as far beyond *B*, as *B* is from *H*, twice as ſmall, both which are likewiſe Contradicted by Fact and Experience; ſince *A C* is ſeen without any Conſiderable Alteration in more Places than *B*, and in Particular, without that Increaſe or Diminution on this Side *B*, or beyond it, which this Reaſoning muſt ſuppoſe.

The Axes of the ſeveral Pencils of Rays, not Prov'd to be in Direct Lines, becauſe there would be ſuch an Increaſe and Diminution of the Object, if this Reaſoning was True, which is not Experienced.

§. 29. Nor does this Objection againſt the preſent Doctrine ſeem Capable of being taken off by an Hypotheſis of a Change in the Figure of the Lens, or of the Diſtance betwixt it and *D F*; ſince however the Lens is made, either Flatter or more Protuberant, *C H D* is ſtill Imagin'd to be one ſtrait Line, as likewiſe *A H F*, and therefore the Elongation of the Retina from the Lens, or it's Approximation to it, will have much the ſame Effect with *H D*, and *H F*, Receding from each other, or Coming together, where *D E F* is ſuppos'd to keep the ſame Determin'd Diſtance from the Lens; that is, the Image will be Proportionably Lengthened and Contracted; beſides it is Impoſſible to Conceive, how the Retina by any kind of Adjuſtment ſhould Retire as *A C* Advances, or on the Contrary, and Eſpecially, ſince, when our Eyes are moſt Fixed and Intent, and do not ſeem to Admit of the leaſt Change, Objects, notwithſtanding that, in very different Diſtances Appear to have the ſame Dimenſions for ſeveral Yards together, whereas, either by the Hypotheſis of the former Section, or the Preſent, they could not Preſerve them Unalter'd and Certain for an Inch.

The Acceſs and Receſs of the Retina to and from the Lens not ſufficient to Expedite the foregoing Difficulty.

§. 30. What ſtill Renders this Way of Reaſoning more Obſcure is, that *A H* and *C H* are not Conceiv'd to ſuffer any Refraction, whilſt other Rays, which do not Senſibly Fall with a greater Obliquity, are Subject to a very Conſiderable one, as *C K* and *A G*; or rather in the direct Pencil *B G K E*, Eſpecially in the Rays, which Fall the neareſt to the Axis *B H*, ſeveral are ſuppos'd to be Refracted, that are Apparently Incident with a leſs Obliquity, than the Axes of the Oblique Pencils, which are ſuppos'd not to be Refracted; And in General, it will Exceed an Ordinary Comprehenſion, how the different Pencils, as well Oblique as Direct, ſhould Equally Terminate in their ſeveral Points behind the Lens in *F*, *E*, and *D*, where the Conditions of their Incidence are ſo Various and ſo Diſagreeable to each other; For if the Rays from *B* are ſo Refracted, as to Arrive at the ſame Point in *E*, how the

Not Explain'd, how it comes to paſs, that the Axes of the Oblique Pencils are not Refracted, or how the Rays in different Incidences ſo exactly Arrive at a Point.

Rays from *A* ſhould be Collected in *F*, or thoſe from *C* in *D*, where there is not one Ray that Falls with a like Incidence with thoſe from *B*, will be a Difficulty not eaſily Surmounted; or which is all one, becauſe *B H* Proceeds Directly to *E*, why *C H*, which has a different Incidence, ſhould to *D*, and Conſequently, why the other Rays of the ſame Pencil ſhould, becauſe thoſe from *B* do, or are ſuppos'd to do, will want a better Explanation.

The preſent Scheme is Irreconcileable with the Paralleliſm of Rays Aſſerted in the Refracting of a Lens according to §. 15.

§. 31. Again, it has been before Affirm'd §. 15, that Rays are to be Conſider'd, as Falling in a manner Parallel, which tho' it ſhould give an Account, how the Axes of the ſeveral Pencils Obſerve the ſame Law, in their Tranſmiſſion thro' the Lens, of not being Refracted at all, and how the Rays belonging to each Pencil follow the Conditions of the Direct; yet on the other Hand, it will be altogether Inconſiſtent, that Rays, when we Conſider different Pencils, ſhall be Parallel, and when we Conſider the ſame, ſhall not be, but are ſuppos'd to make up a double Cone, Radiating from one Point, and Collected to another; For there can be no Reaſon Aſſign'd, why *A H*, *C H*, and *B H*, ſhould be Conſider'd as Parallel, and *B G*, *B H*, and *B K*, or *C G*, *C H*, and *C K*, ſhould be Diverging; nor is it eaſily Reconcil'd to common Senſe, how Lines, which Conſtitute an Angle *A H C*, ſhould be Equidiſtant from one another; and yet the Paralleliſm of Rays is made Neceſſary, according to §. 15, to the Refraction of them, thro' a Lens, to a Point.

A Lens does not Collect the Rays to a Point, but to a Circular Focus, which would not Delineate the Image, which is ſuppos'd.

§. 32. Farther, the Diverging Rays are ſuppos'd to be Collected after their Tranſmiſſion to a Point, which is not Manifeſt, or any ways Confirm'd from the Property of the Lens; on the Contrary, we Plainly Experience, that in the Active and Vivid Rays of the Sun, they are only Determin'd to a Circular Focus, which could never be ſufficient to Delineate ſo Perfect and Exact an Image as *D E F*, whoſe ſeveral Points are thought to be Expreſſed from the Radiation of thoſe in the Object *A B C*.

This Reaſoning ſuppoſes the Object in one certain Poſition, whereas it is ſeen in Various ones.

§. 33. Moreover, this Doctrine ſuppoſes *A C* to be Objected to the Lens in a Perpendicular Situation to it, whereas if *A C* is Placed ſeveral Yards Obliquely on one Side, or another, it will by Experience be Equally Viſible, as in that direct Poſition.

The ſecond Principle Alledg'd in §. 27. is, that the Points of the Object are in an Inverted Order to thoſe Impreſſed on the Eye in Reſpect of Length only, whereas the Inverſion is made every Way.

§. 34. Another Argument againſt the preſent Account of Opticks is, that the Points of the Object, and thoſe of the Image, are in an Inverted Order only in Reſpect of Length, that is, *A* is Refracted to *F*, and *C* to *D*, and the like Arrives to the Intermediate Spaces betwixt *A*, *B*, and *C*, *B*; Suppoſing *A B C* a Plane, with a ſmall Plane *e f* croſſing it, the Parallel Lines in the Plane *A B C* will Obſerve the ſame Laws of Refraction with the former, and Conſequently the whole Plane will be Inverted, in Reſpect of it's Length, but it's Breadth will Continue in the ſame Poſition, in the Image, as in the Object; that is, *f* will be Refracted to *f*, and *e* to *e*, and not on the Contrary *e* in the Object to *f* in the Image, or *f* in the Object to *e* in the Image; or, according to this Theory, there will be only an Inverſion one way, unleſs a Ray from *e* or *f* can be Refracted in a different Plane from that of it's Incidence, that is, in Planes not Parallel to *A G H C*; which is contrary to the Fundamental Law of Refraction; whereas in Fact, and by Experiments, made from the Refraction of Light, thro' a Lens oppos'd to a hole made in a Window-Shut, and the Room being Prohibited any other Light, than what paſſes thro' the Foramen, and is from thence caſt by the Lens upon a White Paper, which is held behind it; if there is any Effect Produced by the Lens, Objects Appear to be every way Inverted, and at all Angles, and Determinations, ſo that in every part it is the very Reverſe in the Image, of what it is in the Object, of which it is the Repreſentation.

This Inverſion every Way of the Image upon the Paper is likewiſe Perform'd without the Interpoſition of a Lens.

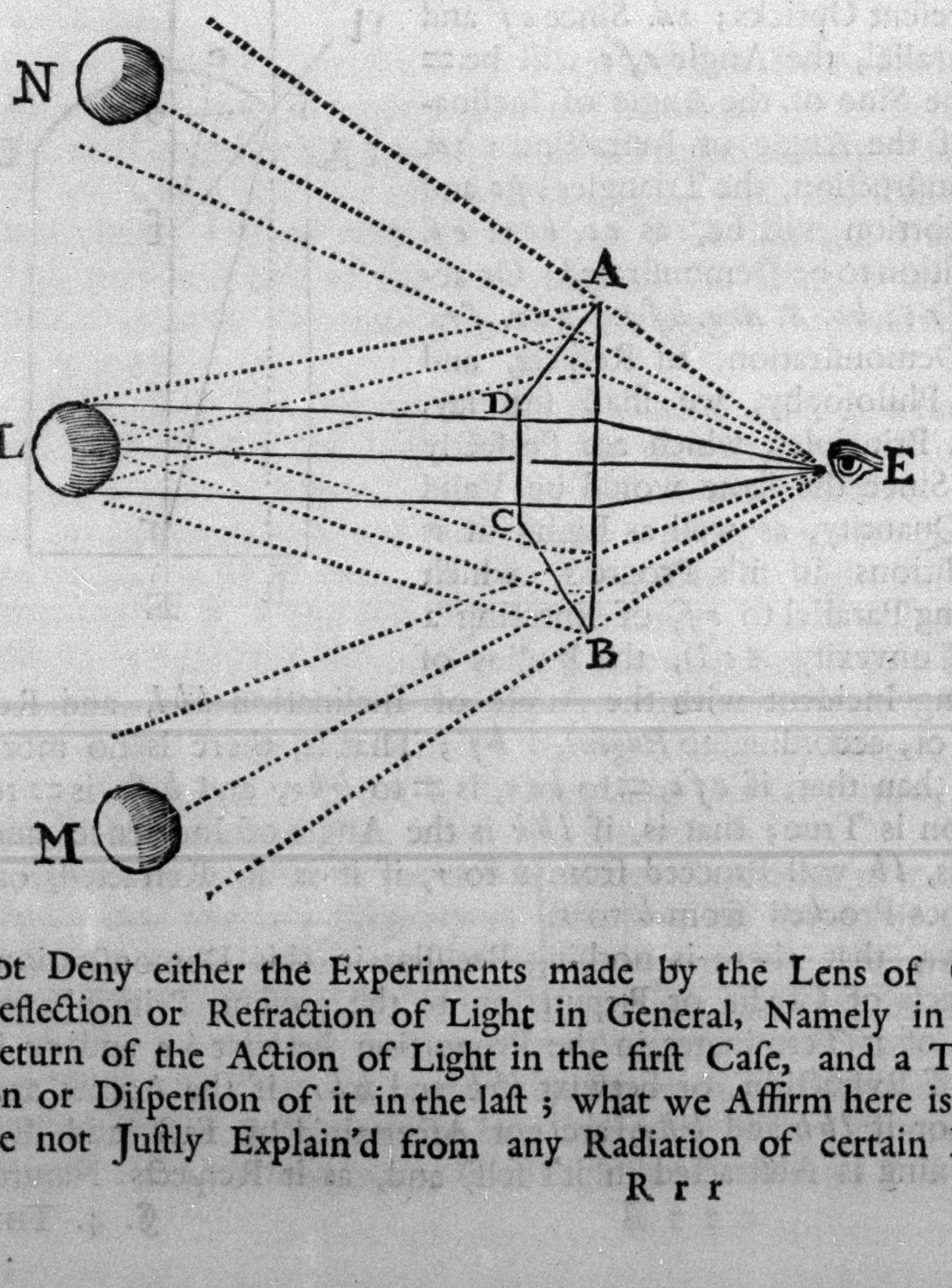
§. 35. What we here juſt now ſaid, is upon the Suppoſition, that the Lens Inverts the Image, but that which is Wonderful in this Doctrine is, that the Refraction of the Lens ſhould be here Aſſign'd, as the Cauſe of this Inverſion, when it has nothing to do in it; ſince it is eaſy for any one to find a Tree, for Example, or any other Object, Imprinted thro' the Foramen upon a White Paper with it's Branches downwards, and thoſe on the Right Hand in the Object will by a Tranſverſe Direction Proceed to the Left Hand of the Image and thoſe on the Left in the one to the Right of the other, without the Interpoſition of a Lens, as this Doctrine ſuppoſes; and if a Lens be Interpos'd, it makes no Alteration, unleſs in Collecting the Image, which is Diffus'd, into a narrower Compaſs, and in Repreſenting it with a greater Clearneſs and Diſtinction.

The Collection of the Rays to a Focus and Divergeing from thence not Agreeable with an Inverſion of the Image by a Lens.

§. 36. Besides, if a Lens had the Property of Inverting the Rays, which Fall upon it, it would be hard to Reconcile it with that other Property, which is Affirm'd of it, of Collecting the Rays to a Focus, which Diverging from thence are at laſt Inverted; but if they were Inverted at the Time, when they came to their Focus, after their Croſſing each other there, they ought to Reſume their Natural and Original Order, which they could not do, if they were Inverted before-hand; From all which it is Evident, what little Reaſon we have to Truſt to, and Confide in Experiments, made to ſerve an Hypotheſis, which we are Reſolv'd to Maintain, ſince theſe of the Lens, which are Fundamental to the preſent Theory of Opticks, are ſo ſtrangely Perverted to ſupport it.

The laſt Principle Aſſum'd §. 27. is, that the Eye Purſues the Rays back in the Direction in which they were Incident. Rohault Phys. Cap. 33. §. 2. Par. 1.

§. 37. Lastly, it is Inconceivable, how the Eye ſhould Recoil upon the Action, which is Impreſſed, ſo as to Purſue the Rays according to the Direction, in which they Fell; It is a known Experiment, that in Polyedral Glaſſes, of ſeveral Cuts or Faces, as *AD*, *DC*, *CB*, and where the Rays Fall upon each of them from *L*, and are Refracted to the Eye in *E*, by it's following back the Rays according to the Direction with which they were Incident, it ſees three Objects, for Inſtance, inſtead of one; And for the ſame Reaſon, in the Preceding Figure, by the Eye's Referring it's ſelf back in the ſame Line *CHD*, in which the Action was Communicated to it, *D* is ſeen in *C*, *F* in *A*, by the Line *FHA*, and *E* in *B*, by *EHB*; But why it ſhould not make Uſe of the ſame Reference according to the Direction of *FG*, *FK*, *EG*, *EK*, and *DG*, *DK*, and Conſequently ſee the Object much beyond the Limits of *A* and *C*, as well as Reſign it's ſelf to the Conduct only of the Axes of the Pencils is not, and I believe cannot be Explain'd; If the Eye does Purſue the Rays, which are Refracted, as well as thoſe, which are not, this Philoſophy is not Juſt; becauſe in ſuch a Caſe *AC* would appear much bigger at leaſt, than it is ſuppos'd to do, that is, we ſhould ſee an Object as large as the Extent, which from *AC* would lie betwixt *DK* and *FG* Prolong'd; however this is, ſince notwithſtanding, theſe Principles take no Notice of any Rays beyond thoſe which reach to *A* and *C*, it is plain the Eye is not ſuppos'd to Refer it's ſelf back in *DK* and *FG*, which why it ſhould not do, as well as in *DH* and *FH*, we want to be Inform'd; What has been ſaid Concerning the Rays *DK* and *FG*, in the former Figure, is Applicable to *EA* and *EB*, as alſo *DE* and *CE* in the Preſent, which if the Eye Purſu'd back according to their Incidence, the Object would not be only Multiply'd into three, but each of them would be Exceedingly Increaſed in their Magnitude and Dimenſions, which yet is not Affirm'd in theſe Opticks but rather the Contrary, and is Confuted by Experience, by which we find Objects to be not Increaſed but Diminiſhed in ſuch Polyedral Glaſſes.



The Phænomenons in Opticks not Deny'd but the Principles, upon which thoſe Appearances are Explain'd.

§. 38. In Concluſion, whilſt we Conteſt the Principles of Opticks, we do not Deny either the Experiments made by the Lens of Collecting Light, or the Reflection or Refraction of Light in General, Namely in one Senſe, that there is a Return of the Action of Light in the firſt Caſe, and a Tranſmiſſion and Contraction or Diſperſion of it in the laſt; what we Affirm here is, that theſe Phænomenons are not Juſtly Explain'd from any Radiation of certain Points either Reflected or

Refracted, and which we ſhall Endeavour farther to Prove from the other Phænomenons, which remain in Opticks.

Besides which we ſhall only at preſent Add, that, if the Doctrine, Propos'd in the foregoing Section, was True, there is no Account given by what Law or Property of Nature the Eye Purſues back the Rays, which Affect it, and in that Direction, in which they Fall; This is a Myſtery not thought fit to be Dived into, and therefore we muſt not Require an Explanation of it. But if we are willing to admit of the Principles, I have Advanced, the Expanſive Force will Furniſh us with a Solution of the Eye's Reacting upon the Rays and of their Diſperſion, and the Contractive with one of their Collection and Contraction.

Chap. VI.

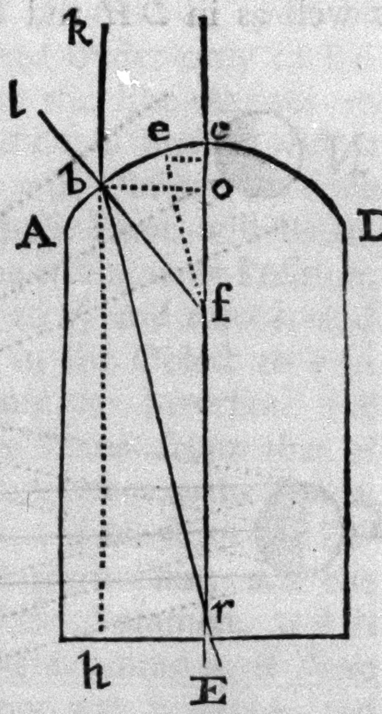
Concerning the Propoſitions in Opticks, which Relate to Glaſſes, which are either double Convex, or Plano-Convex Ones.

Opticks, in the Demonſtrative Part of them, nothing elſe than a Science of Mathematick Quantities, Abſtracted from Things as they Exiſt.

§. 1. After Conſidering the Foundations of the preſent Science of Opticks, we ſhall, in this Chapter, Proceed to the Propoſitions Advanced in it; Which ſince, as we have Prov'd, the Hypotheſes, on which it Depends, are not Anſwerable to Nature, the Deductions from them can be no other, than the Aſcertaining the ſeveral Proportions of Quantities in their Compariſon with each other, which were Originally Abſtracted by the Mind, without any Conformity of them to the Real Eſſences of Things, or the True Properties of Light, and the Mediums of Glaſs, or Water, or Air, with which it's Action is Compared.

The Determination of a Ray, Falling on a Spherical Glaſs, Conſider'd and Explain'd. Molyneux's Dioptric. Prop. 1. Part 1. Hugens's Dioptric. Prop. 8. Probl. 5. and Prop. 2.

§. 2. The firſt Propoſition therefore in this Science is; That when a Ray *kb* Falls on a Spherical Glaſs *AcD*, and after it's firſt Refraction Croſſes the Axis *cE*, it will be; As the Sine of the Angle of Refraction to the Sine of the Angle of Inclination, So the Radius of the Glaſs's Sphere to the Refracted Ray. For, 1*ſt.* ſince *kh* and *cr* are Drawn Parallel, *brc* will be Equal to the Angle *hbr*, which is that of Refraction, according to the Definition us'd in the preſent Opticks; 2*d.* Since *ef* and *br* are likewiſe Drawn Parallel, the Angle *efc* will be = to *brc*, and *bo* will be the Sine of the Angle of Inclination, and *ec* the Sine of the Angle of Refraction; 3*d.* Since therefore, by this Conſtruction, the Triangles *efc* and *bro* are Similar, the Proportion will be, as *ec. bo :: ef. br.* which was the Propoſition to be Demonſtrated; Or according to *Hugens*, as *cr. fr :: bo. S. Ang. bfe :: br. fr.*



This Demonſtration only Affirms what is to be Prov'd.

§. 3. Against which Demonſtration, in Reſpect, and with Relation to a True Philoſophy, we ſhall firſt ſay, that it only Proceeds upon Principles, which are Perfectly Abſtracted by the Mind; Since the ſame would be Valid Concerning any Body or Quantity, as well as Light, if it Obſerv'd the ſame Conditions in it's Progreſs, which are here ſuppos'd, of Falling Parallel to *cf*, of Entering a Medium, which has the Convexity *AcD*, the Radius of which is *ef*, and of being Incident with the Angle of Inclination *kbl*, and Refracted in the Angle *hbr*, or, according to *Hugens*, *rbf*; That is, there is no more ſaid in this Demonſtration, than that, if *efc*, = to *brc*, is = to *hbr*, and *bfc* is = to *lbk*, the preſent Propoſition is True; that is, if *lbk* is the Angle of Incidence, and *hbr* of Refraction; that is, *lb* will Proceed from *b* to *r*, if it is ſo Refracted, or, which is the ſame, if it does Proceed from *b* to *r*.

From whence it follows, that there is nothing Peculiar in this Demonſtration, which ſhould give any Acceſs of Credit or Reputation to the preſent Principles of Opticks; Since the whole of it Terminates in the Proportion betwixt *bo* and *ec* if the Sines are the Meaſure of Refraction, or betwixt *lbk* and *hbr*, if the Angles are.

If therefore *ec* and *bo*, or if *lbk* and *hbr*, are not Aſcertain'd by Fact and Experience, this whole Reaſoning is Abſtracted in it's ſelf, and, as it Reſpects Nature, Invalid and Inconcluſive.

§. 4. The Meaning of the former Section in Part is this, that in the Glaſs Cylinder ADE, in which the Refraction is made, the Medium ADE Continuing the ſame, and the Convexity of it's Superficies AcD, and Conſequently the Radius of it cf, it is altogether Inconceivable, that the Incident Ray kb Falling with different Forces, as well as Inclinations, ſhould be Refracted to the ſame Point r, and therefore that the Rays which are Parallel to the Axis and Refracted in Glaſs, ſhould be United with the Axis at a Certain and Determin'd Diſtance of near a Diameter and half of the Convexity, ſuppoſing the Glaſs to be a Portion of no more, than of about 30 Degrees of it, which Certain and Determin'd Diſtance, notwithſtanding, this Propoſition Affirms; If therefore it is found by Experience, that there is a Determination of the Focus at the Diſtance of near a Diameter and half of the Convexity of the Glaſs, it is yet Certain, that this Demonſtration does not give any Account of it; If it is not found by Experience, it is nothing elſe but an Abſtracted Reaſoning, and Fails of that Accuracy, which might be Expected from it.

The Rays of Light may be different in their Force, and Conſequently the Sine ec *or the Angle* efc.

§. 5. If yet the Refractions of all Kinds of Light, whether more or leſs Vivid and Strong, Reacht to r, we ſhould Probably find ſome Difference in the Glaſs at r from any other Part of it; There would be ſome Particular Brightneſs or other, we may Imagine, in that Place, which as this Doctrine of Opticks does not Aſſert, ſo we may Reaſonably Conclude it does not Appear; and, that Conſequently this is nothing more than a Reaſoning from Abſtracted Notions, and not a Conviction of the Reality of Things, as they are found to Exiſt without us.

There would be ſome Remarkable Brightneſs in that certain Point, if this was True.

§. 6. For moreover, according to this Demonſtration, ſuppoſing the Glaſs was to Terminate in f, there will be a whole Diameter of it's Convexity, in which the Light would have Proceeded in the ſame Medium had it been Continu'd, and Conſequently would have Force enough to be Collected to a Focus in a different one as Air, notwithſtanding the Thickneſs of the Glaſs cf; which yet is not Manifeſt from Experience, that a Glaſs, whoſe Thickneſs is the Semidiameter or the Diameter of it's Curvity, will Cauſe, or any ways Produce, ſuch a Collection of the Rays to a Focus, ſince half a Globe, or a Globe, of Glaſs will not do it, unleſs it is of a Determined Magnitude; Whereas this Demonſtration ſuppoſes the Radius cf to be as large as we Pleaſe.

Globes of Glaſſes or the half Globes of them will not Probably Collect Light to a Focus, if their Diameters are very Large.

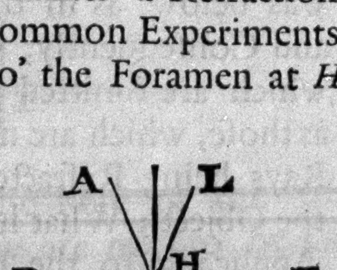
§. 7. Lastly, if $ec.\ bo :: ef.\ br$, which this Demonſtration Preſumes, ſeeing ec and bo will be chang'd according to the Incidence of kb being nearer or farther off from cf, the Length of the Line br will be Proportionably Alter'd, and Conſequently that of cr, which will not in the leaſt Agree with the ſettled Rule in Opticks of the Rays being United with the Axis, at a Certain and Determin'd Diſtance of near a Diameter and half of the Convexity of the Glaſs, as is Affirm'd in §. 4. But that which is more Surprizeing is, that in *Hugens*'s Demonſtration $cr.\ fr :: br.\ fr$, that is, cr is $= br$.

The Rays of Light may be different in their Incidence, upon which Account the Sine bo *may be Alter'd.*

§. 8. And here we cannot but in General Add, that the Refraction in Glaſs, or Water, in the Paſſage of Light from Air thro' it, or the Contrary, Wants a farther Explanation; It is Affirm'd, that the Firſt, Namely Glaſs or Water, makes a Refraction to the Perpendicular, and the Laſt, to wit Air, from it; And the common Experiments to Aſſure us of this are, 1ſt. that AH a Ray of Light Paſſing thro' the Foramen at H Proceeds to B, if the Box $DFEG$ be Empty, but if Filled with Water, is Refracted to C; 2d. *Fig.* §. 9. that ſuppoſing DCE a Baſon, which is Empty with a Piece of Silver, or any other Object, Placed at C, it will be ſeen in B, or M, but if Filled with Water, in A, A, and Conſequently the Refraction is, in this Caſe, made from the Perpendicular.

The Law of Refraction in the Paſſage of Light from Air thro' Glaſs or Water, in General Conſider'd.

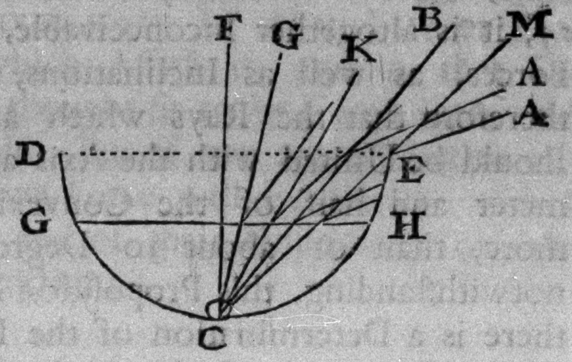
Rohault. Phyſic. Cap. 27. §. 38. 39. 40. Par. 1.



§. 9. In Anſwer to the firſt of which Experiments it is Evident, that this is a Property or Affection of the Solar Light, when it Falls in Full and Pointed Beams on the Foramen H, (*Fig.* §. 8.) and tho' it ſhould be True in ſuch an Obliquity of AH to DH, this yet is not a Standard, by which we can either Meaſure the Actions of the Diffus'd Light of Day on Glaſs, or Water, or even of the Perpendicular Rays of the Sun; 1ſt. As to common Light, it is ſuppos'd to be Directed to H in Columns of all Incidences, which Indeed are only Imaginary, ſince Light is a Continued Diffuſion of Æther Actuated by the

The Refraction of Light thro' Glaſs or Water not Explain'd from Experiment the Firſt of the foregoing Section.

the Sun's Expanſive Force, and if they were Real, would Confute a Refracted Radiation; For in Reſpect of the firſt, who can Conceive Rays of Light, which ſeem to Float Undiſtinguiſht in the Enlighten'd Medium, who can Imagine them like ſo many Active Lines Tending from *A*, and *L*, to *H*, *Fig.* §. 8. without any Apparent Cauſe in *A* or *L*, which Impells them; If this notwithſtanding were Eaſy to Apprehend, it would Follow ſecondly, that Light in it's Refraction, would take up leſs Space than in it's Incidence, and if *D F* was wholly Illuminated, *G E* either ought not to be, and therefore ſome Part of the Latter would not be Tranſparent Contrary to Experience, or elſe by the ſeveral Contractions of *M B* to *N C*, that is, by the Gradual Approaches made thereby to a Focus, each Part of the Line *G E* would be more Bright and Luminous than that of *D F*, which likewiſe is Contrary to what we Obſerve, ſince after Refraction the Light is rather Impaired and Weakened.

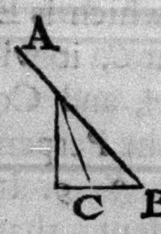


2d. As to what Concerns the Perpendicular Rays of the Sun, or of Light in General, which Fall Directly upon the Surface *D F*, it is Impoſſible for them to be Refracted to a Perpendicular, and yet as Improbable they ſhould Proceed in one; for the Conditions of the Mediums, thro' which they Paſs, being Different, it is not Agreeable to Think they ſhould Obſerve the ſame Laws; Thus a Stone, let Fall Perpendicularly into Water, will find it's ſelf Carried in another Line, than what it Deſcrib'd before it's Entrance into it. All which Difficulties are Accounted for, §. 32. *Chap.* 4.

The Refraction of Light from Glaſs or Water thro' Air, not Explained from Experiment the Second. §. 8.

§. 10. The other Experiment, by which it is Prov'd, that Light is Refracted from the Perpendicular in it's Tranſit from Water to Air, tho' it is True, is not Concluſive of that, which is Intended to be Evidenc'd by it, that is, the Refraction of Particular and Diſtinct Rays, which are ſuppos'd to be Emitted from the Object *C*, *Fig.* §. 9. For beſides what has been already Argu'd Concerning the Hypotheſis of Radiation in the foregoing Chapter, we ſhall here Subjoyn; That if all the Rays, which are ſent from *C* towards *B*, are Bent and Inclin'd to *A*, that we ſee the Object at all when the Eye is Poſited in *A*, muſt Depend upon the Suppoſition of the Eye's Purſuing back the Rays according to the Direction, in which they Fall, which, if it is Matter of Fact, is however, as we have already ſaid, not Explain'd; But moreover ſuch a Refraction could not Alter the Number of the Rays, and only Change the Poſition of them to the Object; Whereas by Refraction we do not only ſee Objects in a different Situation, but likewiſe in a different Magnitude, and even when the Eye is in *F*, where the Rays are ſuppos'd to be Perpendicular; Laſtly, if the Refraction of Particular Rays was only to be Conſider'd, we ſhould ſee the Object in *C* no other ways, than by filling the Baſon with Water up to *D E*; for ſince we looſe the Sight of the Object *C* by the Interpoſition of the Side of the Baſon *E C*, that is, when the Eye is below *C M*, it would be Impoſſible for *C M* to be Refracted to the Eye, unleſs the Surface of the Water was even with the Margin or Brim of the Baſon; for if it was below, as in *G H*, the Rays would be Refracted to the Sides of it, betwixt *E* and *H*, and Conſequently, would never Reach the Eye, contrary to Experience, ſince the Rays, which are Emitted from *C* nearer to *C F*, as *C G* and *C K*, will be Refracted above *A*, as thoſe, which are nearer to *C M*, are below it; Not to Mention the Abſurdity of fewer Rays being Refracted to the Eye, Cauſing an Increaſe in the Magnitude of the Object; What has been Propos'd in the foregoing Sections, is equally Applicable to the Experiments of *Kepler*, Particularly of a Cube of Chryſtal Adapted to an Opake Body, by which it appears, that the Shadow, made by the Glaſs, is Contracted, and the Adjoyning continued beyond it, as far as the Rays without Refraction Extend, and may be thus Expreſs'd, as before in Section 8*th*, *A B* is the Unrefracted Ray, and *A C* the Refracted. Which Difficulties are likewiſe Solv'd, §. 32. *Chap.* 4.

Kepler. Dioptric. Problem 4.



The Determination of a Ray Falling on the Convex Side of a Plano-Convex, Conſider'd and Explain'd.

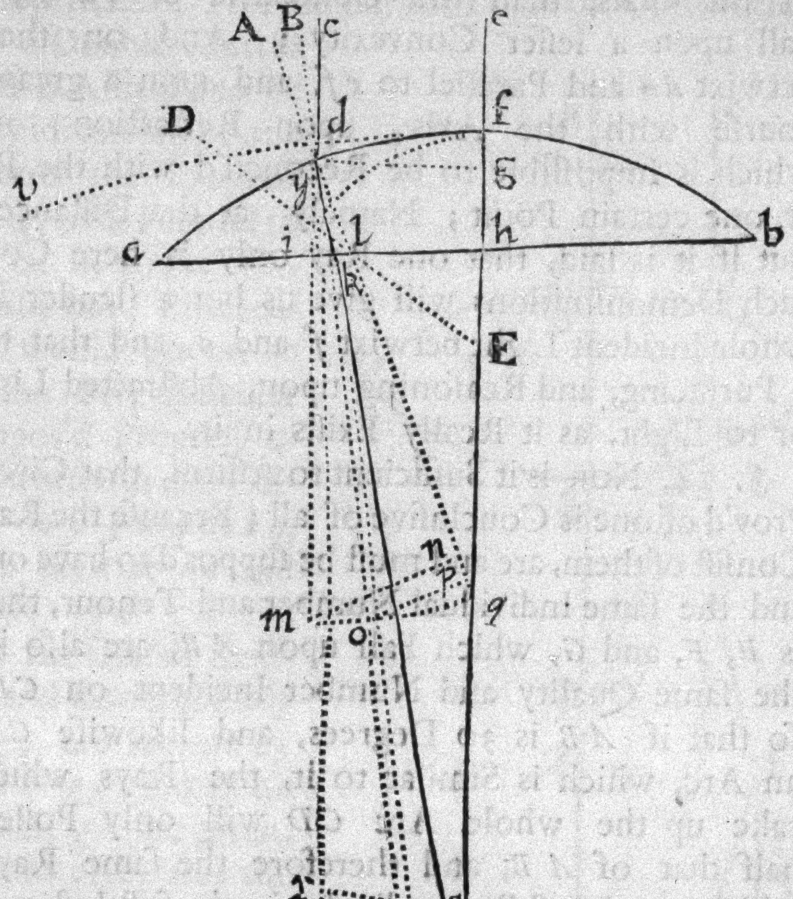
Molyneuxs's Dioptricks Prop. 2. Par. 1. and Schol. Hugens's Diopt. Prop. 14.

§. 11. We ſhall therefore, after having Premis'd theſe general Conſiderations, Proceed to the Second Propoſition in the Science of Opticks or Dioptricks, which is; That ſuppoſing a Glaſs to be Terminated by two Superficies, the one a Convex as *a f b*, *Fig.* §. 12. and the other a Plane *a h b*, and the Incident Ray to be *c d* Parallel to *e f*, the Courſe of the Ray upon Refraction will be Determin'd to *z*; That is, Parallel Rays from any Object to the Axis are United with it about the Diſtance of a Diameter of the Convexity from the Pole of the Glaſs, if the Segment be but 30 Degrees;

In Order to Prove which, there is nothing more Conſider'd, than that *c d* will be Refracted to *k*, and Conſequently the Sines of Incidence and Refraction will be Repreſented by *m n* and *o p*, if *d* is the Center of the Arc, *m q*, and the one is made to the other as 14 to 9, and *d k* will be likewiſe Refracted to *s*, ſo as *t u* the Sine of Incidence, on the Contrary, may be to *r s* the Sine of Refraction, as 9 to 14, *k* being the Center of the Arc *x s*, by which Means *k s* will meet the Axis in *z*; from whence it is alſo Prov'd, that the Point of Concourſe *z* is leſs or more Remov'd from the Pole of the Convexity *f*, as the Glaſs is leſs or more Thick, but cannot Exceed a Diameter and half, ſince it would meet with the Axis at that Diſtance, if there was only one Refraction, by the firſt Propoſition.

This Demonſtration does not give an Account, why the Rays ſhould be Parallel.

§. 12. That this Demonſtration alſo is only an Abſtracted one, and does not Extend to the Reality of Things, as they Exiſt, is Manifeſt on ſeveral Accounts; The firſt is, that the Rays are ſuppos'd to be Parallel, which are Incident, and yet to Proceed from an Object, whoſe Points were before Preſum'd to Emit certain Pencils of Rays in the Form of Cones, and therefore not Parallel, but Diverging; upon the laſt Hypotheſis this Demonſtration will not be True, and upon the former, that the Incident Rays are Parallel from the Diſtant Objects, from which they are Deriv'd, it muſt be Enquir'd, how Objects at a Diſtance Radiate in Parallel Lines, which we have already ſhewn, there is no Reaſon to Affirm.



Nor yet how different Curvities of the ſame Convex ſhould Refract Rays to a Certain and Determin'd Point.

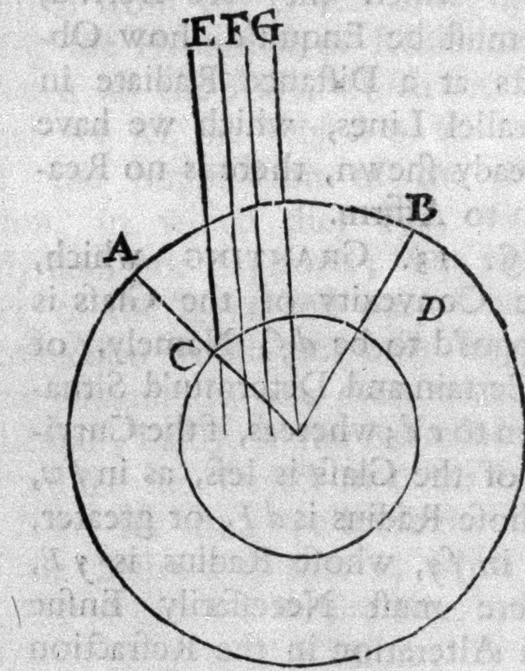
§. 13. Granting which, the Convexity of the Glaſs is ſuppos'd to be *d f*, Namely, of a Certain and Determin'd Situation to *c d*; whereas, if the Curvity of the Glaſs is leſs, as in *f v*, whoſe Radius is *d F*, or greater, as in *f y*, whoſe Radius is *y E*, there muſt Neceſſarily Enſue an Alteration in the Refraction of the Incident Rays; ſince, in the firſt Caſe, they will be little different from ſo many Perpendiculars *e f*, which are not ſuppos'd to be at all Refracted, and therefore will not meet the Axis *e f* in any Diſtance from the Pole of the Glaſs, much leſs at the Diſtance of the Diameter from it; if yet they are not to be Conſidered in this Caſe as Perpendiculars, at leaſt *B d c*, which will be the Angle of Inclination, will be very different from *A d c* Aſſum'd in this Demonſtration, that is, it will be much leſs, and Conſequently, the Sine of it will be much leſs than *m n*, the Sine of the Angle *A d c*, by which Means the Point of Concourſe of the Refracted Ray with the Axis will be much farther Remov'd from the Pole of the Glaſs, than is here Defin'd; On the other Hand, in the ſecond Caſe, where the Curvity of the Glaſs is greater, *D y c* will be the Angle of Inclination greater than *A d c*, the Angle Aſſum'd, and Conſequently the Sine of it will be greater than *m n*, by which Means the Concourſe of the Refracted Ray with the Axis will be much nearer the Pole of the Glaſs than is Expreſſed in the preſent Figure; Or, according to this Demonſtration,

monſtration, ſince the Radius *fE* of *yf* the greater Convexity is ſhorter than the Radius *qf* of the Convexity Aſſum'd *af*, and the Radius *fF* of the Leſſer Convexity *vf* is longer than the ſame Radius, and ſince the Concourſe of the Refracted Ray with the Axis is at a Diſtance of a Diameter of the Curvity of the Glaſs from the Pole of it, it is Neceſſary by the Acknowledgements of this Philoſophy, that the more Convex the Glaſs is, on which the Incident Ray Falls, the ſooner it ſhould meet with the Axis upon Refraction, and the leſs Convex, that the Point of Concourſe ſhould be farther Remov'd.

If therefore what foregoes is either True in it's ſelf, or Acknowledg'd by this Philoſophy, it is as certain, that the Rays, which Fall betwixt *fd* and Parallel to *ef*, will either be like Perpendicular ones, and therefore not Refracted at all, or at leaſt their Concourſe with the Axis, upon Refraction, will be farther Remov'd from the Pole of the Glaſs, than the Concourſe of *cd* is, upon it's Refraction, Becauſe they Fall upon a leſſer Convexity; And on the Contrary, the Rays which Fall betwixt *da* and Parallel to *ef*, and upon a greater Convexity, will have their Concourſe with the Axis, upon Refraction, nearer to the Pole of the Glaſs, which is Impoſſible to be Reconcil'd with the Rays Meeting, upon their Refraction, at one certain Point; Namely, at the Diſtance of a Diameter of the Convexity; But if it is ſaid, that one Ray only is here Conſider'd, as *cd*, it is Evident, that ſuch Demonſtrations will give us but a ſlender Information of the Properties of the whole Incident Light betwixt *f* and *a*, and that the Science of Opticks is no other than a Purſueing, and Reaſoning upon, Abſtracted Lines, without any Regard to Nature, or to Light, as it Really Exiſts in it.

The different Convexities of the ſame Circle cannot have the ſame Individual Number of Rays Incident upon them, which yet ſeems to be ſuppos'd.

§. 14. Nor is it Sufficient to Affirm, that Circles are Similar, and therefore what is Prov'd of one is Concluſive of all; Becauſe the Rays of Light, whilſt Light is Aſſerted to Conſiſt of them, are and muſt be ſuppos'd to have one and the ſame Individual Number and Tenour, that is *E*, *F*, and *G*, which Fall upon *AB*, are alſo in the ſame Quality and Number Incident on *CD*, ſo that if *AB* is 30 Degrees, and likewiſe *CD* an Arc, which is Similar to it, the Rays which take up the whole Arc *CD* will only Poſſeſs half that of *AB*, and therefore the ſame Rays, which are almoſt Perpendicular in the firſt Inſtance, may be Extremely Oblique in the Second, and therefore muſt admit of thoſe Variations in Refraction, which are Expreſſed in the foregoing Section; On the Contrary, to make what is Demonſtrated of one Circle Applicable to all others, it is Preſum'd, that there are as many Rays Fall on a Leſſer as on a Larger Convexity, that is, that in *AB* Light is Diffus'd, and in *CD* Contracted, Proportional to the Arcs, which it Illuminates; An Hypotheſis, which it is Impoſſible ſhould be any where elſe, than in the Head, that Forms it.



The Confining this Demonſtration to an Angle of 30 Degrees is a Proof, that an Abſtracted Reaſoning, and not Nature is Conſidered.

§. 15. We muſt farther Add, that the Segment of the Circle in this Doctrine is ſuppos'd to be only 30 Degrees, and not to Exceed it; The Reaſon of which is Alledg'd to be this, becauſe ſo far the Sines and their Angles are nearly Proportional, a Double Angle having a Double Sine, and a Triple in like Manner a Triple one, Anſwerable to it; All the Refractions of Light beyond ſuch an Arc are either therefore Reducible to this Demonſtration, or not; If they are, there is no Occaſion for Specifying the Particular Segment Requiſite to it; If not, it is Evident, that ſuch a Proof is a Thing of Abſtraction in the Mind, and not Adapted to the Explanation of Nature.

This Demonſtration Contradicts a Receiv'd Axiom in Opticks.

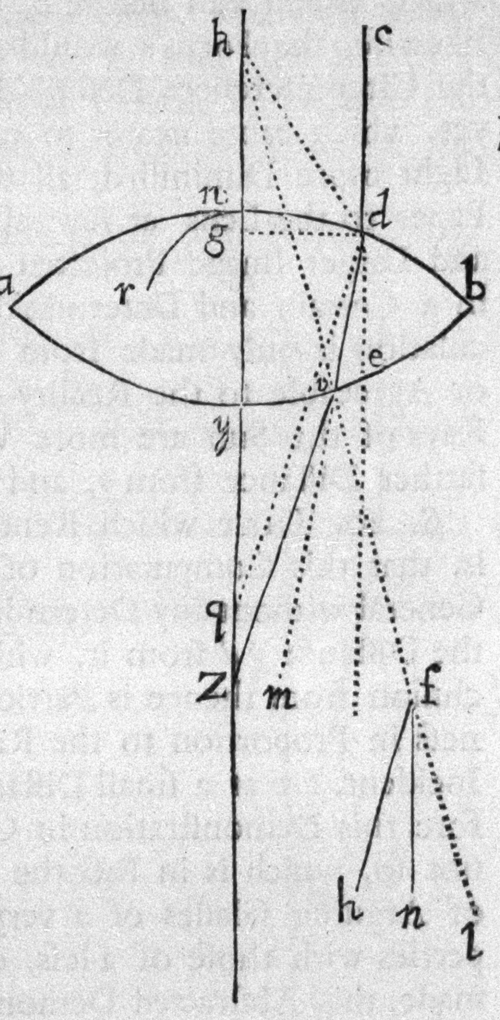
§. 16. Besides which, If the Refraction of Light is in General to be Meaſur'd by the Sines, let the Angles be Proportional to them, or not, the laſt of which is Plainly Aſſerted by this Philoſophy, Namely, that they are not, there can be no Reaſon to Confine a Demonſtration to a certain Segment only, in which the Sines and Angles are Proportional.

§. 17. Lastly,

What the Concluſion from the Diſtance of the Point of Concourſe being different, as the Glaſs is more or leſs Thick.

§. 17. Lastly, it is ſaid, that the Point of Concourſe z is nearer to, or farther from f, as the Glaſs is of a leſs or a greater Thickneſs; Becauſe, as the Glaſs is Thicker, dk will be the longer before it is Refracted, and therefore will not meet fz ſo ſoon, as it would otherwiſe do; From whence it is Manifeſt, and Acknowledg'd by that Demonſtration, which is given, of the Point of Concourſe being at the Diſtance of about a Diameter of the Convexity from the Pole of the Glaſs, that it may be at near a Diameter and half from it; Which is a ſufficient Conviction, that ſuch a Demonſtration only Purſues the Evidence of it's own Principles, and is This or the Other, as They Direct, without any Reſpect had to the Real Principles of Philoſophy, or the Affections of Nature; Since for the ſame Reaſon, if the Glaſs is Thinner, the Point of Concourſe may be leſs Remov'd, than at the Diſtance of the Diameter of the Convexity.

The Determination of a Ray Refracted thro' a double Convex Conſider'd and Explain'd. Molyneux's Dioptricks Prop. 3. Part. 1. Hugens's Dioptricks Prop. 16.



§. 18. A Third Propoſition in Dioptricks is, To Find out the Point of Concourſe of the Ray cd, after it's Refraction thro' a Double Convex Glaſs ab, with the Axis kz; Which is Affirm'd to be at the Diſtance of the Radius or Semidiameter of the Convexity ab; As to what Concerns the Progreſs of the Incident Ray cd, which is ſuppos'd Parallel to the Axis kz, it is Refracted to i, that is, to the Perpendicular qd, for otherwiſe it would have Proceeded to e; After it's Arrival at i, it would go on to m, if it were not Refracted from the Perpendicular kil to z; This therefore is the ſuppos'd Progreſs of the Ray cd; The Determination of which in z is Calculated in the following Manner; $ky-wy+wq=kq$, and gd is the Perpendicular Diſtance betwixt cd and kg; But in the Rectangle Triangle qgd, gd the Diſtance and qd the Semidiameter of the Glaſs's Convexity being given, the Angle gqd, and Conſequently qde which is $=$ to it, and the Side gq, are known by Trigonometry, and therefore kg, $=$ to $kq-qg$; Likewiſe in the Rectangle Triangle kgd, kg and gd are given, and Conſequently the Angle gkd, $=$ to cdk, and the Side kd, are known; Farther, as 300. 193 :: *S. Ang.* qde. *S. Ang.* qdi, which is therefore alſo found out by this Proportion, but $qde-qdi$ is $=$ to edi, and $180-kdc-edi=$ to kdi; It is alſo Evident, that in the Triangle kdi, the Angle kdi, and the Sides ki, and kd, being known, the Angle $kid=mil$ is found, and the Angle ikd, and by Conſequence qki, which is $=$ to $qkd-dki$; Again, 193. 300 :: *S. Ang.* kid. *S. Ang.* zil, which therefore is Determin'd by that Analogy, as is kzi likewiſe, being $=zil-qki=hfl-nfl=hfn=kzi$, which is Manifeſt by drawing hf Parallel to zi, and nf to kz; Laſtly, in the Triangle zki, the Side ki and the Angles kzi and ikz, being known, the Line kz will be found, and therefore yz, $=kz-ky$, the Quantity Deſir'd; That is, the Diſtance betwixt the lower Convex, and the Focus, in which the Rays are Collected, will be Determin'd by this Calculation, which, if the Thickneſs be Inconſiderable in Reſpect of the Radius of the Convexity of the Glaſs, and the Diſtance of the Point of Incidence from the Axis be likewiſe Small, will be about the Length of the Radius of the Glaſs's Convexity from it; whereas in a Plano-Convex the Diſtance of the Focus from the Glaſs was near double that Length.

If this Demonſtration was True, the Rays would be Collected to Various Focus's, and not in Circles but in Points.

§. 19. As to what Concerns the Simple Progreſs of the Rays, Expreſſed in the Beginning of this Demonſtration we have already Conſider'd it, and ſhewn the Incongruity of ſuch an Hypotheſis of Rays, upon which this Calculation is Entirely Form'd; We ſhall now therefore Examine into the Calculation of the Particular Diſtance of yz here Aſſerted, which is by finding out from the Data, *viz.* gd, the Diſtance of the Incident Ray from the Axis, the Right Angle at g, and the Radius of the Glaſs's Convexity qd, qg the unknown Line, from whence kg is known; And by that Means all the Lines and Angles of kgd are Manifeſt; qdi by the Proportion of Refraction is likewiſe Determin'd, and Conſequently the Angle kdi, and therefore all the Angles and Sides of that Triangle, as well as the Angle mil, which is Vertical to kid, and by the Proportion of Refraction,

ion, zil, from which the Angle kzi is found and in Particular the Side kz, and yz that Portion of it, which is ſought. From which ſeveral Steps of Inveſtigating yz it is Plain, that one certain Ray cd only will Arrive at the Point z, and another on the Oppoſite Side of the Axis ky, which is Incident at an Equal Diſtance with gd; ſince, as the Diſtance gd is Continually Chang'd from b to the Axis ky, ſo will the Angle gkd be Alter'd, and the Conditions of all the Triangles, by the Intervention of which yz at laſt comes to be known, and which will therefore be Variable in every Approach of cd to the Axis, or it's Removal from it; And Conſequently, ſuppoſing the Triangle kgd to be Revolv'd upon it's Axis kg, and ab to be the Portion of a Sphere, the Point d would Deſcribe a Circumference on that Sphere, on which the Parallel Rays cd Falling would be Collected to z, and no others; From whence it would follow, that gd, by being Enlarg'd or Diminiſh'd, would Proportionably Deſcribe different Circumferences on the Sphere ab, of a longer or ſhorter Diameter, from which the Rays would be Collected to Various Focus's, ſome of which would Fall below z, and none Riſe above q; According to this Reaſoning likewiſe, the Focus's would end in Abſolute Points, one Brighter than another, as the Circumferences Deſcrib'd by gd were Larger, or if they were Circular, thoſe yet, which came nearer to q, would be leſs and their Light Equal, or Equal and their Light more Diminiſh'd, all which is Contrary to Experience; Since if we Oppoſe a Paper to the Lens at ſeveral Diſtances from y to z, there is Evidently a Circular and Larger Image Projected nearer y, and a Leſſer and Brighter farther from it, within a Certain and Determin'd Diſtance; which is a Plain Conviction, that ſuch a Calculation is only made from an Abſtraction of the Mind, and is not any ways Suited or Agreeable to the Reality of Nature, or the Phænomenons of it; Moreover as the Rays of the Sun are more Vivid and Strong, the Image or Focus is Projected at a farther Diſtance from y, and They are more Weak and Languid at a ſhorter.

The Convexity of the Glaſs and the Diſtance of the Incident Ray from the Axis not Determin'd.

§. 20. THAT which Renders this kind of Philoſophy ſtill more Incomprehenſible is, that this Computation of the Diſtance of the Focus from the Glaſs Proceeds in General without any Determination of the Convexity adb, or of the Radius ky, or of the Diſtance gd from it, which are left for us to chooſe at Large; Whereas the Concluſion from thence is Particular; Namely, in Glaſſes, that have little or no Thickneſs in Proportion to the Radius of their Convexity, and where the Rays, which are Incident, are at a ſmall Diſtance only from the Axis of the Convex: Either therefore this Demonſtration in General is not True, or elſe the Deductions from it are not ſo, which is in Fact the very Caſe; for if there were no Reſtrictions, in this way of Arguing Glaſſes of a very great Convexity as wr, would have the ſame Properperties with thoſe of a leſs, contrary to Experience; and yet, if any Limitations were made, this Abſtracted Demonſtration would not know, where Preciſely to Fix them, and is therefore Oblig'd to Reaſon upon gd and ky, without any Determination of what they are, unleſs of their being in General Given.

This Demonſtration if True, muſt allow that, the greater the Convexity is the Incident Rays will be the nearer to the Axis, and yet Aſſumes a nearneſs of Rays and a leſſer Convexity.

§. 21. BUT that which is ſtill more Wonderfull to Comprehend is, that in the Particular Concluſion from this General Propoſition, the Convexity of the Glaſs is Requir'd to be leſs, and the Incident Rays to be at a ſmall Diſtance from the Axis, whereas, if the General Propoſition Obtain'd, the greater the Convexity is ſuppos'd to be, the Incident Rays would be of Conſequence nearer to it; It is therefore ſtrange that after a General Propoſition is Affirm'd and Prov'd, which ſuppoſes and Neceſſarily Inferrs, that the greater the Convexity of the Glaſs is, the nearer the Incident Rays are from the Axis, in the Concluſion from thence we ſhould Aſſume a nearneſs of Rays from the Axis and a leſſer Convexity, than which nothing can be more Arbitrary.

This Demonſtration ſeems to Conſider all Parallel Rays which Fall within 30 Degrees in the ſame Manner as the Ray cd.

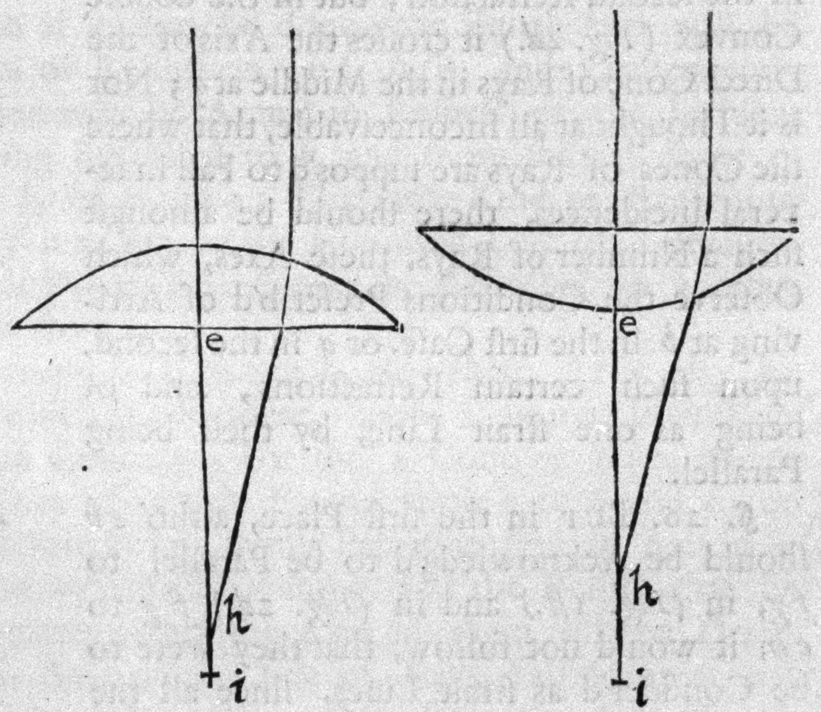
§. 22. IF yet we grant the Convexity of the Glaſs to be as little as Poſſible in Reſpect of the Radius of the Convexity, the Diſtance gd will be Determin'd and Certain, and cannot be Diminiſh'd, as the Radius is Increas'd; that is, the ſeveral Rays, which are Incident betwixt ky and b can never be brought to ky, nearer than they Really are, ſo as to make the Difference of their Diſtances from the Axis of the Glaſs of little or no Account; for altho' by the Diſtance of the Glaſs from the Sun, the Rays may be Conſider'd as Parallel, and not Inclin'd to each other, yet thoſe Parallel Rays, in Reſpect of the Glaſs, cannot be Conſider'd as nearly one without any Diſtinction of Diſtance, which will be to the Full as Large, as is Expreſſed by the ſeveral Sines *g d*, which are Capable of being drawn betwixt the Axis *k y* and *b*, and yet this Demonſtration ſeems to ſuppoſe, that all Parallel Rays are to be Conſider'd, as *c d*, without any Reſpect had to their different Diſtances.

Some other Principles Neceſſary, than what have been hitherto Propos'd, for the Solution of Dioptricks. Hugens's Dioptr.

§. 23. FROM all which, and from what has been ſaid before of the Progreſs of Rays thro' a Lens, and of Radiation in General, we cannot but Think it Reaſonable to Conclude, that whatever Pompous Demonſtrations have been found out Concerning the Invention of the Focus's of Rays Tranſmitted thro' Glaſſes, they only Reſt in Abſtractions of the Mind, and are not any ways Serviceable in Conducting us to a True Knowledge of the Laws of Nature, or of Opticks in Particular, and that therefore there are ſome Principles behind, and as yet Undiſcover'd, which muſt give us a better Solution of theſe Problems, and which, as it ſeems, can be no other than thoſe of the Expanſive and Contractive Forces. See *B.* 1. *C.* 5. §. 43. and *B.* 3. *C.* 1. §. 15.

The Focal Depths of Convex and Plano-Convex Glaſſes Conſider'd.

§. 24. LASTLY, according to the Calculation Propos'd in §. 18. and Suitably in the 11*th*, the Focal Depth *h i* will be leſs in a Plano-Convex Glaſs, where the Convex is Oppos'd to the Incident Rays, than where the Plane is, and ſtill leſs in a double Convex than in either; and the Reaſon of this Focal Length is, becauſe the Rays, which Fall nigher the Axis, are not United thereto ſo near the Pole of the Glaſs, as thoſe that Fall farther from it.



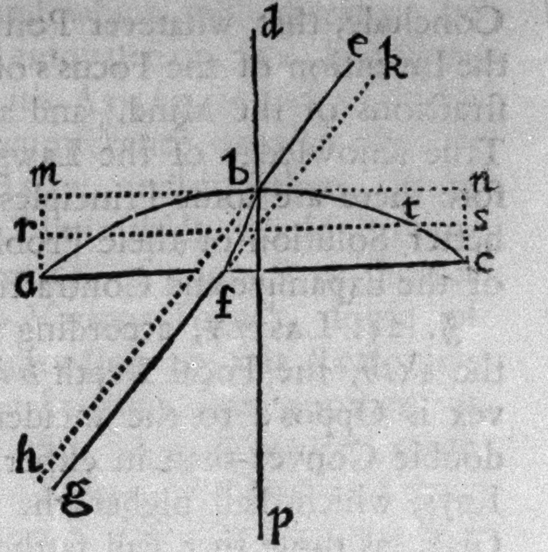
THUS in the preſent Figures, according to the Calculations in this Science, the Rays will be Collected in a much ſhorter Lineal Focus in the firſt Figure than in the ſecond, the one being to the other (where the Proportion of Refraction is ſuppos'd, as 3. 2) as $\frac{700}{1000}$ to $\frac{2770}{10000}$. of an Inch, and (where the Refraction is ſuppos'd to be as 300. 193), as $\frac{650}{1000}$ to $\frac{2698}{10000}$ of the ſame Meaſure; And the double Convex will in the firſt Caſe be, as $\frac{156}{10000}$ and in the ſecond, as $\frac{100}{10000}$ of an Inch; from whence it is Infer'd, that in Viewing an Object by a Plano-Convex Glaſs, it is better to Turn the Spherical ſide to the Object, or in Burning to the Sun: On the other Hand it is Plain in Experience, that Rays are not Collected to a Point in *h* or *i*, which this Philoſophy Affirms, or to any other Determin'd one, but in a Circular Plane; It is likewiſe Evident, that the Circular Focus is Diminiſh'd or Enlarg'd in every Point of it's Receſs from, or Approach to, the Refracting Medium; and Conſequently, if all the Luminous Circles betwixt *h* and *i* may be Accounted the Focus, all the Luminous ones, betwixt *h* and the Refracting Medium, may with as much Juſtice be Term'd ſuch, and the Focal Depth will not be *h i*, but *e i*, which will be very Inconvenient to the Doctrine of Focus's to Acknowledge; Beſides in (*Fig.* 2*d.*) the Focal Depth by Calculation is more than four times greater than that in (*Fig.* 1*ſt.*) and if the Length of the Focus is a Diſadvantage to the Clearneſs of our Views, or the Force of it in Burning, it muſt be four times more ſo in the one Caſe than in the other, whereas in Experience the difference is almoſt, if not altogether, Inſenſible. The Caſe in Nature Appears to be This; that the Expanſive Force of the Rays is Collected to a Closer Focus, by the Contractive Force of the Convex Glaſs tending towards *h* in the 1*ſt. Fig*, than in the 2*d.* where the Contractive of the Convex tends from it, and ſtill to a Cloſer in a Double Convex, where the Contractive Forces of both theſe Figures are United to Collect and Condenſe the Expanſive; And theſe Focus's will be nearer to the Glaſs, as the Expanſive Force of the Rays is leſs, or the Contractive Force of the Glaſs greater; or farther from it, as the Expanſive Force of the Rays is greater, and the Contractive of the Glaſs leſs.

Concerning the Determination of Oblique Rays Falling on a Plano-Convex or a Double Convex. Molyneux. Dioptr. Premiſſes to Prop. 4th. Par. 1ſt. Hugens Dioptr. Prop. 21.

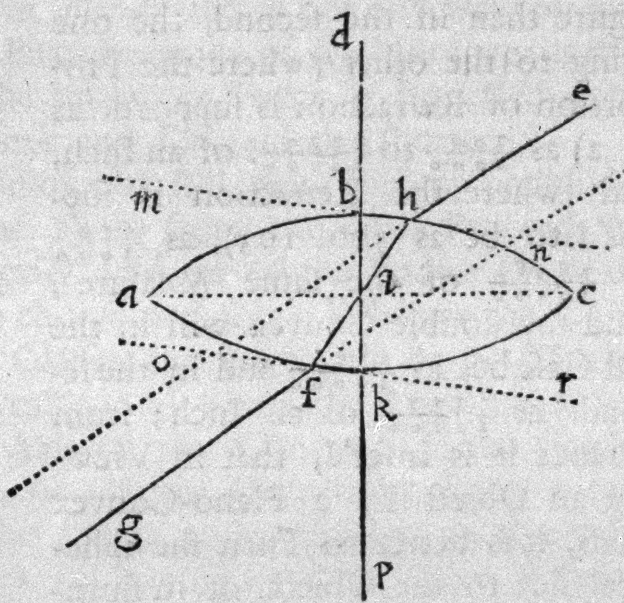
§. 25. A Fourth Propoſition is, that the Parallel Rays, which Proceed from each Collateral Point of an Object, and Fall Obliquely on a Glaſs, are United with their

Axis at the ſame Diſtance, as the direct Parallel Rays are United with their Axis, *viz.* by a Plano-Convex about the Diſtance of the Diameter, and by a Double Equal-Convex about the Diſtance of the Radius.

In Order to Prove this, it is Premis'd, that the Axes of the Oblique Cones of Rays are not Refracted, but may be Conſider'd as ſtrait Lines, as well as the Axes of the Direct ones; *dp* (*Fig. 1ſt.*) is the Axis of a direct Cone of Rays, and *e b* of an Oblique, which Refracted to *f*, and from thence to *g*, makes *fg* Parallel to *e b*, and therefore *e b f g* is ſuppos'd to be the ſame, as one ſtrait Line; and on the Contrary, if the Axis of the Cone is Incident from *g* to *f*, *g f b e* for the ſame Reaſon will be as one Direct Line.



Also *e h f g* (*Fig. 2d*). is ſuppos'd upon the like Account to be in Directum, becauſe *f g* is Parallel to *e h* in a Double Equal-Convex; So that the Rule in theſe Caſes is, that in a Plano-Convex, as in (*Fig. 1ſt.*) where the Oblique Axis Falls upon the Convex, it is Incident on it's Vertex in it's firſt Refraction, and where it Falls upon the Plane part of the Glaſs, it is Incident on the ſame Vertex in the ſecond Refraction; but in the double Convex (*Fig. 2d.*) it croſſes the Axis of the Direct Cone of Rays in the Middle at *q*; Nor is it Thought at all Inconceivable, that where the Cones of Rays are ſuppos'd to Fall in ſeveral Incidences, there ſhould be amongſt ſuch a Number of Rays, theſe Axes, which Obſerve the Conditions Preſcrib'd of Arriving at *b* in the firſt Caſe, or *q* in the ſecond, upon ſuch certain Refractions, and of being as one ſtrait Line, by their being Parallel.



The Incident Rays cannot be Eſteem'd as Strait Lines with the Refracted, upon the Account of their being Parallel, and that they are Parallel, not Certain.

§. 26. But in the firſt Place, altho' *e b* ſhould be Acknowledg'd to be Parallel to *f g*, in (*Fig. 1ſt.*) and in (*Fig. 2d.*) *f g* to *e h*, it would not follow, that they were to be Conſider'd as ſtrait Lines, ſince all the Points betwixt *e* and *d* would be ſeen at *g* on the Right Hand of their Real Poſition, which, if Granted, would prove, that *e b*, or *e h* and *f g* were not ſtrait Lines, nor to be ſo Conſider'd; And that thoſe Points are Viſible on the Right Hand of their Real Situation is Evident from the Object's being Magnified and Enlarg'd, by which Means they are Extended from *e* towards *k* and *n* in each Figure; Moreover, that the Incident Rays *e b* and *e h* are Parallel to the Refracted *g f*, is not Evinced, unleſs the Angle *e b f* is = to *g f b*, or *h b f* to *k f b* in (*Fig. 1ſt.*) and in (*Fig. 2d.*) *e h f* to *g f h*, or *o h f* to *n f h*, which we ſhall Examine in the following Section; For altho', if *eb*, *eh* are Parallel to *g f*, the Points betwixt *d* and *e* would Verge to the Right Hand in the preſent Figure, and which in Fact they do by the Augmentation of the Object, yet they may Tend to the Right Hand of their Real Poſition by an Enlargement of the Object, without a Neceſſity of ſuppoſing ſuch a Paralleliſm; Since there will be in that Caſe the ſame Number of Rays *g f*, *&c.* Refracted, as there are Incident ones betwixt *e b* or *e h* and *d b*, from whence by the Paralleliſm the Entire Situation of the Object will be Chang'd, but the Object it ſelf not Magnified, whereas Really the Poſition of the Object ſeen thro' a Convex is not Chang'd, or the Object Remov'd from one Place to another, but the Whole is Magnified; Suppoſing yet *e b* (*Fig. 1ſt.*) to be Parallel to *g f*, if upon that Account they are to be Eſteemed as Direct Lines, *e b h* will be the ſame as *k f g*, which tho' True in Reſpect of any ſtrait Lines, which may Interſect them as *b f*, will not be ſo, where a Curve, as *b c*, does; for in ſuch a Caſe the Conditions of the Parallels will be Different and Various, and *g f* or *f k* will not be the ſame in Reſpect of *bc*, as *e b* or *b h* is.

The Incident and Refracted Rays not Prov'd to be Parallel from the Angles of Refraction being Equal.

§. 27. In the ſecond Place, as to what Concerns (*Fig.* 1*ſt.*) It is ſuppoſ'd, that the Refraction back from *g* to *f*, *b* and *e* is the ſame, as from *e*, to *b*, *f*, and *g*, or, that the Angle *bfk* is = to the Angle *hbf*, and Conſequently, that the Curve *abc* Refracts the Rays in the ſame Manner, as the ſtrait Line *afc*, and that a Plano-Convex in it's Refractions of Light will be the ſame, as the Plane *amnc*; ſince as *mn* is a Tangent in one Point, ſo is *rs* a Secant in *t* another, and therefore the Refraction at *t*, ſuppoſing a Ray to be Incident upon it, will Obſerve the ſame Laws, whether it is Conceiv'd to Fall on *atc*, or *rts*; As to the double Convex, where *mn*, *or*, are the Tangents, the Reaſoning will not be Different; From whence it will follow, that the Refractions of a Plano-Convex or a Double Convex are the ſame with thoſe of a Plane Glaſs, and that the Point of a Curve has no Peculiar Properties in it's Refractions from that of a ſtraight Line, which is Abundantly Confuted by Experience, and the whole Doctrine of the preſent Opticks.

If it is ſaid, that the Curve *kf* (*Fig. 2d.*) is Equal to *bh*, and Poſited in the ſame Manner to the Incident Ray *gf*, as the other is to *eh*, and therefore the Refraction on both Hands will be the ſame, and the Angles of Refraction *hfn*, *ohf* Equal, and Conſequently *he* Parallel to *gf*, it muſt be Anſwer'd, that this is only taking for Granted, what was to be Prov'd; For altho' *eh* will be Refracted in the ſame Manner at *h*, as *gf* is at *f*, where *hb* and *fk* are Equal Portions of the Lens Computed from the Axis, yet it is not Certain, nor can be Demonſtrated, that *eh* will be Refracted Preciſely to *f*, ſo as *fk* ſhould be Equal to *hb*, and that it may not be Refracted to ſome other Point betwixt *k*, *f* or *f*, *a*, which if it may be done, the Equal Arcs will be Deſtroy'd, as well as the Equal Angles of Refraction *ohf*, *hfn*, and Conſequently the Aſſum'd Paralleliſm; And the ſame may be Argu'd in Reſpect of the Incident Line *gf*, which is not Certain of being Refracted to *h*, any more than to ſome other Intermediate Point betwixt *b* and *h*, or *h* and *c*; The ſame Kind of Reaſoning will alſo Extend to (*Fig.* 1*ſt.*) ſo as *bf* may not be the Common Line of Refraction to *eb* and *gf*, but *eb*, for ought that is Demonſtrated, may be Refracted to another Point than *f*, and *gf* to one Different from *b*.

Impoſſible, if the Incident Rays are the Axes of Cones, that the Refracted ſhould be.

§. 28. Besides which, however ſtrongly it is Imagin'd, that theſe Axes may be found in the ſeveral Cones of Rays, which are Incident, upon a nearer Inſpection our Reaſon will Aſſure us of the quite Contrary; For tho' a Cone of Rays, which Falls from *d* or *e*, in (*Fig.* 1*ſt.*) the Baſes of which Extend from *a* to *b* and *c*, or thoſe, which Fall from any other Intermediate Points, ſhould Perchance find an Axis in *b*, yet where theſe Cones are Incident on *afc*, from *p* and *g*, and the Points that lie betwixt them, it is far from being Apparent, that the Axes of them will be Determin'd in *b*; for how *gf* ſhould be an Axis of a Cone of Rays Falling upon *ac*, when *f* is much nearer to *a* than it is to *c*, is Inconceivable, ſince the Axis of any Cone is always ſuppoſ'd to Terminate in the Center of it's Baſis; unleſs therefore *af* is = *fc*, it is Impoſſible, that *gf* ſhould be that Axis; If it is ſaid, that any Ray is the Axis of the Cone which is Refracted ſo, that *be* ſhould be Parallel to it; That cannot be called the Axis of the Cone, but that Ray, which is ſo Refracted, whether it be the Axis, or not.

Only one Refracted Axis would in it's Regreſs meet with the Incident one.

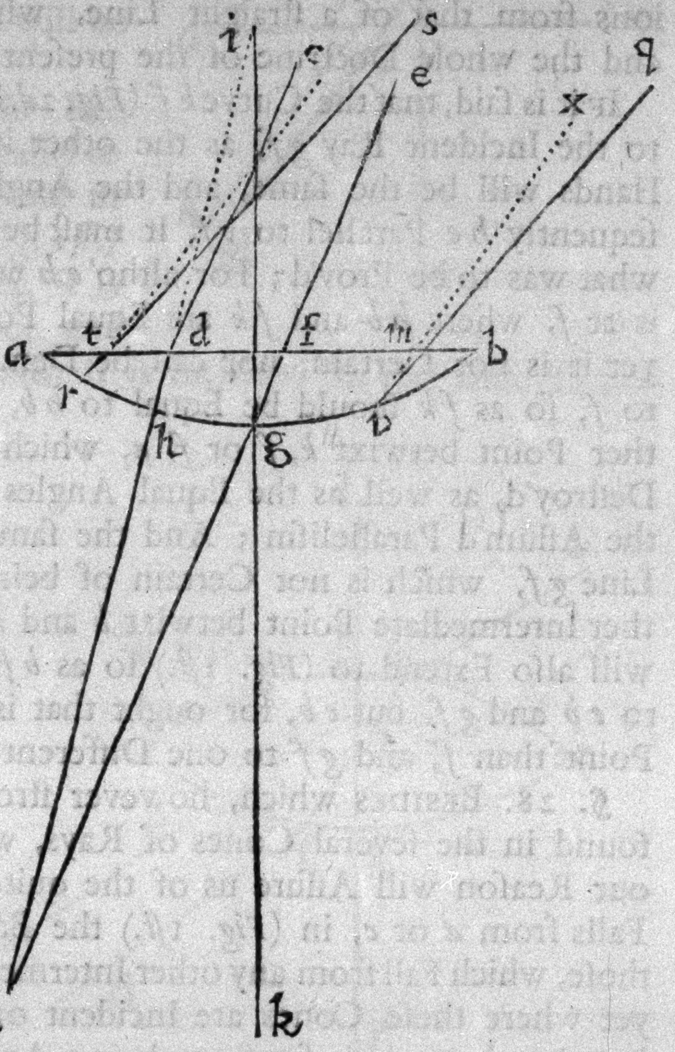
§. 29. We muſt farther Add, that it is altogether Impoſſible in this Doctrine of Refractions to think, that any other Ray, Excepting *gf*, ſhould be Refracted to *b*; for if the Axis drawn from any Intermediate Point betwixt *g* and *p* is, in different Incidences there will be the ſame Refraction, which is Contrary to this Philoſophy, if not, there will be no Axis of Radious Cones, as they are Term'd, Refracted to *b*, Excepting *gf*, which will be of little Advantage to the Science of Opticks.

The Diſpoſition of the Oblique Axes upon Refraction Perfectly Arbitrary.

§. 30. To Conclude, if all this were Granted, in (*Fig.* 2.) it will be Unaccountable, how the Axes of the Cones ſhould End in the middle of the Refraction, whereas before they were Determined in the Beginning or End of it, unleſs we are willing to Receive every Thing for Truth, which is Manifeſtly only a Contrivance; For why ſhould we Imagine, that the Axis of the Cone of Rays in (*Fig. 2d.*) ſhould be Terminated in *q* in the Midſt of the Glaſs, when it is ſuppoſ'd in (*Fig.* the 1*ſt.*) to Center in *b* the Beginning or End of the Refraction? Or, which is the ſame, it is very Arbitrary, where the Incident Rays, before they Fall upon the Glaſs, are alike, and when the Convexity in (*Fig.* 1*ſt* and 2*d.*) is ſo, to make the Axis in the laſt Caſe upon Refraction to Croſs *bp*, and in the firſt to be Refracted beyond it.

The Fourth Propoſition Mention'd §. 25th. Conſider'd and Explain'd. Molyneux's Dioptricks Pag. 32. Hugens's Dioptricks Prop. 21.

§. 31. This Philoſophy having Premiſ'd, what has been already Conſider'd, the Demonſtration of the Propoſition Mention'd §. 25. Proceeds as follows, according to the Common and Uſual way of Proving it; *agb* is a Plano-Convex, on the Plane Side of which the Rays *cd*, *ef*, are ſuppos'd to Fall Obliquely, and Parallel to each other, and therefore the Angles of Inclination *efb*, *cdb* to be Equal, and Conſequently the Correſpondent Angles of Refraction within the Glaſs will be likewiſe Equal, and *dh* Parallel to *fg*; Farther, becauſe the ſame Point in the Object, from whence theſe Rays are ſeverally Deriv'd, Emit an Infinite Number of others, which Poſſeſs the whole Circuit of the Glaſs, ſome one of theſe Rays, after the firſt Refraction, being Produc'd backwards, will Paſs thro' the Center of the Convexity *i*, as *hd*, which is alſo ſuppos'd to be Produc'd the Contrary way, Namely to *l*; for being Perpendicular to the Convexity *agb*, it ſuffers no Refraction in it's Egreſs from the Glaſs to *l*; As to the other Refracted Ray *fg*, the Angle of Inclination at the Point *g* is $fgi = gih$, and the Refracted Angle *kgl*, which by Experiment ought to be $\frac{1}{3}$ more than that of Inclination; But in the Triangle *igl*, *li. lg :: S L. lgi = S L. lgk. S L. lig*; and in theſe ſmall Angles the Sines are as the Angles, and therefore, the Angle *lgk* being $\frac{1}{3}$ more than the Angle *lig*, the Side *li* is $\frac{1}{3}$ more than the Side *lg* or *lh*; for *lg* and *lh* may be Reputed as Equal, in Glaſſes of ſmall Segments, and large Spheres, wherefore *hi* being the Semidiameter, *hl* will be the Diameter *Q. E. D.*



Other Demonſtrations Alledg'd of this Propoſition and why not Conſider'd.

§. 32. There are other Demonſtrations of this by Inveſtigating the Line *hl* from given Angles, and their Sides; But as we have before Conſider'd that kind of Proof, §. 18. ſo we ſhall now do this, which is the moſt Common and Familiar to the Writers in Opticks; There are alſo other Caſes put, where the Convex part of the Glaſs is turn'd to the Object, or elſe where the Glaſs is Equally Convex on both Sides, or laſtly where one Convex is different from and Unequal to the other; All which we ſhall Omit, ſince they Proceed upon the ſame Principles, and ſhall only Examine into the Demonſtration Alledg'd.

The Rays are ſuppos'd to be Parallel in this Figure and to meet in a Point in the Former.

§. 33. In which it is ſuppos'd, that *cd*, *ef*, are Parallel to each other, and by that Means that the Angles of Inclination *cdb*, *efb* are Equal; whereas Granting *e* or *c* to be the Point of an Object, whoſe Rays are Scatter'd and Diſpers'd on the whole Surface of the Glaſs *ab*, the other which is Parallel to it cannot be Drawn from the ſame Point; Albeit therefore we ſhould allow ſuch a Suppoſition to be Eaſy in Reſpect of the Sun's Rays, which Fall at ſo Immenſe a Diſtance from us, it will not yet be any ways Tolerable, where we come to Conſider the Radiating Points of near Objects, unleſs we can Imagine *e* and *c* to Coaleſce at the ſame time, that they are Preſum'd to lye at a certain Diſtance from each other in the Parallel Lines *cd* and *ef*; And we cannot but here take Notice, that the Repreſentation, which is given us in this Figure of the Progreſs of the Rays, which Conſtitute the Point *l* after their Refraction thro' the Glaſs *agb*, and which Trac'd backwards are carried to Diſtinct Points in *c* and *e*, is very different from what is made to us in other Figures, before Deſcrib'd, as in (*Fig. 1ſt.* and *2d.* §. *25th.*) where they Terminate in one Point *e*; which is an Evident Sign, that this is nothing elſe than an Abſtracted Reaſoning upon Hypotheſes, which are Arbitrary, and have not a Foundation in Nature; However, the Paralleliſm of Rays, flowing from a Near or from a Remote Object, has been already Examin'd.

§. 34. What

As alſo in the Preſent.

§. 34. What more Confirms this, is the ſecond Hypotheſis Neceſſary to this Demonſtration, that not only *cd* and *ef*, which were before ſuppos'd to be Parallel, but alſo all the other Rays, which Fall betwixt *a* and *b*, tho' Parallel, are preſently after (See §. 31*ſt*.) ſuppos'd to flow from one and the ſame Point of an Object; which how it is Poſſible that all the Rays, which are Deriv'd from one Point in a nigh Object to the whole Surface or Extent of *a b* ſhould be Parallel, is as Inconceivable, as how Rays Propagated from the ſame Center ſhould be.

Not Certain that one of the Refracted Rays will meet the Center of the Glaſs's Convexity.

§. 35. Granting yet, that *c*, *e*, and the reſt, are an Infinite Number of Points and at the ſame time no more than one Point, this Demonſtration Labours under a third Suppoſition, for Suppoſitions the preſent Philoſophy always takes Care to Introduce, as any new Difficulty Ariſes, which is, that amongſt an Infinite Number of Rays, we may Juſtly ſuppoſe there is ſome one, which if Produc'd backwards after it's firſt Refraction, as *c d*, will meet with the Center of the Glaſs's Convexity *i*, and Conſequently Form the Radius *h i*; But even this Suppoſition is ill Grounded, ſince in a very large Object there may be a Point in it ſo much Inclin'd to the Surface *a b*, that none of it's Rays Produc'd after the firſt Refraction ſhall meet with the Center *i*; for Inſtance, ſuppoſing *e* to be Remov'd to the Right Hand, as far as it is Poſſible for it's Ray to be Refracted (without being Reflected) upon it's Incidence on *a b*, which Ray let be *qm*, ſeeing *r t* is Parallel to *v m*, and *m q* to *t s*, it is Evident, that no Rays from *q* to *s* will ever, if Produc'd after their firſt Refraction, Arrive at *i*, but fall below it, as *t c*, *m x*, the Angles *c t s*, *i d c* being yet Equal, or if *c t s* upon the Account of the greater Obliquity of *s t* ſhould be allow'd to be greater than *i d c*, *c t* would yet Fall below *i*, as is Manifeſt from this Scheme.

The Incident Rays Undetermin'd, and may meet in another Point, than what is here Aſſign'd.

§. 36. If, notwithſtanding, one Ray would certainly Obtain that Property, of being a Radius of the Circle *a g b* upon it's being Produc'd backwards after the firſt Refraction, it cannot yet be Conceiv'd, how *c d* ſhould be that Ray, no more than *e f*, till the Points, *e*, *c* are Determin'd, which are here taken at Large; and if they are not Determin'd, as they are not, ſince the Incidences of *f g*, *d h* upon the Curve *h g* may be different from what is here Expreſſed, the Progreſs of the Rays after a ſecond Refraction will be likewiſe different, and will not Tend to the ſame Point *l* in their Concourſe; and which may in General be Affirm'd of all the Rays, Excepting *c d*, and *e f*.

It is not Certain that Rays, which Fall Perpendicular, are Unrefracted, or if they were, that this Demonſtration is True.

Molyneux Dioptricks, Pag. 2. Experiment 3*d*. Bacon's Natutural Hiſtory Century 8*th* Experiment 761.

§. 37. Let this be likewiſe Granted, which has been Diſprov'd, that *c d* is the Particular Ray, which after it's firſt Refraction upon it's being Produced backwards, will make *h i* the Radius of the Convexity *a g b*, it is in this Demonſtration from thence Affirm'd, that *d h* will not be Refracted, but Proceed in a ſtrait Line to *l*. Whereas, whatever Experiments may be Pretended for this Aſſertion, there is nothing more Evident than that every Point of an Object is Magnify'd by Refraction from a Groſſer to a Finer Medium; But how it is Poſſible for Rays to be Deriv'd in a Perpendicular from each Point of the Object to the Eye, which is Oppoſite to it, when each Point of the Object Appears Larger than it is in Reallity, is Inconceivable; Beſides which, although *d h*, if it Radiated from *i* to *d*, and ſo to *h*, might be ſuppos'd not to be Refracted, as Falling in a Perpendicular, it would not be Certain, that *d h* would be likewiſe unrefracted, if the Ray was Incident from *c* to *d*, which is here ſuppos'd, unleſs a Ray Falling from *c* to *d* is the ſame, as one from *i* to *d*, which is Impoſſible.

To make the Angle h i g *the Meaſure of Incidence is Arbitrary.*

§. 38. In the latter Part of the Demonſtration Alledg'd §. 31. which Concerns the Refracted Angle *l g k*, it is moreover Suppos'd, that the Sines of the Angles *l g k*, *l i g*, are Proportional to the Angles themſelves, becauſe they are ſo Extremely Minute, that there is no Senſible Diſtinction betwixt them; on the other Hand it is not Evident, that *l i g* is ſo Small, as is Repreſented, unleſs it be firſt Determin'd, amongſt an Infinite Number of Rays which Fall upon the Surface *a b*, which it is, that when Refracted, and Produc'd backwards, meets with the Center of the Glaſs's Convexity; but this, as is before Obſerv'd, is left altogether Looſe and Undefin'd; Since, if the Ray Falls from a Point more Remov'd from the Axis than *e* is or nearer to it, *f g i* will not be Equal to *h i g* the Angle of Incidence in the ſecond Refraction; Beſides, it has been likewiſe before Remark'd, that to make the Sines ſometimes Proportional to the Angles of Refraction, and at other times not, is a Manifeſt Indication of the Arbitrary Methods, which this Philoſophy Takes to Demonſtrate.

§. 39. Lastly,

Not Prov'd that more Rays than two are Collected to a Focus, upon all the Suppoſitions which can be Deſir'd.

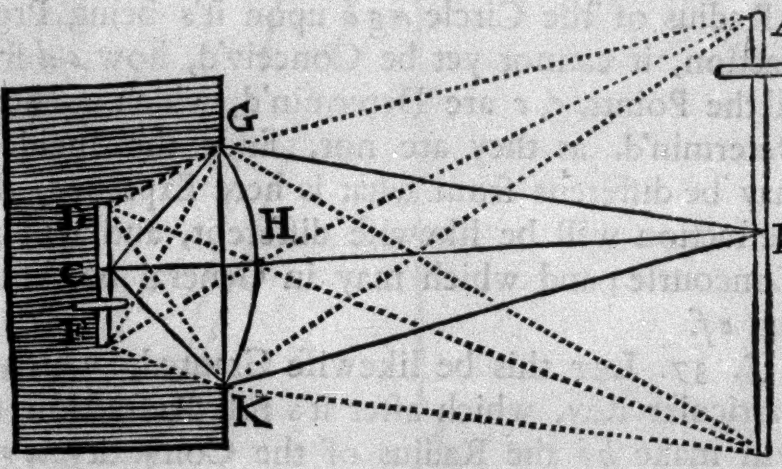
39. Lastly, if all theſe Suppoſitions were True, ſo Unhappy is this Kind of Reaſoning, it would not yet Prove what was Originally Aim'd at and Deſign'd by them, that is, that the Oblique Rays, which Flow from any Point of an Object, are Collected to a Focus at the ſame Diſtance, as the Direct ones are; ſince here is only one Ray Prov'd to be ſo, Namely, That, which, after it's firſt Refraction, and Reproduc'd, Forms the Radius of the Glaſs's Convexity; For either all the Rays, which meet in *l*, meet likewiſe in *i*, or not; If they do, it is not only one Ray, as *c d*, amongſt an Infinite Number, which are Incident on *a b*, and is what this Demonſtration Requeſts or Demands, which Reproduced meets in *i*, but all of them do, which is Impoſſible; If they do not, the Solitary Collection of theſe two Rays will not be to the Purpoſe of Proveing the Collection of all.

Too much would be Prov'd, if this Doctrine was True, viz. *that the Repreſentation of the Object would be at one Certain and Immutable Diſtance, and in a Concave and not in a Plane Surface.*

§. 40. We muſt alſo Add, that, if even this was Evinc'd, too much would be Prov'd, *viz.* that an Object, after the Refraction of it's ſeveral Rays, would be Inflexibly Determin'd, in it's Repreſentation in *k*, *l*, to one certain Diſtance of a Diameter of the Glaſs's Convexity from *g*, and would therefore be only found in the Portion of a Circle's Perimeter, or of a Sphere, and in one Immutable Poſition; Whereas an Object is Diſtinctly Projected thro' a Lens on a Plane; and is alſo very Accurately ſeen by the Eye, which is a Refracting Lens, at ſeveral varying Intervalls, without the leaſt Alteration of it's Figure from any Imaginary Elongation or Depreſſion of it by the Muſcles, that is, when it is Perfectly Steady and Unmov'd; which any one may at Pleaſure Experience.

The Repreſentation of Objects in a Dark Room, Not Explain'd from theſe Principles. Molyneux's Dioptricks Prop. 4*th*. Schol.

§. 41. From theſe Four Propoſitions, and it is the chief Uſe, which is made of them, the Repreſentation of outward Objects in a Dark Chamber by a Convex Glaſs is Endeavoured to be Explain'd, which therefore we need not Particularly Conſider; ſince, as we have ſhewn, it is not Prov'd, that the Rays from *A* are Collected to *F* after their Refraction, nor thoſe from *C* to *D*; However to Evade what has been ſaid, it is Affirm'd, that the Rays, Conſidering the Diſtance of the Object *A C* and the Smallneſs of the Glaſs *G K*, which are Incident from the one upon the other, may be Eſteem'd, as Parallel; If they were Parallel, we have Demonſtrated this Reaſoning to be Falſe, but that they cannot be ſuppos'd to be Parallel, either from a Near or from a Diſtant Object, has been before Evinc'd and Confirm'd.



The Image not Inverted by the Lens till after the Rays have croſſed each other in the Focus, this Reaſoning ſuppoſes before.

§. 42. Besides which, the Image Form'd by the Refraction of a Lens is not Inverted, and is only Enlarg'd, before the Decuſſation of the Rays in the Focus, upon and after which the Object appears Inverted; whereas the Inverſion of the Image here is made in the Focus upon the Decuſſation of the Rays in the Pole of the Glaſs's Convexity *H*, which is not the Property of a Lens, as any one may eaſily Experience; For if the Eye is made to Approach from *D F* towards the Lens *G K*, it will ſee the Object in it's Natural or Erect Situation, which it could not do, if the Rays croſſed each other in *H*, and not in the Focus; If the Eye is Remov'd from the Convex *G K* to the Focus *D F*, or beyond it, the Object will be Inverted, becauſe the Rays there Change their Order into a Contrary one.

The Image Inverted without the Aſſiſtance of a Lens.

§. 43. But that which is ſtill more Diſadvantageous to this Philoſophy is, that without a Lens, or without any Refractions of Glaſs, the ſame Inverted Image of an Object is Impreſſed upon an Oppoſite Paper, with a ſmall Variation in the Diſtinctneſs of the Image, or the Diſtance of the Focus; which may be Experimented by a Foramen in a Window-Shut, and firſt Obſerving what a Picture is Cauſed, and at what Diſtance the Paper Receives it without a Lens, and afterwards what the Effects are with one, which are very nearly the ſame; from whence it is Manifeſt, that theſe Four Propoſitions are Alledg'd to little Purpoſe in Opticks, unleſs they can Explain the Properties likewiſe of ſuch a Foramen, which Produces Similar Effects with the Lens.

§. 44.

§. 44. NOR is the Phænomenon of the Speculum better Explain'd, which is this, that if a Speculum is plac'd in *DCF*, the Image Reflected from it to the Eye will Appear very Faint and Dilute, but if it is Situate betwixt *GK*, and *DF*, with it's Reflecting Surface towards *GK*, and a Paper is held Oppoſite to it, betwixt *GK* and it, at the ſame Diſtance from it, as the Speculum is from the Diſtinct Baſe *FCD*, it will Imprint upon the Paper a Clear and Exact Repreſentation of *ABC*, alike Inverted, as it is in the Diſtinct Baſe; The Reaſon given for this Appearance is, the Imperfect Poliſh of the Glaſs, which therefore is Thought Incapable of Reflecting the Rays with that Diſtinction, that it ſhould; But it is more Evident that this is an Imperfect Reaſon; For it is not Prov'd, that by Poliſhing a Glaſs all the Aſperities of it are not Taken of, or if they were not, that there are fewer Inequalities in the Surface of the Paper, when it is Abundantly Manifeſt, that there are more, and which in the Modern Philoſophy are made to be the very Cauſe of it's being White, by Reflecting the Rays of Light, upon that Account, to all Angles. The Truth ſeems to be, that the ſhaded Light of the Image, which is of a Contractive Force, is more Viſible in the Brisker Expanſive of White Paper, than in the Duller one of Poliſhed Glaſs, becauſe their Actions being more Different, their Diſtinction is more Apparent. And in General the Images here Formed are the Shadows of Light, either Intercepted by Bodies of a Contractive Force, or Projected from them, by giving new Impreſſions from their Expanſive or Contractive Forces in Various Degrees to the Light or Æther, which is Incident, as Fire, which is one Degree of the Expanſive, does; So far are Opticks from being Explained by the Collection of the Rays of Light to a Focus, or a Diſtinct Baſe, that the Images are only differently Coloured Shades of it.

The Phænomenon of the Speculum not Accounted for, becauſe White Paper ſeems to have a leſs Poliſh and more Inequalities than Glaſs. Molyneuxs's Dioptricks Prop. 4. Schol. Pag. 40.

The True Reaſon of this Phænomenon.

What the Images in Viſion are.

§. 45. IF the foregoing Solution was True in Reſpect of the Glaſs's unequal Poliſh, it would be yet Impoſſible to Aſſign a Reaſon in this Philoſophy, why the Image, which is Diluted on the Speculum, ſhould be Diſtinct on the Paper; For, if the Rays on the Speculum are Diffus'd and Diſpers'd from the Inequalities of it, the ſame Effect will be found on the Paper; unleſs the Rays, which are Diſpers'd upon their Incidence on the Glaſs or Speculum, can Collect and Range themſelves by a Willfull and Obſtinate Determination of their own to a more Compact and a Cloſer Order in their Second Reflection, which, however Willfull, would yet ſhow a Good Diſpoſition in them, whilſt they Endeavour'd to make Amends for the Errors of the Firſt.

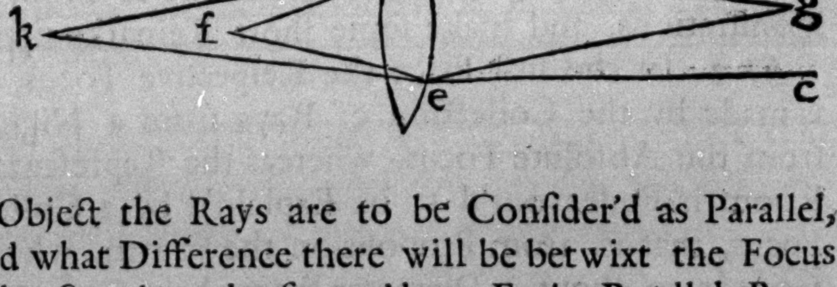
Impoſſible, that the firſt Image which is Diluted ſhould Recover it's ſelf into a cloſer Reflection in the ſecond by a Difference only in the Poliſh of the Reflecting Surfaces.

§. 46. THAT the Image is Diſtinctly Reflected upon the Paper, this Philoſophy Perhaps might Account for, ſince according to it's Principle of the Angle of Reflection being Equal to the Angle of Incidence, it would be the ſame, whether the Rays Proceeded to the Diſtinct Baſe, or were Reflected to the ſame Diſtance from the Intercepting Surface, as the Diſtinct Baſe is, becauſe the Vertical Angle to the Angle of Incidence will be the ſame in the Progreſs of the Ray, as the Angle of Reflection in it's Return, both being Equal to the Angle of Incidence; This however will not Solve the Difficulty of the Image being more Diluted upon the Glaſs than on Paper; nor will it Inform us, how this Appearance upon the Paper is Viſible at other Diſtances of the Paper from the Glaſs, or the Glaſs from the Focus, than what is here Preſcrib'd; nor Laſtly, will it Explain, how that, which is call'd the Diſtinct Baſe it's ſelf, or the Focus, is found at Various and Differing Diſtances from the Lens, which is here ſuppos'd Immutable; All which are Evident in the Experiment; that is Diſtinct Appearances are Obſerv'd at ſeveral Diſtances, tho' in different Degrees of Diſtinction, whereas according to this Scheme, if the Glaſs or Paper were Placed in the leaſt out of the Diſtinct Baſe, All would go into Diſorder and Confuſion.

The Diſtance of the Focus Determinate in this Solution tho' Various in the Experiment.

See theſe Phænomenons Explain'd above §. 44. and 24. at the End.

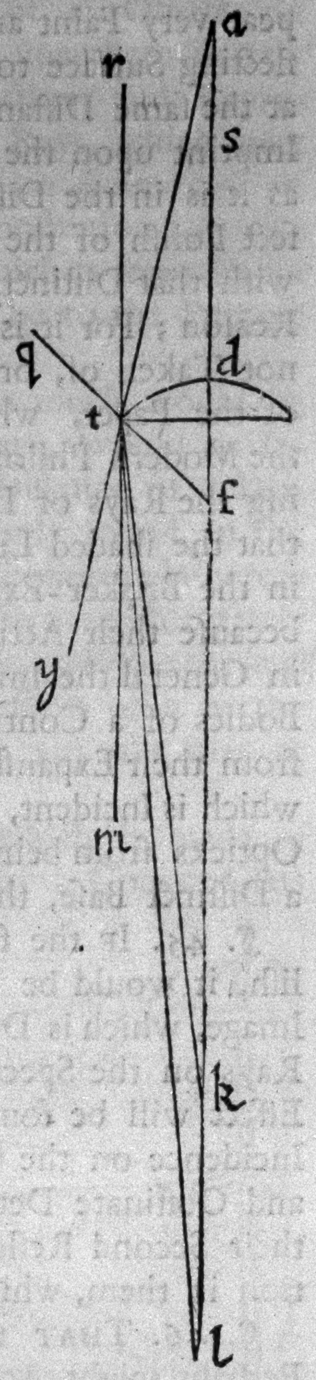
§. 47. CONCERNING Parallel Rays from Remote Objects and Diverging ones from Near, it is ſaid in this Philoſophy, that the firſt will be Refracted to a Focus *f* at a much Leſs Diſtance from the Lens *be* than the other will, which are not Determin'd in a Focus, unleſs at *k* which is a Conſiderably Greater; But if we are willing to Reaſon Juſtly, we ought to know at what Diſtance of the Object the Rays are to be Conſider'd as Parallel, and where they become Diverging, and what Difference there will be betwixt the Focus of one and the other; becauſe if the Sun is only ſuppos'd to Emit Parallel Rays, there



Concerning Parallel and Divergeing Rays. Molyneux Dioptr. Prop. 4. Schol.

there will be only ſuch Rays Collected to the Focus f, and if nearer Objects have their Diverging Rays Aſſembled at k it will be Conſequent, that the Focus of Near and Diſtant Objects is vaſtly Different, whereas the four Preceding Propoſitions Endeavour to Prove, that the Refractions of Rays thro' a Lens have their Focus at the Diſtance of a Semidiameter of the Glaſs's Convexity, even in near Objects as well as thoſe, which are farther Diſtant, as the Sun, and upon one and the ſame Suppoſition, that the Rays are Parallel; So likewiſe in Experiment, the Focus is Alter'd by the Approach to, or Removal of the Lens from the Object, inſomuch, that when a Candle is placed in a Dark Room, the Flame will be Expreſſed in an Inverted Image, Diſtinctly, in a much nearer Diſtance from the Lens, when the Candle is Remov'd farther from it; and in a much greater Diſtance, when it is nearer to it; and yet the Focus of the Sun's Rays is not at a much greater or leſs Diſtance from the Glaſs, than that of a Terreſtrial Object: Nothing of which is Solv'd by theſe Principles.

Phænomenon See this Explained §. 24. at the End.



Concluſion of this Chapter, that the Principles of Opticks are Abſtracted Hypotheſes, and not Sufficient to Solve the Phænomenons of Nature.

§. 48. To Conclude this Chapter, we have thus far Conſider'd the Four Eſſential Propoſitions to the Doctrine of Opticks, and we do not find any Juſt or Solid Reaſoning in the Demonſtrations of them, as to what Concerns a Real Knowledge of Viſion and the Phænomenons of it; They are Plainly no more than an Abſtracted Arguing upon Hypotheſes, which are laid down for the Solution of what is Experimented in Opticks, and it is as Manifeſt, that thoſe Hypotheſes of Rays Expreſs'd by Lines, and the Various Refractions, and Reflections of them, which are Induc'd, are neither Equal to the Explaining of Nature nor their own Reaſonings upon it. And therefore we Conclude again as in §. 44. That theſe Images are not made by the Refractions and Collections of Rays, but that they are Shadows of Bodies, which Intercept the Light, or which Alter and Change it by their Expanſive or Contractive Forces into a Shaded.

CHAP. VII.

Concerning the Reſpective and Vertual Focus's of Glaſſes.

The Fifth Propoſition Explain'd Concerning Reſpective Focus's. Molyneux Dioptr. Prop. 5. Part 1.

§. 1. THIS Philoſophy Proceeds to a Fifth Propoſition, in which it is Affirm'd; That, As the Difference between the Diſtance of the Object and Focus is to the Focus or it's Length, So the Diſtance of the Object from the Glaſs, to the Diſtance of the Reſpective Focus or Diſtinct Baſe from the Glaſs; Thus, *(Fig. Preced.)* td is a Plano-Convex, whoſe Abſolute Focus is about a Diameter of the Convexity, which is ſuppos'd to be sd, a is a Radiating Point, f the Center of the Convexity, ft the Radius of the Convexity, Produc'd Directly to q, and at a Ray Falling on the Glaſs Produc'd directly to y; and therefore the Angle of Inclination or Incidence of the Ray at is qta; From this Figure and the Suppoſitions upon it, it is Affirm'd, that $as.\ sd::\ ad.\ dk$, the Reſpective Focus, or the Diſtinct Baſe from the Glaſs.

The Way we ſhall Proceed in, in Examining the following Propoſitions.

§. 2. HAVING Examined in the Preceding Chapter into the Fundamental Propoſitions of the preſent Opticks, we cannot Think it worth the while to Conſider the ſeveral Particulars in this Demonſtration, or in others, which follow in the ſame Doctrine, we ſhall only Briefly Recount the Hypotheſes, which are Neceſſary to theſe Demonſtrations, and make ſome ſhort Remarks upon them.

The Suppoſition of a Reſpective Focus being different from the Abſolute Contradicted by theſe Opticks.

§. 3. IN the firſt Place the Reſpective Focus, which is Defin'd to be that, which is made by the Collection of Rays from a Nigh Object, is ſuppos'd to be different from the Abſolute Focus, whereas the Repreſentation of outward Objects in a Dark Room is Endeavour'd to be Explain'd by a Collection of the Rays to the Abſolute Focus, that is, from Propoſition the Fourth, where the Rays are Collected in a Plano-Convex to about a Diameter from their Convexity.

§. 4.

§. 4. We may Add likewiſe in the ſecond Place, that ſince this Demonſtration is Concerning the Reſpective Focus *d k*, (*Fig. Preced.*) and Proceeds upon the Conceſſion of the Abſolute Focus *s d*, it by Conſequence ſuppoſes the Truth of the foregoing Propoſition, which we have already Evidenc'd to be only a Reaſoning upon Abſtracted and Notional Principles without any Conformity of them to Nature.

The Demonſtration Concerning a Reſpective Focus, ſuppoſes the Truth of that Concerning an Abſolute one.

§. 5. Thirdly, it is ſuppos'd, that the Ray, which is Incident from *a* to *t* is Capable of a double Refraction, firſt towards *m*, and then to *k*, which is altogether Inconceivable, how a ſingle Point in the Glaſs ſhould Admit of two ſuch different Refractions, or if it could, how any other different Ray which is Incident betwixt *t* and *d*, ſhould have the ſame double Refraction.

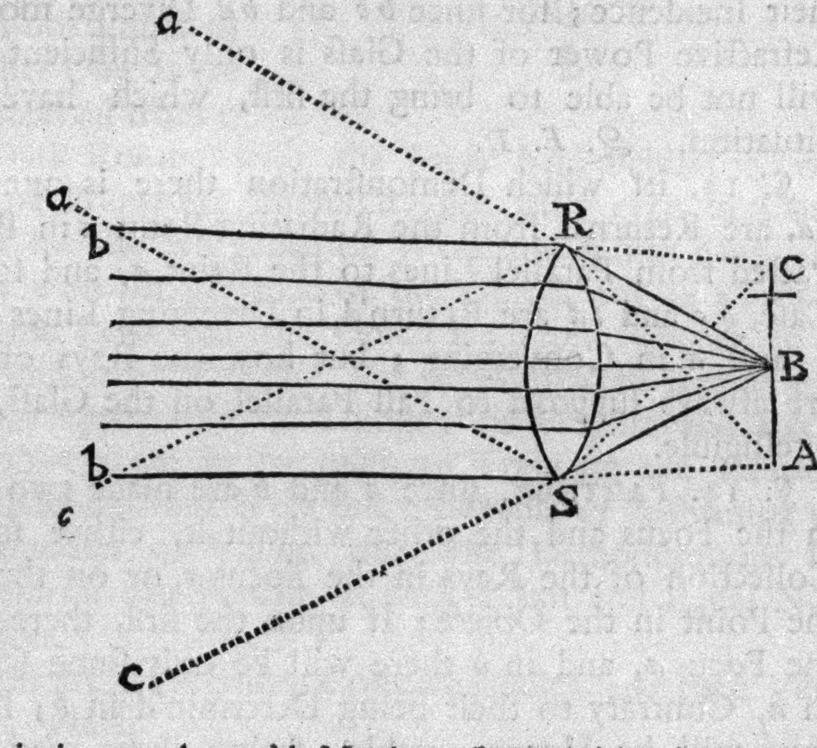
It likewiſe ſuppoſes a double Refraction in a Single Point.

§. 6. It is farther Obſervable, that *l t* and *l d* are ſuppos'd to be Equal, becauſe it is taken for Granted, that the Glaſs is of the leaſt Thickneſs Imaginable, and the Segment, that of a large Sphere. Therefore here is no Account given us of the Actions of Light in Thicker Glaſſes, and ſmaller Spheres, in this Doctrine of Opticks, tho' in a True Philoſophy that ought to be done; and even in Glaſſes of the leaſt Thickneſs, and the largeſt Spheres, *l t* and *l d* cannot be ſuppos'd Equal, unleſs the Radius's of different Circles are ſo; Beſides it will follow, that *a t* is nearly the ſame as *a d*, and will be as little Refracted by the Glaſs, ſince it is Neceſſary, that the Extreme ſmall Curvity of *t d* ſhould be nearly the ſame in Reſpect of *a d*, and *a t*, which will not Agree with this Demonſtration; Laſtly, it is alſo ſuppos'd, that the Angles of Incidence are ſo ſmall, that the Angles and Sines are Proportional, which has been before Conſider'd and Refuted.

Another Suppoſition is that Radius's of different Circles are Equal and that the Sines are Proportional to the Angles.

§. 7. After this we are carry'd to a Sixth Propoſition; That, an Object being plac'd in the Focus of a Convex Glaſs, the Rays from each Point thereof after Paſſing the Glaſs become Parallel; that is, let *A B C* be the Object, Situate in the Abſolute Focus of the Lens *R S*, ſince *a a*, *b b*, *c c* are Parallel Lines, which will be Reſpectively Collected to *A*, *B*, and *C*, therefore *A*, *B*, and *C*, will be Reciprocally Diffus'd backwards, thro' the Lens *R S*, to *a a*, *b b*, *c c*.

Propoſition the Sixth, that Rays Refracted from the Focus thro' the Lens become Parallel.
Molyneux's Dioptric. Prop. 6. Par. 1.



§. 8. It is Really Admirable, that Men can Satisfy themſelves with ſuch a kind of Reaſoning, if they did not Think there was a Sort of Inherent Right to Truth in Mathematick Lines and Figures; for why ſhould *C B A* return theſe Rays juſt in the ſame Manner, in which they are Incident, be they Direct or Oblique, or in whatſoever Angle they Fall, when it is an Avow'd Maxim of Opticks, that Rays are Reflected in thoſe Angles, in which they are Incident, which, if True, would cauſe an Inexpreſſible Confuſion in the preſent Figure, inſtead of that Accuracy and Diſtinction of Lines, which it Repreſents? Thus, ſince *A*, *B*, *C*, are Refracted to *a*, *b*, *c*, becauſe *a*, *b*, *c*, are Refracted to *A*, *B*, *C*, when an Object is not Interpos'd at *A B C*, and ſince *a*, *b*, *c*, Refracted to *A*, *B*, *C*, will be Reflected from thence in Angles Equal to thoſe of their Incidence, when an Object is Interpos'd, therefore *A*, *B*, *C*, when an Object is Interpos'd, muſt be the Points of Rays, which Proceed not from the Points *A*, *B*, *C*, Originally, but of Rays which are Incident upon *A*, *B*, *C*, in thoſe Angles, in which they were Reflected after their Refraction thro' *R S*; Conſequently thoſe, which are Incident upon *A*, *B*, *C*, in different Angles, and which will have an Equal Title to the being Reflected, will be Refracted back thro' *R S* in all Inclinations to *a*, *b*, and *c*, and will not therefore be Parallel, but Cauſe a Confuſion not to be Expreſſed, unleſs in the Imagination only.

The Rays Incident on the Object ought to be Reflected in Angles Equal to thoſe of their Incidence.

§. 9. Besides which, it is not any ways Prov'd, which is here ſuppos'd, that the Action or Reaction of *C B A*, is Equal to the Action from *a a*, *b b*, *c c*, or that the Vivacity of Rays Emitted from an Object is Equal to that of thoſe, which are Im-

This Demonſtration ſuppoſes that the Rays Emitted from an Object are as Vivid and Strong as thoſe from the Sun.

preſs'd from the Sun, which it ought to be, ſince the Effects of both are ſuppos'd to be Reciprocal, and there is as much Force Requir'd to Conquer the Reſiſtence of the Glaſs in a backward Refraction, as a forward.

This Demonſtration Conſiſts of Reciprocal Suppoſitions.

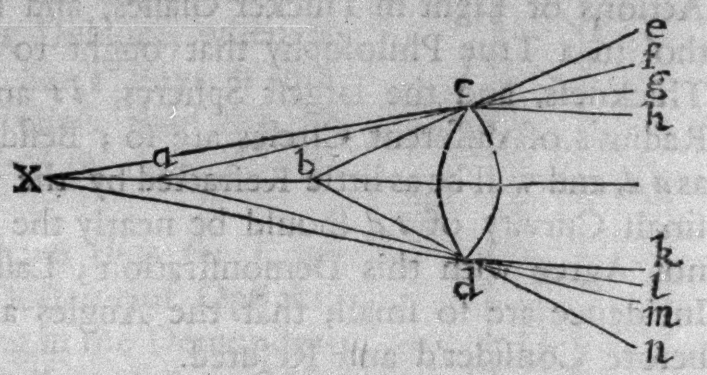
§. 10. But that which is ſtill farther Exceptionable in this Demonſtration is, that *C B A* is ſuppos'd to Refract it's Rays in Parallel Lines, becauſe it is ſuppos'd, that Parallel Lines are Refracted to *C B A,* which Demonſtration cannot but carry with it a wonderful Force of Conviction, which Conſiſts of Suppoſitions, for they are no more, that Mutually and Reciprocally Infer each other.

A Reſpective Focus Inconſiſtent with Parallel Rays.

§. 11. We ſhall laſtly Add, that if *A B C* is the Abſolute Focus, there ought in the Return of the Rays, according to the Preſent Doctrine of Opticks, to be a Reſpective one by Propoſition the Fifth, which is Impoſſible, if *A B C* is Refracted backwards in Parallel Rays; Becauſe a Paralleliſm and a Focus are Inconſiſtent.

Propoſition the ſeventh Explain'd. Molyneux's Dioptricks Prop. 7. Part. 1.

§. 12. Another Propoſition is, that if an Object be Placed nigher a Convex Glaſs than it's Focus, the Rays from each ſingle Point thereof, after they have paſs'd the Glaſs do Proceed onwards Diverging, but do not Diverge ſo much, as before they Enter'd the Glaſs. Thus, Suppoſing *a* to be a Radiating Point in the Focus of the Glaſs *c d*, it's Rays *a f*, *a m*, are by Refraction reduc'd to a Paralleliſm, in the Lines *h c*, *k d*; but if we take a Radiating Point, which is nearer to the Lens than it's Focus, as *b*, it's Rays *c e*, *d n* will not be Refracted to *k*, *h*, but to *g*, *l*, and therefore will be ſtill Diverging, tho' in a leſs Degree than before their Incidence; for ſince *b c* and *b d* Diverge more than *a c* and *a d*, and ſince the Refractive Power of the Glaſs is only Sufficient to draw the laſt to a Parallel, it will not be able to bring the firſt, which have a greater Divergence, to the ſame Situation. *Q. E. D.*



If this Demonſtration was True, the Rays of the Sun would be Converging, which are ſuppos'd to be Parallel.

§. 13. In which Demonſtration there is one Thing very Extraordinary, that *h c*, *k d*, are Return'd from the Radiating Point *a* in Parallel Lines, becauſe they are Refracted from Parallel Lines to the Point *a*, and for the ſame Reaſon, as in the ſecond Caſe, *g c* and *l d* are Return'd in Diverging Lines from *b*, they ſhould be Incident on *c* and *d* in Converging; but how the Rays of the Sun ſhould Converge, which are always ſuppos'd to Fall Parallel on the Glaſs, is altogether Unexplain'd and Unintelligible.

Either there will be no Radiation excepting in the Focus or no Reciprocal Refractions.

§. 14. Farther, ſince *a* and *b* are made two Radiating Points, the one Situate in the Focus and the other without it, either ſuch a Radiation Depends upon the Collection of the Rays in the Focus *a*, or on the Univerſal Diffuſion of them from the Point in the Object; If upon the firſt, there will be no Radiation Excepting in the Focus *a*, and in *b* there will be only ſome ſcatter'd Rays found which are United in *a*, Contrary to their being Determin'd in *b*; If upon the laſt, Reciprocal Refractions will be Unaccountable, ſeeing there may be a Hundred Radiating Points as *x*, *a*, *b*, *&c.* in which the Rays are Collected, whereas in Refractions backwards there is only one Point, in which the Rays can be Collected, Namely in *a*.

If the Rays are Refracted to a Paralleliſm or a Divergence there can be no Focus contrary to other Propoſitions in Opticks.

§. 15. There is another Material Objection againſt this Demonſtrarion, that in all theſe Points as *a*, *b*, *&c.* which are thoſe of near Objects, and which therefore ſhould by the Preceding Doctrine have a Reſpective Focus at ſome Diſtance or other beyond the Lens *c d*, the Rays, which are Emitted from them and afterwards Refracted, become either at the moſt Diverging, or at leaſt Parallel, which every one is Senſible can never Converge, and therefore never Aſſemble to a Focus, contrary to the very Fundamental Propoſitions of this Philoſophy; So little Reaſon have we to Confide in this Kind of Reaſoning, which is Vulgarly Handed about for Invincible Demonſtration.

The Calculations Differ. Molyneu'xs Dioptricks Prop. 8. Par. 1.

§. 16. As for the Imaginary Focus, which is Determin'd by the Producing of *g c*, *l d*, till they Croſs each other in *x*, and is Endeavour'd to be Aſcertain'd in the next Propoſition, we ſhall Prove in §. 23. 24, *&c.* that it cannot Signify any Thing to the Explaining of Nature, which is the Aim we chiefly Propoſe in Examining into theſe ſeveral Allegations; And it is Evident in the Calculus it's ſelf, which belongs to this

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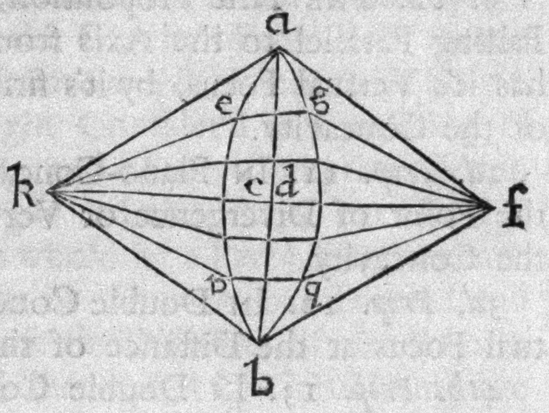
this Eighth Propoſition, and the Foundations, on which it Proceeds, that thoſe, who have Eſpous'd this way of Reaſoning, Differ to that Degree, that the ſame Line, which one makes to Conſiſt only of 7200 Parts, another Thinks he has Demonſtrated to take in 10560, and whereas a ſecond Line is Affirm'd to be 8566, it is by a very Celebrated Writer in Opticks made to be 12484.

That Rays Proceeding from a Point, are Collected to a Point, by a DoubleConvex, at the ſame Diſtance. Molyneux's Dioptricks, Prop 9 Par. 1.

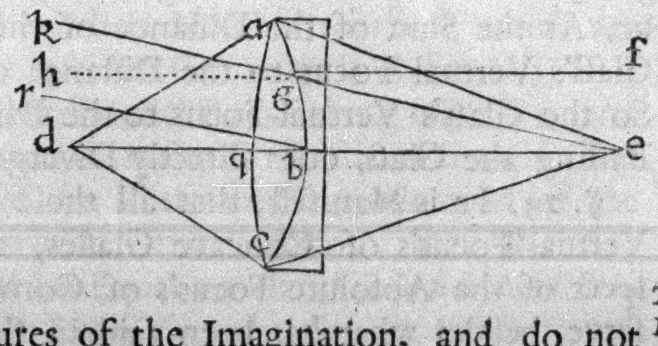
§. 17. Omitting therefore this Extravagant Difference even in an Abſtracted Calculation, which is a Sufficient Proof of the Inaccuracy of this kind of Demonſtration in Reſpect of a Real Philoſophy, we ſhall go on to another Propoſition, which in the preſent Doctrine of Opticks, is the Ninth; Namely, that in Double Convex Glaſſes of Equal or Unequal Convexities, the Rays Proceeding from the Diſtance of a Diameter of one Convexity are United at the Diſtance of the Diameter of the other Convexity; The Truth of which is Endeavour'd to be Evinc'd by Dividing the Convex Glaſs into two Plano-Convex Glaſſes by the Line *ab* (*Fig. §.* 18.) For *ka*, *ke*, *kc*, *kp*, *kb*, being Refracted in Parallel Directions thro' the Glaſs, they will be Incident on *ab* in Parallel Rays, and therefore Refracted to a Focus at the Diſtance of a Diameter $df = kc$.

Several Anſwers Alledg'd againſt the foregoing Propoſition.



§. 18. In Anſwer to which we ſay, that the Propoſition, on which this is Founded, has been already Conſider'd, and therefore that it is not Neceſſary to give a Particular Confutation of it, unleſs we Add, that to make the Rays run Parallel after Refraction, and to Divide the Glaſs by the Line *ab*, ſeem both of them to be Arbitrary Preſumptions, and farther, that no ſuch Appearance, which we know of, Exiſts in Nature, which this Reaſoning ſeems to Affirm; ſince in the Collection of the Rays of the Sun, the Sun's Diſtance from the Glaſs is not Equal to that of the Focus, nor even in nigher Objects are the Radiating Points at the ſame Interval from a Lens oppos'd to a Foramen in a Dark Room, with the Paper held behind it, on which the Images or Focus's of thoſe Points are Impreſs'd; nor laſtly even in Convex Glaſſes, which are made Uſe of for the Aſſiſtance of Weak and Infirm Eyes, do we find that Equality of the Focus with the Diſtance of the Object, for in all theſe Caſes the Focus is much nearer to the Lens than the Object is; From whence it is again Notorious and Manifeſt, that this is nothing elſe, but an Abſtracted Reaſoning, and Perfectly Inadequate to the Real Appearances of Things; 'tis a Sport and Exerciſe of the Mind, which ſerves to Divert, or, at the moſt, to Train up and Educate our Thoughts to Rational Contemplations, but is no more Capable of Informing us in any True Knowledge of the Properties of Viſion, than the Demonſtrations in Geometry, and the Affections of Figures, can Furniſh us with a Juſt and Equal Notion of the Real Nature of Extenſion.

Concerning Vertual Focus's.



§. 19. The following Propoſitions are Concerning Concave Glaſſes, and their Vertual Focus's or Points of Divergence, which are Determin'd by the Refracted Ray Produc'd backwards, till it meets with the Axis of the Glaſs's Concavity; Thus *kg* is Refracted from *fg*, and being Prolong'd croſſes the Axis *de* in *e*, which is Term'd the Point of Divergence or the Vertual Focus; Concerning which we ſhall take Notice, that ſuch Focus's, as theſe, are Perfectly Creatures of the Imagination, and do not ſerve to the Explication of any Real Phænomenon in Nature.

The Vertual Focus a Creature of the Mind.

The Vertual Focus of no Uſe to the Explaining the Real Nature of Refraction or the Action of Rays and the Glaſſes which Tranſmit them.

§. 20. This is Manifeſt, ſince a Ray *fg*, which is Refracted to *k*, cannot be Really return'd to *e*, becauſe it is Impoſſible that *fg*, which by it's Refraction to *k* is ſuppos'd to tend from *g* to *k*, ſhould have a contrary Direction from *k* to *g*; If that notwithſtanding could be done, whilſt *fg* is Refracted to *k*, it would be Abſurd to Think, that *kg* would Proceed in a Straight Line to *e*.

§. 21. Besides, no one ever Obſerv'd one of theſe Points of Divergence in the Rays of the Sun, after their Refraction thro' a Concave, of which it is the Evident Property to Diſperſe the Rays, whether, in a Plano-Concave, the Concave or the Plane be turn'd to the Incident Beams; and therefore it is altogether Impoſſible, that the Rays, which are diffus'd from *g* to *k* one way, ſhould be Contracted from *g* to *e* the other, ſince either ways they ought to Diverge; From whence it is Clear, that the Propoſitions, which follow in this Doctrine, are only made to Trace an Imaginary Point, which is no where Exiſting in Nature, and which it is Impoſſible ſhould, Exiſt in it; That they are Problems in Opticks Invented only to Diſplay the Skill, and to ſhew the Art and Contrivance of the Geometer, who Explains them; and which, by the Similitude of the Reaſoning us'd in the Solution of them with what goes before in Convex Glaſſes, makes it ſtill more Certain and Undoubted, that the whole Doctrine of Focus's, both in Concave and Convex Glaſſes, is nothing elſe than an Amuſement of the Mind, and an Entertaining of it with Abſtracted Truths Concerning Lines and Angles, and the Properties of them, without giving us any Explanation of the Real Nature of Viſion, or the Action of Glaſſes, or the Refractions of Light thro' them: We ſhall therefore Enumerate the Propoſitions which Fall under this Head, without Entering into a Particular Confutation of them, both becauſe the Vertual Focus is Manifeſtly an Imaginary or Notional Point; and becauſe the ſame kind of Reaſoning obtains againſt theſe, as was Alledg'd againſt the former.

An Enumeration of the Propoſitions, which concern Vertual Focus's.

Molyneux Dioptricks Prop. 10. 11. 12. 13. 14. 15. and Corol. to Prop. 15.

§. 22. The Firſt Propoſition, which in this Philoſophy is the Tenth, is, that a Ray, Falling Parallel to the Axis from the Air on the Concave Surface of a Glaſs Medium, has it's Vertual Focus, by it's firſt Refraction, at the Diſtance of a Diameter and half of the Concavity.

2*d. Prop.* 11. In Plano-Concave Glaſſes, when the Rays Fall Parallel to the Axis, the Point of Divergence or Vertual Focus is Diſtant from the Glaſs the Diameter of the Concavity.

3*d. Prop.* 12. In Double Concaves of Equal Cavities, Parallel Rays have their Vertual Focus at the Diſtance of the Radius of the Concavity.

4*th. Prop.* 13. In Double Concaves of Equal or Unequal Concavities, It will be, As the Sum of the Radii of both Concavities to the Radius of either Concavity, So the Double Radius of the other Concavity to the Diſtance of the Vertual Focus.

5*th. Prop.* 14*th.* In Concave Glaſſes and Rays, which Converge, if the Point, to which the Incident Ray Converges, be Diſtant from the Glaſs farther than the Vertual Focus of Parallel Rays, It will be, As the Difference between the Diſtance of this Point from the Glaſs and the Diſtance of the Vertual Focus from the Glaſs is to the Diſtance of the Vertual Focus, So the Diſtance of this Point of Convergence from the Glaſs, to the Diſtance of the Vertual Focus of theſe Convergeing Rays.

6*th. Prop.* 15. In Concave Glaſſes and Rays, which Converge, if the Point, to which the Incident Ray Converges be Nigher to the Glaſs than the Vertual Focus of Parallel Rays, It will be, As the Exceſs of the Vertual Focus more than this Point of Convergence, to the Vertual Focus, ſo the Diſtance of this Point of Convergence from the Glaſs, to the Diſtance of the Point, where this Ray Croſſes the Axis.

7*th. Coroll.* In Concave Glaſſes and Rays, which Diverge, let the Point of Divergency be farther from or nigher to the Glaſs, than the Glaſs's Vertual Focus, it will be, As the Sum of the Diſtance of the Point of Divergency from the Glaſs, and the Glaſs's Vertual Focus to the Diſtance of the Point of Divergence from the Glaſs, So the Glaſs's Vertual Focus to the Diſtance of the Point, from which the Ray, after Paſſing the Glaſs, does directly Diverge.

If the Vertual Focus is Deſign'd to Aſcertain the Inclination of the Refracted Rays, yet of no Significancy.

§. 23. It is Manifeſt, that all theſe Propoſitions are much the ſame in Reſpect of the Vertual Focus's of Concave Glaſſes, as were before Examin'd and Refuted, in Reſpect of the Abſolute Focus's of Convex ones; We ſhall therefore only farther Obſerve, beſides what has been ſaid in §. 19, 20, 21, That the Deſign of theſe Vertual Focus's, tho' they no where Really Exiſt, is to aſcertain the Particular Inclination of the Rays upon their Refractions thro' Concaves, which cannot be done, unleſs by Referring them to a Point to which they Converge.

Vertual Focus's Impoſſible in nigh Objects and where the Rays are not Parallel.

§. 24. As therefore we have before Evinc'd, that the Actual Exiſtence of ſuch Focus's is Impoſſible, ſo we ſhall now Endeavour to Prove, that the Deſign and Intention of them is Chimerical; For if this Doctrine of Vertual Focus's was True, it would be Conſequent, (*Fig.* §. 19.) that from a certain Point *e* if an Infinite Number of Radii are Drawn, as from a Center, Namely *e k*, *e d*, *&c.* The Portions of the Rays, which are beyond the Glaſs will be the Refracted Rays, as *k g*, *d b*, *&c.* Suppoſe now the Glaſs *a b c* to be Remov'd farther from *e* or Brought nearer to it, either the

Vertual Focus *e* will be Chang'd or not; If it is not Chang'd, the Refractions of the Rays will be different, as *kg*, and the reſt will be differently Inclin'd to *a b c*, even where the Incidences *fg*, *&c.* are Parallel, and the ſame, which is Abſurd; If the Vertual Focus *e* is Chang'd, by the Approach or Receſs of the Glaſs, ſo that, as *a b c* Advances, *e* Retires, or the Contrary, it muſt be Preſum'd the Incident Rays are Parallel, otherwiſe there would be the ſame Refractions, where the Incidences are Different; and therefore in Rays, which are Diverging, a Vertual Focus cannot be ſuppos'd, (notwithſtanding it is Aſſum'd in all the Preceding Demonſtrations Concerning ſuch Rays) and can never be Apply'd to nigh Objects, whoſe Rays in the ſame Demonſtrations are Affirm'd to Diverge.

§. 25. But that, which is ſtill farther Obſervable is, that Granting the Rays to be Parallel, a Vertual Focus cannot be Prov'd, unleſs all the Rays, which are Refracted in *a*, *c*, *&c.* have ſuch an Adjuſtment as Produc'd backwards to Terminate in one and the ſame Point *e*, which cannot be Imagin'd in a Sphere, let the Diameter of it be what it will, Excepting the Refracted Lines *g k*, *&c.* are Parallel to ſo many Lines Drawn from *b*, as *b r*, *&c.* and Conſequently, Excepting Rays, which are Refracted nearer to *a*, have a greater Force of Refraction from the Perpendicular, than others which are farther from it, which is wholly Unaccountable in this Philoſophy; For how thoſe, which have a greater Renitency from the Curvity of the Glaſs, ſhould be able to Diſperſe themſelves more, than thoſe which have a leſs, is not to be Solv'd.

Vertual Focus's where the Rays are Parallel Unaccountable.

§. 26. Besides in this way of Reaſoning from Light Conſider'd as Conſiſting of Individual Rays, ſince all the Rays Incident betwixt *f g* and *e b* are Comprehended in their Vertual Focus betwixt *g e b*, or *r b d*, Equal to *k g h*, all the Rays betwixt *k g*, *b d* may be Repreſented by their Parallels *r b d*, that is the whole by a Part; from whence it is Evident, that Light is not Refracted by ſingle Rays or Threads, but Diluted and Attenuated in it's whole Subſtance; becauſe in ſuch a Caſe the Diffuſion of the Rays in *k e d*, may be Equal to the Conſtipation of them in *r b d*, that is, not by Conſidering the ſingle Rays, which are in each of the Spaces, ſince both are filled with them, and therefore in Number Equal, but by Conſidering the Space of the one to be filled with a more Diluted Light, and the other with a more Denſe and Solid.

If Rays were Return'd in ſingle Lines to a Vertual Focus a Part might be ſaid to be Equal to the whole.

§. 27. Farther according to this Doctrine, ſeeing all the Rays which are Incident betwixt *f g* and *e b* are more or leſs Refracted, and the Refracted Rays Produc'd backwards are Determin'd in *e*, to which the Lines in *r b d* are Parallel, it will follow, that there are ſome Lines Refracted betwixt *r b d* or not; If there are, the ſame Difficulty will ariſe as in §. 26; If not, *r b* will be the firſt Refraction Parallel to *e g*, and Conſequently the Angle of Refraction of the Line, which is neareſt to the Axis, will be as great as that, which is the fartheſt from it, Namely $hgk = fge = geb = rbd$; ſo that in a greater or leſs Curvity of the Glaſs, the Refraction will be the ſame, that is, Lines of different Incidences will have the ſame Refraction.

If this Doctrine is True Rays which are Incident on Different Curvities of the Glaſs will have the ſame Refraction.

§. 28. Lastly *e a*, *e g*, *e b*, *e c*, if Prolong'd ſo as to make Equal Radius's, will Conſtitute a Circle, as well as *d a*, *d g*, *d b*, *d c*, Form the Concave of the Glaſs; in which Caſe the Refracted Rays, which are Propagated to the Circumference, which is ſuppos'd to be Drawn from *e*, will be Equal in their Number to thoſe which are Incident on *a b c*, which is Deſcrib'd from *d*, or elſe the Contrary is Affirm'd; If it is Affirm'd, that they are Equal in their Number, the Parallel Rays Falling from *f*, *e*, and the reſt of the Points Parallel to them, will have ſo many Refractions in *a*, *b*, and *c*, whoſe Center after their Refraction is in *e*, and the Ultimate Determination of the Arc in *a* and *c* or near thoſe Points; from whence it will follow, that the Arc *a q c*, which is Drawn from the Center *e*, will be Equal to the Arc *a b c*, which is Drawn from the Center *d*; ſince all the Parallel Lines *f g*, *e b*, *&c.* are Incident upon every Point of the Arc *a b c*, to which *d a*, *d b*, *d c*, are Drawn, and it is ſuppos'd, that there is an Equal Number of Refracted Rays, which Deſcribe ſo many Points in *a q c*, and therefore an Arc from *e* Drawn betwixt the Points *a*, and *c*, is Equal to one Drawn from *d* betwixt the ſame Points, altho' the Radius's of the Arcs are very Different, which is Impoſſible.

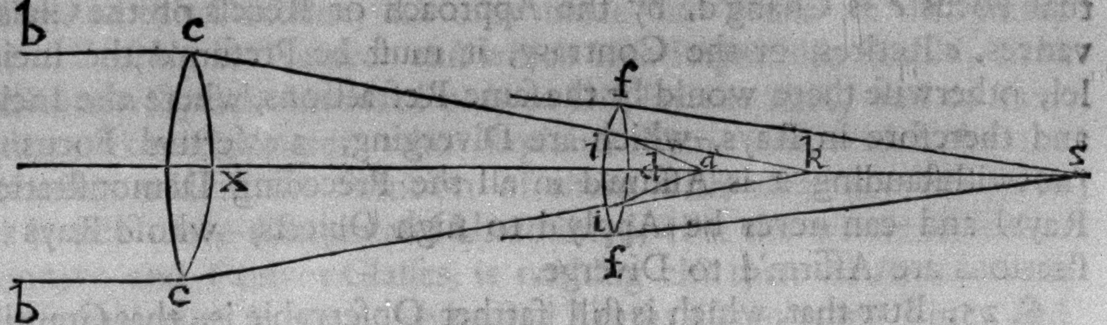
If this Doctrine is True, Arcs Perform'd by different Radius's and Drawn betwixt the ſame Points may be Equal, which ſeems to be Impoſſible.

If it is Affirm'd, that the Rays from *e* are not Equal in Number to thoſe from *d*, that is, that the Refracted Rays are not Equal to the Incident, this Philoſophy would do well to Inform us, how Rays ſhould be Refracted, where they are not Incident, or how thoſe, which are Incident, ſhould not be Refracted, they Appearing to be according to this Doctrine Perfectly Reciprocal.

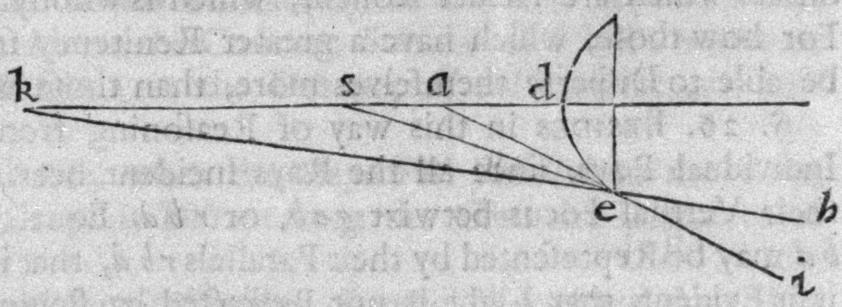
§. 29. Proposition

A Propoſition to Determine in what Point the Rays will meet with the Axis ſuppoſing they are Refracted thro' two Lens's and the ſecond Lens is placed betwixt the Glaſs and Focus of the firſt Lens. Molyneux's Dioptricks Prop. 16. and Prop. 8.

§. 29. Proposition the Sixteenth is, that the Focal Lengths ds, xk, of two Convex Glaſſes ff, cc, being given, *(Fig.* 1.*)* and the longer xk plac'd before the ſhorter at any given Diſtance dx, leſs than the longer Focal length xk, it will be; as $ds+dk.\ dk::ds.\ ad$, to Determine the Diſtance of the Diſtinct Baſe from the ſhorter Glaſs; for by *Prop.* 8*th* of *Molyneux*'s Dioptricks, which we have not before ſet down, and ſhall therefore here Inſert;



In Convex Glaſſes, when an Object is plac'd nigher to the Glaſs than the Focus, the Rule for finding the Imaginary Focus *(Fig.* 2*d.)* is, as $as.\ sd::ad.\ dk$, that is, As the Difference between the Diſtance of the Object from the Glaſs, and the Glaſs's Focus, is to the Glaſs's Focus, So the Diſtance of the Object from the Glaſs to the Diſtance of the Imaginary Focus Explain'd §. 16. Therefore in that *Prop.* da is given and dk Requir'd, on the Contrary in the Preſent dk is given, and da ſought, and ſince by the ſame 8*th Prop.* it is as, $as.\ ds::ad.\ dk$, by Alternation it will be as, $ds.\ dk::as.\ ad::ds-da.\ ad$, and by Compoſition $ds+dk.\ dk::ds-da+da=ds.\ ad$, and Conſequently, ds and dk being given, ad will be found.



The Application of Propoſition the Eighth to the Preſent Conſider'd.

§. 30. In Examining into this Demonſtration, it will be Neceſſary to Compare the Lines, and the Expreſſions of them made Uſe of in both the Figures; In *(Fig.* 2*d.)* as is the Difference between the Diſtance of the Object from the Glaſs, and the Glaſs's Focus, that is, the Object is ſuppos'd to be plac'd in a, and the Focus of the Rays in s, and on the ſame ſide with the Incident Rays from a; sd therefore is the Diſtance of the Real or Abſolute Focus from the Glaſs, ad the Diſtance of the Object from it, and dk the Imaginary Focus, which is made from the Refraction of the Incident Line ae into eh, which Produc'd backward is Determin'd in k; on the other Hand in the preſent *Prop. (Fig.* 1.*)* as is the Difference betwixt the Diſtance of the Diſtinct Baſe a from the Glaſs ff, which is ad, and the Glaſs's Focus ds, which is on the contrary Side to the Incident Rays, which Fall from bb; and the Diſtinct Baſe in this Propoſition is ſuppos'd to Repreſent the Object in the other; Conſequently, as before, sd is the Diſtance of the Abſolute Focus, ad the Diſtance of the Object from the Glaſs, and dk the Imaginary Focus, which here Falls betwixt the Abſolute Focus s and the Object a, notwithſtanding it before Fell beyond the Abſolute Focus.

Such an Application Inconſiſtent unleſs two Oppoſite Directions of Rays are one and the ſame.

§. 31. There are therefore three Things Principally to be Enquir'd into in Reſpect of the Application of Propoſition the Eighth to the Preſent, Namely ad the Diſtance of the Object in the one Caſe and the other, ds the Abſolute Focus, and dk the Imaginary; As to the firſt, a in the Eighth Propoſition or *(Fig.* 2*d.)* is ſuppos'd to be the Radiating Object, which Emits Rays, and in the Preſent Propoſition or *(Fig.* 1.*)* it is that Point, to which the Rays are Refracted; and by Conſequence, unleſs the Courſe of the Rays, and the Force of their Direction, from a and to it, is the ſame, that is, unleſs in *(Fig.* 1*ſt.)* it is the ſame Thing, whether i is carry'd by a certain Impulſe from thence to a, or on the Contrary from a to i, theſe two Demonſtrations are ſo far from ſupporting, that they Contradict each other; and that two Oppoſite Directions are the ſame we cannot eaſily Imagine.

Nothing Demonſtrated in this Propoſition but what ſuppoſes the Thing which is to be Demonſtrated.

§. 32. It is farther to be Obſerved, that as the Diſtinct Baſe, which is Produc'd from the Refraction of Rays Deriv'd from any Object is, in this way Reaſoning, made to be the ſame with the Object, from whence the Rays are Deriv'd, which ſeems to be both Incongruous and Abſurd; So it is likewiſe very Unhappy in this Philoſophy, that it takes *a* for a certain and known Point, whereas it is the very Focus that is Requir'd, and to be Determin'd; for by *(Fig. 2d.)* it is as *as. ds* :: *ad: dk,* in which *a* is known, as being the Radiating Point of the Object, but how the ſame Proportion ſhould be made Uſe of to find out *a* in the Preſent Propoſition *(Fig. 1.)* unleſs by ſuppoſing it, that is, unleſs by ſuppoſing the Point to be Demonſtrated, we are not Inform'd; ſince the Throwing out *a* from the Proportion, and Reducing it to *ds*+*dk. dk* :: *ds. ad* the Quantity Requir'd, is upon the Suppoſition, that *as. ds* :: *ad. dk.* that is, upon the Suppoſition, that *a* is known, when it is the Point Deſir'd.

If this Reaſoning was True, a Plano-Convex Glaſs would have it's Focus nearer to the Glaſs than a Double Convex has which is Contrary to other Demonſtrations in Opticks.

§. 33. THE ſecond Inquiry is Concerning *s* the Abſolute Focus, which in the 2*d.* Scheme is on this Side *k* the Imaginary Focus, and in the 1*ſt.* beyond it, the Reaſon of which is not Aſſign'd, nor can be; ſince the Refraction of a Plano-Convex Glaſs to a Focus is about the Diſtance of the Diameter of the Convexity, and the Refraction of a Double Convex to a Focus at the Diſtance of a Semidiameter only; from which it will follow, that a Double Convex will have it's Focus nearer to the Glaſs than a Plano-Convex has, whereas according to this Reaſoning the Contrary will Enſue, and in *(Fig. 1ſt.)* the Focus *s* from *ff* is at a greater Diſtance than that from *de* in *(Fig. 2d.)*

This Reaſoning depends upon very Arbitrary Suppoſitions.

§. 34. IF *s* in *(Fig. 1.)* is not farther Remov'd from the Glaſs than in *(Fig. 2.)* it muſt be upon one of theſe Accounts, that either the Glaſſes Occaſion ſome Difference in their Refractions by their Different Thickneſs, which this Philoſophy Diſclaims, ſince it only Conſiders the Different Curvities of the Glaſſes, and ſuppoſes the Glaſſes themſelves to be as to their Thickneſs, of little or no Conſideration; or elſe becauſe the Rays are Preſum'd to be Parallel in their Incidence on *ff,* whereas *he* is not ſo, but Divergeing from the Axis, which is at leaſt an Hypotheſis, which is Perfectly Arbitrary, and ought not to be made Uſe of in a Juſt and True Reaſoning.

OR Laſtly it may be ſaid, that *s* in *(Fig. 1ſt.)* is not farther Remov'd from *ff,* than *s* in *(Fig. 2d.)* from *ed,* becauſe the Imaginary Focus *k* may be ſooner Determin'd to a Point in the firſt Caſe, than in the laſt; in Reſpect of which we ſhall here ſay, that ſuppoſing *ds* in *(Fig. 1ſt.)* to be Equal to *ds* in *(Fig. 2d.)* which yet muſt be leſs, if there is any Inequality, by what has been ſaid in the Preſent Section and the Foregoing, becauſe the Incident Rays ought in both Caſes to be ſuppos'd Parallel, and if ſo, a Double Convex will Refract them at a leſs Diſtance from the Glaſs than a ſingle one will, but ſuppoſing them Equal, it is Impoſſible, that *k* ſhould be Determin'd ſooner to a Point in (*Fig. 1ſt.)* where *ci, ci* are Convergeing to the Lens *ff,* than where *he* is ſuppos'd to be likewiſe Convergeing, unleſs it can be ſhewn us, what their Difference of Convergeing is, which ſeems at Preſent to be Entirely Arbitrary.

It alſo ſuppoſes an Incident and Refracted Ray to be the ſame.

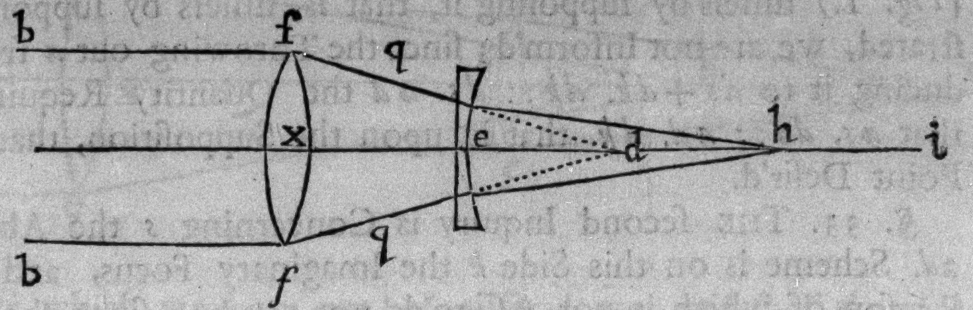
§. 35. THE third Inquiry is Concerning the Imaginary Focus, which is *k,* and in *(Fig. 2d.)* is made by the Production of the Refracted Ray *eh*; Whereas in (*Fig.* 1.) it is Formed by the Production of the Incident *ci*; but it is Impoſſible, it ſhould be the ſame Thing to Prolong the Refracted, or the Incident, Ray, in Order to Determine the Point *k*; ſince, as the Refracted Ray has a Different Inclination to the Axis from the Incident, the Continuation of them muſt Neceſſarily meet the Axis in Various and Diſtinct Points, and therefore cannot Repreſent one and the ſame Imaginary Focus.

No Experiment Deny'd in Opticks but the Reaſoning upon them.

§. 36. WE do not however Deny, that a ſecond Lens will Collect the Rays to a nearer Focus, than one of it's ſelf will without the Aſſiſtance of another; and Conſequently *a* will be nigher to *cc (Fig. 1ſt.)* than *k* is; That Poſſibly may be a Phænomenon in Nature and Confirm'd by Experience; what we Affirm is, that this kind of Reaſoning, which is Eſpous'd by the Preſent Philoſophy, does not give a Juſt Account of that, which it Propoſes to Demonſtrate. All theſe Difficulties are Avoided by the Doctrine of the Expanſive and Contractive Forces, that is, by Conſidering Light, as an Expanſive Force Spread and Diffuſed thro' the Æther from the Sun, and Glaſs as a Contractive one, Acting in the Convex and Concave, as before Explained, *Chap.* 6. §. 23. 24.

Propoſition the ſeventeenth Explain'd
Molyneux Dioptricks, Prop. 17. Part 1.

§. 37. OF the ſame Nature with the Preceding Propoſition is the Seventeenth, which ſays, that a Convex Glaſs being given with a Concave of a larger Sphere, the Concave being Plac'd behind the Convex at any Diſtance, leſs than the Focal Length of the Convex, it will be, if the Diſtance of the Glaſſes is Subſtracted from the Focal Length of the Convex, ſo as to make the Difference *de*; As the Focal Length of the Concave *ie* leſs by the Difference *de*, to the Focal Length of the Concave, So the Difference to the Diſtance of the Diſtinct Baſe from the Concave, Namely *he*; That is, the Focus, made by a Concave after the firſt Refraction is Perform'd thro' a Lens, is farther off from *ff*, than it would have been, if the Concave had not been Interpos'd; which Perhaps may be True in Fact, but the Demonſtration of it not ſo.



The Eighteenth Propoſition. Molyneux's Dioptricks Prop. 18. Par. 1. Tab. 18. Fig. 4.

§. 38. The Eighteenth Propoſition is, the Focal Lengths of a Convex Glaſs, and a Concave being given, and the Concave plac'd towards the Object, Diſtant from the Convex, ſo that the Sum of the Concave's Focal Length and the Glaſſes Diſtance may be greater than the Focal Length of the Convex, and this Diſtance of the Glaſſes being given alſo, if the Focus of the Convex is Subſtracted from the Sum of the Focus of the Concave and the Diſtance of the Glaſſes, ſo as to make the Difference *sa*; It will be, As the Difference *sa* to the Focus of the Convex *sd*, So the ſum of the Concave's Focus and Glaſſes Diſtance *ad* to the Diſtance of the Focus from the Convex *dk*; that is, if *nb*, *mc* are firſt Refracted by a Concave, by which they Diverge, and afterwards by a Lens, the Focus *k* will be farther off from *ef*, than if they had been Refracted by the Lens alone, which alſo may be True in Experience, but that this Demonſtration Proves it, we are not willing to Allow.

Concluſion in General againſt this Kind of Reaſoning.

§. 39. To Conclude this Chapter, all theſe Demonſtrations fix the Collection of the Rays to Particular and Certain Points, which is Evidently Falſe in Experience, ſince they Terminate in Circles; and therefore we may Rationally Infer, that all this Buſtle and Ado about Lines and Angles is nothing elſe than an Abſtracted Reaſoning of the Mind, without being any ways Agreeable to the Real Forces of Nature; They are the Diverſion and Exerciſe of a Mathematick Genius, and not the Reſult of a Judicious and Philoſophick Reaſoning. But it is with Geometers in Conſidering the Great Volume of Nature, as it is with ſome Criticks in Explaining Particular Authors; They both Endeavour, rather to ſhew their Skill in the Science of Criticiſm or Geometry, than to give us the True Meaning of the one, or the Senſe of the other.

CHAP. VIII.

Concerning Meniſcus Glaſſes and the Propoſitions thereto Belonging.

What a Meniſcus Glaſs, and why ſo Nam'd.

§. 1. WE have hitherto Conſider'd the General Propoſitions in this Science, in Reſpect of Concave or Convex Glaſſes; We ſhall now Proceed to Enquire into thoſe, which Concern what are Term'd Meniſcus ones, Namely, ſuch as are Convex on one Side, and Concave on the other, as in *(Fig. §. 6.)* They have their Name from the Greek Word Μηνίσκος, which Signifies Lunula, whoſe Figure theſe Glaſſes ſeem to Imitate and Reſemble.

Some General Obſervations Concerning Meniſcus Glaſſes, and Particularly it is propos'd to Determine the Focus of a Meniſcus, where the Semidiameter of the Convexity is leſs than that of the Concavity. Molyneux Dioptr. Pag. 81. 82. Part 1.

§. 2. In Meniſcus Glaſſes, where the Semidiameters of the Convexity and Concavity are Equal, the Rays, that are Incident on them Parallel to their Axis, are ſaid to Proceed after Refraction in Parallel Lines; where Unequal, if the Semidiameter of the Convexity is leſs than that of the Concavity, the Convexity is Affirm'd to have the greateſt Force of Refraction, and the Glaſs to Poſſeſs a Real Focus; if the Semidiameter of the Concavity is leſs than that of the Convexity, the Concave is ſuppos'd to Prevail, and the Glaſs to have only a Vertual Focus; Conſequent to this, the Focus of a Meniſcus, (the Semidiameter of whoſe Convexity is leſs than that of the Concavity, and theſe two Semidiameters being given) is Thought to be eaſily Determin'd by *Prop.* 17. Since in that two Glaſſes are given, the one a Convex, and the other a Concave, and the Focal Length of the Convex is ſhorter than that of the Concave; and farther, by the ſame Propoſition, the Diſtance betwixt the Glaſſes is given; which in this Meniſcus is Preſum'd to be nothing, or not Conſiderable, in Reſpect of the Thickneſs of the Glaſs, or the Diſtance of the Convex Side from the Concave, Compar'd with the Focal Length of each; Therefore, ſince by *Prop.* 17. It is, As the Focal Length of the Concave − the Focal Length of the Convex + the Glaſſes Diſtance, to the Focal Length of the Concave, So the Focal Length of the Convex − the Glaſſes Diſtance to the Diſtance of the Diſtinct Baſe from the Concave, The Diſtance of the Concave and Convex being Conſider'd as nothing, the Proportion is Chang'd to the following; As the Focal Length of the Concave − the Focal Length of the Convex, to the Focal Length of the Concave, So the Focal Length of the Convex, to the Diſtance of the Focus of the Meniſcus.

And the Semidiameters of the Convexity and Concavity of a Meniſcus being given, the Focal Lengths of the Plano-Convex, and Plano-Concave, of which the Meniſcus is Compounded, are alſo given; Wherefore, the halves being as their wholes, inſtead of the Focal Lengths, the Diameters of the Concavity and Convexity may be Aſſum'd; and the Proportion will be, As the Difference of the Semidiameters of the Convexity and Concavity, to the Semidiameter of the Concavity, So the Diameter of the Convexity to the Focal Length, which is the Proportion Aſſerted by the Writers in Opticks.

The Determination of the Focus of the foregoing Meniſcus not Reducible to the Caſe Alledg'd in Propoſition the Seventeenth for three Reaſons.

§. 3. As to the firſt Part of the Preceding Section, we ſhall afterwards Conſider it in the following Propoſitions, which are Alledg'd to Prove it, and at Preſent take it for Certain and Demonſtrable; It is Affirm'd in Conſequence of it, that a Meniſcus, the Semidiameter of whoſe Convexity is leſs than that of the Concavity, may be Reduc'd to the Caſe in *Prop.* 17. where there are two Glaſſes given, the one a Convex, and the other a Concave, and the Focal Length of the Convex is ſhorter, than that of the Concave, (See *Fig.* §. 37. *Chap.* 7.) To make this Analogy therefore True; 1*ſt.* It is Requiſite to Prove that Propoſition, which has not been done; 2*d.* It is Neceſſary to Evince, that where the Vertual Focus ie of the Concave is longer than the Abſolute Focus dx of the Convex, it is always Certain, that the Semidiameter of the Concave is alſo greater than that of the Convex, which has been Endeavour'd, Indeed, but by Propoſitions and Demonſtrations, which have been already Refuted, Namely, which make the Vertual Focus of a Concave in Equal Curvities to be Determin'd at the ſame Diſtance with the Abſolute one of a Convex, that is about the Diſtance of the Diameter of the Concavity or Convexity, and therefore the Larger the Concavity or Convexity is, the Focus will be the Longer; 3*d.* Even thoſe Demonſtrations Concern Plano-Convex and Plano-Concave Glaſſes, But *Prop.* 17. is Concerning a Double Convex Glaſs, which Collects it's Rays at half the Diſtance of a Plano-Convex, to wit, at about the Diſtance of the Radius of the Convexity, whereas the Meniſcus is Compounded, by what is before Affirm'd, of a Plano-Convex and a Plano-Concave, which therefore cannot be Reduc'd to two Glaſſes, in *Prop.* 17. one of which is a Double Convex, and the other, either a Double Concave or a Plano-Concave.

Not Reducible to the Caſe Alledg'd in Propoſition the Seventeenth for a fourth Reaſon.

§. 4. If this were Allow'd, ſince by *Prop.* 17. it is, as $ie - dx + ex.\ ie :: dx - ex.\ he$, or as $id.\ ie :: de.\ he$, and ſince ex the Diſtance of the Glaſſes is ſuppos'd to be Exterminated out of the Proportion in the Meniſcus, it is ſaid, it will be, as $ie - dx.\ ie :: dx.\ he$, the Focal Length of the Meniſcus; But why ex the Diſtance of the Glaſſes ſhould in the Meniſcus be Conſider'd as nothing, when it is ſuppos'd to be Divided into a Plano-Convex, and a Plano-Concave, which Diviſion will make a much greater Difference in the Refractions than the Intermediate Diſtance betwixt a Double Convex and a Concave, is Unaccountable; for the Plano-Convex and Plano-Concave have their Abſolute and Vertual Focus's at Double the Diſtance of what a Double Convex or Concave have, whereas the Diſtance betwixt the Glaſſes ex which is Undetermin'd, and may be more or leſs, as it is given, will in twenty Caſes Cauſe not half the Difference in the Vertual and Abſolute Focus's, which the other does.

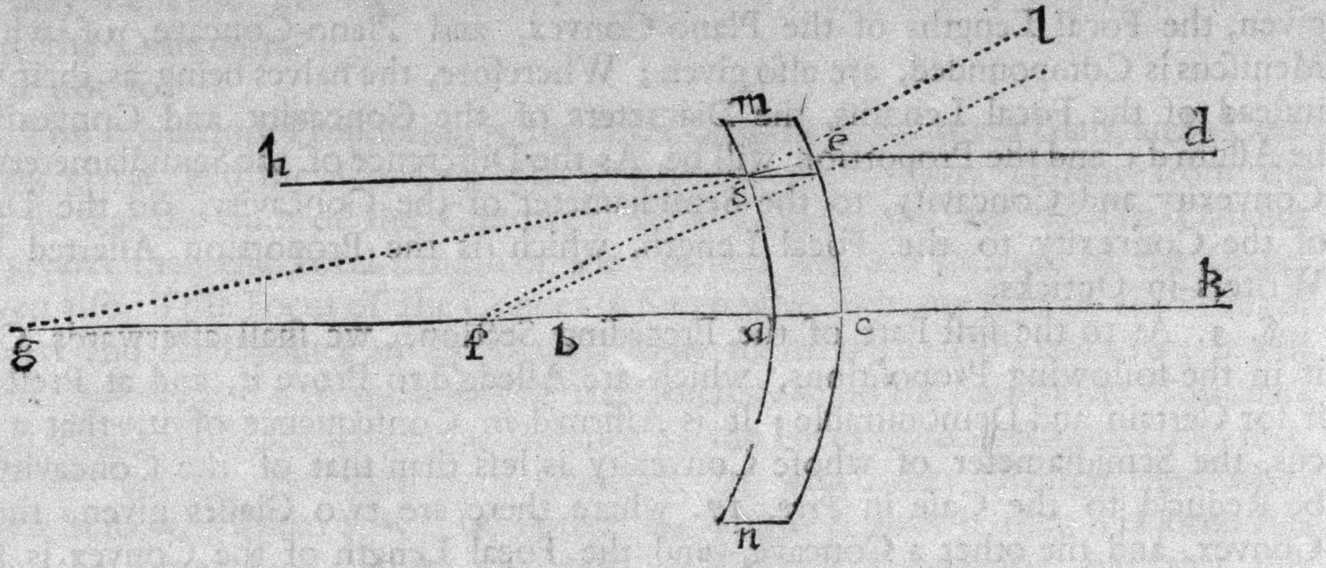
Not Reducible to the Caſe Alledg'd in Propoſition the Seventeenth for a Fifth Reaſon.

§. 5. Moreover it is not Evident or Plain, that upon the Diminution of the Diſtance of the Glaſſes to an Infinity, the ſame Proportions will hold, unleſs, by Enlarging it to Infinity within the Limits Preſcrib'd in the Propoſition, they likewiſe will, which this Philoſophy will not, I Preſume, Venture to Affirm; ſince if the

Concave Approaches Infinitely near to *d*, it will be in the very Focus of the Convex, and Conſequently in ſuch a Caſe, and by this way of Reaſoning, the Proportion will be as *id. id* : : *de. he*, which will therefore be Equal, and the Diſtinct Baſe of the Concave the ſame with the Focus of the Convex; that is, the Concave and Convex will Refract the Rays alike, which is Impoſſible, or elſe the Concave will not Refract them at all, which is Contrary to Experience; To Wit, *df* will Exactly Coincide with *hf* after it's Refraction in *e*; So little can we Depend on the Proportions Aſſign'd by the Writers in Opticks.

Propoſition the Nineteenth Explain'd, that if the Spherical Superficies's of a Meniſcus have the ſame Diameter, the Ray which is Incident Parallel to the Axis, Emerges Parallel. Molyneux Dioptr. Prop. 19. Part 1.

§. 6. PROPOSITION the Nineteenth, it is Aſſerted, in a Meniſcus, if both the Spherical Superficies have the ſame Diameter, the Ray, which is Incident upon it, Parallel to the Axis, after it's ſecond Refraction Proceeds likewiſe Parallel. To Prove which



it is ſuppoſ'd, that the Thickneſs of the Glaſs is Inconſiderable, and that the Sines, and Angles, and Sides, in theſe ſmall Angles are Proportional; If ſo, let *gc* = *ga* be made Triple of *bc* = *fa*, the Semidiameters of the Convexity *ec* and the Concavity *sa*, the Incident Ray *de* at *e* is Refracted towards *g*, and *esg* is a Right Line, at *s* it is again Refracted to *h*, and as *lse* is the Angle of Inclination or Incidence, which is = to *fsg*, the Angle of Refraction is *hsg* = *sgf*, if *hs* and *gf* are Parallel; Conſequently by Experiment the *5th*. the Angle of Refraction *hsg* = *sgf*, will be half of *gsf*, as it is Evident it is by this Conſtruction, becauſe *sf* is the half of *gf*, and it is ſuppoſ'd, that the Angles and Sides are Proportional, and therefore *hs* and *gf* are Parallel.

Argument the firſt againſt this Demonſtration.

§. 7. IN Anſwer to this Demonſtration we ſay, that if *fa* and *bc* are Equal, *fs* and *be* will be ſo too, and ſince the Diſtance betwixt *a* and *c* is Conſider'd as nothing, *f* and *b* will Fall into the ſame Point; From whence it will follow, that *fe* and *fs* are Equal, and that there is no firſt and ſecond Refraction, which this Demonſtration ſuppoſes, but that they are both of them Reſolv'd into one in *s* or *e*, which will Deſtroy the very Nature of a Meniſcus, which Conſiſts of a Concave and a Convex; Since by taking away the Diſtance betwixt them, the Meniſcus muſt either be a Concave or a Convex, and Conſider'd as ſuch, muſt either entirely Contract the Rays, or make them Diverge.

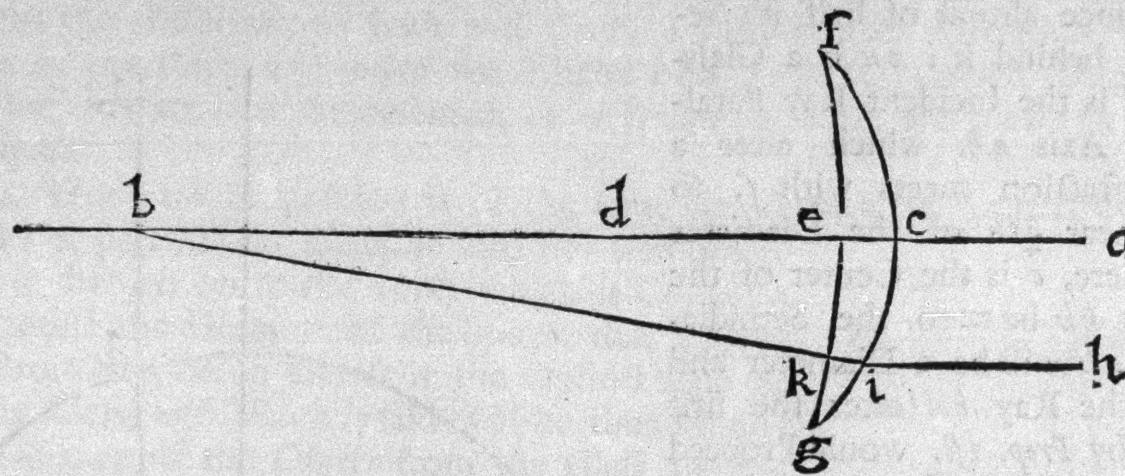
Argument the Second.

§. 8. IT is likewiſe Affirm'd, that the Sines and Angles are Proportional, which is not True, unleſs in very ſmall Segments of a Sphere, and where the Diameters of them are very Large, whereas it is not, that I know of, Evident in Fact, or from this Demonſtration, that if Rays are Refracted Parallel in ſmaller Segments, they are not ſo in much Larger, the Diameters of their Curvity Continuing the ſame.

Argument the Third.

§. 9. GRANTING which, it is not Prov'd, that *hs* is Parallel to *gf*, unleſs by ſuppoſing in the Conſtruction of the Scheme, that *gf* is the Double of *fs*, or which is the ſame, that *sgf* is half of the Angle of Inclination *fsg*; To Evidence therefore, that *sgf* is half of the Angle of Inclination *fsg*, by *gf* being ſuppoſ'd in the Conſtruction, to be Double of *fs*, or *fs* half of *fg*, is no other, than Proving by ſuppoſing the very Thing, which is to be Prov'd; For tho' it ſhould be Granted by Experiment the *5th*, that *hsg* is half of *gsf*, it will not follow that *fgs* is ſo, unleſs by this Conſtruction, that is, by ſuppoſing *fs* to be half of *gf*, or *fgs* Equal to *hsg*, or which is the ſame, it will not be Conſequent, that *hs* and *ga* are Parallel, unleſs by ſuppoſing, that they are Parallel; On the Contrary, if we take gf the Triple or Quadruple of fs, which it may as well be, as the Double of it, the Angle gsf will be the Triple or Quadruple of fgs, whereas it is only Double of hsg by Experiment; and therefore ſince fgs is not Equal to hsg, ga and hs will not be Parallel; why therefore this Demonſtration Aſſumes gf to be the Double, rather than the Triple, or any other Multiple, of fs wants to be ſhewn; ſeeing the Angle of Inclination or Incidence gsf may be Double the Angle of Refraction hsg, according to the Experiment, let the Proportion of gf to fs be what it will.

§. 10. Proposition the Twentieth is, that in a Meniſcus if the Semidiameter of the Concavity be Triple the Semidiameter of the Convexity, the Focal Length is Equal to the Semidiameter of the Concavity; For let dc be the Semidiameter of the Convexity and $be = 3dc$ the Semidiameter of the Concavity, let the Axis be ab, and the Incident Ray hi; by *Prop.* 1. *Schol.* hi after it's firſt Refraction will tend to a Point, which is Diſtant a Diameter and half of the Glaſs's Convexity, that is to b, but b is the Center of the Concavity, and therefore ik Falls Perpendicular on the Concave Surface fkg, and Conſequently is not Refracted at all, Proceeding, after it's Emerſion from the Glaſs, directly forwards in the Line $ikb =$ to the Semidiameter of the Concavity.



Propoſition the Twentieth Explain'd, that in a Meniſcus if the Semidiameter of the Concavity, is Triple the Semidiameter of the Convexity, the Focal Length is Equal to the Semidiameter of the Concavity. Molyneuxs's Dioptricks Prop. 20. Par. 1.

§. 11. This Demonſtration therefore Depends upon Propoſition the firſt, which has been before Examined, and farther ſuppoſes that ikb is $=$ to be, which is the Diameter and half of the Convexity, the Diſtance Requir'd for the Focal Length of hi after it's firſt Refraction; but eb is $=$ to bk, and therefore bk is $= bi$ a Part to the whole; if ki is Conſider'd as nothing, then there will be no firſt and ſecond Refraction, and the Original Suppoſition of the Semidiameter of the Convex being $\frac{1}{3}d$ of that of the Concave is deſtroy'd; ſince i and k will every where Coincide, and the Concave and Convex will be the ſame.

Argument the Firſt againſt this Demonſtration.

§, 12. Besides which, hi after the firſt Refraction is ſaid to Proceed from i to b, becauſe it is Equal to eb, which is a Diameter and half of the Glaſs's Convexity by Conſtruction; 'Tis True the Radius of the Concavity Reaches from b to k, but it does not from thence follow, that ib muſt Neceſſarily be drawn thro' k; for it is not Evident, that it may not Fall upon any other Radius, which is made from b betwixt k and e, in which Caſe ik would be a ſecond time Refracted, contrary to what is here Affirm'd, and yet might Preſerve it's Focal Length at the ſame diſtance of b.

Argument the Second.

§. 13. We Say it is not Manifeſt from this Way of Reaſoning, that the Ray does not Admit of a ſecond Refraction, ſince the only Proof, that it does not, is, that after the firſt Refraction it is Incident upon bk which is a Perpendicular to the Concavity, whereas it will not be therefore certain that ik is a Perpendicular to the Convexity; if it was, ie, or i drawn to any Intermediate Point betwixt e and k, would be likewiſe a Perpendicular; ſince it would Fall upon a Radius, which is Perpendicular to the Concave from the Center b; and in General there could be no Secondary Refraction in a ſecond Convex $fekg$, or even in the firſt $fcig$; ſince even hi is Incident upon ſome Radius or other Produced from d, and yet in this Demonſtration all this is ſaid to be moſt Evident.

The foregoing Argument farther Explain'd.

§. 14. Proposition the Twenty Firſt, it is Affirm'd, that in a Meniſcus the Semidiameter of whoſe Convexity is Triple the Semidiameter of the Concavity, the

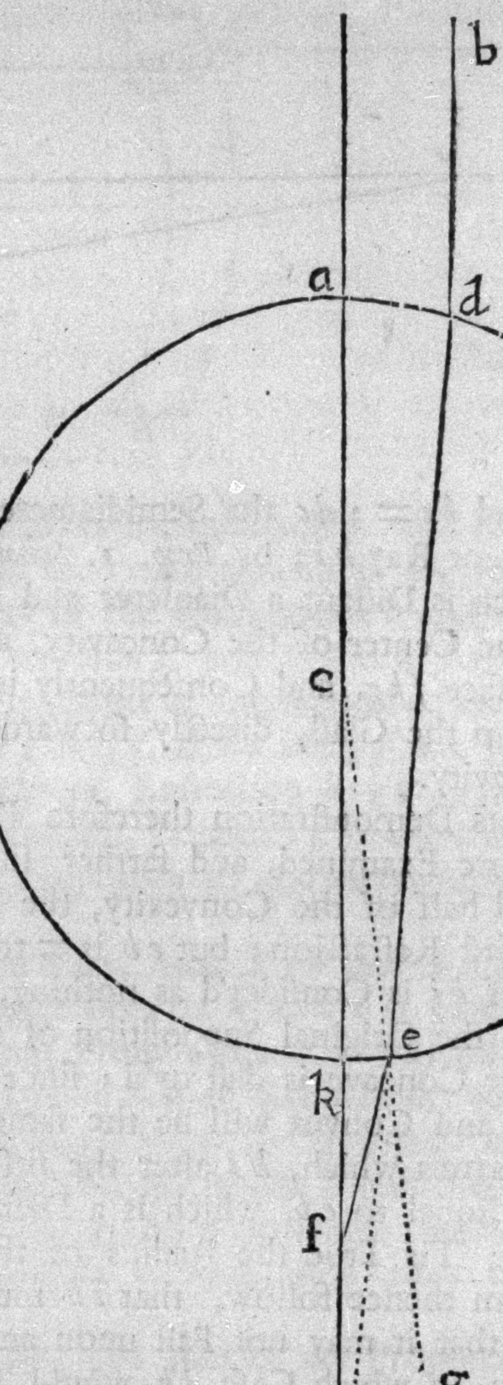
Propoſition the Twenty firſt Conſider'd.

Molyneux's Dioptricks, Prop. 21. Par. 1.

Vertual Focus is diſtant the Semidiameter of the Convexity; The Demonſtration of which is Exactly the ſame with that of the Preceding Propoſition, (See *Fig.* 4. *Tab.* 20. which it is not here Neceſſary to Tranſcribe) unleſs that the Vertual Focus is here Conſider'd, whereas the Abſolute was in the Former, and as to what Concerns the other Propoſitions, which Determine the Real or Vertual Focus's of Meniſcus Glaſſes in General, where the Semidiameters of their Convexity or Concavity are likewiſe Unequal, we ſhall wholly Omit them, as their Demonſtrations are Founded upon a Reaſoning, which is alike Abſtracted, and therefore Signifies nothing to the Explaining the True Nature or Plain Phænomenons of Viſion, and is Equally Falſe and Erroneous.

Propoſition the Twenty Fourth that a Glaſs Sphere Unites the Parallel Rays at the Diſtance of almoſt half it's Semidiameter behind the Glaſs Conſider'd and Explain'd. Molyneux. Dioptr. Prop. 24. Par. 1.

§. 15. From Propoſition the Twenty Firſt, we ſhall therefore Proceed to Propoſition the Twenty Fourth, which is, That an Entire Glaſs-Sphere Unites the Parallel Rays at the Diſtance almoſt of half it's Semidiameter behind it; *ak* is a Glaſs-Sphere, *bd* is the Incident Ray Parallel to the Axis *ak*, which after a double Refraction meets with *f*, ſo that *fk* is near $\frac{1}{4}$*th* of the Diameter of the Sphere, *c* is the Center of the Sphere; let *kh* be = to the Semidiameter *ck*, *ah* will be a Diameter and half, and the Ray *bd* after the firſt Refraction by *Prop. 1ſt.* would Proceed in a Strait Line to *h*, if the Glaſs reacht ſo far, but at *e* it Emerges from Glaſs into Air; Draw the Perpendicular *ceg*, the Angle of this Second Inclination or Incidence is *ced* = *geh*, and the Ray is Refracted from the Perpendicular *eg* in the Angle *feh*, which ought by Experiment the *5th* to be half of *ced* = *geh*, and ſo it is ſaid to be; For *ced* is = *ech* + *ehc* by 32. 1. Elem. but *ech* = *ehc* nearly, becauſe their Subtenſes *eh* and *ec*, or *hk*, are ſo, and therefore *ehc* is half of *ced*; Again *ehc* is = *feh* nearly, ſince *fe* the Subtenſe of the one is nearly = to *fh* the Subtenſe of the other, becauſe *fh* is = to *fk* by Conſtruction, and there is little or no Difference betwixt *fk* and *fe*, and by Conſequence the Angle *feh* = to *ehc* is half the Angle *heg* = *ced*, the Angle of Incidence or Inclination.



This Demonſtration ſuppoſes Lines, which are plainly different, to be Equal, and by that Means Concludes againſt a ſecond Refraction, tho' it is brought to Confirm one.

§. 16. As to the firſt Refraction of the Incident Ray *bd* to *h*, that Depends upon *Prop. 1ſt.* and as to *feh* the Angle, which is made by the ſecond Refraction, being half of the Angle of Incidence or Inclination, the Proof of it is Referr'd to Experiment the *5th*, both which Articles of Demonſtration have been Refuted; What Remains therefore to be Conſider'd is only this, the near Equality of *fk* and *fe*, as likewiſe of *hk* and *he*; For if *fk* and *fe* may be Conſider'd as Equal, and *hk* and *he* may too, then *hf*, *fe* may be likewiſe Conſider'd, as = to *he*, and *he*, *ec*, = to *hc*, which will not only Contradict a Plain Propoſition in *Euclid*, where the two Lines of any Triangle are Greater than the Third, (and *efh* is in this very Demonſtration ſuppos'd to be a Triangle,) but alſo the Lines *ef*, *fh* being the ſame, or being Conſider'd as the ſame, with *eh*, the Point *f* may be Imagin'd to be in the Line *eh*, and therefore the Ray *bd* after the ſecond Refraction ought to Proceed to *h*, ſince it Continues in the ſame Line from the firſt Refraction, in which it would Advance, if there were no Second.

The whole Sphere to be Conſider'd in the Refractions of Light and not an Arc of 30 Degrees only.

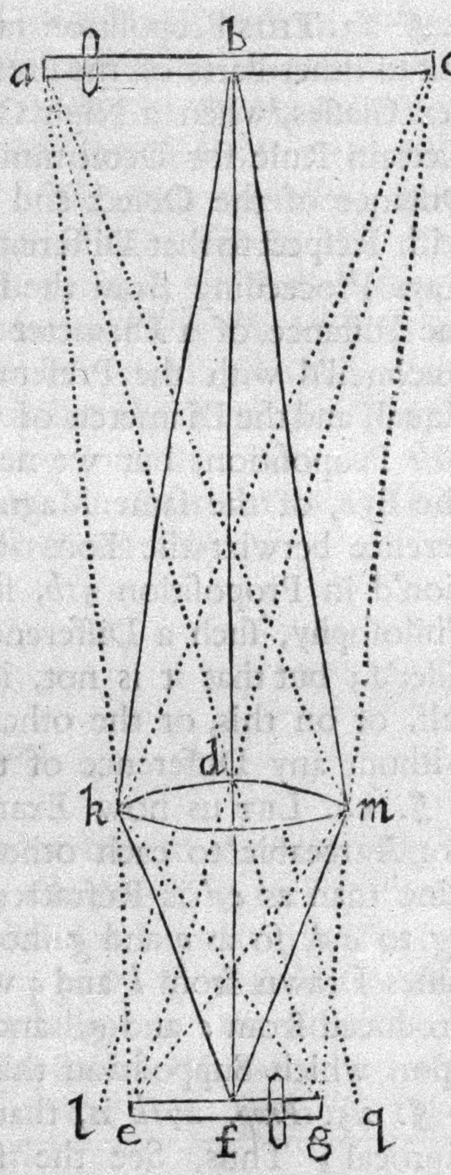
§. 17. But what is more is, that the firſt Propoſition ſuppoſes an Arc not Greater than 30 Degrees, that the Angles are Extremely Small, and the Lines and Sides of them therefore Proportional, whereas the Properties of a whole Sphere are here to be Conſider'd in Reſpect of the Refractions of Light, and Conſequently Angles that are Large, and whoſe Sines and Sides are not Proportional.

This Demonſtration ſuppoſes a Certain Convexity of the Sphere, and therefore cannot be Extended to Spheres of all Diameters.

§. 18. Lastly in that Propoſition the Convexity is Preſum'd to be Large, and the Incidences of the Rays not Extremely Differing, altho' in the Preſent the Diameter of the Sphere is not Specify'd, and whether it is Greater or Leſs, or it's Convexity; There is no Exception made, nor any Precautions us'd, notwithſtanding it is Evident the Difference of both will Cauſe a Conſiderable Alteration in the Refractions of the Rays, and make the Angles *ced*, *feh* very much Diſproportion'd from what they are here Repreſented, as well as *eck*, *ehk*, and *feh*, *fhe*, far from being Equal.

The Twenty ſixth Propoſition Explain'd, Concerning the Diameter of the Image in the Diſtinct Baſe. Molyneux Dioptricks Prop. 26. Part. 1.

§. 19. The 25*th* Propoſition is of the ſame Nature with the former, and therefore we ſhall not particularly Examine into the Demonſtration of it, but go on to the 26*th*, which is Contriv'd to Determine the Diameter of the Image, that is made by the Projection of an Object in the Diſtinct Baſe of a Convex Glaſs, and which is ſaid will be thus found; As the Diſtance of the Object from the Glaſs to the Diſtance of the Image from the Glaſs, So the Diameter of the Object's Magnitude to the Diameter of the Image; *abc* is an Object, *kdm* a Convex Glaſs in the Foramen of a Dark Room, *efg* the Image Form'd by this Glaſs on a White Paper in the Diſtinct Baſe; It will be, as *cd* or *bd* to *ed* or *fd*, ſo *ac* to *ge*, which is thus Prov'd; For by the Premiſſes to the 4*th* Propoſition, the Axes *ad* and *cd* of the Luminous Cones *kam*, *kcm* Paſs thro' the Glaſs, as if they were not Refracted at all, becauſe their Refractions are Parallel to their Incidences, and therefore in Glaſſes of large Spheres, and ſmall Segments, the Thickneſs of the Glaſs, being Inconſiderable in Reſpect of the Focal Diſtance, may be Neglected; From whence it will Follow, that *adc*, *edg* will be two Similar Triangles, and Conſequently *cd. ed*∷ *ac. ge.*



This Reaſoning not Agreeable to Nature.

§. 20. The Premiſſes to the 4*th* Propoſition have been Conſider'd, and a Confutation of them need not be here Repeated; we ſhall Add, that ſuppoſing the Triangles *adc*, *gde*, even in a greater or a leſs Thickneſs of the Glaſs to be Similar, this very Demonſtration is an Abundant Proof, that this Philoſophy of Rays and their Refractions is only an Abſtracted Reaſoning, which is neither Agreeable to Nature, nor to other Parts of the ſame Doctrine, nor to the preſent Aſſertions; It is not Agreeable to Nature, ſince in Proportion, as the Diſtance of the Object from the Lens is Diminiſhed, the Diſtance of the Image in the Diſtinct Baſe from the Lens ought to be Increas'd, and the Magnitude of it, by which Means if *ac* Approaches the Lens *km* to the ſame Nearneſs, as the Image *eg* is Refracted from it, *eg* will be as far Diſtant, and Obtain the ſame Magnitude, as *ac* is ſuppos'd to have in the Preſent Figure, which is not Confirm'd by any Experiment in Nature, but the Contrary; For if an Object, which is in Length about four or five Yards, is Projected thro' a Lens in the Foramen of a Dark Room at the Diſtance of Thirty eight or Forty Yards, the Image will not be above one or two Inches at the moſt, at about the Diſtance of a Foot from the ſame Lens; which will not Agree with this Doctrine of the Image's being Increaſed or Diminiſhed in Proportion to the Acceſs or Receſs of the Object; for ſince, where the Length of the Object is four Yards, for

Inſtance, and the Diſtance from the Lens Thirty Eight, the Magnitude of the Image will be but about an Inch, and the Diſtance of it from the Lens about one Foot, the Diſtances will Conſequently be of the Object and the Image from the Lens as 114 to 1, whereas the Magnitude of the Object and Image will be as 144 to 1.

Contradicted by what we every Day ſee and Experience by our Eyes.

§. 21. It is alſo Manifeſt from this Doctrine, that the Eye is Conſider'd only as a Lens, and Conſequently as *a c* comes nearer to it, the Diſtance of the Diſtinct Baſe at the Retina muſt be Proportionably Enlarg'd, as well as the Image, which is Impreſs'd upon it, which is altogether Impoſſible, unleſs we can ſuppoſe the Retina to be Extended beyond the Lens of the Eye to the Length of *b d* on the one Hand, or to the Breadth of the whole Object *a c* on the other, which will be above the Power of any Muſcles to Effect, whatever Faculty they may be Reputed to have that way.

Objects not Increaſed or Diminiſhed in Proportion to their Diſtance.

§. 22. Besides which, where the Eye is Fix'd and Unmov'd, and cannot with the leaſt Reaſon be Thought to be Alter'd in it's Figure by the Compreſſion of it's Muſcles, this Proportion does not Obtain, for we do not ſee an Object Double at Half the Diſtance and Triple at one Third of the Diſtance, *&c.* But it Continues Viſible in much the ſame Magnitude for a Conſiderable Space together.

This Reaſoning Contrary to that which is Alledg'd in other Propoſitions.

§. 23. This Propoſition farther is not only Contrary to the Appearances in Nature, but to other Parts of the Preſent Philoſophy; for by Propoſition the *5th* in Plano-Convex Glaſſes, when a Nigh Object is Placed more Diſtant than the Focus, there is one Certain Rule for Determining the Diſtinct Baſe, in which the Difference betwixt the Diſtance of the Object and the Focus is Conſider'd, and the Proportion Aſcertain'd with Reſpect to that Difference; by Propoſition the *9th.* In Double Convex Glaſſes, the Rays Proceeding from the Diſtance of a Diameter of one Convexity are United at the Diſtance of a Diameter of the other Convexity; which Propoſitions cannot be Reconcil'd with the Preſent, for *c d* being to *e d : : a c. g e,* whenever *c d* and *e d* are Equal, and the Diameters of the Convexities Equal, *a c* and *e g* will be ſo likewiſe by the *9th* Propoſition, but we never find the Images of Objects in a Dark Room, or in the Eye, of the ſame Magnitude with the Objects themſelves; And as to the Difference betwixt the Focus and the Diſtance of the Object in Plano-Convexes Mention'd in Propoſition *5th,* ſince a Double Convex is a Double Plano-Convex in this Philoſophy, ſuch a Difference ought in ſome Proportion or other to be here Conſider'd; but that it is not, is Evident, ſeeing, whether the Object is in the Focus it ſelf, or on this, or the other Side, of it, the Diſtance of the Object is only Affirm'd without any Difference of that Kind, to Determine the Diſtinct Baſe *e g.*

Contrary to the Aſſertions in this.

§. 24. Let us now Examine into the Preſent Aſſertions, which we ſaid were not Agreeable to each other, for *c d e* is no otherwiſe Preſum'd to be one Direct Line than as *e d* is Refracted Parallel to *c d*; let therefore *e d* be Parallel to *c d*, and *d g* to *a d*, ſo as *e* and *g* ſhould be Determin'd in *l* and *q*, it is Manifeſt, that the Lines Drawn from *l* and *q* will not Comprehend the ſame Angle with thoſe, which are Produced from *e* and *g*, and therefore cannot make a Similar Triangle with *a d c*, upon which Suppoſition this Reaſoning Depends.

Propoſition the Twenty Seventh Explain'd, that the Object and it's Image are Reciprocal. Molyneux Dioptricks Prop. 27. and Corollary.

§. 25. *Prop. 27th* is, that the Object and it's Image in the Diſtinct Baſe are Reciprocal; Thus, See the foregoing Figure, *a b c* being the Object, *e f g* is the Image in the Diſtinct Baſe; And ſince the Object and it's Image are Reciprocal, the Image may be Conſider'd as the Object, and *a b c* on the Contrary will be the Image; In like Manner, let the Sun Project his Image in the Focus of a Convex, that Image Reciprocally may be Conceiv'd as a Real Sun, which will have it's Diſtinct Baſe as Large, and as Remote, as the Real Sun is, ſo that every Convex will have two Focus's, one in the Image, and the other in the Object.

Argument againſt this, becauſe it is the ſame, as to ſuppoſe, that becauſe the Cauſe Produces the Effect therefore the Effect Produces the Cauſe.

§. 26. This ſort of Reaſoning is, as if we ſhould Affirm the Fountain and the Stream, which is Deriv'd from it, is Reciprocal, and Mutually Infer each other, and therefore Conclude, that becauſe the Stream Runs down from it's Source, it is carry'd up to it in a quite contrary Current; or as if we ſhould Pretend to Demonſtrate, that ſince an Adequate Cauſe Produces it's Proper Effect, in Return the Effect will Produce it's Cauſe; The Caſe is the ſame here, the Sun, or any other Object, by a Derivation of Light from either, and a Refraction of it to a Focus, Expreſſes an Image in the Diſtinct Baſe, and where there is an Object, in ſuch a Circumſtance of it, there will be an Image, and Reciprocally; But from thence to Infer, that there is a ſort of Reaction in the Image, by which it Returns the Rays back to a Focus in the Object, is to ſuppoſe the Rays Actually Refracted in two Contrary and Oppoſite Directions, Namely, from and to the Object; or, which is all one, that they Reſult back to the Source, from whence they Flow, at the ſame Time, that they are Deriv'd in a Direct Stream from it.

The former Argument farther Explain'd.

§. 27. It is Plain and Evident, that all, which can be ſaid in Reſpect of theſe Reciprocal Focus's, is only this, that the Object, as the Cauſe, has at ſuch a certain Diſtance from the Glaſs, ſuch a certain and Determin'd Focus, as it's Effect, or on the Contrary, the Focus, Conſider'd as the Effect, will have an Object at ſuch a Diſtance for it's Cauſe; ſince it is Extremely Unreaſonable to Think, that Rays ſhould have the ſame Properties, when they are Conſider'd as Active, as they have, when they are only Paſſive; for whilſt they are Impreſſed from the Object upon the Lens, they are Active towards the Lens, and Paſſive, or Unactive, from it towards the Object, and ſo likewiſe from the Lens to the Focus; But if the Diſtinct Baſe is now Imagin'd to be Active, the Rays on the Contrary will be Active that way, in which they were before Unactive; and Conſequently, unleſs Action, and Paſſion or Inaction, will Produce the ſame Effects, the Object and Image cannot be in the Senſe, which this Philoſophy ſuppoſes, Reciprocal.

This Reaſoning Invalid, becauſe the Dimenſions of the Object are never Alter'd by an Approach to or a Receſs from the Lens, whereas thoſe of the Image are.

§. 28. Besides, the Falſehood of theſe Reciprocal Focus's is Manifeſt from Hence, that the Object *a b c*, as it Approaches nearer to the Lens, makes an Alteration in the Dimenſions of the Image, which is Proportionably Increaſed to the Approach of the Object, whereas on the Contrary, the Approach of the Image has not the like Effects, ſince the Object continues Immutably the ſame. We ſhall therefore here Conclude the Preſent Chapter, having Conſider'd all that is Material in the Abſtracted Reaſoning and Demonſtrations of this Science, from which we ſhall Proceed to the Explanations given us of the more Evident Phænomenons, it Undertakes to Solve. After we have ſaid in General, that the Properties of Meniſcus Glaſſes are only, and no otherwiſe to be Explained, than by Aſcertaining the Expanſive Forces of Light and the Contractive of the Glaſſes, and Examining into their Mutual Actions upon each other; And the ſame Doctrine Obtains both in Relation to the Refractions of a Sphere, or the Determining the Diſtinct Baſe of an Object.

CHAP. IX.

Concerning Simple or Plain Viſion.

Propoſition the Twenty Eighth Explain'd in Part. Molyneux's Dioptricks, Prop. 28. Part 1.

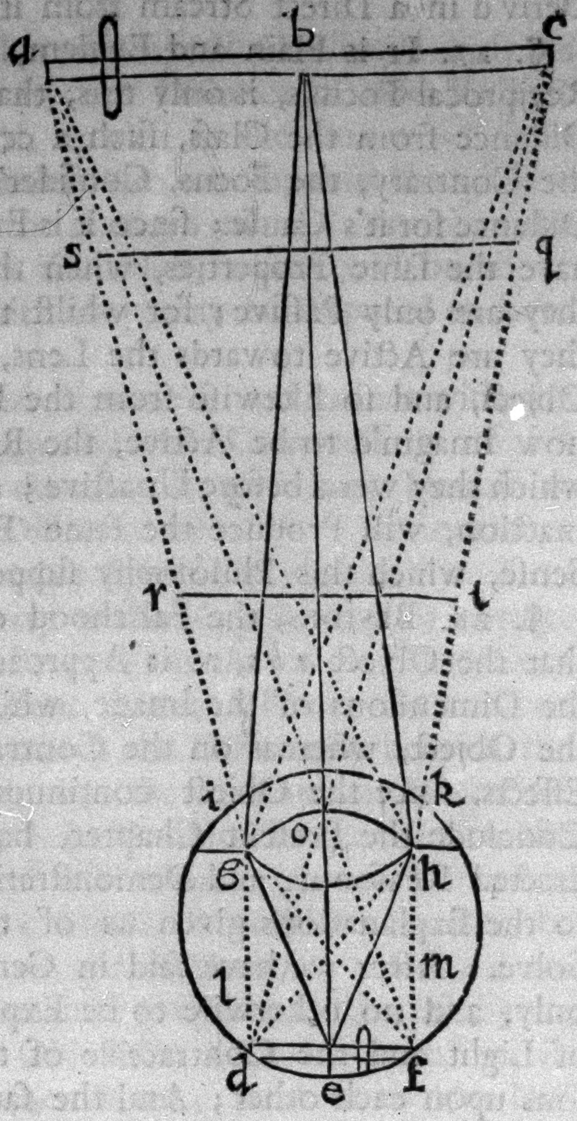
§. 1. THE firſt Appearance, which this Philoſophy Endeavours to Account for, is the Manner of Plain Viſion with the Naked Eye; *a b c (Fig. §. 3)* is an Object, *i k l m* is the Globe of the Eye, in which *i k h g*, Comprehended betwixt the Tunica Cornea *i k*, and the Convex *g h*, is the Aqueous Humour; *g o h*, Contain'd in the Lens, the Chryſtalline; *g h m l*, which lies betwixt the Chryſtalline and the Bottom of the Eye, the Vitreous; but in the Preſent Opticks the Chryſtalline Humour is only Conſider'd, as Subſervient to Viſion, and the Refraction of the Rays from the Object *a b c* thro' it to the Bottom or Retina of the Eye *d e f*.

The Chryſtalline Humour only made Uſe of to Explain Viſion, whereas the Aqueous and Vitreous are without any Reaſon Neglected.

§. 2. It is therefore to be Obſerv'd upon the former Section, that the Writers in Opticks Expreſly Mention three Sorts of Humours, the Aqueous, Chryſtalline, and Vitreous, and yet one Refraction of the Rays thro' the Chryſtalline is only Conſider'd, which at leaſt is a Poſtulatum, that may be Juſtly Diſputed; for ſince they are all Acknowledg'd to be different Mediums, why in the 1*ſt*. Refraction a Meniſcus, or Concavo-Convex, or in the 3*d*. a Concave Medium, ſhould not have their Particular Laws of Refracting, as well as in the 2*d*. a Double Convex ſhould, will not be eaſily Solv'd; Eſpecially, where by no Experiments, which have been made, the Aqueous, Chryſtalline, or Vitreous, Humours are any ways Aſcertain'd in their Refractive Force, as they Lye in the Eye, and therefore not the one to be Prefer'd to the other; unleſs we are willing to Allow, that this Philoſophy may give a Particular Indulgence to the Humour it Eſpouſes, without Aſſigning the leaſt Reaſon, why that ſhould be more Important or Conſiderable than the Reſt. On the Contrary, the Doctrine of the Expanſive and Contractive Forces Requires, that we ſhould know, what Force of Expanſion or Contraction belongs to Each, which is more Significant to the Explaining of Viſion than any Convex, or Concave, Figure of the Humours whatſoever.

A farther Account of Plain Viſion. Molyneux's Dioptricks Prop. 28. §. 1. 2. 3. Part. 1.

§. 3. Farther, this Explanation of Viſion Propoſes, that from each Point of the Object ſhould Flow the Rays *gah*, *gbh*, *gch*, *&c.* which, being Refracted by the Chryſtalline Humour *gh*, are again United at the Bottom or the Retina of the Eye in *f*, *e* and *d*, and ſeeing it is Neceſſary, that the Pencils of Rays ſhould have their Preciſe Apices in the Points *d*, *e*, *f*, to make Viſion Diſtinct, 'tis likewiſe Requir'd, that there ſhould be a Power in the Eye of Adapting it's ſelf to Nigh and Diſtant Objects, the Rays of the former of which, Proceeding from the Luminous Points, Diverge more than thoſe, which are Deriv'd from Remoter ones; but how this Conformation of the Eye to the different Incidences of Rays is Accompliſhed, ſo as to Collect them to certain Points in the Retina without any Manner of Confuſion, is Undetermin'd; however two ways are Thought of, by which this may be done; Either by the Approach and Receſs of the Chryſtalline to and from the Retina, or by the Change of it's Convexity, but which of them it is, is Reſign'd to the Anatomiſts to Inquire, and at Preſent is only Conjectur'd, that they Attend each other, Namely, that a leſs Convexity of the Chryſtalline is Accompany'd with an Elongation of the Eye, and a greater with a ſhortening or a Contraction of it; It is alſo Obſervable from this Account of Viſion, that the Rays from each Point of the Object are Confus'd together on the Pupil of the Eye *gh*, ſo that, if it were not for the Refraction of them by the Chryſtalline or Lens, they would Proceed to the Retina in the utmoſt Diſorder, ſince the Rays from the ſeveral Points of the Object, *a*, *b*, and *c*, are there ſuppos'd to be Mixed and United into one.



The foregoing Propoſitions in this Science of no Uſe to the Explaining what Viſion is.

§. 4. In Anſwer to this whole Account of Viſion we muſt firſt Say in General, that it is very Surpriſing, that Twenty Seven Propoſitions ſhould be Spent in this Science of Opticks without being able to Apply one of them to the Explanation of the firſt Principle in it, the Cauſe of Sight, and how it is Perform'd in the Eye; ſince there are not any of them Cited to Confirm this Doctrine, which Undoubtedly they ought to be, if they were of any Significancy to the Proof of it, and which is therefore a farther Conviction, that all, which has gone before, is nothing elſe than an Abſtracted Reaſoning, and is of no greater Service to the Demonſtrating, by what Methods we ſee, than the moſt Abſtracted Propoſition or Axiom in the Mathematicks, that the three Angles of a Triangle are Equal to two Right ones, or that 90, 60, and 30 Degrees are Equal to 180.

The foregoing Propoſitions Contradict this Account of Viſion.

§. 5. But that, which is more Unhappy in this Doctrine of Opticks is, that ſeveral of the Propoſitions, before Demonſtrated, are ſo far from Complying with the Preſent Account of Viſion, that they Contradict it; for by the Preceding Propoſitions the Lens Collects the Rays at a Definite Diſtance from it; And if the Object is nearer or farther off, there are only four Caſes; Either, where the Object is Placed at a Greater or at a Leſs Diſtance than the Focus, in which Suppoſition there is a certain Rule to find out the Diſtance of the Focus, which is before Alledg'd, Propoſition *5th* and *7th*; Or, where it is Placed in the Focus, in which Caſe after Refraction the Rays become Parallel Propoſition *6th*; Or Laſtly, where it is Placed at the Diſtance of the Diameter of one Convexity, in which Circumſtance the Rays will be Refracted to a Focus at the Diſtance of the Diameter of the other *Prop. 9th*; But in the Preſent Account the Focus of the Refracted Rays will not be at two or three Determin'd Diſtances, but at a Thouſand or a Million Varying ones, which let the Convexity of the Chryſtalline Humour, or the Elongation of the Eye, be what it will, can never be Solv'd; ſince by thoſe Caſes the Abſtracted Part of this Philoſophy only Determines two or three different Focus's, and upon the Suppoſition of Parallel or Diverging Rays in General, as well as of one, that is, a large Kind of Convexity in the Glaſs; but the Preſent Doctrine Aſſerts an Infinite Number of Focus's, and Rays to have their Particular Angles of Incidence, as the Convexity of the Chryſtalline Humour, or it's Remoteneſs from or Approach to the Retina, is Alter'd.

From this Doctrine we ſhould only ſee a Part of the Object and not the Whole.

§. 6. We ſhall yet Omit this Compariſon betwixt the Preſent Account of Viſion, and the Abſtracted Reaſoning in Opticks, which Precedes it, and Conſider, whether the Account it's ſelf of Viſion is Juſt, or what a Rational Mind may Acquieſce in; As to what Concerns the Radiation of the ſeveral Points of an Object, we have already Conſider'd that way of Reaſoning, and have Shewn it to be altogether Unſatisfactory in an Unprejudiced Inquiry into the Nature of Things; we ſhall only Add, that the Expreſſion of the Point *a* in the Object in the Point *f* of the Image is either a Full or only a Partial one from this ſuppos'd Derivation of the Rays from *a*; If it is a Full and Compleat one, all the Rays ſhould be Collected in *f*, which Radiate from *a*, whereas there are only thoſe, which Fall upon the Pupil *g o h*, and are but a ſmall Part of thoſe, which are Incident from *a*; If it is only a Partial Expreſſion of the Point *a* in the Retina, it will Follow, that as we only ſee one Part of the Points, *a*, *b*, and *c*, ſo we Receive one Part of the Object only *a b c* at the ſame Time, that we are Imagin'd and ſuppos'd to ſee the whole.

The Different Convexity of the Chryſtalline Humour Conſider'd in Order to Produce Viſion.

§. 7. That Moreover, which this Doctrine Depends upon is, that the Rays, which are Emitted from *a*, *b*, or *c*, ſhould be Collected to ſo many ſeveral Points in *f*, *e*, and *d*, which is ſaid muſt be Perform'd either by a different Convexity of the Chryſtalline Humour, or elſe by it's Approach to, or Receſs from, the Retina of the Eye, or both together; As to the different Convexity of the Chryſtalline Lens, which we ſhall firſt Conſider, that does not ſeem to be Capable of Producing the Effect, which is Deſired, Namely, of Collecting the Rays, which Flow from the Object *a b c*, when it is Carry'd to ſeveral Diſtances from the Eye, at the ſame Diſtance nearly of the Retina from the Lens *g o h*, and in much the ſame Points *d*, *e*, *f*; for the Convexity of the Lens *g o h*, if it is Imagin'd to be Flatter, would Refract the Rays at a farther Diſtance than *d e f*, and in a larger Circuit, if more Round and Protuberant, it would on the Contrary Refract them to a leſs Diſtance than *d e f*, and in a leſſer Circuit, Granting the Diſtance of the Object from the Eye to be the ſame.

The Different Convexity of the Chryſtalline Humour Conſider'd in Order to Produce Viſion.

§. 8. If the Diſtance of the Object is Vary'd, and the Convexity of the Lens is not, the ſame Effect will Follow, that the Rays at a Greater Diſtance of the Object will be Collected to a nearer Focus than *d e f*, and at a leſſer Diſtance of it will be to a farther, by what is Subjoin'd to *Prop.* 4. Concerning Nigh Objects or Diverging Rays, Where it is ſaid, that Rays, which leſs Diverge from a Point in the Object are ſooner Collected than thoſe, which Diverge more; From which it is Conſequent, that either a leſs Convexity of the Lens, ſuppoſing the Diſtance of the Object to be the ſame, or a leſs Diſtance of the Object, ſuppoſing the Convexity of the Lens to be the ſame, will Collect the Rays at a farther Diſtance than *d e f*, and in a larger Circuit, and on the Contrary, that a Greater Convexity of the Lens, ſuppoſing the Diſtance of the Object to be the ſame, or a Greater Diſtance of the Object, ſuppoſing the Convexity of the Lens to be the ſame, will Collect the Rays to a nearer Focus than *d e f*, and in a leſſer Compaſs.

The Different Convexity of the Chryſtalline Humour Conſider'd.

§. 9. If therefore we now Imagine, both the Convexity of the Lens and the Diſtance of the Object to be Alter'd, either the Chryſtalline muſt grow Flatter, as the Object Removes from it, and more Gibbous and Rounder, as it Approaches it, or, on the other Hand, it muſt grow Rounder, as the Object Removes from it, and Flatter, as it Approaches it; If the laſt, both the greater Convexity of the Lens above what is Expreſs'd by *g o h*, and the Increaſe of the Diſtance beyond *b o*, will Jointly Contribute to Collect the Rays to a much nearer Focus than *d e f*, as on the Contrary, a leſſer Convexity and Diſtance than *g o h*, and *b o*, will Unite in Producing one and the ſame Effect, Namely, of Removing the Focus to a Greater Diſtance from the Lens than *d e f*.

The Different Convexity of the Chryſtalline Conſider'd.

§. 10. THE firſt therefore, if any, muſt be Affirm'd, that the Convexity of the Lens grows Flatter, as the Object Removes from it, and more Gibbous and Round, as it Approaches it; If ſo, we cannot but Ask the Reaſon why, or on what Account, or upon what Manifeſt Proofs and Demonſtrations it is, that the Convexity of the Chryſtalline Humour ſhould be Leſs, as the Diſtance of the Object Increaſes, rather than it ſhould be Greater; If there is no Argument, that can be Produced for it, as it is Certain there cannot, it is Evident, that the Diminiſhing the Convexity of the Chryſtalline in Proportion to the Diſtance of the Object Increaſing, is only a Principle Arbitrarily Aſſum'd to Explain a Phænomenon in Viſion, which could not be done without it, and therefore all the Reaſon of it's being Introduced into the Preſent Philoſophy, is becauſe it is Neceſſary to it, a Method of Solution which an Impartial Inquiry into Nature may be Juſtly Aſham'd of, however Frequently it is Practiced by our Modern Reaſoners; For there Certainly can be no Connexion, in the Nature of Things, betwixt an Increaſe and Decreaſe of the Convexity of the Chryſtalline, and a Decreaſe and Increaſe of the Diſtance of the Object, which is in this Philoſophy Neceſſary to Viſion, or at leaſt no more Connexion, than there is betwixt the Increaſe and Decreaſe of the Convexity of the Chryſtalline and the Increaſe and Decreaſe of the Diſtance of the Object, which would Deſtroy all Viſion.

The Different Convexity of the Chryſtalline Humour Conſider'd, and the Receſs from and Approach of it to the Retina.

§. 11. MOREOVER, in this way of Argueing, there muſt be ſome Certain Diſtance of the Object as *bo*, where the Rays are Refracted to the Bottom of the Eye by a Particular and Determinate Convexity of the Lens *goh*, if therefore the Object is farther Remov'd than *abc*, the Convexes of *goh* will Proportionably come nearer together; On the other Hand, if the Object is Placed any where betwixt the Eye and *abc*, the Convexes will Recede in Proportion from each other; which Contrary Properties in the Lens are Inconceivable, that when the Object is on the farther Side of *abc* the Lens ſhould Contract it's ſelf, and when on this be Dilated to a greater Convexity; For as to what Concerns the Muſcles of the Eye, they do not ſeem to be Contriv'd for the Alteration and Change of it's Figure, but for the Suſpenſion of it in it's Proper Place, and for it's Rotation and Motion, Suitable to what we find to be the Office and Deſign of all other Muſcles, which are not for an Inflation or Depreſſion of the Parts, to which they belong, but for their Action.

Rohault.
Cap. 28.
§ 11. 12.
Par. 1.

The Different Convexity of the Chryſtalline Conſider'd, and the Receſs from and Approach of it to the Retina.

§. 12. LASTLY, it is Notorious, that when the Eye is Fixed and Unmov'd, and when the leaſt Alteration in the Figure of it cannot be Reaſonably Suppoſ'd, that even then the Rays are Refracted to *def* at ſeveral Varying Diſtances of the Object, and that we ſee Clearly and Diſtinctly thoſe Objects, which are Expoſ'd to our View, at very different Intervals, at the ſame Time that we cannot Imagine there is a Diverſity in the Figure of the Eye, which ſhould be Capable of Producing ſo Conſtant an Effect, as that of Clear Viſion; And for the like Reaſon it is, that we may Juſtly Conclude, that the Remoteneſs or Acceſs of the Retina has no Influence in this Phænomenon.

The Figure of the Eye Conſider'd in Reſpect of the Various Poſitions of Objects to it.

§. 13. FOR if the Figure of the Eye in General was to be Conſider'd in an Explanation of Viſion, we ſhould only ſee Objects in a Direct Poſition to it, according to the Account of this Philoſophy, Namely, in *abc*, or in ſome Situation, or other, Parallel thereto; whereas it is Matter of Fact, and our Senſes will Juſtify us in the Aſſertion, that we ſee Objects at one View in a Thouſand different Poſitions to the Eye, and not only, when they are Parallel to *abc*, but when they are Placed in Right, Obtuſe, or Acute, Angles, to what is here Repreſented, that is at leaſt in a Hemiſphere Round it, if not more.

The Refractions of the Chryſtalline it's ſelf Conſider'd.

§. 14. BESIDES, as the Object may be Placed in thoſe Situations, which the Preſent Opticks do not Account for, ſo if we take it in that Situation, which is here Propos'd to us, it is Inconceivable how the ſame Lens, which Collects the Rays from *b* in the Object to *e* in the Retina, ſhould have the ſame Faculty in Collecting the Rays from *a* and *c* in the Object to *f* and *d* in the Retina, and the ſame may be ſaid of the other Intermediate Points; for it Appears to be Impoſſible, that the ſame Lens ſhould Collect Rays in that different Manner, which is here ſuppoſ'd; Farther in the ſecond Place, the Rays upon their Refraction are Determin'd in *def* the Chord of the Arc or in the Arc it ſelf; If in a ſtrait Line, there will be only the two Extremities *d* and *f* Viſible ; If in the Arc it ſelf, the Repreſentation of a ſtrait Line will be in a Curve one, nothing of which is Accounted for ; Laſtly, it is Fact, that the Object *abc* will be Repreſented in *def* without any Refractions of a Lens at all, as is Evident by the known Experiment of Objects being Repreſented on Paper by Admitting the Rays thro' a Foramen in a Window-Shut into a Dark Room, without any Interpoſition of a Lens, or any Imaginary Refractions of the Rays thro' it.

The Impreſſion of the Rays upon the Eye and their Return back from it, Conſider'd.

§. 15. WE are Moreover to Obſerve, that if the Rays could be Collected to the ſame Points, *d, e, f,* in the Retina, at Differing Intervals or Diſtances of the Object, or in the ſame Diſtance of the Object, yet this Scruple will ſtill Remain, how one Point in the Object ſhould be only ſeen, where a Million of Rays are ſuppos'd to be Emitted from it ; For either each ſingle Ray has an Effect upon the Organ of Senſation, or not ; If not, if each ſingle Ray has no Force in it's Impreſſion upon the Retina, all of them together cannot have ; If each Ray Obtains it's Separate Force, as *eh, eo,* and *eg, &c.* by the Property Alledg'd in this Philoſophy of the Eye's Purſuing the Rays in the ſame Direction, in which they are Incident, the Point *b* will Appear to be Larger than the whole Length of the Object *abc,* ſince *eh, eo,* and *eg,* Prolong'd in Direct Lines from *e,* will Fall at a greater Diſtance from *b,* than *a* and *c* do, in the ſame Strait Line *abc* Produced ; And the ſame may be ſaid of every other Point as *d* and *f,* which, if their Rays are Purſued in the ſame Manner, would make the ſeveral Points in the Object Longer than the Object it's ſelf, and Coincident with each other, both which are Contrary to the Preſent Hypotheſes, as well as to all Senſe and Reaſon ; If it is ſaid, that the United Forces of all the Rays from one Point in the Object Refracted to another in the Retina are only Capable of Producing Viſion, that is Manifeſtly Falſe, ſince Viſion by Experience may be Effected in *e,* when the Pencil of Rays is greater or leſs by the Nearneſs or Diſtance of the Object, which could not be, if the Collection of a certain Number of Rays was Neceſſary to cauſe a Senſation in us, and when there is not ſometimes a Tenth or a Hundredth Part of the Rays, which Flow from the Point of any Object, Refracted to the Retina, and never all of them ; But if it is Acknowledg'd, that ſometimes more, and ſometimes fewer Rays can Produce Viſion, it is at the ſame time Confeſſed, that *geh,* for Inſtance, may Conſiſt of ſo Large a Number, that a Tenth or Hundredth Part of them may Cauſe that Senſation in us, and if ſo, by what Foregoes, the Eye muſt Purſue each of thoſe Parcels of Rays in the ſame Direction, in which they Fall, which, as before has been Prov'd, is Abſurd.

Prop. 28. Part 1. §. 4.
Rohault. Cap. 32. Part 1. §. 13. Cap. 34. §. 18. Cap. 33. §. 2.

Concerning the Confuſion of the Rays upon the Pupil of the Eye.

§. 16. IT is farther to be taken Notice of, that in *goh* the Rays are in the Utmoſt Confuſion, which are made to be ſo by an Arbitrary Drawing of them from the ſeveral Points of the Object *a, b, c,* in Compliance with the Situation of the Lens *goh,* or not ; If the firſt, and as the Eye comes nearer, the Rays are in a greater Inclination to *abc,* ſo that the Angles *bco, bao,* are Leſs, and *aoc* Larger, or as the Eye Removes farther off from *abc,* the Rays are in a leſs Inclination, ſo that the Angles *bco, bao* are Greater, and *aoc* Smaller, two Abſurdities will Follow ; *1ſt.* That *co* and *ao* ſhould meet at all the Points betwixt *b* and *o,* or elſe there will be ſome Point *o* betwixt *o* and *b,* in which the Rays are not in the utmoſt Confuſion, Contrary to the Hypotheſis ; If they meet in every Point betwixt thoſe two Terms, they cannot be Propagated in Cones, or Parts of a Sphere, from *a* and *c* Conſider'd as Centers, or, which is the ſame in this Plane, in Circles from *a* and *c,* as Centers of thoſe Circles, which likewiſe is Contrary to another Hypotheſis in this Philoſophy ; For let the Circumference be Drawn thro' *o,* it is Evident, that the Rays, whilſt they are Extended from the Centers *a* or *c,* will Interſect the Line *bo* at Irregular Diſtances, and therefore not in every Point of it, unleſs a ſtrait Line as *bo* can be Imagin'd Commenſurate to the Circumference from *a* or *c* ; But it is Abſurd in any Philoſophy to Introduce Hypotheſes for the Explanation of the ſame Thing, which Contradict and Deſtroy each other ; *2d.* A ſecond Abſurdity is, that if *ao* and *co* are Arbitrarily Drawn, ſo as to meet each other in any Point betwixt *o* and *b,* ſince the Angle *dof* is Equal to *aoc,* it will be Conſequent, by the Approach of *o* to *b,* that *df* may be Ten times as Great as *ac,* but ſince *ac* could never be Expreſs'd at it's full Length in the Retina, it is Impoſſible, that That, which is ten times more Extended, ſhould be : On the other Hand, if the Rays are not Arbitrarily Drawn from *a* and *c,* but are Fixt and Determin'd in one certain Point *o,* which is here Expreſs'd, it is Abundantly Manifeſt, that if the Lens *g o h* is Remov'd farther off from *a b c*, or brought nearer to it, there will not be a Confuſion of Rays from all the Points of the Object *a b c*, in one at *o* in the Chryſtalline, which this Philoſophy Builds and Depends upon ſo Entirely in it's Preſent Demonſtrations; For if it did not, ſuppoſing the Lens *g o h* to be Placed in *r t* or *s q*, what Terrible and Puzzleing Work ſhould we have to make theſe Rays Unite with all the Uniformity Imaginable in the Points *d*, *e*, and *f*? And yet for all that we can at Preſent ſee, the Lens may be as well Placed in thoſe Poſitions, as in that, in which it is, unleſs we may be Allow'd to Draw the Rays from *a* and *c* with what Arbitrarineſs we Pleaſe, which, we have Prov'd, cannot be done without the greateſt Abſurdity and Contradiction.

Concerning the Image of the Object being Inverted in the Bottom of the Eye. Molyneux's Dioptricks Prop. 28. Par. 1. §. 4. 5. Rohault. Cap. 31. §. 2. Par. 1.

§. 17. THE next Thing we are to Conſider in Viſion is, that the Repreſentation of the Object *a b c* on the Bottom of the Eye *f e d* is Inverted, as it is likewiſe on a Paper in a Dark Room, from whence this Inquiry may be Naturally made, how it comes to Paſs, that we ſee Objects Erect, when by this Account they are Inverted in their Impreſſion upon the Retina; To Solve this Difficulty, Thoſe, who Profeſs an Abſtracted Opticks, are willing to Diſclaim the Solution of it, as not belonging to their Province, becauſe it Encroaches upon another Queſtion, that is, what the Properties of the Soul are in ſuch Circumſtances of the Image of an Object being Imprinted on the Bottom of the Eye; However, they have not Thought Fit wholly to Omit the Aſſigning a Reaſon for this Phænomenon, which is, that the Eye Purſues the Rays back in the ſame Direction, in which they Fall, or, which is the ſame, that, the Eye in the Point *f* Receiving a Stroke or Impulſe by the Protruſion forwards of the Luminous Axis *a o f*, the Mind muſt Neceſſarily be Directed to Conſider this Percuſſion, as Deriv'd from the Point *a*, rather than from the Point *c*, or any other.

The Solution of the foregoing Phænomenon not to be Neglected, becauſe it Terminates in an Inquiry into the Nature of the Soul.

§. 18. As to the Willingneſs, which Opticians ſhew of being Excus'd from Solving this Difficulty, it ſeems to Proceed from a Conſciouſneſs they have of the Inſufficiency of their Principles to do it: On the Contrary, we Inſiſt, that it as Properly belongs to a True Doctrine of Opticks to give an Account of this Appearance as any other, which it Conſiders; For if we might Neglect it, becauſe it Borders too much upon an Inquiry into the Nature of the Soul, we may Equally Diſcard the Magnitude, or Apparent Places, or the Diſtances, or Indeed any other Circumſtances, in which Objects are ſeen, becauſe we do not, nor are ſuppos'd to know the Paſſions of the Mind, which Reſult from ſuch Impreſſions; that is, the whole Science of Real Opticks will be Foreign to the Purpoſe, and we are Adviſ'd to Acquieſce in a Vain Knowledge of Rays, and their ſeveral Progreſſes thro' Glaſſes of different Figures, or, for it is the ſame, in an Abſtracted and Notional Reaſoning upon them, tho' we ſhould not be able to Apply it to one Explicit Phænomenon of Nature, or of Viſion.

No more Reaſon that the Eye ſhould Return the Action of the Rays back in the Direction of the Axis than of any other Ray.

§. 19. NOR is the Reaſon, which is here Alledg'd for this Appearance much better than the Excuſe for not Explaining it; for why a Point in the Object ſhould be ſeen by the Axis rather than any other Refracted Ray we can Name, is not ſaid, nor can be, unleſs one Ray of a Force not much Greater than another may be ſuppos'd to Enjoy a Prerogative almoſt Infinitely Superior to it; much leſs can we Imagine, that the Axis ſhould Aſſert a Diſtinction to it's ſelf in Preference to the Force of a Million of Rays Acting together, each of which has one, not much Inferior at leaſt to that of the Axis, which yet in this Philoſophy is only Conſider'd and Eſpous'd; If the Axis and Refracted Rays are nearly Equal in their Force of Percuſſion or Impulſe upon the Eye, for the ſame Reaſon that the Mind Refers it's ſelf back from *e* to *o*, it ought to React from *e* to *g* and *h*, by which Means the Point *b* would be Diffus'd into a Line much longer than the Object it's ſelf.

Allowing the Eye to React according to the Direction of the Axis, it is not Certain or Prov'd where ſuch a Reaction ſhall Determine it's ſelf.

§. 20. BESIDES, altho' we ſhould Allow, that the Axis is only Concern'd in Exciting the Mind to Obſerve it's Direction, it will yet be Unaccountable, why it ſhould not Purſue that Direction, either to a Leſs or to a Farther Diſtance than that of the Object from the Eye; that is, notwithſtanding we ſhould Acknowledge the Impreſſion, which is made upon *e* from *b* or *o*, to be Return'd in the ſame Line, yet from any Property in the Mind, or in the Object, it is not Certain, where the Reaction of the Eye, or at what Diſtance, it will be Determin'd; If it is ſaid in the Object, from which the Ray Flows, or is Deriv'd, that cannot be Prov'd from the Force of the Mind, upon which the Impreſſion is made, unleſs it were firſt Aſcertain'd, how far that Force Extends in it's Reacting upon any Impulſe, which it Feels ; Nor can it be Evidenc'd from the Object it's ſelf, becauſe it cannot Determine to what Place the Rays, which it Emits, ſhall either Proceed or Return, ſince it Reſigns them to the Power of other Beings, on which they Fall ; Thus in Tranſparent Bodies they are Refracted, and in Opake Reflected, to different Angles, Lengths, and Diſtances.

Suppoſing what foregoes it is not Prov'd from the Experiment of the Image upon a Paper in a Dark Room or upon an Artificial Eye, that the Image in the Real is Inverted.

§. 21. WE muſt farther ſay, that this Inverſion of the Image in the Eye is Grounded upon a Falſe, or at leaſt a Precarious Reaſoning, which is, that, becauſe we Obſerve the Object Inverted in a Dark Room, or even in the Bottom of an Artificial Eye made Uſe of in Experiments, therefore it is ſo in Real Viſion or in the Retina of the Eye, which the Viſive Faculties of our Minds Employ for their own Information ; Whereas it is not Certain, that an Object does Paint the Image Inverted upon the Paper or the Artificial Eye, becauſe it Appears ſo to the Natural, unleſs we Firſt and Previouſly know, that what is Erect in the Object is Inverted in the Eye, or which is the ſame, what is Inverted in the Object is Erect in the Eye, which is the Thing to be Prov'd ; for if this Principle is not known, we are not Certain, but what is ſeen Inverted in the Object, which in this Caſe is the Image upon the Paper, or the Artificial Eye, may be alſo Inverted in the Eye, and what is ſeen Erect in a Natural Object may be alſo Painted Erect in the Eye ; But according to the Reaſoning of this Philoſophy, that which is Painted Inverted in the Eye may Really be Erect in the Object, and that which is Erect, Inverted ; from whence it will Follow, that altho' the Image upon the Paper, or the Artificial Eye, may Appear Inverted, yet it may be in Truth Erect, according to the Reaſoning of this Philoſophy, becauſe what is Inverted in the Eye may be Erect in the Object, which is here the Image upon the Paper, or Artificial Eye, and the Object, which Appears Erect, may likewiſe in Truth be ſo, as in a Natural Object, ſince the Principle is unknown ; but if it is ſaid it is known, that is Said, which is to be Prov'd.

This Doctrine not True becauſe Invert the Eye or take it in it's Natural Situation the Poſition of the Object is not Chang'd.

§. 22. LASTLY, when we Stand Upright upon our Legs, the uppermoſt Part of the Object is ſuppos'd to Fall on the lowermoſt Part of the Retina, and the lowermoſt Part of the Object on the uppermoſt Pa rtof it ; On the Contrary, when our Heads are downwards, the uppermoſt Part of the Object Falls on the uppermoſt Part of the Retina, and the lowermoſt Part of the Object on the lowermoſt Part of it, that is, not in Reſpect of the Center of the Earth, but in Reſpect of the Parts of the Eye, or, which is the ſame, the Object in the laſt Caſe will be Painted on the Retina in a quite Contrary Situation to what it was in the Firſt ; And therefore the Mind, which only Conſiders, and takes Notice, as we may Reaſonably Think, of the Internal Poſture of the Parts of the Eye, and how they are Affected, muſt ſee the Object Erect in one Circumſtance, and Inverted in the other, which is not Agreeable to Experience, ſince either ways it Appears Erect.

Concerning the Explanation given of the Magnitude of the Object, Argument the firſt againſt this Explanation.

Molyneux's Dioptricks Prop. 28. Part 1. §. 6.

§. 23. AFTER the Inverſion of the Object in the Eye, we are to Examine into the Explanation given of it's Magnitude, which is ſuppos'd to be Eſtimated by the Optick Angle $aoc = fod$; But how the Eye ſhould Determine the Dimenſions of the Object by the Angle, under which it is ſeen, is Inconceivable, for ſince it is not Preſum'd to be Commenſurate to def, it will be Impoſſible to Aſſign a Reaſon, why it ſhould be the Exact Length or Bigneſs of abc, rather than of any other Line Drawn Parallel to it, within the Angles aoc, dof, either betwixt the Retina df and the Object, or even beyond the Object it's ſelf ; ſeeing the Eye is Imagin'd to have no other Direction for the Forming it's Judgment of the Magnitude of an Object, unleſs that of the Optick Angle, which will be the ſame, when the Lines, which Subtend it, are Infinitely Different.

Argument the Second.

§. 24. IT is likewiſe to be Obſerv'd, that the Optick Angle is Undetermin'd, ſome Placing the Point o in the Center of the Eye, others in the Vertex of the Chryſtalline, and a Third in that of the Cornea, which muſt certainly Cauſe a great Variety in the Optick Angle it's ſelf, and Conſequently of the Magnitude of the ſame Object, even ſuppoſing we could Fix upon one certain Line, which Subtended it, and which might be the Meaſure of the Viſible Dimenſions of the Object, which according to the foregoing Section is not done.

§. 25.

The farther Explanation of Simple or Plain Viſion Conſidered. Molyneux's Dioptric. Prop. 28. Par. 1. § 7.

§. 25. 1*ſt.* THE Preſent Opticks Proceed to Inform us, that the Rays, which Flow from the Points of the Object Diverge in their Progreſs from thence to the Eye, and that if the Object was Infinitely Diſtant, or ſo far Diſtant from the Eye, that the Breadth of the Pupil was Inſenſible in Compariſon of ſuch a Remoteneſs of the Object, the Rays would Fall in a manner Parallel, where before they were ſuppos'd to Diverge, and that by this Means they would be United at the Retina in the Points *d*, *e*, *f*.

2*d.* FARTHER, that if the Diverging Rays *b g*, *b h*, which are Deriv'd from the Point *b*, meet a Convex Glaſs, and are thereby made to Converge, and in their Refraction thro' the Chryſtalline are made to Converge yet more, they will Croſs in a Point before they reach the Retina *d f*, and Conſequently Paint the Image of the Point *b* Confuſedly, ſince it is Diffuſed into a certain Space, whereas in Diſtinct Viſion it is Expreſſed in a Correſpondent Point on the Retina.

3*d.* FROM wence it is Inferr'd, that Convex Glaſſes, when us'd by thoſe, who are call'd Myopes or Purblind, or by thoſe, who are ſhort Sighted, inſtead of Aſſiſting the Sight, are an Occaſion of a Confuſedneſs in it; Becauſe the greater Convexity of the Chryſtalline, or the greater Diſtance of the ſame Chryſtalline from the Retina, which in this kind of Eyes is Preſum'd to be join'd with the Convexity of the Lens, which is Interpos'd betwixt the Eye and the Object, Unite the Rays before they Arrive at the Retina, and therefore Concave Glaſſes on the contrary are found to be Serviceable, which make the Rays Diverge ſo, that the greater Convexity of the Chryſtalline ſhall be only Sufficient to Unite them at the Bottom of the Eye, or the greater Diſtance of the Retina from the Chryſtalline ſhall be at a Juſt Interval only to Receive and Obſerve their Union; It is upon this Principle alſo, that by bringing the Object nearer to thoſe, who are ſhort Sighted, it is Painted upon the Retina with a greater Clearneſs and Diſtinction; Becauſe the Rays, which Fall from each Point of the Object, Diverge more, when the Object is nearer the Eye, than when farther off, and Conſequently the too great Convexity of the Chryſtalline does but Suffice to bring them together on the Retina, as on the Contrary, when the Object is farther off, it Unites them too ſoon, becauſe of their leſſer Divergence in their Incidence upon the Eye.

4*th.* ON the other Hand, the Eyes of Old Men have their Chryſtalline too Flat, and therefore the Aſſiſtance of an Adventitious Convexity in a Glaſs Lens is Requiſite to Unite the Rays at the Retina, which otherwiſe would Proceed beyond it; from whence it is ſaid, that Spectacles are an Advantage to Old Men, not by Magnifying an Object, but by making it's Image Diſtinct; ſince Old Men cannot Read the Largeſt Characters without Spectacles, and yet with them can Read the Smalleſt, though theſe with Spectacles do not ſeem ſo Large, as thoſe without them.

5*th.* MOREOVER, there is an Oppoſition made betwixt a Clear and Faint Viſion, or betwixt a Diſtinct and Confus'd; Namely, either, when the Points of the Object are Collected to a Point in the Image, or not, which makes the Difference betwixt our Seeing Diſtinctly or Confuſedly; Or, when there is a greater Quantity of Rays in the ſame Pencil Illuminating the Correſpondent Points of the Image, or a leſſer Quantity of them, which is ſuppos'd to make Objects Appear either Clearer or Fainter.

6*th.* To which we muſt Add, which is Propos'd in this Philoſophy, that to the Diſtinctneſs or Clearneſs of Viſion, it is only Neceſſary to Conſider a Point in the Object, whereas the Inverſion of it in the Image upon the Eye Requires an Examination of the whole Object, or the ſeveral Points of it.

Not an Infinite nor a Comparative Diſtance of the Object in Reſpect of the Eye, nor a Paralleliſm of the Rays Requiſite to Impreſs a Diſtinct Image of the Object upon the Eye.

§. 26. As to the firſt Part of the foregoing Section, it is Evident, that neither an Actual Infinite Diſtance is Requir'd to make an Object Seen Diſtinct in the Eye, becauſe ſeveral Objects at a much Leſs Diſtance than what is Infinite by Experience Diſappear, nor is a Comparatively Infinite Diſtance in Reſpect of the Pupil of the Eye Requir'd to make an Object Seen Diſtinctly, becauſe when the Diſtance of the Object from the Eye is Inconſiderable in Reſpect of the Pupil of the Eye, the Object yet is fully and Clearly Diſcern'd; For this Paper, which I Write on, whether it is brought nearer to my Eye, or Carry'd farther from it, Cauſes much the ſame Impreſſion; and for the like Reaſon it is, that we cannot Suppoſe the Paralleliſm of Rays, which is there Aſſerted, unleſs we can Imagine, that Rays which Flow from a very a very nigh Object, are Parallel at the ſame Time, that they Manifeſtly Diverge, and are Confeſſed to do ſo in this Philoſophy; for we See Objects at almoſt an Infinitely ſmall Diſtance from us, in which Caſe it is Impoſſible to Conceive, that the Rays, if ever, do not then Diverge, and are far from being Parallel, albeit the Contrary is ſaid to be Requiſite to the Painting the Image upon the Retina with a Sufficient Diſtinction.

Short Sight not Explain'd by theſe Opticks.

§. 27. Nor is the Account given of their Manner or Way of Viſion, who are ſhort Sighted or Purblind, any Confirmation of the Truth of theſe Opticks, for they Suppoſe either the Chryſtalline to be more Convex in ſuch Perſons than in others, or elſe that the Diſtance of the Retina from it is Greater, both which are Hypotheſes Aſſum'd at Large without the leaſt Proof, that the Figures of theſe Kind of Eyes are Really ſuch in Fact, Excepting that by this Means it is Thought thoſe Phænomenons of Viſion may be Explain'd, which otherwiſe could not be, but we have already ſaid, that ſuch a Procedure is Perfectly Arbitrary and Unwarrantable in a True and Juſt Philoſophy.

Confus'd Viſion not Explain'd.

§. 28. That which is yet Worſe is, that if theſe Hypotheſes were Granted, they are not Capable of Solving the Appearances, which they are Deſign'd to Explicate, for if the Rays are not Collected to ſeveral Determinate Points in the Retina, but are Diffus'd into Spaces, as is Suppos'd, *Art. 2d. §. 25th.* we ſhould be ſo far from Seeing the Object Confuſedly, that there would not be the leaſt Picture or Reſemblance of it Deſcrib'd upon the Eye; ſince the Rays, which are Deriv'd from one Point of the Object, would be Intermixt and Blended in the Retina with thoſe, which Flow'd from another, the Conſequence of which muſt be, that *c* would be partly Seen in *b* and *a*, *b* in *a* and *c*, and *a* in *b* and *c*, which could not but Entirely, not only Confound, but Eraſe and Obliterate the Object, and Exhibit a quite different Appearance from it; for no Rules or Methods of Thinking could, in ſuch a Caſe, ever Reduce a Space to bear any Analogy to a Point, or make a Thouſand Lines, laid Promiſcuouſly upon one another, form a Quantity Agreeable to ſo many Points Placed in Directum.

The Explanation given of ſhort Sight, Contradicted by the preſent Opticks.

§. 29. Let us however Allow even this, it is Evident from §. 26. that the Paralleliſm of the Rays does not Contribute any Thing to a Diſtinct Viſion, but that, whether the Object is nearer or farther off, within certain Limits, the Repreſentation of it is not, as to it's Diſtinction, Conſiderably Vary'd in the Eye; it is likewiſe Demonſtrated in this Philoſophy, that the greater the Convexity of the Lens is, and the nearer it Approaches to a Sphere, the Focus will be likewiſe leſs Remov'd from the Lens; the nigher therefore any Object is to the Eye, and the greater the Convexity of the Chryſtalline, the ſooner the Rays will be Determin'd to a Focus; Conſequently, thoſe who are Purblind, or ſhort Sighted, ſhould See Objects leſs Diſtinctly, as the Convexity of the Chryſtalline is greater, which is contrary to what the preſent Hypotheſis Affirms, and if the Diſtance of the Retina from it is likewiſe ſuppos'd Greater, which is Conjectur'd in this Philoſophy to Accompany the other, the Confuſion in Viſion will be ſtill more Senſible.

Short Sighted Men can See Objects at Various Diſtances when there is no Reaſon to Think the Figure of the Eye is Chang'd.

§. 30. It is alſo to be Obſerv'd, that, at ſeveral Changing Diſtances, both Purblind and ſhort Sighted Men can See Objects very Diſtinctly, and it has been before Prov'd, that there is no Reaſon for us to Believe, where the Eye is Fixt and Unmov'd, that there is any Alteration either in the Convexity of the Chryſtalline or it's Diſtance from the Retina, and Conſequently the Diſtinctneſs of Viſion can Proceed from neither. As to the Eyes of Purblind or ſhort Sighted Men growing Flat by Age, and their Seeing Objects more Diſtinctly at a greater Diſtance, when Older, it is Contradicted by all Experience.

Since a Convex Glaſs is Prejudicial to a Short Sight there is one Reaſon to Think a greater Convexity of the Chryſtalline is ſo too.

§. 31. We do not yet Deny, that Convex Glaſſes are Prejudicial, and Concave Serviceable, to ſuch Perſons, becauſe if a greater Convexity of the Chryſtalline is Prejudicial, as we have Evidenced, a Lens Interpos'd muſt be ſtill more ſo, as it Contributes to the ſame Effect; and on the other Hand, ſince a Lens held betwixt the Object and the Eye cauſes a greater Confuſion in Viſion by Experience, why may we not Think a greater Convexity in the Chryſtalline ought in Reaſon to do ſo too? Likewiſe, as a Concave makes the Object Appear more Diſtinct, why ſhould we not Imagine, that there muſt be ſomething Analogous in the Chryſtalline, or the Eye, which Occaſions the ſame Diſtinct Perception, when no ſuch Glaſſes are us'd? Which, what it is, ſhall be afterwards Explain'd.

§. 32. ANOTHER

A greater Divergence of the Rays ſuppos'd which is not Prov'd.

§. 32. Another Argument againſt this way of Reaſoning in Opticks is, that as it ſuppoſes the Convexity of the Chryſtalline, or the Diſtance of the Retina from it, Extraordinary, ſo it likewiſe Preſumes a Divergence of the Rays to be more than Uſual, which this ſuppos'd Convexity is to Collect; for if the Divergence of the Rays was not Increaſed by the Approach of the Object, there would be no Occaſion to make the Chryſtalline more Convex in Order to Refract them to that Degree, which is Requir'd, but that the Divergence of the Rays is not Increaſed by the Approach of the Object to the Eye is Manifeſt from §. 26. ſince, if it is, Plain Viſion cannot be Accounted for, which ſuppoſes the Rays to be nearly Parallel, and it is by that Section Evident, that we See Objects Clearly and Diſtinctly at a very near Diſtance.

Concerning the Sight of Old Perſons.

§. 33. The next Solution we are to Examine is that, which is given us of the Viſion of thoſe Perſons, who are Old, whoſe Chryſtalline is ſuppos'd to be leſs Convex than that of the Myopes, and therefore not Capable of Uniting the Diverging Rays ſo ſoon as is Neceſſary for their Diſtinct Impreſſion on the Retina, and Conſequently are not Collected to a Focus, Excepting in a Point beyond the Bottom of the Eye.

Not Explain'd, becauſe Diverging Rays are ſuppos'd Neceſſary.

§. 34. But here it is likewiſe Remarkable, as before, that Diverging Rays are Introduced, when it is ſaid they ought to be Conſider'd as Parallel to Cauſe a Diſtinct Viſion, and even in an Oppoſite Caſe to the former; for in thoſe that are Purblind, it is Requir'd, that the Object ſhould be placed at a very near Diſtance to be Seen without Confuſion, whereas in Old Perſons the Contrary is Requiſite; if therefore in an Eye of a Common and Ordinary Figure the Rays are to be Conſider'd as Parallel to Produce a Diſtinct Perception, as alſo in a Long or more Convex one, tho' they are Granted to be Diverging, it is Impoſſible they ſhould be ſo, where a greater Diſtance of the Object is Neceſſary to a Diſtinct Perception, than what is ſuppos'd in Common Viſion; If it is ſaid Diverging Rays are not ſuppos'd, it is only Looking upon §. 7. 8. *Prop.* 28. to Convince us of the Contrary.

Why may not Old Mens Eyes have a greater Convexity rather than a Leſs, and ſhort Sighted Mens a Leſs Convexity rather than a greater?

§. 35. The Confuſion of the Rays in the Retina, likewiſe, will be as Unaccountable as before in the Caſe of the Myopes, and ſince the Interpoſition of a Lens is Serviceable to this Kind of Eye, we may Reaſonably ſuppoſe there is ſomething in the Chryſtalline, which is Anſwerable to it, where it Sees Objects Diſtinctly and Clearly, rather than juſt the Contrary; For ſeeing it is by Experience found, that a Convex is Beneficial to theſe Eyes, why ſhould we not Conclude, that the Reaſon, why Old Men See Diſtinctly at Certain Diſtances without theſe Glaſſes, is, becauſe of the greater Convexity of the Chryſtalline, rather than becauſe of a Leſs, it ſeeming very Incongruous to Aſcribe a greater Convexity of the Eye to thoſe, who are Plainly Aſſiſted by Concaves, and a leſs to thoſe, who are Aſſiſted by Convexes, for why ſhould not Men See Originally by thoſe Methods, by which their Sight is Aſſiſted? I may at leaſt be Allow'd to Ask the Queſtion, tho' I do not Affirm, that they do ſo.

To Conclude in Relation to this Hypotheſis of Different Convexities in the Chryſtalline, it is not Impoſſible upon the Suppoſition of a greater Convexity of it in Old Men to Account for the Phænomenons of Viſion in them, for in ſuch a Caſe nigher Objects may not be Seen Diſtinctly by them, becauſe the great Convexity of the Lens may yet not Collect the Rays ſoon enough, to Produce a Clear and Determin'd Image in the Eye, whereas by Removeing the Object farther off, as the Rays have a leſſer Divergence, they may be Collected to a juſt Focus on the Retina, this may be done if the Eyes of Old Men are Flatter than thoſe of others, as they are ſuppos'd to be, and Conſequently the Diſtance betwixt the Chryſtalline and the Retina Leſs; Again, if a nigher Object cannot Collect it's Rays to a Focus in the Retina by the Greater Convexity of the Chryſtalline, being farther Aſſiſted by another Lens it may do it; On the other Hand, ſhort Sighted Men, if the Convexity of the Chryſtalline is Leſs, may See nigh Objects more Diſtinctly, if the Figure of the Eye is more Oblong and Protuberant; Becauſe the leſs Refractive Force of the Lens is Compenſated by the greater Diſtance of the Retina from the Chryſtalline, ſo that the Rays in a longer Eye will be juſtly Refracted to the Retina by a leſſer Convexity of the Lens, and Remoter Objects on the Contrary may be better Diſcern'd by the Interpoſition

poſition of a Concave, becauſe the leſs Diverging Rays would be Collected too ſoon, and therefore ſhould be made to Diverge, or Diſperſe, by ſuch a Concave, in Order to their Exact Collection in the Retina; Theſe Things may be, as Truly Suppos'd, as thoſe, which are Alledg'd by the Preſent Philoſophy, and will Perhaps as well Account for the foregoing Phænomenons, and yet theſe Suppoſitions are Perfectly Contrary to thoſe, which are made Uſe of by theſe Opticks; which is at leaſt an Argument, that they are Precarious and Arbitrary, and can be no more Depended upon for a Solution of Nature, than thoſe, which have been Nam'd, and which are Diametrically Oppoſite to them.

The Diſtinctneſs of Viſion in Old Men, Occaſion'd by Convexes, not owing either to the Magnifying the Object or to the Collection of Rays into Diſtinct Points on the Retina.

§. 36. We are alſo to Obſerve, that Old Men cannot See Diſtinctly the Largeſt Characters without Spectacles, and yet with, they can the Smalleſt, which is a Demonſtration, that the Lens, which is Interpos'd betwixt the Eye and the Object has ſome other Property (Namely, of Increaſing the Expanſive Force of Light) than that of Magnifying, which would Signify nothing to make Viſion Diſtinct, as is Evident from the Preſent Phænomenon; nor is it the Collection of Rays to Certain and Determin'd Points, which Produces this Effect, ſince with Spectacles the ſame Diſtinct Viſion is Caus'd, where the Diſtances of the Object are Chang'd, and without any Imaginable Alteration of the Figure of the Chryſtalline or the Eye, unleſs that, which is Hypothetical and Chimerical.

The Diſtinctneſs and Confuſedneſs of Viſion may be as Probably Attributed to the different Temper of the Humours of the Eye as to the Different Convexities or Figures of it.

§. 37. But that, which gives a full Confutation to this Sort of Opticks is, that the Diſtinction and Confuſion of Viſion is ſo far from Depending upon the Exact or Irregular Collection of the Rays on the Retina, that, where the Object is at the ſame Diſtance, the Figure and Poſture of the Eye the ſame, the Appearance of the Object may be either Diſtinct or Confus'd according as the Organ of Senſation is in a Different Temper to Receive the Impreſſion; Thus in Men, who are Lunatick, or who, by an Exceſs or Intemperance in Wine, find a Diſorder in their Senſations, Objects at the ſame Diſtance, and where the Collection of the Rays is the ſame, may Appear Confus'd and Indiſtinct, which at other Times they would not do; and why we may not ſuppoſe as great a Difference betwixt the Humours of an Old Man's Eye and a Young one's, or betwixt thoſe of a ſhort Sighted Man, and the Contrary, will not be eaſily Shewn.

The Solution of the foregoing Difficulties from the Doctrine of the Expanſive and Contractive Forces.

§. 38. In Fine, all theſe Difficulties are Solv'd by the Doctrine of the Expanſive and Contractive Forces; Simple Viſion is Perform'd by the different Degrees of the Expanſive Forces Communicated from Bodies thro' the Æthereal Plenum to the Eye, from whence they Appear to be Variouſly Colour'd, The Magnitude of Bodies is Diverſify'd according to the Latitude or Extenſion of the Action; and their Diſtinction or Clearneſs Ariſes from a Juſt Proportion or Quantity of it to the Contractive Force of the Optick Nerves, which is alſo Various in different Perſons; ſo that, if the Expanſive Force from Bodies is too Weak to Excite the Contractive Force of the Nerves, a Faint or Obſcure Viſion Enſues, if too Strong, a Confuſed and Diſordered; an Example of the firſt we have in looking upon Dark Objects, which have not Light enough, to Render them Perfectly Viſible to us, and of the laſt, in looking upon the Sun, which Overturns and Confounds the Organ of Senſation; And from hence it is, that Eyes, which want Convex Glaſſes, have their Optick Nerves more Cold and Contractive in their Force, as thoſe of Old Men, and of Younger Men likewiſe, whoſe Eyes are Phlegmatick and Watery; a ſtronger Expanſive Force of Light is therefore Aſſiſtant to them, and which is Effected by the Collection of the Rays in the Convex, and they ſee Things better at a greater Diſtance, becauſe more Luminous and Brighter; On the Contrary, thoſe Eyes, which want Concave Glaſſes, have their Optick Nerves more Warm, Lively, and Expanſive, and therefore the Rays Emitted from Bodies are too ſtrong in their Expanſive for them, which Exceſs of the Expanſive is Moderated by Concaves, which Diſperſe and Weaken the Force of the Expanſive, and therefore ſuch Glaſſes are Beneficial to them; and they ſee Things better at a leſs Diſtance, becauſe they are leſs Illuminated and are Obſcurer; Hence alſo ſuch Eyes will ſee better in a Duskiſh Light, than any others.

Chap. X.

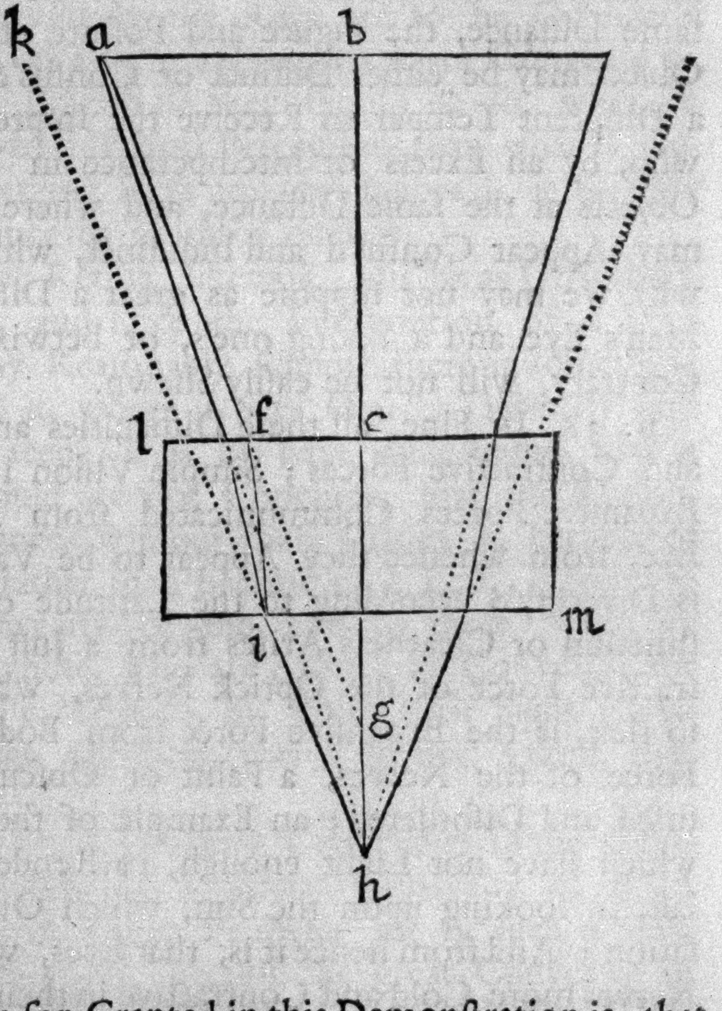
Concerning Viſion, as Perform'd thro' Single Glaſſes, Plane or Convex, which are Interpos'd betwixt the Object and the Eye.

Propoſition the Twenty Ninth Explain'd, that an Object Seen thro' a Plane Glaſs is Magnify'd thereby. Molyneux Dioptr. Prop. 29. Part 1.

§. 1. THIS Philoſophy, haveing Endeavour'd to Explain Viſion in General, Proceeds to Conſider, how we are Affected with Objects, when we look upon them thro' Glaſſes, as it had before Examin'd into them, when they were View'd by the Naked Eye; It therefore Affirms in the Twenty Ninth Propoſition, that an Object Seen thro' a Plane Glaſs, whoſe Surfaces are Parallel, is Magnify'd thereby, altho' *Honoratus Faber*, another Famous Optick Writer, Aſſerts the Contrary, and that the Viſual Angle is not Alter'd, but the Apparent Place of the Object only Chang'd from ſuch an Interpoſition; However that the Viſual Angle is Enlarg'd, and Conſequently the Object Magnify'd, is thus Demonſtrated; the Incident Ray *a f* is Refracted to *i*, and afterwards to *h*, the Place of the Eye, let *h i* be Produced to *k*, and a ſtrait Line Drawn from *h* to *a*, *k h b* will be the Viſual Angle after Refraction, and *a h b* the Natural one, which would have been, if there had been no Refraction, the Former of which is Evidently Greater than the Latter.

Not Prov'd, that there is any Difference betwixt the Natural and the Refracted Optick Angle.

§. 2. Those, who have Affirm'd any thing in Opticks, as it ſeems *Honoratus Faber* has done, in Contradiction to this Propoſition, we ſhall Leave to the Mercy of this Philoſophy to be Refuted by it, and Inquire, whether it's own Demonſtration is as Valid, and Sufficient, as it Apprehends it to be; For, tho' *h a* with *h b* makes a leſs Angle than *h k* with *h b*, yet, ſince in this Doctrine of Opticks it is an Axiom, that what is Demonſtrated of one Ray, and the Axis, holds Good in all other Rays in their Incidence from any Point of an Object, on the Contrary, there is an Equal Reaſon to Affirm, that *k h a* Containing a leſs Angle in the Return of thoſe Rays from the Ocular Point, than what is Comprehended betwixt Rays, which are Deriv'd from any Certain Point in an Object, ſhould make as little Diſtinction, and therefore *k h a* in the Return of the Rays from the Eye ſhould be Equivalent to *h a g* in their Incidence upon it, that is, ſhould Cauſe no Variation, Suitable to what other Opticians Aſſert, that *a g b* is = *a h b* = *k h b*.



Not Prov'd that the Magnitude of the Object is to be Meaſur'd by the Optick Angle.

§. 3. But that, which is Moreover taken for Granted in this Demonſtration is, that Objects are Magnify'd in Proportion, as the Optick or Viſual Angle is Enlarg'd, which, as we have before Evidenced, wants to be ſhewn, or if it did not, *a h b* is not the Natural Optick Angle, where *a f* is the Incident Ray, but *a g b*, which is Equal to *k h b*, the Viſual Angle from Refraction.

Not Prov'd by any Experience of Looking thro' a Window Glaſs. Molyneux Compar'd with Dr. Clark's Annotation. Rohault. Phyſ. Cap. 27. §. 47. Par. 1.

§. 4. Lastly, we do not find this Propoſition Confirm'd by Experience, ſince it is Confeſſed not to Obtain in Glaſs Windows, the only Reaſon for which is ſaid to be the Thinneſs of them, by which Means the Refraction becomes Inſenſible; and yet in other Parts of this kind of Opticks they are ſuppos'd Thick enough to Alter the Poſition of the Object from *a* to *k*, that is, to the left of it's Natural Situation; Indeed this Reaſoning here ſeems to be Agreeable to the Reſt, which we find in this Doctrine, which, by Conſidering Things Abſtractedly, Argues ſeveral Appearances into Nature, which are not found in it, and then Wiſely Endeavours to Contrive a Solution, why they are not Viſible, when the True Account is, becauſe they do not any where Really Exiſt, and are only Creatures of the Mind Form'd from ſuch Abſtractions;

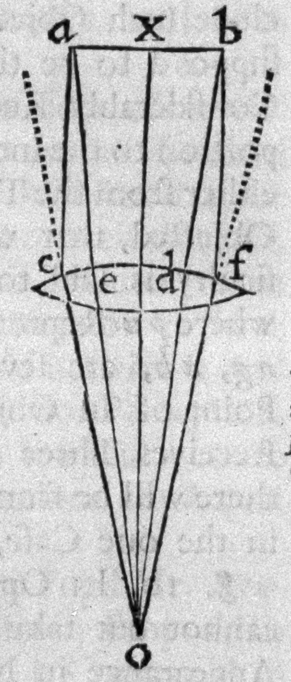
ſtractions; and tho' in Thicker Plates Objects Perhaps are Magnify'd, we ſtill are Aſſur'd it cannot Proceed from the Increaſe of the Viſual Angle from what has been ſaid before, but from the ſame Reaſon, that all Objects Appear Larger, which are View'd thro' a Thicker or a more Denſe and Contractive Medium.

The Thirtieth Propoſition Explain'd, that all Objects Seen Erect thro' Convex Glaſſes are Magnify'd thereby. Molyneux Dioptricks, Prop. 30. Part 1.

§. 5. It is in a following Propoſition Affirm'd, that all Objects Seen Erect thro' Convex Glaſſes are Magnify'd thereby, the Demonſtration of which is, *a x b* (*Fig. §. 6.*) is an Object, *o* the Eye, *a e o*, *b d o*, are Rays Drawn Directly from the Extremities of the Object to the Eye, and the Angle *a o b*, which is Comprehended betwixt them, is the Natural Optick Angle; let the Glaſs *c f* be Interpos'd, which will Haſten the Concourſe of the foremention'd Rays before they Arrive at *o*, wherefore the Eye in *o* will not Perceive the Extremities of the Object *a* and *b* by the Rays *a e o*, *b d o*, but by other Rays, which are Extrinſicall to them, from whence *c o x*, the Optick Angle after Refraction, will be greater than *e o x* the Natural one, and the Object will appear Bigger in the firſt Caſe, than in the Laſt.

Upon the Interpoſition of the Lens, no Rays would be ſeen by the Eye in the Place which is here Imagin'd.

§. 6. We ſhall not here Anſwer to this Demonſtration, that the Solution of the Increaſe of the Object's Magnitude by Refraction is here, as well as in the Former Propoſition, Preſum'd to Depend upon the Optick Angle's being Augmented, ſince we have before Inquir'd into ſuch a Way of Reaſoning; what we ſhall Offer in Oppoſition to this Proof is, that ſuppoſing *a e* to be the Ray, which is Extended from thence to the Eye in *o*, ſo as to make the Continued Line *a e o* from the Object to the Eye, ſince, upon the Interpoſition of the Glaſs, the Ray *a e* will be Refracted nearer to the Lens than *o*, and ſince all the Rays from the ſame Point of an Object are Refracted to the ſame Point in the Eye, that is, have the ſame Effect upon it, *a c* will likewiſe be Refracted to a nearer Point than *o*, and therefore cannot Cauſe a Senſation in the Eye different from what *a e* Refracted would do, which will not Affect it all, or, which is the ſame, all the Rays Incident from the ſame Point of an Object are Collected by a Lens to nearly the ſame Focus.



The ſame Doctrine Confuted from other Conſiderations.

§. 7. But what is farther Obſervable is, that the ſame Ray is not Conſider'd in the Natural, and the Refracted Optick Angle, which ought to be done, and if we do take the ſame Ray, the Solution will not be Valid; ſince if ſo, *a e* will be Refracted nearer to the Lens than the Poſition or Situation of the Eye in *o*, and Conſequently there will be no Erect Appearance at all of the Object from ſuch a Refraction; if Different Rays are Aſſum'd, and the Ray Falls nearer to the Axis than *a e*, it will be Refracted nigher to the Lens than *a e* would be, if it Falls farther from the Axis than it, which is here ſuppos'd, as *a c*, let the Glaſs be taken away, and there will be no Natural Optick Angle from ſuch a Ray, with which we may Compare the Refracted one; But if we are Allow'd at Pleaſure to Compare the Natural Optick Angle made by one Ray with the Refracted Optick Angle made by another Ray, it is Evident, 1*ſt.* that Seeing there may be a Hundred or a Million Different Rays Incident betwixt *a c* and *a e*, there will likewiſe be as many Different Viſual Refracted Angles Form'd by them, which, when Compar'd with one Certain Optick Angle *e o x*, will have a Different Proportion, and therefore the Object will not be Seen by *o c* in Preference to any other Refracted Ray, that is, by this Reaſoning, it may as well Appear in a Hundred or a Million Different Magnitudes above it's Natural, as in that one, which *o c* Preſcribes; 2*d.* It is likewiſe Evident, that if the Object *a b* is Enlarg'd to a much greater Proportion in Reſpect of the Lens, that is, ſuppoſing *a b* to be 3 or 4 or 20 Times as Great, as it is here Repreſented, and the Lens to be the ſame, there cannot be any Imaginable Reaſoning like that, which is Propoſed to us from this Scheme, ſince the Rays, drawn from the Extremities of the Object, would lie beyond, and External to, any Refracted Rays whatſoever.

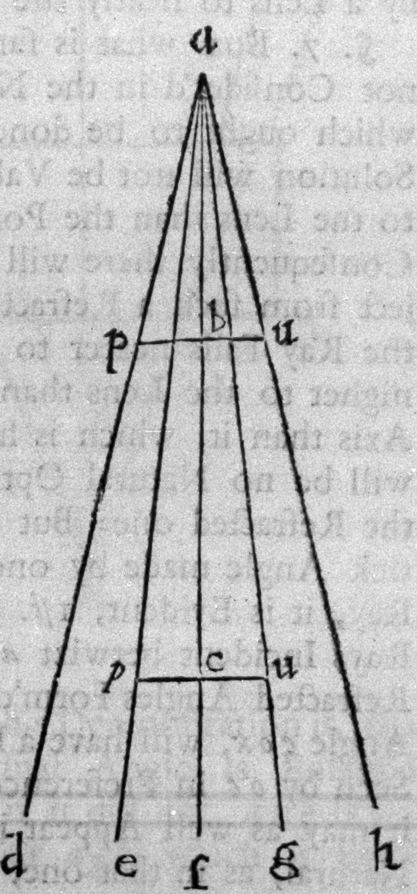
An Object according to this Philoſophy may as well Appear Magnify'd, when Inverted, as when Erect.

§. 8. Another Objection againſt this Demonſtration is, that if it were True, it only Proves, that Objects Seen Erect thro' Convex Glaſſes are Magnify'd by it, which at the ſame Time Inſinuates, that if Objects are not Erect, when Seen thro' a Convex, they are not Magnify'd, and yet that is not any ways Apparent from this Doctrine, but on the Contrary it rather ſeems Plain, that they would be; for the

Rays, Croſſing each other in the Focus *o*, are ſaid to Cauſe an Inverſion of the Image, but ſince the Lines *fo, co,* Produced beyond the Focus *o*, are not Alter'd in their Direction after their being Refracted in the Lens at *f* and *c*, and ſince the Eye is ſuppos'd to Purſue the Rays backwards in that Direction, in which they Fall, it is Manifeſt, that, whether it is Placed in *o*, or in any Situation beyond it, ſo as the Object may be Seen, the Action of the Viſive Faculty cannot Return it's ſelf in any other Direction than that of *co, fo,* from whence it will follow, that the Inverſion of the Image will be Attended with an Increaſe of the Magnitude of the Object, as well as the Erection of the Image will be.

Concerning the Apparent Place of Objects ſeen thro' Convex Glaſſes. Molyneux Dioptricks Prop. 31. Part 1.

§. 9. The Thirty Firſt Propoſition in this Philoſophy is Concerning the Apparent Place of Objects Seen thro' Convex Glaſſes, of which we have this Account; That in Plain or Simple Viſion (Eſpecially, when Objects are ſo far Remov'd, that the Interval between our two Eyes bears no Senſible Proportion to the Diſtance of the Object, or when the Object is looked upon by one Eye only) the Diſtance Obſerv'd is rather the Act of our Judgment than of Senſe, and is Acquir'd by Exerciſe and a Faculty of Comparing one Object with another; for Diſtance, as it is a Line, is only Perceiv'd by one Point of it in the Eye, and therefore muſt owe it's Impreſſion upon our Minds to certain Interjacent Bodies, as the Earth, Mountains, Fields, Trees, *&c.* If Diſtance is not, or cannot be, Solv'd this Way, there is another Propos'd, which is by Conſidering and Comparing the Different Magnitude of Objects, or their Different Colours, for if we Perceive them leſs in Bulk, or their Colours Fainter, we Conclude ſuch Objects are at a greater Diſtance from us than otherways; And this is ſuppos'd to be the Method of Judging Concerning the Diſtance of Objects, that are Conſiderably Remote; But if they are Nigh, and their Diſtance bears a Senſible Proportion to the Interval of the Eyes, it is then ſuppoſed to be Apprehended by the Mind, either from the Turn of the Eyes, or by the Angle of the Optick Axes; to which if it is Objected, that we Perceive Diſtance by the Help of one Single Eye only, that Difficulty is ſaid to be Solv'd from the Figure Annexed, where *pu* Repreſents the Pupil of the Eye, *ad, ae, af, ag, ah,* are ſeveral Rays Emitted from the Radiating Point of an Object *a,* and ſince *pu,* in the Situation *c,* Receives Three of theſe Rays, and in *b* the whole Five, there will be from hence a Different Perception of Diſtance in the one Caſe, from what there will be in the other.



The different Suppoſitions made Uſe of to Explain the ſame Phenomenon of Diſtance an Objection againſt the Solution.

§. 10. In Oppoſition to the foregoing Account we cannot but take Notice, that to Explain the very ſame Appearance in Nature, Namely Diſtance, ſix or ſeven Diſtinct Hypotheſes are thought Neceſſary to be Introduced; *1ſt.* That it is not an Act of our Senſe but our Judgement; *2d.* That the Magnitude of Objects Increaſing or Decreaſing is one Method, by which we Judge of their Varying Diſtances; *3d.* That the Livelineſs or Faintneſs of Colours is another; *4th.* That the Interpoſition of Bodies Occaſions this Preception in us; *5th.* That the Diſtances of Remote and Nigh Objects are owing to Different Cauſes of our Perceiving them; *6th.* That the Turn of the Eyes, by which the Optick Axes are made to Concur and Center in Various Points, as the Object Approaches or Recedes from us, is a Means likewiſe, by which we Form our Notions of Diſtance; Laſtly, That the Greater or Leſſer Number of Rays, and their Leſſer or Greater Divergence from each other in their Incidence on the Pupil of the Eye, is a Direction to the Mind to Conceive Objects Placed at ſeveral Changing Intervals from it; Each of which Hypotheſes we ſhall Examine, and in the mean Time Obſerve, that it is Impoſſible ſuch a Philoſophy ſhould be True, where ſo many Various Suppoſitions are made, and Cauſes Aſſign'd, for the Explication of the ſame Determinate and Individual Effect.

The Perception of Diſtance not an Act of Judgement but of Senſe.

§. 11. As to the Firſt, that Diſtance is an Act of the Judgement, and not of the Senſe, we cannot Acknowledge; ſince every one as Fully and Really Perceives the Interval, which lies betwixt the Object and the Eye, as he does the Object it's ſelf; but to Imagine an Actual Perception to be owing to an Inference in the Mind, or a ſhort and Summary Concluſion of the Underſtanding, (let it be Thought to be as Habitual as it will) is as Irrational and Abſurd, as to Suppoſe, that our Ideas of Senſation may be Deriv'd into us from bare Reflection only, or that we can See or Feel in General, becauſe we can Judge and Infer upon thoſe Perceptions.

The Varying Magnitude of Objects no Reaſon for the Mind to Judge of their Varying Diſtances.

§. 12. In Particular, to Collect the Greater or Leſs Diſtance of Objects from their being Reciprocally Leſs or Greater in their Apparent Magnitude, is Inferring from certain Perceptions, which the Mind has, to others, which have not the leaſt Connexion with them, unleſs any one will Affirm, that it is Neceſſary for the Mind to Judge the Diſtance is Greater, when the Object is Leſs, and the Diſtance Leſs, when the Object is Greater, rather than the Contrary; for if the Mind did Actually Perceive nothing, Excepting the Differing Magnitude of the Object, and we ſhould likewiſe Preſume, that, when it was at a farther Diſtance from us, we by Experience could tell the Magnitude was Leſs, and when nearer, Greater, yet why from ſuch a Perception, and ſuch an Experience, the Mind by it's own Power, and by the Judgement it makes, ſhould be rather carry'd to Perceive the Diſtance Longer in the firſt Caſe than in the laſt, will be Impoſſible to Say.

The Difference of Colours, as to their Vivacity in Nigh and Remote Objects, no Solution of the Apparent Place or Diſtance of them.

§. 13. And for the ſame Reaſon the Difference Obſervable in Colours cannot be an Argument to our Minds of Perceiving Diſtances as it does, for tho' the Colours of Diſtant Objects, as to their Vivacity, are Fainter, and thoſe of Nigher, more Vigorous and Strong, yet why the Mind ſhould join an Idea of Greater Length to a more Languid Colour, and of a Shorter to a more Forcible and Active, will be in this Philoſophy Equally Unaccountable with the former.

The Interpoſition of Objects tho' in Fact it does Increaſe the Diſtance yet no Solution in Philoſophy of the Preſent Phænomenon.

§. 14. A Fourth way of Judging is ſaid to be from the Interpoſition of ſeveral Objects betwixt the Eye and the Diſtant One, and this is ſuppos'd to be the Cauſe of the Sun's or Moon's Appearing Greater near the Horizon, than when it is Obſerv'd in the Meridian; Againſt which there is the ſame Invincible Objection, the Inconceivableneſs, how the Mind ſhould rather join the Idea of a Greater than a Leſs Diſtance to an Object in ſuch Circumſtances, when it is ſuppos'd only by Actual Perception to have an Impreſſion from the Object it's ſelf, and thoſe, which are Interpos'd betwixt it and the Eye; that is, when the Intervalls betwixt each of them Cauſe no Perception, how the Mind ſhould by it's own Force or Choice rather Conceive the Diſtances to be Longer than Shorter, ſince there muſt be an Actual Cauſe to Produce an Actual Effect in the Mind.

An Objection to what we have before Alledg'd Propos'd.

§. 15. It was very Ingeniouſly Urg'd ſome Years ago by my Excellent Pupil Mr. *Newſome*, ſince Fellow of this College, but now Dead, in a Theſis of Philoſophy Propos'd by Him in the Schools in Defence of this Doctrine; That our Judgement is Concern'd in Aſcertaining, both the Diſtance of Objects, and the Magnitude of them, ſince if we look upon any Large Object thro' a Caſement of a Window, which is much Leſs than it, yet we ſeem to Perceive it in it's full Latitude and Extent, which Appears to be Impoſſible we ſhould do, if this Perception was not owing to our Judgement and not to our Senſe.

The foregoing Objection Anſwer'd.

§. 16. It is perhaps the beſt Argument for this Notion, that any Philoſopher has hitherto Offer'd; but, beſides what we have ſaid, we ſhall Add, that as the Preſſure, or Expanſive Force of the Air or of Æther is known to be the ſame, whether in a Room or without it, ſo it is Juſt to Think, that all the other Affections and Properties of it are ſo, and Conſequently thoſe, by which the Impreſſions of Objects upon our Sight are Convey'd to us.

The Three foregoing Accounts of Diſtance Conſider'd farther.

§. 17. But farther, in Contradiction to theſe three Accounts of our Perception of Diſtance, the Diminution or Increaſe of the Object, the Faintneſs or Vivacity of it's Colour, and the Interpoſition of other Objects betwixt it and the Eye, or the Non-Interpoſition of them, it is Evident; That, in the Firſt Caſe, the Magnitude of an Object is not Diminiſhed in Proportion to it's Diſtance from the Eye, and Conſequently not Increas'd in Proportion to it's Approach to it, for if we ſet up a Lamp, or a Candle, and the Socket, in which it is held, there is nothing more Manifeſt, than that the one is Seen at ſeveral Diſtances, where the other Diſappears, altho' the Flame is of no greater a Magnitude, than the Socket, which Contains it, or the other Part of the Lamp, or Candle, which is not Burning; and in Common Objects, which have not that Brightneſs, we See the ſame Magnitude of them at ſeveral Changeing Diſtances. In the ſecond Caſe, the Faintneſs or Livelineſs of Colours, as the Object is Remov'd from or Comes nearer to the Eye, can be no Reaſon from Experience, why the Mind ſhould Judge, that Objects are more or leſs Diſtant from us, becauſe White, or Red, is Seen at a much Greater Diſtance than other Colours are, and therefore if the Mind was to Judge of the Diſtance of Objects by the Intenſeneſs or Remiſſneſs of the Colour, it would Apprehend White and Red Objects to be leſs Diſtant than others, which it does not do. In the Third Caſe, it is likewiſe Manifeſt from Experience, that, where there are ſeveral Objects Interpos'd, the Diſtance to our Apprehenſion is not Increaſed by it, but the Contrary, ſince any Eminent Place upon the Surface of the Earth Appears to be at a Greater Diſtance, if a large Portion of Air without any Interruption is Expos'd to the Eye, than if that View is Broken and Intercepted by other Intermediate Objects, as in an High and Lofty Edifice, which tho' at a Great Diſtance from a Hill or ſeveral Hills, which are Interpos'd, yet Appears in a manner Cloſe to them. And the ſame may be ſaid of the Sun or Moon Riſing or Setting behind one or ſeveral Mountains Interpos'd, for it will ſeem to Riſe or Set Cloſe behind them, be they nearer or farther from us.

The Diſtance of Nigh Objects not Solv'd from the Concourſe of the Optick Axes.

§. 18. That which is moreover Abſurd in this Philoſophy is, that the Diſtance of Nigh Objects and Remote is Accounted for by two as Different Hypotheſes as can be Poſſibly Contriv'd, the laſt is ſaid to be owing to one of the three abovemention'd Cauſes, the firſt to the Angle Form'd by the Axes of the Eye, or to the Greater or Leſs Number or Divergence of the Rays, which are Incident on the Pupil; but what Connection or Alliance is there betwixt Colour, Magnitude, or the Interpoſition of an Object with the Divergence of Rays, or an Angle made by the Direction of our Eyes to a Point? and yet one and the ſame Effect or Appearance of Diſtance is ſuppos'd to be Produc'd in the Mind by thoſe Cauſes, which have not the leaſt Reſemblance one to another; tho' it is altogether as Unreaſonable, as to Believe, that Extenſion it's ſelf is owing to them, which every Body Acknowledges to be Impreſſed upon us by ſome Individual and Unvary'd, tho' unknown Principle, which therefore we ſhall here Preſume to Supply; Namely by the Expanſive Force of the Æthereal Plenum; and Diſtance is Perceiv'd in the ſame Manner, as all other Bodies are, that is by the Expanſive Forces of them.

The Diſtance of Nigh Objects not Solv'd by the forement'd Hypotheſis.

§. 19. Notwithstanding, this Philoſophy is not only able to Digeſt all theſe ſeveral Abſurdities, but it likewiſe Proceeds to Eſpouſe a Sixth, that Diſtance is Meaſur'd and Perceiv'd by the Direction of the Eyes to a Point, that is, by the Angle, in which the Axes of them meet; againſt which there is this Evident and Common Argument, to wit, the Appearance and Senſation of the ſame Diſtance, or nearly the ſame, to one Eye only, without any Imaginable Concourſe of the Optick Axes at all, which is Experienced, when we look upon an Object with both; However it may Poſſibly be ſaid, that, where one Eye is ſhut, the Mind Refers it's ſelf back in that manner, as to direct the Axis of the Eye, which is ſhut, to the ſame Point, with that of the Axis of the Eye, which is open; or at leaſt the Axis of the Eye, which is open, may be ſo variouſly Inclin'd at different Diſtances of the Object to it, as not to Require a Concurrence of the other Axis to Inform us of the Changeing Interval: If the firſt is affirm'd, it is neceſſary, that the Eye, which does not ſee the Object, ſhould be carried to the ſame Point in the Object with that, which does ſee it, or, which is the ſame, that, ſince the Rays are purſued back by the Eye in the Incidence, in which they Fall, thoſe, which are not Incident at all, ſhall be Purſued back with the ſame Direction, with thoſe, which are Incident; If the laſt is Affirm'd, it is a Confeſſion, that no Concourſe of the Optick Axes, and Conſequently no Angle Comprehended by them, is Requiſite to our Different Perception of Diſtance, which yet is an Abſolute Aſſertion of this Doctrine.

The Diſtance of Nigh Objects not Solv'd by the foregoing Hypotheſis.

§. 20. If nothing in the former Section was Juſtly Argued, it would be yet Certain, that neither by the Different Concourſe of the Optick Axes, nor by the Different Inclination of one of them, Different Diſtances could be Accounted for, becauſe in both Caſes the Mind is ſuppos'd to Act and Determine it's ſelf Agreeably to the Diſtance of the Object, inſtead of the Object, or the Diſtance of it, Determining the Mind, whereas Diſtance, as it is a moſt Evident Perception in us, ought to Proceed from an Action without us, and in which the Mind is Paſſive, and not from an Action within us, and in which the Mind is Active.

The Diſtance of Nigh Objects not Solv'd by the forementi'd Hypotheſis

§. 21. Besides, if the Mind was ſuppos'd to be Paſſive only in the Information it Receiv'd of Diſtance from External Nature, how an Angle ſhould Give it, or the Different

Inclinaclination of a Ray, any more than the Extreme Point of a Ray, which is own'd to be Incapable of doing it, is altogether Inconceivable.

The Diſtance of Nigh Objects not Solv'd from the abovemention'd Hypotheſis.

§. 22. LASTLY, it is notorious in the Diſtance of nigh Objects to any one, who will give himſelf leave to obſerve it, that, if one Eye is nearer to an Object than the other, the Diſtance is ſenſibly different, and will be perceiv'd ſo, whether we make uſe of one Eye, and then the other, or of both of them together, which will be Impoſſible to be Explain'd by the Concourſe of the Optick Axes, which ſuppoſes one and the ſame Object to Appear at an Equal Diſtance from both our Eyes.

The Diſtance of Nigh Objects not Solv'd from the Divergence of Rays.

§. 23. THE Weakneſs and Infirmity of the laſt Solution of the Diſtance of nigh Objects from the Concourſe of the Optick Axes, (which our ſeeing the ſame Diſtances by one Eye was Sufficient to Confute) ſeems to be a Reaſon of Introducing a Seventh Hypotheſis to Solve Diſtance, Namely, that Different Diſtances may be Inferr'd by the Mind, where one Eye only is Concern'd, from the uncertain Divergence of the Rays on the Pupil, and by that Means Falling in a greater or leſs Number upon it.

The Diſtance of Nigh Objects not Solv'd from the Divergence of Rays.

§. 24. IN Anſwer to which we Alledge, that, if Diſtance Depends upon the Angle made by the Optick Axes, it will be but a Lame Vindication of ſuch a Doctrine to Aſſert, that it does not Depend upon that Angle, but upon the Divergence of the Rays, which Fall on the Pupil of the Eye, or, which is the ſame, it is a New and Surprizing Way of Defending an Hypotheſis by Relinquiſhing it, and having Recourſe to another, which is Different from it.

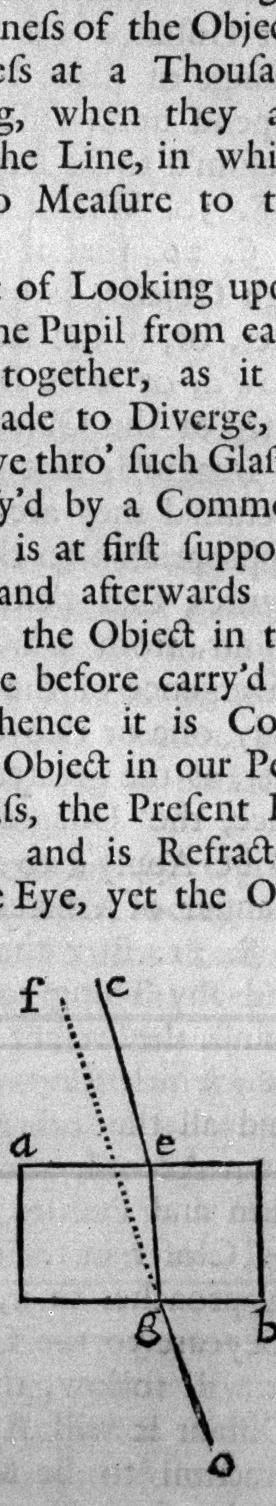
The Diſtance of Nigh Objects not Solv'd from the Divergence of Rays.

§. 25. HOWEVER, that the Uncertain and Undetermin'd Divergence of the Rays, or a greater or leſs Number of them being Incident on the Eye, cannot Produce a Perception of Diſtance in us, is Manifeſt from this Philoſophy, which Acknowledges, that the Effect, which Reſults from thence, is the Clearneſs or Faintneſs of the Object; But Objects Appear with a Diſtinguiſhing Brightneſs and Clearneſs at a Thouſand different Diſtances, and do not grow Faint and Obſcure, Excepting, when they are ready to Vaniſh from our Senſes, and are near the Extremity of the Line, in which the Eye is Capable of Seeing them, and Conſequently can be no Meaſure to the Mind of Judging Concerning the Diſtance of them.

Concerning the Diſtance of Objects when View'd thro' Glaſſes.

Molyneux's Dioptricks Prop. 31. Part 1. §. 4. ſ. 6.

§. 26. FROM the foregoing Principles it is Affirm'd in Reſpect of Looking upon Objects thro' Glaſſes, that, if the Parcel of Rays, which Falls on the Pupil from each Point in nigh Objects, is by Refraction made to Flow as Cloſe together, as it is from Diſtant Objects, or, if the Rays from Diſtant Objects are made to Diverge, as much, as if they Flow'd, or were Deriv'd, from nigh Objects, the Eye thro' ſuch Glaſſes will Perceive the Place of the Object Chang'd; This is Exemplify'd by a Common Experiment, which has been before Conſider'd, of a Baſon, which is at firſt ſuppos'd to be Empty, and a Piece of Silver Placed at the Bottom of it, and afterwards to be Fill'd with Water, by which the Rays of Light, Emitted from the Object in the Baſon, are ſuppos'd to be Bent and Inclin'd to the Eye, which were before carry'd above it, and could not be Perceiv'd without Refraction; from whence it is Concluded, that Refraction Cauſes a different Place or Diſtance of the Object in our Perception of it; To Reduce this Inſtance to Objects ſeen thro' a Glaſs, the Preſent Figure is made Uſe of, where, tho' *c e* is the Ray from the Object, and is Refracted thro' the Glaſs *a b* to *g*, and from thence to *o*, the Poſition of the Eye, yet the Object is Seen in *o g* Continued to *f*, and therefore in a different Situation from what it would have been Obſerv'd in without Refraction; Laſtly from (*Fig.* §. 28.) the Rays *c d*, *c e*, will after a double Refraction Arrive at *i* and *k*, which, being Receiv'd by the Pupil of the Eye, will be Return'd in ſtrait Lines to *f*, and the Object Seen at a nearer Diſtance than it Really is; ſo that Refraction in this Caſe does not only Alter the Place of the Object, but Changes the very Diſtance, in which it Appears to us in a ſtrait Line or Direction from the Eye.

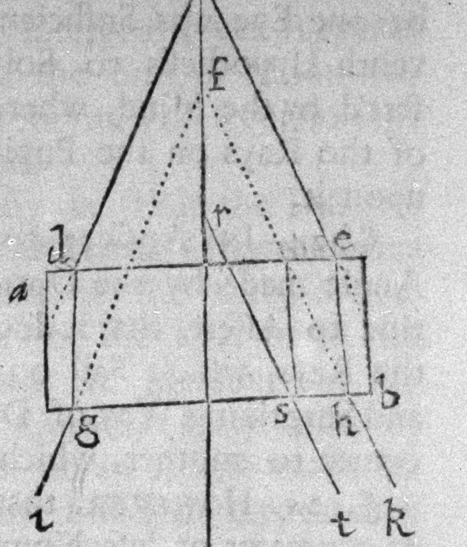


The uncertain Tenour of this Philoſophy.

§. 27. IT is Admirable to Obſerve the Strange and Deſultory Genius of this Philoſophy, and which is a Juſt Argument of the Uncertain and Wavering Principles, on which it Depends; Firſt Diſtance is to be Meaſur'd by the Increaſe or Decreaſe of Objects in their Magnitude; Then by the Livelineſs and Faintneſs of their Colours; Afterwards by the Interpoſition of other Objects; From thence we are carry'd to a Solution of it by the Concourſe of the Optick Axes; When at laſt the Whole is made to Turn upon the more or leſs Divergence of the Rays, or a leſſer or greater Number of them Falling on the Pupil of the Eye, by which the Diſtance of Objects View'd thro' Glaſſes is ſuppos'd to be Accounted for.

The greater or leſs Number of Rays do not Cauſe the Senſation of Diſtance in Objects View'd thro' Glaſſes.

§. 28. As to the greater or leſs Number of Rays Cauſing our Senſation of Diſtance we have already Conſider'd that Solution in §. 25. which, if it were True, would not be of any Service to the Explaining of the Diſtance, or the Change of it, Obſervable thro' Glaſſes; ſince in the Figures Annexed there is no ſuch Thing, as a greater or leſs Number of Rays ſuppos'd, but all is Reſolv'd into the Affections of one Ray in (*Fig.* the 1*ſt.*) and in two only in (*Fig.* the 2*d.*)



The Alteration of Place by Refraction, no Solution of the Diſtance of an Object.

§. 29. MOREOVER in (*Fig.* the 1*ſt.*) there is an Alteration of Place only, Namely, from *c* to *f*, but, why the Lateral Change from one Place to another ſhould be Solv'd by the ſame Cauſe, by which a Change of the Direct Diſtance is, cannot be Explicated in this Philoſophy, becauſe there is no Analogy betwixt them; unleſs the Change of Diſtance in Looking upon Objects from Reflection, as well as Refraction, will Perhaps Aſſiſt us in Aſcertaining the Theory of Diſtance, that is, unleſs in Reflecting Glaſſes, as well as Refracting, there is alſo an Alteration of Place, which whether there is or not, yet no one ever Imagin'd, that Diſtance was therefore to be Explain'd by it; What we Mean is, that the Change of Place by Refraction, and of Diſtance or Remoteneſs from the Eye, may Depend upon different Cauſes, for any Thing, that has been Prov'd, altho' they are Preſum'd to be Reciprocal.

The Cloſeneſs or Divergence of Rays, no Solution of differing Diſtances of Objects ſeen thro' Glaſſes.

§. 30. WHAT we may farther Obſerve is, that, altho' it is ſaid in the Beginning of §. 26. that if the Parcel of Rays, which Falls on the Pupil from each Point in nigh Objects, is by Refraction made to Flow as cloſe together, as it is from Diſtant ones, or, if the Rays from Diſtant Objects are made to Diverge as much, as if they Flow'd, or were Deriv'd, from nigh ones, the Eye thro' ſuch Glaſſes will Perceive the Place of the Object Chang'd, yet in the foregoing Figures no ſuch Caſe is Suppos'd or Imagin'd, that Rays ſhould be Refracted with that Cloſeneſs, with which it is Preſum'd they are by the Eye in Diſtant Objects, nor with that Divergence, which is Appropriated to the Refractions of the Eye in Nigher; Since it is Evident in thoſe Figures, that neither Objects being Nigher or Farther off is Conſider'd, where the Points, from which the Rays Flow are taken at Pleaſure; Nor is the Cloſeneſs or Divergence of the Rays, by which their Number is Magnify'd or Leſſen'd, where only one or two are made Uſe of in the Demonſtration; It therefore ſeems Plain, that, as the greater or leſſer Divergence of Rays was Introduced to Account for Diſtance, tho' Diſagreeable to all the other Hypotheſes of it, ſo it is here Endeavour'd to be Apply'd to the Diſtance of Objects Obſervable thro' Glaſſes, tho' there is no manner of Correſpondence or Relation betwixt them.

The Cloſeneſs or Divergence of Rays, no Solution of Differing Diſtances.

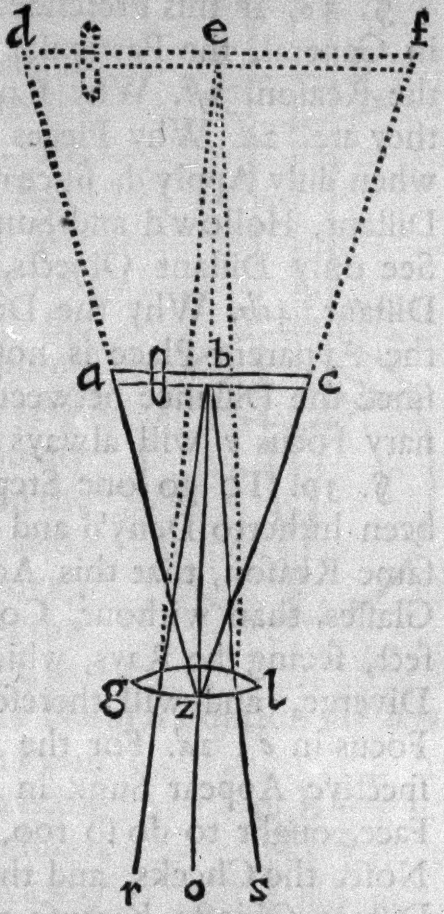
§. 31. BUT that which is ſtill more is, if a different Divergence of Rays Signify'd any Thing to Diſtance, it would Furniſh us with an Argument, not for, but againſt, the Preſent Doctrine; For ſuppoſing the Radiating Point of an Object in *c*, (*Fig.* §. 28.) which Emits Rays from a Center to *d* and *e*, to be now Poſited in *f*, either *c e*, *c d*, and all the other ſeveral Rays from *c*, will Diverge, and Dilate themſelves of their own Accord, which is Impoſſible and Abſurd, or elſe they will keep the ſame Station and Poſture to each other in *f*, as before in *c*; If ſo, it is Evident, the Rays will be Cloſer in their Incidence upon the Plane *a e*, the nearer the Radiating Point Approaches to it, becauſe in any Circle the Rays are more Conſtipated, the nearer they are to the Center, and leſs ſo, the farther they are from it, by which Means it will follow, that the nearer any Object is to the Refracting Glaſs, the farther Diſtant it will Appear from it; For the Incident Rays from *e* are ſuppos'd by Refraction to be ſeen in *k h*, and the Incident Rays from *f* by a like Refraction to be ſeen in *t s*, but the Incident Rays from *c* Diverge more, and are leſs

Cloſe and Denſe, than the Incident Rays from *f*, and therefore the Refracted Rays *t s* will be more Conſtipated and Diverge leſs than the Refracted Rays *k h*, and Conſequently the Object ſeen at a greater Diſtance in *r*. than in *f*, which is Directly Contrary to what is Affirm'd and Maintained in this Philoſophy.

Concerning the Apparent Place of Objects Placed Nigher a Convex Glaſs than it's Focus.

Molyneux's Dioptricks Prop. 31. Par. 1. §. 7.

§. 32. FROM a Solution in General of Diſtance, the Preſent Opticks Proceeds to Determine the Apparent Place of an Object in Particular, which is Situate nigher a Convex Glaſs than it's Focus; in Order to this, it is ſaid we are to have the Power of the Glaſs, the Diſtance of the Object from the Glaſs, and the Length of the Object given. Thus *a b c* is an Object, the Diſtance from the Glaſs *b z*, the Glaſs it's ſelf being of the leaſt Thickneſs, that is Poſſible, that the Optick Angle at the Immerſion and Emerſion may be the ſame, the Middle Point *b* is likewiſe ſuppos'd to Radiate upon the Glaſs, ſo as the Rays ſhould be Refracted, as if they came Directly from *e*; From the forementon'd Data, the Point *e*, being the Imaginary Focus, is Determin'd by *Prop.* 8. Through *e* Draw the Indefinite Line *d e f*, Parallel to *a b c*, and as the Apparent Place of the Point *b* is at *e*, ſo the Places of the Collateral Points *a*, *c*, will be in the Strait Line *d e f*, unleſs the Convexes are of very ſmall Spheres, that is, in *d* and *f*; ſince the Glaſs is ſuppos'd to be of an Infinitely ſmall Thickneſs, and the Vertex of the Immerſion and Emerſion *z* to be the ſame; and therefore, if the Eye is in *z*, the Optick Angle would be *a z c*, and the Point *b* would Appear, as at *e*, and Conſequently the Object *a b c* as in *d e f*.



The Optick Angle being the Meaſure of Diſtance before Confuted.

§. 33. WE cannot but in the firſt Place Obſerve, that all the ſeveral Hypotheſes, made Uſe of to Explain the Apparent Interval of Objects, are here laid Aſide; The Concourſe of the Optick Axes is of no Significance; The greater or leſs Divergence or Number of the Rays of no Importance; The more or leſs Vivacity of Colours is not Material; The Interpoſition of Objects it ſelf is Diſcarded; And we are now only to Conſider, it ſeems, the Greater or Leſs Magnitude of the Optick Angle, which at the ſame time has been already Confuted, as to it's being a Juſt and Exact Rule, by which we ſhould take our Meaſure of Diſtance.

This Account from the Optick Angle Contradicted by Glaſſes of ſmall Spheres and a great Convexity.

§. 34. GRANTING which, it is Acknowledg'd, that if the Convexes are of very ſmall Spheres, *d e f* will not be a ſtrait Line, but a Curve one, which Phænomenon of Diſtance this Philoſophy Thinks fit to Solve by Neglecting it, whereas it is as Neceſſary to be Accounted for, as any other Refracted Diſtance; for why ſhould large Spheres be more Conſiderable, or more Important to us to Underſtand, than ſmall ones, where Nature is the Subject of our Inquiries, and is always as fully Expreſs'd, and as Perfectly and Compleatly Exiſts, in a Glaſs of an Inch, as of a Yard Diameter?

Contradicted by Reaſon.

§. 35. WE ſhall alſo Add, that Allowing both theſe Suppoſitions of the Optick Angles being the Meaſure of Diſtance, and of the Sphere of the Glaſs being Extremely Large, tho' it is Manifeſtly Introduc'd to Square with the Preſent Reaſoning, Allowing which, it would no ways follow, that *f* and *d* ſhould, in the Return of the Refracted Rays to their Imaginary Focus's, be in the ſame Line with *e*; becauſe there is no Reaſon Aſſign'd, or can be, why *c*, which is more Diſtant from *z* than *b*, ſhould be Referr'd back by the Eye thro' a greater Lenth *c f*, than *b* is, which Determines it's ſelf at a ſhorter Interval, *b e*; for *c*, which Affects the Eye at a greater Diſtance, one would Think, ſhould Cauſe a leſs Force of Reaction in it than *b*, which is Nearer, and Conſequently *f* ſhould Appear leſs Remov'd from the Eye than *e*, and *d e f* in a Bent, or Concave, Situation.

Contradicted by Experiment.

§. 36. MOREOVER, the Eye is ſuppos'd to be in *z*, which is Contrary to Experience, that is, ſuch an Hypotheſis will not be Capable of Solving, what we find to be True in Matter of Fact; ſince we Plainly Perceive Objects, Refracted thro' a Lens, to Appear at a greater Diſtance than other ways they would do, Albeit the Eye is Placed at a Conſiderable Diſtance from *z*, or the Glaſs, in Reſpect of the Diſtance of the Object from it, that is, where the Diſtance of the Object and the Eye from the Glaſs, bear a very Senſible Proportion to each other, which is here Preſum'd to be none at all.

Contradicted by this Philoſophy.

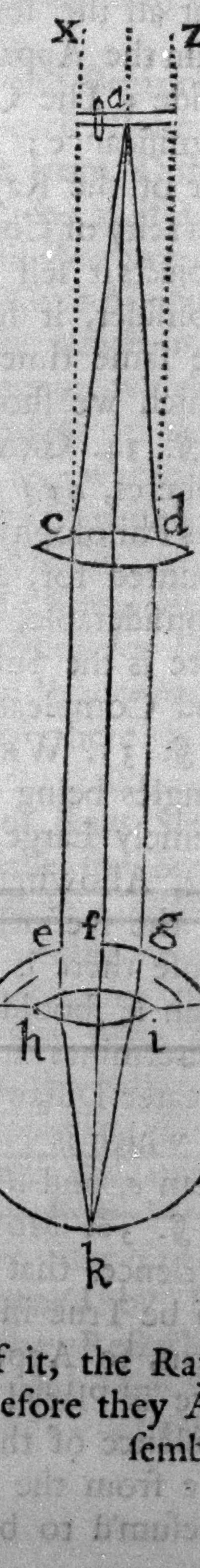
§. 37. Lastly, if we likewiſe Reſign this Suppoſition to the Preſent Doctrine of Opticks, and Place the Eye in *z*, as near to the Glaſs, as is Poſſible, either *g z l* will Perform the Office of the Eye, or not; if it does, *b g*, *b l*, will be Refracted towards *o*; if not, the Glaſs and the Eye will Repreſent a double Lens, and the Rays be made to Approach ſooner to the Axis *z o* than before; in both which Caſes, *g r*, *l s*, inſtead of Divergeing, as they are Imagin'd to do Contrary to the whole Tenour of this Philoſophy, will Converge to a Point on this ſide *z*, towards *o*, and therefore it will be altogether Impoſſible for them to have any Imaginary Focus at *e*, *d*, or *f*; ſince in their being Referr'd backwards according to the Direction of the Refracted Rays, they will Neceſſarily Diverge and Divaricate from each other, ſo far are they from being Determin'd to any of the foremention'd Points in *d e f*.

The Inferences from this Doctrine Invalid.

§. 38. If this Pretended Demonſtration is Inſufficient to Solve Refracted Diſtance in General, the Particular Inferences, made from it, will be Equally Invalid, Namely the Reaſon, 1*ſt*. Why Convex Glaſſes Repreſent Objects as farther off than Really they are. 2*d*. Why Pieces of Perſpective Appear very Natural and Strong thro' them when duly Apply'd, ſince they make the Parts of the Perſpective ſeem, by being more Diſtant, Hollow'd and Sunk in. 3*d*. Why Convex Glaſſes help the Eyes of thoſe, that See only Diſtant Objects, as Old Men, ſince they make nigh Objects Appear, as Diſtant, 4*th*. Why the Diſtance between the Object and Glaſs Continueing the ſame, the Apparent Place is not Alter'd, tho' the Eye be Remov'd to and from the Glaſs, ſince the Diſtance between the Object and Glaſs Continueing the ſame, the Imaginary Focus *e* will always be the ſame.

A farther Confutation of the Preceding Doctrine.

§. 39. To go one Step farther in the Confutation of this Doctrine, all that has been hitherto Deny'd and Diſprov'd we will Suppoſe to be True; Yet, 1*ſt*. For the ſame Reaſon, that this Accounts for the greater Diſtance Obſervable thro' Convex Glaſſes, than without, Concaves likewiſe will have the ſame Effect, ſeeing the Rays, which are Refracted thro' them, do Evidently Diverge, and will therefore be Referr'd back to an Imaginary Focus in *e*; 2*d*. For the like Reaſon, that the Parts of a Perſpective Appear Sunk in and Hollow'd, thoſe of a Picture, or a Face, ought to do ſo too, which yet Appear Prominent, as the Noſe, the Cheeks, and the Eye-Brows; 3*d*. If Old Men only See Diſtant Objects, becauſe their Chryſtalline is Flat, as this Philoſophy Teaches, it is not Conſequent, that they ſhould See Objects, as Diſtant, becauſe their Chryſtalline is more Convex, or the Rays are Refracted, as if it were ſo, by the Aſſiſtance of the Lens Apply'd to it; 4*th*. If *a c* is in the ſame Poſition from the Glaſs, tho' the Eye is Remov'd nearer to *z*, or farther from it, the Imaginary Focus being the ſame at *e*, the Object, it is ſaid, will Appear at the ſame Diſtance, which is not Probable, or at leaſt not Manifeſt; For if the Eye is Situate in *z*, and Referrs the Ray *b z* back to *e*, if it is afterwards Placed in *o*, by §. 35, It is Unreaſonable to Think, or at leaſt it is no ways Evident, that the Eye will Determine it's Reaction to the ſame Point; Eſpecially, ſince at ſeveral Diſtances of an Object, without any Refraction, the Eye is ſuppoſed to React to different Points.

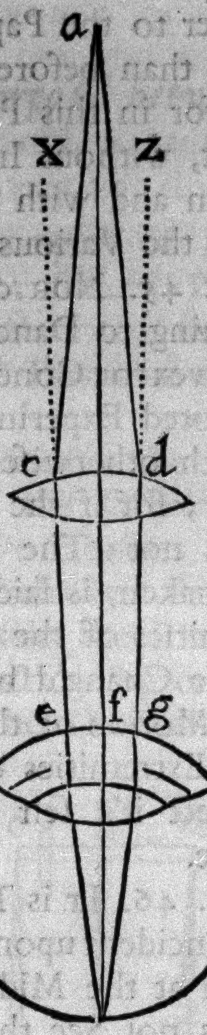


Concerning the Place of an Object Expos'd to a Convex Glaſs in it's Focus, &c. Molyneux's Dioptric. Prop. 31. Par. 1. §. 8. 9. 10.

§. 40. 1*ſt*. The next Solution, which is offer'd to us, is that of the Place of an Object, Expos'd to a Convex Glaſs in it's Focus, being Undetermin'd; For if the Radiating Point *a* is Placed (*Fig.* §. 40.) in the Focus of the Convex *c d*, the Rays *c e*, *d g*, after paſſing the Glaſs, run Parallel, and being Produced backwards to *c x*, *d z*, will not any where Converge to a Point, and therefore the Place of the Object will be at a Vaſt and Prodigious Diſtance from us, or elſe be wholly Unaſcertain'd.

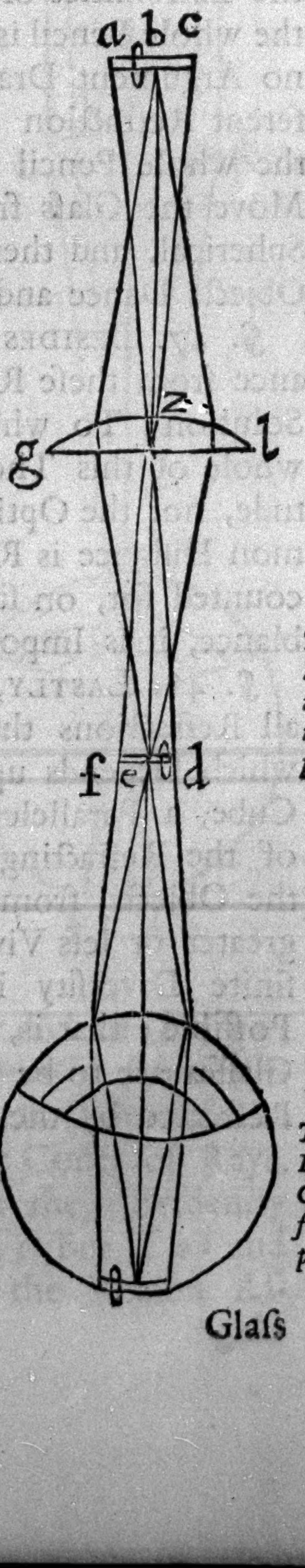
2*d*. A like want of Determination of Place is Aſcrib'd to an Object, Poſited beyond the Focus of a Convex Glaſs, when Seen by the Eye, Situate between the Glaſs and Diſtinct Baſe; *a* (*Fig.* §. 41.) is the Radiating Point farther Remov'd from the Glaſs than the Focus of it, the Rays *a c*, *a d*, after Refraction Converge, let *e f g* therefore Receive them, before they Aſſemble to a Point, and Conſequently *e c*, *g d*, being Converging to the Eye, in their Production backwards will Retire from each other to *c x*, *d z*, by which Means no Imaginary Focus can be Form'd, which is here ſuppos'd Neceſſary to Determine the Place of the Object.

3*d*. If an Object be more Diſtant from a Convex Glaſs than it's Focus, and the Eye beyond the Diſtinct Baſe, the Apparent Place of the Object is in the Diſtinct Baſe; the Object *a b c* (*Fig.* §. 42.) is Projected by the Glaſs *g z l* to the Diſtinct Baſe *d e f*, the Place of each Point in the Object is in the Correſpondent Point of the Image in the Diſtinct Baſe, thus *a* Appears at *d*, *b* at *e*, and *c* at *f*.



The firſt and ſecond Articles Contradicted by Experiment according to the Confeſſions of Dr. Barrow. Molyneux's Dioptricks Prop. 31. Par. 1. §. 9. Rohault Phys. Cap. 33. Par. 1. Dr. Clark's Annotat. §. 7. Dr. Barrow. Lect. Optic. 18.

§. 41. As to the firſt and ſecond Articles it is Acknowledg'd, there are Difficulties lie againſt them, which are Unanſwerable; for, whereas the leſs Rays Diverge in their Incidence upon the Eye, the Remoter the Object will Appear, and where the Rays Fall Parallel upon it, the Object is Judg'd to be Extremely Remote, or it's Place altogether Undetermin'd according to this Doctrine, and ſtill more ſo, if the Rays are Converging, the Contrary of all this is found in Experience, or at leaſt nothing is Apparent in Nature Agreeable to ſuch a Reaſoning; for (*Fig.* 2*d*.) *a*, a Luminous Object, is Apprehended in Fact to be Variouſly Diſtant according to the Various Situations of the Eye between the Glaſs and Diſtinct Baſe, and ſcarcely ever, if at all, is Seen more Diſtant, than it would Appear to the Naked Eye, and ſometimes much Nigher; or rather, the greater the Convergence of the Rays is, the Object in Proportion Approaches; ſo that, if the Eye is Placed, as Nigh the Glaſs, as can be, the Luminous Object Appears in it's Natural Poſition; if it is farther Remov'd from the Glaſs towards the Diſtinct Baſe, the Point *a* Obſerves a leſs Diſtance from the Glaſs; till at laſt, the Eye being Placed in a certain Station, as at the Diſtinct Baſe, the Luminous Object, or Point *a*, Appears very near, when it begins to Vaniſh away in mere Confuſion; which is a Sufficient Confutation of *Cas.* 1. and 2. in the foregoing Section;



This kind of Reaſoning Contradicted farther by Experience.

§. 42. On the other Hand, if the Glaſs is held, as near to the Luminous Object, as is Poſſible, the Diſtance will be no other, than what it Appears in to the Naked Eye, tho' the Rays then are moſt Diverging; and as the Glaſs is carry'd from thence towards the Eye, the Diſtance will Decreaſe, till it comes to about a middle Station betwixt both, and then will Increaſe, and at the Eye is the ſame, as before at the Object; ſo that, when the Rays are ſuppos'd to be the moſt Converging, and the moſt Diverging, the Diſtance will Appear the ſame.

This kind of Reaſoning Contradicted farther by Experienſe.

§. 43. Farther, when the Glaſs is Placed next to the Eye, and is kept in the ſame Poſition, whilſt the Eye Removes from it, or is Placed next the Object, and the Luminous Object is Remov'd from it, in both Caſes the Diſtance is leſſen'd, and the Magnitude of the Object is Continually Increaſed, till it is too Large for the Glaſs to Comprehend it, and the Diſtance is none at all, but the Light, as ſuppoſing that of a Lamp, or of a Candle, is Diffus'd upon the Body of the Glaſs it's ſelf, by which it's ſelf becomes one Bright and Luminous Object; And this does not happen with any Reſpect to the Focus, as *Cas.* 3. Affirms, but as the Proportion lies betwixt the Magnitude of the Glaſs and the Object.

This kind of Reaſoning Contradicted farther by Experience.

§. 44. Besides, if, inſtead of a Bright and Reſplendent Object, we take a Darker and more Obſcure one, as the Letters we now Write, and we lay the Glaſs upon them, they will Appear at the ſame Diſtance nearly, and of the ſame Magnitude, as others, which are not Refracted by it, but if, on the other Hand, the

Glaſs is made to Approach the Eye, as near as Poſſible, they become into one White Proſpect Intermixt with a Confuſion of Black; and if the Glaſs and Eye are brought nearer to the Paper, the Letters will be of ſomething a greater Magnitude and Diſtance than before, when it was laid on the Paper; nothing of which can be Accounted for in this Philoſophy, if we only Conſider the Refractions of Rays in the Abſtract, without Inquireing into the Nature of the Objects, which Emit them, in Relation and with Reſpect to the Glaſs, which Refracts them, that is, unleſs we Conſider the Various Expanſive and Contractive Forces of each.

Concerning Objects ſeeming to Dance and Move, when Seen thro' Spherical Glaſſes. Molyneux's Dioptricks Prop. 31. Schol.

§. 45. NOR does this Philoſophy better Explain that Common Effect of Objects ſeeming to Dance and Move, when they are ſeen thro' Spherical Glaſſes, whether Convex or Concave, Nimbly ſhaken between the Eye and the Object; 'Tis, it ſeems, a Noted Experiment, by which a Glaſs, when Form'd upon a large Sphere, and which otherwiſe cannot be eaſily Diſcover'd, is known, whether it is Plane or Spherical; for if the Latter, the abovemention'd Phænomenon is Obſervable, if the Former, not; The Reaſon of which Deſultory Motion in the Object, when the Sphere is Shaken, is ſaid to be this; That the Rays, being Refracted more towards the Extremities of the Glaſs than towards the Middle, the Apparent Place of the Object is more Chang'd by Refraction in the firſt Caſe, than in the laſt; ſo that the Object by the Motion of the Glaſs, Appearing ſometimes thro' the Middle, and ſometimes thro' the Extremities of it, the Apparent Place of the Object is Vary'd, and therefore the Object it's ſelf ſeems to Move, but in Plane Glaſſes the Refractions are the ſame.

The Danceing of Objects Seen thro' Spherical Glaſſes not Explain'd.

§. 46. IT is True, if this Doctrine of Opticks is Certain, that the Rays, which are Incident upon the Spherical Glaſs, are Refracted more, or leſs, towards the Sides of it than at the Middle, but it is likewiſe as True, upon ſuch a Conceſſion, that the Eye does not See the Object by the Rays, which are Refracted at the Sides, in Diſtinction from thoſe, which are Refracted in the Middle; ſince the whole Pencil of Rays from one Point in the Object, ſome of which are Refracted at the Middle, and others at the Extremities of the Glaſs, and at the Intermediate Diſtances betwixt them, I ſay, the whole Pencil is Requir'd to Produce Viſion in us, and Conſequently there can be no Argument Drawn for a different Perception of Place or Diſtance from the different Refraction of the Rays of the ſame Pencil; on the other Hand, if we take the whole Pencil or Cone of Rays, there will be different Refractions, when we Move the Glaſs from one Pencil of Rays to another, in a Plane one, as well as a Spherical, and therefore not only Spherical, but Plane Glaſſes, ſhould likewiſe make Objects Dance and Move, which will Deſtroy the Grand Criterion betwixt them.

The Solution of Refracted Diſtance ſhould have ſome Analogy with that of common Diſtance.

§. 47. BESIDES which, all, that has been ſaid againſt the Determination of Diſtance from theſe Refractions of Rays thro' a Convex, will be Equally Valid againſt this Solution; To which we ſhall Add, that an Imaginary Focus is that, into which the whole of this Theory of Refracted Diſtance is Reſolv'd, which is neither Magnitude, nor the Optick Angle, nor the Interpoſition of Objects, *&c.* Into which common Diſtance is Reſolv'd; But, why Common and Refracted Diſtance ſhould be Accounted for, on ſuch Principles, betwixt which there is no Connexion or Reſemblance, it is Impoſſible to Affirm.

An Imaginary Focus not always to be had in Refractions, where yet there is a Refracted Diſtance.

§. 48. LASTLY, an Imaginary Focus cannot be had in all Refractions, and yet in all Refractions there is a Diſtance, which Varies according to the Nature of them, which Depends upon the Figure of the Glaſs, whether a Priſm, a Lens, Globe, Cube, a Parallelepipid, *&c.* Or on the Conſiſtency and Temper of it, in Reſpect of the Refracting Medium, and in Reſpect of the Rays, which are Refracted, upon the Objects, from which they Flow, as the Sun, Common Light, Colours, and the greater or leſs Vivacity of the Actions of Bodies in General, all which Cauſe an Infinite Diverſity in Refractions without any Imaginary Focus ſuppos'd, or even Poſſible; that is, the Expanſive and Contractive Forces of the Objects and of the Glaſſes are to be Conſidered, and not the Abſtracted Rays Emitted from the one, or Refracted by the other.

CHAP. XI.

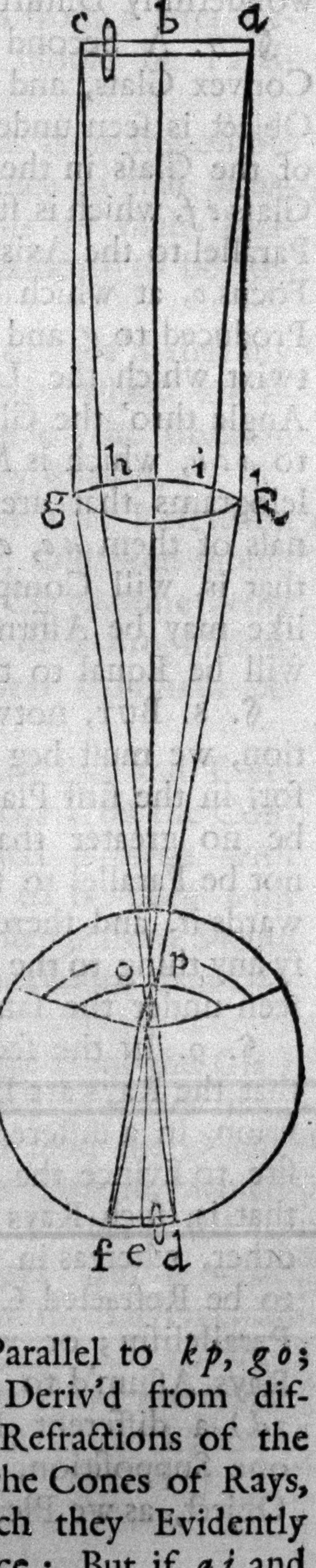
Concerning other Propoſitions, which Relate to Convex Glaſſes, Interpos'd betwixt the Object and the Eye.

§. 1. THE Preſent Doctrine of Opticks having Explain'd the Principles of Refracted Diſtance, and the Reaſons of it, Proceeds to Form ſeveral Propoſitions Concerning it, but, which is the conſtant Miſchief of it's Solutions, without the leaſt Adherence to any of it's Hypotheſes, or even to it's laſt Favourite one of an Imaginary Focus.

The unſteady Procedure of this kind of Philoſophy.

§. 2. THE Firſt Propoſition it gives us, is, that an Object being Placed in the Focus of a Convex Glaſs, and the Eye on the other Side of the Glaſs, the Object is Seen Diſtinctly and Erect; For it's being Seen Diſtinctly, there are two Reaſons Aſſign'd; 1*ſt*. From (*Fig.* §. 40. *Pag.* 546.) becauſe *ac*, *ad* being Refracted Parallel to the Eye, they will from thence Tend to ſeveral Diſtinct Points in the Retina of it, for where Rays Fall Parallel, that will be their Law of Refraction by other Propoſitions before Alledg'd; 2*d*. From the Preſent Figure, becauſe the Chryſtalline of the Eye *op* (being in the Place, where ſome of the Rays from every Point in the Object are Mixt together) is in the Place of the greateſt Confuſion of the Rays, and therefore by a Former Propoſition the Viſion is Diſtinct in the Retina; As to the Object's being Erect, it is thus likewiſe Prov'd from this Figure, *c*, *b*, and *a*, each of them, Emits a Cone of Rays, which Spreads it's ſelf on the Glaſs *gk*, of which *gch*, *iak* (which are but Parts of thoſe Cones) are only Conſider'd; but the Rays, which lie betwixt *i*, *k*, will be Refracted by the Eye to *f*, and thoſe, which are Incident betwixt *g*, *h*, to *d*, which therefore will Paint the Object Inverted in the Eye, and Cauſe it to be Seen Erect in *cba*. *Q. E. D.*

The Reaſons given why an Object being Placed in the Focus of a Convex Glaſs and the Eye on the other Side of the Glaſs is Seen Diſtinctly and Erect. Molyneux's Dioptricks Prop. 32. Par. 1.



§. 3. As to what Concerns the Propoſitions Referr'd to, we have before Examin'd into them, and have at leaſt ſhewn the Precariouſneſs of them; we ſhall therefore now only Inquire into the Validity of the Reaſons Aſſign'd, and the Application of thoſe Propoſitions in Order to ſtrengthen them; In (*Fig.* §. 40. *Pag.* 546.) *ce* and *dg* are ſaid to be Refracted to a Point in the Retina *k*, becauſe they are Parallel; but if this Argument was a Juſt one, ſince, after their Refraction in the Lens *cd*, they are ſuppos'd to Continue Parallel, without any farther Alteration in their Direction, it would Follow, that, wherever the Eye was Placed, whether nearer or farther off from the Glaſs, the Object would Appear equally Diſtinct, which is Contrary to our Plain and Manifeſt Experience; For the Object in ſeveral Diſtances from the Convex will Appear Confus'd, and at length Vaniſh, and make no Impreſſion at all upon us.

Confuted by Experience in Reſpect of Figure the Firſt Referr'd to.

§. 4. ANOTHER Reaſon given for Objects Appearing Diſtinct to the Eye, when Placed in the Focus of a Convex, is from the Preſent Figure, where the Rays *ch* and *ai* are thoſe, which are now ſuppos'd to Cauſe this Confuſion in the Chryſtalline, or the Cornea, that is, at the beginning of the Eye's Refraction, Namely ſuch, which after Refraction ſhall be Parallel to *kp*, *go*; on the Contrary, in Propoſition the 28*th*, the Rays, which were Deriv'd from different Points of the Object, and Caus'd a Confuſion in the firſt Refractions of the Eye, were the Axes of the Cones; Unleſs therefore the Axes of the Cones of Rays, which Proceed from *c* and *a* are the ſame with *ai* and *ch*, which they Evidently are not, this Demonſtration cannot be of any Force or Significance; But if *ai* and *ch* are not thoſe Rays, which are Refracted Parallel to *ak* and *cg*, the Reaſon Aſſign'd

The Reaſon Alledg'd from Figure the ſecond Confuted.

See Scheme Chap. 9. § 2. 17, *&c.*

ſign'd in the foregoing Section cannot be Valid, which ſuppoſes the Rays, that Fall on the Eye, to be Parallel.

The Contrivance of this Philoſophy Evident but not the Reaſon of it.

§. 5. We muſt likewiſe Obſerve, that in this Figure the Confuſion of the Rays is in the Cornea, and which Divaricate upon the Chryſtalline, whereas by Propoſition the 28*th.* they meet in the Chryſtalline, and Diverge at the Cornea, which difference in the Incidence muſt Manifeſtly Cauſe a difference in their being Diſtinctly Refracted; But the Contrivance is Evident, Namely, to Preſerve the Paralleliſm of the Rays after their Refraction thro' the Convex, that it may Agree with what is Affirm'd in (*Fig. §. 40. Pag. 546.*) and at the ſame time to Aſcertain the Confuſion of Rays from all the Points of the Object, which, by the 28*th.* Propoſition, is thought to be Neceſſary to their Juſt and Punctual Refraction to the Retina.

Another Objection againſt this way of Reaſoning.

§. 6. The next Thing Aſſerted in this Demonſtration is, that the Object Appears Erect, but why it ſhould ſo, is not Evident by any very Flagrant Proof, and certainly, not from an Arbitrary Aſſumption of one Part of the whole Cone of Rays, which is here done; For ſince it is Acknowledg'd, that from *a* and *c*, the Extreme Points, the Rays are Diſperſt, as from a Center, upon the whole Surface of the Convex, it will be Impoſſible to Aſſign a Reaſon, why *gch*, or *iak*, ſhould rather make an Impreſſion upon the Retina of the Eye at *d* and *f*, than the Other, or Remaining, part of the Cones, *hck*, *iag*, which yet, if taken into the Account, would wonderfully Diſturb the preſent Reaſoning.

Concerning an Object being Placed in the Focus of a Convex Glaſs, and the Eye in the Focus on the other Side of it. Molyneux Dioptricks Prop. 33. Part. 1.

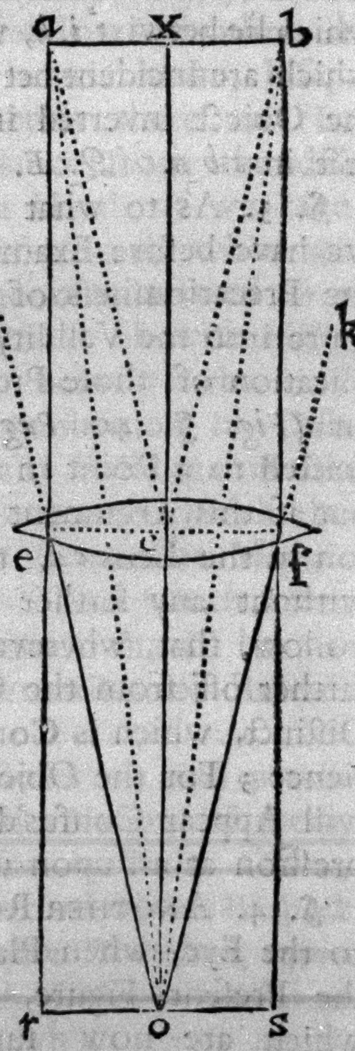
§. 7. A Second Propoſition is, that an Object being Placed in the Focus of a Convex Glaſs, and the Eye being Placed in the Focus on the other ſide of it, the Object is ſeen under the ſame Angle, as if the Naked Eye was Placed at the Station of the Glaſs in the preſent Figure; *axb* is an Object Placed in the Focus of the Glaſs *ef*, which is ſuppos'd to be of the leaſt Thickneſs Imaginable; The Rays *ae*, *bf*, Run Parallel to the Axis *xco*, and therefore are United in the Focus *o*, at which Point the Eye is Situate; *oe*, *of*, are Produced to *g* and *k*, and *ecf* is Parallel to *axb*, betwixt which the Lines *ac*, *bc*, are Drawn; The Optick Angle thro' the Glaſs is *eof*, and is to be Prov'd Equal to *acb*, which is Manifeſt; ſince *axce*, *ecro*, are Parallelograms that are Equal, and Conſequently, the Diagonals of them *ac*, *eo* will be Equally Inclin'd to *xc*, *co*, that is, will Comprehend the ſame Angle, and ſince the like may be Affirm'd of *xbcf*, *cfſo*, the Angle *acb* will be Equal to the Angle *eof*. *Q. E. D.*

This Reaſoning not True, becauſe it does not obtain in General.

§. 8. But, notwithſtanding this Infallible Demonſtration, we muſt beg Leave to ſay, there is nothing in it; for, in the firſt Place, the Object *ab* muſt be ſuppos'd to be no greater than the Glaſs, otherwiſe *ae*, *bf*, could not be Parallel to the Axis *xc*, but would Converge towards it, and therefore this Demonſtration cannot ſignify any thing to the Explaining of Objects, in General, being ſeen under the ſame Angle *eof*, or *acb*.

Another Objection againſt this Reaſoning.

§. 9. In the ſecond Place, it is Obvious to Obſerve, that the Rays are Diſpos'd, in Order to Prove this Propoſition, in a different Manner, from what is found Requiſite to Evince the former; there, only *aik* is Conſider'd, that is, ſuch Rays as may be Refracted Parallel to each other, whereas in the Preſent Figure, *bf*, *ae*, are ſuppos'd to be Refracted Converging to the Eye, without any ſuch Paralleliſm; or, which is the ſame, in (*Fig. 1ſt.*) there is one Portion of the Cone of Rays Aſſum'd to Evidence the former Propoſition, to which it belongs, and in (*Fig. 2d.*) a different Portion to Evince the laſt, which is not Reconcileable, unleſs upon one Suppoſition, that we may uſe the Cone of Rays, which is Emitted from any Object, as we Pleaſe, which is Abſurd.

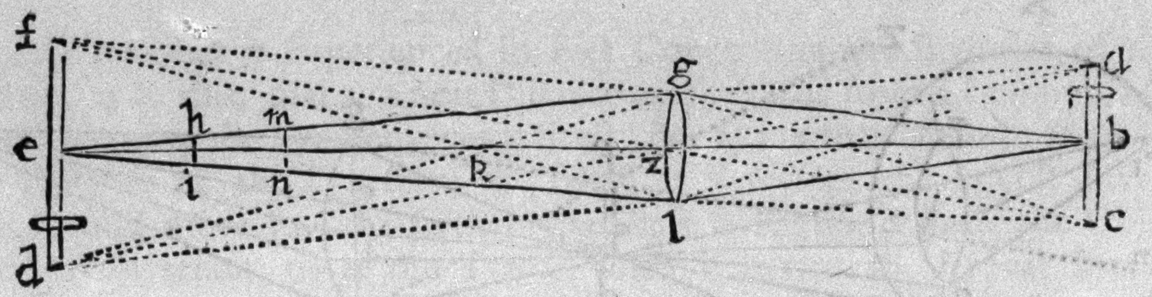


A third Objection againſt this Reaſonings.

§. 10. Besides what we have ſaid, *ecf* is ſuppos'd Parallel to *axb*, that is, that the Glaſs is of an Infinitely Small Thickneſs, or that the Curve of the Double Convex, and the ſtrait Line Drawn betwixt them, are the ſame; if ſo, there can be no Imaginable difference betwixt a Plane, and a Convex Glaſs, and the ſame Demonſtration, which is here made Uſe of, will be Valid in the one, as well as the other, which is not Conſiſtent with this Philoſophy; ſince, according to ſuch a Reaſoning, the Convex *ecf* will have no different Properties from a Plain Parallelogram.

Concerning an Object Placed more Diſtant from a Convex Glaſs than it's Focus and the Eye Placed nigher the Glaſs than the Diſtinct Baſe. Molyneux Dioptricks Prop. 36. Part 1.

§. 11. A Third Propoſition is, that an Object being Placed more Diſtant from a Convex Glaſs than it's Focus, and the Eye being Placed on the other ſide of it, nigher to the Glaſs than the Diſtinct Baſe, the Object is ſeen Erect and Confus'd; *abc* is an Object Refracted to the Diſtinct Baſe *def*, and *k* is the Eye; 1*ſt*. the Object Appears Erect, because the Rays from *a* Tend, after their Tranſmiſſion thro' the Convex *gl*, to *d*, and thoſe from *c*, to *f*; 2*d*. It Appears Confus'd, becauſe the Rays from *b* after Refraction are Converging, whereas to make Viſion Diſtinct, they ſhould be Parallel. *Q. E. D.*



Firſt Reply to the Preceding Demonſtration.

§. 12. To which Demonſtration, as it is Term'd, we Reply, that it is Impoſſible the Eye in *k* ſhould Perceive the Object otherwiſe than Diſtinct, ſince in that Situation there will be a perfect Confuſion of the Rays from all Points in their Incidence on the Cornea, or the Chryſtalline, which before has been Affirm'd to be Requiſite to a Clear and Diſtinct Viſion.

Second Reply.

§. 13. If we now Remove the Eye to *mn*, or *hi*, it is not Certain, or Prov'd, that any Rays are Incident upon it, Excepting thoſe from *b*, but how the Rays, which are Emitted from one Point of an Object, ſhould give us an Impreſſion of the Erection or Inverſion of the whole, cannot be Solv'd by any Method, or any Reaſon.

Third Reply.

§. 14. As to the Object's Appearing Confus'd, becauſe the Rays after Refraction are Converging; for the ſame Reaſon, according to this Philoſophy, Old Men ſhould not be Aſſiſted in their Viſion by Convexes; If it is ſaid, their Chryſtalline being Flatter wants them, that can be no Objection to a Confutation of a Propoſition, which in it's Demonſtration in General Aſſerts, that Objects Appear Confus'd, becauſe the Rays are Converging upon the Eye.

Fourth Reply.

§. 15. But that, which Overthrows all this Reaſoning, is, that in Fact and Experience Objects do not Appear ſo Confus'd, as this Doctrine Repreſents; on the Contrary, when the Eye is Placed the neareſt to the Glaſs *gl*, the Object is Seen with the greateſt Diſtinction, when by the Courſe of the Rays it ought to be Seen with the leaſt; and as the Eye removes farther off, by § 13. It will be ſo far from Seeing the Object Confuſedly, or Diſtinctly, that it is not Prov'd, it will See more than one Point of it.

Concerning an Object being more Diſtant from the Glaſs than it's Focus and the Eye Placed in the Diſtinct Baſe. Molyneux's Dioptric. Prop. 38.

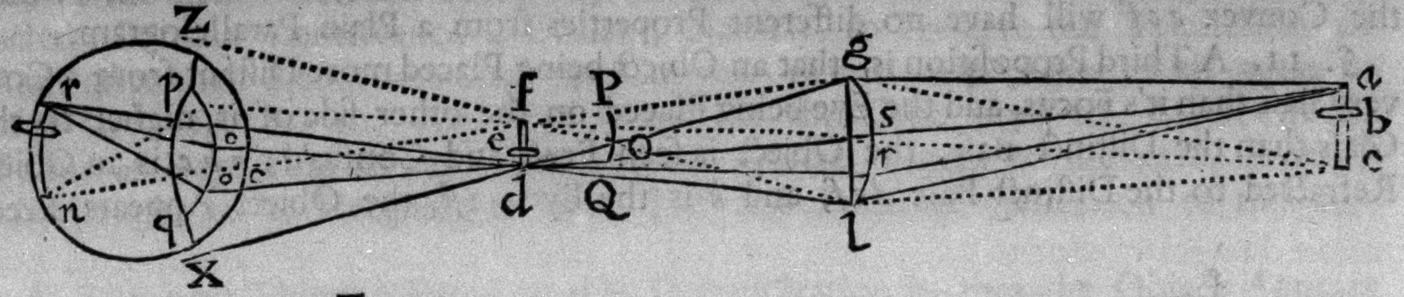
§. 16. A Fourth Propoſition is, that an Object being more Diſtant from the Glaſs than it's Focus, and the Eye Placed in the Diſtinct Baſe, the Object Appears moſt Confus'd; Thus in the foregoing Figure, when the Eye is Placed in *fed*, the Object in *abc* will ſeem more Confus'd, than if the Eye is in any other Poſture, which has been Conſider'd in §. 13. And does not Require any farther Anſwer; that is, the Space, which the Eye takes up, is to be Determin'd, before we can Affirm what Rays will Fall upon it, and what will not, as alſo the Compaſs of the Divergence of each Pencil of Rays.

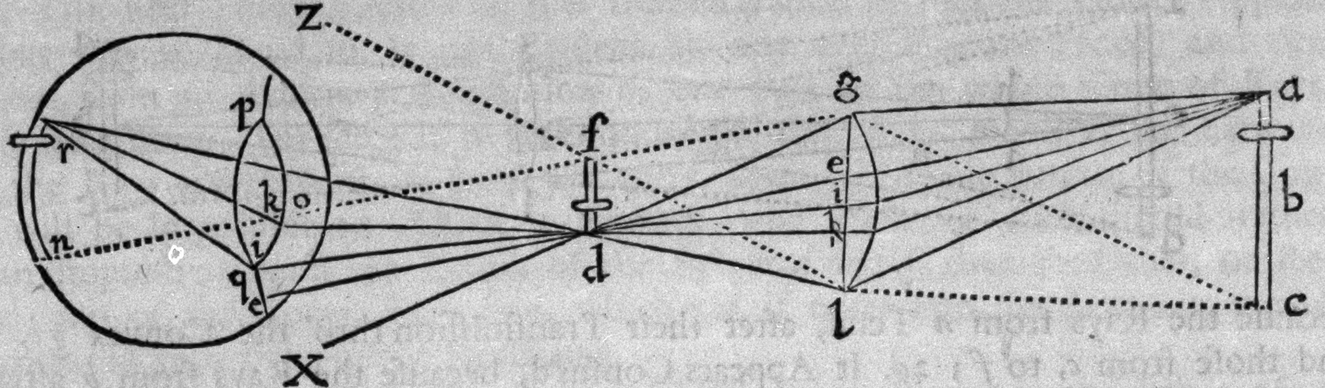
Concerning an Object being more Diſtant from the Glaſs than the Focus and the Eye farther from the Glaſs than the Diſtinct Baſe.

§. 17. A Fifth Propoſition is, that, the Object being more Diſtant from the Glaſs than the Focus, and the Eye farther from the Glaſs than the Diſtinct Baſe, the Object begins to be Perceiv'd Inverted, and, at a Proper Diſtance, Diſtinct. This is Prov'd from the Preſent Figure the Firſt, *abc* is the Object, *gsl* the Convex, the Diſtinct Baſe *fed*, *o* the Place of the Eye; from *a* the Cone of Rays *agl* is Emitted, but ſince half of this Cone is ſuppos'd to Diverge to *qdx*, which does not Fall upon the Eye, the

Molyneux, Dioptricks Prop. 39 §. 1. 2. 3.

the other half *a s l* is only Conſider'd, which does; *a s l* therefore being Refracted to *o q* on the Chryſtalline of the Eye, and *c g s* to *p o*, they will be Refracted to the Retina Inverted, in Reſpect of the Diſtinct Baſe *f e d*; and Conſequently, ſince the Object *a b c* (when Erect, and is Painted upon the Eye Inverted) Appears Erect, Seeing the Diſtinct Baſe is an Inverſion of the Object, and that is again Inverted in it's Impreſſion upon the Eye, it ought to Appear Inverted; and as the Object in ſuch Circumſtances is ſaid to be Seen in a contrary Poſition to it's Natural, ſo there will likewiſe be a Diſtinction in it's Appearance, if the Eye is ſo far Remov'd from the Diſtinct Baſe, as to be able to Correct the Divergence of the Rays, that come to it from each Point in the Diſtinct Baſe, and to Form them into Correſpondent Cones, which Determine their Apices on the Retina.





The First Objection againſt this Demonſtration.

§. 18. THE firſt Thing we ſhall Obſerve upon the foregoing Section is, that half the Cone of Rays is only made Uſe of in the Preſent Demonſtration, the Conſequence of which muſt be; either that *a* is not fully Expreſſed in *d*, and therefore not in *r*, which may be ſaid of all the other Points in the Object, and if ſo, that the Object is only Partially Repreſented to the Eye, when it is ſuppos'd to be Wholly Impreſſed upon it, and Confuſedly, when the Repreſentation is Imagin'd to be Diſtinct; or elſe, half a Cone of Rays will make the ſame Impreſſion upon the Eye as a whole one would, that is, in Reallity half is Equal to the whole, which is Impoſſible and Abſurd.

The Second Objection.

§. 19. NOR does the Account, which is here given us of the Inverſion of the Object, carry the utmoſt Evidence with it, which we could Deſire, for if the Eye was Placed betwixt the Convex and Diſtinct Baſe, ſome where in the half Cone of Rays *s d l*, ſince it is a Rule in this Philoſophy, that the Eye Purſues the Rays back according to the Direction of the laſt Refraction, *s d l* will be return'd to *c*, and *f s g* to *a*, that is, *a* will Appear in *c*, and *c* in *a*, and Conſequently the whole Object in a Poſture, which is the Reverſe of it's True one.

The Third Objection.

§. 20. Suppoſing yet, that the Eye in *s d l* would See the Object in it's true Poſition, it is not Neceſſary to Infer, that beyond *d* it will Apprehend it in a Contrary Situation, becauſe *d o q* is in a Contrary one to *d s l*, ſince *l d o*, *s d q* are Lines Continued from the laſt Refraction in the Convex *g l*, and by the foregoing Rule the Rays are Purſu'd back to the Object according to the Preſcription of the Refraction in *g l*, from whence it will follow, that if the Object is Seen Erect in *s d l*, it ought to be ſo in *o d q*.

The Third Objection farther Explain'd.

§. 21. If not, it muſt be upon one of theſe two Accounts; either becauſe *s d* and *d l* are in an Inverted Poſition to *d q*, and *d o*, that is, *s d q*, *l d o* Croſs each other in *d*; or elſe, becauſe *s d l*, when the Eye is Poſited in it, is Refracted towards *n*, and *g f s* towards *r*, whereas, when it is Poſited in *o d q*, it is Refracted towards *r*, and *g f s* towards *n*; As to the firſt, it is very Manifeſt, that the Croſſing of the Particular Rays, which Flow from one and the ſame Point of the Object can Signify nothing to the Altering the Poſition of the Object, ſince no one Affirms, or can with any Reaſon do it, that the Point *a* or *c* it ſelf is Inverted; for Suppoſing the Points *a*, *b*, and *c*

Inverted, that would not Cauſe ſuch an Inverſion of the Object, as is here Alledg'd and Maintain'd, Namely, that the Line, which Croſſes *bo*, ſhould be Transferr'd to the ſame Diſtance at *c* from *b*, as it is at *a* from *b*, in it's Natural Poſture; Nor, as to the Second, is it Reaſonable to Grant, that the Rays *ſal*, *odq*, are Refracted to Contrary Situations in the Retina; all the Difference, which can be Juſtly Imagin'd betwixt the Refractions of them in the Eye, is only this, that the one are Refracted by it Converging, the other Diverging, which would Produce one Effect indeed according to this Philoſophy, that, the Chryſtalline Continuing the ſame, if the Rays in *sdl* were Refracted to *n* with a greater Exactneſs, thoſe in *odq* would be with a leſs, and on the Contrary, if the Rays in *sdl* were Refracted to *n* with a leſs Accuracy, in *odq* they would be with a greater, Suitable to the Doctrine, which is Affirm'd, in this Philoſophy, of a Lens Aſſiſting the Sight of ſome Men, and a Concave of others, where the Rays are made to Converge, or to Divaricate; But what is this to Inverſion?

What is Matter of Fact and Experience in Reſpect of this Propoſition.

§. 22. As to what is Experienced in Fact Concerning the Inverſion of the Object by a Convex, it is this, that at a near Diſtance of the Glaſs from a Luminous Object, as a Lamp or a Candle, the Eye Perceives it Erect, but at a farther Diſtance from it, and at the ſame Diſtance of the Eye from the Glaſs, it Perceives the Object Inverted and Diminiſhed, and more ſo, the farther the Glaſs is Remov'd from the Object; tho' upon the firſt Inverſion, or at the Time, when it begins to Appear Inverted, it is much Larger than the Object it's ſelf; If the Diſtance of the Glaſs from the Object is the ſame, and the Poſition of the Eye is Alter'd, the Object will likewiſe Appear at firſt Erect, and Larger, then Larger, and Inverted, as we Remove farther from the Glaſs, afterwards Inverted and Diminiſhed, then, as we Remove to a greater Diſtance, it will Appear on the ſame Side of the Glaſs with the Eye, at ſome Interval from the Glaſs; and as we go ſtill farther from the Glaſs, will ſeem to Flame in it.

These Experiments not Explain'd by the Preſent Opticks.

§. 23. This Philoſophy on the other Hand Suppoſes, that the Diſtance betwixt the Eye and the Glaſs is very different, when an Object is Seen Erect, and when Inverted; in the firſt Caſe, it is Requir'd, that the Eye ſhould be Placed ſomewhere in *gdl*, and in the laſt, in *odq*, whereas the Erection and Inverſion of the Object is Perceiv'd at the ſame Diſtance of the Glaſs from the Eye, that is, when there is a different Diſtance of the Object from the Glaſs; If it be therefore ſaid, that by the Increaſe of the Object's Diſtance, the Rays become more Parallel in their Incidence, and Conſequently are ſooner Refracted, by which Means an Object, which Appears Erect in *sdl* may by a farther Remove of it from the Glaſs ſeem Inverted in the ſame *sdl* by the Croſſing of the Rays before they Arrive at *d*; we Anſwer, *1ſt.* That the Focus of Rays Emitted from a Nigh Object, by this Philoſophy, is Suppos'd to a Demonſtration to be very nearly the ſame, whether ſuch an Object be leſs or farther Remov'd from the Glaſs; *2d.* That, the Rays, which Fall from *a* or *c*, do not Approach nearer to a Parallel by the Removal of the Object from the Glaſs, unleſs upon a Suppoſition, which has been already Confuted, that the Rays, which Flow from a Point, do not keep the ſame Station from each other, at whatever Diſtance they are Emitted, which they ought to do, Excepting they are Conceiv'd to be ſelf-Moving, and to Dilate and Contract themſelves, as Occaſion ſhall Serve; that is, they will be always Equally Diverging from the ſame Point, when the Glaſs is Placed in any one Part of the ſame Circumference from ſuch a Point, as the Center; and if the Circumference is Drawn, at a greater Diſtance, where the Glaſs is now Suppos'd to be, they will be ſo far from growing Parallel by it, that they will Diverge the more; *3d.* If the Rays *sal* could be Imagin'd to grow more and more Parallel, by which Means they would be Refracted ſooner, yet that they ſhould Exactly Repreſent the Diſtinct Inverſion of the Object at one and the ſame Diſtance is Unaccountable, in what Manner ſoever the Eye and Glaſs, Preſerving the ſame Diſtance in Reſpect of each other, are Remov'd from it; Laſtly it is Obſerv'd, that the Object is very much Diminiſh'd in this Inverſion of it, which is likewiſe not Solv'd, ſince Magnitude is ſaid to Depend upon the Angle, made by the Axes of the Rays Meeting on the Eye, whereas there are no Axes Conſider'd at all in theſe Partial Cones, which are Alledg'd, or if there were, the ſooner the Rays were Refracted, the greater would the Angle of the Axes be, and Conſequently the Object would be Increaſed in it's Appearance.

It's own Experiments not Solv'd and Explain'd.

§. 24. It is no Wonder however, that theſe Opticks do not Explain thoſe Phænomenons, which they do not ſeem to have known, ſince they do not Endeavour to Account for the Inverſion of the Object, where the Diſtance of the Eye and the Glaſs is the ſame, and the Diſtance of the Object and Glaſs different, but, on the Contrary, where the Diſtance of the Eye and Glaſs is different, and that of the Glaſs and Object the ſame; And yet it does not give us a Solution even of this; for 1*ſt.* In ſuch a Circumſtance of the Eye, the Glaſs, and the Object, the Object will Appear Larger than it's ſelf and Erect, and at a little farther Diſtance of the Eye from the Glaſs, Larger than it's ſelf, and, Inverted, and after that, Inverted and leſs than the Object, but here we are only Inform'd of the Inverſion, without giving any Account of it's Increaſe or Diminution, which for the Reaſons Affirm'd in the foregoing Section cannot be done; 2*d.* After this, by a farther Removal of the Eye from the Glaſs, if it is a Luminous Object, it will Appear like a Flame Darting from the Convex, and, in the Proportion of this Figure, betwixt *gl*, *fd*, ſo that the Eye ſhall be much farther Remov'd from the Flame, than the Flame is from the Glaſs, but which way this Phænomenon can be Explain'd, is Inconceivable, if we only Conſider the Refractions of *sal*, or *gcs*, the Rays Croſſing each other in *d* and *f*, their being Refracted again in the Eye to *r* and *n*, and from thence Referr'd backwards in the Incidence, in which they Fell, which are the ſtanding Principles of this Philoſophy, and no other are at Preſent Introduced; And for the ſame Reaſon it is Unaccountable, how the Luminous Object ſhould ſeem, at a farther Diſtance of the Eye, to Flame in the Glaſs it's ſelf, and, at ſtill a greater Diſtance of it, to Appear beyond it.

The Diſtinction of the Object not Explain'd. Molyneux Dioptricks, Prop. 39. Part I. §. 4.

§. 25. Moreover, in Reſpect of the Diſtinction of the Object, it is ſaid, That is Obtain'd, when the Eye is ſo far Remov'd from the Diſtinct Baſe *fed*, as to be able to Collect the Rays to Certain and Determin'd Points in *r* and *n*, which, as has been before Prov'd, is not in very many Diſtances of the Eye and the Object, or, as here, of the Diſtinct Baſe, which is Conſider'd as an Object, whereas almoſt in any Poſition of the Eye to the ſaid Baſe, where the Diſtance is not extremely Great, the Clearneſs and Diſtinction of the Object is Suppos'd by this Demonſtration to be, and in Fact and Experience is, fully Preſerv'd.

Concerning the Object's Moving upwards and downwards in a Convex. Molyneux Dioptr. Prop. 39. Part I. §. 5.

§. 26. Another Phænomenon, which this Figure Propoſes to Solve, is, that if the Eye be Mov'd upwards towards *z*, (*Fig.* 1. and 2. §. 17.) the Object ſhall Appear to Move downwards, and if the Eye be Mov'd downwards towards *x*, the Object ſeems to Move upwards; To Explain which, the Point *c* is Conſider'd, and how the Appearance is Brought to and Form'd in the Eye, which is thus; the Rays Refracted from *c* are *gcl*, which Proceed to *f*, and afterwards to *oz*, half of which are only Suppos'd to Fall on the Eye in the Preſent Situation of it, Namely *pfo*, and *zfp* to Eſcape it; if therefore the Eye is Mov'd higher towards *z*, it then Receives the Rays *zfp*, and the Point, from whence theſe Rays Flow, will Conſequently be Perceiv'd in a lower Situation than *c*; thus alſo, if the Eye be Mov'd downwards to *x*, *a* will Appear in a higher Situation for a like Reaſon; and by this Means it is Thought, that the Locus Apparens of the Object is Evidently Proved to be in the Diſtinct Baſe, as was before Alledg'd in a foregoing Propoſition, for by Raiſing the Eye, the Point *f* is Depreſſed, and by Depreſſing the Eye, the Point *f* is Rais'd, juſt as *c* in the Object ſeems to Move Contrary to the Motion of the Eye; From hence it is likewiſe ſaid to follow, what we in Reality Experience, that if the Glaſs be Mov'd upwards, the Object Appears to Move upwards, and if the Glaſs be Mov'd downwards, the Object Appears to Move downwards, for Moving the Glaſs upwards is the ſame, as Moving the Eye downwards, and therefore the ſame Effect of the Object's Appearing to Move upwards will be Produced from either; Or, ſince the Diſtinct Baſe always Follows the Glaſs, as the Glaſs Moves upwards, the Baſe will too, and Conſequently the Object, which is here Repreſented by it.

Molyneux Dioptric. Prop. 31. §. 10.

The firſt Objection againſt this Account.

§. 27. But, if as the Eye Mov'd upwards towards *z*, *c* would Appear to Move downwards, for the ſame Reaſon *a* would not be Seen at all, becauſe, as the Eye Receives the Rays from *c* in *z*, which before Eſcap'd it, ſo *do* being the utmoſt of the Refracted Rays from *a*, there will be none above it, which, Falling upon the Eye, ſhould Repreſent *a* or *d* in a lower Situation than before; and the ſame may be Argued in Reſpect of the Deſcent of the Eye to *x*; ſince, in ſuch a Caſe, there would be no Rays below *fo* to make *c* Appear in a higher Station to the Eye.

A Second Objection.

§. 28. SUPPOSING this yet could be done, the Raiſing of the Eye to *z* could Signify nothing to the making the Point *f* Appear higher or lower, for if we only Conſider that Individual Point, whether it is lookt upon from *z* or from *o*, it will be the ſame, as the Center of a Circle, if View'd from any Part of the Circumference, will be; On the other Hand, if the Eye at *o* Purſues the Ray to *g*, and at *z* to *l*, in the laſt Caſe *l* indeed will Appear much lower than *g*, but then it muſt likewiſe be Affirm'd, that this can be no Argument for Seeing the Apparent Place of the Object in the Diſtinct Baſe, which is made a Conſequence of this Kind of Reaſoning, and therefore Suppoſes, that the Eye Terminates it's Reference in *f*, Contrary to the Preſent Hypotheſis, which makes the Eye Purſue the Ray to *g*.

A Third Objection.

§. 29. BESIDES which, if the Object, when the Eye is at *z*, is not Seen in the Point *f*, but in *fsl*, it will not Appear in *fl*, in Preference to *fs*, which will Terminate in *c*, and will therefore make no Alteration in the Place of it, but each Line in the Triangle will have an Equal Claim to carry back the Notice of the Eye to the Object, which Impreſſed it; if the whole Triangle *fls* is Concern'd in Returning the Action of the Eye, the Point *c* will Appear in a Line Parallel to *sl*, Drawn betwixt *fs* and *fl* Produced, and the ſame will obtain in all the other Points of the Object *abc*, from whence it is Conſequent, that *abc* will ſeem Larger and Confus'd, notwithſtanding it is Certain in Fact, and is Endeavour'd to be Prov'd by this Philoſophy, that the Object is Leſs in it's Inverſion, and is Painted on the Eye with a Peculiar and Remarkable Diſtinctneſs.

A Fourth Objection.

§. 30. FARTHER, as to the Object's Appearing in the Diſtinct Baſe, for which Propoſition 31. §. 10. is Alledg'd, and which Depends upon Principles, which have been already Confuted, it is Falſe in Experiment, nor is it Confirm'd by this Reaſoning; It is not Confirm'd by this Reaſoning, as is Evident from the foregoing Sections, and more Particularly from §. 28. and 29. and that it is Falſe in Experiment, will be Notorious to any one, who is willing to make the Trial; for, if a Luminous Object is Placed beyond the Glaſs at a Certain and Determin'd Diſtance, and the Eye is made to Retire from the Focus of the Convex, where the Object Appears Inverted, the Eye will yet be far from Perceiving the Object in the Focus; If the Eye ſtill Recedes, there is no ſuch Phænomenon Produced, but for a Conſiderable Diſtance together, the Object Appears Inverted and Diminiſhed, and on the other Side of the Glaſs, till at Length, when we are Remov'd ſeveral Yards from it, a Pointed Flame Darts out on this Side of the Glaſs towards the Eye, at about half a Yard from the Convex, which, as the Eye goes farther from it, Retires back to the Glaſs, and is at laſt Scatter'd and Diffus'd on it's Surface; from which it is Apparent; 1ſt. That there is no Expreſſion of the Object at all, either Erect or Inverted, on this Side the Glaſs, But only a Bright and Brillant Flame; 2d. That if there were, it is not in the Focus, becauſe it Appears in ſeveral Diſtances from the Glaſs, whereas, the Diſtance of the Glaſs and Object being the ſame, the Focus ſhould likewiſe Preſerve the ſame Station.

A Fifth Objection.

§. 31. LET this Reaſoning be, as Precarious as it will, it is yet ſaid, that from hence it follows, that, if the Glaſs is Mov'd upwards, the Object Moves upwards, if downwards, the Object Appears to do ſo likewiſe; becauſe it is the ſame Thing for the Glaſs to be Mov'd upwards, as for the Eye to be Mov'd downwards, or the Contrary; but we muſt take Leave to Say, that, if ſo, there is no more Reaſon to Exact the Phænomenons of the Glaſs's Moving upwards, to the Account, which is given of thoſe, that Reſult from the Eye's Moving downwards, than, on the other Hand, to Accommodate the Phænomenons of the Eye's Moving downwards to the Account of thoſe of the Glaſs's Moving upwards; that is, if, according to what is Acknowledg'd in the Preſent Opticks, the Diſtinct Baſe, or the Object, of which it is a Repreſentation, follows the Motion of the Glaſs, there is no more Reaſon, why we ſhould Aſcribe the Phænomenons Produced by the Glaſs's Moving upwards to the Poſition of the Rays, whoſe Cauſe is ſaid to be the Baſe's following the Glaſs, than why we ſhould Aſcribe the Phænomenons Produced by the Eye's Moving downwards to the Motion of the Glaſs upwards, which Relatively, in ſuch a Caſe, is

Mov'd upwards, whoſe Cauſe is yet Affirm'd to lie in the Poſition of the Rays; that is, there does not ſeem to be any Connexion betwixt the one and the other; for, if *d* is Mov'd upwards to *f*, the Eye would Purſue the Rays back in a more Direct Line than before, and Conſequently *a* would Appear in a lower Situation inſtead of a more Elevated, whereas on the other Hand, if the Eye is Mov'd downwards, it is ſaid to Purſue the Rays from *a* back to a higher Situation.

A Sixth Objection.

§. 32. Farther, the Various Poſition of the Rays is the Cauſe Inſiſted upon, by which the Phænomenons of the Eye Moving upwards or downwards are to be Explain'd; But, if the Glaſs is Mov'd upwards or downwards, the Phænomenons, Ariſing from it, are Aſſign'd to the Diſtinct Baſe, or the Object, following the Glaſs; and ſince the latter Explanation is made a Conſequence of the former, the different Poſition of the Rays muſt be a Reaſon, why the Diſtinct Baſe, or the Object, follows the Motion of the Glaſs; if therefore the Eye is Remov'd from *o* to *q*, and the Glaſs from *l* to ſome other Point above it, which ſhall be in Proportion as far Diſtant from *l*, as *q* is from *o*, the Queſtion is, whether the Angle *odq*, which is Suppos'd to make the Object Appear in one certain Poſition of *d* in the Diſtinct Baſe, or of *a* in the Object, and *qdx*, which we ſuppoſe Equal to the former, and to Repreſent the Diſtinct Baſe and the Object in a different and a higher Situation, the Queſtion is, whether either of theſe two Angles are Exactly Equal to the Angles, made by the Motion of the Glaſs according to the Conditions before Preſcrib'd, which it is Impoſſible they ſhould; for altho' in the Preſent Figure *gds* is = to *qdx*, and *odq* to *sdl*, yet if *l* is Remov'd to *s*, on the Contrary *gds* will become = to *sdl* = to *odq*, which will not be Equal to *qdx*, and therefore the Depreſſion of the Eye, and the Elevation of the Glaſs, will not Cauſe the ſame Angles, nor the ſame Appearance.

A Seventh Objection.

§. 33. It is alſo Obſervable in the Preſent Figure, that the Eye is Contriv'd to be Placed ſo, that the Depreſſion of the Eye, and the Elevation of the Glaſs, might Appear the ſame, but if the Eye is three or four Times the Diſtance from the Diſtinct Baſe, than it is Suppos'd to be in this Scheme, the Matter will Appear quite otherwiſe than is here Repreſented; for, in ſuch a Caſe, a Third, or Fourth, Part of the Cone *odq*, Namely *odc*, will be Sufficient to Fill and Poſſeſs the Space *oq*, ſince *do*, *dc*, Continually Divaricate, as the Eye Removes from *d*; if therefore at that Diſtance the Eye is Mov'd downwards to *q*, the Space it Deſcribes will be = to a Third, or Fourth, Part of *odq*, Namely *oc*, Conſequently upon the Depreſſion of the Eye to *q*, the Glaſs would be Elevated only from *l* to *r*, whereas upon the Elevation of the Glaſs to *s*, as far as the Eye was Depreſſed to *q*, the Eye in Order to Apprehend it, muſt be Depreſſed Three, or Four, Times the Space it was before, Namely to the Triangle *qdx*, or quite through the Triangle *odq*, and by the Hypotheſis *odq* is Three, or Four, Times Bigger than *odc*, which is Contrary to this Doctrine, which Affirms, that it is the the ſame; whether the Eye is Depreſſed, or the Glaſs Elevated; from whence it is Evident, that this Philoſophy of Rays is Incapable of Solving the Preſent Phænomenons. We ſhall only farther Add in this Section, that by the Eye's Moving downwards, the Object will not always Move upwards, even according to the Preſent Way of Reaſoning; for, as the Eye is carry'd from *o* to *c* in *cr*, the Object will Appear in a lower Situation than *a*, as may be more fully Seen in (*Fig. 2d.*) for there even in *kk*, *ii*, it will be Depreſſed, and will not Riſe, till the Eye comes below *ee*, which ſhews, how Uncertain this Kind of Arguing from the Poſition of the Rays is, and that the former Figure is Contriv'd on Purpoſe to Solve the Phænomenon, not Drawn to Explain it; ſince the Eye in any other Poſture will Receive the Rays differently, by which the Object will be made ſometimes to Move downwards, and ſometimes upwards, whilſt the Eye is yet all the Time Moving downwards, which will not in the leaſt Agree either with the Phænomenon it's ſelf or the Preſent Explication of it.

A Phenomenon not taken Notice of by this Philoſophy, and altogether unexplain'd by it in Reſpect of Objects Moving upwards and downwards.

§. 34. Lastly, there is another Phænomenon, which theſe Opticks do not take Notice of, and which wants to be Explain'd; that is, if the Eye is Placed betwixt the Diſtinct Baſe and the Glaſs, ſo as the Object ſhall Appear Erect, every Thing happens Contrary to what was ſaid before, that if the Eye is Mov'd upwards, the Object Moves upwards, and if the Eye is Mov'd downwards, the Object Moves ſo too; On the other Hand, if the Glaſs is Mov'd upwards, the Object Moves downwards, and if the Glaſs is Mov'd downwards, the Object Moves upwards.

How the foregoing Phænomenon cannot be Juſtly Accounted for from this Philoſophy.

§. 35. Let the Eye be Poſited in *P O Q*, and afterwards Rais'd from *Q* to *O*, as it Gradually Aſcends, the Rays, which are Refracted from *a* Succeſſively Vaniſh, till at Length they are Perfectly loſt in *O*; and thus, if the Eye Moves downwards from *P* to *O*, the Rays, which are Emitted from *c*, will likewiſe by Degrees Eſcape the Eye, till they Finally Diſappear in the ſame Point *O*; which will be far from Explaining how *a* or *c* Move upwards, or downwards, according to the Motion of the Eye, ſince even thoſe Rays, which Remain, Preſerve the ſame Incidence nearly, and therefore ſhould Conſtantly Repreſent the Points in *a* and *c*, or, if any Thing, *a* Lower, and *c* rather Higher, than in their Natural Poſture, ſince the firſt is lookt upon from a more Elevated Station, and the laſt from a more Depreſſed.

How farther the foregoing Phænomenon is not Accounted for.

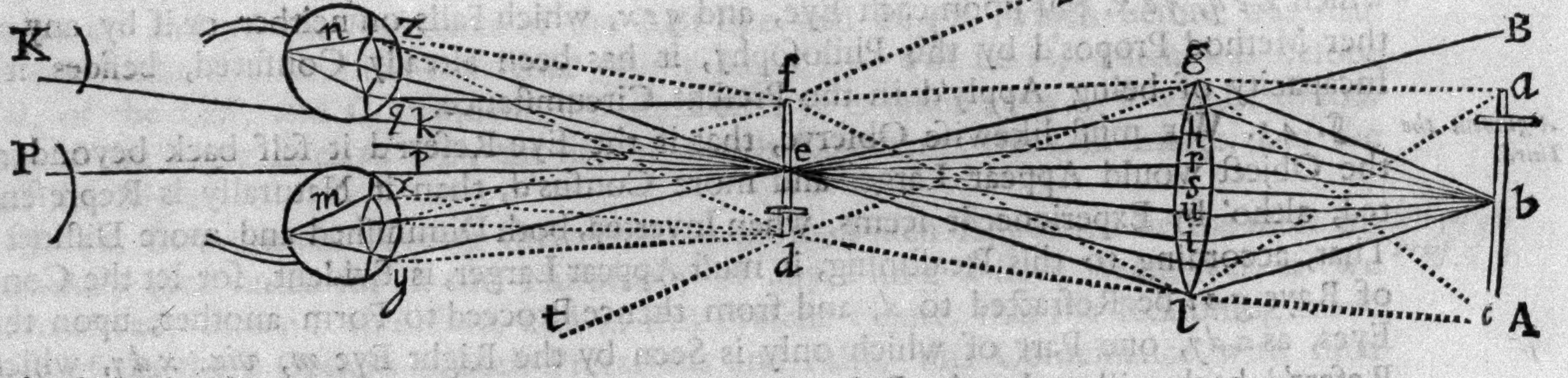
§. 36. Also, if the Glaſs is Mov'd from *l* to *s*, it will then only Receive the higher Rays, which Flow from the Points *a* and *c*, to wit, *g a s*, *g c s*, and therefore, if this can be Imagin'd to Cauſe any Alteration, it ſhould rather ſeem to Elevate the Object than Depreſs it, ſince, if the Eye Purſu'd back the upper Refracted Rays differently from the lower, it is Prepoſterous and Abſurd to Think, it would do it in the Direction of the lower; We ſaid, if this Occaſion'd any Alteration, becauſe it is not Moreover Evident, that Rays, which Flow from the ſame Point, and are Refracted thro' the ſame Glaſs, will Produce a different Poſition of the Point to the Eye; from all which it is Plain, that this Phænomenon was Wiſely Suppreſſed by the Preſent Opticks.

Concerning our Seeing with both our Eyes an Object Double through a Convex Glaſs. Molyneux's Dioptric. Prop. 39. Par. 1. § 6.

§. 37. Concerning the Double Viſion of an Inverted Object thro' the Convex, the Appearance is this, that, if the Glaſs is Placed any where, ſo as to Repreſent the Object Inverted, if the Eye is near the Glaſs, the Inverted Object will Appear Double, or, if it Retires farther from it to ſome Yards Diſtance, it will ſtill Appear Double, but Diminiſhed, and in a nearer Approach or Situation to each other, till by Degrees they ſeem to Coaleſce into one; moreover, if the Right Eye is clos'd, the left Image Diſappears, and if the left Eye is Shut, the Image on the Right Hand likewiſe Vaniſhes.

The Explanation of the foregoing Phænomenons from theſe Principles.

§. 38. The Account or Explanation of theſe Phænomenons, which this Doctrine Supplies us with, is from the Preſent Figure; *a b c* is the Object, *g l* the Glaſs, *f e d* the Diſtinct Baſe, the Eyes *n* and *m*, which Meet the Cone of Rays *z e y*; the Eye *n* therefore is ſaid to Perceive the Point *e*, or *b*, by the Rays *z e q*, which Flow Forward from *i e l*, and Conſequently ſomewhere betwixt *s* and *l*; for the ſame Reaſon the Eye *m* Perceives the Point *e*, or *b*, ſomewhere betwixt *s* and *g*, ſince the Rays, by which the Impreſſion is made, are *x e y*, or *g e h*; from whence it follows, that the Point *e*, or *b*, will be Seen in two Diſtinct Places; and as the like may be Demonſtrated of any other Point, the whole Object will Appear Double; it will likewiſe Unite into one ſingle Repreſentation, at a greater Diſtance of the Eyes from the Glaſs, becauſe, the Rays *e k*, *e p*, Diverging Continually, they will at laſt be Incident upon them, *viz.* *e k* on the Eye *n*, and *e p* on *m*, by which Means the Eye *n* will Perceive *e* at *u*, *k e u* being one right Line, and the Eye *m* will Perceive the ſame Point *e* at *r*, *p e r* being likewiſe one Continued Line, ſo that the Difference of the Places *r* and *u* becomes leſs than before, till at laſt by a farther Removal of the Eyes, this Difference of the Apparent Places is ſo Small, that it is altogether Inſenſible to the Sight. We muſt alſo Add, which the Preſent Opticks does not Obſerve, tho' it ſeems to be a mighty Confirmation of this Account, that if the Eye is Remov'd to ſome Place betwixt the Diſtinct Baſe, and the Convex, the Object will then likewiſe Appear Double, and if we Shut the Right Eye, the Image on the Right Hand will Vaniſh, and if



the left Eye is Clos'd, the Left Image will in like Manner become Inviſible, becauſe *g e h*, in this Caſe, falls upon the Left Eye, and *u e l* on the Right.

Argument the firſt againſt the foregoing Explanation.

§. 39. FIRST, as to our Eyes Seeing the Point *e* by the different Cones, or Pencils, *z e q*, *x e y*, and therefore *e*, or *b*, Appearing in two ſeveral Places; We Anſwer, that either the Rays are Purſu'd by the Eye no farther than *e*, or they are Purſu'd beyond it; If the former is Affirm'd, ſince the Optick Axes do Concur and are Terminated in *e*, the Point *e* ſhould Appear Single, by what is afterwards Declared in this Philoſophy; beſides, if *e* could in ſuch Circumſtances be Seen Double, on the Account only of *z e q* being Diſtinct from *x e y*, tho' our Sight ſhould be Determin'd in *e*, it would follow that all Objects whatſoever, for they alſo ſend different Cones to the Eyes *m* and *n* from the ſame Points, ſhould Exhibit a double Repreſentation of themſelves to us; Moreover, if the Eye Determin'd it's ſelf in *e*, it would be Impoſſible to Explain, how *e* comes to be Obſerv'd towards *g* and *l*, and even beyond the Convex *g l*; If the laſt is Affirm'd, the Eye *m* is carried in it's References towards *g h*, and the Eye *n* towards *i l*, but then how the Rays, which Terminate in a Line, as *g h*, *i l*, ſhould Expreſs to the Eye no more than a Point, is Inconceivable; for ſince, when the Eye is Remov'd to a greater Diſtance from *e*, the Rays *x e y*, or *z e q*, are more Divergeing, and Conſequently, a 4*th*, or 5*th*, or a 10*th*, Part of them Perhaps are not Incident upon it, and yet upon their Refraction to the Retina, they are Capable of Exciting a Senſation in us, it will follow, that if *g e h* is Divided into 4, or 5, or 10 Parts, each of them will be Sufficient to Affect the Eye; and if the Point in the Eye, whoſe Senſation is Caus'd by a Complication of the Various Parts of *g e h*, is not Referr'd back to a Point in *e*, but in the different Parts of *g e h*, the Point in the Eye, by ſuch a Reference, will be Diffus'd into the Parts of *g e h*, and *b*, or *e*, will not Appear in a Point, but will be Diſperſed and Scattered into ſeveral.

Argument the Second.

§. 40. AGAIN, if the Eye Refers it's ſelf back beyond *e*, it will be hard to Say, where it ſhould Determine it's Purſuit; if at *s*, that is, in the Convex it's ſelf, the Apparent Place of the Object ſhould be Seen there, whereas it Evidently is not; if we Affirm at *b*, the Plain Diſtance of the Object from the Eye would be the ſame as the Refracted, which is Contrary to Experience and this Philoſophy; Laſtly, if the Diſtance of the Object is to be Aſcertain'd by an Imaginary Focus, here is none, if by the Concourſe of the Optick Axes, they do not Meet but Diverge, if by the Optick Angle *a e c*, or *g e l*=to *z e y*, it is Impoſſible, ſince that is Divided into three Parts, Two of which *z e q*, *y e x*, Fall upon each Eye, and *q e x*, which Falls on neither, or if by any other Method Propos'd by this Philoſophy, it has been already Confuted, beſides it's Incapacity of being Apply'd in the Preſent Circumſtance.

Argument the Third.

§. 41. WE muſt likewiſe Obſerve, that if the Eye Referr'd it ſelf back beyond *e*, the Object would Appear Larger and more Confus'd, than it Naturally is Repreſented, altho' by Experience it ſeems, when Inverted, both Diminiſhed and more Diſtinct; That, according to this Reaſoning, it muſt Appear Larger, is Evident, for let the Cone of Rays *g a l* be Refracted to *d*, and from thence Proceed to Form another, upon the Eyes, as *z d y*, one Part of which only is Seen by the Right Eye *m*, *viz.* *x d y*, which Referr'd back will make the Point *A* Appear about *c*, the middle Point *b*, by the Reference back of *x e y* to *g e h* beyond the Convex, will be Seen at *B*, ſo that in this Deſcription, *b a*, which is half the Object, will Appear in *B A*, which is much Larger than the whole; Again, if *g c l* is Refracted to *f*, and from thence a Cone of Rays is Form'd upon the two Eyes, Namely *z f y*, Part of which, *x f y*, will only Fall upon the Right Eye *m*, which will be Referr'd back in *f s* to *C*; ſo that the Image will Appear above twice as Long, as the Object it ſelf, when it ought to be rather Diminiſhed in that Proportion, and the ſame Argument Obtains in Reſpect of the Left Eye at *n*; nor can a greater Diſtance of the Eyes *m*, *n* Solve this, for where they are at a ſhort Interval from the Diſtinct Baſe, the Object is thus Diminiſhed; nor can a greater Diſtance of the Object from the Glaſs, becauſe in a very little ſhorter Interval of the Eye and the Glaſs than before, the Object is very much Increaſed, even where it Appears Inverted.

Argument the Fourth.

§. 42. BESIDES, *f s* Falling on one Side of the Glaſs, and ſeveral of the Rays, Conſequently, betwixt *c* and *b*, upon their being Refracted to the Points between *f* and *e*, and their Reference from *f e* towards *s g*, it will follow, that one Part of the Object will be Seen Refracted, whilſt the other is not, which is Contrary to Experience, and muſt Evidently alſo Produce a Confuſion or Irregularity in the Repreſentation, which is not Acknowledg'd by this Philoſophy.

Argument the Fifth.

§. 43. THIS is Argu'd upon the Suppoſition, that xdy is the Cone, which Falls upon the Eye from a, and is Referr'd back from d towards lg; but there is ſtill another Scruple Ariſes in this Doctrine, which is, that gal is Refracted to d, and Forms the Cone gdl, which Proceeding Tranſverſely, according to the Laws of the Cones gel, zey, will Conſtitute the Cone xdt, from whence it is Conſequent, that no Rays from a, after their Tranſverſion, will Reach the Left Eye n, ſince thoſe, which lie moſt to the Left, as xdy, are Poſſeſſed by the Right Eye, and therefore the Remainder ydt will Fly off without being Perceiv'd at all, and the ſame may be ſaid of the Rays, which Flow from the Point c; that is, thoſe, which are Deriv'd from the Left Point of the Object, will not be Seen by the Left Eye, and thoſe, which come from the Right Point of the Object, will Eſcape the Right Eye; or, which is the ſame, the Left Point a will be only Seen by the Right Eye m, and the Right Point c, only by the Left Eye n; For, if dz can be ſuppos'd to Reach the Left Eye n, either zdt muſt be Equal to xdt, that is a Part to the whole, or elſe a Cone of Rays may be Bigger after the outmoſt Rays have Croſt each other in a Point, than before, which is Impoſſible.

Argument the Sixth.

§. 44. IF yet dz could Reach the Eye n, zdq, when Referr'd back by the Eye, would be Directed to a Point, which would Fall at a very great Diſtance on the Right Hand of c, ſo that, the other Points betwixt a and b Obſerving a like Direction, ab, when Seen by the Left Eye, would Appear vaſtly Larger, than when Seen by the Right, whereas by Experience they are nearly of an Equal Magnitude, and their Poſition is only Inverted.

What ſeems very Favourable to this Doctrine.

§. 45. THERE yet Perhaps may ſeem to be ſomething Coincident with this Explanation, that in the Inverted Object, if the Right Eye is Clos'd, the Left Image Diſappears, and on the Contrary, if the Left Eye is Shut, the Image on the Right Hand alſo Vaniſhes; as on the other Side, in the Erect Object, if the Right Eye is Shut, the Right Image Diſappears, if the Left Eye, the Left Image; from whence the Diſpoſition of the Rays, Aſſerted in this Scheme, ſeems to be Neceſſary; becauſe gh, being the Reference of the Right Eye m, and Repreſenting the Left Image, and il being the Reference of the Left Eye n, and Repreſenting the Right Image, if m is taken away, gh is, and if n, il is; So in the Erect Object, where the Eyes are Placed betwixt e and gl, the Reference of the Left Eye will be gh, which before was that of the Right Eye, and the Reference of the Right Eye will be il, which was before that of the Left, and Conſequently, if the Left Eye is taken away, gh will be ſo too, and if the Right Eye, il will, the firſt of which Repreſents the Left Image, and the laſt the Right.

Not yet any Proof of it.

§. 46. BUT notwithſtanding theſe Phænomenons, which Appear to be the moſt Ingeniouſly Accounted for in this Philoſophy of any Thing I have Seen, yet, if we have already ſhewn both before, and in this Preſent Figure, that the Reaſoning made Uſe of in theſe Opticks is both Precarious and Falſe, this Coincidence with ſuch an Explanation, which is Falſe in it's ſelf, cannot be an Argument, why we ſhould Believe it to be True; on the other Hand, what is Coincident with a Falſe Reaſoning will be a Juſt Perſuaſion to us to Think, that what is ſo Coincident is likewiſe Falſe, ſo far as it Depends upon the ſame Principles.

Argument the Seventh.

§. 47. WHICH is the Preſent Caſe before us, for altho' the Explanation given in §. 45. might Poſſibly hold Good, if we Conſider'd the whole Object abc, as one Point b, which that Section only does, it will not yet be Juſt or Sufficient, where the Object is Suppos'd to Conſiſt of ſeveral Points, and thoſe ſeveral Points Impreſſed upon the Eye by Diſtinct and Independent Cones of Rays, for the Reaſons before Alledg'd, and Particularly in §. 43. where, even tho' the Cones cannot be Incident upon the Eye, as is Imagin'd, the Tranſverſe Poſition of the Images is notwithſtanding Preſerv'd; thus Albeit there are no Refractions of the Rays from the Left Point a upon the Left Eye n, and no Refractions of the Rays from the Right Point c upon the Right Eye m, yet there being Refractions from a to m, and from c to n, the foregoing Phænomenon may be True, tho' the Reaſoning in this Philoſophy is Falſe.

Argument the Eighth.

§. 48. Lastly, it is Evident, that if by our Principles of the Expanſive and Contractive Forces we can give a Reaſon, why the Eye in ſuch Circumſtances does Refer it's ſelf, as is Preſcrib'd by theſe Phænomenons, without the Aſſiſtance of Rays being Refracted in the Manner, which is Suppos'd, theſe Appearances can be no Plea for ſuch a Doctrine, which has been Prov'd to be Falſe ; And this we Say may be done.

Argument the Ninth.

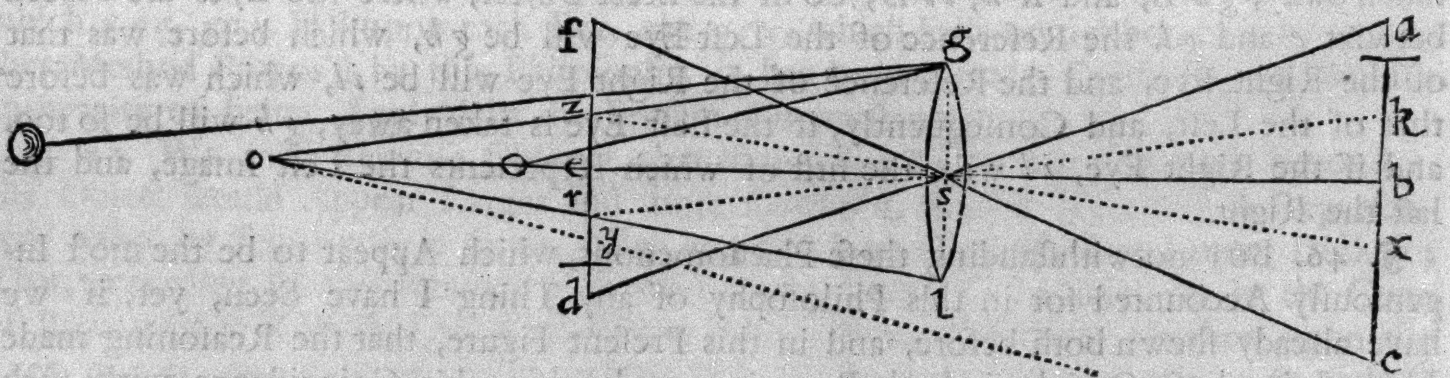
§. 49. As to both our Eyes Receiving, at a farther Diſtance from the Diſtinct Baſe, the Rays only k and p, by which Means h and i will Approach to r and u, and at Length will Coaleſce in s, which is Suppos'd to be Agreeable to what we Experience, that theſe Inverted Objects, which Appear Double, will by a farther Diſtance of the Eye from the Diſtinct Baſe Unite into one ; I ſay, as to this Phænomenon, we Anſwer, that this Explanation of the Approach of the Images only Reſpects the Point b, but what is Demonſtrated Concerning that Point, is, as has been Evinced, far from being Applicable to the whole Object.

Argument the Tenth.

§. 50. Let us now however Suppoſe, that K and P are the Places of our Eyes, in Right Lines Continued from k and p, and let us Conſider the Reſult from ſuch a Situation ; it is Manifeſt therefore, that if the Rays from a are but Juſt Refracted to m, in the Cone xdy, they cannot be Refracted to P at all, and ſince the Rays from c are but Juſt Refracted to n, they cannot be Carried by any Imaginable Refraction to K ; from which it Follows, that when the Eyes are Poſited in K and P, all the Rays from the Points a and c will Equally Eſcape them, as, when they are Placed or Situate in m and n, k and p are Confeſſed to do, by not being Incident on them, but if this Reaſoning is True, r and u will be ſo far from being nearer s, than h and i, that they will not in any Reference of the Eyes back to the Rays, which are Incident, be Perceiv'd at all.

Why the next Propoſition is not Particularly Conſidered, and only the Corollary of it. That the farther the Eye is Remov'd from the Diſtinct Baſe it Perceives the greater Area of the Object, but can never See more than what is Projected in the Breadth of the Glaſs. Molyneux, Dioptricks Prop. 40. Corol. Par. 1.

§. 51. The next Propoſition we have no Reaſon to Conſider, becauſe, as ſeveral others, which we have Omitted, it Depends upon an Abſtracted Reaſoning, that has been Confuted ; the Corollary of it however we ſhall Examine, which is, that the farther the Eye is Remov'd from the Diſtinct Baſe, it Perceives the greater Area of the Object, but can never See more, than what is Projected in the Breadth of the Glaſs ; that is, zr can never be greater than gl, nor ever Equal to it, unleſs the Eye o be at at an Infinite Diſtance ; This is Concluded from Suppoſing abc the Object, fed the Diſtinct Baſe, and the Eye in o ; for the Eye can See no more of fed than zr, which is Intercepted betwixt gol, and therefore no more of the Object than kx, which lies between $ksx = zsr$.



The Reaſons againſt the foregoing Corollary.

§. 52. If therefore gol is the Meaſure of what the Eye is to See of fed, and Conſequently ksx, of what it Perceives of the Object abc, the farther the Eye is Remov'd from the Diſtinct Baſe, as at O, the more it will take in, both of it and the Object, and at Length, at an Infinite Diſtance, Og will Fall Perpendicular to gl, which would Interſect a Portion of fed Equal to the Diameter of the Glaſs gsl; This Perhaps may be True upon the Suppoſition of the Doctrine of Rays, but it is not any ways Confirm'd by Experience, and therefore is only a Right Concluſion from a Wrong or Falſe Hypotheſis ; ſince, if the Object was Larger than the Glaſs, it would be never, that we could See it Entire, whereas it is Evident we do, at ſeveral Various and Changing Diſtances, or Removes, of the Eye from the Diſtinct Baſe, that is, if the Glaſs is Placed Two, or Three, Yards from the Object.

§. 53. ON the Contrary, if *o* is made to Approach to *f e d*, it will Intercept a leſs Portion of it than before, and Conſequently will See leſs of the Object *a b c*, whereas in Experience the Object Appears much Larger than the Glaſs, and Inverted, when the Eye is Extremely near to the Focus, if the Glaſs is but Remov'd at ſome Diſtance from the Object.

The Reaſons againſt the foregoing Corollary.

§. 54. ANOTHER Propoſition in this Philoſophy is, that an Object being Placed nigher a Convex Glaſs than it's Focus, and the Eye on the other Side of the Glaſs, at any Diſtance within the Eye's Power, it Sees the Object Diſtinct and Erect. The Demonſtration of this Depends upon Propoſitions, which have been before Conſider'd, and therefore does not Require a Particular Examination here; we ſhall Add to this Propoſition, that, on the other Hand, if the Eye is Placed betwixt the Focus and the Glaſs, it is alſo Sufficiently known, that the Object will Appear Erect and Diſtinct; and therefore, ſince, whether the Eye or the Object is Placed betwixt the Focus and the Glaſs, the Appearance will be the ſame, and ſince the former Caſe has been Conſider'd, upon that Account alſo, there is Scarcely any Occaſion to Speak to the Latter.

Concerning an Object being Placed Nigher a Convex Glaſs than it's Focus, and the Eye on the other Side of the Glaſs at any Diſtance, within the Eye's Power and it's Seeing ſuch Object Diſtinct and Erect. Molyneux's Dioptric. Prop. 41. Par. 1.

§. 55. HOWEVER, we ſhall Alledge one Thing more Peculiarly in Anſwer to this Reaſoning upon the Rays Emitted from the Object, which is, that in the Demonſtration Annexed to this Propoſition, it is ſaid the Rays ought to be Moderately Diverging, that is, not to Diverge too much, otherwiſe, by ſome former Propoſitions, the Eye will not be able to Collect them; Whereas the nearer the Object is Placed to the Glaſs, that is, the more Diverging the Rays are upon it, the Clearer and more Diſtinct it is Seen, and on the other Hand the farther from the Glaſs it is Situate, and the nearer to the Focus, that is, the more Moderately Diverging the Rays are, the Object will Appear the more Indiſtinct and Confus'd, which is quite the Reverſe in Experience of what is Affirm'd in the Demonſtration; ſo likewiſe, the nearer the Eye Approaches to the Glaſs, the more Diſtinct the Object Appears, tho' the Rays Diverge the moſt then.

Quite the Reverſe in Experience from what is Affirm'd in the Demonſtration.

§. 56. THE next Propoſition is, to Determine the Viſible Area and the Optick Angle, or Apparent Magnitude of an Object, Placed betwixt the Focus and the Glaſs, from an Abundance of Conceſſions, as the Power or Focal length of the Glaſs, the Diſtance of the Object or Eye from the Glaſs, and the Breadth of the Glaſs, but as the Determination of the Viſible Area of the Object, and the Optick Angle, or the Apparent Magnitude of it, has been before Examined and Confuted in Reſpect of the Account, which this Philoſophy gives of both, ſo the Data or Conceſſions, which are Suppos'd of the Power or Focal Length of the Glaſs, the Diſtance of the Object from the Glaſs, *&c.* are all Chimerical, and not Correſpondent with Nature or Experience by the former Section; ſince it is Plain by that, that Objects are not Seen according to the Power or Focal Length of the Glaſs, or the Diſtance of the Object from the Glaſs, or the Diſtance of the Eye from the Glaſs, or according to the Breadth of the Glaſs, but according as the Eye or the Object Moves betwixt the Focus of the Glaſs, and the Glaſs it's ſelf, and the Appearances are not much Alter'd at any other Diſtance either of the Eye, or the Object from the Glaſs.

How the Viſible Area and the Optick Angle, or Apparent Magnitude of an Object Placed betwixt the Focus and the Glaſs is Determined from theſe Principles; This Account Chimerical and not Conformable to Nature or Experience. Molyneux's Dioptricks Prop. 42. Part 1.

§. 57. WE ſhall therefore Conclude this Chapter with a moſt Particular Admiration, that Men will Suffer their Reaſon to be Inſulted by a Magnificent Show of Lines, and Angles, and, as they are Thought, Elaborate Calculations upon them, which, by what we have yet Seen, Alledge no more to Convince us of their Truth and Certainty, than that they Indeed make a Figure, and which may be Juſtly ſaid, not to Inform but to Amuſe us; and if ſuch a Temper ſhould ſtill Prevail in the Learned World, we cannot but Think and Pronounce, that, as Humane Nature, in the Common Occurrences of Life, is too much Influenced by Pomp and Grandeur, ſo in Matters of Reaſon and Speculation, it is likewiſe.

Concluſion of this Chapter with an Admiration, that Men ſhould be more Apt to Reſign their Senſe to Lines and Angles than to Nature and Reaſon.

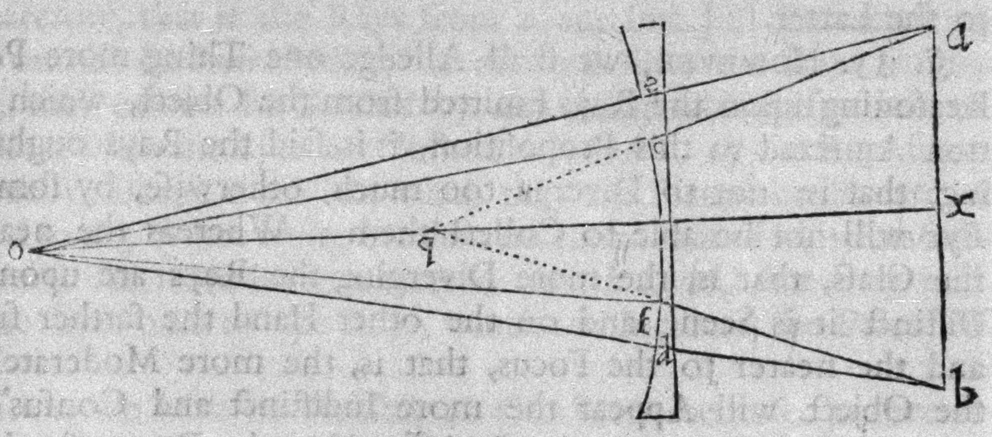
I may Add, that in Geometrick and Algebraick Performances, Men Reaſon by Certain Rules of Art, and their Principles are Contracted and Narrow, which, when once Learned and Attain'd, make Geometry and Algebra an Eaſy Science, and therefore, whilſt we are ſo Fond of this Kind of Reaſoning, in Preference to all others, which are many Times more Difficult and Hard, tho' equally Demonſtrative and True, it may not be owing to our Vanity only, but to our Idleneſs.

Chap. XII.

Concerning the Properties of Concave Glaſſes, Placed betwixt the Object and the Eye.

Concerning Objects Seen thro' Concave Glaſſes Appearing Erect and Diminiſhed. Molyneux Dioptricks Prop. 43. Par. 1.

§. 1. THE next Sort of Glaſſes, we are Led to Examine the Properties of, are Concaves; The firſt of which Properties or Affections is, that all Objects, Seen thro' Concave Glaſſes, Appear Erect and Diminiſhed; They Appear Erect it is ſaid, becauſe there is no Diſtinct Baſe Form'd by them from a Collection of the Rays, and therefore no Inverſion of the Image, Excepting on the Retina of the Eye, which by what has been already Advanced, will Cauſe the Object to be View'd in it's Natural Situation; They are Affirm'd to Appear Diminiſhed from *Tab.* 32. *(Fig.* *1ſt.)* *axb* is an Object, *o* the Eye, *aeo*, *bdo*, are Drawn Strait from the Extremities of the Object to the Eye, betwixt which the Natural Optick Angle *aob* is Contain'd, the Concave *cf* is now Interpos'd, by which Means the Concourſe of the two Lines *ae*, *bd*, will be Refracted beyond *o*, from whence the Eye will not Perceive the Extremities of the Object by the Rays *ae*, *bd*, becauſe they will be Refracted beyond *o*, and therefore will not be Incident upon it, nor by any Rays External to *ae*, *bd*, ſince they will be ſtill more Refracted beyond the Eye in *o*, and Conſequently the Rays, which are Refracted to the Eye, muſt be Internal to *ae*, *bd*, as for Inſtance *ac*, *bf*, and which Continued in Direct Lines would Meet in *q*, but by the Refractive Power of the Concave are made to Diverge from thence, and at Length to Concur in *o*, in which Caſe *cof* is the Refracted Optick Angle, and is leſs than *eod* the Natural one, and by Conſequence the Object in the firſt Circumſtance will Appear of a Diminiſhed Magnitude, in Reſpect of what it will do in the Laſt.



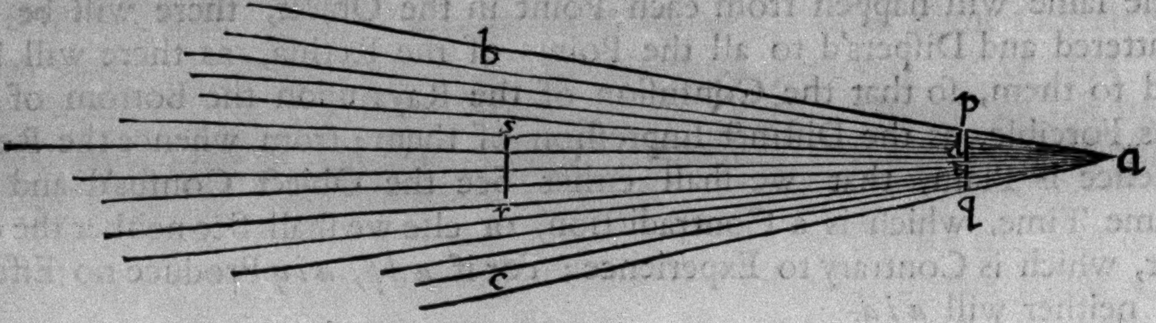
In this Chapter, Pag. 565. 566. The Fig. of §. 12. is to be Placed in §. 16. and the Fig. of §. 16. in §. 12.

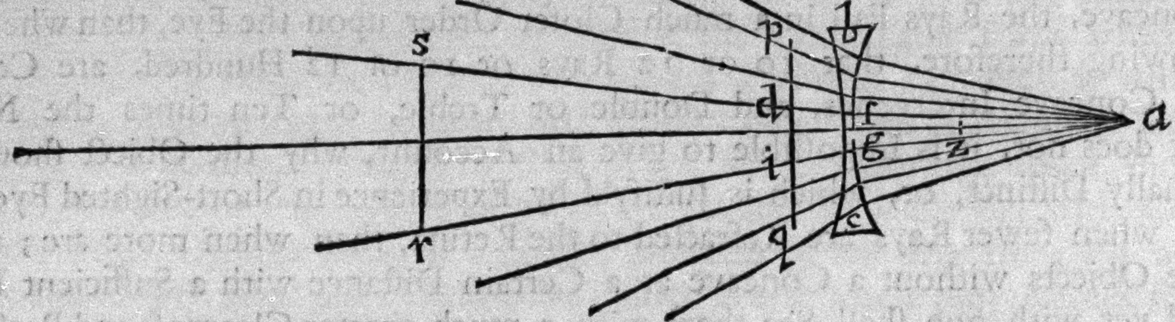
The foregoing Reaſoning not Valid.

§. 2. As to the Erectneſs of the Object, where there is no Diſtinct Baſe Produced, or the Divergence of the Rays Refracted thro' a Concave, by which they are far from being Collected to a Focus, that has been before Conſidered; and as to the Account, which is given of the Object's being Diminiſhed, it Proceeds likewiſe upon the ſame Reaſoning, which was made Uſe of to Prove, that all Objects, Seen thro' Convex Glaſſes, are Magnify'd, *Prop.* 30. of this Philoſophy, and therefore, a Juſt Allowance being made for the Convergence of the Rays in the one Caſe, and a Divergence in the other, the ſame Arguments will be Valid againſt this Account of the Diminution of the Object, which were us'd in the Former againſt the Increaſe of it.

Concerning the Diſtinctneſs and Confuſedneſs of an Object, as alſo of it's Faint and Obſcure Appearance thro' Concaves. Molyneux's Dioptricks Prop. 44. Par. 1.

§. 3. AFTER the Erectneſs of the Object and it's Diminution thro' Concave Glaſſes, the Diſtinctneſs and Confuſedneſs of it, as alſo it's Faint and Obſcure Appearance, is Propos'd to be Solv'd, which is done in the following Manner; To make an Object Appear Diſtinct it is Requir'd, that the Rays from each Point ſhould be Exactly Collected to ſeveral Points on the Retina, which cannot be Effected, unleſs the Rays Diverge ſo, as the Eye may be able to Unite them; *(Tab.* 32. *Fig.* 2.*)* *sr* is the Pupil of the Eye, Placed at ſuch a Diſtance from the Radiating Point *a*, as to Receive the Incident Rays in a Proper Divergence to be Punctually Refracted to the Bottom of the Eye; if now the Pupil is Set at a nearer Diſtance from *a* in the Poſition, which *pq*,=*sr*, has to it, thoſe Rays, which are Refracted with a Juſt Diſtinction to the Retina, will be Confin'd to *di*; *ad*, *ai*, being of an Equal Divergence with *as*, *ar*, which are Suppoſ'd to be ſo Refracted; the ſame alſo will be True, when Viſion is Perform' d thro' a Concave Glaſs from (*Tab.* 32. *Fig.* 3.); where the Breadth of the Pupil is *pq*, and is Placed in a near Situation to the Glaſs, the Rays *ap*, *aq* will Diverge ſo much, that it will be Impoſſible for them to be Refracted Diſtinctly to the Eye; but, the ſame Pupil being Poſited in *s r*, *a s*, *a r* are Suppoſ'd to have a more Moderate Divergence, and by that Means to admit of a Collection in the Retina, and Conſequently *a d*, *a i*, which have the ſame Divergence; This is moreover Thought to be Confirm'd by an Experiment on a Concave of a Small Sphere, which if Placed near the Eye, the Appearance of Diſtant Objects thro' it is Confuſ'd, if farther, more Diſtinct; and even, where Objects Appear Confuſ'd, Contract the Pupil by Placeing before it a Small Foramen, made in the Midſt of a Paper by a Needle, and the Object will Appear more Diſtinct, tho Obſcure, and not ſo Lightſome, or ſo much Illuminated, as if the Paper was not Interpoſ'd; and the ſame is ſaid to happen if an Object, without the Intervention of a Glaſs, Appears Confuſ'd, by too near an Approach of the Eye to it, Namely, that ſuch a Foramen will Cauſe the Object to Appear more Diſtinct; Farther, if the Imaginary Focus of, *ſd*, *ri*, is in *z*, the Object may be Seen Diſtinct in *z*, ſince the Divergence of the Rays is ſtill the ſame; and as to the Strong or Faint Appearance of Objects thro' Concaves, Seeing the Concave Scatters and Diſperſes the Rays, the Object will Appear thro' it more Faint and Obſcure; a Strong or Luminous Viſion Proceeding from a greater Quantity of Rays, and a more Faint and Obſcure, from a leſs; What is Laſtly Remarked in this Doctrine is, that in Convex Glaſſes, the farther the Eye is from them, or the nearer it Approaches to the Focus, ſince the Rays Converge the more, the Object Appears more Confuſ'd; on the Contrary in Concaves, the farther the Eye is from the Glaſs, the more Diſtinct the Object Appears, but ſtill more Faint, becauſe the Rays in ſuch a Caſe have a greater Divergence.





The Firſt Argument againſt the foregoing Account.

§. 4. In Anſwer to which we ſhall firſt Conſider, what is Propoſ'd to us from (*Tab.* 32. *Fig.* 2*d.*) where *ſr* and *di* Receive Rays of the ſame Divergence, the Pupil of the Eye *s r* and *pq* being Equal; if therefore the Eye is Placed in *pq*, either the Rays, which Flow from *a* betwixt *p*, *d* and *q*, *i*, will have the ſame Effect upon the Eye with thoſe, which are Deriv'd from *a* to the Space betwixt *s*, *b* and *r*, *c*, or not; If not, the Eye, Situate in *pq*, will not be under the ſame Circumſtances, as when it is Poſited in *s r*, and Conſequently will not See the Object with the ſame Diſtinction, which is Affirm'd; If they have the ſame Effect, the Rays *aiq*, *adp*, which Fall on the Pupil of the Eye in *pq*, will make no greater Impreſſion upon the Sight than *arc*, *asb*, which do not Fall upon the Eye at all, which is Impoſſible; For it is altogether Unavoidable, that in Truth, *adp*, *aiq*, which are Conſidered as nothing, ſhould not Obtain a very Conſiderable Influence in Rendering the Image Extremely Indiſtinct and Confuſ'd, or rather in Entirely Obliterating and Eraſing it; ſince either of them are much greater than *adi*, and altho' they cannot be Refracted to the ſame Point with it, yet they will be Diffus'd at leaſt and Spread on the Retina, and Seeing the ſame will happen from each Point in the Object, there will be as many Rays Scattered and Diſpers'd to all the Points of the Retina, as there will be Rays Collected to them, ſo that the Confuſion of the Rays upon the Bottom of the Eye will be as Forcible, as the Diſtinct Impreſſion of them; from whence the Reſult and Conſequence is Plain, that we ſhall either See the Object Confus'd and Diſtinct at the ſame Time, which is a Contradiction, or elſe we ſhall See neither the one nor the other, which is Contrary to Experience; For if *adp*, *aiq* Produce no Effect upon the Eye, neither will *aid*.

Argument the Second.

§. 5. What has been ſaid Concerning (*Fig. 2d.*) is eaſily Apply'd to (*Fig. 3d.*) Since the Reaſoning upon the former Figure is Introduced to Explain the Phænomenons of the latter, that which is ſtill farther Obſervable is in Reference to (*Fig. 3d.*) Compar'd with (*Fig. 2d.*) That in both Caſes the Rays, which Flow from *a* are Equally Suppos'd to Cauſe Diſtinct Viſion, tho', where there is no Interpoſition of a Concave, the Rays Fall in a much Cloſer Order upon the Eye, than where there is; Allowing therefore, that 10 or 12 Rays, or 10 or 12 Hundred, are Collected, where a Concave Intervenes, and Double or Treble, or Ten times the Number, where it does not, it is Impoſſible to give an Account, why the Object ſhould Appear Equally Diſtinct, or, which is Juſtify'd by Experience in Short-Sighted Eyes, more Diſtinct, when fewer Rays are Refracted to the Retina, than when more are; for Myopes See Objects without a Concave at a Certain Diſtance with a Sufficient Diſtinction, and yet with one ſhall See them with a much greater Clearneſs and Perſpicuity.

Argument the Third.

§. 6. As to the Experiment of a Small Foramen made thro' a Paper, I have found the Contrary, that if the Eye is made to Approach very near to what is here Written, ſo that it can hardly Diſcern the Letters Inſcrib'd upon it, if a Paper with ſuch a Foramen is Interpos'd, it is far from Aſſiſting us in a Diſtinct Perception of them, on the Contrary it makes them only Larger and more Confus'd; If this were not yet Certain, why either the Pupil of the Eye ſhould be Contracted, becauſe it Looks thro' a Small Foramen, rather than Enlarged, that it might Receive all the Rays Poſſible, which are Tranſmitted thro' it from the Object, which ſeems to be the Caſe, or, if it were Contracted, how it ſhould be a Confirmation of this Solution, where the Pupil is not Suppos'd to be Contracted, but the Rays only to Fall upon a leſs Space of it, Excepting the Contraction of the Rays and the Contraction of the Pupil is the ſame, which it is not by the Reaſoning from the Preſent Figures, will be hard to Determine.

Argument the Fourth.

§. 7. It is likewiſe Aſſerted in §. 3. that the Object will Appear Diſtinct, if it is Placed in *z*, which cannot be, unleſs the Imaginary Focus is the ſame, when the Concave is Poſited in *bfc*, and the Object in *a*, as it is when Placed in the ſame *bfc* and the Object in *z*, whereas in ſuch a Caſe it muſt Neceſſarily Aſſume a different one.

Argument the Fifth.

§. 8. Another Affirmation is, that the Object thro' Concaves Appears more Faint and Obſcure, becauſe the Rays are Diſperſed and Diffus'd in their Refraction thro' them, whereas to make an Object ſeem Bright and Vivid, a greater Multitude of Rays is Required; This has been in ſome Meaſure already Conſidered, and we ſhall only Add, that if the Brightneſs or Livelineſs of an Object was to be Aſcribed to the greater Number of Rays, which are Refracted to the ſame Point, there would be no Reaſon, why a Poliſhed Piece of Steel, or Silver, ſhould Appear more Luminous than that of a White or Black Marble, ſince we See both with an Equal Diſtinction, that is, the Rays from the Points of both are Equally Refracted to ſeveral Points in the Retina, and there cannot be any Difference in the Number of Rays, unleſs upon the Suppoſition of an Inequality in their Surfaces, according to the Account of the Corpuſcular Philoſophy, which has been before Abundantly Confuted.

Argument the Sixth.

§. 9. But that which is ſtill more is, that this Part of §. 3. Endeavours to Account for a Phænomenon, which is not in Nature; Namely, that Objects Appear more Luminous and Lively, if they are Seen without a Concave Glaſs, than if they are beheld with it, whereas the Contrary is Evident, for whether we Obſerve Objects thro' Concaves or Convexes, it is Manifeſt, that at a due Diſtance we Perceive them, let our Eyes be longer or ſhorter in their Views, with a fuller Clearneſs and Brightneſs, than when we Obſerve them thro' Air; thus, thoſe Pictures, which thro' the

Air Appear to be Flat, and only bare Repreſentations of what we Imagine, by Convexes are Impreſſed upon us, both before and after an Inverſion in the Focus with a wonderfull Life and Vivacity, and in Concaves to the ſame Sight Objects are made to Appear, either by Refraction or Reflection with a greater Force and Strength, than they would do in common Air or Æther, which cannot be owing to any Thing, but to the different Expanſive and Contractive Force in Glaſs, from that in Air, to Communicate the Impreſſions.

Argument the Seventh.

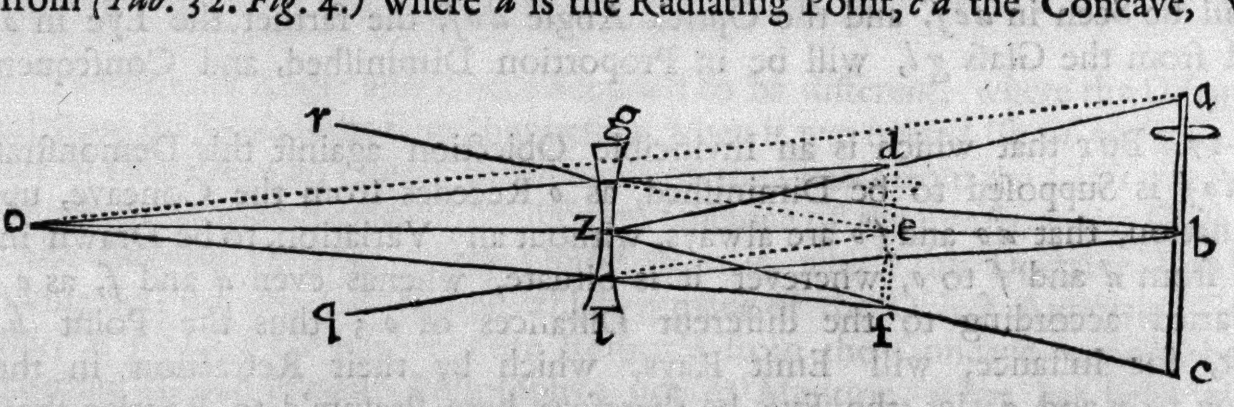
§. 10. MOREOVER, it is ſaid, that in Convex Glaſſes the farther the Eye is Remov'd from them, within the Diſtance of the Focus, the Object Appears more Confuſed, and in Concaves, the farther the Eye Recedes, the Object on the Contrary Appears more Diſtinct; This, if there were nothing elſe, would be a ſtrong Objection againſt the Philoſophy of Rays; for how in the ſame Eye, which Sees Objects Diſtinctly, both when the Rays are Converging, and when they are Diverging, a greater Divergence of Rays ſhould have the Preference, in Repreſenting an Object Diſtinct, to a greater Convergence, is not eaſily Apprehended; that is, let there be a fixt Station of the Eye, in which it exactly Refracts to the Retina the Converging Rays, which are Incident upon it; let there be likewiſe one certain Poſition, in which it will Refract the Diverging with an equal Accuracy and Diſtinction; when it Removes from theſe Stations, it will be Inexplicable, why thoſe Rays, which Converge more, ſhould in a very ſhort Diſtance of the Eye from it's firſt Situation be Refracted with Confuſion, and thoſe, which Diverge more at a greater Diſtance of the Eye from it's firſt Situation, ſhould be Refracted with Diſtinction; ſince Rays, where they Recede from each other, as well as where they Approach each other, ought to be Equally Incapable or Equally Capable of being Refracted with a Juſtneſs to the Retina, if the Eye is Placed beyond the Suppoſed Station of Diſtinct Viſion.

Argument the Eighth.

§. 11. What we have ſaid in the former Section Naturally Leads us into another Inquiry, how Diverging and Converging Rays in General can Poſſibly Repreſent an Object Diſtinct; for, if the Eye is of one Determin'd Figure, be it Longer or more Depreſſed, according to the Difference made in it's Figure by this Philoſophy, it ſeems Unaccountable, that the ſame Eye, which Sees Objects Diſtinct by Converging Rays, ſhould do it by Diverging, or the Contrary; unleſs the ſame Figure of the Eye can Refract Rays, which Flow from different Points, as in the Converging, and Rays, which Flow from the ſame Point, or the Imaginary Focus, as in the Diverging, to the ſame Individual Point in the Retina, which ſeems to be Abſurd and a Contradiction.

Concerning the Apparent Place of Objects Seen thro' Concave Glaſſes.
Molyneux's Dioptric. Prop. 45. Par. 1.

§. 12. WE are after an Account of the Diſtinction, in which Objects Appear thro' Concaves, led to the Aſcertaining the Apparent Place of them, which is Explain'd from *(Tab. 32. Fig. 4.)* where *a* is the Radiating Point, *c d* the Concave, which Cauſes the Rays to Diverge, in that Manner and Inclination to *e g*, upon the Eye, as, if Produced Directly backwards, ſhould Determine the Imaginary Focus in *b*; from whence, after ſuch a Refraction of the Rays, it is ſaid to be Conſequent, that the Apparent Place of the Object will be nearer than in it's Natural Poſition, and that thoſe, who are Short-Sighted, therefore See Objects Diſtinct thro' theſe Glaſſes, becauſe they make them to Approach Nigher to the Eye.



Argument the Firſt againſt the Preceding Solution.

§. 13. IF this Reaſoning is True, on the ſame Account, that Rays Diverging and Return'd back to an Imaginary Focus, Determine the Apparent Place of an Object, Rays, which Converge upon their Refraction thro' a Convex, and are Return'd back to no Focus at all, ſhould Aſſign no Apparent Place to the Object, from whence they are Emitted; And therefore this kind of Arguing is either Falſe, or elſe Objects thro' Convex Glaſſes are not Seen in any Poſition, which is Contrary to Experience.

§. 14. AND

Argument the Second.

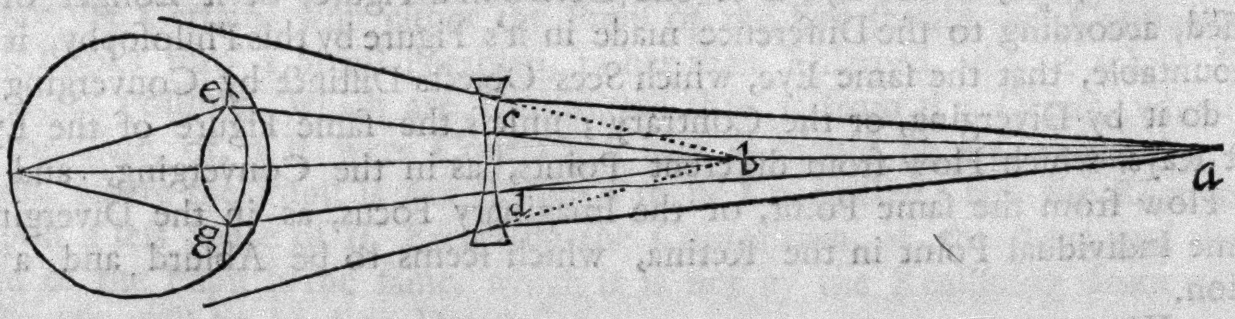
§. 14. AND as to the way of Accounting for Short-Sighted Perſons Seeing better with theſe Glaſſes than without them, becauſe they make Objects Appear nearer; it is a wonderful Method of Solution, to Infer, becauſe we See Objects nearer, as the Optick Angle is greater, without the Interpoſition of a Glaſs, which this Philoſophy owns, that therefore we ſhall See Objects nearer, as the Angle of the Imaginary Focus is greater, where a Concave Intervenes; for, unleſs we can Reconcile the Angle of the Imaginary Focus of a Concave, which is Determined by the Rays, that Proceed from one Point of an Object, with the Optick Angle, which is made by Rays, that Proceed from different Points of an Object, which is Impoſſible, ſuch a Concluſion will be Invalid, and Falſe.

Argument the Third.

§. 15. BESIDES, it is very Unreaſonable to Preſume, that, where there is a Short Sight, an Approach of the Object ſhall be always Beneficial to it, ſince an Object by ſeveral Means may be made to Approach, where it is not Beneficial to it; as if without a Concave it comes too near, every Thing Appears Undetermined and Confuſed, if with a Concave, tho' according to this Reaſoning Objects are nigher in all Caſes than otherwiſe, yet it is Experience and Fact, and Acknowledged likewiſe by this Doctrine, ſince it Preſcribes a certain Diſtance from the Concave to certain Eyes, that they are not Seen in all Caſes with any Clearneſs or Diſtinction; and Laſtly, Objects may be made to Approach nearer to Short-Sighted Men, where it is of no greater Advantage to them, than to thoſe of a longer Proſpect, as in any Eminency, which is Situate beyond another, which, tho' there may be the Diſtance of ſeveral Yards, yet will Equally Appear to a Short Sight or a Long one, to be Immediately behind it; from which Inſtances we may Reaſonably Conclude, that a Short Sight does not always See Objects, as they are at a ſhorter Diſtance, or which is the ſame, that a Short Sight and a Short Diſtance are not always Reciprocal.

Concerning the Object Appearing the Leſs, the farther the Eye is Remov'd from the Concave. Molyneux Dioptricks Prop. 47. Part. 1.

§. 16. WE muſt therefore Omitting the Forty Sixth Propoſition, go on to the Forty Seventh which is, that the farther the Eye is Remov'd from the Concave Glaſs, the Object Appears the leſs, and is Prov'd from (*Tab.* 32. *Fig.* 5.) becauſe the Object *a b c* by (*Fig.* 4.) will be Seen in *d e f*, and the Optick Angle *d o f*, the farther the Eye in *o* is Remov'd from the Glaſs *g l*, will be in Proportion Diminiſhed, and Conſequently the Object.



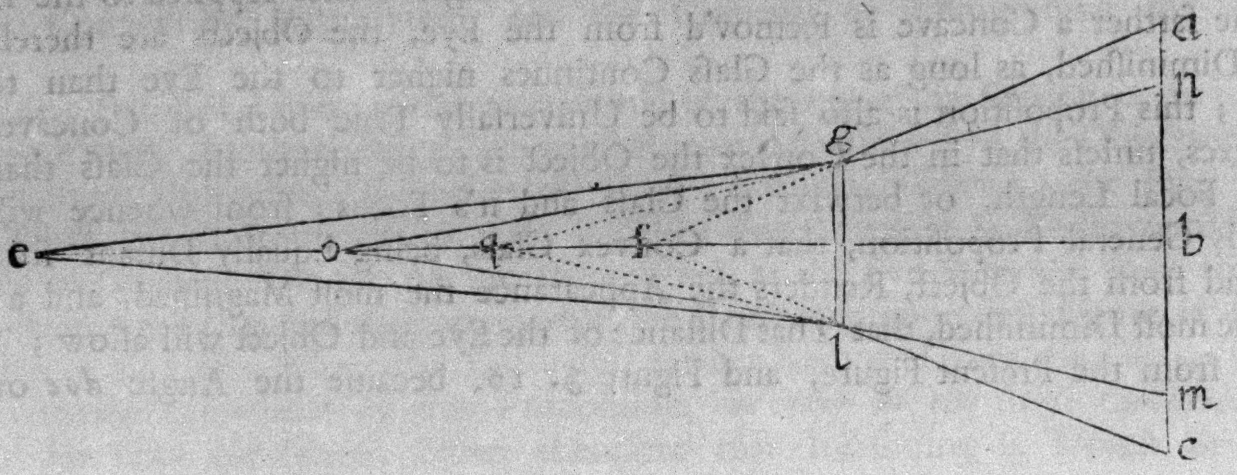
An Invincible Objection againſt this Demonſtration.

§. 17. BUT that which is an Invincible Objection againſt this Demonſtration is, that *d o f* is Suppoſed to be Diminiſhed, as *o* Recedes from the Concave, upon this Imagination, that *d o* and *f o* are always, without any Variation, to be Drawn in Direct Lines from *d* and *f* to *o*, wherever it is Situate, whenas even *d* and *f*, as *e* is, will be Varied according to the different Diſtances of *o*; thus the Point *b* in the Object, for Inſtance, will Emit Rays, which by their Refraction in the Glaſs Diverge to *r* and *q*, let the Eye be therefore here Preſum'd to Receive thoſe Rays, and to Return them back to *e*, if the Eye is now Placed in *o*, it will Receive Rays, which are leſs Diverging, and therefore will Return them to a Point betwixt *e* and *b*, and the ſame Reaſoning Obtains in the other Points *a* and *c* Referr'd to *d*, *f*; from whence it Follows, that *d e f* will Appear Larger, inſtead of being Diminiſhed, from a greater Diſtance of the Eye in *o*.

Concerning the Eye being Remov'd from the Concave, and not Seeing ſo Large an Area of the Object as before. Molyneux's Dioptricks Prop. 48. Par. 1.

§. 18. PROPOSITION the Forty Eighth is, that if the Eye be Remov'd from a Concave, ſo Large an Area or Space of the Object cannot be Seen thro' it, as before, (*Tab.* 33. *Fig.* 3.) *a b c* is an Object, *g l* the Glaſs, *o* a nearer Poſition of the Eye, and *e* a more Remote, *f* is the Point of Divergence in the former Caſe, and *q* in the latter; if therefore Direct Lines are Drawn from *f* to *a* and *c*, the Eye in *o* will See the whole Extent of the Object *a b c*, if from *q* to *n*, *m*, the Eye in *e* will only Perceive ſo much of it, as lies betwixt thoſe two Letters, Namely *n b m*, which is Manifeſtly leſs than *a b c*.

§. 19. IN



Argument the First againſt this Demonſtration.

§. 19. In this Demonſtration it is to be Obſerv'd, that *f* and *q*, the Points of Divergence, are Preſum'd to be betwixt the Glaſs and the Eye, and Conſequently, the Rays to Diverge from the Glaſs to the Object, whereas in the former Propoſition which we Conſidered, (*Tab.* 32. *Fig.* 4. and 5.) the Points of Divergence on the Contrary were betwixt the Object and the Glaſs, and Conſequently the Rays Diverged from the Glaſs to the Eye; but if we can make the Rays Diverge from the Glaſs to the Object, or from the Glaſs to the Eye, we can make them do what we Pleaſe; ſince, if the Rays in the former Propoſition are Incident Converging to the Point, from which they Fall, and are Refracted Diverging, ſo that the Refracted Diverging Rays ſhould meet in a Point in an Imaginary Focus betwixt the Glaſs and the Object, it is Impoſſible, that they ſhould Diverge to the Object, and have their Point of Convergence betwixt the Glaſs and the Eye.

Argument the Second.

§. 20. Besides, Suppoſing the Point of Divergence might be on this, or the other Side of the Glaſs, as the Former or the Preſent Solution ſhould Require, it will be hard to Conceive, how the Eye in *o* or *e* (*Tab.* 33. *Fig.* 3.) ſhould Determine it's Viſion of a greater or leſs Extent of the Object by the Imaginary Focus's in *f* and *q*, which do not ſo much as Reach the Eye in either of it's Situations, and Conſequently cannot, as it yet Appears, have any Influence upon it.

Argument the Third.

§. 21. If notwithſtanding the Eye at *o* was Placed in *f*, or the Eye at *e* was Placed in *q*, ſo that it might Effectually be Govern'd by the Optick Angle in it's Judgment of the Extent of the Object, yet the Eye in *f* would Perceive the Object Larger and in *q* Smaller, where the Diſtance of the Object from the Glaſs is the ſame; which is not Evident in Experience, that in the ſhort Interval betwixt *f* and *q* in the Poſition of the Eye from the Glaſs, there is ſo great an Increaſe or Diminution of the Object to our Sight, as the Proportion betwixt *m n* and *a c* Repreſents.

Argument the Fourth.

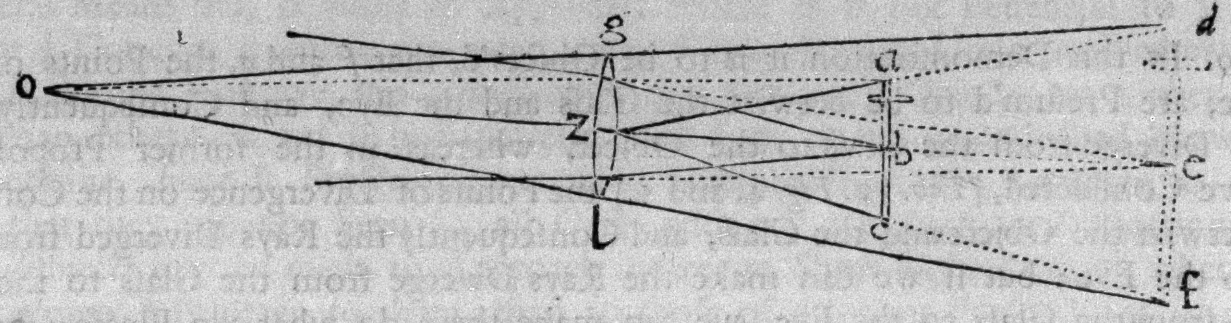
§. 22. The Optick Angle alſo is here Suppos'd to be different, where the Diſtance of the Object from the Glaſs is the ſame, but before, when it was made Uſe of to Explain the Magnitude or Diſtance of an Object, the Diſtance of the Object from the Glaſs was Suppoſed to be different; unleſs therefore the different Diſtance of the Eye from the Glaſs is the ſame, as the different Diſtance of the Object from it, which this Doctrine Denies in the following Section, the Concluſion which is made of the Object Appearing Greater or Smaller, as *f* and *q* are leſs or more Remov'd from the Concave, cannot be Juſt even according to the Preſent Philoſophy; not to Mention, what has been already Obſerv'd Concerning the Incompetency of the Optick Angle to Solve the Phænomenons of the Object's Magnitude or Diſtance.

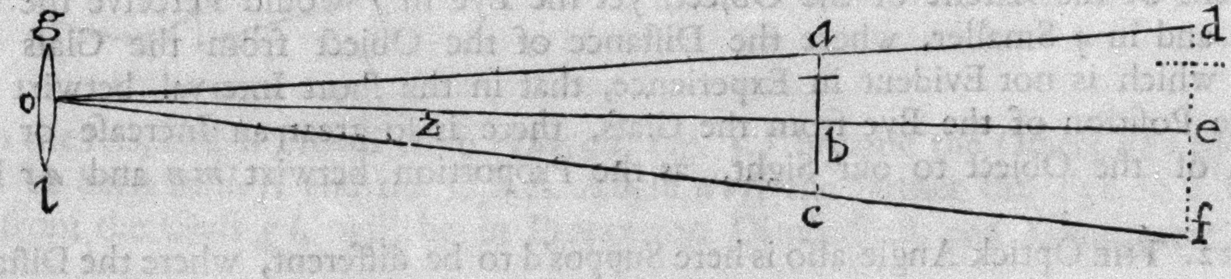
Argument the Fifth.

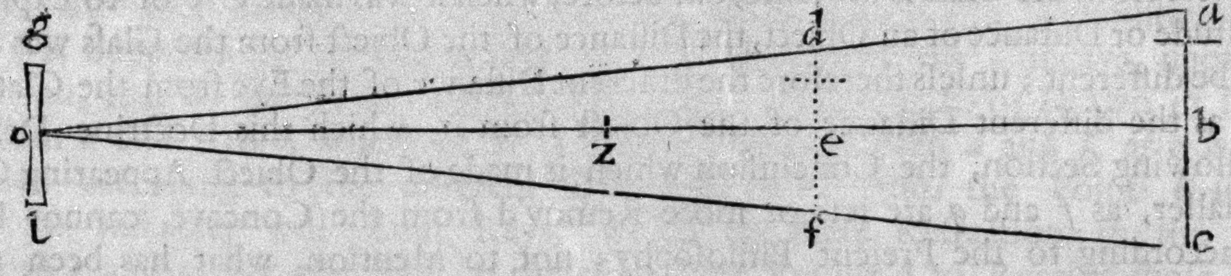
§. 23. Lastly, it is Acknowledg'd, that it is not the ſame for the Eye to be Remov'd from the Glaſs, as for the Object, ſince in Moving the Glaſs from the Eye, the Glaſs Approaches the Object, and the Apparent Place of the Object is Chang'd, but in Moving the Eye to, or from the Glaſs, the Apparent Place of the Object never Alters; and yet a Famous Optician, and one of the moſt Sagacious Men of the Age, he Lived in, the Great *Kepler*, Endeavours to Prove the ſame of the Glaſs being Remov'd from the Eye, which is here Affirm'd of the Eye being Remov'd from the Glaſs, which Shows, how Complying this Doctrine of Rays is, which can Demonſtrate both the Sides of a Contradiction, that the Rays are Refracted in the ſame manner, where the Apparent Place of the Object is Changed, and where it is not Changed.

Concerning Objects being more Diminiſhed, as long as the Glaſs Continues nigher to the Eye than to the Object, the farther the Concave is Remov'd from the Eye. Molyneux's Dioptricks Prop. 49. Part 1. Tab. 31. Fig. 3.

§. 24. THE Laſt Propoſition, which Concerns Single Glaſſes Applied to the Eye, is, that the farther a Concave is Remov'd from the Eye, the Objects are thereby the more Diminiſhed, as long as the Glaſs Continues nigher to the Eye than to the Object; this Propoſition is alſo ſaid to be Univerſally True both of Concaves and Convexes, unleſs that in the Convex the Object is to be nigher the Glaſs than the Glaſs's Focal Length, or betwixt the Glaſs and it's Focus; from whence will Reſult this General Propoſition, that a Convex Glaſs, being Equally Diſtant from the Eye and from the Object, Renders the Appearance the moſt Magnified, and a Concave the moſt Diminiſhed, that That Diſtance of the Eye and Object will allow; This is Prov'd from the Preſent Figure, and Figure §. 16. becauſe the Angle *doe* or *dof* (which is the Angle, under which the Object Appears thro' the Glaſs) where the Glaſs is Equally Diſtant from the Eye and the Object, is greater in the Convex and leſs in the Concave, than where the Diſtance between the Eye and the Glaſs is greater or leſs than between the Glaſs and the Object; and therefore, when the Glaſs is Equally Remov'd from the middle towards the Eye, or towards the Object, the Appearance is Equally Magnify'd by the Convex, and Diminiſhed by the Concave, for the Angle *doe* is Equal, where the Diſtance between the Eye and the Glaſs is greater or leſs than between the Glaſs and the Object; This Doctrine is farther Demonſtrated from the Magnify'd Appearances of Objects thro' Convexes, and their Diminiſhed Appearances thro' Concaves, Deduced from the Apparent Places of Objects thro' thoſe Glaſſes; for it is ſaid, we may eaſily Conceive, that in the Convex Glaſs from (*Tab.* 34. *Fig.* 2.) and in the Concave from (*Tab.* 34. *Fig.* 3.) Suppoſing either of the Glaſſes *gl* to Touch the Eye in *o*, and that the Apparent Place of the Object *abc* is *def*, the Eye Perceives the Object under it's own Natural Optick Angle, neither Magnify'd nor Diminiſhed by the one or the other Glaſs; likewiſe if in the ſame Figures we Conceive the Glaſſes (which are Suppos'd of the leaſt Thickneſs Imaginable) to Touch the Objects, the Object and Apparent Place of it are the ſame, and Conſequently the Optick Angle in this Poſture can neither be Increaſed nor Diminiſhed by either of the Glaſſes, wherefore it Remains, that Seeing the Glaſſes do not at all Exert their Effects in either of the Extremes, that is, either Touching the Eye or Touching the Object, and Seeing they Exert their Effects Equally, being Equally Remov'd from the middle Point *z* between the Eye and the Object, it is Affirm'd, it will follow, that they Exert their Effects moſt Powerfully







fully, being Placed juſt in the middle Point *z* between the Eye and the Object, that is, the Convex by Magnifying, and the Concave by Diminiſhing the Appearance.

§. 25. THE firſt Thing, which is Affirm'd in the Preceding Section is, in the Propoſition it's ſelf, that the farther a Concave is Remov'd from the Eye, the Objects are thereby the more Diminiſhed, as long as the Glaſs Continues nigher to the Eye, than to the Object, which from (*Fig.* §. 18.) is Falſe; ſince there it is Endeavoured to be Proved in General, that Objects are Diminiſhed, as the Eye Removes from the Concave, from *n m* being leſs than *a c*, and if the Reaſoning upon that Figure is True, it Obtains in all Caſes, where the Eye is ſtill farther Removed from the Glaſs, without the Exception Mention'd in this Propoſition, *as long as the Glaſs Continues nearer to the Eye than the Object*, either therefore that Reaſoning is Unjuſt or this is not True.

Argument againſt this Demonſtration.

§. 26. NOTWITHSTANDING which, the Univerſality of this Propoſition both in Reſpect of Concaves and Convexes is next to be Conſidered, and the Reaſons Alledg'd for the Proof of it; The General Propoſition is, that a Convex Glaſs being Equally Diſtant from the Eye, and from the Object, Renders the Appearance the moſt Magnify'd, and a Concave, being Equally Diſtant from the Eye and from the Object, the moſt Diminiſhed; and the Reaſons for it are from (*Fig.* 1. §. 24.) and (*Fig.* §. 16.) becauſe the Angle *d o e* or *d o f*, where the Glaſs is Equally Diſtant from the Eye, and the Object, is greater in the Convex and leſs in the Concave, than the ſame Angle is, either where the Diſtance between the Eye and the Glaſs is greater than between the Object and the Glaſs, in the Convex or Concave, or where the Diſtance between the Eye and the Glaſs is leſs than the Diſtance between the Object and the Glaſs, in a Convex or Concave.

The Univerſality of this Propoſition Conſidered.

§. 27. IT is Evident this Reaſoning Depends upon that, which is Uſed in (*Fig.* 1. §. 24. and *Fig.* §. 16.) which ſince we have Particularly Conſidered and Examin'd, it will not be here Neceſſary to Repeat the Confutation of it; And we ſhall only farther Obſerve, that *d e f* is Aſſum'd for the Object, which is yet no other than the Apparent Place of it, but how the Glaſs can be ſaid to be nearer to the Object than to the Eye or farther from it, becauſe the Apparent Place of the Object is ſo, wants an Explanation, Eſpecially, ſince in (*Fig.* §. 16.) *d o f* in the Concave, and the ſame may be ſaid of the Convex, is Adjuſted, as to it's Magnitude, by the Object *a b c*, being more or leſs Diſtant from the Glaſs, without any Conſideration of the Diſtance of the Eye from it, which by what goes before, and is Argu'd upon (*Fig.* 1. and 2. §. 3.) will at a greater or leſs Diſtance Receive the Rays in the ſame Divergence.

This Reaſoning Depends upon Principles and Propoſitions before Confuted.

§. 28. OMITTING therefore this Demonſtration, which Refers us to Figures that have been already Refuted, we ſhall now Inquire into another, which is Propoſed to us from (*Fig.* 2. 3. §. 24.) Namely, that Suppoſing the Convex or Concave Glaſs *g l* to Touch the Eye in *o*, and that the Apparent Place of the Object *a b c* is *d e f*, the Eye Perceives the Object under it's own Natural Optick Angle, neither Magnify'd nor Diminiſhed, if alſo the Convex or Concave Glaſs are Suppoſed to Touch the Object, the Object and it's Apparent Place are the ſame, and Conſequently the Optick Angle in this Poſition is neither Increaſed nor Diminiſhed; therefore, Seeing the Glaſſes do not at all Exert their Effects in either of the Extremes, that is, either Touching the Eye or Touching the Object, and Seeing they Exert their Effects Equally, being Equally Remov'd from the middle Point *z*, between the Eye and the Object, it will Enſue, that they Exert their Effects moſt Powerfully, being Placed in the middle Point *z* between the Eye and the Object, that is, the Convex by Magnifying, and the Concave by Diminiſhing the Appearance.

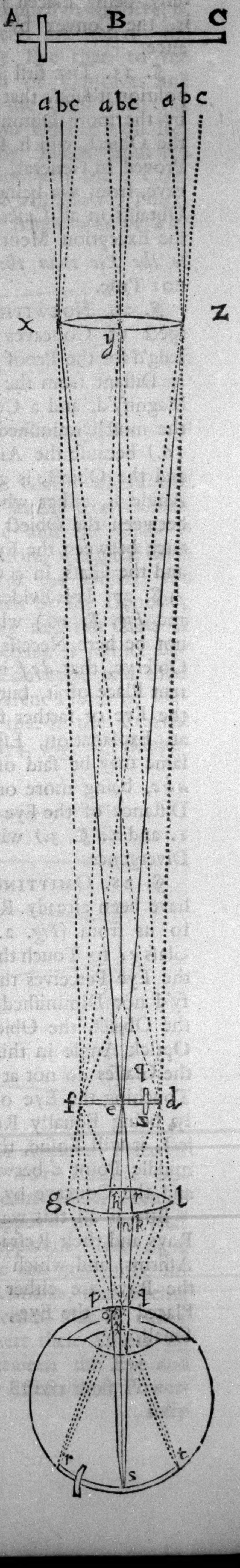
Another Demonſtration of this Propoſition and the ſeveral Reaſons againſt it.

BUT if all this was True, the Solution would not be owing to the Doctrine of Rays and their Refractions to a Convergence or a Divergence, which this Philoſophy Affirms, and which we are at Preſent only Concern'd to Oppoſe, on the Contrary, the Rays are either very little, or not at all, Refracted, where the Glaſſes are Placed at the Eye, and where their Thickneſſes are Imagin'd to be Infinitely Small.

Nor can this Reaſoning be Valid in it's ſelf; for if the Rays are not Refracted, or are Inſenſibly ſo, there can in this Philoſophy be no difference betwixt the Object and the Apparent Place of it, and *abc* and *def* will Coaleſce, which are Preſum'd to be in very different Situations.

Moreover, if, when the Glaſs Touches the Eye or the Object, there is no difference in the Magnitude of it, inſtead of Concluding from thence, that in the middle Diſtance of the Convex betwixt the Eye and the Object, the Object will be Enlarged, and in a Concave Diminiſhed, we ought to Infer the Contrary, that, ſince in the greateſt Remove of the Eye from the Glaſs, or in the leaſt, the Magnitude of the Object is not Changed, it will not be in any Intermediate Diſtances.

Lastly, if this was Granted, that in Intermediate Diſtances it's Magnitude ſhould be Increaſed in the Convex and Diminiſhed in the Concave, yet it will be Impoſſible to give a Reaſon in this Philoſophy of this Exertion of the Glaſs's Power, and of the Glaſs's Power it's ſelf, unleſs by Attributing both to a certain Occult Quality, which tho' it does not Offer to Explain, yet in other Caſes it would make us Believe, it very much Deſpiſes the Aſſuming any ſuch unknown and Fantaſtick Principles.



CHAP. XIII.

Concerning the Propoſitions, which Relate to Teleſcopes and Microſcopes.

After Single Glaſſes the Compoſition of them is Conſidered.

§. 1. SINGLE Glaſſes Apply'd to the Eye was the laſt Thing, which this Philoſophy Conſidered, the next is a Compoſition of theſe Glaſſes in Teleſcopes, where the Object-Glaſs is Defin'd to be the Glaſs, which is Placed towards the Object, and the Eye-Glaſs that towards the Eye; but if there be more than one, the firſt Eye-Glaſs is that next the Object-Glaſs, the ſecond is that next the firſt, and ſo on.

Concerning Teleſcopes which Conſiſt of a Convex Object-Glaſs and a Convex Eye-Glaſs of a leſs Sphere or greater Convexity. Molyneux Dioptricks, Prop. 50. Part. 1.

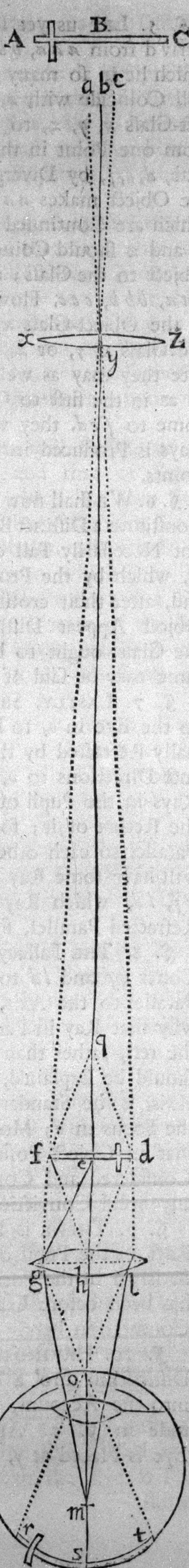
§. 2. After which Definitions, the firſt Propoſition is to Explain a Teleſcope Conſiſting of a Convex Object-Glaſs and a Convex Eye-Glaſs of a leſs Sphere or greater Convexity, (*Tab.* 35. *Fig.* 1.) *ABC* is a Diſtant Object, which Emitts Rays from the Left Hand-Point *A*, *ax*, *ay*, *az*, from the middle Point *B* *bx*, *by*, *bz*, and from the Right-Hand Point *C*, *cx*, *cy*, and *cz*, to the Object-Glaſs *xyz*, where they are Refracted to the Diſtinct Baſe *fed*; *ghl* the Eye-Glaſs is Placed ſo, that *fed* and the Eye in *o* may be nearly in it's Focus, but the Eye rather beyond it, and Conſequently the Diſtance betwixt the Glaſſes is the Aggregate of their Focal Lengths; The Object, the Glaſſes, and the Eye, being thus Diſpoſed, the Eye, it is ſaid, will Perceive the Object *ABC* Diſtinct, Magnify'd, and Inverted.

1*ſt*. It will Appear Diſtinct, becauſe the Rays from each Point, being made by the Object-Glaſs to Converge towards the Image *fed*, Proceed forward from thence Diverging, and ſo Fall on the Eye-Glaſs; thus *bx*, *by*, *bz*, Converge into *xe*, *ye*, *ze*, and Croſſing at *e*, Diverge to *h*, but *e* being in the Focus, the Rays at *h* will be Refracted Parallel to the Eye in *o* by a former Propoſition, and from thence are brought together in the Point *s* on the Retina, according to what is before Deſcrib'd Concerning the Refractions of the Chryſtalline, and the other Humours; In like Manner *ax*, *ay*, *az*, Converge to *d*, from thence Diverge to *l*, afterwards are Refracted Parallel to *o*, and at laſt meet in *r* on the Retina, and the ſame may be ſaid of *cx*, *cy*, *cz*, which after their ſeveral Refractions to a Convergence,

vergence, then a Divergence, and ſo to a Paralleliſm, at length Arrive in *t*.

2*d*. THE Object will be Magnify'd from (*Tab.* 35. *Fig.* 2.) ſince if the Naked Eye were in the Place of the Object-Glaſs at *y*, the Object would Appear to it under the Angle *a y c*, = *f y d*, that is, the Object would Appear to the Eye at *y* under the ſame Angle, as the Diſtinct Baſe *f e d*, were it an Object, would Appear to the ſame Eye at *y*; let therefore now the Diſtinct Baſe *f e d* be Conſider'd as an Object, this Appears to the Eye at *o* through the Eye-Glaſs under the ſame Angle *g o l*, as were the Naked Eye Viewing it at *h* by a former Propoſition; Make *e q* = to *e h*, and Draw *h f*, *h d*, and *q f*, *q d*, the Angle *f h d*, by another Propoſition, is Equal to *g o l* the Optick Angle thro' the Teleſcope, and the ſame Angle *f h d* is Equal to *f q d*, but *f q d* is much greater than *f y d* = *a y c*, the Natural Optick Angle to the Eye ſuppos'd at *y*, and yet much greater than the Natural Optick Angle would be to the Eye at *o*, becauſe *o* is yet farther from the outward Real Object than *y*, by the whole Length of the Teleſcope.

3*d*. THE Object is Inverted from (*Fig.* 1. Preſent §.) ſince the Diſtinct Baſe *f e d* is Inverted in Reſpect of the Object, and therefore *r s t* in the Retina is Erect in Reſpect of it, which will Conſequently make, by the foregoing Doctrine, the Object Appear Inverted.



This Philoſophy does not Explain how the Object Appears Diſtinct; Argument the Firſt.

§. 3. As to the Object Appearing Diſtinct, we may be Juſtly Allow'd to ſay in Confutation of it, that the Original Diſpoſition of the Rays Falling from the ſeveral Points of the Object *A*, *B*, *C*, ſo as to Poſſeſs each Point of the Object, and to ſeem to Flow from it, is ſomething Extraordinary; for if *A* can Emitt Rays in that manner, that they ſhall Appear to be Deriv'd from *a*, *a*, *a*, there is no Reaſon can be Aſſign'd, why it may not ſend Rays likewiſe to the Points *b*, *c*, *b*, *c* which lie betwixt *a* on the Left Hand and that on the Right, and Conſequently, why *A* ſhould not have as great an Influence in the Refraction of it's Rays, thro' the Object-Glaſs and the Eye-Glaſs, to the Eye or the Retina of it, as *B* or *C*, betwixt the ſame Left-Hand and Right-Hand *a*; for if it has not the ſame Influence, *A* has a Particular Impreſſion of it's own, which is Inferior to that of *B* or *C*, tho' they are Points of the ſame Object; if it has the ſame Influence, it will be Impoſſible to Say, how *b b*, *c c* have their Particular Influence upon the Eye in Diſtinction from it, which this Scheme ſuppoſes; and what has been Argu'd Concerning the Point *A*, is Valid in Reſpect of *B* or *C*.

Argument the Second.

§. 4. IF yet the Points *A*, *B*, and *C*, could be ſo Wiſe and Sagacious, as to Emitt their Rays in Diſtinction from one another, ſo as *A* in Particular (and the ſame may be ſaid of the Reſt) ſhould Chooſe it's Place of Refraction to the Left Hand of *b* and *c*, and then to the Right Hand of the ſame *b* and *c*, afterwards to the Right of the ſame *b* and *c*, and then to the Left of them, and ſo on, Suppoſing this Wonderful Sagacity in the Rays, they will be hard put to it to Contrive, how they ſhall come with any Order or Diſtinction to the Eye; *a a a*, *b b b*, *c c c*, are therefore now Suppoſed to be Deriv'd from the ſeveral Points of the Object *A*, *B*, *C*, notwithſtanding which, if the Left-Hand *a* is Refracted thro' the Object-Glaſs to *d*, why the middle or the Right-Hand *a* ſhould be Refracted to the ſame Point, rather than *b*, which is Refracted to *e*, or *c*, which is Refracted to *f*, is Unaccountable; ſince *b* and *c* are both to the Left and Right of *a*, which is yet Refracted to a Point to the Right of what either of the others are Refracted to.

Argument the Third.

§. 5. Let us yet Imagine, that the Rays are by good Fortune of good Conduct Arriv'd from aaa, bbb, ccc, to the Diſtinct Baſe def, why the Rays aaa, bbb, ccc, which lie in ſo many Planes, that Terminate at length in A, B, and C, and therefore will Coincide with a, b, c, ſhould rather be Collected from ſeveral Points in the Object-Glaſs x, y, z, to one Point in the Diſtinct Baſe d, e, or f by Converging, than from one Point in the Object-Glaſs x, y, or z be Diſperſed to ſeveral, in the Diſtinct Baſe, d, e, f, by Diverging, will be Impoſſible to Affirm; for as the great Diſtance of the Object makes aaa Coincide, as alſo, bbb, and ccc in Reſpect of the Rays, which are Continued from the Glaſs to the Object, ſo likewiſe on the Contrary x, y, and z ſhould Coincide too in Reſpect of the Rays, which are Continued from the Object to the Glaſs; and Conſequently there is no more Reaſon to make the Rays aaa, bbb, ccc, Flow from Single Points in the Object A, B, C to ſeveral Points in the Object-Glaſs x, y, z, than to make abc, abc, abc, Flow to one Point of the Glaſs x, y, or z, Suitable to what any one of them Separately does, and therefore they may as well in the laſt Caſe be Suppoſed to Diverge from either x, y, or z, as in the firſt to Converge from x, y, and z; from whence tho' the Rays ſhould come to fed, they would not yet be Capable of forming a Diſtinct Baſe, which always is Produced in this Philoſophy by a Collection of Rays to ſeveral Diſtinct Points.

Argument the Fourth.

§. 6. We ſhall now however Preſume, that the Rays come to fed, and that they there Conſtitute a Diſtinct Baſe, thoſe notwithſtanding, which are Refracted from aaa to d, will not Neceſſarily Fall on the Glaſs at l, but may, for all that is Demonſtrated, paſs by it, which by the Proportion of this Figure ſeveral of them ſeem Actually to do, and, after their croſſing each other at d, to Proceed towards m; whereas to make an Object Appear Diſtinct, all the Rays, which are Refracted from one Point thro' the Glaſs, ought to be Tranſmitted to another in the Retina of the Eye; and the ſame may be ſaid of the Rays from ccc.

Argument the Fifth.

§. 7. Lastly, Suppoſing lo, ho, go, that is, the Rays Refracted from the Eye-Glaſs to the Eye in o, to Poſſeſs or Engroſs in their Progreſs all thoſe, which were Originally Refracted by the Glaſs xz, and that they are Parallel in their ſeveral and Diſtinct Directions to o, yet this Doctrine cannot Account for the Confuſion of the Rays in the Pupil of the Eye, which is ſaid to be Neceſſary to a Diſtinct Viſion in the Retina of it; for to make the Rays Mix together in o, fg, eh, dl ought to be Parallel to each other, which is thus Prov'd, becauſe the Cone of Rays zfx or xdz will have ſome Ray or other Parallel to ey, and Conſequently the Cone of Rays gf, ld, which Ray therefore will be Refracted to o, and the reſt of them will be Refracted Parallel, ſince f and d are in the Diſtinct Baſe.

Wherein the Fallacy of this Reaſoning lies.

§. 8. The Fallacy of this Reaſoning lies in the Reducing all the Rays of the Cones gf and ld to the ſame Laws of Refraction, which are Obſerved by the Rays Parallel to the Axis, which are Acknowledged to be one only in each Cone, but why that Ray in Particular, which is Parallel to the Axis, ſhould Preſcribe a Rule to the reſt, rather than the reſt, or any one of them, to it, is not, nor is it Poſſible it ſhould be Explain'd, but if any of the other Rays, beſides the Parallel one to the Axis, is the Standard for the Refractions of the Reſt, they will not be Refracted to the Focus in o; Moreover the Foundation, upon which this Demonſtration Proceeds, that an Object Poſited in a Focus will Refract the Rays Parallel, has been already Conſidered and Confuted, and therefore cannot have a Place here, ſo as to give us any Solid Conviction of what it is Deſigned to do.

Another Explanation offer'd for the Confuſion of the Rays in the Pupil of the Eye.

§. 9. There is likewiſe another Explanation offered us of this Confuſion of the Rays in the Pupil of Eye, which ſhows, that the Preſent Doctrine was not fully Satisfy'd in the Former; but this Depends upon *Prop. 5th.* of this Philoſophy, which has been before Examined, and does not upon that Account Require a Particular Confutation here.

An Account how the Magnitude of the Object is Increaſed.

§. 10. Having taken Notice of the Arguments Alledged for a Proof of the Object's Diſtinction thro' a Teleſcope in this Philoſophy of Rays, we are next to Inquire into the Account, which is propoſed for the Solution of the Increaſe of it's Magnitude in §. 2. Art. 2. where it is Affirm'd, that if (*Fig.* 2. §. 2.) the Naked Eye is Placed at y, the Object will Appear to it under the Angle $ayc=fyd$, that is, the Object will Appear to the Eye at y under the ſame Angle, as the Diſtinct Baſe fed, were it an Object, would Appear to the ſame Eye at y.

Argument the Firſt againſt this Account.

§. 11. In this firſt Part of the Account of the Object's Magnitude, it is ſuppos'd, that ayc is the Natural Optick Angle, and Conſequently, that abc is the full Extent of the Object ABC, and therefore that aaa, bbb, ccc in (*Fig.* 1*ſt.* §. 2.) are Ultimately Reſolved into a, b, c; if ſo, what is Argued in §. 5. is Valid, and we have here a Confeſſion of what we there Endeavoured to Prove to be an Apparent Weakneſs and Infirmity in this Hypotheſis of Rays.

Argument the Second.

§. 12. Besides which, if ayc was the Natural Optick Angle, it is moſt Evident, that a and c muſt be the utmoſt Extremities of the Object, and therefore, that no Rays can Poſſibly Fall from ſuch an Object, unleſs to y; for if they could Extend themſelves to x or z, and be there Refracted, it would be Impoſſible for ayc to be the Natural Optick Angle, ſince that would be Determin'd by the Refractions of ax, az, and the Purſuit of them back in a Direct Line, but if no Rays can be Refracted beyond a and c, the whole (*Fig.* the 1*ſt.* §. 2.) is ſet aſide, where aaa, ccc, are ſuppoſed to be Diſtinctly Refracted; either therefore that Figure, and the Reaſoning upon it, or the Preſent, is Triffling and Abſurd.

Optick Demonſtrations like thoſe of the Corpuſcular Philoſophy.

§. 13. This way of Proceeding in the Optick Demonſtrations will Oblige us to Draw a Compariſon betwixt the Preſent Philoſophy of Rays, and that of Small Corpuſcles, or Particles of Matter, the Abſurdity and Folly of which has been Sufficiently Evidenced; For, as in the Corpuſcular Philoſophy, ſometimes the ſame Body or Subſtance is made to Conſiſt of Oblong Parts, then of Globular, afterwards of Pointed, and ſo on, according to the Exigence of the Phænomenon, which is to be Explain'd, ſo in the Philoſophy of Rays, at one Time they ſhall Proceed in a Direct and Parallel Courſe, at another in an Oblique, Juſt as the Nature of the Solution Requires, without any Regard had to the Reality of Things; which is not only Manifeſt from Comparing theſe two Figures together, but from almoſt all the Solutions, which are given us in this Doctrine.

Argument the Third.

§. 14. Let now $ayc = fyd$ be granted to be the Natural Optick Angle; That gol is Equal to $fhd = fqd$ cannot be Prov'd, whatever former Propoſitions are Alledged to Evince it, of which Propoſitions we ſhall not Repeat the Confutation; we ſhall only farther Say, that if gol is Equal to fhd, ſince fg, by what goes before, is the Ray, which is Parallel to the Axis he (for upon that Suppoſition fg is Refracted to o by *Fig.* 1*ſt.*) gfh will be $=$ to $fhe =$ to goh, and therefore gfh will be $=$ to $goh =$ to geh, and Conſequently he is $=$ to gf, that is, the Direct Line betwixt the Diſtinct Baſe and the Glaſs gl, and the Oblique one betwixt the Glaſs and the ſame Diſtinct Baſe, which is Impoſſible.

Argument the Fourth.

§. 15. Besides, if fg was not Suppoſed Parallel to eh, gol could not be Equal to fhd, for ſhould the Object be greater or leſs than ABC, the Natural Optick Angle would be greater or leſs than $ayc =$ to fyd, and Conſequently the Sides, which Comprehend it, would Suitably Retire from or Approach to each other; if therefore goh, which is half of gol, ſhould be Imagin'd Equal to fhe the half of fhd, in this Situation of the Rays, ſince fg is not Parallel to he but Diverges from it, in any other Poſition of the Rays they could not be Equal, becauſe if the Ray yfg comes nearer to the Axis or Removes farther from it, in both Caſes, it will Intercept a a leſs Portion of the Diſtinct Baſe ef than of the Glaſs gh, wherefore oh, he being Equal, the Line, which is Drawn from o to g, which is at a greater Diſtance from h, than f from e, will with the Axis Contain a greater Angle goh, than the Line Drawn from h to f will with the ſame Axis; For which Reaſon we may alſo Think, that goh, even in the Preſent Poſture of the Rays, will not be Equal to fhe, unleſs o, being ſomething farther Diſtant from the Glaſs than e, can Compenſate for this Inequality, which if it can, it yet cannot for the Irregular Interſections of fe, gh, by the Line yfg Moving upon the Center y, Compar'd with the Variations of the Angles fhe, goh, Form'd by the Motion of the Lines og, hf upon their ſeveral Centers o and h, which Motions, according to what we have before Explain'd, will Reſult from Suppoſing the Object Bigger or Leſs than ABC.

Argument the Fifth.

§. 16. Moreover it is ſaid in §. 2. that $gol = fhd$ is Equal to fqd by Construction, but fqd is greater than fyd the Natural Optick Angle of the Eye Placed at y, and therefore much greater than the Natural Optick Angle, when the Eye is Preſumed to be in *o*, becauſe it is then farther Diſtant from the Object than in *y*, by the whole length of the Teleſcope; in Reply to which we muſt Obſerve, that if *ayc* is the Natural Optick Angle to the Eye Situate in *y*, that is, if *ay*, *cy* are the outmoſt Rays, which Fall from the Extremities of the Object *A* and *C*, it is Impoſſible there ſhould be any Natural Optick Angle in *o*, becauſe they would not be the outmoſt, ſince *ao*, *co*, would be External to them; if *ay*, *cy* are not the outmoſt Rays, which Flow from the Extremities of the Object, there can be no Reaſon given, why Rays may not be Drawn from *A* and *C* to *y*, ſo as to Comprehend a greater Angle than *ayc*, as well as to *o*, ſo as to Comprehend a leſs, and Conſequently *ayc* will not be the Natural Optick Angle to the Eye Placed in *y*.

Argument the Sixth.

§. 17. If yet *aoc* was the Natural Optick Angle to the Eye in *o*, it would Prove too much, Seeing ſuch an Angle would be in a manner Infinitely Small in Reſpect of the Refracted Optick Angle *gol*, whereas Objects are not by any Teleſcopes Magnify'd in that, but in a Manifeſtly Definite and a much leſs Proportion; for Suppoſing *gol* to be 30 Degrees, and *aoc* a ſecond of a Minute, which ſeems to be the moſt, the Object will be Increaſed to 10800 Times it's Natural Bigneſs, which is not Experienced in the Largeſt and Beſt Teleſcopes, which make the Diameter of the Object Appear only 48 Times as Great, as when it is View'd by the Naked Eye, and the Circular Surface 2304 Times as Great.

Argument the Seventh.

§. 18. On the other Hand, ſince in (*Fig. 1ſt.*) *ax*, *cz* are Refracted by the Glaſs from the Extreme Parts *A* and *C*, the Natural Optick Angle ſhould not be, what is Expreſſed in (*Fig. 2d.*) *aoc*, but *axozc* in (*Fig. 1ſt.*), which are widely Different from each other.

Argument the Eighth.

§. 19. Laſtly, in (*Fig. 2d.*) as the Object, if this Reaſoning was True, Approached, it would be leſs Magnify'd by the Teleſcope, and as it Receded, more, which ſeems to be Abſurd and Impoſſible; for in the firſt Caſe, the Natural Optick Angle *aoc* would be Enlarged, and in the ſecond Diminiſhed, whilſt *fhd* = *gol* Continued nearly the ſame, that is, in the former Caſe there would be a leſs Proportion betwixt the Natural Optick Angle, and the Refracted, and in the latter a greater.

The Inverſion of the Object not Neceſſary to be Conſidered.

§. 20. As to the Inverſion of the Object, mention'd in §. 2. by the Image's being Inverted in the Focus of the Convex *xyz*, that has been already Conſidered, and does not Require a Re-examining, nor yet the Apparent Place of the Object, which is Subjoined, as a Scholium to this Demonſtration.

An Account of the Apparent Diametral Magnitude of an Object, View'd thro' the foregoing Teleſcope, in which the Lemmas are firſt Conſider'd.
Molyneux's Dioptricks Prop. 51. 52. 53. Par. 1.

§. 21. The next Propoſition is to Aſcertain the Apparent Diametral Magnitude of an Object view'd thro' the former Teleſcope; in Order to which it is thought Neceſſary to Premiſe two Lemmas; the firſt is, that if the Eye directly Approaches to or Recedes from an Object, it will be as the Tangent of the Semi-Optick Angle of one Station, to the Tangent of the Semi-Optick Angle of the other Station, ſo Reciprocally the Diſtance of the Eye from the Object in this latter Station, to the Diſtance of the Eye from the Object in the former; which is Prov'd from (*Tab. 35. Fig. 3.*) in which *acb* (See Scheme §. 23.) is an Object, from whoſe middle Point *c*, *ce* is a Perpendicular to *ab*, the firſt Station of the Eye is at *e*, and the ſecond at *d*, and the Proportion will be, as *ce* to *cd*, ſo the Tangent of the Angle *adc*, to the Tangent of the Angle *aec*; for let *ca* be Infinitely Produced towards *z*, and *ez* be drawn Parallel to *ad*, the Angle *zec* is = to the Angle *adc*, and *ec* being put for the Radius, *zc* is the Tangent of the Angle *zec* = *adc*, and *ac* is the Tangent of the Angle *aec*; but as *ce*. *cd* : : *zc*. *ac*, which was to be Demonſtrated.

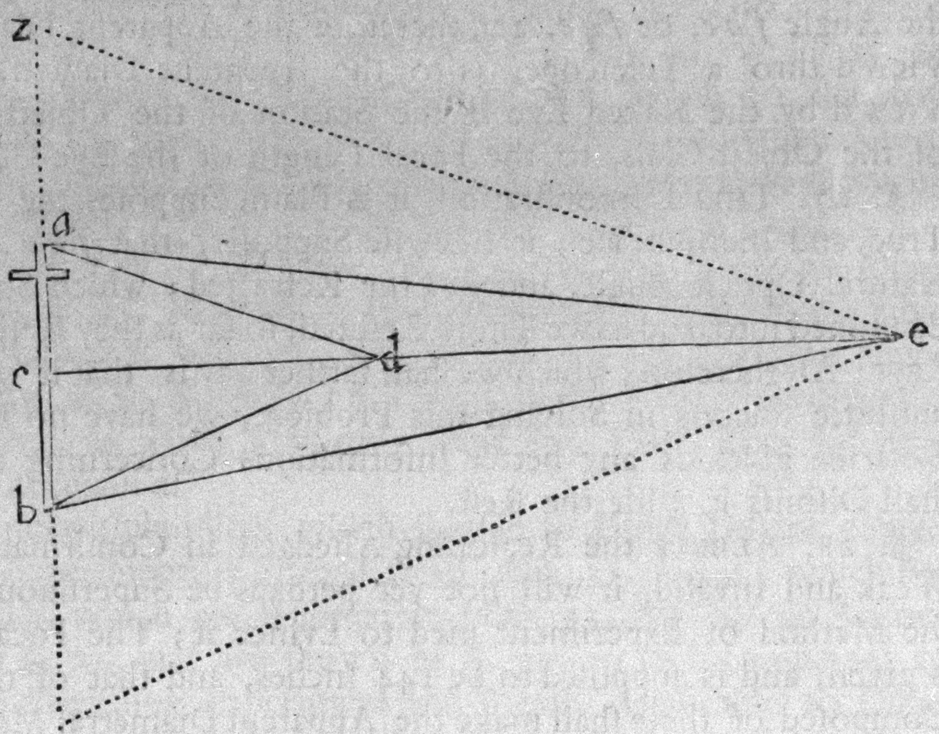
Argument against the Firſt Lemma.

§. 22. But this Demonſtration ſuppoſes the Philoſophy of Rays to be True, that the Object is ſeen by *ad*, *bd*, and *ae*, *be*, and under thoſe Angles, which they Comprehend, whereas it has been before Prov'd, that all theſe Hypotheſes are either Falſe or Precarious, and therefore, altho' the Mathematick and Abſtracted Reaſoning upon theſe Lines may Poſſibly be Juſt, yet, if the Lines themſelves are Arbitrarily Introduced, ſuch a Demonſtration will not be any Thing to the Purpoſe of a Real Philoſophy.

Argument against the Second Lemma.

§. 23. And for the ſame Reaſon the ſecond Lemma will Signify as little, which is, that if the Eye directly Approaches to, or Recedes from an Object, it's Apparent bigneſs Increaſes or Diminiſhes, as the Tangents of the Semi-Optick Angles at the one and the other Station; This is ſaid likewiſe to be Manifeſt from the ſame Figure; be-

becauſe the Eye at d Sees ac as Big, as the Eye at e would See zc, ſince the Angles adc, zec are Equal, and it is a Maxim or Axiom in theſe Opticks, *that thoſe Objects, which Appear under an Equal Angle ſeem Equal*, therefore the Eye advancing from e to d Sees ac as much Bigger, as if continuing at e the Object ac had Increaſed to cz.



Argument the Second.

§. 24. ALTHO' this Reaſoning is Anſwered by what goes before, we ſhall yet Conſider the Particular Foundation of it, as it Depends upon the Equality of the Appearance of an Object, and the Equal Angle of it's Viſion; which if it were True, a Brighter or more Luminous Object would have the ſame Conditions in the Repreſentation of it's Magnitude with one that is leſs ſo, whereas the Contrary is found by Experience, and is Acknowledged in the Preſent Philoſophy, where in Proportioning Teleſcopes to the Eye, not only the Deepeſt Charge is Conſidered, that is, ſuch an Object Glaſs, which will bear an Eye Glaſs of the greateſt Convexity, but alſo the Object we Contemplate; for Objects of a Sedate Light, as *Saturn*, *Jupiter*, &c. will allow deeper Charges than thoſe of a more Lively and Strong, as *Venus*, *Mercury*, and *Mars*.

Argument the Third.

§. 25. BESIDES, even in Common Objects, with which we are Daily Converſant, it is Evident, that upon ſeveral Removals of them from the Eye, where the Angle is Manifeſtly Changed, the Appearance is not, as well as in Luminous Objects, where the Angle is not Changed, the Appearance is; ſo far is the forementioned Maxim from Obtaining in a Real and not a Notional and Abſtracted Philoſophy.

The Propoſition it's ſelf Explain'd.
Prop. 53.

§. 26. WE ſhall therefore now, after this ſhort Conſideration of the Lemmas, Proceed to the Propoſition it's ſelf, which is; That the Apparent Diametral Magnitude of an Object, View'd thro' the Teleſcope before Deſcribed, is to the Apparent Diametral Magnitude of the Object View'd by the Naked Eye at the Station of the Object-Glaſs, as the Focal Length of the Object-Glaſs to the Focal Length of the Eye-Glaſs; This Propoſition is Affirm'd to be Evinced by no Dioptrick Writers, neither by *Kepler*, *Galileus*, nor *Faber*, and as to *Dechales*, He was ſo far from Thinking it Capable of a Demonſtration, that He Believed the Propoſition it's ſelf Falſe; what is ſaid to be done is, that they have Endeavoured to Prove it by Experiments and there Leave it, which is taken up by theſe Opticks and Pretended to be Fully Cleared and Explained from the following Demonſtration; The Object-Glaſs xyz (*Fig.* 2. §. 2.) Projects the Image of the Object ABC to the Diſtinct Baſe fed; ghl is the Eye-Glaſs, he the Focal Length of it, to which eq is made Equal, and the Lines hf, hd, qf, qd are Drawn, the Eye at o is Placed in the Exteriour Focus of the Eye-Glaſs, and the Rays yfg, ydl, are Refracted by the Eye-Glaſs and Croſs at o; Wherefore gol is the Angle, under which the Object Appears thro' the Teleſcope, but the Angle gol is $=$ to $fhd =$ to fqd, if therefore the Naked Eye were at y, it would Perceive the Object under the Angle $fyd = ayc$, and thro' the Teleſcope under the Angle fqd, the halves of which Angles are fye, fqe, which may be made Uſe of Inſtead of their wholes, as alſo fe Inſtead of fd; by Lemma 2*d.* the Apparent Bigneſs of the Object fe at the Station q, is to it's Apparent Bigneſs at the Station y, as the Tangent of the Angle fqe, to the Tangent of the Angle fye, and by Lemma 1*ſt.* the Tangent of the Angle fqe is to the Tangent of the Angle fye, as ey, to eq; from whence it follows, that the Apparent Bigneſs of the Object fe at the Station q is to it's Apparent Bigneſs at the Station y, as ey to eq, that is, as the Focal Length of the Object-Glaſs, to the Focal Length of the Eye-Glaſs; but goh the Angle, under which the Eye Sees half the Object through the Teleſcope, is Equal to the Angle fhe, or fqe, and therefore the Apparent Diametral Bigneſs of an Object View'd thro' a Teleſcope, is to the Apparent Diametral Magnitude of an Object View'd by the Naked Eye at the Station of the Object-Glaſs, as the Focal Length of the Object-Glaſs, to the Focal Length of the Eye-Glaſs. *Q. E. D.*

The foregoing Demonſtration Invalid.

§. 27. THIS Demonſtration, it is Plain, Suppoſes the two Preceding Lemmas to be True, and Indiſputable; it likewiſe Suppoſes, that fhd is $=$ to gol, that ayc is the Natural Optick Angle, and gol the Refracted; which Suppoſitions have been already Examined into, and as we Think Refuted, ſo far as they Reſpect the Preſent Philoſophy of Rays; All therefore, which we ſhall farther Say is, that if other Optick Writers have had but little Succeſs in Solving this Problem, we have no Reaſon to Believe the Preſent Doctrine gives us any better Informations Concerning it, upon which Account we ſhall Diſmiſs it with the Reſt.

The Method of Experiment made Uſe of to Evince the Truth of the Demonſtration.

§. 28. ALBEIT the Reaſoning Alledged in Confirmation of this Propoſition ſeems Weak and Invalid, it will not yet perhaps be Superfluous or Impertinent to Conſider the Method of Experiment uſed to Evince it; The Focal Length of an Object-Glaſs is given, and is ſuppoſed to be 144 Inches, and that of the Eye-Glaſs 3; A Teleſcope Compoſed of theſe ſhall make the Apparent Diametral Magnitude of an Object, to the Apparent Diametral Magnitude of the ſame Object View'd by the Naked Eye, as 144 to 3, or as 48 to 1, and in the Circular Superficies of ſuch an Object, as the Square of 48 to the Square of 1, or as 2304 to 1; From a Convenient Scale therefore take one Part and Deſcribe a Circle with it, and from the ſame Scale take 48 Parts and Deſcribe another Circle, let theſe two Circles be Cut out in Paper, or any Conſpicuous Object, and Placed at 3 or 4 Feet from each other on a Wall, at ſuch a Diſtance, as will Require the Length between the Glaſſes in the Teleſcope but juſt $144 + 3 = 147$ Inches to ſhew theſe Objects Diſtinctly, then with one Eye through the Teleſcope, Obſerve the ſmaller Circle, and at the ſame Time with the other Eye Naked Look upon the greater Circle, theſe two Circles ſhall Appear Equal to both Eyes.

An Anſwer to the Recited Experiment.

§. 29. IN Anſwer to which Experiment we muſt Obſerve, what this Philoſophy it's ſelf Apprehends is a Difficulty which lies againſt it, that the Compariſon is not Juſt between both Appearances; becauſe the Propoſition Suppoſes the Naked Eye at the Station of the Object-Glaſs, whereas the Experiment ſets the Naked Eye Diſtant from the Object-Glaſs the whole Length of the Teleſcope; This is likewiſe Own'd to be a Material Objection, were not the Diſtance of the two Circles from the Eye vaſtly greater than the Length of the Teleſcope, ſo that the one may not bear any Senſible Proportion to the other; ſuch a Solution therefore of the Objection, altho' it will poſſibly obtain, where we Contemplate the Moon or Planets, yet does not ſeem well Adapted to the Preſent Caſe, where we View a Nigh Object, as Paper; on the Contrary, if the two Circles of Paper Appear Equal in the Circumſtances before Deſcribed, when there is a Senſible Proportion betwixt the Diſtance of the Object, and the Length of the Teleſcope, the Moon or Planets cannot Appear Equal in the ſame Circumſtances, when there is no Senſible Proportion betwixt the Diſtance of the Object, and the Length of the Teleſcope.

Argument the Second againſt the aforeſaid Reaſoning.

§. 30. THIS is Wiſely foreſeen in the Preſent Doctrine, and therefore it Affirms, that there is no Senſible Proportion betwixt the Diſtance of the Paper and the Length of the Teleſcope, becauſe there is the ſame Diſtance of the Object-Glaſs and Eye-Glaſs in Viewing it, as there is in Viewing an Infinitely Diſtant Object; which is ſo far from Clearing the Matter, that a freſh Difficulty ariſes from it, Namely, why we ſhould See Objects at a leſs Diſtance, and at an Infinitely greater, in the ſame Diſtance of the Object-Glaſs and Eye-Glaſs, which Conſiſts of the Aggregate of both their Focal Lengths, and not only with a ſufficient Clearneſs and Diſtinction, but alſo in the ſame Increaſed Magnitude with Reſpect to the Natural Appearance of them; However, ſuppoſing theſe Difficulties Surmounted, becauſe 147 Inches bears no Proportion to the Diſtance of the Moon from the Earth, which Diſtance is no leſs than 60 Semidiameters of it, or 240000 Miles nearly, it can be no Reaſon why the ſame 147 Inches, becauſe they are only 147 Inches as before, ſhall bear no Proportion to ſome Yards, at which Diſtance a Nigh Object is Seen.

Argument the Third.

§. 31. Besides which, this Experiment takes not the leaſt Notice of the Brightneſs of an Object, which yet, as the Preſent Philoſophy Affirms, ought to come into the Account; if therefore it were True in Reſpect of theſe Circles of Paper, the following Calculation, which Reduces the Appearance of the Moon to the ſame Proportion of 48 in it's Diameter, when Seen thro' a Teleſcope to 1, when View'd by the Naked Eye, cannot be Juſt, unleſs the Brightneſs of the Moon is no more, nor greater, than that of an Object, which is barely Diſtinct and Viſible, as Paper.

Argument the Fourth.

§. 32. Lastly, it is Evident, that if we take one Convex, and View an Object thro' it at a certain Diſtance, it will Magnify it in a Determin'd Proportion, which at a farther Diſtance of the Object it will not; why therefore Two Convexes ſhould have ſuch a Peculiar Property, as to Magnify an Object in the ſame Proportion at many Thouſand Miles Diſtance, as when it is only ſome few Yards Remov'd from us, is Inconceivable, and Unexplain'd.

Argument the Fifth.

§. 33. The Calculation notwithſtanding, which is uſed to Prove Diſtant Objects Reducible to the foregoing Experiment, is Form'd from (*Tab.* 35. *Fig.* 4.) which Proceeds upon the ſame Principles, which are Aſſerted in (*Tab.* 35. *Fig.* 2.) an Account of which has been given in §. 26. and are ſhown in §. 27. to be Incompetent and Invalid; ſo that, if even the Preceding Experiment was Really Applicable to Diſtant Objects, yet that Proportion, which it Affirms of 48 to 1, could not be made out by a Juſt and fair Computation, ſince the Grounds, upon which it Depends are not either Accurate or True, we ſhall therefore now go on to Examine the Corollaries, which are Deduced from this Propoſition.

The firſt Corollary of the foregoing Propoſition Explain'd.

§. 34. The firſt Corollary, which is Inferred from it is, that if the ſame Object-Glaſs xyz (*Fig.* 2. §. 2.) is at one time Combin'd with an Eye-Glaſs, whoſe Focus eh or oh is 1, and at another time with an Eye-Glaſs, whoſe Focus eh or oh is 2, the firſt Teleſcope will Magnify twice as much, as the Latter, I Suppoſe, becauſe the Angle $fhe = goh$ or the Subtenſe $gh = fe$ will be Double in the one Caſe of what it is in the other, when on the Contrary he or oh is Reciprocally but half.

Argument the Firſt againſt the Reaſoning in this Corollary.

§. 35. Let therefore he and ho be Increaſed to twice their Length, that is, let the Glaſs be Placed in o, and the Eye as far Diſtant from the Glaſs, as the Glaſs from the Diſtinct Baſe, and let us then Conſider the Conſequences of ſuch a Situation; and firſt fg, and dl, would not be Propagated in a ſtrait Line from y to the Object-Glaſs ghl, for ſince yfg, ydl Continually Diverge, if ghl is Poſited at a greater Diſtance than in the Preſent Scheme, it muſt Neceſſarily fall within the Rays, which Recede more from each other than before, when they were Incident upon it, and Conſequently fg, and dl, Continued will not touch the Glaſs ghl, and the Natural Optick Angle $fyd = ayc$ will be wanting, with which the Refracted one is Suppoſed to be Compar'd; if yfg, ydl are not ſtrait Lines, they muſt be Bent and Refracted to the Eye-Glaſs in o, from the Points f and d, which is Impoſſible, unleſs the Diſtinct Baſe fed has a Power of Refracting, as well as the Object-Glaſs xyz, which will not, I Preſume, be Acknowledged; or Laſtly, if in (*Fig.* 1.*ſt*) ſome of the Innermoſt Rays of fg, dl, may Poſſibly Fall on the Eye-Glaſs at o, the Object will not be Diſtinctly Repreſented to the Eye, where all the Rays from the Diſtinct Baſe are not Collected to it.

Argument the Second.

§. 36. Moreover, Suppoſing the latter Poſition of the Glaſs and Eye to be Expreſſed by gql, where qh is the Double of oh, and the Angle at the Circumference gql, half of the Angle at the Center gel, if the Object is Seen Single at the Circumference, and Double at the Center; in the Firſt Place, g and l muſt be Preſum'd to be the ſame Unvaried Points, which by the Preceding Section, it is Impoſſible they ſhould be, and in the ſecond, the Received Axiom in Opticks muſt likewiſe be Allowed, that Objects Seen under the ſame Angle are Equal, and Conſequently under Double ones Increaſed in a Double Proportion, which Axiom has been already Diſproved.

The ſecond Corollary Explained.

§. 37. The Second Corollary is, that in two Teleſcopes of different Lengths, if the Focus of the Eye-Glaſs of the ſhorter bears the ſame Proportion to the Focus of it's Object-Glaſs, as the Focus of the Eye-Glaſs of the longer bears to the Focus of it's Object-Glaſs, theſe two Teleſcopes Magnify Equally; which is indeed a Neceſſary Conſequence of the Preſent Reaſoning, becauſe what is Demonſtrated in (*Fig.* 2. §. 2.) will Obtain in a greater or leſſer Length of the Scheme, where the ſame Proportions are Obſerv'd.

Argument the Firſt againſt the Reaſoning in this Corollary.

§. 38. On the Contrary, as ſuch a Reaſoning has been Evidenced to be Falſe, ſo it is alſo Confuted from ſo Abſurd a Deduction from it; for if this were True, it is very Evident, that there would be no Occaſion to Enlarge Teleſcopes to ſo Prodigious a Length, as 30, 40, 50, or 100 Feet, *&c.* However, to Solve this Objection it is Affirm'd, that Object-Glaſſes of a ſhorter Focus will not bear Proportionally Eye-Glaſſes of ſuch ſhort Focus's, without Colouring the Object, and Rendering it Dark, as Object-Glaſſes of longer Focus's; thus, if a good Object-Glaſs of 12 Foot Focus will Receive an Eye-Glaſs of no ſhorter Focus than 3 Inches, with any Clearneſs and Diſtinction, an Object-Glaſs of 24 Foot Focus of the ſame Goodneſs or Perfection ſhall Receive an Eye-Glaſs of leſs than 6 Inches with an Equal Clearneſs and Diſtinction, and perhaps, it is ſaid, it may take an Eye-Glaſs of 5 or 4 Inches Focus; if ſo, an Object-Glaſs of 12 Foot with an Eye Glaſs of 3 Inches Magnifies but 48 Times, whereas an Object-Glaſs of 24 Foot with an Eye-Glaſs of 4 Inches Magnifies 72 Times, Namely, $\frac{1}{3}$ more than the former; it is Confeſſed indeed, that the longeſt Teleſcopes do generally Render the Objects more Dark and Obſcure, yet when ſhorter Glaſſes have Proportionably, as ſhort Eye-Glaſſes, and as Cloſe Apertures, they are more Obſcure than the longer Teleſcopes.

Argument the Second.

§. 39. In the Beginning of the former Solution we have this Aſſertion, that Object-Glaſſes of a ſhorter Focus will not bear Proportionably Eye-Glaſſes of ſuch ſhort Focus's, without Colouring the Object and Rendering it Dark, as Object-Glaſſes of longer Focus's, and this is the very Reaſon given, why longer Teleſcopes are better than ſhorter, notwithſtanding which, at the latter End of the ſame Solution, Truth and Matter of Fact Extorts a quite different Confeſſion from this Doctrine, that the longeſt Teleſcopes do generally Render the Objects more Dark and Obſcure.

Argument the Third.

§. 40. However, it muſt be Acknowledged, that this laſt Affirmation is with ſome Reſtrictions, for it is Added, where ſhorter Object-Glaſſes have Proportionably as ſhort Eye-Glaſſes, and as cloſe Apertures, they are more Obſcure than longer Teleſcopes; but as the Apertures of Glaſſes are Introduced, which are not yet Explain'd, it does not ſeem to be any ways Pertinent here, where the Length and Shortneſs of the Teleſcope is only Conſiderable, and if that is done, by the foregoing Section the Longeſt Teleſcopes Render Objects more Dark and Obſcure altho' the Reaſon Alledged, why the Longeſt are Preferrable to the Shorteſt, is, becauſe the Shorteſt Render Objects more Dark and Obſcure; beſides, it is afterwards Acknowledged in this Philoſophy, that the Viſible Area of an Object is not Increaſed, or Diminiſhed, by the Greater or Leſſer Aperture of the Glaſs.

Argument the Fourth.

§. 41. But that, which is ſtill more, is, that by this Propoſition Longer or Shorter Teleſcopes, in the forementíon'd Circumſtances, ſhould Magnify Equally, and the Reaſon Aſſign'd, why this does not Prevail in Experience, is, becauſe Objects Appear more Dark and Obſcure, or elſe Coloured, in Shorter ones than in Longer, when we are not at all Inform'd, why the Obſcurity, or Colours, of an Object ſhould Diminiſh the Magnitude of it, and it is Impoſſible to tell us, why they ſhould; ſince it is Notorious and Manifeſt, that Objects of one Colour, or another, may Preſerve their Proper Dimenſions.

Argument the Fifth.

§. 42. Lastly, ſuppoſing the Obſcurity, or Colours, of an Object Changed the Dimenſions of it, this Conceſſion it ſelf would Prove, that Shorter Glaſſes, whoſe Focal Lengths have the ſame Proportion to each other as Longer, do not Magnify Equally, which is the very Thing, which the Preſent Propoſition, and this ſecond Corollary of it, Affirms.

Concerning the Second Solution Offered of the Difficulty §. 37.

§. 43. A Second Solution, which is Offered of the Difficulty mention'd in §. 37. is, that the Image of the Moon, or other Object, in the Diſtinct Baſe of the Object-Glaſs of 24 Foot, is twice as long as the Image in the Diſtinct Baſe of an Object-Glaſs of 12 Foot, and therefore the Picture in the Former muſt be much more Diſtinct and Perfect than in the Latter.

Contradicted by Experience.

§. 44. Which Solution comes to this, that by the foregoing Propoſition, and the Corollary of it, Longer or Shorter Glaſſes, if they bear the ſame Proportion in the Length of their Focus's, will Repreſent the Object in the ſame Magnitude, but the Reaſon in Fact, that they do not, is, becauſe an Object-Glaſs of 24 Foot Magnifies twice as much as one of 12, that is, the Reaſon, why this Propoſition, and it's Corollary, does not hold is, becauſe it is Manifeſtly Contradicted by Experience, which if it is not a Confutation of it, nothing elſe can be.

The Third Corollary Explain'd.

§. 45. THE Third Corollary to this Propoſition is, that, if the Object-Glaſs be Form'd on a leſs Sphere, for Inſtance, whoſe Radius is 6 Inches, than the Eye-Glaſs, whoſe Radius is 12, the Appearance of the Object will be Diminiſhed, and in this Caſe will be half the Natural Appearance of the ſame Object; becauſe, Suppoſing the Object-Glaſs xz in q, (*Fig.* 2. §. 2.) the Radius eq 6 Inches, and oh the Radius of the Eye-Glaſs gl likewiſe 6 Inches; fed will be the Diſtinct Baſe, ſince in a Double Convex of Equal Convexities, the Focus is at the Diſtance of the Radius or Semidiameter of either; the Natural Optick Angle will alſo be $fqd=fhd=gol$ the Refracted Optick Angle, and therefore the Object will Appear Equal, when Looked upon by the Bare Eye, or thro' the Teleſcope; Let the Eye be Remov'd to m, ſo that mh may be 12 Inches, = to the Semidiameter of the Eye-Glaſs's Convexity, and Double of oh or eq the Semidiameter of the Object-Glaſs's Convexity, gml will be the Refracted Optick Angle, which, by what goes before, is ſaid to be the half of $gol=fqd$ the Natural Optick Angle; from whence it follows, that the Object, thro' ſuch a Teleſcope, will ſeem Leſs by half, than when View'd by the Naked Eye.

Argument the Firſt againſt this Reaſoning.

§. 46. IF this is the Demonſtration Intended, for we have been Obliged to Supply what was wanting in this Philoſophy, and we do not at Preſent See any other; We Anſwer, that the Propoſition Referr'd to of Double Convexes, Determining their Focus at the Diſtance of their Semidiameter, as well as all the other Particulars of the Preſent Propoſition, and the Scheme Annexed to it, have been before Examined, as likewiſe gml being only the half of gol, all which lie againſt what is Advanced in the Preceding Section.

Argument the Second.

§. 47. TO which we may Subjoin, that if the Glaſs is Suppoſed to be in q, it is Impoſſible, that qf, qd ſhould ever Arrive at g or l, unleſs they are Broken and Refracted in f and d to a Prodigious Degree, which it is not Allowed to be a Property in the Diſtinct Baſe to Effect; Moreover, that any Teleſcope ſhould Repreſent an Object in the ſame Magnitude, in which it is Seen without the Interpoſition of Convexes, cannot be Conceiv'd; not even in the Preſent Caſe; for if fqd is the Natural Optick Angle, it is not to be Imagined, that $gol=fqd$ can be the Refracted one, unleſs the Rays, which Flow from fed, which is here Conſidered as the Object, are not Refracted by the Convex gl, or, if Refracted, do not Form a different Angle from what they would do, if they were not; which on either Hands is Abſurd, according to the very Notions and Principles of this Philoſophy.

Hugens's Doctrine of Teleſcopes not more Convincing than the former.

§. 48. To Conclude upon this Propoſition, and it's Corollaries, *Hugens* is Thought to have Demonſtrated, that an Object, Seen thro' a Teleſcope, is to that, which is Perceiv'd by the Naked Eye, in the Diameters of both, as the Diſtance of the Focus of the Object-Glaſs to the Diſtance of the Focus of the Eye-Glaſs, which, if True, would be an Admirable Propoſition Indeed; ſince by making the Diſtance of the Focus of the Object-Glaſs Infinitely Great in Reſpect of the Diſtance of the Focus of the Eye-Glaſs, Objects would be Infinitely Magnify'd, and there would be nothing more wanting to the Perfection of Teleſcopes; whereas it is Sufficiently known, that there is a great deal wanting to Complete that Sublime Theory; and therefore this, as well as the reſt of that Ingenious Perſon's Opticks, ſince they Proceed upon the ſame Foundations as the Preſent, are nothing more than an Abſtracted Reaſoning.

Nor If. Newton's.

§. 49. NOR is it enough to Say, what another Philoſophy Offers, that the main Cauſe of the Imperfection of Teleſcopes is owing to the different Refrangibleneſs in the Rays of Light; Seeing the Chimerical Contrivance of ſuch a Doctrine has been already Shewn, and the Project of Reflecting Teleſcopes has had ſo little Succeſs.

The Theory of the laſt a Contradiction to that of the Firſt.

§. 50. BESIDES, if Colours were Introduced in Teleſcopes of a larger Size by the different Refrangibleneſs of the Rays, it would be an Evident Confutation of *Hugens*; who only Conſiders one Sort of Rays to be Refracted in his Demonſtration of this Propoſition, as well as in all his Opticks.

Not Prov'd, that Rays are Difform, or Coloured, in their Refractions thro' Teleſcopes.

§. 51. FARTHER, it is Acknowledged in Experience, that Convex Glaſſes, whether Lights are differently Refrangible, or not, do Refract Rays ſo, as to make them of the ſame Uniform Appearance; how therefore the ſame Convex Glaſſes in Teleſcopes ſhould Produce Colours, or Rays of a Difform Appearance, ſhould be Demonſtrated; as alſo, why common Window-Glaſſes, and all others of what Figure or Dimenſions ſoever, do not Produce the ſame Effect of Colours, which, by the Manuſcript I have Seen that undertakes it, is not done; And nothing, that I know of, has been Offered to the World from the Author of this Hypotheſis upon that Subject, which can be ſaid to be in the leaſt Convincing; which is not Wonderful, ſince, as we have Prov'd, the Hypotheſis it ſelf is Falſe; and therefore we ſhall not Trouble our Selves Particularly to Inquire into the Explanations of that Doctrine, as it is Apply'd to Teleſcopes.

Concerning the Angle Receiv'd by a Teleſcope of the foregoing Combination. Molyneux's Dioptricks Prop. 54. Par. 1.

§. 52. THE next Thing Obſervable in this Philoſophy of Rays is, the Rule which is given for Finding out and Determining the Angle Receiv'd by a Teleſcope of the foregoing Combination, which is (*Fig.* 2. §. 2.) as hy the Diſtance of the Glaſſes, to gh half the Breadth of the Eye-Glaſs, ſo the Radius, to the Tangent of the Angle gyh, which is half of the Angle $gyl =$ to the Angle Receiv'd by the Teleſcope; That is, the Eye at o ſhall Perceive no more of the Object than Subtends this Angle at the Object-Glaſs.

Argument the Firſt againſt this Reaſoning.

§. 53. IF this Demonſtration is True, Either $gol = fqd$ is Equal to $gyl = fyd$, which is both Impoſſible by the Elements of *Euclid*, and Contradicts this Philoſophy, which Aſſigns it as a Reaſon for the Object's being Magnify'd, becauſe $fqd, =$ to the Refracted Optick Angle gol, is greater than the Natural Optick Angle $= fyd$; Or elſe, gol, which is greater than gyl, Repreſents the Object in the ſame Magnitude, which is again Contrary to this Philoſophy; ſince Objects are greater or leſs in Appearance, according to the different Angles, under which they are Seen; that is, the Object in the Angle gol is View'd under the Angle gyl, which is Abſurd.

Argument the Second.

§. 54. IF yet gyl was the Standard for the Viſion of the Object, ſo that there ſhould be no more Seen of it, than what it Preſcrib'd, and was therefore $=$ to gol, all the Difficulties are to be Solved, which are before Mention'd, of gyl being the Natural Optick Angle.

The Firſt Scholium Annexed to the Preceding Propoſition Explain'd.

§. 55. THE firſt Scholium annexed to the foregoing Propoſition is, That if the Eye Approaches nigher to, or Recedes farther from, the Eye-Glaſs ghl, (§. 2. *Fig.* 1.) than it's Focus at o, it ſhall Perceive a leſſer Area of the Object, tho' what it Sees ſhall be as Diſtinct as at o; for if the Pupil of the Eye is at m, the Rays go, lo, do not enter the Eye, and Conſequently the Points in the Object Anſwerable to f, d, in the Diſtinct Baſe ſhall not be Viſible; The ſame, it is ſaid, may be Conceiv'd if the Eye Recedes farther from the Eye-Glaſs than o, becauſe all the Rays from the ſeveral Points in the Object are Mixt together and Interſect each other at o, in the Focus of the Eye-Glaſs, and from thence Flowing forward Separate and Diverge; but then the Eye at m Receives the Rays, that enter it Parallel, or very little Diverging, and Conſequently the Viſion is Diſtinct.

Argument the Firſt againſt this Reaſoning.

§. 56. IT is ſaid, that the Object Appears leſs, when the Eye is at m, or beyond o towards s, becauſe go, lo, or, ot, ro, do not Fall on the Pupil of the Eye in ſuch a Situation; but if ſo, the Object, by this Explanation, will not Appear leſs, but Curtaild, and Maimed of the Extreme Parts of it; ſo that fed will not be Seen in it's Perfection, and only Diminiſhed in Reſpect of it's Dimenſions; but, as f and d are loſt to the Sight by this Account, ſo will ſeveral other Intermediate Points betwixt f and e, and d and e; from whence it will follow, that zz, the Diſtinction of the Preſent Object from other Parts of it, may likewiſe Diſappear, which will be far from Informing us, how the Object may be Diminiſhed and yet Diſtinct, where ſeveral Parts are Obliterated and Eraſed out of the Image.

Argument the Second.

§. 57. MOREOVER, if any of the Rays of go, which are neareſt the Axis, or any next towards them betwixt g and h, or any from i and k, which are near the Rays ho, ſhould be Refracted to m, they will make a greater Angle than gol, and Conſequently the Object will not Appear Diminiſhed, but Increaſed; if none but the Rays ho enter the Eye at m, they will not form any Angle at all, becauſe they are Suppoſed to be Refracted Parallel, and therefore will not Repreſent much more of the Object than ſome few Points of it on each Side of e, that is, ſo much as the Rays *ho*, Referr'd back in ſtrait Lines, will Interſect; from whence the Object will not Appear Diminiſhed in it's Dimenſions, but, as before in §. 56, Curtaild and Imperfect.

The Second Scholium Explain'd.

§. 58. **The** ſecond Scholium is, that the Angle Receiv'd, or Viſible Area of the Object, is not Increaſed or Diminiſhed by the greater or leſſer Aperture of the Object-Glaſs, that is, whether there is more or leſs of it uncovered, becauſe the Angle gyl Continues the ſame, tho' the Object-Glaſs were all Covered to the very middle Point y, all that is Effected by the greater or leſs Aperture is the more Bright or Obſcure Appearance of the Object.

Argument the Firſt againſt this Reaſoning.

§. 59. **This** therefore likewiſe Depends upon gyl being the Natural Optick Angle, and our Seeing Objects Magnify'd or Diminiſhed according to the greater or leſs Proportion of the Viſual Angles in their Bigneſs; it alſo Aſſumes the whole Second Scheme for Granted; all which have been Confuted and Diſproved; what we ſhall farther Alledge is, that if $gyl = ayc$ (*Fig.* 2. §. 2.) is the Pencil of Rays, which is only Neceſſary to make an Object Appear in a certain Magnitude and Diſtinction, the ſame will Obtain (*Fig.* 1. §. 2.) and axc, azc will be Perfectly Uſeleſs; on the Contrary, if axc, azc in the laſt Figure are Subſervient to the Painting the due Diſtinctions of an Image upon the Eye, they will be ſo in the firſt; and if x and z are Covered, and the Intermediate Parts betwixt them and y, the Image too muſt be Obſcured, and the Portraiture of the Object upon the Retina Marr'd and Defaced, which is yet Suppoſed of a Juſt Magnitude, and to be Diſtinct.

Argument the Second.

§. 60. **Farther**, if $gyl = ayc$ are the Rays Principally Requiſite to Form a Diſtinct Viſion, an Object may be Seen Diſtinct, and in the Proper Magnitude, which Appertains to it's Refraction, by one Ray from each Point, whereas the Collection of ſeveral from each Point has been hitherto thought Neceſſary to that Diſtinction; If therefore this were now True, that one Ray from each Point could have Force enough to Awake our Attention to it's Impreſſions, what would be the Conſequence of it in other Parts of this Philoſophy? Namely, that, where ſeveral Rays are Collected to one Point in the Retina, each of them will have a Diſtinct Force to Repreſent the Point in the Object, from which they are Deriv'd, and being Referred back in almoſt Infinite Directions, will Cauſe one Point in an Object to Appear, as, very near, an Infinite Number of them would do, which, at leaſt, is not Confeſſed, or Own'd, in the Preſent Opticks.

Concerning the Apertures of Object-Glaſſes Molyneux's Dioptric. Prop. 55. Par. 1.

§. 61. **Concernng** the Apertures, however, of Object-Glaſſes, by them is Meant that Part of the Glaſs, which is left Open and Uncovered, which is ſaid ought to be Various, according as we would have more or leſs Light admitted, or according to the differing Focal Lengths of the Object-Glaſſes; ſince a 10 Foot Object-Glaſs, that is, whoſe Focal Length is 10 Foot from the Glaſs, will bear a greater Aperture than an Object-Glaſs of one Foot, and a 20 Foot Glaſs yet greater than a 10 Foot; but at what Rate or Proportion the Apertures of Glaſſes Alter in Reſpect of their Focal Lengths is not yet, it ſeems, well Settled; All that is done in this Caſe is, that Monſieur *Auzout* has found, that the Apertures, which Glaſſes can bear with Diſtinctneſs are in, or about, a Subduplicate Ratio to their Lengths, or as the Square Roots of them.

The Theory of Apertures not Juſt.

§. 62. **But** that this Theory of Apertures is not Juſt, is from hence Evinced, that the ſame Glaſs will Endure greater or leſſer Apertures, according to the leſſer or greater Light of the Object; thus to Obſerve the Sun, or *Venus*, or the Diameters of the Fixt Stars, ſmaller Apertures are Requir'd, whereas the Moon in Day-Light, *Saturn*, *Jupiter*, or *Mars*, expect the Largeſt; And if this is certain in theſe Diſtant Objects, the ſame in Proportion muſt hold in Nigher, where the Force of their Lights or Colours are different, which is a ſufficient Confutation of the foregoing Rule.

Concerning Teleſcopes, which Conſiſt o a Convex Object-Glaſs and three Convex Eye-Glaſſes. Molyneux's Dioptricks Prop. 56. Part. 1.

§. 63. A following Propoſition in this Doctrine is to Explain the Teleſcope conſiſting of a Convex Object-Glaſs and three Convex Eye-Glaſſes; but this Depending entirely on Propoſition 50 §. 2. and being only a double Teleſcope of that kind, which is there Deſcrib'd and we have fully Conſidered, there is no occaſion to Examine it any farther, nor yet the Theory of Combining ſeveral Convex Glaſſes in Various and differing Numbers.

The Method of ſuch Compoſitions.

§. 64. However, we ſhall Juſt Recount the Method of ſuch Compoſitions or Combinations of Convexes, and then make one ſhort Remark or two upon them; we are firſt, it is ſaid, to Obtain, by ſome Practical Rules to be afterwards Delivered, the Focal Length of each Particular Glaſs by it's ſelf; then we are to Conſider the Diſtances of each of theſe Glaſſes, as they lie in the Tube, from the Glaſſes before and behind it; afterwards, where the Diſtinct Baſes are Form'd by theſe ſeveral Glaſſes, and how they are Projected, as to Amplification or Diminution, which is Affirm'd to be eaſily found by what has been before Delivered; and then, how the Eye-Glaſſes Affect theſe Diſtinct Baſes, as how they Confound them, Rectify them, Invert ot Magnify them; the Reſult of all which this Philoſophy Aſſerts is, that the Rays from the ſeveral Images in the ſeveral Diſtinct Baſes ſhall be Confuſed on the Pupil of the Eye in Order to be Rectify'd by the Chryſtalline, which is to be Conſidered, as a Glaſs, whoſe Focus is on the Retina; from whence it is, Laſtly, ſaid it will Appear, why ſome Teleſcopes Conſiſting of Convex Glaſſes Repreſent the Object Erect, others Inverted, in ſome one Glaſs is to be taken as one, and in others two Glaſſes likewiſe Perform the ſame Effect.

Argument the Firſt againſt this Method of Compoſition.

§. 65. In Anſwer to this Method we muſt firſt Say, that Practical Rules are Originally Requir'd in it, which is, or ſeems to be, a Plain Confeſſion; that the Theoretick will not do, which have been before Endeavoured to be Demonſtrated in this Philoſophy; for if they could, in this firſt Part of Opticks it ſhould at leaſt have been ſhown us, how the Focal Length of each Particular Glaſs by it's ſelf is to be Obtain'd, without having Recourſe to the ſecond which is Experimental; ſince either the Practical Rules Agree with the Theoretick, or not; if they do, there was no Reaſon to Referr us in the Theoretick Part of this Doctrine to the Practical Part of it, but rather to thoſe Propoſitions which went before, and which, in all other Caſes, it Conſtantly does; if they do not, it is an Evident Demonſtration, that what is Propoſed to us in Theory is not Anſwered, or Juſtify'd, by Matter of Fact or Experiment, and that therefore the whole Doctrine is no other than a Laboured and Abſtracted Hypotheſis of the Brain, without being any ways Conformable to Nature.

Argument the Second.

§. 66. Besides which, the foregoing Propoſition being the main Foundation, on which this Method of Combining Glaſſes Proceeds, and having likewiſe Conſidered the way, by which the Diſtinctneſs, Magnitude, Inverſion, and Erectneſs, of an Object is Explain'd in this Preſent Philoſophy, we need not Enlarge in the Confutation of a Doctrine, which Endeavours to Account for thoſe Effects upon the ſame Principles.

Concerning Teleſcopes of a Concave Eye-Glaſs and Convex Object-Glaſs of a Larger Sphere. Molyneux's Dioptric. Prop. 57. Par. 1.

§. 67. We ſhall therefore go on to the next Propoſition, which is, to Explicate the Teleſcope Compoſed of a Concave Eye-Glaſs and Convex Object-Glaſs of a larger Sphere; in this Teleſcope, the Diſtance of the Glaſſes is to be the difference of their Focus's, or the Concave Eye-Glaſs is to be Placed ſo much nigher the Object-Glaſs than the Focal Length of the Object-Glaſs, as is the Vertual Focus of the Eye-Glaſs, and the Eye Poſited as near as Poſſible to it; in which Caſe the Object will Appear, Diſtinct, Erect, and Magnify'd; (*Tab.* 37. *Fig.* 2. or *Fig.* §. 68.) the Diſtinct Baſe *f e d*, is Form'd, as before in the Preceding Propoſition, and let *e h* be the Virtual Focus of the Concave, the Rays *x i*, *y h*, *z k*, in their Incidence on the Eye-Glaſs, Converge towards the Focus *e*, and after Paſſing the Glaſs become Parallel in *i m*, *h e*, *k n*, which is Suppoſed to have been already Prov'd in this Philoſophy; and the ſame may be Conceiv'd of the Rays, which Flow from the Collateral Points, and which Converge to the Focus's *f* and *d*, but by Refraction are made to Diverge to a Paralleliſm, as if they Proceeded from *p*, *h p* being by Conſtruction = to *h e* = to the Length of the Virtual Focus.

Argument the Firſt againſt this Account of Teleſcopes.

§. 68. In the firſt Place therefore, all which has been ſaid in the former Propoſition, (§. 2. *Fig.* 1.) againſt that Part of the Scheme, which lies betwixt *a a a*, *b b b*, *c c c*, and *f e d*, is Valid likewiſe here; it is farther to be Obſerved, that the Rays are Suppoſed to be Refracted Parallel from Corollary to Propoſition the Thirteenth, which Says, that a Ray Falling on a Concave Glaſs, and tending towards the Virtual Focus, after it has Paſſed the Glaſs, ſhall Proceed Parallel to the Axis, becauſe the Progreſs of Light thro' Glaſſes is Reciprocal; but that Reaſoning from the Virtual Focus's and the Reciprocal Progreſs of the Rays has been already Examined; to which we ſhall only Add Concerning the Preſent Figure, that if ſuch a Reaſoning were

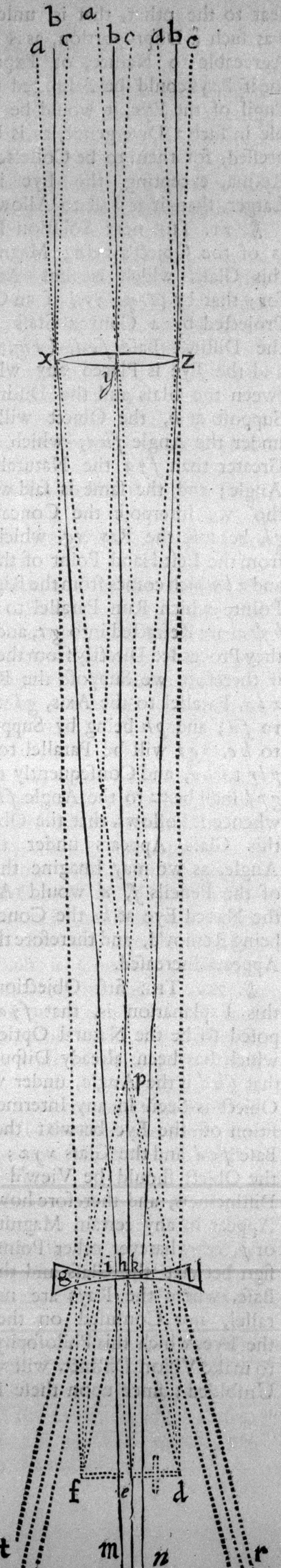
True, to make it Applicable to this Scheme; 1*ſt*. the Concave *g l* muſt have ſeveral Virtual Focus's, becauſe *x i k z* do not tend to the ſame Point with *x l z*, or *x g z*; 2*d*. thoſe different Virtual Focus's are Preſum'd to lie in the Diſtinct Baſe *f e d*, that is, that the Virtual Focus of a Concave Eye-Glaſs ſhall be in the ſame Point with the Real Focus of the Object-Glaſs 3*d*. that *g t*, *l r* are Refracted Parallel to the Axes, Namely, to *y f*, *y d*; the two Firſt are not Prov'd, and the Laſt is Evidently Falſe.

Argument the Second.

§. 69. The next Thing, which is done in this Propoſition, is to ſhow, how the Diſtinctneſs of the Object would be Conſequent upon this Explanation; for the Rays from each Single Point after Refraction thro' the Concave, Falling Parallel on the Eye amongſt themſelves, by Propoſition 28*th*. each Point is Diſtinctly Repreſented in the Retina of it; On the other Hand, we Say, that the Propoſition Referred to has been fully Conſidered, and moreover, that not only the Paralleliſm of the Rays Incident on the Pupil of the Eye, but the Confuſion of them likewiſe are, by the foregoing Doctrine and Propoſition, alſo Thought Neceſſary to a Diſtinct Perception; Whereas, Suppoſing, as in (*Fig*. 3. *Tab*. 37. or *Fig*. §. 71.) the Eye as cloſe to the Concave Glaſs, as we can Poſſibly Imagine, the Rays from the ſeveral Points of the Object do not in the leaſt Intermix, or Blend, with each other; If therefore at one Time a Confuſion of the Rays in the Pupil of the Eye (*Prop*. 28. §. 3.) from each Point is Deem'd Requiſite to Solve the Diſtinctneſs of Viſion, and at another Time not, what is it Poſſible for us, to Conclude, unleſs that ſuch a Doctrine Depends more on the Fancy, and Imagination, than on any True or Juſt Reaſon? for it is Impoſſible, if the Axes of the Rays Converge to a Point in the Pupil of the Eye, in Order to Produce Diſtinct Viſion in the Retina, that the Axes when they do not Converge, but Diverge, ſhould; Nor can we Conceive why this Particular Closeneſs of the Eye to the Glaſs ſhould be Required, excepting with the Intention, that the Rays might be Confuſed in it's Pupil, which are not even then; and if it is not upon that Account, why they might not be Juſtly and Punctually Refracted to the Retina, at a greater Removal of the Eye from the Glaſs, ſince they will ſtill be Parallel, is not Explain'd.

Argument the Third; of the Object's being Erect.

§. 70. As for the Object's being Erect, which is Prov'd from (*Prop*. 28.) we ſhall not Say any more Concerning it, than what has been Alledged under that Propoſition; excepting, that it ſeems Impoſſible the Rays ſhould Paint the Object Erect, ſince, Conſidering the great Divergence of them, they do not Appear Capable of Expreſſing the full Extent of it at all; for we find in (*Fig*. 3. *Tab*. 37. or *Fig*. §. 71.) that the Eye is made Extremely Large, and Placed very near to the Glaſs, in Order to Receive theſe Diverging Rays, which it could not do, unleſs there was ſuch a Monſtrous and Unnatural Diſproportion betwixt the Glaſs and the Eye, tho' the one ſhould be Poſited never



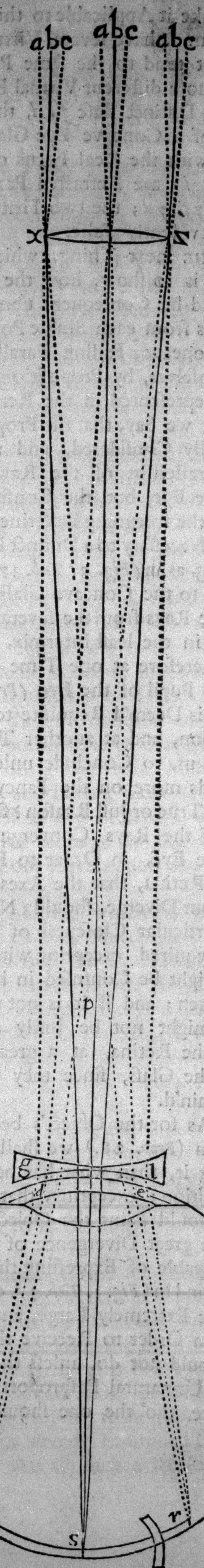
near to the other, that is, unleſs there was ſuch a Diſproportion, as is no ways Agreeable to Nature, or Experience; for if Rays could be Admitted into the Pupil of the Eye, it would be Impoſſible in ſuch a Divergence, as is here Expreſſed, for them to be Collected in the Retina, excepting the Eye is made Larger, than it is Juſt to Allow.

A Solution Propoſed of the Objects being Magnify'd by this Glaſs.

§. 71. THE next Solution Propoſed is of the Object's being Magnify'd by this Glaſs, which is thus Accounted for; that by (*Prop.* 37.) if an Object be Projected by a Convex Glaſs xyz in the Diſtinct Baſe fed, (*Fig.* §. 68.) and the Eye is Placed any where between the Glaſs and the Diſtinct Baſe, Suppoſe at h, the Object will Appear under the Angle fhd, which is much Greater than fyd the Natural Optick Angle; and the ſame is ſaid will hold, tho' we Interpoſe the Concave Glaſs gl, becauſe the Ray xg, which comes from the Left-Hand Point of the Object and zl which comes from the Right-Hand Point, which Run Parallel to the Axis $byhe$, are Refracted into gt, and lr, as if they Proceeded Directly from the Point p; if therefore we Suppoſe the Rays xgf, zld, Parallel to the Axis, gl will be $=$ to fd; and ph being by Suppoſition $=$ to he, pgt will be Parallel to hf, and plr to hd, and Conſequently the Angle gpl ſhall be $=$ to the Angle fhd; from whence it Follows, that the Object thro' this Glaſs Appears under the ſame Angle, as we may Imagine the Apices of the Pencils f, d would Appear to the Naked Eye at h, the Concave Glaſs being Remov'd, and therefore the Object Appears Increaſed.

Argument the Firſt againſt this Explanation.

§. 72. THE firſt Objection againſt this Explanation is, that fyd is Suppoſed to be the Natural Optick Angle, which has been already Diſputed; and that fhd is the Angle, under which the Object is Seen in any Intermediate Poſition of the Eye betwixt the Diſtinct Baſe fed and the Glaſs xyz; but how the Object ſhould be View'd with any Diſtinctneſs, and therefore how it ſhould Appear in any certain Magnitude in h or p, or whatever other Point we Aſſign betwixt the Glaſs and the Diſtinct Baſe, where the Rays are neither Parallel, nor Confuſed on the Pupil of the Eye, which this Philoſophy Requires to make Viſion Diſtinct, will want to be Unfolded; ſince upon theſe Principles, it



it is Impoſſible to be done, excepting by Introducing a New Scheme according to *Prop.* 37. which is not Anſwerable to the Preſent, and Conſequently by altering the Courſe of the Rays, in Order to Explicate another, that is This, which is different from it.

Argument the Second. §. 73. We are farther to Obſerve, that if *fhd* was always the Angle, under which the Object thro' the Glaſs *xyz* is Seen, as *h* Approaches to *y*, the Object will Appear the Leaſt and as to *e*, the Greateſt, whereas this Philoſophy Acknowledges, that when the Eye is Placed in the Focus of the Glaſs, as at *fed*, every Thing is Repreſented in the utmoſt Confuſion, without being of any Determin'd Bigneſs, and if on this Side the Focus towards the Glaſs, that there is not, according to the foregoing Section, any Juſt Rule of the Refraction of the Rays Aſſerted, by which we ſhould know the Object will either be Perceiv'd with Diſtinction or any certain Magnitude.

Argument the Third. §. 74. It will alſo be Difficult, if not Impoſſible, to Explain, why *fhd* ſhould be rather Terminated in *h* than in any other Point betwixt *fed* and *xyz*, before the Interpoſition of the Glaſs, which is Suppoſed, ſince the Intervention of it is Affirm'd to Cauſe no Alteration, *gpl* being Prov'd to be = to *fhd*, and ſince the Angle, which is therefore thought to be made by the Glaſs, is *gpl*.

Argument the Fourth. §. 75. We may Add, that if *dhf* is the Angle, under which the Object is Seen, yet *gpl* is not Evinced to be Equal to it, unleſs by taking it for Granted, that *gp*, *lp*, are Parallel to *fh*, *dh*, or which is the ſame, that the Refracted Rays *tg*, *rl*, are ſo, but this is Proving *fhd*, *gpl*, are Equal, by Suppoſing it.

Argument the Fifth. §. 76. Such a Suppoſition is Moreover Impoſſible, becauſe if in the Preſent Situation *tg*, *rl*, were Refracted Parallel to *fh*, *dh*, they could not be in any other; for *xg*, *zl* are Imagin'd in this Scheme to be Parallel to the Axis *yh*, and Conſequently will be always Refracted, whether the Glaſs Approaches to, or Recedes from, *xyz*, into *gt*, *lr*, Seeing the Refracting Power of the Concave *gl* will Continue the ſame, whereas *fh*, *dh*, are Conſtantly Vary'd in their Inclinations, as *h* comes nearer to, or is farther from, the ſame *xyz*.

Argument the Sixth. §. 77. Again *xg*, *zl*, are made Parallel to *yh*, and therefore the Concave *gl* ought at leaſt to be Equal to the Convex *xyz*, for if it is leſs, *xg*, *zl*, will neceſſarily Converge towards the Axis, and if greater, Divaricate from it, but all the Phænomenons of the Teleſcope are Equally the ſame, in Reſpect of the Diſtinction and Magnitude, with which Objects are Repreſented by it, whether the Apertures of the Glaſſes are Greater or Leſs, according to the Confeſſions of this Philoſophy.

Argument the Seventh. 78. Lastly, this Account ſuppoſes, that Concaves are Capable of Magnifying the Object, or not; If not, we ſhould See it in the ſame or greater Dimenſions without the Concave *gl*, than with it, which is not Experienced; If they are Capable of Magnifying the Object, a Property is Aſcribed to them, which is Contrary to all that has been Endeavoured to be Demonſtrated and Proved concerning them; and that this is done is Evident, ſince the Wider the Rays *tg*, *rl*, are Refracted from each other, the Greater the Object will Appear, and therefore according to this Doctrine, the Divergence of the Rays will be a Reaſon for the Increaſe of an Object, which was One before for the Diminution of it, and it will have the ſame Effect, as was Explained to belong to the Contraction of them; and *gpl*, where the Vertex of the Angle is from the Eye, and the Baſis *gl* lies upon it, will Produce the ſame Appearance, as a Convex, where the Vertex of the Angle lies upon the Eye, and the Baſis, which Subtends it, is in an Averſe and Reciprocal Poſture.

The Corollaries not Conſidered. §. 79. We ſhall not here Conſider the Corollaries, which are Deduced from this Propoſition, ſince by what goes before it is Manifeſt, this is only an Arbitrary Diſpoſition of the Rays Contrived by the Mind, without any Foundation in Nature, to Solve, as well as it could, the chief Appearances of this Teleſcope; we ſhall therefore Proceed to the Doctrine of Microſcopes, of which we ſhall Treat in the Following Sections of this Chapter.

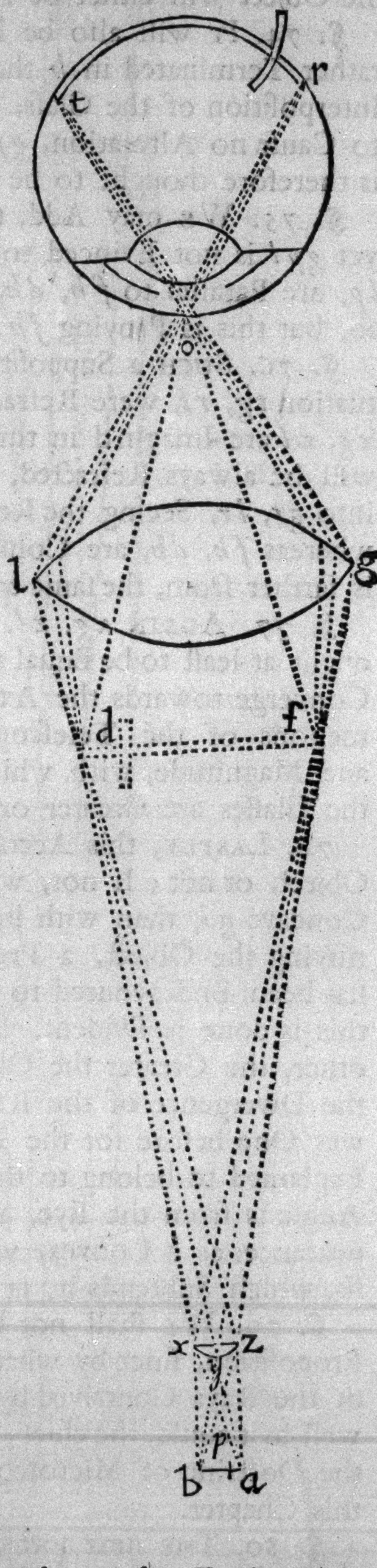
An Account of Microſcopes Conſiſting of a Single Convex. Molyneux's Dioptric. §. 80. The next ſort of Glaſſes, which are Endeavoured to be Explain'd, are Microſcopes, Conſiſting of a ſingle Convex-Glaſs, in which, the Object is uſually Placed, either in the Focus of the Glaſs, or a little Nigher the Glaſs than the Focus, and the Eye is Placed in or about the Focus on the other Side of the Glaſs, in which Caſes, the Phænomenons are ſaid to be Solved by *Prop.* 31. 32. 33. 34. 35; which Propoſitions, as to the Subſtance of them, have been already Examined.

This Account Contrary to Experience.

§. 81. We ſhall only therefore Add, that this is contrary to Experience, ſince if we take a Glaſs, which is Convex on each Side, the Larger the Convexity is, the Greater Difference there is betwixt the Eye's and the Object's being in the Focus, and the Smaller it is, the Leſs; for if in a Glaſs of a Large or Extraordinary Convexity we look upon a Print, the Character ſhall Appear the Greateſt, when the Eye is 5 or 6 times more Removed from the Glaſs, than the Glaſs from the Object, which Diſproportion will be Diminiſhed, as the Convexity of the Glaſs Decreaſes; Not to mention, that the Focus, when Determin'd upon a Paper from a Candle, or any other Luminous Object, is different from that of a Printed Character.

An Account of the Double Microſcope Conſiſting of a Convex Object-Glaſs, and Convex Eye-Glaſs. Molyneux's Dioptric. Prop. 58. Par. 1.

§. 82. The double Microſcope, compoſed of a Convex Object-Glaſs, and Convex Eye-Glaſs, is, after the former, Propoſed to be Solved; (*Tab.* 38. *Fig.* 1.) *a b* is a Minute Object expoſed before the Object-Glaſs *x y z*, the Segment of a very ſmall Sphere, the Focus of which is *p*, the Object therefore being ſomething more Diſtant from the Glaſs than it's Focal Length *y p*, the Diſtinct Baſe will be Formed on the other Side of the Glaſs in *f d*, according to *Prop.* 5; the largeneſs of the Image in the Diſtinct Baſe is likewiſe Determined by *Prop.* 26; Let now the Eye-Glaſs *g l* be Placed, ſo far Diſtant from the Diſtinct Baſe *f d*, as is the Focal Length of this Eye-Glaſs, and the Eye *o r t*, ſo as the Rays *l o*, *g o*, may be Confuſed together on the Pupil of the Eye, which will be about the Focus of the Eye-Glaſs, and the Effects of this Microſcope are Explained in all Things, as in the Teleſcope of *Prop.* 50. as to the Magnify'd, Inverted, and Diſtinct Appearance of the Object; It is yet to be Remarked in Reſpect of the Extraordinary Magnifying of theſe Teleſcopes, that whereas, for Viewing a Minute Object, a good Eye does uſually Approach thereto about the Diſtance of eight Inches, could we make the Eye come to it, and View it Diſtinctly at the Diſtance of half an Inch, the Optick Angle would be extremely Magnify'd, by *Prop.* 51. 52. that is, the Apparent Magnitude of the Object would be Increaſed at the Rate of Sixteen to one; we are therefore deſired to Suppoſe the Eye at *y* Viewing the Object *a b*, which will then Appear under the Angle *a y b* = *d y f*, which is ſaid to be the ſame, as were the Object Increaſed to the Bigneſs *d f*, and Viewed by the Eye at the ſame Diſtance from it, as *y* in the Figure is Removed from *f*; wherefore if by the help of the Eye-Glaſs *g l*, the Eye can Approach to the Diſtinct Baſe *d f*, which is in this Philoſophy now Reputed the Real Object, Suppoſe 10 times Nigher than *y* is to *d f*, the Apparent Magnitude of *d f* ſhall again be Increaſed ten times more than before, and the Object by all theſe Helps will be Magnify'd 160 times in Length, or in it's Diametral Magnitude; It is farther Affirmed, that there are various Combinations of Glaſſes in this kind of double Microſcopes, there being in ſome two Object-Glaſſes, one of a very deep Convexity, and the other of a leſſer, placed Nigher the former, than the Projection of the Diſtinct Baſe, according to *Schol.* 1. *Prop.* 16. which is ſometimes called a middle Glaſs, and in others two Eye-Glaſſes, *&c.* but the Theory of all theſe is owned to Depend on the Explanations and Principles of the Foregoing.



Argument the Firſt, againſt this Account.

§. 83. The Grounds of *Prop.* 5. 26. and 50*th.* have been before Proved to be Unjuſt and Abſurd, and therefore the preſent Explanation of Microſcopes cannot be Rational or True, what we ſhall more particularly Examine into now is, the Solution, which is given of the extraordinary Magnitude Obſervable in Objects, View'd thro' this ſort of Glaſs.

Argument the Second.

§. 84. IN the firſt Place this Explanation ſays, that in Viewing a Minute Object, the Eye Approaches to the Diſtance of about 8 Inches, and if it could See it Diſtinctly at the Interval of half an Inch, which is the 16*th* Part of 8 Inches, the Object would Appear 16 times as Large as before, according to *Prop.* 51. 52. But as the Lemmas contained in thoſe Propoſitions have been already Refuted, ſo likewiſe, which was then Obſerved, it is contrary to the plaineſt Fact and Experience in Nature, that Objects ſhould be Seen Magnify'd by our Naked Eye in Proportion to their leſs Diſtance from it; ſince the Letters I now Write, and the ſmalleſt Points, which belong to them, are View'd at 18 or 20 Inches, at 4 or 5, and in all the intermediate Spaces, with the ſame Magnitude and Diſtinction; and when they are Placed nearer to the Eye than the leaſt Diſtance, or farther from it than the greateſt, of their being Perfectly Repreſented to us, they are not in the firſt Caſe Increaſed, or in the laſt Diminiſhed, only look Confuſed and Uncertain.

Argument the Third.

§. 85. SUPPOSING this were yet True, that Minute Objects, or any other, were Increaſed or Diminiſhed, in Proportion as they Approach to, or Recede from the Eye, what this would Signify to the Explaining the preſent Microſcope, we cannot eaſily Apprehend; for when the Eye is Preſumed to be Placed in *y*, and there to View the Object *a b*, it is at the Diſtance of half an Inch, or not; If it is, it Contradicts the Acknowledgments of this Philoſophy, which owns, that a good Eye Sees Objects Diſtinctly at about 8 Inches Diſtance from it, and cannot do it at $\frac{1}{2}$ an Inch, the laſt of which is alſo Manifeſt from common Obſervation; If it is not, the Object will not be Magnify'd in the Proportion of 16 to 1, and Conſequently by a 10 times nearer Diſtance, not as 160 to 1 in it's Length, or the Diametral Magnitude of it.

A farther Explanation of the Preſent Reaſoning.

§. 86. GRANTING farther the Eye at *y*, being $\frac{1}{2}$ an Inch Diſtant from *a b*, to receive the Object Diſtinctly; *a y b* is alſo = *d y f*, which is ſaid to be the ſame, as if the Object was Increaſed to the Bigneſs *d f*, and Viewed by the Eye in *y* or *o* at the ſame Diſtance from it, as *y* in the Figure is now Removed from *d f*, and Conſequently *d y f* = *d o f* = *a y b* will Repreſent the Image *d f* at a greater Diſtance, as *a b* at a leſs, in an Equal Magnitude, that is, on both hands 16 times Bigger than before, and if ſuch a Diſtance is ſhortened to a 10*th* Part of the whole, both the Image *d f*, and the Object *a b*, will be Increaſed to 160 times the Greatneſs of what it was.

Argument the Firſt againſt this Reaſoning.

§. 87. The Fallacy of which Reaſoning will Appear in the firſt Place from this, that when the Eye is Placed in *y*, it is Impoſſible it ſhould See *d f* at all, ſince the Courſe of the Rays tend from the Object *a b*, and cannot return from *d f* to the Parts, from whence they are Deriv'd; if the Eye is Situate in *o*, it is likewiſe Impoſſible, that it ſhould See *d f* at all, without the Interpoſition of the Glaſs *g l*, becauſe the Rays *d l*, *f g*, would Proceed directly forwards, and Fall beyond the Limits of the Eye in *o*; On the Contrary, if the Interpoſition of the Glaſs is Neceſſary in this Firſt Caſe of making the Object Appear 16 times Bigger, how comes it to be only Introduced in the Second, where it is to Magnify it to 160 times it's Dimenſions?

Argument the Second.

§. 88. BESIDES, the Interpoſition of the Glaſs cannot be Conſidered in the firſt Caſe, where the Object is to Aſſume 16 times it's Bigneſs, Seeing that is Proved, when the Eye is in *o*, upon the Suppoſition that *d o f* is = *d y f*, which is Impoſſible, if the Glaſs is Interpoſed; and if not, it will be on the other Hand Impoſſible, that any Rays ſhould be carried from *d* or *f* to *o*, ſince they would Proceed in the Direction from *d* to *l*, and from *f* to *g*, that is, Diverging from *o*, and not Converging to it.

Argument the Third.

§. 89. IF yet *d o f* was = to *d y f*, and upon the Interpoſition of the Glaſs *o*, was 10 times nearer to *d f* than before, it would not be Neceſſary, that *d f* ſhould Appear 10 times Bigger; 1*ſt*. Not according to the Preſent Scheme, where the Rays are Refracted to *o* from *g* and *l*, whereas in the Angle *d o f* the utmoſt Rays, which could be Refracted, would be *d o*, *f o*, which Fall within the aforementioned *d l* and *f g*; 2*d*. Becauſe the Refractions of *d o*, *f o*, however the Diſtance betwixt *o* and *d f* was Contracted by them, would not be Anſwerable to the Eye's Approaching Regularly from *o* to *d f* without ſuch Refractions, which is made to be the Reaſon in the Firſt Caſe, why the Object is Magnify'd in a certain Proportion, and therefore ought to obtain in the Second; 3*d.* Becauſe, as has been before Proved, the Approach it's ſelf of the Eye to the Object is not a ſufficient Reaſon for it's Viewing it Increaſed in Proportion to it's Proximity or Nearneſs to it.

Argument the Fourth.

§. 90. Moreover, it is Remarkable, that this whole Demonſtration ſuppoſes a Conſiderable Diſproportion betwixt the Object-Glaſs and the Eye-Glaſs, and that the Laſt is much Leſs than the Former, otherwiſe the Diſtance of 10 to 1 could not be Obtained, which is Requiſite to Magnify the Object from 16 times it's Bigneſs to 160, whereas in moſt Microſcopes the Object-Glaſſes and the Eye-Glaſſes are Extremely Small, and do not much Exceed the Diameter of the Object-Glaſs *xz.*

Argument the Fifth.

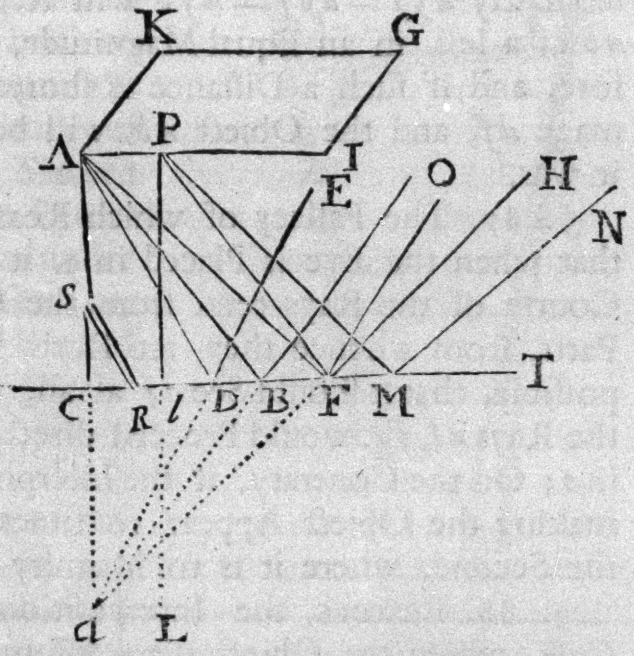
§. 91. Lastly, it is known, that if 160 is Squar'd, the Product will be 25600, the whole Latitude of the Object, as 160 is the Diameter of it, which is not certain in Experience from the foregoing Suppoſitions, but may be more or leſs, for all that we are yet Informed concerning the Real Dimenſions of it; or, if the Abſolute Meaſure of the Object, when Magnify'd by this Microſcope, was truly Aſcertain'd in this way of Reaſoning, it would be hard to Reconcile it with the other of Single Microſcopes, where one Convex only ſhall Magnify as much, if not more, than ſeveral put together.

CHAP. XIV.

Concerning the Propoſitions Relating to Catoptricks.

The Fundamental Propoſition of Catoptricks Explain'd. Dr. Gregory's Elementa Catoptricæ. Prop. 2.

§. 1. After an Examination into the Principles of Dioptricks, we ſhall Proceed to an Inquiry into thoſe of Catoptricks, or the Doctrine concerning ſuch Glaſſes, which, inſtead of Refracting, Reflect the Rays, which are Incident upon them; The Firſt Propoſition, which this part of Philoſophy undertakes to Explain is, how the Phænomenons, which Reſult from common Speculums, are, or can be, Accounted for; which is done in the following Manner, by Proving that *a* after Reflection is the Focus of thoſe Rays, whoſe Focus was *A* before their Incidence; For in the Triangles *ACD*, *aCD*, *AC* and *Ca* are Suppoſed to be Equal by Conſtruction, the Angles *ACD*, *aCD*, are alſo Equal, becauſe they are Right ones, and the Side *CD* is common to both, from whence it follows, that the Angle *ADC* is = to the Angle *aDC*; But the Angle *aDC* is = to the Angle *EDF*, as being Vertical, and Conſequently *ADC* = to *aDC* is alſo = to *EDF*, and *DE* is the Reflected Ray, ſince the Angles of Incidence and Reflection are Equal; The ſame is ſaid to be Demonſtrated concerning any Ray, which Proceeds from *A*, as *AB*, *AF*, and therefore the Rays, which Diverge from *A*, after Reflection Appear to Diverge from *a*; To this it is added by way of Scholium, that wherever the Eye is Placed, as at *O*, it Receives the Rays, as if they came from *a*, and therefore Sees the Image of the Object in that Place; ſince the Rays, which Diverge from *a*, are Affirm'd to Affect the Eye in *O*, as if they were immediately Derived from *A*.



From this Account of our Perceiving Objects by Reflection in Plane Glaſſes, it is Inferr'd; 1*ſt.* That the Rays *ED*, *OB*, which Converge to the Focus *a*, after Reflection Converge to the Focus *A*; 2*d.* That the Image of any Radiating Point, as of *A*, is Seen in the Concourſe of the Reflected Line *OB* Produced, with the Perpendicular *ACa* to the Glaſs *CD*, and therefore if the Part of the Speculum *B* is covered, the Object Diſappears; 3*d.* That the Images, Reflected from the Speculum, are in the ſame Inclination to it, as the Objects, from which they are Impreſſed, and therefore Glaſſes in an Horizontal Situation make Objects Appear in a direct Oppoſition to a Vertical one, ſince what is Demonſtrated of a Point, obtains in a whole Object, which Conſiſts of ſeveral Points; *4th.* That a Radiating Plane and it's Image in the Glaſs are Equal and Alike, but in a different Poſition, as a Seal is from the Character Imprinted on the Wax; *5th.* That the Diſtance of the Image in the Glaſs from the Eye is Equal to the Incident and Reflected Ray taken together; *6th.* That whatever is Affirm'd Concerning the Image of an Object, may be alſo Aſſerted of the Image of that Image, *&c.* Suppoſing ſeveral Glaſſes to be made Uſe of; in which, it is Principally to be Obſerved, that the Diſtance of any Image from the Eye is Equal to the Ray, Incident from the Primary Object, and from thence Propagated thro' all the Various Reflections, which are Conſidered.

Argument the Firſt againſt this Reaſoning in Catoptricks.

§. 2. In Reſpect of the firſt Part of this Reaſoning, we are to Obſerve, that *AC* is made Equal to *Ca*, and Conſequently the Angle *ADC* = to the Angle *aDC* = to the Angle *EDF*, that is, the Angle of Incidence to that of Reflection, which is the ſame, as if Reciprocally, the Angle of Incidence ſhould be ſuppos'd to be Equal to the Angle of Reflection, and by Conſequence *AC* = *Ca*; But it has been before Proved in the beginning of this Fourth Part of the Principles of Philoſophy, that the Angle of Incidence is not always Equal to that of Reflection, and therefore ſuch a Demonſtration cannot be Juſt; On the other Hand, if we take the Original Suppoſition of *AC*, being Equal to *Ca*, it muſt be Confeſſed to be an Arbitrary one; ſince, why we ſhould rather make *AC* = to *Ca*, than of different Lengths, is not Explain'd, unleſs to Prove the Angle *EDF* = to the Angle *ADC*; which is in Effect to Suppoſe, what is to be Evidenced, Namely, That the Angle of Incidence is always Equal to the Angle of Reflection.

Argument the Second.

§. 3. The ſecond Thing Aſſerted is, That the Eye in *O* will Receive theſe Rays upon their being Reflected, which it is not Manifeſt, that it will; for if *DF* is the Latitude of the Incident Rays on the Glaſs, *EH* cannot be that of the Pupil of the Eye, unleſs we ſuppoſe, that it does not only bear a Senſible, but a very Great Proportion to the Reflecting Surface, when in Fact the Eye is Inconſiderable in Reſpect of it; beſides, granting *DF* to be Infinitely Small, and by that Means abundantly Capable of being Receiv'd into the Eye at *O*, which is the greateſt Conceſſion, that can be made in Favour of this kind of Reaſoning, yet if we Conſider the Point *A* Diffuſed into a Plane, *AKGI*, as likewiſe the Line *CF*, into a like Plane, ſince every Point in the Plane, which is Objected to the Glaſs, will have the ſame Conditions with *A* in it's Reflection, according to the Preſent Demonſtration, it will be Neceſſary, that there ſhould be ſo many Diſtinct Points *O* in Reflection, as there are Points *A* in the Object; from whence it will Follow, that the Pupil of the Eye ſhould be at leaſt of as great a Capacity, as the Object Repreſented to it from Reflection, which is Impoſſible.

The Propoſition hitherto Conſidered and the Scholium upon it.

§. 4. Thus far we have Conſidered the Propoſition it's ſelf, and the Scholium upon it, Excepting what Concerns the Place of the Object's Appearing, which is ſaid to be in *a*, for that alſo is unexplained; ſince it is not yet Proved, why the Eye in *O* ſhould rather juſt Extend it's Viſive Faculty to *a*, which is at the ſame Diſtance from the Reflecting Speculum, as the Object it's ſelf is, than to a greater or a ſhorter Diſtance; This however, we ſhall not at Preſent Inquire into, becauſe it falls under a following Conſideration in one of the Corollaries to the Preſent Propoſition.

The Corollaries next to be Examined into.

§. 5. After the Propoſition, and it's Scholium, we are to Examine into the Conſequences, or Corollaries, which are Drawn from it; as to what Reſpects which we muſt Premiſe, that they being Derived from the foregoing Scheme, if any of them Prove Falſe in Fact, and are not Anſwered in Experience, it is a Manifeſt Conviction, that ſuch a Scheme is not an Adequate Repreſentation of Nature.

The firſt Corollary Refuted.

§. 6. The 1*ſt.* Corollary is, that *ED*, *OB*, which Converge to *a*, after Reflection Converge to *A*; But this Reaſoning Depends upon a Principle, which we have before Endeavoured to Refute, Namely, that becauſe *AD* is Reflected to *DE*, and *AB* to *BO*, therefore, if the Rays were Returned back from *E* and *O*, they would be likewiſe Reflected from *D* and *B* to *A*; which would be Impoſſible, unleſs the Rays are Suppoſed to Riſe from the Glaſs Exactly with the ſame Force, with which they Fall; for if *DE* Reſults with a leſs or a greater, from the Action of the Glaſs, than what is in the Incident *AD*, *EDF* will not be Equal to *ADC*, which is the very Foundation of the Preſent Inference; and ſince there is an Attractive Power in the Glaſs Acknowledged by the beſt Syſtemes of Philoſophy Extant, it will by Conſequence follow, that the Incident and Reflected Rays cannot have the ſame Force, ſince AD will be Accelerated in it's Motion, and ED Retarded.

The Second Corollary Refuted.

§. 7. The 2*d*. Corollary is, that the Image of the Point is Seen in the Concourſe of the Reflected Ray OB, with the Perpendicular ACa; If this were True, it would not only Obtain in a Single Point, but in all the Points of the Object; Suppoſing therefore, as before, A to be Spread into one Uniform Plane $AKGI$, the Points K and G will Require a Reflection to ſeveral other Points, which are as Diſtant in their Latitude from O, as K and G are from A, I, which lie in the ſame Plane with C, D; and for the like Reaſon, G, I, will Require a Reflection to ſeveral Points, which ſhall be as far Removed in their Length from O, as G, I, are from A, K; from whence it will follow, that the Eye, to be Rendered Capable of Receiving theſe Diſtant Reflections, muſt be at leaſt as Large, as the Object it's ſelf $AKGI$, which is ſo Monſtrous an Abſurdity, as will be enough to Sink the beſt Philoſophy in the World, however, or by whatever Arguments, it may be otherwiſe Supported.

A farther Objection againſt it.

§. 8. If yet the Eye could Receive all theſe Reflections, without being as Large, as the Object it's ſelf, the Image of it cannot be Seen, by any Thing which is here Demonſtrated, in the Concourſe of the Reflected Ray OB, with the Perpendicular AC, in a, unleſs AC is Suppoſed Equal to Ca, which is Directly Contrary to Experience, as we ſhall Evidence, when we come to Conſider the 5*th*. Corollary.

The Third Corollary Refuted.

§. 9. The 3*d*. Corollary from this Propoſition is, that the Images, Reflected from the Speculum, are in the ſame Inclination to it, as the Objects, from which they are Impreſſed, and therefore thoſe, which are either in a Perpendicular or Horizontal Poſition to the Glaſs, Appear in a Perpendicular or Horizontal one behind it; In Anſwer to which we ſay, that Suppoſing PL to be a Line Parallel to Aa, P will be Reflected from M to N, that is, PM will be Parallel to AB, and Conſequently, ſince the Eye is Placed in O, will not be Seen at all according to the foregoing Sections; or elſe P will be Reflected from F to O, and the Point will Appear in a, at an Equal Poſition from the Glaſs with a, which is Contrary to Experience; and the Angle of Incidence will not be Equal to the Angle of Reflection.

Another Objection againſt it.

§. 10. That, which moreover lies hard againſt this Doctrine is, that what is here Demonſtrated of Objects, which are Parallel to the Glaſs, is Obſervable of thoſe, which are in a Right Angle to it, but how that, which is in a Parallel Situation, and that, which is in a Perpendicular, can be Explained by the ſame Scheme, is Unintelligible; ſince what is Affirmed of AI does not ſeem to Obtain in AC; for let us take ſome of the Points, which are the neareſt to C, and Draw the Lines SR Parallel to AB, which will therefore be Reflected Parallel to BO, and Conſequently will never Arrive at the Eye in O; If the Angle SRC is made leſs than ABC, the Rays will be Reflected below the Eye in O, nearer to BM, if greater, above it, nearer to AC; In both which Caſes, the Rays will not Reach the Eye in O; and indeed, whilſt the Eye is Fixt in one certain Poſition, as in O, it is Impoſſible it ſhould See the ſeveral Points in AC, by Rays Reflected in the ſame Angle, with which they Fell; For Suppoſing A is Seen by the Angle $ABC = OBM$, the Reſt of the Points betwixt A and C muſt either be Seen in Angles of Incidence, which are Equal or Greater, and of Reflection, which are Leſs, or elſe in Angles of Incidence, which are Leſs, and of Reflection which are Greater, which is Contrary to the Fundamental Rules of this Philoſophy.

A Third Argument againſt it.

§. 11. Besides, if we take a Glaſs, and Place it Perpendicular to the Paper, on which I Write, all the Characters will Appear Inverted, and in a Contrary Order to their Natural and Real, whereas, by this Scheme and the Reaſoning from it, every Point ſhould be Expreſſed in the Glaſs, as it lies in the Object, A in a, K in k, G in g and I in i, without any other Alteration, which yet by the Preſent Inſtance is not Experienced.

A Fourth Objection.

§. 12. Farther, this Demonſtration gives us at the beſt an Account only of Objects Appearing in the ſame Poſition or Inclination to the Glaſs, which they have, when they are Parallel to it, but in Experience we find, that if the Glaſs, from a Perpendicular Situation to the Horizon, is made to Turn upon an Axis, till it has Deſcribed a Semi-Circle, the Image of the Object, which is Suppoſed likewiſe to be in a

Perpendicular Poſition to the Horizon, Suitably Changes it's Inclination from being Parallel to the Glaſs, to the being in a Direct Line with it, with all the Intermediate Variations, and the ſame is True, if the Glaſs is Fixt, and the Object is Revolv'd upon an Axis; Thus, if *CD* is the Speculum, *AIGK* the Object, and *AK* the Axis, as *GI* Recedes from the Glaſs, the Image will ſo too, on the other Side of *CD*, and at Length will come to be in the ſame Continued Plane; whereas on the Contrary, ſince *G*, *I*, *P*, and the other Points of the Object, do conſtantly Retire from the Glaſs *CDM* by this Revolution, the Perpendiculars from them to the Glaſs will be Increaſed in their Length, and Conſequently the Angles of Incidence *PFD*, *&c.* will be Greater or Leſs than before, and the Angles of Reflection *OFM*, either Leſs, if the Incident Ray Falls betwixt *F* and *D*, or Equal, if on *F*, or elſe the Angles of Incidence, as *PMD*, will be Equal to the Angles of Reflection *NMT*; If the laſt, the Rays will not be Reflected to the Eye in *O*, but much below it; If the firſt, the Image will Appear at a Leſs or Equal Diſtance behind the Glaſs, as before, contrary to Experience; and the Angles of Incidence will not be Equal to the Angles of Reflection, contrary to the Original Hypotheſis of the Preſent Doctrine.

A Fifth Argument againſt it.

§. 13. LASTLY, in Fact it is Evident, that by the Reflection of the Speculum we See Objects almoſt in the whole Circuit of a Room, and very nearly in all manner of Poſitions at once, which does not ſeem Capable of being Explained by ſo Short and Conciſe a Figure, as what is here Offered; For if the Eye at *O* could View Objects at *AKGI*, which are directly Oppoſed to the Glaſs, how it ſhould from thence be Manifeſt, that it will Take in and Comprehend thoſe, which lie on the Right, or the Left, which are Situate above, and below, and all around it, is not ſo readily Conceiv'd; And when any Optician will Draw us a Scheme in Lines, which Way it may be done, it will be then time enough to Confute it.

The Fourth Corollary Refuted.

§. 14. THE *4th.* Corollary is, that a Radiating Plane, and it's Image in the Glaſs, are Equal and Alike, but in a contrary Poſition, as a Seal is to the Character it Impreſſes; Let us therefore Take the Point *P*, the Rays, which Fall from it, and are Reflected to the Eye in *O*, muſt make the Angles of Incidence and Reflection Equal, and there- *PFD* is = to *OFT*, which is greater than the Angle $OBT = ABC$, from whence the Angle *CAB* will be greater than the Angle *lPF*, and the Lines, which Subtend them, *CB* greater than *lF*, ſince *AC* is $= Pl$; But the Force of the Incident Ray from *P* is = to that from *A*, that is, the Diagonal Force *AB* is = to the Diagonal Force *PF*, and therefore the Forces $AC+CB=Pl+DF$, or *CB* is $= lF$, tho' before it was Proved, that *CB* was greater than *lF*, which is Impoſſible; The Caſe is this, if this kind of Demonſtration, which the Preſent Opticks makes Uſe of, is True, we muſt Suppoſe different Inclinations of the Rays, ſo as the Eye may Return one to *a*, and the other to *L*, and that yet theſe Rays, which are differently Inclined, ſhall have the ſame Conditions, as if they were not ſo, which is a Contradiction, ſince the different Inclination is it ſelf a different Condition; but that this Philoſophy does Preſume all Rays alike, however Incident, is Plain, becauſe the ſame Demonſtration, which ſerves to Prove *AB* will be Determin'd in *a* beyond the Glaſs, ſhows, that *AF*, *AD*, or any other Ray, will Arrive at the ſame Point; If therefore this Reaſoning is not Juſt, *OF* will not Arrive at *L*, that is, unleſs it is Certain, that *AD*, or *AF*, as well as *AB*, will Arrive at *a*, and Conſequently *P* will not be Seen at *L*, nor the Object either Equal, or in a Contrary Poſition.

The Fifth Corollary Refuted.

§. 15. THE *5th.* Corollary is, that ſince *aB* is $= AB$, *aBO* will be $= AB+BO$, that is, the Diſtance of the Image from the Eye is = to the Incident Ray, and the Reflected; If this is a True Conſequence from the foregoing Reaſoning, it is very Manifeſt, that the whole of it is Falſe; For let the Object *AKGI* Continue in the ſame Situation to the Glaſs *CB*, if the Eye at *O* Approaches Nearer to, or Removes Farther from it, the Object will Appear at the ſame Diſtance behind the Glaſs, that is, at the Diſtance of a certain Line or Interval beyond it, as Suppoſe *aB*, and not of *aBO*, Seeing, when *BO* is Continually Altered, there is no Alteration of ſuch an Interval.

Another Argument againſt it.

§. 16. BESIDES Matter of Fact, ſince *PF* is Reflected to the Eye in *O*, which is not Equal to *AB*, and ſince there are Ten Thouſand Rays betwixt *A* and *I*, which Fall from the ſeveral Intermediate Points, which are not Equal to $AB = aB$ (and the ſame may be ſaid of the whole Surface *AIKG*) it is Impoſſible to give a Reaſon, why the Diſtance of the Reflected Image ſhould be rather Meaſured by $AB+BO$, than by

$PF + FO$, or any other Incident and Reflected Rays, which are Extremely different, according to their Various Inclinations to the Reflecting Surface.

The Sixth Corollary Refuted.

§. 17. The *6th.* Corollary is, that whatever has been Affirm'd or Demonſtrated Concerning the Image of any Primary Radiating Point, as *A*, is alſo certain Concerning a Secondary Radiating Point from the Glaſs, which Reflects it's Rays upon another, that is Objected to it, or a *3d. 4th.* or *5th.* Point, which Radiates from a *2d. 3d.* or *4th.* Glaſs, and is Reflected from a *3d. 4th.* or *5th.* But, that this is not True, is Evident from what has been already Propoſed; ſince there is no Juſt Account given of the Primary Radiating Point of an Object being Reflected to the Eye from the Glaſs; Granting which, the Secondary Radiating Point muſt either be in *B* or in *a*; It cannot be in the laſt, becauſe that is Determined by the Reaction of the Eye upon a Reflection from *B* to *O*, nor yet in the firſt, that is, in *B*, becauſe the Rays are Diſperſed from the Primary Radiating Point in *A*, and therefore Impoſſible to be Collected in *B*; from whence it is Evident, that the Point *B* cannot Emit the Rays with the ſame Conditions as *A* does; ſo far from it, that, of the Infinite Number of Rays, which are Suppoſed to be Derived from *A*, there can only one of them Arrive at *B*, which is Reflected to a Second Glaſs, and from thence to the Eye; But how an Infinite Number of Rays Flowing from *A*, and being Reflected to the Eye in *O*, ſhould Cauſe the ſame Appearance with one Solitary Ray Emitted from *B*, and Reflected from a Second Glaſs to the Eye is Inconceivable, and cannot be any ways Explained; And the ſame may be Argued Concerning a *3d. 4th.* or *5th.* Radiating Point, which is here Demonſtrated of a Secondary.

The Reaſon why we do not Enter any farther into the Doctrine of Catoptricks.

§. 18. Having Conſidered the Reaſoning Alledged for the Solution of the Phænomenons of Plane Speculums, we ſhall Deſire an Indulgence from the Preſent Philoſophers, if we do not Enter into a Particular Confutation of the Hypotheſes, Contrived to Explicate Thoſe, which are Obſerved to Reſult from Concave or Convex ones; For if this Philoſophy cannot Account for the Phænomenons, which Ariſe from the former, it is not likely it ſhould do it in Reſpect of the latter, whoſe Theory muſt Neceſſarily be more Intricate and Perplexed than that of common Reflecting Surfaces; Beſides, as Convex Refracting Glaſſes, and Concave Reflecting, have the ſame Effects of Collecting the Rays to a Focus, and Magnifying an Object, which is Preſented to them, and on the Contrary, Concave Refracting Glaſſes and Convex Reflecting, of Diſperſing the Rays, and Diminiſhing the Object, their Explanation in ſeveral Particulars muſt be much alike, and will Depend on Principles, that have a near Alliance and Agreement with each other; and Conſequently, ſince we have Examined into thoſe, which are Offered to Solve the Difficulties, that Occur in Refracting Spherical Glaſſes, we have the leſs Reaſon to Oblige our Selves to a Tedious Inquiry into the Hypotheſes, made Uſe of, for the Solution of the Appearances of Spherical Reflecting ones; we ſhall therefore think our Selves Juſtly Excuſed, if we Omit theſe, and Proceed to the Examination of one or two Opinions more, which this Science Eſpouſes, and ſo Conclude our Diſquiſitions Concerning the Preſent Opticks, which have been hitherto Received; The Firſt is, That of the Optical Axes, which Notion, as it is Managed, is Thought of Singular Service in this Philoſophy, and very Deſervedly, if True; The Second is, That of the Prejudices, which we are Apt to Entertain in ſeveral Caſes, by which Means we are ſaid, not to See Things, as they are Really Repreſented to us, and more Eſpecially in Reſpect of Diſtance.

Concerning the Optical Axes.

§. 19. As to what Relates to the Optical Axes, there are Three Inquiries Produced by the Reverend Dr. *Laughton*, a Late Worthy and Learned Fellow of this Colledge, in his Philoſophical Queſtions, which rather ought to be Named a Compendium of the Modern Philoſophy; The *1ſt.* is, When an Object is Seen with both our Eyes, Whether it is owing to the Concourſe of the Optick Axes, that we See it Single, and not Double; The *2d.* Whether ſeveral Objects, Seen by both Eyes, ought to Appear Double, ſince the Optical Axes, in ſuch a Caſe, are not Particularly Directed to any one Object; *3d.* Whether the different Inclinations of the Optical Axes, upon Various Removals of the Object from the Eye, have any Influence in giving us a Perception of it's Diſtance.

No Optical Axes, Argument the Firſt.

§. 20. In Anſwer to all which Queſtions, we ſhall Endeavour to Prove, that there are no ſuch Imaginary Viſual Lines, as thoſe, which are call'd the Optical Axes; Firſt, Becauſe it is Notorious, that the Eye Sees a Million of Points at once in any

Object, which is Propoſed to it, but it is Impoſſible the Optick Axes ſhould Concurr in all thoſe ſeveral Points, unleſs they can Concurr in a Million of Points at the ſame time, that they can only Meet in one; There is nothing more Confirmed by Experience, than that we See Objects both on the Right, and the Left, Hand of us at the ſame time, and if the Optical Axes are Concerned in Reforming and Adjuſting our Viſion of ſuch Objects, it muſt be by their Divergence, and not by their Concourſe, but of what Service the Divergence of the Optick Axes is in any Part of this Science, we are not yet Informed.

Argument the Second againſt Optical Axes.

§. 21. A Second Objection againſt the Optical Axes is, That if we look upon any Object with one Eye only, the Diſtance of it is the ſame, which is a Manifeſt Confutation of the Optick Axes being any ways Concern'd in Impreſſing that Idea upon us; or if it were not, this very Doctrine of Opticks Endeavours to Explain our Notion of Diſtance by other Methods, that is, by the Optick Angle, by the greater or leſs Magnitude, or the more or leſs Brightneſs of the Object, which little Agrees with the Optick Axes; And therefore ſuch Explanations, whether the one or the other, Seem to be Equally Inſufficient and Invalid; We ſhall only farther Say, in Reſpect of the Optick Axes, that the Inclination of them cannot be any Meaſure of the Diſtance of Objects, where there is no Inclination of them at all, as there is not, when one Eye only Receives the Impreſſion.

We muſt alſo Subjoin, that in Refracted Diſtance, as thro' a Lens, or a Concave, it is Impoſſible for the Optick Axes by any Inclination to Deſcribe the Interval of an Object from us; becauſe, tho' our Eyes were Suppoſed Capable of Directing their Axes to any Point in a Common and Ordinary Medium, yet they could not do it in a Refracting; ſince to what Point ſoever the Axes were Directed before the Interpoſition of the Glaſs, the Refracting Medium would, after it, moſt certainly Interrupt that Direction, and Oblige us to See Objects according to it's own Refracting Power, and not according to the Arbitrary Reference of our Eyes.

Argument the Third againſt ſuch Axes.

§. 22. But That, which lies, as an Invincible Objection againſt theſe Imaginary Axes, is, that there is no Connection betwixt the Cauſe, which they are Suppoſed to be, and the Effect Produced from them, that is, betwixt the Perception of an Object Single, or of Diſtance, and the Concourſe of the Optick Axes, and the Various Angles, which they Comprehend; It is Manifeſt, that the Optick Axes meet, and have a certain Inclination to each other, is an Act of our Wills, and of our own Minds; and That Perception is a Paſſion in us; But how it is Poſſible for the Mind to Perceive by a Reſult from it's ſelf, or how it can be Paſſive by being Active, is a Problem, which wants to be Reſolved; Thus in the firſt Inſtance, that we See Objects Single, is a Perception in the Mind, or an Impreſſion made upon it, that the Axes Concurr, is an Action of the Mind, or a Direction and Impreſſion from it, and Conſequently there can be no Manner of Reaſoning from the one to the other, ſince there is no Agreement betwixt them, unleſs an Impreſſion upon the Mind is the ſame, as an Impreſſion from the Mind, which is altogether Impoſſible; And the like Argument obtains in Reſpect of Diſtance, betwixt which Perception, and the Inclination of the Axes, there is not the leaſt Congruity; If it is ſaid, the Mind Supplies what is wanting in it's Actual Perceptions by it's Judgment, it is the ſame, as to Affirm, the Mind can Perceive by Judging, which is Abſurd and Ridiculous; and we may as Juſtly Aſſert, that the Mind can Hear by Seeing, or Taſte by Smelling, or Apprehend any Thing by one Faculty, which is only Cogniſeable by another, as that it can Perceive by Inference and Reaſoning.

Concerning the Prejudices which the Mind is Suppoſed to Receive, in Reſpect of Viſion, and the Confutation of that Doctrine.

§. 23. The Laſt Thing we ſhall take Notice of in the Preſent Opticks is, the Prejudices, which the Mind is Suppoſed to Receive, by which Means it Oftentimes is Thought to See Objects at a Different Diſtance from that, in which they are Really Repreſented to it; But, that this is not ſo, is Evident from the foregoing Section, ſince it is Impoſſible our Judgments ſhould Impoſe upon our Senſes, however our Senſes may upon our Judgments; For tho' we may Falſely Collect, that an Object is nearer than it in Truth is, becauſe our Senſes Perceive it ſo, yet our Senſes cannot Perceive an Object nearer than it is, in Conſequence of our Mind's Judging it to be ſo.

1ſt. Because

1ſt. BECAUSE, if the Judgment of our Minds could Determine us in our Perceptions, it could make us See Objects farther off, as well as nearer, than we Really Perceive them; and ſince we know the Sun to be ſeveral Millions of Miles Removed from us, Conformable to that Judgment, we ſhould Apprehend it at that Diſtance, as well as in a Reflecting Concave Glaſs, we ſhould See an Object at a very near Interval, whoſe Image is ſaid to be at an Infinite one from us, upon the Account of our Knowing, and having before-hand Judged, it to be nigh the Glaſs, which Reflects it.

2d. BECAUSE, if the Judgment of our Minds could Determine us in our Perceptions, it muſt be Affirm'd, that what we Remember has a greater Force, and makes a ſtronger Impreſſion upon our Thoughts, than our Actual Senſations; Thus in the Inſtance above-mentioned, the Image, which is Reflected from the Concave, is Suppoſed to be at an Infinite Diſtance from it, and ſo Repreſented to the Eye, that is, that the Rays come to the Eye, as they would from an Object, which is Actually Placed at an Infinite Diſtance, and yet the Image Appears at a very near one, becauſe we had before Judged, or it had been already Impreſſed upon us, that the Object was ſo; which is as much as to Say, that a Former Impreſſion made upon our Minds is more Forcible than a Preſent; But, ſince that Former Impreſſion was once Preſent, for that very Reaſon, the Preſent cannot be leſs Forcible than the Former.

3d. IF our Pre-judging could have any Influence upon our Actual Perceptions, and Particularly in the Caſe we have Named, it would be a Reaſon, why we ſhould never Perceive any Thing at all; For if we do not Perceive Things, as they are Impreſſed upon us, becauſe we Perceived them otherwiſe before, for the ſame Reaſon we ſhould not Perceive them at all, becauſe there was a Time Prececedent to our Perceiving them, when we Really did not Perceive them.

4th. IF Pre-judging can Alter our Preſent Perceptions, or Senſations, our Preſent Judging may with Equal Reaſon Alter our former Perceptions, that is, if we See an Object at a nearer Diſtance, becauſe we had before Judged it to be ſo upon another or a former Perception, we ought upon the Preſent Perception, which if it Obtained it's due Force, would make the Object, or Image of it, Appear more Remote, to Judge, that the Object was more Remote in our firſt Perception, than it Appeared, and accordingly, whenever we look on it again, after ſuch a Perception, it muſt by this Reaſoning Appear more Remote.

5th. IF Pre-judging has any Thing to do in our Perceptions, we muſt Aſſert this, that becauſe an Object Appears at a certain Diſtance in one Circumſtance, Namely, where no Glaſs is Interpoſed, therefore where a Glaſs is, and the Circumſtance of the Object is entirely Chang'd, the Mind is ſo Irrational to Conclude, that the Diſtance will be ſtill the ſame; It ſeems to be a much quicker Reaſoning for the Mind, and a more Natural, into which theſe Prejudicate Perceptions are Reſolved by the Preſent Philoſophy, to Imagine, that where the Mediums, thro' which we See Objects, are Altered, the Preceptions of them muſt be ſo likewiſe.

6th. IT is alſo to be Obſerved, that, if Prejudice could Conquer and Subdue our Actual Perceptions, it ſhould do it in one Caſe, as well as another, and the Image Reflected from a Concave Speculum ought to Appear in the ſame Dimenſions, as the Object, as well as the Diſtance from it; For why we ſhould See an Object Reflected from a Concave Speculum, much greater than it is, and it's Diſtance the ſame, when our Prejudice would Equally carry us to think the Object no greater, and the Diſtance no more, than what we before Perceived them, is not to be Explained; It is therefore Evident from what has been Alledged, that this Notion of Prejudices in the Mind being able to Alter, and even to Efface, and Obliterate, our Actual Senſations, is nothing elſe than a Fineſs, and a Prevarication in Philoſophy, to which Recourſe is always had, where it is not able to Account for our Real Perceptions; It being very Eaſy to Say, we are Prejudiced, where we do not See Things, as theſe Philoſophers would have us.

Concluſion upon Opticks, Dioptricks, and Catoptricks.

§. 24. To Conclude this Fourth Book of our Examination into the Principles of the Preſent Philoſophy, we do only Requeſt of the Rationaliſts, and Free-Thinkers of our Age, to Judge more Modeſtly of their own Reaſoning, if, what has been Propoſed to us under the Name of Demonſtration, we have Proved comes far ſhort of it; This of Opticks is that Part of Philoſophy, which has been Thought more Eſpecially to Arrive at a Mathematick Certainty, whereas if the Contrary has been Evinced, the World will do well to Conſider, how much they are to Confide in ſuch Pretences of Reaſon, and how much they ought to Value their Religion, in Preference to whatever the Wiſdom of Men can Offer in Affront to it.

It is ſomething Strange and Unaccountable, that, tho' we are every Age Convinced of the Vanity, and Falſeſhood, of the ſeveral Syſtems of Philoſophy, which have been Produced to the World, by One, Like an *Ignis Fatuus*, Diſappearing, and Another Riſing in it's Room, We ſhould yet always be ſo Fond of the Prevailing, as if That Undoubtedly muſt be Right, when None before had ever Succeeded, but Each had been Baffled in it's Turn; And one would be Amazed, that Men, who make Great Scruples and Difficulties in Believing the Doctrines of Religion, ſhould not take Warning from their Predeceſſors, in being too Credulous in theſe Matters; But ſo it is, that as in Enthuſiaſm, ſo in Philoſophy, there is no Convincing Men of their Errors, 'tis a Kind of Frenzy, which Poſſeſſes the Mind for the Preſent, and muſt have Time to Work it's ſelf off, and it is by Degrees, and by Patient Waiting, not by the Application of Reaſon and Argument only, that they at Length Recover their Senſes.

Had *Ariſtotle* been once ſo Uſed, as he is now, Perſons would Scarcely have been Thought to have been in their Witts, whereas, ſhould any one go about to Defend his Opinions of Nature at this Time of Day, he would ſoon be Concluded to be out of Them; And why may we not Think the ſame of the Modern Syſtems; That They alſo may in Future Times be Diſparaged and Decryed, as much as They are now Eſteemed and Valued? Human Nature is the ſame, as it was, and the Genius's of Men are, at leaſt, not Improved; Nor do we find Brighter, or Nobler, or Sublimer, Faculties in thoſe, who Commence Authors in this Age, than in thoſe who Wrote many Hundred Years ago.

We may farther Affirm, without any Claims to Inſpiration, or a Power of Divining, That the Modern Syſtems of Philoſophy cannot ſtand Long, if we are True in our Explanations of Nature, from our Doctrine of the Expanſive and Contractive Forces; And if, likewiſe, Men will put themſelves to the Trouble of Examining, and Determining according to the Truth and Reality of Things, and not according to Intereſt, Paſſion, Prejudice, or Authority.

THE

PRINCIPLES

OF THE

PHILOSOPHY

OF THE

EXPANSIVE and CONTRACTIVE

FORCES.

BOOK V.

Concerning the METAPHYSICKS and LOGICKS, or the
SYSTEME of IDEAS, of Mr. *LOCKE*.

THE PRINCIPLES of the PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK V.

CHAP. I.

Concerning the Deſign and Intention of the Eſſay on Human Underſtanding.

Concerning the Deſign and Intention of the Eſſay on Humane Underſtanding. Mr. Locke's Introduction.

§. 1. THE Purpoſe, it ſeems, of this Eſſay is to Inquire into the Original, Certainty, and Extent, of Humane Knowledge, together with the Grounds and Degrees of Belief, Opinion, and Aſſent, that is, in two ſhort Words to Inform us, what Faith and Reaſon are, and how far we may be Convinced by Either of them; A Deſign, which is Truly Great and Noble, but was Illy, and too Raſhly undertaken by a Philoſophy, which had not Conſidered the Evidences, which Ariſe from the One or the Other; Had this Philoſophy Impartially, and Sincerely, Examined into all the Sciences, which are Subject, and do Properly belong, to the Province of Reaſon, or the Connection of Ideas, and at the ſame time made a Juſt Inquiry into thoſe Evidences, which might Rationally Convince us of any Hiſtorical Fact, whether Sacred or Profane, Neither of which it has done, becauſe It Underſtood Neither, we might have had ſome mighty Expectations from ſo great a Performance; But, as it is not found, nor does it ſtand upon Record in it, that it Knew one Rational Science in Nature, or was much Acquainted with the Hiſtories of the Ancients, or the Reaſons, which we have to Believe them, we may Juſtly Think, and be Aſſured, that it is an Unfit, and a very Improper, Judge of Either.

The Qualifications, in this Philoſophy, of Reaſon and Hiſtory, too Low to make it a Judge of the Evidences of either.

§. 2. As to Reaſon, it was very Extravagant for this Philoſophy to Think to ſet Bounds to the Celebrated Inquiries of Sir *Iſ. Newton*, and all the Philoſophers, and Mathematicians, who ſhould Succeed him; When, by what Appears, it Knew little more than the Corpuſcularian Scheme of it, then in Requeſt, and in Mathematicks, two Common Propoſitions in *Euclid*, That the three Angles of a Triangle are Equal to two Right ones; And That, in a Rectangle Triangle, the Square of the Hypotenuſe is Equal to the Squares of the Sides, without, for all that is yet Evident, it's Underſtanding the Demonſtrations of Them: And as to it's Knowledge in Hiſtory, it was, ſo far as can be Collected from the Tenour of it's Writings, Confin'd pretty much to the Modern Travels and Voyages, to the Accounts of the Bay of *Soldania*, and the *Hottentotts* of *Africk* and the *Cape* of *Good Hope*, and the Inhabitants of the *Caribbees*, and to ſuch Kind of Stories, which are Affirm'd, and Refuted every Day:

Mr. Locke's Introduction.

AND with theſe Grand and Magnificent Qualifications of Knowing little in Philoſophy, or the Mathematicks, or being Acquainted with any Rational Science, which is Extant, and of being a Marvellous Proficient in the Hiſtories of the *Romiſh Miſſionaries*, and other *Voyagers* to the *Indies*, this Philoſophy is Pleaſed to ſet up for a *Preſcriber* of Limits to Human Underſtanding, for a *Settler* of the Boundaries of our Knowledge, and for a *Tetherer* to the Mind of Man, ſo far as to let it Know, how far it can, or ſhall, go, and where it muſt ſtop; This is one Part of it's Province: Upon the other Branch of it's vaſt Attainments, from the Reading of the Reports and Stories of Travellers, it Riſes into an Inſtructor, and a Dictator, of what we are to Believe, Concerning all the Ancient Hiſtories, even the Beſt and the Trueſt of them, whether Sacred or Civil, and which all of them, therefore in it's Senſe, are only Probable at the moſt; becauſe it's Converſation lay amongſt thoſe, which had little Authority, and leſs Integrity, to Recommend them; And Conſequently it too haſtily Imagined, that the Ancient ones were much upon the ſame Level with them, without giving it's ſelf the Trouble to Conſider the different Atteſtations belonging to them.

The Human Underſtanding Boundleſs and Unlimited and not to be Confined to the Narrow Sentiments and Attainments of this Philoſophy.

§. 3. THE Real Deſign therefore of this Philoſophy of Ideas, whatever it's Gloſſes, or Pretences, may be, is, Firſt to Propoſe it's ſelf, as Judge in all Matters of Reaſon, and to Tell us, how far our Underſtandings can Reach in Mathematick and Philoſophick Inquiries, which, if we Reflect upon the Small Portion it was Poſſeſſed of, in Either of thoſe Studies, cannot but Raiſe in every Learned Mind a Kind of Contempt and Indignation, at ſo Dareing and Arrogant a Preſumption; Muſt all the Glorious Atchievements of the Men of Speculation and Science be laid Aſide, becauſe It Pronounces, we can only Extend our Knowledge to ſuch certain Limits and Degrees, which It has been Pleaſed to Preſcribe to it, from it's own Want of it? Or muſt we Think, that it was Impoſſible for Sir *Iſ. Newton* to make any Advances in Natural Knowledge, in Fluxions, or the Infinite Series, becauſe this Philoſophy had never the leaſt Notion, or Apprehenſion of them?

THIS Philoſophy ought to have Known, that the Underſtanding of Man is Boundleſs, and that there might Poſſibly be ſuch Principles of Reaſon and Knowledge, which were as much above it's own, as it's own were Superior to thoſe of an Ideot; and of which, when it was not Apprized, it would Diſcourſe of Reaſon, and the Powers of the Mind, with as much Ignorance, as one, who never ſo much as Read *Euclid*, would do of *Apollonius*, or *Archimedes*.

Mr. Locke's Eſſay.

THERE is nothing more Plain, than that this Philoſophy of Ideas muſt either give us the Limits and Bounds of Human Knowledge, from what it's ſelf was Informed of, or not; If it Meaſured the Extent of all Knowledge by it's own, it was a Falſe and Injurious, as well as a Scanty Meaſure; If it did not, it Diſcourſed out of the Bounds of it's own Knowledge, and *Lighted* up the *Candle* of it's *own Reaſon* to thoſe, who *Walked* in *Clear Sunſhine*.

This Philoſophy Confeſſes it's ſelf Ignorant of the Nature of the Mind, and only Propoſes an Hiſtory of it.

§. 4. BUT, being Conſcious of it's own Defects, and Frailties, in theſe Matters, it Tells us, that it ſhall not at Preſent Meddle with the Phyſical Conſideration of the Mind, or Trouble it's ſelf to Examine, wherein it's Eſſence Conſiſts, or by what Motions of our Spirits, or Alterations of our Bodies, we come to have any Senſations by our Organs, or any Ideas in our Underſtandings, and whether thoſe Ideas do any, or all of them, Depend on Matter or no; It ſays, it only Intends to give us an Hiſtory of the Mind, and it's Faculties, and by that Means it Hopes to be very Serviceable to our Knowledge of it.

No True Hiſtory of the Mind without underſtanding ſomething of the Nature of it.

§. 5. BUT, how this Philoſophy of the Mind, and of it's Ideas, ſhould give us any Account, or Hiſtory, of it, without knowing ſomething of it's Eſſence, and Nature, whether it is Material, or not, tho' it Inclines to the Latter in the Progreſs of this Eſſay, is Inconceivable.

Any One would be Eſteemed to be a very Lame, and Imperfect Hiſtorian of Light and Colours, or of Plants and Vegetation, or of any Being in Nature, who did not know ſo much, at leaſt, of the Eſſence of that Being, as to Inform us, whether it was Matter, or not Matter; Becauſe if Light and Colours, or if Plants, and the Vegetation, which Attends them, have a Principle Appertaining to them Diſtinct from Matter, all the Hiſtorical or Experimental Properties, which Reſult from them, will be but ſo many Names, without our Underſtanding, what thoſe Properties are, or wherein their Nature Conſiſts; Thus there are a Million of Plants and Vegetables, which Grow upon this Earth, which are ſo many Hiſtorical Facts in Nature, and yet, if we Knew nothing more than, that there were ſuch Plants or Vegetables, and the Names of them, without Underſtanding their Qualities, Natures, or Virtues, we ſhould only have a Vocabulary in our Minds, and ſhould be very Wiſe in our Knowledge of Names, and at the ſame time very Ignorant in Things themſelves.

And this is the Caſe of the Preſent Philoſophy of Ideas; It Pretends to Inform us of the ſeveral Properties of the Mind, which do in Fact Exiſt in it, that it Wills, Remembers, Reaſons, and Judges, but will not be ſo kind as to tell us, what it is that does ſo, whether it is Matter, or not, or what the Nature of that Being is, which has all theſe Great and Sublime Qualities; But if it is ſo Doubtful and Cautious in a Thing ſo Certain and Clear, as that of the Mind's Immateriality, how can it be ſo Punctual and Poſitive, in Reſpect of our Ideas being Innate or not? Becauſe, if the Mind is a Being of it's ſelf, and Independent of Matter, it may have Notions and Conceptions Originally of it's own, without Thanking this Philoſophy for Introducing them into it; For how can this Philoſophy Say, when it does not Enter into the Nature of the Mind according to it's own Profeſſions, whether it may not be Furniſhed with Certain Ideas Previous to thoſe, which it Receives from Senſation.

Any One therefore, who would give us a True and Impartial Hiſtory of the Mind of Man, ſhould at leaſt be Informed of the Nature and Kind of that Being, upon which he is willing to Comment; Otherwiſe we ſhall be Entertained with a very Defective Hiſtory of it, and ſhall be only Inſtructed what the Mind is, which this Philoſophy Means, but not Furniſhed with a Real and Juſt Account of what is Meant by Human Underſtanding.

But, as this Philoſophy does not Profeſs to Inquire into the Nature of the Mind, whether it is Immaterial, or not, ſo we ſhall Excuſe it upon that Head, and ſhall therefore Examine, whether it has given us a Juſt Hiſtory of it, in Reſpect of it's ſeveral Operations, and of Abſtracting and Compounding it's Ideas, which we Say and Affirm, it has not done; And as that is, what it Acknowledges to be it's Deſign, upon that only, I ſhall put the whole Streſs of the Controverſy in Relation to Reaſon; Becauſe, Indeed it Knows nothing more, and therefore was not Capable of Judging any farther.

§. 6. A Second Province, which this Philoſophy Undertakes, is, to Inform us, what Faith and Opinion is, and what Degree of Aſſent we are to Yield to Things, which Depend upon Teſtimony, and Report, All which Evidences, in it's Notion, Riſe no Higher than to ſome Kind and Degree, or other, of Probability; So that the moſt Certain and Undoubted Matters of Fact, that there is ſuch a Country, as *Spain*, or the *Indies*, that there were ſuch Cities, as *Athens* and *Rome*, and ſuch Hiſtorians, as *Thucydides* and *Livy*, that there was ſuch a Perſon, as our Saviour, ſuch a Doctrine, which he Taught, and ſuch Miracles, which he Performed, according to the Account and Eſtimation of this Philoſophy, are not Certain, and only highly Probable; And Who is it, that tells all the Learned World, and all the Divines of it, this Wonderful Story? A Dealer in Voyages and Romances, and as to the Ancient Hiſtories, whether Eccleſiaſtical or Profane, and the Undoubted Credit of them, as may be Juſtly Imagined, a Perfect Novice.

What Faith and Opinion is, according to this Syſteme.

Thus a Perſon, who Knew little or nothing of Nature and Philoſophy, ſets up for a Dictator to the Wiſeſt Men in thoſe Sciences; And One, who had Cultivated but a ſparing Acquaintance with the Learning of Former Times, Propoſes himſelf for an Umpire and a Judge to all the Excellent Perſons of the Preſent, in the Knowledge of thoſe Times, how far they are to Value, and to Regard it.

THIS, in Plain and Open Terms, is the Real Deſign of the Preſent Philoſophy of Ideas, and theſe the Qualifications, as it Appears, with which it was Endued, for the Perfecting and Accompliſhing of it; Of which in what Manner it has Acquitted it ſelf, ſhall be the Inquiry of the following Chapters.

CHAP. II.

Concerning no Innate Ideas, and the Mind's being a Raſa Tabula.

That the Mind is a Raſa Tabula, *not Proved, becauſe this Philoſophy owns it's ſelf Ignorant of the Nature and Conſtitution of it.* Mr. Locke's Eſſay, Chap. 2. 3. 4. Book 1. and Chap. 1. Book 2.

§. 1. THE firſt Attempt in this Philoſophy is, to Clear the Mind of any Innate Ideas or Principles, and to make it a *Raſa Tabula*, or to Reſemble a Piece of Blank Paper, without any Original Characters, or Inſcriptions, Engraved upon it; Whereas it is Impoſſible for it to Prove this, whilſt it Profeſſes it's ſelf Ignorant of the Nature and Conſtitution of the Mind; For, if it is a Being Independent of the Body, and had an Exiſtence of it's own before it's Union with it, it is not certain, but it might have Ideas Peculiar to it's ſelf, Previous to any Received from Senſation; This Philoſophy will not Diſown, that there are Spirits and Intelligences, which Exiſt in the World, Separate and Diſtinct from Body, and that Almighty God does Exiſt in that Manner; How then can it be Evidenced, that Human Minds may not have, not only Powers and Faculties, but Ideas alſo, Derived from thoſe Powers, which were not owing to any External Impreſſions? How can it Demonſtrate, that our Souls did not Pre-exiſt to their Union with our Bodies? All which is to be done, before it can Affirm, that the Mind has no Innate Ideas, no Proper Notions, belonging to it, which are the Immediate Reſult and Conſequence of it's own Immaterial Nature.

The Mind may have Ideas of it's own, without being Derived from Senſation, and therefore not a Raſa Tabula.

§. 2. BESIDES, if the Mind had it's Riſe, and Origin, at the ſame Time with the Body, if yet it is of a Diſtinct Nature from it, if it is Suppoſed to be a Perceptive, Intelligent Being of it's ſelf, it might, and would, have Ideas and Notions of it's own, tho' none had ever been Communicated to it from Senſation, and it might Reflect upon it's own Operations and Powers, tho' it had been never joined and United to a Corporeal Subſtance.

There muſt be ſome Perception or other in the Mind Previous to any Impreſſion upon it, and therefore ſome Innate Ideas in it, and the Mind not a Raſa Tabula.

§. 3. NAY farther, if the Mind is only Allowed to have a Capacity of Perception at the ſame Time, that an External Impreſſion is made upon it, and it would be Abſurd not to allow it that Capacity, ſuch a Faculty of Perceiving muſt either Include Actual Perception and Senſe, Originally in the Percipient, or not; If it does, then there is Senſe and Perception in the Mind, Previous to the External Impreſſion; If it does not, Senſe and Perception is Communicated to that, which had no Senſe and Perception before, from an External Impreſſion, which has no Senſe or Perception belonging to it; Which is as Irrational, and as much Impoſſible, as that Inanimate Subſtances, or mere Corpuſcles of Matter, by a Mutual Concourſe, ſhould Produce Thought, or Perception, which, this Philoſophy does Acknowledge, cannot be; But, if there is Senſe, and Perception, in the Mind, before the External Agent ſtrikes upon it, it has ſomething Innate, ſome Ideas, or Notions, or, by whatever other Name we Pleaſe to call them, ſome Senſe, ſome Perception, or other, Peculiar to it's own Nature.

The Mind not a Raſa Tabula, Evident from the Different Faculties, Attainments, and Genius's of Men, and the Senſations and Reflections of Brutes.

§. 4. FOR Moreover, if the Mind was a mere Raſa Tabula, if it was only a Plain Piece of Paper to Write on, what Difference could there Poſſibly be in Fact in One Man's Underſtanding, and Another's? The Characters Engraven on each, would be much the ſame, Deriv'd from thoſe Senſations, which are Common to all; ſince according to this Philoſophy, what is Originally Writ upon our Minds, is from our Converſation with External Objects, and then Reflecting upon the Operations of the Faculties and Powers of our Underſtandings; Theſe therefore would be nearly Alike in all Men, and there would, in General, be little or no Room left for that Prodigious Variety and Diſtinction, Obſervable in the ſeveral Thoughts, Reaſonings, Opinions, and Fancies of Men, and in the Forces and Strength of their Proper Genius's, and Complexions of Mind; If the Ground-work of all was nothing elſe than a Raſa Tabula, and the Pencils Employed to Embelliſh it, were no other than our Senſes, which are the ſame in moſt, and the Paintings and Portraitures made upon it, the Conſtant and Unvaried Objects of Nature, an Ideot, according to ſuch an Account of the Mind, may be as Wiſe, as a Philoſopher, and a Brute as Knowing, as a Man; There is one Raſa Tabula belongs to them all, the ſame Senſes to Inſcribe the Various Characters of External Nature upon it, and it is not to be Doubted, but they have Reflection.

There ſeems to be nothing more Evident, on the Contrary, than that there is a Peculiar Nature, and Genius, a Various Force, and Power, which belongs to every Individual and Animate Being, and that even in Human Minds, there is ſome Diſtinction, by which One Perſon comes to have Sentiments and Notions, entirely Different from thoſe of Another; that therefore the Mind is not a Raſa Tabula, upon which any Thing may be Inſcribed, but can only Receive ſuch Notions and Apprehenſions of Things, either from Senſation or Reflection, as are Suited to it's Particular Nature; which ſeems to be the Riſe of all the Various Reaſonings, and Opinions, we find, either in common Life, or amongſt Men of Science and Speculation.

Nor is it Unreaſonable to Think, that from this Vigour, Force, and Strength of Mens Minds, in which they are Infinitely Superior to any other Animate Beings, the Natural Immortality of them may be One way Derived; But this is not our Preſent Inquiry.

§. 5. **Lastly**, Suppoſing the Mind was not an Immaterial Subſtance, Grant it to be a Material one, if it has yet any Peculiar Nature or Conſtitution of it's own, it could not be a Raſa Tabula, upon which any Thing might be Imprinted: This Paper, for Inſtance, on which I Write, is Suſceptible of thoſe Characters, which I Draw upon it, becauſe it's Nature is ſuch, as to Receive the Impreſſion of the Ink, which Falls from the Pen, but Fire, or Flame, would not Admit of the ſame Characters, Deſcribed in the ſame Way, nor would Oil, or Spirit of Nitre, do it, nor, on the Contrary, would it be Poſſible to Write theſe Characters upon this Paper with thoſe Subſtances; That is, the Paper I Write on, as much Concurrs by it's Nature to the Production of theſe Characters, as the Ink does, or the Hand which Writes them; And thus it is in the Mind, if it is any Thing at all; It cannot Receive any Characters, whatever we Pleaſe, but ſuch only, as are Suitable to it's Being and Conſtitution; and therefore, if ſuch Being and Conſtitution in Different Men is Different, the Inſcriptions made upon their Minds will be Different likewiſe; and Conſequently, what may ſeem Good to ſome Men, may Appear Vicious and Ill to others, what may Produce Pleaſure in one Man, may in another Cauſe Pain and Uneaſineſs, and which is more, what one Perſon ſhall Eſteem Juſt and Reaſonable, another may Think has the utmoſt Iniquity and Baſeneſs in it; And all this may be Derived from the Various Genius's and Complexions of Men's Minds, and the Natural Powers and Faculties, which belong to them; A Good Temper will, in the ſame Inſtance, Judge Mercy, and Kindneſs, and Compaſſion, the moſt Reaſonable Method of Procedure, in which an Auſtere and Rigid One will Prefer Harſhneſs and Severity; Which ſeveral Kinds of Reaſoning, in the ſame Caſe, cannot well be Accounted for, excepting from a Fundamental Difference in Men's Minds, and upon the Conceſſion, that they are not, nor can be Conſidered, as ſo many Raſæ Tabulæ.

If the Mind was Material, which this Philoſophy ſeems to Defend, it yet could not be a Raſa Tabula.

§. 6. **But** be our Minds Raſæ Tabulæ, or not, it will be the Part of Wiſe Men to Inculcate and Impreſs upon them, when they are Young and Tender, the moſt Noble Sentiments, of which they are Capable; This ſeems to have been the Practice of the moſt Antient and Eaſtern Nations, from whom the *Greeks* and *Romans*, and the Preſent *Europeans*, Derive all their Principles of Reaſon, Philoſophy, Religion, and Learning; For they in their very Alphabets, which they Taught their Children, Appear to have Contrived them for their Inſtruction, in the Grand and Sublime Dictates of Religion and Reaſon, at the ſame Time, as they Learned and were Informed, how to Write and Read; So that the Alphabet of the Eaſtern Countries, and of the Old *Egyptians*, in all Probability, was a Summary of Divine and Moral Precepts, Convey'd to them with the very Rudiments of their Language and from their Infancy, Expreſſed by Hieroglyphicks, and Explained by their *Magi*, or their Inſtructors; And from whence, the Fables of the Antients Poſſibly, and Particularly thoſe of *Æſop*, were Modelled and Framed.

Whether the Mind is a Raſa Tabula or not, it becomes every Wiſe Man to Inculcate the beſt Principles upon it, Namely, thoſe of Religion, and of Reaſon; This Confirmed from the Deſign of the Antient Alphabets; The Difficulty of Inventing which is Explained.

Thus *Aleph*, or *Alpha*, which Signifies an Ox, in all Likelihood, was the Repreſentation of God, to ſhew his Stability and Strength, his Majeſty and Greatneſs, and Force; And this is the Firſt Letter in the Alphabet to Inſtruct us, That God is Principally to be Regarded by us, and that he is to be Firſt in our Adoration and Eſteem.

Bath is a Daughter, and *Beth* an Houſe, to Inform us, That our Domeſtick Relations ought to Claim our next Veneration and Reſpect; Which two Precepts are exactly Agreeable to what we find in the Writings of *Moſes* and *Pythagoras*, who both were in *Egypt*, and were Inſtructed in all the Learning of it; The Firſt in Effect Tells us, That a Molten Calf, or Ox, was the God, which the *Egyptians* Worſhipped, *Exod.* 32. which is alſo Confirmed by all the Accounts of their *Oſiris*, or *Miſraim*, or *Meres*, or *Apis*, being Worſhipped in the Form of a Bull; The ſame Inſpired Perſon alſo Tells us, that God was firſt to be Worſhipped, according to the Commandments of the firſt Table, *Exod.* 20. And, That our Parents were in the next Place to be Reſpected by us, according to the firſt Commandment of the ſecond, in the ſame Chapter; The Laſt in the χρυσᾶ Ἔπη, whether they were his, or one of his Diſciple's, is not Material, Gives us the following Precepts.

Ἀθανάτους μὲν πρῶτα Θεοὺς, νόμῳ ὡς διάκειται,
Τίμα, ϗ σέβου Ὅρκον· ἔπειθ' Ἥρωας ἀγαυούς.
Τούς τε καταχθονίους σέβε δαίμονας, ἔννομα ῥέζων,
Τούς τε γονεῖς τίμα, τούς τ' ἄγχιστ' ἐκγεγαῶτας.

Which Perfectly Correſponds with the Precepts of *Moſes*, That the Deity is firſt to be Worſhipped, and our Parents in the next Place to be Honoured by us.

Gimel, or *Gamal*, Imports a Camel, and therefore Poſſibly Denoted Induſtry and Patience in their ſeveral Vocations and Employs, which they ought to Purſue with Conſtancy and Labour, and of which that Creature was a Glyphick or a Characteriſtick, and an Example and Monitor to them.

Daleth has the Signification of a Gate or Door of an Houſe, which by the *Magi* and the Doctors of the Antient *Egyptian* Learning, even before the Times of *Moſes*, might be Explained, as an Inſtruction to keep within, and to give a Cloſe Attendance to the Affairs of the Family.

He, from *Hiſah*, to be Silent, might with the ſame *Magi* Signify, that a Family ſhould be Governed and Ordered without Noiſe or Tumult, that each ſhould Perform the Buſineſs of his Station with Silence, and that Oeconomy Conſiſts in ſuch a Regular Diſcharge in every one of his Duty, without Quarrels and Contentions, and even without the Speaking of a Word; And it is not Impoſſible, that it might not likewiſe farther Intimate all the Advantages of Silence in Reſpect of our Conduct and Behaviour in our common Converſation, as alſo in all the Solemn Offices of Religion, which always adds a kind of Reverence and Grandeur to both, there being more Eloquence many Times in Silence, than in the greateſt Performances of Human Wit and Oratory; And from hence it is not unlikely, that the Pythagorick Silence was Derived, as alſo That amongſt the *Greeks* in their Engagements with the *Trojans*, according to *Homer*, as That likewiſe amongſt the Eaſtern Nations to their Princes, and in all their Grand Solemnities, and which is Preſerved amongſt the *Turks* and *Mahometans* to this Time, and with us *Chriſtians*, in the moſt Important and Serious Affairs; Nature ſtill Dictating to us, what was Thought to be a Juſt and Rational Precept with the moſt Learned and moſt Knowing of the *Egyptians*, their *Magi*, even before the Great Law-giver of the *Jews* Appeared.

Vau, which Signifies the Hinges, on which a Door Turns, might be Interpreted by the ſame *Magi*, to be an Emblem, that we ſhould not be always Recluſe, but that the Door ſhould be Opened for Hoſpitality, for Acts of Love and Kindneſs, and to Keep up a Commerce with others, and to Cultivate the Duties, which belong to a Community.

I might go thro' the Remaining Part of the *Hebrew* Alphabet, from which the *Greek*, and others, are Evidently Derived, in the ſame way of Conjecture; And, Tho' Perhaps it would be Difficult to Aſſign a Meaning to every Word in it, both becauſe there may be many Words Loſt to the *Hebrew* Language, which were then in

Being, when the Alphabet was Formed, and becauſe we may not know the full Extent of that Language; As alſo, becauſe ſeveral might be Derived from the Old *Egyptian* or *Coptick*, which is entirely unknown to us; Yet it is not Improbable, but that this was the firſt Deſign of Alphabets, to make Children Learn their Letters, and at the ſame Time ſome Wholeſome Precepts Annexed to them, both as it is a Natural Way of Inſtruction, and as Words are made Uſe of in them; Whereas mere Letters might have been as well Expreſſed by Syllables, as *A*, *B e*, *Ce*, *De*, *E*, *e F*, *&c.* According to what is done in our Modern Languages.

The Order of theſe Eaſtern Alphabets is likewiſe from hence Determined, Namely, from the Importance of the Precepts, as *Aleph* is Placed before *Beth*; Becauſe our Duty to God is to be Preferred before our Duty to our Parents or Family, *&c.* For there is nothing in the Nature of the Letter, why the one ſhould be ſet before the other, and why it might not be *B*, *A*, *&c.* as well as *A*, *B*, *&c.* Only that we follow the Order of the Old *Egyptian* and *Hebrew* Alphabet, without Retaining the Senſe and Intent of it.

I ſhall therefore here give, Suitable to my Conceptions and Apprehenſions of theſe Matters, a *Philoſophick* and *Chriſtian* Alphabet, Agreeable to the Deſign of the Primitive and Eaſtern ones, which was very Good and Laudable, and which will Contain the Sum of our Religion, as we are Chriſtians, of Philoſophy, as we are Wiſe Men, and of Learning, as we are Scholars; And which, if Lodged in the Memory, when we are Young, may ſerve for an Alphabet to our Reading, and for a Compendium of our Knowledge in Religion and in Reaſon; In Both which, we ſhould Endeavour to be Informed thro' the whole Courſe of our Lives.

Which *Philoſophick* and *Chriſtian* Alphabet is as follows, and which will make the Letters to be in the Order Subjoined, *viz.*

F. C. S. M. R. O. L. H. E. P. A. G. N. K. D. B. Q. V. Z. W. T. I. U. X. Y.

The Explanation of which Alphabet is this.

F.	Father, The Supreme, Eternal, and Self-Exiſtent Being.	The Objects of our Divine Worſhip.
C.	Chriſt, The Eternal Son of an Eternal Father.	
S.	Spirit, The Eternal Proceſſion from Both.	
M.	Mankind, That we ought to Conſult the Good of the whole Species in General, Abſtracted from Countries, Particular Perſuaſions, Relations, or Friends.	The Objects of our Affection and Duty, and of our Care and Solicitude.
R.	Relatives, 1*ſt.* As to Chriſtianity, which is Firſt to be Preferred, we are to Love one Another, and to Deſpiſe all this World, in Reſpect to, and in Compariſon with a Future State of Perfection and Bliſs; 2*d.* As to our Country, from the foregoing Principle, we ſhould be Ready to Die for it, where it's Intereſts do not Interfere with the Good and Happineſs of Mankind; 3*d.* As to our Parents; or our Bretheren, Children, Pupills, and Friends; or our Servants; we are to Pay the greateſt Duty and Reverence to the Firſt; the greateſt Love and the moſt entire Affection to the Second; and the utmoſt Condeſcenſion and Kindneſs to the Laſt.	
O.	Ourſelves, That we ought to be Temperate and Sober, and Averſe to all the Vices, which Deſtroy our Faculties, or Ruin our Conſtitutions, that we ſhould know Religion and Morality, to be both Contrived and Deſigned for our Well-being here, and for our Happineſs hereafter.	

L.	Language,	Engliſh, Latin, Greek, Hebrew, *&c.* Containing all our School, or Claſſical Learning.	Objects of our Memorial, or Elementary Knowledge.
H.	Hiſtory,		
E.	Eloquence,		
P.	Poetry,		

A.	Algebra,	Objects of our Rational, or Academical Knowledge.
G.	Geometry, or Mathematicks in General,	
N.	Nature, or Philoſophy,	
K.	Catoptricks, or Opticks and Dioptricks in General,	
D.	Diſſection, or Anatomy,	
B.	Botany,	
Q.	Chymiſtry,	
V.	Phyſick, or Medicine in General,	
Z.	Zodiack, or Aſtronomy in General,	
W.	World, or Geography,	
T.	Time, or Chronology,	
I.	Inſtitutes, or Civil Law,	
U.	Univerſal, or Catholick Doctrine,	
X.	Schiſms, or Heterodox Opinions,	
Y.	Years, or Antiquity, Hebrew, Greek, Roman, Engliſh, *&c.* and Philology.	

IF ſuch an Alphabet was in Uſe among us, inſtead of the Modern, all theſe Names would be Imprinted early upon Children, and they would have their Letters, and a Monitor to their Duty, and an Incentive to Learning, at the ſame Time; And even Thoſe, who ſhould afterwards Want an Education in Learning, would yet Retain the Alphabet of it, and would therefore, either be Deſirous of Proceeding farther themſelves in the Knowledge, it Propoſed, or Reſpect and Eſteem Thoſe, who did; Which would bring Religion and Learning into Honour and Reputation, which now are too often Received with Mirth and Ridicule, or Looked upon with Contempt; becauſe Men do not in the General, ſo much as know the very Names of the Studies and Sciences, which ſo highly Deſerve our Regard and Cultivation.

AND that, what I here Write concerning Alphabets, may not ſeem to be upon a Low and Trivial Subject, I ſhall Venture to Say, that the Invention of them was one of the greateſt Efforts of the Human Genius; Namely, to Reduce all the Various Sounds in any Language to 24, or 25, Characters, which by the different Combinations of them ſhould be Capable of Expreſſing, whatever was Poſſible to be Uttered by the Voice; This muſt be a Work of Ages, not of one Man; And therefore, we find *Cadmus* of *Thebes*, and *Palamedes* at the Siege of Troy, who is ſaid to be the Inventor of *Cheſs*, Celebrated for Introducing new Letters into the Alphabet: *Ariſtotle*, has been Famous for Telling us, how many Ways we may Conclude Juſtly in a Categorick Syllogiſm; But the Method, by which he found this great Secret out, is eaſily Diſcovered from Combining the Letters *A, E, I, O*, in all the Poſſible Variations of them; On the other Hand, how we ſhould Arrive at a certain Number of Characters, as 24, or 25, which by a different Diſpoſition ſhould Anſwer all the Articulate Sounds in all Languages, I leave as a Problem to Philoſophers to Explain, and at Preſent Conclude, it muſt be by Innumerable Trials, and that it muſt have Exerciſed the Wit of many Ages, and of the greateſt Virtuoſo's in it, till the Alphabet came to that Perfection, that when ſucceeding Genius's made their Attempts that Way, they found them Unſucceſsful, on Account of there being no Character wanting to Expreſs, what we were Deſirous to Pronounce; and ſo that Kind of Learning Ended, and Poſſibly with *Palamedes*; But to Return to our Preſent Subject of no Ideas being Innate.

The Mind not a Raſa Tabula, for the Reaſons above, and which are Summ'd up in this Section.

§. 7. To Conclude this Head of no Ideas being Innate, or of the Mind's being a Raſa Tabula: If it is ſo, it muſt be firſt Suppoſed, according to §. 1, That the Mind is not a Being Independent of the Body, and that it did not Pre-exiſt to it's Union with it; Second, Agreeable to §. 2, That it has no Diſtinct Spiritual Powers, and Faculties of it's own, nor any Proper Ideas from thence Ariſing; Third, according likewiſe to §. 3, That it has not ſo much as Senſe or Perception, before an External Impreſſion is Actually made upon it; Fourth, According alſo to §. 4, That all Men's Minds, tho' allowed to be Immaterial, are Alike, and that they are even upon the Level with thoſe of Brutes themſelves; And Laſtly, according to §. 5, That if they are Material, they yet have no Peculiar Genius, Nature, or Complexion, belonging to them, they are all no more than one Similar Blank Paper, upon which any Impreſſions may be Made, or Characters Drawn, or Engraven; All which Suppoſitions are either without Grounds, and Unproved, or elſe are Falſe in Fact and Reaſon; And yet, if all theſe Suppoſitions are not True, this Doctrine of no Innate Ideas, and of the Mind's being a Raſa Tabula, is entirely Unſupported, and has as little of Innate Senſe in it, as it is Preſum'd there was in the Mind, that Contriv'd it.

How this Doctrine of Ideas not Innate, Agrees with the Aſſertion of the Materiality and Mortality of the Soul.

§. 8. It is no Wonder yet, that a Philoſophy, which Entered into an Eſſay upon the Mind with ſuch Notions of it, that it ſhould in the Progreſs of it Declare, that it could not Tell, whether it might not Reſult from a Syſteme of Matter fitly Diſpoſed, and from whence that Power of Thinking, which we Perceive in our Selves, might be Derived; It was Eaſy for a Philoſophy, which had Diſclaimed all Proper Senſe, or Intelligence, to belong to the Mind, and that it had any Ideas or Perceptions of it's own, and which had Affirm'd, that all it's Notices were Conveyed into it from Senſation, and afterwards had Informed us, that the Soul did not always Think, it was Eaſy for it to Maintain farther, that it was either in it's own Nature Material, or that it was only a Property, which aroſe from a Syſteme of Matter fitly Diſpoſed, and which would Conſequently Vaniſh, and be no more, when that Syſteme was Deſtroyed, and therefore, that as the Soul was Material, ſo it was Mortal likewiſe; Theſe Sentiments do Naturally Cohere with one another; and what Religion, either Natural or Revealed, we are to Expect from ſuch Perſuaſions, any one may with Facility enough Conclude, and what Notions we are to Entertain of ſuch a Syſteme of Ideas, when it Comments on the Scriptures, or Diſcourſes to us of the Being of a God, or of Morality, or of the Sanctions of the Divine Law, or of the Rewards and Puniſhments of another World, or when it Solemnly Appeals to the laſt Tribunal: This is no Wonder.

Third Letter to the Biſhop of Worceſter.

This Philoſophy Contradicts it's ſelf, whilſt it Profeſſes not to Inquire into the Nature of the Mind, and yet Determines it to be Matter, or Matter fitly Diſpoſed.

§. 9. But this muſt needs be a Wonder, that a Philoſophy, which had Profeſſed, it ſhould not Inquire into the Nature of the Mind, ſhould Engage in Queſtions, which ſo Immediately Concern it, and a little farther Openly Avow it's being Matter, or elſe a Quality, which Reſulted from a due Diſpoſition of it, and therefore, that it is not much more than the Vegetation of a Plant, which Riſes and Dies with it; for according to it's own Declared Opinion, a Fœtus in the Mother's Womb Differs not much from the State of a Vegetable.

The Reaſon of ſuch a Contradiction.

§. 10. And yet Perhaps this Wonder may alſo Ceaſe, if we Conſider with what Deſign and Intention it Profeſſed this, which was, that it would not ſeem to know any Thing of the Nature of the Mind, but the Hiſtory of it; But, as it Evidently knows nothing of the one, ſo it will I Believe Appear, that it has not any Notion of the other; That it has not given us a True Account of Human Underſtanding, and that it's only Real Buſineſs was to Clear that Spacious and Magnificent Room of all it's Proper Furniture, that it might afterwards fill it with an Ornament and Garniture of Ideas, Suitable to it's own Fancy, and that therefore, Inſtead of a Real and Impartial Hiſtory of the Mind, we muſt reſt Satisfy'd with a Romance of it.

Whether the Soul always Thinks, or not.

§. 11. There is one Thing, which I ſhall take Notice of in this Chapter, as Relating to the Nature of the Mind, as well as to the Hiſtory of it, and which therefore I ſhall not have an Opportunity of Conſidering ſo fully in another Place; which is, Whether the Soul always Thinks, or not.

That the Soul does not always Think Connected with this Propoſition, that the Soul is not of it's ſelf, and in it's own Nature, an Intelligent Being.

§. 12. As to the Nature of the Mind, whether it is a Material or an Immaterial Being, if it is not of it's ſelf an Intelligent one, it may not always Think; becauſe it will only do ſo, when it's Thoughts are Excited in it by ſome Impreſſions made upon it; and therefore, if the Preſent Philoſophy is of this Opinion, that the Mind is a Raſa Tabula, a Perfectly Unactive and Unintelligent Being in it's ſelf, and in it's own Nature, if it only Perceives, as Impulſes are made on it, in like Manner, as this Paper Receives no Characters, but what are Written thereon, it will not always Think and Perceive; And it may, if it Pleaſes, upon ſuch a Worthy Sentiment of our Minds, Affirm, that they do not always Think, or have no Senſe at all Appertaining to them; That is, that our Underſtandings, and our Minds, are no more, than Motion is to Matter, a mere Accident of it, and that Thought, Perception, Senſe, or Intelligence, is nothing elſe, but a Quality of ſome unknown Subſtance, which may in ſuch a Caſe be Matter, as well as any other Subſtance, and which it Thinks it may be, when fitly Diſpoſed.

§. 13. But,

If the Mind is of it's ſelf, and in it's own Nature a Senſible and Intelligent Being, it muſt always Think.

§. 13. But, if on the Contrary, the Mind is of it's ſelf, and in it's own Nature, an Intelligent and Senſible Being, it will always Think, that is, will have always Senſe, Intelligence, and ſome Kind of Perception or other; becauſe it is Impoſſible, that a Being, which has Intelligence, Senſe, or Perception, in it's own Nature, ſhould ever be Deſtitute of it, and therefore, in ſuch a Notion of the Mind, it muſt always Think, that is, be always Endued with an Actual Senſe, and Perception, and not with a Capacity of it only, ſo that it is very Evident and Plain; That thoſe, who Affirm the Soul does not always Think, do it upon this Principle, that it is not an Intelligent Being in it's own Nature, but may be Matter, or any other Subſtance, for it is all the ſame, if it is an Unintelligent one; and on the Contrary, that Thoſe, who Affirm, the Soul does always Think, muſt do it upon a Principle Perfectly Different, Namely, that it is, in it's own Nature, a Senſible, Perceptive, and Intelligent Being.

This Philoſophy yet Profeſſes, it will not enter into the Phyſical Conſideration of the Mind, tho' at the ſame Time it Pretends to Determine ſuch Queſtions, which do moſt Intimately Relate to it, and which Determination of it does very Plainly Conclude, that it was of Opinion, that the Soul either was Material, or if it was Immaterial, was notwithſtanding an Unintelligent Being, in it's own Nature and Conſtitution; if ſo, it ſhould ſurely have brought us ſome Arguments, ſome Conviction, that it was ſo; Whereas, we do not find one Word to that Purpoſe, becauſe it muſt be very Conſcious to it's ſelf, that it could Produce no ſuch Arguments: On the other Hand, there are very Notorious Proofs, that the Mind is an Intelligent, and Perceptive Being in it's own Nature, which I have given in other Parts of this Philoſophy, and ſhall only at Preſent Say, that it cannot Prove the Being of a God, unleſs it Allows the ſame Arguments for a Human Intelligence, Separate and Diſtinct from Matter, or any Unintelligent Subſtance, which it does for a Supreme and a Divine One.

The Mind in Fact always Thinks, that is, always Perceives.

§. 14. As to the Hiſtorical Part of this Diſpute, whether the Mind does in Fact always Think or not? 'tis Anſwered; That, Altho' we may not have in View any one or other Particular Idea, that is, Altho' the Mind may not be Determined to the Contemplation of any Ideas in Preference, and in Diſtinction, to others, for in ſuch a Caſe the Mind is Active, 'tis Buſyed and Employed about Objects of it's own Chooſing, and in that Meaning of the Word it will not always Think, Yet it does not from thence follow, that it may not have a General Senſe and Perception; Thus I Believe any one may every Day in his Life Experience, that whilſt he is Walking, Sitting, or Standing, he many Times has no Certain and Explicit Ideas, which he can the next Minute give any Account of, and yet he can Remember this, that he had, all that while, Senſe and Perception in General, as he was Sitting, Walking, or Standing, and that he was Conſcious of it, all the Time he did ſo; Thus alſo, even in Study, where the Mind is Intent, and does then certainly Think, there is an Interruption, and ſome Kind of Interval betwixt our Actual Ideas, which Succeed one another, and ſometimes that Interruption is Continued for a Conſiderable Time, when it cannot find out the Idea, which it wants, and yet, notwithſtanding that, we are Conſcious to ourſelves of a Strong and Eager Senſe and Perception in the Mind, tho' we cannot Aſcribe to ſuch Interval any Idea at all; So that Hiſtory, and Fact, ſeems to be on that Side of the Queſtion, that the Mind always Thinks, that is, has always Senſe and Perception belonging to it; for ſurely this Philoſophy cannot Mean, by the Mind's not always Thinking, that it does not always Study, that it does not always, on ſet Purpoſe, and by it's own Choice, Contemplate ſome certain Ideas or other, or that it does not always Remember thoſe Ideas, which it had; If ſo, as the Aſſertion is Triffling and Impertinent, it might be Aſſured, it would never meet with any Adverſary upon that Head; The Queſtion is, whether the Mind of Man has always in Fact an Actual Senſe and Perception? Thus far we have Endeavoured to ſhew it has; In *Reſveries* it Acknowledges it, whilſt it Defines them to be a State of the Mind, in which Ideas Float thro' it, without our Notice or Obſervation; However we ſhall not be willing to take it's Acknowledgements, unleſs we find it Really is ſo; In that Condition and Circumſtance of the Mind now Mentioned, it does not Appear, that it has any Ideas Floating in it, but rather, that it has only then the General Senſe and Perception before Expreſſed, that it is in a Perfect State of Quieſcence, as to any Particular or Explicit Ideas, and is little more than Conſcious of it's

Exiſting; For, if we Ask ſuch a Perſon, what he has been Thinking of, he cannot Inform us, but if we Ask him, whether he was Alive, or had Senſe, or not, he will readily Anſwer to the Inquiry; In Sleep, which is the laſt State of the Mind to be Conſidered, every one will eaſily own, that Dreams are an Inſtance of our Thought, and Perception; And that there is this General Senſe, and Perception, in the Mind, when we do not Dream, or do not Remember, that we did, is Evident from Hence.

1ſt. BECAUSE, we Perceive, when others are in the Deepeſt Slumbers, that there are certain Motions and Agitations of the Human Body, which are as Real Indications of this General Senſe, and Perception, as our Sitting, Standing, or Walking, is, even when our Minds may be Void of any Explicite, or Actual Ideas.

2d. BECAUSE, in the Deepeſt Slumbers, if any Pain is Applyed, the Perſon is Immediately Senſible of it; which is as much an Argument of this General Senſe, and Perception, and the Mind's always Thinking, in that Acceptation of the Word, as it is an Argument, that the Fire always Burns, tho' I do not always Perceive it, becauſe, whenever I am Deſirous to make the Trial, I ſhall always find it does.

3d. BECAUSE, without this General Senſe, and Perception, it is not Conceivable, how the very Fluids of the Body ſhould obtain their due Action and Circulation; for, if upon the Abſence of Senſe and Perception, a Perfect Stagnancy Enſues, and the whole Maſs of them is Congealed into a Cold and Contractive Subſtance, as in Death, the Preſence of it muſt Certainly be Concluded from their Activity and Motion.

4th. BECAUSE, there are an Hundred Inſtances, where we are Certain we have a General Senſe, and Perception, and yet are not Conſcious of, or do not Remember, any Determined or Diſtinct Ideas, which we had; and therefore, it is Equally Reaſonable for us to Think, that ſuch is the Circumſtance of the Mind in the Deepeſt Slumbers.

CHAP. III.

Concerning Simple Ideas.

§. 1. IF we Conſider the Images, or Ideas, of External Objects, which are only Mentioned in the Chapter of Simple Ideas, how they Enter into the Mind, it is Plain, that they are Impreſſed upon it at once, and together, ſo that it is not one Single and Diſtinct Image, which Strikes us, and then another, but we See the Colour, the Extenſion, the Solidity, the Opakeneſs, or Tranſparency, the Brightneſs, or Obſcurity, and all the Viſible Qualities of an Object at the ſame Time; We likewiſe Feel Solidity and Extenſion, Hardneſs or Softneſs, Warmth or Cold, and all the other Properties of any Body, which Fall under the Senſe of Feeling at one Inſtant; And ſo it is, in Reſpect of our Hearing, Taſting, or Smelling: That is, our Ideas Enter into the Mind, Complex and United, according to the Variety, with which our Senſes are Capable of being Affected, or according to the Variety, with which one Senſe is ſo; And after theſe Impreſſions are made upon them, when we have Acquired them in the Groſs, we can then, if we Pleaſe, Sit down and Reflect upon theſe Complex Ideas, and Diſtinguiſh them into the ſeveral Parts, of which they Conſiſt, and Contemplate them, as ſo many Different Properties, Exiſting together in the ſame Subject; Therefore, if we allow the moſt to this Philoſophy, that the Mind has no Ideas Originally in it's ſelf, but that they are Derived into it from Senſation, All that we can Affirm is, that the Complex Ideas, which it Receives from External Nature, are the Foundation and Materials of it's Knowledge, And not that it's Simple Ideas are.

Our Ideas Enter into the Mind Complicated, and are Separated into Simple Ones, by Reflection and Abſtraction. Mr. Locke's Eſſay, Chap. 2. Book 2.

§. 2. THAT is, as to External Beings, our Knowledge Depends, and is Founded, upon thoſe ſeveral Informations, we Receive from our Senſes, Concerning Bodies, and their Exiſting in that Aſſemblage and Conjunction of Properties, which we Obſerve in them; Thus the Heat of the Fire, it's Burning and Conſumeing Quality of the Fuel, which is laid upon it, or of the Finger, Applyed to it, and the Pain, which is

The Knowledge of External Beings Depends, and is Founded, upon thoſe Complex Ideas, which we Receive in by our Senſes. Chap. 2. Book 2.

Conſequent thereupon, it's Brightneſs and Flame, and it's Colour, do all Unite together in one and the ſame Subject; So likewiſe, the Whiteneſs and Softneſs of Snow, it's Coldneſs, and it's Capacity of being Condenſed upon a Compreſſion of it, or of Melting upon an Acceſſion of Heat or of Warmth, are Ideas which Appear under one Collection; And the ſame may be Affirmed of all other External Beings in the Univerſe; That they all of them lie in the Underſtanding, like ſo many Diſtinct Faciculi, or Congeries of Properties, which the Mind has Obſerved from it's Senſations to belong to each, and which, when it is willing to Survey, it Diſtinnguiſhes into the Various Ideas, of which every Individual Complexion of them is Compos'd.

The True Hiſtory of our Ideas of External Beings, and the Conſequences from thence. Chap. 2. Book 2.

§. 3. This ſeems to be the True Hiſtory of our Ideas of External Beings, how they Enter the Mind, and how upon a Critical Examination of them afterwards, we come to Divide and Diſtinguiſh them into thoſe, which are here Term'd Simple ones.

From whence it will Follow;

1ſt. That the Foundation of our Knowledge of External Beings, and the Materials of that Knowledge, are the Ideas, which we in Fact Receive from thoſe Beings, and according to that Complexedneſs, and that Conjunction of Properties, which they Obtain in Nature.

2d. That we Receive ſuch Ideas by the Mediation of our Senſes.

3d. That we cannot Frame any new Being in our own Minds, any new Congeries or Faſciculus of Properties, which we have not Obſerved in Fact to Exiſt, unleſs a Chimerical one.

4th. That we cannot Divide theſe Complex Ideas, which we Receive from External Beings into Simple ones, ſo as to make them Exiſt Diſtinctly and Separately, tho' we may View in our Minds, both the Difference of theſe Simple Ideas, and can Recollect, by which of our Senſes they were Impreſſed upon us; That is, in Short, our Knowledge of Beings without us Depends upon, and is Derived from, what we Experience Concerning them, and what we Experience Concerning them is owing to thoſe Informations, which we Receive from our Senſes, all which is nothing leſs than a Plain and Evident Axiom.

On the other Hand, this Philoſophy Propoſes thoſe, which it calls Simple Ideas of Senſation, and afterwards thoſe of Reflection, for the Foundation of all our Natural Knowledge, and that they are the Materials of it; In which Doctrine two Things are Affirm'd, which are either Falſe, or not Prov'd.

1ſt. That our Senſations are the Original Perceptions in our Minds, and that, what is Diſtinct from them, Proceeds only from our Reflections upon thoſe Senſations; Which Suppoſes the Mind to be a *Raſa Tabula*, to be an Unintelligent Being of it's ſelf, and in it's own Nature, and that Senſe, or Perception, is Excited in that Being, which had no Senſe or Perception before, by a Being likewiſe, which had Neither; All which Suppoſitions are at leaſt not Prov'd, and by what has been ſaid in Chapter the Second are All of them Falſe.

2d. The Second Thing Affirm'd in this Doctrine of Simple Ideas is, that they are the Materials at leaſt of our Knowledge of External Beings; 'tis True they are ſo, when theſe Simple Ideas are Joined and Connected together, Agreeably to what Nature Preſcribes, and to what we by our Senſations Experience from it; but if we Diſſolve this Union of the Real Properties, which are found to Exiſt together, and Separate them into Single Ideas according to this Philoſophy, we lay the Foundations of a Chimerical Knowledge of Nature, but not of a Real one.

Thus we know, that Heat, that Light, and Brightneſs, and a Particular Kind of Redneſs, and a Power of Conſuming the Fuel, which is thrown upon it, of Fuſing Metals, and Raiſing an Ebullition in Water, of Melting Wax, and Hardening Clay, are Properties, which belong to Fire, and we have the Diſtinct and Simple Ideas of all theſe, and of their being United in one Subſtance; Thus alſo we know, that Whiteneſs, and Softneſs, and Cold, and a Capacity of being Diſſolved into Water by Heat, of being Condenſed by Compreſſion, *&c.* are Properties, which belong to Snow, the ſeveral Simple Ideas of which our Minds are Furniſhed with from our ſenſes, and we at the ſame Time find, they Exiſt together in one Individual Subſtance; Thus far therefore we Proceed according to Nature.

But, If we Reſolve theſe Subſtances into their Simple Ideas, and Suppoſe them to Lie Single and Separate in the Mind, and Reſign it to the Underſtanding to Put what Ideas, and what Number or Collection of them it Pleaſes, into one Complex One, and Imagine our Simple Ideas to be the Materials of our Knowledge in this Senſe, We then Depart from Nature, and Form ſo many Chimeras in our own Fancies, which will be of little Uſe or Service to any Real Knowledge of Things; For if we Confound the Simple Ideas of one Subſtance, and Mix and Blend them with thoſe of another at Pleaſure, we to all Intents and Purpoſes as much Triffle in our Compounding them, as when we Frame the Idea of a Griffin, or a Centaur, upon Members and Parts, Taken from Animals of Various Kinds, and Differing Denominations; And therefore, what is Affirmed by this Philoſophy, and which is the Foundation of the Whole, is Intirely Falſe, and Wrong; That, when the Underſtanding is once Stored with theſe Simple Ideas, It has the Power to Repeat, Compare, and Unite them, even to an almoſt Infinite Variety, and ſo can make at Pleaſure new Complex Ideas.

Whereas, It cannot make one Complex Idea from theſe Simple ones, otherwiſe than what is Experienced from Nature, and when it Recedes from that, whatever Complex Idea it Forms, it is as much Chimerical, as that of a Griffin, or a Centaur.

The ſeveral Ways, by which the Mind is ſaid to Obtain it's Simple Ideas. Mr. Locke's Eſſay, Chap. 3. Book 2.

§. 4. Thus far we have Conſidered Simple Ideas, and the ſecond Chapter, which belongs to that Head; This Philoſophy from thence Proceeds to Ideas of one Senſe, Chap. 3. Book 2. After having Recounted the ſeveral Ways, by which the Mind Obtains it's Simple Ideas; Which are ſaid to be Impreſſed upon the Underſtanding,

1ſt. Either by one Senſe only.
2d. Or by more Senſes than one.
3d. Or by Reflection only.
4th. Or by all the Ways of Senſation, and Reflection.

As to the Ideas of one Senſe, Light and the Various Colours, which we Perceive, are Gained by the Eye; All Kinds of Noiſes, Sounds, and Tones, by the Ears; The ſeveral Taſtes and Smells, by the Noſe and Palate; And the moſt Conſiderable of thoſe belonging to the Touch, are Heat, Cold, and Solidity.

The Inferences and Concluſions from this Doctrine Conſidered. Chap. 3. Book 2.

§. 5. But, after all this Enumeration of our Simple Ideas, which Enter the Mind by one Senſe only, what is the Diſcovery, which is made to us? Unleſs, that External Beings cannot be Perceived by us, but by the Mediation of our Senſes, which no Philoſopher, who was ever in them, would go about to Deny; 'Tis a Truth, as Plain, as that the Mind, and the Beings External to it, are Diſtinct and Different Beings.

If any Thing farther is Inſinuated, Either;

1ſt. That the Mind is a Raſa Tabula, ſince it would not have one of theſe Ideas, if it's Organs of Senſation were Stopt, or Diſabled, from Informing it, we have already Conſidered that Pretence; And it may be here Added, that even in this Caſe, the Mind is no more Proved to be a Raſa Tabula, from the Neceſſity it finds of Senſations to Furniſh it with the Ideas of External Nature, than External Nature is a Raſa Tabula, from the Neceſſity it finds of the Mind to the being Perceived; That is, Perception is as Neceſſary to our Senſations, as our Organs of Senſe are; And till it is Evinced, that the Mind is nothing in it's ſelf, and that Perception is only a Quality, Reſulting from a Syſteme of Matter fitly Diſpoſed to Produce it, all the Enumeration of Ideas in the World, and the Senſations, which are ſaid to be the Cauſes of them, will not be Sufficient to Prove, that the Mind is nothing elſe, than a Raſa Tabula, or a Piece of Blank Paper, good for little, but to Write on.

2d. Or, if it is farther Inſinuated, That theſe Simple Ideas Enter the Mind Singly, and not in Conjunction with others, or, That, altho' they do Enter in Conjunction with others, the Mind may afterwards Conſider them, as Single and Independent Ideas, and Unite and Compound them at Pleaſure; The Firſt is Falſe in Fact, and the Laſt a mere Chimera.

3d. Or, If it is Inſinuated, That theſe Simple Ideas from Senſation are Firſt in the Mind, becauſe this Philoſophy has Placed them Firſt in this Exact and Accurate Hiſtory of it, it is, what is taken for Granted, without the leaſt Shadow or Appearance of a Proof of it.

The Idea of Solidity not a Simple Idea Gained to the Mind from Feeling, but a Complex and Abſtracted one. Mr. Locke's Eſſay, Chap. 4. Book. 2.

§. 6. As to the Ideas, which we Receive from the Eye, the Ear, the Taſte, or the Smell, This Philoſophy Paſſes them over, it may be Juſtly Suppoſed, as Underſtanding little or nothing of the Matter, and very Unfortunately Chooſes to Take the Idea of Solidity into it's Examination, which is Affirmed to Proceed from our Touch.

Whereas, Firſt, If by Solidity is Meant, according as it is Defin'd, That Unſurmountable Force, which Hinders the Approach of two Bodies, when they are Moving One towards Another, it is not an Idea of one Senſe only; or rather it is not Properly any Simple Idea at all, but a Complex and Abſtracted one; It is not an Idea of one Senſe only, becauſe we can as well by our Eyes be Informed, that ſome Bodies are Supported by others from not Sinking downwards, that they cannot have a Mutual Acceſs to each other, without the Removal of the Intermediate one, that one cannot Enter into the Place of another, unleſs upon the Latter's Relinquiſhing the Space it Poſſeſſes, all this we may be as fully Informed of by our Sight, as by our Feeling, and therefore, as we ſhall afterwards Prove, it cannot be a Simple Idea, ſince that can only Enter by one Senſe, §. 7. 8.

It is not farther any Simple Idea at all, but an Abſtracted and a Complex one; It is an Abſtracted one, becauſe it is a Force in General, which is Aſſerted in this Idea, and not any Particular and Definite one, which is Neceſſary to any Diſtinct and Simple Senſation; And it is a Complex Idea, becauſe it takes in the Mutual Relations of ſeveral Bodies to each other, or of two at leaſt, ſo that there is the Force Proper to each Body, the Space which it Poſſeſſes, the Removal out of that Space before the other can Enter into it, to be Conſidered in this Idea, which therefore is far from being a Simple Idea of one Senſe, or of being a Simple Idea Derived from our Feeling in Particular, unleſs we can be Allowed to Feel Space, as well as Solidity.

Second, If by Solidity is Meant only, what we Actually Feel from Body, upon the Application of our Hands to it, in Reſpect of Reſiſtence, for we muſt not take into this Idea any other Tangible Qualities, as Heat or Cold, Roughneſs or Smoothneſs, Gravity or Levity, but only thoſe Perceptions, which we Plainly Experience, from the Preſſure of our Hands upon the Body we Touch or Feel; in ſuch a Caſe, our Perceptions are not of the Abſtracted Solidity, which has been Named, we do not Feel the Space, which our Hand Enters, and the Body Leaves, but our Senſations are, in Reſpect of Reſiſtence, either that a Body is Hard, and therefore Reſiſts more, or is Soft, or is Fluid, and Reſiſts leſs; That is, all which we Experience in Relation to the Reſiſtence of Bodies, from our Preſſing our Hands upon them, is only the Various Degrees of ſuch Reſiſtence, and which are the ſame in Regard to our Feeling, as the Various Kinds of Colour are to our Sight; But if, becauſe we Actually Perceive Blue, or Red, or Green, in the ſeveral Objects we Converſe with, we are therefore Minded to make Colour in General, which Certainly is an Abſtracted Idea, a Simple one of Senſation, we ſhould do Juſt, as Wiſely, as this Philoſophy has done, in making Reſiſtence in General a Simple one of Feeling: As to theſe Various Kinds of Reſiſtences, which we in Fact Experience, 'tis the Part of Natural Philoſophy to Account for them, and which has been already done from the Doctrine of the Expanſive and Contractive Forces.

No Simple Ideas Enter the Mind by more Senſes than one, which this Philoſophy Aſſerts. Mr. Locke's Eſſay, Chap. 5. Book 2.

§. 7. The next Thing, which this Philoſophy Conſiders, is the Simple Ideas of Diverſe Senſes, and here likewiſe it is far from giving us a True Hiſtory of the Mind; ſince there never was any Simple Idea, which Entered the Mind by more Senſes than one, becauſe it is in the Nature of the Thing Impoſſible.

No one ever Imagined, that Colour could be Gained by the Ear, or Sound by the Eye, or that we could Smell, what was Sweet or Bitter to the Taſte, or on the Contrary, Taſte the Fragancy of a Roſe or a Violet, tho' the Smell and the Taſte have as near an Affinity to each other, as any Senſes, which we have, becauſe thoſe Organs of Senſation are Perhaps of the Neareſt and moſt Similar Contexture of any we are Poſſeſſed of, and why we ſhould not Affirm the ſame in Reſpect of our Sight and our Feeling, will be hard to Say; Can any one Suppoſe, that what I Actually See, I Feel? Is Sight, Feeling? If this cannot be Affirmed or Maintained, our Real and Simple Ideas, which are Derived from Sight into our Minds, muſt be as Different from thoſe of our Touch, as they are from Sounds, which we Receive by our Ears, or from Taſtes and Smells, which we have from thoſe Senſes, which are Proper to them.

§. 8. Besides,

Evident from the Different Contexture and Formation of our Organs of Senſe.

§. 8. BESIDES, the very Formation and Contexture of our Organs of Senſe, which is Various and Different in each of them, does Sufficiently Evince, that it is Impoſſible, the ſame Simple Idea ſhould be Impreſſed upon the Mind from different Organs of Senſation, where the External Object is the ſame, which is the Occaſion of ſuch an Impreſſion; It is Impoſſible, if the Contexture of my Eye is Different from that of my Hand, or of my Ear, or any other Organ of Senſation, that my Mind ſhould Perceive the ſame Simple Idea from the ſame Object, where the Impreſſion muſt needs be Different, from the Different Quality in the Organ of Senſation to Receive it.

The Ideas Mentioned in this Philoſophy to Enter the Mind by Diverſe Senſes are Space, or Extenſion, Figure, Reſt or Motion; Space or Extenſion does not.
Chap. 5. Book 2.

§. 9. As to the Ideas Mention'd, which are ſaid to Enter the Mind by Diverſe Senſes, they are thoſe of Space, or Extenſion, Figure, Reſt, or Motion, which are Affirm'd to be Deriv'd into it, both by our Senſes of Sight and Feeling.

IN Reſpect of Space, which Conſiſts of Length, Breadth, and Thickneſs, which is neither Solid, nor Coloured, nor Tranſparent, and is only Room, in which Bodies Move or Exiſt, or are Felt or Seen, it is Impoſſible our Minds ſhould Receive it, from either our Eyes, or our Touch, becauſe it is an Idea Abſtracted from all our Senſes, and Formed by the Mind in Excluſion of their having any Share in it; 'Tis True, we Feel the Extenſion of Body or of Matter, but that is not Space, and we See the Extenſion, or Colour of it, or a Diffuſion of Light or Darkneſs, where we cannot Perceive any Groſſer Subſtance to Exiſt, but nothing of this is Space neither; ſo that Space in the Abſtracted Notion of it is a Creature Shaped and Contrived by the Mind, without any Derivation of it from our Eyes or our Feeling.

AND as to that Real Extenſion, which our Eyes Perceive in Bodies, it is as Diſtinct from that, which we Feel in them, as Colour is from Extenſion, or any Viſible Quality of Matter from a Tangible one of it, or as our Eyes are from our Fingers; And thus in the Extenſion of Light or Darkneſs, where no Groſs Body is Viſible to them, what do our Eyes Receive, unleſs the Impreſſions of the one, or the other? And if we come to Feel that Space, thro' which both are Diffus'd, we are Apprehenſive of nothing, but of Heat, or a Lively Action from the one, or a Coolneſs, and more Languid Senſation from the other.

Nor Figure.
Chap. 5. Book 2.

§. 10. THUS alſo in Relation to Figure, we See the Terms and Boundaries of Bodies, and likewiſe Feel them, but yet notwithſtanding, this Philoſophy will own, that a Perſon, who is Born Blind, and afterwards Receives his Sight, cannot Diſtinguiſh a Globe from a Cube, tho' he had been never ſo well Verſed in Diſcriminating the one from the other by his Feeling; which is an Evident Argument, that the ſame Idea of a Globe or a Cube does not Enter the Mind by both Senſes, ſince, if it did, the Eye would ſoon Acknowledge (even upon the firſt Impreſſion made upon it) the Idea, which the Mind had been ſo Long, and ſo Familiarly, Acquainted with, by it's Organs of Feeling.

IT is therefore Manifeſt, that this Idea of Figure, which is ſaid to Proceed from both Senſes, is not an Idea of Senſation, but one of Abſtraction, Made and Form'd by the Mind; and it is only an Abſtracted Termination of an Abſtracted Space, which would be a True Idea, tho' there was no ſuch Thing, as a Real Figure Exiſting in Nature; for this is the Senſe of Philoſophers, that a Globe, or a Cube, a Circle, a Square, or a Triangle, would be the ſame, whether there were ſuch Figures Actually Exiſting or not, and the Properties of them would be Alike and Invariable, which they call Eternal Truths; But what is this but a Confeſſion, that ſuch Ideas are not the Ideas of our Senſes, but the Abſtractions and Creatures of our own Minds? And what are ſuch Eternal Truths Different from a Chimera, and the Properties of it, being Diſtinct, and bearing ſuch a Proportion and Analogy to thoſe of a Centaur, which they will always do, tho' ſuch Beings were never found in Nature? Such Truths would be Eternal indeed, but would not be worth any Wiſe Man's Knowing.

IN Fact, what we Experience from our Eyes in Reſpect of Figure, is, that we find Bodies Variouſly Terminated and Bounded, and we likewiſe Feel them ſo, but the Impreſſions, which are made upon our Minds from ſuch Terminations by our Eyes, and by our Feeling, are as Different, as the Feeling of a Surface of any Portion of Matter is from the Seeing of it; Or again, as before, as our Eyes are from our Fingers; This Indeed, according to what I have Mentioned in the firſt Part of the Principles of Philoſophy, may be done; we may Traverſe a Figure with our Eyes with the ſame Direction of Motion, as we do with our Hands, and by that Means a Man, who is Born Blind, may know how to Diſtinguiſh a Globe from a Cube upon his firſt Seeing them; This however is not from an Impreſſion upon the Organ of Sight, but from giving it the ſame Motion, which Arrives to the Hand in ſuch an Incident, and is not an Argument that we have the ſame Idea of Figure from both our Senſes, but the Contrary; ſince ſuch a Notion of Figure is not Produced in the Mind from it's Actual Senſations, but from it's Reaſonings upon them, and therefore cannot be Term'd an Idea of two Senſes.

Nor Motion and Reſt. Chap. 5. Book 2.

§. 11. THE Laſt Idea, which is ſaid to be Deriv'd into our Minds by two Senſes, is that of Motion, or it's Contrary Reſt; But how? What is this Motion, which is Impreſſed upon us, both by our Senſes of Sight and Feeling, or this Reſt, which is Contended for? Do I See a Body Move in the ſame Manner, as I Feel it Move? I call Indeed both of them Motion, by one General Term; But are the Senſations the ſame? If they are, my Sight is the ſame Perception, as my Feeling is; In this Caſe therefore, as in the others beforemention'd, The Real Senſations of Motion, or Reſt, in Body, from our Sight and Feeling are Different, but the General and Abſtracted Idea of them is the ſame, and ſuch an Idea is not Convey'd into the Mind by two Senſes, nor yet by one, becauſe it is Form'd and Made by our Reaſoning and Abſtractions upon ſuch Different Perceptions.

That our Abſtracted Ideas Formed by the Mind from our Ideas of Senſation, are no Proof that the Mind is a Raſa Tabula, but the Contrary.

§. 12. IF yet it ſhould be Argued, that altho' theſe Abſtracted Ideas are not the Immediate Impreſſions upon our Senſes, or Deriv'd into our Minds from them, they are notwithſtanding Originally owing to them, becauſe without them we could not make ſuch Abſtractions, and if from thence it is Concluded, that the Mind is nothing elſe but a Raſa Tabula, and Receives all it's Impreſſions from External Objects, it is Anſwered, That the Powers of Reaſoning and Abſtracting, and by that Means of Making new Ideas Diſtinct from thoſe, which the Mind has from Senſation, are as Juſt Proofs, that there is ſomething Originally in the Mind different from the Impreſſions of Senſe, as our Ideas of Senſation are an Argument, that there are Beings External to us; Beſides all the other Powers and Faculties of our Minds, of Willing, Judging, and Remembering, and even of Perception it's ſelf, are no ways Deriv'd from Senſation, nor the Ideas which belong to them, but are Own'd to be Ideas of Reflection; All which Ideas of Reflection, do as fully Evince the Mind not to be a Raſa Tabula, as it is Suppos'd the Ideas of Senſation do Prove it is one.

The next Sort of Simple Ideas in this Philoſophy are thoſe of Reflection, which are Suppos'd to be Derived from thoſe of Senſation. Chap. 6. Book 2. Mr Locke's Eſſay,

§. 13. THE next Sort of Simple Ideas, which this Philoſophy Conſiders, are thoſe of Reflection, as being Suppos'd to be Deriv'd into the Mind from the Former, Namely, when the Mind turns it's View Inwards upon it's ſelf, and Obſerves it's own Actions about thoſe Ideas it had Received from External Objects; From whence a new Set of Ideas Ariſes, which are Term'd Ideas of Reflection, which the Underſtanding can Reaſon upon and Contemplate as Fully and Exactly, as thoſe, which are Furniſhed to it from any Foreign Cauſes; And the two Main and Principal Actions of the Mind are ſaid to be Perception or Thinking, and Volition or Willing; Remembrance, Diſcerning, Reaſoning, Judging, Knowledge, Faith, *&c.* Being only ſeveral Modes of theſe Simple Ideas of Reflection.

This Account of the Ideas of Reflection Suppoſes the Mind to be a Raſa Tabula.

§. 14. THIS Account of our Ideas of Reflection does therefore Suppoſe in the firſt Place, which hath been already ſaid, is far from being Prov'd, or True, that the Mind is of it's ſelf a Raſa Tabula, and that it Derives it's Original Ideas from External Impreſſions; No one can Affirm, that an Embryo, or an Infant, does not Perceive, that it does Perceive, and Will, as Early as it is Senſible; It cannot indeed Think of theſe Operations of the Mind in the ſame Manner, as a Philoſopher does, nor can it Diſtinguiſh it's Senſations into thoſe Orders and Claſſes, which a Perſon of Reaſon and Speculation would do; But as this is no Argument of it's wanting theſe Ideas of Senſation, ſo is the other none of it's being Deſtitute of thoſe other Ideas, of Perceiving and Willing, from it's very firſt beginning to Exiſt.

Ideas of Reflection can be no more Properly Termed ſuch than our Ideas of Senſation may be.

§. 15. THIS Account does likewiſe farther Suppoſe, that theſe Ideas are gain'd by a Reflection upon the Operations of our Minds, in Reſpect of it's other Ideas, Accrueing to it from Senſation; Whereas it is not Manifeſt, that they are any more Ideas of the Mind's Reflecting upon it's ſelf, than our Ideas of Senſation are; That my Mind is a Perceptive, Willing, Reaſoning, Being, is as much an Actual

Perception, as that I See this Paper, that is, my Idea of Perception, or Volition, is not ſo Properly an Idea of Reflection, as of Inward Conſciouſneſs; Or, which is the ſame, my Ideas of Senſation are Impreſſed upon my Mind from External Objects; That I have Reaſon, Will, and Perception, I Know, and have an Actual Perception of, from the Internal Impreſſions of my own Mind; I do not Perceive, I have Percepception by Reflecting, but by the Actual Senſe and Experience I have of it, as I do not Perceive any Senſation by Reflection, but by the Real Incourſe of it upon my Mind.

How far Ideas of Reflection may be called ſo, and in that Senſe our Ideas of Senſation may have that Name.

§. 16. THUS far Indeed they may be called Ideas of Reflection, when we come to Reflect upon and Conſider what Ideas the Mind has, of which theſe are one Sort and Kind; but in this Acceptation of the Word, our Ideas of Senſation will be likewiſe Ideas of Reflection, becauſe they are another Sort, of which the Mind is Poſſeſſed from it's Actual Experience.

Ideas Derived from the Mind Deſerve no more to be called Ideas of Reflection, than thoſe Derived from External Objects.

§. 17. FARTHER, If the Ideas, Derived from the Mind in Diſtinction from thoſe, which are Derived from External Objects were Ideas of Reflection, it is yet Suppoſed in this Account, that they Reſult only from the Mind's Reflecting upon it's Ideas of Senſation, and the Operations of the Mind about them, which is alſo Unprov'd from what goes before, and is in Fact Falſe.

All the Qualities and Properties of the Mind as much Actual Ideas there without Reflection, as our Ideas of Senſation are; we Actually Perceive both, and may Equally Reflect upon both.

§. 18. FOR Moreover, all the Qualities and Properties of the Mind, which are not to be found in Matter, as that I Doubt, I Hope, I Fear, and the ſeveral other Paſſions, and Affections belonging to me, of Love, Hatred, Anger, and Averſion, and the Like, or of Perception, Willing, Judging, *&c.* Are as much Real and Actual Ideas in my Mind, as Sound, Colour, or Extenſion are; Nor are they Ideas, which are Produced in me from Reflecting upon the Ideas of Senſation and the Operations of the Mind about them, ſince they are Impreſſions, which I Feel and am Senſible of from within, without having the leaſt Reſpect or Regard to any Foreign Objects, or the Ideas of Senſation, which Ariſe from them; I may Doubt of the Faculties and Properties of my own Mind, or of other Mens, I may Hope Concerning them, I may be Afraid of them, I may Love them, I may Hate them, or be Angry with them, and may Employ my Paſſions about them, without any Regard to Material Objects, or the Senſations, which they Produce in me; I may likewiſe Exerciſe all theſe Paſſions in Reſpect of Spirits, and Immaterial Subſtances, as Good or Bad Angels, which this Philoſophy does ſeem to Acknowledge, the Notices of which Beings cannot be Deriv'd from our Senſations; And Laſtly, in Relation to Almighty God, I may Experience ſeveral of them, as my Fear and Dread, or my Love of him, my Hope or Joy in him, and the like: Theſe Properties therefore or Operations of the Mind in all theſe Caſes, are not Employed about any Ideas of Senſation, nor are they Deriv'd from them, becauſe all, Excepting the Firſt, are Own'd by this Philoſophy to be Immaterial, and the Being of Almighty God in Particular is Endeavoured to be Prov'd to be ſo.

AND, What is ſaid in Reſpect of the Paſſions of the Mind, of Love, Fear, *&c.* Equally Obtains in the other Faculties, which belong to it, of Perception, Willing, Judging, Reaſoning, or Remembering; Namely, that they are in themſelves Original and Actual Ideas in the Mind, Deriv'd from thoſe Inward Impreſſions, which we have of them, as our Ideas of Senſation are, which we Receive from External Objects, and that they may be, and are, Equally Employed about Immaterial and Intelligent Beings, Abſtracted from any Ideas of Senſation, as they are about External Objects, and the Senſations which Reſult from them; and all theſe Ideas, both thoſe, which are Originally in my Mind, and thoſe which are Derived from Senſation, may become Ideas of Reflection, that is, when I Reflect upon them.

Perception and Volition, no more Eſſential to the Mind than Remembering, Reaſoning, or Judging.

§. 19. As to what is ſaid in the Thirteenth Section, that Perception, and Volition, or Thinking, and Willing, are the Principal Actions of the Mind, and that the others Mentioned, are only Modes of thoſe Actions, that likewiſe does not Appear to be True; becauſe all the Faculties and Operations of the Mind have an Equal Claim to belong Primarily and Originally to it, they are all of them Fundamental Properties of it, and do Eſſentially Reſult from it; 'tis Confeſſed indeed, that there is Thinking, and Willing, in Remembering, in Reaſoning, and Judging, but theſe Laſt are no more Modes of the Former for that Reaſon, than becauſe there is Extenſion in Solidity and Colour, and in all Material Properties whatſoever, that therefore all the Beings in the Univerſe, and all the Qualities, which Appertain to them, are only ſo many Modes of ſuch Extenſion.

Concerning Simple Ideas, which in this Philoſophy are ſaid to be Derived from Senſation and Reflection, as Pleaſure, Pain, Power, Exiſtence, Unity, and Succeſſion; no Simple Ideas Enter into the Mind from theſe two Sources. Mr. Locke's Eſſay, Chap. 7. Book 2.

§. 20. Another Kind of Simple Ideas, which this Philoſophy Conſiders, are thoſe, which are Derived into the Mind from Senſation and Reflection, ſuch are ſaid to be Pleaſure or Delight, and Pain or Uneaſineſs, Power, Exiſtence, Unity, and Succeſſion.

Whereas there are no Ideas, which Enter into the Mind, the ſame by Senſation and Reflection, but they are Perfectly Different; Thus the Pleaſure, which is Created in the Mind from the Taſte of a Nectarine, or the Smell of a Roſe or a Violet, is Perfectly of another Nature and Kind, than what is Experienced from the Satiſfactions of Friendſhip, or from the Delight of Doing Good, or from the Joys of Religion; We may call all theſe by one Name Pleaſure, but the Ideas are Different, and vaſtly Diſagreeing; And therefore Pleaſure in this Senſe, is not an Idea either of Senſation or Reflection, but is a General and Abſtracted Term, under which we Comprehend, whatever is Pleaſing and Delightful to the Mind; Thus the Particular Colours of Red, Orange, Yellow, Green, Blue, or Purple, are Ideas of Senſation, but no one I hope will therefore Affirm, that Colour in General is ſo, which is an Abſtracted Idea, Made and Formed by the Mind from thoſe Diſtinct and Various Senſations.

What has been Affirm'd of Pleaſure, is likewiſe True of Pain and Uneaſineſs; ſince the Pain, which is Produced from the Point of a Needle, is Different from the Uneaſineſs, which Ariſes from the Ill Uſage or Ingratitude, which we Receive from Others, in Return for any Good Offices Perform'd to Them.

Concerning the Religious Notions, which this Philoſophy Entertains of Pleaſure and Pain, being Annexed by Almighty God to moſt of our Ideas; and the Falſity of ſuch Notions.

§. 21. As to the Pleaſure or Pain, which is Suppoſed to be Annexed by Almighty God to moſt of the Ideas, which we have, with a Deſign Mention'd in this Philoſophy, to Incite us to Purſue ſuch certain Actions and Thoughts, and to Deter us from others, I ſhall only Say, That whatever Pretences this Philoſophy may make to the giving us due Sentiments of the Wiſdom and Goodneſs of God, and Inſpiring into our Minds a Juſt Knowledge and Veneration of him, it will be a hard Matter for it to Prove, that Almighty God only Intended, that we ſhould follow our Pleaſures in this Life, and that, what we ſhould moſt Avoid, ſhould be the Pains of it; 'Tis a Syſteme of Divinity, or of Morality, well fitted for an Epicure, but how it will Suit with the Rigorous Diſcipline of the Chriſtian Scheme, which Teaches us to Bear and to Suffer, and that Pain is ſometimes our Duty, Thoſe would do well to Conſider, who Profeſs that Religion, and yet Eſpouſe this Philoſophy.

Besides, in the Nature of Things, Pleaſure and Pain are no Standards to Adjuſt our Actions by; ſince we ought not to Depart from what is Becoming us, as Rational Creatures, for any Fear of Pain, or any Proſpect of Pleaſure, and we are not to Conſider, what we are to Feel from, but what Reaſon we have for, our Actions: But Perhaps it may be Thought I Strain this Matter too Far, when I Charge the Preſent Philoſophy with making Pleaſure and Pain the Meaſure of what is Good, or what is Evil; Which yet will not be found ſo, in the Proceſs of this Eſſay, ſeeing it does Expreſly Aſſert This; And that the Preſent Account of Pleaſure and Pain being Annexed to our Actions and Thoughts by God, in Order to Raiſe us to the Purſuit of ſomething or other, has a Direct Tendency to the Aſſerting the ſame Doctrine, is what I here Affirm; Since, If Almighty God intended to Excite our Minds into an Activity, by the Pleaſure and Pain Affixt to certain Ideas, it was his Deſign, that Pleaſure and Pain ſhould be the Motives, according to which our Actions and Thoughts ſhould be Directed; and Conſequently, when we did any Thing, which was Uneaſy or Painful to us, we Acted Contrary to God's Deſign and Intention, and when we Purſued our Pleaſures and Delights, we Preſerv'd the beſt Conformity to it; How far this is Rational, I Believe I need not Say, and how far it is Pious, or *Gives us due Sentiments of the Wiſdom and Goodneſs of the Soveraign Diſpoſer of all Things, the Knowledge and Veneration of whom ſhould be the chief End of all our Thoughts, and the Proper Buſineſs of all Underſtandings,* I will not Say, unleſs ſo far, that it muſt give every Conſiderate Man a very Mean Opinion of the Deity it's ſelf: The Caſe ſeems to be this; That Almighty God has been Pleaſed to Create all Things in that Manner, that theſe External Beings, with which we Converſe, are Fitted and Adapted ſo to our Organs of Senſation, that as we Receive the Perceptions of Pain, or of Pleaſure, from them, it in Prudence becomes us to Avoid and Shun, or

Mr. Locke's Eſſay.

to Uſe and Embrace them, ſo far as is either Conſiſtent with our Duty, or our Love to him; but he has not Annexed theſe Ideas of Pleaſure and Pain, as Motives or Incentives to our Thoughts and Actions, and as a Rule and Standard, by which we ſhould Conduct them; The Firſt is an Inſtance indeed of his Wiſdom, his Power, and Goodneſs; but to do the Laſt, would be an Argument, that he Originally Deſigned to Encourage all the Pleaſures of Vice, Licentiouſneſs, Lewdneſs, and Debauchery in the World, which is a much Worſe Notion than *Epicurus* ever had of the Deity, who by him was Suppoſed not to Concern himſelf at all in the Affairs of it.

Concerning Unity and Exiſtence, that they are not Simple Ideas Derived from Senſation and Reflection, but Abſtracted ones Formed by the Mind.

§. 22. CONCERNING Exiſtence and Unity we ſhall Say no more, than what has been already Alledged in Reſpect of Pleaſure and Pain, that they are Abſtracted Ideas, and ſuch as the Mind Forms to it's ſelf, from what it in Fact does Experience; Would it not be an Odd Fancy to Affirm, that Number in General was an Idea of Senſation and Reflection, which is not any Certain or Determined one? for it is neither 1, nor 2, nor 10, nor 100, and therefore cannot be an Actual Idea of Senſation, nor is it an Idea of Reflection, becauſe what is ſuch, is ſaid to be Derived from our Reflecting upon our Actual Senſations, but there can be no Reflection upon a Senſation, which never was, that is, of Number in General; And the ſame is True of Unity and Exiſtence, which are not Ideas either of Senſation or Reflection, but are Created by the Powers of the Mind and Abſtracted by it.

THE like Reaſoning does alſo Equally Extend to Power, and Succeſſion.

This Philoſophy in the next Place Affirms, that ſome of our Simple Ideas may Proceed from a Privative Cauſe, which is likewiſe Saying without Proving.

Mr. Locke's Eſſay, Chap. 8. Book. 2.

§. 23. AFTER having Settled the foregoing Points, this Philoſophy Proceeds to ſome farther Conſiderations Concerning Simple Ideas; And the firſt Aſſertion, which we meet with is, that whatſoever is able by Affecting our Senſes to Cauſe any Perception in the Mind, doth thereby Produce in the Underſtanding a Simple Idea, which would be a Poſitive one, altho' the Cauſe of it were a mere Privation; The Queſtion here therefore is, whether a mere Privation can Affect the Senſes, ſo as to Produce an Idea in the Underſtanding, for if it can, there is no Manner of Doubt, but ſuch an Idea will be as Poſitive, as any other.

ON the Contrary, what Idea is it Poſſible for a mere Privation to Produce? This Paper, which I Write on, does Impreſs a Certain and Determined Idea upon my Mind; Remove it, and I have None, Excepting of the Space, which it Left, and which does not Ariſe from the Abſence of the Paper, but from the Preſence of a new Cauſe to Produce that Idea in me.

THUS I have the Idea of Matter and Space, Annihilate the one, and Suppoſe with *Borellus* a Privation of the Dimenſions of the other, and he will be a Shrewd Metaphyſician Indeed, who can find any Poſitive Idea in his own Mind from ſuch Privations, and a much Shrewder, if he can Perceive an Ability in theſe Privations to *Affect* his *Senſes*.

THIS ſeems to be too Notorious and Plain, that Privations of any Kind can only Infer the Abſence of an Idea, where it is not Impreſſed, but not the Preſence of a Contrary one to that, which would have been Impreſſed, had there been no Privation.

LET us take the Inſtances Mentioned in this Philoſophy, of Heat and Cold, Light and Darkneſs, or Shadow, Motion and Reſt, which are all Confeſſed to be Poſitive Ideas; but that the Cauſes of any of them are Privations is Aſſumed without Proof, and cannot be Granted for Asking only, and we may in General Affirm, that no Privation can Produce a Poſitive Idea, or any Idea at all.

The Idea of Cold not Produced in us from a Privative Cauſe, but from a Contractive Force, as Heat from an Expanſive.

§. 24. As to what Reſpects Heat, there is no Queſtion Concerning it, every one will allow it to Produce a Poſitive Idea in us, and that Cold does ſo likewiſe, but that Cold is barely a Privation of Heat, and has not a Poſitive Cauſe, by which the Mind becomes Senſible of it, is more than this Philoſophy Undertakes to Prove, or can Prove.

FOR if we Conſider Cold in Relation to External Beings, it is Inveſted with Properties as Actual, and as Poſitive, as thoſe of Heat are; It Congeals and Cements Fluid Bodies together, as the other Diſſolves and Diſunites them; It has Various Degrees of Force, as the other has; Lifts and Raiſes Ponderous Subſtances; and Strikes and Penetrates into Earth and Stones to a Conſiderable Depth.

But the Privation of one Force, as of Heat, cannot be Productive of another, which is Contrary to it, it can only Infer the Ceſſation of that Force, without Introducing a New and Oppoſite one.

And thus in Relation to our ſelves, the Cold, which we Feel, is not a bare Privation of Heat, but Proceeds from the Acceſs of a Contrary Force, Acting upon our Organs of Senſation; That is, Heat Proceeds, according to what has been before Explained, from an Expanſive Force, and Cold from a Contractive, which are both of them Equally Poſitive, and Equally Poſitive Cauſes of thoſe Ideas in us.

The Idea of Darkneſs or Shadow not Produced in our Minds from a Privative Cauſe, no more than Light is, the one Proceeding from the Expanſive Force of the Medium, and the other from the Contractive.

§. 25. Light, and Darkneſs or Shadow, is another Inſtance Propoſed by this Philoſophy, Light is Confeſſed to be a Poſitive Idea, and to be Derived from a Poſitive Cauſe, whereas Darkneſs and Shadow are ſaid to be Privations, but 'tis Affirm'd without any Conviction of their being ſo.

For the moſt, which can be with Certainty Alledg'd is, that as the Rays of the Sun Impreſs a Brightneſs upon the Medium, thro' which they Paſs, and therefore Communicate a Kind of Vivid Colour to it; ſo the Abſence or Interception of them Deprives the Medium of that Brightneſs; but the Darkneſs or Shadow, which Succeeds, may be owing to the Native and Proper Color of the Medium in ſuch Circumſtances of the Light's Privation.

Thus, if Water is Shone upon with the full Rays of the Sun, it Receives from thence a Beauty and Luſtre, which is loſt in a Kind of Darkneſs and Obſcurity, when thoſe Rays are Intercepted by a Cloud; which Complexion and Hue is not yet owing to the Privation of Light, but to the Colour, Nature, and Temper of the Medium in ſuch a Circumſtance, which in ſtill other Circumſtances is Continually Vary'd and Altered, that is, as the Expanſive Force is Diminiſhed, or the Contractive Increaſed, which Forces we have ſhewn to be the Occaſion of all the Colours we Experience.

So likewiſe Black, does not Proceed from a Privation of White, ſince Blew, Yellow, Green, and every other Colour is a Privation of White, as well as Black is, and yet every other Colour is not Black; Light and Darkneſs, or Shadow, as alſo White and Black, ſeem to come under no other Conſideration, than all other Colours do; They Equally with them Produce Poſitive Ideas in us, and are Equally owing to Poſitive Cauſes Ariſing from the Nature of the Mediums, and the Various Affections of them, in which theſe Viſible Qualities are; That is, Light Proceeds from an Expanſive Force, by which the Optick Nerves are Acted upon by Impulſe; and Darkneſs Proceeds from a Contractive Force, by which the ſame Nerves are Attracted towards the Medium, and from whence we Thruſt out our Hands in the Dark, in Obedience to ſuch Attraction of our Nerves, and our Eyes are more Intent on, and Protrude themſelves forwards in it; The Contrary to which happens in an Exceſs of Light; The Eye Declines it from the too Great Impulſe upon it, and the Motions of our Hands and Body are Alert and Vigorous, not Stupid, and Heavy or Contractive.

The Idea of Reſt not Produced from a Privative Cauſe, no more than Motion, the Firſt Proceeding from the Contractive Force, as the laſt from the Expanſive.

§. 26. The laſt Inſtances Mentioned in this Philoſophy are Motion and Reſt, the laſt of which is ſaid to be a Privation of the other, and yet to Produce a Poſitive Idea, which is in ſo many Terms Deny'd; For as Motion Produces a Poſitive Idea in the Mind, that is, of a Continual Alteration of Place in Reſpect of the Body, which is Mov'd, ſo a Privation or Ceſſation of ſuch Motion can only Cauſe a Privation or Ceſſation of ſuch an Idea, without any Poſitive one Impreſſed in the Room of it; The Poſitive Idea of Reſt, on the Contrary, is as much Derived from the Poſitive Continuance of the ſame Body in the ſame Place, as that of Motion is from the Continual Change and Alteration of Place, unleſs a Variation of Place is a Poſitive Cauſe of an Idea, and the Continuance of the ſame Place is not, which cannot be Affirm'd; Moreover, if we Examine into the Cauſes themſelves of Motion and Reſt, they will Appear to be Equally Poſitive, the Firſt Proceeding from an Expanſive Force, and the Laſt from a Contractive.

The Diſtinction of Ideas as they are Perceptions in our Minds from what they are in Bodies a Triffling one. Chap. 8. Book 2.

§. 27. The Diſtinction of Ideas, as they are Perceptions in our Minds, from what they are in Bodies, is a Triffling one, ſince there can be no ſuch Thing as Ideas in Bodies, no more than there can be Perceptions in them; Qualities there are Indeed Inherent in Bodies, which Raiſe ſuch Certain and Determined Ideas in us, and the Queſtion will be, whether our Ideas are Exact Images and Reſemblances of ſuch Qualities.

THIS Philoſophy Aſſerts, that in ſome Caſes they are, and in others not, and accordingly Divides theſe Qualities into two Sorts, Primary or Original, and Secondary or Imputed; the Firſt Sort, as Solidity, Extenſion, Motion or Reſt, Number and Figure, are ſaid to Exiſt in Bodies themſelves, and to be the Exact Patterns of thoſe Ideas, which we Tranſcribe from them, and of which therefore our Ideas are the True and Juſt Reſemblances.

THE Laſt Sort, as Sweetneſs, Blueneſs, or Warmth, and Colours, Taſtes, Odours, and Sounds in General, are Affirm'd on the Contrary, not to Exiſt in Bodies themſelves, nor are our Ideas of them the Portraitures or Draughts of any Real Qualities, which are Inſeparable from them.

1ſt. BECAUSE, Solidity, Extenſion, Motion or Reſt, Number and Figure, will be in Bodies, whether we Perceive them or not, whereas Colours, Taſtes, Odours, and Sounds are not there, but when we Perceive them.

2d. BECAUSE, when we do Perceive them, they are not Really there, but in our Perceptions only, which Ariſe from certain Powers in Bodies fit to Excite them.

3d. BECAUSE, theſe Powers in Bodies are Ultimately Reſolved into the Primary Qualities of Extenſion, Figure. Motion, *&c.*

The Primary Qualities of Bodies either Abſtractions and Creatures of the Mind, or elſe upon the ſame Level with the Secondary ones, as to their being Reſemblances of Things themſelves.

§. 28. As to Solidity, Extenſion, Motion or Reſt, Number and Figure; They are all of them General and Abſtracted Ideas, and therefore are their own Archetypes, and are not the Real Reſemblances of any Thing but themſelves; But let us Conſider them, as the Properties and Affections of Particular Bodies, and we ſhall find them to be no ways Diſtinguiſhed from the other Qualities of Bodies, which are Termed Secondary or Imputed, that they are all of them Equally Powers in thoſe Bodies to Raiſe ſuch Determined Ideas in our Minds, and that there is no Likeneſs betwixt the Idea and the Quality in the one Caſe, more than in the other.

THUS the Solidity, or Extenſion, of Marble or Gold, ſo far as it is not an Abſtracted Idea, but is Derived into the Mind from an Actual Feeling of each Body, or from the Viewing of it, is as much an Effect of a Power in either to Produce thoſe Explicite Senſations in the Mind, as the Whiteneſs, the Blackneſs, or the Variegated Colours of the one, or the Yellowneſs of the other; And the Ideas or Senſations of ſuch Solidity or Extenſion are, as unlike the Qualities or Powers, which Impreſs them, as Perception is unlike the Body, which is Perceiv'd, or as the Smell, or Taſte, or Colour of a Roſe, or a Violet, is unlike thoſe Qualities, which Cauſe thoſe Ideas in us.

THE Solidity or Extenſion of Marble or Gold, ſo far as they are Ideas of Senſation, and not of Abſtraction, are alſo upon the Level with thoſe of their Colours, that is, they no longer Exiſt in Bodies than they are Perceived, Excepting in their Powers of Producing at any Time thoſe Senſations in us.

FOR in all theſe Caſes, the Perception of Solidity or Extenſion, and all the other Ideas, which Ariſe from the Primary Qualities of Bodies, are no more thoſe Primary Qualities, which are only ſo many Powers of Producing ſuch Actual Senſations in our Minds, than the Heat or Warmth, which we Receive from the Fire, or the Pain from the Point of the Needle, are the Real Qualities in Fire, and the Needle, which Cauſe thoſe Perceptions in us.

As to the Laſt Reaſon Mention'd in the Foregoing Section, wherefore the Secondary Qualities of Bodies are Different and Diſtinct from the Primary, Namely, becauſe they are Ultimately Reſolved into the Latter, it ſo Entirely Depends upon the Corpuſcular Scheme of Philoſophy, that we ought to be very Certain of the Truth of that Syſteme, before it can be Allow'd to have any Validity or Force in it; For if we are not willing to Grant, which a Rational Inquirer into Nature will not eaſily do, that the Phænomenons of Colours, Taſtes, Smells, and the other Secondary Qualities of Bodies, are not to be Solv'd by the Various Figures of the Parts, and the Contexture of them, 'tis an Arbitrary Aſſumption to Say, that all the Secondary Qualities are Ultimately Reſolved into ſuch Corpuſcles, and this *little Excurſion into Natural Philoſophy* might well enough have been Spar'd, unleſs theſe Celebrated Metaphyſicks had known ſomething more of it; To Conclude this Chapter, we need Say no more, unleſs, that in the Former Parts of this Philoſophy we have Shewn, how the ſeveral Qualities of Bodies, both Primary and Secondary, may be Accounted for from the Doctrine of the Expanſive and Contractive Forces, and that in Nature there

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there is no Real Diſtinction betwixt them; and which Diſtinction is only made Neceſſary by the Abundant Follies and Abſurdities of the Corpuſcular Syſteme, or the Syſteme of a Similar and Homogeneous Matter.

CHAP. IV.

Concerning Perception.

The Account or Hiſtory, which this Philoſophy gives of Perception. Mr. Locke's Eſſay, Chap. 9. Book 2.

§. 1. THIS Philoſophy having Treated of Simple Ideas in General, and afterwards of thoſe of Senſation, or of Senſation and Reflection together, Goes on to the Ideas, which more Particularly belong to Reflection.

1ſt. AND the Firſt is Perception, in which the Mind is ſaid to be for the moſt Part Paſſive, ſince what it Perceives it cannot Avoid Perceiving, and what this is, we cannot otherwiſe Know, than by Reflecting on what Paſſes in our Minds, when we See, Feel, Hear, *&c.*

2d. IT alſo Informs us, that Impreſſions made on the outward Parts, if they are not taken Notice of within, Cauſe no Perception, as is Seen in thoſe, whoſe Minds are intently Buſied in the Contemplation of Certain Objects; ſo that, where there is Senſe or Perception, there ſome Idea muſt be Actually Produced, and Preſent in the Underſtanding.

3d. WE are farther Told, that the Ideas, we Receive from Senſation, are often in Grown People Altered by the Judgment, without our taking Notice of it; Thus a Globe of any Uniform Colour, as of Gold or Jet, being ſet before our Eyes, the Idea thereby Imprinted is of a Flat Circle Variouſly Shadowed, but being Accuſtomed to Perceive, what Kind of Appearances Convex Bodies are wont to make in us, the Judgment Alters the Appearances into their Cauſes, and from that Variety of Shadow or Colour Frames to it's ſelf the Perception of a Convex Figure of one Uniform Colour; and this by a ſettled Habit is Performed ſo Readily, that we take that for the Perception of our Senſation, which is but an Idea Formed by the Judgment, ſo that one ſerves only to Excite the other, and is Scarce taken Notice of it's ſelf; as a Man, who Reads or Hears with Attention, takes little Notice of the Characters or Sounds, but of the Ideas, that are Excited in him by them; Thus Habits come at laſt to Produce Actions in us, which often Eſcape our Obſervation.

4th. IT is likewiſe Alledged, that Perception ſeems to be That, which puts the Diſtinction between the Animal Kingdom, and the Inferior Parts of Nature, as Vegetables, whether Senſitive or other, *&c.* And that, even in an Oyſter or Cockle, there is ſome Degree of it, tho' they have not, it is likely, ſo Many nor ſo Quick Senſes, as a Man, or ſeveral other Animals.

5th. LASTLY, Perception is Affirm'd to be the firſt Step and Degree towards Knowledge, and the Inlet of all the Materials of it; ſo that the fewer Senſes any Man has, and the Duller the Impreſſions made by them are, the more Remote he is from that Knowledge, which is to be found in other Men.

The Mind not Paſſive only in it's Perceptions.

§. 2. THE Mind in Reſpect of Perception is here Affirm'd to be for the moſt Part Paſſive, and the Reaſon given for it is, becauſe it is ſo in Reſpect of it's Senſations, as thoſe of Seeing Feeling, Hearing, *&c.*

IF therefore That is Meant, that Senſation and Perception are Reciprocal Terms, Namely, that we have no other Perceptions than thoſe of Senſation, and our Senſations are our Perceptions, It will not be True, according to the Profeſſions and Acknowledgements of the Preſent Philoſophy, which Owns, that we have Perceptions of a Different Nature from thoſe of Senſation, to wit, thoſe of Reflection; And Farther, if our Perceptions are all Reſolv'd into our Senſations, it could not be ſaid that the Mind is for the *moſt Part* Paſſive in it's Perceptions, ſince it would be *always*, and *in all Caſes*, or at no Time, nor in any Caſe, ſo.

IT is Plain, what this Philoſophy Aims at, Namely, to Reduce all Perception to Senſation, and to make the Mind Paſſive in the Impreſſions it Receives, that ſo the Fond Notion of it's being a Raſa Tabula, or a Blank Paper, may be the better Eſtabliſhed in the Preſent Syſteme of Ideas.

On the Contrary, it is Evident, that the Mind is both Active and Paſſive in it's Perceptions.

That it is Active, is Evident from all the Voluntary Ideas, which it Raiſes and Excites in it's ſelf, by Study, Contemplation, or Reflection, and by it's Intent and Eager Purſuit of thoſe Ideas in Preference to any others, ſo far, that very Forcible Impreſſions, made upon it from External Objects, ſhall not be Obſerv'd by it; That it is alſo Paſſive in it's Perceptions, is from hence Manifeſt, that it cannot Avoid in certain Circumſtances, the Senſations, which are Impreſſed upon it from the Force and Violence of External Objects; and yet it is likewiſe Plain too, that in that Paſſion of the Mind from Foreign Cauſes, it is at the ſame time Active, ſince it does not only Receive Pain, or any other Involuntary Idea from Senſation, but at the ſame Time Reſents it; It Recoils and Reflects upon it with a Force Equal, if not Superior, to the Action, which Diſturbs it; And any one may as well Affirm, that Fire is merely Paſſive, when any Impreſſion is made upon it, as that the Mind is ſo.

There is another Property, which belongs to the Mind in Reſpect of Perception, when it does not Appear to be, either Active or Paſſive, but in a certain Kind of Æquilibrium; Namely, when we have either a General Senſe or Perception without any Explicit Ideas, or when our Explicit Ideas Float in our Minds without any very Particular Obſervation, as in what is call'd a Reverie.

The Phænomenons of Perception Solved upon the Hypotheſis of the Mind's being an Active and not a Paſſive Principle; If the Innate Action of the Mind is greater than the Impreſſion, the Mind will be Active.

§. 3. In Order to Solve theſe Various Phænomenons of our Perceiving, let us Suppoſe the Mind to be an Active, Intellectual Principle, Conſiſting of Thought, Senſe, or Intelligence, which does not Imply any Thing of Matter, unleſs Thought or Senſe is an Avow'd Property of it, which no Philoſopher does Pretend to Shew, or to Prove.

First, if therefore there is an Action of this Senſitive and Intelligent Principle, which is Greater than the Impreſſion, which is made upon it from the Body, or from Matter, or from the Action of other Mens Minds, the Force of the one will be Superior to the other, and in that Caſe the Mind will be Active, and the Body, or Matter, or other Mens Minds Paſſive.

Thus, where the Mind is Active, as in Study and Contemplation, it is little, or not at all, Affected with it's own Senſations; and even the External Impreſſions from other Mens Minds, tho' Attended with a very Senſible Pleaſure or Pain, as thoſe of Praiſe and Reproach, carry in this Circumſtance of the Mind a very Inconſiderable Force with them, and the Aggreſſor in Calumny and Reproach often Proves the Patient; And it is the ſame in the Natural and Innate Action of the Mind, which the Greater it is in it's Real Conſtitution, the Leſs Impreſſion do any External Powers or Forces make upon it; So that he, who is by Genius and Complexion Endu'd with the moſt of this Action, is the moſt Magnanimous, and finds the leaſt Concern from any External Violences, whether of Senſitive Pain or of Pleaſure, or of Reproach or Injury, or even of Commendation, and Conſequently is the moſt Couragious and Undaunted, and the moſt Reſolute in his Purſuits or Atchievements; This Action of Thought, Senſe, or Intelligence, ſeems farther to Conſiſt in a Force of Exertion or Expanſion, by which it Dilates it's ſelf to the whole Material Syſteme, which is United thereto, and by Conſequence to all External Nature by the Mediation of it.

§. 4. From this Account of the Action of the Mind, it is Manifeſt.

The Conſequences from the Foregoing Hypotheſis, that the Mind always Thinks, that it's Ideas are not Originally from Senſation, that there is an Innate Intelligence and Action belonging to it, and that it's Various Properties are Derived from thence.

1*ſt*. That the Mind does always Think, that is, has always Senſe and Intelligence, and is Endu'd with a Native Action belonging to it, that it is in a Continual Diſpoſition to have Ideas Impreſſed upon it from outward Cauſes, or to Conceive them from it's own Capacity of Acting, and that, either of theſe Ways, Perceptions may be Produced in the Mind.

2*d*. That all our Ideas are not Originally from Senſation, and then Reflection, ſince we have Equal Reaſon to Think, the Mind was firſt Endued with an Active Power of it's own, as well as it is Certain, it now Exerciſes ſuch a Power, in it's Reflections, Diſtinct from Senſation.

3*d*. That it would be no Objection againſt this Original Action of the Mind, and this Innate Intelligence in it, altho' we could find no other Ideas there, Excepting ſuch as were Deriv'd from Senſation; Since our Original Ideas may be Worn out and Effaced from the Continual Importunity of our Senſes, as well as our Preſent

Senſations may be, by Future ones; Thus Suppoſing any one had Liv'd for ſome Years in an *European* Country, and was Remov'd from thence to an *Indian*, where many of theſe Objects did not Occurr to his Senſes, there is no Doubt, but they would in a Courſe of Time be Obliterated from the Mind, which yet would be no Proof, that it never had any Notion of it's former Ideas.

4th. THAT therefore, if there is an Innate Intelligence and Action belonging to the Mind, it is not certain, but it might Pre-exiſt to it's Union with the Material Syſteme; it being wholly in the Pleaſure of it's Creator, whether it ſhould Exiſt before, or Immediately upon, it's Conjunction with it.

5th. THAT from this Innate Action and Intelligence of the Mind are in a great Meaſure Derived the Various Properties, which belong to Animate Beings; Namely, in Men, the Natural Pride or Elation of Spirit, or the Humility and Depreſſion of it, and the other Qualities Obſervable in Humane Nature, both in Reſpect of it's Temper, Complexion, and Duration; and in Brutes, the Fierceneſs and Courage, or Stupidity and Tameneſs, and the ſeveral different Affections and Properties, which we behold in them.

THIS is the Foundation of that Pſycheometria, Mentioned in the Firſt Part of the Principles of the Preſent Philoſophy Examined; To Wit, an Innate Action and Intelligence of the Mind, by which one Perſon is Diſtinguiſhed from another, and by which one Species of Animals has it's Peculiar Difference from all others whatſoever.

AND I do not much Queſtion, by a Juſt Conſideration of the Animal World in this Way, but there may be a Philoſophical and Mathematick Account given of the Various Properties of Senſible Beings, of Virtue and Vice, and of the Immortal Duration of our own Minds, as well as of the Periſhableneſs of all other Creatures; but when I Speak of Virtue and Vice, I Speak of them in a Natural Senſe, Religion being of a Different Genius and Complexion, and Depending upon Different Arguments, Namely, thoſe which are Derived from Teſtimony and Fact.

Second, if the Innate Action of the Mind is Leſs than the Impreſſion, it is Paſſive.

§. 5. SECOND, If the Action of this Senſitive and Intelligent Principle is Inferior to the Force, which is Impreſſed upon it from the Body, or from Material Beings, or from the Action of other Mens Minds, the Mind will in ſuch a Caſe be Paſſive.

THUS, in all Involuntary Ideas or in Painful ones, where the Impreſſions are Greater than the Action of the Mind, it Suffers and is Paſſive; and ſo, if the Action of other Mens Minds is Superior to that of our own, we likewiſe Submit, and become Tractable and Obedient to the Impreſſions Received from them.

FROM whence it Follows,

1ſt. THAT the Force of the Mind is Weakened and Impaired by the External Violences upon it, and even by our Common and Ordinary Senſations. Thus Blind Men are Generally found to be Quicker and Livelier and more Vigorous in their Intellectual Faculties than others; becauſe they are not Liable to many of thoſe Impreſſions, which are ſtruck upon the Mind from External Objects, and in which the Mind is Principally and Chiefly Paſſive.

THUS likewiſe, thoſe who Reſign themſelves to the Pleaſures of Luxury and Debauchery, and to the Impreſſions of their Senſes, are Weaker in their Intellectual Faculties than others; becauſe the Mind is Paſſive, and the Proper Action of it Diminiſhed by ſuch Inſults made upon it.

ALSO the Native Action of the Mind is Impaired by Pain Ariſing from Corporeal Beings, or from the Inſolence or Action of other Mens Minds upon it; becauſe in all theſe Caſes, the Action of other Beings, ſo far as they Affect the Mind, is Superior to the Action of it; but theſe, being only Temporary Impreſſions or Violences upon the Action of the Mind, have only a Suitable Effect, that is, a Temporary one, the Mind ſtill Reſtoring it's ſelf to it's Priſtine and Native Action.

2d. THAT the Immortality of the Soul is from hence Demonſtrable; ſince it has an Action Peculiar to it's ſelf, which is ſo far from being Extinguiſhed by the Separation of the Body from it, that the Senſes do Actually Impair and Weaken it in moſt of the Inſtances, which can be Nam'd.

3d. THAT, as to it's Nature, it is alſo Immaterial; ſince the Properties of Matter are Directly Repugnant and Oppoſite to thoſe of the Mind, the Mind being often the moſt

Active, when it Suffers the leaſt from Matter, and the moſt Paſſive, when Matter is the moſt Active upon it.

Third, when the Innate Action of the Mind is Equal to the Impreſſion, there is an Æquilibrium of Forces, and it is neither Active nor Paſſive.

§. 6. THIRD, there is another Property of the Mind, which we find by Experience, when it is neither Active nor Paſſive, as in Reveries, or in the Floating of Ideas in it without any Particular Obſervation of them, or when there is a General and Confuſ'd Senſation, without Perhaps any Particular or Explicit Ideas at all; And in this Caſe, the Action of the Mind ſeems to be in an Equal Poiſe, with that of the Material Syſteme United to it.

1ſt. FROM hence Reſults the Union of the Body with the Soul, which Depends upon this exact Balance betwixt them; and the Separation of the one from the other Ariſes from an Exceſs of the Action of the one above the other, by which the Divulſion is made; Thus any Great Diſtempers in the Body, or any Extraordinary Pains in it, or any Violent Emotions of the Mind, are follow'd by Death: So that all the Science of Phyſick muſt Rationally Determine in this, *viz.* In a Knowledge of the Proper Force and Action of the Mind and of the Body, and by what Methods an Æquilibrium may be Preſerv'd betwixt them.

2*d*. FROM hence likewiſe it Follows, that all Senſible Creatures are Endu'd with Free Will in their own Nature; ſince the Mind, and the Material Syſteme Annexed to it, are in an Æquilibrium, and yet we find a Power in all Animate Beings to Exert an Action, which ſhall make an Over-Balance.

Fourth, not Impoſſible, that the Motive Force of the Mind may be in a Certain Proportion to the Force of Perception, from whence the Phænomenons of all Animate Syſtemes might be Mathematically Explained.

§. 7. FOURTH, it is not Impoſſible, but the Motive Force of the Mind may be in ſome certain Proportion, or other, to the Force of Perception in all Animate Beings; and that theſe Forces or Intenſions of Perception may be Infinitely Various in the ſeveral Claſſes and Orders of them; and that the Force or Intenſion or Intenſeneſs of Perception is Infinitely Greater in Men, than in any other Animals.

FROM Different Suppoſitions of the Force or Intenſeneſs of Perception in each Animate Being, and of it's Proportion to the Motive Force of the Mind, as alſo of the Active Force of the Material Syſteme upon it, might be Derived a Perfect and Compleat Pſycheometria, by which all the Properties of any Poſſible Syſteme of Mind and Body would be Mathematically Aſcertain'd and Explain'd.

IF to this were Added the Proportions of Time, in which one Syſteme of Mind and Body Exerted it's Actions, in Compariſon with another, there might be a Determination made of the Degrees of Knowledge, and Attainments, which one would Have and Arrive to, in Reſpect of the other.

AND in General, we ſhould Firſt find, That, If the Force or Intenſeneſs of our own Perceptions is Infinitely Greater than that of an Inſect, and if the Motive Force is Proportional to the Force or Intenſeneſs of Perception, and if farther the Action of the Material Syſteme is in an Equal Poiſe or Balance with the Animal one, it would Follow, that if a Human Mind was Annexed to the Material Syſteme of an Inſect, it would Diſſipate and Deſtroy it, the firſt Moment of it's Union, becauſe it would not be able to Suſtain the Immenſe Force and Action of a Human Soul; From whence there would be Reaſon to Admire the Wiſdom and Contrivance of our Maker, who had ſo Exactly Suited the Temper and Action of each Body, to the Action of the Soul, which Informs it, that it would be Impoſſible for them to Subſiſt together, unleſs with thoſe very Proportions of Action in Both, which he has Allotted and Aſſign'd to Each.

Fifth, from the Preceding Hypotheſis the Various Knowledge and Underſtanding of Animate Beings might be Determined, as alſo in what the Identity of any Mind Conſiſts.

§. 8. FIFTH, If farther the Intellect, the Knowledge, or Underſtanding, of Animate Beings, is Suppoſ'd to be Increaſed in a Reaſon Aſſigned of the Force or Intenſion of Perception in thoſe Beings in a Time Given, let the Different Forces or Intenſions of Perception be Multiplyed into Times, which are Reciprocal, and the Intellect, the Knowledge, and Underſtanding, will be on both ſides Equal; And on the Contrary, if the Intellects, the Knowledges, or Underſtandings are Equal, and they are Multiplyed into Times, which are Reciprocally Proportional, the Intenſion or Force of Perception in each Caſe will be Known.

THUS, Suppoſe the Intenſion or Force of Perception in one Animate Being, is to that in another, as 1 to 2, and the Knowledge in each in a Given Time is Increaſed in that Proportion, it will be therefore in that Time as 1 to 2; But if the Time, in which the firſt is Increaſed, is as 2, and in which the laſt is, as 1, the Knowledge in each will be, as 2 to 2, that is, in a Ratio of Equality.

And from this Propoſition only it would Follow, That Thoſe, who are more Advanced in Years, and who are of a Duller Apprehenſion, may Underſtand more than Thoſe who are Younger, and who are Endued with a Livelier and Brisker Genius; And on the Contrary by a Change of the Proportion, That Thoſe, who are of a Quicker Apprehenſion, may in leſs Time Underſtand more than Others, who are of a more Groſs and Languid, can in a Longer Period of it.

Hence likewiſe, If other Ratios were Aſſumed, it would not Perhaps be Impoſſible to Aſcertain the Various Differences, Degrees, and Proportions, betwixt the Underſtandings of Men and Brutes, and of the whole Animal Creation.

It might alſo Probably be this way found out, why Brutes and Inſects can never Arrive at the Knowledge and Underſtanding of Men; becauſe their Force or Intenſion of Perception is Infinitely below that, which belongs to Human Minds, which would likewiſe Furniſh us with a Good Argument for the Periſhableneſs of the one, and the Duration of the other.

It is moreover Conſequent from ſuch a Doctrine of Intenſions or Forces of Perception, that the Identity of any Mind does not Conſiſt in it's Self-Conſciouſneſs, but in that Force or Intenſion of Perception, which is Peculiar to it; and that there are as many Various Intenſions in the Animate World, as there are Extenſions in the Material one, and all of them Reducible to a Mathematick Reaſoning.

Sixth, the Phænomenons Reſulting from the Union of the Mind with Matter Explained from the ſame Hypotheſis.

§. 9. Sixth, If Laſtly we Conſider the Mind, as United to it's Material Syſteme; Firſt, If the Motive Forces of the Mind are in a Ratio of Equality with the Intenſions of it's Perception, and the Organs of the Material Syſteme, upon which the Motive Force of the Mind is Exerted, are, as to their Re-Action or Reſiſtence to that Exertion, in the ſame Ratio of Equality, as may be Suppoſed in ſome Inſtances or other, the Perceptions will be, as the Motive Forces of the Mind, or as the Reſiſtences from the Corporeal Syſteme to them; In which Caſe, there will be an Æquilibrium on all Hands, and the Mind will be in a Reverie, with a General Senſation or Perception, without any Particular or Explicit Ideas: But if the Reſiſtence or Contra-Action of the Material Syſteme, and the Motive Force of the Mind are in a Ratio of Inequality, and the Intenſion of Perception is as the Motive Force;

Then Second, If the Motive Force of the Mind is Increaſed by the Contra-Action of the Material Syſteme being Greater than it, the Intenſion of Perception will be Proportionably Greater.

Thus we find Men's Perceptions are more Lively and Vigorous in Proportion, as the Corporeal Syſteme is ſo, and as the Blood and Animal Spirits Circulate more Briskly and Strongly; and from whence, if in a Great Degree, may Proceed Madneſs.

Third, on the Contrary, If the Motive Force of the Mind is Diminiſhed, by the Contra-Action or Reſiſtence of the ſame Syſteme being Leſs than it, the Intenſion of Perception is in like Manner, and in Proportion Leſs; which is alſo Confirmed by Experience, that the more Languid and Flaccid the Circulation of the Blood and Animal Spirits is, the Intenſion and Force of Perception is the leſs Elate and Alert; and from whence, if in a great Degree, may Proceed Lethargy and Death.

The Former of theſe Suppoſitions is Confirm'd by Inſtances, where Things of a Volatile Nature, as thoſe which Cauſe an Intenſe Pleaſure and Satisfaction, Add to the Volatile Force of the Mind; and the Latter by Sickneſs, or Sleep, or Melancholy, or Labour, and Fatigue, which Detract from it: And if this is True, it will be the Province of Phyſick, as was before ſaid, to Enquire into the Motive Force and Intenſion of Perception in the Mind, as well as into the Contra-Action or Reſiſtence of the Material Syſteme, in Order to make it's Preſcriptions Juſt or Succeſsful.

Fourth, If the Motive Force of the Mind is Increaſed by the Contra-Action of the Material Syſteme being Leſs than it, the Intenſion of Perception will be Proportionably Greater; which is alſo Experienced in Study and Contemplation, where the Leſs the Mind is Acted upon by Senſual Pleaſure, or by Pain, the Stronger, the more Vigorous, and Active, it becomes.

Fifth, on the Contrary, If the Motive Force of the Mind is Diminiſhed, by the Contra-Action of the Material Syſteme being Greater than it, the Intenſion of Perception will be Proportionably Leſs; which is alſo Confirmed by Experience, since, Thoſe, who Indulge themſelves in Senſuality and Pleaſure, do Weaken and Impair the Force of their Underſtandings, and many Times Degenerate into Brutiſhneſs and Stupidity.

These ſeem to be the only Caſes, which are Poſſible in the Union of the Mind with it's Material Syſteme; For they cannot be Diverſify'd; Unleſs by Suppoſing the Intenſion of Perception is Greater or Leſs than the Motive Force of the Mind, or that the Contra-Action of the Material Syſteme is not Correſpondent or Agreeable to the Conſtitution and Temper of that Syſteme, both which Suppoſitions ſeem to be Unreaſonable, and will not, I Imagine, be found to be Anſwered by Experience.

And from theſe Caſes now Mention'd, the Natural Immortality of the Soul Appears to be Evinced; for they all Depend upon the Mind's having a Peculiar Force and Intenſion of Thought belonging to it's ſelf, and Independent of Matter.

In the Firſt Caſe, the Force of Perception is Equal to the Reaction of the Material Syſteme upon it.

In the Second, the Action of the Material Syſteme is Greater than the Motive Force of the Mind, and Increaſes it's Perception; which if done in a great Degree, as in a Fever or in Diſtraction, Occaſions Death, even whilſt the Perception is Augmented; which therefore Deſtroys the Union of the two Syſtemes, and Drives the one from it's Cohabitation with the other, but does not Diſſipate or Diſſolve it.

In the Third, the Action of the Material Syſteme is Leſs than the Motive Force of the Mind, and Diminiſhes it's Perception by Hanging, as a Weight upon it, by which it Sinks and Depreſſes it's Proper Vigour; and which if done in a great Degree, the Union is again Deſtroyed betwixt theſe two Powers, and upon the Falling off of this Weight by Death, the Mind Recovers it's Priſtine and Native Force and Energy.

In the Fourth, the Action of the Material Syſteme is Leſs than the Motive Force of the Mind and Increaſes it's Perception; and if the Motive Force of the Mind is too Great for the Reaction of the Material Syſteme, as in the Caſe of Intenſe Study and Contemplation, the Union is likewiſe Diſſolv'd; The Thought of the Soul is too Mighty for the Body, which it Informs, and therefore Diſengages it's ſelf from it.

In the Fifth, the Action of the Material Syſteme is Greater than the Motive Force of the Mind, and Diminiſhes it's Perception; not as before, as by a Weight Hanging Heavy upon it, but as a Superior Force Overpowering it, by which at Length the Union is Deſtroy'd, till the Mind, Releaſed from the Over-bearing Force by a Disjunction from it, Returns to it's former State of Activity and Perception.

We may be Senſible of Impreſſions made upon our Minds in the Deepeſt Attention of them, tho' perhaps we may not Remember them.

§. 10. As to the Second Article Mentioned in §. 1. it is no leſs than an Axiom, that Impreſſions made on the outward Parts, if they are not taken Notice of within, Cauſe no Perception, if by being Taken Notice of within, is Meant our Perceiving them, becauſe it is Impoſſible we ſhould have any Perception without our Perceiving.

But the Queſtion is, whether we may not be Senſible of an External Impreſſion, when it is made, tho' we do not Retain or Remember it one Minute after; the Mind, as in Study, being Diverted to other Thoughts in the Purſuit of which it is Eagerly Carried, ſo far as to Reject, or not to Regard thoſe other Impreſſions, tho' it may at the ſame Time Perceive them; Thus any one, I Believe, does and muſt Experience, that, in the Midſt of the greateſt and moſt Intenſe Thoughts, he is by a Fire, or he is not, that he is Warm or Cold, that he Sits in the Profoundeſt Silence, or Surrounded with Various and Tumultuary Noiſes, tho' he may not Regard or Take Notice of any of theſe at the ſame Time that he Perceives them, but Proceeds in the Purſuit of his own Thoughts and Ideas in Neglect of them.

Our Ideas of Senſation not Altered by our Judgments.

§. 11. The Third of thoſe Articles, into which that Firſt Section is Divided, ſeems to be Entirely Falſe and Wrong, Namely, that the Ideas, which we Receive from Senſation, are often in Grown People Altered by the Judgment, without our taking Notice of it; for it does not Appear Poſſible, that we ſhould have any other Senſations than what are Agreeable to, and which Naturally Reſult from, the External Impreſſions, which Cauſe them.

As to the Globe of Gold, or of Jet, Imprinting only the Figure of a Flat Circle upon our Eyes Variouſly Shadowed, the Caſe is this, that thoſe different Shadows are Intimations to the Mind, that the Impreſſions Received are of the ſame Body from different Diſtances and Situations of the Parts; and therefore the Action of Light, or of Colour, Produces the ſame Senſation of Yellow or Black, as being the ſame Kind of Action upon the Mind; but the Diſtances of Action being Various, and Shadows being thereby Made and Form'd in the Circle, the Mind has Natural Impreſſions of thoſe Varying Diſtances, and therefore from ſuch a Circle has the Real Senſation of a Globe, that is, of Circles of Changing Diſtances from it; On the Contrary, that this Appearance ſhould be an Effect of the Judgment is not Conceivable, becauſe, tho' our Judgments Inform us, that the Sun is ſeveral Millions of Miles above us, yet we can See no other Diſtance, than what our Senſes Impreſs upon us; and in the Preſent Caſe, if we make Uſe of our Sight only, we cannot Perceive any Convexity, unleſs that, which is Deriv'd to us from the Organs of it; The Judgment cannot Interpoſe here, ſince if our Senſe of Seeing does Repreſent a Convexity Wrongly to us, our Judgment has nothing to Correct that Error by; we muſt See Convexity, juſt as it is Impreſſed, and we have no other Methods of Determining Concerning it, but by the Sight, which Imparts it to the Mind.

IF we make Uſe of more Senſes than One, as of Sight and of Feeling, ſince there can be no Connexion betwixt the Ideas of one Senſe, and thoſe of another, ſo as that our Feeling the Convexity of a Globe ſhould make us See it, it cannot be Imagined, how our Senſe of Feeling, in this Particular, ſhould Correct the Errors of our Senſe of Seeing, or how our Judgment has any Thing at all to do in the Matter; Becauſe, tho' we were never ſo well Informed, what Convexity was by our Feeling, we could not from thence Learn, what Idea would be Rais'd in our Minds from our Seeing it, by any Judgment, which we could make from one Senſe or the other.

BESIDES, how a Judgment of our Minds ſhould give us an Actual Senſation, for no one will Deny, that he Actually Sees, what we call Convexity in a Globe, is as Unintelligible, as that the Mind ſhould Think it's ſelf into a Real Perception of Fire, without any External Impreſſion from it; or, becauſe it finds a Grain of Sand is in it's ſelf Tranſparent, if looked upon by a Microſcope, that it ſhould by it's Judgment Correct the Abſolute Senſation, which it has of it, into another, which it has not.

As to what is ſaid in Reſpect of our Hearing or Reading with Attention, that we take little Notice of the Characters or Sounds, but of the Ideas, which are Excited in us by them, it is no Proof of our Altering our Ideas by Judgment, but only of the Quickneſs and Activity of the Mind, which can have ſeveral Ideas in a Train, and that in an Inſtant; Thus, when we Read, and the like may be ſaid of Hearing, the Words, and even the Letters, cauſe an Impreſſion upon us, and from thence a Senſation of them, if they did not, we might Read without Seeing; theſe Words Produce thoſe Ideas in us, which have been Annexed to them, and yet the Mind taking Notice Principally of the Ideas, the Words Vaniſh, as ſoon as they are Imprinted; But this is not Changing Ideas of the Firſt Impreſſion into Ideas of the Laſt by our Judgment, but ſhews a Quick Progreſs of the Mind from the Firſt to the Laſt, without any great Regard had to the Former; And, that this is ſo, is Confeſſed by this Philoſophy, which Acknowledges, that the Words and the Letters, which we Read, do Excite the Ideas Affixed to them in us, but how they ſhould Excite any Thing in us, is Inconceivable, without our Actual Senſations of them.

LASTLY, as to Habits Producing Actions in us, which often Eſcape our Obſervation, this likewiſe can be no Argument of our Changing our Ideas by our Judgment; Becauſe, when it is ſaid they often Eſcape our Obſervation, it is Own'd, they many Times do not, nor yet do they ever Entirely do it, tho' they may be forgot the very Inſtant after they are Perform'd; There is no one, I believe, who ever did any Action, who was not Senſible of it at the very Time and Moment he did it, tho' it might not make any other than a Temporary Impreſſion upon him, and it is Certain, that when we Uſe Attention, we always Diſcover theſe Habits.

BUT our Altering our Ideas by our Judgment is ſo Unobſervable, that it does not only often Eſcape our Obſervation, but never Falls under it, tho' we Uſe never ſo great an Attention; No one, if he ſhould Watch his own Thoughts and Ideas with the utmoſt Nicety and Exactneſs, could Say, how he came to See a Globe Convex when his Eye only Repreſented it as Circular: And therefore, as this laſt and the other Arguments are Inſufficient to Prove this Alteration of our Senſations by Judgment, and as there are Reaſons given on the Contrary againſt the Poſſibility of it, and Laſtly, as there is an Account Propoſed, how that, which is a Shaded Circle in the Eye, may Produce the Actual Senſation of a Globe or a Convexity in our Minds, we have no Occaſion to Suppoſe, or to Believe, for there is no Demonſtration of it, that our Judgments in any Caſe have a Power of Altering or Changing our Actual Senſations.

§. 12. In the Fourth Diviſion or Article of §. 1. It is Affirm'd, that Perception is That, which puts the Diſtinction between the Animal Kingdom and the Inferior Parts of Nature, as Vegetables, whether Senſitive or other, and that even in an Oyſter, or Cockle, there is ſome Degree of it, tho' they have not, it is likely, ſo Many nor ſo Quick Senſes, as a Man, or ſeveral other Animals.

Not Certain, that Perception is That, which puts the Diſtinction between the Animal Kingdom and the Vegetable.

But this may be Doubted, whether all Kinds of Vegetables are not Endued by the All-Wiſe Creator with a Certain Portion of Senſe and Perception, tho' Perhaps as much Inferior to that of an Oyſter, as that of an Oyſter is to a Man's; an Oyſter, it is Probable, has no other Senſe than that of Sight and Feeling, to Diſtinguiſh it's Nouriſhment from what is Noxious to it; and Plants and Vegetables no other than that of Feeling; of which if there were not ſome Degree, it is not Conceivable, how there ſhould be thoſe Strange Affections Obſervable in them, of a Sort of Sulleneſs and Dejection, of Enmities, and Antipathies, and Averſions, on the one Hand, and on the other, of a Certain Kind of Pleaſure and Complacency, of Friendſhip, Harmony, and Agreement, which Naturaliſts have Reported Concerning them; 'Tis ſaid Trees do Senſibly Contract themſelves upon the firſt Stroke of the Ax, which Fells them, as an Oyſter does it's Shell, when it Perceives a Violence Uſed to Disjoin and Diſunite it.; And the Phænomenons of the Senſitive Plant are well Known.

They are Form'd from Seeds, as Animals are from Eggs, and there ſeems to be as much Care taken for Preſerving and Defending the New Offspring amongſt Plants and Vegetables, and Encloſing the little Seeds for a Future Production in Choice Cabinets Contrived on Purpoſe for them, and many Times very Curiouſly and Artificially Wrought, and with a Proper Mucilage Lodg'd there for their Nouriſhment, as there is amongſt Bees, in their Fine and Delicate Hexangular Cells, and in the Repoſitories, which they make of Honey, for the Education and Bringing up of their New Fœtus.

Add to this, that Almighty God may be as Juſtly Suppoſed to give theſe Innate Impreſſions, or Inſtincts, to Plants and Vegetables, to Propagate their own Species, as he is Confeſſed to do in Relation to theſe Small Inſects, and to all the Various Ranks and Orders of the Animal Creation.

This is likewiſe Suitable to the Wiſdom and Providence of God, to Aſſign to every State and Condition of his Creatures the Province, which belongs to them, and to Enact by his High Will and Pleaſure, that all of them ſhould have, from the Greateſt to the Meaneſt of them, their Proper Employments Agreeable to their Natures, and to the Stations, in which he has Fixt them.

On the Contrary, Perception (if we Reſpect the Intenſion of it, as Explain'd in *§. 2. 3. 4, &c.*) Does as much Diſtinguiſh us from all the Animate Creation, which is below us, as Almighty God's does the Divine Nature from the Human; For, as there is no Compariſon betwixt the Infinite Energy and Force of the Perception of the Deity, and our Finite one, ſo is there none betwixt the Finite Energy and Force of Human Perception, and that of any Being, which is Inferiour to us; The Cauſe is Prov'd from the Effect in both Caſes, and in the ſame Manner; Since all the Arts and Sciences, the Wiſdom and Prudence of Forming and Modelling Societies, and Governing the Affairs of this World we Inhabit, and of all the Creatures in it, is an Argument of this Force and Intenſion of Perception being Infinitely beyond that, which Brutes are Endowed with, in ſome Meaſure Analogous and Correſpondent to that, which we Uſe, for the Infinite Intenſion and Force of Perception in the Deity beyond our own, by which all Things in the whole Univerſe are Ordered and Diſpoſed according to the utmoſt Knowledge, the moſt Accurate Skill and Contrivance, and the Greateſt and moſt Conſummate Prudence, Foreſight, and Penetration.

§. 13. The Laſt Thing Mentioned in the above Nam'd Section is, that Perception is the Firſt Step and Degree towards Knowledge, and the Inlet of all the Materials of it; ſo that the Fewer Senſes any Man has, and the Duller the Impreſſions made by them are, the more Remote he is from that Knowledge, which is to be found in other Men.

The Fewer Senſes any Man has, and the Duller the Impreſſions made by them are, no Standard of Human Knowledge.

But by this Account a Man is in no Degree above an Oyſter, unleſs that he has More Senſes, and Quicker, which is a low Diſpenſation, that Human Nature is Reduced to Indeed, if it ſhould happen to be True, and we ſhould have no great Cauſe to be Fond of the very Beings we Poſſeſs.

However, to our Satisfaction it is Plain and Evident, that the Force and Intenſion of Perception in Men is Infinitely Greater than what is found in an Oyſter, and if a Man had no more Senſes, than what an Oyſter is Suppoſed to be Endow'd with, and an Oyſter had all the Senſes, which a Man is Confeſſed to have, there would be yet a great Diſparity betwixt the Underſtanding of the one and the other, and much in Favour of the Latter.

For it cannot but be Own'd, that Brutes have all the Senſations, which Men have, they Feel, they See, they Hear, they Taſte and Smell, and yet with all theſe Senſes they are not Capable of Arriving at the ſame Degree of Underſtanding and Knowledge, which a Man can do, who has no other Senſe leſt to him than his Hearing and Feeling; for with theſe two Senſes only, (ſince Taſte and Smell will not be Affirmed to be very Conducing to Knowledge) there may be Numerous Examples Produced, where Men have been Blind, and notwithſtanding that Defect, have ſhewn a Force of Genius and Perception, which has ſet them, in Reſpect of Science, Learning, and Sagacity, not only above the Brutes, which were Endued with all the Senſes, but even above the Greateſt of their own Species, who were alike Defective in None.

Nor can it be ſaid, that this is owing to their Senſes of Feeling and Hearing, being Naturally more Vigorous than in other Men, becauſe their Organs of Feeling and Hearing are in all Reſpects like thoſe of Others, there is nothing that was ever Obſerved, which Diſtinguiſhed a Blind Man's Fingers from any other Perſon's, or was the Tympanum of his Ear ever found to be of a Different Make or Contexture; and there can be no Reaſon Alledged, for any Superiority of Knowledge in ſuch a Caſe, Excepting what Terminates in the Superior Force and Intenſion of the Mind's Perception.

Since Eſpecially, on the other Hand, thoſe who have the Quickeſt Senſes in Reſpect of Seeing, Feeling, and Hearing, are not always the Quickeſt in their Apprehenſions and Underſtandings; a Man may See at a Great Diſtance and very Clearly, and yet want a True Diſcernment of Things in his own Mind; he may Feel the leaſt Impreſſion from an External Action upon his Senſes, and yet not be Able to take the Impreſſion of the Plaineſt Propoſition, which ſhall be Delivered to him in Algebra or Geometry; he may Hear very Accurately and Exactly, and not Know the Senſe, the Coherence, and Meaning, of Ten Words, which are Spoken; So that there is not, nor can be, any Poſſible Connexion betwixt the Vigour and Quickneſs of our Senſes, and that of our Intellectual Faculties, which we only now Conſider, tho' there may be in the Senſe, which we have Expreſſed in §. 8. and 9. That is, our Organs of Senſation, as to the Vigour of them, may be Adapted to the Force and Intenſion of our Perception, which will be ſo far from Contradicting, what we have here ſaid, that it will Confirm it, Namely, that as the Force and Intenſion of Perception is Greater in Men than in Oyſters, ſo the Organs of Senſation are Correſpondent and Anſwerable to thoſe Forces.

CHAP. V.

Concerning Retention, and of Diſcerning, Comparing, and Compounding, our Ideas.

Of Retention. Mr. Locke's Eſſay, Chap. 10. Book 2.

§. 1. From Perception this Philoſophy carries us to Retention, which it Divides into Contemplation and Memory.

The Firſt is, when we keep the Idea, which is brought into the Mind, for ſome Time Actually in View, Concerning which I ſhall Say no more; Unleſs, that this Faculty of Retaining our Ideas is an Argument of the Proper and Native Action of the Mind, and makes it ſomething Different from a Raſa Tabula, or a Being, which is only Paſſive of Perception, or of Ideas being Imprinted upon it.

The Second is, when we Revive thoſe Ideas in our Minds, which have Diſappear'd, and have been, as it were, laid out of Sight; which is another Inſtance of it's Active Principle, and even of a Power of Creating Ideas to it's ſelf; for thoſe Ideas, which we Remember, being Different from thoſe Ideas, which we have from Senſation, are as much the Creatures and Production of the Mind, as the Paintings of Artiſts, or the Deſcriptions of Poets are; which yet, like our Ideas of Memory, are Tranſcribed from, as they are the Imitations of, Nature.

§. 2. That, which is to be Conſidered here is, what this Recollection of Ideas is owing to, and from whence it is Derived; It is Plain it cannot ſo much as Subſiſt in the Mind, if the Mind was no more than a Raſa Tabula; Becauſe then there would be Ideas Impreſſed upon it, of which it would be Paſſive, without any Power of Reviving them; They would be Imprinted upon it, and there Continue, or elſe by Degrees Wear off and Die, and ſo there would be an End of them, and this whole Chimerical Syſteme of them; Whereas, if we Conſider the Mind, as an Active Principle, and which has a Share in Forming and Conſtituting even thoſe Ideas, which are alſo in Part owing to an External Agency, it is no Wonder, that it can Recollect, and New Form Ideas from it's ſelf, which it was Capable of doing from Exterior Objects; It will then Act Conſiſtently with it's Nature; And Perception and Recollection will have ſome Kind of Affinity to each other, and will Appear to be Properties, which muſt Naturally belong to the ſame Subſtance.

How Perception and Retention are Connected upon the Suppoſition of the Mind's being an Active Principle, and not a Raſa Tabula.

But, if we make the Mind to be Paſſive only in it's Perceptions, it can go no farther than the Impreſſions it Receives, and there will be no Principle in the Mind, from which we ſhall be able to Derive it's Power of Recollecting, or even of Judging, Abſtracting, or Compounding; All which are Affections and Properties, which muſt Reſult from an Active Principle in it.

§. 3. As therefore the Intenſion and Force of Perception, and the Native Action of the Mind is That, which Principally Diſtinguiſhes Men from Brutes, as well as from One Another; ſo Retention, both that of Contemplation and that of Memory, being Active Forces of the Mind, are Analogous and Correſpondent in ſome certain Proportion, or other, to the Innate Force and Intenſion of Perception; and which will be another Ground and Foundation for a Pſycheometria, or a Reduction of the Forces of the Mind to a Mathematick Reaſoning.

How Perception and Retention may be Reduced to a Pſycheometria or to the Proportions of Geometry.

Thus, if the Intenſive Force of Perception, the Force of Keeping or Retaining an Idea in our Minds, and that of Reviving one, are in a Ratio of Equality;

1ſt. The Greater the Quickneſs of Apprehenſion is, the Greater will be the Capacity of Retaining the Idea in our Minds, and of Reviving it upon Occaſion.

2d. The Leſs the Quickneſs of Apprehenſion is, the Leſs will be the Capacities of Retaining or Reviving our Ideas. If the Intenſive Force of Perception, of Retaining an Idea, and Reviving it, is in a Proportion of Inequality; Then,

3d. The Apprehenſion may be Quicker, when the Faculty of Retaining an Idea may be Weaker, or of Recollecting it Slower.

4th. Or, The Apprehenſion may be Slower, when the Faculty of Retaining is Stronger, and that of Recollecting is Quicker.

5th. Or, The Faculty of Retaining may be Stronger, when the Apprehenſion may be Slower, and the Faculty of Recollecting Slower.

6th. Or, The Faculty of Retaining may be Weaker, when the Apprehenſion may be Quicker, and the Faculty of Recollecting Quicker.

7th. Or, The Faculty of Recollecting may be Quicker, when the Apprehenſion may be Slower, and the Faculty of Retaining Weaker.

8th. Or, The Faculty of Recollecting may be Slower, when the Apprehenſion may be Quicker, and the Faculty of Retaining Stronger. Which ſeem to be all the Caſes, which can Poſſibly Happen; and Each of which may be Conſidered in a Thouſand Different Proportions to one another.

It is not Impoſſible but Moſt, if not All, of theſe Caſes may Obtain in Human Nature; or, if they did not, they would Furniſh abundant Matter for Mathematicians to Employ their Speculations about; and which would as much Anſwer the Purpoſes of a True Knowledge, and a much more Important one, as the Proceeding (which is at Preſent done) in the Inquiry only into the Relations and Proportions of Abſtracted Quantities in Extenſion; For Each of theſe Caſes will Admit, as I have ſaid, of a Thouſand,

ſand, and I may Say, of a Million of Different Proportions and Relations, which will Open a wide Field of a New Mathematick Knowledge, not hitherto Propoſed to, or ſo much as Imagin'd by, Philoſophers: And to ſhew, that theſe Caſes in General do Obtain in Humane Nature, we ſhall Conſider them Separately and Singly.

1ſt. SOME Perſons ſeem to be Equally Quick in their Apprehenſions, Firm in Keeping and Retaining their Ideas before them, and as Vigorous and Lively in Recollecting them; which is the Utmoſt Happineſs of a Rational Mind, and the Greateſt Inſtance of the Force, and Capacity of it.

2d. ON the Contrary, where theſe Faculties are Equally Dull, it is a Mark and Character of Stupidity.

3d. THERE are Others, whoſe Genius's are Quick in Apprehending, what is Propoſed to them, either from External Objects, or from other Men's Notions, and yet cannot be either ſo Intent in their Contemplation of any one or more of their Ideas, or ſo Sudden in Recollecting them.

4th. NOR are there Wanting in Humane Nature ſuch, who may be Dull in Apprehending Things, and, notwithſtanding that, may be very Studious and Induſtrious in the Contemplation of them, and very Ready likewiſe in Recollecting the Ideas, which they have thus Conſidered or Apprehended.

5th. IT is alſo Obſervable, that Men may Retain Ideas in their Minds, and Engage their Contemplations upon them, when yet their Apprehenſions are Languid and Weak, and their Memories Short and Imperfect.

6th. IT is likewiſe Known, that other Men, who cannot Study, or Bring any of their Ideas under an Intenſe Contemplation, Have yet a Quick and Lively Apprehenſion, and as Sudden a Memory and Recollection.

7th. WE are farther Inform'd from Experience, that Men may have good Memories, when their Apprehenſions are not Extraordinary, nor their Application to their Conſidering their Ideas, or their Intenſe Contemplation of them, in any Kind Remarkable.

8th. NOR do we leſs Obſerve, that, when Perſons are Short, and Fallible, and Defective in their Memories, or in their Recollection of Ideas, they yet may be very Tenacious of the Ideas, which they have, may Contemplate and Study upon them with an Intenſeneſs of Mind, and may be very Quick and Ready in the Receiving the Impreſſion of them.

THESE ſeem to be the Various Denominations of Men, according to their ſeveral Particular Faculties, and which may again be Diverſified by an Infinitely Different Number of Proportions.

AND the Caſe of Brutes, at leaſt in Compariſon with Men, ſeems to be that, which is Expreſſed in the Second; where all theſe Faculties are Equally Dull, and which Denotes, and is a Mark of Stupidity: But the Faculties of an Intelligent Being, Conſidered in this loweſt Diſpenſation of them, may be likewiſe Infinitely Varied; from whence Ariſes the Different Capacities Obſervable in thoſe Species of Creatures, which are Below us; As the Firſt Caſe Expreſſes the Capacities of thoſe Orders of Beings, which are Above us; And all the Six following Ones may, in a Higher, or Lower Degree, than what belongs to Human Nature, be Applied to them.

IT would Perhaps be Difficult to find Proper Names, or Appellations, for theſe ſeveral Sorts of Undeſtandings, amongſt the Terms, which are now in Uſe; and the Trueſt Way to Diſtinguiſh them would be, by Denominating them from their Particular Natures.

THUS the ſeveral Kinds of our Underſtandings might Fall under the Subſequent Appellations;

THE Firſt might be Called, The Apprehenſive-Retentive-Recollective.
THE Second, The Non-Apprehenſive-Non-Retentive-Non-Recollective.
THE Third, The Apprehenſive-Non-Retentive-Non-Recollective.
THE Fourth, The Retentive-Recollective-Non-Apprehenſive.
THE Fifth, The Retentive-Non-Apprehenſive-Non-Recollective.
THE Sixth, The Apprehenſive-Recollective-Non-Retentive.
THE Seventh, The Recollective-Non-Apprehenſive-Non-Retentive.
THE Eighth, The Apprehenſive-Retentive-Non Recollective.

And by theſe Means, and this Procedure, we might Form a Thouſand Mathematick Propoſitions, and Draw Concluſions from an Abſtracted Reaſoning; And then by Comparing thoſe Concluſions, with what we Obſerve in Humane, or any other Intelligent Minds, we might know, which of them Agreed with and were Correſpondent to Nature, and which of them were to be Conſidered, as Abſtracted Theorems and Contemplations only.

A farther Explanation, how Memory and Perception are Connected in the Mind, from the Mind's being an Active Principle.

§. 4. It muſt be farther Said, that as the Mind is Active in Perception, and not Paſſive only, ſo is it in Retention and Recollection, which is a Secondary Kind of Perception, or a Perception of a Perception; And, as in our Senſations, External Objects do not barely Act upon our Minds, but our Minds are likewiſe Active in Receiving their Impreſſions, thus, when we Apply Water to Fire, Water does not only Act upon it, but it Acts again in Receiving it's Impreſſion, ſo is it in our Memories, when any Idea Lies in them, and is View'd by them, the Mind is Active, and is Exerted in the Contemplation of it; that is, of an Idea, which in this Circumſtance is not Properly an Idea of Senſation, but a Picture of ſuch an Idea, made by the Mind from it's Firſt and Original Perceiving it.

How Memory Reſults, or is Derived from this Active Principle of Primary or Original Perception.

§. 5. But it may be ſtill Inquired, how Memory Reſults from this Active Principle of Primary or Original Perception, which ſeems to be from a Greater Proportion of Action in the Mind in Receiving it's Senſations, than in the External Agents, which Impreſs them; For where the Mind is moſt Excited to Obſerve it's Senſations, either from Pain or Pleaſure, or any other Paſſion or Emotion Produced in it, and to which it is, as it were, Animated and Rouz'd, it always Remembers ſuch Senſations, with a more Particular Emphaſis and Regard, 'tis more Quick and Lively in Recollecting them, and more Apt and Ready to Retain them; And, that this Pain or Pleaſure, or any other Paſſion or Emotion of the Mind, cannot Proceed from the mere Impreſſions of External Agents, and their Action upon it, but from it's own Innate Action likewiſe, is Evident; Since the ſame External Actions will Produce Different Reſentments of Pleaſure or Pain, or any other Paſſion, where the Action of the Mind, which Receives them, is Different, and ſince without any External Agency, where a Man Thinks within his own Breaſt, ſome Thoughts will Cauſe him more Pleaſure or Pain, or other Paſſions or Emotions of Mind, which no External Agents were ever able to Produce in him.

On the Contrary, when the Action of the Mind in Receiving an Impreſſion is not much Greater, or Leſs, than the Action of the External Object upon it, ſuch an Impreſſion Ends in a bare Senſation, and the Mind will Retain but a Faint and Languid Memory at the moſt, or Perhaps none at all, of it; Which is alſo Confirmed by Experience, for when the Mind is not much Mov'd by it's Ideas, they ſoon Vaniſh, and it Remembers little or nothing of them: And what has been ſaid, in Reſpect of our Ideas of Senſation being the Subject of our Memories, or of our Secondary Perceptions, is alike Applicable to our Ideas of Reflection.

The Ideas of Memory may Proceed in Infinitum, and is an Argument of the Innate Force and Action of the Mind, and of it's Diſtinction from Matter.

§. 6. Farther we may Obſerve, that the Mind cannot only Remember, that it once Actually Perceived ſuch an Idea, but it can alſo Remember, that it Remembered it, or Remember that it Remembered, that it had a Memory of it; So that, as our Ideas of Memory are Secondary Perceptions in Reſpect of our Primary and Original Ones, ſo likewiſe, there may be Secondary Ones in Reſpect of theſe Secondary Conſidered as Primary, and ſo on; Or which is the ſame, there may be Secondary, Ternary, and Quaternary Perceptions, *&c.* In Reſpect of our Primary Ones; 'Tis True, the Mind does not Proceed far in this Way, becauſe of the Uſeleſsneſs and Inſignificancy of ſuch Perceptions, either in Common Life, or in the Purſuit of Knowledge, or of any Rational Inquiries; but that it has this Capacity is Evident in Fact and Experience.

And the Perceptions of this Sort, I Mean all theſe Secondary Ones, are Perfectly Different from either our Ideas of Senſation or Reflection; They are in a Manner Entirely owing to the Native Action of the Mind, and the farther we Proceed in them, the farther we are Retir'd from our Primary Ones, and Conſequently from thoſe of mere Senſation only.

So that the Mind Thinks and Acts within it's Self, without any Relation to, or Dependence upon External Objects, which is the Caſe of Study and Contemplation, which always makes Uſe of Memory and it's Secondary Ideas of the Firſt Sort.

Since therefore the Mind is thus Active, even in it's Primary Ideas, and much more ſo in it's Secondary Ones, which Action is Innate to it, and Independent of Matter, we have all the Reaſon in the World to Think, and be Aſſured, that the Mind will Survive the Body, and Preſerve ſtill it's Proper Action, upon a Diſſolution from it.

Concerning the Diſcerning our Ideas, and that an Exactneſs of Judgment does not Ariſe from thence. Mr. Locke's Eſſay, Chap. 11. Book. 2.

§. 7. The next Faculty of the Mind, which this Philoſophy Proceeds to, is That of Diſcerning between it's Ideas; Which need not be Diſtinguiſhed, as is here done, from it's Faculty of Perceiving; For upon the very Account, that we Perceive our ſeveral Ideas, for that very Reaſon we have a Faculty of Diſcerning between them, and the One is a Neceſſary Conſequence and Reſult of the Other.

And this Faculty of Diſcerning is Common to all, who have the Faculty of Perceiving their Ideas, and therefore is not what is Peculiarly Requiſite to an Exactneſs of Judgment, and to a Clearneſs of Reaſon, according to the Affirmations of this Philoſophy; Nor is Wit, in it's Diſcerning of Ideas, Oppoſite to Judgment, there being an Equal Faculty of Diſcerning and Diſtinguiſhing Ideas in both Caſes; For to Form an *Ilias*, or an *Æneis*, will Require as Accurate, and as Juſt a Diſtinction of our Ideas, as to Demonſtrate with *Euclid* or *Archimedes*.

From whence an Exactneſs of Judgment.

§. 8. On the Contrary, an Exactneſs of Judgment, or a Clearneſs of Reaſon, does Suppoſe Indeed, that we can Separate and Diſtinguiſh one Idea from another, with the utmoſt Nicety; but then beſides this, all the Faculties of the Mind Have and Obtain their Proper Share and Province; For we muſt likewiſe be able to Compound, to Compare, and Abſtract our Ideas with a like Juſtneſs and Accuracy, and above all to Remember and Retain the ſame Ideas, which we have Originally Fixt in our Minds, without Suffering them to Vary and Change into others, in the Proceſs of our Reaſonings; To which Infirmity, the Greateſt and Wiſeſt Men are Subject, as is Evident from the Writings of Philoſophers, and Divines, whoſe Failures in Argument do Generally Proceed from an Unwary Altering of thoſe Ideas, Concerning which they Diſcourſe, into others, which are Entirely Different from them; from whence there muſt Neceſſarily Reſult an Inconſiſtency in their Concluſions.

And, that this is a Natural Frailty in the Mind is Manifeſt from our Dreams, when the Mind in Sleep is Reſigned to it's own Actings without any Government or Diſcipline Exerciſed over it by us, or any Rules Preſcribed, whereby it's Thoughts may be Conducted, for then our Ideas Change their Shape and Appearance every Minute.

And for this Reaſon it is, that the more Lazy and Oſcitant we are in our Conſidering of Things, the nearer we Approach to that State of Mind, in which we Dream, and the leſs Accurate we are in our Reaſonings; Becauſe our Ideas are leſs Conſtant to themſelves, and are more Prone to Change into others; From whence it Follows, that to an Exactneſs of Judgment, and a Clearneſs of Reaſon, the Utmoſt Diligence and Intenſeneſs of Thought is Required, and a Watchful and Vigilant Eye is to be kept upon our Ideas, that they do not Gradually Transform themſelves into Different Ones.

Thus, when Philoſophers Diſcourſe of Matter, which they have Laid Down to have certain Invariable Qualities belonging to it, by which it becomes a Similar and Homogenous Subſtance; Carry them a little farther into their Syſtemes, and we ſhall find That Similar and Homogenous Matter has Chang'd it's Shape and Form, and is every Thing elſe but Homogenous, and has a Thouſand Various Powers, which at Firſt were all Excluded from it; 'Tis Hard, 'tis Fluid, 'tis Colour'd, 'tis Sonorous, 'tis all that we find in Matter, and yet at Firſt it was only Impenetrable, Diviſible, Moveable, and Extended; This is a Frailty in Reaſon and Judgment, and Inſtead of Philoſophizing, is only a Learned Kind of Dreaming.

What Wit is.

§. 9. On the other Hand, Wit Relaxes the Mind from this Severity of Thinking, and tho' it has a Sufficient Diſcernment betwixt one Idea and another, and can Accurately enough Diſtinguiſh them, and tho' with a Juſt Application and Attention it might therefore keep it's Ideas Firm and Unaltered, ſo as from thence to make an Exact Judgment of Things, it yet Chooſes a quite Contrary Method, that is, to Change and Diverſify it's Images of the ſame Thing, as much as Poſſible, ſo as to Strike us with Surprize, and with Variety; And whereas Reaſon and Judgment Connects our Ideas Cloſely together, and Shews the Dependence of one upon another, Wit

Breaks every Link of the Chain, Throws off ſuch Fetters of Thought, and is never better Pleaſed, than when it Soars with a Licence Uncontrouled, and Looks Down upon Coherence, as a Thing below it; which is Evident from the Pindarick Writings of the Poets, Compared with Thoſe of the Mathematicks and of Philoſophy.

Judgment and Wit Compared together.

§. 10. REASON and Judgment is a Scrupulous Exactneſs of Thought, and Wit an Unbounded Liberty of it; The One is, Agreeable to the Subject on which it is many Times Employ'd, Like the Nice Deſcription of a Figure, where the Lines and Angles are Punctually Drawn by Rule and Compaſs; The Other, Like the Bold and Maſterly Strokes of the Painter's Pencil, which muſt not be Guided by any Thing, but by Genius and Fancy.

AND therefore it is, that to Judge well, the utmoſt Seriouſneſs and Attention is Neceſſary, but to be a Wit, nothing Leſs; The Invention muſt be let Looſe, and no Reſtraints laid upon the Management of our Ideas, but all muſt be Reſigned to the Pleaſure of the Mind to Set them in what Light, or to Give them what Form or Colour, it Thinks Proper; Whereas Judgment and Reaſon Dare not Stir one Step beyond what Nature Preſcribes, and always Conforms it's ſelf to her Directions.

Of Comparing our Ideas one with another, in Reſpect of Extent, Degree, Time, Place, or any other Circumſtances.

§. 11. THIS Philoſophy farther Tells us; that the Comparing of our Ideas one with another, in Reſpect of Extent, Degree, Time, Place, or any other Circumſtances, is another Operation of the Mind about it's Ideas, which is the Ground of Relations, and that Brutes ſeem not to have this Power in any Great Degree; and that they Probably have not any Power at all of Comparing General Ideas.

IT would be Hard Indeed, if a Philoſophy, which has Conſidered Ideas with ſuch Accuracy, ſhould be Miſtaken in every Point; And it may Poſſibly be True in this, with one Reſtriction, in Reſpect of the firſt Part of what is here Affirm'd, that our Comparing Ideas with one another, as to Extent, Degree, Time, Place, *&c.* is the Ground of Relations, Namely, if it is Meant, ſo far as we Apprehend thoſe Relations; But if it is Affirm'd in General, it is a Miſtake; Becauſe not only our Comparing our Ideas, but the Different Genius's, Tempers, and Complexions of our Minds, by which we firſt Receive, and then Compare them, comes into the Account, which this Philoſophy does not at all Regard; For it's whole Syſteme Aims at this, to make the Furniture of every Perſon's Mind Alike, their Reaſon and Faculties the Same, and which Garniture, after it has made it a Raſa Tabula, muſt be of it's own Supplying; 'Tis an Empty Room, without any Thing to Set it off or Adorn it, till this Philoſophy has taken Care to put into it, what Ideas and Faculties it Thinks Proper for it's Ornament and Embelliſhing; And amongſt the Reſt, is this of Comparing Ideas; as if it was the ſame in all Men, and was not to be Diſtinguiſhed by the Various Forces, Powers, and Capacities of the Mind, which it Certainly Ought to be.

THUS in Reſpect of Extent, by Comparing our Ideas, 'tis True, we find that one Thing is Greater or Leſs, and that one Mind is more or Leſs Extenſive or Capacious than another; In Reſpect of Degree, that one Subſtance is Hotter or Colder, Harder or Softer, Whiter or Blacker, *&c.* than another; In Reſpect of Time, that the Exiſtence of certain Perſons or Things has been of a Longer or Shorter Continuance; and that, in Reſpect of Place, ſome Beings are at a Farther, and others at a Nearer Diſtance from us; And ſo it is in Reſpect of any other Circumſtances, in which Things are Conſidered by us.

THIS is very Obvious and Plain, and does not Require any Great Depth of Philoſophy to Know, or Underſtand, however Pompous theſe Aſſertions may Appear.

The Relations Ariſing from the Compariſon of our Ideas are Different according to the Different Complexions of the Minds, which Form them.

§. 12. BUT, If we Say theſe Relations are the ſame in all Men, becauſe all Men Gain them by Comparing their Ideas in this Way, That does not ſeem to be True; Since Men's Ideas themſelves Differ, according to the Various Complexions of their Minds, and Conſequently the Relations, which Ariſe from a Compariſon of thoſe Ideas.

THUS One, who has been Uſed to Large and Boundleſs Proſpects upon the Ocean, will Eſteem that a Narrow and Contracted View, which Another, not Accuſtom'd to Greater, will Reckon Sufficiently Wide and Spacious; One, who is Poſſeſſed of an Ample Eſtate, or of Immenſe Riches, will Look upon that, as an Inconſiderable Revenue, or as a Small and Triffling Sum, which Another will Think a very Great and Extraordinary One.

And a Perſon ſhall be Regarded by the Vulgar, as One of a Great and Extenſive Genius, who to Wiſe and Knowing Men ſhall Appear to have a very Small and Narrow One; So that in Reſpect of Extent, our Ideas of Greater or Leſs Vary and Change according to the Different Minds, which Compare, and Form the Relations of Things from ſuch a Compariſon.

That Temperature of Air or Climate, which is Exceſſively Hot or Cold to One, is not in the ſame Degree ſo to Another; He, who has been Laid in Down and Silks, will Eſteem that Hard, which is Soft enough for a Labourer's Repoſe; And an *American*, who never Saw, what Snow is, will Admire the Whiteneſs of an *European* Skin.

And even in Time, One, who has Lived to Fourſcore or a Hundred, will call his Son of Threeſcore a Young Man, whom his Grandſon of Thirty will call an Old One.

Thus alſo in Diſtance, that will Appear to One Perſon to be but a Short Journey, who has been Uſed to Travel a Longer, which to Another will ſeem a Long and Tedious One, who has not been Uſed to any at all.

And in other Circumſtances of Comparing our Ideas; That is Beauty, Symmetry, and Proportion to One, which to Another is Deformity; And in General, what is Rational or Agreeable to One Man is not ſo to Another.

The Genius and Temper of the Mind therefore, or the Innate Force of it, is to be Conſidered in our Forming the Relations of Things; ſince it Influences our Ideas, and Mixes it's ſelf in all our Compariſons of them.

Brutes ſeem to have no Power at all of Comparing Ideas, either General or Particular.

§. 13. As to the Second Part of the Aſſertion in the Beginning of §. 11. that Brutes do not ſeem to have this Power of Comparing in any Great Degree, nor to have any Power at all of Comparing General Ideas, it is Anſwer'd, That they Appear to have an Equal Incapacity for both. Comparing our Ideas Suppoſes a Certain Kind of Study and Contemplation, for when we Compare our Ideas one with another, we ſet them together in Order to Conſider and Examine their Difference, which is a Studious and Contemplative Faculty, and quite of another Kind from that of Perceiving a Diſtinction in our Ideas; Thus I may know, that Fire is Hot, and Snow Cold, by my bare Senſations, without ſetting theſe Ideas together in my Mind, and then Comparing them; The One is the Faculty of a Perceptive Being, the Other of a Rational and a Conſidering One; The Firſt of which Brutes have, but I Think I may Affirm, they have not the Laſt, tho' they may ſeem to make ſome Approaches towards it: For let us Examine a little into this, ſince it is of ſome Importance to us, to Know it, and to be Satisfy'd about it.

Men, who can Study, Contemplate, and Compare their Ideas, are likewiſe Furniſhed with Abilities, by Speech and by Writing, to ſhew, they have ſuch Faculties, for if they were not, they would be in Vain Poſſeſſed of them; It is therefore Reaſonable to Think, that Brutes are not Endowed with them, ſince they would have them to no Purpoſe, if they lay Hid in their Minds, without a Poſſibility of Shewing or Explaining them; But 'tis a Common Axiom with Philoſophers, that Nature Works nothing in Vain; Nay, even in Perſons, who are Dumb, we find ſuch a Capacity of Expreſſing their Thoughts by Signs and Geſtures, which is in a Manner, as Significant, as any Language could be, and which we do not Experience to be in Brutes in any Degree.

Apes or Monkeys have Indeed ſome Surprizing Actions, which ſhew a Quickneſs of Perception, and a Readineſs in Diſtinguiſhing their Perceptions, and Conſequently of Remembering them, and the ſame may be ſaid of Dogs; But there does not ſeem to be any Footſteps of Study or Contemplation in them, which there muſt be, if they Actually Compared their Ideas or Perceptions with one another, and did Set them together in their Minds in Order to Conſider them; Whereas the Ridiculous and Unthinking Tricks of Apes and Monkeys ſhow no ſuch Faculty or Power.

On the other Hand Parrots, which make ſome Kind of Efforts towards Language, can only Expreſs themſelves in ſome few Particular Things, which Evidences, that they Diſtinguiſh their Perceptions and Remember them, but not that they Study or Compare them, not even in the Famous Inſtance of Prince *Maurice*'s, given by this Philoſophy, Suppoſing that Inſtance True.

And Thus, it is, even in the Fine and Curious Cells or Neſts of Bees or of Birds, which ſeem to be Formed by a Mechanical Operation, ariſing from this Quick and Lively Diſtinction of Ideas, and the Memory, which Reſults from it, but does not any Ways

Evince, that they Compare their Ideas, or that they Study upon them; no more, than a Mechanick Building a Houſe, by what he has Learnt from mere Perception, and Memory, does Prove, that he has Studied upon, and Compared all the ſeveral Ideas, which belong to his Building, or, that he is the beſt Philoſopher, or Architect in the World.

From hence an Argument for the Immortality of the Soul.

§. 14. But, If Brutes have not the Power of Comparing Particular Ideas, it will be very Certain, they have it not, in Reſpect of General Ones, To Compare which is ſtill more Operoſe, and will Require a much Greater Study and Contemplation.

And, If they have it not in Either of theſe Particulars, If Brutes can only barely Diſtinguiſh their Perceptions, and juſt Remember what they Diſtinguiſh, without being Able to Proceed any Farther; And If on the Contrary, Men cannot only Diſtinguiſh their Perceptions, and Remember them, with an Infinitely Greater Exactneſs and Quickneſs than they can, but can alſo Study and Contemplate upon their Ideas, and Compare them to an Infinite Variety; for the ſame Reaſon, that Almighty God, who Infinitely Exceeds us in the Comprehenſion of his Mind, is Eternal, we may Preſume, that the ſame Infiniteneſs of Comprehenſion in Reſpect of Brutes, will Entitle Our's to a Natural Immortality; whilſt the Minds of Brutes are Periſhable and Mortal.

Add to this Farther, that a Faculty, which is merely Paſſive of Perception, and 'tis Poſſible, That is all, which Brutes have, and Memory, or a Recollective one, which Reſults from it, (which is no more than a Kind of Reſiliency in the Mind upon an Impulſe made upon Senſation) in it's own Nature ſeems to be as Capable of Ceaſing and Vaniſhing, as the Percuſſion made upon an Harp or a Violin; But, when we find in ourſelves, that there is not only a Perceptive Faculty, and a Memory, which is Derived from it, but an Active Principle in thoſe Perceptions and in Memory, as alſo in Comparing our Ideas, in Studying of them and Contemplating upon them, Abſtracted from Senſe, it gives us a Juſt Impreſſion, that there is ſomething in ſuch a Mind, which will Continue after all it's Perceptions from Senſation are no more, that is, after the Diſſolution of our Minds from the Bodies, which Encompaſs and Attend them; Laſtly, tho' Brutes have not the Faculty of Comparing Ideas, this Philoſophy cannot ſhew, why they have not, whereas, it may be done by the Pſycheometria Explained above, that is, from the Different Intenſions of Perception.

Concerning the Compoſition, and Abſtraction of Ideas; a Faculty not Reſideing in Brutes.

§. 15. The next Operation of the Mind, which this Philoſophy takes Notice of, is Compoſition, a Faculty, with which Human Minds are certainly Endu'd; Thus we can Form a Centaur, or Chimera, and Join and Unite Ideas together at our Pleaſure; But in Brutes, this Faculty ſeems alſo to be Perfectly wanting; and a Dog does not Appear to know his Maſter, any other way, than by Diſtinguiſhing his Ideas, and by Memory, both which he muſt, of Courſe and Neceſſity, have from his bare Faculty of Perception.

Abstraction is a quite Contrary Operation of the Mind, by which we Frame General Ideas, and Separate and Diſunite every Thing from them, which ſhall Render them Particular.

Thus, we can Remove Solidity, Figure, and other Properties, from Body in our own Minds, and Conſider only the Colour of them; or Colour, *&c.* and Conſider only the Extenſion of them.

This Faculty is Confeſſed not to Reſide in Brutes in any Degree, but why they ſhould be Able to Set and Combine Ideas together, as this Philoſophy Thinks, and not to Separate them, which it does not Allow, is Inconceivable.

The Diſtinction of the Minds of Brutes from thoſe of Men, and why the Firſt are Mortal, but not the Laſt.

§. 16. The whole Caſe therefore ſeems to be This, in Reſpect of Human Minds, and Thoſe of Brutes;

That the Latter are only Capable of Perception, by External Objects Acting upon their Senſes, and in which their Minds are barely Paſſive, or Suſceptible, of thoſe Impreſſions, and, by a Kind of Natural Reſult or Reaction, of Remembering them likewiſe; That, by theſe Means they Diſtinguiſh one Thing from another, and accordingly are Directed to Purſue, or Avoid it; That they do not Contemplate, Compare, Compound, or Abſtract their Ideas, becauſe their Minds are only Paſſive of the Impreſſions, which are made upon them, whereas thoſe Operations Conſiſt in Action; and Conſequently, and for the ſame Reaſon, That they have no Ideas of Reflection; And upon all theſe Accounts that they have no Conſcience of their Actions, and are Capable of no Moral Rules to Guide them; Since therefore their Minds are merely Paſſive from the Strokes of External Objects, it is Reaſonable to Think, that when thoſe Impulſes Ceaſe, their Perceptions will alſo Vaniſh; And ſince they have no Actions to Anſwer for, as they have no Senſe or Conſcience of them, nor no Reflection upon the Operations of their own Minds, it is not to be Imagined, that they will be Summon'd to a Future Audit; If therefore we Conſider Nature, or Religion and Morality, there is every Thing, which can Convince us, that the Souls, or Minds, of Brutes are Mortal.

That, yet Human Minds are not ſo, is as Evident on the Contrary; Since (which has been already Shewn) their Faculty of Perception is Active, and their Recollective Powers, not a Natural Recoiling, or Reſult, from a bare Impreſſion upon them, made by External Objects, as in Brutes, but an Active Force likewiſe; from which Principle of Action Proceeds their Ability of Comparing, Compounding, and Abſtracting, and Reflecting upon the Operations Produced in them; That they are Endued with a Conſcience of their Thoughts and Actions, and can Form Moral Truths for their Direction and Conduct; and Conſequently we may Juſtly Preſume from Nature, that this Principle of Action, which is Innate in theſe Human Minds will never Ceaſe, ſince it grows Stronger with Age, and is Fortified by Time and Experience; and we may Preſume from Religion, that ſuch Beings muſt, ſome Time or other, Anſwer for their Actions, and be Rewarded for their Good Ones, and Puniſhed for their Ill; And for both theſe Reaſons we may Pronounce, that Human Minds are Immortal, that they will Survive the Bodies, to which they are at Preſent Joined, and will Continue in a State of Activity, which is Innate to them, for Everlaſting Ages.

To be Satisfy'd and Apprized of the Infinite Difference betwixt Human Minds and Thoſe of Brutes, there is nothing more Requiſite, than to Conſider the Vaſt, the Boundleſs and Immenſe Reaſonings of Philoſophers, and Mathematicians, with their Various Comparing, Compounding, and Abſtracting their Ideas, the Excurſions of their Thoughts, and Contemplations, thro' the Diſtant Regions of the Univerſe, and thro' all the Ranks and Orders of Created Beings, and then Compare them, with the Short and Narrow Intellects of a Dog, a Monkey, or a Parrot; For from thence we may be Convinced of the Great and Stupendous Powers and Faculties of the One, and of the Small and Minute, and Inconſiderable, Forces of the other; The One makes Us Approach to God himſelf, the Other Scarcely Raiſes Them above the Energy of Matter. If we therefore Regard only the Relations of Things, we ſhall be Led to Think, and to Believe, that as Almighty God is Eternal, ſo We are Immortal; and as Matter is Periſhable, ſo the Souls, or Minds, of Brutes, which have a near Alliance with it, are nearly in the ſame State of Corruption, Change, and Alteration with it.

No Objection againſt this Diſtinction betwixt the Minds of Brutes and thoſe of Men, that there are ſome Men, who ſeem to be little Superior to Brutes themſelves.

§. 17. **Nor** can it be Objected, that there are ſome Men, who ſeem to be very little Superior in Knowledge to Brutes themſelves; Since, altho' the Improvements of their Faculties are not Equal to thoſe of Others, yet they have the ſame or like Faculties of Activeneſs in Perceiving, and Remembering, of Comparing, Compounding, and Abſtracting, and of Reflecting upon their own Thoughts and Perceptions; All which Brutes Want; And 'tis the Faculties, not the Uſe or Improvement of them, which muſt Denominate and Diſtinguiſh One Intelligent Being, as Such, from Another: There is not the Meaneſt Perſon in Underſtanding, who has not Language, and there is no one, who has Language, who has not, in a Greater or Leſs Degree, the Diſcurſive Faculty; which ſhews they have an Active Perception, and a Memory, that they Compare, Compound, and Abſtract Ideas, or elſe they could not Talk in the ſame Language with others, which even Ideots or Madmen can do; Tho' Perhaps they may not Connect their Ideas Juſtly; But this is an Imperfection or Diſorder in the Mind, and not a Proof, that they have not the ſame Kind of Faculties with other Men; On the Contrary, Brutes have no Diſcurſive Faculty at all, and what is Alledged from Parrots Extends only to ſome few Words, or even Sentences, which might be Learned by a bare Diſtinction of Ideas, and a Paſſive Memory, Reſulting from ſuch a Perception.

The Story of Prince Mau-*rices's Parrot, an Imperfect, and Improbable Relation, or if True, no Objection againſt the foregoing Diſtinction.* Mr Locke's Eſſay, the Supplement to it. Pag. 181. l. 19.

§. 18. **But** altho' we Argue upon a Conceſſion, that the Story of Prince *Maurices*'s Parrot is True, Mentioned by this Philoſophy, yet we are no more Obliged to Believe, that it is ſo, much leſs to Form and to Ground our Perſuaſions in Philoſophy, Contrary to all the known Faculties of Brutes, upon ſuch a Story, which is Alledged to Prove, or, at leaſt, to Countenance the Suppoſition of a Rational Parrot; Than this Author of Great Note, who is Quoted, or the Great Author, who Quotes him, would have Thought themſelves Obliged to Believe the Immortality of the Soul, becauſe Prince *Maurice*, with his Uſual *Plaineſs and Dryneſs* in Talk, or *Shortly and Coldly*, for it's much the ſame, as to the Truth of the Story, had Informed them, that he had Met with a Tall Black or Tawny Ghoſt in *Braſil*, which had Told him as much; For the Evidence Amounts to no more, than what we have for every Ghoſt, that has been Talked of for this Hundred Years, or as many Ages, which Generally is Confined to the Teſtimony of One or Two; and therefore we may as fully Believe any Story, which we have Heard of an Apparition, as this of the Rational Parrot; But Thoſe, who are ſo willing to Believe This, would take it to be an Affront to their Reaſon and Underſtanding to Believe the Other.

BESIDES, Is it not Incredible, that of ſo many Millions of Parrots, which either have Belong'd, or do Belong to the Species, there never was one beſides Prince *Maurice*'s, that was Old, or Big, enough to come to the Uſe of it's Reaſon, that ever any one Heard of? And would not One be Apt to Think, that this was only ſome *Cold* and *Dry* Jeſt of the Prince, to Divert himſelf with the Credulity of thoſe about him?

Is it not alſo ſomething Surprizing, that, when Prince *Maurice* had the Curioſity to ſend for this Rational Parrot, tho' at a Good Diſtance off, he ſhould only give us one Poor Dialogue for the Proof of it's being Rational, and that the ſame Curioſity did not Tempt him to Inquire farther, and Examine very Strictly into the Profound Abilities and Attainments of this Wonderfull Bird? Here is nothing, only a ſhort Story of coming from *Marinnan*, of belonging to a *Portugeze*, and looking after the *Chickens*; The Prince's Invention muſt be either very Low, that could not Prompt him to Ask any more Queſtions, or very Cunning to Put ſuch, which he Thought might be the Propereſt, and not the moſt Unlikely, for a Parrot to Anſwer; For if it had the Diſcurſive Faculty, it might have Anſwered a Thouſand Queſtions beſides; From whence it is Plain, that this was only a Leſſon, which the Parrot had Learned, and therefore is Reſolved into the Ideas of bare Memory and Perception, which is Allowed to be in all Animals, in ſome Degree or other.

BUT, what is very Extraordinary in this Dialogue is, that it was betwixt the Prince and the Parrot; The Firſt Spoke in *French* or *Dutch*, for by his own Confeſſion, he did not Underſtand the Language of *Braſil*, and the Laſt Anſwered in *that* Language; and tho' by the Method, which the Prince took of an Interpreter, he might Underſtand, what the Parrot ſaid, yet the Parrot could not know, what he ſaid, unleſs by a like Interpreter, which yet is not in this Account Affirm'd; Nay Farther, it is not ſo much as ſaid, that the Prince Inquir'd of his Interpreters, upon every Anſwer of the Parrot, what was Meant by it; But it is rather Repreſented ſo, as if after the Dialogue, which Paſſed Currently betwixt them, the Prince took his Interpreters Apart to Know, whether they both Agreed in giving the ſame Senſe of the Parrot's Anſwer; which if ſo, is an Evident Conviction, that the Story is all, a mere Contrivance; ſince it is Impoſſible the Prince ſhould Propoſe his Queſtions in *French* or *Dutch*, and the Parrot ſhould Anſwer in the *Braſil*; However it is ſo Imperfect, and at the ſame Time ſo Improbable a Relation, and if it were not, ſo little Proves that Diſcurſive Faculty which there is in Men, that we have no Reaſon to Build any Opinions or Notions in Philoſophy upon it, and with this I ſhall Diſmiſs it, and Conclude the Preſent Chapter, after having Conſidered the ſeveral Faculties of Human Minds in a Mathematick Way, and as a Foundation for a Geometrick Reaſoning upon them, and after having Reduced them to the Doctrine of the Expanſive and Contractive Forces.

The Different Combinations of the Faculties of Intelligent Beings, in Reſpect of their Comparing, Compounding, and Abſtracting.

§. 19. THE Three Operations of the Mind, not yet Conſidered in this Way, are Comparing, Compounding, and Abſtracting our Ideas, in Reſpect of which, the ſeveral Combinations are as Follow,

FIRST, Comparing-Compounding-Abſtracting.
SECOND, Non-Comparing-Non-Compounding-Non-Abſtracting.
THIRD, Comparing-Non-Compounding-Non-Abſtracting.
FOURTH, Compounding-Abſtracting-Non-Comparing.
FIFTH, Compounding-Non-Comparing-Non-Abſtracting.
SIXTH, Comparing-Abſtracting-Non-Compounding.
SEVENTH, Abſtracting-Non-Comparing-Non-Compounding.
EIGHTH, Comparing-Compounding-Non-Abſtracting.

THE Firſt Caſe is of thoſe Intelligent or Animate Beings, which have an Equal Faculty and Quickneſs in Comparing, Compounding, and Abſtracting their Ideas.

THE Second of thoſe, who Entirely want theſe Faculties, as it is Probable Brutes do, or elſe have them in a very low Degree.

THE Third is of thoſe, who Compare their Ideas with Vivacity and Quickneſs, but who Compound and Abſtract them with Difficulty.

THE Fourth is of thoſe, who Compound and Abſtract them with Eaſe, but Compare them with more Trouble and Pains.

THE Fifth is of thoſe, who Compound them with Readineſs and Facility, but Compare and Abſtract them more Slowly.

THE Sixth is of thoſe, who Compare and Abſtract with Greater Briskneſs and Activity, but Compound with a Leſs.

THE Seventh is of thoſe, who Abſtract more Readily, and Compare and Compound more Difficultly.

THE Eighth is of thoſe, who Compare and Compound with more Readineſs and Eaſe, but Abſtract with a Leſs.

ALL theſe Various Faculties will give ſo many Different Denominations to Intelligent Minds, as the ſeveral Combinations of Perception, Retention, and Recollection, before did; and all theſe Faculties in Animate Beings, if they are at all in them, may be there in a Thouſand Different Degrees and Proportions.

AND Farther, theſe Three laſt Mentioned Faculties may be Differently Combined with the Three Former; From whence an Infinite Variety of Intelligent Beings will Ariſe, Sufficient to Account for the whole Animate Creation, and the ſeveral Orders, Claſſes, and Diſtinctions of them which are known, or can be Conceived or Imagined in the Univerſe; and will Infinitely Inlarge the Bounds of a Pſycheometria, and Give Scope and Room for a Mathematick Genius to Extend it's ſelf to a Million of Propoſitions, hitherto Unknown to Geometry.

The Faculties and Powers of the Mind Accounted for from the Doctrine of the Expanſive and Contractive Forces: Senſation belongs to the Expanſive Force, and Reflection to the Contractive.

§. 20. As to the Doctrine of the Expanſive and Contractive Forces, Perception ſeems to Participate of both; It has the Greateſt Expanſive Force, when it Exerts it's ſelf into Buſineſs and Action, and the Greateſt Contractive, when it Retires from it, and Reſigns it's ſelf to Study and Contemplation.

THUS alſo, when our Senſations are Vigorous, Lively, and Strong, in Exerting themſelves Outwards, and Purſuing the Objects, which Impreſs them, it is an Inſtance of the Expanſive Force of the Mind, or of Perception; when our Senſations are Vigorous, Lively, and Strong, in Exerting themſelves Inwards, ſo as to make us Reflect upon them in our own Minds, it ſhews the Contractive.

So that Senſation in General Appears to Demonſtrate the Expanſive Force of the Mind, and which Agrees with what we have ſaid before, that the Mind is not only Paſſive, but even Active in Senſation; Reflection, on the other Hand, does not Explain an Expanſive Force of the Mind, by which it Exerts it's ſelf towards External Objects, but a Contractive, by which it Exerts it's ſelf Inwards towards it's own Perceptions.

The Expanſive Force, the Source of Vice and Folly, the Contractive of Wiſdom and Virtue: Retention, Recollection, Compariſon, Compoſition, Abſtraction, all Inſtances of the Contractive Force.

§. 21. FROM whence it will Follow, that as Senſation, or the Expanſive Force, is the Source of Vice and Folly in Men's Minds, ſo Reflection, or the Contractive Force, is That of Wiſdom and Virtue; Suitable to what has been Aſſerted, in other Parts of this Philoſophy, of the Expanſive and Contractive Forces.

NEXT to Senſation and Reflection, we are to Conſider, what Retention and Recollection is, in Reſpect to the Doctrine of the Expanſive and Contractive Forces; That Force, by which we Retain Ideas in our Minds, and Fix them down in it, Appears to be Evidently a Contractive One; as alſo That, by which we Recollect to our View, thoſe which are Scattered and Diffuſed in it, or by which we Revive the Dormant and Latent Ones; As Retaining and Fixing, or Recollecting, Reflecting, or Reviving them, are Inſtances of Contraction on the one Hand; and their being Volatile and Deſultory, their being Scattered and Diffuſed, or Tranſient and Periſhing, is an Example of Expanſion on the other; This Recollection muſt be Diſtinguiſhed from the Memory of Brutes; this being an Actual bringing Ideas into our Minds by this Contractive Force, the other only a Repeated Perception, with a Power of Diſcerning, that it was before Perceived, and therefore is an Affection of the Expanſive.

As to Comparing, or Setting our Ideas together, in Order to Conſider them, it is Manifeſtly the Reſult and Conſequence of the Contractive Force, as is that of Compounding or Uniting them into One ; In like Manner, as the Diſſipation of them, the Disjoining, or Diſſolving them, Infers and Suppoſes an Expanſive.

Abstraction, or that Faculty, by which we Separate Ideas from each other, may be Thought to belong to the Expanſive Force ; But if we Conſider, that this Operation of the Mind Ariſes from Reflection, which has been Proved to be Derived from the Contractive ; and if we farther Conſider, that when we Abſtract, we Contract ſeveral Ideas into One, as Red, Orange, Yellow, Green, Blue, Purple, *&c.* into one Abſtracted Idea of Colour, and the ſame be ſaid of Sound, Taſte, Solidity, and Extenſion, which are all Abſtracted Ideas, we ſhall Eaſily be brought to Acknowledge, that this Power of Abſtraction is to be Aſcribed to the Contractive Force of the Mind, as it's True Cauſe and Origine.

§. 22. This Opinion of the Principal Faculties of our Minds being owing to the Contractive Forces of them, is farther Proved from Hence ; That theſe Faculties of Diſtinguiſhing our Ideas, of Retaining, Reviving, Comparing, Compounding, and Abſtracting them, are Impaired and Diminiſhed by Debaucheries, Exceſſes, and Lewdneſs, or by Vice in General, which is of an Expanſive Force, and Enervates and Diſſipates the Contractive Force of the Underſtanding ; This alſo Agrees with the Account, which we have given in Anatomy of the Nervous Syſteme, which we made to be of a Contractive Force.

Another Argument, that the Faculties of our Minds are of a Contractive Force.

§. 23. From hence it Follows, that the Minds of Brutes, being Solely or Chiefly of an Expanſive Nature, and Conſiſting only in bare Perception and a Memory, which is no more than a Repeated Perception, with a Perception, that it was before Perceived, are not Capable of Virtue or Vice, which Requires Reflection, Thought, Choice, and Conſideration, that is, Faculties of a Contractive Force and Conſtitution ; And Conſequently, they are not Capable of Rewards and Puniſhments, and therefore it is Unreaſonable to Imagine, that ſuch Minds will have any other Period of their Exiſtence, than what is Allotted to their Material Syſtemes.

From hence the Minds of Brutes Mortal, and not Capable of Virtue or Vice, Rewards or Puniſhments.

Besides, as ſuch Minds are of an Expanſive Force ; without the Contractive, they, by their Expanſion, and in their own Nature, Tend to a Diſſolution, as Fire and Flame, which are Expanſive, do.

§. 24. On the other Hand, the Human Mind Depends upon Properties and Affections, which, as we have ſhewn, are of a Contractive Nature, Excepting it's Senſitive Faculty, which is of an Expanſive ; and therefore ſuch a Mind is Capable of Virtue and Vice, and of Rewards and Puniſhments, and for that Reaſon muſt be Suppoſed to be Immortal, ſince Virtue is not always Rewarded, nor Vice Puniſhed, During it's Exiſtence here ; And, tho' the Senſitive Powers may Ceaſe their Action, with the Diſſolution of the Material Syſteme, which may again Exert themſelves upon the Reunion of the Body to them ; Yet it is Juſt and Rational to Think, that the Contractive Forces of the Mind will Remain and Subſiſt in their full Strength and Vigour, or in a Greater Degree of Both, after ſuch Diſſolution ; And, as Blind Men Remember and Reaſon more Strongly, by a Deprivation of one Senſe only of Seeing, ſo we may Imagine, that the Human Mind, when Deprived of all, may Reflect upon, Retain, Recollect, Revive, Compare, Compound, and Abſtract, it's Ideas with more Quickneſs, Life, and Vivacity, than at Preſent it does, whilſt United to this Syſteme of Senſe and Matter.

The Human Mind, from the ſame Doctrine or Principles, Immortal, and Capable both of Virtue and Vice, and of Rewards or of Puniſhments.

These Contractive Forces of the Mind alſo ſeem in their own Nature to be Averſe to a Diſſolution, quite Contrary to the Expanſive Forces of it ; As, in Body, the Contractive Forces Tend to a Cloſe Cement and Connection, and to the Eſtabliſhing the Perpetuity of the Material Syſteme, whether of Marble, Flint, or an Adamant ; Whereas the Expanſive Force, of Fire, Percuſſion, or a Proper Menſtruum, Diſſolves the Continuity, and Diſſipates and Deſtroys the Compages.

That the Expanſive and Contractive Forces of the Mind and of Matter are Analogous, tho' not the ſame, and therefore Juſtly Prove the Immortality of the Human Soul, and Mortality and Diſſolution of the Brutal.

§. 25. When I Speak of the Expanſive and Contractive Forces of the Mind, and Aſſert the like Forces to belong to Bodies, I do not make Mind and Matter the ſame ; I only Affirm, that they have Properties and Powers, which are Analogous, when the Beings, in which they Reſide, are Perfectly Different ; The One being a Thinking and Intelligent Subſtance, The Other a Material and Inanimate.

Thus we may Affirm, that there are Motions in the Mind, as well as in Matter, that there is Gravity, Levity, Solidity, Diſſipation, Complexion, *&c.* in the one, as well as the other ; But every one knows, that theſe are only Properties, which we find in the Mind, Anſwering and Correſpondent to others, which we Obſerve in Matter,

Matter; but, that they are not Punctually and Literally the ſame, and only ſo far, as the differing Natures of each, of Intelligence and Matter, will Bear, and only ſo far, as they will Allow of the Compariſon.

In like Manner we ſay, there is an Expanſive Force in the Mind, by which it's Ideas are Diſſipated, Diſſolved, or Diſunited, and a Contractive, by which they are Held and Continued together, and Cohere with each other; Agreeably to what we Experience in Matter, in which we Obſerve two Different Forces Reſiding, an Expanſive, by which it's Parts are Diſſolved, and a Contractive, by which they Cohere; Tho' we do not therefore Affirm, that Mind is Matter, or that the Ideas of the one are the Parts of the other; However, this is no Reaſon, why we may not Juſtly Conclude, that the Expanſive Force is the Principle of Diſſolution, and the Contractive of Continuance and Perpetuity in both; And ſince the Firſt belongs to the Minds of Brutes, and the Laſt to thoſe of Men, That therefore, there is a Natural Principle of Mortality in the Former, as there is one of a Perpetual Duration in the Latter.

The Elements of the Pſycheometria from hence Advanced.

§. 26. I ſhall here only farther Add, that the Different Proportions of theſe Expanſive and Contractive Forces in the Minds of Men, or of Brutes, will Furniſh us with a New Set of Mathematick Propoſitions, and Give a farther Advance to the Elements of a Pſycheometria.

CHAP. VI.

Of Complex Ideas, and Firſt, of Simple Modes of Space and Duration.

The Account, which this Philoſophy gives of our Complex Ideas and the ſeveral Ways, by which we Compound them, and firſt of Modes. Mr. Locke's Eſſay, Chap. 12. Book 2.

§. 1. As to what Relates to Simple Ideas, and whether the Mind is barely Paſſive, or not, in Receiving them, both have been already Conſidered; Complex Ideas are ſaid to be Thoſe, which Ariſe from that Faculty, which the Mind has, of Compounding it's Simple ones; We muſt therefore Inquire, what this Faculty in the Mind of Compounding is, and how far, and in what Inſtances it is made Uſe of by us.

This Philoſophy gives no Account of what the Nature of it is, or whence it Ariſes, and only Mentions Three Inſtances of our Compounding.

First, in Reſpect of Modes; Secondly of Subſtances; and Thirdly of Relations; As to Modes, which are ſaid to be ſuch Complex Ideas, which Contain not the Suppoſition of Subſiſting by themſelves, but are Conſidered, as Dependences on, and Affections of Subſtances, as Triangle, Gratitude, Murder, *&c.* They are, it is Alledged, of two Sorts; Firſt, Simple, or ſuch as are the Combinations of the ſame Simple Idea, as a Dozen, Score, *&c.* Which are but the Ideas of ſo many Diſtinct Units put together; Second, Mixed, which are Compounded of Simple Ideas of ſeveral Kinds, as Beauty, which Conſiſts in a Certain Compoſition of Figure and Colour, cauſing Delight in the Beholder; Theft, which is the Concealed Change of the Poſſeſſion of any Thing without the Conſent of the Proprietor.

Modes, not Ideas made by the Compoſition of the Mind, but by the Abſtraction of it, otherwiſe they may be Conſidered as Simple Perceptions.

§. 2. As to theſe Ideas, ſo far as they are Defined not to Contain a Suppoſition of Subſiſting by themſelves, but to be Dependences on, and Affections of Subſtances, they are Abſtracted and General Ideas; and therefore, if we Conſider them as Modes, or ſuch Affections and Dependences, they are owing to the Abſtracting Power of the Mind, and not to the Compounding; for, when we Abſtract a Triangle, Gratitude, Murder, Beauty, or Theft, from any Subjects, in which they Inhere, we do not Compound them, unleſs Abſtracting and Compounding are the ſame Things; and ſo in the Repetitions of the ſame Ideas, as of a Unit, when we Speak of 100, or 1000, or any Number in General, without Conſidering the Things, which are Numbered, we do not Compound but Abſtract.

If, on the other Hand, we Conſider theſe Ideas, as made up of Several Combined together, we Receive them at once, and as much by a Single Perception, as the Simpleſt Ideas whatſoever; To wit, ſo far, as they are Ideas of Senſation, or Reflection, that is, Real and not Fantaſtick ones, Made and Formed by the Will and Pleaſure of the Mind, in which Caſe only the Mind Compounds, as it does in all other Chimeras.

Thus, If we take a Triangle, as it any where Exiſts in Nature, or is Drawn in a Mathematick Scheme, it is an Idea of Senſation, which we no more Compound than we do any one Line of it, or any other Senſation of Red, or Green, or Blue, or the moſt Simple Idea we can Imagine; If we Conſider it in a Notional Way, as Abſtracted from it's being Drawn in a Mathematick Scheme, or Exiſting in Nature, it is an Idea of Abſtraction; And the only Compoſition of the Mind can be in Taking at it's Pleaſure Three Lines, and Joining them together, to make a Triangle, and to Give what Properties, or Aſſign what Conditions ſuch Lines and Angles ſhall be Conſidered Under, as thoſe of mere Length and Breadth, which is an Arbitrary Conſideration and Compoſition of them, and does not Fall Short of any other Chimera.

Thus alſo, If we Take Gratitude, Murder, Beauty, or Theft, and Conſider them as Real Actions, or Properties of the Subſtances, in which they Exiſt, they are Ideas of Senſation, and are no more Compounded by the Mind, than any other of our Ideas, which Ariſe from Simple Impreſſions; Thus we See one Man Cutting another's Throat, and the Two Men, the Knife, the Throat, and the Baſeneſs of the Action, as much at once, as we See a Table, the Extenſion, Colour, and Situation of it, at one and the ſame Time, and at one View.

If again we Abſtract theſe Ideas from the Subſtances, in which they Exiſt, they are Ideas of Abſtraction, and not of Compoſition.

And Laſtly, If we Compound theſe Ideas any Way, it is by taking Arbitrarily ſeveral Ideas and Uniting them together, as we Think Fit and Proper, and then giving them ſuch certain Denominations, which alſo is the Procedure of the Mind in Forming any other Chimera; For if we give Definitions of them Agreeable to what we Really find in Nature, we do not thereby Compound thoſe Ideas, but only Deſcribe them from what we in Fact Experienced by the Intervention of our Senſes.

The ſame may be ſaid of Subſtances.

§. 3. Secondly, as to Subſtances, we do not Compound them, ſo far as they are Real Beings Exiſting in Nature, as a Man or a Stone, they are Ideas of Senſation, and are Impreſſed upon our Minds in the ſame Manner, as our Simple Ideas are; we See the Shape, the Figure, and Compoſition at Once; And, tho' we may Arrive at other the Various Properties of theſe Subſtances one after another, and at Different Times, yet they are ſtill the Reſult of our Obſervation, and do not Meet together from the Mind's Combining or Compounding them, but from it's Obſerving them to Exiſt United; And if we Conſider them as General Ideas, and Separate them from Real Exiſtence, they are Ideas of Abſtraction, and never become Ideas of Compoſition, but when the Mind Arbitrarily takes ſeveral Properties and Unites them together, and then Denominates ſuch a Compoſition, a Man or a Stone, which is not Correſpondent to, nor is an Exact Deſcription of Nature, but a Chimera in it.

Thus alſo an Army, an Heap, or an Hundred, as they are Ideas of Senſation, or Abſtraction, are not Compounded by the Mind, and only are ſo, when the Mind Arbitrarily Puts ſo many Men or Things together, whether there ever were ſo many Put ſo together in Nature, or not; that is, where we do not Join Things together as they Exiſt, but according to our own Pleaſure; For, where they are Joined together, as they Exiſt, that Compoſition or Combination is made Ready to our Hands, and cannot be Properly Termed a Compoſition or Combination of the Mind; Rather, what the Mind does in ſuch Caſes, is Dividing, what is thus already Combined, and Conſidering the ſeveral Parts of the Idea, of which the Whole is Form'd and Compoſed.

The Like may be Affirmed of Relations.

§. 4. And the ſame may be ſaid in Reſpect of Relations, that they in Fact Exiſt, or elſe are Abſtractions of the Mind, and are only Compoſitions of it, when they are Arbitrarily made; So that we may ſay in General, that our Complex Ideas are either Ideas ſo Compounded in their Exiſtence and in Nature, and Conſequently are Ideas of Senſe and Obſervation; or elſe are General Ones, and therefore Ideas of Abſtraction; or elſe are Compounded Ideas, which the Mind Divides into the ſeveral Parts, of which they Conſiſt; In none of which Inſtances does the Mind Compound them; And ſo far as they are not Conſidered in One, or Other, of theſe Ways, that is, where they are Formed by the Arbitrary Power of the Mind, they become Creatures of it, and on that Account Chimerical.

Of Space in General; not Rightly Defined by this Philoſophy on three Accounts. Mr. Locke's Eſſay, Chap. 13. Book. 2.

§. 5. The Firſt Sort of Complex Ideas, which this Philoſophy Proceeds Particularly to Conſider, are thoſe of the Simple Modes of Space.

As to Space it's ſelf, it is ſaid to be a Simple Idea, which we get both by our Sight and Touch; when it Lies in Length betwixt two Bodies, it is called Diſtance; when it Fills the Dimenſions of Length, Breadth, and Thickneſs, 'tis Named Capacity; and when Conſidered between the Extremities of Matter, which Repleniſhes Space with ſomething Solid, 'tis Termed Extenſion.

There are therefore Three Things Aſſerted here, which are not True.

1*ſt.* That Space is a Simple Idea, ſince it is an Abſtracted One, for what we Feel is not the ſame Senſation with what we See, nor is Extenſion, which we Feel, the ſame with the Diſtance, which we See; Space therefore is Abſtracted from both, and is only Room for Bodies to Move or Exiſt in, without Conſidering our Senſations of it; and therefore is not a Simple Idea, it being not ſo much as an Idea of Senſation.

2*d.* That we get this Simple Idea of Space both by our Sight and Touch; For, if it is an Abſtracted Idea, we get it by neither; nor can we Obtain the ſame Idea of Senſation from our Sight and Feeling, unleſs our Eyes are the ſame with our Hands, and the Conſtitution and Formation of thoſe Organs of Senſe in no Reſpect Different.

3*d.* That Extenſion of Body is the ſame with Space; Whereas Space, as it is an Abſtracted Idea, is neither Capable of being Seen nor Felt, but Extenſion of Body, or Matter, is Subject to both thoſe Senſes, and is Different to our Sight, and to our Feeling.

What the Simple Modification of Space is, according to this Philoſophy.

§. 6. The next Thing, which this Philoſophy of Ideas Conſiders, is, the Modification of Space; and it is ſaid, that each Different Diſtance is a Different Modification of it; and that each Idea of any Different Space, is a Simple Mode of ſuch an Idea, Namely of Space, as an Inch, a Foot, or a Yard, *&c.* and when the Mind has got Ideas of theſe Meaſures, it is Capable of Repeating them, as often as it Pleaſes, and by that Means, at Length Arrives at the Idea of Immenſity.

The Simple Modification of Space no Complex Idea.

§. 7. But, as we have Proved, that Space is an Abſtracted Idea, and therefore no Simple One of Senſation, which this Philoſophy Affirms, ſo neither is the Modification of this Idea into Different Lengths and Meaſures a Complex One; For an Inch, a Foot, or a Yard, is not an Idea made by Compoſition, but by Abſtraction likewiſe; ſince, if we take an Inch, or a Foot, we do not Conſider it, ſo far as it is an Inch, or a Foot, as Conſiſting of Parts, but of one Certain and Determined Length, Taken and Abſtracted from Univerſal Space or Extenſion.

'Tis True, we may Conſider a Foot, to be Made up and Conſtituted of ſo many other Lengths, as Inches, and a Yard to Correſpond to ſo many Feet; But this is Foreign to the Real and Determinate Meaſure of a Yard, or a Foot, and only Ariſes from a Compariſon of them with other Lengths; For a Foot is no more Compounded of 12 Inches, or a Yard of 3 Feet, than of any other Number of Parts, we can Mention, into which they are Capable of being Divided.

The Length, or Meaſure, whether of an Inch, a Foot, or a Yard, is one Preciſe, Individual, and Uniform Portion of Space, Abſtracted by the Mind from the Reſt of it; And therefore, ſo far only, as it is a Modification of Space, is not an Idea of Compoſition but Abſtraction.

After the Mind is Furniſhed with theſe Lengths, it cannot be Denied, but that it has a Power of Repeating them, from whence Indeed will Ariſe Certain Complex Ideas of Space; Thus, if the Abſtracted Length of a Yard is made by a Repetition of 3 Feet, or a Foot by the Repetition of 12 Inches, or an Inch by the Repetition of a Certain Number of Barley Corns, they will then become Complex Ideas, but ſtill, they will not, as this Philoſophy Affirms, be Compounded of Simple Ideas of Senſation, but of a Repetition of the ſame Abſtracted Idea of Length; And it ſeems, as if the Mind had it's Original Meaſures from Abſtraction, and that the Repetition of them does Obtain only in Uſe and Practice.

The Idea of Immenſity not gained by the Repetition of any Finite Meaſure, as this Philoſophy would Perſuade us, but by the Negation of it.

§. 8. As to the Idea of Immenſity, it does not Appear to be Gained to the Mind, by a Repeating any Finite Meaſure, becauſe that only would Supply us with the Idea of a Greater Finite, but by a Negation of Finite, be it as Great as it will; This yet does not make Immenſe Space a Negative Idea, becauſe it is a Negation of Finite, no more than Finite is a Negative Idea, becauſe it is a Negation of Infinite; On the Contrary, Infinite Space is a Poſitive Idea, and Includes every Thing, which is Finite in it; Finite is nothing more than the Mind's taking a Portion or Part of what is Infinite, and we cannot have the Idea of Finite Space, without the Suppoſition of an Infinite one; Becauſe, ſince we may take our Finite Portion of it, Where and as Great as we Pleaſe, 'tis Manifeſt, we could not do it, unleſs it were in Fact Infinite.

THE Idea of an Immenſe Space is therefore Acquired, not from a Repetition of any Finite Meaſure of it, but from Reaſoning, and a Plain and Evident Concluſion from the Nature of it, which Receives no Bounds, and is not Capable of any Limitation.

THERE are alſo other Ways, by which we come by this Idea, that is, from the Various Theorems and Demonſtrations in the Mathematicks, Concerning the Angle of Contact, Incommenſurables, the Aſymptotes of the Hyperbola, *&c.*

Mr. Raphſon de Spatio Infinito.

BUT this Idea of an Infinite Space is Formed by Men of Reaſoning and Speculation; The Common and more Uſual Way of Gaining it ſeems to be from the Boundleſs Proſpects of the Sky and Æther, which we every Day See, and gives an Impreſſion of the Immenſity of Space to the moſt Ignorant, as well as to the moſt Learned, becauſe we See no End of ſuch Proſpects, that is, They are Infinite; But I dare Affirm, that no One ever got his Idea of it by the Method, which this Philoſophy Aſſerts, Namely, by the Repetition of any Finite Meaſure of it whatſoever, for the Reaſon before Alledged, that it is Impoſſible, that the Repetition of any Finite ſhould Produce any other Idea in us, than that of a ſtill Greater Finite, which will always be Infinitely ſhort of what is Unlimited and Immenſe.

§. 9. ANOTHER Modification of Space is ſaid to be That of the Different Terminations of it, which is Called Figure; But according to what has been before Alledged in §. 1. Figure is either an Idea of Senſation or Abſtraction, and is not an Idea of the Mind's Compounding, but when it becomes a Chimerical One, and is not Made up of Simple Ideas of Senſation.

Figure not a Complex Idea, but an Idea of Abſtraction, or elſe a Simple Idea of Senſation.

§. 10. THE Laſt Mode of Space Mentioned in this Chapter is Place, which may be Conſidered, as Relative, or Poſitive and Abſolute; Relative Place is, when we Conſider any Thing with Reference to it's Diſtance from ſome Fixed and Certain Points, but then this is not a Modification of Space, nor an Idea, which Ariſes from Compoſition, but from Comparing the Situation of any Body in Reſpect of ſuch Points, and which does not Inferr any Modification of Space, not even as to Length, becauſe ſuch Modifications are always Preciſe, and Determinate, as in an Inch, a Foot, or a Yard, according to the Confeſſions of this Philoſophy; Whereas in Relative Place, we only Conſider any Bodies in General, being Nearer to or Farther off from the Point, to which it is Referr'd, and the Manner of Poſition, which it Obtains in Reſpect of it; For if we Examine into the Real Diſtance, and Take the Exact Number of Yards or Miles, or Semidiameters of the Earth, into the Account of Relative Place, Relative Place will not be Diſtinguiſhed from the Modifications of Space in Reſpect of Diſtance, which Fell under another Head, and which has been already Inquired into.

Place no Complex Idea, but an Idea of Abſtraction.

POSITIVE or Abſolute Place muſt therefore, if any, be a Modification of Space, nor yet is This, becauſe it is an Idea of Abſtraction, and not of Compoſition; For, if we Take any Body or Portion of Matter, which Exiſts, the Place is an Abſtracted Idea, which Excludes all other Properties, but mere Space or Dimenſion; Whereas the Real Place is no Other than that Body or Portion of Matter, which does Exiſt; And ſo it may be ſaid in Reſpect of the Univerſe, that it does not Exiſt in Univerſal Space, or Place, but that it Exiſts in it's Self, whatever the Nature of that Exiſtence is; Which we have Proved to be a Plenum of Action, Worthy of an Infinite and Almighty Agent.

§. 11. THE next Kind of Simple Modes are ſaid to be thoſe of Duration, which are ſeveral Meaſures, or Diſtances, of Duration, as Hours, Days, Years, *&c.* Time and Eternity.

THE Idea of which Duration is Affirmed to be Derived from the Fleeting and Perpetually Periſhing Parts of Succeſſion, and the Idea of Succeſſion, from Reflecting on

How we Gain the Idea of Duration from Succeſſion, and the Idea of Succeſſion from Reflecting on that Train of Ideas, which Conſtantly Follow one another in our Minds, whilſt we are Awake, as Explained by this Philoſophy.

on that Train of Ideas, which Conſtantly Follow one another in our Minds, as long as we are Awake.

AND the Continuation of the Exiſtence of ourſelves, or any Thing elſe Commenſurate to the Succeſſion of any Ideas in our Minds, is ſaid to be, what we call our own Duration, or that of another Thing, Co-exiſting with our Thinking; That this is ſo, is Proved from Hence.

1ſt. BECAUSE we have have no Perception of Succeſſion, or Duration, when that Succeſſion of our Ideas Ceaſes, as in Sleep; The Moment we Sleep and Awake, how Diſtant ſoever, ſeems to be Joined and Connected, Excepting, when we Dream and there is a Manifeſt Succeſſion of our Ideas.

2d. BECAUSE, whilſt we are Awake, and our Thoughts are Intent upon one Thing, we let Slip out of our Account a good Part of that Duration, and Think that Time Shorter than it is.

3d. BECAUSE Motion, which is Commonly Suppoſed to be a Meaſure of Duration, is no farther ſuch, than as it Produces in our Minds an Idea of Succeſſion, or a Continued Train of Diſtinguiſhable Ideas in us.

AND therefore, it is ſaid, Motions, which are very Slow, are not Perceived by us, becauſe the Change of Diſtance is ſo Slow, that it Cauſes no New Ideas in us, but after a Long Interval; As in the Hand of a Clock, or the Shadow of a Dial.

AND the Same Happens in Things, which Move very Swift, which, not Affecting the Senſe with ſeveral Diſtinguiſhable Diſtances of their Motion, Cauſe not any Train of Ideas in our Minds, and Conſequently are not Perceived.

THUS any Thing, which Moves Round in a Circle, in leſs Time than our Ideas are wont to Succeed one another in our Minds, is not Perceived to Move, but ſeems to be a Perfect and Entire Circle of that Matter, which is in Motion.

AND ſuch a Part of Duration, as takes up the Time of only one Idea in our Minds, wherein we Perceive no Succeſſion, is Called an Inſtant.

WHEN we have thus got the Idea of Duration from a Succeſſion of our Ideas, it is ſaid, we can Apply it to Things, which Exiſt, whilſt we do not Think, and thus we Meaſure the Time of our Sleep.

The Succeſſion of Ideas no Standard or Meaſure of Duration, for Three Reaſons.

§. 12. THE Firſt Thing therefore, which is to be Conſidered is, whether the Succeſſion of our Ideas is that, from which we Derive our Idea of Duration, which it is not.

1ſt. BECAUSE the Succeſſion of our Ideas does Neceſſarily Suppoſe an Interruption of them, as there is in Shadows upon a Wall Succeeding each other; there muſt be ſome Kind of Diſtance and Interval betwixt them, or elſe there would not be a Succeſſion; Beſides which, it is Plain in Fact and Experience, that our Ideas are as Diſtinct and Separate from each other, as the Words are, which I now Write, and by which I Expreſs the Ideas, which I have; But if this is ſo, how is it Poſſible, that we ſhould Form an Idea of Duration from a Succeſſion of Ideas, which Includes, as to our Conſciouſneſs of it, a Non-Duration in it, by the Chaſms and Intervals made in Duration, from the Chaſms and Intervals made in the Succeſſion of our Ideas, from their being Diſtinct, Separated, and Divided, from each other?

2d. BECAUSE the Succeſſion of our Ideas is Various in Reſpect of the Velocity and Quickneſs, or the Slowneſs of them, with which they Paſs thro' the Mind, and therefore upon that Account muſt Imprint very Different and Uncertain Ideas of the Duration, which they give us.

FROM whence, it is not Impoſſible, that other Creatures may have been ſo Formed by their Creator, that what Appears to us an Hour or a Day only, may to them have the Length and Period of an Year, or an Age, and a Minute the Duration of an Hour or a Day; And the Creature, which *Tully* makes Mention of, upon the River *Hypanis*, Whoſe Term of Life was only for a Day, might ſeem to it's ſelf to Live as long, as we do; And on the Contrary, what Appears to us to be an Hour or a Day, to other Creatures more Perfect than we are, and the Pace of whoſe Ideas is Swifter and Quicker than ours, may ſeem to be no more than a Minute; For thus, when our Thoughts Move Duller and Heavier, and with a leſs Vivacity, Time ſeems to be Proportionably Lengthened; and to become Shorter, as they are Carried on with more Fire, Spirit, and Activity.

3*d*. Because, the Succeſſion of our Ideas Receives an Alteration, in Reſpect of Duration, from the Pleaſure or Pain, with which they are Attended; Thus if our Ideas are Accompanied with Pain, the Time ſeems Long and Tedious to us, but if with Pleaſure, Short and Tranſient; Whereas our Idea of Duration is that of one Uniform Flux of Time, and therefore cannot be Derived from the Succeſſion of our Ideas, which is Broken and Interrupted in every Interval of one Idea Succeeding another, and is Alterable and Various in the Quickneſs and Slowneſs of their Succeſſion, and by the Impreſſions of Pleaſure or Pain; the one or other of which does Generally go along with them.

The Arguments for the Succeſſion of our Ideas being the Meaſure of Duration, and how Anſwered; the Firſt Argument Inconcluſive.

§. 13. As to the Arguments made Uſe of to Prove, that we have our Idea of Duration from this Succeſſion of Ideas;

The Firſt is, becauſe we have no Perception of Duration; when that Succeſſion of our Ideas Ceaſes, as in Sleep; Which does not Evince, that we Obtain our Idea of Duration from the Succeſſion of our Ideas, but that we cannot Obtain it from our having no Ideas at all, of which we are Conſcious; But no One ever Affirmed, that we could know what Duration was, without a Conſciouſnes of our own Exiſtence; It is not Pretended, that we can Form an Idea of Duration from our having no Ideas at all, of which we are Conſcious, or which at leaſt we can Remember, as in Sleep; It yet does not from thence Follow, that ſuch an Idea muſt be owing to the Succeſſion of our Ideas, when we are Awake, and not to our having Ideas in General, of which we are Conſcious, without Conſidering the Succeſſion of them.

The Second Argument Explained, and Refuted.

§. 14. The Second Argument, that becauſe whilſt we are Awake, and our Thoughts are Intent upon one Thing, and there is no Succeſſion of our Ideas, we Think Time or Duration Shorter than it is, is as little to the Purpoſe;

For Firſt, if we have not our Idea of Duration from the Mind's Intent Contemplation of one Idea only, it is not yet therefore Conſequent, that we muſt Derive it from the Succeſſion of our Ideas, and not from ſeveral Ideas in General, Compar'd together, without Attending to the Exact and Minute Succeſſion of them.

Second, if Time or Duration Appearing Shorter than it is, is a Reaſon, why we cannot have our Idea of it from that, which makes it Appear ſo, the ſame will Equally hold againſt the Succeſſion of Ideas; ſince thoſe ſmall Diſtances, which lie betwixt our Ideas in their Succeeding each other, will make us to Slip out of our Account one Part of Duration, by which it will Appear Shorter than it is, as thoſe Intervals are Greater, or Longer, as they are Leſs.

Third, again likewiſe, when the Succeſſion of our Ideas is Accompanied with Pleaſure, or when they Move Swifter than Ordinary, Duration will Appear Shorter, and when they are Attended with Pain, or the Motion of them is more Languid and Slow, it will Appear Longer; From all which it is Evident, we cannot have an Idea of Duration from the Succeſſion of our Ideas, no more than we can from the Intent Contemplation of one Idea only; Since in all theſe Circumſtances Time or Duration will be Altered from what it Really is.

Fourth, beſides which, this Inſtance of the Mind's Intent Contemplation of one Thing, to Prove that our Idea of Duration is Derived from Succeſſion, is wrongly Choſen; for either by one Thing is Meant one Idea, or elſe one Subject, Argument, Propoſition, or Demonſtration, or the like;

If the Firſt is Intended, as it muſt be, and in which Senſe we have taken it, becauſe otherwiſe there would not be that Oppoſition betwixt one Single Idea, and the Succeſſion of ſeveral, which the Preſent Argument ſeems to Require, it will be ſo far from being True, that Duration Appears Shorter, whilſt the Mind is Employ'd about one Idea only, that it Manifeſtly is Longer; Thus, if a Perſon was Confined to Hold a Feather upon his Finger for an Hour together, and not to take his Eye off it, but to View it all that Time, tho' there would be no Pain in the Object, either to his Sight or Feeling, it is not yet to be Doubted, but it would be a Long and Tedious Hour, where he had no Variety of Thought, or Action, to Relieve him; and the like may be ſaid of any other Idea, and which, it is not Improbable, may be one Reaſon, why Pain becomes ſo Irkſome to us, not only becauſe of the Violent Action upon the Mind, but alſo, becauſe the Mind is thereby Forced Continually to Attend to one and the ſame Idea, which Cauſes it's Affliction; for if we have Courage and Reſolution enough to Draw our Minds off to other Ideas, the Pain is

Rendered more Tolerable, and the Time of it's Continuance ſeems to be Diminiſhed.

If, by the Mind's being Intent upon one Thing, is Meant our Intenſe Application to Conſider any Subject, Argument, Propoſition, or Demonſtration, which Lies before us, It is an Inſtance Foreign to the Intention, for which it is Produced; ſince in the Study and Contemplation of any Subject, there is an Infinite Succeſſion of Ideas; and thoſe Hours, in which the Mind is Engaged in a Deep Attention, are Crouded with a Greater Variety and Multiplicity of Ideas, than thoſe which we Commonly Paſs in the Other, and more Idle Part of our Lives, and in which our Ideas may be Thought to keep their Uſual Pace in their Succeſſion to each other; Nay it may be farther with Truth Affirmed, that the moſt Tedious Part of our Time, and when Duration Appears the Longeſt, is, when the Mind is wholly Unemployed, and Conſequently, when there is the Leaſt Succeſſion of our Ideas in it.

'Tis yet True, that at ſuch Times of Study and Intenſe Thinking, Duration ſeems to be Shorter, tho' the Number of Ideas, which Enter the Mind, is Greater, than when we do not on Purpoſe Set our Minds to Think and Contemplate; The Mind in Study is in the moſt Active State, which it is Poſſible to be in, and Turns and Revolves it's Ideas about, with more Life and Quickneſs, than at any other Time.

And the Reaſon of Duration's Appearing Shorter is, not from our Reſting upon one Idea, tho' that is ſometimes done too, when we do not at Firſt fully See and Comprehend it, or the Relation, which it has to another, with which it is Compared, but from the Pleaſure, which we Receive from finding out Truth, and from the Satisfaction we Experience in the Managing, Ranging, and Conducting our Ideas in that Manner, as to Produce a Conviction in us, that we have Attained it; And for this Cauſe, it is, that to thoſe, who cannot thus Command their Ideas, ſo as to find any Succeſs in their Inquiries, Contemplation and Study becomes Tedious and Irkſome, and the Time ſeems Long, which they Employ in it: Another Thing, which makes Duration ſeem Shorter in Study and Contemplation is, that the Mind is Entertained with an Infinite Variety of Ideas and Speculations, which are New to it, and which are like ſo many Freſh Pictures and Landſkips of Things, which Raiſe a Perpetually Succeſſive Scene of Delight to the Studious and Inquiſitive.

That we cannot have an Idea or Meaſure of Duration from a Succeſſion of our Ideas from hence farther Evident, becauſe the very ſame Number of Ideas, which are Repeated by one, and Heard by another, and whoſe Succeſſion muſt therefore be the ſame, will Produce Different Ideas of Duration in each.

But that we cannot have an Idea of Duration from a Succeſſion of Ideas, will be from Hence farther Manifeſt; That the very ſame Number of Ideas, which are Repeated by One, and Heard by Another, and whoſe Succeſſion muſt therefore be the ſame, ſhall Produce Different Ideas of Duration in Each.

'Tis no Uncommon Thing for a Perſon, who Reads, or who Speaks, any Certain Number of Ideas, to Think the Time much Shorter, in which he Delivers thoſe Ideas, than he Imagines it to be, who Hears them, and this Perhaps may not be only Generally, but Always True; Not only in Reſpect of Compoſitions Made by thoſe, who Read them, and of Diſcourſes Held in Converſation by thoſe, who Maintain them, but likewiſe, in Reſpect of the Compoſitions or Diſcourſes of Others, when Read or Heard; The Time or Duration, I believe I may Say, is always, in theſe ſeveral Caſes, Shorter to the Perſon, who Reads, or Speaks, the ſame Number of Ideas, than to him, who Hears them.

If therefore it is Matter of Fact, in all or any of the aforementioned Inſtances, that the Reader or Speaker of the ſame Number of Ideas Thinks the Time Shorter, and he, who is the Hearer of them, Longer, it is Sufficient to Invalidate the Preſent Doctrine of Duration from the Succeſſion of Ideas; And, that the Hearer is at leaſt ſometimes the Patient, and does Think the Time Longer, than he, who Reads or Speaks, is Matter of Fact, and beyond Contradiction.

That it is ſo in General, I have ſome Reaſons to Convince me, from a Conſideration of what is too much the Frailty of Humane Nature, to be very well Pleaſed with what each of us does in Particular, and to be too Fond of it, even in the Smalleſt and Minuteſt Matters; By which Means, the Duration, or Time, may Probably Appear Shorter from the Pleaſure and Satisfaction, which we Conceive from our own Performances, when the Hearer of the ſame Number of Ideas Imagines it Longer, becauſe it Looks like a Sort of Inſult upon his Mind, and a Kind of Dictating to his Underſtanding; he, at leaſt, many Times Receives not the ſame Pleaſure, for the very Reaſon, that there is nothing of his own in it; Self-Love being a Ruling Principle in our Natures, and that, which Recommends, or Diſparages, Actions in our Eſteem, according as they are Approv'd, or Rejected, by it.

ANOTHER Argument, why the Hearer may Think the Time Longer, and the Speaker, or Reader, of the ſame Ideas, Shorter, is, that the Mind is in the Firſt Caſe Paſſive in ſome Degree, and in the Laſt Active, by which it Approaches to Affliction in the Former, and in the Latter to Pleaſure; from whence likewiſe, the one State of the Mind becomes more Tedious than the other.

A Third Argument is, that the Hearer is in a Certain Kind of Pain many Times, either for want of Underſtanding the Ideas, which are Delivered to him from the Speaker or Reader, or becauſe they are not Agreeable to his Apprehenſions, or elſe becauſe they are too Quick, or two Slow, for the Pace, which his own keeps in their Succeſſion; which either Overwhelms and Oppreſſes the Mind of the Hearer, or Creates an Uneaſineſs, Ariſing from an Heavineſs and Dulneſs in the Speaker or Reader; All which Pains in the Hearer, in the Speaker or Reader, often Times are Turn'd to ſo many Pleaſures; he Underſtands Frequently his Ideas, they are Agreeable to his Apprehenſions of Things, and are Delivered with a Velocity and Impetus, or a Sedateneſs and Gravity, Natural to his own Mind: And therefore, we may in General Affirm, that Duration ſeems to be Shorter, or Longer, according to the Pleaſure, or Pain, which Attends our Ideas, and not according to the Succeſſion, or Non-Succeſſion of them.

'TIS True, ſometimes the Duration may Appear the ſame, both to the Hearer and to the Speaker, but never unleſs, where their Pleaſures and Satisfactions are Equal, that is, where the Hearer is as much Delighted with the Subject, as the Speaker, where he Alike Comprehends the Ideas, and the Succeſſion of them is Alike in both; From all which it is Manifeſt, that the Succeſſion of Ideas is no Meaſure of Duration, nor is it that, from which we Derive the Idea of it.

§. 15. THE Third Argument, for our Gaining our Idea of Duration from the Succeſſion of Ideas, is, becauſe Motion, which is Commonly Suppoſed to be a Meaſure of Duration, is no farther ſuch, than as it Produces in our Minds an Idea of Succeſſion; for the Proof of which, two Inſtances are made Uſe of, and which we ſhall Conſider.

The Third Argument for the Succeſſion of Ideas being the Meaſure of Duration not Valid. for if Motion is not the Meaſure of Duration, it is in Vain to Alledge it in the Preſent Caſe, if it is, it will be ſo, Independent of the Succeſſion of our Ideas.

BUT, Firſt let us Grant the two Inſtances Alledged, yet, if Motion is only a Suppoſed Meaſure of Duration, and is not Really One, it is of no Importance to Explain, how it becomes a Meaſure of that, of which it is no Meaſure; If it is a Mea- of Duration, then it is not a Meaſure of Duration, as it Produces in our Minds an Idea of Succeſſion, but as it is Motion, which will Continue Invariable and the ſame, even when the Succeſſion of our Ideas Alters, and even, when there is no Succeſſion of them at all.

A Pendulum will Perform it's Vibrations Regularly, whilſt the Ideas of the Mind may Succeed One Another in a Tumult and Hurry, or whilſt they are, in a Manner, Quieſcent, without any Senſible Succeſſion; The Pace of our Ideas does not keep the ſame Conſtant Tenour with the Returns of a Body, which Oſcillates, the One Moving Faſter or Slower, whilſt the Other Swings with an Exact Uniformity.

NAY Farther, Suppoſing any One were Set on Purpoſe to Obſerve the Progreſs and Regreſs of a Pendulum for an Hour together, without Suffering his Ideas to Wander to any Thing elſe; In this Caſe, if in any, the Succeſſion of his Ideas would Anſwer and Correſpond to the Succeſſion of Motion, and to that of every Inſtant in Duration; But yet the Mind is not Able to do this; It can Follow the Progreſs and Regreſs in the ſeveral Oſcillations, and Finds a Continual Motion in the Pendulous Body; But it's Ideas cannot Succeed ſo Faſt, as the Body Moves, which Alters it's Situation in every Single Point of it's Itus and Reditus, and which Points are Infinite, and all is, in ſome Pendulums, Performed in a Second of Time, or the *60th* Part of a Minute; And therefore the Mind can no more make it's Ideas Succeed in that Velocity, than it can Count an Infinite Number of Points or Units in that Time.

FARTHER, Let us Take our Conceptions of the Moving of a Pendulum in the Groſſeſt Way, Namely, that we See a Continual Motion, and the Diſtinct Progreſſes and Returns of it; If any One were, for an Hour together, Set to Obſerve and Count them, and the Same Perſon ſhould Spend ſo many Vibrations in a Succeſſion of Ideas in Study, or in Reading, he would Apprehend the Duration to be very Different; The Firſt would be Long and Tedious, and the Laſt Short and Tranſitory, at leaſt, if the One was Painfull and Irkſome, and the Other Agreeable and Pleaſant; Which ſhews, that there is no Connection at all betwixt Motion, and the Succeſſion of our Ideas, or betwixt the Succeſſion of our Ideas, and Duration; Beſides, as is before Said, the Succeſſion of Ideas is Different in Different Perſons, as to their Slowneſs, or Velocity, and therefore cannot be a Standing and Determinate Meaſure of Duration; In Some they are Carried on with a Certain Kind of Rapidity, in Others they Proceed in a more Solemn Pace, and in a Third, they Move ſo Slowly, that we Eaſily Infer and Conclude from thence the moſt Groſs Stupidity.

The Inſtances of Motion in this Philoſophy to Prove, that our Idea of Duration is Derived from a Succeſſion of our Ideas. Inſtance the Firſt with it's Confutation.

§. 16. As to the Inſtances Abovementioned, by which this Doctrine of the Succeſſion of Ideas is Endeavour'd to be Prov'd from Motion being no otherways a Meaſure of Duration, than as it Impreſſes upon us ſuch a Succeſſion of Ideas;

THE Firſt is, that Motions, which are very Slow, are not Perceiv'd by us, becauſe the Change of Diſtance is ſo Slow, that it Cauſes no New Ideas in us, but after a Long Interval, as in the Hand of a Clock, or in the Shadow of a Dial; Which is as much as to Say, *1ſt.* That Motion is not a Meaſure of Duration, unleſs ſo far, as ſuch Motion is Senſible to us; And *2d.* That no Motion is Senſible to us, unleſs, that which we can Trace by our Ideas in every Point of it's Period; And *3d.* that there is no Duration, where there is no Senſible Motion; All which are not only Paradoxes, but are Propoſitions, which are in themſelves, both Falſe and Abſurd; For *1ſt.* The Motion of Pendulums, and the Index of a Clock, which is Carried Round by it, will Deſcribe a Minute, or an Hour, or a Day, whether we Look upon it's Motion, or not; And *2d.* As for our Tracing it's Motion in every Point of it's Period, even tho' the Motion is Senſible to us, it is Directly Impoſſible to do it; And to Affirm, *3d.* That there is no Duration, where there is no Senſible Motion, is to Say, that there would not be Duration in the World, that is, the World would not Continue to Exiſt, if all the Parts of it were Quieſcent, which is Abſurd.

The Second Inſtance Confuted.

§. 17. THE Second Inſtance is, that for the ſame Reaſon, when any Thing Moves Round in a Circle in leſs Time than our Ideas Succeed One Another in, it's Motion is not Perceived by us, but it ſeems to be One Continued Circle; But if this were the Reaſon, why we could not Diſcern it's Motion, Namely, becauſe it was too Swift for our Notice, as Probably that may be the Reaſon, yet this would Signify nothing to our Notion of Duration; Becauſe, when we See a Body Quieſcent, and to Continue in that State of Quieſcence, as in the Circle Mentioned, we have as much an Apprehenſion of what Duration is, as we have from any Body Continuing in Motion.

THESE Inſtances have therefore no Relation to Duration, or to the Meaſure of it, and are only Accounts, why a Body, which Moves, Appears to be Quieſcent to us; whereas Duration Equally belongs to Bodies, which are at Reſt, or in Motion, and what the Meaſure of this Duration is, is what is Required, which cannot be the Succeſſion of our Ideas for the Reaſons before Alledged.

What the Truth is in this Matter, and how we Gain the Idea of Duration. Firſt, the Succeſſion of our Ideas may give us a Notion of our own Duration, and ſo the Succeſſion of the Ideas of other Intelligent Beings may Furniſh

§. 18. WE are therefore now, after Inquiring into the Preſent Doctrine, and the Arguments for it, to Propoſe, what ſeems to be the Truth in this Matter.

AND Firſt, ſince the Various Quickneſs and Slowneſs of our Ideas, with which they Paſs thro' the Mind, ought to be Conſidered in the Succeſſion of them, and ſince the Pain and Pleaſure, with which they are Attended, makes Duration ſeem to be Longer, or Shorter to us, it is Evident, that the Succeſſion of our Ideas, tho' it may be a Meaſure of Duration to ourſelves, yet cannot be One of a Real, or rather, a Rational, or Abſtracted Duration, which is the ſame in all Beings, and does not Differ, or Vary, tho' our Apprehenſions of it may.

OTHER Creatures may Differ from us in their Ideas of Duration, as Thoſe upon the River *Hypanis* Mentioned by *Tully*, and which we Referred to before, whoſe Period of Exiſting is ſaid to be only that of a Day, which yet might be, if any ſuch Creatures ever were, either from the Slow Pace of their Ideas, or from the Dull and Languid Pleaſure, which Attended them, as much to Them, as many Years are to Us; And it is not Certain in other Animals, and even Inſects, tho' their Term of Life is in Reality Shorter than ours, but it may to them Appear as Large and Extenſive, as ours does to ourſelves; And if we go into the Higher Order of Beings, which are above us, we may Juſtly Think, that a Thouſand Years, to Them, is but as One Day, upon the Account of the Extreme Vivacity of their Faculties, and the Pleaſure they Continually Enjoy, ſince we are Aſſured by the Scriptures, it is ſo with the Almighty.

them with an Idea of theirs, but neither of them can give us an Idea of Real, or rather a Rational and Abſtracted Duration, Conſidered by Philoſophers.

If we have Recourſe to Religion in this Caſe, it will Tell us, that it is very Agreeable to the Goodneſs of God to Adapt the Lives of his Creatures, and the ſeeming Duration of them, to the Nature of their Ideas, and the Quickneſs and Slowneſs of their Succeſſion, which he has Implanted in them; That they may Appear to themſelves Equally Happy in their Preſent Duration in this Life; And as to Rational Beings, whatever Afflictions or Difficulties they Meet with in it, in Reſpect of Others of the ſame Denomination with them, there is no Queſtion or Doubt to be made, but they will be Abundantly Compenſated by the Beneficence of their Creator in a Future One.

If we have Recourſe to Philoſophy, what can be more Reaſonable to Think, than that, as Men Differ amongſt themſelves in the Paces, which their Ideas Take in their Minds, and in the Pleaſure or Pain, with which they are Accompanied, ſo, that the ſeveral Species of Animals do ſo too; and, that the Inferior Species of them, as an Oyſter, or a Muſcle, or a Fiſh, have their Term of Life, which is many Times Short, Proportioned to the Slowneſs and Stupidity of their Ideas, and the Dull Pleaſures Annexed to them, and which yet to them may ſeem as Long, as that to other Creatures, which Live to many Years beyond them?

On the Contrary, we find that Birds, as Ravens, *&c* are Eſteemed to Live to a Great Age, 'tis alſo at the ſame Time Evident, that their Faculties and Powers of Perception, and their Ideas, are Quicker and Stronger, than thoſe of an Oyſter, or of any Kind of Fiſh; ſo that, as their Ideas have a Quicker and more Lively Turn and Succeſſion, and are Attended with a Variety of Pleaſure, accordingly we may Think the Duration of their Life does not Appear Greater to them, than that of an Oyſter or a Muſcle to it.

Let us now Conſider Rational Beings, in Reſpect of their Exiſtence here, Ideots and Men, who have but a Stupid Senſe of Things, or whoſe Ideas are leſs Pleaſing to them, are Generally more ſhort-Lived than Others are, who Abound in the Quickneſs and Livelineſs of their Parts and Intellects, and in the Velocity or the Strength of their Ideas, or whoſe Ideas are Agreeable and Delightful to them; and yet it ſeems Rational to Believe, that the Duration of each of their Lives does not Appear to them Extremely Different, for the ſame Reaſon, that an Hour Spent with a Quick and Lively Succeſſion of our Ideas, and with Delight and Satisfaction, does not Appear half ſo Long, as one Conſumed in a Slow and Languid Succeſſion of them, and in Dullneſs, Stupidity, and Unenjoyment; However This is, This yet is Certain, that the Succeſſion of Ideas cannot be a Meaſure of Real Duration, or a Rational and Abſtracted one, which is one Conſtant and Unvaried Succeſſion of Time; whereas the Succeſſion of Ideas is Different in Different Perſons, both in their Natural Velocity, with which they Paſs thro' the Mind, and in the Pleaſure or Pain, which belongs to them.

How we Conclude, that there is a Real, Rational, or Abſtracted Duration, which is Conſtant and Unvaried.

§. 19. Second, there is an Interruption in the Train of our Ideas, as in Shadows upon a Wall, whereas the Idea of Duration is that of a Continued and Uninterrupted Succeſſion of Time; And Conſequently the moſt Rational Account, which can be given of our Idea of Duration, or of a Continued Succeſſion of Time is, that, Obſerving we Exiſt in ſeveral Parts or Portions of it, and likewiſe Concluding, that we have always Exiſted, whether we have been Conſcious to the Succeſſion of our Ideas or not, ſince we cannot Imagine ourſelves to Non Exiſt, and then again Revive into Being, we from thence Inferr, that there muſt have been a Continued Duration, whether we have been Senſible of every Part or Minute of it, or not, and which Goes on and Proceeds with Conſtancy and Equability, and is not Beholden to our Ideas, or the Succeſſion of them, for it's Reality; or elſe Obſerving, that other Beings, as a Tree, or a Stone, or the whole Univerſe, ſtill Continue to Exiſt, whether we Particularly Contemplate the One, or the Other, or not, we likewiſe again are Aſſured from thence, that there muſt have been a Continual Duration of them; becauſe we cannot Conceive, that they ſhould Exiſt at one Time, then Ceaſe to be, and afterwards Return into their Former Exiſtence.

From both theſe Ways we Juſtly Conclude, that there is an Uninterrupted and Equable Succeſſion of Duration, in which our Ideas, or the Succeſſion of them, have no more Concern, than any other Exiſtence whatſoever; 'Tis an Idea not Formed by any ſuch Succeſſion, but from the Standing Reaſon and Relation of Things, and from the Evident Convictions of our own Minds.

From which Perſuaſion, it was Reaſonable for Philoſophers to Inquire what was an Adequate Meaſure of ſuch an Equable and Uniform Succeſſion in Duration.
Dr. Keill's Introduct. ad Aſtronom. Lect. 26. De Temporis Æquatione. Hugen's Horolog. Oſcillator.

§. 20. And, when we are fully Satisfy'd of ſuch a Conſtant and Continued Duration, which all Philoſophers have been hitherto Convinced of, it was very Proper for them to Enquire, as they have done, what was the moſt Equable, and the Exacteſt Meaſure of it; Aſtronomers have Sought after it in the Motions of the Heavenly Bodies, and the Mechanical Philoſophers in thoſe of Pendulums, and Both without the Deſired Succeſs, as is Evident from the Theory of a Fictitious Sydus, Propoſed by the One, for the Æquation of Time, and from the Anomalies, and Irregularities, Daily Obſerved in Watches, in Clocks, and Common Pendulums, by the Other.

And altho' theſe are Noble and Laudable Attempts, and altho' they Approach towards an Equation of Time, yet we muſt, I am Afraid, Deſpair of a Perfect One, for this Reaſon; Becauſe, if we Conſider the Motions of Heavenly Bodies, or of Pendulums, we Conſider them, as Conſiſting of the ſame Kind of Matter, and Moving in much the ſame Medium, that is, we Conſider them in an Abſtracted Way, and Agreeably to our own Abſtracted Reaſonings upon them, which is, what the Different Natures of Things will not Bear or Allow of; And upon this Account it is, that a Perfect Equation of Time, by any Motions whatſoever, is Perhaps a Problem, as Incapable of being Explained by all the Philoſophick, or Mathematick Genius's in the World, that either now are, or have been, as a Perpetual Motion, or Sailing to the Moon, which will Scarcely ever Meet with a Solution; Becauſe ſuch Notions Appear Contrary to the Standing Laws of Nature, and of Being.

The Aſſertion in this Philoſophy, that it is not Neceſſary, that Time ſhould be Meaſured by Motion Conſidered, Motion the beſt Meaſure of it.

§. 21. However this is, Let us now Inquire into that Opinion of the Preſent Philoſophy, which Maintains, That it is not *Neceſſary*, that Time ſhould be Meaſured by Motion, That any Conſtant Periodical Appearance in ſeemingly Equi-Diſtant Spaces, may as well Diſtinguiſh the Intervals of Time, as what we make Uſe of; For Suppoſing the Sun to be Lighted up, and then Extinguiſhed every Day, and that in the Space of an Annual Revolution, it ſhould Senſibly Increaſe in Brightneſs, and ſo Decreaſe again, ſuch a Regular Appearance would ſerve to Meaſure out the Diſtances of Duration to all that could Obſerve it, as well without, as with Motion; The Freezing of Water, or the Blowing of a Plant, Returning at Equi-Diſtant Periods in all the Parts of the Earth, would ſerve for the ſame Purpoſe; 'Tis farther ſaid, that, in Effect, we find a People of *America* Counted their Years by the Coming and Going away of Birds at Certain Seaſons.

But whatever this Philoſophy may Think *Neceſſary*, it could be very much Wiſhed, that there was ſuch a Motion found out, that would give us an Exact Equation of Time, that is, which would Keep an Uniform and Equable Pace with the Flux of each Minute and Inſtant of it; but, as we are not yet Informed of any ſuch Motion, and as in all Likelihood, None ſuch there is, or can be found out in Nature, 'tis a Laudable Undertaking to Come as Near ſuch a One, as we Can.

On the other Hand to Affirm, that any Conſtant Periodical Appearance, in ſeemingly Equi-Diſtant Spaces, would ſerve the Purpoſe of Meaſuring Time as well, is Suitable to the Reſt of this Philoſophy; For, how ſhould we know, what was a ſeemingly Equi-Diſtant Period, if there was no Uniform Motion, nor no Meaſure of Time by it? Hours, and Minutes, and Days, would ſeem Longer or Shorter, according to the Various Sentiments of Pleaſure and Pain, we Perceive in them, and according to the Various Occupations, and Employs, in which Men are Ingaged; And if the Sun was to be Lighted up and Extinguiſhed, at ſeemingly Equi-Diſtant Periods to us, it would find it's Illuminations, and Extinctions, to be very Irregular; nor would there be Perhaps Three Men in the World, who would Agree to have it Lighted up and Extinguiſhed at the ſame Time; This Illumination and Extinction would Run thro' all the Hours of Day and Night, and thro' all the Seaſons of the Year, and like the

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Mahometan Moons, for which ſome of the Diſciples of this Philoſophy ſeem to have no Small Regard, Make the Hotteſt Day in *July* Fall in the Midſt of Winter; This Contrivance for the Meaſure of Duration would do much the ſame, and be an Equal Proof of it's Ignorance in Aſtronomy, and of it's Contempt of the Sacred Writings; Which Aſſure us, that Almighty God Created the Sun and Moon for Lights, to Rule the Day and the Night, and for the Diſtinctions of Seaſons, of Days, and Months, and Years; And by the Conſideration of their Motions, and Endeavouring to Aſcertain an Equation of Time from them, Learned and Inquiſitive Men have been Satisfy'd, that it is a much better Way, than if it had Pleaſed Almighty God to have Lighted up and Extinguiſhed them at Certain Times, in Compliance with the Humour and Fancy of this Extravagant and Vain Opinion.

But Apart from Religion, what can be more Plain, than that we have a Notion of an Equable Succeſſion of Time or Duration? The Queſtion is, how we ſhall Meaſure that Succeſſion? That we cannot do it by that of our Ideas, I have Endeavoured to Shew, nor yet by ſeemingly Equi-Diſtant Periods to us, which comes to the ſame Thing; ſince thoſe ſeemingly Equi-Diſtant Periods to us muſt be ſeemingly ſo by our Ideas, which Succeed one Another, and which Succeſſion of Ideas is Varied, as to the ſeeming Duration of Time, by the Pleaſure and Pain, and other Circumſtances, which Attend them.

On the other Hand, not to Mention the Motions of the Heavenly Bodies, which are little, or not at all, Underſtood by this Philoſophy of Ideas, we ſhall Point it out to a Common Inſtance of Watches, which Gentlemen are Particularly Acquainted with; Suppoſing a Perſon was in the utmoſt Gaiety of Company, and Injoy'd all the Pleaſures and Satisfactions of it, which he could Wiſh for, or Deſire, or was Intent upon Studies, which were the moſt Agreeable to him, Let him look upon his Watch for the Hour, and he will find it is gone beyond the Period he Expected.

But, If the ſame Perſon is under a Fit of the Stone, or the Gout, or under any other Pain, or Indiſpoſition, which makes every Minute, and every Hour, ſeem Tedious to him, the Hand of the Watch will Move Slower, than he Thought for, and he will Fix the Period of his Hour on this Side the True One; And the ſame would Happen, If it was in his Power to Light up and Extinguiſh the Sun at ſeemingly Equi-Diſtant Periods; Or, Changing the Senſe of Seeing for that of Hearing, the Striking of a Clock at ſeemingly Equi-Diſtant Periods would Serve the Turn as well; and yet we find by Experience, that tho' the Hour is Pretty Exactly Meaſured by the Motions of a Pendulum, the Sounds, which are Repeated at much the like Intervals of Time, come Faſter or Slower upon us, as we Imagine the Duration of it to be Shorter or Longer.

Motion the beſt Meaſure of Duration, which we can Attain to.

§. 22. On the Contrary, how do we Correct this Error of our Imaginations, but by the Regular Motions of the Celeſtial Bodies, or of Pendulums? I Mean ſo far Regular, as they are Capable of being ſo.

That is, we find Pendulums, Allowing for ſome Small Irregularities Ariſing from Foreign Cauſes, to Perform their Oſcillations in much the ſame Time, as alſo the Heavenly Bodies, their Revolutions; and we have this Reaſon to Conclude they do, becauſe their Nature, and the Motion, which Reſults from it, Continues Unvaried; and what Effect Proceeds from them one Minute, it is Juſt to Think, will Nearly Obtain the Next.

It is therefore Rational enough to make ſuch Motions a Meaſure of a Conſtant and Uniform Succeſſion of Duration, becauſe they come the Neareſt to it; but the Succeſſion of our Ideas is Deſultory and Various, full of Irregularity and Caprice, and Conſequently a very Unfit and Improper Standard for a Succeſſion, which goes on without any Interruption, Change, or Alteration.

From all which it Follows, that theſe Ideas of Duration, and the Meaſure of it, are not ſo Truly Ideas of Senſation or Reflection, as they are Ideas, which are Formed by our Reaſon, and Made and Collected by the Mind's Abſtracting, Comparing Things together, and Arguing upon them, and much more ſo is That of Eternity.

Concerning Duration and Expanſion Conſidered together.

§. 23. As to Duration and Expanſion Conſidered together, the Notion and Apprehenſion, which this Philoſophy of Ideas has of the Firſt, has been already Examined, and the Laſt, to Wit, Space or Expanſion, we have Shewn in our Former In-

Duration be fore Examined into, and Space or Expanſion is an Abſtracted Idea, or a Creature of the Mind.

quiry into the Principles of Natural Philoſophy, is Nothing Elſe than an Abſtracted Idea, and a Creature of the Mind, and that there is no Real Space, or Length, Breadth, and Thickneſs, Exiſting by it's ſelf in Nature.

But, ſince that Opinion has given a Diſtaſte to ſome, tho', for the Reaſons there Alledged, I Think it True, I ſhall Explain my Self here a little Farther upon that Head.

And Firſt, Thoſe, whom I Oppoſe, are Such Philoſophers, who make Space Infinite and Eternal, and who give, in a Manner, all the Attributes to it, which belong to God himſelf, I Affirm this is Nothing Elſe than a Creature of our Imaginations, and that there is no ſuch Real and Abſtracted Space in Being.

On the Contrary we Affirm a Plenum in Nature, which is here farther Explained.

On the Contrary, it has been Told me, that I make Matter Eternal and Infinite, ſince it is ſaid, that I Suppoſe all Space to be Filled with it; If it were ſo, I hope there is no Greater an Abſurdity in Aſſerting, that the whole Univerſe was from all Eternity Filled and Repleniſhed with Something Derived from God, than in Aſſerting, that it was from all Eternity, till about Five or Six Thouſand Years ago, Filled and Repleniſhed with nothing but Space, Underived from Him; as if the Deity had lain Idle and Unemploy'd, till that Time, and, as ſome Maintain Concerning our Saviour, his God-Head had been, for a Certain Term, Quieſcent.

It is more Reaſonable to Believe and Think, that the Almighty Author and Creator of the Univerſe did from all Eternity Exert his Power in the Production of ſomething, that he for Ever Communicated and Diſpenſed his Influences thro' the Immenſe and Boundleſs Space, and at Length, in his own Good Time, Formed and Created the World, we See.

And this is the Eternal Plenum, in Oppoſition to an Eternal Vacuum, which I Maintain; and in this, I do not Affirm an Eternity of Matter, according to the Vulgar Conceptions of it, Namely, of it's being a Paſſive, Solid, and Extended Subſtance; But thus far I Affirm an Eternity of Matter, as I Affirm an Eternity of an Almighty Action of the Deity, Spreading and Diffuſing it's ſelf thro' all Nature; And in which Action and Exertion of the Infinite Mind, I take the very Eſſence and Being of Matter to Conſiſt, as it was Formed and Created by him, and without Whom it cannot Subſiſt one Moment.

We likewiſe Affirm, Theſe Exertions and Actions of the Alwiſe Creator are Infinitely Varied and Diverſify'd by him, according to his own High Will and Pleaſure; from whence Ariſes that Heterogeneouſneſs of Matter, which is Obſerved in the World; ſo that the Real and Intrinſick Nature and Properties of Æther are as Different from thoſe of Air, as Air from Water, or Water from Earth, or Earth from Minerals or Vegetables, or thoſe from Metals, or Gold, Agreeably to what we have Propoſed Concerning the Expanſive and Contractive Forces.

See Chap. 1. Book 1. of this Philoſophy.

As for a Plenum in General, in Contradiction to a Vacuum, I have not only moſt of the Ancient Philoſophers on my Side, as *Thales*, the Wiſeſt and Earlieſt of them, and the Reſt, Excepting *Democritus* and *Epicurus*, but alſo even *Des-Cartes* himſelf, tho' it is Inconſiſtent with his own Principles of a Similar Matter; And it is very Evident, that the Celebrated Sir *Iſaac Newton* has, ſince the Printing of that Inquiry, of which I did my ſelf the Honour to make him a Preſent upon the Firſt Publication of it, it is Plain, that he has alſo come into the Notions of an Univerſal Plenum or Æther, in what he has Subjoined to the Second Edition of his Opticks; notwithſtanding in his Principles he had Endeavoured by Reaſonings and Experiments to Demonſtrate, there was a Vacuum, which in a Second Edition of them, he has Changed to this Affirmation, That All Things are not Equally Full.

For the Caſe is this with all the Corpuſcularians, that their Principles, which are Contrivances of their own Minds, in Shaping Matter into what Figures they Pleaſe, do Certainly Lead them to Aſſert a Vacuum; becauſe there muſt be Pores and Interſtices betwixt thoſe Figures; But, as Nature, and the Common Reaſon of Mankind, does Direct us to a Plenum, *Des-Cartes* was Obliged to Form a Romantick One, tho' in Oppoſition to his own Avowed Tenets; And which Plenum Sir *Iſ. Newton*, and other *Engliſh* Philoſophers, have Sufficiently Confuted.

On the Contrary, the ſame Learned Perſons, Building their Schemes upon the Corpuſcular Doctrine, Retained the Errors of their *French* Maſter, and Rejected the Truth, which his Great Wit and Genius had taken Abundance of Pains to Reconcile to Them; So Unfortunate is it for us to be Philoſophers upon Truſt, and in Obedience, and Complaiſance, to the Dictates of Thoſe, for Whom we Profeſs a Veneration.

CHAP. VII.

Concerning the Complex Ideas of Number, Infinity, Motion, Colour, Taſte, and Smell.

The Complex Ideas of Numbers, how Gain'd from Reaſon and Abſtraction, and not Intirely, as this Philoſophy Repreſents it. Mr. Locke's Eſſay, Chap. 16. Book 2.

§. 1. AS to our Complex Ideas of Numbers, they are Gained by our Obſerving ſeveral Diſtinct Beings to Exiſt in a Variety of Individuals, ſo that the Mind, by a Proſpect of Nature in General, Obtains in Groſs the Idea of Multitude, which Conſiſts of ſeveral Combinations of Particulars, and each Particular gives us the Idea of Unit; from thence we Proceed to Abſtract Number from Beings, as they Exiſt, and Conſider every Number, as Formed from a Collection of Single Units; Thus Multitude in General, and as it is Preſented to us in Conjunction with the Beings which we Survey, ſeems to be the Firſt Idea in the Mind, and the Arithmetical Progreſſion of Units is a Work of Abſtraction and Reaſon, as it is the Foundation of a Science, which is Certainly owing to thoſe Powers and Faculties of our Underſtandings; ſince it is by the ſame Powers, that we Arrive at our Notions of Decimals, and their Progreſſion, which we could not have from a Bare Contemplation of Things Exiſting.

The Ideas of Number not more Preciſe or Diſtinct, than thoſe of Extenſion.

§. 2. WHAT is ſaid in Reſpect to the Preciſeneſs and Diſtinction of our Ideas of Numbers, as if, upon that Account, the Demonſtrations Concerning them were more General in their Uſe, and more Determinate in their Application, than thoſe of Extenſion, it is not True in Fact.

FOR Firſt, the Ideas of Numbers are not more Diſtinct than Thoſe of Extenſion; we Preciſely and Exactly know, what 1, 2, and 3, are, and ſo likewiſe we Preciſely know and Comprehend, what 1 Inch or Foot, or 2 or 3 Inches or Feet are.

BUT, if it is ſaid, that we are not ſo fully Informed, what Various Properties lie within the Extenſion of an Inch, or a Foot, and therefore do not ſo Perfectly Conceive, what the Nature of Extenſion is, as we do what Unit is, the Contrary muſt be Affirmed; Since, whatever Properties an Inch or a Foot has Different from that Certain Length or Meaſure, they Ariſe from Different Compariſons and Relations; Thus if a Line of an Inch is the Diagonal of a Square, it becomes, in Compariſon with them, Incommenſurable with the Sides of it; If it is the Radius of a Circle, or the Abſciſs of a Parabola, it likewiſe Obtains the ſame Property, with Reſpect to thoſe Curves; And ſo, in other Relations, it Undergoes Different Forms, and Aſſumes a Different Nature, from the mere Length and Meaſure of an Inch, which Compariſons or Relations being Infinite, it is Impoſſible for us to Know the whole Extent of them.

AND the ſame alſo Obtains in Unit, for it, in Compariſon with the Surd Roots of Numbers, is Incommenſurable, as with the Root of Two, or the Root of Three, and with the Root of a Thouſand other Different Numbers; It likewiſe, as the Extenſion of an Inch, is Diviſible to Infinity, and Admits of a Million-Million of Fractions, whoſe Names we know not, and whoſe Properties and ſeveral Relations we as little Comprehend, as thoſe of the Infinitely Various Diviſions of Extenſion.

Nor the Demonſtrations Concerning them more General in their Uſe, or more Determinate in their Application.

§. 3. SECOND, and as the Ideas of Numbers are not more Diſtinct and Preciſe than thoſe of Extenſion, ſo neither are the Demonſtrations Concerning them, more General in their Uſe, or more Determinate in their Application; They are not more General in their Uſe; Algebra Indeed is ſo, becauſe it Equally Reſpects all Quantities, whether Number or Extenſion, or Motion, or whatever elſe, which is Capable of a Compariſon or Relation; But all the ſeveral Propoſitions in *Diophantus* or *Euclid*, which Peculiarly Concern Numbers, I Preſume, will not be ſaid to be more General in their Uſe, which are moſt of them only Contemplations of the Properties of Numbers, and Reſt in ſo many Theoretical Truths, without any farther Advantage, than the Knowing them; Theſe Surely will not be ſaid to be more General in their Uſe, than the Demonſtrations Contained in the Elements of Geometry, which are Uſefull and Serviceable in all the Arts and Sciences of Human Life; whereas, thoſe Reſerved and Abſtruſe Properties of Numbers are of no Kind of Service to it.

Perhaps this Philoſophy when it Speaks of the Demonſtrations in Numbers, may only Mean thoſe, which Concern the Operations in Arithmetick; If ſo, it is yet Miſtaken.

§. 4. BUT Perhaps, I may Preſs this Philoſophy too Far in this Particular, ſince it might not Think or Mean any more by it's Demonſtrations in Numbers, as Poſſibly it might Underſtand no more, than the Common Operations in Arithmetick, of Addition, Subſtraction, Multiplication and Diviſion, 'tis not Improbable alſo, that it might Reach ſo far, in it's View of theſe Studies, as the Extraction of the Root, and the Rule of Three, and from thence might Determine, that the Common Arithmetick was more General in it's Uſe, than Common Geometry.

IF that is the Queſtion, this Philoſophy will yet be in the Wrong; For tho' Arithmetick is Taught to moſt, and Geometry not to ſo many, yet the Generality of the Uſe of the One in Preference to that of the Other, cannot be from thence Concluded; Thus, tho' Algebra is of a more General Uſe than Either, ſince it Includes the Demonſtrations of Both, yet it has Fewer Diſciples, Perhaps, than any Study whatſoever.

BESIDES, it is alſo Plain, that Geometry does Aſſiſt us in a Thouſand Difficult Problems, which are of Univerſal Uſe to Mankind, to which the bare Contemplation of Numbers could never Lead or Guide us; And in a few Words, a Philoſopher may as well Affirm, that a Writing Maſter, and a Teacher of Accounts, is a more Uſefull Man, than he who Inſtructs us, how we may Perform our Navigations with Security to all Parts of the Earth, how we may Pretty Nearly Aſcertain the Motions of the Heavenly Bodies, for the Uſe and Service of Men in General, in their Diſtinctions of Time, and how we may Propoſe thoſe Theorems to the World, which in Reſpect of Philoſophy, and all the Mechanical Arts and Sciences, are of the Higheſt and Utmoſt Advantage to it.

The General Uſefullneſs of any Thing no Conviction of the Clearneſs, Preciſeneſs, or Diſtinction of the Idea.

§. 5. BUT if this was not True, in Reſpect of Vulgar and Common Arithmetick, Compared with the Demonſtrations of Geometry, there would yet be no Proof from Hence, that the Ideas of Numbers were more Preciſe and Diſtinct than thoſe of Extenſion; becauſe the General Uſefullneſs of any Thing is no Conviction of the Clearneſs, Preciſeneſs, or Diſtinction of the Idea; we may have a very Confuſed Idea of Fire or Water, or any other Subſtances in Nature, which notwithſtanding does not hinder the General Uſefullneſs of them, and I may have a very Diſtinct and Perfect Idea of Twenty Propoſitions in *Diophantus*, without being Able to do one Good or Uſefull Thing with them.

The Axioms and Principles of Euclid's *Elements of Geometry, of more Univerſal Uſe, than thoſe of Addition and Subſtraction in Arithmetick; And Algebra more Comprehenſive than both.*

§. 6. LASTLY, in the Preſent Queſtion Concerning Number and Extenſion, we are not to Conſider the Common Axioms or Demonſtrations of Either, but the Sciences of Both, as to their Uſefullneſs and Advantage, which yet if we were to do, the Axioms and Principles of *Euclid's* Elements of Geometry are of more Univerſal Uſe, than thoſe of Addition and Subſtraction in Arithmetick, for the ſame Reaſon, that this Propoſition in Geometry, If Two Quantities are Separately Equal to a Third, They are Equal to One Another, is of more General Uſe than this in Arithmetick, If One and Two is Equal to Three, and Two and an Half + Half is Equal to Three, One and Two, and Two and an Half + Half are Equal to One Another; Becauſe This Laſt is Included in the Former; And for the like Reaſon, Algebra is of more Univerſal Uſe than either Arithmetick or Geometry, ſince both of them are Comprehended in it.

Sturmius's Matheſis Enucleata.

THUS alſo, if we Proceed Farther, the Demonſtrations in Geometry are more General than thoſe in Numbers, as is more Particularly Evident from the Second Book of *Euclid's* Elements, where, what is Demonſtrated in Extenſion, does by Conſequence Obtain in Numbers, and Algebra gives a Demonſtration of the ſame Propoſitions, which is Equally Applicable to Both.

Mr. Kerſey's Algebra, Dr. Wallis's Commercium Epiſtolicum.

AND it is not Unknown of what little Uſe or Service, ſeveral of the Problems of *Diophantus*, Monſieurs *Fermat*, *Bachet*, and *Frenicle*, are, in Reſpect of the mere Properties and Relations of Numbers; Tho it muſt be alſo Confeſſed, that Geometers likewiſe have Exceeded in that Point; However this does not Evince, but that the Demonſtrations in Geometry are more Univerſal, and therefore of more General Uſe, than thoſe of Numbers.

That the Demonſtrations in Numbers are more Determinate in their Application either Unintelligible or Abſurd.

§. 7. **What** is ſaid in Reſpect of their being more Determinate in their Application is Unintelligible; For what Demonſtration is not Determinate? and to what is it, or can it be Applied, unleſs to what it Demonſtrates?

Let us make Uſe of a Favourite Demonſtration of this Philoſophy; That the Three Angles of a Triangle are Equal to Two Right Ones; Is it not Determinate? and is it not as Determinate in it's Application, where there is ſuch a Triangle in Nature, which Geometry Conſiders, as any Demonſtration in Numbers, but as there is none, which Conſiſts of Lines without Breadth, and Surface without Depth, ſo it is not Applicable to any Thing, unleſs under thoſe Abſtracted Conſiderations.

And what is the Difference betwixt theſe Demonſtrations, and thoſe of Numbers? which alſo only Center in themſelves, and are not Capable of being Applied to Things, as they Exiſt, unleſs as they Exactly Correſpond to the Abſtractions of the Demonſtration; which they do not; For as before in Geometry, ſo in Arithmetick, tho' Two and Three are Equal to Five, yet Two and Three Exiſtences are not Equal to Five Exiſtences, Two and Three Stars, are not Equal to Five Stars, Excepting upon the Suppoſition, that they Differ from One Another, no more than Unit from Unit; and ſo it may be ſaid of all other Beings, which we Number in the Univerſe.

The Miſtake, which this Philoſophy ſeems to lie under, is, that it Imagines Geometry to be an Abſtracted Science, and that of Numbers not ſo; whereas, they are both of them Equally Abſtracted Parts of Knowledge, and have no Relation to Nature any farther, than as it is Submitted to thoſe Abſtractions.

What Infinity is According to this Philoſophy. Mr. Locke's Eſſay, Chap. 17. Book 2.

§. 8. **Infinity** is another Idea, which this Philoſophy Conſiders; The Firſt Thing, which it Aſſerts is, that Finite and Infinite are Modes of Quantity, and are Attributed Primarily to Things, that have Parts, and are Capable of Increaſe or Diminution by the Addition or Subſtraction of any the leaſt Part, as Space, Duration, and Number; And when we Apply this Idea to the Supreme Being, it is ſaid, we do it Primarily, in Reſpect of his Duration, and Ubiquity, and more Figuratively, when to his Wiſdom, Power, Goodneſs, *&c.* which are Properly Inexhauſtible and Incomprehenſible; for when we call them Infinite, we have, it is Affirmed, no other Idea of this Infinity, but what Carries with it ſome Reflection on the Number, or the Extent of the Acts, or Objects, of God's Power and Wiſdom, which can never be Suppoſed ſo Great, or ſo Many, that theſe Attributes will not always Surmount and Exceed, tho' we Multiply them in our Thoughts with the Infinity of Endleſs Number; Thus far this Philoſophy.

Infinite is a Negation of every Thing, which is Finite, and may be Properly, and without a Figure, Applied to God's Wiſdom, Power and Goodneſs.

§. 9 **But** Firſt, tho' Finite is Capable of Increaſe and Diminution, Infinite is not ſo; for if it could be Increaſed or Diminiſhed, it would Ceaſe to be Infinite, and therefore it cannot be Applied to Things Capable of Increaſe and Diminution.

On the Contrary, Infinite, in Reſpect of Duration, Space, or Number, is a Negation of Increaſe or Diminution, or a Negation of the Greateſt Finite; whereas Finite may be Perpetually Increaſed or Diminiſhed; Infinite is not any Thing, which is Finite, but every Thing, which is Finite, is Included in what is Infinite.

Second, by what has been ſaid in the Principles Propoſed for the Advancement of a Pſycheometria, or a Geometry of Minds, we may Conceive an Infinite Variation of them in their Innate Forces and Faculties, and that, as Human Minds in their Powers are Infinitely Greater, than thoſe of Brutes, ſo that Almighty God's is Infinite in Reſpect of Men's; And this Infinity of Force in the Supreme Mind would be the ſame, whether he Exerted Fewer, or More, Explicit Acts in Number; And there ſeems to be as much a Gradation of the Powers and Faculties of Intelligent Beings, down from the Almighty, thro' all the Orders of them, to Men, and from Men, thro' the Intermediate Species of Animals, to an Oyſter, as there is a Variety of Meaſures in Extenſion, from Infinity to Nothing.

That there is a Real and Proper and not a Metaphorical and Figurative Infinity of Wiſdom in God.

§. 10. **And** Conſequently, there may be an Infinite Wiſdom, Power, and Goodneſs, not only in a Figurative, but in an Abſolute and Real Senſe; The Natural Force and Strength of the Mind, and it's Faculties, in Perceiving, Remembering, Judging, *&c.* I Preſume, will be Allowed to be Infinitely Greater in Men, than in Brutes, and that therefore their Wiſdom, Sagacity, and Penetration, is Greater in the ſame Infinite Proportion; This Appears to be Evident, from what has been before Alledged, in Reſpect of the Diſcurſive Faculty in Men, which, even in Ideots, is Infinitely beyond what Obtains in Brutes; For, If Brutes had Language, we may Reaſonably Imagine, they would only Expreſs ſome few Ideas, which they had Learned by Rote, without being able to Connect them, or to Reaſon and Diſcourſe upon them, which yet Ideots can in ſome Meaſure do.

IF therefore there is a Wiſdom, Ariſing from the Natural Force and Penetration of our Faculties, which may be Infinite in Reſpect of that of other Animals, which are below us; It is not Incongruous to Suppoſe, or Irrational to Conceive, that the Wiſdom of God may be Infinite from the Innate Forces of his own Mind, from the Stupendous and Vaſt Comprehenſion, Underſtanding, Conception, and Thought, which he has; and which Reſults from the Infinite Nature and Eſſence of his Intellectual Powers; in which he Exceeds the Higheſt Orders of Created Beings, Infinitely more, than thoſe Higheſt Orders do the Meaneſt Inſect; Becauſe, the Diſtance betwixt the moſt Glorious Spirit, or Seraph, and the moſt Abject Worm, is only Finite; Whereas, That betwixt Almighty God, and the Greateſt and Sublimeſt of his Creatures, is Boundleſs and Infinite.

That there is a Real and not a Figurative Infinity of Power in God.

§. 11. AND, as an Infinite Wiſdom, Ariſing from the Nature of the Infinite Mind, in which it Reſides, may be Juſtly Maintained and Defended, ſo likewiſe, for the ſame Reaſon, may an Infinite Power.

THUS, ſince it is Obſervable in Men, that, from the Infinite Force and Strength of their Faculties, in Reſpect of what Brutes are Poſſeſſed of, they Exerciſe an Unlimited Sway over the ſeveral Animals and Creatures of the Earth, Subject them to their Dominion, and Make all Nature, and the Beings of it, in ſome Meaſure Subſervient to their Wills, which no Creature in the World, beſides them, can do; May we not therefore very Reaſonably Affirm, That they Have and Poſſeſs an Infinite Power, in Reſpect of what a Fly or an Inſect does, and that this Infinite Power is Derived from the Native Force of their Intellectual Capacities and Beings?

IF ſo, it is no hard Matter, to Conceive a Mind, which is more Infinitely Powerfull than our's is, by which it Governs all the Beings in the Univerſe, Submits them to it's Laws and Decrees, and Makes, that, not only Men, but all the Orders and Claſſes of Intelligences above us, ſhould Pay an Obedience to them; For, whatever our Power is, we find one Superior to it, ſince we can only Act within a Certain Sphere, and under Certain Limitations in Nature, Preſcribed to us; Nor is it Hard to Conceive, that this Infinite Power of the Almighty Mind is the Derivative and Reſult of the Superior Force of his Faculties, as Our's is, from a like Superiority of them, in Reſpect of the Creatures, which are Below us.

That there is a Real and not a Figurative Infinity of Goodneſs in God.

§. 12. LASTLY, nor is an Infinite Goodneſs, in it's own Nature, leſs Intelligible, or leſs Agreeable to Reaſon; For do we not find, that Brutes and Animals, which are Inferior to us, have little or nothing of this Quality and Perfection in them? Their Senſe ſeldom, if ever, Extends any farther, than to the Conſervation of their Species, or the Preſervation of themſelves, an Inſtinct, we may Juſtly Think, Implanted by Almighty God in their Natures, for the Continuance of thoſe Beings, which he had Originally Formed and Created.

AND therefore we may Say, that Mankind in General is Infinitely above them in Reſpect of Goodneſs; for here we do not Speak of Individuals, but of the Natures of Each; ſince there are not wanting Thouſands of Inſtances of the Greateſt Inhumanity in our own Species; But then it muſt be Obſerved, that very Corruption and Perverſion of it to a Brutal Fierceneſs and Barbarity is, what is Properly Termed Inhuman by us; From whence it is Apparent, that we Diſclaim it from belonging to us, or from being a Part or Attribute of our Beings.

LET us now come to our own Species, Diſtinct and Separate from a Compariſon with Brutes; and may we not there find a Thouſand Degrees and Diſtinctions of Goodneſs, Ariſing from the Innate Temper, Genius, and Complexions of our Minds? Some have a Goodneſs, which is more Confined, from the very Narrowneſs of their Conceptions, and the Contractedneſs of their Faculties, and Others a more Univerſal and Unlimited, from the Largeneſs and Extenſiveneſs of Them; For it is always Obſerved, that the Greater any One's Mind is, the Gooder it is, and the Leſs it is Hampered and Cramped with the Private Views of Self, of Intereſt, or Ambition; It Exerts it's ſelf in Spite of all theſe, by the Energy and Force of its Faculties, into a Boundleſs and Noble Generoſity of Doing Good for Ill, of Returning Bleſſings for the Curſes of it's Enemies, and Diffuſing it's Beneficence, it's Charity, and Love, to even the Meaneſt Inſects of the Creation, where it is Impoſſible it ſhould Meet with any Retaliations of Gratitude for it's Bounties and Favours.

Thus alſo may we Conceive, that the like Powers and Faculties, Increaſed to Infinity, will Produce a Goodneſs, which is Infinite in it's Native Energy, and by which it Scatters and Diſperſes it's Influences thro' the whole Univerſe, by which every Being in it Subſiſts, by which it Broods over each Animate Creature, and Suſtains it's Life, and Supports it's Species, with the Tendereſt Regard, and by a moſt Indulgent Providence.

From all which, we may not only Apprehend, how there is a Natural Connexion betwixt Wiſdom, Power, and Goodneſs, in Human Minds, Derived from, and Owing to, the Innate Strength and Force of their Capacities, but how there is a like Connexion betwixt Infinite Wiſdom, and Infinite Power, and Infinite Goodneſs, Ariſing and Flowing from the Infinite Comprehenſion and Eſſence of the Eternal Mind.

How we come by the Idea of Infinity according to this Philoſophy.

§. 13. Having hitherto Endeavoured to Shew, that Infinity is not only Applicable to Space, Duration, and Number, but alſo to the Powers and Faculties of Intelligent Beings;

The next Inquiry in this Philoſophy is, which way we come by this Idea of Infinity; which it Informs us is, by an Endleſs Repetition of Stated Lengths in Space, as of a Foot, a Yard, *&c.* which gives us the Idea of Immenſity; and that by the Addition of Certain Stated Lengths in Duration, as of a Minute, a Day, or a Year, with the Infinity of Number, we have our Idea of Eternity; But it Tells us, that we cannot have the Actual Idea of an Infinite Space, and Conſequently not of an Eternal Duration, for the ſame Reaſon, that we cannot have an Actual Idea of an Infinite Number; For an Actual Infinite Number is Affirmed to be a Contradiction, and therefore an Actual Infinite Space, and therefore alſo an Actual Eternity; Tho' it Owns, there is an Infinity of Number, of Space, and Duration.

This Account not Juſt; That there is an Actual Infinite Space, Duration, and Number.

§. 14. But this Fallacy of Arguing Appears from Hence; That, if there was not an Actual Infinite Space, or Duration, and I may alſo Add Number, in One Senſe of it, and which ſhall afterwards be Explained, there could not be Everlaſting and Endleſs Approaches to it; And for the ſame Reaſon, that there is an Infinity of Space, or Duration, or Number, there is an Infinite Space, Duration, and Number; For how is it Poſſible, there ſhould be an Infinity of any Thing, without That Thing's being Infinite? This Philoſophy may Diſtinguiſh, as it Pleaſes, but ſuch Diſtinctions will never make the Real Relations of Things, which are the Same, to be Different.

For, If Space is not Actually Infinite, and the ſame may be ſaid of Duration and Number, it muſt be Actually Finite, and there may be an Endleſs Addition of Stated Lengths of Space, or Duration, or of Unit, in that which is not Endleſs, which is a Contradiction; Or, in other Words, the Poſſibility of an Endleſs Addition in any Quantity whatſoever, without Suppoſing that Quantity to be Actually Infinite, is Abſurd and a Contradiction; For, if it is not Actually Infinite, we cannot be Continually Adding, ſince, where it Ceaſes to be in Reality Infinite, our Addition muſt Stop, and it will be Impoſſible to Go any Farther.

If it is ſaid, that there may be an Infinite Space, or Duration, or Number, but that we have not the Idea of it; That is not at Preſent the Queſtion, but whether there is an Actual Infinite Space, Duration, or Number, as well as an Infinity of them, which we Affirm, there is.

What ſeems to Puzzle and Confound this Philoſophy in it's Reaſonings.

§. 15. What ſeems to Puzzle and Confound this Philoſophy is, that it Unwarily makes an Actual Infinite Space, Duration, or Number, to be, what we have Gone Over and Meaſured in our Minds; That is, it makes Actual, or being in Fact, to Signify nothing elſe than Finite, that is, which we have Actually Gone Over in our Minds; And then there is no Doubt, but an Actual Infinite Space, Duration, or Number, that is, a Finite Infinite Space, Duration, or Number, is a Contradiction.

But, if we make Actual, or being in Fact, to Signify, not what we have in Fact Gone Over in our Minds, which is a very Improper Senſe of it, but what does in Fact Exiſt, which is the True Senſe of it, it is ſo far from being a Contradiction to Say, that there is an Actual Infinite Space, Duration, and Number, that is, that an Infinite Space, Duration, and Number, does and muſt Exiſt, that it is a Plain and Evident Truth; Unleſs this Philoſophy will go one Step Farther, and Aſſure and

Prove to us, that there is nothing, which does or can Exiſt, but what is Finite; And if ſo, then there cannot be that Endleſs Addition of the Meaſures of Space, Duration, or Number, which it Contends for, ſince there cannot be ſuch an Endleſs Addition, unleſs they were in Fact Infinite.

That we have an Idea of an Actual Infinite.

§. 16. If it be Inquired, whether we can have an Idea of an Actual Infinite, and if we can, if it be farther Asked, which way we Obtain it;

'Tis Anſwered to both theſe Inquiries, that we may have ſuch an Idea, and that we Obtain it, not by the Repetition of any Stated Meaſures of Space, or Duration, or Number, but by Evident Deductions from our Reaſon, and from the Nature of thoſe Quantities and Beings.

That we may have, and Really have, ſuch an Idea is Certain, that is, if we Mean by Idea, what the Mind Apprehends, and can Reaſon upon, and not only that, of which it has an Exact and Definite Image or Picture.

If the Latter is only Meant by Idea, 'tis very Plain, that we can have no Idea of an Actual Infinite; becauſe an Exact Image, or Picture, of a Thing is always Finite; But then it muſt be alſo Said, that if this is the Notion of an Idea, we cannot have an Idea of a Thouſand Finite Things; becauſe we cannot have an Image, or a Picture, of them; Thus we have no Image of Ten Thouſand, of a Million, Billions, Trillions, Quatrillions, *&c.* or of a Polygone, whoſe Sides are Expreſſed by thoſe Numbers.

But if by Idea is Meant, which Undoubtedly is the Juſt Meaning, That, which we ſo far Apprehend, as to be Able to Diſcourſe about and Reaſon upon it, we then have an Idea of Thouſands, of Millions, Billions, Trillions, Quatrillions, and farther of an Infinite Number, as alſo of an Infinite Space, and Duration; And in Particular all Surds, as the $\sqrt{2}$, $\sqrt{3}$, *&c.* are in Fact Infinite Numbers.

The Properties of which Infinites we know to be, that None of them are any Thing, which is Finite, that they have the ſame Proportion to any Thing, which is Finite, as any Thing, which is Finite, has to Nothing; Becauſe, Take any Finite Quantity, as a Line, and Multiply it into it's ſelf, and it will Produce a Square, which will be Equal in this Compariſon to an Infinite Line Multiplied into Nothing, that is, to an Infinite Line, without any Multiplication of it; For a Square is as much Infinite in Reſpect of a Line, which has no Breadth, as an Infinite Line is in Reſpect of Nothing, which has no Length.

How we Obtain this Idea of an Actual Infinite.

Mr. Raphſon de Spatio Reali Infinito.

§. 17. Since we have therefore an Idea of an Actual Infinite, it Remains upon us to Shew, how we Obtain this Idea, which is not from a Repetition of any Stated Meaſures of Space, or Duration, or Number, but by Evident Deductions from our Reaſon.

For thus, from the Nature of Incommenſurables and of Surd Numbers, and from the Properties of the Aſymptotes of an Hyperbola, and from the other Various Arguments, made Uſe of by the Learned Mr. *Raphſon*, to Prove there is an Actual Infinite, Philoſophers come to Know there is ſuch a Thing.

So likewiſe they Arrive at the ſame Idea, not by the Repetition of a Finite, but by Reaſoning, and Conſidering, that if there is not an Actual Infinite in Space, Duration, and Number, there is ſome Finite Aſſignable in them, which ſhall be Capable of Comprehending all the Numberleſs Relations of Greater or Leſs, which the Mind can Poſſibly Attribute to them without End, or Determination; But ſince no ſuch Finite is, or can be, there muſt of Conſequence be an Actual Infinite.

Infinite Applicable to Taſte and Colour.

§. 18. And, as This Obtains in Number, Space, and Duration, ſo does it moreover, in the Faculties of the Mind, and in the Properties of Body; By which Means Nature will be every Way Infinite, not only in it's Extent, but in the Variety of it's Forces, Suitable to the Great and Almighty Author of the Vaſt and Amazeing Syſteme.

As to the Faculties of the Mind, we have already Conſidered them; And as to the Properties of Body, and Particularly thoſe of Taſte and Colour, which this Eſſay of Ideas Mentions, we muſt Affirm, that, tho' we cannot with any Propriety of Speech Say, that there is an Infinite Sweetneſs, or an Infinite Whiteneſs, no more than that there is an Infinite Numerouſneſs, or Extenſiveneſs, or Succeſſiveneſs of Time; yet we may Juſtly Say, that One Thing is Infinitely Sweeter than Another, and that there are Infinite Degrees of Taſte betwixt the Sourneſs and Harſhneſs of a Lemon, or an Unripe Grape, and the Sweetneſs and Luſciouſneſs of a Dried Fig, or a Raiſon, by which they ſhall Gradually Approach from the One to the other, and by that Means become leſs Sour, and more Sweet, thro' an Infinite Number of

Diſtinctions; which alſo Holds in our Smells betwixt thoſe, which are the moſt Delicate and Pleaſant, and thoſe, which are the moſt Ungratefull and Offenſive.

AND the ſame may be ſaid in Reſpect of Colour, ſince if we Conſider all the Intermediate Degrees of Colour from Black, Violet, Purple, Blue, Green, Yellow, Orange, to the Solar Red, and if we Conſider all the ſeveral Variations and Shades of Violet, and thoſe of Purple, or Blue, Green, *&c.* and the ſeveral Compound Lights, which Ariſe from the Mixture of theſe, we ſhall be Eaſily Led to Think, that the Degrees of Colour are Endleſs and Infinite; And we have already Shewn in the Principles of Philoſophy, that they are only Infinitely Different Enervations and Diminutions of the Force of the ſame Homogeneous Light.

Infinite Applicable to other Properties of Bodies, as well as to the Faculties of Intelligent Beings, to Space, Duration, and Number, and to Taſte, and Colour, Namely, to Gravity and Levity.

§. 19. THUS far we have Conſidered the Inſtances, which this Philoſophy gives; But this Reaſoning does moreover Prevail in the other Properties of Body, as in Gravity, and Levity, in which we find an Infinite Number of Gradations, ſo far that from the Weight of Gold, which is the Heavieſt Subſtance, which we know, it has been ſo far Diminiſhed, that it has been Juſtly Inquired by Philoſophers, whether there is not an Actual Levity; That there is a Centrifugal or Expanſive Force in Bodies, is not to be Doubted, ſince the Elaſtick or Repulſive Force is ſuch a One; And in Vegetation, what there can be, which ſhall Raiſe Plants from the Ground, Contrary to their Centripetal or Contractive, unleſs a Centrifugal or Expanſive One, no One can Imagine; or how Fire, or Flame, ſhould not be Endued with ſuch a Force, but ſhould Exert it's ſelf with that Violence and Fury ſometimes, by it's Comparative Levity only in Reſpect of other Bodies, which Surround it, is not Eaſy to Conceive; If therefore there are theſe Centrifugal or Expanſive, and Centripetal or Contractive, Forces in Nature, which Perhaps are only other Names for what is Meant by an Actual Gravity, and an Actual Levity, and as theſe Forces may be Infinitely Diverſified, Some having a more Violent Tendency from their Center than Others, and Others a more Violent Tendency to it, and as theſe Forces may be Infinitely Combined, Apart amongſt themſelves, and Conjointly with One Another, we may with Reaſon Aſſert, that theſe Properties of Bodies are Infinite, according to their Various Degrees of Gravity or of the Contractive Force, or of Elaſticity or the Expanſive, Separately, or of Gravity or the Contractive, and Elaſticity or the Expanſive, in Conjunction with each Other; From whence will Ariſe an Infinite Variety of Theorems, in Reſpect of theſe Properties of Bodies, or of Matter, as before in Reſpect of the Properties of the Mind, which will be all Reducible to a Mathematick and Algebraick Calculation; And it is not to be Queſtioned, but by this Method, we might Find out the Variety of Taſtes, of Smells, and Colours; Which ſeem to be all Owing to a Certain Proportion of their Elaſtick or Centrifugal or Expanſive Force, Mixed, tho' Perhaps in a very ſmall One, with their Gravitating or Centripetal or Contractive.

Infinity Applicable to the Various Extenſions of Bodies.

§. 20. LASTLY, As to the Peculiar Extenſions of Bodies, by which they Appear in ſuch a Certain Manner to Fill the Eye with a Greater or Leſs Conſtipation of their Parts, and to our Feeling, to have ſuch a Certain Degree of Roughneſs or Smoothneſs, of Solidity or Fluidneſs, of Rarer or Denſer, and of all the Tangible Qualities of Bodies, by which they are Infinitely Diverſify'd to us; They ſeem to Ariſe from the Filling of their Dimenſional Spaces with Infinitely Different Kinds of Matter, that is, with Different Sorts of Actions, Communicated to our Senſes, and which Different Actions are Owing to the Infinitely Various Compoſitions of the Centripetal and Centrifugal Forces, or the Contractive and Expanſive, from which all the Phænomenons of Matter may be Solved, and the Grand Inquiry of the Subſtratum of it Finally Determined; The Variety of Sounds will be alſo Accounted for from the Infinitely Different Mixtures of the Elaſtick and Gravitating Forces, or of the Expanſive and Contractive, or, which is much the ſame, of the Centrifugal and Centripetal.

ALL which Theory of Matter only Proceeds upon this One Plain Axiom, that it is Impoſſible for us to have any Senſations from Matter, but by ſome Kind of Action, or Other, Impreſſed upon our Minds from it; And that, it is Impoſſible, we ſhould have Different Senſations, but by ſuch Different Impreſſions or Actions.

Of the Simple Modes of Motion.

Mr. Locke's Eſſay, Chap. 18. Book 2.

§. 21. ANOTHER Kind of ComplexIdeas, taken Notice of by this Philoſophy, are thoſe of the Simple Modes of Motion, of which I ſhall here only Mention thoſe of Swiftneſs and Slowneſs, the Meaſures of which are ſaid to be made out of the Diſtances of Time and Space put together; But theſe Modes of Motion have been already Conſidered in our Examination into the Mechanical Philoſophy, Book the Firſt, and in other Parts of our Preſent Inquiry, and therefore I ſhall not Repeat that Trouble here; And ſhall only Say, that it was my Intention to have gone thro' every Part of this Syſteme of Ideas, but finding the Number of Sheets to Grow too Faſt upon my Hands, I was Obliged to Lay Aſide that Deſign; And muſt therefore give, for the Future, ſome ſhort Strictures upon the Following Chapters of this Eſſay.

CHAP. VIII.

Of the Modes of Thinking; Of Pleaſure and Pain; Of Power; Of Mixed Modes; Of our Complex Ideas of Subſtances; Of Collective Ideas of Subſtances; Of Relation; Of Cauſe and Effect; Of Identity and Diverſity; Of Other Relations; Of Clear and Obſcure, Diſtinct and Confuſed Ideas; Of Real and Fantaſtick Ideas; Of Ideas Adequate or Inadequate; Of True and Falſe Ideas; and Of the Aſſociation of Ideas.

Of the Modes of Thinking. Mr. Locke's Eſſay. Chap. 19. Book 2.

§. 1. IN This Chapter, Contemplation, and Intention or Study, are Suppoſed to be Different Operations of the Mind, and yet the Definitions of them are the Same; And, whereas a Re'verie is made to be, when the Mind does not Obſerve it's Ideas Floating thro' it, it is more Rational to Think, that ſuch Re'verie is, where there are no Explicit Ideas Floating in it at all, but it is Reſigned to it's General Senſe or Perception; And if Sleep without Dreaming is, as it is Affirmed, a Reſt from all theſe, it is a Reſt from a Re'verie, or a Reſt from a Reſt, which is Abſurd; On the Contrary, a Re'verie, and Sleep without Dreaming, ſeem to be much the ſame; and the Mind in both Caſes may be Compared to a Stagnant Fluid, which has no Impreſſions made upon it, with this Diſtinction, that as the Fluid Nature is to Water, ſo is the Senſible to the Mind.

Of the Modes of Pleaſure and Pain; That Pleaſure and Pain are not the Meaſure of Good and Evil. Chap. 20. Book 2.

§. 2. IT is here ſaid, that things are Eſteemed Good or Evil, only in Reference to Pleaſure or Pain; But 1*ſt*. If nothing is Good or Evil, but what Produces Pleaſure or Pain in us, Virtue and Vice would be very Changeable in their Natures, according as every One was Pleaſed with the One or the Other; ſo far, that if a Vicious Man was Pleaſed with his Vices, they would be Good to him, and if a Religious or Virtuous Man underwent any Pains or Afflictions upon that Account, his Religion and Virtue would be Evil to him; And in General, according as Perſons were Pleaſed, or Diſpleaſed, with the ſame Thing, it would be a Good or an Evil, which is to the Utmoſt Degree Abſurd; Since there would be no Intrinſick Nature of Things, by which they might be Valued, and Diſtinguiſhed from each Other; The Immutable Reaſons of them would Vaniſh, and all would be Reſolved into Caprice and Humour; 2*d*. This is farther a Syſteme, or Leads to a Syſteme, of Morals, which makes all our Actions Center in a Selfiſh Principle, without Conſidering Things, as they Really Exiſt, which we ought to do, Abſtracted from our own Pleaſure or Pain, which are Accidental, and cannot Enter into the Real Relations of Things, or the Moral Relations of Good or Evil; 3*d*. According to this Doctrine, by Reflecting upon the Thought we have of the Delight, which any Thing is Apt to Produce in us, we have an Idea, which we call Love; Which if ſo, no One can Love his Enemies, tho' Religion Requires it of us, nor any Virtue, which Cauſes him Pain, tho' Morality Enjoins it; 4*th*. The Like may be ſaid of the other Paſſions of Deſire, Joy, Hope, if the Foundation of them is Pleaſure, and of Hatred, Sorrow, Fear, Anger, Deſpair, and Envy, if they are only Referred to Pain.

Of Power. Chap. 21. Book 2.

§. 3. POWER is here Divided into Active and Paſſive, the Former is ſaid to be Derived from Thinking and Motion, the Latter from Senſible Beings and Matter; But if Matter is Active, and not Paſſive only, ſuch Action will as much Inform us of an Active Power, as God, from whom all the Properties and Actions of Matter are Produced, as Motion or even Thinking will.

In the Progreſs of this Chapter, Will is Defined to be a Power, which the Mind has, to Order the Conſideration of any Idea, or the Forbearing to Conſider it, or to Prefer the Motion of any Part of the Body to it's Reſt, and Vice Verſâ; and Preſently after it is ſaid, So far as a Man has Power to Think, or not to Think, to Move, or not to Move, according to the Preference or Direction of his own Mind, So far is a Man Free; Will therefore is a Power to Prefer, and Liberty or Freedom, a Power to Act according to that Preference, and the Firſt belongs to the Mind, and the Laſt to the Man, as an Agent: But this Doctrine makes the Preſent Philoſophy Conclude, that therefore a Man may not be Free, tho' the Action is Voluntary, that is, a Man may have a Liberty or Freedom of Will, at the ſame Time, that he is not Free, as to his Will; For as to a Liberty of Executing or Performing his Will, that is Entirely Foreign to the Nature of the Mind or it's Freedom, which is here only the Queſtion, which will be the ſame, whether a Man is in Chains, or upon a Rack, or Abſolutely Reſigned to his own Actions.

Wherein the Liberty or the Freedom of Will Conſiſts.

§. 4. On the Contrary, the Will, and the Freedom of it, ſeems to be a Power in the Mind to Produce any Action, or to Think of any Determinate Idea from it's ſelf, and from a Principle of it's own, by which it is the Beginning and Riſe of ſuch Actions or Thoughts, and by which it is Diſtinguiſhed from a Neceſſary Agent; And Conſequently, there might be a Will, and a Freedom of it, if the Mind was Confined to One ſingle Idea, and could not Think of any Other in Preference to it, that is, if that Idea was Produced from it's ſelf, and Aroſe from an Innate Principle, Reſideing in it; For thus, if a Ball could Move it's ſelf in One Certain Direction, without any Stroke or Impulſe from a Racket, it would be a Free, and Ceaſe to be a Neceſſary, Agent; And, where the Queſtion is, whether a Man is a Free Agent, or, whether the Will is Free, it is to Inquire, whether the Mind can Act and Think from it's ſelf, Independent of any External Impulſe, either from Almighty God, or from any Material Beings; which Appears Pretty Evident, that it may, and that Almighty God has been Pleaſed to Create it, and to Endue it with that Power.

If therefore the Mind has this Faculty of Determining it's ſelf to this Thought or Action, it will Follow, that it has the Power to Determine it's ſelf to any other Thought or Action, there being nothing in the Nature of One Thought or Action, more than in Another, which ſhould hinder the Minds Determining it's ſelf to it; Thus, if a Ball can Move it's ſelf in One Direction without an Impulſe, it may in Another, there being no more Impediment to it's Motion in One Direction, than in a Contrary, if it has a Principle of Moving, from it's ſelf, and Innate and Internal to it. This brings to my Mind a very Wonderfull and Surprizing Phænomenon, which I lately Obſerved of the Down of a Feather, as it ſeemed to be, which Moved in the Air, near a Glaſs-Window, in the Inſide of a Room, Upwards, Downwards, Backwards, Forwards, and Tranſverſely, in all Kinds of Directions, with an Incredible Celerity, when there was little or no Air ſtirring in the Room, tho' it was ſomething Windy Abroad; If we can Imagine therefore this little Down to have Moved it's ſelf, there is Scarcely any Doubt to be made, but it had a Certain Degree of Volition, and Conſequently of Perception, Annexed to it, as we find in the ſmall Eels of Vinegar, or in the Exceeding Minute Animalcula of Pepper-Water.

But it is Irrational and Ridiculous to Suppoſe, that ſuch a Freedom of Will was ever the Diſpute of the Learned, which Conſiſts in a Power to Think, or not to Think, to Move or not to Move, according to the Preference or Direction of our own Minds; Since, according to ſuch a Notion, a Man ſhall have a Freedom of Will, in Reſpect of his Thoughts, when he is Allowed to Think as he Pleaſes, but no Freedom of Will, when he is Hindered from doing it, and in Reſpect of Action, he ſhall have Freedom of Will, when he may Walk, where he Thinks Fit, but no Freedom of Will, when Loaded with Chains and Fetters; By which Means the ſame Mind ſhall be Endued with a Liberty of Will, or not, according to the Differing External Circumſtances of the Body, which have no Relation to, or Conneixon with, the Properties of the Mind, which in the Diſputes Concerning the Freedom of the Will, are only to be Conſidered.

Of Mixed Modes.
Mr. Locke's Eſſay, Chap. 22. Book 2.

§. 5. Mixed Modes are ſaid to be Combinations of Simple Ideas of Different Kinds, as Virtue, Vice, a Lie, *&c.* and are Diſtinguiſhed from Simple Modes, which Conſiſt only of Simple Ideas of the ſame Kind, Put together by the Mind; Where-

as

Mixed Modes Received in by Obſervation, and then Separated into the ſeveral Parts or Ideas, of which they are made up.

as Mixed Modes ſeem not to be Formed by the Mind from it's Joining and Uniting ſeveral Simple Ideas into One, but they are Received, and Impreſſed upon it, in the Complex from an Obſervation of Human Actions, and then Separated and Divided, by the Thinking and the Contemplative, into the ſeveral Parts, of which they are Made up; Thus, Men's Virtuous or Vicious Actions are ſeen by us at Once, which Leads us to Form the General and Abſtracted Notions of Virtue and Vice; We Hear a Lie in Fact, which Perhaps may Carry us to Conſider the ſeveral Ideas, which Enter into the Nature of it; And ſo of the Reſt.

Of our Complex Ideas of Subſtances. Mr. Locke's Eſſay, Chap. 23. Book 2.

§. 6. SUBSTANCES, in this Philoſophy, are ſaid to be General, or Particular, and Each of them to be a Complication of many Ideas together, which Subſiſt in one Subſtratum, which is the Support of them, by which they are United and Cemented into One Complex Idea, and from whence they are Derived and Connected with each other; Particular Subſtances are alſo ſaid to be Compoſed out of the Obſcure and General One, which have their Particular Combinations of Simple Ideas, and are Suppoſed to Flow from the Unknown Eſſence of their ſeveral Subſtances.

Particular Subſtances not Derived from the General One, but the Latter from the Former.

ON the other Hand, it ſeems to be Evident, that the General or Obſcure Subſtance is Formed by Abſtraction from Particular Ones; and that, as the Preſent Philoſophy has Managed it, it is a mere Notional Thing, an Arbitrary Creature of the Mind, and Conſiſts of Abſtracted Ideas, United together at the Pleaſure of the Will; Thus we Form our General Idea of Matter from the ſeveral Particular Material Subſtances, which Exiſt, as Gold, Lead, Earth, *&c.* and when we come to Inquire into the Subſtance of Matter, we ſhall find it to be only an Aſſemblage of Various Abſtracted Ideas, ſuch as Extenſion, Solidity, Diviſibility, Mobility, *&c.* which have no common Cement, by which they are United, nor any Mutual Dependence upon each Other, and therefore can have no Subſtratum, Subſtance, or Eſſence, from whence they are Derived.

What the Subſtance, Subſtratum, or Eſſence of any Particular and Material Being is.

§. 7. IN a True Philoſophy, and in the Reality of Things, Particular Subſtances are therefore Firſt to be Examined into; Which are not Subſtratums, Conſidered as Beings Diſtinct from the Properties, which are Imagined to Inhere in them; But are One and the Same Nature, or Being, Applied to Different Natures, or Beings, from which Application it's Properties Reſult; So that any One Property of a Particular Subſtance is the Eſſence of it, as much as the Subſtance is, becauſe, One Property being Given, all the Reſt will Follow upon Different Applications; Thus Gold is not a Subſtratum, with Yellow, Weight, Malleableneſs, Fuſion, Extenſion, Solidity, *&c.* Inhereing in it; but it is One Invariable Nature, or Being, which is Suſceptible of ſeveral Qualities, upon it's being ſeverally Related and Referred to Different Natures; If it is Applied to the Eye, it Produces it's Peculiar Yellowneſs in us; If to the Hand or Eye, it's Peculiar Extenſion and Solidity; If to Fire, it becomes Fuſed; If to a Hammer, Malleable; If to Aqua Regalis, a Diſſolution Enſues; And the Nature of Gold, as much Conſiſts in any One of theſe Properties, as it does in All of Them; Becauſe where One is, the Other, upon Different Applications, will be Conſequent; For where there is the Peculiar Yellowneſs, Weight, or Solidity of Gold, there will be the Peculiar Malleableneſs, Fuſibility, or any other Property of it, and Vice Verſâ.

IF therefore a Certain Degree of the Expanſive and Contractive Forces will Produce the Yellowneſs of Gold in us, and the ſame Degree of each will Produce the Weight or Solidity of it, it's Particular Extenſion, it's Malleableneſs, and Fuſibility, ſuch a Degree of the Expanſive and Contractive Force will be the Eſſence, or Subſtratum, of Gold; And which Degree of Force will be as much Expreſſed by One Property, as Another, and, upon Different Applications, may be Productive of a Thouſand more, of which we are at Preſent Entirely Ignorant; And yet we may be ſaid to Know the Entire Nature and Eſſence of Gold; Becauſe, That, as much Conſiſts in One Property, as in a Million of Them.

THUS likewiſe; If Fire is Applied to the Eye, it Cauſes in us the Senſation of it's Peculiar Yellow; If to the Hand, it's Heat; If to Wood, it Burns it; It alſo Melts Wax, Hardens Clay, and Fuſes Metals, by it's Various Applications to Them; And yet this is One and the Same Nature, which is as much Expreſſed by one Property, as by all of them; Becauſe, That, which will Produce One Effect, upon Different Applications, will Produce the Other; And, If a Certain Degree of the Expanſive and Contractive Forces will Anſwer theſe Phenomenons, ſuch a Degree of the Expanſive and Contractive Forces, will be the Eſſence or Subſtratum of Fire, which is as much Exerted in the Production of One of theſe Phenomenons, as in the Production of All of them, and the Nature of any Being, as much Aſcertained by One Property, as by an Hundred.

To Exemplify this Doctrine Farther; The Point of a Needle is Sufficiently Determined, which yet can Perforate Paper, or Rend it, can Cauſe a Pain in the Finger, or a Pleaſure to it, and by Different Applications can Produce other Effects, which may be all ſaid, upon ſuch Applications, to be the Properties of it, when, notwithſtanding, every One knows, what the Point of a Needle is, as well by One of theſe Properties, as by Twenty of them, as to it's Nature, Figure, and Conſtitution.

What the Nature and Eſſence of a Triangle or any other Figure in Geometry.

§. 8. **Let** us now Extend this Reaſoning to Geometry and the Mathematicks, and it may be Inquired, What is the Eſſence of a Triangle? To which it is Anſwered in the ſame Way, That any Certain and Undoubted Property of it, is it's Eſſence, and not any One ſuch Property in Preference to the Other; Becauſe where any Certain and Undoubted Property of a Triangle Exiſts, all the Others will Follow, upon it's being Applied, or Referred, to Different Quantities, and upon it's being Conſidered under Different Relations; Thus Three Lines Inclined to Each Other, ſo as to Include Space, or ſo as to Form Three Angles, may be Termed the Eſſence of it, and Generally is ſo Eſteemed to be; But there ſeems to be no more Reaſon for it, than to Aſſert, that the Three Angles of a Triangle being Equal to Two Right Ones, is it's Eſſence, or, that the External Angle of a Triangle being Equal to it's Two Internal Oppoſite Ones, is that Eſſence, or any other Property of it, which we can Name, when it is Compared with, and Referred to, other Quantities; Becauſe Each of theſe Properties of Figure will Infer One Another; Thus, Suppoſing there is a Figure, which Conſiſts of Three Lines and Three Angles, the other Properties will be Conſequent, when it is Compared with, and Referred to, other Quantities, and if a Figure is Referred to, and Compared with, other Quantities, and has theſe Properties, it muſt be a Triangle; What I Mean is, that all the Properties, which can Reſult from a Triangle, either Conſidered with Reſpect to it's Lines and Angles, or in Compariſon with any other Quantities, are Intimately United and Connected together, and if we have One Property, all the Reſt muſt Follow, whenever the Relation, Reference, or Compariſon is made.

And this is ſo True, that the Late Geometers Diſtinguiſh their Curves, and the Natures or Eſſences of them, by Algebraick Equations; Becauſe, where thoſe Equations are, all the Properties, which can be Imagined or Found out, in Reſpect of thoſe Curves, will be a Neceſſary Reſult and Conſequence from ſuch Equations; As is Evident from *L'Ho'pitals* Conick Sections.

Thus, $yy = px$, Expreſſes the Nature of a Parabola.

Alſo, $yy = \frac{1}{2}pt - \frac{pxx}{2t}$, the Nature of an Ellipſis.

And, $yy = \frac{pxx}{2t} + \frac{1}{2}pt$, the Nature of an Hyperbola.

With Reſpect and Relation to their Axes.

L'Ho'pital's Conique Sect. Livre. 1. Prop. 1. Livre. 2. Prop. 2. Livre. 3. Prop. 2.

Before whom our own Country Man, the Famous Dr. *Wallis*, Late *Savilian* Profeſſor of Geometry in *Oxford*, in his Treatiſe of Conick Sections, and Who Firſt Took the Sections out of the Cone, and Conſidered them, Abſtracted from and Independent of it, Propoſed this Equation for the Parabola, Namely, $p^2 = dl$, which he Says is the Eſſential Symptome, Property, or Affection of it, and from whence all the Reſt may be Deduced;

Dr. Wallis's Opera. Mathem. Vol. 1. De Sect. Conicis. Prop. 12.

Likewise, for the Ellipſis, $e^2 = ld - \frac{l}{t}d^2$, which Expreſſes the Nature of the Ellipſis, and from whence all the Properties and Affections of it may be Derived, which is there done by him;

Prop. 16. 28.

And for the Hyperbola, $h^2 = ld + \frac{l}{t}d^2$, which is it's Eſſence, or Eſſential Property, from which all it's Others may be Inferred; becauſe they are all Intimately United and Connected together.

Prop. 20. 35.

On the other Hand, If we take any One of theſe Properties, which are Derived from theſe Equations, we may call it the Eſſence of theſe Figures, and the Equation will be an Eſſential Property, becauſe it will Neceſſarily Follow from ſuch Property, which is now Suppoſed the Eſſence; That is, every Eſſential Property of any Figure has an Equal Claim to be Eſteemed the Eſſence of it; Becauſe they Mutually Infer and Conclude Each Other.

Thus in the Hyberbola, *1ſt.* If $h^2 = ld + \frac{l}{t}d^2$, it will be Conſequent; *2d.* That $h = \sqrt{}:ld + \frac{l}{t}d^2 = \sqrt{}:\frac{td+d^2}{t}l$, *3d.* That $l = \frac{h^2}{td+d^2}t$; *4th.* That $t = \frac{d^2 l}{h^2 - dl}$ *&c.* Each of which Equations are Linked and United together in that Manner, that, if One is Suppoſed, the Reſt do Neceſſarily Enſue; And therefore the *2d. 3d.* or *4th.* Equation may be as well Termed the Eſſence of this Figure, as the *1ſt.* And this, as we have ſaid before, is the Caſe of Material Subſtances or Eſſences, which will be Equally Expreſſed by any of their Eſſential Properties, becauſe they are Joined and Subſiſt together by an Indiſſoluble Connexion.

What the Obſcure and General Subſtance is.

§. 9. When the Mind has got it's Ideas of Particular Material Subſtances, if it Proceeds to Form a General One, as that of Matter, it is done by Throwing out of it's Idea every Thing, which is Particular, and which Diſtinguiſhes One Portion of Matter from Another, and Retaining only, what is General and Common to all the Parcels of it; Thus if we find, that all the ſeveral Portions of Matter Conſiſt in the Various Degrees of the Expanſive and Contractive Forces, we Abſtract from all Particular Degrees of thoſe Forces, and Conſider the Expanſive and Contractive Force in General, and in which we may Say the General Nature and Eſſence of Matter Lies; and if we farther Abſtract Force from the Expanſive or Contractive, we may then Affirm, that the General Subſtance, Subſtratum, or Eſſence, of Matter is Force, or Action; In all which there is nothing Obſcure or Unintelligible; Tho' in the Preſent Way of Thinking in Philoſophy, All is Involved in Clouds, Darkneſs, and Confuſion; And we are Entertained with little elſe upon this Subject of Eſſences, and Subſtratums, Excepting with an Unknown, Inconceivable, Something, It cannot Tell What, nor has any Imaginable Way of Comprehending.

What the Eſſences of Particular Human Minds are.

§. 10. We may farther, from this Doctrine of Subſtances or Subſtratums, Give ſome Account of the Eſſences of Particular Minds, and from thence of the Eſſence of Mind in General; As to Particular Minds, we ſhall Inſtance in thoſe of Men, with which we are moſt Acquainted, and upon which therefore our Reaſonings in Philoſophy will be the beſt Grounded; For thoſe of Brutes we are not ſo fully Informed of.

The Eſſence therefore of any Particular Human Mind Conſiſts in any One Eſſential Property of it, with which all the Others, which are Eſſential to it, are Connected; Thus, if any One has a Particular Force of Perception, he will likewiſe have a Particular Force of Volition, Memory, Recollection, Comparing, and Compounding his Ideas, a Particular Force of Judgment and Reaſoning, Particular Paſſions and Forces of them, all Joined and United together, ſo that if he has One, he has all of them; And this is ſo True, and ſo Evident and Certain, that it will be never found in Nature, where One Man's Mind was Exactly and in every Reſpect the ſame with Another's, either as to his Force, or Quickneſs and Slowneſs, of Perception, of Volition, of Memory, Judgment, Paſſions, or of any other of his Faculties. This is the Riſe and Cauſe of all the ſeveral Genius's amongſt Men, the Peculiar Force and Diſtinction of their ſeveral Faculties; By which, One becomes a Poet, Another a Philoſopher, a Third an Orator, or a Stateſman; And it would be as Impoſſible, for One Man, who has his Certain Force of Perception, not to have a Force of Volition, Memory and Judgment, *&c.* Peculiar to that Force, and Vice Verſâ, as it would be for a Being, which has the Yellowneſs of Gold, or the Weight of it, not to have the other Qualities belonging to it, and Vice Verſâ; or, as it would be for a Parabola, an Ellipſis, or Hyperbola, not to have all their Properties Appertaining to them, upon the Suppoſition of One, with which all the Reſt are United and Connected; The Eſſence, Subſtratum, or Subſtance, therefore of any Human Mind Conſiſts in the Particular Force of Paſſion or Emotion in that Mind, or in the Particular Force of it's Perception or Volition, or in That of Reaſoning or of Judgment, *&c.* which Mutually Infer each other; and the One may be Taken for the Eſſence, and the Other for the Eſſential Property, or Vice Verſâ.

How the Eſſential Properties of Particular Human Minds are Connected with each other; Namely, how Perception and Volition are Connected.

§. 11. The Only and Principal Difficulty is, how the Properties of any Particular Human Mind are Connected with each other, as the Properties of Gold, or of a Parabola, an Ellipſis, or an Hyperbola, are; Which may be thus Explained; Where there is Perception, there is Volition always Attending it; For Volition is only Beginning an Action of it's ſelf; Thus, if a Ball ſhould Move it's ſelf in any Direction, it would Will that Direction; But Perception is Beginning an Action of it's ſelf, becauſe, tho' there were a Thouſand Impreſſions made upon the Mind from External Objects, they would be no more than ſo many Impulſes upon Body, without Producing any Senſation at all, if the Mind did not of it's ſelf, and from it's own Innate Force begin the Perception; for the Mind therefore to Perceive of it's ſelf is the ſame, as for it to Act of it's ſelf, and where any Thing Acts of it's ſelf, there is Volition; That this is ſo, is Evident even in our Senſations, which are Eſteemed the moſt Neceſſary; As in Seeing an Object, which is Preſented to us, when our Eyes are Open, it is ſaid, we cannot Avoid Perceiving it; but it muſt likewiſe be Affirmed, that we could not Perceive that Object, unleſs the Mind had a Principle of doing it, that is, unleſs it was of it's ſelf Active in Perceiving it, for if it was only Paſſive, there would nothing more Reſult from an Impreſſion on it, than from One Piece of Matter, which has no Principle of Perception, Acting upon another Paſſive Subſtance, which has no Principle of Perception, which at the moſt would be only Motion; Beſides which, we can Shut our Eyes at Pleaſure, and by that Means Reject the Impreſſion from the External Object; that therefore we keep them Open to Receive it, is as much an Act of Volition, as if we had Shut them, and thereby Refuſed the Senſation; And from Hence it is, that even in Dreams, for the ſame Reaſoning will Obtain Concerning them, the Mind is of it's ſelf Active, and therefore has Volition in thoſe Perceptions, which it Experiences in them; And from whence it is, that the Reverſe is True; That, where there is Volition, there is always Perception, for no One ever Doubted, that, where a Man Wills, he Perceives, and why, where a Man Perceives, he ſhould not Will, cannot be ſaid, or with any Juſtice Maintained; And, as this Force of Action or Perception is Weaker or Stronger in the Mind, ſo is that of Volition, and Vice Verſâ.

How Perception and Memory are Connected.

Perception and Volition being thus United, it is Eaſy from thence to Shew, how Memory muſt be Conſequent; becauſe a Recollecting our Perceptions is Manifeſtly Derived from an Active Principle of the Mind, and the Mind, which can Begin Action or Perception in it's ſelf from the Impreſſions of External Objects, muſt Neceſſarily have a Power of Beginning Action or Perception in it's ſelf, from the Impreſſion of it's own Perceptions; For Memory, or Recollection, is only Perceiving a Thing before Perceived, with a Will or an Act of Volition to do it; And as Perception is to External Objects, ſo is Memory to Perception; From whence it will alſo Follow, that as the Force of Perception is Weaker or Stronger in the Mind, ſo will the Force of Memory or Recollection be, and Vice Verſâ.

How Perception and Reaſoning are United, as alſo the Paſſions.

Perception, Volition, and Memory, being thus Connected, Reaſoning Enſues, by which we Diſtinguiſh our Perceptions from each Other, and which Reſults from our very Faculty of Perceiving, and by which we Compare them with each Other, which Follows from Volition and Memory, and from thence make Deductions, Inferences, and Concluſions from them; And as the Force of Perception, Volition, and Memory, is Weaker or Stronger, ſo will the Force of Reaſoning be, and Vice Verſâ.

Thus, alſo the Paſſions of the Mind are Evidently Derived from the Peculiar Force of Perception and Memory in it; Since therefore all theſe Properties of any Human Mind are Connected with each Other, If we have any One of them, we ſhall have All of them, and in that Particular Force and Degree, which the One or the Other Preſcribes; So that, if we have ſuch a Certain Force of Perception, we ſhall have ſuch a Certain Force of Volition, which belongs to it, and is Joined with it, ſuch a Certain Force of Memory, Reaſoning, *&c.* And Vice Verſâ, If we have ſuch a Certain Force of Reaſoning, we ſhall have ſuch a Certain Force of Memory, Volition, Perception, *&c.* I will not Pretend here Exactly to Define what Proportion of Force of Perception there muſt be, or of Volition, or Memory, or

Reaſoning, which will Mutually Inferr each Other; It is Sufficient in General to Shew, how theſe Faculties of the Human Mind are Connected together, and in Different Degrees of Force; And that any One of the Properties of it is enough to Denominate it's Eſſence, Genius, or Nature, ſince all the Others are Connected with it.

What the Eſſences of the Minds of Brutes, and of the ſtill more Inferior Part of Nature.

§. 12. From Hence we may Reaſonably Conjecture, in what the Eſſences and Natures of the Minds of Brutes Conſiſt, Namely, in a very Low Degree or Force of Perception and Volition, and Conſequently of Memory or of Reaſoning; That is, the moſt Lively and Vivacious of them ſeem to be Raiſed little Higher than Matter, their Minds have Juſt Action enough in themſelves to begin Perception; They Perceive Objects, which are Impreſſed on their Minds, but take not much farther Notice of them, and I Believe it may be ſaid in General of them, that they never Perceive them with any Kind of Force, Activity, or Attention; And they are as much Paſſive, as an Active and Intelligent Nature can be; Their Force of Volition therefore is Suitably Low and Depreſſed, for we find no Exerciſes of that Faculty, Excepting to Walk, or to Fly, or to Swim about, for the Preſervation of Themſelves, or the Conſervation of their Species; Their Memory, likewiſe, Conſiſts in a Repetition of the ſame Senſations upon their Minds, by which they at Length Learn to Diſtinguiſh ſome Few Objects, which are the moſt Advantageous or Pernicious to them.

And their Reaſoning ſeems to be Correſpondent and Anſwerable to their Perception, Volition, and Memory, Inconſiderable, Triffling, and Next to Nothing.

If we Deſcend Lower to a Cockle or an Oyſter, we ſhall find, that as the Perception in them is of the Leaſt Degree or Force Imaginable, ſo is the Volition, the Memory and Reaſoning of them, by which they are Placed but little above the Vegetable Kingdom.

If we therefore now go ſtill Lower to Vegetables, there is nothing more Plain, than that they have a Peculiar Action to themſelves, Beginning and Ariſing from them, by which they Exert themſelves in a Thouſand Various Directions, but there can be no Action from it's ſelf without Volition, and there can be no Volition without Perception, and therefore we may Suppoſe very Juſtly, that even Plants and Vegetables have a Degree of Perception and Volition, tho' Infinitely Below, even that of a Cockle or an Oyſter.

Lastly, the Inanimate Part of Nature we have Endeavoured to Shew, Conſiſts of Various Degrees of the Expanſive and Contractive Forces, which Forces, being Active of themſelves, muſt alſo Infer ſome Degree and Force of Volition, and Conſequently of Perception, but this is in ſo Infinitely Small a Degree, that it may be Eſteemed as Nothing, or as mere Stupidity, or as a General and Dull Senſe Diffuſed by Almighty God thro' his Whole Creation.

This Opinion is in ſome Meaſure Confirmed by the Small Animalcula, which are Found in Vinegar, as almoſt in all Nature, which Move in all Directions, tho' theſe little Eels ſhall be Ten Thouſand Times Leſs than the Fineſt Hair, which can be Imagined; And is it not more Rational to Affirm, that ſuch Anguillæ are Minute Parcels of Matter, which are Endued by God with a Principle of Beginning Action, and therefore have Volition and Perception, than to Suppoſe they have all the Organs of an Animal? For how is it more Contrary to the Wiſdom of Almighty God to Impart ſuch a Property of Self-Action, Volition, and Perception to a Small Hair of Matter Unorganized, than to Communicate it to a Syſteme of Matter Organized, that is, to a Brute, an Oyſter, or a Cockle?

What the Eſſences of Angelick Minds.

§. 13. We have hitherto made the Human Mind, as it is that, which we have the moſt Perfect Knowledge of, the Standard of our Reaſoning to all the Parts of Nature Below us; And if we carry our Inquiries to thoſe Intelligent Subſtances, or Eſſences, which are Above us, and to that of the Infinite and Divine Mind, we ſhall find the ſame Reaſoning to Obtain; Thus, if it is Asked, in what the Eſſence or Nature of Angelick Minds Conſiſts, it is Anſwered in the Degree and Force of their Perception, which is as much Superior to the Degree and Force of Human Perception, as that is to the Perception of a Brute, an Oyſter, a Plant, or any Being, as common Matter, which has the Leaſt Degree of it, and in Compariſon is Nothing; This Infinite Degree and Force of Perception, Above what we are Endued with, by which they Apprehend Things with an Infinitely Greater Quickneſs and Force than we do, is Attended with an Infinitely Greater Force of Volition, by which they are Active and Miniſtering Spirits to the Whole Univerſe, as we are to the Creatures Below us, and the Earth we Inhabit, and with an Infinitely Greater Force of Memory, Reaſoning, and Judgment, by which they can Baffle and Confound all the Short and Imperfect Reaſonings of Men.

What the Eſſence of the Infinite and Divine Mind.

§. 14. **And** in Reſpect of the Infinite and Divine Mind, even the Force of Perception, Volition, Memory, and Reaſoning, in the Angelick, is as much Below it, as that of the Human is Inferior to that of thoſe Exalted Beings and Intelligences; So that the Infinite and Divine Mind has an Infinite, Unlimited, and Inimitable Force of Perception, by which he Views and Comprehends all Nature at Once, Penetrates in One Inſtant into all the Beings and Powers of it, and Contemplates Millions of Millions of Worlds, with all the Innumerable Parts and Appendages of Them at one Glance, and with a Quickneſs of Thought, and a Force and Vivacity of Perception, which Infinitely Surpaſſes that, which is Exerted in Human Minds, whoſe Faculties are the moſt Expedite and Lively.

Consequent to this, his Force of Volition is likewiſe Immenſe, by which he Wills the Whole Order and OEconomy of Univerſal Nature, Eſtabliſhes the Laws and Rules of the ſeveral Beings in it, and this with ſo Boundleſs and Irreſiſtable a Force, that nothing is Able to Controul it, but every Thing has it's Effect, and Obtains it's Particular Powers and Conſtitution, Suitable to the Almighty Volition of the Supreme Diſpoſer.

The Force of Memory therefore of this Infinite Mind muſt be likewiſe Incomprehenſibly Great, Vaſt, and Stupendous, ſo that not the Minuteſt Thing has Paſſed, nor even any of the Ideas of his own Mind, in an Eternal Duration, which are not Preſent with him; the Force of Recollection is ſo Exceedingly Quick, that the Time of Recollection is Infinitely Diminiſhed, by which his Memory is not Properly ſuch, but is Reſolved into Actual Perception and Intuition.

Thus alſo, the Force of Reaſoning of this Divine and Infinite Mind is in like Manner Infinite, by which he does not Recollect his Ideas, but Retains them under a Preſent View and Contemplation, nor does he Compare and Reaſon upon his Ideas by a Temporary Deduction of Conſequences, but he at the ſame Time, and in the ſame Moment of it, Beholds, Perceives, and Apprehends, the Premiſſes, and the Concluſions.

Lastly, the Force of his Paſſions likewiſe muſt be from hence Infinite, by which he has an Infinite Deteſtation and Abhorrence of Vice, and an Unbounded Love and Eſteem for Piety and Virtue.

And theſe Properties are ſo Intimately United and Connected together, that where any One of them is, the Reſt muſt be Neceſſarily Inferred and Concluded from it, ſo that Each of them is Perfectly and Fully Expreſſive of the Divine Eſſence and Nature.

A Various Quantity or Force of Perception Aſſerted in this Doctrine of the Eſſences of Minds.

§. 15. I therefore here Affirm a Various Quantity, or Force of Perception, Volition, *&c.* by which the Properties of the Mind may be as Juſtly Reduced to Mathematicks and Proportions, as Extenſion may, and which, as was ſaid before, would Lay the Foundations of a Pſycheometria, as well as the other has Furniſhed us with the Elements of Geometry; For it is very Evident, that Men do Perceive the ſame Things with a Different Force, even in their Common and Ordinary Senſations; Thus, Some ſhall Naturally See Objects with Greater Quickneſs and Life, whilſt Others Behold them with more Dullneſs and Stupidity; ſo far, that the Volition of the Firſt ſhall be Excited to Conſider and Examine into the Objects Preſented, when the Laſt ſhall Sit down Satisfy'd, with only taking a Lazy Notice of the Impreſſion; the Firſt ſhall Immediately Penetrate into all the Circumſtances of ſuch Objects upon the Firſt Sight of them, the Laſt ſhall Apprehend them but Slowly and by Degrees; and there is nothing more Apparent, than that Men's Minds, and the Forces of them, are as Different in the Quickneſs and Slowneſs of their Senſations and Perceptions, as of their Apprehenſions, ſince the Latter is no more than another Term for the Former; And from hence it is, that Men Reaſon with a Different Force, which is a Perception of ſeveral Perceptions, and of the Diſtinction, or no Diſtinction, which Lies betwixt them; As alſo, that ſome Men's Volitions are more Strong and Impetuous, and other Men's more Faint and Languid; Thus Men, who in Learning have a Quick Perception of the Ideas and Studies of it, will be Carried into an Eager

Purſuit of ſuch Ideas and Studies ; But thoſe, whoſe Perceptions are Duller, find their Volitions as Heavy, and they will be Scarcely ever Perſuaded to make any Progreſſes in Knowledge, where the Nature of their Perceptions has given them ſo little Encouragement to Endeavour after it.

How the Eſſence of Mind in General is Obtained.

§. 16. When we have thus Formed our Notions of the Eſſences of Particular Minds, which Conſiſt in Certain Forces and Degrees of Perception, we from thence may Naturally and Reaſonably Proceed to Conſider, what the Eſſence of Mind in General is, which is to be Fixed and Determined by Throwing out of the Particular Eſſences of Minds every Thing, which Diſtinguiſhes the One from the Other; And ſince that is the Peculiar Forces of Perception, Volition, Memory, or Reaſoning, which belong to Each, The Eſſence of Mind in General is a Force of Perception, Volition, Memory, and Reaſoning, without Defining, what that Force in Particular is, whether Greater or Smaller, whether Infinite, Finite, or Next to None at all.

The Reaſon of the Preſent Errors in Philoſophy in Reſpect of Eſſences and Subſtances.

§. 17. The Reaſon, why the Preſent Philoſophy Diſcourſes ſo Obſcurely and Unintelligibly Concerning Subſtances, Subſtratums, or Eſſences, and with ſo little Truth, ſeems to be ; 1*ſt*. Becauſe it Imagines the Subſtratum, or Subſtance, to be ſomething Different and Diſtinct from the Eſſential Properties, which are Suppoſed to be Derived from it ; The one is the Root or Stock, the other the Branches ; The one the Baſis or Foundation, the other the Superſtructure upon it ; The One is the Support, the Other it's Dependences ; But this it is to Philoſophize by Metaphors and Similes, without Inquiring into the Nature of Things themſelves ; For the Eſſence of any Thing is neither the Root, nor the Baſis, nor the Support or Subſtratum, of it's Eſſential Properties ; All which Terms are made Uſe of to Explain it ; Since, as we have Shewn, every Eſſential Property as much Denotes the Eſſence of the Thing, as any Imagined Eſſence it's ſelf can do ; 2*d*. Becauſe it very Prepoſterouſly Conſiders General Subſtances, before Particular Ones, and Forms the Particular out of the General, which it Acknowledges to be the more Obſcure, whereas it ought Firſt to have Conſidered Particular Subſtances, and from thence Formed it's Notion of a General One, Agreeable to what we have here done.

The Falſe Account, which this Philoſophy gives of the Eſſence of the Supreme Mind.

§. 18. As to the Idea of the Supreme Being, which this Philoſophy gives, it is ſaid to be a Complex One of Exiſtence, Power, Knowledge, Duration, Pleaſure, Happineſs, and of ſeveral other Qualities and Powers, *which it is better to have than be without, with the Addition of Infinite to each of theſe* ; And it is very Punctually Affirmed, that we have no other Idea of the Supreme Being than ſuch a Complex One ; But I have already Shewn, that there may be another and a more Rational Idea Propoſed to us of the Infinite and Divine Mind ; On the Contrary, this which is Offered to us, is only a Complication of Abſtracted Ideas, each of which, as they are General Ones, are mere Creatures of the Mind, and Conſequently God himſelf, according to this Philoſophy, is nothing elſe than a Fancy and Imagination of it ; And, that it was of this Opinion, ſeems to be Plain and Manifeſt, ſince it Subjoins with a kind of Ridicule, that there were ſeveral other Qualities and Powers Annexed by us to this Idea, *which it was better to have than be without, with the Addition of Infinite* ; So that all the Idea, which we have of a Supreme Being, is an Idea made up of all the Fineſt Ideas we can Invent, and Put together, from our own Fancies and Imaginations, without any Certainty of the Exiſtence of ſuch a Being in Nature, unleſs as it Exiſts in our own Minds ; It muſt be yet ſaid in Excuſe of this Philoſophy, that it has given as Good a Definition of Almighty God, as it has done of Matter, which is likewiſe made to Conſiſt of a Combination of Abſtracted Ideas, as Solidity, Extenſion, Mobility, Diviſibility, *&c.* in General, which are only ſo many Creatures of the Mind, and ſo many Beings and Imaginations in it.

We cannot but Think in Reverſe of all this, that the Infinite and Almighty Being is a Real and not a Fantaſtical One, that he has in Truth and Reality an Infinite Force of Perception, and Volition, of Memory, Reaſoning, and Thought, which is his Eſſence, according to what has been before Explained, and that this may be Truly Collected from Conſidering the ſeveral Beings and Creatures of the Univerſe, their Nature, Order, Situation, Dependence upon, and Connexion with, each Other, both in the Animal and Material World ; In like Manner, as the Eſſence of Matter does not Conſiſt in the Abſtracted Ideas Abovementioned, but is Collected and Inferred from the Effects and Properties it Produces from the Different Applications of it to the Various Natures of Mind or of Matter.

§. 19. THIS Philoſophy yet Tells us; That the Infinite Wiſe Author of our Beings has Fitted our Organs and Faculties to the Conveniences of Life, and the Buſineſs we have to do Here; That we have alſo Inſight enough into the Admirable Contrivances of Things, and the Wonderfull Effects of them, to Admire and Magnify the Wiſdom, Power, and Goodneſs of their Author; That ſuch a Knowledge as this, which is Suited to our Preſent Condition, we Want not Faculties to Attain; and we have Abilities enough to Provide for the Conveniences of Living.

This Philoſophy Endeavours to Diſcourage any Searches after Truth, tho' it's whole Eſſay is Pretended to Promote it.

WHAT Notion this Philoſophy has of the Infinite and Al-wiſe Author of our Beings, has been Obſerved in the Foregoing Section; and if he has only Fitted our Organs and Faculties to the Conveniences of Life and to Know ſo much of Nature, as to Admire and Adore Him; There are only Two Ends, which any Man ſhould Propoſe, Namely, to Take Care of Himſelf, and to Worſhip the Deity; So that all Endeavours after Knowledge and Truth, are at once Caſhiered, which this Philoſophy ſeems to be ſo Fond of, and the firſt Thing we ſhould do, ſhould be to Throw away and Renounce this Mighty and Celebrated Eſſay on Human Underſtanding; For we have Abilities enough to Provide for the Conveniences of Life without it, and Senſe, and Religion, and Faculties enough, to Worſhip God, tho' we were Deprived of it's Pious and Worthy Inſtructions; Theſe ſeem therefore only to be a Set of Words put together in this Philoſophy to Cover it's Atheiſm, which has made the Deity to Conſiſt in Fancies of our own, and to Draw a Veil over, and to Palliate, it's Ignorance, which knows little of Nature, but the Abſtracted Ideas of the Corpuſcular Philoſophy, and little of Mathematicks, Excepting ſome few Propoſitions in the Firſt Book of the Elements of *Euclid*, and the Common and Ordinary Rules of Arithmetick and Numbers.

§. 20. As to what is ſaid in this Chapter, that we might have Different Ideas of Things, if our Senſes were Acute enough to Diſcern the Minute Particles of Bodies, and that Microſcopes are of Uſe to Aſſiſt them in ſuch an Inquiry; we Anſwer, that, if our Senſes were never ſo Acute, or Microſcopes never ſo Perfect, they could not Inform us of the Various Degrees of the Expanſive and Contractive Forces in Bodies; and as to the Minute Parts of them, we ſhall Leave them to the Corpuſcular Syſteme to Inveſtigate and Trace them out, as well as it can, by the Aſſiſtance of ſuch Valuable Glaſſes, and Deſire, that it would in Particular Search after, and Inquire into, the Minute Particles of Water, and the Pores and Interſtices, which Intervene them; ſince, according to the Preſent Philoſophy, there is 19 Times Leſs Matter in Water, than there is in an Equal Portion of Gold, and therefore, the Particles and Pores muſt needs be very Senſible by the Interpoſition of a Good Glaſs, and One well Contrived for ſo Notable an Invention.

Microſcopes of no Service to the finding out or Explaining the Properties of Nature.

§. 21. COLLECTIVE Ideas of Subſtances, as a Troop, Army, *&c.* do not ſeem to be Gained to the Mind by it's Power of Compounding, but by Actual Senſation, or by a Repreſentation from Others; For no One, I Believe, ever Sat down to Frame a Notion of a Troop or Army, by Putting Fifty, or Fifty Thouſand, Individual Men together; On the Contrary, when he has Seen, or Heard of, a Troop or an Army, he may then Poſſibly Conſider the Number, of which each Conſiſts.

Of Collective Ideas of Subſtances. Mr. Locke's Eſſay, Chap. 24. Book 2.

§. 22. THE Relative Ideas of Father, Brother, Son, Husband, Friend, Subject, General, *European*, *Engliſh-Man*, Iſlands, *&c*, are ſaid to be got by the Mind from the Comparing of One Thing with Another; Whereas they Appear, as the Former, to be rather Produced in the Mind by Experience and Obſervation, and then Diſtinguiſhed by Thinking, and Inquiſitive, Men into their Reſpects and Relations; Thus a Farmer, or a Peaſant, would call One Perſon a Father, Another a Son, as he Sees them Acknowledged in Life to be ſuch, without Formally and Previouſly Comparing or Relating theſe Ideas to each Other, and then Determining them; He may know himſelf to be an *Engliſhman* from Living in this Country, which is Called *England*, or an *Iſlander* from Inhabiting this Iſland, and yet Perhaps be Ignorant, what a Continent is, or whether there is any other Country in the World; As to what is ſaid, that Relations are better known than Things Related, it Proceeds from this Philoſophy's Imperfect Notion of Subſtances, or Subſtance in General, which has been before Explained, and which it is Evident it Knew little of.

Of Relation. Chap. 25. Book 2.

This Philoſophy is rather an Ingenious Romance, than a True and Natural Hiſtory of our Ideas, and how the Mind Gains them.

§. 23. The Caſe is this, in the Preſent Syſteme of Ideas, that ſince the Mind is made to be a Raſa Tabula, this Philoſophy Undertakes to give an Hiſtorical Account, how it is Furniſhed with that Variety of Ideas, which we Experience in it; The firſt Step therefore is to Repleniſh it with a Set of Simple Ideas, and then to Put it into Action, and Set it to Work upon theſe Materials by Abſtracting, Comparing, and Compounding of them; by which it has Formed a Quaint and Pretty Romance of the Mind's Procedure, but has by no Means Given us a Natural and Real Hiſtory of it; For I would Ask any Reaſonable Man, whether he Thinks, or does Imagine, that a Plain Honeſt Shepherd, or a Ploughman, Obtains all his Complex Ideas by the Method here Repreſented? If ſo, he muſt be a very Studious and Contemplative Perſon, and he would be ſo wholly Taken up in Comparing, Abſtracting, and Compounding his Ideas, that he would have little Time left for his Flock or his Furrows; On the Contrary, is it not more Juſt to Suppoſe, that he Gains his Complex Ideas, if he has any, in the ſame Manner, as his Simple, that is, by his Senſes and his Obſervation? All this Syſteme of Ideas therefore comes only to this, not to Inſtruct us, how we Gain our Ideas, and how every Human Mind is Furniſhed with them, but how a Curious and Metaphyſical Head may Divide and Diſtinguiſh them into Various Claſſes, and Give Diverſe Witty and Ingenious Denominations to them.

Of Cauſe and Effect, and other Relations. Mr. Locke's Eſſay, Chap. 26. Book 2.

§. 24. As to Cauſe and Effect, this Philoſophy only Informs us of the ſeveral Ways, by which an Effect is Produced, Namely, by Creation, as Matter, by Generation, as a Man, an Egg, a Roſe, *&c.* and by Alteration, as when any Simple Idea is Produced, which was not in the Subject before, or by Making, as any Artificial Things; However it may be Worth While to take Notice, that tho' this Philoſophy may ſeem to Acknowledge the Creation of Matter, yet it's Notion of Matter is ſuch, that it is beyond any Conceptions, how it is Poſſible for Almighty God to Create or to Annihilate it; For take any Solid, Extended, Particle of Matter, no One can ever Imagine, how that ſhould be made Solid or Extended out of Nothing, that is, out of that, which had no Solid Extenſion, or how ſuch a Solid Extenſion ſhould become not Solid or Extended; And therefore the Philoſophers of the Corpuſcular Perſuaſion very Wiſely make Space or Extenſion, and theſe Solid Particles of Matter, to be Eternal and Unproduced; And Conſequently, as Incapable of Diſſolution; For how can Omnipotence it's ſelf Diſſolve Extenſion into Nothing, or Solidity into a mere Extenſion? It ſeems to be little Leſs than to Suppoſe, Omnipotence can Effect, what is a Contradiction, and that he can make Extenſion to be no Extenſion, and Solidity to be no Solidity; The Truth is this, that Extenſion and Solidity are both Abſtracted Ideas or Creatures of the Mind in the Preſent Philoſophy, and tho' Almighty God can Create and Annihilate a Real and Active Matter, he cannot a Chimerical and a Fantaſtick One, which never was in Nature or in Being.

On the other Hand, if we Preſume Matter to Conſiſt in Force and Action, Derived from an Infinite Agent, there is nothing Inconceivable, how an Infinite Agent ſhould Produce Action, or Surceaſe that Action, that is, how Almighty God ſhould Create Matter, or Annihilate it.

To what is Diſcourſed farther Concerning Relations in this Chapter, we ſhall only Add; That all Nature Conſiſts of Relations, and thoſe Ideas, which we take to be the moſt Abſolute and Poſitive, are yet Relative; Thus Subſtance, as that of Gold, is Yellow with Reſpect to our Eyes, Heavy with Reſpect to the Earth, Solid with Reſpect to our Hands Preſſing it, Malleable with Reſpect to a Hammer, and Fuſible with Reſpect to Fire, according to what we have Propoſed in the Doctrine of Subſtances, §. 7.

Of Identity and Diverſity. Chap. 27. Book 2. *The General Notion of Identity, which this Philoſophy Propoſes Refuted.*

§. 25. In Relation to Identity and Diverſity, it is ſaid, that the Mind here Conſiders the very Being of Things, but that it Determines the Identity of them by Conſidering a Thing, as Exiſting at any Certain Time or Place, and Comparing it with it's ſelf, as Exiſting at any other Time, and Place; when therefore we Demand, whether any Thing be the Same or no, it Refers always to Something, that Exiſted ſuch a Time, and in ſuch a Place, which it was Certain at that Inſtant, was the Same with it's ſelf and no Other; This Looks like the Famous Definition of a Syllogiſm, by that Great Logician *Burgerſdicius, Syllogiſmus eſt Oratio, in quâ, quibuſdam poſitis, Diverſum quid à poſitis, propter ea quæ poſita ſunt, Neceſſario ſequitur*; For what is the *Thing*, which Exiſts at any Certain Time or Place? If we do not know, what this *Thing* is, how ſhall we Compare *it* with *it's ſelf*, at any other Certain Time, and Place? or which way ſhall we be Informed, what *it* is, or how *it* Agrees with *it's ſelf*? What is the *Something*, which Exiſted ſuch a Time, and in ſuch a Place, which it was Certain at that Inſtant, was the ſame with *it's ſelf*, that is, with *we know not what*, till the *Something* is Known and Determined?

It is too Evident, that the Identity of any Thing cannot be Determined from a Certain Time or Place alone, which are External to it, but from it's own Nature, Being, and Conſtitution; Thus, if we take a Particle of Similar Matter, which this Syſteme of Ideas Embraces, the ſame Particle of it may Exiſt in the Same Place, or in Different Places, in Different Inſtants of Time; and if we take a Particle of Heterogeneous or Diſſimilar Matter, it may be Different in the Same Place, or in Different Places, in Different Inſtants of Time, as a Coal, by Fire, may be Turned to a Cinder.

Time and Place have therefore nothing to do in Aſcertaining the Identity of Things, in Excluſion of the Nature of the Things themſelves, but we muſt rather Inquire into the Real Nature of them; This Philoſophy yet was willing to make the moſt it could of the Firſt, as it Underſtood little or nothing of the Latter, and yet could not but Confeſs, that when the Mind Examined into the Idea of Identity, it Conſidered the very Being of Things; See the Beginning of this Section.

Concerning the Identity of Subſtances, and Firſt of the Identity of God.

§. 26. After giving us a General Notion of Identity, this Philoſophy would Fix that of Subſtances, which will be found to be Attempted with as little Succeſs; They are ſaid to be of three Sorts, 1*ſt*. That of God; 2*d*. That of Finite Intelligences; 3*d*. That of Bodies.

As to the Firſt, God being Eternal, Unalterable, and every Where, Concerning his Identity, it is ſaid, there can be no Doubt; moſt Certainly, ſince if Identity is to be Determined by Time and Place, what is Eternal, and every Where, muſt be Unalterable, and therefore the Same; But this has been Shewn to be a Falſe Notion of Identity; and in the Preſent Caſe, a Being may be Eternal, and every Where, and yet not Unalterable, unleſs it is Proved and Evidenced to be ſo from it's own Nature; And Conſequently, the Identity of God muſt be Derived and Concluded from his Infinite Force of Perception, Volition, Recollection, and Thought, of Love and Abhorrence, which are Properties Intimately United and Connected with each Other, as before Explained; But what is thus Infinite, muſt be Unaltered, and Unchangeably the Same. I have here Mentioned Paſſions, as Appertaining to the Divine Mind, Namely, an Infinite Love of what is Virtuous and Good, and an Infinite Deteſtation and Abhorrence of Evil; Nor does it ſeem Derogatory to the Perfection of his Nature, to Admit even of Tranſient and Temporary Paſſions, which are the Reſult of ſuch a Perfection, as an Infinite Love, Delight, and Pleaſure in Particular Men, and Particular Actions of Men, who are Good, and an Infinite Abhorrence and Averſion to ſuch as are Profligate and Ill; Since, as Religion and Revelation does Exhibit and Repreſent ſuch a Being to us, ſo we may as Juſtly Collect and Infer theſe Paſſions or Properties in him, in an Infinite Degree of Force, from what we Obſerve in our Selves, in a Finite, as we may his Infinite Force of Perception, Volition, *&c.* from that, which in us is Limited and Confined; God yet cannot Admit of Fear, becauſe he knows he is Almighty; Nor can he Grieve or Repent, becauſe he is Infinitely Wiſe, Juſt, and Good; Nor is he Capable of any other Paſſions, which Imply a Weakneſs and Imperfection in him, which Love of Good, and an Abhorrence of Ill do not; but on the Contrary, are the Effects and the Inſtances of the Infinite Rectitude of his Will, and Demonſtrations of his Soveraignty and Power, which Raiſes and Supports the Virtuous and Humble, and Confounds and Overturns the Haughtineſs of the Inſolent and Proud.

Of the Identity of Finite Spirits.

§. 27. Second, it is Told us, that Finite Spirits, having had their Determinate Time and Place of Beginning to Exiſt, the Relation to that Time and Place will always Determine to each it's Identity, as long as it Exiſts.

On the other Hand we Affirm, that the Identity of Finite Spirits cannot be Eſtimated from Time and Place only, which are Circumſtances External to them, but muſt be Defined from the ſeveral and Various Forces of their Perception, Volition, *&c.*

If it ſhould be here Objected, that therefore, Suppoſing the Force of Perception, Volition, *&c.* ſhould be Impaired and Diminiſhed by a Diſtemper, or by Age, in a Finite Spirit, the Identity of it would be Deſtroyed, and Conſequently the Soul or Spirit of a Man will not be the Same, when he is in Health, and under the Paroxyſms of a Fever; when he is in the Vigour of Thirty, and in the Dotage of Ninety or an Hundred; It is Anſwered, that what Alterations are made in the Force of any Peculiar Mind or Spirit, and it's Perception, Volition, *&c.* Are Effected by Cauſes External to it, as the Diſtemper of the Body, or the Decay of it, which muſt Neceſſarily make Suitable Impreſſions; But this will not Deſtroy the Identity of the Mind or the Force of it, by which it is Diſtinguiſhed from all Others, which Identity is only here Conſidered, ſince, notwithſtanding theſe Impreſſions, it ſtill Preſerves thoſe Diſtinctions, Agreeable to it's Native Force, which no other Mind or Spirit, in the ſame Circumſtances would; Thus, let us Imagine Two Equal Bodies, to be Endued with their Peculiar Forces or Velocities, let the Velocity of One be as an Hundred, and the Other as Ten; It is Plain, if Another Equal Body with a Velocity of Five Incurs upon Each of them, tho' their Forces are Altered, yet the Alteration will be ſuch, as to Shew, that the One had the Velocity of an Hundred, and the other of Ten, by which the Identities of ſuch Forces of Ten and an Hundred, will be as much Aſcertained and Diſtinguiſhed from One Another, as if there had been no Impreſſion made upon them from the Incourſe of a Third.

On the Contrary, if the Identity of Finite Spirits is to be Determined from the Time and Place alone of their Beginning to Exiſt; It muſt be Suppoſed, that it is Impoſſible for Two Finite Spirits to Begin to Think in the ſame Place, at the ſame Time; Whereas, ſince Thought and Spirit is not Extended, One Finite Spirit may be Conſcious to it's own Perceptions, and Begin to be Conſcious of them in the ſame Place, and at the ſame Time, in which Another Finite Spirit is ſo; Thus, my Beginning to Think, and to be Conſcious of ſuch Thought, in a Certain Time and Place, does not Exclude any other Being from Beginning to Think, and being Conſcious of ſuch Thought, in the ſame Time, and Place; Unleſs it is Affirmed, that no Spirit can Think in the Same Place, where I do, which is Abſurd; ſince it muſt be Owned by this Philoſophy, that Almighty God, or any Angelick Nature may; Or, Unleſs it is Affirmed, that Almighty God cannot make an Angelick Nature, or any other Finite Spirit, Begin to Exiſt at the ſame Time, that I do.

From whence it ſtill ſeems to be Plain, that the Identity of any Finite Spirit is to be Determined from it's own Nature and Force of Perception, Volition, *&c.* and not from the Time Alone, when, or from the Place Alone, in which, it Exiſts; Nay Farther, we may as well, and more Truly, Aſſert the Identity of a Finite Spirit, from the Identity of it's Force of Perception, Volition, *&c.* tho' Beginning to Exiſt in Different Times and Places, as we may Defend the Identity of Finite Spirits from the Time and Place, in which they Exiſt; Since a Finite Spirit, tho' Beginning to Exiſt in the ſame Time and Place, may Change and Alter in it's Nature, for all that we Know of it; and tho' it may be Diſtinct from Others, and have the Principium Individuationis, by which it becomes that Individual Spirit, which it is, yet it may not be Conſtant to it's ſelf, by which it becomes an Identical One; But Identity and the Principium Individuationis ſeem to be not Sufficiently Diſtinguiſhed in this Philoſophy, both which are Fully Aſcertained in what Precedes at the Beginning of the Preſent Section.

Of the Identity of Matter.

§. 28. The Identity of Matter is not Determined from Time and Place Alone, but from it's Internal Being and Conſtitution, and from the Degree of the Expanſive and Contractive Forces, which each Portion Obtains; becauſe the ſame Matter may begin to Exiſt in Different Places, and at Different Times, as Gold, Ivory, Water, *&c.* As to the Inſtance given of a Particle or Atom of Matter, to which no Addition, or Subſtraction is made, that it's Identity is Defined by the Time and Place, in which it Exiſts, it is True, becauſe the Conſtitution of it is Suppoſed to be the ſame and Unaltered; It is a Portion of Similar Matter, which is Imagined to be it's ſelf, and Unvaried.

Of the Identity of Modes and Relations.

§. 29. The Identity and Diverſity of Modes and Relations are ſaid to be Determined after the ſame Manner, that Subſtances are; If therefore this Philoſophy has not given us a Juſt Account of the Identity of Subſtances, it will be Equally Deficient in that of the Identity of Modes and Relations, which I ſhall not, for that Reaſon, Particularly Conſider.

Of the Identity of Vegetables and Brutes.

§. 30. The Identity of Vegetables, we are Told, Depends not on the ſame Maſs, and is not Applied to the ſame Thing; The Reaſon of which is, the Difference between an Animate Body and Maſs of Matter; The Laſt being only the Coheſion of Particles any how United; The Firſt ſuch a Diſpoſition and Organization of Parts, as is Fit to Receive and Diſtribute Nouriſhment, ſo as to Continue and Frame the Wood, Bark, Leaves, *&c.* of an Oak for Inſtance, in which Conſiſts the Vegetable Life; That therefore, which has ſuch an Organization of Parts Partaking of one Common Life, Continues to be the ſame Plant, tho' that Life be Communicated to New Particles of Matter, Vitally United to the Living Plant; And it is ſaid, the Caſe is not ſo much Different in Brutes, but that any One may hence See, what Makes an Animal, and Continues it, the Same.

Identity therefore here is not Defined by Time and Place, but by a Certain Organization of Parts, Partaking of One Common Life, which is Fit to Frame the Parts of the Individual, and the Notion of Identity is Changed by this Philoſophy into Another, which is Different from the Former; Whereas, whatever Being is Denominated the Same, it ſhould Receive that Denomination from One and the Same Reaſon, Otherwiſe the Idea of Identity it's ſelf becomes an Idea of Diverſity.

On the other Hand, Suitable to what has been Alledged before Concerning the Identity of Subſtances, we Say, that the Identity of a Plant or a Vegetable Conſiſts, not only in the Time and Place of it's Exiſting, but in it's Peculiar Degree of it's Expanſive and Contractive Force, and, if Senſe is Allowed to belong to Vegetables, as it is Confeſſed in a Cockle and an Oyſter, in it's Peculiar Degree of Perception and Volition, which yet may be Infinitely Below that of the Laſt Mentioned Species of Beings.

The Identity of Brutes therefore muſt be Determined likewiſe from the Various Degrees of their Expanſive and Contractive Forces, and of their Perception and Volition, which Forces of Perception and Volition are Infinitely below thoſe of Men, by which they are only Lazily Carried into Actions, which Tend to the Conſervation of their Beings, but not into the Brisk, Eager, and Lively Ones of Study, or Contemplation, of Forming Laws, and Inſtituting Societies, and the Eſtabliſhing ſeveral Plans and Models of Religion, Polity, and Government.

Of the Identity of a Man and a Perſon.

§. 31. This Philoſophy Proceeds to Explain to us the Identity of a Man and a Perſon, which it has Thought Fit to Diſtinguiſh, and makes no Scruple of ſtill Varying it's Idea of Identity; which is Different, when Applied to a Man, or an Animal Body, and is ſaid to Conſiſt in a Participation of the ſame Continued Life, in Succeeding Particles of Matter Vitally United to the ſame Organized Body, and when to a Perſon, or an Intelligent Being, that Reaſons and Reflects, and can Conſider it's ſelf the ſame Thing in Different Times and Places, and which is ſaid to Conſiſt in Conſciouſneſs; And as it Adds alſo, when Applied to Subſtances, it is likewiſe Different. But Firſt, as we have already Obſerved, it is very Strange, that a Philoſophy, which Undertakes to Define and Explain to us the Identity of Things, ſhould be ſo Inconſtant in it's Notion of Identity it's ſelf, and ſhould make Identity to Conſiſt ſometimes in One Thing, and at other Times in Another; This is the ſame, as to Endeavour to give us an Idea of Identity, by an Idea of Diverſity, or to make Identity Conſiſt in Diverſity; Second, it has without any Manner of Reaſon, and Certainly without any Precedent, made the Man and the Perſon Different; For whoever Heard, that he who was ſuch a Man, was not ſuch a Perſon? Nay farther, it goes on to Inform us in Purſuance of this Diſtinction, that the Man is the Human Body, and the Perſon is the Human Soul; Tho' every One hitherto has Imagined, and I Believe, does ſtill Imagine, notwithſtanding this Wiſe and Subtile Colluſion, that a Man is a Being, Compounded of a Human Body and a Human Soul, or of Organized Matter and of a Mind and Intelligence; And yet according to this Philoſophy, any One may be a very Rational, Learned, and Underſtanding Man, without the leaſt Thought or Reflection belonging to him, becauſe the Man is nothing but an Organized Body, and Thought and Reflection is Confined to the Perſon; And on the Contrary, a Perſon may be very Handſome, well Shaped, may have

have a Good Aſpect, Walk Gracefully, and be Perſonable enough, without either Lineaments, Feature, or Proportion, or any Form or Motion, which he can Boaſt of; Becauſe the Body is the Property of the Man, and not of the Perſon. I cannot but Speak it with a little Kind of Indignation; Is this to Philoſophize? Is this to Reaſon? Or rather, Is it not to Play the Banterer and Buffoon? Nor do I See what Account can be given of ſo Monſtrous and ſo Abſurd a Diſtinction, as That of a Man from his Perſon, unleſs this Philoſophy, in it's Zeal againſt the Logicks of the Ancients, was willing to Ruin and Overturn that Eſtabliſhed Definition of a Man, that he was *Animal Rationale*, that is, a Being, which Conſiſted of an Organized or Animal Body, and of a Rational and Intelligent Mind; Which it Endeavours to do ſo Effectually, as to make a Man and a Brute the Same, and Both of them to be Nothing more than an Animal Nature.

Of the Identity of a Human Body.

§. 32. Laying aſide therefore this Ridiculous and Senſeleſs Diſtinction of a Man and a Perſon, Let us now Inquire into the Real Natures, which are Intended by thoſe Names, and the Identity of them; By Man therefore is Meant a Human Body, and by Perſon a Human Mind or Soul; According to this Philoſophy.

As to the Identity of a Human Body, or a Man, it is thus Defined, an Animal is a Living Organized Body, and the ſame Animal is the ſame Continued Life, Communicated to Different Particles of Matter, as they Happen Succeſſively to be United to that Organized Living Body, and our Notion of Man is but of a Particular Sort of Animal.

But what is this Organized Body with the ſame Continued Life Communicated to Different Particles of Matter, as they happen to be Succeſſively United to it, When the Organs of the Body are Undetermined, and, for all which this Philoſophy knows, may Change every Minute? The Capillary Veins and Arteries, at leaſt, may do this; The Small Nerves and Lympheducts, and Adipoſe Veſſels, likewiſe, may Undergo a Thouſand Alterations, when yet, it ſeems, the whole Syſteme of an Organized Body may be Preſerved the Same; Nay, a Man may Loſe his Organs of Seeing, or Hearing, of Taſte, or of Smell, may be Deprived of a Leg, an Arm, or any other Member, or Organ of Feeling, and Muſt we yet call this the Same Organized Body? If it is ſaid, the Identity of an Animal or Human Body does not Conſiſt in the Identity of the Organs, but in the Identity or Sameneſs of Life, Communicated to Different Particles of Matter, as they Happen to be United to them; We muſt again Inquire, what this Identity or Sameneſs of Life is? Does it Conſiſt in any Diſtinct Life, Spread and Diffuſed thro' this Organization of Parts, Separate from the Mind? If ſo, what is that Identity of Life, which Ariſes from an Organization, which is Various and Undetermined? To Anſwer the Queſtion, is to Confeſs an Abſurdity; If this Identity of Life is Communicated, not from the Organs themſelves, but from the Mind, from Intelligence, from the Conſcious and Reflecting Being, the Identity of the Organized Body, or of Man, will Conſiſt in the Identity of the Perſon, or of the Mind; And then, he that is the Same Man, is the Same Perſon, and Vice Verſâ, and this Philoſophy will be Obliged to Unite, what it had Took ſuch Pains to Diſtinguiſh. But beſides, what Connexion is there betwixt a mere Organized Body, and Perception, Reflection, and Conſciouſneſs, ſo that a Man's being Conſcious to his Thoughts ſhould give an Identity of Life to the Organs of his Body? All therefore, which this Philoſophy Propoſes, ſeems to be Entirely Romantick, and without any Foundation in Nature or in Reaſon.

On the Contrary, the Identity of a Human Body muſt Conſiſt in the Expanſive and Contractive Forces Peculiar to it, and which are Adapted to the Nature or Degree, and Force of Perception, Volition, *&c.* of the Mind, to which it is United; According to what has been in ſome Meaſure Explained in our Principles of Anatomy; For, let the Organs or Veſſels of the Body be what they will, of One Shape or Another, Fewer or More, we have there Endeavoured to Demonſtrate, that the very Formation of them muſt be Owing to theſe Forces, and to a Particular Degree and Proportion of them; But a Human Body, whilſt Living, cannot Exiſt of it's ſelf without a Mind or Soul to Actuate and Inform it, and therefore the Identity of a Human Body Conſiſts in the Degree of the Expanſive and Contractive Forces, by which it's Organs are Formed, in Conjunction with that Peculiar Force of Perception, Volition, *&c.* which Contributes it's Influences to the Conſtituting and Making the

Same Human Body; For there is no Rational Man will Say, that a Human, Living, and Organized Body has no Manner of Connexion with the Mind or the Soul, tho' this Philoſophy does in Terms Aſſert it.

On the other Hand, it is the Same Human Body, when Dead, which we had, when Alive and Actuated by the Soul; Becauſe, if the Same Soul was Reunited to it, it would Reſume the Same Peculiar Forces and Properties, which it had before, and is no more Different from it's ſelf by an Extinction of thoſe Forces, than a Body at Reſt would be from what it was, when in Motion, or than a Triangle would be Different from it's ſelf, when Circumſcribed by a Circle; For thus, the Human Body is the Same, tho' Conſidered under Different Relations, by which Identity it Obtains ſuch a Degree and Proportion of Force, when Informed by the Mind, and ſuch a Degree and Proportion of Quieſcence of it's Parts, when Separated from it; And by which it is Diſtinguiſhed from any other Human Body, when ſo Separated from the Soul or Mind, which Acts it.

Of the Identity of a Human Mind or Soul.

§. 33. The Identity of a Perſon, or of that Thinking and Intelligent Being, which we Perceive within us, or of the Human Mind or Soul, does not ſeem to Conſiſt in Conſciouſneſs, as this Philoſophy Suggeſts, but in that Particular Force and Degree of Perception, Volition, *&c.* which is Proper and Peculiar to it, by which it is It's ſelf, and Diſtinct from all Others; And tho' this Force and Degree of Perception, Volition, *&c.* may be Varied under Different Circumſtances of Life, as of a Diſtemper, or Eating and Drinking to Exceſs, and the like, yet the Human Mind or Soul is the Same, for that very Reaſon, that it is Suſceptible of Alterations, upon an Alteration and Change of Circumſtances; For if Fire was not Increaſed by Fuel, and Extinguiſhed by Water, it would not be the Same Fire; If a Square was not Divided into two Triangles by a Diagonal, and into two Parallelograms by a Perpendicular to it's Sides, it would not be the Same Square; On the other Hand, Thoſe, who make Perſonal Identity to Conſiſt in Conſciouſneſs, Lead us into many Abſurdities.

For Firſt, If ſo far only, as I am Conſcious of my Thoughts, ſo far only I am the Same Perſon, that is, the Same Thinking, Perceptive, and Intelligent Being, or the Same Human Mind or Soul, the Same Human Mind or Soul will become Different every Day, and almoſt every Hour, and Minute, of our Lives, and every Human Mind or Soul will be Split and Diſtinguiſhed into a Thouſand Different Ones; Thus, Ten Years ago, I was Conſcious to ſeveral Thoughts and Actions, which now I have Entirely Forgot, and cannot by any Ways or Methods of Recollection Recover; Nay even Yeſterday, I had many Thoughts, which I do not now Remember; And it is not an Hour ſince, that ſeveral Ideas Paſſed thro' my Mind, which at Preſent wholly Eſcape me; and it is but a Minute, from the Time, when I Red ſeveral Periods in *Demoſthenes*, of which I was then Conſcious, tho' I cannot at this very Juncture Repeat One of them; I therefore Ask, whether my Perſon, or which is all One, whether my Soul or Mind is the Same, which it was Ten Years ago, Yeſterday, an Hour ſince, a Minute, or not; If it is the Same, Then the Identity of my Soul or Mind does not Conſiſt in what I am Conſcious of; Becauſe I am not Conſcious of a Million of Things, which were at thoſe Times Preſent to my Mind, and of which I now Know nothing; If it is not the Same, Then this Soul or Mind, which I Poſſeſs, is the Same, when yet it is Acknowledged to be not the Same; It is the Same, Becauſe I am Conſcious of ſomething, which Paſſed Ten or Twenty Years ago, which Joins and Connects that Intelligent Being with That, of Which I am now Conſcious; But it is not the Same, Becauſe there have been Intervals of Time betwixt That and the Preſent, of which I am not any ways Conſcious; So that by this Way of Determining the Identity of the Human Mind or Soul, it is Connected with What it was, Ten or Twenty Years ago, and in all the Intermediate Time, of which it is not Conſcious, it is Unconnected with it; That is, the Mind is the Same or not, According, as it Remembers, or Forgets, the Ideas, which have Paſſed in it, which is to the utmoſt Degree Irrational and Abſurd.

Second, If Conſciouſneſs makes the Mind or Soul the Same, if that Conſciouſneſs Reſides in the little Finger, the Soul or Mind would be Same, as it is now, and is what this Philoſophy Confeſſes; And for the ſame Reaſon it might as well Affirm, that if the ſaid Conſciouſneſs Reſided in the Pen, I Write with, it would be the Same Human Soul; Which are ſuch Extravagant Aſſertions, and ſo Contrary to Nature, that no Philoſophy would ever have Taken it into it's Head, or it's Finger, to have Propoſed ſuch Romantick Fancies, unleſs ſuch a One, which is ſo Remarkable for Them; Since it is Unreaſonable to Affirm, that the Perſon, or the Intelligent Being, is the Same, where the Subſtances are Various, to which it is Annexed; For the ſame Properties of Thought, Conſciouſneſs, *&c.* cannot Reſult from Different Principles, or Subſtances, tho' this Philoſophy would Perſuade us, that it will be the ſame, whether Derived from a Material or an Immaterial Subſtance, whether it Reſides in the Head, or in the Finger, Separated and Cut off from the Body; But what can be more Extravagant and Unnatural, than to Take Properties from Things, in which they Really Exiſt, and Affix them to Thoſe, in which they never did, nor could? Conſciouſneſs is only One Property of the Mind amongſt ſeveral Others, and is beſides a General Term, which, when Abſtracted from the Subſtance, in which it Inheres, is the Same, and as Ridiculous, as if we ſhould Abſtract the Heat of the Fire from the Subſtance of it, and Place it in the Subſtance of Water or Ice, or the Yellowneſs and Extenſion of Gold, from it's Subſtance, and Place it in That of a Cork or a Feather.

THIRD, if Conſciouſneſs makes the Mind or Soul the Same, it will be Different under a Diſtraction, an Exceſs of Wine, or in Sleep, from what it is in a State of Wakefullneſs, or of Sobriety; Becauſe in the Latter Caſe, it is not Conſcious of what was done in the Former; To Solve this Difficulty, the Preſent Philoſohy Diſtinguiſhes betwixt the Man and the Perſon, and Tells us, that tho' in theſe Caſes, the Man is the Same, it may not be the Same Perſon; Or, according to it's own Explanations, there is, in theſe Circumſtances, the Same Human Body, but not the Same Human Soul; So that a Man is Mad, or Drunk, or A-Sleep with One Soul, and Awake and Sober with Another; Than which, what can be more Notable, or more Profound?

To Juſtify yet this Way of Reaſoning, we are Referred to ſome *Engliſh* Phraſes, or Ways of Speaking, of a Man's being not Himſelf, or beſides Himſelf, by which it is Inſinuated, that Self is Changed, and the Self-ſame Perſon is no Longer in that Man; But if we ſay, ſuch a Man, or Perſon, is Diſordered in his Mind, I Suppoſe the *Engliſh* is as Good, and the Phraſe as Common; Which yet Preſumes the Man and Perſon to be the Same, and that he has the Same Mind or Soul, tho' it's Powers and Faculties are Diſturbed by ſome Foreign and External Impreſſions.

FOUTRH, if Perſonal Identity is the Object of Reward and Puniſhment, that is, Conſciouſneſs, or the Thinking, Intelligent, Being within us, the Chriſtian Religion is Falſe in One of it's Fundamental Articles; which Teaches the Reſurrection of the Same Body; and that the Perſon, which ſhall be Rewarded or Puniſhed in another World, does not Conſiſt of the Mind only, but of a Soul and a Body, which ſhall be Reunited, and Jointly Receive the Sentence, which ſhall be then Pronounced by the Supreme Judge, of Eternal Happineſs, or Eternal Miſery.

BUT, Revelation Apart, what can be more Incongruous or Diſſonant to Reaſon, than to Imagine, First, that Conſciouſneſs ſhall be Rewarded or Puniſhed, and not the Soul, and all it's Faculties, in which ſuch Conſciouſneſs Reſides, whether ſuch Soul or Intelligence Forgets or Remembers it's Thoughts or Actions? Or Second, that the Soul ſhall be Rewarded or Puniſhed, and not the Whole Man, as Compounded of Soul and Body, ſince not the One in Diſtinction from the Other, but the whole Compounded Being was the Innocent, or the Criminal? Thus, in the Firſt Caſe, it would be but a Lame and Defective Kind of Juſtice amongſt Men, not to Reward or Puniſh Real Actions, according to the Intrinſick Virtue, or Malignancy of them, whether the Perſons, whoſe they are, are Conſcious of, or Remember them, or not; And in the Second, we ſhould Take it to be a Partial Diſtribution of it, to Reward or Puniſh a Man's Mind only, by Imparting Joy or Affliction to it, without making Uſe of thoſe Rewards or Puniſhments, which are, or may be, Termed Corporal, and which Affect the Mind and Body in Conjunction; For Inſtance in Puniſhments, in which Human Laws Chiefly and Principally Exert themſelves, Suppoſing any One has been Guilty of Murder, would it not be a very Ingenious and Philoſophick Doctrine to a Judge, and a very Agreeable One to the Aſſaſſine, to Plead in Open Court, that Perſonal Identity is only the Object of Puniſhment, that is, the Mind or Soul of the Man, that his Body therefore is to be Excuſed from any Penalty to be Inflicted upon it, and tho' he may Deſerve to be Pained and Grieved in his Mind, he does not Deſerve to be Treated in the Rough Manner, which Perhaps Laws, and the Reaſon of Mankind, would, and in Fact do, Preſcribe in ſo Wicked a Crime?

Of Proportional Relations.
Mr. Locke's Eſſay, Chap. 28. Book 2.

§. 34. As to Proportional Relations, This Philoſophy Tells us, that all Simple Ideas, wherein are Parts or Degrees, Afford an Occaſion of Comparing the Subjects, wherein they are, to One Another, in Reſpect of thoſe Simple Ideas, as Whiter, Sweeter, More, Leſs, *&c.* Which, Depending on the Equality and Exceſs of the ſame Simple Idea in ſeveral Subjects, may be Called Proportional Relations; Whereas, it is Impoſſible, that the ſame Simple Idea ſhould Admit of Parts or Degrees, or of Equality or Exceſs, if it is the ſame Simple Idea; Thus White, Sweet, and Quantity or Extenſion, are Simple, General, and Abſtracted Ideas, and as they are Simple, General, and Abſtracted Ideas, do not Admit of any Variation in Parts or Degrees; But when the Mind comes to make it's Compariſon of the Ideas of White, Sweet, Quantity or Extenſion, it Forms it's Relations of Whiter, Sweeter, More, or Leſs, from Obſerving Particular Ideas, which are Different in themſelves, and not from an Increaſing or Diminiſhing it's Simple or Abſtracted Idea of White, Sweet, Quantity or Extenſion, by it's own Power of Framing ſuch an Increaſe or Diminution of them.

Of Natural and Voluntary Relations.
Chap. 28. Book 2.

§. 35. NATURAL Relations, as Father, Son, Brother, *&c.* are ſaid to Ariſe from an Occaſion of Comparing Things, Taken from the Circumſtances of their Origine; Inſtituted or Voluntary, as General, Captain, Burgher, *&c.* from ſome Act, whereby any One comes by a Moral Right, Power, or Obligation, to do Something; The Laſt are Alterable, and Separable from the Perſons, to whom they ſometimes belonged, tho' Neither of the Subſtances ſo Related be Deſtroyed; The Firſt not ſo, but are as Laſting, as their Subjects.

THESE being Little more than Definitions of Terms, there is Little more to be ſaid Concerning them, Excepting, That the Mind does not Form theſe Complex Ideas from it's Comparing of Simple Ideas, but Receives them Complex from Experience and Obſervation, and then by Reflection and Reaſon Diſtinguiſhes them into the ſeveral Parts, of which they are Compoſed. We the rather take Notice of this here; Becauſe this Philoſophy had made the Mind a Raſa Tabula, or a Blank Paper, or an Empty and Void Room without any Furniture, which therefore it was to Supply; And this is done by Storing it with it's Simple Ideas from Senſation and Reflection, and from thence Deriving it's Complex Ones; On the Contrary we ſay, that what this Philoſophy Terms Simple Ideas, are Abſtracted Ones, as Colour, Sound, Extenſion, *&c.* and therefore are not Firſt in the Mind, but are Made by it; And on the other Hand, what it Names Complex Ideas, are Received Whole, and Compounded into the Mind, and are afterwards Separated into the Simple Ideas, or the Particulars, of which they Conſiſt.

Of Moral Relations.
Chap. 28. Book 2.

§. 36. MORAL Relations are Affirmed to be Derived from the Conformity or Diſagreement of our Voluntary Actions to a Rule, to which they are Referred, and by which they are Judged of; This Conformity or Diſagreement of our Actions to ſome Law, whereby Good or Evil is Drawn on us from the Will and Power of the Law-maker, and is what we call Reward or Puniſhment, is That, which Renders our Actions Morally Good or Evil; Of which Moral Rules or Laws there are Three Sorts with their Different Enforcements; Firſt, the Divine Law, whether Promulgated to us by the Light of Nature, or the Voice of Revelation; That there is ſuch a Law is Concluded, from God's Right to give us One, who are his Creatures; from his Goodneſs and Wiſdom to Direct our Actions to what is Beſt; and from his Power to Enforce it by Reward and Puniſhment of Infinite Weight and Duration; And this Law, or Rule, is ſaid to be the only True Touchſtone of Moral Rectitude, by which Men Judge of the moſt Conſiderable Moral Good or Evil of their Actions, and by which they are Denominated Duties or Sins, which are like to Procure them Happineſs, or Miſery, from the Hands of the Almighty. Second, the Civil Law, or the Rule Set by the Common Wealth, to the Actions of Thoſe, that belong to it, which has it's Peculiar Rewards and Puniſhments to Enforce it, as the Protecting, or Taking away, of Life, Liberty, and Eſtate, of Thoſe, who Obſerve or Diſobey it, and by which our Actions are Termed Innocent, or Criminal. Third, the

Law of Opinion or Reputation; Or, which is the ſame, the Law of Faſhion; Which has likewiſe it's Enforcements from the Praiſe or Blame, which Attends thoſe Actions which are Conformable to it, or Swerve and Deviate from it; And by which they are Defined to be Virtues or Vices; For tho' Virtue and Vice are Names Suppoſed every where, to Stand for Actions in their own Nature, Right and Wrong, yet ſo far, as they are ſo Applied, they are ſo far Coincident with the Divine Law; And it is Viſible, that theſe Names in the Particular Inſtances of their Application, thro' the ſeveral Nations and Societies of Men, are Conſtantly Attributed only to ſuch Actions, as in each Country and Society, are in Reputation or Diſcredit; For tho' that Paſſes for Virtue in one Place, which is elſewhere Accounted Vice, yet every where Virtue and Praiſe, Vice and Blame, go together; Virtue is every where, that, which is Thought Praiſe-worthy, and they have ſo Cloſe an Alliance, that they are often Called by the ſame Name; 'Tis yet ſaid, that Virtue and Vice do in a Great Meaſure every where Correſpond with the Unchangeable Rule of Right and Wrong, which the Laws of God have Eſtabliſhed, becauſe the Obſervation of theſe Laws Viſibly Secures, and Advances, the General Good of Mankind, and the Neglect of them Breeds Miſchief and Confuſion.

Morality therefore being Nothing, but a Reference of our Actions to theſe Laws or Rules, and theſe Rules being Nothing but a Collection of ſeveral Simple Ideas, the Conformity thereto is but ſo Ordering the Action, that the Simple Ideas belonging to it, may Correſpond to thoſe, which the Law Requires; from whence it is ſaid to be Plain, how Moral Beings and Notions are Founded on, and Terminated in, the Simple Ideas of Senſation and Reflection; Thus, in the Complex Idea of Murder; From Reflection we have the Simple Ideas of Willing, Conſidering, Purpoſing, Malice, *&c.* Alſo of Life, Perception, and Self-Motion; From Senſation we have the Ideas of Man, and the Action of Killing him; This Collection of Ideas, when Referred to the Firſt Rule or Law, according as it Agrees, or Diſagrees with it, we call Good or Evil, a Sin or a Duty; when to the Second, Lawful or Unlawful, Innocent or Criminal; and when to the Third, Praiſe-or Blame-worthy, Virtuous, or Vicious.

It is farther ſaid, that Moral Actions may be Conſidered two Ways; Firſt, as they are in themſelves a Collection of Simple Ideas, in which Senſe they are Poſitive, Abſolute Ideas; Second, as Good, or Bad, or Indifferent, in which Reſpect they are Relative, it being their Conformity or Diſagreement with ſome Rule, that makes them be ſo; Both which are Comprehended under one Name, which often Occaſions Confuſion, and Miſleads the Judgment, as Stealing a Madman's Sword from him is Stealing, but not Vicious or Criminal; Thus far the Preſent Philoſophy.

In Anſwer to which, we Alledge.

First, that what is here Meant by Moral Relations, is no more than this, that Moral Actions are nothing elſe but Human Actions Referred to ſome Law or Rule, by which they become Moral, whereas, as Human, they are in themſelves Indifferent, whether Murder, Theft, Adultery, *&c.* that is, there is no Intrinſick Good or Evil in our Actions, and the laſt Named are Nothing more than the Turn of a Hand, or the Writing on this Paper, unleſs ſo far as they are Referred to a Rule, by which they are Denominated Duties or Sins, Innocent or Criminal, Virtuous or Vicious, according to their Agreement or Diſagreement with the Divine Law, the Civil, or the Law of Faſhion, to which the Reference is ſeverally Made.

But, Suppoſing no Revelation Promulgated to Mankind, and no Natural Religion Proved, and no Civil Laws Enacted to Preſcribe to our Actions, and Faſhion had Determined nothing in the Caſe, would there be no Intrinſick Difference betwixt Virtue and Vice, betwixt Murder or Malice, and the Preſervation of a Man's Life or Benevolence? And without any of theſe Rules, would not the Former be Odious and Abominable, and the Laſt Virtuous and Good? Human Actions, without any Reference to a Poſitive Law, are as much Diſtinguiſhed into Good or Evil from the Nature of them, as Colours are into Beautiful and Ugly; Or any other Qualities in Bodies into thoſe, which are Agreeable, or Unpleaſant and Diſtaſteful; And we may as Juſtly Affirm, that Malice and Benevolence are the ſame, and that there is no Diſtinction betwixt them, as that the One is as Good as the Other, and as Innocent and Virtuous, if there is no Reference made to a Law, by which we ſhould Judge of them.

Second, if we Refer our Actions to a Law, by which it is to be Determined, whether they be Really Good or Evil, Virtuous or Vicious, Innocent or Criminal, there can be no other than the Divine, or That, which is Acknowledged to be the Revealed Will of God, whether in the Scriptures, or from Nature; God's Will to Mankind, as Revealed to us in the Chriſtian Syſteme, is Definite and Certain, and tho' there have been an Infinite Number of Diſputes Concerning ſome Doctrines in it, thoſe Generally Ariſe from Genius's, who are more Willing to Speculate upon, than to Practice, the Truths of it; The Scope, Deſign, and Intention of it is on all Hands Confeſſed, and it's Fundamental Articles of Faith are almoſt every where Acknowledged; And it is as Clear and Punctual in the Whole, as any Rule or Law can well be; Which yet, were it never ſo Explicit and Plain, might be Perverted and Miſinterpreted by Corrupt, or by Weak Minds, and from which no Rule or Law whatſoever is Secure.

God's Will to Mankind, as it is Revealed in Nature, is much more Intricate and Obſcure, and it will Require Vaſt Attention and Pains to Diſcover it, becauſe it is Impoſſible for us even to Know, that there is ſuch a Being, unleſs from the Nature of the Works and Operations, which are Suppoſed to be Derived from him, that is, unleſs we Enter into the Depths of Reaſon and Philoſophy.

As to the Light of Nature, which is here Mentioned by this Philoſophy, and ſeems to be Preferred to the Voice of Revelation, both as it has the Preference in Place, and as a Man would ſooner Chooſe to be Enlightened in a Thing, than barely Told it, for his Information; I Say, as to the Light of Nature, which this Philoſophy Eſpouſes, it is much the ſame, as the Light Within of the Enthuſiaſts, which Conſiſts in nothing more than in Conceit, and in a Profound Ignorance of what Chriſtianity is on the one Hand, or Nature on the other.

This Light of Nature therefore, or this Law of Reaſon, we Reject from being a Divine Law, becauſe Reaſon, as the Light of Enthuſiaſts, is a Vain and Deſultory Buſineſs, and is little elſe than the Various Fancies of Men, with which they are Pleaſed, and then Stamp the Sacred Character of Divine upon them; Moreover, how can Reaſon, in the beſt Senſe of it, Aſſume the Attribute of Divine? Since the Strictest Reaſon Depends upon Perception, Memory, Judgment, *&c.* which are all Human and Uninſpired; Unleſs therefore, what is Human is Divine, the Law of Reaſon or of Nature is no Divine Law; Laſtly, as Reaſon is Fallible in Ten Thouſand Inſtances, of which we have given ſome Examples in our Inquiry into the Preſent Sentiments of Philoſophers, it would be Unjuſt and Derogatory to the Divine Mind, to Charge all the Errors and Frailties of the Human upon it; which we muſt do, if we make our Poor and Imperfect Reaſon a Light Within, ſet up by Almighty God in our Minds, and to be nothing Leſs than a Ray of the Divinity.

In Reſpect of the Divine Law it is ſaid, that it ſeems Undeniable, that God has given us ſuch a Law; But the Arguments to Prove it are Weak and Inconcluſive; For God may have a Right to do it, as we are his Creatures, and yet may not have done it, ſince it is Confeſſed he has not done it to mere Brutes or Animals, which notwithſtanding are Owned to be his Creatures; He may have Goodneſs and Wiſdom to Direct our Actions to what is Beſt, and yet he may not have Thought Proper for Reaſons Unknown to us, to Exert ſuch Wiſdom and Goodneſs in that Particular, but might Leave us to the Liberty of our own Actions, as is the Caſe, with Reſpect to the Inferior Part of his Creation; He has Power to Enforce his Laws by Rewards and Puniſhments, of Infinite Weight and Duration, but it is not Evinced by this Philoſophy, that he has Determined and Reſolved to Exerciſe ſuch a Power; All therefore, which is here ſaid, to Prove the Certainty and Undeniableneſs of a Divine Law is only this, that becauſe it is in the Power of Almighty God to Enact ſuch a Law, therefore he has in Fact done it. Had Chriſtianity Argued ſo Oſcitantly and Lazily for the Truth of it's Divinity, as this Philoſophy has done for the Light of Nature, and the Law of Reaſon, it would have been Deſervedly Scouted and Ridiculed, for Impoſing upon Men's Faith, without Convincing their Underſtanding.

On the Contrary, the Evidences for the Divine Law of Revelation Depend upon Hiſtorical Facts, which cannot be Conteſted, on Prophecies, and the Completion of them, which cannot be Refuted, and on a Written, not an Oral Tradition, which no Man in his Wits, or his Senſes, would not Aſſent to.

The Evidences for the Divine Law, Collected and Inferred from Nature, are thoſe, which Ariſe from an Induſtrious and Diligent Inquiry into it, from ſome Certain Knowledge or other of it, and not from a Superficial Syſteme of Ideas, which Profeſſes, not to Meddle with, or Inquire into, the very Nature of the Mind, or of any Thing Elſe, only Deſires, it may be Allowed to give an Hiſtory, which Indeed is a Romance, of it; That is, this Philoſophy Takes a Method, which is the Reverſe of a True One, it would Reduce the Divine Law of Revelation to the Evidences of Reaſon, which is to be Aſcertained by Fact and by Hiſtory, and the Divine Law of Reaſon, to the Evidences of Hiſtory, in which yet it is Wonderfully Miſtaken, which ought to be Aſcertained by Nature and Philoſophy.

Third, the Civil Law, or the Law of the Common Wealth, is ſaid to be a Second Rule, to which Men Referr their Actions, by which they are Termed Innocent or Criminal; It is True, if by Innocent and Criminal, is Meant no more than ſuch Actions, which Correſpond with, or Offend againſt ſuch a Rule; For then it is only to Say, that Actions which do Agree with the Rule, do Agree with it, and thoſe which do not, do not, which is a very Wiſe, as well as a moſt Indiſputed, Axiom; But if by Innocent and Criminal is Meant what is Really Good or Evil, Virtuous or Vicious, it is Exceedingly Falſe, becauſe nothing can Determine what is ſo, but the Law of Revelation, or of Nature; Since the Civil Law, or that of a Common Wealth, Depends upon the Uncertain Tempers, Inclinations, and Intereſts of Men, which in Various Countries, Climates, and Circumſtances, are Different, and therefore can be no Certain Standard of what is Really Innocent or Criminal; Laws may be Enacted, which are Unjuſt and Unrighteous in themſelves, and which, it would be ſo far from being Innocent, that it would be Criminal, to Obey; And therefore the Civil Law, or the Law of a Common Wealth, cannot be a Juſt Rule for our Actions; Which is ſo far True, that even the very ſame Societies of Men Repeal and Diſannul their own Laws, or Decrees.

Fourth, the Law of Opinion, Reputation, or Faſhion, is Affirmed to be a Third Rule, to which we Refer our Actions, and by which they are made to be Virtuous, or Vicious; To which it is Anſwered, in like Manner, as before; That, If Virtue and Vice do not Signify Actions in their own Nature Good and Evil, this Philoſophy only Says, that thoſe Actions Meet with Blame, which Meet with the Diſlike of the Society, and thoſe with Praiſe, which Meet with the Approbation and Applauſe of it, which is a very Certain Truth, but a very Silly One; If Virtue and Vice do Signify Actions, in their own Nature, Good and Evil, it is not True, that Virtue and Praiſe, Vice and Blame go together; Becauſe in a Greater, or a Leſs, Society of Men, a Really Good Action may be Blamed and Reproved, and an Ill One Countenanced and Encouraged.

Fifth, Moral Beings and Notions are not Founded on, and Terminated in, the Simple Ideas of Senſation and Reflection; Becauſe, in the Inſtance of Murder, we do not Compound ſuch an Idea from ſeveral Simple Ones already Lodged in the Mind, and then Refer it to a Rule, but we Receive the whole Complex Idea at Once, and afterwards by Conſideration and Reflection, Separate and Divide it into it's ſeveral Parts, of which it is Made up; And we ſay it is a Bad Action, tho' we do not Immediately Think of God's or Men's Diſpleaſure or Diſlike to it; and we ſhould Pronounce it the Action of a Baſe, a Villainous, and Ill Temper, tho' the Laws of God, or the Decrees and Opinions of Men, were Silent in the Matter, for the ſame Reaſon that Malice is not Benevolence, nor Black, White, nor a Triangle, a Square or a Circle.

Of Clear and Obſcure, or Diſtinct, and Confuſed, Ideas.
Mr. Locke's Eſſay, Chap. 29. Book 2.

§. 37. We ſhall not ſay much of this Chapter, it being little more than Naming our Ideas, and Making a Diſtinction of them without a Difference; For there ſeems to be no Reaſon for the Difference Alledged betwixt Clear and Diſtinct, and Obſcure and Confuſed; ſince, I Believe, it may be Generally Affirmed of them, that thoſe, which are Clear, are Diſtinct, and Vice Verſa; and that on the Contrary, thoſe, which are Obſcure, are Confuſed, and Vice Verſa; Thus a Clear Idea is Defined to be, when it is ſuch, as the Object it's ſelf, from whence it is Taken, did in a well Ordered

Senſation or Perception Preſent it, which is done in a Diſtinct Idea, a Diſtinct Idea, That, wherein the Mind Perceives a Difference from all Others, which it does in a Clear Idea.

As to what is Mentioned of a Chiliaedrum, being Clear and Diſtinct in Reſpect to the Number of it's Sides, but Obſcure and Confuſed as to it's Figure; we ſay, 1000 in Number is Diſtinct from 999, as they are Abſtracted Collections of Units, but a 1000 Sides in the Chiliaedrum is not Diſtinguable from 999 in a Polygone of thoſe Sides, and therefore the Numbers, ſo far as they are Applied to Polygones, are Equally Obſcure and Confuſed with the Figures.

Of Real and Fantaſtical Ideas. Mr. Locke's Eſſay, Chap. 30. Book 2.

§. 38. Real Ideas are ſaid to be ſuch, as have a Foundation in Nature, ſuch as have a Conformity with the Real Being and Exiſtence of Things, or with their Archetypes.

Fantastical, ſuch as have no Foundation in Nature, nor any Conformity with that Reality of Being, to which they are Referred, as to their Archetypes.

All our Simple Ideas are Affirmed to be Real, not that they are Images or Patterns of what does Exiſt, but as they are the Certain Effects of Powers in Things without us, which are the Cauſes of them, whereby we Diſtinguiſh the Qualities, that are Really in Things themſelves.

Of Complex Ideas, Mixed Modes and Relations, having no other Reality, than what they have in the Minds of Men, nothing elſe, 'tis Alledged, is Required to make them Real, but a Poſſibility of Exiſting Conformable to them; and being themſelves Archetypes, they do not Differ from their Archetypes, and ſo cannot be Chimerical, unleſs we Join in them Ideas, which are Inconſiſtent, or Affix a Name to them, which has not Uſually that Signification.

Complex Ideas of Subſtances are alſo ſaid to be Real, ſo far as they are ſuch Combinations of Simple Ideas, as are Really United and Co-exiſt in Things without us; Thoſe Fantaſtical, which are made up of ſeveral Ideas, that never were found United, as Centaur, *&c.* Thus far the Preſent Syſteme.

On the Contrary; If we will Diſtinguiſh our Ideas into Real and Fantaſtical; All thoſe ſeem to be Real Ideas, whether Simple or Complex, which we Receive from Nature, either from Things without us, or from the Operations of our own Minds, and according, and in the ſame Manner, as they are Impreſſed, that is, all our Real Ideas are only Real Effects Produced in the Mind from Real Cauſes; Thus, a Table, a Fire, a Plant, or a Tree, a Man, the Mind, Volition, Perception, Memory, Father, Brother, Gratitude, Murder, *&c.* are Real Ideas, or any Other Idea, which comes into the Mind, as Received and Derived from Nature; Fantaſtical Ideas, on the other Hand, are ſuch, which are made by the Pleaſure of the Will, or the Strength and Force of the Imagination and Fancy.

Thus, If we make any Simple Idea to be the Pattern, Image, or Repreſentation, of the Thing it's ſelf, it is a Fantaſtical Idea; Becauſe we only Receive from Nature the Impreſſion of the Idea, and the Power of Impreſſing it; If we make a Simple Idea to Enter into the Mind, Separate and by it's ſelf, which always is United and Combined with Others in the Impreſſion, ſuch a Simple Idea is a Fantaſtical One, becauſe it is Owing to our own Fancy and Imagination, and not to Nature; If alſo this Simple Idea is an Abſtracted One, it is a Creature of the Mind, and not the Product of Nature, and therefore again Fantaſtical; If Laſtly, we make this Simple Idea to be Impreſſed upon the Mind by External Objects, and the Mind to be a Raſa Tabula only, and not as Active in Receiving the Impreſſion as the External Object in Impreſſing it, it is a Fantaſtical One, and not Agreeable to the Natural Forces, which Produce it. And yet theſe are the Ideas of this Philoſophy.

And in Reſpect of Complex Ideas; Mixed Modes, and Relations of Things, are Real Ideas, as Gratitude, Murder, Father, Brother, *&c.* becauſe they are Ideas Derived to us from the Real Exiſtence of ſuch Actions and Perſons, and not from a Poſſibility of their Exiſtence; For, if they had no Other Reality, than what they have in the Minds of Men, if they were only Archetypes there, that is, Complex Ideas of our own Creating and Forming, without any other Foundation in Nature, they would be Fantaſtical Ideas; Since, if a Poſſibility of Exiſtence was only Required to the Reality of ſuch Ideas, a Centaur or a Chimera might be a Real Idea, becauſe no One will Say it is Impoſſible for a Monſter to be Produced in Nature, tho' it is Acknowledged, that ſuch Ideas are Fantaſtical; Beſides, it is Abſurd, to Make the Reality of an Idea, in the Caſe of Simple Ideas, to Conſiſt in it's having a Foundation in Nature, and in the Caſe of Complex, to Conſiſt in it's having no Foundation in it, but wholly to Conſiſt in it's Exiſtence in the Mind; The Reaſon yet of this is Plain; Becauſe this Philoſophy of Ideas was to Form all it's Complex Ideas out of Simple Ones, and to Make both of them Real, otherwiſe the whole Contrivance, and all the Ideas of it, had been Fantaſtical; But whether, or no, it has not Incurred this Imputation, by the very Way, which it Endeavours to Avoid it, may be eaſily Seen, by what we Juſt now Offered to Prove and to Evince it; Since all it's Ideas, both Simple and Complex, for the Reaſons Alledged above, as they are the Creatures of the Mind, and not the Effects and Impreſſions of Nature, are Really Chimerical and Fantaſtical.

THUS likewiſe, our Ideas of Subſtances are Real, ſo far as they are Impreſſions Derived from Nature, and the Exiſtence of Things; But, when the Mind makes Subſtances to Conſiſt in Certain Subſtratums or Eſſences, which it Knows nothing of, and then Joins a Parcel of Qualities to them, which, it is Suppoſed, Flow from them, but how, or in what Manner, is Undetermined, our Ideas of Subſtances become Fantaſtical; and ſuch are the Ideas of Subſtances in the Preſent Philoſophy.

Of Ideas Adequate or Inadequate. Mr. Locke's Eſſay, Chap. 31. Book 2.

§. 39. AFTER this Philoſophy had Diſtinguiſhed it's Ideas into Real and Fantaſtical, which we have Conſidered in the foregoing Section, it Divides it's Real Ideas into Adequate or Inadequate; Adequate are ſaid to be Thoſe, which Perfectly Repreſent thoſe Archetypes, which the Mind Suppoſes them Taken from, and which it makes them to Stand for; Inadequate, Thoſe, which are ſuch, as do but Partially or Incompletely Repreſent thoſe Archetypes, to which they are Referred.

ALL our Simple Ideas are ſaid to be Adequate, as being the Effects of Certain Powers Ordained to Produce them in us, which cannot but be Adequate and Correſpondent to ſuch Powers.

ALL Complex Ideas of Modes, as Virtue, Vice, a Lie, Murder, being Voluntary Collections of Simple Ideas put together by the Mind, without Reference to any Real Archetypes, muſt be Adequate, and can only be Inadequate, when we would Conform them to Thoſe of another Perſon, to which yet they do not Agree; Our Ideas of Subſtances, if by that is Meant Certain Eſſences of Things, which are Unknown, or a Collection of Senſible Qualities, which will be always Imperfect, are Inadequate; Thus far the Preſent Philoſophy.

ON the other Hand, Simple Ideas are ſaid to be Adequate for the ſame Reaſon, for which they were before Affirmed to be Real, and therefore Real and Adequate, in this Caſe, are not Diſtinguiſhed; and in one Suppoſition they are not, for if the Idea is a Real Effect of a Cauſe which Produces it, it muſt be an Effect Adequate to the Cauſe; That is, the Idea, which is Produced in the Mind, as an Effect, by the Mind it's ſelf and the External Impreſſion upon it, as the Joint Cauſe of ſuch an Effect, muſt be Adequate, the One to the Other; But this is not the Meaning or Intention of this Philoſophy, which Suppoſes the Mind to be a Raſa Tabula, or Merely Paſſive in it's Perceptions, and External Objects or Bodies to be Active upon, and to Imprint or Engrave their ſeveral Characters in it; We ſay therefore in ſuch a Suppoſition, that the Idea or Effect is not Adequate to the External Object or the Cauſe; ſince it is Impoſſible, that External Body ſhould Produce any Idea or Senſation in us, unleſs there was ſome Original Perceptive and Active Force in the Mind, which Concurred, and was Aſſiſtant to the Production; And it is Surprizing in this Philoſophy, that it makes Body Active, which is Defined, and Eſteemed by it, to be Paſſive, and the Mind, or Principle of it, Paſſive, which is known to be Active; and all to Favour it's New Syſteme of Ideas, which is to Explain and Unravel the Grand Myſtery, or the Great Falſehood, to us, how the Mind, as a Raſa Tabula, becomes Replete with all it's Furniture; and which is done by a Thouſand Contradictions; Of which this is One, which we have Mentioned.

AGAIN, if we Refer our Simple Ideas, as they Lie in the Mind to the Objects, which are Suppoſed to Produce them, in Reſpect of the Similitude, or Likeneſs, of the One to the Other, they are Inadequate; our Ideas not being the Repreſentations or Reſemblances of Things themſelves, but only the Effects of Certain Powers in the Mind and Body to Produce them, and in which Particular only they become Adequate to each Other; So that, ſo far as Simple Ideas are Adequate, they are Real, and on the Contrary; And ſo far as they are Inadequate, they are Fantaſtical, and Vice Verſâ.

Complex Modes, if they are only Voluntary Collections of Simple Ideas, as they are Affirmed to be, can neither be Adequate nor Inadequate; becauſe they are not Capable of being Compared with any Thing, as being their own Archetypes, they are Originals Themſelves, and therefore cannot be Referred to Any; But would another Philoſophy, beſides the Preſent, make Virtue, Vice, Gratitude, a Lie, Murder, *&c*, only Voluntary, or Arbitrary, Collections of Simple Ideas? Is there no Foundation in Nature of Them, to which when they Correſpond, they may be ſaid to be Adequate, and when they do not, Inadequate? Thus, in it's own Inſtance of Murder, it ſeems Unreaſonable to make a Collection of all the Ideas of Benevolence and Goodneſs, and then give it that Denomination, this would be far from being an Adequate Idea of it; and yet, according to this Syſteme, we may do this, becauſe it is a Voluntary Collection of Ideas, to which we may Affix, what Name we Pleaſe; There is no One Doubts, but Names are Arbitrary, and we may, if we Think Fit to be ſo Fooliſh, call Virtue Vice, and Benevolence Murder; But, this will not yet Evince, that the Ideas themſelves are Arbitrary, and that they are not Settled and Determined by Nature; There is now Probably ſome where, or has been at leaſt, ſuch an Action amongſt Men, by which a Period is put to another Man's Life, tho' Innocent and Undeſerving of ſuch an Uſage, with the Great Injury, and a Malice Prepenſe, in the Aggreſſor; If we Take in all the Circumſtances and Qualities of this Action, as it Really and Truly Exiſted, we have an Adequate Idea of it; But if we Omit any, or Add others, which did not belong to it, it becomes Inadequate to it; And if we call this Action Murder, we may in this Senſe have an Adequate, or Inadequate, Idea of it; If farther we Strike out of this Idea any Immaterial Circumſtances, which Attend it, as the Manner or Way, in which ſuch Action was Performed, or by Whom, *&c.* our Idea of Murder then becomes General, and it is Adequate or Inadequate, as we Retain all the Eſſential Properties of it, or not; From whence it is Apparent, that our Ideas of Murder, or of any other Virtuous or Vicious Action, are not Voluntary or Arbitrary Collections of Ideas, but are Ideas, which are Derived from the Real Exiſtence of Human Actions, and are Adequate or Inadequate, as they Agree, or Diſagree, with the Reality and Exiſtence of ſuch Actions, and have as much a Foundation in Nature, as our Ideas of Subſtances themſelves; It is alſo Evident, that of theſe Complex Modes, Thoſe, which are Adequate, are Real Ideas, and Thoſe, which are Inadequate, are Fantaſtical; We muſt here Add, that, when we Refer our Ideas of Complex Modes to, and Compare them with, other Men's, which are only Voluntary Collections of ſeveral Simple Ideas, they may be Adequate Fantaſtical Ideas, as all Voluntary or Arbitrary Collections are Fantaſtical; whereas Real Ideas are in this Philoſophy ſaid to be only Adequate or Inadequate, but ſuch Real Adequate Ideas they cannot be.

As to our Ideas of Subſtances, which this Philoſophy Propoſes, becauſe their Eſſences are either Unknown, or the Collections of their Properties Defective, it is Plain in both Caſes, ſince they are not Agreeable to Things, as they Exiſt, they are Inadequate, and for the ſame Reaſon Fantaſtical; and are not Adequate, and Conſequently not Real Ideas, Excepting in the Senſe, which we before Explained, when we Endeavoured to ſhew, what were the Real Eſſences of them.

Of True and Falſe Ideas. Mr. Locke's Eſſay, Chap. 32. Book 2.

§. 40. In Reſpect of True and Falſe Ideas, it is ſaid, that Truth and Falſehood, Really Speaking, belongs only to Propoſitions, and can no more be Applied to Ideas, as they are bare Perceptions and Appearances in the Mind, than to Single Names of Things, and the Idea of Centaur has no more Falſehood in it, than the Name of it has; Our Ideas therefore are only True or Falſe, when the Mind Paſſes ſome Judgment on them, that is, Affirms or Denies ſomething Concerning them, and Refers them to ſomething Extraneous to them, to which it Suppoſes them Conformable or not; As ſuch Suppoſition therefore is True, or Falſe, ſo the Ideas themſelves come to be Denominated.

Thus, Firſt, when the Mind Suppoſes it's Idea Conformable to that in other Men's Minds, Called by the ſame Name, as Juſtice, Virtue, Green, Blue, Sweet, Bitter, *&c.* it may be True or Falſe, according to the Truth or Falſehood of ſuch a Suppoſition; When we Judge of our Ideas by their Conformity to thoſe of other Men, they may be any of them Falſe, but Simple Ideas, as Green, Blue, Bitter, Sweet, *&c.* are the leaſt Liable to be Miſtaken; Much leſs do we Confound the Names belonging to Different Senſes, and Call a Colour by the Name of a Taſte; Complex Ideas are much more Liable to Falſehood, and thoſe of Mixed Modes, more than Subſtances, the Firſt being Men's Voluntary Combinations of their Simple Ideas, whereas the Laſt are Diſtinguiſhed by their Senſible Qualities.

Second, when the Mind Suppoſes any Idea Conformable to ſome Real Exiſtence, it is True or Falſe, as that Suppoſition is ſo; Thus, That of a Man is a True Idea, and That of a Centaur, a Falſe One; And when we thus Refer our Ideas, it is ſaid, that none can be Termed Falſe, but our Complex Ones of Subſtances; our Simple Ideas being nothing but Perceptions in us, Anſwerable to Certain Powers in External Objects Capable of Producing them; Which can Neither be Falſe, whether we Judge theſe Ideas to be in the Things themſelves, or no, that is, whether Blue is in the Violet, or in the Mind; The Name Blue, Noteing Properly nothing, but that Mark of Diſtinction, that is in a Violet, Diſcernible only by our Eyes, whatever it Conſiſts in; Nor can they be Falſe, if by the Different Structure of our Organs it were ſo Ordered, that the Same Objects ſhould Produce in ſeveral Men's Minds Different Ideas, for this could never be Known, ſince Objects would Operate Conſtantly after the ſame Manner; it is yet Probable, that the Ideas Produced by the Same Objects in Different Men's Minds, are nearly and Undiſcernibly Alike; Names of Simple Ideas may be Miſ-applied, as a Man, Ignorant of our Language, may call Purple Scarlet, but this makes no Falſehood in the Ideas.

Complex Ideas of Modes cannot, it is ſaid, be Falſe in Reference to any Thing Really Exiſting, becauſe they have no Reference to any Pattern Exiſting, or made by Nature.

Third, when the Mind Refers any of it's Ideas to that Real Conſtitution and Eſſence of any Thing, whereon all it's Properties Depend, and Suppoſes them Conformable to ſuch Real Eſſences, it's Ideas are Falſe, which is ſaid to be the Caſe of the Greateſt Part, if not, of all our Ideas of Subſtances, as being all Referred to Patterns in Things themſelves; Thus, when we Look upon our Ideas, as Repreſentations of the Unknown Eſſences of Things; or when we Put together Simple Ideas, which, in the Real Exiſtence of Things, have no Union, as in a Centaur; or when from any Collection of Simple Ideas, which always Exiſt together, we Separate any One Simple Idea, as the Colour of Gold from it's Extenſion, Solidity, Fixedneſs, Malleableneſs, Fuſibility, *&c.* they may be Juſtly Eſteemed Falſe Ideas; Tho' the Laſt Sort are Thought to be rather Inadequate and Imperfect, than Falſe; Since there are none Put together, which do not Really Exiſt together. Upon the whole, it is Deemed, that our Ideas, either in Reference to the Proper Signification of their Names, or to the Reality of Things, may more Truly be Called Right or Wrong Ideas, according as they Agree, or Diſagree, to thoſe Patterns or Archetypes, to which they are Referred; Ideas, as they Lie in our Minds, without being Referred, cannot be Wrong, unleſs Complex Ideas, wherein Inconſiſtent Parts are Jumbled together; All other Ideas are Right, and the Knowledge about them Right and True Knowledge, unleſs when Referred to Patterns or Archetypes, with which they Diſagree. Hitherto this Philoſophy.

As to the Firſt, we ſay, our Ideas cannot be Properly Termed True or Falſe, with Reference to other Men's; For, tho' they Agree never ſo Exactly with Thoſe of other Men, and upon that Account are True according to this Syſteme, yet if Thoſe of other Men are Falſe, our Ideas, which are Conformable to Them muſt be Falſe likewiſe; from whence it is Evident, that the Conformity of our Ideas to other Men's can be no Standard or Meaſure of the Truth of them, becauſe, if ſo, they may be True and Falſe at the ſame Time, which is a Contradiction; The Truth or Falſehood of Ideas muſt therefore be Determined the ſame Way, as the Reality or Fantaſticalneſs, or the Adequateneſs and Inadequateneſs of them, that is, by Comparing them with the Real Exiſtence of Things, to which, if they Correſpond, they are True Ideas, if not, they are Falſe Ones; Thus, all Simple Ideas are True, as Blue, Green, Sweet, Bitter, *&c.* nor are they made Leſs ſo, tho' what is Green to me ſhould be another Man's Blue; ſince my Idea and his are both Derived from Real Powers in External Nature and in the Mind, which are Adapted to Produce thoſe ſeveral Ideas in us; The Force of his Perception may be Different from mine, and therefore the ſame External Object may Cauſe Different Senſations, but yet both the Senſations will be Equally True, becauſe Derived from Real, tho' Differing, Forces in Nature. All Complex Modes, ſo far as they are Voluntary or Arbitrary Collections of Simple Ideas, which do not Agree with Men's Real Actions, are Falſe; And if I Call that Human Action, by which a Poor Man is Relieved from his Wants, the ſame as that, by which a Period is Put to his Life, it is a Falſe Idea, whether the One or the Other is Named Charity or Murder; Or, If I Collect a Certain Number of Ideas together, which never did Exiſt together in Human Actions, it is a Falſe Idea; For Complex Modes are the ſame in Reſpect of Real Human Actions, as Subſtances are in Reſpect of Real Beings; And if Voluntary Collections of Ideas are made with Relation to Either, which do not Agree with the Exiſtence of Human Actions, or Real Beings, they are Falſe Ideas.

As to the Second Article, it is Told us, that Man is a True Idea, and Centaur a Falſe One; and yet it Immediately after Subjoins, that no Ideas can be Termed Falſe, Excepting our Complex Ideas of Subſtances, but Man is a Complex Idea of a Subſtance, and therefore ſhould be a Falſe Idea, whereas it was Juſt before Affirmed to be a True One; And there is no other Way of Expediteing ourſelves from ſuch a Contradiction, which this Philoſophy has Incurred, but by Aſſerting, what we have done; That ſo far as our Ideas are Anſwerable and Correſpondent to Nature, they are True Ideas, and ſo far as they Diſſent and Deviate from it, they are Falſe Ones; Thus, however we Define him, a Man is ſomething Really Exiſting in Nature, a Centaur not, the Former therefore is a True Idea, the Latter a Falſe; But, when we Speak of the Complex Subſtance of a Man, we Mean by it a Certain Unknown Eſſence, from whence all his Properties of Mind and Body are Derived, and therefore is a Falſe Idea; becauſe we cannot Affirm any Thing of that, of which we Know Nothing, and which is not Derived into our Minds from Nature, but from our own Fancies and Amuſements.

Another Thing Obſervable in this Article is, that all our Simple Ideas are Affirmed to be True, as if it were Impoſſible they ſhould be Falſe, whereas we have given ſeveral Inſtances in §. 38. in which our Simple Ideas may not be Anſwerable to Nature, and therefore according to the Preſent Article Falſe; As in that Section they were Proved Fantaſtical.

But it is Surprizing, that this Philoſophy ſhould Aſſure us, that our Simple Ideas cannot be Falſe, whether we Judge or Suppoſe them to be in Things themſelves or no, that is, whether the Blue is in the Mind, or the Violet, in an Article, which Expreſly Maintains, that when the Mind Judges or Suppoſes any Idea Conformable to ſome Real Exiſtence, it is True or Falſe, as that Suppoſition is ſo; If therefore the Mind Judges or Suppoſes the Blue to be in the Mind, when it is in the Violet, or the Contrary, and that Judgment or Suppoſition is not True, is it not Plain and Notorious, that ſuch an Idea muſt be Falſe according to it's own Confeſſion? And the ſame may be Argued in Reſpect of the Different Structures of the Organs, for if Different Ideas are Produced from what we Judge or Suppoſe to be, ſuch Simple Ideas muſt be Falſe Ones.

Lastly, Complex Ideas of Modes, it is ſaid, cannot be Falſe, whereas we have already ſhewn, they may be True or Falſe, according as they Correſpond, or not, with the Real Exiſtences of Human Actions, to which they are to be Referred, and not to themſelves, as their own Archetypes.

As to the Third, it is Manifeſt, that our Ideas of Subſtances are Falſe, becauſe not Agreeable to Nature.

In Concluſion therefore upon theſe Three laſt Chapters, we ſay, that True and Falſe Ideas are in the Main the ſame with Adequate and Inadequate, and with Real and Fantaſtical; That Ideas may be Juſtly ſaid to be Real, Adequate, or True, when they Exactly Correſpond and Anſwer to the Real Adequate, and True Cauſes, from which they are Derived, that they are then Fantaſtical, Inadequate, or Falſe, when they are made and Created by the Mind, without any Dependence on a Real Nature to Produce them; And therefore, First, Simple Ideas are Real, Adequate, and True, ſo far as they are Effects Cauſed in the Mind, by the Natural Powers in External Objects in Conjunction with thoſe of the Mind to Imprint them there; But if we Affirm any Thing of theſe Simple Ideas, either that they are Repreſentations and Patterns of Things themſelves, that is, that the Blue in the Mind Exiſts in the Violet; or that they Enter the Mind Separately and not Combined with Others, which this Philoſophy Teaches; or if we make them Abſtracted Ideas, and General Ones; or Suppoſe them to be Introduced into the Mind by the Action of Material Beings, without any Action of the Mind it's ſelf, and by a mere Paſſion of it, Conſidered as a Raſa Tabula, which is alſo done by this Philoſophy; or if we Suppoſe any Thing of theſe Simple Ideas, which is not Agreeable to the Nature of them; Such Simple Ideas are Fantaſtical, Inadequate, and Falſe.

Second, Complex Modes made by the Mind, by a Voluntary or Arbitrary Collection of Simple Ideas, which this Philoſophy only Takes Notice of, and Affirms to be Real, Adequate, and True, are Fantaſtical, Inadequate, and Falſe; becauſe they are Creatures of the Mind, and are not Suppoſed or Imagined to be Referred to any Real Exiſtence in Nature; On the Contrary, Complex Modes, which are Derived from, and are Conformable to, the Real Exiſtence of Human Actions, with the Principles and Motives of them, are Real, Adequate, and True, by whatever Names we Call them.

Third, Complex Ideas of Subſtances are in Reſpect of the Eſſence of them, which no One Knows what it is, and is therefore only a Thing Contrived by the Mind, Fantaſtical, Inadequate and Falſe; But in Reſpect of the Collection of Properties, which Really Exiſt in Nature, Real, Adequate, and True; If yet we make this Collection of Qualities or Properties to be a Juſt Idea of the Subſtance, in which they Inhere, it is a Fantaſtical, Inadequate, and Falſe Idea, becauſe ſuch a Collection is at the beſt Defective and Imperfect; And the only Real, Adequate, and True Idea of a Subſtance is, when we know the Eſſence and Eſſential Properties of it, which we have before Explained from the Doctrine of the Expanſive and Contractive Forces.

Concerning the Aſſociation of Ideas. Mr. Locke's Eſſay, Chap. 33. Book 2.

§. 41. This Philoſophy, to Account for the Extravagant Fancies, Opinions, and Perſuaſions of Men in Matters of Truth and Falſehood, Proceeds to Tell us, that beſides a Natural Correſpondence and Connexion of our Ideas with one another, which it is the Office and Excellency of our Reaſon to Trace, there is an Accidental One, which is wholly Owing to Chance or Cuſtom; by which, Ideas, which are not at all of Kin, come to be ſo United in ſome Men's Minds, that it is very Hard to Separate them; they always keep Company, and the One no ſooner comes into the Underſtanding, but it's Aſſociate Appears with it, and if they are more than Two, the whole Gang, always Inſeparable, ſhew themſelves together; This Strong Combination of Ideas, not Allied by Nature, the Mind makes in it's ſelf, either Voluntarily, or by Chance; and hence it comes in Different Men to be very Different, according to their Different Inclinations, Educations, Intereſts, *&c.* Cuſtom ſettles Habits of Thinking in the Underſtanding, as well as of Determining in the Will, and of Motions in the Body; all which ſeem to be but Trains of Motion in the Animal Spirits, which once ſet agoing, Continue on in the ſame Steps, they have been uſed to, which by often Treading are Worn into a Smooth Path, and the Motion in it becomes Eaſy, and as it were Natural.

This Combination or Aſſociation of Looſe and Independent Ideas in our Minds, is of ſo great Force to ſet us Awry in our Actions, as well Moral, as Natural, and in our Paſſions, Reaſonings, and Notions themſelves, that Perhaps there is not any one Thing, that Deſerves more to be Looked after.

Thus, the Ideas of Goblins and Sprights, have Really no more to do with Darkneſs than Light, which yet by a Fooliſh Maid may be ſo often Inculcated and Raiſed together in the Mind of a Child, that he may not be able to Separate them again, as long as he Lives; But Darkneſs ſhall ever after bring with it thoſe Frightfull Ideas; If a Man has Suffered Pain or Sickneſs in any Place, if he Saw his Friend Die in ſuch a Room, when the Idea of the Place or Room Occurs to his Mind, it brings that of the Pain and Diſpleaſure with it.

THUS, in Intellectual Habits, let the Ideas of Being and Matter be ſtrongly Joined, either by Education or much Thought, what Notions, what Reaſonings will there be about Separate Spirits? Let Cuſtom from very Childhood have Joined Figure and Shape to the Idea of God, and what Abſurdities will that Mind be Liable to about the Deity? Let the Idea of Infallibility be Joined to any Perſon, and then One Body in Two Places at Once ſhall be Swallowed for a Certain Truth, whenever that Infallible Perſon Dictates and Demands Aſſent without Inquiry.

THIS Wrong Combination or Aſſociation of Ideas Eſtabliſhes the Irreconcileable Oppoſition between Different Sects of Philoſophy and Religion; for we cannot Imagine every One of their Followers to Impoſe Willfully on himſelf, and Knowingly Refuſe Truth offered by Plain Reaſon; Intereſt may do Something, tho' that cannot Work whole Societies of Men to ſo Univerſal a Perverſeneſs; Some Independent Ideas are by Education, Cuſtom, and the Conſtant Din of a Party, ſo Coupled in their Minds, that they always Appear there together, and they can no more Separate them in their Thoughts, than if they were but One Idea, and they Operate, as if they were ſo; This gives Senſe to Jargon, Demonſtration to Abſurdities, and Conſiſtency to Nonſenſe, and is the Foundation of the Greateſt, if not, of All the Errors in the World.

THIS is a Summary Account of the Preſent Doctrine of the Aſſociation of Ideas, in which it muſt be Firſt Inquired, whether there is in Reality ſuch an Aſſociation or Combination of them; and Second, whether the Inſtances Alledged do Prove it.

AND Firſt, that there is no ſuch Aſſociation or Strong Combination of our Ideas, by which One Enters Forcibly into the Mind in Conſequence of the Other, but only that it is a Quick and Lively Recollection of Ideas One after Another, and which the Mind at Pleaſure can Separate and Disjoin, ſeems Plain from Hence.

FIRST, Becauſe otherwiſe, ſince this is Eſteemed by the Preſent Philoſophy, to be the Foundation of the Greateſt, if not, of All the Errors in the World, Men would not be Chargeable with, or Accountable for, thoſe Errors, which would be Neceſſary, and not in their Power to Avoid; But it is Unreaſonable to Suppoſe, that we ſhall not be Anſwerable to Almighty God for what is Awry in our Moral Actions, if not in our Natural, as well as in our Paſſions, Reaſonings, and Notions themſelves; ſince it is Told us, that Perhaps there is not any one Thing, that Deſerves more to be Looked after; But it is likewiſe Unreaſonable to Imagine, that we ſhall be Anſwerable for an Aſſociation of Ideas, which, by Chance or Cuſtom, unheeded by us, are ſo Strongly Linked and United in the Mind, that we ſhall never, it is Poſſible, be able to Separate them, no more, than if they were but One Idea, and which Operate, as if they were ſo.

SECOND, Becauſe, if we may be Allowed to Reaſon, as this Philoſophy does in it's Chapter of Identity, from *Engliſh* Phraſes, and Ways of Speaking, we Generally Say in ſuch Aſſociations, that ſuch a Place, or Thing, or Notion, Puts us in Mind of Another, to which it has been Aſſociated, as ſuch a Room, of a Friend that Died there, Darkneſs, of Goblins and Sprights, *&c.* Which at leaſt ſhews, that in the Strongeſt Combinations of our Ideas, we Take and Apprehend them only to be Ideas of Memory and Recollection.

THIRD, Becauſe there cannot well be a Stronger Aſſociation of Ideas than there is in the Leſſon of a School-Boy, which he Perfectly Retains, and can Repeat without the leaſt Heſitation, and with the utmoſt Quickneſs and Fluency; And yet theſe Ideas are only Ideas of the Memory or Recollection, and were never Imagined to Enter Forcibly into the Mind by a Mechanical Aſſociation, ſo that if One makes it's Appearance there, the whole Gang, always Inſeparable, ſhew themſelves together; Since he can Stop at every Word, Verſe, Line, or Period, which Evinces, that they are Entirely under the Direction and Conduct of the Will, and are as much Separable by the Pleaſure of the Mind, as any other Ideas whatſoever.

FOURTH, Becauſe, That there is a Power in the Mind of Perceiving, of Comparing, Compounding, and Dividing, and of Recollecting it's Ideas to a Great Degree of Quickneſs, is Owned and Confeſſed; But it is Inconceivable, that there ſhould be a Power in it of Aſſociating them in the Senſe of this Philoſophy, that is, Really a Power of not Aſſociating them, but Receiving them already Mechanically Aſſociated, and Joined together; This ſeems to be, to make the Mind an Active Principle, and yet a mere Paſſive and Mechanical One at the ſame Time, which is little Leſs than a Contradiction; If therefore a Power Acknowledged in the Mind, as that of Memory and Recollection, is Sufficient to Solve theſe Imaginary Aſſociations, it is Irrational to Introduce a Mechaniſm into it, which is not Known to be there, in Order to Account for them.

Fifth, Becauſe, which was to be our Second Inquiry, None of the Inſtances Alledged do Prove or Conclude ſuch a Mechanical Aſſociation or Combination of our Ideas.

For Firſt, it does not from this Neceſſary Aſſociation Come, that ſuch a Neceſſary Aſſociation of Ideas in Different Men is very Different according to their Different Inclinations, Educations, Intereſts, *&c.* Unleſs ſuch a Neceſſary Aſſociation is Proved, and therefore this is no Inſtance of the Thing, but a Preſumption of it; And we may as well and more Juſtly Say, that there is no ſuch Aſſociation of Ideas, but that Men Judge Differently of Things, according to the Different Powers in their Mind of Perceiving, Recollecting, Compounding, Comparing, *&c.* and that Inclination, Education, and Intereſt, are only Accidental to the Mind, but can never Overbear the Real Powers of it, of Perceiving, Recollecting, and Judging.

Second, As to Cuſtom Settling Habits of Thinking in the Underſtanding, as well as of Determining in the Will, and of Motions in the Body, and all Ariſing from Certain Trains of Motion in the Animal Spirits; How is this a Proof of ſuch an Aſſociation? A Man may Acquire a Habit of Thinking without Suppoſing his Ideas are Inſeparably Linked and United together; Beſides, what is here Called a Habit of Thinking, may in Reality be no more than a Habit of Remembering, and Memory is often Explained by a Train of Motion in the Animal Spirits, which once ſet agoing Continue on in the ſame Steps, they have been Uſed to; But what theſe Animal Spirits are, or whether there are in Reality any ſuch Spirits, is yet Unknown, and therefore cannot much Clear this Matter of the Aſſociation of our Ideas.

Third, the Ideas of Goblins and Sprights are not ſo Connected with Darkneſs, that the Idea of the Latter ſhould Neceſſarily Raiſe in us the Idea of the Former, for the Weakeſt and moſt Timorous Mind is under no ſuch Apprehenſions, whilſt it is in Company with Others; And if Alone, by a Moderate Courage and Reſolution can Throw off and Divert them; On the Contrary, even in Day-Light Minds, which are Subject to ſuch Fears and Terrors, will not be without them in Retirement and Solitude, and Eſpecially in a Room, which has been Reported to be Haunted, or in the Body of a Church, with the Doors Locked, and Amidſt the Graves and Sepulchres of the Dead. The Caſe is this, upon any of theſe Occaſions, the Mind Recollects ſeveral Stories, which it has been Told in Relation to Sprights, or Goblins, or Ghoſts Appearing after the Deceaſe of thoſe, who have been Interred, by which it is Frighted and Scared; but it may by a Steady Reliance on the Divine Goodneſs, or by a Fixed Reſolution and Intrepidity, Maſter ſuch Groundleſs Fears and Separate thoſe Ideas, which it's Memory Suggeſts; Theſe Inſtances therefore are Reſolved into Memory and Recollection, and are not in the Leaſt any Proof of the Aſſociation Pretended. 'Tis True, there is ſomething in Darkneſs Peculiar to it, which Naturally gives us a Horror, and Dejection of Mind, and therefore a Kind of Cowardice from the Contractive Force of it, as Light is Connected with Chearfullneſs, Vigour, Alacrity, and therefore Courage, from the Expanſive Force; But this is no more, than what we in ſome Meaſure Obſerve in Birds and Animals, and even in Vegetables and Plants, which Droop and Languiſh in the Night, but are Briſk and Lively with the Riſing of the Sun; Which alſo Furniſhes us with the Account of another Phænomenon in Nature, why Studious Men, if they Spend the Night in Contemplation and Attention, and the Day in Repoſe, Contract ſuch a Dejection of Mind, and ſuch a Fear and Deſpondency in it, as is always Infinitely Prejudicial to Them; However this is Plain, that ſuch a Connexion of Fear and Horror with Darkneſs is Natural, and is not Owing to any Imaginary Aſſociation of Ideas, Derived from the Stories and Tattle of a Nurſe.

Fourth, when a Man has Suffered Pain or Sickneſs in any Place, or Saw his Friend Die in ſuch a Room, the Place or the Room only gives Occaſion to the Mind to Recall or Revive thoſe Ideas of Pain or Diſpleaſure in it; Which does not

Prove a Mechanical Aſſociation of theſe Ideas, but a Power in the Mind to Recollect them upon ſuch Intimations. This brings to my Thoughts the Artificial Memory, Deſcribed by the Author to *Herennius*, Practiced amongſt the Ancient *Romans*, which the Orators made Uſe of to Enable them to Repeat *Memoriter*, and *Verbatim*, or Nearly, what they had before Premeditated for their Audience, and by the Aſſiſtance of which no Doubt it was, that *Hortenſius* Pronounced his Oration, for *Meſſala* Word for Word, as he had Contrived and Set it Down in his own Mind, without any Tables or Writing; and which *Tully* gives us, as a Great Inſtance of the Memory of Words in that Orator, as he Informs us, *Lucullus* was ſuch an Example for the Memory of Things; who, tho' he was wholly Unacquainted with the Art of War, when he went as General againſt *Mithridates*, yet by Reading only the Hiſtories and Exploits of other Generals before him in his Paſſage from *Italy* to *Greece*, and in his Expedition againſt that Prince, became the moſt Accompliſhed General of his Age; And what was this Artificial Memory? It Conſiſted in making Choice of a Certain Number of Loci, or Places, which were Diſtinguiſhed from each Other by their Order, of Firſt, Second, *&c.* by Various Spaces, Figures, and Intervals, and by Certain Marks and Characters, which were Affixed to every Fifth, or Tenth Place of them; Theſe Places were Inculcated on the Mind ſo often, and ſo Strongly, by a Frequent Repetition of them, that a Perſon, Complete in his Art, could Recite theſe Places in Order, Backwards, or Forwards, or by any other Ways of Combining them; Theſe were Conſidered and Eſteemed in the ſame Manner as Paper, or a Raſa Tabula, on which any Characters were to be Writ, or Engraven; And which were to Serve all Occaſions of Writing, as a Book of Vellum, which, upon the Daſhing out of the Former Impreſſions made by a Pencil, is Fit to Receive any New Ones; When the Mind was Furniſhed with theſe Loci or Places, whatever was to be Repeated by the Memory, was Conceived in Images and Repreſentations of Things, which were Set down in Order, the Firſt Image in the Firſt Place, the Second in the Second, and ſo on; By theſe Means the Orator by going over the Places, which were Familiar to his Mind, Raiſed in himſelf the Images, which he had Affixed to thoſe Places, as in the Inſtance, which we are now Conſidering of a Room, or a Place, which Excites Certain Ideas in us Peculiar to thoſe Places; This I Believe is the Strongeſt Inſtance, which can be given, of an Aſſociation of Ideas, which, it is ſomething Wonderfull, ſhould Eſcape the Great Learning and Reading of this Philoſophy of them, which to ſhew it's Acquaintance with Antiquity, ſometimes makes Mention of *Alexander*, and *Bucephalus*, and *Socrates*, tho' it may be almoſt Queſtioned, whether it Knew much more, than that the one was a Man, the other a Horſe, and the Laſt a Philoſopher; in the ſame Manner, as it Aſſumes in it's Explanations of it's Doctrine, that the Three Angles of a Triangle are Equal to Two Right Ones, and yet One may Reaſonably Think it's Knowledge of ſuch a Propoſition was rather from Hear-ſay, than from a Demonſtration of it; Omitting which, this Greateſt Inſtance of an Aſſociation of Ideas is Plainly Reſolved into the Ideas of Memory and Recollection, and the Mind can Separate and Disjoin them, as it Pleaſes; For it could Remember theſe Places in Order, Backwards, or Forwards, or by other Ways of Combining them; It could Diſpoſe Certain Images in theſe Places, and then Expunge them, and Affix others in the Room of them; All which ſhews, that there was no Mechanical Aſſociation of theſe Ideas, but all of them were under the Management and Government of the Mind and the Will, which made Uſe of them to thoſe Deſigns and Purpoſes, which it Intended.

FIFTH, Intellectual Habits are given, as another Inſtance of this Aſſociation; But can any One Perſuade himſelf to Think or Believe, that the *Panhyliſts*, or *Anthropomorphites* Aſſerted the One, that all Being, and all Nature, were Matter, the Other, that Almighty God was in the Form of a Grave and Venerable Old Man, from an Aſſociation of Ideas, and not from a Falſe Connexion of, and a Groſs and Abſurd Reaſoning upon, them? Or can we Suppoſe the *Romaniſts* to Embrace their Doctrine of Tranſubſtantiation from the Infallibility of their Guide, when thoſe, who Diſown and Renounce the Latter, and Place it in the Church, and in General Synods and Councils, yet Patronize and Defend the Former? Theſe are not therefore Examples of the Aſſociation of Ideas, but of the Weak and Erroneous Concluſions of Human Reaſon; And as to what Reſpects Tranſubſtantiation, which we take to be a Docrine, neither Juſtify'd by Scripture, nor Agreeable to Nature or Philoſophy, yet this Syſteme of Ideas ſhould not too Haſtily Determine, what is Poſſible, or not, in the Change or Alteration of Subſtances, which is ſo Entirely Ignorant, of what they Are, or wherein they Conſiſt.

SIXTH, In the Irreconcileable Oppoſition, which is found between Different Sects of Philoſophy and Religion, it is not Probable Indeed, that whole Societies of Men ſhould be Wrought to ſo Univerſal a Perverſeneſs, as that every One of them ſhould Knowingly Maintain Falſehood. This Irreconcileable Oppoſition does not yet Appear to be Eſtabliſhed from ſuch Wrong Combinations of our Ideas, which by Education, Cuſtom, and the Conſtant Din of a Party, are ſo Coupled in our Minds, that we can no more Separate them in our Thoughts, than if they were but One Idea, but from Men's Different Faculties and Powers of Perceiving Things, of Viewing and Comparing them, and of Judging and Reaſoning upon them, which may be ſometimes as Various as their Complexions, and by which each Perſon ſhall Imagine himſelf to be in the Right. Thus in the Diſputes of a Plenum and Diſſimilar Matter, or a Vacuum and Similar Matter, which I have ſhewn muſt be Connected together, Philoſophers have Diſagreed; which Diſſenſion cannot be Owing to a Mechanical Aſſociation of Ideas, but to Men's Different Apprehenſions of Nature, even Perhaps after the Stricteſt Inquiry and Scrutiny into it; They Look upon it in a Different Light, and he who has Formed his Ideas Concerning it with the moſt Exactneſs, and Reaſons upon it with the greateſt Accuracy, Bids the Faireſt for Truth, and is the moſt likely not to be Deceived: And the ſame may be ſaid in Reſpect of Religion and the Scriptures; The Two Grand Diſtinctions in Chriſtianity are Thoſe of Proteſtantiſm and Popery, and in Proteſtantiſm, Thoſe of the Orthodox, and the Socinian or Arian Faith; it would be too Hard an Imputation not to Think, that there may be Men of Integrity in all theſe Perſuaſions, and that That, which makes ſuch Men Differ, is not a Wrong and a Mechanical Aſſociation of Ideas, but a Falſe and Undue Connexion of Ideas from Falſe Views and Notices of Things; Their Powers and Faculties of Perceiving, Reaſoning, and Judging, are not the Same, but Every One Concludes Right according to his own Premiſſes, and Every One is Convinced by the Force of his own Demonſtrations; But he who Examines his Ideas in Religion with the Greateſt Care, and Argues with the Greateſt Strength and Force upon Them, and Penetrates the moſt into the Nature and Senſe of Revelation, will Come the Neareſt to Truth, and will Prefer Proteſtantiſm to Popery, and rather Chooſe to be in the Orthodox, than in the Arian or Socinian Scheme. This Philoſophy of Ideas was Unwilling however to Charge all theſe Errors upon Human Reaſon, which is only Anſwerable for them, and would have been Glad to have Shifted off the Load upon an Imaginary Aſſociation of Ideas of it's own Inventing; That Reaſon might ſtill be an Infallible Guide, a Light ſet up by Almighty God in our Minds, and a Ray of the Divinity. But it is too Notorious in Fact and Experience to be Concealed, and of which all the Modern Philoſophy, as well as this Syſteme of Ideas, is a Standing Evidence; That Reaſon is a Deſultory and a Vagrant Term, and that, That is Reaſon to Whole Societies of Men, which is not ſo to a Thouſand Others.

CHAP.

CHAP. IX.

Of Words or Language in General; Of the Signification of Words; Of General Terms; Of the Names of Simple Ideas; Of the Names of Mixed Modes and Relations; Of the Names of Subſtances; Of Particles; Of Abſtract and Concrete Terms; Of the Imperfection of Words; Of the Abuſe of Words; Of the Remedies of the Foregoing Imperfections and Abuſes.

Of Words or Language in General. Mr. Locke's Eſſay. Chap. 1. Book 3.

§. 1. SINCE There is a Cloſe Connexion betwixt Ideas and Words, it is ſaid it will be Neceſſary to Conſider Language, which Man, being a Sociable Creature, was Furniſhed with by God, as the Great Inſtrument and Common Tye of Society; and therefore had his Organs ſo Faſhioned, as to be Fit to Frame Articulate Sounds, which he might be Able to Uſe, as Signs of Internal Conceptions, in Order to make them Known to Others.

BUT it was not Sufficient for the Perfection of Language, that Sounds might be made to Stand for Marks of Ideas in our Mind, unleſs they could at the ſame Time Comprehend ſeveral Particular Things; For the Multiplication of Words would have Perplexed their Uſe; And hence Aroſe General Terms, which are Expreſſive of General Ideas, as Particular Ones are of Particular.

THERE are, it is Told us, other Words, which Signify the Want or Abſence of Ideas, as Ignorance, Barrenneſs, *&c.* which Relate to Poſitive Ideas, and Signify their Abſence.

LASTLY it is Obſerved, that the Words, which Stand for Actions and Notions, quite Removed from Senſe, are Borrowed from Senſible Ideas, *v. g.* to Imagine, Apprehend, Comprehend, Underſtand, Adhere, Conceive, Inſtill, Diſguſt, Diſturbance, Tranquillity, which are all Taken from the Operations of Things Senſible, and Applied to Modes of Thinking; Spirit is no more than Breath, and Angel a Meſſenger; ſo that Nature is Suppoſed, even in the Naming of Things, Unawares to have Suggeſted to the Beginners of Languages, the Originals of all their Knowledge; whilſt, to make Known to Others, any Operations they Felt in themſelves, or any other Ideas, that came not under their Senſes, they were Fain to Borrow Words from the Ordinary and Known Ideas of Senſation. To which we Anſwer.

Anſwer to the Foregeing Account.

As to the Meaning and Deſign of Language, That is Confeſſed on all Hands, tho' it muſt be Said, that Letters, and Words Written, or Dumb Signs, are likewiſe Expreſſive of our Thoughts and Ideas, as well as Articulate Sounds, tho' not to ſo Great a Perfection; And Society might be Preſerved even thoſe Ways; Of which Laſt very Surprizing Stories are Told, and which Shew, that there is ſomething Infinitely Superior in the Mind of Man, to That of Brutes, even, where Both of them are Deprived of Language.

BUT it is not Certain, that our Ideas are Originally Derived from Senſation, becauſe we make Uſe of Words Taken from the Operations of Things Senſible.

FIRST, Becauſe there is no Connexion betwixt the Naming our Ideas, and the Origine of them; For tho' they were Derived from one Source or another, we might Uſe Metaphorical Expreſſions Borrowed from Either, where there is an Analogy betwixt the Operations of the Mind and Body. Thus we often Transfer the Operations of the Mind to thoſe of Body, as well as thoſe of Body to the Mind, where there is a Similitude and Agreement with each Other; For we Call Colours, Lively, or Faint, and Languid, a Sound Vigorous and Strong, or Weak and Feeble; we Aſcribe the Paſſions of Love, Fear, and Abhorrence, to Inanimate Beings; and the Beautifull Expreſſions of Language, Invented by the Wits and Genius's of it, can never be made an Argument in Philoſophy, for the Formation of One, from the Origine of our Ideas.

SECOND, Becauſe there is nothing more Plain, than that there are Operations in the Mind, as Doubting, Fearing, Hopeing, Memory, Perception, Judging, Reaſoning, *&c.*

&c. which have no Kind of Reſemblance with the Operations of Senſible Beings, and therefore muſt Originally have ſome Arbitrary Names Invented to Denote them, which do not Depend upon any Operations without us.

Third, Becauſe all which is in our Mind, and all the Ideas of it, are not Originally Derived from the Impreſſions of External Objects, ſince there is an Innate Senſe in it Previous to any ſuch Impreſſions; For, if there were not, it would be as Impoſſible for Them to Produce Senſe in the Mind, as to do it in any Piece of Matter, which is Acknowledged to have no Senſe; Therefore Senſe muſt be Originally Exiſting in the Mind, otherwiſe it could never be Produced in it by the Actions of Things Inſenſible: Nor is it Sufficient to Say, that there may be a Capacity of Senſation in the Mind, but no Explicit and Real Senſe, till Excited by External Impreſſions; Becauſe a Capacity, for Inſtance, of Burning in Fire does Actually Infer a Real Quality of Burning in the Fire; And the ſame may be ſaid of Senſe or the Mind, that it's Capacity of Senſation muſt Conclude an Actual Senſe in it; And therefore the Ideas of Perception, Memory, Hope, Fear, and other Operations of the Mind, may from that Innate Senſe, which it has, as well, for all that we Know, Exiſt before, as after, Senſation from External Impreſſions.

Fourth, Becauſe there is Really an Analogy betwixt the Operations of the Mind and Body, Founded in Nature, as we have ſhewn in our Doctrine of the Expanſive and Contractive Forces, and therefore Expreſſions to Denote the Operations of Each may be Promiſcuouſly Borrowed from One Another; For there is a Real Gravity and Levity in the Mind, with other Affections, which Attend them, as there is a Real Gravity and Levity in Bodies, with their Peculiar Qualities, which belong to them; The Firſt being the Reſult of the Contractive Forces in Each, and the Laſt of the Expanſive. This Philoſophy Proceeds.

Of the Signification of Words. Mr. Locke's Eſſay, Chap. 2. Book 3.

§. 2. Words then are made Uſe of by Men, as Signs, or Senſible Marks, of their Ideas, yet not upon the Account of any Natural Connexion between Articulate Sounds and Certain Ideas, for, if ſo, there would be but one Language amongſt all Men, but by a Voluntary and Arbitrary Impoſition, and ſuch Words Stand for Nothing more but the Ideas in the Mind of him, that Uſes them; For, when a Man Speaks to Another, it is, that his Sounds may make Known his Ideas to the Hearer; Thus a Child, that takes Notice of Nothing more in Gold than the Yellow Colour, Calls the ſame Colour in a Peacock's Tail Gold; Another, that hath better Obſerved, Adds it's Weight to it's Colour, and then the Sound, Gold, Stands for a Complex Idea of Yellow, and Great Weight;

Words, being Voluntary Signs, cannot be Impoſed on Things we Know not, this would be to make them Sounds without Signification; A Man cannot make his Words the Signs either of Qualities in Things, or of Conceptions in the Mind of Another, whereof he has None in his Own.

Tho' Words Signify Nothing, but the Ideas in Men's Minds, yet they are in their Thoughts Secretly Referred to Two other Things.

First, They Suppoſe their Words to be Marks of Ideas in the Minds of other Men, with whom they Communicate, elſe they could not Diſcourſe Intelligibly with One Another; And in this Caſe they Think it enough, that they Uſe the Word in the Common Acceptation of that Language; and do not Examine, whether Their Ideas, and Thoſe of Other Men, be the ſame.

Second, They Suppoſe their Words alſo to Stand for the Reality of Things; And by a Conſtant Uſe and Connexion, the Sounds, or Names Heard, almoſt as Readily Excite Certain Ideas, as if the Objects themſelves were Preſent to the Senſes; and becauſe we do not Preciſely Examine into the Signification of Words, which we Frequently Learn before the Ideas they Stand for, we often ſet our Thoughts more on Words than Things, and ſometimes Speak them, as Parrots do, without any Meaning at all; But if they are of Uſe and Signification, there is always a Connexion between the Sound and Idea.

Since therefore Words Signify our Ideas by an Arbitrary Impoſition, every Man has an Inviolable Liberty to make Words Stand for what Ideas he Pleaſes; it is True, Common Uſe by a Tacit Conſent Appropriates Certain Sounds to Certain Ideas in all Languages, to which if we do not Conform, we cannot Speak Properly, and unleſs our Words Excite the ſame Ideas in the Hearer, which we make them Stand for, we cannot Speak Intelligibly. Thus far This Doctrine.

Anſwer to the Foregoing Doctrine, That the Langnage it Propoſes is Chimerical.

THIS Philoſophy, by the foregoing Account of the Signification of Words, ſeems to Vary it's Sentiments; Sometimes Language is to Maintain Society amongſt Men, and then it muſt Signify ſomething more than the Ideas in a Man's own Mind, that is, the Ideas, which are in Other Men's; Afterwards it Denotes only our Own Ideas, yet with a Secret Reference to thoſe of Others, and to Things themſelves; Then it is a Mere Arbitrary Impoſition of Words and Terms, but yet again, we Tacitly Conſent to Thoſe, which Others have Impoſed, and Uſe has made Common, and which will Raiſe the ſame Ideas in Other Mens Mind's, which we have in our Own; In ſhort, if we were to make Uſe of that Arbitrary Language, which this Syſteme of Ideas Says, we have a Right to, it Owns we ſhould neither Speak Properly, nor Intelligibly, which yet ſeem to be Eſſential to the Deſign of Language according to it's own Confeſſions. No one I Believe will Deny, that we might, if we Pleaſed, make Uſe of a Triangle to Signify Perception, a Square might Denote Memory, a Circle Recollection, and a Pentagone might be a Good Sign or Mark for Judgement, as well as any Combinations of Letters in the Alphabet; But this would be ſuch a Chimerical and Fairy Language, that no one would either Think it worth his while to Learn, or to Diſcourſe, or Diſpute, about it.

What a Real, and not a Chimerical Language is.

REAL, and not a Chimerical, Language, therefore does not only Conſiſt of Words, which Stand for Nothing more than the Ideas in the Mind of him, that Uſes them, but which alſo Stand for the Ideas of them, with whom we Converſe, and ſo far as our Knowledge Reaches, for the Ideas of Things themſelves. If it were not ſo, if Language only Terminated in Marking and Noteing our own Ideas, I would Ask, how it would be Poſſible for us to Communicate our Ideas to Others, which yet is the Principal Intention of it? This is a Problem, which I Apprehend the Preſent Philoſophy could not Solve, and therefore Left it; For this is the Chief Thing to be Explained in Real Language, how it is ſo Contrived, as to Expreſs our own Thoughts or Ideas by Articulate Sounds, which Articulate Sounds ſhall Raiſe the ſame Thoughts or Ideas in Others, which we have: This, which is the Grand Difficulty, is Wholly Unaccounted for here, and all the Anſwer is, that every One may Name Things for himſelf, and when Men come to Converſe with One Another, they Uſe Words in the Common Acceptation without Examining, whether their Ideas be the ſame.

The Problem Solved, how Men Arrive at a Real Language, by which they can Expreſs not only their Own, but Other Men's Ideas. Firſt in Reſpect of External Objects.

ON the Contrary we Say, that Men do Endeavour in Language, as far as they can, to Expreſs the ſame Ideas by the ſame Words; and the Queſtion or Problem is, Firſt, How, when Ideas are Latent only in our own Minds, we ſhould make Uſe of ſuch Articulate Sounds, which have no Connexion with thoſe Ideas, and yet ſhall Communicate them to Others; Second, it will be alſo Proper to Inquire, How Dumb Signs can Impart the ſame Ideas to Others, which we Have.

As to the Firſt, it muſt be Said, and cannot be Denied, that the Ideas of Senſation, or Thoſe, which Proceed from the Impreſſions of External Objects, Lie in Common to any Two or More Obſervers, Who Live in the Same Community with each Other, and therefore they may Call, What they Jointly See and Obſerve, by One Certain Name; Thus, if Two, or Three, or More, See the Same Subſtance of Gold, They All Call it by the Same Name, and as They have much the Same Faculties of Perceiving, They Conclude their Intrinſick Perceptions and Ideas of it are the Same; For it is no more Probable, that the Yellow of Gold in the Idea of One Man, ſhould Produce the Idea of Blue or Violet in Another, than that the whole Idea of the Subſtance of Gold in one Man, ſhould be the Idea of Indigo in Another, or that the Idea of the Weight of a Pound in One, ſhould be the Idea of an Ounce in Another: The Powers of Perception from External Objects are very nearly Alike in all, as is Evident from their Agreeing to Call their Perceptions of them by the ſame Name; For, if what was One Man's Blue, was Another Man's Yellow, or what was One Man's Gold, was Another Man's Indigo, or what was One Man's Pound, was Another's Ounce, it would be Impoſſible for Men to Talk or Diſcourſe with any Kind of Conſiſtency with each Other. 'Tis True, from the Different Force of the Perceptive Faculties in Different Men, the Same Kind of Ideas may be Received or Apprehended with a Greater, or Leſs, Quickneſs and Vivacity, and One Man may Perceive the Yellow of Gold, or any other Quality of Body, with Greater Spirit, Life, and Attention, than Another, but the Nature or Kind of the Idea it's ſelf will be much the Same.

Second in Reſpect of the Properties of our own Minds.

THIS therefore is not ſo Hard to Determine; But, How we ſhall Explain the Latent Ideas of the Operations and Paſſions of our own Minds, as Perception, Memory, Hope, Fear, Joy, Sorrow, *&c.* So as by Language to Communicate them to Others is the Inquiry, ſince they do not, as External Objects, Lie Open to Three or More Obſervers, but are Internal Actions of the Mind, in which they Subſiſt; It is Conſequently to be Known, how we Judge, that other Men have the ſame Affections and Properties in their Minds, which we Perceive in our own, which ſeems to be done by the following Method.

Of Perception.

As to Perception, it is Evident, that Every Man has that Faculty, which is Agreeable to Mine, by our Senſations of External Objects, Correſponding with each Other, and by Obſerving, that Other Men Act, and Speak, and Diſcourſe, in the Same Manner, as I would do, in ſuch Circumſtances.

Of Memory.

So I find my ſelf to have an Active Power of Remembering, but how ſhall I Declare to Another, what I Mean by that Word Memory, unleſs I Know there is a like Faculty, which I Have, which alſo Exiſts in his Mind? This therefore we are Convinced of, by Finding, that other Perſons can Repeat ſeveral Lines or Words in a Book, which have their Ideas belonging to them, and which they could not do, when the Book was Abſent, unleſs they had the Same Faculty, which we Perceived in our ſelves upon the Same Occaſion; and by Conſequence we may well enough Agree to Call the Same Faculty by the Same Name of Memory.

Of Reaſoning.

THUS, if I can Demonſtrate a Propoſition in *Euclid*, *Apollonius*, or *Archimedes*, and I Perceive Another can do it likewiſe, I may Fairly Say, there is a Faculty of Reaſoning, in Both of us, and we ſhall not be Miſunderſtood by Each Other, if we Call it by the Same Name.

Of Paſſions.

I am Conſcious to my Self, of the ſeveral Paſſions of Hope, Fear, Joy, Sorrow, Complacency, Anger, Pleaſure, Pain, *&c.* Which I Expreſs by Certain Natural Indications of them, by an Aſſurance in my Behaviour, or a Dejection and Deſpondency in it, by an Air of Delight and Satisfaction, or by One of Grief and Trouble, which is alſo ſometimes Explained by Tears, *&c.* When therefore I find the Indications or Signs of the ſame Paſſions in other Men, I Conclude they are there, and may Juſtly Call them by the Same Name, which I give to my own, by which Means I become Intelligible; ſince they Actually Experience, they are Endued with ſuch Paſſions.

THIS would be the Method, if we were to Begin and Form a Language A-new, by which Two or Three, or More Perſons, or a Whole Community of Men, might Communicate their Thoughts to Each Other, that is, Agree upon Certain Words or Articulate Sounds, which ſhould Expreſs and Signify the Same Idea or Perception, whether in the One, or the Other; But Languages being already Framed, we have nothing to Do, but to Learn the Meaning of Articulate Sounds, which either do, or ought to, Expreſs and Signify the Same Ideas in the Minds of all the Members of the Community, which Uſes it; And tho' there may be ſome Words of a Doubtfull Senſe, yet Generally Speaking, I Believe we may Say in all Languages, that they are ſo Contrived, as to be Certain and Determinate Marks and Characters of Ideas, Common and Known to All of that Society or Clan of Men, which have Conſented together to Adopt ſuch Sounds, to Expreſs their Ideas.

THIS Philoſophy on the other Hand, becauſe there are ſome Ambiguous and Undefined Words, which muſt be in all Languages, and which Proceeds from the Difficulty of the Ideas, which are Annexed to them, in which Men may not fully Agree, would Extend this Obſcurity to a Whole Language, and therefore Aſſerts that Language is only an Expreſſion of our own Ideas, and not of other Men's, which is Exceedingly Partial and Abſurd, and Something More.

FROM what we have here Argued it Follows, that Language is not only an Expreſſion of our own Ideas, but of other Men's, as well as of Things themſelves, ſo far as our Knowledge, or Comprehenſion of them, Reaches; As alſo, that our Ideas may be Expreſſed by Dumb Signs, as well as Articulate Sounds, ſince Articulate Sounds themſelves are Impoſed from ſuch Dumb Signs, or the Natural Indications of the Operations and Paſſions in our own Minds. This Philoſophy Proceeds.

§. 3. ALL

Of General Terms. Mr. Locke's Eſſay, Chap. 3. Book 3.

§. 3. ALL Things, it is ſaid, Exiſting in Particulars, it might be Expected, that Words ſhould be ſo too in their Signification, whereas moſt of the Words, which make all Languages, are General Terms, which is the Effect of Reaſon and Neceſſity; For.

FIRST, It would be Impoſſible to Retain the Name of every Particular Thing, with it's Peculiar Appropriation to the Idea of that Thing.

SECOND, It would be Uſeleſs, unleſs all could be Suppoſed to have theſe ſame Ideas in their Minds; Becauſe Names Applied to Particular Things, whereof I alone have the Ideas in my Mind, could not be Intelligible to Another, who is not Acquainted with all thoſe Particular Things, which had fallen under my Notice.

THIRD, It would be of no Great Service for the Improvement of Knowledge, which, tho' Founded in Particular Things, Enlarges it's ſelf by General Views, to which Things, Reduced into Sorts under General Names, are Properly Subſervient; For, when we Diſcourſe of Individuals, we Uſe Proper Names, as in Perſons, Countries, Cities, Rivers, Mountains, *&c.*

WE are afterwards Informed, how General Words come to be Made, which is by being made Signs of General Ideas, and Ideas become General by Separating from them the Circumſtances of Time, Place, or any other Ideas, that may Determine them to this or that Particular Exiſtence; By this way of Abſtraction they are Capable of Repreſenting more Individuals than One, Each of which having a Conformity to that Abſtract Idea, is of that Sort; Thus, the Firſt Ideas of Children are only Particular, as of the Nurſe, or Mother, but afterwards finding other Beings like to them, they Leave out that, which Diſtinguiſhes them, and Form their General Idea of Man; Then Leaving out Shape, and ſome other Properties, and Retaining only a Body with Life, Senſe, and Spontaneous Motion, we Frame the Idea of Animal; And by the ſame way the Mind Proceeds to Body, Subſtance, and at Laſt to Being, Thing, and ſuch Univerſal Terms, which Stand for any Ideas, whatſoever; From whence it is Evident, that the whole Myſtery of Genus, and Species, is nothing elſe but Abſtract Ideas, More or Leſs Comprehenſive, with Names Annexed to them.

GENERAL therefore and Univerſal, it is ſaid, belong not to the Real Exiſtence of Things, but are Inventions of the Underſtanding made by it for it's own Uſe, and Concern only Signs, either Words or Ideas.

THESE General Terms, farther, do not Signify a Plurality, for then Man and Men would Signify the ſame Thing, but a Sort of Things; and this they do, by being made Signs of Abſtracted Ideas; And the Eſſences of the Sorts or Species of Things are nothing, but theſe Abſtracted Ideas.

IT is Allowed, that Nature makes Things Alike, and ſo lays the Foundation of this Sorting and Claſſing, but the Sorts or Species themſelves are the Workmanſhip of Human Underſtanding, ſo that every Diſtinct Abſtracted Idea is a Diſtinct Eſſence; Thus, Oval, Circle, Rain, Snow, are Eſſentially Different; The Word Eſſence may be Taken two Ways.

FIRST, For the Real, and Internal, tho' Unknown Conſtitution in Subſtances.

SECOND, In the Schools, it has been almoſt wholly Applied to the Artificial Conſtitution of Genus and Species, which are nothing elſe but Abſtracted Ideas; The Firſt of theſe Eſſences may be Termed Real, the Laſt Nominal.

BETWEEN the Nominal Eſſence, and the Name, there is ſo Near a Connexion, that the Name of any Sort of Things cannot be Attributed to any Particular Being, but what has the Eſſence, whereby it Anſwers that Abſtracted Idea, whereof that Name is the Sign, as Man or Brute has the Eſſence of an Animal.

THE Real Eſſences of Corporeal Subſtances are either.

FIRST, Thoſe, according to which all Natural Things are Suppoſed by Some to be Made, and of which they Equally Partake, and do become of this or that Species, and of which there is Thought to be a Certain Number; But the Monſtrous Productions in Nature are Imagined to be a Sufficient Confutation of ſuch a Suppoſition.

SECOND, Or they are Suppoſed by Others to be the Real, but Unknown Conſtitution of the Inſenſible Parts of Bodies, from whence their Senſible Qualities Flow.

It is alſo farther Obſerved, that the Nominal and Real Eſſences of Simple Ideas and Modes are always the Same, but in Subſtances always quite Different; Thus the Real and Nominal Eſſence of a Triangle is the Same; But the Real Eſſence of Gold, that is, the Real Conſtitution of it's Inſenſible Parts, on which all it's Properties Depend, is Different from it's Nominal Eſſence, that is, a Collection only of it's Properties, as of Colour, Weight, Fuſibility, Fixedneſs, *&c.* which is our Abſtracted, and Complex Idea of it.

That Eſſences are but Abſtracted Ideas, Appears alſo, from their being held Ingenerable and Incorruptible, ſince all Things in Nature are Liable to Change, but as they are Ideas Eſtabliſhed in the Mind, they are Immutable, and whatever becomes of *Alexander*, or *Bucephalus*, the Ideas of Man and Horſe Remain the Same.

Anſwer to the Foregoing Account of General Terms.

How far this Philoſophy is Right, and in what it is Miſtaken, may be Eaſily Judged, by what has been Diſcourſed and Argued before, in Relation to General, Abſtracted, or Complex Ideas, and Particularly thoſe of Subſtances, and by the Preceding Section, Concerning the Signification of Words, and therefore it would be Needleſs to Repeat the Trouble of a farther Examination here of the Particulars Alledged.

I ſhall only Add, Firſt, that Men do not Form their General Ideas, and Affix to them General Names, which Expreſs ſuch Ideas, becauſe they could not Remember Particulars, but becauſe they could not otherwiſe be Intelligible to One Another; For I Ask, whether ever any Man, or Child, made his General Idea of Gold, or a Triangle, to ſave himſelf the Trouble of Remembering every Particular Parcel of Gold, or every Triangular Figure, which has Occurred to him? On the Contrary, I See a Piece of Gold, or a Triangle, which Another does, and a Third, or a Fourth, *&c.* we Call each by their Different Names, and when we See any Subſtance, or any Figure of a Like Kind, tho' we do not Know all the Properties of Either, we Call them by the Same Names.

Thus, any Ordinary Tradeſman may nearly Know, what Gold is, as well as the beſt Refiner of it, by it's Weight and it's Colour, and a Common Meaſurer of Lands and Acres, what a Triangle is, as well as the Profoundeſt Geometer; ſo Thought, Perception, Judgment, are General Terms, which are made, not to Save the Memory a Trouble, but to Expreſs Ideas without the Particularities of them, which are Common to All, and from whence we Know what Each other Man's are, according to what has been Explained before in the Signification of Words.

Second, As to the Improvement of Knowledge, General Ideas are ſo far from being Serviceable to it, that they are the very Bane of it, and if we would Inquire into Particular Ideas, or Particular Things, with more Accuracy and Attention, the Learned World would find a Greater Advantage from it; Thus, if I, or any other Perſon could Tell what the Nature of any One or More Particular Beings was, their Subſtance and Eſſence, I ſhould do more towards the Informing of it, than all the Volumes, which have been Writ in Logicks, of Genus, and Species, or than what even the Preſent Syſteme of Ideas has done; Which is Spent in Sorting and Claſſing it's Ideas, without Inſtructing us in any Uſefull Truths in Philoſophy, or Reaſon.

As to the General or Nominal Ideas of Genus and Species, they are Certainly nothing but Creatures of the Mind, and can give no Advance to Knowledge, nor can the General or Real Ideas of Subſtances, whilſt thoſe Subſtances are Unknown; And as to the Diſtinction of Simple and Complex Ideas, of what Uſe is it to the Knowledge of Things? This all only Reſts, as the Ancient Logicks did, in General and Abſtracted Terms, without Leading us into the Solution of any one Appearance in Nature.

Third, We are Told, how we come by theſe General Ideas from Children to Men, as before Recounted; Whereas the Mind does not ſeem to Proceed in that Way, but it rather Forms it's General Ideas by mere Senſe, and the Apparent Likeneſs and Similitude of Things, and not by Abſtraction, which is a Philoſophick Way of doing it, and which can never Enter into the Minds of Children, no more than the Genus or Species of a Logician; Thus, when we See ſeveral Men together, or One after Another, we find they Agree in Shape, Figure, in Diſcourſing, and ſeveral Actions, which are Common to them, or when we See ſeveral Pieces of Gold, we alſo find them to Agree in Colour, and to be Like Each Other in Diverſe Particulars; And if it be Inquired of a Child, What that Perſon is, or the Other, or a Third, he will Anſwer a Man, or What that Piece of Metal is, or a Second, Third or Fourth, his Anſwer will be Gold; Becauſe he Concludes them to be the ſame Sort from their Similitude, without Knowing the Exact Properties of Either; Which we afterwards Learn, and how to Abſtract them, by Obſervation, Reaſoning, and Philoſophy; And that this is ſo, is Manifeſt, ſince we Acquire our Native Language, and even the General Names in it, when we are yet Exceeding Young, and are Incapable of Abſtracting.

As to Genus and Species, they are Evidently the Contrivance, not of Children, but of Subtile, Studious, and Ingenious Men, and are no other than ſeveral Abſtracted Ideas, which are More or Leſs Comprehenſive; and yet Genus and Species are General Terms or Ideas.

NAY, it is farther Allowed by this Philoſophy, that tho' Nature makes Things Alike, and ſo lays the Foundation of this Sorting and Claſſing, yet the Sorts or Species themſelves are the Workmanſhip of Human Underſtanding; If ſo, the Firſt therefore belongs to Children, *viz.* to Apprehend Things as they are Alike; But the Laſt, that is, the Forming Abſtracted and General Ideas, to the Philoſopher; According to what we have before Affirmed.

FOURTH, The Suppoſition, that there are a Certain Number of Eſſences, which are Patterns and Standards, according to which all Things in Nature are Formed, is Derived from the Nominal Eſſences, or the Abſtracted Ideas, of the Logicians of Genus and Species, which are Creatures of the Mind, and do not Really Subſiſt in Nature.

FIFTH, The Real Eſſences of Things, which are the Unknown Conſtitution of the Inſenſible Parts of Bodies, from whence their Senſible Qualities Flow, are in Truth nothing but Nominal Eſſences, or Abſtracted Ideas, made by the Mind, and do not Exiſt any where, but in the Underſtanding; Thus Matter, Abſtracted from it's Particular Qualities, by which it is made Similar and Homogeneous in all Corporeal Beings, tho' it is Imagined by Philoſophers to be Real, is nothing but a General Idea, an Abſtraction of the Mind, and a mere Nominal Eſſence, or a Genus of the Logicians, as I have Shewn in the Firſt Part of our Principles of Philoſophy, Chapter the Sixth.

SIXTH, Nominal and Real Eſſences of Simple Ideas and Modes, are not always the Same, but in Subſtances always Different, as this Philoſophy Affirms; For as Simple Ideas or Modes are General and Abſtracted Terms or Ideas, as Colour, Sound, Red, Green, Blue, Grave, Acute, Gratitude, Murder, *&c.* they have a Nominal Eſſence, but as Colour, Sound, Red, Murder, *&c.* are Real Properties in Nature, or are Real Human Actions, they have a Real Eſſence, and So of all Others. Abſtractions of the Mind have only a Nominal Eſſence, and no Real; Real Properties of Nature, or Real Human Actions, have a Real Eſſence Diſtinct and Different from the Nominal; Thus a Triangle, as it is a Genus or General Term, has a Nominal Eſſence, but not a Real; as a Triangle Exiſts in Nature, it has a Real Eſſence, and not a Nominal; And the ſame may be ſaid of Gold, or any other Subſtance; And what the Real Eſſences of a Triangle or Gold are, we have before Explained.

Of the Names of Simple Ideas. Mr. Locke's Eſſay, Chap. 4. Book 3.

§. 4. THO' Words Signify nothing Immediately, but the Ideas in the Mind of the Speaker, yet, it is ſaid, the Names of Simple Ideas, Mixed Modes, and Natural Subſtances, have each of them ſomething Peculiar.

FIRST, The Names of Simple Ideas, and Subſtances, with the Abſtracted Ideas in the Mind, Intimate ſome Real Exiſtence, from which was Derived their Original Pattern, but the Names of Mixed Modes Terminate in the Idea, that is, in the Mind.

SECOND, The Names of Simple Ideas and Modes Signify the Real, as well as Nominal Eſſences of their Species, the Names of Subſtances Signify Rarely, if ever, any Thing, but Barely the Nominal Eſſences of thoſe Species.

THIRD, The Names of Simple Ideas are not Capable of Definitions, Thoſe of Complex Ideas are; Becauſe a Definition is nothing elſe, but ſhewing the Meaning of one Word by ſeveral other *not Synonymous Terms*; The Names therefore of Simple Ideas are Incapable of being Defined, ſince the ſeveral Terms of a Definition Signifying ſeveral Ideas, they cannot Repreſent an Idea, which has no Compoſition at all.

And from not Obſerving this Difference in our Ideas, it is ſaid, Proceed thoſe Trifling Definitions of the *Ariſtotelian* Philoſophy; That Motion is the Act of a Being in Power, as far forth as it is in Power; and of the *Atomical*; That it is the Paſſage of a Body, from One Place to Another; or of the *Carteſian*; That it is the Succeſſive Application of the Parts of the Superficies of One Body to thoſe of Another; That Light, in the Peripatetick or *Ariſtotelian* Philoſophy, is the Act of Perſpicuous, as far Forth, as Perſpicuous; and in the *Carteſian*; That it is a Great Number of Globules Striking Briskly on the Bottom of the Eye; Theſe are ſaid to be Trifling Definitions, becauſe he that has not before Received into his Mind by the Proper In-let the Simple Idea, which any Word Stands for, can never come to Know the Signification of that Word, by any other Words or Sounds whatſoever.

But in Complex Ideas, it is ſaid, Definitions take Place; For Words Standing for thoſe ſeveral Ideas, that make up the Compoſition, may Imprint Complex Ideas in the Mind, that never were there before, and ſo make their Names be Underſtood; So in the Rain bow, by Enumerating the Figure, Largeneſs, Poſition, and Order of the Colours, it might be ſo well Defined, as to be Perfectly Known and Apprehended.

It is farther Said, that the Names of Simple Ideas, Subſtances, and Mixed Modes, have this Difference, that Thoſe of Mixed Modes Stand for Ideas Perfectly Arbitrary; Thoſe of Subſtances are not Perfectly ſo, but Refer to a Pattern with ſome Latitude; and Thoſe of Simple Ideas are Perfectly Taken from the Exiſtence of Things, and are not Arbitrary at all; All which Aſſertions we ſhall now Conſider.

Anſwer to the Foregoing Account.

And Firſt, this Philoſophy had Endeavoured to Prove, Chap. 2. that Words Signify Nothing, but the Ideas in the Mind of the Speaker, againſt which we Argued, §. 2. it now Retracts that Notion and Tells us, that the Names of Simple Ideas and Subſtances, with the Abſtracted Ideas in the Mind, Intimate Some Real Exiſtence, from which was Derived their Original Pattern, that is, that Words Signify ſomething more than the Ideas in the Mind of the Speaker, which we Affirmed; and Thoſe, who are not willing to be Convinced by the Arguments Alledged in that Section, I hope will Pleaſe to Acquieſce in the Acknowledgements of That Philoſophy, Which had Advanced the Contrary; 'Tis True, the Names of Mixed Modes Terminate in the Idea, that is in the Mind, but we have Evinced, that thoſe Mixed Modes are mere Creatures of it, and if not, that they Terminate in the Real Exiſtence of Human Actions.

Second, If the Names of Simple Ideas, and of Modes, Signify the Real, as well as Nominal Eſſences of their Species, they are upon the Level with each Other as to their Reality; Whereas before, Simple Ideas Intimated ſome Real Exiſtence, but Modes Terminated in the Idea, which was in the Mind; On the Contrary, if the Names of Subſtances Signify only the Nominal Eſſences of thoſe Species, ſince Nominal Eſſences are only General and Abſtracted Ideas, they Terminate in the Mind, and ought to be Joined with Mixed Modes, whereas before, they were Joined with Simple Ideas, as to their Intimating ſome Real Exiſtence.

To Conclude upon this Head, Simple Ideas, Subſtances, and Mixed Modes, have nothing Peculiar and Diſtinct from each Other, but do all Terminate in the Mind, when Conſidered as General and Abſtracted Ideas, under which all of the ſame Kind are Ranked and Sorted; And on the other Hand, they all Intimate ſome Real Exiſtence, from which thoſe Abſtractions are Made, and from which was Derived their Original Pattern, when they Signify thoſe Real Exiſtences, as Gold, *&c.* in Subſtances; Red, Yellow, Green, *&c.* in Simple Ideas; and Gratitude or Murder, in Mixed Modes; that is, when theſe are Suppoſed in Fact to Exiſt in Nature, or in Human Actions, and are not Merely General or Abſtracted Ideas, which only Terminate in the Mind.

From Hence it is, that all Simple Ideas, Subſtances, and Mixed Modes, have their Nominal and their Real Eſſences; The Firſt Concerns the Logician, who is willing to Rank and Claſs his Ideas under ſeveral Diſtinctions, which is Common to this Syſteme, with that of Predicaments and Predicables, and which is of little Importance or Significance to a Juſt Inquiry into Things; The Second Concerns Nature, and

Thoſe, who Love and Value the Studies of it, and which we have Endeavoured to Shew, and Evince, what they are, in the foregoing Chapter.

ANY Apparent Properties of Gold, a Man, a Triangle, *&c.* which are Sufficient to Diſtinguiſh one Sort of Beings from Another, will Serve for their Nominal Eſſences, but we muſt Inquire into the Nature of Gold, Man, or a Triangle, to Know the Real Eſſences of Them.

THIRD, Complex Ideas of Subſtances do not ſeem Capable of a Definition, no more than Simple Ideas, ſo far as to give us a True Conception of Them; Nor are they Known, but by Obſerving and Seeing Them.

FOR Firſt, they cannot be Defined by any *Synonymous Terms*; Becauſe, that is Explaining *Idem per Idem*, and they cannot be Defined by Terms, which are *not Synonymous*, or Equivalent to the whole Complexion, becauſe that is Explaining *Idem per Diverſum*; Or Thus, if Simple Ideas cannot be Defined, it is Impoſſible Complex Ideas, which Conſiſt of thoſe Simple Ones, ſhould be; unleſs we can Define an Aſſemblage of Simple Ideas, by an Aſſemblage of Simple Ideas, that is, by *Synonymous Terms*, or an Aſſemblage of Ideas, by that which is not an Aſſemblage of the ſame Ideas, that is by Terms, which are *not Synonymous*.

SECOND, Moreover, in Complex Ideas of Subſtances, the ſeveral Peculiar Ideas, which Enter into their Compoſition, Want Names to Diſtinguiſh them from all Others; Thus, the Infinite Variety of Colours and Shades of the ſame Colour, the Innumerable Kinds of Extenſion, and Sorts of Brightneſs or Dulneſs, of Smoothneſs, or Roughneſs, are Deſtitute of Terms to Explain them; And this is the Reaſon, why it is Impoſſible fully to Conceive the Deſcription of a Plant, or a Tree, a Metal, Stone, or an Ædifice, or of Landſkips and Countries, which we have not Seen; Nor can a Rainbow, which this Philoſophy Mentions, be ſo Defined to One, who never Saw it, by the Order of the Colours and Figure of the Bow, as to give him the ſame Idea, which he would have upon a View of it.

FIRST, Becauſe, a Bow Painted on Paper, with all the Diſtinctions of Colour with that in the Heavens, will not Produce the ſame Idea in us, with that in the Heavens; It will Want of it's Peculiar Hue, which Proceeds from the Sun's Various Impreſſions on the Æther, of it's Particular Agreeableneſs, Life, Vivacity, and Brightneſs, which will be Entirely Different from the Fineſt and moſt Delicate Colours Laid on Paper, or a Canvas; And yet no One can Deſcribe or Define to Another the Rainbow, with the ſame Advantage, as a Skilfull and Accurate Pencil may.

SECOND, Becauſe, we have not Names for the Different Reds, Blues, Greens, *&c.* which are as Diſtinct from each Other, as Red from Green, or Blue.

THIRD, Becauſe, a Blind Man may as well Know, what Light is, as a Man, who never ſaw the Rain bow in the Air, can Form a Notion of the Different Modifications of Light in the Air, from a Deſcription of them; A Rain-bow may, it is Likely, be ſo Defined or Deſcribed, as that we ſhall Know it, when we See it, that it is, what was Deſcribed; And ſo it is not Impoſſible, but Light may be ſo Defined to a Blind Man, as that he ſhould be Able to Diſtinguiſh it from Darkneſs, when he Receives his Sight, tho' the Idea of it may be Perfectly New to him; For a Blind Man knows, what Alacrity, Pleaſure, and Chearfulneſs is; And on the Contrary, what Melancholy, Unpleaſantneſs, and a Gloomineſs of Mind are; If therefore he ſhould be Told, that Light Imparts Alacrity and Pleaſure, and Darkneſs the Reverſe of it, all Spread thro' the Space he Walks in; It is not Improbable, but he might know the One from the Other, altho' his Conceptions of them are not any Thing Like what his Ideas would be upon the Actual Impreſſion of them.

WHAT we have ſaid of Subſtances, may be Equally Applied to Mixed Modes, which may be Indeed Defined, and Sufficiently Deſcribed, ſo long as they are Conſidered only as General, or Abſtracted Ideas, or Creatures of the Mind, and ſo may a Chimæra; That is, what Lies and Terminates in One Man's Mind, may be Exactly Communicated and Imparted to Another's, as a Triangle or Murder in General, or a Globe, or a Cube, or any Proportions in the Mathematicks, which are Abſtractions from Things, as they Really Exiſt; From whence a Blind Man is Equally Capable of ſuch Notions, and of having Juſt Conceptions of them, with One who has the moſt Abſolute, and Perfect Uſe of his Sight; But when we Conſider Mixed Modes, as Real Human Actions, which are the Foundation of thoſe Abſtractions, they are not, nor can they be Clearly and Fully Defined, for Inſtance, Murder; 'Tis True, we know it is Killing a Man with a Malice Prepenſe, and Extends to all Murders in General, but this is not to Repreſent any Particular Murder, which is Attended with that Horror from the Singular Behaviour, Innocence, Looks, and Features of the Perſon Suffering, and the Barbarity and Cruelty, and the Grim and Ghaſtly Countenance of the Aggreſſor, and the Diſtinguiſhing Paſſions of Each, that the whole of the Action, and all the Circumſtances of it, are no More, but much Leſs, to be Deſcribed or Defined, than they can be Fully and Exactly Painted.

Fourth, As to what is ſaid, Concerning the Trifling Definitions of Philoſophers, that they are Occaſioned from a Want of Obſerving, that Simple Ideas cannot be Defined, but that Complex Ones may; It is no ſuch Matter; For theſe Definitions of the Philoſophers, are not the Definitions of the Simple Ideas of Light and Motion; They Suppoſe both to be Known, as to the Ideas of them, and only Endeavour to Explain their Nature; And therefore the Abſurdity and Trifling Lies on the Side of the Logician, and not on the Side of the Philoſopher; The Truth here ſeems to be, that tho' this Syſteme of Ideas had Entirely Diſclaimed the Entering into Natural Philoſophy, yet it was willing to Shew, it was not out of an Ignorance, but a Contempt of it; And from this Example, I am Apt to Believe, it will be Pretty Evident, that it had Good Reaſons for the Latter, becauſe it had ſo Much of the Former.

Of the Names of Mixed Modes and Relations with the Anſwer. Mr. Locke's Eſſay, Chap. 5. Book 3.

§. 5. It is Eaſy from what we have already ſaid, Concerning Mixed Modes, to Determine againſt the Preſent Syſteme of Ideas.

First, That They are not made by the Underſtanding, and that They do not in this Reſpect Differ from Simple Ideas, which are not made by the Underſtanding; Becauſe They Depend upon the Real Exiſtence of Human Thoughts and Actions, as much as Simple Ideas do upon the Real Properties of the Mind or Body.

Second, That They are not made Arbitrarily without Patterns or Reference to any Real Exiſtence, and therefore do not in this Caſe Differ from our Ideas of Subſtances, which are not made without ſuch a Reference; And which is afterwards Confeſſed, when we are Told, that Mixed Modes are not made at Random, and Jumbled together without any Reaſon at all, but as Men have Frequent Occaſion to Mention them, that is, we Say, as Human Actions Require them; And if ſo, They are not made Arbitrarily, and without Patterns or Reference.

In Fine, Mixed Modes in all Particulars ſeem to Lie under the ſame Laws and Conditions with Thoſe of our Simple Ideas, and our Ideas of Subſtances; They are Equally Taken from Nature, and have as much a Foundation in it, as thoſe Others have; All the Difference is in the Patterns, from whence they are ſeverally Derived; The Simple and Uncompounded Properties of Beings are the Foundation of our Simple Ideas, Beings themſelves of our Ideas of Subſtances, and Human Thoughts and Actions of our Mixed Modes, which Human Thoughts and Actions are as Real in Nature, as Beings, or the Properties of Them; With this Diſtinction, that the Laſt are Neceſſary, and the First Voluntary only, and therefore may not Exiſt in Fact, but always muſt, and will, in the Power we have to Produce them.

On the Contrary, we Own, that All theſe Ideas, both Simple, Thoſe of Subſtances, and Mixed Modes, may be Conſidered as only Abſtracted Ideas, under which we Rank and Sort Things, without having any other Real Exiſtence; But then we Affirm, they are ſtill upon the Level, and are alike General Terms, and Logical or Metaphyſical Notions.

Nor is it True, which this Philoſophy of Ideas Aſſerts towards the End of this Chapter, that the Names of Mixed Modes are Commonly Got, before the Ideas, they Stand for, are Perfectly Known, whereas in Simple Ideas, and thoſe of Subſtances, the Names or Ideas are Got the One before the Other, as it Happens. For the Names of Mixed Modes, as well as of Simple Ideas and Subſtances, are Got before or after the Ideas of them are Perfectly Known, according as it Happens; Thus, we may as well See a Human Action Committed, as Murder, before we have a Proper Name for it, as we may See a Simple Idea, or a Subſtance, before the Name, by which it is Expreſſed, is Known to us; Nay it is Evident, that we are as much Taught in our Infancy, what to Call Human Actions, as by what Term to Signify our Idea of Red, Green, Blue, Sound, or any other Simple Idea; But we have 'Shewn all our Mixed Modes have their Foundation in Human Actions, and Depend upon them;

If yet we Mean by Mixed Modes, Abſtracted and General Ideas Formed by the Mind, it has been Proved, that they are of the ſame Nature with our Simple Ideas, or Thoſe of Subſtances, when made General and Abſtracted.

It is Said, what has been Affirmed of Mixed Modes, is Applicable to Relations, which we therefore need not Conſider.

Of the Names of Subſtances. Mr. Locke's Eſſay, Chap. 6. Book 3.

§. 6. This Chapter, Excluding all Unneceſſary Ambages and Circumlocutions, which this Philoſophy is not a little Fond of, is Contrived to give us an Account of what a Nominal and a Real Eſſence is, and the Diſtinction betwixt them.

The Meaſure and Boundary of each Sort of Beings, whereby it is Conſtituted that Particular Sort, and is Diſtinguiſhed from Others, is ſaid to be, what we Call it's Eſſence.

Which Eſſence is nothing but that Abſtracted Idea, to which that Name is Annexed, ſo that every Thing Contained in that Idea is Eſſential to That Sort, This is Called Nominal Eſſence.

But Real Eſſence is Affirmed to be that Real Conſtitution of Subſtances, on which this Nominal Eſſence, and all the Properties of That Sort Depend.

Thus, the Nominal Eſſence of Gold is that Complex Idea, the Word Gold Stands for, for Inſtance, a Body, Yellow, Weighty, Malleable, Fuſible, and Fixed; But it's Real Eſſence is the Conſtitution of it's Inſenſible Parts, on which all it's other Properties Depend, and which is Affirmed to be wholly Unknown to us; And the ſame is ſaid of Matter, it's Nominal Eſſence is Extenſion, or Solidity, or Both, according as Philoſophers Define it, but it's Real Eſſence is, what we are Entirely Ignorant of; And therefore it is Alledged, that we Rank and Sort Subſtances and Things by their Nominal, and not by their Real Eſſences, of which we Know nothing; And it is from hence Concluded, that it is in Vain to Think of Ranking or Sorting of Things by their Real Eſſences, whoſe Fine Contrivances are Infinitely beyond our Comprehenſion; Since the Workmanſhip of the Al-wiſe Powerful God in the Great Fabrick of the Univerſe, and every Part thereof, farther Exceeds the Comprehenſion of the moſt Inquiſitive and Intelligent Man, than the Beſt Contrivance of the moſt Ingenious Man doth the Conceptions of the moſt Ignorant of Rational Creatures.

This Philoſophy of Ideas Proceeds to Tell us, that Nominal Eſſences of Subſtances are not made ſo Arbitrarily, as Thoſe of Mixed Modes, but that it is Neceſſary in a Nominal Eſſence,

First, That the Ideas, whereof it Conſiſts, have ſuch an Union, as to make but One Idea, how Compounded ſoever; The Mind here only Follows Nature, and Puts none together, which are not Suppoſed to have an Union in it.

Second, That the Particular Ideas ſo United be Exactly the Same, neither More nor Leſs, otherwiſe they make Two Different, and not One and the Same Nominal Eſſence; And here, tho' the Mind never Puts together any Ideas, that do not, or are not Suppoſed to Co-exiſt, yet the Number it Combines, Depends upon the Various Care, Induſtry, or Fancy of him, that makes it; In Vegetables and Animals, which are Propagated by Seeds, and are Organized Bodies, the Shape is the moſt Characteriſtical Part that Determines the Species; in moſt other Bodies, which are not Propagated by Seeds, and are not Organized Bodies, it is the Colour we Chiefly Fix on, and are moſt Led by.

The Nominal Eſſences of Subſtances, tho' Suppoſed to be Copied from Nature, are very Imperfect, and we may Conclude, that theſe Boundaries of Species are as Men, and not as Nature makes them, if at Leaſt there are in Nature any ſuch Prefixed Bounds; And the Sorting of Things by us being in Order to Naming and Comprehending them under General Terms, it is Thought, it cannot Properly be ſaid, that Nature Sets the Boundaries of the Species of Things, but if it be ſo, that our Boundaries of Species are not Exactly Conformable to Nature; And in the whole Buſineſs of Genus and Species, the Genus, or more Comprehenſive Term, is but a Partial Conception of what is in the Species and the Species but a Partial Idea of what is to be Found in each Individual; and this is done without any Conſideration of Real Eſſences.

Anſwer to this Doctrine.

As to Nominal and Real Eſſences, we have before Diſcourſed Concerning them, and we ſhall only here farther Take Notice.

First, That this Syſteme of Ideas Owns, what we Endeavoured to Shew in our Principles, Chap. 6. Book 1. that Matter, as Received and Embraced by the Preſent Philoſophy, is an Hypotheſis or Creature of the Mind, and is nothing more than the Nominal Eſſence of it; and therefore It might as well Take the Genus and Species of the Logicians to Explain Nature, as that Nominal Matter, which it Aſſumes; And Conſequently Thoſe, who will not be Convinced by what we have Writ on that Head, we Hope will at Leaſt Subſcribe to the Authority and Confeſſions of ſo Oraculous and Infallible a Guide.

Second, That the Judgement, which is here made Concerning the Real Eſſences of Things, and the Fine Contrivances of them, being beyond our Comprehenſion, and the Pious Inference Drawn from thence, is only upon the Suppoſition of the Truth of the Corpuſcularian or Atomical Scheme: And we may Venture to Say, That from ſo Abſurd and Irrational a Plan, it would not only be Impoſſible for the Greateſt Genius's, which ever were in the World, to Sort Things according to their Real Eſſences, but for Infinite Wiſdom it's ſelf to do it; Becauſe it would be Endeavouring to Aſcertain the Reality of Beings from Chimæras and Fantaſtical Eſſences, which never had any Exiſtence in Nature, Excepting in our own Dreams and Imaginations.

Third, From hence likewiſe it is, that is, from it's being only Acquainted with the Corpuſcularian Scheme, that this Syſteme of Ideas Thinks, that the Boundaries of the Species of Things are as Men, and not as Nature makes them, and at the ſame Time Doubts, whether there are any ſuch Prefixed Bounds in Nature; that is, it only Knows the Nominal Eſſences of Things, which are ſaid to be Various in Different Men, and therefore it Concludes, or is Apt to Believe, there are no Real Eſſences of them; Whereas Nominal Eſſences may be Various, and may Conſiſt of One, Two, Three, or more Properties, which yet may Denote the ſame Real Eſſence; But becauſe One Perſon Signifies his Idea of Gold by it's Colour and Weight, Another by it's Colour, Weight, and Fixedneſs, and a Third Adds Fuſibility, and therefore their Nominal Eſſences Differ, Would any One from hence Infer, that it was Uncertain, whether there was ſuch a Real Eſſence in Nature, as that of Gold, which Intrinſically Diſtinguiſhed it from all other Metals or Subſtances whatſoever? The Nominal Eſſence with Men, let it Conſiſt of Fewer or More Properties, Diſtinguiſhes it from all other Subſtances, and the Real, in the Nature of Things; The Firſt is Contrived for the Mutual Communication of our Ideas, and for the Convenience of Converſation and Commerce, the Laſt is the Labour and Province of the Philoſopher. And the Same may be ſaid of Vegetables and Animals, that, Albeit the Nominal Eſſence is Varied, it is yet Intended to Signify and Denote the Same Thing or Real Eſſence, which tho' Unknown, yet may Poſſibly be Traced out and Diſcovered by the Diligence and Sagacity of the Studious and Inquiſitive.

Fourth, It is very Probable, as we before Hinted, That, where there are Two or Three Ideas, which Diſtinguiſh a Subſtance from all Others, all it's other Properties are likewiſe Diſtinct, and Mutually Infer Each Other.

Thus in Gold, which we Juſt now Mentioned, it's Peculiar Yellowneſs and Weight, will have a Particular Extenſion, Fuſibility, *&c.* Attending it, in Diſtinction from all other Subſtances; And ſo the Nominal Eſſence may be a Juſt Repreſentative of the Real; For a Triangle may have Ten Thouſand Properties by being Differently Related to and Compared with Different Quantities; And yet it's Two Properties of Three Lines, and Three Angles, which are Plain and Viſible, and which Diſtinguiſh it from all other Figures, are Thought to be a Sufficient Deſcription of it's Eſſence; It is the ſame in Subſtances, where the Real Eſſence of any One of them, as well as Nominal, may be Expreſſed by Two or Three Plain Properties, which ſhall Diſtinguiſh it from all other Subſtances, and the Numerous other Properties will Ariſe from the Various Applications of Different Subſtances to it, by which their Peculiar Expanſive and Contractive Forces will be Exerted, in which their Real Eſſences Conſiſt.

Of Particles. Mr. Locke's Eſſay, Chap. 7. Book 3.

§. 7. Particles are ſaid to be Words, which are made Uſe of by the Mind to Connect it's Ideas into Propoſitions, as *is, is not*, are Marks of Affirming or Denying; or to Unite whole Sentences with One Another, to make a Coherent Diſcourſe; or to Expreſs what Connexion, Reſtriction, Diſtinction, Oppoſition, Emphaſis, *&c.* it gives to the ſeveral Parts of it; Which cannot be Rightly Underſtood without a Clear View of the Poſtures, Stands, Turns, Limitations, Exceptions, and other Thoughts of the Mind, of which there is a great Variety, much Exceeding the Number of Particles, that moſt Languages have to Expreſs them by, from whence moſt Particles have Diverſe, and ſometimes Oppoſite Significations; Thus in the Particle *But*; *But to ſay no more,* Intimates a Stop of the Mind in the Courſe it was Going, before it came to the End of it; *I ſaw but Two Planets* Shews, that the Mind Limits the Senſe to what is Expreſſed with a Negation of all Other; *You Pray, but it is not, that God would bring you to the True Religion, but that he would Confirm you in your own*; The Former Intimates a Suppoſition in the Mind of ſomething otherwiſe than it ſhould be, the Latter Shews, that the Mind makes a Direct Oppoſition between that, and what goes before; *All Animals have Senſe, but a Dog is an Animal, Ergo a Dog has Senſe,* which Signifies the Connexion of the Latter Propoſition with the Former; And in the Right Uſe of theſe Particles it is Thought, that more Particularly Conſiſts the *Clearneſs* and *Beauty* of a *Good Stile.*

Anſwer to this Doctrine of Particles.

THAT there are Affirmations and Negations in the Mind of it's Ideas is Manifeſt, and that they are Expreſſed in our own Language by *is, and is not*, is as Evident; but the Queſtion in Philoſophy will be, which Way we make theſe Particles, or any Others, to be Significant of ſuch Affirmations, or Negations in the Mind, or of the Connexions, Reſtrictions, Diſtinctions, Poſtures, Stands, Turns, and Limitations in it, ſo as Others ſhould Underſtand them to be ſuch; Which is not here Explained; And which cannot be by making Uſe of Words, which only Signify our own Ideas, which yet this Philoſophy in Mixed Modes Affirms, but which ſhall Expreſs and Communicate the ſame Ideas to Others; This Difficulty notwithſtanding will be Eaſily Solved by what has been Alledged before, upon the Paſſions and Affections of the Mind, and the Way of Imparting them to Others by Names and Marks, upon which the ſame Community Agrees, and therefore there will be no Occaſion at Preſent to Enlarge upon this Head.

THE Second Thing to be Inquired into is, whether a Right Uſe of theſe Particles is That, in which Conſiſts the Clearneſs and Beauty of a Good Stile; Which is far from being True, becauſe the Clearneſs of a Stile does not ſo much Conſiſt in the Particles, as in the Explicit and Determined Ideas of the Mind, Expreſſed by the Same Terms always Annexed to the Same Explicit and Determined Ideas, and the Beauty of it is Owing to the Lively and Agreeable Images Raiſed in us by it; Which is Evident from what we find in *Homer*, whoſe Particles of ῥα, γὲ, νὺ, τὸι, tho' we do not fully Underſtand, yet his Images are ſo Determined and Juſt, and at the ſame Time ſo Lively and Agreeable, that we cannot but Pronounce his Stile to be both Clear and Beautifull; and the like may be Affirmed of *Demoſthenes*, tho' we do not ſo Perfectly Comprehend, what Perhaps is Meant by καὶ μὴν ὅπι γε, οὐ γὰρ δή πȣ μὲν, *&c.*

NAY farther in thoſe Particles, which are of Greater Importance in Language, and which this Philoſophy Principally Intends, which Connect and Range our Ideas into Sentences and Diſcourſe, there is many Times a Kind of Negligence and a Want of Strict Propriety and Grammar, where yet the Language is Sufficiently Intelligible and Clear, and the Notions Beautifull, Magnificent, and Sublime, which is Manifeſt in the Nobleſt Productions of the Ancients, as well as Moderns; For no One will Deny, that the Language and Stile of *Homer* is Infinitely Superior to that of *Apollonius Rhodius*, tho' the Laſt is a Critical Obſerver of theſe Particles, and the Rules of Conſtruction, whereas the Firſt is more Unreſtrained and Unconfined, and ſeems to Soar above them. It is the Same in *Pindar*, *Horace*, and our Own *Milton*, Compared with other Writers.

WE do not, it is True, come up to the Elegance and Fineſs of the Spirit of any Language as it was, when Spoken, Writ in, and Exactly Underſtood, and which, ſince that Time, we Call a Dead One; But yet no One will Say, that there is not both Clearneſs and Beauty in the Stile of the Claſſicks, tho' the Life and Spirit of them may be very much Loſt, from a Want of Underſtanding thoſe Small Expletives or Particles, we have Mentioned; It ſeems to be like a Fine and Beautifull Picture, which has Colour, Lineaments, Symmetry, and Proportion, but the Life and Spirit is wanting, Excepting ſo much as Subſiſts in the Grand, Excellent, and Divine Sentiments of the Authors; Tho' I do not Think it Impoſſible, by a Strict Application to the Study of them, and of *Demoſthenes*, and *Homer*, and *Sophocles* in Particular, to Renew this Life and Spirit to them, by an Explanation of thoſe Particles or Expletives, which have been hitherto Deſpaired of, and of which, in thoſe Great and Celebrated Genius's, I have given ſome Inſtances Formerly to my Pupils, in Reading of Them.

For the Trueſt Way of Interpreting the Mind and Senſe of an Author, and the Particles he makes Uſe of is, by Conſidering, and Endeavouring to Enter into, the True Poſture, Genius, and Force of his Mind, who Writes to us in a Dead Language; To Know and Inquire into all the Circumſtances of it at that Time, and from Comparing his Various Paſſions and Thoughts, and Collating the ſeveral Expreſſions in him with each Other, To Place ourſelves, as much as Poſſible, by all the Informations we can Get or Acquire Concerning him, in the Same State; This will give Life and Vigour to the Dead Language, and the Particles of it, which otherwiſe will be quite Loſt, and Appear as Flat and Inſipid, as the Reading or the Conſtruing of a Claſſick does to a School-Boy; Becauſe by this Means, that is, by Conſidering, what We ſhould Say in ſuch a Poſition of Circumſtances and Paſſions, we often, as Human Nature is much the Same in all, do Find out and Diſcover, what Really was ſaid in thoſe Circumſtances of Things, and under thoſe Affections of the Mind; If therefore we would Explain any Author, our firſt Buſineſs muſt be to Get into the Man, and we muſt do like the Famous *Roſcius*, Mentioned by *Tully* with Applauſe, who, to Expreſs a Character to the Life, was Obliged to Put himſelf into the Place and Circumſtance of the Character, he Perſonated or Explained.

Thus, if we would Explain *Demoſthenes*, we muſt Put on all the Paſſions and Vehemence of an *Athenian*, Defending the Liberties of *Greece*, againſt the Encroachments and Uſurpations of a *Barbarian*; If *Sophocles*, we muſt Place our ſelves in the Perſons and Circumſtances of all the Characters he Introduces; If *Homer*, we muſt do the Same in Reſpect of his ſeveral Heroes, and thoſe other Characters, which he Tranſmits to us; And What is Singular and Particular to each Author, will be beſt Explained, by Endeavouring to Know the Genius and Character of his Mind.

This alſo is the Trueſt Method, which any One can Purſue in Giving the Real Sentiments of St. *Paul*, or any Other Apoſtle, Evangeliſt, or Sacred Writer, Allowing for their Inſpiration; that is, the Mind muſt be Warmed with the Same Integrity and Zeal for the Chriſtian Faith and Perſuaſion, which Obtained in the Primitive Times of it, and then the Expoſitor will in all Likelihood Fall into the Same Notions, Paſſions, Way of Reaſoning, and Senſe, with the Divine Author he Expounds; But if Men will Explain the Scriptures without the Spirit of Them, by a Dull and Fantaſtick Way of Reaſoning upon them, which is the Caſe of our Preſent Syſtematiſt, in his Comments upon thoſe Writers, and of moſt of the *Socinians* and *Arians*, they will do much the Same, as if a Philoſopher ſhould Endeavour to Explain to us, what a Man is without his Soul, which in Fact is done by this Syſteme of Ideas, or what Fire is without any Heat belonging to it, which is done by it's Notable and *French* Maſter, or Inſtructor, *Des-Cartes*.

Of Abſtract and Concrete Terms. Mr. Locke's Eſſay, Chap. 8. Book 3.

§. 8. As to Abſtract and Concrete Terms it is Told us; That each Abſtract Idea being Diſtinct, ſo that the One can never be the Other, the Mind will by it's Intuitive Knowledge Perceive their Difference; and therefore in Propoſitions, no Two whole Ideas can ever be Affirmed One of Another, nor does the Common Uſe of Language Permit it; All our Affirmations are only in Concrete, which is the Affirming One Abſtract Idea to be Joined to Another, not that One Abſtract Idea is Another.

All our Simple Ideas, it is likewiſe Said, have Abſtract, as well as Concrete Names, as Whiteneſs, White, Sweetneſs, Sweet, *&c*; alſo our Ideas of Modes and Relations, as Juſtice, Juſt, Equality, Equal, *&c*; but as to our Ideas of Subſtances, we have very Few Abſtract Names at all, Excepting *Animalitas*, *Humanitas*, *&c.* which bear but a Small Proportion to the Infinite Number of Names of Subſtances, and which could never get Admittance into Common Uſe, or to be Current amongſt Underſtanding Men.

Anſwer to this Doctrine of Abſtract and Concrete Terms.

As to what is Said, that no Two Abſtract Words or Names of Abſtract Ideas can be in the Common Uſe of Language Affirmed One of Another, and that all our Affirmations are only in the Concrete, which is the Affirming One Abſtract Idea to be Joined to Another; It is no more than to Tell us, that we never Affirm, that a Subſtance, as Gold, is Yellowneſs, but that a Subſtance, as Gold, is Yellow, which is a Wonderfull Diſcovery, and Worthy of the Sagacity of this Grand Vocabulary of our Ideas.

As to what is afterwards Said, it ſeems to be a very Great Miſtake, and a Diſtinction is made without a Difference; For we have as many Abſtract Ideas of Subſtances, as we have of Simple Ideas, or Mixed Modes, or of Relations, and as many Concrete Terms belonging to thoſe Abſtract Ideas; Thus Man, Animal, Body, Gold, Iron, Stone, are all General, and Abſtract Ideas, and have Abſtract Terms Appertaining to them, and their Concretes are Human, Animate, Corporeal, Golden, Iron, and Stony; And it is as Impoſſible, that Subſtances ſhould not have Abſtract Ideas and Terms, and Concrete Ones belonging to them, as well as any Modes, Relations, or Simple Ideas, as that a Subſtantive in Grammar ſhould not have it's Adjective Derived from it.

Nor, becauſe the Schools have Formed but a Few Abſtract Names of Subſtances, as Animality, Humanity, Corporeity, *&c.* does it Follow, that a Thouſand more might not have been as Juſtly Adopted into Language, as Aureity, Ferreity, Lapideity, or Goldneſs, Ironneſs, Stoneneſs, *&c.* ſince they are all of them either the Same with the General Ideas Expreſſed by Animal, Man, Body, Gold, Iron, Stone, and therefore may be Equally Admitted into the Propriety of Speech, or elſe they are Abſtractions of Abſtractions, or General Ideas of General Ideas, which have no Particular Ones belonging to them, and therefore ought to be Equally Rejected as Mere Jargon, and the Creatures of Soleciſm and Barbarity.

Of the Imperfection of Words. Mr. Locke's Eſſay, Chap. 9. Book 3.

§. 9. The Uſe and End of Words is Said to be, either to Record our own Thoughts, or to Communicate them to Others; The Firſt is for the Help of our own Memories, for which any Words may ſerve Turn, which being Arbitrary Signs, we may Uſe, which we Pleaſe, for this Purpoſe, and there will be no Imperfection in them, if we Conſtantly Uſe the Same Sign for the Same Idea; The Second, Namely, for the Communication of our Ideas, has a Double Uſe; Firſt, a Civil Uſe, or ſuch a Communication of them, as may Serve in Common Converſation and Commerce about the Ordinary Affairs and Conveniences of Civil Life; Second, a Philoſophical Uſe, or ſuch as may Serve to Convey the Preciſe Notions of Things, and to Expreſs Certain Truths in General Propoſitions; Theſe Two Uſes are very Diſtinct, and a great deal leſs Exactneſs will Serve in the One, than in the Other.

The End of Language in Communication is, to be Underſtood, that is, to Excite by Sounds in the Hearer the Same Ideas, which they Stand for in the Mind of the Speaker; And the Doubtfullneſs and Uncertainty of their Signification, which is the Imperfection of Language here Meant, Ariſes more from the Ideas themſelves, than from any Incapacity in the Sounds to Signify them; For in that Regard, they are all Equally Perfect; That then, which makes the Difference is, the Difference of Ideas they Stand for, which muſt be Learned and Retained by thoſe, who would Diſcourſe together Intelligibly, which is Difficult in Theſe Caſes,

First, Where the Ideas, they Stand for, are very Complex, and Hence the Names of Mixed Modes are Liable to great Uncertainty and Obſcurity in their Signification; For here the Idea being made up of many Parts, it is not Eaſy to Form and Retain it Exactly; Of this Sort Chiefly are Moral Words, which have Seldom in Two Different Men the Same Preciſe Signification.

Second, Where the Ideas, they Stand for, have no Certain Connexion in Nature, and therefore no Settled Standard to Rectify and Adjuſt them by, which is again the Caſe of the Names of Mixed Modes, which are Aſſemblages of Ideas Put together at Pleaſure; Common Uſe Indeed Regulates the Meaning of Words Pretty well for Common Converſation, but it is not Sufficient to Adjuſt them to Philoſophical Diſcourſes, there being Scarce a Name of any very Complex Idea, which is not made the Sign of far Different Ideas.

The Way of Learning theſe Names is ſaid, not a little to Contribute to the Doubtfullneſs of their Signification; For Children are Taught the Names of Simple Ideas and Subſtances, by having the Things Shewn them; And then they Repeat the Name, that Stands for it, as White, Sweet, Milk, Sugar, *&c.* but in Mixed Modes, the Sounds are Learned Firſt, and Men are to Learn afterwards their Signification by their own Obſervation and Induſtry, or the Explication of Others, which is the Reaſon, that theſe Words are little more than bare Sounds in the Minds of Moſt; Becauſe Few are at the Pains to Settle their Ideas and Notions Preciſely, and Thoſe, who are, Make them the Signs of Ideas Different from what Others Underſtand by them, which is the Occaſion of moſt Diſputes.

THIRD, Where the Signification of a Word is Referred to a Standard, which is not Eaſily Known, as in the Names of Subſtances, which being Suppoſed to Stand for their Real Eſſences, which are Unknown, muſt needs be of Uncertain Application; And the like may be Said of the Names of Subſtances, when they Stand for the Various Collections of Simple Ideas, which are found to Co-exiſt together, and which are Put together Differently by Different Men.

FOURTH, Where the Signification of the Word, and the Real Eſſence of the Thing, are not the Same, which is ſtill the Caſe of Subſtances.

FROM Hence it is Said, Firſt, that the Names of Simple Ideas are leaſt Liable to Miſtakes, both becauſe they Stand for Single Perceptions, and becauſe they are not Referred to any Other Eſſence.

SECOND, That the Names of Simple Modes, are next the leaſt Liable to Doubt or Uncertainty, Eſpecially thoſe of Figure and Number.

THIRD, The Names of Mixed Modes, when they are Compoſed of a Few and Obvious Ideas, are Clear and Diſtinct enough, otherwiſe Doubtfull and Uncertain.

FOURTH, The Names of Subſtances, that are neither the Real Eſſences nor Exact Repreſentations of Things, are Liable yet to a greater Imperfection.

Anſwer to this Doctrine.

THE Grand Problem therefore Propoſed to be Solved in this Chapter is, from whence Ariſes the Imperfection, Doubtfullneſs, or Uncertainty of Language; In Order to do which it is Premiſed, that One End of Names and Words is to Record our own Thoughts or Ideas; and Conſequently in that Reſpect they cannot be Doubtfull, Uncertain, or Imperfect, if we always Uſe the ſame Sign for the ſame Idea; becauſe theſe Signs are Arbitrary, and we may Impoſe them upon our Ideas, as we Pleaſe.

But, Whoever Heard of a Language, the End of which was to Record our Ideas Separate and Diſtinct from our Communicating them to Others? Or did ever any One Sit down to Name his own Ideas Arbitrarily, and Aſſume Signs to that Purpoſe of his own Chooſing, which no One Underſtood beſides Himſelf, and which he did not, either by Defining them, or Shewing the Things which they Repreſented, Intend any One ſhould? For Inſtance in Botany, which is for the moſt Part a Mere Science of Words, and only Conſiſts in giving Names and Terms to Plants, which are very Frequently of little Uſe or Significance; Suppoſing a Perſon had Found out Twenty Four New Simples, Unknown before to the moſt Learned and Skilfull in the Knowledge of Vegetables, it is not to be Queſtioned, but he might Name them by the Twenty Four Letters of the Alphabet, as Signs are Arbitrary, if he did not Deſign to Communicate his Thoughts to Others; But it muſt be at the ſame Time Said, that he might as well not Name them at all; Since he would neither Diſtinguiſh them better, nor would he Retain them longer in his Memory by ſuch Denominations, than by a bare Contemplation of the Ideas themſelves; And any One may Perfectly Diſtinguiſh all the Plants in Nature from each Other, and Record their ſeveral Diſtinctions in his own Mind, by a Carefull and Conſtant Review of them, tho' he may not Know the Name or Term, which is Affixed to any of them; And there is nothing elſe, which makes that Knowledge ſo Delightfull to us, Excepting that we are Able by Names to Communicate our Ideas to Others; Which is a Remarkable Inſtance of the Innate Philanthropy of Human Nature; For if we were only to Name them for our Selves, there would be no more Satisfaction in it, than there would be to a Man, who is Ignorant in Botany, to Contemplate the ſeveral Species of Plants without Underſtanding the Terms, by which they are Diſtinguiſhed.

THIS therefore muſt be Laid down, as a Certain Maxim, that Words or Language, are never Intended to Mark and Record our Ideas, unleſs with a Deſign to Communicate them to Others; and he who ſhould go about to Invent Arbitrary Signs for his own Ideas without a Regard had to the making them Intelligible to Others, would not only Uſe an Imperfect, Doubtfull, and Uncertain Language, whatever this Philoſophy may Think, but a Chimerical, Whimſical, and a Romantick One; And that the Deſign of Words and Language, is to Communicate our Ideas, not to Record them, is Evident, becauſe all the Languages, which we at any Time Learn, are in Order to the One, and are of no Uſe or Service to the Other.

THE next Thing, which is Premiſed is, that Language, when Intended to Communicate our Ideas, may be either Termed Civil, that is, which we Uſe in Common Converſation and Commerce, or elſe Philoſophical, which is to Convey the Preciſe Notions of Things to us, and is therefore more Accurate and Exact than the Former; But I would Beg Leave to Ask, whether a Mechanick, and a Philoſopher, do not Mean the Same Thing by Gold, and whether a Triangle, or a Square, is not the Same Figure to a Peaſant or a Geometer? If ſo, Civil or Common Language does not Differ from the Philoſophick, ſince they both Expreſs the Same Things; If not, the Same Terms will be Applied to Different Ideas, or Different Terms to the Same Ideas, which will Render Civil Language Unintelligible to the Philoſophick, and Vice Verſâ; And Conſequently a Mechanick could not Converſe with a Philoſopher, or the Contrary, which is Contradicted by all the Mutual Commerce of the World.

'TIS True, a Philoſopher may Underſtand more Properties of the One, and a Geometer of the Other, than a Mechanick or a Peaſant is Capable of Knowing; But I would Preſume to Affirm, that neither Gold, nor a Triangle, become of Different Natures, by there being more Known of them in the Firſt Caſe, than in the Laſt; Thus in the Vegetable World, every One is Acquainted with as many Properties of an Oak, as will Serve to make him Diſtinguiſh it from an Aſh, or a Willow, and yet the Anatomy of Plants, and the Application of Microſcopes, may Furniſh us with ſeveral Qualities and Affections, which are Unknown to the Vulgar; Notwithſtanding which, I Hope this Philoſophick Oak, is no other than the Civil One, and that the Same Individual Oak is Meant and Expreſſed by the Same Name.

Dr. Grew's Anatomy of Plants.

BESIDES which, it has been already Shewn, that any Thing or Subſtance may be Sufficiently Known by One or Two of it's Leading Properties, upon which the Reſt Depend; and therefore thoſe Common Ones, by which Subſtances or Figures are Diſtinguiſhed from each Other in Civil Commerce and Converſation, as Gold or a Triangle, will as much Cauſe Men to Call the Same Things by the Same Name, as any other more Latent and Reſerved Properties will; For it is not to be Doubted, but there are ſtill Others in all Subſtances and Figures, which are yet not Diſcovered, and which it is Impoſſible ever ſhould be; Becauſe the Properties of every Subſtance, and of every Figure are Infinite, as their Compariſons with, and their Relations to, Other Subſtances and Figures are Infinite; the Ignorance of which Properties Notwithſtanding no more Hinders Philoſophick Men from Agreeing in their Language, than the Ignorance of them Hinders the Civil or Common Language from Agreeing with the Philoſophick; From whence it Follows, that the Philoſophick Language is in Vain Diſtinguiſhed in this Syſteme of Ideas from the Common or Civil.

AFTER theſe Premiſſes and Diſtinctions, which are Falſely Alledged, we are with an Equal Prevarication Told, that the End of Language in Communication is to be Underſtood, that is, to Excite by Sounds in the Hearer the Same Idea, which they Stand for in the Mind of the Speaker, which is True, if we Mean by the Same Idea, the Same Thing, as Gold or a Triangle, but if we Mean by the Same Idea, the Same Knowledge of all the ſeveral Properties of Gold or a Triangle, which this Philoſophy does, it is Falſe; For, to be Barely Underſtood it is Sufficient to Denote the Same Figure, Subſtance, or Thing, by the Same Name, which Others Uſe for the Same Figure, Subſtance, or Thing; Which is very Different from our Knowledge of all the ſeveral Properties of ſuch Figure, Subſtance or Thing, which is not to be Gained, unleſs by Study, Application, and Thought; and we may as Juſtly Say, if this is Intended by the Same Idea, That the End of Language is to be Underſtood, that is, to Excite by Sounds in the Hearer, the Knowledge of all the Abſtruſe Properties and Relations of Things, which are in the Mind of the Speaker, of which the Hearer is Ignorant, than which Nothing can be more Abſurd; becauſe it is to Affirm, that the End of Language in Communication, is to be One of the Greateſt Philoſophers in the World, which is the End of Labour and Study, and not of Language; Language gives us the Names of Things, which are Common to the Illiterate, and the

Learned; and Application and Study Furniſhes and Imparts to us the Knowledge of them, which is Peculiar to the Laſt.

And from Hence we are Carried to the Solution of the Problem, Namely, from whence Ariſes the Imperfection, Doubtfullneſs, or Uncertainty of Language, that is, from whence Ariſe all or moſt of our Diſputes; The Way of Solving which is, by Diſtinguiſhing our Ideas according to the Preſcriptions of this Syſteme of them; But if it has a Wrong Notion of Mixed Modes and Subſtances, as we have before Shewn it has, the Uncertainty and Imperfection, Charged upon Language, is Owing to it's Chimerical Opinions of theſe Matters, and to the Corpuſcular Scheme of Nature, which this Philoſophy Defends, and from which it's Sentiments of Subſtances in Particular are Derived; And Language is ſaid to be Imperfect and Uncertain, becauſe it's own Ideas are ſo.

And Firſt, as to Mixed Modes, which are General Ideas Formed from Human Actions, as Virtue, Vice, Gratitude, Murder, this Philoſophy only Conſiders them as Arbitrary Collections of Ideas, and which have no Settled Standard to Rectify and Adjuſt them by; Whereas they are in that Reſpect upon the Level with our Simple Ideas, or our Ideas of Subſtances; Thus, if in Simple Ideas, we Take any Particular Idea as Red, it is Explicite, Certain, and Determined, but the General, which Includes ſeveral Particular Reds, is not; Thus alſo in Subſtances, any Particular Idea of Matter, as Gold, or Marble, is Determined and Certain, but the General One not; And the Same may be Said of Mixed Modes, that Human Actions are Determined and Certain, but the Genus's under which they are Ranged, are not ſo; And the Particulars always Anſwer to ſome Originals, whereas the Generals are Abſtracted Ideas, and only have their Exiſtence in the Mind: And in Reſpect of the Principle, from whence theſe Ideas Flow, they ſeem to be alike Certain, and Determined; For, as in Simple Ideas, Red or Blue, *&c.* or in Subſtances, the Properties of Gold, are Actions of Bodies Derived from a Peculiar Force, which is Capable of Producing thoſe Senſations in us, ſo in Mixed Modes or Human Actions, all Particular Ones are Derived from a Principle of the Mind, from whence they are Denominated what they are; For Inſtance, when we Look into our Minds, we find a Force or Quality in it, which Inclines us to the Stricteſt and Severeſt Dealing with Others in all Occurrences of Life, which makes us Punctual, and Exact, both in every Thing, which we Say, and every Thing which we do; Human Actions, which Reſult from ſuch a Principle are Juſt; In like Manner, we find another Force or Principle in our Minds, which Inclines us to Love and Compaſſion, and thoſe Human Actions, which Derive from it, are Properly Termed Benevolent; So that the Nature of all theſe Ideas, whether Simple, or of Subſtances, or of Human Actions, or Mixed Modes, is Equally Known to us, and there is no more Doubtfullneſs, Imperfection, or Uncertainty in the One, than the Other; And if we Conſider them as General Ideas, they are only then Names, by which we Claſs and Sort Things or Actions.

As to the Way of Learning the Names of the ſeveral Ideas, whether Simple, or thoſe of Mixed Modes and Subſtances, which this Philoſophy before Mentions, that can Signify nothing to the Clearneſs or Obſcurity of the Ideas, in which it is Said the Imperfection and Uncertainty of Language, or the Contrary, Conſiſts; For it will be the Same, whether we Learn the Articulate Sound of Red or Blue, before we Know the Proper Idea which belongs to it, and are afterwards Shewn it, or whether we Learn the Terms Red and Blue, and the Ideas Annexed to them together; And the Same is True in Mixed Modes; I ſhall not have a Clearer Idea of what Juſtice is, by Learning the Term, and the Idea together, than if I Learned the Term before the Idea; And the Imperfection, Doubtfullneſs, or Uncertainty of Terms, or of Language, Proceeds from the Imperfection, Doubtfullneſs, or Uncertainty of the Nature of the Ideas themſelves, which we have before Conſidered, And not from the Way that we Learn them.

It is farther Affirmed in this Philoſophy of Ideas, that Simple Ideas are leaſt Liable to Miſtakes; becauſe they are each of them but One Single Perception, whereas the more Complex Ones of Subſtances and Mixed Modes are not; And becauſe they are not Referred to any other Eſſence, but barely that Perception they Immediately Signify. On the Contrary we Say, Particular Subſtances, or Particular Human Actions,

ions, are as much One Single Perception, as Red or Blue is, and are Equally, not Referred to any other Eſſence, but Barely that Perception, which they Immediately Signify; But if General and Abſtracted Ideas are Formed from Particular Subſtances, or Particular Human Actions, the Same may be done in the Simple Ideas of Red or Blue, which may likewiſe be made General by the Abſtractions of the Mind, and therefore they will be ſtill upon the Same Foot, and not Diſtinguiſhed in this Reſpect from each Other.

It is Moreover here Alledged, that in Mixed Modes, when they are Compoſed of a Few and Obvious Ideas, their Names are Clear and Diſtinct enough, otherwiſe Doubtfull, and Uncertain; But this Allegation Proceeds upon Mixed Modes, being Suppoſed to be Arbitrary Collections of Ideas in the Mind; Whereas on the other Hand, Mixed Modes are General Ideas made by Abſtraction upon Human Actions, and which are not Diſtinguiſhed from each Other, by the External Circumſtances, which this Philoſophy ſeems to Apprehend, but from the Nature and Principle of Them; Thus Modeſty is Sufficiently Diſtinguiſhed from all other Human Actions, not by the External Paſſions of the Body, as by Bluſhing, or Looking Pale, but from the Internal Principle of Diffidence and Humility, from which it Proceeds; For Different Paſſions of the Body may Proceed from the Same Principle or Paſſion in the Mind; As the Same Paſſions in the Body may likewiſe from Different Principles or Paſſions in the Mind, for Inſtance, Bluſhing from Pride and Indignation, or from Humility and Modeſty; And therefore, as in Simple Ideas, and Subſtances, ſo in Mixed Modes, the Perceptions are Diſtinguiſhed enough, but the Nature of all of them Obſcure and Undetermined, which it is the Province of Reaſon and Philoſophy to Diſcover; ſo that Simple Ideas, Subſtances, and Mixed Modes, are ſtill upon the Same Level in that Particular; Which Conſideration alone is Sufficient to Invalidate all Mr. *Wollaſton's* Scheme of Natural Religion, ſince the Principles of Men's Actions are ſo very Different, when the Explanations of them are the Same, and Vice Verſa.

The Concluſion of this Chapter is with a Preference of Natural Religion to a Revealed, becauſe there is ſuch an Uncertainty in Words, in which the Scriptures and Written Relations are Delivered to us, whereas Natural Religion is Thought to be Clear and Plain; But it has been Proved, that Words are Certain enough, and all the Difficulty is in our not Underſtanding the Real Nature of Things, in which Difficulty Human Actions are Involved, as much as even Subſtances themſelves, and which as much Want an Explanation from Philoſophy.

This Arguing againſt Revealed Religion Amounts therefore to This, that becauſe we do not Underſtand the Nature of Things, we are not to Depend upon any Hiſtory of Fact, that ever was in the World, which is Exceedingly Wiſe, and moſt Wonderfully Sagacious!

Of the Abuſe of Words. Mr. Locke's Eſſay, Chap. 10. Book 3. *The Firſt Abuſe.*

§. 10. Beside the Natural and Unavoidable Imperfections of Languages, it is Said, there are Willfull Faults and Neglects, which Men are often Guilty of in their Uſe of Words.

First, it is Affirmed, they Uſe Words without Clear and Diſtinct Ideas, or, which is Worſe, Signs without any Thing Signified; Such are for the moſt Part Introduced by Sects of Philoſophy and Religion, either out of an Affectation of Singularity, or to Support ſome Strange Opinion, or to Cover the Weakneſs of their Hypotheſis; Theſe it is Thought may be Juſtly Called Inſignificant Terms, and are often to be Met with in School-Men and Metaphyſicians; Others Learn Words, which the Propriety of Language has Affixed to very Important Ideas, and often upon Occaſion Uſe them without any Diſtinct Meaning at all; From whence their Diſcourſe muſt be Filled with Empty, Unintelligible, Noiſe and Jargon, and Eſpecially in Moral Matters; Amongſt which Inſignificant Terms, Glory and Grace are Mentioned by this Philoſophy.

Anſwer.

This Paragraph therefore ſeems to be Little Elſe than a Short and Comprehenſive Satyr, or Invective, againſt Philoſophers, and Divines, without any Particular Charge againſt Either, unleſs in General Terms, and Propoſes the Author, as Supreme Judge of both theſe Profeſſions of Men, when himſelf muſt be Conſcious, how Unequal he was to ſuch a Province, who was Scarcely Acquainted with the Firſt Rudiments and Principles of Revelation, or of Nature.

It is Said, School-Men and Metaphyſicians are here only Deſigned, but it is too Plain and Evident, that Revelation it's ſelf is Aimed at, by the Firſt, under the Words Glory and Grace, and the Terms of Singularity and Strange Opinions; and that Philoſophy and Nature are ſo too by the Laſt; That is, the Preſent Syſteme of Ideas, Intending to Pave a Way for the Reception of a Natural Religion, that is, for the Ancient Heatheniſm, Deſpiſes the Arguments and Diſputes about a Revealed One, or about the Explanations of Nature and Philoſophy, and Repreſents them, as nothing more than an Abuſe of Words, and the Fancies and Chimeras of Metaphyſicians and School-Men; Whereas in Reality, the Main and Principal Conteſts are not Concerning Words and Terms, but the Truth or Falſehood of Things, and which Ariſe from the Difficulties, in which they are Entangled, when we come to Conſider them; Thus, there is as Great an Abuſe of Words amongſt the Rationaliſts or Deiſts, or the Defenders of this Syſteme of Ideas, as there is amongſt Philoſophers or Divines; And the Queſtion upon the whole will be, not which of them Abuſes Words Moſt, but which of them Underſtands Things Beſt, and which upon Inquiry, will be Determined in Favour of the Latter.

A Second Abuſe.

Second, It is Said, another Abuſe is Inconſtancy in the Uſe of Words, the Wilfull doing whereof can be Imputed to nothing but Great Folly, or Greater Diſhoneſty; And a Man in his Accounts with Another, may with as much Fairneſs make the Characters of Numbers Stand, ſometimes for One, and ſometimes for Another Collection of Units.

Anſwer.

In Obſervation upon which Remark I ſhall only Say, that this Chapter Treats of the Wilfull Faults and Neglects of Men in their Uſe of Words, of which this is Eſteemed One of them; Whereas, it is Evident in Fact and Experience, that Men are Naturally Prone to Change their Ideas, to which they Affix the Same Word or Term, and it Requires the utmoſt Caution and Attention not to do it; Thus in Dreams, where the Mind is not any ways upon it's Guard, the Idea, which is Affixed to the Name of Man ſhall Change into a Multitude of Shapes, when yet we Think we are Dreaming of the Same Perſon, and ſo of other Ideas; And which ſeems to be the Chief Occaſion of that Confuſion, which we find in thoſe Deluſory Sports of Nature; It is in ſome Degree the Same, when we are Awake, as is Manifeſt in the Reaſonings both of this Syſteme of Ideas, and of the Philoſophy of *Des-Cartes*, and even in thoſe of the Great and Incomparable Sir *Iſ. Newton*, and in Particular of their Notion of Matter, which is ſometimes One Thing, and ſometimes Another, juſt as it Suits the Preſent Emergence of Explaining this or the other Phænomenon, by which it becomes at Length a Subſtance Entirely Different from what it was at Firſt Thought and Aſſerted to be; Inſomuch, that what was at Firſt Inactive, and was Endued with a Perfect Inertia, ſhall by Degrees Riſe into Action, and Find it's ſelf Changed in it's Properties into all the Varieties of Motion, Gravitation, or Attraction, which we can Poſſibly Imagine; This however does not Appear to be a Wilfull Fault or Neglect, but a Natural Infirmity of the Mind, which is not to be Avoided without the utmoſt Guard and Caution againſt it, and is therefore neither Chargeable with Folly nor Diſhoneſty.

A Third Abuſe.

Third, Another Abuſe of Words is an Affected Obſcurity, either by Uſing Old Words in New Significations, or by Introducing New and Ambiguous Terms without Defining them, or by Putting them together, ſo as to Confound their Ordinary Meaning; The Peripatetick Philoſophy, and the Admired Art of Diſputing, are Loaded with this Charge.

Anſwer.

But this Cenſure may be Attributed to the Author's Diſlike of the Worthy Univerſity, of which he had the Honour to be a Member, and of the Philoſophy, which is Profeſſed there, rather than to any Solid Reaſon for it; For, notwithſtanding what has been Alledged againſt *Ariſtotle*, and his Syſteme of Nature, there will not be Found a more Affected Obſcurity in the Peripatetick Account of it, than there is in the Atomical or Epicurean, which is at Preſent Eſpouſed by our Modern Reaſoners, and with what Juſtice we have already Shewn.

A Fourth Abuſe and the Anſwer.

Fourth, Another Abuſe is, the Taking Words for Things; Thus in the Peripatetick Philoſophy, Subſtantial Forms, Abhorrence of a Vacuum, *&c.* are Taken for ſomething Real. Here alſo the Same Philoſophy is Attaqued, but without any One Argument againſt it; Since Subſtantial Forms are nothing more than ſome Certain

Eſſences, from whence the Properties of Particular Bodies are Derived, and are therefore as Real as the Unknown Subſtratums or Subſtances of the Preſent Syſtems; and an Abhorrence of a Vacuum, is no other than Affirming the Neceſſity in Nature of a Plenum; Conſequently to make this Remark Significant, it ſhould be Proved, that there is no Real Eſſence of Things, nor any Plenum Exiſting, which will in the Iſſue be a Difficult Task to Perform.

A Fifth Abuſe.

FIFTH, Another Abuſe is, the Setting Words in the Place of Things, which they can by no Means Signify; As in the General Names of Subſtances, whereof the Nominal Eſſences are only Known to us, when we Affirm or Deny any Thing about them, we Tacitly Suppoſe or Intend, they ſhould Stand for the Real Eſſence of a Certain Sort of Subſtances; Thus, when a Man Says Gold is Malleable, he would Inſinuate ſomething more than This, that what I Call Gold is Malleable, (tho' Truly it Amounts to no more,) Namely, that what has the Real Eſſence of Gold is Malleable, that is, that Malleableneſs Depends on, and is Inſeparable from, the Real Eſſence of Gold; But, a Man not Knowing, wherein that Real Eſſence Conſiſts, the Connexion in his Mind of Malleableneſs is not Truly with an Eſſence, he Knows not, but with the Sound Gold, he Puts for it.

Anſwer.

IN Anſwer to which we Say, that Something more is Meant than the Nominal Eſſence of Gold, which is a General Term, and a Creature of the Mind, when we Affirm, that Gold is Malleable, Namely, that Real Gold is ſo; of which tho' we have not an Idea, as to it's Real Eſſence, or Principle, from whence it's Properties Flow, yet we Know as many of it's Real and Eſſential Properties, as are Sufficient to Diſtinguiſh it from all other Bodies or Subſtances whatſoever, and which muſt Include and Suppoſe a Real Eſſence Different from that of any other Subſtances, as it's Eſſential Properties are ſo; And by no other Ways is a Triangle Diſtinguiſhed from a Square or a Circle; But Concerning the Real Eſſences of Subſtances or Figures, we have already Diſcourſed, and Shewn what they are, and wherein they Conſiſt.

What is farther Alledged under this Fifth Head.

IT is farther Alledged in this Fifth Article, that in Mixed Modes, any Idea of the Complex One being Left out, or Changed, it is Allowed to be another Thing, that is, to be of another Species, as is Plain in Chance-Medley, Man-Slaughter, Murder, *&c.* becauſe the Complex Idea Signified by that Name is the Real, as well as Nominal Eſſence, and there is no Secret Reference of that Name to any other Eſſence but That; But in Subſtances it is not ſo, for tho' in That Called Gold, One Puts in his Complex Idea, what Another Leaves out, and Vice Verſâ, yet Men do not Uſually Think the Species Changed, becauſe they Refer the Name in their Minds to a Real Immutable Eſſence of a Thing Exiſting, on which thoſe Properties Depend.

Anſwer.

To which we likewiſe Reply, that in Human Actions, or Mixed Modes, the Caſe is much the Same with that of Subſtances; becauſe, as in Subſtances we have Eſſential Properties or Qualities enough to Diſtinguiſh One Subſtance from Another, and when we have ſo, we Know them, and Name them as Different Subſtances; So in Mixed Modes or Human Actions, tho' the Fact in all theſe Caſes Mentioned is Killing a Man, yet, ſince ſuch an Action is Derived from a very Different State and Principle of the Mind, and ſince the Mind, or the Active Principle, is as much to be Conſidered in ſuch Actions, as the Hand, which Commits the Fact, theſe Human Actions are as much Differenced by the Real Properties of them, as Subſtances are; In Chance-Medley, the Mind does not Act with an Intention, in Man Slaughter it does, but with a Sudden One, in Murder with a Premeditated and a Malicious; It is True, the Killing of a Man belongs to all of them, and ſo does Extenſion to all Subſtances, which are Material; But the Killing a Man upon Different Principles of the Action, is as much Diſtinguiſhed by the Peculiarities of Doing it, as the Extenſion of Suſtances is by the Particular Properties which Attend it, as Difference of Colour, Denſity, Weight, Brightneſs, Smoothneſs, *&c.*

NOR is the Difference, Affirmed above, betwixt Mixed Modes and Subſtances True; Becauſe we may Add a Million of Properties to our Idea of Gold, by Applying it to the Infinitely Various Actions of other Bodies and Subſtances upon it, and yet not Change the Nature of it, or our Idea of that Nature; in the Same Manner as we may in a Triangle, or any other Geometrick Figure, which, in it's Relation to other Mathematick Quantities or Figures, will Admit of an Infinite Number of Properties, when yet the Nature of the Triangle is not from thence Altered, but Confirmed and Aſcertained; And the Same may be ſaid in Mixed Modes, or Human Actions, that the Nature of the Action, as of Murder, ſhall be the Same, tho' there may be Various Properties Annexed to it, from it's Compariſon with and Relation to a Million, or an Infinite Number of Actions belonging to Human Beings, which yet will not Alter the Principle of the Mind, from which it Proceeded; Thus Murder, or the Killing of a Man with a Premeditated and a Malicious Deſign, is Sufficient to Shew the Nature of ſuch an Action, and yet we may Add the Perſon of the Man, the Way and Manner of Killing, we may Compare this Action with Gratitude, or Benevolence, or with Theft, Whoredom, or Drunkeneſs, and a Thouſand other Human Actions, in Reſpect of which it will have it's Particular Properties and Relations, but yet will ſtill Continue an Act of the Same Nature.

What is Farther Alledged under this Fifth Head.

It is Farther Alledged under this Fifth Article, that the Reference of the Name to a Thing we have not the Idea of, is Grounded upon this Suppoſition, Namely, that the Same Preciſe Internal Conſtitution goes always with the Same Specifick Name, in which are Contained theſe Two Falſe Suppoſitions.

First, That there are Certain Preciſe Eſſences, according to which Nature Makes all Particular Things, and by which they are Diſtinguiſhed into Species.

Second, That this Tacitly Inſinuates, as if we had Ideas of theſe Eſſences.

Anſwer.

To which we Anſwer, that there are no Two ſuch Suppoſitions, as are Imagined.

First, Becauſe we do not Suppoſe any ſuch Chimerical Eſſences, ſince we only Suppoſe ſome Certain and Eſſential Properties, with which the Eſſence of Things is Connected, or rather, which is the Real Eſſence of them, as before Explained, and which Eſſential Properties every One may, and does Know, ſo as Really to Diſtinguiſh One Subſtance from Another.

Second, Becauſe, tho' we ſhould not Pretend to any Knowledge of the Internal and Real Eſſences of Subſtances, yet we can Fully Know ſome Real and Eſſential Properties, which are as Expreſſive and Significant of the Eſſences, and which as much Diſtinguiſh their Subſtances from all Others, as the Eſſences themſelves.

A Sixth Abuſe.

Sixth, Another and more General Abuſe of Words is Said to be, that Men, having by Long and Familiar Uſe Annexed to them Certain Ideas, they are Apt to Imagine ſo Near and Neceſſary a Connexion between the Names, and the Significations they Uſe them in, that they forwardly Suppoſe, One cannot but Underſtand what their Meaning is, when they are no more than the Voluntary and Unſteady Signs of their own Ideas.

Thus Life is a Term, none more Familiar; And yet if it Comes in Queſtion, whether ſuch a Thing has Life, or not, it is Eaſy to Perceive, that a Clear, Diſtinct, Settled, Idea does not always Accompany the Uſe of ſo Known a Word.

Anſwer.

On the Contrary, we Say, that Men are Generally Agreed in their Uſe of Words, and Underſtand each Other well enough by the Marks or Signs, by which they Diſtinguiſh their Ideas; and the Chief and Principal Difficulty Lies in the Nature of Things; Thus every One Means the Same Thing by Gold, but yet the Real Nature and Eſſence of it may not be Known to Any, and there may be Diſputes and Differences about it; The Same may be ſaid of Vacuum, Plenum, Matter, *&c.* which Terms are Sufficiently Intelligible to all, and yet the Controverſies about the Things themſelves are Endleſs upon Account of the Perplexity and Obſcurity of them; So likewiſe in the Inſtance Mentioned, we do not Miſtake One Another in what we Mean by Life, or almoſt in any other Term, to which we Affix an Idea; We all Know, that an Animal has Life, and that a Vegetable has, according to the Common Acceptation of the Word, that a Plant is Alive; But then, wherein the Life of the One, or the Other Conſiſts, is a Diſpute of the Nature of Things, and not of Words, or the Meaning of them; It is an Inquiry of Philoſophy, and a Difficulty in it, and is not a Want of Affixing Certain Ideas, to Certain Terms; Thus where we See in Animals a Motion, which is Voluntary, and Contrary to a Neceſſary Impulſe, as of a Ball Struck by a Racket, and there are ſeveral Indications of Thought and Senſe, we Call it Life; If in Vegetables we Obſerve a Verdure and Sprightlineſs of Growth, Contrary to what we Find in Groſs and Dull Matter, which Continues the Same with Little or no Alteration, we Call it alſo Life, and there is no Diſagreement in the Terms or the Signification of them; But what is the Cauſe of either of theſe Phenomenons, or what the Nature of them, may be a Queſtion, and which Reaſon and Philoſophy is to Reſolve.

The Seventh Abuſe.

SEVENTH, It is Affirmed, that Figurative Speech is an Abuſe of Language, which is True, where it is Intended to Enſnare and Deceive, otherwiſe it is Unjuſtly Condemned; Becauſe, if we can Infuſe into Others our Real Ideas by Metaphors, it will be the Same, as if we did it by Simple and Plain Expreſſions; Excepting, that the One Conveys the Same Ideas to us with Pleaſure, whereas the Other has not the like Advantage of Elegance and Beauty to Recommend it; And I Hope no One will Deny, that as Great and Important Truths in Morality are Impreſſed upon us from the Divine Parables of our Saviour, and the Exquiſite Fables of *Æſop* and the Antients, as if they had been Delivered in a Dry Syſteme of Precepts, and in the Stiff Garb and Dreſs of an Heathen Philoſophy.

IF yet Figurative Speech is an Abuſe of Language, what Pity it is, that there ſhould be ſo many Thouſand Abuſes of it in the very Philoſophy, which Condemns and Explodes it? and who would Think, we ſhould, Preſently after ſuch a Cenſure, find it Talking in this Manner? "Language being the Great Conduit, whereby Men Convey their Diſcoveries, Reaſonings, and Knowledge, from One to Another; He that Makes an Ill Uſe of it, tho' he does not Corrupt the Fountains of Knowledge, which are in Things themſelves, yet he does, as much as in him Lies, Break or Stop the Pipes, whereby it is Diſtributed to the Publick Uſe and Advantage of Mankind. Which Language, I Suppoſe, is ſomething Figurative.

HOWEVER, ſince, according to *Laertius*, *Epicurus* had Profeſſed the Same Doctrine, See Chap. 1. Book 1. it was Neceſſary for a Philoſophy to Eſpouſe the Sentiments of One, whom in ſo many other Particulars it had Acknowledged to be it's Maſter; And, I Preſume it is more Pardonable, when Figures and Tropes are Uſed in a Philoſophy, which Diſclaims them, than when they are Adopted into the Lively Eloquence of the Divine, or the Pulpit, where, I Humbly Conjecture, it Chiefly Blames the Force and Beauty of them.

Of the Remedies of the Foregoing Imperfections and Abuſes, and the Anſwer. Mr. Locke's Eſſay, Chap. 11. Book 3.

§. 11. As to the Remedies of the Foregoing Imperfections and Abuſes, we ſhall Say Little to them, Seeing we have already Shewn, that they are either Imaginary, or Falſely Repreſented, or Derived from the Wrong Notions, which this Philoſophy has taken up of Subſtances and the Real and Nominal Eſſences of them, and which Generally Terminate in this Suppoſition, that Men do not Underſtand Each Other's Terms, and the Language, in which they Diſcourſe; Whereas it is pretty Evident from a Thouſand Inſtances in Mathematicks, Philoſophy, and Divinity, that our Diſputes are, not ſo much about Words and Terms Illy Underſtood, as about Things and the Real Difficulties, with which they are Invironed and Surrounded. Inſtinct, Sympathy, and Antipathy, which are here Mentioned, are well enough Known, as to the Properties and Affections in Animals or Bodies, which are Expreſſed by them, and they are far from being Words without a Meaning, as is Affirmed; But what is the Cauſe of them, or from whence they Ariſe, or wherein the Nature of the One or the Other Conſiſts, is Hard to Determine; and ſo it is of moſt of our Other Terms, and the Ideas Annexed to them; That is, Men, who are Educated and Trained up in the Same Language, in the General Know, what Ideas, or Things, are Marked out, and Pointed at, by ſuch and ſuch Words or Terms, tho' they may not Know the Entire Nature of thoſe Things or Ideas, which yet are Confounded and Made to be the Same by the Preſent Syſteme; Thus, when Gold, or Silver, or Wood, or Stone, is Named, every One is Carried in his Mind to the Same Subſtance, and no One has ever Miſtook ſo, as to Call Stone Metal, or Wood Stone; But if we come to Inquire into the Real Nature and Conſtitution of any of Theſe, we are then at a Loſs; Clouds and Obſcurity begin to Ariſe, we Feel about in the Dark, and often Claſh and Interfere with each Other; It yet ought not to be Said, that we do not Affix the Same Ideas to the Same Terms, becauſe we Reaſon and Philoſophize Differently about our Ideas themſelves, no more than it ſhould be Said, that we cannot Converſe Intelligibly together, unleſs we are all Philoſophers, and Philoſophers of the Same Syſteme and Perſuaſion; On the Contrary, Men could not ſo much as Reaſon and Philoſophize Differently about their Ideas, if thoſe Ideas, about which they Diſpute, were not the Same: And therefore it is nothing elſe than a Piece of Affectation and Vanity to Aſſert, which is done, that if all Diſputes about Words and Terms were Cut off and Abridged, many of the Philoſophers, (not to Mention thoſe of Divines) as well as Poets Works, might be Contained in a Nutſhel; Since the Principal Difficulty is not to Underſtand Each Other's Language, but to Determine Concerning the Truth and Reality of the Things, Expreſſed and Intimated by ſuch Language.

NOTWITHSTANDING which, we are made to Believe, that, whilſt Men are Diſcourſing in Plain *Engliſh* to Each Other, they are yet ſo Uncertain in the Ideas, which they Affix to the Terms they make Uſe of, that they might as well Propoſe their Thoughts or Sentiments in the *Ruſſian* Dialect, and Write or Speak in the *Turkiſh*, or the *Sclavonian*; Which is ſo Monſtrous a Poſition, that this Philoſophy muſt have been well Acquainted with the Art of Rhetorick and Inveigling, and how to Cheat and Delude Men with it's Tropes and it's Figures, which could be able to Gain ſo Many Proſelytes to ſo Great an Abſurdity.

AND from whence does all this Flow? Namely, from this Syſteme of Ideas Conſidering only the Terms and Appellations of Things, without Entering into a Real Diſquiſition of them, which it Diſclaims, and from Ranking and Claſſing it's Ideas under Various Denominations, and in Short, from Profeſſing a Science of Words in Contempt of every Rational Science now in Being, before it had Inquired into any One of them; From which, it is Natural to Conclude, what would be the Reſult, to Wit, a Valuing of Words in Preference to Things, and Making Terms, and Signs, and Marks, of our Ideas the moſt Important Part of our Knowledge: The Conſequence of which is, the Filling our Heads with Trifling Niceties and Diſtinctions about Language, and Forms and Manners of Speaking, and Turning the Solid Reaſonings on Nature, Religion, and Philoſophy, into Mere *Chicaning* and Sophiſtry, into Weighing of Points and Syllables, and Canvaſing of Expreſſions and Phraſes, and in Fine, Converting Senſe and Argument into a Levity of Thought, and into Ridicule and Buffoonery; Of which this Syſteme of Ideas is a Standing Inſtance, in it's Defenſes of it's ſelf againſt the Attaques of Thoſe, whoſe Religion, Integrity, Gravity, and Learning, has been much Superior, and who Wanted nothing to make them Victorious in their Cauſe, even with the Friends of this Syſteme, unleſs a Few Small Arts, and Little Cavils, which they were Above, and Thought it their Wiſdom to Deſpiſe.

Biſhop Stillingfleet and Others. Mr. Locke's Letters to the Biſhop.

CHAP. X.

Of Knowledge in General; Of the Degrees of our Knowledge; Of the Extent of Human Knowledge; Of the Reality of our Knowledge; Of Truth in General; Of Univerſal Propoſitions, their Truth and Certainty; Of Maxims; Of Trifling Propoſitions; Of our Knowledge of Exiſtence; Of our Knowledge of the Exiſtence of a God; Of our Knowledge of the Exiſtence of other Things; Of the Improvement of our Knowledge; Some Farther Conſiderations Concerning Knowledge; Of Judgement; Of Probability; Of the Degrees of Aſſent; Of Reaſon; Of Faith and Reaſon, and their Diſtinct Provinces; Of Enthuſiaſm; Of Wrong Aſſent, or Error; Of the Diviſion of the Sciences.

Of Knowledge in General. Mr. Locke's Eſſay, Chap. 1. Book 4. *Four Sorts of Knowledge.*

§. 1. KNOWLEDGE Is Said to be Nothing, but the Perception of the Connexion and Agreement, or Diſagreement and Repugnancy, of any of our Ideas; And where it is not, there, tho' we Fancy, Gueſs, or Believe, yet we always come Short of Knowledge; Thus we Know, that White is not Black, becauſe we Perceive theſe Two Ideas do not Agree, and that the Three Angles of a Triangle are Equal to Two Right Ones, becauſe they do; This Agreement or Diſagreement, it is Said, may be Reduced to theſe Four Sorts.

The Firſt Sort Confuted.

FIRST, Identity, or Diverſity; that is, Ideas are the Same, or elſe they Differ; or, which is the Same, they Agree or Diſagree; So that the Firſt Sort, it ſeems, of Agreement, or Diſagreement, is Agreement or Diſagreement, which is a very Accurate and Wiſe Diſtinction.

The Second Sort Confuted.

SECOND, Relation; that is, the next Sort of Agreement or Diſagreement the Mind Perceives in it's Ideas, may be Called Relative; But I would Ask, whether there can be any Agreement or Diſagreement without a Relation, or a Comparing One Idea with Another? This therefore is not a Diſtinct Sort; but Identity and Diverſity Suppoſes Relation, and Identity and Diverſity Signifies no more than Agreement and Diſagreement; Here therefore is a Second Diſtinction without a Difference.

The Third Sort Confuted.

THIRD, a Third Sort of Agreement or Diſagreement in our Ideas is Co-exiſtence, or Non-Co-exiſtence, in the Same Subject; And this is Said to belong Particularly to Subſtances, as when we Pronounce Gold to be Fixed, or to Remain in the Fire Unconſumed; But this Falls under the Relative Agreement or Diſagreement of our Ideas, when we Compare One Property of Gold with the Other Properties of it, and therefore does not Deſerve a Diſtinction here.

The Fourth Sort Confuted.

FOURTH, The Fourth Sort is, that of Actual and Real Exiſtence Agreeing to any Idea, as that God is; Which is no more than to Say, that the Idea of God Agrees with our Idea of Exiſtence, or that Comparing One Idea with Another they Agree, which ſtill comes under Relation, and likewiſe did not therefore want a Particular Diſtinction.

Anſwer to this Doctrine of the Agreement or Diſagreement of our Ideas.

ALL that can be Said, is Conſequently only this, that the Perception of the Agreement or Diſagreement of our Ideas, upon any Compariſon or Relation of them whatſoever, is the Sole Foundation of all our Knowledge; To which we Anſwer.

FIRST, That the Bare Perception of the Agreement or Diſagreement of Ideas is no Certain Foundation of Real Knowledge, unleſs our Ideas are Accurate and Exact; Becauſe a Man may Think he Perceives ſuch Agreement or Diſagreement, when yet in Truth there is no ſuch Thing; Thus the Viſions of a Madman, or an Enthuſiaſt, will be Equally Certain with the Truths and Concluſions of Geometry; and if a Perſon Sees an Agreement betwixt the Idea of himſelf and a Prince, as in the Caſe of Madneſs, or if *Mahomet*, or *Fox*, does, betwixt that of himſelf and an Apoſtle of God, as in the Caſe of Enthuſiaſm, or if this Philoſophy finds One betwixt it's own Reaſon, and a *Ray* of the Divinity, or a *Candle* Set up by Almighty God in the Mind, which I Suppoſe is little Leſs than a *Light within*, tho' there is no ſuch Real Agreement, it will be as Certain and as True, as that the Square of the Hypotenuſe is Equal to the Square of the Sides.

THIS Doctrine will likewiſe Juſtify all the moſt Whimſical Reaſonings in the World; For if Men Think, or Imagine, they Perceive a Connexion betwixt Certain Ideas, let thoſe Ideas be never ſo Wrongly Connected, it will be all Irrefragable Truth and Demonſtration, and there is not the Greateſt Falſehood, or Ignorance, in the World, which may not be Patronized upon ſuch a Conceſſion; Every One is a Philoſopher, as well as Prieſt, to himſelf, and Every One is in the Right according to his own Connexion of Ideas, tho' Every One Differs in his Philoſophy and his Religion, according to that Connexion.

WE therefore Say, it is not Every One's Particular Connexion of Ideas and his Perception of that Connexion, or Agreement and Diſagreement, of them, which is a Standard of Knowledge, or of Truth and Certainty; But it muſt be the Concurrence of Many and of Thoſe of the Wiſeſt, moſt Judicious, Studious, and Sober Men; And if their Connexions of Ideas Agree, we may then Rationally Conclude, that Truth and Knowledge is on their Side, which we ſhall in Vain Seek for amongſt the Connexions, or rather Fancies, of Some, who Think fit to make their own Private Perſuaſions a Rule for Others to Form their Sentiments upon, in Reſpect of Religion, Nature, or of Government.

Mr. Locke.

SECOND, The Bare Perception of the Agreement or Diſagreement of our Ideas, tho' they ſhould be Accurate and Exact, cannot be a Foundation of our Real Knowledge of Things, unleſs our Abſtractions are True; Thus, tho' all our Ideas of Lines, Surfaces, and Solids, ſhould be Exact and Punctual, and all the Connexions of the Ideas Relating to them, ſhould be ſo too, ſo that every Propoſition in the Elements of *Euclid* was True and Certain; Yet None of them would be ſo in a Real Knowledge, Excepting we Grant the Abſtractions of Length without Breadth, Breadth without Depth, and Depth without Thickneſs, to be Juſt and Conformable to Nature, which they are not, according to what we have Shewn in our *Geometria Solidorum, ſive Materia*, and in the Preceding Books of this Philoſophy.

THIRD, If the Perception of the Agreement and Diſagreement of our Ideas was the only Foundation of our Knowledge, all our Senſitive and Natural Knowledge would be Excluded, which yet is, and is Confeſſed by this Syſteme of Ideas, to be a Real One; Becauſe there is no Connexion betwixt our Ideas, and the Exiſtence of Things, of which we have the Ideas, till ſuch Exiſtence is Proved; and this muſt be done by other Mediums than the Connecting of Ideas in our own Minds, which cannot Extend to any Thing, which is Foreign and Extraneous to it.

FOURTH, Hence alſo all Hiſtorical Knowledge of the Plaineſt and moſt Notorious Facts in the World would be Diſcarded, and even the Evidences of Chriſtianity it's ſelf Precarious and Uncertain.

ON the Contrary, the Perception of the Agreement or Diſagreement of our Ideas is the Baſis only of one Part of our Knowledge, Namely, of That, which is Abſtracted or Mathematical, and which would be True, if there was no Being, nor any Exiſtence Suppoſed, in the whole Circle of Nature; Since a Triangle would not be a Square, or Black White, upon any Suppoſitions whatſoever.

As to what is Said afterwards of Actual and Habitual Knowledge, the One by an Actual Perception of the Agreement or Diſagreement of our Ideas, the Other by Remembering ſuch a Perception, it will not be Material to Offer any Thing in Relation to that Head, Excepting, that ſince we may Know from Remembering, we may Arrive at Knowledge another Way, than by a Perception of the Agreement or Diſagreement of our Ideas, unleſs Remembering is Perception, or Habitual is Actual Knowledge, and there is Conſequently no Foundation for the Foregoing Diſtinction; Which cannot be, ſince the Immutability of the Same Relations, between the Same Immutable Things is Owned to be the Idea, that Shews us, that if a Propoſition was Once True, it will always be ſo, and which is Said to be the Foundation of our Habitual Knowledge, and is Different from the Connexion of Ideas, the Baſis of our Actual.

Of the Degrees of our Knowledge. Mr. Locke's Eſſay, Chap. 2. Book 4. *Intuitive Knowledge nothing more than Perception*

§. 2. IN Reſpect to the Degrees of our Knowledge, the Firſt is Affirmed to be Intuitive, when the Mind Perceives the Agreement or Diſagreement of Two Ideas Immediately by Themſelves, without the Intervention of any Other, as One and Two are Three, or Five is not Equal to Six.

INTUITIVE Knowledge therefore, according to this Account of it, is no Other, nor More, than a Faculty in the Mind of Perceiving; For if we Grant, that the Mind Perceives, it muſt by Conſequence have thoſe Perceptions, which it has, and therefore muſt Diſtinguiſh, where they Differ or Agree; But to Aſſert, that we have a Faculty of Perceiving is no very Great or Profound Matter, much Leſs does it Inform us, in what Science, or where, it Prevails, and only Acquaints us in a Chapter about Knowledge, that One Idea is, or is not, Another, according to our Perceptions of it.

Demonſtrative Knowledge a Definition only, and an Imperfect One.

THE Second Degree of Knowledge is Called Demonſtrative, where the Mind Perceives not the Agreement, or Diſagreement, Immediately, or by the Juxta-Poſition, as it were, of the Ideas, but is Fain to Diſcover the Agreement, or Diſagreement, which it Searches, by the Intervention of Other Ideas, and this is that, which we Call Reaſoning, which is Exemplified by the Demonſtration of *Euclid*, that the Three Angles of a Triangle are Equal to Two Right Ones; Thoſe Intervening Ideas, which Serve to Shew the Agreement of any Two Others, are Called Proofs, and where the Agreement or Diſagreement is by this Means Plainly and Clearly Perceived, it is Called Demonſtration; A Quickneſs in the Mind to Find theſe Proofs, and to Apply them Right, Is, it is Suppoſed, that which is Called Sagacity.

THIS Account of Demonſtrative Knowledge is only Defining, without Proving, that all Knowledge, which comes under the Character of Demonſtration, is Mathematical or Abſtracted, and Conſiſts in the Connexion of our own Ideas; From whence all Senſitive and Hiſtorical, all Chriſtian and Civil, Knowledge is Excluded, as before, from being Demonſtrative and Certain; But this is only Mere Definition, and Calling Things by thoſe Names, which we are Pleaſed to Give them.

Some Sorts of Senſitive and Hiſtorical Knowledge, as Certain, as what is Termed Demonſtrative.

ON the other Hand we Affirm, till the Contrary is Evinced, that there is as much Demonſtration in Matters of Senſe and Hiſtory, as there is in the Connexion of Abſtracted Ideas; If there are Plain and Manifeſt Propoſitions in *Euclid*, both in Reſpect of Figures and Numbers, which no One can Diſpute, There are alſo Propoſitions in Reſpect of Senſe and of Fact, which every Body in his Right Mind muſt Allow and Aſſent to; If there are Propoſitions in Reſpect of Senſe and of Fact, which are Found Falſe and Untrue, there are the Like in the Connexion of our Ideas; And let any One Shew, that he is Leſs Deceived in the Reaſonings of Mathematicians and Philoſophers, who Proceed in the Abſtracted Way of Conſidering Nature, of whoſe Errors there may be Ten Thouſand Inſtances Produced, Nay it may be Proved, that even all Mathematicks and Philoſophy are upon a Wrong Baſis, as they are now Profeſſed; I Say, let any One Shew, that he is Leſs Deceived in this Kind of Knowledge, than he is in theſe Propoſitions, That there are ſuch Places as *China*, and the *Indies*, as *Borneo*, *Japan*, or the *Caribee* Iſlands, or That there are or were ſuch Cities as *Rome*, *Athens*, and *Thebes*, or ſuch Perſons at Preſent, as his moſt Excellent and moſt Sacred Majeſty King *George*, and the *Pretender* to his Crown and Dominions; or Formerly ſuch, as *Epaminondas*, *Pericles*, *Themiſtocles*, and *Demoſthenes*, *Scipio Africanus*, *Cato Uticenſis*, *Brutus* and *Cicero*, or *Piſiſtratus*, *Æſchines*, and *Philip* of *Macedon*, *Marius*, *Catiline*, and *Julius Cæſar*, and then I ſhall Readily Acquieſce, that there is more Knowledge and Demonſtration in the Connexion of Ideas, than there is in Common Senſe, and in Hiſtory of Fact; and Conſequently, that the Chriſtian Religion, with the Author of it, which is a Piece of Hiſtory, is not a Part of Knowledge, which is Demonſtrative, and only comes under One Degree of Probability; But till That is Evidenced, I muſt Beg Leave to Think, that Probability Lies on the Side of the Connexion of Ideas, Eſpecially, ſince the Connexion of them is here Owned to Require Pains and Attention, and a Steady Application of the Mind; and that Demonſtration Lies on the Side of Senſe and of Fact, in Oppoſition and Contradiction to the Aſſertions of this Syſteme of Ideas, into which we are now Inquiring.

HOWEVER, it would be ſome Comfort and Satisfaction to an Inquirer into the Truth of Things, to find this Definer and Claſſer of the ſeveral Degrees of Knowledge, to Furniſh us with ſome Inſtance of his Own, which even in the Demonſtrative, ſeems to Riſe little Higher, as I have Said, than that the Three Angles of a Triangle are Equal to Two Right Ones, or that in a Rectangle Triangle, the Square of the Hypotenuſe is Equal to the Squares of the Sides.

As to what is Said of the Juxta-Poſition of Ideas, and Diſcovering their Connexion by the Intervention of other Ideas, and Calling it Reaſoning, it is nothing elſe, but an Idle and Trifling Comment upon the Way, by which we come to be Convinced, that the Three Angles of a Triangle are Equal to Two Right Ones; and, that the Mind Arrives at Conviction in ſuch Caſes by thoſe Methods, will not be Denied by any One.

AND as to what is Said, that Sagacity Conſiſts in a Quickneſs of the Mind to find out the Intervening Ideas, by which we come to a Proof of the Propoſition, it is a Wrong Affirmation; Becauſe Sagacity does not Conſiſt in a Quickneſs of the Mind, to find out ſuch Intervening Ideas, but in a Faculty of the Mind, for which we have no other Name than That of Penetration; A Quickneſs of the Mind to find out theſe Intervening Ideas, is rather an Act of it, which may be Derived from Memory, and which belongs to a Parrot, but Sagacity or Penetration is that, which Dives into Hidden and Remote Things from Common Obſervation, whether the Mind Acts Quicker or Slower in ſuch a Reſearch; The Inſtance of the Former is That of Philoſophers, who can only be Sagacious on other Mens Writings, which ſeems to be the Caſe of the Preſent Syſteme of Ideas, and of the Latter, That of Thoſe, who can Enter into the Nature of Things with a Sagacity and Penetration of their Own, as Sir *Iſ. Newton* has Endeavoured to do, and with an Applauſe, which is in a great Meaſure Due to him; That is, ſo far as he is an Advocate of the Forces of Nature; But, whether thoſe Forces are Juſtly Aſſerted by him of Gravitation or Attraction, will Perhaps be ſtill a Queſtion; Since, on the Contrary we Affirm thoſe of Contraction and Expanſion, which by the Preceding Books of Philoſophy, are Shewn to be Different Principles from the other, and in ſome Caſes Diametrically Oppoſite to them.

IT is True, this Philoſophy after takes Notice, that other Parts of Learning, it is Poſſible, are as Capable of Demonſtration as Mathematicks and Numbers, but it does not Inform us, what they are; Beſides, for the Reaſons before Alledged, by the Doctrine of the Connexion of Ideas, which cannot Extend beyond the Ideas of our own

Minds, all Senſitive and Natural Knowledge, and all Hiſtories of Fact, as well as Thoſe of Chriſtianity, are Excluded from Demonſtration and Certainty, and Terminate only in Faith and Opinion, which has been Shewn to be Abſurd, and that Faith and Opinion is rather the Reſult of the Connexion of Ideas, than of Fact, Senſe, and Experience.

Of the Extent of Human Knowledge. Mr. Locke's Eſſay, Chap. 3. Book 4.

§. 3. In this Chapter, there is Little more than a Repetition of what goes before, Excepting where it Speaks of the Agreement or Diſagreement of our Ideas in Co-exiſtence, in which, it is Said, our Knowledge is very Short, tho' in this Conſiſts the Greateſt and moſt Material Part of our Knowledge Concerning Subſtances, which are nothing but Certain Collections of Simple Ideas, Co-exiſting in One Subject; Upon which I ſhall only Obſerve,

Anſwer.

That, what is here all along Repreſented, as a Defect in our Knowledge of Subſtances, is no other than ſo many Proofs of the Ill and Abſurd Contrivance of the Modern Syſtemes of Nature, that is, of the Corpuſcularian or Atomical Philoſophy, which is now almoſt Univerſally Eſpouſed, and which has been already Inquired into, in Relation to it's Notion of Subſtances, and does not Want any farther Confutation.

Another Aſſertion Anſwered.

Another Aſſertion, which is Remarkable, is, that we have no Knowledge farther thanwe can have a Perception of the Agreement or Diſagreement of our Ideas, either by Intuition, Demonſtration, or Senſation; For what Agreement or Diſagreement of our Ideas can there be by Senſation? I Perceive Colour, for Inſtance, Sound, Extenſion, or Solidity, *&c.* but where is the Agreement or Diſagreement of Two Ideas, when only One is Perceived by the Mind, by the Intervention of it's Senſes? *Des-Cartes,* I Suppoſe, was Aware of this, and therefore, when he made Certainty to Conſiſt in the Connexion of the Ideas of our own Minds, very Juſtly Rejected Senſation from any Claim to it, which the Preſent Syſteme of Ideas has, as Weakly, Reſumed.

A Third Aſſertion Anſwered.

Third, It is Told us, that the Idea of a Supreme Being Infinite in Power, Goodneſs, and Wiſdom, whoſe Workmanſhip we are, and on whom we Depend, and the Idea of our ſelves, as Underſtanding, Rational Creatures, would, it is Suppoſed, if Duly Conſidered, Afford ſuch Foundations of our Duty and Rules of Action, as might Place Morality among the Sciences Capable of Demonſtration, and it is not Doubted but from Principles, as Inconteſtable, as thoſe of the Mathematicks, by Neceſſary Conſequences, the Meaſure of Right and Wrong might be made out to any One, that would Apply himſelf with the Same Indifferency and Attention to the One, as he does to the Other of theſe Sciences; For the Relations of other Modes may Certainly be Perceived, as well as thoſe of Number and Extenſion, and where there is no Property, there is no Injuſtice, is a Propoſition as Certain, as any Demonſtration in *Euclid,* as alſo, that no Government Allows Abſolute Liberty; It is Farther Alledged, that, What has given the Advantage to the Ideas of Quantity, and made them Thought more Capable of Certainty and Demonſtration, is;

First, That they can be Repreſented by Senſible Marks, which have a Nearer Correſpondence with them, than any Words or Sounds.

Second, Becauſe Moral Ideas are Commonly more Complex than Figures.

From hence the Three Cauſes of our Ignorance are Said to be,

First, Want of Ideas.

Second, Want of a Diſcoverable Connexion between the Ideas, we have.

Third, Want of Traceing and Examining our Ideas.

Revelation Preferable to Natural Religion in Point of Demonſtration.

As to what is Alledged Concerning a Supreme Being, and Morality, being Capable of Demonſtration, it only Shews, how Willing this Philoſophy is to Prefer the Reaſonings of a Natural Religion, and the Refined Heatheniſm of *Socrates,* or *Plato,* to the Evidences of Revelation, and the Doctrines and Precepts of a *Meſſiah*; By making the Firſt Demonſtrable, and the Laſt only Probable, and the Former to be the Reſult of Truth and Certainty, the Latter of Bare Faith and Opinion. But whatever it's Will, or it's Inclinations, may be, or it's Prejudices, It is Wiſe enough not to give us any Principles, upon which we might Demonſtrate the Being of a God, or Morality, which will not be more Liable to Exception than the Arguments and Proofs for a Revelation; by which Means the Religion of Nature, or of a Philoſopher, will Appear to be more the Effect and Product of Faith and Opinion, than That of Revelation or of a Chriſtian; For the very Being of a God is not afterwards

Proved by this Philoſophy, which muſt be the Baſis and Foundation of the whole Syſteme; Or if it were Proved, an Obligation of Duty upon us can never be Inferred from the Nature of God, unleſs we are Obliged to Imitate the Power and Wiſdom of God, as well as his Juſtice and Goodneſs, which it does not ſeem Reaſonable, becauſe not Poſſible, for us to do; Nay it will Want ſome Force of Argument to Evince, that I am Obliged to Imitate God at all in any of his Excellencies or Perfections; It may, till it is Revealed to us, that it is his Will, it may be Pride, and Preſumption in us to Affect the Attributes, and Array our ſelves in the Qualities and Properties of our Creator.

The Being of a God, and Morality, not Proved.

Farther, the Meaſure of Right and Wrong being the Thing to be Made out, it Looks, as if all Morality was Suppoſed to Conſiſt in Aſcertaining, What was every One's Property, and that, therefore, the whole Terminated in Juſtice; Whereas Benevolence, and Generoſity, are Principles in our Nature, which Carry us to a Higher Degree of Morality, than a Mere Principle of Juſtice would do; and to Give or Impart to Another more than his Right, which is Munificence and Charity, is a Nobler and Sublimer Act of Morality, than Mere and Strict Juſtice, or Giving to Another, What of Mere Right Appertains to him.

Morality does not Conſiſt in Juſtice only, but in Benevolence.

Moreover, the Axioms here Mentioned are Little Better, than what are afterwards Termed Trifling Propoſitions, amongſt which are all Identical Ones; For, To Say, where there is no Property, there is no Injuſtice, is little more than to Affirm, that where there is no Property, no One has any Thing to Claim, or there is no Property; And to Tell us, that no Government Allows Abſolute Liberty, is the Same as to Pronounce, that no Government will Permit any One to do what he Pleaſes, that is, to Live without Government; And the Great Diſputes about Property and Government are not any ways Cleared by ſuch Small Maxims in Either.

The Axioms Mentioned are Identical and Trifling Propoſitions.

The Reaſon, why we cannot ſo well and ſo Demonſtrably Argue upon any Moral Subjects, as upon Mathematical Ones, is not, becauſe we cannot have Characters Engraven of thoſe Ideas, as we have in the Figures of Geometry, nor, becauſe they are more Complex; Since the Firſt is Remedied by Exact Definitions, which Definitions, when Set down in Writing, may be Eſteemed Juſt and Standing Characters of thoſe Moral Ideas; And as for their Complexedneſs, a Triangle or a Square, or a Cyclois, or a Parabola, Hyperbola, Ellipſis, or Paraboloeid, do Equally Conſiſt of a Number and a Variety of Simple Ideas, of Space, Lines, and Angles, of which the Complex is made up.

The Reaſons of our Uncertainty in Moral Matters, and why Revelation Neceſſary.

The Reaſon therefore of the Difficulties in the Science of Morality is, the Diverſity of Temper and Complexion in *Human* Nature, which makes it a very Unfit Judge of Virtue and Vice, or of Good and Evil, and Conſequently Evinces the Neceſſity of the *Divine* Interpoſing to Fix and Preſcribe Rules for our Actions and Conduct.

A Triangle, or any other Mathematick Figure, is Certain and Unaltered, and ſo is Gold, and other Material Subſtances, and for that Reaſon we may Argue upon them with ſome Kind of Truth and Conſiſtency; But the Tempers of Men are Different, Deſultory, and Uncertain, and therefore, tho' we ſhould have never ſo Determinate Ideas of Moral or Human Actions, the Rules and Methods of Judging of them would be Various, and Gratitude would be Eſteemed a Virtue with One Man, which would be a Weakneſs and Vice with Another, Charity would be a Laudable Action in One Perſon's Opinion, which with Another would be little better than Extravagance and Folly; And there is Scarcely any Doubt to be made, but that the ſeveral and Particular Complexions of Men's Minds, Derived from the Different Expanſive and Contractive Forces of them, are the Occaſion of the Continual Diſputes about ſuch Matters.

As to what is Mentioned of the Cauſes of our Ignorance, and that our Want of Ideas, of which we are Capable, is One of them, as for Inſtance, of the Bulk, Motion, and Figure of the Particles, of which all the Bodies in the Univerſe are Suppoſed to Conſiſt, there is little or nothing in ſuch an Aſſertion; Becauſe we do not want to See the Minute Figures of the Particles of Bodies, in Order to Inform us of the Nature of them; For, if there were ſuch Microſcopes, as could Shew us the leaſt Atoms of Bodies, they could no more Acquaint us with the Primary Qualities of Bodies, and the Cauſes of them, than our Modern Ones can, which only Preſent us with a Different View of

The Cauſes of our Ignorance not Explained.

the Parts of Bodies, without giving us any ſuch Information; and we may as well Say, when a Microſcope Repreſents to us little Plants in a Spot of Mold, or Hairs in a Mite, or Feathers in a Fly, that therefore we better Know the Nature of Mold, a Mite, or a Fly, as that by Seeing the Laſt Atoms of Bodies, we ſhould be better Inſtructed in the Nature of them, which does not Conſiſt in a Various Configuration of the Parts and the Pores, as is here Imagined, but in the ſeveral Degrees of the Expanſive and Contractive Forces, according to what has been before Alledged.

Of the Reality of our Knowledge. Mr. Locke's Eſſay, Chap. 4. Book 4.

§. 4. This Philoſophy begins now to Doubt, that the Reader by this Time may be Apt to Think, that it has been all this While only Building a Caſtle in the Air, and may be Ready to Object, If it be True, that all Knowledge Lies only in the Perception of the Agreement or Diſagreement of our own Ideas, the Viſions of an Enthuſiaſt, and the Reaſonings of a Sober Man, will be Equally Certain; It is no Matter, how Things are, ſo a Man Obſerve but the Agreement of his own Imaginations, and Talks Conformably; It is all Truth, all Certainty; That an Harpy is not a Centaur, is by this Way, as Certain Knowledge, and as much Truth, as that a Square is not a Circle; But it is Said, of what Uſe is all this Knowledge of Men's own Imaginations to a Man, that Enquires after the Reality of Things? To which it Anſwers.

That, if our Knowledge of our Ideas ſhould Terminate in Them, and Reach no Farther, where there is ſomething farther Intended, our moſt Serious Thoughts would be of little more Uſe, than the Reveries of a Crazy Brain; That 'tis Evident, the Mind Knows not Things Immediately, but by the Intervention of the Ideas it has of them, and that our Knowledge therefore is Real, only ſo far, as there is a Conformity between our Ideas, and the Reality of Things; It is farther Alledged, that there are Two Sorts of Ideas, which, we may be Aſſured, Agree with the Reality of Things.

First, Simple Ideas, which, ſince the Mind can by no Means make to it's ſelf, muſt be the Effect of Things Operating upon the Mind in a Natural Way, and therefore are not Fictions of our Fancies; Thus the Idea of Whiteneſs, as it Lies in the Mind, Exactly Anſwers that Power, which is in any Body to Produce it there.

Second, All our Complex Ideas, Except thoſe of Subſtances, being Archetypes of the Mind's own Making, and not Referred to the Exiſtence of Things, as to their Originals, cannot Want any Conformity Neceſſary to Real Knowledge, for that, which is not Deſigned to Repreſent any Thing, but it's ſelf, can never be Capable of a Wrong Repreſentation; Thus the Mathematician Conſiders the Truth and Properties belonging to a Rectangle or a Circle, only as they are Ideas in his own Mind, and yet his Knowledge is not only Certain, but Real; Becauſe Real Things are no farther Concerned, nor Intended by any ſuch Propoſitions, than as Things Really Agree to thoſe Archetypes in his Mind.

Hence it is Said to Follow, that Moral Knowledge is as Capable of Real Certainty, as Mathematicks; Becauſe our Moral Ideas, as well as Mathematical, being Archetypes themſelves, and ſo Adequate or Complete Ideas, all the Agreement or Diſagreement, we ſhall find in them, will Produce Real Knowledge, as well as in Mathematical Figures: That, which is Requiſite to make our Knowledge Certain, is the Clearneſs of our Ideas; And That, which is Required to make it Real is, that they Anſwer their Archetypes.

Third, But the Complex Ideas, which we Refer to Archetypes without us, may Differ from them, and ſo our Knowledge about them may come Short of being Real; and ſuch are our Ideas of Subſtances, which, to be Real, muſt be made up of ſuch Simple Ones, as have been Diſcovered to Co-exiſt in Nature, whether they are Perfect Collections of ſuch Simple Ideas, or not.

Anſwer to the Foregoing Account of the Reality of our Knowledge.

Thus far the Preſent Philoſophy, in Reſpect of the Reality of our Knowledge; But, notwithſtanding all the Fine Turns, which are given to this Account of the Reality of our Knowledge, it will, I Believe, upon a Juſt Examination be found to be nothing more, than *Building a Caſtle in the Air;* than the *Viſions* of a *Rational Enthuſiaſt,* than an *Agreement of his own Imaginations,* and *Affirming, that an Harpy is not a Centaur, and that a Square is not a Circle.*

For, in General, This Syſteme of Ideas, This Hiſtory, or Philoſophy, of the Mind is, as we have Shewn, little Elſe than a Romance of it; which, in the Firſt Place, Diſfurniſhes the Mind of all it's Ideas and Forces, by Making it a Mere *Raſa Tabula,* and then by it's own Art and Contrivance Endeavours to Place it's Ideas in it, according to it's own Fancy and Pleaſure; That is, by Introducing Simple Ideas Firſt into the Mind, and afterwards Complexing and Combining thoſe Ideas, in Order to give us Materials enough for the whole Extent of our Knowledge; Whereas the Real Hiſtory of the Mind is the Reverſe of all this, which Originally Receives it's Ideas in a Complex Form, and then Separates and Divides them into the ſeveral Diſtinct or Simple Ideas, of which they are Compounded.

THUS in our Infancy, we Receive the Ideas of Subſtances, and of Human Actions, United, by a Groſs Experience and Obſervation of Both, which, when afterwards we come Philoſophically to Inquire into, we Diſtinguiſh into Extenſion, Solidity, Colour, Figure, *&c.* on the One Hand, and into the ſeveral Species and Kinds of them on the Other; And whether they are to be Eſteemed Good or Bad, Virtuous or Vicious; For what Children, or Youths, tho' Grown up to ſome Degree of Maturity in Years and in Judgement, were ever Known to Diſcourſe of theſe Matters in the ſame Manner, as this Philoſophy? And yet they ought to do, if it is the Natural Way, by which the Mind is Originally Furniſhed with it's Ideas, and we ſhould not Meet with theſe Notions in the Privacies of the Studious or Learned in theſe Ideas, but in the very Plays and Diverſions, and the Converſation, of School-Boys; who would be as Good Philoſophers, as the Patrons of this Syſteme, which yet we do not Find, as, I Preſume, the Greateſt Favourers of this Eſſay will Acknowledge.

BUT to Omit Generals; Firſt, as to Simple Ideas, which, it is Said, we may be Aſſured Agree with the Reality of Things; How is this Proved, Suppoſing, that Simple Ideas Enter firſt into the Mind, and that the Mind afterwards Complexes and Combines them, which has been before Diſputed? Why Truly, by only Telling us, that, ſince the Mind can by no Means make theſe Simple Ideas to it's ſelf, they muſt be the Effect of Things Operating upon the Mind in a Natural Way, and therefore are not Fictions of our Fancies; Thus the Idea of Whiteneſs, as it is in the Mind, Exactly Anſwers that Power, which is in any Body to Produce it there. But, I would Ask, What is the Effect of Things Operating upon the Mind in a Natural Way, Unleſs a Suppoſition, that Things do Operate upon the Mind? And What is it for an Idea to Anſwer that Power, which is in any Body to Produce it there, Excepting to Take it for Granted, that there is ſuch a Power in Body to Produce it? Which is the very Thing Conteſted by Philoſophers; And which this Philoſophy, at the ſame Time, that it would Shew the Reality of our Knowledge, does not Prove, but Preſume, and Evinces, how much it is Uſed to Play with Words, without Entering into the Truth and Evidences of Things. On the Contrary, we have Endeavoured to Demonſtrate in the Firſt Part of our Principles of Philoſophy, that there is ſuch a Being as an External Nature, which Acts upon our Minds, and to Overthrow all the Cavils and Objections againſt it, either from Dreams, or Deliriums, or any Reputed Imaginary Pains, upon the Amputation of an Arm, or any other Member.

SECOND, Complex Ideas, ſuch as Mixed Modes, which are Archetypes of the Mind's own Making, and are not Referred to the Exiſtence of Things, it is Said, cannot want any Conformity Neceſſary to Real Knowledge, and for which an Inſtance is Produced from the Mathematicks; But, if any Mixed Modes, or any Mathematical Figures, do not Agree to the Reality of Things, and have only their Exiſtence in the Mind, and are the Mere Creatures and Fictions of it, which has been Shewn, we on the other Hand Affirm, that this is, as much a Chimerical and Fantaſtical, and not a Real Knowledge, as if our Simple Ideas did not Anſwer the Powers of Nature, or, as if there were no External Powers in Nature to Produce them, or, as if we ſhould make Certain Concluſions about a Sphinx, or a Centaur, when there were no ſuch Things in Being. Reality is a Term, which is Applied to Things, as they Really Exiſt, and of which we therefore have a Real Knowledge, or it is a Vagrant One, and may Signify any Thing, which either does, or does not Exiſt, according to the Pleaſure of the Philoſopher; And as to Mathematicks and Philoſophy, we have Shewn in Reſpect of Both, that they are not Agreeable to the Nature and Reality of Things, nor, that the Reaſonings upon Morality, or Natural Religion, are ſo.

Third, It has been alſo *Shewn*, that our Knowledge of Subſtances, as it has been hitherto Propoſed, is no ways Agreeable to the Reality of Things, no more than That of Mixed Modes; And Indeed, how an Imperfect Collection of Simple Ideas, which Co-exiſt in Nature, ſhould be Anſwerable to the Reality of it, is Unaccountable, and yet, That is here Aſſerted.

I might Proceed to Examine into the Remaining Chapters of this Book, in ſo many ſeveral Sections, but it will be a Superfluous Work, Eſpecially to One, who is already Tired with the Fatigue, he has Undergone in theſe Diſquiſitions, and ſince I have given my Sentiments and Opinion in Relation to all of them, either by what Foregoes, or in the Book which Follows, Concerning Natural Religion.

I ſhall only Add, that if any Thing has Eſcaped me in my Inquiry into this Philoſophy of Ideas, which may ſeem too Harſh and Severe, I would Deſire it may be Attributed to my Real Love and Affection for Truth in Philoſophy on the One Hand, and for Chriſtianity on the Other, Againſt both which It ſeems to me, in many Points, to have Offended; For, as to the Perſon of It's Author, whom I never Knew, and therefore cannot be Suppoſed to have Contracted any Prejudice againſt, ſo if he were now Living, I ſhould Value and Eſteem upon ſeveral Accounts, as a Curious, Inquiſitive, Ingenious, and Underſtanding, tho' I Believe, no very Learned, or Judicious Man; And if the Heat of Youth, when many of theſe Papers were Written, as alſo ſeveral in the Following Book, has Carried me rather too Far in Warmth and Zeal, in the Eagerneſs of the Contention, I Hope, if the Reaſoning is Juſt, as I Think it is, it will be Allowed and Indulged me by every Candid and Ingenuous Inquirer into theſe Matters.

THE

THE PRINCIPLES OF THE PHILOSOPHY OF THE EXPANSIVE and CONTRACTIVE FORCES.

BOOK VI.

Of the ETHICKS, or Natural RELIGION, of *DES-CARTES'S* MEDITATIONS, Mr. *LOCKE'S* Eſſay, of Dr. *CLARKE*, and Mr. *WOLLASTON*.

Yyyy 2

Thus, it has been also Shewn, that our Knowledge of Substances, as it has been hitherto Proposed, is no ways Agreeable to the Reality of Things, no more than That of Mixed Modes; And indeed, how an Imperfect Collection of Simple Ideas, which Coexist in Nature, should be Answerable to the Reality of it, is Unaccountable, and yet, That is now Asserted.

I might Proceed to Examine into the Remaining Chapters of this Book, in so many several Sections; but it will be a Superfluous Work, Especially to One, who is already Tired with the Fatigue he has Undergone in These Disquisitions, and since I have given my Sentiments and Opinion in Relation to all of them, either in the Foregoing, or in the Book which Follows, Concerning Natural Religion.

I shall only Add, that if any Thing has Escaped me in my Inquiry into this Philosophy of Ideas, which may seem [illegible] Severe, I would Desire it may be Attributed to my Real Love and Affection for Truth in Philosophy on the One Hand, and for Christianity on the Other; Against both which It seems to me, in many Points, to have [illegible] and therefore [illegible] were now Living, [illegible] upon several [illegible] Invention, and Understanding, tho' I believe, not very Learned or Judicious Man; And if the Heat of Youth, when some of these Papers were Written, and in several in the following Book, has Carried me some-what too far in Warmth and Zeal, in the Treatment of the Controversy, I Hope, if the Reasoning is Just, as I Think it is, it will be Allowed and Indulged me by every Candid and Ingenuous Inquirer into Truth.

THE
PRINCIPLES of the PHILOSOPHY
OF THE
EXPANSIVE and CONTRACTIVE FORCES.

BOOK VI.

CHAP. I.

Concerning the Little Importance of Natural Religion, after the Revelation of the Chriſtian.

Natural Religion Propoſed to Convince Atheiſtical Men of the Truth of Chriſtianity.

§. 1. THERE is no Argument, which I Know of, that has been ſo Carefully, and with ſuch Dexterity, Managed and Propoſed to us, in the Preſent, and ſome Late Times, as that of a Natural Religion; Which, upon it's firſt Appearance, ſeemed with ſome Kind of Modeſty, only to Take the Merit upon it's ſelf of being an Aſſiſtant to Chriſtianity, nor did it Pretend to have any other Deſign, Excepting to Convince Atheiſtical Men of the Truth of thoſe Doctrines by Rational Principles, which they were not willing to Embrace upon Chriſtian Ones.

And to Confirm Good Men in the Sentiments, which they had already Entertained Concerning it. Such a Deſign truly Commendable, but Perverted to Ill Ends.

§. 2. IT alſo farther Surmiſed, that ſuch a Method might not only Prevail upon Atheiſts and Deiſts, but upon Men of Learning and Thought, who, Notwithſtanding they had Entertained a moſt Entire Veneration already for Chriſtianity, yet might thereby be Confirmed in the Truths it Delivered, as well from the Undoubted Teſtimony of their Revelation from God, as from the Aſſent, which Reaſon it's ſelf Gave to Them.

§. 3. THESE Intentions of Thoſe, who Endeavoured to Promote a Natural Religion, if Sincere, were moſt Certainly Commendable and Good, and could not Fail of Gaining a Number of Proſelytes, where Chriſtianity and Reaſon were Inculcated to us, as Terms, in a great Meaſure Reciprocal and Synonymous; But ſo it has Fallen out, That, whatever ſuch Intentions were, the Conſequences have not been Anſwerable, ſince, Inſtead of Converting Atheiſts to Chriſtianity, we find Chriſtianity is not with ſome Men, who Profeſs it, many Degrees Removed from Atheiſm; Whilſt Reaſon

ſon, which at firſt Preſumed nothing more than to Aſſert it, Sets up for it's ſelf, and is at Length Reſolved to be it's own Conducter.

Natural Religion, as Propoſed by Philoſophers, and Set up in Oppoſition to Chriſtianity, Precarious or Falſe.

§. 4. As therefore Reaſon, under a Shew and Pretence of Religion, has Acquired the Reputation it at Preſent Enjoys, and, notwithſtanding which, has been ſo Kind to Deſert it, we ſhall Endeavour to Evidence the Weakneſs and Infirmity of Natural Religion in General, upon thoſe Principles which have been hitherto Eſpouſed, That all the Reaſonings, which we are Entertained with upon this Subject, are either Precarious or Falſe, and That they are rather Arguments of the Contrivance and Wit of the Philoſopher, than of any Religion, which is, or can be, Concluded from them.

Natural Religion, as now Propoſed, Superfluous and of no Uſe.

§. 5. If we had nothing more to Offer, in Vindication of our ſelves, for Diſputing the Principles of Natural Religion, as Propoſed by the Philoſophers of Ideas, beſides it's Unworthy Treatment of the Chriſtian, by Affirming it in Pretence and in Reality Rejecting it, it would be Sufficient; But we have more Reaſons to Juſtify us in it. For, If according to §. 1. Men are Atheiſts, Reaſon, as it is now Managed, will have as little Sway in Convincing them of the Truth of Religion, as Chriſtianity it's ſelf; If, according to §. 2. they are Confirmed in the Belief of their Chriſtianity, a farther Confirmation from Reaſon is Superfluous and Unneceſſary, however Agreeable it may be to our Inquiries after Truth, or to every Man's Particular Curioſity, or Private Perſuaſion.

Natural Religion as now propoſed, Unable to Support it's own Arguments.

§. 6. It may be Poſſibly Said in Anſwer to the Preceding Section, that we Divide all Men, who have Heard of Chriſtianity into Atheiſts or Chriſtians, as if there was no Medium betwixt the One and the Other; And That is, What we Affirm, They muſt be, who Embrace this Ideal Philoſophy; For, if we Believe the Chriſtian Religion from the Abundant Evidences, we have for it, and which are ſuch, as no Matter of Fact in the World is Capable of more, we are at the ſame Time Certain, that there is a God; and that, That Gracious, Omnipotent, and All-wiſe Being, Revealed his Will to Mankind; On the Contrary, if this is not Allowed, but Reaſon and the Connexion of Ideas is Recurred to, which is what is now Generally Meant by Reaſon, we ſhall Abundantly Prove, that, as it can be of no Service to the *Chriſtian* Religion, which Depends upon the Evidence of Fact, ſo it is Inſufficient to Support or Defend it's *Own*.

No Danger of Diſcarding a Natural Religion Derived from the Connexion of Ideas. Demonſtration of the Chriſtian Religion.

§. 7. Nor are we at all Apprehenſive of any Ill Conſequences, which ſome Good Men may Probably Think, will Follow from ſuch a Procedure; Becauſe, in the Firſt Place, we have already Evinced our Chriſtianity to be True, and to be Derived from Almighty God himſelf, by Arguments, which Depend on Matter of Fact, and which are as Clear and as Full, as Thoſe which Convince us, that there ever was ſuch a Place, as *Athens*, or *Rome*, or ſuch a Perſon, as *Alcibiades*, or *Cæſar*; Second, Becauſe, beſides this Demonſtration, we have Another, which Confirms the Divinity and Truth of the Chriſtian Religion from the ſeveral Doctrines, It Propoſes to our Belief; A Third, from the Nature of the Morals it Aſſerts; A Fourth, which we ſhall Purſue, as far as we are Able, from an Explanation of the Scriptures themſelves; And a Fifth from the Affirmations and Senſe of the Primitive Writers.

Natural Religion may be Proved from Truer and Juſter Principles than what have been hitherto Aſſerted.

§. 8. We are likewiſe under no Apprehenſions, in the Second Place, of any Ill Conſequences from ſuch a Procedure; Becauſe, if Natural Religion was Requiſite to the Conviction of Chriſtianity, we Dare be Certain, that we have Proved, the Higheſt and moſt Important Doctrines it Pretends to Demonſtrate, upon Juſter and more Rational Principles, than Any, It has been hitherto Pleaſed to *Reveal* to us.

The Demonſtration of the Being of a God of no Great Importance to us, unleſs our Souls are Immortal.

§. 9. We cannot alſo but Add, that if the Chriſtian Religion is not True, which to any Wiſe or Juſt Inquirer into Things, is an Impoſſible Suppoſition, whatever Atheiſts, or Deiſts, may Think of it, it is not worth the while to Prove a Natural One is; Since the Being of a God, if our Souls are not Immortal, will not Deſerve our Conſideration, any more than any other Being, which Exiſts in the Univerſe; And that our Souls are Immortal, the Preſent Syſtemes of Natural Religion, Derived from Metaphyſicks, as we ſhall afterwards Shew, are not Capable of Proving; It being the Revelation of Almighty God himſelf to us, in that Holy and Divine Religion we Profeſs, Which brought Life and Immortality to Light thro' the Goſpel; Notwithſtanding which, we do not Affirm, but both the One and the Other may be Demonſtrated from Principles, Unknown to the Preſent Philoſophy.

§. 10.

§. 10. LASTLY, It is Plain, that Natural Religion Abſtractedly Speaking, is nothing elſe than the Philoſophy of the Heathens, and we cannot Think, That Chriſtianity and Religion can be Supported by That, Which it was Intended to Annul and Deſtroy, and Which the Apoſtle to the *Corinthians* in Expreſs Terms Explodes, as Carnal, and Contrary to the Divine Spirit, and Genius, of the Doctrine, He Preached to Them; Which was to Inſtruct them in the Knowledge of God.

Natural Religion little better than the Philoſophy of the Heathens.

§. 11. THE Method therefore, which we ſhall Follow, in Conſidering the Evidences of Natural Religion, ſhall be; *1ſt.* To Compare Teſtimony with Knowledge, and Faith with Reaſon; To Aſſign the Provinces, which belong to Each of Them, and to Examine into the Degrees of Certainty, or Probability, which may be Expected from Both, as alſo into the Obligations, in Reſpect of our Duty, which Reſult or Ariſe from Them; *2d.* We ſhall Enter into all the Particular Doctrines of Natural Religion, and Endeavour to Shew, that there is no Satisfaction to be had from any Reaſonings, which have been hitherto Propoſed to us on that Subject; The Firſt of which Heads we ſhall Treat of in the Two Enſuing Chapters.

The Order and Method, in which we ſhall Proceed.

CHAP. II.

Concerning the Evidences from Self-Information.

§. 1. THERE is Nothing more Manifeſt, than that, whatever comes to our Notice in a Natural Way, muſt either be, from our own Informations, or from other Men's; That, which is Derived into our Minds by the Firſt of theſe Methods, I Call Self-Information, and That, which is Imparted to us by the Laſt, Teſtimony, the Evidences of which we ſhall Diſtinctly Conſider.

All our Knowledge Ariſes from our own or other Men's Informations.

§. 2. WE ſhall not therefore, as Others have Prepoſterouſly done, Define What our Notion of Probability is, Namely, that it is a Likelineſs to be True, and then Arbitrarily Rank under that Head All thoſe Propoſitions, in which we cannot Find an Immediate Connexion of Ideas, wherein Certainty is Said to Conſiſt; Nor on the other Hand, ſhall we Firſt Explain, what we Mean by Certainty, Namely, That in all the Parts of it, There is Intuition, and Each Idea, Each Step, has it's Manifeſt and Viſible Connexion, and after we have Happily Settled this Point, with great Satisfaction and Contentment of Mind, Receive no other Propoſitions into the Number of Certain Ones, which do not Agree with that Definition; And yet this is all, which is Atchieved by theſe Sagacious Reaſoners, that is, Thoſe Propoſitions are Termed only Probable, and not Certain, which they Define to be ſo, and Thoſe are Certain and Infallible, which they have Pronounced ſuch; Becauſe they have ſo Named them.

We are not Informed from this Philoſophy of Ideas, what is Really Certain, and what is Probable only, but what it Defines to be ſo.
Mr. Locke's Eſſay. Chap. 15. 16. 17. Book 4.

WHEREAS the Probability, or Certainty, of Propoſitions cannot be Inferred or Concluded from a Definition of Probability and Certainty, but from a Real Proof of What Probability and Certainty are, and wherein they Conſiſt, and then Examining What Kind of Propoſitions have a Claim to the One, or the Other.

§. 3. AGREEABLE yet to this Method, we are Told, That all Matters of Fact, however well Atteſted, may Produce Aſſurance in us, but cannot Certainty, That they may be Highly Probable, but do not Carry Demonſtration with them; Alſo, That Conſequently ſuch Matters of Fact we Aſſent to, we Believe, and have no Manner of Doubt Concerning them, but do not Know them to be True, nor are Infallibly Certain of them.

Matters of Fact tho' never ſo well Atteſted, are in this Doctrine only Probable.

§. 4. FROM which Doctrine we muſt Obſerve, that we are Led by it into very Improper Ways of Expreſſion; For Firſt, we muſt Affirm, that Probability and Aſſurance are in General the Same; Aſſurance being only a Kind of Probability. Thus, altho' we are as Sure, as we can be, of any Thing, that there is ſuch a Place as *France*, or *Spain*, Philoſophically Speaking, we muſt Only Say, it is Probable there is; and whilſt I am Writing this, altho' I am Sure, that my Friend Lies Dangerouſly Ill, by Twenty Meſſages, which I have Received Concerning it, according to the Phraſe of our Modern Reaſoners, This is only Probable.

Great Improprieties in Language Follow from this Doctrine.

§. 5. ON the Contrary, Certainty, Knowledge, and Aſſurance, are made to Differ in this Way of Reaſoning; For it ſeems, tho' I may be Aſſured of a Thing, I may not

Great Improprieties in Language Follow from this Doctrine.

not be Certain of it, nor Know it to be, What I am Aſſured it is; Thus, as in the Former Inſtances, I may be Sure, that there is ſuch a Country as *France* or *Spain*, or that my Friend is Nine Miles from this Place, in a Weak and Languiſhing Condition, and yet I ſhall not Know, or be Certain, of the One or the Other; Such Methods of Thinking, it is Plain, therefore are either not True, or elſe, that they make Uſe of a Language, which is Directly Contrary to what they Propoſe to Speak in.

The Reaſon of theſe Improprieties and Contradictions, becauſe all Certainty is made to Conſiſt in the Viſible Connexion of Ideas.

§. 6. What we have Said in the Foregoing Sections will be more Evident, if we Shew the Reaſon of theſe Improprieties, which is Manifeſt; For, as the Preſent Doctrine had Originally Fixt the Viſible Connexion of Ideas, as the only Teſt of Certainty, it was Neceſſary for it to Reject all other Propoſitions from being Certain, which did not Depend upon that Evidence, and to Make them only Probable Ones, whatever Convictions beſides, they were Surrounded with; Let us therefore Conſider the Full Senſe of §. 3. if Put into a Language Suitable to the Ideas, this Philoſophy has Entertained, and Stript of the Equivocal Terms of Aſſurance, Probability, Certainty, and Knowledge, and we ſhall Find it to be, no more than This, that Matters of Fact, however well Atteſted, do Depend upon other Evidences, than Thoſe, which Ariſe from the Viſible Connexion of the Ideas in a Propoſition; and if we are Deſirous to Call One Aſſurance, and the Other Certainty, and the One Probability, and the Other Knowledge, it will be an Argument, how Eaſy it is to Impoſe Names of Diſtinction, but not One, that we Know the True Nature of the Evidences themſelves, which Reſult from Both; For, Albeit we ſhould Affix or Attribute any other Terms to them, the Queſtion would ſtill Return, which is to be Diſcuſſed, that is, What the Real and Proper Evidences are, which the Mind Receives from Teſtimony, and what Thoſe are, which it has from Reaſon, or the Connexion of Ideas, and how far they are Each of Them Adapted to Produce a Full and Certain Perſuaſion in our Minds; All therefore, which we Obtain from this Celebrated Philoſophy, is only an Arbitrary Nomenclature of Things, or rather a Miſnaming of Them, as will be Evident to any One, who Reads over the Chapters Cited.

The Method Propoſed Reſumed.

§. 7. SINCE then we cannot be Satisfy'd in the Preſent Doctrine, we ſhall Conſider Things in the Way, we at Firſt Propoſed, of their being Informations from our own Minds, or of their being Thoſe from other Men's; The Firſt of which we ſhall Examine into in the Following Sections, and the Laſt, in the Succeeding Chapter.

Self-Information Reſolved into Senſitive, Intuitive, Experimental, Mathematical, and Philoſophical Knowledge or Certainty.

§. 8. As to what Reſpects thoſe Informations, we Receive from our ſelves, They are either Reſolved into Senſation, or into the Intuition of our own Ideas, or into our Arguments from Either, or Both. The Evidence, which the Mind has from Senſation, we may Call Senſitive Knowledge, or Certainty; and That, which it has from the Intuition of it's Ideas, Intuitive; What it has from Arguing upon Senſe, Experimental; and That from it's Abſtraction of Ideas, we may Term Mathematical; and laſtly, Where we Reaſon both from the Intuition of our Ideas, and from Senſation together, we cannot give a Juſter Denomination to it, than That of a Philoſophical One; We ſhall Speak Diſtinctly to Each of Theſe, and Conſider the ſeveral Aſſurances or Convictions, which the Mind is Capable of Receiving from them.

Knowledge or Certainty Conſiſts in a Conviction of the Mind from Juſt and Rational Premiſſes.

§. 9. WE have Mentioned Knowledge, Certainty, and Aſſurance, in all theſe Caſes, as the ſame Thing; If it be therefore Inquired, what we make Knowledge, or Certainty, or Aſſurance to Conſiſt in, in General; we Anſwer, in a Conviction of the Mind from Juſt Premiſſes, which may be Different, as in the ſeveral Kinds of Knowledge above Named, where the Certainty is Equal; If it be farther Asked, how we ſhall Know, when the Premiſſes are Juſt, and what Fixed and Determined Criterion there is to Diſtinguiſh them from Falſe Ones, from which a Mind, that is Leſs Intent and Cautious, may be ſometimes Convinced, and Conſequently Deceived; I do not yet Know of any other Reply, than that in Order to Aſcertain this Point, we are to Examine into the Nature of the Premiſſes or Evidences themſelves, which being Different, cannot be Reduced to One Certain Rule of our Judging of them; and we may as well Say, what Nature is in General, which Conſiſts of the ſeveral Diſtinct Species and Orders of Beings, which it Comprehends, as what Certainty is in General, which ought likewiſe to be Conſidered in the ſeveral Particulars, it Includes.

The Evidences of Senſitive Knowledge Produce Certainty in the Mind.

§. 10. As to the Evidences of Self-Information, the Firſt to be Examined into, are Thoſe, which we Receive from our Senſes, and which, in our Principles of Natural

Philoſophy we Endeavoured to Shew, were as Forcible and Strong, as the Evidences, or the Arguments, we have for our own Perceptions, or our Thinking; Since all the Errors, which are Committed in our Senſations, do not Proceed from our not being Really Senſible of External Objects, as they Appear to us, but from our Falſe Judgments and Concluſions, which we make upon thoſe Senſations; Thus, we as Fully See and Perceive that Diſtance, which Appears to be betwixt us and the Sun, and it is as Juſt an Effect Produced in our Minds from the Sun, and the Intermediate Æther, which Lies betwixt our ſelves and it, as any other Perception of Willing, Judging, Remembering, *&c.* which Terminates in our own Minds; We have only a Certain and Determined Perception in either of theſe Caſes of what we do Perceive, but what the Real Cauſe of thoſe Perceptions is, whether of Diſtance, why it Appears Different from what it in Fact is, or of Willing, Judging, Remembering, why, or in what Way, it is Impreſſed upon our Minds, we are Equally at a Loſs to Know; This is Evident, that our Senſations, as they are Offered to our Minds, however Falſe our Judgments or Concluſions upon them may be, are as Certain and Infallible, as our Perceptions themſelves are, and we have, according to thoſe Principles of Philoſophy, as Juſt an Evidence for the Reality and Certainty of the One, as the Other.

So do the Evidences of our Intuitive Knowledge.

§. 11. We have in the ſame Principles alſo Shewn, that Intuitive Knowledge is likewiſe Certain, that is, That which Properly and Peculiarly Reſults from the Contemplation of our own Ideas; Thus we are as Certain, that we Perceive, we Will, and Judge, and Remember, and that there are thoſe ſeveral Faculties in the Mind of which we are Conſcious, as we are, that there are the ſeveral Powers in External Objects, of which we are Senſible; For we are Equally Sure, that we Perceive Thought in our ſelves, and Heat, or Light, or Solidity, *&c.* from Beings, which are without us.

Senſation and Intuition, the Foundations of our Natural Knowledge.

§. 12. Here are therefore Two Foundations of Certainty, which are Infallible; The One from our Senſations, that, what we See, or Feel, or Hear, *&c.* we do See, Feel, or Hear, *&c.* The Other from our Intuition, that, what we Perceive from a Reflection upon our own Minds, as that we Will, Judge, and Remember, we do Perceive by ſuch a Reflection; Theſe Two we Affirm to be the Sources of all our Natural Knowledge, and the only Criterions of what is Certainty in thoſe Sciences, which Proceed upon Self-Information.

Original Certainty Conſiſts in the Senſation or Intuition of our Ideas, and not in the Connexion of them.

§. 13. If what we have Said is True, Original Certainty does not Conſiſt in the Connexion of our Ideas, or our Perceptions, but in the Intuition, or Senſation, of them; We are Certain, that we Perceive our own Thoughts, or the Impreſſions of External Objects, as they Appear to us, but the Connexion of Ideas, and the Compariſon of them with one another, is Made and Formed by the Mind, according as it Apprehends their Diſtinct and ſeveral Relations; and therefore, we may be Deceived in the Connexion of Ideas, tho' we cannot be in the Ideas or Perceptions themſelves. But this will Lead us to Conſider after Intuitive, and Senſitive, Certainty, what a Mathematick, or Geometrick, One is.

Mathematick Certainty Conſiſts in the Connexion of ſuch Abſtracted Ideas, whoſe Agreement or Diſagreement, the Mind as Plainly Perceives, as it does the Ideas themſelves.

§. 14. A Mathematical, or Geometrick, Certainty Ariſes from our Comparing Ideas together, which are Abſtracted by the Mind, and betwixt which it as Fully and Clearly Diſcerns the Agreement or Diſagreement, as it Sees and Views the Perceptions, which thus Agree or Differ, and ſo far as this is done, our Certainty is as Great, as that which we have from our Reflections upon our Minds, or from the Impreſſions made upon our Senſes; Notwithſtanding which, this Certainty Alters according to the Nature of the Ideas we Conſider, and the Greater or Leſſer Number of them, which we Compare. Number and Extenſion are the Principal Quantities, whoſe Proportions and Reſpects to each Other, Mathematicks Demonſtrates.

As to Number, ſo far as it Conſiſts of Units, or any Determined Parts of a Unit, our Perceptions are Diſtinct and Determined, and we may Reaſon with Certainty upon them; and even in Surd Numbers, of which we have no Clear or Exact Conception, we may do it; for we as well Know, and are as Sure, that the $\sqrt{3}$ is Equal to the $\sqrt{3}$, as that 3 is Equal to 3; that the $\sqrt{2}$ is not Equal to the $\sqrt{3}$, as that 2 is not 3; Thus, alſo in Extenſion, we are as Certain and Definitive, that a Triangle is not a Square, or a Circle an Octogone, or that a Triangle is Equal to Another of the Same Dimenſions, as we can be of our own Proper Perceptions of ſuch Quantities: For, where only Two Ideas are Compared, whether in Number or Extenſion, the Mind Perceives their Agreement, or Diſagreement, upon the very Account, that it Diſtinctly Perceives them, and therefore, thus far we are as Certain, as we are from our Perceptions, or our Senſes; Theſe being, what we Call, Axioms, or Self-Evident Principles.

Mathematick Certainty, when it Proceeds to what is Termed Demonſtration, begins to Diminiſh.

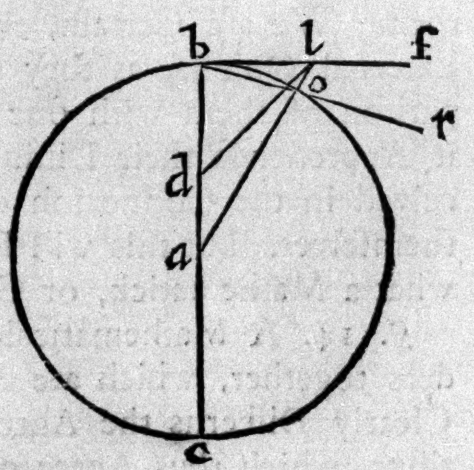
§. 15. IF afterwards the Mind is Carried to Conſider ſeveral Perceptions in their Compariſon with each Other, Certainty begins to Diminiſh; Becauſe it does not Reſult from the Immediate Contemplation of our Ideas, but Memory is alſo Required to Connect the Laſt Idea with the Firſt, ſo that in all the Intermediate Steps, from the One to the Other, the Mind ſhould Perfectly Retain the Progreſs, which it has Made, in the Compariſon of it's ſeveral Ideas, as well as, by it's Judgment, Diſcern the Connexion or Agreement betwixt the Compariſon of every Part, and of the Whole, which is, what is Termed Demonſtration; That is, Mathematick Demonſtration Conſiſts of Five Things; 1*ſt.* Of the Apparent and Viſible Agreement, or Diſagreement, betwixt the Ideas made Uſe of, and therefore in that Reſpect is Reſolved into Perfect Axioms, according to §. 14. 2*d.* of Axioms Compared with One Another, or the Agreement, or Diſagreement, of Two Certain Ideas, Compared with the Agreement, or Diſagreement, of Two other Ideas, and That Compared with a Third, *&c.* in the Compariſon of which, Memory is Particularly Requiſite; 3*d.* Of Connecting theſe ſeveral Axioms together, ſo as to make one Conſiſtent Demonſtration, which is the Province of Judgment, and to which it is Reſigned; 4*th.* Of the Agreement betwixt One Propoſition and Another, which may Proceed upon Different Suppoſitions, and ſometimes Contrary Ones; 5*th.* Of the Adequateneſs of our Perceptions to the Real Nature of Extenſion, where we make theſe Mathematick Demonſtrations Abſolute Ones, and not Relative to the Principles Suppoſed or Abſtracted.

ACCORDING therefore to the Accuracy of our Connecting Ideas, or according to the Firmneſs of our Memories, or the Strength and Force of our Judgments, the Certainty of this Kind of Evidence, which is Called Demonſtrative, Varies; From whence we find ſome Perſons Miſtaken, who Argue even upon theſe Abſtracted Principles, whilſt Others Produce to us the moſt Undoubted and Infallible Concluſions from them.

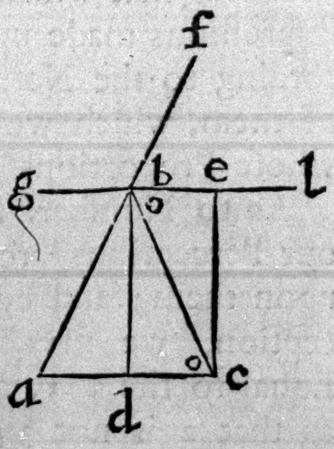
WE have an Inſtance of the Falſe Reaſoning in Reſpect of each of theſe Heads, in the 16*th.* Propoſition of *Euclid's* Elements, Book 3*d.* and Schol. Art. 1. 2. *Tacquet's* Edition.

An Inſtance of the Fallibleneſs of Mathematick Demonſtration. Euclid. Elem. Prop. 16. Lib. 3. Schol. Artic. 1. 2. Tacquet's Edit.

§. 16. THE Propoſition is, That if a Right Line *bf* is Drawn Perpendicular to the Diameter *bc*, it will not Enter into any Part of the Circle, but Touches it in one Point only in *b*; And that no other Right Line can be Drawn to the Point of Contact *b*, betwixt the Circle *bo*, and the Tangent *bf*, without Cutting or Interſecting the Circle.



FOR let us Take the Neareſt Point Imaginable to *b*, Namely *l*, ſince by Suppoſition, *lba* is a Right Angle, *bla* will be an Acute from Coroll. 5. Prop. 32. Par. 2. Book 1. Which Propoſition is, That all the Three Angles of a Triangle are Equal to Two Right Ones, from whence the Coroll. Cited is Evident; But all the Three Angles of a Triangle are Equal to Two Right Ones, that is, to 180 Degrees, becauſe Fig. 2. the External Angle *fbc*, is Equal to the Two Internal Oppoſite Ones *bca*, *bac*, by the Firſt Part of this Propoſition, and *fbc*, with the Angle *abc*, are Equal to Two Right Ones, or to 180 Degrees; Alſo, that the External Angle *fbc* is Equal to *bca*, *bac*, is Proved from *bl* being Drawn Parallel to *ac*, in which Caſe, by Prop. 27. Book 1. *lbc* is Equal to *bca*, which are Alternate Angles, and *fbl* to *bac*, which are Oppoſed, the External to the Innal, *ad Eaſdem Partes*; The Firſt of which is likewiſe Demonſtrated from Hence, that *bd* is Equal to *ec*, and *be* to *dc*, and therefore the Triangles *ebc*, *dcb*, being Equal, the Angles, Expreſſed by the ſame Letters *oo*, will be ſo too; The Second is Proved from *fbl* being Equal to *gba*, which are Vertical Angles, and *gba* being Equal to *bac*, which are Alternate; From which Series of Propoſitions it is Evidenced, that *alb*, Fig. 1. is an Acute Angle.



THEREFORE *a l*, Fig. 1. which is Oppoſed to the Greater Angle *a b l*, is Greater than *a b*, which is Oppoſed to the Leſſer Angle *a l b*, by Prop. 19. Book 1. For *a l* is not Leſs than *a b*, if it was, by Prop. 18. the Angle *l b a* would be Leſs than the Angle *b l a*; Nor is *a l* Equal to *a b*, ſince, if ſo, by Prop. 5. Book 1. the Angles *l b a*, *b l a*, would be Equal, and Conſequently, *a l* is Greater than *a b*.

FARTHER, that the Angle *l b a* is Greater, which is Oppoſed to a Greater Side, and the Angle *b l a* Leſs, which is Oppoſed to a Leſſer, according to Prop. 18, is thus Proved; Since the Angle *lba* cannot be Equal to the Angle *b l a*, otherwiſe by Prop. 6. Book 1. the Sides, which are Oppoſed, would be Equal, Contrary to the Preſent Suppoſition; And the Angle *l b a* cannot be Leſs than the Angle *b l a*, for, if ſo, by a Line Drawn from *d* to *l*, *d l b* may be made Equal to the Angle *l b a*, and therefore, according to Prop. 6. *d l* and *d b* will be Equal, and *a b* = to *a d* + *d l*; But *a b*, by Suppoſition, is Leſs than *a l*, and therefore *a l* will be Greater than *a d* + *d l*, Contrary to the Notion of a Right Line, which is the Shorteſt of any Drawn from the ſame Points *a* and *l*; from whence it Follows, that ſince the Angle *l b a* is not Leſs than the Angle *b l a*, nor Equal to it, it muſt be Greater.

LASTLY, the 5*th*. and 6*th*. Propoſitions Cited, the Firſt of which is, that the Angles at the Baſis of an Iſoſceles, or Æquicrural, Triangle are Equal, the Second, that if Two Angles in a Triangle are Equal, the Sides, which are Oppoſed to them, will be likewiſe Equal, are ſtill to be Proved; Both which Propoſitions Depend upon the 4*th*. Prop. of *Euclid*, Book 1*ſt*. Which is, that of Two Triangles *d b c*, and *e c b*, Fig. 2. if one Side *b d* is Equal to one Side *c e*, and another Side *d c* Equal to another Side *e b*, and the Angles *b d c*, *b e c*, made by thoſe Sides, are Equal, the Baſes *b c*, *c b*, and the whole Triangles *c d b*, *b e c*, and the Remaining Angles at the Baſes, which are Oppoſed to Equal Sides, *e c*, *e b*, and *d b*, *d c*, will likewiſe be Equal; And This is Evinced from the 7*th*. and 8*th*. Axioms; The Firſt of which is, that thoſe Quantities or Things, which Mutually Correſpond or Agree with Each Other, are Equal; The Second, that if any Right Lines, or any Angles, are Equal, they will Mutually Correſpond or Agree with Each Other.

IT is alſo to be Obſerved, that the Definition of a Line is That of the Flowing of a Point, and only Conſiſts of an Abſtracted Length, that a Superficies Conſiſts of Length and Breadth, and is Produced from the Flowing of a Line, and that an Angle is the Inclination of One Line to Another, and Includes a Space made up of Length and Breadth only; This is Suppoſed in all the Preceding Demonſtrations.

ALL that has been Premiſed, being therefore Granted, it is Conſequent, that if Fig. 1. *a l* is Greater than *a b*, and *a b* Reaches only to the Circumference, any Point, as *l*, muſt Fall out of it, and the Line *f b* will Touch the Circle in one Point only.

THE Second Part of this Propoſition is, That no Line can be Drawn betwixt the Tangent *f b*, and the Circumference of the Circle *b o*, without Interſecting the Circle, which is Manifeſt from the Former Demonſtration; ſince, if any Point in *b f*, Excepting *b*, Falls out of the Circumference, there cannot be another Line Drawn, as *b r*, unleſs it makes an Angle *r b c*, Leſs than a Right One, and therefore muſt Interſect the Circle, ſince *b f* makes a Right One, by Juſt Touching it.

IF this therefore is True, it will Follow; 1*ſt*. That the Angle of Contingence *f b o* is Leſs than any Acute Angle, as *f b r*; 2*d*. That the Angle of the Semi-circle *c b o*, altho' it is Leſs than a Right Angle, is Greater than any Acute One; Which ſeems to be no Leſs than a Contradiction, whatever Evaſions are made to Elude the Difficulty. And from Hence is Derived the Famous Controverſy in Mathematicks, Concerning the Angle of Contact *f b o*, Some Affirming it not to be a Quantity, as *Peletarius*, and Dr. *Wallis*, and Others as Strenuouſly Maintaining, that it is not only a Quantity, as *Clavius*, but that there are Infinite and Different Series of Quantities Included in it, as Sir *Iſaac Newton*, Schol. Lem. 11. Sect. 1. Lib. 1.

An Examination into the Preceding Mathematick Reaſoning.

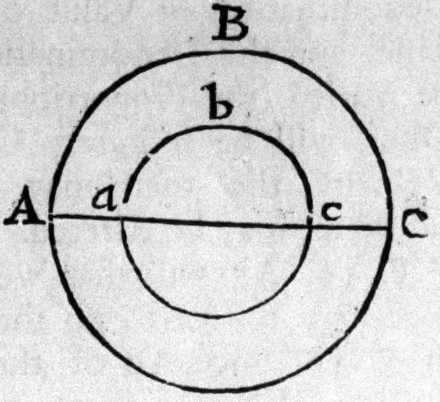
§. 17. THE Firſt Thing therefore to be Conſidered in this Abſtracted Reaſoning, in Order to Judge Concerning the Certainty of it, is, whether, in this Long Series of Ideas, we have Connected them A-right, ſo as there is a Viſible Agreement betwixt them in Each Step of the Demonſtration, for if in any Part that Agreement Fails, the Whole Ceaſes to be Demonſtrative, or Certain; And ſuch is the Infirmity of the Mind in the Connecting of Ideas, that in the Firſt Advance it makes in the Preſent it is Miſtaken; For *bla* is not Proved to be an Acute Angle, unleſs upon the Suppoſition, that *bl* is a Strait Line, and not a Curve, or the Portion of a Circle, for if it is a Portion of the Circle, the Three Angles of ſuch a Triangle are not Evidenced to be Equal to Two Right Ones, if it is not a Part of the Circle, *bl* is Suppoſed not to Coincide with the Circle in the Demonſtration Uſed to Prove it, or, which is the Same, *bl* does not Touch the Circle any where, Excepting in one Point, becauſe it does Touch it only in one Point; For it is not Certain, or Evident, in it's ſelf, that an Infinitely Large Curve may not Coincide with a Tangent, as well, at Leaſt, as an Infinitely Small Subtenſe is Acknowledged to Coincide with a Curve; and if ſo, in the Smalleſt Circle, there may be likewiſe a Coincidence betwixt a Strait Line and it, ſince a Large Tangent is no more in Relation to a Large Circle, than a Leſſer One is to a Smaller; On the Contrary, if there is no Coincidence betwixt a Circle and a Tangent, Excepting in one Point, it is yet not Proved by Suppoſing it, which the Preſent Demonſtration does.

An Examination into the Mathematick Reaſoning, which Foregoes, and Applied to the ſeveral Parts, of which Mathematick Certainty was ſaid to Conſiſt. §. 15.

§. 18. The Second Step, made in this Demonſtration, is to Prove, That all the Three Angles of a Triangle are Equal to Two Right Ones; For 1*ſt.* there being, by what is Proved before, a Connexion of Ideas Separately, betwixt *fbl*, *bac*, and *lbc*, *bca*, and betwixt *abc* and it's ſelf, Fig. 2. The Mind Immediately Concludes, 2*d.* That there muſt be an Agreement betwixt *abc* + *bac* + *acb*, and *fbl* + *lbc* + *abc*, becauſe the Parts are Equal to the Wholes, which we Called the Agreement of Axioms, §. 15. or the Agreement of the ſeveral Separate Connexions Compared with One Another in the Demonſtration, in which Memory has it's Province; and therefore in this Caſe we may be Deceived, where the Compariſons of our Ideas are Long and Tedious, tho' we ſhould Plainly Diſcern all the Diſtinct Connexions of them: Farther, 3*d.* Judgement is alſo here Concerned, for altho' the Mind ſhould Retain by it's Memory the ſeveral Separate Connexions of the Ideas, and the Connexions of thoſe Connexions, *&c.* yet, if it ſhould Fail in making a Right Judgment from them, the Demonſtration would be again Defective, that is, if the Mind did not by it's Judgement Connect theſe Ideas ſo, as they ſhould be Anſwerable to Each Other in every Part, and in the Whole, and thus far the Preſent Propoſition Proceeds Juſtly; 4*th.* Another Requiſite in Demonſtration is, that the Principles and Reaſonings of One Propoſition ſhould be Agreeable to, and Correſpond with any Other, to which it is Compared; And in this it Fails, *viz.* in the Application of the Three Angles of a Triangle, being Equal to Two Right Ones, to the Triangle *alb*, Fig. 1. before it is Known, or Demonſtrated, to be a Triangle, which Admits of that Property; ſince, till the Contrary is Evidenced, it may be a Curvilinear One, according to what has been Obſerved in §. 17. 5*th.* We are to Conſider, whether our Perceptions, in the Abſtraction of them, are Adequate and Juſt; For, if they are not, Mathematick Demonſtration will, in ſuch a Caſe, Fall Short of Certainty; Thus, Albeit the Three Angles of a Triangle are Equal to Two Right Ones, Relatively to the Principles, upon which our Ideas in that Propoſition are Connected, yet it is not Abſolutely and Univerſally True; Since this Propoſition, as well as moſt, if not all, in the Elements of *Euclid*, are Contrived upon a Uniform Space and Extenſion. Accordingly, in the Preſent Caſe, Two Right Angles are Conſidered to Take up the Uniform Space, which Lies on one Side of a Right Line, which always Subtends a Greater or Leſs Semi-circle, whoſe Degrees are Suppoſed to be 180 Immutably, and without any Alteration; It is alſo Suppoſed Reſpectively, that *fbl*, *lbc*, *cba*, are Portions of ſuch an Abſtracted Extenſion or Space, to which *bac*, *bca*, *cba* are Equal, and therefore are Conſidered, as Portions of a like Abſtracted Space; So far therefore it is True and Certain, That the Three Angles of a Triangle are Equal to Two Right Ones; But if theſe Principles are not Adequate to the Real Dimenſions of Space, according to Chap. 4. §. 11. Part 1. of our Principles of Natural Philoſophy, but Extenſion is Variable, This Propoſition would not be True in General, and only upon the Preſent Suppoſition of this Particular Abſtraction of Space, which it Affirms; For Inſtance, if the Degrees of a Circle were Meaſured, not by the Equal Spaces, which Thoſe of a Greater and a Leſs, Comprehend at the Center, upon which Suppoſition, both the Semi-circles are 180 Degrees, but by the Celerity, or Slowneſs, of their Motion, by which thoſe Spaces are Performed, which would be more Reaſonable, there may be 180 Degrees Deſcribed in One

Circle, which is Double of Another in it's Circumference, whilſt there are only 90 Deſcribed in That, which is Half of it.

THUS, if *ABC*, Fig. 3*d*. is Double of *abc*, it will Require a Double Velocity to Deſcribe *ABC*, in the ſame Time, that *abc* is Deſcribed; Let us now therefore Suppoſe *ABC* = *abc*, and the Velocity, by which One is Performed, Double to the Velocity of the Other, *ABC*, or 180 Degrees, will be Deſcribed, when only 90 of *abc* are; From whence it will Follow, that 180 Degrees in *ABC* will be Double of 180 in *abc*; Or, there will be a Greater Force Compreſſed into the Same Space on the One Hand, than on the Other.



AND Conſequently it would Follow, that a Right Angle, made by the Perpendicular to the Diameter, would not be Equal in both Caſes; What I Mean is, that in Fig. 2. if the Extenſion on the Side of *fba* may be Variable and Different, it will not be Certain, that, becauſe *bac*, *acb*, *cba*, are Equal to the ſaid Space, that therefore they are Equal to Two Right Angles, or 180 Degrees; Becauſe the Velocities, with which any Quantities are Deſcribed, may be as Juſt a Meaſure of their Values, as the Spaces, which are Deſcribed, ſince the Effect may be Truly Eſtimated from the Cauſe, which Produces it; and where we Conſider Space, or Extenſion, as one Individual Thing, it is only an Abſtraction of the Mind, without Taking the Reality of it's Cauſe and Production into the Account; And This is the Fault of the Firſt and Original Generation of Quantities in Mathematicks, That it only Conſiders the Length, and Breadth, and Depth, of Lines, or Superficies, or Solids, as already Generated, ſo as to Determine their Value or Quantity from their Abſtracted Length, or Breadth, or Depth, without Taking any Notice of the Power, Force, or Velocity, by which they are Generated and Produced.

WHEREAS, there is nothing more Plain, than, if One Force, which is Double of Another, will Produce a Line Double in Length to Another, as *A* |———|———| *B* is Double of *a* |———| *b*, ſo, that if the Line is the Same, and *AB* is = to *ab* in Length, the Quantity, or Value, of the Line muſt be Meaſured by the Force, which Produces it, and tho' the Length is Equal, the Effect of the One will be Double of the Effect of the Other, as the Cauſe is; I have Mentioned ſomething of this Nature in the *Geometria Solidorum ſive Materiæ*, and tho' this Notion is not at Preſent Apprehended, or Underſtood, by either the Philoſophers, or Mathematicians, yet I will Venture to Affirm, That, Neither the One will Form a Juſt Scheme of Elements in the Mathematicks, Such as ſhall be Agreeable to Things, as they Exiſt, Reaſon Indeed they may, and Truly, upon their own Abſtractions, Nor will the Other give us a Conſiſtent Syſteme of Philoſophy, Till this Notion, as New and as Uncommon, as it may Appear, is better Conſidered by Them; For then They will find, that a True Mathematicks does Exactly Correſpond with a True Philoſophy, and that there will be an Intimate Connexion betwixt the One and the Other; Whereas, according to the Preſent Management in Both, no One can See the Leaſt Agreement betwixt the Elements of *Euclid*, and the Arguments and Reaſonings upon *Nature*; Both are, at Preſent, Studies, which Terminate in Abſtraction, and There they End, without Informing us of what either Extenſion is in the One, or what the Real Properties of Beings are in the Other; They will alſo then Perceive more Fully the Infiniteneſs of the Agent from the Infinite Variety of the Agency, and Confeſs from a Contemplation of Nature it's ſelf, that there is, and muſt be, a God.

AND yet, as the Philoſophick World now is, We are Apt to Admire the Wit, Contrivance, and Invention, of Men of Abſtraction, and Speculative Opinions, more than Nature it's ſelf, or the Infinite and Divine Author of it; It was Once a very Great Portion of Knowledge to be Able to Underſtand *Des-Cartes*, and Nature Lay Deſpiſed and Neglected, whilſt That Celebrated Commentator upon it was Eſteemed the only Perſon Fit to Inſtruct us in the Secrets of it, Notwithſtanding which, we now find our Miſtake, and that, whilſt we were Thought to be Purſuing the Knowledge of Nature, we were only Following the Sentiments of the Philoſopher; And the Same may be Said of the Modern Philoſophy, whatever High Conceptions Men may have Formed Concerning it; But This I ſhall Refer to a farther Diſquiſition, which has been made upon theſe Subjects, and to thoſe Real Principles of Nature and Philoſophy, which we have in the Preceding Books Propoſed to the Learned World.

We ſhall only Add in this Section, that the Three Angles of a Triangle, upon Different Suppoſitions of the Space, Lying on one Side of the ſame Line, and according to the ſeveral Denominations, Values, or Quantities, of that Space, may be Equal to 2, 3, 4, 5, *&c.* Right Angles, or to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* of Two Right Angles, or in any other Proportion, and ſo in Infinitum; And, on the other Hand, Suppoſing the Denomination or Value of the Abſtracted Space to be the Same on the Side of one Line, and the Denominations and Values of the ſeveral Angles of the Triangle to be Varied, the Proportions betwixt the Three Angles of a Triangle, and Two Right Ones, will be Infinitely Changed likewiſe; I have Mentioned here only the Force of Velocity, tho' there may be other Forces alſo, by which Spaces, and the Contents of them, may be Altered.

How we are to Judge, whether a Mathematick Reaſoning is True.

§. 19. After what we have Alledged in the Foregoing Section, it will not be Neceſſary for us to go thro' the ſeveral other Steps of the Demonſtration Produced in §. 16. ſince All of them may be Examined by the Same Methods; *1ſt.* By Inquiring, Whether there is a Viſible Connexion betwixt the Ideas we make Uſe of; *2d.* Whether we Perceive the Connexion of the Connexion of thoſe Ideas, and in the Whole are not Miſtaken in our Memory, in Retaining the ſeveral Parts of it; *3d.* Whether our Judgement has been Firm and Sincere in Diſcerning the Connexion of the Parts, and the Whole, and This Concerns only One Propoſition; *4th.* When we farther Compare One Propoſition with Another, and That with a Third, or a Fourth, the Same Memory and Judgement is ſtill more Requiſite, and the Viſible Connexion of the Simple Ideas, which belong to Each, and the Connexion of thoſe Connexions, *&c.* ſo that One Propoſition, neither in the Whole, nor it's Parts, ſhould Interfere or Diſagree with Another, either in the Principles, they Each Lay Down, or in the Connexion of their ſeveral Parts; *5th.* we are to Inquire, Whether thoſe Principles are Particular, or Univerſal; If Particular, as in One Abſtraction of Space, the Propoſitions, which are Demonſtrated, can Extend no Farther, and are Falſe, when Applied to a Space, which has not the Same Abſtraction of the Mind; Which is the Chief Cauſe of all the Errors and Obſcurities, or Uncertainty, in Mathematick Reaſoning; If They are not Particular, but Univerſal, that is, if One Particular Abſtraction of Space is not only to be Conſidered, but Several may be, All the Elements of *Euclid* will be far from giving us Demonſtrations, which are Abſolutely, or Univerſally True, as we have Inſtanced in the Three Angles of a Triangle, being Equal to Two Right Ones.

A State of the Queſtion, Concerning the Angle of Contact, being a Quantity or no Quantity.
Euclid. Elem. Lib. 3. Prop. 16. Corol. and Schol.
Dr. Wallis de Angulo Contactûs, Vol. 2, Pag. 605.

§. 20. We ſhall therefore Conclude this Head of Mathematick Certainty, by Examining into the Celebrated Controverſy Concerning the Angle of Contact, being a Quantity, or being None; Which is Eaſily Solved; For if *a b l*, Fig. 1. is a Right-Angle Triangle, that is, if *b l* is a Strait Line, (which is not Proved, unleſs upon the Suppoſition, that what is Called a Tangent, Touches the Circle in one Point only,) it is Manifeſt, that *a b o* makes a Leſs Angle, and Conſequently, the Angle of a Semi-circle is Leſs than a Right Angle, and the Angle of Contact *l b o* is a Quantity; On the Contrary, if *l b* is not a Strait Line, but a Curve, *a b o* will be Equal to *a b l*, and therefore the Angle of Contact *l b o* will be no Quantity, and the Angle of the Semi-circle *a b o* will be Greater than any Acute Angle, becauſe it will be Equal to a Right One.

The Foregoing Queſtion Reſolved.

§. 21. The Queſtion Therefore is not, whether the Angle of Contact is a Quantity, or no Quantity, but whether a Curve Line Coincides with a Strait, Drawn from the Same Point; If it does, the Angle of Contact is no Quantity; If it does not, the Angle of Contact will be a Quantity; And in this Terminates the whole Diſpute, which can never be brought to a Deciſion; Since the One Side muſt Affirm a Curve, Drawn from the Same Point, is Coincident with a Strait, and the Other, that it is not Coincident with it.

Wherein the Certainty of Experimental Knowledge Conſiſts.

§. 22. Another Branch of Cerrainty, which Ariſes from Self-Information is, That which we Named Experimental; Thus, ſince all we Experience is Reſolved, either into the Properties of our own Minds, or thoſe of External Beings, and ſince we are Certain of Both, This Kind of Evidence or Knowledge is as Infallible, as the Geometrick is; For I may as well, and with as Great Certainty, Affirm, that Doubting

is

not Believing, or Remembering is not Judging, *&c.* in Reſpect of the Operations of the Mind, or, in Reſpect to the Properties of External Beings, that Colour is not Sound, or Taſte Smell, as I can, that Three is not Equal to Four, or a Triangle the Same Figure with a Circle; So far therefore, as we Reaſon upon Fact and Experience, without Entering into the Nature or Cauſe of the Properties they Preſent to us, The Principles, upon which we Proceed, are Equally Certain with Mathematick or Abſtracted Ones; And upon theſe Principles it may be as Evidently Proved in Relation to the Mind, that the Properties of it, ſo far as they Appear to us, or are Experienced by us, when Compared together, do Agree amongſt themſelves, or Differ, as that the Three Angles of a Triangle are Equal to Two Right Ones, or not; for Inſtance, a Dejection of Mind, That is, that Uneaſineſs and Oppreſſion of Spirit, which we Feel in our ſelves, does Agree with Fear, and That with the Apprehenſion of an Apparent Miſchief, which is likely to Befall us from the Malice, or Ill Will, of our Enemies; On the other Hand, a Dejection of Mind does Agree, and is Connected with a Remorſe, and That with a Conſciouſneſs of Faults, or Crimes, which we have Committed; From whence we may Conclude, that Innocence and Blame may be Equally the Cauſe of Deſpondency; On the other Hand, a Sprightlineſs and Gaiety of Temper is Connected with the Eaſe and Satisfaction of the Mind, and That with Innocence, as likewiſe, the Same Sprightlineſs and Gaiety of Temper will Agree with Pleaſure, Pleaſure, in ſome Tempers, with the Oppreſſion of Thoſe, whom they Hate or Diſlike, and Oppreſſion with Injury; From whence it will Follow, that Innocence and Blame may be Equally the Cauſe of ſuch an Alacrity; So that from the whole Connexion, the Mind may be Dejected, when Innocent, or Debonair, when Conſcious of it's own Crimes, and on the Contrary, Dejected when Conſcious of it's Guilt, and Eaſy and Satisfied in it's Innocence; It may alſo, from what Foregoes, have a Mixture of Satisfaction and Diſcontent from it's own Innocence on the One Hand, and from the Injuries, it finds it's ſelf Oppreſſed with, on the Other, or elſe, in ſome Tempers, from the Pleaſure of Oppreſſing Thoſe, whom they Diſlike, and the Inward Regret they may have of the Injury, they Commit, in doing it; Such Reaſonings, as theſe, in Reſpect of the Properties of the Mind, and what is Experimentally found in it, as Men are Good or Ill, Innocent or Miſchievous, are as Demonſtrable, as Mathematicks themſelves; We only Produce This as One Inſtance; and where we make Uſe of the ſame Kind of Arguing upon what is Experienced in Humane Minds, Such a Science is not Inferior to a Geometrick One; But when we Apply theſe ſeveral Caſes to Particular Inſtances of Perſons, being Chearfull or Dejected, ſo as to Pronounce the One Innocent, and the Other Conſcious of his Faults, or the Contrary, There we may be Deceived.

Also, in Reſpect of what we Experience from outward Objects, where we Reaſon only upon what we Perceive, or are Senſible of, our Demonſtrations are as Juſt, as any in *Euclid*; For we as fully Know, that the Sun is not the Earth or Moon, and that Light is not Heat, or Colour is not Sound, as we are Sure, that Three is not Equal to Five, or a Line or a Surface is not Equal to the Double of it; We therefore in this Sort of Reaſoning Know, that a Red Light is an Impreſſion upon our Eyes, that Heat is One upon our Organs of Feeling, and Theſe we Experience in the Sun, We likewiſe Perceive, in ſome Degree, the ſame Senſations from Fire; and Conſequently may as Juſtly Conclude, that the Sun and the Fire are of the Same, or of a like Nature, in thoſe Reſpects, as that the Three Angles of a Triangle are Equal to Two Right Ones: So in Glaſs, Water, or any other Tranſparent and Uncoloured Subſtance, where we find Correſpondent Effects in Senſation, we may Rationally Infer, they Agree in that Particular. And to this Head of Knowledge all Matters of Fact are Reducible.

The Method which is to be Uſed in our Reaſoning, upon Experimental Knowledge.

§. 23. But, as the Foundations of Experimental Knowledge are as Certain, as the Principles of the Mathematicks, and the Connexions of Ideas are as Plain and Viſible in the One Caſe, as the Other, yet we muſt have Recourſe, in Order to Produce Certainty, to the Same Method, which was Requiſite in Mathematick Demonſtrations, which, according to §. 19. was, Firſt, to Inquire, whether there is a Viſible Connexion betwixt the Ideas we make Uſe of; Second, whether we Perceive the Connexion of the Connexion of thoſe Ideas, *&c.* That is, Memory and Judgement are here likewiſe Concerned in the Connecting of Ideas, and we muſt farther Examine, whether ſuch Ideas, which we Receive, in what we Call Experimental Knowledge, are Agreeable to the Nature of Things, and to Facts themſelves, as they Exiſt; In which Reſpect, This Knowledge is Superior to the Mathematick; Becauſe, in the Preſent Circumſtance, we Argue upon Fact, and what we Experience; Whereas, in the Abſtractions of Figure and Extenſion, The Will is Employed, which does not always Conform it's ſelf to the Reality of Things, but to it's own Pleaſure; When, on the Contrary, in Experimental Knowledge, the Mind only Reaſons upon what it Finds, without Forming and Shaping to it's ſelf it's own Notions; That is, Mathematick Knowledge Proceeds upon the Arbitrary Abſtractions of the Mind, and Experimental, upon what is Really Offered or Propoſed to it from Nature.

Wherein Philoſophick Certainty Conſiſts.

§. 24. The Laſt Head of Certainty, which Proceeds from Self-Information is, what we Termed a Philoſophick One, where we Reaſon upon what we Experience in Nature, or our own Minds, and Join our Abſtractions to it, in Order to Explain the Phenomenons, which Occur to us from Both; Thus we Perceive Extenſion, Solidity, *&c.* in all Bodies, upon which the Mind, by Abſtracting, Forms to it's ſelf One Invariable Perception of Each; and the Same may be ſaid of Matter in General; The Uncertainty of which Kind of Reaſoning we have Endeavoured to Shew in our Principles of Natural Philoſophy, wherein moſt of the Arguments upon Nature are Evinced to be only a Reaſoning, upon Arbitrary Abſtractions, made by the Mind, without being any ways Agreeable to the Truth and Reality of Things; From whence it Follows, that the Chief Cauſe of our Errors Proceeds from the Depravity of our Wills, even in Speculative Matters, as well as in Practical; For by theſe Abſtractions, which are the Reſult of our Pleaſure, rather than of our Underſtandings, we are Led into moſt of the Miſtakes we Commit in Philoſophy or Mathematicks; Beſides which, here likewiſe is Requiſite, Memory and Judgement, to Retain and Connect our Ideas A-right thro' the whole Progreſs of the Demonſtration; This therefore may be Said, that if our Abſtractions in the Mind are Agreeable to the Qualities and Properties of Being, and if upon thoſe Abſtractions, we Connect our Ideas Juſtly, and Retain the ſeveral Ideas we Connect, Philoſophy is as much Capable of Demonſtration, as Geometry is, and we may be Equally Certain of Both.

Recapitulation of the ſeveral Kinds of Certainty from Self-Information, or Private Perſuaſion.

§. 25. To Conclude this Chapter of the Certainty, we Receive from Self-Information or, Private Perſuaſion; Intuitive and Senſitive Knowledge are Infallibly Certain; Experimental, as it is Explained in §. 22. is the next Certain Knowledge we have, which Proceeds from Reaſoning only upon the Intuitive and Senſitive; Mathematick and Philoſophick Knowledge may be Certain, with Due Caution and Judgement, but Falls Beneath That, which we Receive from the Other Three; Becauſe in theſe Two Laſt Caſes, the Mind Reaſons upon Hypotheſes of it's own Contriving, which are not often, and Perhaps never, either in Mathematicks or Philoſophy, Agreeable to the Nature of what it Conſiders, whether of Extenſion and Quantity in the Firſt Caſe, or of the Properties of a Spirit, or of Space, or of Matter, in the Laſt.

CHAP. III.

Concerning the Evidences of Teſtimony, or Thoſe Informations, we Receive from Others.

Knowledge and Certainty from Teſtimony, or the Informations of Others, may be as Great, as that which is Derived from Experimental Knowledge.

§. 1. We have in the Foregoing Chapter Conſidered the Certainty, which may be had from our own Informations, we ſhall now Inquire into Thoſe, which we Receive from Others; And in this Caſe it is Manifeſt, that we may Arrive at a Certainty and Knowledge, Equal to the Experimental, which, we Said, was the Higheſt our Minds were Capable of; For in Matters, whoſe Evidence Proceeds upon Teſtimony, the Foundations, upon which we Reaſon, are the Same, without any Arbitrary Abſtractions of the Mind; in which Abſtractions we before Obſerved, we are Principally Deceiv'd, and from whence Ariſe all the Various Hypotheſes of Philoſophy and Mathematicks; which Demonſtrate Indeed the Humour and Pleaſure of the Philoſopher, or the Geometer, rather than the Strength or Force of his Judgement, or his Making his Notions Agree with the Truth and Reality of Things.

Teſtimony of Fact Excludes Hypotheſes and Abſtractions, the Chief Cauſes of our Errors.

§. 2. In Arguing therefore from the Teſtimony of Others, all Abſtractions of the Mind are Excluded, which are the Chief Cauſe of our Errors, that is, we Reaſon upon Fact, and not upon an Hypotheſis Framed by us, or which is the Same, we Reaſon upon Fact, which is Neceſſary, and which we Experience, in the One Caſe, whereas in the Other, our Arguments are Formed upon the Abſtractions of the Will, which is Deſultory and Capricious, or, which is all one, Uncertain.

A Compariſon betwixt the Certainty, which Ariſes from Teſtimony, and that which Ariſes from Mathematicks and Philoſophy.

§. 3. Since then the Foundation of our Reaſonings upon Teſtimony Imports the Higheſt Evidence that can be, and That which is Superior to What we find in the Abſtractions of Philoſophy, or Mathematicks, to Wit, the Same, which is Found in Experimental Knowledge, which we Conſidered before, (For Thoſe, who Atteſt Things to us, do it upon the Evidence of Fact and Experiment,) the Inquiry, which Remains, is, what Methods we Uſe in Reaſoning upon ſuch Teſtimony, and whether we have not a Superior Evidence alſo from it, to what Mathematicks or Philoſophy can Afford to, or Supply us with.

Certainty upon a Single Report, Requires only the Sincerity of the Atteſter.

§. 4. It has been Said already, that in Order to Produce a Mathematick or Philoſophick Certainty, we muſt be Sure, that our Abſtractions are Juſt and Agreeable to the Properties of the Extenſion or Body Conſidered; and in this Particular Both Fail, in Reſpect of Certainty, to which Error Matter of Fact or Experience is not any ways Liable or Obnoxious; Farther, in Mathematick or Philoſophick Certainty, the Connexion of Thoſe Abſtracted Ideas is Requiſite, in which the Mind cannot be Deceived, where there is a Viſible and Apparent One; And thus in Teſtimony, if a Perſon, who Affirms any Thing to us upon his Experimental Knowledge is Sincere, and of the utmoſt Veracity, we have as Great a Certainty and Evidence from the Report of ſuch a Perſon, as if we our ſelves had been Experimentally Convinced of That, which he Affirms, and which for Diſtinction's Sake, we ſhall Call Certainty on a Single Report; All the Difficulty, which Lies in ſuch a Caſe is, by what Methods we ſhould know, where a Perſon is Sincere and Veracious, for there is as Real a Connexion betwixt Speaking Truth, and being Sincere, as there is betwixt any Ideas, which we have.

How we may Judge of, and Diſtinguiſh, the Sincerity of the Reporter.

§. 5. And this may be Known ſeveral Ways; *1ſt.* If the Whole Courſe and Tenour of the Perſon's Actions is according to the Rules, not only of Juſtice and Honeſty, but of Charity, Generoſity, and Love; *2d.* If his Apprehenſions and Thoughts, in all his Converſation, Explain an Inward Principle of Integrity and Goodneſs; *3d.* If he is Willing to Sacrifice his Intereſt and Advantage in this World, his Life, Reputation, or any Thing elſe, which is Dear to him, in the Juſtification and Defence of what he Affirms, and Knows to be True. Any One, who has theſe Characters, may be as Juſtly and Experimentally ſaid to be Sincere, as we may Pronounce it Impoſſible for a Pious and Good Man to be a Wicked or an Ill; He, who is ſo far from Injureing or Wronging Another, that he would always, and upon any Occaſion, do him all the Services and Good Offices, which Lie in his Power, and which Property is Rooted and Fixed in his Temper; He, who is ſo far from Impoſing upon the Weakneſs and Credulity of thoſe he Converſes with, that he is rather Willing to be Patient under thoſe Fallacies and Abuſes Practiced towards himſelf, and who on the Contrary Speaks, howver Illy it may be Received, what ſeems to be the Trueſt, and the moſt Conformable to an Exact Honeſty; He, who to This is Ready to Undergo Reproaches, or Death, in the Vindication of That, which he Atteſts to us; In Any, or All of Theſe Caſes, we may be as Sure, that Such a Perſon is Sincere, and Conſequently, that What he Reports to us is True, as we can be Certain, that we Hear him Speak, or that he Relates it.

Other Characters, by which we may Judge of, and Diſtinguiſh, the Sincerity of the Reporter.

§. 6. Besides theſe Three, there are other Marks alſo, by which we may Diſtinguiſh the Sincerity of one, who Reports his own Experimental Knowledge, or which is the Same, his Knowledge of Fact and Experience; as *1ſt.* When there can be no Worldly or Temporal Advantage Accruing to him from his Affirmations, but on the Contrary, Calamities, Afflictions, and Oppoſition, are the moſt Likely and Probable Conſequences of them; *2d.* When he has a Full Impreſſion of the Rewards and Puniſhments of another World, the Former of which, he is Convinced, will not be Diſpenſed to the Aſſertors of a Falſehood, and that they will, as Certainly In-

Incurr the Latter; Theſe are ſuch Invincible Ties upon Men, who are Confirmed in ſo Juſt a Belief, that all the Intereſts of this World can never Induce them to Relinquiſh their Integrity; And therefore upon ſuch we may as Intirely Depend in what they Solemnly Report to us, as we can upon our own Experimental Knowledge, that is, upon our Perceptions, or our Senſes.

Other Characters likewiſe, by which we may Judge of the Sincerity of the Reporter.

§. 7. THERE may be ſeveral Characters more of a Perſon's Sincerity, which we cannot Perhaps now Recollect; we ſhall only Mention the Simplicity and Plainneſs of Converſation and Behaviour, Meaneſs and Deſpicableneſs of Fortune, and the Contempt, which Attends it, Application and Induſtry in all the Affairs and Circumſtances of Life, and an Averſion to any Thing, which is Trifling and Ludicrous, All which Generally Infer the Integrity of the Perſon, in whom they are; For, If any One is Innocent and Harmleſs in his Commerce with Others, if he would not therefore, and upon that Account, Injure them in the Leaſt Reſpect, he, no Doubt, would not Deceive them; If the Same Perſon is Mean and Deſpicable, the Contempt, he Lies under, will make him Regardleſs of every Thing, but what he Thinks and Knows to be True; Application and Induſtry likewiſe, in whatever Employ, Produces a Severity in the Mind, which is Contrary to the being Ludicrous and Trifling, and is an Argument of Sincerity; ſince, where the Mind is Grave and Severe in it's own Temper and Conſtitution, it cannot Raiſe it's ſelf into the Lightneſs and Levity of Impoſing Tricks and Fancies upon Others of it's own Inventing.

The Nature of the Fact, whether Miraculous, or Agreeable to Reaſon, has nothing to do in that Certainty, which Ariſes from Teſtimony.

§. 8. As to what Concerns the Fact which is Related, in Reſpect of Certainty from Teſtimony, it does not Enter into the Account; For, if it is of ſomething Agreeable to our Reaſon, Teſtimony Imports no Evidence at all to that, which we can Conclude from thence; Thus, Suppoſing any One ſhould Inform me, that the Three Angles of a Triangle were Equal to Two Right Ones; If I was Capable of Collecting the Same Propoſition from Reaſon, the Evidence, which I had from Teſtimony, would be of no Force or Significance; If I was not, my whole Acquieſcence would be Reſolved into the Veracity of the Perſon, who Affirmed it.

The Evidences from a Single Report Compared with thoſe, which Reſult from an Abſtracted Reaſoning.

§. 9. THE Only Queſtion therefore, which Remains, when we Compare the Evidences we have from Self-Information, and from the Teſtimony of a Single Report, is, whether the Arguments upon the Abſtractions of Any other Part of Self-Information, Excepting the Experimental, which Admits of None, and therefore is not ſo Fallible upon that Account, are Really more Concluſive than Thoſe we have of any Perſon's Sincerity; On the One Hand, there is the Connexion of Ideas, the Connexion of the Connexion of thoſe Ideas, the Connexion betwixt One Propoſition and Another, and the Exacteſt Memory and Judgment Required, which is Poſſible, to make a Conſiſtent Demonſtration from the Whole; On the Other Hand, the Arbitrary Abſtractions of the Mind are Entirely Superſeded in Experimental Knowledge, and when we come to Conſider the Evidences of a Single Report, we are only to Examine into the Sincerity of the Perſon, who makes it to us, which alſo is Known from Fact and Experience, that is, from the Probity Expreſſed in his Words, and from the Tenor of his Life and Actions, and the other Characters we Mentioned of Sincerity; So that, as he who Reports any Thing to us, does it from his own Experimental Knowledge of what he Relates, ſo from the Same Experimental Knowledge we are Convinced of the Honeſty and Integrity of the Reporter, without any Fallible Abſtractions, or Connexions of Ideas; And Conſequently, ſuch an Evidence is Superior to a Mathematick One, according to §. 23. of the Preceding Chapter; For, if it be Said, a Perſon may have all the Characters Mentioned of Sincerity, and yet in Reality be Falſe and Inſincere in any Solemn Report he makes to us, we only Anſwer as before, that it is as Impoſſible, as that a Truly Good Man, ſhould be a Truly Ill One.

The Perfect Acquieſcence and Certainty which a Man may Juſtly have from a Single Report.

§. 10. WE have hitherto Conſidered the Report of a Single Perſon, and ſhall only Add ſome Inſtances, to Shew the Unreſerved Aſſurance and Confidence we may have in ſuch a Teſtimony; Thus, if the Preſent Great and Surprizing Genius of the Mathematick World Sir *Iſ. Newton*, or the Excellent and Learned Profeſſor of Aſtronomy in this Univerſity Mr. *Cotes*, whoſe Names are Sufficiently Known and Celebrated for Integrity and Goodneſs, as well as their Uncommon Penetration and Judgment, ſhould Affirm, and Punctually Declare, that they had Found the Square of the Hypotenuſe was Equal to the Squares of the Sides in a Right-Angle Triangle, I could

could be as Fully Satisfied, and as Certain, that they did See that Connexion, from whence they Apprehended ſuch a Concluſion might be Drawn, as if I had Seen and Viewed that Connexion in the ſeveral Parts of it; Likewiſe in Experimental Philoſophy, if they had Aſſured me, that they had made ſeveral Experiments upon the Pendulum, or upon the Tranſmiſſion of Light thro' a Glaſs Priſm, which were ſuch, as they Deſcribed them, I ſhould not make the Leaſt Scruple of giving my Perfect Aſſent to them; Where they did not Plainly Tend to the Advancing any Hypotheſes, and were only Delivered, as Matters of Fact; And in Common Life, if any Single Perſon, whoſe Integrity, from the whole Series of his Conduct, was Evident to me, and who had moreover all the Characters, above Recounted, of Sincerity, ſhould Aſſert he had Seen any Accidental Occurrence, which to my Apprehenſion might, or might not, be True, I could be as Juſtly Certain of it, as if it had Happened in my own Notice and Obſervation, tho' it might Appear to me very Surprizing and Extraordinary.

The Concurrent Teſtimony of ſeveral Perſons, who are Sincere, Produces a Certainty, which is Indiſputable.

§. 11. THIS therefore is the Undoubted and Infallible Evidence, which we have from a Single Report; If farther, 12, 60, or 500, or Some Thouſands, ſhould Averr the ſame Thing to us, All of them Perſons Inveſted with the ſame Characters of Sincerity; Who were Ready to Die in Atteſtation of what, they were either Convinced of from their own Experimental Knowledge, or from their Experimental Knowledge of the Sincerity of Thoſe, who Reported Their's to them; Who Firmly Believed the Rewards and Puniſhments of another World; Who were Endued with the Utmoſt Affection to Mankind, were in the whole Tenor of their Lives Upright and Diſintereſted, and Exactly Honeſt and Punctual in their whole Converſation and Behaviours; Who did not Regard the Applauſe of this World, or the Honours of it; Who were Simple and Plain in their Tempers, and Deſpicable and Mean in their Fortunes, Sedulous and Diligent in their Employments, and in the Complexion of their Minds Grave and Severe, according to §. 5. 6. and 7; What would the Concurrent Teſtimony of Such Perſons Produce in us, but the Utmoſt Aſſurance, which it was Poſſible for the Mind of Man to Entertain or Receive? And This we may Call a Certainty from the Concurrent Teſtimony of Men, Who are Sincere.

When this Concurrent Teſtimony is Committed to Writing, and which Written Atteſtation has all the Characters of Integrity and Probity in it, the Utmoſt Certainty is Produced.

§. 12. IF after This, Theſe very Perſons do not only Report Matters of Fact, but Commit thoſe Facts to Writing, and in the Books, which they Wrote, if there are the ſame Characters of Sincerity; This would be ſtill a farther Confirmation of ſuch an Evidence; So that hitherto Certainty is as Great, as That, which Reſults from Experimental Knowledge, which is the Higheſt, that can be, according to §. 22. and 23. Chap. 2. And This is the Evidence, which the Primitive Times had for the Truth and Divinity of the Chriſtian Religion, and which therefore we may Affirm to be Superior to a Mathematick One, as it was an Experimental, and therefore not Liable to the Errors, which are Committed by the Mind in the Abſtraction of it's Ideas, and the Connexion of Them.

The Concurrent Teſtimony of Men, who in other Matters Diſagree, Produces Certainty.

§. 13. BESIDES That Certainty, which is Derived from the Report of a Single Perſon, Who is Sincere, or from the Concurrent Teſtimony of Several, Who are ſo, there is Another, which Reſults from a United Teſtimony of all Sorts of Men, Who are of Different Intereſts, Perſuaſions, and Deſigns; For Example, That there is ſuch a Place as *France* or *Spain*, as *Paris* or the *Hague*, or That there is ſuch a Prince as *Charles* the 3*d*. or *Lewis* the 14*th*. Every One is ſo far from Thinking it only highly Probable, that he is Sure and Certain of it, not from the Connexion of Ideas, but from the ſeveral Steps, or Degrees, in Experimental Knowledge, from which we Conclude it; For, altho' we are not Convinced of the Sincerity of Thoſe, who Tranſmit Weekly, and almoſt Daily, Accounts Concerning them, yet the Concurrence there is in their ſeveral Reports, when the Reporters themſelves Diſagree, in their Intentions of Writing, in their Principles, and their Party, is ſo Strong a Conviction to us, that there are ſuch Places and Perſons, that we can be as Certain of it, or more Certain of it, than of the Abſolute or Univerſal Truth of any Mathematick Propoſition whatſoever; And This Kind of Evidence we may Name a Certainty from the Concurrent Teſtimony of Perſons, Who in other Matters Diſagree, and which is alſo Derived from Experimental Knowledge, that is, from the Report of Matter of Fact from One to Another; This Evidence alſo the Chriſtian Religion has, as we have Endeavoured to Prove in the *Demonſtration* of the *Truth* and *Divinity* of it; Diſcourſe the Laſt, §. 28. 29. 30. 31. 32.

That there is an Hiſtorical Certainty, and which belongs to Chriſtianity.

§. 14. ANOTHER Kind of Certainty, which Ariſes from Experimental Knowledge, is, what we may Term Hiſtorical, which is Derived to us, either from Written, or from Oral Tradition; The Firſt of which is as Certain, as our Experimental Knowledge it's ſelf; Thus, If it was Certain, that Chriſtianity was True in the Primitive Times, it is as Demonſtrable, that it is ſo in Ours; Since there are Books Extant in all Ages, from it's being Firſt Propoſed, which Prove, that the Scriptures were Written in thoſe Times, according to the Demonſtration we have given of the Chriſtian Religion; That the Perſons, who Wrote Them, Died in Defence of Them, and had all the Characters of Truth and Sincerity, which we before Explained; And, as the Eccleſiaſticall Writers in Hiſtory are the moſt Numerous of any Other Hiſtorians, ſo it is alſo Evident, they were the moſt Sincere, as they were Ready to Sacrifice their Lives in Defence of what they Wrote; Beſides, Thoſe Principles of Future Rewards and Puniſhments, and the Severe Doctrines and Rules, which the Chriſtian Religion Preſcribed to them, would of Neceſſity make them more Impartial in their Hiſtories, than any other Perſuaſions could do; If therefore there is any Hiſtorical Certainty, which no One will Refuſe to Acknowledge, unleſs he who Places all in the Fantaſtick Connexion of his own Ideas; If there was ſuch a Perſon, as *Pompey* or *Cæſar*, as *Alexander* or *Philip*, or ſuch Places, as *Athens* or *Rome*, and if there were thoſe Actions, which are Recounted to be done by thoſe Perſons or States, which Depends upon Hiſtory, We may as Juſtly Affirm, that the Chriſtian Religion is True, and the Experimental Knowledge, given us of it, by the Evangeliſts and Apoſtles, and which has been Tranſmitted Down to us thro' all the Ages of the Church.

Hiſtorical Certainty farther Explained, and Applied to Chriſtianity.

§. 15. To Draw the Parallel more Particularly; We Know there was ſuch a Perſon as *Cæſar*, and that he Performed thoſe Actions againſt the *Gauls*, *Helvetians*, and *Germans*, which his Commentaries Affirm to us; We are likewiſe Certain, that he Gained the Victory over *Pompey*, at the Battle of *Pharſalia*, and Subdued the *Roman* State to his Obedience; This, in that Time, was not Proved by a Connexion of Abſtract Ideas, but was Evident and Manifeſt from an Experimental Knowledge of it; We have his own, if the Criticks will Allow it, and other Hiſtories, Extant to Prove it; And the Experimental Knowledge, which is Derived from Matter of Fact to the Reporter, and from the Sincerity of the Reporter to a Second or a Third, and ſo on, is Thought a Sufficient Conviction, that we may be Certain of all this, as Indeed we may; On the other Hand, our Bleſſed Saviour never Writ any Commentaries of his own Life and Actions, and therefore the Hiſtory, we have of Him, is not Liable to the Partiality, which might be Expected from One, who did; His Diſciples, who Saw and were Witneſſes of every Thing, have done it, and therefore may be Juſtly Imagined to give us a more Sincere Account of them, than *Cæſar* would do of his own; Not to Mention the ſeveral Marks of Sincerity, which are in the Hiſtories, and the Perſons, of the Evangelick Writers. Let us now Proceed Farther, from the Times, in which the ſeveral Commentaries were Wrote, and it is Manifeſt, that the Evangelick Writers upon the Account of the very Genius and Temper of their Chriſtianity, which was, To be Sincere and Honeſt, To be, as Thoſe, who were to give an Account of all their Words and Actions to a Supreme Tribunal, it is Evident, That they had Juſter Principles to make their Hiſtories Exact and Impartial, than any Heathen Writers could Pretend to; And Conſequently, we have a Greater Certainty from the Degrees of Experimental Knowledge, that is, from the Tranſmitting it from One to Another, for the Truth of Chriſtianity, than we have for any other Hiſtory, which is Extant; To which we may Add, beſides the Sincerity of the Writers in Eccleſiaſtical Hiſtory, which muſt be Greater upon the Principles of their Holy Religion, than That of Heathen ones, who had not the ſame Perſuaſions to Awe them, The Succeſſion of theſe Atteſters is ſo Conſtant, and have been ſo Numerous, that no Experimental Knowledge whatever has been Delivered Down to us with a like Certainty and Conviction.

Chriſtianity has Oral, as well as Written Tradition, on it's Side.

§. 16. LIKEWISE by Oral Tradition, as well as Written, Chriſtianity is Affirmed to us, which no Experimental Knowledge has beſides it; For No One by Oral Tradition, ever Knew, there was ſuch a Perſon as *Pompey*, or *Cæſar*; And yet, even this Evidence, whatever it is, which we ſhall afterwards Conſider, Chriſtianity Claims.

A Farther Account of Hiſtorical Certainty, and Applied to Chriſtianity.

§. 17. We ſhall therefore now in General, after a Compariſon betwixt the Hiſtorical Evidence of Chriſtianity, and That of any Heathen Writer, Conſider the Certainty we have from Hiſtory it's ſelf, either Written or Oral; As to what Reſpects Written Hiſtory, If the Experimental Knowledge, Originally Expreſſed in any Book, is True, the Methods of Judging of which we have already Deſcribed, it is not Weakened or Enervated by a Long Series of Ages, thro' which it Paſſes, Suppoſing That Book has been always Publick; Thus, if the Hiſtory of the *Peloponneſian* War by *Thucydides* was of Real Matter of Fact, and it was Publickly Handed about from his Time down to the Preſent, we are as Certain of ſuch a War betwixt the *Lacedæmonians*, and *Athenians*, as That Age was; becauſe we are Sure it is the Same Book, which was Writ in Thoſe Times, and Gave an Account of It; That is, This Certainty does not Fall Short of an Experimental One; For this Book was then Known Experimentally to be a Deſcription of that War, the next Age Experimentally Knew this to be that Book, and ſo on to the Preſent, Namely, as to the Main and the Groſs of it, ſince we are not to Anſwer for all the Critical Variations, which may have been made in the Tranſcribing it, or even any Falſe or Diſingenuous Interpolations, which may Poſſibly have been made in Favour of the One State or the Other, from the Firſt Writing of it; But, if farther, there are ſeveral Hiſtories Written in the Same Time, and by Different Perſons, and All of them come Publickly Tranſmitted to Poſterity, ſuch a Derivation of Experimental Knowledge is in it's Evidence Irreſiſtable, and is far from being Weakened by Time or Continuance; Which is the Caſe of the Chriſtian Religion, and the Hiſtories and Goſpels, which Affirm it, at this Diſtance from their Firſt Promulgation; And therefore the *Principia Mathematica Chriſtianæ Religionis* of Mr. *Craig*, I Suppoſe, in Imitation of Sir *Iſ. Newton's Principia Mathematica Naturalis Philoſophiæ*, in which he Endeavours to Shew, that the Evidences of the Chriſtian Religion, which, it ſeems, are only Probable Ones, muſt be Weakened and Impaired by a Long Progreſs of Time, are Vain and Abſurd Imaginations of his Own, and neither Juſtified by Common Senſe, or Common Reaſon.

Of Certainty from Oral Tradition, and how Applied to Chriſtianity.

§. 18. As to Oral Tradition, That is another Way likewiſe of Conveying Experimental Knowledge to us; But This is very Fallible, and can Scarcely be Depended upon, unleſs it is nearly Univerſal amongſt Thoſe, who may be Suppoſed Capable of Delivering ſuch a Knowledge to us, in which Circumſtance the Evidence is as Forcible and Convincing, as That of Hiſtory; Becauſe, where the Truth of any Matter of Fact is ſo Notorious, and Plain, that it is not Changed by the Variety of Men's Inclinations and Complexions who Report it, in Different Ages and Countries, it is an Indiſputable Mark of the Certainty of it; And This is the Nature of the Earlieſt Traditions Concerning Chriſtianity and it's Author, and the Miracles Wrought by him, and the Doctrines, which he Preached to the World, and which Oral Traditions were Publickly Committed to Writing in the Primitive Times, and from thence are Tranſmitted to us by Written Hiſtories of them; The Evidences of which we have already Conſidered. Thus we Know and are Aſſured of Several Tranſactions, I do not ſay All, in the Time of King *Charles* the 1*ſt*. *Edward* the 6*th*. *Harry* the 8*th*, *&c*. and which we could have Entirely Depended upon from Tradition, without any Hiſtories Writ to Inform us of them.

A Civil War.
Reformation.
Diſſolution of Monaſteries.

Where Written and Oral Tradition Agree, there is the Higheſt Evidence which can belong to Experimental Knowledge, and which are United in Chriſtianity.

§. 19. Lastly, where Written and Oral Tradition Unite and Agree, we may Juſtly Call That the Higheſt Evidence, which belongs to Experimental Knowledge, at any Diſtance of Time from That, in which the Matter of Fact Related was Experimentally Known; And This Evidence alſo belongs to our moſt Holy and Divine Religion.

The Evidences of Experimental Knowledge Superior in Chriſtianity to thoſe, which we have of Philoſophy or Aſtronomy.

§. 20. To Conclude therefore this Head of Experimental Knowledge, we ſhall Draw a Parallel betwixt That, which we have for Religion, and for Philoſophy, as we have done betwixt It or That of any Profane or Heathen Writer in Hiſtory; It is Evident then, in all the Syſtemes of Experiments, which are given us in Philoſophy, that they Depend upon the Teſtimony of a Few, ſince they are not Many, who are Furniſhed with the Opportunities and Conveniences of the Same Trials; In Aſtronomy, the Evidence is ſtill Leſs, becauſe in ſeveral Hundreds of Years, from the Time of *Hipparchus* of *Rhodes*, or *Ptolomy*, to *Tycho Brahe*, *Caſſini*, *Hevelius*, Mr. *Flamſteed*, and Dr. *Halley*, how Inconſiderable is the Number of Obſervers, we Meet with in that Science? and

Sir Iſ. Newt. Des-Cartes, Rohault, &c. Dr. Gregory's Aſtronom. Prop. 29. Lib. 2.

yet all our Arguments Concerning Nature, or the Planetary World in Particular, are Formed upon ſuch a Teſtimony, which is Infinitely Leſs in Reſpect of the Multitude of the Atteſters, and the Credibility of them, than That, which Affirms Religion; Not to Mention the Difficulty and Uncertainty in the Experiments and Obſervations themſelves.

A Great Error in Religion and Reaſon, that Faith and Reaſon are not Oppoſite to each Other. Mr. Locke's Eſſay, Chap. 17. §. 24. Book 4.

§. 21. Having hitherto Examined into the ſeveral Foundations of Certainty, we cannot but take Notice of Two Errors, which have of Late been Induſtriouſly Promoted by Men, who ſeem to have Little or no Regard to Religion, unleſs ſo far as it Agrees with their Reaſon, or, which is the Same, with their own Humours and Fancies; The Firſt is, that Faith and Reaſon are not Oppoſite, ſince we cannot Believe the Scriptures themſelves, but upon Good Reaſon, and upon an Evident Conviction of their Truth and Sincerity. The Fallacy of which Lies in the Equivocal Uſe of the Word Reaſon, which is ſometimes made to Signify our Connecting Ideas together, as in the Rational Sciences of Philoſophy and Mathematicks, and in that Senſe, Faith is Oppoſed to Reaſon; ſince it is Impoſſible for us by any Methods of Connecting Ideas to Prove, there is ſuch a Place as *France*, or the *Indies*, or that 1711 Years Ago, *Jeruſalem* was a City in *Judea*, and that *Jeſus* the *Meſſiah* was afterwards Crucified in it; becauſe there is no Viſible Connexion betwixt my Idea of *France*, or the *Indies*, and of the Perſons, or their Sincerity, who Report it to me, as there is betwixt my Idea of 3 and 2 making 5, or the Half not being Equal to the Whole; Another Senſe of Reaſon is, when we Conclude any Thing upon thoſe Evidences, which the Nature of the Subject will Bear, according to the ſeveral Ways, by which we Arrive at Certainty, and which we have before Deſcribed; Thus, in Matter of Fact, or Experimental Knowledge, Faith is not Oppoſed to Reaſon, becauſe Reaſon, in other Terms, is the Evidence we have of Fact, and Faith or Belief is the Same Evidence of Fact likewiſe.

Another Error in Religion and Reaſon, that Revelation cannot be Admitted againſt the Clear Evidence of Reaſon. Mr. Locke's Eſſay, Chap. 18. §. 5. Book 4.

§. 22. A Second Error is, that Revelation cannot be Admitted againſt the Clear Evidence of Reaſon; Or, in other Words, there is nothing in Religion, which we ought to Aſſent to, Excepting what is Agreeable to Reaſon, that is, to our Connexion of Ideas in a Rational or Scientifical Way, as is Plain and Obvious from the Section, to which we Refer, and is the very Foundation of ſome Men's Rejecting all Myſteries from Chriſtianity; Whereas we have already Proved, that Certainty may be Attained to by other Methods, than That of the Connexion of Ideas, that is, by Senſe and Experiment, and Undoubted Hiſtories of Fact, and on which Evidences Chriſtianity Depends: For no One by a Connexion of Ideas can Prove, that there ever was ſuch a Perſon as *Jeſus*, that he wrought thoſe Miracles, which are Recounted of him, or that there was an Effuſion of the *Holy Ghoſt* upon the Primitive *Chriſtians*, and upon theſe the Myſteries of Religion are Founded, which if we Diſcard, becauſe not Deducible by Reaſon, or a Scientifical Connexion of Ideas, which is made Uſe of in Mathematick or Philoſophick Knowledge, we may as Juſtly Deny the whole Hiſtory of our Religion, which is Equally Incapable of ſuch an Evidence, according to the Foregoing Section, Albeit, as we have Proved, whilſt it comes up to an Experimental Certainty, the Evidences are Stronger and more Forcible than the moſt, if not all, which we Meet with, in thoſe Rational Sciences.

An Internal Evidence from Writings themſelves Produces Certainty.

§. 23. Besides what we have Said Concerning the External Teſtimony of any Writing, there is alſo an Internal One, which is Capable of Producing the Higheſt Degree of Certainty in us; Thus, When we Conſider the *Olynthiacks* and *Philippicks* of *Demoſthenes*, and that Noble Concern, which he Expreſſes in them, for the Safety of his Country, againſt the Invaſions and Encroachments of *Philip* upon the Liberties of *Greece*; When we find him, by all the Turns and Paſſages of his Orations, to Speak to, and Harangue, his Audience, as any One would have done in thoſe Times; When his Arguments and his Paſſions are all Adapted to the Genius and Temper of the *Athenians*, to the Circumſtances in which they were, and to all the Minute Occurrences of that State, when the *Olynthians* were to be Supported, and the War againſt the *Macedonians* to be Carried on in a Foreign Country, in Oppoſition to Thoſe, who were for Staying, till *Philip* came into *Attica*, as *Phocion*, who was the Ax of *Demoſthenes*'s Periods according to *Plutarch*, and his Party, were Willing to do; When we Hear him Declaiming againſt *Æſchines*, and the Orators of his own Age, and againſt *Philip*, with that Warmth and Vehemence; When we Perceive him Tracing all *Philip's* Actions and Intentions, and the Methods he Uſed to Enſlave *Greece*, by Entering the *Pylæ* upon a Peace, and under a Pretence of Defending the *Amphictyones* and Redreſſing the Sacrilegious Injuries done by the *Phocians*, in Pillaging the Temple of *Apollo* at *Delphi*, and after he had Poſſeſſed himſelf of *Phocis*, by Falling in with the *Thebans*, *Argives*, and *Meſſenians*, againſt *Sparta*, and *Athens*; What can we Conclude from Such Particulars, which are Agreeable to the Hiſtories of thoſe Times, unleſs, that *Demoſthenes* was the Real Author of theſe Orations? And This Internal Evidence Chriſtianity likewiſe has, according to the Aforeſaid Demonſtration given of it.

Philip. 2.

Therefore the Demonſtration of Religion, Superior to that of any Rational Science.

§. 24. WE ſhall here only Add, that therefore that *Demonſtration*, which we have Propoſed of the *Chriſtian Religion*, may be more Juſtly Termed ſuch, than Thoſe, which we ſo often Call by that Name, in Philoſophy and Mathematicks; Becauſe the One has all the Evidences, which are Poſſible, to Produce an Experimental Knowledge in us, and on the Contrary, the Preſent Philoſophy and Geometry even Want thoſe Evidences, which are Required in a Juſt and Rational Connexion of Ideas, which we have, as we Hope, Abundantly Shewn in our *Principles* of *Natural Philoſophy*, and our *Geometry* of *Solids*, or of *Matter*, as well, as in the Preceding Books of this *Inquiry*.

Objections againſt the Demonſtration of the Chriſtian Religion Anſwered.

§. 25. AFTER which we cannot but be Surprized at ſome Good Men, who have ſeemed to Endeavour to Weaken the Arguments, we have Formerly Alledged for Religion, by Telling us, that Hope, which is Seen, is not Hope, that Faith is a Chriſtian Virtue, but if there is Demonſtration for Religion, that Virtue will be Superſeded, from whence they Conclude, that Religion is only, in a Certain Degree of Probability, True; But does not every Know, that the Hope, there Referred to, Reſpects the Joys of another Life, which Chriſtians might Expect hereafter, the Nature of which was not Fully Comprehended, and therefore they with Patience Waited for the Full Revelation of them? yet this Hope did not Exclude a Certainty of what they Hoped for, ſince St. *Paul* Says, *for we* Know, *that when this Earthly Tabernacle is Diſſolved, we have a Building of God Eternal in the Heavens*, and it is Manifeſt by the whole Tenor of the Scriptures, that the Primitive *Chriſtians* did not Hope for a Future Joy and Immortality, as if they had the Leaſt Doubt or Heſitation Concerning it, as if it was only Probable; They were Certain and Aſſured of it, and wanted upon all Occaſions to Shew their Full and Abſolute Perſuaſions therein, by Undergoing the Greateſt Tortures, and Submitting to the moſt Ignominious Deaths, that they might Enter into thoſe Bleſſed Manſions they Expected; And of the Same Kind was their Faith, which was the Foundation of their Settled and Certain Hope, and ſo it is Expreſly Defined by the Author to the *Hebrews*, that it was the *Aſſurance* and *Confidence* of what they *Hoped for*, and the *Evidence of Things not Seen*; And all the Inſtances, Alledged in that Chapter to the *Hebrews*, do Evidently Explain Faith to be a Fixed and Certain Perſuaſion in the Mind, by which the ſeveral Patriarchs were Excited to the moſt Heroick Actions, which could not Proceed from a Principle of Doubt and Probability; Beſides, was it only Probable in the Firſt Times of the Goſpel to the Diſciples, who Converſed with our Lord, that he Wrought thoſe Miracles, which they Saw him Perform? Was it only Probable, that he Roſe from the Dead? If ſo, what becomes of all Religion, and how is it Proved not to be a Fiction? or how ſhall we Believe our own Senſes? If it was not only Probable, but Certain, I have Endeavoured to Shew by a Continued Atteſtation of Authentick Hiſtories, that we may be as Certain of all this, as the Primitive Diſciples and Apoſtles of our Lord were; How therefore do we Deſtroy the Virtue of Faith by ſuch a Procedure? Was it a Virtue then, how does it Ceaſe to be a Virtue now, if we Believe upon nearly the Same Evidences which they did? Moreover, what Virtue can it be to Believe a Thing, for which we have no, or no Competent, Grounds? If it is, we may as well Reſign our Selves, and our Aſſents, to any Impoſture in the World, and it ſhall be Put to the Account of our Virtues for ſo doing; And Indeed, the Virtue of Faith does not ſeem to Conſiſt in Believing without Reaſon, but in Submitting to the Plain Evidences of Chriſtianity, Notwithſtanding all the Prejudices of the World, or our own Minds, which might Lie againſt it, Notwithſtanding the Virulent Oppoſitions made to it, and the Inveterate Perſecutions Raiſed to Suppreſs it.

That the Evidence of Chriſtianity does not only Depend upon the Re-

§. 26. We muſt likewiſe Obſerve another Way, which has been made Uſe of to Enervate the Forementioned Demonſtration of the Chriſtian Religion, Namely, by ſhewing, that the whole Evidence of it Depends upon the Reſurrection of our Lord, and *ſurrection of our Lord, but upon the Miracles Wrought by him, the Greateſt of which was his Reſurrection.*

and not upon the Truth of his Miracles; But is not the Reſurrection Juſtly to be Eſteemed One of the Greateſt Miracles he Wrought? And is it not One of thoſe Facts, Related in Scripture Concerning him, which how is it Poſſible for us to be Certain of the Truth of, unleſs by a Previous Demonſtration of the Truth and Validity of the Scriptures, which Relate them? We have yet on Occaſions Proved the Certainty of our Lord's Reſurrection, and which we have Shewn to have all the Evidence, which is Poſſible, for a Truth ſo Divine, as well as ſo Important to us.

But Laſtly, the Famous *Huetius*'s *Demonſtratio Evangelica* might, I would Hope, Sufficiently Juſtify me in ſuch a Procedure, and the Rather, when we find Demonſtrations of a God Propoſed to us, and of Natural Religion and Philoſophy, which have not Half ſo many Evidences on their Side, as the Chriſtian Religion has.

CHAP. IV.

Concerning the Being of a God, and the Demonſtration of it, from Des-Cartes's *Connexion of Ideas.*

The Chriſtian Religion gives us other Notions of a God, than Reaſon and Philoſophy does.

§. 1. We Have Said in our Preface to the Demonſtration of the Chriſtian Religion, that the Being of a God is there Affirmed with thoſe Attributes and Perfections of Juſtice, Goodneſs, *&c.* as no Reaſon could ever Furniſh us with the like Apprehenſions of him; For, what Connexion of Ideas, without a Revelation from God himſelf, could Lead us into an Imagination of his Sending his Own Son into the World for our Redemption? That he ſhould Take upon him Human Nature, and Unite it with his God-Head? That he ſhould Paſs thro' a Series of Miſeries and Afflictions, and at Length Die the Death of a Malefactor? That he ſhould Riſe again, and Aſcend into Glory, for the Salvation of Mankind? This gives us ſo High and Elevated a Notion of the Infinite Goodneſs of God to us, and of his Severity againſt the Sins of the World, that if it had not been Revealed to us, it had been Impoſſible for us to have Imagined or Thought of a Diſpenſation ſo Wonderful and Amazing; The Utmoſt Fathom of Human Reaſon could not have Entered the Depths of ſuch a Knowledge, by any Connexion of Ideas of the moſt Penetrating and Sagacious, and yet all This is Strictly Demonſtrable by an Experimental Knowledge, that is, by Plain Evidences of Fact.

Des-Cartes's *Notions of the Being of a God.*

§. 2. On the other Hand, the very Being of a God, when we come to Natural Principles, is very Difficult to be Demonſtrated, and much more, all thoſe Shining and Glorious Attributes, with which we are Sure he is Inveſted from the Truth and Certainty of our Religion. The Firſt Demonſtration of an Omnipotent, All-Wiſe, and Perfect Being, we ſhall Inquire into, is That, which is Derived from the Idea we have of ſuch a Being. In Order to Prove which, It is Suppoſed,

Des-Cartes's Meditat. 3.

Art. 1*ſt.* That all our Thoughts ſhould be Abſtracted from External Objects, and from our Senſes, and That we ſhould Conſider Nothing, but the Properties of our own Minds, as Thoſe of Thinking, Willing, Judging, Remembering, *&c.*

Art. 2*d.* That we are Certain of Nothing, but what we Clearly and Diſtinctly Perceive, and that, what is Impreſſed upon us by our Senſes, has not that Clearneſs and Diſtinction in our Ideas of it, which is Requiſite to ſuch a Certainty; becauſe we Unwarily Join the Idea of Things, Exiſting without us, to our Perception of them, for it is not Evident, that they Really do Exiſt without us, upon the Account only of our Perceiving them.

Art. 3*d.* That we are Certain, that Two and Three make Five, unleſs it is Poſſible for Almighty God to Deceive us in the Plaineſt Matters; and, according to the Prejudice we have in Reſpect of his Omnipotence, we are Apt to Think, it is not Impoſſible, but he may do it, and therefore, that we cannot be Infallibly Certain, that Two and Three do make Five; Yet, when we Recurr to the Clearneſs and Diſtinction of Ideas, we cannot but be Confirmed in this, that, whatever Omnipotence can do to Impoſe upon us, ſuch Propoſitions muſt be True; Eſpecially, ſince we are ſo

Far from Knowing, whether Almighty God will Deceive us, that we are not Sure, whether ſuch a Being in Reality Exiſts.

ART. *4th.* That an Almighty Being is in the Firſt Place to be Proved, and in the Second, that he will not Deceive us, and before this is done, that we cannot be Certain of any Thing.

ART. *5th.* That to Effect the Proof of an Almighty Being, it is Farther Laid Down, that our Thoughts, or Perceptions, ſhould be Divided into their ſeveral Kinds, and an Inquiry made, to which of them Truth or Falſehood belongs.

THE Firſt Sort are thoſe, which, as it is Said, may be Properly Called Ideas, and are only Images of Things, as when we have the Conception of a Man, of Heaven, of an Angel, or of God.

A Second Sort, which are Different from the Other, are thoſe of Willing, Fearing, Affirming, Denying, *&c.* in which Caſes, the Mind has always ſomething, which is the Subject of it's Operations, and does not Terminate it's ſelf in the Bare Perception of the Similitude or Likeneſs of Things with it's own Ideas, but Refers it's ſelf ſtill Farther, and of this Kind of Thoughts, or Perceptions, Some are Termed our Wills or Affections, and Others our Judgments.

DIVIS. *1ſt.* As to our Ideas, for Inſtance, of a Man or a Chimæra, *&c.* it is Impoſſible they ſhould be Falſe, if they are not Referred to any Thing beyond the Perceptions of them in our own Minds.

DIVIS. *2d.* As to our Wills and Affections, there is no Falſehood to be Apprehended from Them, becauſe, altho' we may Wiſh for Things, which either are Ill, or which do not any where Exiſt, yet it is True, that we do Wiſh for them.

DIVIS. *3d.* Our Judgments therefore are Thoſe only, of which we are to Beware, Leſt we ſhould be Led into any Error by them; And our Chief Miſtake in them is, that we too Frequently Conclude our Ideas are Anſwerable and Correſpondent to Things, as they Exiſt without us; For if we only Conſidered them, as Certain Modes or Faſhions of Thinking, without Referring them to any Thing Farther, it would be almoſt Impoſſible for us to be Deceived.

DIVIS. *4th.* Our Ideas may be either Termed Innate, as Thought; or elſe Adventitious, as when we Hear a Sound, or See the Sun, or Feel Fire; or elſe Fictitious, and are Made and Produced by the Mind, as a Syren, or a Centaur; or they may be all Innate, or all Adventitious, or all Made and Formed by our Selves, their Origine or Cauſe being yet Unknown.

DIVIS. *5th.* Concerning Adventitious Ideas, we are Apt to Think them ſuch, becauſe they are Involuntary, and many Times are Impreſſed upon the Mind without it's Conſent or Approbation, as Heat; and we Eaſily are Led to Conclude, that ſuch an Impreſſion from an External Cauſe will rather Repreſent That, from which it Proceeds, than any Thing elſe, which is Different from it; Whereas, *1ſt.* a Natural Propenſity, or Inclination, to Think any Thing is not an Argument, that we are in the Right, on the Contrary, we ſhould be the more Suſpicious, by how much Stronger ſuch an Inclination is, which often Carries us in a Direct Oppoſition to what is True and Rational; *2d.* Altho' theſe Ideas do not Depend on our Wills, it is not from thence Manifeſt, that they Ariſe from an External Cauſe, becauſe, as there may be Natural Impetus's in our Minds, which Run Counter to our Wills, ſo it is not Impoſſible, but there may be a Faculty in them, which we do not Underſtand, which may be the Efficient Cauſe of theſe Adventitious Ideas in us, Eſpecially, ſince in Dreams we have the ſame Repreſentations without the Aſſiſtance of External Objects to Produce them; *3d.* Altho' theſe Adventitious Ideas did Proceed from External Cauſes, it does not Follow, that they ſhould be Conformable to the Things themſelves; Thus, That Idea, which we Receive of the Sun from our Senſes, is Different from That, which Ariſes in our Minds from the Computations of Aſtronomers, by which it is Proved to be ſeveral Times Bigger than the Earth.

DIVIS. *6th.* The Way therefore to Evince, that any Beings Exiſt without us, the Ideas of which we have in our own Minds, is, by Conſidering, that ſo far as thoſe Ideas are only ſo many Modes of Thinking, there is no Difference or Inequality betwixt them, and they all Proceed from our ſelves in the ſame Manner, but ſo far, as One Idea Repreſents One Thing to us, and Another Idea Another Thing, there is an Apparent Inequality or Difference of them; For thoſe Ideas, which Exhibit Subſtances

ſtances to us, are ſomething Greater, and have more of an Objective Reality in them, than thoſe, which only Repreſent Modes or Accidents of Thinking; And again, That Idea, by which we Apprehend ſome Certain Supreme, Eternal, Infinite, Omniſcient and Almighty Being, the Creator of all Things, has more of an Objective Reality in it, than Thoſe, by which Finite Subſtances are Repreſented to us, but it is Evident from Reaſon, that there is at Leaſt as much in the Cauſe, as there is in the Effect, and Conſequently as much of Reality in the One, as in the Other; For upon this Principle it is, that Something cannot be Produced from Nothing, nor That, which is more Perfect, (or which has more of Reality in it), from That, which is Leſs ſo. And this is not only Evidently True Concerning thoſe Effects, whoſe Reality is Actual or Formal, but likewiſe in Reſpect of Ideas, in which an Objective Reality is Barely Conſidered; Thus, for Inſtance, As a Stone, which was not before, could not begin to Exiſt, unleſs it was Produced by Something, in which was Contained, either Formally or Eminently, all that, which is in the Stone, and As Heat cannot be Induced into any Subject, which was not Hot before, unleſs by that, which at Leaſt has as much of Perfection in it's Kind, as Heat has, So likewiſe, I cannot have the Idea of Heat in me, or of a Stone, unleſs it is Impreſſed upon my Mind from ſome Cauſe, in which there is as much of Reality, as I Conceive there is in Heat it's ſelf, or in a Stone; For, altho' that Cauſe does not transfuſe any of it's own Actual or Formal Reality into my Idea, yet there is no Reaſon to Think, that it is therefore Leſs Real, but rather, that the Nature of the Idea it's ſelf is ſuch, as not to Require any other Formal Reality, Excepting what it Receives from my Thought, of which it is a Mode; And, whereas this Idea Contains One Certain Objective Reality, by which it is Diſtinguiſhed from other Ideas, this Ariſes, and is Derived, from ſome Cauſe, in which there is as much Formal Reality at Leaſt, as there is Objective in the Idea, ſince, if there is any Thing in the Idea, which is not in it's Cauſe, it will be Produced from Nothing; But let the Mode be, as Imperfect as it will, by which any Thing is Objectively in the Mind, in the Form of an Idea, it cannot be Nothing, and therefore cannot Proceed from Nothing, nor ought we to Suſpect or Imagine, ſeeing the Reality, which we Conſider in our Ideas, is only Objective, that therefore it is not Neceſſary, that the Same Reality ſhould be Formally and Actually in the Cauſes of thoſe Ideas, and that it is Sufficient, if it is Objectively alſo in ſuch Cauſes; For, as that Objective Mode of Being, or Exiſting, is Suitable to the Nature of our Ideas, and Properly belongs to them, ſo a Formal or Actual Mode of Being, or Exiſting, is Proper and Peculiar to the Cauſes of our Ideas, and the Nature of them, inſomuch, that it is Manifeſt by the Light of Nature, that thoſe Ideas, which are Painted in our Minds, are there, as the Images and Reſemblances of Things, which may fall Short of the Perfection of the Things, which they Repreſent, but do not Contain any Thing Greater, or more Perfect than the Things, which they Repreſent; From whence, if the Objective Reality of any of our Ideas is ſo Great, that we may be Certain it is neither Formally, nor Eminently, in us, and Conſequently, that we our ſelves are not the Cauſes of ſuch an Idea, it Follows, that ſomething Exiſts in the World Diſtinct from us.

Divis. *7th.* Of theſe Ideas, whoſe Cauſes are External to us, there are Five Sorts, either Thoſe, which Repreſent Corporeal and Inanimate Beings, or Thoſe, which Repreſent Animals, or Men, or Angels, or God, and as to That, which Exhibits to us our own Exiſtence, it is Manifeſt and Obvious to every One.

In Relation to the Ideas, we have of Angels, Animals, or other Men, beſides our Selves, Theſe are all Said to be Compounded of Thoſe, which we Form from our own Exiſtence, from That of Corporeal Beings, and That of God, and therefore might Eaſily be Underſtood, or Collected, altho' there were no Men, Animals, or Angels, Exiſting in the World.

Divis. *8th.* As to the Ideas of Corporeal Beings, there is nothing, it is Said, in them, which we may not Eaſily Conceive to Proceed from our Selves, becauſe, altho' Extenſion, Figure, *&c.* cannot be Formally in a Thinking Being, yet as they are only Certain Modes of a Subſtance, and a Thinking Being is a Subſtance, they may be Eminently Contained in it.

Divis. *9th.* Therefore the Idea of God, it is Said, only Remains to be Inquired into, that is, of a Subſtance, which is Infinite, Independent, All-Wiſe, All-Powerful, and from which we our Selves, as well as every Thing elſe, if any Thing elſe Exiſts, is Produced and Derived, which are Attributes or Perfections of that Nature, as not to be Capable of Proceeding from our Minds Alone; For, Notwithſtanding we have the Idea of a Subſtance from our own Exiſtence, yet we have not of an Infinite Subſtance, ſince we are Finite, and which, Conſequently, muſt be Impreſſed upon us from an Infinite One; Nor are we to Think, that we cannot Perceive, what is Really Infinite by an Expreſs Idea of it, and that we can only do it by a Negation of what is Finite, becauſe, on the Contrary, we Plainly Apprehend there is more of Reality in an Infinite Subſtance, than there is in a Finite, and Therefore our Perception of what is Infinite is Previous to our Perception of what is Finite, that is, our Perception of a God is Previous to that of our own Exiſtence; for we could not Know, what it was to Doubt, or Deſire, or that ſomething was Wanting to us, or that, in General, we were Imperfect, unleſs we had an Idea of a Being more Perfect than our Selves, from a Compariſon with which we Found our own Defects and Failings; Nor can it be Said, that Poſſibly this Idea of God is Materially Falſe, and therefore may Proceed from Nothing; Since on the other Hand, as it is the moſt Clear and Diſtinct of any which we have, and Contains more of an Objective Reality, there is no Idea, which has more Truth, and leſs Suſpicions of Falſehood in it; It is True, becauſe, altho' we might be Capable of Thinking or Imagining ſuch a Being did not Exiſt, yet it is Impoſſible for us to Affirm, that ſuch an Idea does not Exhibit ſomething Real to us; It is Clear and Diſtinct, becauſe, whatever we Clearly and Diſtinctly Perceive, which is Real and True, and which Carries any Perfection with it, is all Contained in this Idea; Nor is it any Objection, that we cannot Comprehend, what is Infinite, or that we cannot ſo much as Reach in our Thoughts ſeveral of the Attributes, which belong to an Almighty Being; Since it Appertains to the Eſſence of what is Infinite, that it ſhould not be Comprehended by us, who are Finite, and all which is Requiſite in this Caſe is, that we Underſtand and Judge, that all thoſe Things, which we Clearly and Diſtinctly Perceive, and have any Perfection in them, and, it may be, Innumerable others, of which we are Ignorant, are Formally or Eminently in God; Farther, it cannot be Affirmed, that all theſe Perfections may be in our own Minds Potentially, tho' not Actually and in Effect, becauſe the Idea of God has nothing Potential in it; For, tho' our own Knowledge may Admit of Gradual Increaſes, yet to Suppoſe it in God, is an Argument of Imperfection, and therefore cannot Enter into our Idea of a moſt Perfect Being, whom we Conceive to be Actually Endowed with all Perfections, and to which Nothing can be Added; Beſides which, the Objectiveneſs of an Idea in the Mind, cannot Proceed from what is Potential, which is Nothing, but from what is Real and Actual; Notwithſtanding which, ſince we are Apt to Think, it is not Neceſſary, that the Idea of a Being, more Perfect than our Selves, ſhould Proceed from a Being, which is Really more Perfect, it will be farther Requiſite to Examine, whether we who have ſuch an Idea, could Poſſibly Exiſt, if ſuch a Being did not; If we could, our Exiſtence muſt be either Derived from our Selves, or from our Parents, or from Something elſe, which is Leſs Perfect than God; If we were from our Selves, we ſhould not Experience the Imperfections of Wiſhing, Doubting, or any Other, ſince all the Perfections, which our Ideas Repreſent to us, and are Objectively in the Mind, we Impart to our ſelves, and Conſequently ſhould be Almighty, Omniſcient, Eternal, Infinite, *&c.* nor is it Reaſonable to Think, that, if we were from our Selves, it would be more Difficult for us to Supply, what we Want, than to Give, what we have; On the Contrary, it is Evident, that it is more Difficult to Exiſt from our Selves, or for a Thinking Subſtance to Emerge, and Riſe, out of Nothing, than to Acquire the Knowledge of many Things, of which we are Ignorant, our Informations being only the Accidents of ſuch a Subſtance, and the Same may be Said of the other Perfections of an Infinite Being; Nor can we farther Evade the Force of this Reaſoning, by Suppoſing, that we always Exiſted, what we find our Selves to be at Preſent, that is, that our Minds did always Pre-exiſt, becauſe, tho' we do now Exiſt, there can be no Reaſon Aſſigned, why we ſhould Exiſt the next Minute, which is Conſequent upon a Former, unleſs from a Cauſe, which Creates us every Moment, and therefore the Inquiry will be, whether we are Endued with any ſuch Efficacy or Power, by which we, who now Exiſt, can Produce, or Create, our Selves a Moment from our Preſent Exiſting; For, Seeing we only Conſider our Selves, as Thinking Beings, if we were Poſſeſſed of any ſuch Efficacy, or Power, we ſhould be Conſcious of it, which ſince we are not, it is Manifeſt, that we muſt Proceed from ſome Cauſe, or other, Different from our Selves; Which yet, it may be Said, is leſs Perfect than God; But there being, as much at Leaſt in the Cauſe, as there is in the Effect, and ſince we are Thinking Beings, and have a Certain Idea of God in us, whatever is the Cauſe of our Exiſtence, it muſt at Leaſt be a Thinking Being, and muſt have an Idea of all thoſe Perfections, which we Attribute to God, and it will be then Inquired, whether this Cauſe is Derived from it's Self, or Something Elſe; If from it's Self, it will be God; If from ſomething Different from it's Self, That will alſo either be Derived from it's Self, or from ſomething Elſe, till at Length we Arrive at the Firſt and Original Cauſe, which is God, for there cannot be a Progreſſion to Infinity; Eſpecially, ſince the Cauſe, which Firſt Produced us, is not at Preſent Conſidered, but That, which now Preſerves us in Being, and Produces us every Moment; Nor can it be Imagined, that ſeveral Partial Cauſes may Poſſibly Concurr to the Production of us, as Thinking Beings, and that we Receive the Idea of One of the Perfections, which we Aſcribe to God from One Cauſe, and a Different from a Different, ſo that all thoſe Perfections, ſome-where or other, Exiſt in the Univerſe, but are not Joined and United in God; For, on the Contrary, Unity, an Uncompounded Eſſence, and an Inſeparableneſs of all thoſe Attributes, which we Conceive in God, is One of thoſe Perfections, which we Apprehend, and Underſtand, to belong to him; nor would it be Poſſible for us to Conceive ſuch a Unity and Conjunction of Perfections in God from any Cauſe, which did not Impreſs them Jointly and Inſeparably upon us.

As to what Concerns our Beings, Conſidered as Thinking Ones, Deriving from our Parents, there is no Neceſſity of Confuting ſo Abſurd a Thought or Opinion.

The only Queſtion therefore, which Remains is, which Way, and in what Manner, we Receive this Idea from God; For it is neither Derived from our Senſes, as Material Objects are, nor is it Contrived or Invented by our Selves, ſince we cannot Add to it, or Take from it, and therefore muſt be Innate, and is, as it were, a Character Impreſſed upon our Minds by God himſelf, ſo that, whenever we Reflect upon them, we are at the ſame Time Carried to an Acknowledgement of this Infinite Being, of which there is ſo Plain a Signature in our Own.

Des-Cartes's Meditat. 5.

It is farther Added, in Reſpect of the Exiſtence of a God, that there are Innumerable Ideas in us of Certain Things, which, altho' they do not Exiſt without us, cannot be Accounted nothing, and Notwithſtanding they may be Conceived by the Mind at Pleaſure, yet are not Abſolutely Formed by it, but Aſſert a Nature to themſelves, which is Real and Immutable; Thus in a Triangle, which we Imagine, altho' there never had been ſuch a Figure in Being, Excepting in our own Thoughts, it is yet a Certain Determined Nature and Eſſence, or an Immutable and Eternal Form, which is not Contrived or Feigned by us, nor Depends upon our Minds, ſince there may be ſeveral Properties Demonſtrated Concerning it, Namely, that it's Three Angles are Equal to Two Right Ones, that the Greateſt Side Subtends the Greateſt Angle and the like, which Properties, whether we will or no, we Diſtinctly Perceive, Albeit we did not Apprehend them, when we had only a Bare Notion of a Triangle, and therefore were not Contrived and Forged by our Minds; Beſides which, what we Clearly and Diſtinctly Perceive is Something and not a Mere Non-Exiſtence; for what is Truth is Something, and that, which we Clearly and Diſtinctly Perceive is Truth; Conſequently, if, becauſe we can from our Thoughts Draw out the Idea of any Thing, it Follows, that all thoſe Things, which we Imagine to Appertain to ſuch an Idea, Really do belong to it, we are Farther to Conſider, whether we may not from Hence Derive another Proof of the Being of a God; In Order to which, it is Said, we have as Clear and as Diſtinct an Idea of a Being Infinitely Perfect, as we have of a Triangle, or any other Figure, and we with an Equal Clearneſs and Diſtinction Perceive, that Eternal Exiſtence belongs to ſuch a Being, as we do, that any Property, which we Demonſtrate of a Triangle, belongs to it, for, however it is in other External Beings, the Exiſtence of God is Inſeparable, and cannot be Diſtinguiſhed from his Eſſence, nor do our Thoughts Impoſe a Neceſſity of ſuch an Exiſtence, becauſe we cannot have the Idea of a God without it; But we have a Clear and Diſtinct Idea of him, which we cannot Avoid, and therefore it is True, that God Exiſts.

Our Modern Reaſoners, are only Diſciples of their French Maſter.

§. 3. We have Tranſcribed, as Exactly, as we Poſſibly could, this Demonſtration of *Des Cartes*, becauſe, altho' it is now almoſt every where Exploded, yet it Once had as Great a Reputation for the Juſtneſs of it's Concluſions, as any Thing, which has been ſince Propoſed to us from the Connexion of Ideas; Nor was the Sagacity, or the Penetration, of the Author in the Leaſt Inferior to That of our Preſent Reaſoners, whoſe Accuracy in the Connecting of Ideas, which they ſeem to be ſo Fond of, we ſhall afterwards Examine, when we have Firſt Diſmiſſed their Great Maſter, and Perhaps, in theſe Late Ages, the Chief Patron and Aſſertor of them, in Reſpect of Religion.

Article the Firſt, Section the Second of this Chapter, not Neceſſary to be Granted.

§. 4. To Prepare us for this Demonſtration, it is Preſcribed to us, that we ſhould Abſtract our Selves from every Thing elſe, and Conſider only the Properties of our own Minds; which, Indeed, is a Neceſſary Lemma to a Demonſtration, which Proceeds Wholly upon the Properties of our own Minds, but is not therefore Neceſſary to be Granted; For, if we are not Willing to make ſuch Abſtractions, and if we are as Fully Satiſfied of the Truth and Exiſtence of other Beings, as well as our own, which we have Proved, we may be, in our Principles of Natural Philoſophy, there is no more Reaſon to Confine our Thoughts to Thoſe Perceptions, which we Experience from the Operations within our Selves, and to Reject Thoſe Perceptions, which are Impreſſed upon us from External Objects, than to Conſider any External Operations upon our Minds, and to Neglect Thoſe, which Proceed from our own Reflections.

Article the Second, Section the Second of this Chapter not Proved.

§. 5. After this Requeſt from us, that we ſhall, in Matters of Reaſon, only Think of our Selves, without Regarding any other Part of the Creation, which is rather an Inſtance of our Pride, than of an Humble Inquiry after Truth, we are Told in the Second Place, that we are Certain of Nothing, but what we Clearly and Diſtinctly Perceive, which is True Indeed, if we Abſtract our Selves from every Thing elſe but our own Minds; But, if we have as full an Evidence of Beings Exiſting without us, as that we our Selves Exiſt, which has been Evidenced in our Principles of Philoſophy, the Whole of our Knowledge will not Reſt in the Connexion of Ideas, or in Joining the Abſtracted Perceptions of our own Minds; For Inſtance, there is no Connexion in the Ideas of our own Minds, betwixt the Perception of Fire, and the Fire it's ſelf, which Cauſes that Perception, and which is not an Idea in the Mind, but a Being Diſtinct from it, and yet we are as Certain, that an Igneous Subſtance Exiſts, which Produces the Senſations in us, of Light and Heat, as that we have the Ideas of thoſe Senſations; Becauſe there cannot be any Greater Reaſon given, why we are Sure we Perceive them, than there can be, that they are Perceived; There is no Connexion of Ideas betwixt Twenty, or a Hundred, Perſons Telling me, that there is ſuch a Place as *France* or *Paris*, and my having a Notion, or an Idea, of them in my own Mind, much Leſs than betwixt the Fire and the Perception of it, and yet I can be as Fully Satiſfied, that there is ſuch a Country, and ſuch a City, as of any Connexion of Ideas in a Philoſophick, or even a Mathematick Reaſoning, according to the Compariſons, which we have already made betwixt an Experimental, and a Notional Certainty.

Article the Third, Section the Second of this Chapter not Evinced.

§. 6. Third, what we do Clearly and Diſtinctly Perceive, it is Said, we may be Aſſured of, as that 3 and 2 make 5, and it is Impoſſible for Almighty God himſelf to Deceive us in ſuch Explicit and Diſtinct Perceptions; But how Different the Joining of our Ideas of 2 and 3 together, or even Inferring the Evident Property of any Mathematick Figure, as that the Three Angles of a Triangle are Equal to Two Right Ones, is from Reaſoning in General upon the Ideas of our own Minds, whether Abſtracted or not, is Plain and Manifeſt; ſince, tho' in the Firſt Caſe we ſhould be Incapable of Erring, yet in the Laſt it is too Notorious, that we are often Abuſed and Miſled into very Wrong and Unjuſt Concluſions, both in Mathematicks, and in Philoſophy; Beſides which, 3 and 2 are not Abſolutely Equal to 5, but upon a Suppoſition only, Namely, of Individual Units, for if a Unit may have ſeveral Values, ſo as One Unit ſhould Contain 100 Parts, and another 1000, 3 and 2 will not be Equal to 5, in that Reſpect or Relation, becauſe $\frac{3}{100}$ Added to $\frac{2}{1000}$ will not Pro-

Euclid Prop. 11. 12. Lib. 6. Sturm. Math. Enucl. Prop. 34. Shol. 2. Lib. 1. Dr. Wallis's Arith. Infinit. and Geometr. Solidor.

duce any ſuch Number; and that Unit, even in Mathematicks, and in the Connexion of Ideas, has a Different Value, is Evident from the Method of finding a Third, or Fourth, Proportional in Lines, where Unit is Repreſented by One Aſſumed at Pleaſure; as alſo, from One and the Same Individual Line in the Abſciſſes of Curves Exhibiting a Million of Different Quantities; And the Same may be Said of the Three Angles of a Triangle being Equal to Two Right Ones, which is not Abſolutely True, and only is ſo, under Certain Circumſtances and Conditions, according to what has been Evinced in our *Geometry* of *Solids*, or of *Matter*.

Article the Fourth, Section the Second of this Chapter, not Certain.

§. 7. Fourth, It is Told us, that an Almighty Being is Firſt to be Proved, and then, that he will not Deceive us, and before this, that we cannot be Certain of any Thing; To which we Reply, *1ſt*. That this Aſſertion very Illy Agrees with, as it Perfectly Contradicts, the Former, which Affirmed to us, that we might be Certain, where our Ideas were Clear and Diſtinct, and that it was Impoſſible for Almighty God himſelf to Deceive us in ſuch a Caſe; *2d*. We Anſwer, that if we can be Certain of Nothing, till we are Certain of a Deity, and that he will not Deceive us, our Ignorance is Invincible; Becauſe it will be Impoſſible for us to be Certain of the Being of a God, if the Being of a God is Neceſſary and Previous to any Certainty; It will therefore Perhaps be Inquired, how this Philoſophy ſhould Set Two Propoſitions together, which ſo Evidently Deſtroy each other; The Reaſon of which is Plain, becauſe it Affirms the Idea of God to be Innate, and Impreſſed Originally upon our Minds, and yet that Certainty Depends upon the Clearneſs and Diſtinction of our Ideas, and the Connexion of them; Whereas, if there is an Impreſſion of ſuch an Idea upon our Minds, we muſt be Certain of it, without any Connexion of Clear and Diſtinct Ideas to Prove it; That is, Certainty is Said to Conſiſt only in the Agreement of Ideas, and yet we are Certain of the Being of a God, without any Agreement or Compariſon of Ideas, Namely, from a Bare Impreſſion only.

Article the Fifth, Section the Second, Diviſion the Firſt of this Chapter, not Manifeſt,

§. 8. Fifth, We are next Adviſed, in Order to Prove the Being of a God, to Diſtinguiſh our Perceptions into their ſeveral Claſſes, which are Divided, Firſt into Ideas, that lie in the Mind, as they are Repreſentations of Things, and not Referred; Thus we have the Imagination of a Man, of Heaven, of an Angel, or of God; and theſe Ideas, it is Said, it is Impoſſible ſhould be Falſe, if it is not from thence Concluded, that Things Exiſt; That is, what we Think of, is That Thought and no Other, which is an Axiom; But Notwithſtanding this, There may be a Falſehood in our Thoughts or Conceptions, altho' we do not Refer them to any Exiſtence, Excepting That, which they have in our own Ideas; For, if I Take a Triangle for a Circle or a Square, and Imagine the Space Comprehended by the One to be That, which Appertains to Another, I am Deceived, Albeit I ſhould not Refer theſe Perceptions to any Thing External to me; So likewiſe, if I ſhould, in my Idea of a Chimæra, make the Head of a Lion the ſame with That of the Body of a Goat, or the Tail of a Dragon, my Perceptions would be Falſe, which yet is That, which often Happens in Dreams, wherein, whilſt we Dream of a Man, we Frequently have the Ideas of a Chimæra Enter into his Compoſition, and Conceive him to be made of a quite Different Shape, than what belongs to that Idea; And thus it is in Madneſs or a Phrenzy, that Men Think themſelves to be Wolves, to have Horns, to be Glaſs, Butter, *&c.* according to what we have Mentioned from *Mallebranche* in our Principles of Philoſophy; Which is not by a Reference of the Mind from it's Ideas to Things without us, but by Falſely Apprehending One Idea to be Another, which is Different from it; Nor does this Proceed from any Undue Connexion of Ideas, which would, if ſo, be Reſolved into our Judgments, but Merely from a Falſe Appearance in our Ideas themſelves, by which Means we Actually See and Perceive One Idea Inſtead of Another; From whence it is Conſequent, that we may be Deceived in the Proof of a God, not only in our Connecting our Ideas, but in the Ideas we Connect; So that it can never be Evinced, which this Philoſophy only Endeavours to do, that we have the Idea of an All-Wiſe, Perfect, Omniſcient, and All-Powerful Being, and that ſuch an Idea does not Proceed from our Selves, but from ſome External Cauſe, and that ſuch a Cauſe is no other than God himſelf, if that Idea is a Falſe One, or the ſeveral Parts of it are not True, becauſe all the Arguments, Concerning the Being of a God from the Idea of him, Depend upon this very Suppoſition, that we cannot be Deceived in our Ideas, or in the Bare Repreſentations of our Minds, which it is Evident we may; Thus if our Idea of God does not Expreſs all the Attributes, which belong to him, or Comprehends more than do, if it Exhibits that, as a Perfection in the Deity, which is none, or Unduly Apprehends thoſe Perfections of Omnipotence, Omniſcience, *&c.* which it Aſcribes to him, it will be a Wrong and Miſtaken Idea, tho' it ſhould not be Referred to any Being, which Exiſts without us, and yet all this is Preſumed in the Preſent Demonſtration; Namely, that our Ideas of Infinite, Eternal, Omniſcient, Almighty, *&c.* are Accurate and Juſt, that there are no other Attributes belong to God, and that all theſe do; for if there is any Thing Defective or Redundant in the Repreſentation, whatever is Falſe, or Illy Pictured in the Mind, will be as much an Argument againſt the Exiſtence of ſuch a Being, as what is Poſitive and Determined there, can be an Argument for it.

§. 9. A Second Kind of Ideas are Affirmed to be thoſe of our Affections and Wills, in which, it is Said, there can be no Falſchood, becauſe tho' we may Wiſh for Things, which either are Ill, or do not Exiſt, yet it is True, that we do Wiſh for them, which is as much as to Say, becauſe it is Certain and True, that what we Wiſh for, we do Wiſh for, therefore our Wiſhes never Incline us to Form Wrong and Unjuſtifiable Ideas in our Minds; whereas the Contrary is moſt Evident, that our Inclinations and Wills have as Great an Influence and Sway in Producing very Fantaſtical and Falſe Ideas in us, without Conſidering the Connection of them, as any other Property of our Minds; Thus in the Famous Impoſture of *Mahomet*, or *Mohammed*, his Senſual Paradiſe, his Deſcription of Heaven, of the Angels, and of God, his Changing Virtue into Vice, and Vice into Virtue, (all which was Calculated for his Darling Paſſions of Luſt and Ambition) are Sufficient Inſtances, what Strange and Chimerical Ideas a Man may Paint upon his Mind, who Reſigns himſelf to the Conduct of his own Brutal Appetites and Deſires; No one I Believe will Think theſe Errors of his were Owing to his Want of Judgment, or the Connexion of Ideas, but to his Will, to his own Pleaſure and Caprice, which was the Author to him of theſe Fancies in Plain Oppoſition, to his Certain Knowledge and Conviction, ſince he muſt be Conſcious to himſelf, that all he Propoſed of this Nature was an Impoſture, tho' it is very Probable, there was a Mixture of Enthuſiaſm in it.

Article the Fifth, Section the Second, Diviſion the Second of this Chapter, not Evident.

Dr. Prideaux's Life of Mahomet.
Mr. Ockley's Hiſtory of the Saracens.

§. 10. After this it is Told us, that our Judgments are only Capable of Deceiving us, and that the Chief Reaſon of our Errors in them is, our Referring thoſe Ideas, we have in our Minds, to Things, as Exiſting without us, when there is no Connexion betwixt our Ideas, and any Exiſtence Foreign to them; in which Aſſertion, there are Three Things, which are Falſe; 1*ſt*. That our Judgments are only Capable of Deceiving us, Seeing it is already Proved, our Ideas themſelves, and our Inclinations and Paſſions, may do it; 2*d*. That, in our Judgments, the Principal Occaſion of our Errors is our Attributing the Ideas, we have in our Minds, to ſome External Cauſe of thoſe Ideas, whereas we have Evidenced before in our Compariſons of an Expemental and Notional Knowledge, that in Abſtracting our Ideas from any Thing External to them, the Connexion of them may be Fallible in ſeveral Reſpects; 3*d*. It is Said, that there is no Connexion betwixt our Ideas, and that which is Foreign to them, if by that is Meant, that there is not the Same Connexion betwixt the Ideas of our Mind, as thoſe of 4 and 4, or the Three Angles of a Triangle being Equal to Two Right Ones, as there is betwixt my Perception of Light or Heat in the Fire, and that which Cauſes thoſe Perceptions in me, it is True, but at the ſame Time it is no more than to Say, that the Connexion betwixt any Two Abſtracted Perceptions is not the Same, as the Connexion betwixt a Perception in the Mind, and that which Produces it, but it is no Argument or Proof, that becauſe there is a Connexion in the Ideas of our Minds, betwixt 4 and 4, *&c.* that therefore there is no Connexion or Dependence betwixt my Idea, or Perception, of Heat, or of Light, and that, which Produces it; to Wit, we cannot Infer, that the Cauſe of Heat or Light may not Exiſt Diſtinct from our Perceptions of them, and that there is not a Connexion or Dependence betwixt them, becauſe the Cauſe, which Produces them is not an Abſtracted Idea in the Mind, unleſs it is Firſt Proved, that there is no Knowledge, but in ſuch Abſtractions, the Contrary of which is Evidenced in our Principles of Philoſophy; or, which is the Same, we cannot Conclude, that our Perceptions of Light or Heat are not Owing to Something External to us, *viz.* becauſe we are Sure, that 4 is Equal to 4, or that the Three Angles of a Triangle are Equal to Two Right Ones.

Article the Fifth, Section the Second, Diviſion the Third of this Chapter, not True.

From what goes before, it Appears how Little this Philoſophy is Acquainted with the Real Evidences of Things.

§. 11. From Hence therefore it is Plain, how Little this Philoſophy Underſtands what belongs to the Real Evidences of Things, which makes all Certainty to Conſiſt in the Clearneſs and Diſtinction of Ideas, and the Connexion of them, that is, in the Abſtracted Ideas of our own Minds; for there is no Proper Connexion, Excepting betwixt ſuch Ideas, there being None, as it is Affirmed, betwixt our Ideas, and the Exiſtence of the Things, which they Repreſent; by which Means, all Experimental Certainty, all Hiſtorical Certainty, all Certainty from Teſtimony, and, Indeed, all the Maxims of Prudence, by which we Conduct the ſeveral Actions of our Lives, are Diſcarded, and there is nothing Certain in the whole Maſs of Human Affairs, bateing, what we Find and Experience in our own Abſtracted Notions; The Contrary of which has been Fully Evinced. But, However Falſe ſuch an Opinion may be, this Philoſophy is in ſome Meaſure Conſiſtent with it's ſelf, for whilſt it Derives all Certainty from a Clearneſs and Diſtinction of Ideas, and the Viſible Connexion of ſuch Ideas, it Wiſely Rejects External Beings, betwixt which, and our Perceptions of them, there is no Connexion from that Certainty, and Endeavours to Prove the Exiſtence of a God from our Idea of him, which is Impreſſed upon us; On the other Hand, our Preſent Celebrated Philoſopher of Ideas has taken Care to Tranſcribe into his Syſteme this Doctrine, and to Vend it for his Own, without Underſtanding the Scope and Intention of it, whilſt he Argues, for the Certainty of External Beings, betwixt which, and our Abſtracted Ideas, there is no Connexion of Abſtracted Ideas, and for the Being of a God from the Nature of Motion, when we are not Certain of Motion it's ſelf, if Certainty Depends Solely upon the Abſtracted Ideas of our own Minds, and the Agreement or Diſagreement of them; and, which is ſtill more Unfortunate to this *Gentleman*, he begins his Philoſophy with a Confutation of any Innate or Impreſſed Ideas of God, which *Des-Cartes* well Knew, or any One elſe might Know, was Neceſſary to the Demonſtration of One, if Certainty was Confin'd to our Ideas, Independent and Diſtinct from all Foreign Conſiderations, and yet Profeſſes to Demonſtrate the Exiſtence of a God from Things, which are Foreign to our Ideas as they Lie, without any Reference, in the Mind, Notwithſtanding he has Adopted that Notion of Certainty into his Writings, which Excludes every Thing from being Certain, which is not Reſolved into thoſe Ideas, and the Plain Connexion of them; So Unhappy always is the Caſe of ſuch, who Borrow their Notions from Others, and Live *Lazily* upon the *Scraps* of *Begged Opinions*, the Riſe, and Occaſion, and Deſign of Propoſing which, they have not been Poſſeſſed of Sagacity, or Diligence, enough to Examine.

Mr. Locke's Eſſay, Epiſt. to the Reader.

Article the Fifth, Section the Second, Diviſion the Fourth of this Chapter not Plain.

§. 12. From hence we are Preſcribed to Divide our Ideas into Innate, as Thought, Adventitious, as Sound, or Heat, *&c.* or Fictitious, as a Syren, or a Centaur; whereas this Deſcription of our Ideas into their ſeveral Claſſes is not Juſt; For Thought cannot be Proved to be Innate, unleſs it is Firſt Shewn, that our Reflections, or the Bare Operations of our Minds, are not Conſequent upon our Senſations from External Objects, and therefore not Derived from them; On the Contrary, if Thought and our Perceptions are Innate, it cannot be Explained, how any Perceptions are Adventitious, Eſpecially, if we Acknowledge, that we are Certain of Nothing, but the Plain Ideas of our own Minds; For if ſo, we have no more Reaſon to Call our Ideas of Heat, or Sound, Adventitious, than our Ideas of Memory or Perception, that is, if our Reaſon or Knowledge can Proceed no farther than the Repreſentations made to us by our Ideas themſelves; Laſtly, as to what Reſpects Fictitious Ideas, ſince they are Reſolved into Thought, without any Reference to External Beings, it is not Determined in this Philoſophy, why they may not be Innate too, ſince Thought is Said to be ſo, or if they they are not, and thoſe Compoſitions Depend upon our Wills, it is not Certain, but they may be as True Ideas, as thoſe of God, or an Angel, according to the Preſent Reaſoning; ſince they are not Referred to any Thing without us, and it is Suppoſed, there can be no Error in our Affections, or our Wills, from §. 2. Art. 5. Divis. 2. of this Chapter.

Article the Fifth, Section the Second, Diviſion the Fifth of this Chapter not Demonſtrated.

§. 13. However this is, we ſhall now Particularly Conſider the Nature of thoſe Ideas, which are Called Adventitious, which is Said to Conſiſt in their being Involuntarily Impreſſed upon the Mind, by which they are Diſtinguiſhed from all Others; if therefore they Ariſe from Something Different from our Selves, we are Apt, it is Said, to Think, that they Bear a Reſemblance rather of that, which is the Cauſe of them,

them than of any Thing elſe; whereas, we are Told, as to the Firſt, that Involuntary Ideas may Start up in our Minds, without an External Efficient to Produce them, for ſuch we Actually Experience in Dreams; and as to the Second, that it is not Neceſſary, altho' they ſhould be Impreſſed from without, that they ſhould be the Pictures or Images of the Things, from which they are Taken; thus the Sun, of which we have a Conception, is not the Same in it's Magnitude with that which Really Exiſts, according to the Computations of Aſtronomers.

But, 1*ſt.* if Involuntary Ideas may Proceed from the Mind, ſince the Idea of God is Affirmed to be Innate, that is, Originally Impreſſed upon it, and therefore Involuntary, it is not Certain, but the Same Efficient Faculty in the Mind, which is Capable of Cauſing One Involuntary Appearance in it of Objects, which are Suppoſed to be External, may Occaſion another, as of God; For, both being Explicit Ideas in the Mind, and Repreſenting, as it were, Certain Signatures and Characters Engraven upon it, in the Senſe of this Philoſophy, the Difference of Finite and Infinite is of no Conſideration; ſince, if they are both Ideas, which in Fact Exiſt in, and are Produced upon the Mind, and are alike Involuntary, there can be no Reaſon Aſſigned, why the Suppoſed Efficient Faculty of it may not Introduce into it's ſelf the Idea of the One, as well as of the Other, that is, the Idea of God, as well as that of Corporeal Beings; Beſides which, there are ſome Involuntary Ideas, as Particularly thoſe, we Receive from Objects, which are Suppoſed External, that cannot be Derived into the Mind from it's ſelf, by what we have already Evidenced in the Principles of Philoſophy. Chap. 2. Part. 1.

As to what Concerns the 2*d.* Aſſertion, that, altho' theſe Involuntary Ideas may Proceed from External Impreſſions, it is not yet Neceſſary, that the Effects in the Mind ſhould be a Bare Tranſcript upon it, of the Cauſe of them; It is Owned; But it cannot be Concluded from Thence, which is done in this Philoſophy, that therefore, there is no Connexion betwixt the Cauſe and Effect; for, tho' Heat in One Body Produces Heat in Another, that is, Heat Original, or Transferred, Produces the Same Kind of Perception in the Mind, yet it is not Conſequent, that Heat in Body ſhould be the Perception of it in our Minds, no more, than that Thought ſhould be Matter, and therefore the One will not be a Tranſcript of the Other; and yet it cannot be Inferred, that our Perception of Heat is not Derived from Something, which Cauſes that Perception, Albeit that, which Cauſes it, is not that Perception.

Moreover, If it is Acknowledged, that Involuntary Ideas may Proceed from External Impreſſions, be they Anſwerable to thoſe Impreſſions or not, in Reſpect of the Idea Impreſſed, and the Cauſe which Impreſſes it, the Argument Uſed for the Setting Aſide all Ideas, Except the Abſtracted Ones of our own Minds, is Invalid; Becauſe, in the Preſent, it is Suppoſed, that there are External Objects, tho' they are not Conformable to our Ideas of them; and even, as to what Reſpects the Inſtance of that Idea, which we have of the Sun in our Minds, and that, which we have from the Computations of Aſtronomers, I would Ask, is it to the Purpoſe of Proving, that we are not Certain of External Objects? or, that External Objects are not Anſwerable to our Ideas of them? If the Firſt, it is Evidenced to the Contrary in our Principles of Philoſophy; If the Second, Of what Advantage is it to the Aſſertion of all Ideas, and the Cauſes of them, being in our Minds, when it is a Plain Confeſſion, that the Cauſes of them are Without, and External to us?

Article the Fifth, Section the Second, Diviſion the Sixth of this Chapter, not Clear.

§. 14. Notwithstanding what we have Objected againſt that, which this Philoſophy Propoſes Concerning Adventitious Ideas, we ſhall now Relinquiſh it, and Conſider, whether the Proof of External Beings is Juſt, which it Alledges from our Ideas themſelves; In which, there is ſuch a Jargon of True Nonſenſe, and ſo Unſuitable to the Celebrated Wit, and Genius of *Des-Cartes*, that I cannot Attribute it to any Thing elſe, Excepting, that, as he had made the Exiſtence of External Beings Uncertain, ſo he was Willing to make the Exiſtence of a God ſo too; that is, This Wiſe Philoſopher had Diſcarded the Chriſtian Religion, by Believing Nothing, but what the Abſtracted Ideas of his own Mind Preſented to him, but then he was Obliged to Aſſert ſome Religion or other, that the World might not Conſider him in the Quality of an Atheiſt; he therefore Thought it was Neceſſary to Prove the Being of a God, which yet was done in ſo Unaccountable a Manner, that, if Men were Satisfied to Leave their own Religion, it was at their Pleaſure, whether they would be of his or of any, that is, as he was not Much Concerned about Chriſtianity, ſo was he as Little about the Being of a God it's ſelf, any farther, than as it might Serve him in his Romance of Philoſophy, and to Divide his Similar Matter, into what Elements he Pleaſed.

This Farther Proved.

§. 15. WHAT we have Said in the Foregoing Section, we ſhall Prove in This, and the Following Ones, whilſt we Shew, that *Des-Cartes*'s Notions are the moſt Abſurd in this Particular, that can Poſſibly be Imagined, and which he himſelf could not Think were True, tho' he Hoped at Leaſt, they might be well enough Digeſted by thoſe, who were Apt to Believe the Chriſtian Religion was a Slavery upon Men's Minds, and therefore Willing to Entertain the Firſt, which ſhould be Propoſed in Oppoſition to it.

§. 2. Art. 5. Divis. 6. of this Chapter.

IT is Said therefore, that the Way to Evince, that any Beings Exiſt without us, the Ideas of which we have in our own Minds, is, by Conſidering, that ſo far, as thoſe Ideas are only ſo many Modes of Thinking, there is no Difference or Inequality betwixt them, and they all Proceed from our Selves in the Same Manner, but ſo far, as One Idea Repreſents One Thing to us, and Another Idea Another Thing, there is an Apparent Inequality or Difference of them; For thoſe Ideas, which Exhibit Subſtances to us, are ſomething Greater, and have More of an Objective Reality in them, than thoſe, which only Repreſent Modes or Accidents of Thinking; and again, That Idea, by which we Apprehend ſome Certain Supreme, Eternal, Infinite, Omniſcient, and Almighty Being, the Creator of all Things, has more of an Objective Reality in it, than Thoſe, by which Finite Subſtances are Repreſented.

THIS is laid Down, as a Fundamental Axiom, for the Proof of External Beings, whereas, there are ſeveral Exceptions to be made againſt it.

FIRST, that Thought, in General, is Suppoſed to be an Abſtracted Idea, that is, that whether we Think of a Triangle, or a Square, or of the Sun, it is ſtill Thought, or Perception; When yet we do not Perceive, or Think, of any of theſe without their Particular Diſtinctions; That is, Thought or Perception, in General, is nothing but an Abſtraction of the Mind, which is not Owing to our Experience, but our Wills; ſince no One can Tell, what Thought Abſtractedly is, without Conſidering the Thing, which we Think, or have the Idea, of.

SECOND, Suppoſing this Abſtracted Notion of Thought, without Conſidering the Particular Thing, which we Think of; In what, I would Ask, does the Inequality of our Thoughts or Ideas Conſiſt, Excepting in Thought or Idea? If ſo, there is no more Reaſon, why the Difference of Thought ſhould not Proceed from our Selves, than the Identity, or the Sameneſs, of it ſhould; and therefore, the Difference, or Inequality, of our Perceptions cannot be an Argument for External Beings, ſince thoſe Different Perceptions may, as well Proceed from the Mind, as Thought or Perception may.

THIRD, Thought, in General, where we Exclude any Particular Ideas, being an Abſtraction of the Mind, is an Idea, which Depends upon the Will and Pleaſure of it; any Particular Idea, which we have, on the Contrary, as of a Man, or an Angel, or of Doubting, Willing, *&c.* is not made by ſuch an Abſtraction, but is the Immediate Senſe, which the Mind has of thoſe Ideas; from whence it is Certain, that, becauſe the Mind is Pleaſed to make Abſtractions of it's Own, it is no Inference, that, where it does not make thoſe Abſtractions in it's Ideas, they may not Equally Proceed from it; Unleſs the Will is a Standard for our Minds, and thoſe Perceptions only, which are Derived from it, ſhall be Received as the Genuine Product of them, which is as Abſurd, as Arbitrary, to Suppoſe.

FOURTH, as to the Inſtance Alledged of Subſtances, having a more Objective Reality in them, than Modes or Accidents, it muſt be Inquired, what is Meant by thoſe Subſtances and Accidents; If by that is Intended the Subſtances and Accidents of Matter, which is External to us, it is a Confeſſion, that there is Something External to us, in the Argument, which is brought to Prove, that we are Certain of Nothing, but what we Perceive in our own Ideas; If, by Subſtances and Accidents, is Meant thoſe of our Thoughts; What is the Subſtance of Thought, or the Accidents of it, unleſs what we have before Mentioned, that is, Thought in General, or the Abſtracted Notion of it, and the Particular Diſtinctions of that Abſtracted Thought into our Real Perceptions? and if ſo, we have no Reaſon to Think, there is more of an Objective Reality in the Subſtance of Thought, than in the Accidents of it, but the Contrary, by what Foregoes; becauſe the Subſtance of Thought is an Abſtraction of the

Mind at Pleaſure, and therefore has no Objective Reality in it, whereas the Accidents of it are Real and Actual Perceptions.

FIFTH, what is Meant by a Greater or Leſs Objective Reality is Unintelligible; for that is Said to be an Objective Reality, where there is a Repreſentation made in the Mind of an Object, which is Really Exiſting, either in our Minds, or out of it, and which is the Cauſe of Impreſſing that Repreſentation; But, if there is any Idea in our Minds, it has a Real Cauſe of it's Exiſtence by Impreſſing it upon us, or not; If it has not, it is a Compoſition of the Will, and a Chimæra, and has no Objective Reality in the Mind; If it has, that Cauſe cannot be more or Leſs Real, and therefore not the Objective Reality of it in the Mind.

SIXTH, This very Doctrine of Things, being Objectively in the Mind from a Real Cauſe, Proceeds upon a Suppoſition, that all Ideas, which are Involuntary, are Impreſſed upon it, as the Idea of God, and of External Beings, Notwithſtanding this Philoſophy had before Affirmed, that it was not Certain, but Involuntary Ideas might Riſe up in the Mind by an Innate Faculty of it's own, without any External Cauſe to Produce them; for, if the Involuntary Ideas of God, and of External Beings, are not Suppoſed to be Impreſſed upon the Mind, they may Start up in it by a Peculiar Power, which belongs to it, and therefore are not Objectively in the Mind, by any Real Cauſe, which Produces them, External to it.

SEVENTH, An Objective Reality in the Mind cannot be Proved, but by Suppoſing, that Something Exiſts External to us, as the Cauſe of that Objectiveneſs in the Mind, which is an Expreſſion of that, which is External to it; For otherwiſe, what is an Objective Reality more than any Common Appearance in the Mind? If therefore we Argue for the Reality of External Beings from an Objective Reality of thoſe Beings in the Mind, we Argue for the Exiſtence of External Beings from a Suppoſition and Preſumption of their Exiſtence; If we Conſider an Objective Reality in the Mind, only as Diſtinct Perceptions in it, and as they are Differenced from Thought in General, it is Confeſſed by this Philoſophy, that any Repreſentations, which are made in the Mind, have no Connexion with the Objects, they are Imagined to Repreſent, and therefore are no ways Concluſive of them.

EIGHTH, As to the Idea of an Infinite Being, having more of an Objective Reality in it, than that of a Finite, ſuch a Notion is already Confuted, or if it were not, as there is no Reaſon Aſſigned, why it ſhould have, there would not be a Neceſſity of Alledging One, why it ſhould not; However we ſhall Proceed even ſo far, and Shew, that there is no more Objective Reality in an Infinite Being, than in a Finite; Becauſe it is Evident, the Idea of God is not Proved to be an Innate Character Engraven upon the Mind, and therefore is not Derived to us from One Uniform Signature or Impreſſion upon it, and yet the Ideas of Finite and External Objects are Manifeſtly ſo; From whence it will Follow, that there is more Objective Reality in Finite Subſtances, than in an Infinite One, ſince there is an Objective Reality in the One, and not in the Other, Contrary to this Philoſophy.

NINTH, We ſhall Moreover take Notice, that in this Doctrine of an Objective Reality, the Impreſſions, which are made upon the Mind, are Suppoſed to Differ in nothing from the Things, which are Impreſſed, Excepting, that the One are in an Objective Repreſentation to it, whereas the Other are in Form and Exiſtence; Thus my Idea of a Stone has all that Objectively in the Mind, which is Eminently or Formally, or, which is the Same, which is Truely and Really Contained in the Stone; Namely, making an Allowance for the Different Natures of our Minds, and of External Objects, there is all that in the Mind Agreeable to it's own Nature, which is in External Objects Agreeable to theirs.

IN Anſwer to which, we ſhall 1*ſt*. Say, that this Philoſophy Endeavours to Shew, there is no Kind of Connexion betwixt our Perception of Heat, Colour, Sound, *&c.* and the Cauſes of them, no more, than betwixt the Point of a Needle, and the Pain it Produces; in all which Caſes, it Suppoſes, that the Objection, or Repreſentation, in the Mind is no ways Anſwerable, or Analogous, to the Cauſe, which Impreſſes it, as the Point of the Needle is not to the Pain, which it Produces, and yet, according to the Preſent Doctrine of an Objective Reality, the Point in the Needle, ſhould not Cauſe a Pain, but ſhould Object, or Repreſent, a Point to the Mind, as a Stone is only Suppoſed to Impreſs it's Signature, or Character, upon it.

2*d.* To Suppoſe or Imagine, that an External Being ſhould Transfuſe it's ſelf, as it were, into the Mind, ſo far as it is Capable by it's Nature of Receiving it, is at the Beſt a very Difficult Hypotheſis to Apprehend; Eſpecially, before we are Acquainted either with the Nature of External Beings, or That of our own Minds, and the Propriety of thoſe Actions, which belong to each, which this Philoſophy is ſo Wiſe, as to Conceal, or ſo Weak, as not to Underſtand.

3*d.* It is farther the moſt Unaccountable Thing, that is Poſſible, to Introduce Examples and Reaſonings from External Beings, in an Argument, which is brought to Prove, there are ſuch Beings, and before which Proof it is Suppoſed to be Uncertain, whether there are any, or not.

Unneceſſary to Purſue this Doctrine any Farther.

§. 16. Having thus far Conſidered the Import and Real Senſe of this Doctrine, as it is Propoſed in the Beginning of the Foregoing Section, it will be Unneceſſary to Purſue the Reaſoning from it in the Remaining Part of Divis. 6. Art. 5. §. 2. and ſince, upon the Arguments Alledged in that Diviſion, the *7th. 8th.* and *9th.* Diviſions, the Laſt of which Particularly Concerns the Proof of a God, do Entirely Depend, that is, upon an Objective Reality, it will be likewiſe Superfluous to Conſider the ſeveral Turns, if I may ſo Call it, and Doubles, made in them; However, becauſe at the Latter End of the *9th.* Diviſion, there is a New Argument Propoſed from *Des-Cartes*'s Fifth Meditation, we ſhall now Inquire into that.

Article the Fifth, Section the Second, Diviſion the Ninth, where Des-Cartes's *Meditation the Fifth is Cited, not Convincing.*

§. 17. The Argument is, that there are ſeveral Ideas in our Minds of Things, which do not Exiſt without us, and yet thoſe Ideas are not wholly Chimerical, but have Something of Reality in them Independent of the Mind; Thus the Triangle and it's Properties are Something Real, Notwithſtanding no ſuch Figure ſhould Exiſt in Nature, and for the ſame Reaſon, the Idea of God is not an Arbitrary Compoſition of the Mind, becauſe it is Said, we Certainly have ſuch an Idea, One of the Properties of which is, that he muſt Neceſſarily Exiſt.

In Anſwer to this, we muſt Firſt Say, that the Idea of a Triangle, and the Properties of it, is a Creature of the Mind, according to §. 15. 16. 17. 18. Chap. 2. becauſe a Triangle may have other Properties than thoſe, which are Aſſigned to it, ſince the Three Angles of a Triangle may be Equal to 4, or 5, or a 100, Right Angles, as well as to 2, if the Suppoſitions of an Abſtracted Extenſion are Different.

Second, Allowing this, Namely, that the Three Angles of a Triangle are Equal to Two Right Ones, and that it is a Property which is Invariable, and belongs to it, it is not from thence Neceſſary, that it ſhould not be an Abſtraction of the Mind, and Conſequently a Creature of it in that Reſpect; for, if there is no Triangle in the World, it only Exiſts in the Mind, as an Idea of it's Own, and the Truths, which Reſult from it, are only ſuch, which are the Conſequences of it's being ſuch an Idea, and no Other, for Inſtance, That of a Square or a Circle; for, whilſt it is Conceived to be a Figure, which is Terminated by Three Lines, all the Properties of it are Included either Apparently, or by Conſequence, in it; Therefore ſuch Immutable Relations are no more than to Affirm, that our Idea of a Triangle is that Individual Idea, and is neither the Idea of a Square, nor of a Circle, for That will be Equally a Property of it with the Reſt; and ſuch Immutable Truths are likewiſe to be found in our Ideas of a Chimera and a Centaur; ſince, after the Mind has Fixed each of thoſe Ideas, it will be as Certain and as True, that the One is not the Other, as that a Triangle is not a Square, and there may be ſeveral Properties Affirmed and Proved of Both of them, Diſtinct from each Other.

Third, The Application of this Doctrine to the Proving the Being of a God is ſtill more Exceptionable; For it does not Follow, becauſe the Idea, or Conception, of a Triangle is it's ſelf, which is all, as we have Proved, which can be Meant by the Immutable Truths, it Comprehends, and becauſe the Idea of God is it's ſelf, that therefore God Exiſts; which yet is all, which can be Concluded from this Analogy, or Compariſon, betwixt the Idea, which we have, of the One or the Other.

Fourth, Altho' the Exiſtence of a God ſhould be Included in the Idea of him, as Three Angles are in our Idea of a Triangle, yet ſince in the laſt Caſe, ſuch a Property is only Immutable and Certain, as it Reſpects that Idea, without Suppoſing any Real Exiſtence of a Triangle in Nature, ſo, tho' Exiſtence ſhould be Contained in the Idea of God, as it Lies Explicitely in our Minds, it would be only a Property of that Idea, and could not, by being ſo, Infer the Neceſſary Exiſtence of that Being, of which it is an Idea.

Fifth, The Original Suppoſition, that there is ſuch a Character, or Idea, Impreſſed upon the Mind, that it is an Innate Signature, Engraven, as it were, upon it, is, and Perhaps may be Juſtly, Diſputed; and yet all this Reaſoning Depends upon that Suppoſition, and the Principal Endeavour in this Philoſophy is to Explain, which Way we come by ſuch an Idea.

Laſtly, If this likewiſe was Granted, that the Idea of an Infinite Being Lay, as an Expreſs Character, or Signature, in the Mind, the Way to Prove it Imprinted upon us from an External Exiſtence, could not be by Comparing it with Ideas, which are in the Mind, as that of a Triangle, the Origine of which Ideas are Acknowledged not to be Derived from any External Exiſtence, but may be there, if there was no ſuch Thing, as a Triangle in the World; ſince, in ſuch a Caſe, the only Method, which could be Rationally Taken, muſt be by Conſidering the Nature of the Mind, which Receives the Impreſſion, and That of the External Agent, which is Suppoſed to Impreſs it.

Concluſion upon this Chapter.

§. 18. **Having** gone thro' the ſeveral Parts of this Demonſtration of a God from the Connexion of Ideas, and Shewn, how Defective it is in every Branch of it, it might be Unneceſſary to Proceed any Farther in the Confutation of thoſe other Philoſophers, who have Succeeded *Des-Cartes* in the Same Doctrine, but have Diſallowed his Demonſtration of a God by it; ſince, if their Great Maſter and Leader, and who has been Thought to Poſſeſs a Genius Superior to That of any, who Followed him, is ſo Evidently and Apparently Miſtaken, it is not to be Imagined, that they ſhould be more Fortunate in their Connexion of Ideas, than One, who Taught them the Connexion it's ſelf; However, we ſhall, in the Following Chapter, Examine into the Demonſtration given us by the next Great Patron of the Connexion of Ideas, Mr. *Locke*, and Evince, how Imperfect it is, in Order to Produce a Juſt Conviction in the Mind of a Truth ſo Valuable to us.

CHAP. V.

Concerning Mr. Locke's *Demonſtration of the Being of a God.*

Concerning **Mr. Locke's** *Notion of a God.*

Mr. Locke's Eſſay, Chap. 10. Book 4.

§. 1. **In** This Demonſtration of the Being of a God it is Affirmed, that there are no Innate Characters Imprinted upon the Mind of him, but, that his Being may yet be Collected, by a Due Attention and Application of our Faculties, and from ſome Part, or other, of our Intuitive Knowledge, ſo that we may be as Certain of it, as of any Thing in the Mathematicks; if not, it is Said, we ſhall be as Uncertain and Ignorant of this, as of other Propoſitions, which are in themſelves Capable of Clear Demonſtration.

Art. 1. Firſt, the Way therefore to Shew, there is a God, is, by *Affirming*, that it is Certain we Exiſt, which is Proved by *Saying*, that it is beyond all Queſtion, that One would Argue no more with a Man, who Denies it, than One would with Nothing, or, than One would Endeavour to Convince Non-Entity was Something; For the Pains, or the Pleaſures, we Feel, are a Sufficient Conviction of it.

Art. 2. Second, as we Know by the Foregoing *Argument*, it is Said, that we our Selves Exiſt, ſo it is as Manifeſt by our Intuitive Certainty, that Bare Nothing can no more Produce any Real Being, than it can be Equal to Two Right Angles; for, if a Man Knows not, that Non-Entity, or the Abſence of all Being, cannot be Equal to Two Right Angles, it is Impoſſible he ſhould Know any Demonſtration in *Euclid*; If therefore we Know there is ſome Real Being, and that Non-Entity cannot Produce any Real Being, it is an Evident Demonſtration, that from Eternity there has been Something; ſince, what was not from Eternity had a Beginning, and what had a Beginning muſt be Produced by Something elſe.

Art. 3. Third, it is Evident, that, what had it's Being and Beginning from Another, muſt alſo have all that, which is in, and belongs to it's Being, from Another too; all the Powers, it has, muſt be Owing to, and Received from, the Same Source; this Eternal Source then of all Being muſt alſo be the Source and Original of all Power, and ſo this Eternal Being muſt be alſo the moſt Powerfull.

ART. 4. Fourth, we find in our Selves Perception and Knowledge, from whence we are Certain, that this Being is ſome Knowing and Intelligent Being, or other, in the World.

WE muſt alſo Say, that there was a Time, when there was no Knowing Being, and when Knowledge began to be, or elſe, there has been a Knowing Being from all Eternity; If the Firſt, it is Impoſſible, there ever ſhould have been any Knowledge, it being as Impoſſible, that Things wholly Void of Knowledge, and without any Perception, ſhould Produce a Knowing Being, as it is Impoſſible, that a Triangle ſhould make it's Three Angles Bigger than Two Right Ones; For it is as Repugnant to the Idea of Senſeleſs Matter, that it ſhould Put into it's ſelf Senſe, Perception, and Knowledge, as it is Repugnant to the Idea of a Triangle, that it ſhould Put into it's ſelf Greater Angles than Two Right Ones.

ART. 5. And from hence it is Concluded, that there is an Eternal, moſt Powerful, and moſt Knowing Being, Which, whether any One will Pleaſe to Call God, it Matters not, ſince the Thing is Evident; and from this Idea will Eaſily be Deduced all thoſe other Attributes we ought to Aſcribe to this Eternal Being.

IF, Nevertheleſs, any One ſhould be found ſo Senſeleſly Arrogant, as to Suppoſe, Man alone Knowing and Wiſe, but yet the Product of Mere Ignorance and Chance, and the Reſt of the Univerſe Acted only by that Blind Hap-Hazard, this Philoſopher will Leave with him that very Rational and Emphatical Rebuke of *Tully*, Lib. 2. *de Legibus*, to be Conſidered at his Leiſure. *Quid eſt enim verius, quàm neminem eſſe oportere tam ſtultè arrogantem, ut in ſe mentem & rationem putet ineſſe, in cælo mundoque non putet? Aut ea, quæ vix ſummâ Ingenii ratione comprehendat, nullâ ratione moveri putet?*

This Notion Inquired into.

§. 2. THIS is the Famous Demonſtration of the Being of a God, which is Thought by the Rationaliſts, or Free-Thinkers, of the Age, Sufficient, not only to Convince us of the Truth of his Exiſtence, but likewiſe to make us ſet Aſide all the Evidences of a Revelation of him, from the Wonderful and Admirable Conviction it Contains.

§. 1. Art. 1.

IT will be therefore Neceſſary to Inquire into the ſeveral Parts of it; and Firſt it is Told us, that, tho' the Idea of a God is not an Innate Character, Imprinted upon our Minds, yet his Being may be Collected by a Due Application and Attention to the Arguments for it; Which is a Confeſſion from this Philoſopher of Ideas, that his Maſter *Des-Cartes* was in the Wrong in his Connexions, who Affirmed ſuch an Innate Character; we ſhall therefore now farther Inquire, whether this Perſon's are more Juſt and Concluſive.

IT is *Affirmed* therefore in the Firſt Place, that we Exiſt, that is, that we are Thinking, Perceptive Beings; 'tis True, but to Affirm, is not to Prove; Suppoſing we Dream, that we Perceive and Think, and nothing more, and Conſequently, Suppoſing we only Dream, that we Exiſt, what is there Offered to Convince us of the Contrary? not one Word, Excepting, that this Grave Philoſopher will not Talk with you, that he Deſpiſes you, as much as he does Non-Entity, *&c.* we have, in our Principles of Natural Philoſophy, Anſwered all Objections, which can be Alledged againſt our Exiſtence, and Endeavoured to Conſider every Part of the Argument, which, ſince this Perſon had not done, he was in the Right to Contemn thoſe, who ſhould Queſtion him about it.

§. 1. Art. 2.

GRANTING however, that we Exiſt, to this Demonſtration, for it is not Evinced, but Preſumed; the next Step is, that Something cannot be Produced from Nothing, that is, that our own Perceptive, Thinking, Exiſtence cannot; and therefore, it is Said, there muſt be Something from all Eternity, becauſe, what was not from Eternity, had a Beginning, which muſt be Produced by Something elſe.

BUT, how Weak and Infirm this Connexion of Ideas is, may be Eaſily Shewn; for it is Firſt Said we Exiſt, that is our Perceptions do; It is, in the Second Place, Affirmed, that Something cannot be Produced from Nothing; But what is that Something, unleſs, that which we are Convinced of, as to the Truth and Reality of it's Exiſtence, which is, that we Think and Perceive, and therefore Perception cannot be Produced from Nothing? It is, Thirdly, Affirmed, that Something muſt Exiſt from all Eternity, becauſe, what was not from Eternity, had a Beginning, which muſt be Produced by Something elſe; Whereas again, we being Certain of Nothing, but that our Minds Exiſt, this Aſſertion can Signify no more, than that, ſince our Minds Exiſt, and are

Produced from Something elſe, therefore That, which Produces it, is Eternal, which is the very Thing to be Proved.

So that, Excluding the General, and Inſignificant, Words of Something, Nothing, What, That Which is, *&c.* the Preſent Demonſtration of the Being of a God will Amount to this; We are Certain, we Perceive; That Perception cannot be Produced from it's ſelf, therefore it is Produced from Something elſe; Which, What it is, we are Ignorant of, and therefore is Eternal. If any One can find a Connexion of Ideas here, he may, if he Pleaſes, be as Enthuſiaſtick in Philoſophy, as Others are Said to be in Religion; for it is Nothing elſe than a Reſolution and Impetus of the Mind, which Carries Men to Take up with ſuch Convictions, and not the True Reaſon, which they find in them.

LET us yet Conſider each Particular of this Demonſtration; the Perceptions of our Minds Exiſt; it is Owned; but, Notwithſtanding which, how is it Certain, that the Perceptions of our Minds are not Original in them? If they are, the whole Argument Falls of their being Produced by Something elſe; If they are not, it is Firſt to be Proved, that our Ideas of thoſe Things, which are Accounted External to us, cannot be Produced in the Mind by it's own Faculties, which *Des-Cartes* Thought they might; for, if our Involuntary Ideas may Proceed from our Selves, there is no Reaſon to Think, our Voluntary Ones may not, as thoſe of Willing, Judging, *&c.* and therefore our Minds, and the Perceptions of them, which are our Minds, may not be Owing to any Thing, External to them.

IF this Difficulty was Surmounted, it is to be Proved in the Second Place, that, if our Ideas are Produced from Something elſe, That Something is God; this Philoſophy will not, I Believe, Affirm, that our Ideas from Senſation are Produced by God, becauſe it Endeavours to Shew, that any Material Being cannot be God; and it only Owns another Sort of Ideas, which are thoſe, which we Perceive in our Selves, by Reflecting upon our own Minds; But thoſe Ideas, which we Perceive by Reflecting upon our own Minds, that is, the Faculties and Powers of them, as Willing, Judging, Remembering, *&c.* ſeem to have as little Apparent Pretenſe to be Derived from God, according to the Preſent Philoſophy, as our Ideas of Senſation; ſince it is as Unaccountable, or as much Unknown, in the Preſent Doctrine of Nature, how we Will, or Judge, or Remember, or Doubt, or Fear, as how we Perceive, or have the Senſations of External Objects; that is, our Senſations from External Objects are Matter of Fact, and ſo are our Reflections upon the Operations of our Minds, but as the Preſent Philoſophy cannot Shew, which Way thoſe Senſations are Produced, or what the Nature of Matter is, or how it Operates upon us, ſo it is Equally Defective in Demonſtrating to us, what the Faculties and Powers of the Mind are, and from whence, or from what they Proceed, and whether they may not Ariſe from a *Syſteme of Matter Fitly Diſpoſed.*

Mr. Locke's Eſſay, Chap. 3. Book 4.

THIRD, Either our Actual Perceptions, whether of Senſation or Reflection, Proceed from God, or the Powers of our Minds, by which we have thoſe Perceptions; the Firſt will not be Affirmed by thoſe, who Deny, that we See all Things in God; If yet the Power of Perceiving, *&c.* was Immediately Derived from God, our Actually Perceiving any Thing, which is a Neceſſary Reſult and Conſequence of that Power, would be ſo too, Contrary to what this Philoſophy Maintains, it being very Cautious of Admitting, what may Appear like Enthuſiaſm, which Indeed is no more than to Say, that it will have as little to do with an Almighty Being, as is Poſſible, and ſo far only, as is Juſt Neceſſary to it's Deſigns.

Mr. Locke againſt Mallebranche.

FOURTH, If the Powers and Faculties of our Minds are Derived from God, there was a Time, when each of them Exerted the Firſt Act of thoſe ſeveral Powers; thus, if the Power of Willing was Originally from God, when we firſt Willed, it was from his Influence, or, when we Firſt Perceived or Judged, *&c.* if ſo, there can be no Reaſon Aſſigned, why any Subſequent Acts of thoſe Powers ſhould not Proceed from the Same Impulſe, which is a Degree of Religion, to which this Philoſophy, I Fear, will not be Willing to Riſe.

FIFTH, How is it Proved, that our Souls are not Pre-Exiſtent to their Being in our Bodies? for, if they are, what Point of Time can be Affixed to them, when they began to Exiſt, which may not be Carried Higher, and Conſequently, why may they not be Eternal? If this Philoſophy Alledges, that all our Ideas are from Senſation, and from our Reflection upon thoſe Senſations, and therefore, that our Souls are not Pre-Exiſtent to our Bodies, it will yet I Preſume Acknowledge, that thoſe Powers of them, which are Said to be Derived from God, are Pre-Exiſtent to their Senſations and Reflections; and if ſo, it will be Hard to Demonſtrate, when they began to Exiſt; For, if they Exiſted before our Senſations and Reflections, they might Exiſt before our Minds were United to our Bodies, and ſince no Time can be Determined, when they began to Exiſt, they might, for all that is Proved at Preſent, Exiſt from Eternity; If they did not Exiſt before our Senſations, and our Reflections upon thoſe Senſations, how will it be Evinced, that they did not Ariſe from ſome Unknown Affections of Matter, as Heat, and Ebullition, from the Mixture of an Acid and an Alkali, which ſeems Indeed to be the Senſe of this Philoſophy, when it Affirms, that a *Syſteme of Matter Fitly Diſpoſed* may, it is Poſſible, be Endued with a Power of Thinking.

THIS we Argue only againſt the Preſent Demonſtration, tho' we have in our Eſſay upon the Mind, or our *Pſycheometria*, Shewn, that it is Diſtinct from Matter, that it's Faculties are Independent of it, and that both the Powers of the Mind, and of Matter, do Equally Proceed from God.

§. 1. Art. 3. THE Third Thing, which is Said to be Evident in this Demonſtration is, that What had it's Being and Beginning from Another, muſt alſo have all that, which is in and belongs to it's Being from Another too; all the Powers it has, muſt be Owing to, and Received from, the Same Source and Original of all Power; and ſo this Eternal Being muſt be alſo the moſt Powerful.

IN Anſwer to which, it would be Sufficient to Say, that if any Thing is Unintelligible, this Way of Writing is ſo; For all that can be Meant by *what had it's Being*, and *all that which is in*, and *belongs to it's Being*, &c. is only this, That, if the Mind, with it's ſeveral Faculties and Perceptions, was Produced from Something elſe, it muſt Derive all the Faculties and Perceptions, which belong to it, from that, which Produces it, and becauſe all it's Powers are Received from ſome Source and Original Diſtinct from it, therefore ſuch a Being, from whence our Minds Flow, muſt be All-Powerful.

BUT Firſt, it is not Evident, that all That ſhould be in the Cauſe, Which is in the Effect; For, tho' Heat Diſſolves Wax, and Hardens Clay, Heat is, Notwithſtanding, neither Hard nor Liquid; To come nearer ſtill to the Preſent Caſe, Heat Cauſes the Senſations in us of Pleaſure and Pain, and yet Heat has nothing of that in it, which it Cauſes; that is, Effects are Produced in Nature, Suitable to the Proportions of Action, which Lie betwixt One Being and Another, and the Kinds of them; and therefore, how is this Certain, that, becauſe the Mind is Produced, That, which Produces it, muſt be Mind to?

SECOND, It is leſs Clear, that, if the Mind, and the Powers of it, were Derived from ſome other Intelligent Being, that, therefore, ſuch a Being muſt be All-Powerful; For, tho' it ſhould be Granted, that there is all That in the Cauſe, which is in the Effect, it would not from thence Follow, that there was more in the Cauſe, than in the Effect, or that the Intelligent Being, which Produced our Minds, was any Thing more than they, Excepting that he was Prior in his Exiſtence; Beſides which, there is no Connexion betwixt the Infinite Power of Almighty God, and the Powers or Faculties of an Intelligent Being; First, becauſe it is not Proved from our Finite Faculties, being Produced by ſome Intelligence, that ſuch an Intelligence is Endued with Powers of the Mind, Infinitely Superior to thoſe of our Own; Second, becauſe, if thoſe Powers were Infinite in ſuch a Being, which were Finite in us, it would not be a Conviction, that he was All-Powerful; ſince, to have the Faculties of Perceiving, Willing, Remembering, Judging, &c. in an Infinite Degree, (which is Said to be Unintelligible, Namely, what it is to Perceive or Will Infinitely), would not be the Same, as to be Almighty, that is, to Create the World, and all Things in it, and to Rule and Govern the Whole by an Abſolute and Uncontroulable Empire over it; For, in this Demonſtration, we are not Certain of any Thing, but our own Exiſtence, and therefore not of any World, or any Thing External to us; In Fine, this Wonderful Proof of the Almighty Power of God, from the Powers of our own Minds, is, in the laſt Reſult, nothing elſe, Excepting a Chime, or Jingle, upon the Word, Powers, which, if we Alter into That of the Faculties, or Properties, of an Intelligent Being, the Entire

Force of the Argument is Loſt, and it will Appear, as Weak and Inconcluſive, as it Really is.

What Follows in the 4*th.* and 5*th.* Articles of this Demonſtration is of the Same Kind of Reaſoning with what Foregoes, and which has been Particularly Conſidered, and therefore it will be Unneceſſary to Enter into any Farther Examination of it.

Another Demonſtration of the Being of a God.

Mr. Locke's Eſſay, Book 4. Chap. 10. §. 8. 9. 10, &c.

§. 3. The Foregoing Demonſtration is not ſo Perfectly Convincing, but that this Philoſophy Thought it Neceſſary to Subjoin Another; Which is thus Summed up in Short; If we Suppoſe Nothing Firſt or Eternal, Matter can never Begin to be; If we Suppoſe bare Matter without Motion Eternal, Motion can never Begin to be; If we Suppoſe only Matter and Motion Firſt or Eternal, Thought can never Begin to be; Therefore there is, and muſt have been, a Thinking and Intelligent Being from all Eternity.

The Foregoing Demonſtration Conſidered in General.

§. 4. Upon which it will be Requiſite to Obſerve; Firſt, that, tho' this Philoſophy Calls it going over the Former Demonſtration again, yet it is a Different One; ſo Eaſily is it able to Miſtake it's own Notions with all it's Infallible Pretenſes; For, whereas the other Demonſtration was from our own Exiſtence, this is from the Exiſtence of Things without us, as well as our Own.

Second, It is Impoſſible, even according to this Philoſophy, that we ſhould be Certain of this Demonſtration, becauſe it's only Standard of Certainty is from the Agreement or Diſagreement of our Ideas, that is, from the Connexion of them; but, whenever we make one Step from our Selves, and from that, which we Perceive in our own Minds, Abſtractedly from every Thing elſe, the Connexion of Ideas is Loſt and Vaniſhes; Becauſe there is no Connexion, Properly Speaking, betwixt the Idea of Matter, and Matter as it Really Exiſts, betwixt the Idea of Motion, and Motion it's ſelf; and the Same may be Said of the Ideas of our Minds in General, and of the External Objects they Repreſent: But this, as we have before Mentioned, is the Weakneſs of the Preſent Philoſophy, that, whilſt it Tranſcribed from the *Carteſian* it's Opinion of Certainty, from the Connexion of Ideas, it Departed from it, in Proving Things to be External to us without ſuch a Connexion, whereas, to Adopt the One into it's Syſteme, and not the Other, is Inconſiſtent, and a Contradiction; For to Say all Certainty Depends upon the Connexion of Ideas, and to Say at the Same Time, that there is a Certainty, which does not, is no Leſs.

The Firſt Step of this Demonſtration Conſidered.

§. 5. Thus far we have Conſidered this Demonſtration in the Groſs, and ſhall now Enter into the Particulars of it; Firſt, it is Said, if we Suppoſe Nothing Eternal, Matter could never Begin to be; Which is very Evident, becauſe Something muſt have always Exiſted, and Nothing is only a Relative Term, which Suppoſes Something, according to what has been Shewn in the Principles of Philoſophy, Chap. 4. §. 2. Part. 1. But then it muſt likewiſe be Said, that this Philoſophy does not Prove it, becauſe, if we Inquire what Nothing is, it is Silent, if what Matter is, it can as little Inform us; but to Argue upon Principles, which are Unknown, is, at the Beſt, an Ignorant Kind of Demonſtration.

The other Steps Examined.

§. 6. The next Thing in it is, that, if we Suppoſe Bare Matter, without Motion, Eternal, Motion could never Begin to be; But this Argument Depends upon a Suppoſition, which has been Confuted in the Principles of Philoſophy, of Matter's being Similar and Homogeneous, that is, a Paſſive, Extended, Solid, Diviſible, Subſtance; which, if True, it would be Impoſſible for Actual Motion to Enter into that Definition of Matter, which in Terms Excludes it; if yet, on the Contrary, ſuch an Hypotheſis of Matter is neither Agreeable to the Nature of it, nor to any Appearances, which are Obſervable in it, if, Farther, Matter is Endued with a Certain Force and Action, it will not be Manifeſt, that Motion, ſome way or other, may not be Produced from it, as it in Fact is from that one Principle only of Gravitation, with which it is Inveſted. The Third Step in this Demonſtration is, that, if we Suppoſe Matter and Motion Eternal, Thought could never Begin to be; Whereas this likewiſe is Uncertain; becauſe this Philoſophy does not Underſtand, what the Nature of Matter is, and therefore cannot Preciſely Aſſure us, whether Thought, as well as Motion, may not belong to it; we have already Shewn, that the Firſt cannot; and what we Say now is only this, that the Preſent Philoſophy does not Prove it, whilſt it makes an Arbitrary Abſtraction of Matter, and Throws out of it's Eſſence ſeveral Properties, which, it is not Certain, may not Appertain to it, by making Extenſion, Solidity, *&c.* the Definition of it; None of which Properties do Really Enter into it's Nature, ſince they are only Abſtracted Ideas, according to what has been Evinced in the Principles of Philoſophy, Chap. 6. and 7. Part. 1.

The Laſt Advance made in this Demonſtration is, that, from what Foregoes, there muſt be a Thinking, Intelligent Being, from all Eternity; Suppoſing it, yet it no ways Follows, that, if Thought is Eternal, therefore Thought is God; to Wit, that it is Endued with all the Perfections and Attributes of an Almighty, Omniſcient, and which is more, of an Infinitely Good, and Beneficent, Being; Since Thought, for all this Philoſophy Knows, might be from Eternity, and yet be no more than Mere Thought, without either any Great Power, or Wiſdom, or Goodneſs, or Infinity, belonging to it.

Tho' the Demonſtration of a God is Diſputed with this Philoſophy, yet ſuch a Being is Demonſtrable on Juſt and Rational Principles.

§. 7. To Conclude this Chapter, altho' we have Diſputed the Being of a God with this Connexion of Ideas, that is, with a Philoſophy, which Fails in every Part of the Connexion, which it Suppoſes Neceſſary to Certainty, yet we have, and ſhall Farther Evidently Prove, there is ſuch a Being from the Infinite Variety and Wiſe Adjuſtment of the Expanſive and Contractive Forces. Next to which, we Say, that the beſt Demonſtrations of One are from Matter of Fact; Namely, From the Obſervation of the Wiſe Oeconomy and Order in the Animal and Material World; From the Various Providences, which are Manifeſt in Reſpect of Particular Perſons, as well as Whole Nations and Communities; and From the Chriſtian Religion it's ſelf.

The Firſt is to be had from a bare View of all thoſe Beings in the Univerſe, which Surround us; which, when we Look upon only, we are, without any Laboured Deductions of Reaſon, as fully Carried into an Immediate Acknowledgement of a Supreme Author, as when we Look upon an Exquiſite Picture, or a Landskip, we are Satisfied, it was Drawn by ſome Skillfull Hand; and yet, if we were to ſet our Selves to the Proving of the Latter by the Method of Connecting Ideas, it would be Impoſſible.

The Second is to be Deduced from Hiſtory, and the ſeveral Occurrences Obſervable in the World; Which are not only Incapable of being Solved from Natural Principles, but do Neceſſarily Infer an Almighty and Superintending Agent; Such are the Strange Revolutions of States and Empires, and the Surprizing Accidents, which Arrive to Particular Perſons and Societies, Perfectly Unforeſeen, and, upon Reflection, as Unaccountable.

The Being of a God not Previouſly Neceſſary to a Demonſtration of Chriſtianity.

The Third, which is the Greateſt and moſt Convincing of them all, is That, which is Derived from our Religion, and the Certainty and Truth of it; For it is a Great Miſtake to Think, that the Being of a God is Previouſly Neceſſary to the Demonſtration of Chriſtianity.

Since, if the Chriſtian Religion is True, that is, if our Saviour came from God, and was the Son of God, and Wrought thoſe Miracles, which are Recounted of him, by the Power of God, and was God himſelf United to our Human Nature, it is very Evident, that there is a God, without any other Proof of it; Beſides which, can we Think, or Imagine, that the Firſt Chriſtians Wanted to have a God Proved to them, before they Believed their Religion, or was it ever Attempted? If they did, why are not all the Chriſtian Writers full of ſuch Demonſtrations, why do they always take it for Certain and Concluded? If they did not Want a Proof of ſuch a Being, it is as Evident, that their Religion Inferred it, that the very Facts of it Teſtified and Aſſured it to them, and to bring a Rational Demonſtration of it, was in Effect to Queſtion the Truth of thoſe Facts, and that Faith, of which they were Convinced.

On the other Hand, if the Chriſtian Religion is Doubted of, or is not True; in the Firſt Caſe, the Demonſtration of the Being of a God would be of little or no Service to it; For, if it was Manifeſtly Certain, that there was an Infinite, Almighty, and All-Wiſe Being, of what Importance would that be to a Religion, which Owns, that this Being is not only ſo, but Infinitely Juſt and Good, that he Sent his own Son into the World for the Redemption of Mankind, and that he Suffered him to Die the Death of a Malefactor for their Sakes? This cannot be Proved from the Being of a God, but from the Plain Evidences of Hiſtory and of Fact.

In the Second Caſe, if the Chriſtian Religion is not True, a Demonſtration of a God will not be Previouſly Neceſſary to Prove it, becauſe it will not be in the Leaſt Significant to it; From whence it is Plain, that thoſe, who Proceed in this Way, take a Method to Evince the Chriſtian Religion in Pretenſe, by Diſcarding it in Reality, which ſince ſeveral Good Men have Unwarily Followed, we ſhall now Conſider the Riſe and Original of this Procedure; which ſeems to be Derived from that Great Inventor *Des-Cartes*, who when, tho' not in Shew, yet in Truth, had Relinquiſhed his Religion, by making all Certainty to Conſiſt in the Connexion of the Ideas of his own Mind, to which it was Impoſſible that Religion ſhould be ever Exacted, was Pleaſed to Propoſe the Demonſtration of a God; This Indeed was Neceſſary to Juſtify a Philoſophy, which had nothing elſe to Recommend it, but a Colour or Appearance at Leaſt of ſuch a Demonſtration; Mr. *Locke* alſo Following the Steps, ſo far, of this Demonſtrator, as to Think the Proof of a God to be the Neceſſary Foundation of Morality, as the Other's was of his Philoſophy, and *Grotius* Premiſing One to his Proof of Chriſtianity, the World began to be Perſuaded, that this was likewiſe Requiſite in Religion, whereas, as we have Shewn, there is nothing Leſs ſo.

All that we ſhall Say farther is only, that this Demonſtration of a God is Said to be, by the Preſent Philoſophy, the Foundation of Morality; If therefore it is not a Juſt One, all the Mighty Rules of Morality, which it Affirms, Fall to the Ground, Notwithſtanding which, we ſhall in the Following Chapter Conſider Them likewiſe.

Concerning Dr. Clarke*'s Demonſtration of the Being of a God.*

§. 8. But before we do This, we ſhall make a Short Inquiry into what has been Propoſed by another Celebrated Philoſopher upon this Subject, and which is Drawn out into ſeveral Propoſitions.

Dr. Clarke, Prop. 1.

The Firſt of which, Namely, that Something has Exiſted from all Eternity, is Owned and Confeſſed on all Hands; the Deiſt Affirms it, and the Atheiſt does not Deny it, and every Chriſtian will Readily Subſcribe to the Truth of the Aſſertion; The Grand Queſtion is, What this Eternal Something is, whether it is a Supreme, Infinite, Intelligent, and Independent Being, Diſtinct from Nature and the Univerſe, which the Chriſtian and the Deiſt Aſſert, or whether it is the Univerſe and Nature it's ſelf, which is the Doctrine of the Atheiſt.

Dr. Clarke, Prop. 2.

The Second Propoſition is, that there has Exiſted from Eternity ſome One Unchangeable and Independent Being; If not, it is Said, there muſt have been an Infinite Succeſſion of Changeable and Dependent Beings Produced One from Another, in an Endleſs Progreſſion, without any Original Cauſe at all, which is Abſurd, and Plainly Impoſſible, and Contradictory to it's ſelf. It ſhould have been here Subjoined, Or elſe, there has been an Infinite Succeſſion of Changeable and Dependent Beings One upon Another, Joined and Cemented together by a Mutual and Neceſſary Dependence of Cauſe and Effect, Ariſing from the Fixed and Unalterable Laws of their Natures, which Act one amongſt another by Certain Eſtabliſhed Rules, and according to which, they always did Act, and ever will; That is, there is a Neceſſary Dependence of One Thing upon Another, Owing to the Intrinſick Nature of Things ſo Depending, which Nature of Things is Neceſſary and Eternal, and by which Neceſſity all theſe Dependent Beings upon each Other are United together into One Syſteme, ſo that any Dependent Being could not Exiſt without That, on which it Depends, and on the Contrary, That on which it Depends could not Exiſt without That, which is Dependent on it; They Mutually Infer and Suppoſe each Other, and That which Connects them is, the Original Neceſſity of their Natures; Or, which is the Same, there is a Series of Beings Dependent upon each Other, which has a Neceſſity and Cauſe, and therefore a Reaſon, of it's Exiſtence from the Nature of the Things themſelves, which Conſtitute this Series, and from which Neceſſity of their Nature they Obtain that Relation of Dependence upon each Other, which we Obſerve in them; So that, Notwithſtanding this Propoſition, and the Preſent Demonſtration of it, Atheiſm Stands where it did, and the Queſtion ſtill Returns, and is not yet Solved, whether the Nature of Things is in it's ſelf Eternal, or whether it is Derived from a Supreme and Intelligent Cauſe, Diſtinct from it; The Preſent State of the World and the Univerſe is That of a Mutual Dependence of One Thing upon Another, and the Queſtion only Lies, whether this Mutual Dependence is Owing to the Intrinſick Nature of Things themſelves, as they Exiſt in that Dependence upon each

Other, or whether to the Will and Pleaſure of an Infinite, All-Wiſe, and Super-Intending Being; Which cannot be Reſolved by ſuch Metaphyſical Reaſonings, as we are Entertained with upon this Propoſition, but by Inquiring into Nature it's ſelf, and Shewing, as we have Done, the Neceſſary and Indiſſoluble Connexion betwixt the One, and the Other, that is, betwixt an Infinite Action, or an Infinity of Expanſive and Contractive Forces, of which Nature Conſiſts, and an Infinite Agent, who is the Author of ſuch Action.

For Thus, if we *à Priori* Suppoſe, that Almighty God is an Infinite, Eternal, and All-Wiſe Agent, which is but a Juſt and Reaſonable Suppoſition, it will be Conſequent, that he Exerted himſelf into Action from all Eternity in the Way, which was the moſt Suitable to his Wiſdom and Power, and therefore, that he Created and Produced an Infinite Action from all Eternity, and that the World, or the Univerſe, was always Full of ſuch an Infinite Action, which by the ſame Infinite Power, and by his Uncontroulable Will, he can Alter, Diverſify, and Change, as he Pleaſes, but that there will ſtill be, by the Neceſſity of his Agency, ſome Action or other Diſperſed and Diffuſed thro' the whole Expanſe of Space, and the Unlimited and Boundleſs Extent of Being. On the Contrary, it has been Shewn, that there is no Vacuum in Nature, but that all Things Conſiſt of a Plenitude of Action, and that it is Diverſified and Changed into an Infinite Variety of Expanſive and Contractive Forces, into which Æther, Fire, Air, Water, and Earth, and the ſeveral Planets, Sun, and Stars, are Separated and Divided; and by Conſequence, or *à Poſteriori*, there muſt be an Infinite, Eternal, and All-Wiſe Agent; And the Particular Diſpoſition of Things, or of the Expanſive and Contractive Forces, or of this Infinity of Action, muſt be the Reſult of his Will and Pleaſure, tho' the Infinite Action or Plenitude of Being is Neceſſary from his Infinite Agency.

And by Conſidering Things in this Way, there is a Plain Connexion betwixt the Author of Nature, and that Nature, which is Produced and Derived from him; and they Mutually Infer each Other; We Conclude, what the Effect muſt be from the Cauſe, and what the Cauſe muſt be from the Effect, and that there muſt be an Infinite Action from an Infinite Agent, and, Reciprocally, an Infinite Agent from an Infinite Action; Which has not been done by any Philoſophy; and the Reaſon of it is, becauſe no One has hitherto been Able to Reduce all Matter, and the Properties of it, to Action, which has been always Eſteemed to be a Mere Paſſive Subſtance, whereas we have Evinced, that it Conſiſts Entirely in Action, which is the Eſſence of it, and that all it's Qualities and Affections are Owing to Different Degrees of it's Expanſive and Contractive Forces.

Dr. Clarke, Prop. 3.

The Third Propoſition is, that That Unchangeable and Independent Being, which has Exiſted from Eternity, without any External Cauſe of it's Exiſtence, muſt be Self-Exiſtent, that is, Neceſſarily Exiſting.

But it may be Said by thoſe, who Defend a Self-Exiſtent Nature againſt a Supreme Being as the Author of it, that this Propoſition might be True, if there was no Neceſſity in the Nature of Things themſelves, from whence a Mutual Dependence of Beings amongſt one another Ariſes, and from which Neceſſity of Nature, which is Self-Exiſtent, an Endleſs Progreſſion of Cauſe and Effect is Derived; But, if there is ſuch a Neceſſity in Nature, All, they will Say, which can be Granted is, that That Neceſſary Connexion, which there is betwixt Cauſe and Effect in a Mutual Dependence upon each Other, Ariſing from the Nature of Things themſelves, is Self-Exiſtent, or is Neceſſarily Exiſting; That therefore One Neceſſary, Unchangeable, and Independent Being is not Proved, nor it's Self-Exiſtence; They will Own, That Unchangeable and Independent Exiſtence is Self-Exiſtence, or a Neceſſary Exiſtence, but that this belongs to one Certain Being in Diſtinction to Nature, and the Mutual and Neceſſary Dependence of Things upon each Other, is what they Want to See Demonſtrated, and upon which the whole Controverſy Turns, and which will Require a Greater Depth in Philoſophy to Prove, than, I am Afraid, this Demonſtration is Maſter of.

As to what is Said afterwards, that, to be Self-Exiſtent, is, not to be Produced by it's Self, but to Exiſt by an Abſolute Neceſſity, Originally in the Nature of the Thing it's Self; Be it ſo, tho' what any One can Mean by Self-Production, but Self-Exiſtence, I do not Eaſily Apprehend; The Queſtion yet Remains, hitherto Undetermined,

mined, and Undecided, betwixt the Deiſt and the Atheiſt, Namely, whether this Self-Exiſtence Appertains to God, or to Nature, and there has been nothing yet Alledged, which Proves it of the One, or Diſproves it of the Other. In the Proceſs of this Propoſition it is Inferred,

1ſt. That the only True Idea of a Self-Exiſtent, or Neceſſarily Exiſting, Being is the Idea of a Being, the Suppoſition of whoſe Non-Exiſtence is an Expreſs Contradiction, which by any Thing, which has been hitherto Shewn, may be as well Affirmed of a Self-Exiſtent Nature, as of a Self-Exiſtent Being, Different and Diſtinct from it; For, if Nature is Self-Exiſtent, and therefore Neceſſarily Exiſting, which is ſtill to be Proved by this Demonſtration, that it is not, it muſt be a Contradiction to Suppoſe it not to Exiſt; And by Nature, the Defenders of it againſt a Supreme Being, Diſtinct from it, will not Mean Matter or Motion, but thoſe Eternal Laws of Beings, which, tho' Unknown to us, have always Obtained in the Univerſe, and ever will Obtain.

But, as no Patrons of Atheiſm can ever Shew, what thoſe Eternal Laws, and Self-Exiſtent Ones, of Nature are, ſo their Cauſe is Deſperate and Indefenſible; On the Contrary, as the Aſſertors of a Supreme Being can Evidence, that there are Marks and Characters of an Infinite and All-Wiſe Contrivance in the Conſtitution, Order, and Harmony of the whole Fabrick, they may from thence Shew, that there is an Almighty, Eternal, Self-Exiſtent, and All-Wiſe Author of it.

2d. It is Inferred, that there is no Man whatſoever, who makes any Uſe of his Reaſon, but may Eaſily become more Certain of the Being of a Supreme, Independent Cauſe, than he can be of any Thing elſe, beſides his own Exiſtence.

This yet is not Certain, or Proved, from any Thing, which Foregoes, only, that there is ſomething Eternal, Self-Exiſtent, and Independent of any other Cauſe, but whether it is God, as the Deiſt Affirms, or Nature, as the Atheiſt, is Entirely, as yet, Undetermined.

3d. It is Inferred, that *Des-Cartes's* Proof of a God from an Idea of him, is not Juſt or Concluſive; Which has been, in a Former Chapter, Particularly Inquired into, and Shewn to be Weak and Abſurd.

4th. It is Inferred, that the Material World cannot Poſſibly be the Firſt and Original Being, Uncreated, Independent, and of it's Self Eternal.

In Order to Evince which, Nature is Reſolved into the Form or Diſpoſition of the World, into the Motion Obſervable in it, and into the Matter of it; None of which are ſaid to be the Firſt Original Being, Uncreated, Independent, and of themſelves Eternal.

But if this Demonſtration is altogether Unacquainted with what Nature, or a True Philoſophy, is, it only Argues and Aſſerts in the Dark, and nothing can be Produced by it, in Conviction of an Atheiſt.

For Firſt, as to the Form or Diſpoſition of the World, no Atheiſt will Maintain, that the Preſent, or any other Particular, Form is Neceſſary, but will Affirm with the Deiſt, that the Univerſe may have Undergone Various Forms and Changes, which yet, he will Say, do all of them Reſult from Certain Eternal and Self-Exiſtent Laws in Nature; Thus, he will not Deny, that there are Various Seaſons in the Year, in which Plants and Inſects Riſe, Come to Perfection, and Decay, Revive again, and ſo Run the Round of their Exiſtence; That Stars and Planets may do the Same; And yet, that all theſe Viciſſitudes and Alterations may be Owing to ſome Eternal, Self-Exiſtent, and therefore Neceſſary, Laws of their Being and Acting; This, tho' he cannot Prove, he may Imagine and Surmiſe, and there is nothing Produced in this Demonſtration, which ſhould, or can, Convince him of the Contrary; Becauſe it does not Enter into the Reality of Things, or into an Inquiry of what Nature is, but Diſcourſes Metaphyſically, and Generally, about theſe Matters, with little Knowledge of, or Concern about them.

Second, as to Motion, whatever the Author Mr. *Toland*, who is Cited, Affirms, Concerning a Conatus to Motion, or whatever his Character is, I have nothing at Preſent to Say to Either, whoſe Tenets and Principles I have Conſidered upon other Occaſions; I ſhall only Say here, that an Atheiſt is not to be Convinced by an Argument from any Particular Motion, not being Neceſſary; Becauſe, tho' any Certain, or Determined, Motion is not Neceſſary, or Self-Exiſtent, it is not Shewn, that the

Principle and Cauſe of it may not be ſo, in the Real Nature of Things; Eſpecially, if we Allow the *Gravitation* of Matter, that is, of every Part of Matter to every Part of Matter, which Sir *Iſ. Newton* Affirms, and is little more than Mr. *Toland's Conatus to Motion*; For if ſo, that Principle of Gravitation may be Self-Exiſtent and Eternal, as that of *Epicurus's* Atoms was Suppoſed to be; and, where there is a Preponderancy of One to the Other, Motion will be Neceſſarily Produced.

THIRD, as to Matter, That, in this Demonſtration, is likewiſe not Proved not to be Neceſſary, unleſs upon two Suppoſitions, which are Precarious, and therefore not Sufficient to Convince an Atheiſt; The First Suppoſition is, that there is a Vacuum in Nature, which, as I have before Shewn, is not, nor can be, Proved by any Philoſophers, unleſs by Suppoſing the very Thing in Queſtion, Namely, that there is a Vacuum, Part the 1*ſt*. of this Philoſophy, in the Chapter Concerning a Vacuum; The Second Suppoſition is, that, if Matter Neceſſarily Exiſts in one Place, it muſt therefore Neceſſarily Exiſt in every Place, which is not Juſtified by any Reaſon or Argument; For, tho' the Eternal Laws of Nature might, by Certain Mutual Attractions, Amaſs ſeveral Proportions and Quantities of Matter to the Sun or Stars, to the Planets or Comets, it would not from thence Follow, that thoſe Laws ſhould Diſperſe and Diffuſe it every where, but rather the Contrary; But, which is more, this Argument from Matter Proceeds upon an Entire Ignorance of what the Nature of it is, which if it ſhould Happen to be Different from what the Preſent Demonſtration Conceives it to be, the whole Reaſoning on this Head is Weak and Inconcluſive; and This we have Shewn to be the Caſe, as we have Evidenced it to Conſiſt in Action, which is here Imagined to be Merely Paſſive, and that ſuch Action is Made up of the Various Degrees of the Expanſive and Contractive Forces.

As to what *Spinoza* is Cited afterwards to Say, that there is no Difference of Subſtances, and that One Subſtance cannot Produce Another, and therefore, that an Almighty, Independent, and Supreme Being, cannot Produce a World or a Univerſe, Diſtinct from it's ſelf; It is ſaid Entirely without Proof, as it is likewiſe Affirming the very Thing in Queſtion, that every Subſtance is Neceſſarily Infinite, and therefore the Univerſe, which is a Subſtance, is Neceſſary and Infinite, and Includes all Being within it, and Conſequently, that there is no God Diſtinct from the World, or the Univerſe; Which are all of them Aſſertions only in Different Words of what was to have been Shewn, that Nothing, No Being, or Intelligence, Exiſts Diſtinct and Independent of Mere Nature.

So that hitherto, here is no Demonſtration on the One Side or the Other, the Matter, as yet, is wholly Undetermined, and all Centers and Ends in Bare Affirmation; On the Side of the Deiſt, that there is a Supreme Self-Exiſtent, and Independent Being, Diſtinct from the Univerſe, and who is the Cauſe and Author of it, and on the Side of the Atheiſt, that there is no ſuch Being, but that the Univerſe it's ſelf is the only Self-Exiſtent, Neceſſary, and Independent Subſtance, and that Things could not have Exiſted in a Manner Different from what they do. *Ocellus Lucanus* is afterwards Quoted under this Propoſition, and whoſe Authority and Antiquity Mr. *Blount*, it ſeems, is Pleaſed to Oppoſe to That of *Moſes*; Whereas he does not Appear to have been an Atheiſt, but of *Plato's*, and *Ariſtotle's* Opinion, and his Followers, to Wit, that the Univerſe was Eternal, tho' Created, Produced, or Impreſſed, by an Infinite Mind, which is a Doctrine not much, if any Thing Different, from that of *Moſes*, Rightly Underſtood, as we have Formerly Shewn; Princip. of Philos. Part. 1*ſt*; And that theſe were the Sentiments of *Ocellus Lucanus* is Evident, becauſe tho' in one Place he Says, Ἀγένητον τὸ πᾶν, ϗ ἀνώλεθρον, in another, that it is, Ἄναρχον ϗ ἀτελεύτητον, and in a Third, that κόσμος αὐτὸς ἐξ ἑαυτȣ̃ ἀΐδιός ἐςι, ϗ αὐτοτελὴς ϗ διαμένων τὸν πάντα αἰῶνα, yet in other Places he tells us, that τὸ ἀεικίνητον θεῖον μὲν, ϗ λόγον ἔχον, ϗ ἔμφρον, that συνέχει τὸν κόσμον ἁρμονία, ταύτης δ' αἴτιος ὁ Θεός. From whence it is Manifeſt, that he both Aſſerts the Eternity of the World, or the Univerſe, and that there is a Wiſe and Intelligent Being, which Actuates, Informs, and Supports, the whole Fabrick or Frame of Nature.

HOWEVER, be *Ocellus Lucanus's* Opinion, or *Plato's*, or *Ariſtotle's*, what it will, in Reſpect of Atheiſm or Deiſm, a Deciſion of the Diſpute is not from thence in the leaſt Advanced by it; the Controverſy Remains, where it was, and there is nothing yet Urged in the Preſent Demonſtration, which is Concluſive of the Latter; In fine, there does not ſeem to be any other Way of Proving the Being of a God from Nature, and the Univerſe, but by knowing what Nature is, and in what it Conſiſts; If we are Ignorant of the Laſt, we cannot Arrive at a Knowledge of the Firſt; we are only Entangled and Confounded with Idle, and Metaphyſical Reaſonings, without being Able to Affirm any Thing with Solidity or Certainty; All is Cobweb, and Fine Spun Thoughts, and Nice Speculations, and we ſhall find nothing on either Hand, unleſs Abundance of Abſtracted Terms, Abſtruſe Words, and Mere Sophiſtry and Chicaning.

BUT, if Nature is Proved to Depend upon Action, and that Action is Owned to Subſiſt in an Infinite Variety and Extent of Expanſive and Contractive Forces; From an Infinite Action, and the Order and Diſpoſition of ſuch Action into it's ſeveral Forces, we Immediately come to an Infinite and All-Wiſe, and All-Powerful Agent, and Vice Verſâ; which therefore Mutually Evince and Infer each other, according to what has been before Repreſented.

PROPOSITION the Fourth is, That, what the Subſtance or Eſſence of that Being, which is Self-Exiſtent, or Neceſſarily Exiſting, is, we have no Idea, neither is it at all Poſſible for us to Comprehend it. Dr. Clarke, Prop. 4.

THIS will be Allowed by the Atheiſt, as well as the Deiſt; For, tho' the Former Aſſerts, that the World, or the Univerſe, or Nature, is that Self-Exiſtent, or Neceſſarily Exiſting Being, he will not Affirm, that he Underſtands what Nature is, or that he Knows the Subſtance or Eſſence of Matter or the Mind, or that it is at all Poſſible for us to Comprehend either of them; So that here is no farther Argument Alledged for the Proof of Deiſm, than of Atheiſm, and is a Conviction, that both the One and the Other Reſt in General and Abſtracted Terms, without Inquiring, what God, or Nature is; On the Contrary, in our Doctrine of the Expanſive and Contractive Forces, we have Endeavoured to Explain, what Nature is, and what the Author of it.

HOWEVER, from this Propoſition is Inferred the Weakneſs of ſuch, as have Preſumed to Imagine Infinite Space to be a Juſt Repreſentation or Adequate Idea of the Eſſence of the Supreme Cauſe; This is the Notion of Mr. *Raphſon, de Spatio Reali & Infinito*, which, in the Firſt Part of the Principles of Philoſophy, we have Shewn to be Falſe, and that Space is nothing in Reality, unleſs an Abſtracted Idea; Beſides which, Infinite Space can neither be the Eſſence of God, nor the Attribute of him, becauſe he is an Infinite Agent, whereas Space is an Empty, Unactive, Nothing; On the Other Hand, the *Ariſtotelians*, and School-Men, much better Define his Eſſence to be *Purus Actus*, that is, if they Mean by That, Mere *Agency* or *Intelligence*.

PROPOSITION the Fifth is, that tho' the Subſtance, or Eſſence, of the Self-Exiſtent Being is it's ſelf Abſolutely Incomprehenſible to us, yet many of the Eſſential Attributes of his Nature are Strictly Demonſtrable, as well as his Exiſtence; Thus, in the Firſt Place, the Self-Exiſtent Being muſt of Neceſſity be Eternal; In Explanation of which it is Added, that becauſe Something muſt of Neceſſity be Eternal, therefore it muſt be Neceſſarily Self-Exiſtent; and becauſe it is Impoſſible, but Something muſt be Self-Exiſtent, therefore it is Neceſſary, that it muſt likewiſe be Eternal. Dr. Clarke, Prop. 5.

As to the Propoſition, Thoſe, who Affirm Nature to be the Self-Exiſtent Being, will Equally Own the Main Parts of it, Changing *his* for *it's*, with thoſe, who Affirm, that a Supreme Being, Independent of Nature, is that Self-Exiſtent Being; There is therefore nothing Proved in this Propoſition to make us Believe the One, more than the Other, and the Atheiſt and Deiſt are ſtill where they were, Notwithſtanding the Force and Evidence of this Propoſition.

As to what is Said of the Connexion betwixt being Eternal and Self-Exiſtent, there is this Diſtinction in our Ideas of them, Namely, that every Thing, which is Self-Exiſtent, is Eternal, but every Thing, which is Eternal, is not Self-Exiſtent, as is Acnowledged by thoſe Divines, who Believe the Eternity of the Son, and his Generation, or Origination from the Father, amongſt whom is the Preſent Ingenious Perſon, and by thoſe Philoſophers, as the *Platoniſts*, and *Ariſtotelians*, who Held the Eternity of the World, and yet the Eternal Production of it from an Eternal Cauſe.

THE Sixth Propoſition is, that the Self-Exiſtent Being muſt of Neceſſity be Infinite and Omnipreſent. Dr. Clarke, Prop. 6.

THIS Propoſition will likewiſe be Affirmed by the Aſſertors of a Self-Exiſtent Nature, Independent of a Supreme Being, that is, that Nature is Infinite, and every where, or, that the ſame Laws of Being and Acting, Exiſt thro' the whole Univerſe, and in every Part or Portion of it.

NOTWITHSTANDING which, from this Propoſition it is Inferred.

1ſt. THAT the Infinity of the Self-Exiſtent Being muſt be an Infinity of Fulneſs, as well, as of Immenſity, that is, it muſt not only be without Limits, but alſo without Diverſity, Defect, or Interruption, and therefore not the World, or Matter, but a Supreme and All-Wiſe Intelligence, Diſtinct from Either, muſt be this Self-Exiſtent Being.

BUT it may be Replied to this, that if Infinity belonged to the Self-Exiſtent Being, not only in Reſpect of Immenſity, but in Reſpect of Fullneſs, a Supreme and All-Wiſe Intelligence, if he is the Self-Exiſtent Being, muſt have that Infinity of Fullneſs, that is, he muſt have no Aſſignable Vacuities in himſelf, but muſt be One Solid, Conſiſtent, Compacted Maſs, without Diverſity, Defect, or Interruption; which is Giving us the Image of a Supreme Being, Compoſed of Denſe and Groſs Matter, and may be therefore Juſtly Reſembled to that Image, which *Aaron* Made, both as to the Clumſineſs of the Materials, and the Ridiculouſneſs and Abſurdity of the Repreſentation.

ON the other Hand, Nature, for all that is yet Proved, may be Infinite, tho' with Various and Infinite Degrees of Fullneſs, (Nay, if there was only One Certain Degree of Fullneſs, upon that Account it would be Finite,) and therefore it may be Said, that it is Infinite as to it's Immenſity, and as to it's Degrees of Fullneſs; Thus, One Part of Space may be Full of a Stronger or more Conſtipated Force and Action, and Another of a Weaker or more Diffuſed and Diluted; And all may be Derived from the Infinite Mind or Agent, as an Infinite Effect from an Infinite Cauſe.

2d. THAT the Self-Exiſtent Being muſt be a moſt Simple, Unchangeable, Incorruptible Being, without Parts, Figure, Motion, or Diviſibility; Whereas, Infinity does not Suppoſe, that, what is Infinite, muſt be a moſt Simple, Unvaried, and Uninterrupted Being, without Parts, Figure, Motion, or Diviſibility, which are the Deſcriptions of an Abſtracted Space; Becauſe, if Space is Infinite, as it is Acknowledged to be, the Univerſe may be Infinite, which Conſiſts of that Space, with ſeveral Corruptible, Changeable, and Moveable Subſtances in it, which Univerſe, will be Said, is not Leſs Infinite for having theſe Additional and Acceſſory Beings in it; Since no One, it is Imagined, can Shew, that Nature, or the Univerſe, is not Infinite, becauſe a Fire, or a Sun, Burns, or a Planet Revolves within it's Verge or Extent; So that we may ſtill Conclude, that there is nothing yet Alledged by this Metaphyſical Demonſtration, which can Inform us, whether God or Nature is this Self-Exiſtent Being, which is the only Point in Queſtion.

Dr. Clarke, Prop. 7.

PROPOSITION the Seventh is, that the Self-Exiſtent Being muſt of Neceſſity be but One, which is Said to Follow from his being Neceſſarily Exiſtent; Since therefore That Propoſition has been Examined, there is no Occaſion to Inquire into the Evidences of This, Eſpecially, Seeing the Atheiſt will, if he has any Reaſon, Acknowledge, that Nature is but One Thing, if it is Self-Exiſtent, that is, that it is one Infinite Syſteme Governed and Actuated by the Same Laws of Being and Acting, which are, have been, and always will be, Neceſſary and Eternal, and this is the Thing to be Refuted.

Dr. Clarke, Prop. 8.

PROPOSITION the Eighth is, that the Self-Exiſtent and Original Cauſe of all Things muſt be an Intelligent Being.

THIS I have Affirmed to be the Principal Queſtion, in which, I have ſaid, Lies the whole Streſs of the Controverſy betwixt the Atheiſt and the Deiſt, and which, upon Some Occaſions, has been Diſputed with me; when yet, this very Demonſtration does in Fact Own it, which Says, in this Propoſition Lies the Main Queſtion betwixt us and the Atheiſts; For, that Something muſt be Self-Exiſtent, and that That, which is Self-Exiſtent, muſt Neceſſarily be Eternal and Infinite, and the Original Cauſe of all Things, will not Bear much Diſpute; Notwithſtanding which, it has Spent Seven Propoſitions, and it's Arguings upon them, to Contradict and Confute the Atheiſts, in which Propoſitions yet it Owns, that both the Atheiſt, and the Deiſt, do Agree, and in which I have Endeavoured to Shew, there is no Diſſenſion betwixt them; which is

Something Admirable, and has an Appearance of Artifice, rather than of any Solid or True Reaſoning.

WHAT afterwards Follows to Prove the Being of a God Independent of Nature, and that the Self-Exiſtent Being is a Wiſe and Intelligent One, is from the Same Topicks, which have been always made Uſe of by the Aſſertors of ſuch a Being, that is, from the Harmonious and Beautiful Structure of the World, and all Things in it, and are Celebrated in all Chriſtian Writers from St. *Paul*, down to the Late Eminent *Hugo Grotius*, the Excellent Mr. *Ray*, in the Wiſdom of God in the Creation of the World, and Others, and which, as we Acquieſce in, ſo we have no Reaſon to Contradict, but Affirm.

IT was my Intention to have Examined into the Notions of a New Patron of the Connexion of Ideas, as to his Truths, Relating to the Deity; but they ſo Entirely Depend upon what has been already Conſidered, that there is no Manner of Occaſion to Enter into a Diſquiſition of him upon this Head, however, to Satisfy his Admirers, we ſhall not Refuſe that Trouble.

THE firſt Article, or Propoſition, of this Section is, That, where there is a Subordination of Cauſes and Effects, there muſt Neceſſarily be a Cauſe in Nature, Prior to the Reſt, Uncauſed; Or thus, where there is a Series, in which the Exiſtence of One Thing Depends upon Another, the Exiſtence of This again upon ſome Other, and ſo Upwards, as the Caſe ſhall be, there muſt be ſome Independent Being, upon whom it doth Originally Depend.

Mr. Wollaſton's Religion of Nature Delineated §. 5.

THIS is Exemplified in the Following Manner; If Z (Some Body) be Put into Motion by Y, Y by X, and X by W, it is Plain, that X Moves Y, and Y Moves Z, only as they are Firſt Moved, X by W, and Y by X: that Z, Y, X, are *Moveds*, or rather $Z+Y+X$ Taken together, are *One Moved*: that W Stands here, as the Firſt Mover or Author of the Motion, Unmoved by any Other: That therefore, without W, there would be a *Moved*, without a *Mover*, which is Abſurd: And Laſtly, that of what Length ſoever the Series may be, the Caſe will be ever the Same, *i. e.* if there be no *Firſt Mover* Unmoved, there muſt be a *Moved*, without a *Mover*.

BUT this Grand and Oſtentatious Way of Reaſoning of $Z+Y+X$ Depending upon the Firſt Mover W, as if there was ſomething Mathematical in the Demonſtration, is nothing more than what is Aſſerted in the Second Propoſition of the Foregoing Demonſtration, that there has Exiſted from Eternity ſome One Unchangeable and Independent Being; if not, there muſt have been an Infinite Succeſſion of Changeable and Dependent Beings Produced One from Another, in an Endleſs Progreſſion, without any Original Cauſe at all; And the Anſwer, which may be made to ſuch a Kind of Reaſoning is the Same, and therefore need not be here Repeated; It alſo Falls in with Mr. *Locke's* Way of Conſidering theſe Matters, which has been before Inquired into, and Conſequently on that Account alſo does not Require a Particular Confutation.

WHAT I ſhall here farther Obſerve, in Relation to this *Algebraick* Proof of a Supreme Being, is, that if Z is not put into Motion by Y, Y by X, and X by W, or $Z+X+Y$ by W; but if Z Moves Y and Reciprocally Y Moves Z, Y Moves X and X Moves Y, *&c.* or $Z+X+Y$ Moves W and W Moves $Z+X+Y$ by any Neceſſary, Eternal, and Fixed Laws of Nature, or Attraction, or Gravitation, this whole Reaſoning Falls to the Ground.

BESIDES, to Prove there muſt be an Original Mover, as W, it is Suppoſed, that Z is Moved by Y, Y by X, and X by W; that is, if we go no Higher than Y, Y is the Firſt Mover, if no Higher than X, X is the Firſt Mover, and if no Higher than W, W is the Firſt Mover; ſo that to Prove an Original or a Firſt Mover, it is Suppoſed there is One; which is a very Emphatical, as well as a Sagacious, Way of doing it. What is afterwards ſaid of a Chain, with it's Links, Hanging in the Air, it well enough Explains, what Notion this Philoſophy has of an Eternal Progreſſion of Motion, or Things, without an Original Cauſe, but is not Significant to the Proving, either for or againſt it.

AND now, after thus Freely Inquiring into the Demonſtrations of a God by the Preſent Philoſophers, it will not, nor can be, Imagined, that I Deny ſuch a Being, but the Preſent Philoſophick, or Metaphyſical, Demonſtrations of One; Since I have Shewn, that a Supreme and Intelligent Being is Strictly Demonſtrable from a True Underſtanding of Nature, and the Infinite Forces, and the Infinite Variety of Actions in it, Diſpoſed and Adjuſted in the Wiſeſt Order, and according to the Exacteſt Oeconomy; That the Chriſtian Religion, of the Truth and Divinity of which I have Formerly given a Demonſtration, does, Independent of Reaſon and Philoſophy, Afford us another Unconteſted Proof of a like Being; And that the Providences of God, Exerciſed by him over Nations and Kingdoms, and Particular Perſons, are full and Convincing Evidences of his Exiſtence.

I have only Impartially and Sincerely Inquired into the Cauſe betwixt the Atheiſt and the Deiſt, as it has been hitherto Managed; and upon the Sum, I cannot but Conclude, that Neither of them are much Acquainted with the Merits of it; becauſe they Underſtand little or nothing of Nature, the Comprehenſion of which is, in ſome Degree, Entirely Neceſſary to the Determining a Controverſy, which Concerns the Author, and Origin, of the whole Syſteme; which yet is Neglected, and all is Reſolved into General, Metaphyſical, and Abſtracted Terms, which a Man ſhall not be Extremely the Wiſer for Knowing.

I have been likewiſe more Open, Explicit, and Unreſerved in this Inquiry, that Men, by Seeing the Difficulties, which Attend ſuch Abſtruſe Speculations, may be more Ready to Acknowledge the Neceſſity of, as well as the Infinite Wiſdom and Goodneſs of God in, his making a Revelation of himſelf to the World, in the ſeveral Ages and Periods of it, by the Patriarchs before and after the Deluge, by *Moſes*, and at Laſt, by his own Son, the Great Fullfiller of the Law, and of Prophecies, the long Expected Meſſiah of Mankind, the Divine Word, or Λόγος, and the Eternal Begotten of his Father.

But what has ſtill Farther Confirmed me in this Way of Inquiry is, that I find very little Difference betwixt the Atheiſt, and the Deiſt, as to their Notions, or their Practices in Moral Matters, they are Equally upon the Foot of Reaſon, or the Connexion of Ideas, Equally Deſpiſers of the Chriſtian Goodneſs, and Equally Strangers to Philoſophy and Nature; Which is not to be Known by Metaphyſical Abſtractions, nor yet, even by ſome Fanciful Experiments made in Chymiſtry, Anatomy, Hydroſtaticks, Mechanicks, or any other Way; but by Conſidering the whole Order, and Syſteme of it, which Lies Plain to every One's View, and is the Subject of every One's Obſervation; For is it not very Abſurd, that Philoſophers ſhould Pretend to give an Account of Theſe, or the Other, Experiments, made by a Few, when they cannot Solve any One Common Phænomenon, or Appearance, in the Whole Fabrick?

I ſhall only here Add, that it is a Lamentable Caſe, and which Deſerves to be Conſidered by Learned and Good Men, that all our Philoſophy is Derived to us from Popiſh Countries, which Abound in Superſtition; Namely, from *Italy* and *France*; and it is not Impoſſible, or Unlikely, but that the Fopperies and Follies of the One, which could not be Reliſhed by Underſtanding and Thoughtful Men, might give an Impreſſion of Atheiſm, or Deiſm, to the Other; and Hence it was, that *Galileus* in *Italy*, and *Des-Cartes* in *France*, it is Probable, Met, the Firſt with ſuch Severe Uſage, and the Laſt with ſo little Encouragement.

Dr. Clarke, Prop. 10.

As to the Freedom of Will, or Man's being a Free, and not a Neceſſary, Agent, nor Determined by any Impulſes from External Nature; It is as Evident, that we have a Power of Beginning Motion, and Directing our Actions this or that Way, or in this or that Manner, as we Pleaſe, as that there is ſuch a Thing as Motion, or Thought, and there Wants no more, nor other, Arguments to Prove it.

And, as to what is Alledged by Mr. *Hobbs*, or *Spinoza*, that the Will may Poſſibly be Determined by ſome Impulſes from External Nature; If it were ſo, it would Render it ſo Whimſical and Ridiculous a One, which ſhould by Fits and Starts, upon every Determination of our Will to Move our Hand Backwards or Forwards, be Suppoſed to make it's Inſults upon our Minds, that no Rational or Conſidering Man can ever Fall in with ſuch an Abſurd Notion of it.

I have all along here Repreſented the Argument, as Lying betwixt the Atheiſt, and the Deiſt, in Order to make the Controverſy General, but am far from Imagining, or Suppoſing, that the Philoſopher, here Referred to, is to be Ranked with Either, for whom, as a Perſon of Religion, and Learning, tho' I Diſſent from many of his Notions in Both, I have a Great Eſteem and Value.

CHAP. VI.

Concerning the Moral Relations of this Philoſophy.

Of the Moral Relations of this Philoſophy.

Mr. Locke's Eſſay, Chap. 28. Book 2.

§. 1. AFTER a Diſtinction of Proportional, Natural, and Inſtituted or Voluntary Relations, we are Directed to thoſe, which are Moral; the Firſt Three Sorts, whether Juſtly Diſtinguiſhed or Explained, we ſhall not now Inquire, ſince they are not of any Concern to the Preſent Argument.

Art. 1. As to Moral Relations, the Account in General is, that they are Human Actions with their Various Ends, Objects, Manners, and Circumſtances, Formed into Diſtinct Complex Ideas, and from thence may be Termed ſo many Mixed Modes, a great Part whereof have Names Annexed to them, as Gratitude, and Polygamy; the Underſtanding the ſeveral Simple Ideas, which Enter into the Whole Compoſition of theſe Mixed Modes, to which we Affix Certain Names, is Neceſſary to Denominate them Clear and Diſtinct.

Art. 2. And their Conformity to, or Diſagreement with, ſome Law, to which they are Referred, that is, whereby Good or Evil, Pleaſure or Pain, is Drawn on us from the Will and Power of the Law-Maker, is Neceſſary to Denominate them Morally Good or Bad.

Art. 3. Which Good and Evil, Pleaſure or Pain, Attending our Obſervance, or Breach, of the Law, by the Decree of the Law-Maker, is, what we Call Reward and Puniſhment.

Of theſe Laws, it is Said, there are Three Sorts, with their Three Different Enforcements, or Rewards and Puniſhments; For, ſince it would be Utterly in Vain to Suppoſe a Rule Set to the Free Actions of Man, without Annexing to it ſome Enforcement of Good and Evil, to Determine his Will, we muſt, whenever we Suppoſe a Law, Suppoſe alſo ſome Reward or Puniſhment Annexed to that Rule.

The Laws, to which our Actions are Referred, are Suppoſed to be either, the Divine Law, or the Civil, or the Law of Opinion and Reputation, to which is Subjoined the Philoſophical Law; by the Relation theſe Complex Ideas, or Mixed Modes, Bear to the Firſt of theſe Rules, Actions are Sins or Duties, to the Second, Criminal or Innocent, to the Third, Virtues or Vices.

Art. 4. Firſt, as to what Reſpects the Divine Law, by that is Meant, what is Promulgated to Men by the Light of Nature, or the Voice of Revelation, neither of which, it is Said, any One can be ſo Brutiſh as to Deny; This Section goes on to Tell us, that God has a Right to give a Rule, that we are his Creatures, that he has Wiſdom and Goodneſs to Direct our Actions to that, which is Beſt, that he has Power to Enforce it by Rewards and Puniſhments, of Infinite Weight and Duration, in another Life, for no Body can Take us out of his Hands; This is Said, however, to be the only True Touch-Stone of Moral Rectitude, and by Comparing them to this Law it is, that Men Judge of the moſt Conſiderable Moral Good or Evil of their Actions, that is, whether as Duties, or Sins, they are like to Procure them Happineſs, or Miſery, from the Hands of the Almighty.

Mr. Locke, §. 8. Chap. 28. Book. 2.

Art. 5. Second, as to the Civil Law, which is the Rule Set by the Common-Wealth to the Actions of thoſe, who belong to it; This is Propoſed, as another Standard, to which Men Refer their Actions, to Judge whether they be Criminal or not; This Law no Body is Said to Overlook, the Rewards and Puniſhments, that Enforce it, being Ready at Hand, and Suitable to the Power that makes it, which is the Force of the Common-Wealth, Engaged to Protect the Lives, Liberties, and Poſſeſſions of thoſe, who Live according to it's Laws, and which has Power to take away Life, Liberty, or Goods, from him, who Diſobeys, which is the Puniſhment of Offences Committed againſt this Law.

Art. 6. Third, as to the Law of Opinion and Reputation; That is, in this Philoſophy, Eſteemed to be the Meaſure of what Men Generally Call Vertue and Vice; ſince, where they Stand for Actions, which, in their own Nature, are Right or Wrong, they are Said to be ſo far Coincident with the Divine Law, and in the

Particular Inſtances of their Application, thro' the ſeveral Nations and Societies of Men in the World, are Conſtantly Attributed only to ſuch Actions, as, in each Country and Society, are in Reputation or Diſcredit; and, that this is ſo, will Appear to any One, who Conſiders, that tho' that Paſſes for Vice in one Country, which is Counted a Vertue in another, yet every where Vertue and Praiſe, Vice and Blame, go together; thus *Virgil* Says, *Sunt ſua Præmia Laudi*; and *Cicero*, *Nihil habet Natura Præſtantius, quàm Honeſtatem, quàm Laudem, quàm Dignitatem, quàm Decus*, which, he Tells us, are all Names for the ſame Thing.

THIS is the Language of the Heathen Philoſophers, who well Underſtood, wherein their Notions of Vertue and Vice Conſiſted; and tho' Vertues and Vices Changed in Different Communities, yet, ſince nothing can be more Natural than to Encourage with Eſteem and Reputation That, wherein every One finds his Advantage, and to Blame and Diſcountenance the Contrary, 'tis no Wonder, that Eſteem and Diſcredit, Vertue and Vice, ſhould, in a great Meaſure, every where Correſpond with the Unchangeable Rule of Right and Wrong, which the Law of God has Eſtabliſhed, there being nothing, that ſo Directly and Viſibly Secures, and Advances, the General Good of Mankind in this World, as Obedience to the Laws he has Set them, and nothing that Breeds ſuch Miſchiefs and Confuſion, as the Neglect of them; whereby, even in the Corruption of Manners, the True Boundaries of the Law of Nature, which ought to be the Rule of Vertue and Vice, were Pretty well Preſerved; ſo that even the Exhortations of Inſpired Teachers have not Feared to Appeal to Common Repute, *whatſoever is Lovely, whatſoever is of Good Report, if there be any Vertue, if there be any Praiſe*, &c. *Phil.* 4. 8. and of this Law, the Enforcements, or the Rewards or Puniſhments, Annexed to it are Commendation and Diſcredit.

THESE Moral Relations are farther Accounted, as Certain and Diſtinguiſhable, as any Perceptions, which we have; for, altho' the Rule were Falſe, to which we Referred and Compared our Actions, yet the Conformity, or Non-Conformity, of our Actions to that Rule, would be Viſible enough.

The Demonſtration of the Being of a God, Neceſſary and Fundamental to any Syſteme of Morality, which Being is not Proved.

§. 2. HAVING given as Short an Account, as we could, of this Syſteme of Morality, we ſhall Conſider every Part of it.

BUT Firſt, we muſt Obſerve in General, that this Philoſophy takes a very Inaccurate Method to Inculcate to us any Principles of Morality at all, before it has Proved the Being of a God, which is not done, Excepting in Book the 4*th*. Eſpecially, ſince it there Acknowledges, that the whole of Morality does Depend upon the Demonſtration of ſuch a Being.

SECOND, if it be ſo, neither the Civil Law, nor the Law of Opinion and Reputation, can be any Rules, by which we can Judge of Moral Actions; ſince, if Morality Depends upon the Being of a God, the Laws of a Common-Wealth, or the Law of Opinion and Reputation, or Plain Nature, cannot be any Rules or Standards, by which our Actions can be Denominated Moral; ſince Men may make Laws and Decrees for the Government of a Community, and they may alſo Think Well or Ill of Certain Actions, and Blame or Commend them, whether there is any ſuch Being, as a God, or not, and yet they ſhall not be Moral upon that Account.

THIRD, If our Moral Actions ſhall be Said to Conſiſt in their being our Natural or Human Actions, and thoſe Referred to ſome Rule, or other, in General, and therefore, that our Moral Actions are to be Diſtinguiſhed from what is Properly Called Morality, which Suppoſes the Exiſtence of a God, the Fallacy will be Plain, that this Philoſophy Confounds Moral, and Natural or Human Actions; and under thoſe Equivocal Terms would Inſinuate, that they are the Same; for, if our Natural or Human Actions, by being Referred to the Eternal Rule of Right and Wrong, or to the Law of God, from thence may be Called Moral, as Morality is Said to Depend upon the Truth of Almighty God's Exiſting, they cannot without the Greateſt Impropriety and Confuſion of Language and Words be Termed Moral, when Referred to the Law of a Common-Wealth, or to That of Opinion and Reputation; ſince the Actions, being only Natural or Human Ones in themſelves, upon Different References to Diſtinct Rules, muſt likewiſe Receive Different Denominations; and if the one are Moral, the other will be more Juſtly Accounted, Agreeable to the Rules, to which they are Referred, Civil or Sociable.

§. 3. ALTHO', what we have Said in the Foregoing Section, would be a Sufficient Confutation of this Doctrine, yet we ſhall not Refuſe to Conſider every Branch of it. It is Said, in the Firſt Place, that Human Actions, with their Various Ends, Objects, Manners, and Circumſtances, are Formed into Diſtinct Complex Ideas, and from thence may be Termed Mixed Modes; In Anſwer to which, we ſhall FIRST Say, that Nothing of this is Certain, according to the Senſe of the Preſent Philoſophy, which makes all Certainty to Conſiſt in the Connexion of Ideas, or in the Agreement or Diſagreement of them, as they Lie, and are Abſtracted, in the Mind; Whereas Human Actions, and the Circumſtances, which Attend them, Depend upon Things, which are External to us, of which we cannot be Certain by any Connexion of Ideas. And Indeed, this is the General Failure in the Preſent Philoſophy, which was Avoided by the *Carteſian*, which Proved the Being of a God by Ideas, as they Lay in the Mind, and from thence, that God would not Deceive us in Reſpect of External Beings, in which there was ſome Kind of Conſiſtency; On the Contrary, there is nothing can be more Unreaſonable, than to make Certainty Conſiſt in the Agreement or Diſagreement of the Ideas of our Minds, and yet at the ſame Time to Speak of Human Actions with their Various Circumſtances, and their Relations to a Rule, as Matters, in which we may Arrive at Certainty, when there can be no Connexion of Ideas in them; becauſe Human Actions, and their Reference to a Rule, are External to the Abſtracted Ideas of our own Minds.

Article the Firſt of Section the Firſt, Conſidered and Examined.

SECOND, Let us Inquire into the Method, which this Philoſophy Teaches in Compounding Mixed Modes; we are therefore Directed, in the Firſt Place, to be Clear and Diſtinct in our Simple Ideas, into which ſuch Modes are always Reſolved, and, afterwards, to be Certain, that we have neither More nor Fewer Simple Ideas in the Complex One, than what Properly belong to it, and to which we have Affixed a Name, as that of Gratitude, or Polygamy, *&c.*

Mr. Locke, Chap. 28. Book. 2. §. 18.

Chap. 28. §. 4.

BUT Firſt, the Mind does not ſeem to take any ſuch Method of Compounding it's Ideas from Simple Ones, which is every where Inculcated to us, thro' this whole Philoſophy; ſince all our Ideas cannot be ſo Juſtly Divided into Simple and Complex, as Voluntary and Involuntary, or Abſtracted and Actual Perceptions; For our Complex Ideas of a Man, of Beauty, Gratitude, Polygamy, of Gold, of a Stone, or the Like, are ſo many Single and Uniform Impreſſions upon the Mind, as much as any Simple Idea of Extenſion, Colour, or any Individual Property of a Man, of Space, or of Matter, is; Thus, if we Actually See a Piece of Gold, it is as much one Image in the Mind, as the Yellow, or the Extenſion of it; if we See a Man, the Picture, or Repreſentation of him, is as Entire and Uniformly One, as that of his Eyes, or his Face; and the Same may be Said of an Army, or of a Multitude of Men Ranged in our View, in Reſpect of a Single Perſon, which has been Affirmed of a Single Perſon, in Reſpect of any Part of him; Thus likewiſe it is in Human Actions, as in Murder, where we Actually behold One Man Cutting Another's Throat; in Gratitude, where we Actually See a Kindneſs done and another Kindneſs Returned for it; in Polygamy, where we Actually See a Perſon Married to, and Cohabiting with, many Wives; Theſe all are Obſerved ſeverally by the Mind, whether Gold, a Stone, a Man, an Army, Murder, Gratitude, or Polygamy, and ſo of the Reſt, as one Picture or Repreſentation, as much as any Simple Ideas we can Mention, as the Yellow, or Ductility, or Weight of Gold, *&c.* and therefore, Really are no more Complex Ideas, the One, than the Other; All, which they Differ in, is, the Variouſneſs of the Repreſentation, not the Uniformity of it, and in the ſame Manner, as One Simple Idea Differs from Another, which is Diſtinct from it.

Mr. Locke, Chap. 12. 13. 14, *&c.* 22. 23. 28. Book 2.

IF it be Said, that the Repreſentations of Extenſion, or Colour, *&c.* may be Termed Simple Ideas, becauſe they are more Single and Undivided than the Entire Images or Pictures in the Mind of Gold or a Stone, we may as Truly Say, that Man therefore is a Simple Idea, tho' it is Reckoned in this Philoſophy amongſt the Number of Complex Ones; ſince it likewiſe, in Reſpect of an Army, is more Single and Undivided too; For it does not Enter into the Nature of Simpleneſs or Uniformity of Impreſſion in our Ideas, the Parts of which any Single Impreſſion is made up, no more than a Collection of Units in Numbers is an Hindrance to our Seeing ſuch a Number at One View; For the Idea we have of 4 or 100 is as Clear and Diſtinct, and is as Simple and Undivided in the Idea, which we have of them, as that of Unit; Seeing, if we Conſider 4 or 100, as Conſiſting of 4 or 100 Units, and it is Imagined to be Complex upon that Account, Unit alſo, if Referred to the Parts of which it is Compoſed, may be Equally Compounded, for $\frac{4}{4} = \frac{100}{100} = 1$; If this were not ſo, but the Parts of any Idea are to be Inquired, to Know, whether it is a Simple One or not, neither any Extenſion, or Colour, or Figure, or Motion, *&c.* which we Actually Beheld, would be a Simple Idea; becauſe it would Conſiſt of Parts, in Reſpect of which therefore, it would Neceſſarily be a Complex One.

Thus alſo, the Idea of a Chimera, or a Centaur, when Painted upon a Paper, or Repreſented in the Mind, is One Uniform Image, or Picture, in Either, and Differs from thoſe we have Mentioned, in nothing elſe, Excepting, that the One is Arbitrary or Voluntary, whilſt the Other is Involuntary, the One Abſtracted from Things as they Exiſt, the Other Actual, or Suitable to their Exiſtence; and all that Philoſophy would have to do, would be to Inquire into the Nature of theſe ſeveral Ideas, how they are Produced in us, what the Various Relations of them, and of what Parts they Conſiſt.

On the Contrary, the Preſent Philoſophy Proceeds in a quite Different Method, and Suppoſes the Mind to Gain Firſt it's Simple Ideas, and Then to make it's Compoſitions from them; whereas, by what Foregoes, it is Certain and Evident, that the Mind Receives thoſe Ideas, which are Termed Compounded, in the ſame Manner, as it does thoſe, which are Named Simple, where they are both Actual or Real Perceptions; But, if they are not, and the One is Actual, and the other a Compoſition made by the Mind, it is ſo far from being True, that the Mind Forms it's Complex Ideas from it's Simple, that it Really Forms it's Simple from it's Complex; For from the Actual Perception of Extenſion, Yellow, and Weight, in Gold, it does not Form the Notional or Complex Idea, which is made up of Extenſion, Colour, and Weight, which Inheres in an Unknown Subſtance; but from it's Actual Perception of Gold, in Seeing and Weighing it, it Forms or Abſtracts the Diſtinct Properties of it, and then makes thoſe General Ones, which it likewiſe finds in other Bodies, of Extenſion, Colour, and Weight; Thus alſo, the Mind does not Form an Army from the Actual View of a Single Perſon, but from the Actual View, or Report, of an Army Divides it into Single Perſons; and the ſame may be Said of Human Actions, as Murder, Gratitude, or Polygamy, which the Mind Obſerves or Hears in Groſs, and afterwards Reſolves, by it's own Abſtractions, into the ſeveral Parts and Circumſtances, which Enter into thoſe Actions, and does not Perceive the Different Parts and Circumſtances of them, and then Form them into Complex Ideas.

This, if Truly Conſidered, would be a Full Anſwer to the Preſent Eſſay upon Human Underſtanding, which is Spent in Rangeing our Ideas into Simple and Complex Ones, and Placeing them in ſeveral Diſtinct Claſſes, and under it's own Arbitrary Denominations; that is, as we before Obſerved, it's Chapters of Certainty and Probability were nothing elſe than a Nomenclature, Suitable to the Definitions it had Arbitrarily given, ſo the Philoſophy of Simple and Complex Ideas, and Conſequently, this whole Eſſay upon the Mind is no other; Namely, than an Adjuſtment of our Ideas under thoſe Names, which it is Pleaſed to make Uſe of, without their being Agreeable to the Reality of Things.

It has been Said, that if our Simple and Complex Ideas are, the one Formed by the Mind, and the other by our Actual Perceptions, the Mind is rather Concerned in Abſtracting the Firſt, than in Compounding the Laſt.

Let us yet now Grant the Contrary, that Simple Ideas are the Firſt in our Knowledge, and that, what are Called Complex Ones, are Deduced from them; It will Evidently Follow, that all this Entire Syſteme, however Celebrated by the Free-Thinkers of the Age, is no more, nor no better, than One Continued Error.

Mr. Locke, Chap. 2. 3. 4. 5. 6. 7. Book 2.

The Simple Ideas from one Senſe only are Light and Colours, with their ſeveral Diſtinctions, from our Sight; all Kinds of Noiſes, Sounds, and Tones, from the Ears; the ſeveral Taſtes and Smells from the Noſe and Palate; the moſt Conſiderable from the Touch are Heat, Cold, and Solidity; the reſt, Conſiſting almoſt wholly in the Senſible Configurations, as Smooth, and Rough, or elſe more or leſs Firm Adheſion, of the Parts, as Hard and Soft, Tough and Brittle; The Ideas from Diverſe Senſes are Space or Extenſion, Figure, Reſt and Motion, from the Eyes and Touch; Simple Ideas from Reflection are Thinking and Willing, the Modes of which are Remembrance, Diſcerning, Reaſoning, Judging, Knowledge, Faith, *&c*; The Simple Ideas from Senſation

tion and Reflection are Pleaſure or Delight, Pain or Uneaſineſs, Power, Exiſtence, Unity; and theſe are Said to be the Materials of all our Knowledge, which the Mind cannot Create to it's ſelf, but may from thence Form Various Combinations, and Compoſitions, at Pleaſure.

IN Reply to which, either all theſe Simple Ideas are Actually Experienced in the Mind Singly, Diſtinctly, and Abſtractedly from all Others, or not; If they are, we Perceive Colours in Body, without Extenſion; Sound, Abſtracted from that, which is Sonorous; Taſte, without Feeling what we Taſte; Smell, without Conſidering, that which is Odorifick, *&c.* which is Manifeſtly Falſe, and Contrary to every Senſe, Chap. 2. §. 1. Book 2. which we have; 'tis True, they are Diſtinct Perceptions in us, as the Warmth of Wax, and the Softneſs of it, but they come Joined together in the Mind, and the One is not Felt, Excepting at the ſame Time with the Other; If they are not Perceived by the Mind Diſtinct, or Abſtracted, from all Ideas beſides, the Way, by which they become Simple Ones from Compounded, is by the Arbitrary Pleaſure of the Mind, in Separating the One from the Other in it's own Thoughts; Contrary to the Affirmations of this Philoſophy, which Reſigns the making of Complex Ideas, not Simple, to the Reſolutions of the Will; for, if the Mind is Allowed the Prerogative to Diſtinguiſh, and Conſider, only the Warmth of Wax, for Inſtance, Separate from the Softneſs of it, tho' they both Enter into the Mind at Once, and from thence they ſhall be Eſteemed the Materials of all our Knowledge, for the ſame or a greater Reaſon, it may Join them together, and more Ideas, as the Extenſion and Colour of it, if they are Perceived at the ſame Time; and Conſequently, not Simple, but Complex Ideas, will be the Materials of it.

FARTHER, if the Simple Ideas of any Object, Preſented to the Mind, may be Abſtracted by it from all Others, and from ſuch Simple Ideas or Abſtractions all our Knowledge Proceeds by the Different Combinations and Compoſitions of ſuch Ideas; that is, if we Conſider Warmth in General, or Softneſs, without any Reference made to the Wax, which Produces it, and the ſame may be Said of Heat or Cold, of Figure, Solidity or Extenſion, or Roughneſs and Smoothneſs, *&c.* they will become Ideas Perfectly Abſtracted in our own Minds, from whence all our Knowledge will be Derived.

ON the Contrary, ſuch Abſtracted Ideas are ſo far from Supplying us with any True Knowledge, that it will be Difficult to Determine, what they themſelves are; They are not the Extenſion we See, or the Solidity we Feel, or the Sound which we Hear, *&c.* for if they were, there would be ſome other Perceptions, which would Enter the Mind together with them; Thus in Extenſion we See Colour, Figure, Reſt, and even a Cloſer or Looſer Extenſion; as in Gold, or in Silver, we See ſuch Extenſion is more Compact and United than in Cork, or a Feather, tho' there is no Senſible Interruption of the Parts; and yet our Abſtracted Idea of Extenſion, in General, throws out all theſe other Perceptions, and only Conſiders it, as one Undiſtinguiſhed Idea; The ſame may be Said of Solidity, or even, of any Particular Colour, as Red, or Blue, or Purple; or of Sound, as a Grave, or Acute, or a Mellow, or a Shrill, *&c.* or of Taſtes, as Sweet, Bitter, Sharp, and the like; and ſo of Smells; All theſe Simple Ideas, when they are Abſtracted from our Actual Perceptions and Senſations, are Names only, and Denominate General Ideas, Made and Created by the Mind, without any Thing Real belonging to them; But to make the Creatures of our Minds the Foundation of Knowledge, and to Propoſe them as the Materials of it, is, at the beſt, Reduceing the Whole of it into a Chimera, the very Notion of which Conſiſts in Compounding Ideas, and Putting them together in an Arbitrary Manner, which have been Formed and Lodged in the Mind by a like Arbitrary Abſtraction.

BESIDES, theſe Simple Ideas, which are Suppoſed to be the Materials of our Knowledge, may be Combined and United together at the Will and Fancy of the Mind, or not; If they may, we may Form a Complex Idea of the Sweetneſs of a Violet, the Redneſs of a Roſe, the Taſte of a Damaſcene, the Solidity of Gold, and the Sound of a Virginal; which will make a very Incongruous and Inconſiſtent Compoſition, and more Abſurd and Chimerical than any Picture or Fiction, which the Poets have hitherto Tranſmitted to us; If theſe Simple Ideas may not be Joined together, as we Pleaſe, in Order to Produce Real Knowledge in us; What other Rule is there to Direct us in Duly Connecting them, Excepting our Experience? That is, we find in Fact, that the Purple, and Sweetneſs, and Softneſs of a Violet, Exiſt together, and therefore we may Combine them into one Complex Idea; that a Solid Reſiſtence, a Yellow Colour, Weight, *&c.* are Aſſembled into one Complex Idea of Gold; and therefore we Juſtly may Form ſuch a Compoſition.

But if this is all, as I am Apt to Think it is, the Preſent Philoſophy will have Taken a great Deal of Pains to little Purpoſe, whilſt it Endeavours to Reſolve all our Knowledge into Simple Ideas, which whenever it comes to Compound, it is Obliged to Put them together again, Juſt as it at Firſt found them; a Method of Proceeding, which is Exactly Repreſented by the Proverb of *Penelope's* Web, Applied to thoſe who Trifle; For what is this, but Unweaving, if I may Call it ſo, and Disjoining the Properties of Things as they Exiſt, in Order to Weave and Connect them into One again? Which Shews at Leaſt the Wonderful Force of Truth and Nature, that, whilſt the Mind of the Philoſopher is Tugging and Labouring to Tear and Rend, by the Power of his Thoughts, One Property of Being from Another, which Exiſts with it, Nature, by a Kind of Invincible Obſtinacy, Recoils, Forcibly Returns to it's own State, and Carries the Proud and Reluctant Mind of the Philoſopher along in Triumph with it.

Article the Second of Section the First Conſidered.

§. 4. Having Conſidered the Nature of Simple and Complex Ideas, and Shewn the Chimerical and Abſurd way of Proceeding in this Philoſophy thereupon, we are now to Conſider the Reference of theſe Complex Ideas, or Mixed Modes, to a Rule, which is Neceſſary to Denominate them Moral, according to their Agreement or Diſagreement with it.

In Anſwer to which, we ſhall Suppoſe our Complex Ideas to be made in that Manner from our Simple Ones, which this Philoſophy Preſcribes to us, altho' that Method has been already Confuted; Let theſe Complex Ideas therefore, or Mixed Modes, be Produced in the Mind one way or other, the Queſtion is, whether their Reference to a Rule in General can Denominate them Moral, or not; Which it is Plain it cannot; For what Connexion is there, or can there be Suppoſed, betwixt a Complex, or even a Simple Idea, and a Rule, which has no Analogy to it? Thus, for Inſtance, let us take, in the Firſt Place, the Simple Idea of the Redneſs of a Roſe, or the Fragancy of a Violet; What Rule can either of thoſe Ideas be Referred to, with which they Correſpond, ſince they are only Anſwerable or Analogous to themſelves; If farther we take the Complex Idea, as it is Called, of a Violet, or a Roſe, or of Gold, it is Plain they cannot be Referred to any Rule, to which they Agree or Diſagree by any Connexion of Ideas; and the ſame may be Said of any Complex Idea of Human Actions, as Gratitude, Murder, or Polygamy; that is, theſe are all Human Actions, which are Expreſſive of themſelves, and nothing more; For if they are Referred to any Thing beſides, as the Divine Law, the Civil, or that of Reputation, there can be no Connexion of Ideas betwixt them, in which Certainty is Said to Conſiſt; Thus in Murder, which is the Action of Killing a Man, there is no Reſemblance or Connexion betwixt That, and the Divine or Human Law, or the Law of Honour and Reputation, as to the Agreement or Diſagreement of it with them; Seeing, when we Say One Idea is Connected with Another, as 4 with 4, or Unit with Unit, the Connexion is Evident, or when 4 is not the Same with 5, the Diſagreement of thoſe Ideas is Manifeſt; But, when we Compare a Complex Idea of Murder, or Polygamy, that is, the Ideas we have of thoſe Actions, with the Divine Law, that is, with the Authority of God over Human Actions, ſo as his Will ſhould be a Rule to them, the Connexion is ſo far from being Certain and Viſible, that there is None.

'Tis True, when theſe Human Actions, for Inſtance, are Prohibited by the Divine Law, and when we at any Time find ſuch Actions Committed, we Perceive they are the Same with thoſe Forbidden; betwixt which, there is an Apparent Connexion; but then, what is the Rule? Not the Action Expreſſed, but the Authority of the Divine Law, which Forbids it, and the Sovereignty and Empire of Almighty God over his Creatures, betwixt which and Murder, or Polygamy, there is not the Leaſt Similitude or Reſemblance; For we may more Agreeably Connect the Ideas of a Mite and an Elephant together, than Human Actions with the Supreme Will of an Almighty Law-Giver; and yet all Certainty is Said to Conſiſt in the Connexion of Ideas, and therefore the Preſent Foundations of Morality muſt be Uncertain. What has been Said of the Divine Law, may be Applied to the others of the Common-Wealth, or of Honour, with Juſt Allowances to the Different Natures of the Law-Givers.

Article the Third of Section the Firſt Conſidered and Examined.

§. 5. The next Thing Propoſed to us in theſe Moral Relations is, that Human Actions are Denominated Good or Bad, according as they Draw on us Pleaſure or Pain, from the Will of the Law-giver; that is, in the Divine Law, Happineſs or Miſery; in the Human, Deprivation of Life Liberty, and Poſſeſſions or a Protection of all of them; in the Law of Honour and Reputation, Applauſe or Diſgrace and Reproach.

If therefore Actions are to be Denominated Good or Evil, by the Pleaſure or Pain, which they bring upon us, the Epicure is the moſt Vertuous and the Beſt Man in the World; the Covetous Man, who Delights in Riches, is a Good Inſtead of an Ill-Natured Perſon; and the Prophane and Atheiſtical, whilſt they take Pleaſure in being ſo, and this Philoſopher himſelf, whilſt he Rejoyces in his Ideas, are all Moral and Excellent Men; For the Queſtion is not here, whether a Man has Reaſon to be Pleaſed, but whether he Really is; and Indeed, where we Recede from the Truths of Religion, what is there Offered to us in this Wiſe Syſteme of Morality, which was not more Accurately Conſidered by the Heathens, and by *Tully*, in Particular, *De Finibus Bonorum & Malorum*, than has been done by this Sagacious Reaſoner? For, if Pleaſure and Pain are the Chief Things to be Purſued or Avoided, what will Reſult from ſuch a Doctrine, unleſs, that the ſeveral Opinions, Recounted by that Incomparable *Roman* Concerning the *Summum Bonum*, are all True? If *Epicurus* Thought, there was the greateſt Satiſfaction in Corporeal Pleaſures, (which yet is to be Doubted, and who rather ſeems to have had the Notions of Mr. *Locke*, that Pleaſure, in General, was what we ſhould Chiefly Aim at), all the Vices of Human Life would be Really Vertues; If there was the Greateſt Pleaſure in Temperance, Sobriety, and Chaſtity, Thoſe would be Vertues; If in Wiſdom and Learning, or in Riches or Honours, *&c.* Thoſe would likewiſe Fall under the ſame Denomination; From whence it is Evident, that, if Human Actions are to be Termed Good or Evil, according as they bring Pleaſure or Pain along with them, Vertue by the ſame Reaſon may be Called Vice, or Vice Vertue; Since, as we before Said, the Inquiry is not, what Rationally ought to Produce a Pleaſure or Satiſfaction in us, but what Really does.

It is farther to be Objected againſt this Philoſophy of Pleaſure and Pain, being the Meaſures of Vertue and Vice, or Good and Evil, that Men may be Vertuous and Good under the greateſt Afflictions, tho' they were not Certain of being Rewarded for their Piety, either in this World, or the next; Becauſe there is ſomething Intrinſically True in the Nature of Things, which muſt, and will, Determine us to Think Agreeably to them, whatever the Conſequences of it are; Thus, if there were no Rewards or Puniſhments in this Life, or another, or if there were, the Mind may Conſider Things Abſtractedly in themſelves, without any Reference to a Rule, which may bring Pleaſure or Pain upon it, and without Regarding, what will Enſue, whether Pleaſure or Pain there-from; For, if I Prove, that the Three Angles of a Triangle are not Always and Abſolutely Equal to Two Right Ones, I may Expect Reproach from Aſſerting it, and all that could Induce me to Affirm it, is nothing elſe, Excepting what Appears to be Truth, without the leaſt Reſpect to any Pleaſure or Pain, which I ſhall Receive from ſuch an Affirmation, being Entirely Regardleſs of Either; and ſo it is in Human Actions; for thoſe, which are Vertuous and Excellent, have ſome Innate and Peculiar Diſtinctions in them, which would Denominate them Good; and on the Contrary, Vicious and Criminal Ones would be Bad; altho' Pleaſure ſhould belong to the Laſt, and Pain ſhould be the Conſtant Reſult of the Firſt, to the Perſon, who Committs them; So little has Pleaſure or Pain, which are the Conſequences of our Actions, to do, in Aſcertaining the True Nature of them.

If therefore a Kingdom was the Reward of Ingratitude or Murder, as it has been ſometimes, even in our own Hiſtories, and the Greateſt Innocence, Piety, and Vertue, was Puniſhed with Death, which once at leaſt Happened in the Perſon of our Lord, the Great and Divine Author of our Religion, it would not be from thence Conſequent, that Ingratitude or Murder were Good Actions, or that Innocence and Vertue were Bad Ones; There is ſomething Unalterable in the Conſtitution of Things themſelves, which cannot be Changed by any External Circumſtances, which Attend them, and Conſequently, not even by the Rewards and Puniſhments of another Life, no more than by thoſe of this.

Article the Fourth of Section the Firſt Conſidered.

§. 6. The Firſt and Principal Rule however of Morality is Said to be the Law of God, which is either that of Revelation or of Reaſon, to which Referring our Actions, they become Good or Bad, according as they are Productive to us of Pleaſure or Pain, that is, of Happineſs or Miſery, from the Will of the Almighty Law-giver, who has a Right, as he is our Sovereign, to Preſcribe a Rule to us, *&c.*

As to Revelation, in the firſt Place, this Philoſophy cannot Aſſure us, that there is any ſuch Thing; For, where it makes Certainty to Conſiſt in the Connexion of the Abſtracted Ideas of our own Minds, to which Matters of Fact, and the Evidences of them, can never be Reduced, it is Plain, that we cannot be Certain of any Revelation from God by the Chriſtian Diſpenſation, which is a Matter of Fact; This the Preſent Philoſophy was Senſible of, and therefore Ranks it under thoſe Truths only, which are Highly Probable; But if Revelation is one Chief Rule, by which we Judge, whether Human Actions are Moral or not, and that Rule is not Certain, but Probable, which way will Morality, I Wonder, be Capable of a Demonſtration, which this Philoſophy Apprehends it is, when one of the Chief Rules of it is not Certain? Unleſs Demonſtrations can be Happily Formed and Contrived from Principles, which are not Demonſtrable.

Omitting which, and Granting Revelation Certain, the Evidences of which this Philoſopher had never ſo much as taken the Pains to Conſider, or Really, what the True Nature of Certainty was, Excepting in the Notion he had Tranſcribed from *Des-Cartes*, and of which, in all his Philoſophy, he is ſo Unfortunate to make Uſe, without Underſtanding, even what is Meant by it, as we have already Shewn; Notwithſtanding this, How are Human Actions Referred to this Rule, ſo as there ſhould be a Viſible Agreement or Diſagreement betwixt them? If the Actions, we Commit, are found Expreſſed in that Rule, or either Allowed or Forbidden in it, the Queſtion will be, which way we are Certain of it; For, whilſt Certainty is made to Conſiſt in the Connexion of the Abſtracted Ideas of our Minds, as in the Mathematicks, which is the Favourite Inſtance of this Eſſay, it will be Impoſſible for us to Know, or be Certain of, any Actions, or any Rule, which we even See or Read, ſince they are External and Foreign to the Abſtracted Ideas of our own Minds.

Farther, let this be likewiſe Acknowledged; Human Actions yet, when Compared with what is Written in the Scriptures, which is the Revelation here Intended, can only Inform us of a Connexion betwixt thoſe Actions, and thoſe which are Deſcribed in the Writings, which are Fixed as a Rule of them; that is, if Adultery or Murder *&c.* are Prohibited, and Innocence, Chaſtity, and Goodneſs, are Enjoined upon us, we Know each of theſe Actions in Practice are Correſpondent, or not, to the Deſcriptions of them in the Rule Revealed to us; But yet the Connexion of Ideas will again Fail us in Referring thoſe Actions, (whether of Murder or Innocence, Deſcribed in the Revealed Rule) to the Will of the Legiſlator, who Preſcribes that Rule to our Actions; ſince there can be no Connexion of Ideas betwixt them and the Will and Power of an Almighty Law-giver, who in this Philoſophy is Suppoſed to have a Right over us as our Sovereign, and therefore may do what he Pleaſes with us, and Set what Limits or Boundaries he Thinks fit to our Actions; But betwixt the Arbitrary Will and Pleaſure of an Almighty Being, and the Human Actions, which are Referred to it, there can be no Connexion.

It is moreover to be Objected, that the Revealed Rule, which is Set to our Actions, is Suppoſed to be wholly Reſolved into the Sovereignty, which God has over us, whereas, I would Hope, the Rational Rule, which we ſhall afterwards Conſider, has ſomething better in it, than what is Derived from Mere Power and Sovereignty; If it has, this Philoſophy would Deal Kindly and Ingenuouſly by us, if it would Inform us, wherein the Revealed Rule is more Arbitrary than the Rational, *i. e.* wherein it is not Superior in Real Goodneſs and Vertue to any other Syſteme of Morality Deduced from Reaſon.

Lastly, if all this were Set Aſide, it could not be Said, that our Actions were Good or Bad, according as they Mett with Pleaſure or Pain, Happineſs or Miſery; Becauſe they would be Good or Bad, according to their Conformity, or not, to this Revealed Rule, and the Power and Sovereignty of the Law-giver, Abſtracted from the Rewards and Puniſhments Annexed to the Law, which his Sovereign Pleaſure had Preſcribed.

UPON the View therefore of the Whole, we cannot but Think, that this Philoſophy has, as little Value for Revelation, as it has a True Senſe of it; It was Neceſſary Indeed to Pretend Something of that Nature in a Country, which Believed it, and was fully Satiſfied of the Truth of it; But, if it was Really and Certainly Perſuaded of the Excellent Doctrines it Contains, it has taken a very Ill Method to Convince us, that it was ſo.

THE next Rule of Morality is That of Reaſon, which is Said to be a Divine One, as Revelation is, by the Bounty of this Philoſophy, Acknowledged to be.

IN Anſwer to which we Say, that Revelation we Know, but what is Reaſon? Is it the Connexion of Ideas, or is it every One's Humour and Opinion?

IF the Laſt, which is too often what Men Call Reaſon in the Preſent Age, it neither is, nor ought to be, any Rule to our Actions, much leſs is it a Sacred or a Divine One.

IF the Firſt, it has been already Proved, that Certainty, and therefore Reaſon, does not Conſiſt Barely in the Connexion of Ideas, and that, even in ſuch a Connexion, Men are as Capable of being Deceived into Appearances of Truth and Certainty, as in any other Way of Conviction, which this Philoſophy is Pleaſed to Term Probable only; But if this was alſo Granted, by what Methods is it Evinced, that Reaſon is a Divine Rule, unleſs by a Affirming it to be ſo? It is not Certain from any Connexion of Ideas, becauſe there is none betwixt Human Reaſon and the Divine, as there is not betwixt Finite and Infinite; If it is not Certain, we may as Juſtly Propoſe another Idea of Human Reaſon, which is Agreeable to that Humble Notion we ought to have of our ſelves, that is, that we are Human Creatures, and not otherwiſe Divine than by the Grace and Spirit of God, which is Imparted to us from our Religion; For by the ſame Argument, that our Human Reaſon is Divine, becauſe Implanted by God in us, we may Equally Prove our other Faculties to be ſo, that our Memory, our Perception, nay our very Eyes, Legs, and Hands, are Divine, ſince they are alſo Derived to us from Almighty God; If yet this Inſpired Philoſopher could Prove, that our Bare and Abſtracted Reaſon was, by a Peculiar Diſtinction from the Reſt of our Faculties, ſome Enthuſiaſtick Ray or Light Within, he would either give us by his Falſe Concluſions, which we have Shewn, a very Low Idea of that Divine Light, which, it ſeems, was ſo Good as to Irradiate and Illuminate his Underſtanding, or Furniſh us with a full Confeſſion, that his Reaſon had no great Reſemblance of any Thing Divine in it.

IN ſhort, what we can Mean by Human Reaſon, is nothing elſe than what is Expreſſed by thoſe Words, which is, has, and may be, Fallible in a Thouſand Inſtances; whereas, this Philoſophy would Induce us to Believe, that it is Infallible and Divine, that is, that Human Reaſon is Human-Divine Reaſon, which is a Contradiction in Terms, and will Require a greater Faith to Aſſent to, than any Propoſition in the Whole Chriſtian Diſpenſation.

§. 7. ANOTHER Rule of Morality, after the Divine, is Said to be That of the Laws of a Community, by Referring our Actions to which, they are Denominated Criminal or Innocent; But, that Human Laws or Injunctions ſhould Change the Nature of Actions themſelves, is altogether Impoſſible; All the Authority in the World can never make Murder an Innocent Action, or Gratitude a Criminal One.

Article the Fifth and Sixth of Section the Firſt Conſidered and Examined.

MUCH leſs can the Law of Opinion or Reputation Denominate our Actions Vertuous or Vicious; For, if Real Goodneſs and Clemency, if Humility and Modeſty, if Juſtice, and Honeſty, and Integrity, were Hooted out of Human Society, they would not therefore become Vices; nor would Malice and Envy, Pride and Inſolence, Injuſtice and Oppreſſion, tho' Supported and Abetted by whole Nations of Men, Commence and Change into Vertues upon that Account; Thus, no One Thinks *Ariſtides*, tho' Baniſhed from *Athens* by the *Oſtraciſm*, was an Ill Man, or that *Alexander* of *Macedon*, amidſt his Oppreſſions and Victories, was a Good.

NOTHING can be more Abſurd or Irrational than to Impoſe Names upon Human Actions in this Arbitrary Manner, and which are only Contrived to Abuſe us into Falſe Opinions of Things themſelves; ſince the Firſt may be more Truly and Juſtly Termed, according to the Rule to which they are Referred, Legal or Illegal, and the

Second, Sociable or Unſociable; by which Means the Mind, being Free from thoſe Equivocal Terms of Criminal and Innocent, Vertue and Vice, Immediately Diſcerns, that Human Actions may be Legal or Illegal, Sociable or Unſociable, without being Suitably Innocent or Criminal, Vertuous or Vicious, Commendable or Blame-Worthy, in their own Nature; from whence we may Conclude, that there is not much in the Real Senſe of this Philoſophy, when we Appropriate to every Thing it's own Name; But That is, what it ſeems Peculiarly Delighted with, to Range and Order Things into Various Claſſes, and then Impoſe what Names it Pleaſes upon them; Thus we have Evidenced, it Proceeds in Certainty and Probability, in Simple and Complex Ideas, in Moral Actions, under which are Comprehended Sins and Duties, Crimes and Innocence, and Vertue and Vice, and now, in making Innocent and Criminal Stand for Legal and Illegal, and Vertue and Vice, Praiſe-Worthy and Blameable, for Actual Praiſe and Blame, or Sociable and Unſociable; And the Same is Obſerveable in this whole Philoſophy, or Eſſay upon the Mind.

The Reaſons given, why Morality is Capable of Demonſtration. Mr. Locke's Eſſay, Chap. 11. Book 3. §. 15. 16.

§. 8. Having Examined into the Moral Relations of the Preſent Eſſay, we ſhall now Proceed to Inquire into the Reaſons it gives, why Morality is Capable of Demonſtration.

For Firſt, it Tells us, that Mixed Modes, Eſpecially thoſe belonging to Morality, being moſt of them ſuch Combinations of Ideas, as the Mind Puts together of it's own Choice, and where-of there are not always ſtanding Patterns to be found Exiſting, cannot, as Simple Ideas, be Expreſſed to us by Shewing, but by Defining them.

Second, upon this Ground it is Bold to Think, that Morality is Capable of Demonſtration; becauſe, theſe Mixed Modes being Combinations of ſeveral Ideas, that the Mind of Man has Arbitrarily Put together, without Reference to any Archetypes, Men may, if they Pleaſe, Exactly Know the Ideas, that go to each Compoſition; and this, if well Conſidered, it is Said, would *Lay great Blame on thoſe, who make not their Diſcourſes about Moral Things very Clear and Diſtinct*; ſince the Preciſe Real Eſſence of the Things, Moral Words Stand for, may be Perfectly Known, and ſo the Congruity or Incongruity of the Things themſelves be Certainly Diſcovered, in which Conſiſts Perfect Knowledge.

The Foregoing Reaſon Anſwered.

In Anſwer to which, it has been already Proved, that Mixed Modes, and even thoſe of Morality, are not Combinations of Ideas, which the Mind Puts together of it's own Choice, unleſs they are very Chimerical Ones, which can never be the Foundation of any Knowledge at all; Thus the Complex Idea of Gold or a Diamond, or of a Man, cannot be made at Pleaſure, tho' on the Contrary, we may Indeed at Pleaſure take the Perception of our Minds, the Yellowneſs and Weight of Gold, and the Hardneſs and Conſiſtency of a Diamond, and Unite them together into One Complex Idea; But what will this be, unleſs the Forming of a Chimera? and the Arguing upon it will be, not Knowledge, but Whimſy, if not an Inſtance of the utmoſt Phrenzy and Diſtraction; 'tis yet the very Same in Moral Complex Ideas, for Human Actions, as they in Fact Exiſt, and to which we Affix Particular Names, are the only Real Complex Ideas in Morality which we have, and are Anſwerable to the Complex Ideas of Subſtances, before Mentioned, of Gold, or a Diamond, or of a Man; So, when we See any one Kill another, we Call it Homicide; if Malice enters into the Action, Murder; if the Relation of a Father to a Son, Parricide; if any One Actually Returns an Ill Office for a Good, we Term it Ingratitude; and ſo of the Reſt; that is, our Denominations of this Sort of Ideas are from the Actions themſelves; On the other Hand, if we may Form theſe Complex Ideas at Pleaſure, without Conſidering, what Really Exiſts in Nature, we may Join the Idea of Malice, which is one Part of the Complex One of Murder, with the Idea of doing a Good Office, which is One of the Ideas belonging to Ingratitude, and which, when United, will be as Chimerical a One, as if in Subſtances, we ſhould, as before, Unite the Yellowneſs and Weight of Gold, the Conſiſtency of a Diamond, and the Perception of our Minds into the Same Complex Idea; From whence it is Evident, that the Mind has not a Liberty to Join, what Ideas it Pleaſes, either in Subſtances, or in Moral Actions, into One Complex Idea, and then Aſcribe a Name to ſuch a Compoſition; It is ſtill Confined and Tyed down to Nature, that is, either to Subſtances, as they Exiſt, or to Human Actions, as they are Performed; and when it Leaves them, it Wanders into Mere Fancies and Chimeras.

Moreover, if Mixed Modes were Reſigned to the Pleaſure of the Mind, the Compariſon of ſuch Complex Ideas, let them be as Determinate and Diſtinct as we will, with their Proper Names Annexed to them, could not Produce any Conviction or Demonſtration of Morality; becauſe, where the moſt Determined Ideas are Compared together, as in the Mathematicks, there can nothing Reſult from that Compariſon, or from their Agreement or Diſagreement, unleſs Certainty and Uncertainty, Truth and Falſehood; That is, ſuch Complex Ideas could Signify no more, by the Comparing of them, to the Aſcertaining, what was Moral or Immoral, than the Comparing the Three Angles of a Triangle with Two Right Ones; They would be only Abſtracted Truths, and nothing more.

Farther it is Confeſſed, that to Denominate any Action Good or Bad, or in General to make it a Moral One, it ought to be Referred to a Rule; if therefore the Connexion betwixt the Action and the Rule is not Evident and Plain, the Demonſtration will likewiſe Fail in this Reſpect, which we have before Proved it does.

Lastly, Granting the Mind could Form Mixed Modes at Pleaſure, in Reſpect of Human Actions, yet, ſince in thoſe Complex Ideas there would Enter the Ideas of Subſtances, both Diſtinct from us, and whoſe Natures and Eſſences we Knew nothing of, upon each of theſe Accounts the Demonſtration of Morality in this Philoſophy would be Impoſſible.

For Firſt, as Human Actions, and the Subjects of them, are External to our Minds, the Connexion of Ideas, which is, in this Philoſophy, the only Teſt of Certainty or Demonſtration, Lies only in the Mind, Abſtracted from every Thing which is External to it, ſo it is Evident, that where Things which are External to the Mind, as Human Actions are, Enter into the Inquiry, they are not Capable of Mathematick Certainty; and Conſequently Morality, which does Conſider ſuch External Actions, is not Capable of Demonſtration.

In the Second Place, if this were Allowed, Namely, that we might Connect with Certainty the Ideas of Things, which Lay External to our Minds (which, Notwithſtanding, is not a Mathematick Certainty) yet if thoſe Things, or Subſtances, are Unknown, it will be Impoſſible to Derive any True Knowledge from them; Thus, if it is a Moral Axiom, that Man is Subject to a Law, I muſt Know, what is Meant by a Man, or by a Law, that is, the Real Nature of the One, and the Real Conſtitution of the Other.

Nor is it Sufficient to Say, that it is not Requiſite to Define, what the Nature of Man is in a Phyſical Senſe, becauſe, be that as Diſputable as it will, it Concerns not, it is Said, the Moral Man, which is this Immoveable Unchangeable Idea, a Corporeal Rational Being; Since 1*ſt*. the Moral Man, Abſtracted from the Real Man, will be only a Creature of the Mind, a Notion and Fancy only, in the Same Manner, as in the Principles of Philoſophy we have Proved Matter is, and in our Geometry of Solids, that Mathematick Spaces and Quantities are, according to the Vulgar Eſtimation of them; and Conſequently, as the Received Philoſophy, and Mathematicks, is from thence Evidenced to be a Notional Thing, without any Real Knowledge of Beings, as they Exiſt, belonging to them, ſo would it be in Morality, if the Moral Man was a Notion Suppoſed, and Different and Abſtracted from the Real; 2*d*. If the Moral Man is Defined to be a Corporeal Rational Being, it will be yet Neceſſary for us to Know, what Body or Matter is, and what is Reaſon or the Mind, how they are Joined together, and what Influences they have upon each Other, whether Reaſon is a Connexion of Ideas, which is not Certain; if it is, from whence, and how that Faculty Ariſes in the Mind, whether from God himſelf, or from the Nature and Conſtitution of it; if from God, what that Communication of himſelf is to us, and in what Manner it is Performed, and whether ſuch a Notion may not be as Enthuſiaſtick, as this Philoſophy ſeems to Think his Imparting himſelf to us in the Graces and Vertues of Religion is; All theſe Things are to be Known and Demonſtrated, before we can be Sure, whether Man, as a Corporeal Rational Being, is Subject to a Law, that is, as it is before Explained, to the Divine One.

Eſſay, Chap. 11. Book 3. §. 16.

For, if the Faculty of Reaſon Ariſes from the Mind alone, or from it's Union with the Body, it will be no more Certain, that Reaſon is Subject to a Law Impoſed, than that our Memory, or our Perception is; and if the Faculty of Reaſon in our Minds Proceeds from God, as our Creator, ſo do our Memories and Perceptions likewiſe,

likewiſe; But, if the Faculty of Reaſon is an Impulſe from Almighty God, or a Particular Ray of the Divinity, which this Philoſophy would ſeem to Inſinuate; Which way is it done? or How is it leſs Enthuſiaſtick, than any Thing in Revealed Religion? or How does it fall Short of that Notion, which this Philoſopher himſelf would Ridicule, and does in his Chapter of Enthuſiaſm, of Men having a Light Within?

No great Loſs, that this Philoſophy has not, as it could not, give us a Demonſtration of Morality.

§. 8. We have thus far Traced the Notions of this Eſſay, and could Proceed to a Particular Confutation of every Part of this Philoſophy of Ideas; What we have already Said, is only to Shew, as before in it's Moral Relations, that there was nothing to be Depended upon, in what it Offered to us, ſo here, that there is nothing, which the World has Reaſon to Lament the Loſs of, when it gave us the Hopes of a Demonſtration of Morality, that it has not done it; ſince, if the Great Friend of it Mr *Molyneux* were now Alive, and would Read with Impartiallity, what has been Propoſed upon this Subject, he would be Aſſured, that the Preſent Philoſophy was ſo far from being Capable of giving us a Demonſtration of Morality, that it did not ſo much as Underſtand, what Belonged ot Appertained to a Demonſtration.

Eſſay, Chap. 11. Book. 3. §. 16.

For, if what we have Said is not enough to Evince this, we ſhall Add, what this Philoſophy Tells us at the End of the Section Cited in the Margin; That the Names of Subſtances, if they be Uſed in them, as they ſhould, can no more Diſturb Moral, than they do Mathematical Diſcourſes, where, if the Mathematician Speaks of a Cube, or Globe, of Gold, or any other Body, he has his Clear Settled Idea, which Varies not, *&c.* From whence it is Plain, that this Perſon Thinks, when the Mathematicians Argue upon the Abſtracted Space, which a Globe, or Cube, of Gold Poſſeſſes, they Reaſon upon the Nature of Things or not; if not, ſince he will not, I Hope, Say Morality is a Notional, and not a Real Science, by the ſame Argument which he brings for the Truth of Mathematick Speculations, which are only Notional, he Concludes Morality to be no other, Contrary to his Opinion; if it is his Opinion, that Morality is a Real Science, and not a Chimerical or Notional One, and Mathematicians, when they Conſider a Globe or a Cube of Gold, do Enter into the Nature of thoſe Bodies, Then they are not Settled and Abſtracted Ideas, but the Nature of the Matter Comprehended in a Globe or Cube is to be Known; and by Conſequence the Nature of thoſe Subſtances, which Enter into our Ideas of Moral Actions, Contrary to the Affirmations of this Philoſophy; Which, may we not Add with ſome Juſtice and Propriety, if well Conſidered, would *Lay great Blame on thoſe, who make not their Diſcourſes about Moral Things very Clear and Diſtinct?*

The Demonſtration of a Natural Religion Mean and Trifling, that of the Chriſtian Solid and Serious.

§. 9. If this therefore is the Caſe of Demonſtrations from Reaſon, or the Connexion of Ideas, in Reſpect of a God and Morality in General; What a Poor, What a Mean and Trifling Religion is here Propoſed to us? When, if all was True, as all is Falſe, the Demonſtration of the Soul's Immortality would be ſtill Unproved, and That of Future Rewards and Puniſhments; and which will be Impoſſible from the Connexion of Ideas; and yet, without ſuch Demonſtrations, the Proof of a God and of Morality, would not be of much Concern to us; For, if we were not to Exiſt hereafter, the Knowledge of the One or the Other would only make us Act our Parts with Decency here, Suitable to the Notions we had of an Almighty and Perfect Being, and to the Rules of Morality, which ſhould be Anſwerable to ſuch Notions; which is no more than a Player Acting his Part upon a Stage, with an Air and Manner becoming the Perſon he Repreſents, and Agreeable to the Rules of the Dramatick.

On the Contrary, we have Demonſtrations of Fact of our Religion, which does not only Aſſure us of a God, and of the Truth of Morality, but of our Exiſtence hereafter, and that our Rewards and Puniſhments ſhall be Proportioned to our Good or Ill Actions here; ſuch Notions, as theſe, will Indeed make us Behave our ſelves Seriouſly, and with a Conſcientious Regard to what we do; whereas, the other Turns our Thoughts and Actions into Folly and Grimace, and all the Grand Duties of Life into a Theatrical Apiſhneſs and Buffoonery.

Section the Firſt of Moral Good and Evil, and the General Anſwer, which may be made to it.

§. 10. It will Perhaps be Expected here, before we go any Farther, that we ſhould Inquire into the Moral Notions of a New Philoſopher, which Aſſert, That Good and Evil, Vertue and Vice, Conſiſt in Acting according to Truth, and the Real Relations of Things, or Contrary to them; That a Man may Declare a Lie or a Truth by his Actions, as well as his Words; And that the Firſt is Vice, and the Laſt

Mr. Wollaſton's Religion of Nature Delineated.

Laſt Vertue; This is a General Repreſentation of this Celebrated Doctrine of Moral Good and Evil; To which it is in General Anſwered;

First, That Barely Saying or Affirming in Words, what is True, or what is Falſe, can neither be Vertuous, nor Vicious; Thus, if any One Affirms, that the Three Angles of a Triangle are Equal to Two Right Ones, or that the Diagonal of a Square is Incommenſurable with the Side of it, and thoſe Propoſitions ſhould be True, he cannot therefore be Denominated from thence, ſo far a Vertuous Man, but a Rational; Nor, if he ſhould Deny them, could he be Termed ſo far a Vicious One, but One, who Joined and Connected his Ideas together Wrongly; Should *Anaxagoras*, or Sir *Iſ. Newton* Aſſert, that the Sun was an Immenſe Globe of Red-Hot Iron, or Something like it, and it ſhould Prove to be a Globe of Liquid, Denſe, Fire, or ſhould *Hugens*, or Dr. *Wilkins*, take the Moon to be Inhabited by Good Philoſophers or Aſtronomers, and it ſhould Happen to be a Vaſt Rock of Adamantine Ice, it would be Hard to Condemn them ſo far for Vicious Perſons, nor, if they were Right, would it be any Commendation of their Vertue; The Reaſon of which is, becauſe there can be no more Vertue or Vice in the Placeing and Setting our Ideas together Rightly or Wrongly, than in the Diſpoſing of ſo many Grains of Wheat or Sand in a Regular and Juſt Order, or with Perplexity and Confuſion; our Ideas, in the Nature of Things, and Abſtracted from all other Conſiderations, being only One Part of Nature, and ſuch Grains of Wheat or Sand, Another; It would be Infinite and Endleſs to Produce Examples in all the Ways of Life and of Science, wherein Men may Connect their Ideas Juſtly, and Affirm Truths from thence, and yet not be therefore ſo far Vertuous, and wherein they may make an Undue Connexion of them, and from thence Affirm Falſehoods, without their being ſo far Vicious or Immoral.

See Chap. 1. Book 1. of this Philoſophy.
Hugens's Coſmotheor.

If therefore this is ſo in our Affirmations in Words, the ſame muſt be Acknowledged by this Syſteme in our Affirmations in Actions, and that Men may Affirm very Innocently, and without any Immorality, ſeveral Things, which are Falſe, and Others, which are True, without any Acceſſion to their Vertue; ſince it Endeavours to Reduce the Actions of Men to their Expreſſions by Words, and to Show, that they are Equally Capable of Truth and Falſehood.

Second, It is not Evinced, that Men are under any Moral Obligation to Speak always what they Think, and therefore not to Declare and Explain always their Thoughts by their Actions; For, if they were, all Prudence and Diſcretion, which have been hitherto Eſteemed amongſt the Cardinal Vertues, muſt be Reſigned, and Caſhiered at Once; And Innocence and Goodneſs muſt be laid Open to the Attaques of Avarice, Ambition, Intereſt, Prejudice, and all the Paſſions and Deſigns of the Inſolent and the Injurious, and Perhaps even of ſome Perſons, who would Promote ſuch a Doctrine, not with an Intention to Practice it themſelves, but that Others ſhould, whoſe Weakneſs yet they would Deride and Ridicule, and upon Proper Occaſions would Know, how to make an Advantage of; What I here Defend is the Chriſtian Moral, Namely, To be Ingenuous, Honeſt, Candid, Charitable, and Free in all our Words and Actions, but withal, to Uſe a Wiſdom and Prudence, not to Expoſe our Thoughts, and the Inward Sentiments of our Minds, to every One, and upon all Events, under a Conviction of ſuch Notions.

Third, It is not Proved, that our Actions are as Expreſſive of Truth or Falſehood, as our Words are in Propoſitions; Becauſe in Verbal Propoſitions, we Formally and Intentionally Affirm or Deny Something, but our Actions are many Times, and moſtly, not Explicite, nor do they Mean, or Intend, any Thing by way of Affirming or Denying any Truth; Thus, when we Walk or Ride, or Viſit a Relation, or a Friend, or in any other of the Actions of Life, we do not Affirm any Propoſitions, which I know of, in Reſpect to Truth or Falſehood; we Perform theſe Actions, as they are Agreeable to us, and Delight us, and which Render this World in ſome Meaſure Pleaſant and Eaſy; And if this Doctrine, of Reduceing Moral Good and Evil to the Truth and Falſehood in Human Actions, is True, it muſt Pronounce and Aſſert, that Men may not, nor can, Act to Indulge their own Innocent Pleaſure, without Affirming ſome Propoſition or other, Relating to Truth or Falſehood, tho' the Perſons who Act, are not Conſcious to themſelves of any ſuch Propoſition; And it would be a Laborious Thing for the Patrons of this Syſteme to Show, what Propoſition every Action Amounts to.

FOURTH, It is Impoſſible to Know all the Relations of Things, and therefore, if Vertue Obliges us to Act up to the Truth of ſuch Relations, Moral Good and Evil will, as to the Practice of it, be an Impoſſible Buſineſs; For who can Determine all the Relations, in which any Particular Man Stands to all the Men and Beings in the Univerſe? And yet, if we do not Act with, or by, ſuch a Perſon, as he is thus Related, we Incurr, according to this Syſteme, the Blame of Vice, and are Cenſured and Condemned, as Guilty of an Immorality; Thus I may Act with or by a Man, as an Honeſt One, and yet as he Stands Related in the World, he may be quite the Reverſe of it, which yet I may Know nothing of; I do not therefore here Act according to the Truth and Reality of Things, Notwithſtanding which, I would Beg the Favour to be Excuſed from being a Vicious and Immoral Man for ſo Doing; On the Contrary, a Man may to Appearance ſeem Gay and Trifling, or Perhaps Extravagant and Fooliſh, tho' in his Inward Sentiments, and the Reſerves of his Thoughts, he may be Grave, Sedate, and Serious; If I ſhould therefore Act with, or by him, according to the Firſt Relation of Things, which are Apparent and Real to me, I ſhould Act, as this Philoſophy Says, Agreeably to Truth, and to Vertue; That is, if I Acted in a Ridiculous Manner with, or by him, When at the Same Time I ſhould Think my Self highly Blameable, and in Reality Injurious and Vicious, if I was Acquainted with his True Character, and the Natural Relations, under which he was to be Conſidered, in Reſpect to Men and Things; So that, if this Doctrine is Juſt, a Man may be a very Good and Vertuous Man, when it Pronounces him a Vicious, and a very Vicious Man, when it Declares him a Vertuous One; That is, Vertue is by this Syſteme Changed into Vice, and Vice into Vertue; There might be a Multitude of Inſtances of the like Kind Produced to the Same Purpoſe, which would Show the Great Abſurdity, and Vanity of the Preſent Doctrine.

FIFTH, In all Moral Good and Evil, or in Vertue and Vice, there muſt be an Intention and Deſign of Acting; otherwiſe our Actions are Merely and Barely Mechanical; But this Syſteme takes no Notice of the Intention of our Actions, only of their Conformity to the Truth and Reality of Things, whether we Intend ſuch a Conformity, or not; As Thus, a Man may Act according to the Truth and Reality of Things, and not Know he does it, and may Act Contrary to them, and yet be Ignorant of it; Notwithſtanding which, it would not Follow, that he would be ſo far a Vertuous Man in the Firſt Caſe, or a Vicious in the Second, of which likewiſe many Examples and Inſtances might be Produced.

SIXTH, by Acting according to the Truth and Reality of Things, and the Relations of them, or the Contrary, we may be Said to Act Properly or Improperly, Rationally or Irrationally, but ſurely not Vertuouſly or Viciouſly, Morally or Immorally; Thus, if I Cut a Stick in Pieces, or Talk and Reaſon with a Poſt; I Uſe the Firſt as it is, and the Laſt as it is not; But I would Deſire, that we may not be from thence Obliged, to Call the One a Vertue, and the Other a Crime.

SEVENTH, The Suppoſition of a God is Neceſſary to Moral Good and Evil, Vertue and Vice; For if there was no God, all our Actions would be only Purely Natural and Human, ſome would be more Agreeable and Beautiful, more Applauded, and better Eſteemed than Others, but there would be no more Vertue or Vice in them, than there is in Colours, for their Pleaſing and Delighting, or Offending the Eye and Senſe of the Beholders; And therefore, there is Something more Required to Vertue and Vice, than Acting, or not Acting, according to the Reality of Things.

EIGHTH, Nor muſt a God be only Suppoſed, but that he Concerns himſelf with the Actions of Men, and that he has Preſcribed and Laid down a Rule for their Behaviour and Conduct; Which muſt either be from his own Adjuſtment of the World, and from the Works of his Creation, that is, from Nature, which therefore ought to be Underſtood by us, or from Revelation; and Conſequently to Know, what Morality is, we ſhould be Good Chriſtians, or Good Philoſophers, or both, neither of which Sciences ſeem to Enter into the Preſent Syſteme.

Lastly, Future Rewards and Puniſhments, and therefore the Immortality of the Soul, are alſo Neceſſary to the Moral Good and Evil of our Actions; For if there was a God, and he alſo Concerned himſelf in the Actions of Mankind, and had farther Impoſed upon them a Law either from Nature, or from Revelation, and did not Enforce it by Suitable Rewards and Puniſhments; It would be no more, than if I, or any other Perſon, ſhould Make and Ordain Rules for the Deportment and Demeanour of Others, over whom I either had not, or would not, Exerciſe any Power or Authority, which would be a Ridiculous Grimace, and a Pride and Affectation, both Irrational and Abſurd; and therefore we muſt ſtill have Recourſe to Revelation, which gives us the Sureſt Proof of ſuch a Futurity, to Complete our Notions of Morality.

These are the Arguments in General againſt this Scheme, but ſince it is Received with an Applauſe, which does not Juſtly belong to it, I ſhall now Particularly Inquire into every Article and Propoſition of this Doctrine.

The Firſt Propoſition of this Doctrine Examined.
Mr. Wollaſton's Religion of Nature Delineated. §. 1. Prop. 1.

§. 11. **The** Firſt Propoſition is, that That Act, which may be Denominated Morally Good or Evil, muſt be the Act of a Being Capable of Diſtinguiſhing, Chooſing, and Acting for himſelf, or more Briefly, of an Intelligent and Free Agent. Which is thus Proved, becauſe, that which cannot Diſtinguiſh, cannot Chooſe; And that, which has not the Liberty of Chooſing for it's ſelf, and Acting accordingly, from an Internal Principle, Acts, if it Acts at all, under a Neceſſity Incumbent *ab extra*, and is in Reality only an *Inſtrument* in the Hand of Something, which Impoſes the Neceſſity, and cannot Properly be Said to *Act*, but to be *Acted*; a Being under the Above-Mentioned Inabilities is, as to the Morality of it's Acts, in the State of Inert and Paſſive Matter, and can be but a *Machine*, to which no ἤθη or *Mores* can be Aſcribed.

Which Proof is nothing more than Affirming the ſame Thing in a Variety of Expreſſions, of *Free* and *Neceſſary*, *Acting* and *being Acted*, of being an *Agent* or an *Inſtrument* and a *Machine*, Namely, that a Moral Action muſt Proceed from a Free Agent. Was this ever Diſputed? or does it Want a Demonſtration? There is no Doubt of it: And we may Add, that not only a Moral Action, but every other Action, which is Voluntary and Free, Proceeds from a Free Agent, which alſo muſt be Aſſerted, and as little Requires a Proof of it, as the Other; And the Queſtion here will be; Whether every Voluntary and Free Action is a Moral One? This Doctrine ſeems to Inſinuate it, but does not Think Fit to Prove it; As it cannot do; For tho' every Moral Action muſt be a Free, and a Voluntary One, yet every Free and Voluntary Action is not a Moral; Since even Brutes are Capable of the Former, and yet no One will Say, they are of the Latter.

And yet this Syſteme Sets out ſo Prepoſterouſly, as to Think, that, if it Shows one Side the Propoſition, it Proves the Reverſe of it; that is, that if it Demonſtrates, that every Moral Action muſt come from a Voluntary and Free Agent, which no One will Contradict, therefore every Voluntary and Free Action muſt come from a Moral Agent, which no One will Aſſent to, unleſs he Eſteems Brutes to be ſuch.

That this Propoſition does yet Inferr and Suppoſe this, is Evident; becauſe, to Prove, that a Moral Action Proceeds from a Free Agent, is nothing, Excepting it is Shewn in this Syſteme, that every One, who Acts Voluntarily and Freely, and even according to the Reality of Things, is a Moral Agent; all which Brutes may do, and yet not be either Vertuous or Vicious; For a Brute may Eat Hay or Graſs, and do it Voluntarily and Freely, and Act according to the Nature of Things, and yet not be a Vertuous Brute, as, I Suppoſe, this Doctrine will Acknowledge.

The Second Propoſition Recited and Examined.
Mr. Wollaſton's Religion of Nature Delineated. §. 1. Prop. 2.

§. 12. **The** Second Propoſition is, that thoſe Propoſitions are True, which Expreſs Things, as they are; Or Truth is the Conformity of thoſe Words or Signs, by which Things are Expreſſed, to the Things themſelves. Is this then a Definition of Truth? Where therefore is the Connexion of Ideas? And what becomes of an Intuitive Perception of ſuch a Connexion? We are Indeed to Expreſs Things, as they are, or to Conform our Signs, by which Things are Expreſſed, to Things themſelves; Thus, we muſt not in Words, Call White Black, nor Apply that Sign or Word, which Expreſſes an Horſe, to a Man; And in Actions, we ſhould not Uſe a Motion of the Muſcles, or a Geſture of the Body, which is Significant of Joy, to Denote Grief or Sorrow, nor Explain any Thoughts or Paſſions of our Minds by thoſe Signs, which may Inſinuate to Others, the Contrary to them; This would be Agreeable to Truth, but it is not all, which is Requiſite to it; Becauſe, a Man may be very Punctual in his Expreſſions, either by Words or Actions, and yet may be Erroneous in his Connecting his Ideas, which he thus Plainly and Honeſtly Expreſſes; That is a Sincere, Downright, Well-Meaning Perſon may neither be a Good Mathematician, a Divine, or a Philoſopher.

BUT here likewiſe is ſomething Deficient, as to the Point of Morality, which is to Shew, that we may not Miſname Things without a Vice, that to Put off Paſſions and Sentiments upon Others, which we have not our ſelves, is a Crime, and that even Error in the Connexion of our Ideas is a Grand and an Unpardonable Fault, and a Great Impeachment to Vertue; which Things will be Hard to Evince, that they may not be Allowed in *any* Caſe *whatſoever*.

The Third Propoſition Inquired into. Mr. Wollaſton's Religion of Nature Delineated. §. 1. Prop. 3.

§. 13. THE Third Propoſition is, that a True Propoſition *may* be Denied, or Things *may* be Denied to be what they are, by Deeds, as well as by Expreſs Words, or another Propoſition.

THIS Propoſition is very Certain, ſo far as it Affirms; ſince it is very Evident, that a Man *may* Signify, and Aſſert, a Propoſition by his Actions, as well as by his Words, they being only Different Ways of Explaining the Inward Conceptions of our Minds; But it will not from thence Follow, that becauſe he *may*, he always *does* do it, or is *Obliged* to do it; a Perſon, who is Deaf and Dumb *may* Expreſs the ſame Senſe, Meaning, and Intention by his Signs, as another does by his Words; But then it is with a Purpoſe to Expreſs it; As much as the other does by his Words; Or elſe they are Idle Signs, without Intention or Meaning; And ſuch may the Caſe be of many Actions in Human Life, which are not *Intended* to Expreſs any Propoſition in Relation to Truth, or to Falſehood, as well as ſeveral Words, which Paſs in Converſation, which are not Deſigned to Affirm or Deny any Truth, but are the Sport and Diverſion of ſuch, who are Willing to Relax and Unbend ſome Times from the Cares of Life, or the Severities of Study and Contemplation; For who ever Imagined, that ſome Propoſition or other in Reſpect of Truth, was always Meant and Intended in a Jeſt, and that we muſt look for Maxims of Reaſon and Philoſophy in a Pun or a Witticiſm?

IF therefore Words and Actions may, or may not, Expreſs a Propoſition in Relation to Truth or Falſehood, how muſt we Know, when they do, or do not do it, unleſs from the Explicit Intention and Deſign of the Perſon who Speaks or Acts? To Interpret other Mens Words and Actions for them, and to Affirm, that they Mean this, or another Propoſition by them, when they Declare, they never Thought or Dreamed of any ſuch Propoſition, is Dealing, not only Abſurdly, but Arrogantly, with Mankind, and Deſerves a Suitable Return of Contempt from them.

HOWEVER, we ſhall Examine this Propoſition Farther, by the Inſtances Alledged.

1ſt. IT is Said, there is a Meaning in *many* Acts and Geſtures; Every Body Underſtands Weeping, Laughing, Shrugs, Frowns, *&c.* Theſe are a Sort of Univerſal Language; *Zeno's* and *Diodorus's* Argument againſt Motion, was Anſwered by getting up and Walking; *Roſcius* Pretended to Expreſs the ſame Sentence, as many Ways by Geſture and Action, as *Cicero* could do by a Variety of Words and Elocution.

To which it is Replied; That if there is a Meaning in *many* Acts and Geſtures, it is a Confeſſion, that in *ſome* there is, or may be, None; That if every Body Underſtands Weeping, Laughing, *&c.* it is, when they are Intended and Deſigned to have a Meaning by the Perſon, who Uſes them, for Men may Weep or Laugh, *&c.* they Know not the Reaſon why, as is the Caſe of ſome Diſtempers, or they may do it upon a Different Account from what is by Others Imagined or Preſumed; That the Philoſopher, who Anſwered the Arguments againſt Motion by his own Motion, did it with that Intention; That, if *Roſcius* Endeavoured to Expreſs the ſame Sentence by Action as Variouſly, as *Cicero* could by Different Forms of Elocution, it was his Meaning and Intention ſo to do.

FROM whence it is, hitherto thus far Evident, that not only Speaking or Acting, but a Deſign and Intention in ſuch Words and Actions, is Requiſite to the making them Significant and Expreſſive of ſome Truth or Propoſition.

2d. IT is Told us, that theſe Inſtances do not come up to the Meaning of this Philoſophy; And that there are many Acts of other Kinds, ſuch as Conſtitute the Character of a Man's Conduct in Life, which *have* in Nature, and would be Taken by any Indifferent Judge to *have, a Signification,* and to *Imply ſome Propoſition,* as Plainly to be Underſtood, as if it was Declared in Words; and therefore, if what ſuch Acts Declare to be, is not, they muſt Contradict Truth, as much as any Falſe Propoſition or Aſſertion can.

To which we again Reply; That there are no ſuch Acts, as are Mentioned above, which *have* in Nature *a Signification,* or *Imply any Propoſition,* unleſs they are ſo Meant and Intended, and therefore, if what ſuch Acts Declare to be, is not, that they do not Contradict Truth, as much as any Falſe Propoſition or Aſſertion can, unleſs they are Deſigned to do it; This ſeems to be ſo Plain, that nothing can be more ſo; For does any One Affirm or Deny in Words a Propoſition, but with an Explicite Intention of doing it? And ſince Actions are put in this Syſteme upon the ſame Level with Words, there is the Same Reaſon, that we ſhould Affirm or Deny in Actions with the Same Explicite Intention; And yet it would Fain Gain this Point, that Mere Actions, without any Intention or Deſign Annexed to them, ſhould Affirm or Deny; And that theſe Actions are not to be Explained from the *Intention* of the Perſon, who Commits them, but by *Indifferent Judges,* by *having a Signification,* which in *Nature* belongs to them, and by an *Implication* of a Propoſition; When yet it would be Impoſſible for the moſt *Indifferent Judges* in the World to Determine, what the *Nature* of an Human Action is, or to Know, what Propoſition it *Implies,* without taking into the Account, the *Intention* of it.

3*d*. It is Inſtanced, that if a Body of Soldiers, Seeing another Body Approach, ſhould Fire upon them, this Action would Declare, they were Enemies, and if they were not Enemies, this Military Language would Declare, what was *Falſe*; This Inſtance goes on to Shew it's Learning about the *Athenians,* in the Attacque of *Epipolæ,* and of the *Carthaginians,* in their laſt Incampment againſt *Agathocles* in *Africa,* but Reſumes the Aſſertion, that Suppoſe, Inſtead of this Firing, ſome Officer ſhould have Said, they were Enemies, when Indeed they were Friends, ſuch an Affirmation would be a Falſe, tho' a Miſtaken One, and what Holds in Words, Obtains alſo in Actions.

To which we likewiſe Anſwer, that here the Intention of the Action is Plain and Expreſs, which, we have Said, is always Neceſſary to an Affirmation of any Thing, in Words, as well as Actions, in Relation to Truth and Falſehood; For Suppoſing an Officer had Said they were Enemies, when they were Friends, and had Acted with them Accordingly, he as much, and Equally Intended to Declare them Enemies by his Actions, as his Words, when yet they were Friends; Notwithſtanding which, he on the other Hand had no Deſign or Intention of Speaking or Acting Falſely; What therefore can be Concluded from hence? What? That ſuch a Perſon Speaks and Acts againſt Truth, and the Reality of Things, and that he is by Conſequence an Ill and Immoral Man? Surely this Syſteme will not Affirm ſo Groſs an Abſurdity, and yet it's Doctrine Directly Leads to ſuch an Aſſertion; Since Mere Truth and Falſehood in Words and Actions, are made the Meaſure and Standard of Vertue and Vice, and of our being Good, or being Immoral, without Conſidering the Intention of the Speaker or the Agent.

If this Philoſophy only Deſigns in this Propoſition to Prove, that ſome Actions are as Expreſſive, as Words, it will not find any Opponent; Every One will Readily Aſſent to the Thing, and not Require a Demonſtration of it; But, if it would Inſinuate, or Evince, that Speaking, or Acting, according to the Reality of Things, without an Intention of doing it, is a Vertue or a Vice, it will not Meet with many Rational or Conſidering Perſons, who will be Patrons of ſuch a Doctrine.

4*th*. It is Inſtanced, that, when *Popilius Lænas* Sollicited to have *Cicero* Proſcribed, and that he might find him out, and be his Executioner, his Carriage Sufficiently *Signified,* that *Tully* was either ſome Bad Man, or had ſome way Injured him, or at leaſt had not Saved his Life; all which Things being Falſe, his Actions were Expreſſive of that, which was Falſe, or were Contradictions to Truth; and herein, it is Said, Conſiſted his Ingratitude.

To which it is again Anſwered, that what theſe Actions of *Popilius Signified* cannot be Explained or Determined, without Knowing the *Intention* of them; They might be Expreſſions of ſome Inward and Latent Malice and Inveteracy towards *Cicero,* upon Incidents and Incentives, which are not come to our Knowledge, and therefore might not Show *Cicero,* but *Popilius* to be a Bad Man; Tho', if he Acted according to the Reality of Things, that is, according to any Juſt Reſentments againſt *Cicero*, he might be in this Syſteme, a very Good One; They might be alſo Expreſſions of his Ambition, and of his Complements to a Great and an Elevated Villain, *Antony*, and Conſequently, might not Declare, that *Tully* was a Bad, but a Worthy and a Vertuous Man; So that *Popilius's* Ingratitude might not Conſiſt in the Falſeneſs of the Action, but in the Truth of it, by his Profeſſed Intention to Gratify his Ambition, and Ruin the State, rather than Side with Vertue Oppreſſed, tho' it had been Formerly ſo Advantageous and Salutary to him.

I have hitherto Called him *Popilius*, in Obedience to this Philoſophy, but his Name was *Popillius Lænas*, according to *Fabricius's* Hiſtory of the Life of *Tully*; And that his Actions did not Declare *Tully* to be a Bad Man, but himſelf to be One, is Plain from the Exit, which that Brave and Noble *Roman* made, according to *Aufidius Baſſus* in *Seneca*, *Paulùm remoto Velo, poſtquam Armatos Vidit, Ego verò Conſiſto, ait, accede Veterane, & ſi hoc Saltem potes rectè facere, Incìde Cervicem.* Our Philoſopher Forgets alſo, that one *Herennius* a *Centurion*, as well as *Popillius* a *Tribune*, was Concerned in this moſt Barbarous and Inhuman Murder, which will likewiſe a little Perplex his Reaſoning upon this Head; Becauſe *Herennius* did not ſtand in the ſame Relation to *Tully*, as *Popillius* did, and therefore, if *Tully* had been an Inveterate Enemy to the Former, he Acted, in the True Senſe of this Philoſophy, according to the Real Relations of Things in Killing of him, and Conſequently, the Same Murder was a Good and Vertuous Action in the Firſt, tho' Perhaps a Criminal and a Vicious One in the Laſt.

5*th*. It is Alledged, that if *A* ſhould Enter into a Compact with *B*, by which he Promiſes never to do ſome Certain Thing, and after this he does that Thing, it muſt be Granted, that his Act Interferes with his Promiſe, and is Contrary to it. Will any One Deny this? Or that an Action may not Affirm and Deny, as well as Words? The Queſtion is, whether any Actions or Words Affirm or Deny, without an Intention in the Agent or the Speaker, and whether Farther an Intention likewiſe is not Neceſſary to any Moral Aſſertion in Words or Actions; If *A* Enters into ſuch a Compact with *B*, and his Act, with Deſign and Intention, Interferes with his Promiſe, he is an Immoral and an Ill Man; if Inadvertently, and Unawares, and without his Deſign, it Chances to Interfere with his Promiſe, we may Safely and Truly Affirm, there is no Vice or Immorality in it.

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As for *Ægon's* Sheep, or *Damætas's*, or *Alphondas's*, or *Melibœus's*, and the Arguing from *Theocritus*, or *Virgil*, it is not very Material to the Point, and only Shews, that this Philoſophy had Once been a little Converſant in thoſe Authors, without Perhaps Knowing the Full Scope and Senſe of them; Excepting, that *Damætas* Underſtood them to be *Ægon's* Sheep, becauſe he had the Uſe and Diſpoſal of them; And from whence it is as Weakly Concluded, that he who has the Uſe and Diſpoſal of a Thing, has the All, or the Property, of that Thing.

6*th*. It is Told us, that in the *Jewiſh* Hiſtory we Read, that when *Abimelek* Saw *Iſaac* Sporting with *Rebekah*, and Taking Conjugal Liberties, he Preſently Knew her to be *Iſaac's* Wife, and if ſhe had not been his Wife, the Caſe had been the Same, as that of *Ægon's* Sheep, which tho' he Uſed and Diſpoſed of them, were not his, but *Alphondas's* or *Melibœus's*; and it is Said, if it is Objected, that ſhe might have been a Miſtreſs, or an Harlot, it is Anſwered, that ſo ſhe might have been, tho' *Iſaac* had Told him by Words, that ſhe was his Wife, as he Told him ſhe was his Siſter; What is Said to be Sufficient for the Preſent Purpoſe, and to make Acts Capable of Contradicting Truth, is, if they may be Allowed to *Expreſs Things, as Plainly and Determinately, as Words can.*

To which it is alſo Replied, that we do not, nor will any One Deny, that *Acts may not Expreſs Things, as Plainly and Determinately, as Words can*; They *may* do it, but do they do it? *Iſaac* might by ſome Act of Fondneſs, Affection, and Endearment, Different from what is Practiced to Lewd Women, Expreſs and Shew *Rebekah* to be his Wife, as much as by any Words, but then here is a Deſign and Intention of doing it; On the Contrary, the taking Conjugal Liberties, as they are Termed, could never Declare *Rebekah* to be his Wife, when they might as well Declare her to be his Harlot; As any Word, which has a Double Senſe or Meaning, cannot be Expreſſive of One only; From whence it is Manifeſt, that when we would Expreſs our Thoughts by Actions, we are to Intend and Deſign thoſe Actions to Expreſs our

Thoughts; but will it be here Said, that *Iſaac* Intended to Shew by his Actions, that *Rebekah* was his Wife, when in Words he Affirmed her to be his Siſter?

DID *Ægon*, becauſe he gave the Sheep to another's Keeping Intend to Declare, they were his Sheep? They might be a Third Perſon's, and *Ægon* Deputed by him to the Care of them; Does he, who has the Uſe and Diſpoſal of a Thing, Declare that he has the All, and the Property of that Thing? And might not *Ægon* Commit the Sheep to *Damœtas's* Inſpection from, and under, an Authority from *Melibœus*?

THIS therefore is what we Maintain, that where Actions are made to Expreſs our Thoughts, they are Choſen on Purpoſe to do it, and with a Deſign and Intention of doing it, as a Dumb Man makes Uſe of Signs, when he is Incapable of Explaining his Sentiments by Language; Theſe Actions muſt be Proper and Significant of what they are Intended to Expreſs, and not of a Dubious and Uncertain Interpretation; And we muſt be as Careful to Convey our Real Meaning by them, as by Juſt and Appoſite Words; For if we take Men's Actions in General, there is Scarcely One of them, which may not Admit of a Thouſand Conſtructions, and can only be Aſcertained from the Principle, that is, the Inward Intention of the Mind, and the Motive, from which it Proceeds; and which does not Lie Open to Common Obſervation.

7*th*. IT is Said, if a Soldier ſhould Take an Oath to *Cæſar*, and afterwards Deſert to the Enemy, this would be as much as if he Denied him in Words, ſince his Actions Abjured him; And if a Chriſtian ſhould Burn Incenſe to *Jupiter*, upon the Command of a Tyrant, tho' he did not in Words Deny Chriſt, yet by that very Fact he would Deny him; And if a Man Lives, as if he had that Eſtate, which he has not, he Acts or Lives a Lie.

ON the other Hand we Say, the Soldier might upon ſome Motives of Terror, and for Self-Preſervation, go over to *Pompey*, and yet Wiſh well to *Cæſar*; The Chriſtian might Love, and Inwardly Adore and Reverence his Saviour, and yet not be Courageous enough to Reſiſt and Encounter the Tortures of an Heathen Perſecutor; And a Man may Live above his Eſtate, and yet Condemn his own Weakneſs, Vanity, and Indiſcretion for it; In all which Caſes, the Man Affirms by his Actions, what in Terms he Denies, and therefore theſe Inſtances are ſo many Proofs, that Actions are not often ſo Expreſſive of the Thoughts and Sentiments of our Minds, as Verbal Propoſitions are; For it is Plain here, that our Aſſertions in Words Declare our Real Opinions of Things, and that our Actions are Entirely Inexpreſſive of them; And from Hence it is again likewiſe Evident, that the Intention of an Action is Neceſſary to make it Significant of our Inward Thoughts or Conceptions.

8*th*. NOTWITHSTANDING which, this Religion of Nature Delineated, goes on to Tell us; That *Words* are but Arbitrary Signs of our Ideas, or Indications of our Thoughts, ſince that Word, which in *Hebrew* Denotes Poverty, in *Arabick* Denotes Riches, whereas *Facts* may be Taken as the Effects of them, or rather as the Thoughts themſelves Produced into Act, as the very Conceptions of the Mind, brought forth, and Grown to Maturity, and therefore as the moſt Natural and Expreſs Repreſentations of them; And beſides this, they bear Certain *Reſpects* to Things, which are not Arbitrary, but as Determinate and Immutable, as any *Ratio's* are in Mathematicks; For the Facts, and the Things they Reſpect, are *Juſt what they are*, as much as any Two given Quantities are, and therefore the Reſpects, Interceding between thoſe, muſt be as *Fixed*, as the *Ratio* is, which One of theſe Bears to the Other; That is, they muſt Remain the Same, and always Speak the Same Language, till Things Ceaſe *to be, what they are*.

THIS therefore is laid Down, as a Fundamental Maxim, that whoever Acts, as if Things were ſo, or not ſo, doth by his Acts Declare, that they are ſo, or not ſo, as Plainly, as he could by Words, and with more Reality; And if the Things are otherwiſe, his Acts Contradict thoſe Propoſitions, which Aſſert them to be as they are.

BUT Firſt, we Say, Words are Arbitrary Signs Indeed of our Ideas, before any Senſe is Affixed to them; Thus we may Take what Articulate Sounds we Pleaſe to Denote our Ideas by, and from Hence, Different Nations and Countries make Uſe of Different Languages; But after the Senſe is Affixed to the Word, it Ceaſes to be an Arbitrary Sign, and we may not, if we will Diſcourſe, Converſe, or Reaſon with Others, Annex to it, what Idea we Think fit; It is Certain and Determined, and is always Expreſſive of the Same Idea, as much or more than any Fact or Action can be.

On the Contrary, Facts are not always the Effects of our Thoughts, or our Thoughts themſelves Produced into Act, *&c.* and therefore more Natural Expreſſions of them; Since they in many Particulars are as Arbitrary as our Words, as this Philoſophy Owns; thus *Chriſtians*, it is Said, when they Pray, Take off their Hats, but the *Jews*, when they Pray, or Say any of their *Berakoth*, Put them on, the Same Thing Expreſſing Reverence with the One, which Denotes Irreverence with the Other; And it will be Hard to find any Acts, which have an Unalterable Signification; and even Killing a Man with One Intention, Expreſſes Malice and Murder, and with another, as in the Execution of a Criminal, Humanity and Juſtice; *Ægon's* Treating the Flock, and Diſpoſing of it, as *if it was his*, did not Inferr, that *it was his*, for he might have been Authoriſed ſo to do by *Melibœus*.

Second, But which is more, if this Doctrine is True, that Facts more Naturally and Unalterably Expreſs our Thoughts, than Words do, and that they are our Thoughts themſelves Matured and Produced into Act, it muſt Affirm that a Perſon, who is Deaf and Dumb, can Maintain a more Intelligible Converſation with thoſe about him, and can Explain his Mind more Fully, Truly, and Naturally, by Actions and Geſtures, than another can by any Force of Language, or the Ambiguity of Verbal Propoſitions; And yet, every one Knows how Difficult it is, and how much Gueſſing and Conjecture it Requires, to come at the Inward Senſe and Meaning of ſuch Dumb Signs, and how Plain and Obvious Words are, and how Eaſily they Inſtill into the Mind of the Hearer the Thoughts and Ideas of thoſe, who Uſe them.

Third, Facts are not our Thoughts *Matured* and Teeming into Actions, becauſe, there is many Times no Connexion betwixt our Thoughts and our Actions, that is, our Actions may not Expreſs our Thoughts; A General of an Army may at this Time Think of Inveſting and Laying Siege to a Place, and yet at that very Time, by a Feint, March his Soldiers a quite Contrary Way; *Hannibal* may Think of Subduing and Conquering *Rome*, and yet Suffer his Men to Diſſolve in Luxury and Eaſe at *Capua*, which was no ways Expreſſive of ſuch a Thought or Intention; And a Chriſtian, or a Philoſopher, may have very Good Thoughts, which are not always Declared or Explained by his Actions.

Fourth, Nay Farther, Men may Perform ſeveral Actions, with which they do not Perceive any Thought is Connected, we do not find in the Common Actions of Life, as Walking, Sitting, Standing, *&c.* that every Step, which we Take, is the Reſult of any Explicit Thought, or that every Moment, we Sit or Stand, is Attended with the Thought of doing it; That is, ſeveral of our Actions, and even thoſe, which come under the Notion of Moral Ones, are Mechanical, and are not the Expreſſions or Indications of any Abſolute or Determinate Thought or Perception; And this is the Caſe of moſt Habits.

Fifth, Nor do Facts bear Certain *Reſpects* to Things, which are as Immutable, as any *Ratio's* are in Mathematicks, nor do they always therefore Speak the Same Language; For, tho' *Things are Juſt, what they are*, and will never Ceaſe, *to be what they are*, yet the Same Facts will not always Bear the Same Relation to them, which ſhould have been Proved, but is very Wiſely Declined by this Philoſophy; For we are to Judge of Mens Thoughts by the Facts, which are Said to Explain them, and therefore the Intention of the Fact cannot be here Taken in, and the Fact it's ſelf is only Conſidered; Whereas the Same Human Fact may be Infinitely Diverſified by the Intention of it; Thus, if any One Kills a Man by Accident, or with Deſign, the Fact is the Same, but I Hope it does not Bear the Same Immutable Reſpect to Things; Becauſe the Intention Alters the Compariſon; If with Deſign, and that is Various, either to Indulge his own Malice, or to Put him out of his Pain, or in Obedience to a Juſt Law, which Condemned him, I would ſtill Believe, that tho' the Fact is the Same, the Principle of it, the One Expreſſing Cruelty, the Second Mercy, and the Laſt Juſtice, will make it ſtand in a Different Relation to Things, and therefore, that *Mere Facts*, without the *Intention* or *Principle* of them, do not Bear Certain *Reſpects* to Things, which as it is Said, are not Arbitrary, but as Determinate, and Immutable, as any *Ratio's* are in Mathematicks; And yet this is what the Preſent Doctrine muſt Maintain, for according to it, we cannot Know the *Principle* or *Intention* it ſelf, but by the *Fact*.

SIXTH, As to the Fundamental Maxim Concluded from Hence, that whoever Acts, as if Things were ſo, or not ſo, doth by his Acts Declare, that they are ſo, or not ſo, as Plainly as he could by Words, and with more Reality, it is, from what Foregoes, One Entire Falſehood.

1ſt. BECAUSE a Man may Act, as if Things were ſo, and yet not Declare by thoſe Acts, that they are ſo; Unleſs he Intends to do it.

2d. BECAUSE Acts do not Declare our Thoughts ſo Explicitly and Fully as Words do, as was Inſtanced from Dumb Signs and Actions, not being alike Intelligible with Words and Verbal Propoſitions.

3d. BECAUSE all the Reaſoning above, from whence this Concluſion is Made, has been Conſidered and Refuted.

IF therefore this Fundamental Maxim Proves to be an Error, What muſt we Expect from a Syſteme of Natural Religion, which is wholly Built and Depends upon it? Nothing Certainly but Romance, and Thouſands of Petitions and Poſtulatums; However, we ſhall Proceed to it's Fourth Propoſition.

The Fourth Propoſition Examined. Mr. Wollaſton's Religion of Nature Delineated. §. 1. Prop. 4.

§. 14. THE Fourth Propoſition is, that no Act, whether Word or Deed, of any Being, to whom Moral Good and Evil are Imputable, that Interferes with any True Propoſition, or Denies any Thing to be, as it is, can be Right; For,

1ſt. IF that Propoſition, which is Falſe, be Wrong, that Act, which Implies ſuch a Propoſition, it is Said, cannot be Right, becauſe it is the very Propoſition it ſelf in Practice. But it ſhould have been here Determined, what is Meant by Right and Wrong; If Truth and Falſehood is Intended by it; The Aſſertion Runs thus, if that Propoſition, which is Falſe, be Falſe, that Act which Implies ſuch a Propoſition, cannot be True; which Perhaps may be a Certain Propoſition, but is a Silly One; If by Right and Wrong is Intended, Vertue and Vice, Good and Evil; The Aſſertion will be; If that Propoſition, which is Falſe, be Vicious or Evil, that Act, which Implies ſuch a Propoſition, cannot be Vertuous or Good, which is wholly Precarious and Unproved.

FOR it is not yet Shewn, that Vertue and Vice, Good and Evil, are Terms Reciprocal with Truth and Falſehood; Nor has it Proved, that *Mere* Acts are as Expreſſive and Significant, as Words, or that they are at all Significant, without a Motive, a Principle, or Intention, Annexed to them.

2d. IT is Told us, that thoſe Propoſitions, which are True, and Expreſs Things, as they are, Expreſs the Relation between the Subject and the Attribute, as it is; That is, This is either Affirmed, or Denied of That, according to the Nature of that Relation, which Relation is Fixed by the Natures of the Things themſelves; To Follow Nature therefore, is not to Comply with our Bodily Inclinations, as ſome Imagine, but to Conform our ſelves to the Truth and Reality of Things, and the Proper and Determined Relations of them; As to what is Said here of Right and Wrong in Nature, that has been before Conſidered.

IN Anſwer to which we Say, that it is not hitherto Evinced, that our Conforming our ſelves to the Relations of Things is either a Vertue or a Duty; It is at the moſt only Proved, that we Act according to thoſe Relations; Which Suppoſing we ſhould not do, it is not Demonſtrated to be a Vice or a Crime. If any One ſhould Toſs a Stone into the Air, or Uſe Gold, as he would a Piece of Cork or a Feather, he would Act againſt the Gravitation of the One, and Confound the Natures of the Others, by making no Diſtinction betwixt them; but, I would Hope, this Philoſophy, the Grand Patron and Aſſertor of Truth, and of the Reality of Things, would not Condemn theſe for Evil and Immoral Actions; Nay, if we ſhould Endeavour to Deſtroy thoſe Relations, which we find Subſiſting in Nature, does it Appear, that we Deſerve Blame; Lead is Solid, and Water Fluid, with Reſpect to each Other, and yet it is to be Preſumed, that we may without any Impeachment of Vertue, Fix the One into a Salt, and Reduce the Other into a Fuſion, and ſo, that we may Alter the Properties of an Infinite Number of Beings, without any great Harm or Moral Evil; Thus likewiſe, a Father may Change his Son into a Husband by Marrying him, into a Brother, by having another Son, into a Magiſtrate, by Procuring him that Dignity, or into a Subject, by Admitting him into any Civil Community, and this with Commendation and Vertue enough; Tho' this is not Acting Conformably to the Reſpects and Relations of Things, as they are, but Changing the Old Ones, and Introduceing New: That is, this Doctrine muſt Oblige us to Receive and Take Things, as they are, and to Conform our ſelves to them, as they are, and Conſequently Forbid our Changeing them, Annulling, or Deſtroying them, or any Relations of them, which is Monſtrous, and Abſurd, by what Foregoes. Beſides which, how is it Shown, it is at Leaſt not yet done, that we are under an Obligation to Follow Nature, or to Conform our ſelves to the Fixed and Determined Relations of Beings in it? Things, and the Relations of them, may be Produced with other Deſigns, for all that we are yet Informed, than for our Following of them, or Conforming our ſelves to them; Perhaps they were Conſtituted and Framed, not for a Standard of our Practice, or for a Rule of our Actions, but for Incitements to us to Admire and Adore the Infinite Wiſdom and Goodneſs of the Almighty Architect; They might be Meant and Intended to Employ our Studies and Speculations, and not to Direct our Behaviour or Conduct; However, it is not Apparent, that we are Obliged to Conform our ſelves to theſe Relations of Things, becauſe God is the Author of them, and has Formed and Created them, till it is Shewn, that any Thing, which he has Created, is and ought to be a Meaſure of, and a Preſcription to, our Actions; Which if ſo, the very Actions of Brutes themſelves on the One Hand, and of the Angelick Natures on the other, will be a Rule and a Guide for our Morality, the One a very Unworthy One, and the Other an Unknown One, but Each, no Doubt, Incompatible with the Other; But, why we ſhould Conform our ſelves rather to the Relations of Things, which he has Created, than to Intelligent Beings, which he has Created, Diſtinct from us, cannot be Shewn.

Lastly, What the Fixed and Determinate Relations of Beings are, is the Hardeſt Thing in the World to Know; The Greateſt and Wiſeſt Philoſophers could never yet Tell, what they were, that is, what Nature is, for That it ſeems, is what we are to Follow; It has heen the Inquiry of many Ages, and of the Nobleſt and moſt Elevated Genius's in each Age, and yet Perhaps Mankind is as far from Arriving at Truth and Certainty in theſe Matters, as when the Search, and the Inveſtigation, Firſt began; Notwithſtanding which, we ſhould and ought to have a Comprehenſive View of all Nature, and of the Laws and Principles of it's Acting, before we can Know the Real and Abſolute Relations of the ſeveral Beings in it, and therefore, before we can Conform our own Actions to ſuch Relations; And Conſequently, if we are to Follow Nature, we are to Follow That, which it is Morally Impoſſible for us Fully and Exactly to Underſtand; If we can at all Underſtand it; Thus, in Particular, we Deal with Brutes, as Intirely Irrational, but till it is Known they are ſo, how can we Say, that we Act according to the Real Relations of Things? And yet, no One has hitherto Scrupled to Hunt a Fox, a Hare, or a Deer, or to Kill a Fiſh, or a Fowl, or any other Creature, tho' it is only to Serve our Palates, or our Diverſion.

All which ſtill Evinces, how Neceſſary it was for Almighty God to make a Clear Revelation of his Will to us in the Chriſtian Diſpenſation, that we might be Informed, which Way, and in what Manner, he would be Served and Obeyed.

3d. It is Said, that if there is a Supreme Being, upon whom the Exiſtence of the World Depends, and nothing can be in it, but what he either Cauſes, or Permits to be, then to Own Things *to be as they are*, is to Own what he Cauſes, or at Leaſt Permits *to be thus Cauſed or Permitted*; And this is to take Things, as he gives them, to go into his Conſtitution of the World, and to Submit to his Will, Revealed in the Books of Nature; To do this therefore muſt be Agreeable to his Will, and the Contrary Diſagreeable, and ſince there is a Perfect *Rectitude* in his Will, as will be afterwards Shown, Certainly *Wrong*; And, as the Owning of Things in all our Conduct *to be, what they are*, is Direct Obedience, ſo on the Contrary, not to Own Things *to be*, or *to have been, that are*, or *have been*, or not *to be, what they are*, is Direct Rebellion againſt him, who is the Author of Nature.

To which we alſo Reply; that here it is therefore Confeſſed, that Conforming our ſelves to Nature, or the Truth of Things, is not a Duty Incumbent upon us, Excepting, as the Exiſtence of the World Depends upon the Supreme Being; that, by Conſequence, ſuch a Conformity to the Real Nature or Relations of Things, is

Taking Things, as he *Gives them*, is *Going into his Conſtitution of the World*, and *Submitting to his Will, Revealed in the Books of Nature.*

First, That Nature is not Self-Exiſtent, but Depends upon the Supreme Being, as the Cauſe and Producer of it, is therefore Originally and Primarily to be Shewn, as the Baſis of all Natural Religion; which, tho' it may be Evidently done, is yet not done by any of the Patrons of ſuch a Religion of Nature; As we have already Evinced.

Second, Suppoſing the Supreme Being to be the Author of Nature, it is not Proved, that our Conforming our ſelves to it, is Submitting, or Conforming, to his Will, unleſs it is Firſt Evidenced, that it is his Will, that we ſhould Conform our Actions to it, which likewiſe Remains without any Proof or Eviction of it, according to what we have Juſt now Alledged; Beſides which, if any Perſon had made a Curious Piece of Clock-Work, and at the ſame Time was Able to make a Fly, and to Inſpire a Senſe into it, by which it ſhould Know the Maker of Both; Would any One Think, that the Laſt was Obliged to Conform it's Actions to the Motions or Syſteme of the Former, unleſs it Knew and was Satisfied, that it was the Will and Pleaſure of the Perſon, who gave it Being, that it ſhould? It might Admire it, and the Author of the Contrivance, and be Carried to an Adoration of him, for the Wiſdom of ſo Exquiſite a Piece of Exiſtence, and to a Gratitude and Thankfullneſs for it's own, but not Imagine it's ſelf any farther Concerned with the Motions of the Syſteme, which were left to go on, as it found them, at leaſt, not Suppoſe, that it was Scrupulouſly to Exact it's own Actions to the Laws or Relations of Things, which it Preſcribed.

As to what is Said of *Taking Things*, as *he Gives them*, and *Going into his Conſtitution of the World*, they are only Ingenious Phraſes and Ways of Expreſſion, which have more of the Pun of a Wit, than of the Argument of a Philoſopher, belonging to them.

Third, It is here alſo Preſumed, that Nature is the Revelation of God's Will to Mankind; It may be ſo; But then it is to be Interpreted in Conjunction with, and in Obedience and Conformity to, the Revelation of Chriſtianity; Unleſs, whilſt we Aſſert the One, we Diſcard and Deny the Other; For God has not, nor can have, Two Diſtinct Wills; The Revelation of his Will by Nature, or by the Goſpel, muſt Speak the ſame Thing, or the One or the Other is Falſe, and an Impoſture.

Here we Conſider Nature, with the Patrons of it, as Separate and Apart from Chriſtianity; And then how is it Proved, that Nature is a Revelation of God's Will? May not an Infinite, All-Wiſe, and an Almighty Agent Create a World with all the Relations and Reſpects of the ſeveral Beings in it, for the Exertion of his Power, and the Diſplay of his Wiſdom, which all other Rational Creatures, as well as Man, which all Superior Intelligences, Angels, and Arch-Angels ſhould Celebrate with their Praiſes, and Fall Down Proſtrate to the Omnipotent Author of, in an Humble Adoration of his Greatneſs and Majeſty, without Suppoſing, that this Grand and Stupendous Scheme of Nature was Intended for a Rule, and as a Preſcription to the Poor and Periſhing Actions of Men, and the Small Mortal Inhabitants of this Globe we Live on, and that it was his Will it ſhould be So? May not the ſame Almighty and All-Wiſe Agent have had a Thouſand other Deſigns and Intentions in his Creation, in Reſpect of other Beings, beſides our ſelves, which are not, nor, in this Imperfect State, ever can, or will, come to our Knowledge, which will not Infer, that *our Actions* ſhould be Governed by *his Nature and Conſtitution of Things?*

Lastly, Granting Nature to be the Revelation of his Will to us, Separate and Diſtinct from the Chriſtian One; It would be almoſt Inſcrutable to, and Unfathomable by us, and Conſequently could be no Direction to our Lives, or our Practice; Since it would not be Sufficient for us to Know the Obvious and Apparent Relations of Things, which may Depend only upon our Fancies, but the Intimate, Inward, and Real Ones; That is, we muſt be Apprized and Fully Informed of all the Depths and Receſſes of Nature, and of thoſe Infinite and True Reſpects, which Things bear to each Other, thro' all the Mazes and Labyrinths of the Vaſt Univerſe of Being; For otherwiſe, we could not Submit to God's Will, nor *Take Things, as he Gives them*, nor *Go into his Conſtitution of the World*; Whereas, all that we do, is to Conſider Things in Reſpect of our ſelves, and to Form our Relations of them, Chiefly, as they have a Reference to us, and many Times with too much Affectation and Vanity, as well as Partiallity to our own Beings and Exiſtence; When therefore this Philoſophy Tells us, that the Owning of Things in all our Conduct *to be, as they are*, is Obedience to the Supreme Being, and not to Own Things *to be*, or *to have been*, that *are*, or *have been*, or not *to be*, what *they are*, is Direct Rebellion againſt him, is all an Unintelligible Chime of Words, upon Abſtracted Notions of *are, be, been, have been*; Till we are Informed, what Things *are*, and what the Relations of them; That is, till we Know, what Real Nature is, and that God Deſigned the Creation of the World for the Rule of our Conduct, and not for a Subject of our Praiſe and Adoration.

As to the Firſt, what Real Nature is, and the Relations of the ſeveral Beings in it, this Philoſophy does not Undertake to Explain, or if it did, it would be Incapable of the Performance.

AND as to the Laſt, Prop. 9. Art. 2. Sect. 5. which Concerns Truths Relating to the Deity, it ſeems to Acquit Almighty God of ſo *Vain-Glorious* an Intention; But That is withall to be Proved there, which is not done, that for an Eternal Being to Conſult his own Glory, would be as Weak and as Frail, as for a *Cæſar*, a *Pompey*, or an *Alexander* to do it; On the Contrary, a Being, which is Infinite and Eternal, and All-Wiſe, would Act as Imprudently, if he did not Conſult his own Glory, as a Man of Common Diſcretion would do, who did not Conſider, or Value, his own Reputation; And the Weakneſs and Folly Lies in Extending our Names beyond the Grave, Suppoſing we our ſelves Ceaſed to be; Which is not the Caſe of a Being, who is Confeſſed Eternal; Tho' Perhaps this Philoſophy, Notwithſtanding it's Natural Religion, may Preſume to be That of us Groveling Mortals.

FOURTH, IT is Affirmed, that Things cannot be Denied to be, what they are, in *any Inſtance* or *Manner* whatſoever, without Contradicting Axioms and Truths Eternal, ſuch as theſe, *Every Thing is, what it is*; *That which is Done, cannot be Undone*, and the Like; And then, if thoſe Truths be Conſidered, as having always Subſiſted in the Divine Mind, to which they have always been True, and which Differs not from the Deity it's ſelf, to do this, is to Act, not only in Oppoſition to his *Government* or *Sovereignty*, but to his *Nature* alſo; Or, in another Way of Expreſſion; If there are ſuch Things, as *Axioms*, which are, and always have been, Immutably True, and Conſequently, have been always *Known to God to be ſo*, the Truth of them cannot be Denied any Way, but the Truth of the Divine Knowledge muſt be Denied too. Which is ſuch a Heap of Jargon, and Metaphyſical Trumpery, that it is almoſt Hard to Anſwer it.

FOR Firſt, tho' Things cannot be Denied to *be* what they *are*, does this Philoſophy Know, what they *are?* Nothing Leſs; How therefore is it Poſſible for us to Deny Things to *be* what they *are*, when we are Ignorant of their *Real Conſtitution?*

SECOND, The Truths or Axioms, which are here Said to Subſiſt in the Divine Mind, are in the Number of Trifling Propoſitions, Namely, *Every Thing is, what it is*, and *that which is done, cannot be Undone*, and to Suppoſe the Divine Mind to be Fraught with ſuch, or a Million of ſuch, Fooliſh and Ridiculous Axioms, is to Suppoſe, what is Unworthy of him; On the Contrary, we may Juſtly Imagine, that the Infinite Mind is Replete with the Ideas of all Nature, whether Intelligent, or Material, and that the Whole and Univerſal Syſteme of it Lies Perfectly Delineated and Drawn out on his Mind in all it's Relations, Properties, and Proportions, which are Truths and Axioms, that are Remote from our Knowledge, and therefore it is Impoſſible, we ſhould Deny them.

THIRD, Suppoſing theſe Truths and Axioms, which are only the Ludicrous Abſtractions of Fanciful Men, to be the Furniture, with which the Divine Mind is Repleniſhed, and I ſhould Deny a Thing to be what it is, which God knows, and Sees, I ſhould not Deny the Truth of the Divine Knowledge, unleſs I was ſome Way or other Informed, that God did Know and See This, and if I was ſo Informed, it would be Madneſs to Think, that I, or any One Elſe, ſhould or would Deny it.

LASTLY, There is no One in Terms or Words would Deny ſuch a Propoſition, That every Thing is, what it is; And therefore it cannot be Imagined, or Preſumed, that any One would do it by Facts, which ſhould Mean and Intend the Denial of ſuch a Propoſition, becauſe the Same Mind would Expreſs the Same Senſe, whether by Facts or by Words, if it was to Declare it's Senſe of ſuch a Propoſition; But, if Mens Facts can Affirm or Deny Things or Propoſitions without their Expreſs Meaning and Deſign to do it, Facts are made to Affirm and Deny, without any Purpoſe or Intention Annexed to them; which is Ridiculous, and againſt this Philoſophy, which Suppoſes, and has Endeavoured to Prove, that Facts are as Expreſſive and Significant, as Words.

Fifth, It is Told us, that Deſignedly to Treat Things, as being, what they are not, is the Greateſt Poſſible Abſurdity; It is to Put Bitter for Sweet, Darkneſs for Light, Crooked for Streight, *&c.* it is to Subvert all Science, to Renounce all Senſe of Truth, and Flatly to Deny the Exiſtence of any Thing; For Nothing does Exiſt, if Things are not, what they are; To Talk to, or Treat a *Poſt*, as if it was a *Man*, would be an Abſurdity, if not Diſtraction; And why ſhould it not be as Bad to Treat a *Man* as a *Poſt?* As if he had no Senſe, and Felt not Injuries, which is what the Cruel and Unjuſt often do.

To which we Anſwer Firſt, that to Treat a *Poſt* as a *Man*, is a very Abſurd Thing, and to Treat a *Man* as a *Poſt*, a very Bad and Cruel One; But I would Beg the Favour to be Excuſed from Thinking, that ſuch a Perſon who does ſo, therefore Denies, that there is any ſuch Being, as a *Man* or a *Poſt* in the World; He may Deal Wrongly by theſe Exiſtences, but does not from Thence Pretend to Annihilate them.

Second, To Treat a *Poſt* as a *Man*, is Said to be Abſurd, and to Treat a *Man* as a *Poſt*, to be Bad or Vicious; But to be Abſurd, and to be Bad and Vicious, are, in the Common Acceptation of Language, Two Different Things; And yet they ſhould be the Same, according to this Doctrine; Since they are both of them Facts, which Deny Things *to be what they are*; And if any One ſhould Talk and Reaſon with a *Poſt*, he ought not to be Eſteemed a Ridiculous or Abſurd, but a very Immoral and Wicked Man; Tho' One would be Apt to Believe, that, if any One had a Mind to Play ſuch a Prank, and Toſs ſome Words in the Air to no Purpoſe, there would be no Occaſion to Condemn him for a Vicious Man, but a Fool; On the Other Hand, if Treating a *Poſt* as a *Man*, is only Abſurd, and not Bad or Vicious, then Treating a *Man*, as a *Poſt*, will be ſo too, and not ſo Blameable, as Prepoſterous and Silly.

Third, Injuſtice and Cruelty does not Conſiſt in Treating a *Man*, as a *Poſt*, that is, as what he is not, which is only a Fancy of this Philoſophy, but in Uſing him Baſely and Illy, tho' he Owns and Acknowledges him to be a *Man*; No Perſecutor, or Tyrant, ever Treated a *Man*, as a *Poſt*, as if he Thought or Imagined he had no Senſe of Pain or Torture; The very Reaſon he Treats him in that Manner is, becauſe he Knows him not to be a *Poſt*, and can Feel Both; And the Vice of the Tyrant's or Perſecutor's Mind does not Lie in Denying Things by his Actions, *to be what they are*, but in Affirming them *to be what they are*, and Explaining at the Same Time the Revengeful, Malicious, and Inhuman, Principle of ſuch Actions; That is, the Perſecutor or Tyrant, by the Tortures he Inflicts upon another, Affirms him, not to be a *Poſt*, but a *Man*, for otherwiſe, he would not have Inflicted them, but then he Affirms his own Rage, Violence, Paſſion, and Injuſtice, which Denominates the Vice; That is, the Principle of the Action, and not the Falſe Aſſertion Implied by it; For the Aſſertion Contained in the Action is True in every Particular, that the Perſon Perſecuted or Tortured is a *Man*, and an Innocent *One*, but that the Perſecutor, or Tyrant, be he as Great, or as Small, as he Will, is a moſt Notorious Raſcal and a Villain.

Sixth, It is Said, that to Deny Things *to be as they are*, is a Tranſgreſſion of the Great *Law of our Nature*, the Law of *Reaſon*, for Truth cannot be Oppoſed, but Reaſon muſt be Violated. To which alſo we Reply **First**, that it ſhould be Proved, that the Great *Law of our Nature* is the *Law of Reaſon*, for it may be the *Law of God*, that is, the *Law of Revelation*, as we find it Delivered to us in the Scriptures.

Second, If it is Said, that the *Law of God*, or *of Chriſtianity*, is the Same with that of *Reaſon*, then *Reaſon* ſhould Submit to *Chriſtianity*, or not Interfere with it; And if ſo, our Diſpute is at an End.

THIRD, What *Chriſtianity* is, we Know; But what is *Reaſon?* Why Truly little more, as the Matter is now Managed, but *Faith, Belief, Fancy,* and *Opinion*; Which we have Shewn to be the Caſe of this, and other Syſtemes of Natural Religion; And which is in another Place Subſcribed to by this Philoſophy, however Fond it is of the *Law of Reaſon* in this; For it Tells us §. 1. Prop. 9. That they, who make *Right Reaſon* to be the Law, by which our Acts are to be Judged, and according to their Conformity to this, or Deflexion from it, Call them *Lawful* or *Unlawful*, Say Something more Particular and Preciſe; But the Manner in which they have Delivered themſelves, is not yet Explicite enough; It Leaves Room for ſo many Diſputes and *Oppoſite Right-Reaſons*, that Nothing can be Settled, while every One Pretends, that *his Reaſon* is Right.

IF this Philoſophy will but Allow, that this Remark may be Applied to it's ſelf, as it very Juſtly may, we need not Diſcourſe much more about the *Law of Reaſon*; And Indeed, it is in Vain to Talk of the *Law of Reaſon*, or *of Nature*, till we Know, what the *One* or the *Other* is, which I much Doubt, whether we ever ſhall; Tho' I Think, and Believe, that the Philoſophy of the Expanſive and Contractive Forces will Lead us into as many Truths in Both, as any Syſteme of Either, which has been hitherto Propoſed by the moſt Celebrated Genius's in Philoſophy, however Mean theſe Performances of Mine may Appear to the Grand Adepts and Profeſſors in that Science, the Science of Nature. As to what is Added Concerning the Amiable Nature of Truth, and the Great Force of it; Every One who Writes, Lays Claim to it; And as to this Philoſophy's Feeling within it's ſelf a Great Reluctance to the not Embraceing it, and Valuing it in the Higheſt Degree, I would In Charity Believe, that all Impartial Inquiries into Nature, or Religion, are Indued with the Same Noble Paſſions and Sentiments.

The Fifth Propoſition Recited and Conſidered. Mr. Wollaſton's Religion of Nature Delineated. §. 1. Prop. 5.

§. 15. THE Fifth Propoſition is, that, what has been Said of Facts Inconſiſtent with Truth, may be Said of many Omiſſions, or Neglects to Act; that is, by Theſe alſo True Propoſitions may be Denied to be True; and then thoſe Omiſſions, by which This is done muſt be Wrong, for the Same Reaſons with thoſe Aſſigned under the Former Propoſition.

IT is Farther Alledged upon this Propoſition, that ſome Things cannot Poſſibly be done, but Truth muſt be Directly and Poſitively Denied, and the Thing will be Clear; But the Caſes Ariſing from Omiſſions are not always ſo well Determined, and Plain; It is not always Eaſy to Know, when, or how Far, Truth is Violated by Omitting; Here therefore more Latitude muſt be Allowed, and much muſt be Left to every One's own Judgment and Ingenuity; But yet there are *ſome Neglects*, or *Refuſals* to Act, which are Manifeſtly Inconſiſtent with Truth, or with ſome True Propoſitions.

To which we Anſwer, that this ſo Entirely Depends upon the Foregoing Propoſitions, that there is no Occaſion to Enter into a Particular Confutation of it here; And we ſhall only Farther Obſerve, that it is Owned Omiſſions to Act are not ſo Explicite in the Denial of Truths, as Actions themſelves are, which have been already Conſidered; And Moreover, that it is Abſurd to Pick and Cull our Omiſſions to Act, and make ſome of them Deny Truth, and Others to be no Ways Significant of ſuch a Denial; However, we ſhall Proceed to Examine the Inſtances Produced; Becauſe, by them the Force of a Doctrine is much better Explained than by Looſe and General Terms, and Reſting in Univerſals.

1ſt. THE Firſt Inſtance is, that if *A* ſhould by ſome Solemn Promiſe, Oath, or other Act, Undertake to do ſome Certain Thing before ſuch a Time, and he Voluntarily, *Oblivione Voluntariâ*, as *Cicero* Expreſſes it, Omits to do it, he would Behave himſelf, as if there had been no ſuch Promiſe or Engagement, which is Equal to Denying there was any; and Truth is Manifeſtly Contradicted; And the Omiſſion to Act therefore, according to this Philoſophy, becomes Criminal and Vicious.

But in Reply, we muſt Beg Leave to Say; That, if *A* does not Perform his Oath, or his Promiſe, out of an *Involuntary* Forgetfullneſs, that is, becauſe he had Actually Forgot it, it is to be Hoped, he is not Blameable; and yet the Mere Fact, or the Omiſſion to Act, Contradicts Truth; For he Promiſed and Swore it, and did not Perform it; If a *Voluntary* Forgetfullneſs, that is a Deſign to Forget, or, which is the Same, an *Intention* not to Perform his Oath, or his Promiſe, is Neceſſary to make the Omiſſion Inſer a Denial of Truth, then mere Acts, or Omiſſions to Act, do not Affirm or Deny, but the *Intention* of them; which is, what we have been hitherto Defending, and is Contrary to the whole Scope and Tenour of the Preſent Doctrine, which Endeavours to Shew, that *Facts themſelves* without the *Meaning* of the Perſon, who Produces or is the Author of them, are Significant of Truth or Falſehood, as they Fall in with, or Diſſent from, the Real Relations of Things, and that they are more Expreſſive than any Verbal Affirmations or Denials of them, tho' the *Intention* is never ſo Obvious, and Manifeſt.

MOREOVER, it is here Suppoſed, that Performing an Oath, or a Promiſe, is a Vertuous Action, and the not doing it a Vicious One, and this Philoſophy only Explains, wherein the Vertue of the One, or the Vice of the other Conſiſts, to Shew the Reaſonableneſs and Juſtneſs of it's own Doctrine; But, if it ſhould be Denied, that the One is a Vice, or the Other a Vertue, it would be at a Loſs for a Proof of Either; And yet, when we Propoſe to *Demonſtrate* a Natural Religion, or to *Delineate it in a Series of Propoſitions,* we ſhould be Able Firſt to Prove, what is Vertuous, or what is Vicious, and then Evidence, that all Vertuous Actions are ſo many Aſſertions of the Truth and Reality of Things, and all Vicious Ones ſo many Contradictions to it.

THUS, that *A's* Performing an Oath is a Vertue, is not Shewn, unleſs upon the Conceſſion of the Being of a God, which by Atheiſts will not be Allowed or Granted, till it is Proved; And which, I have Evinced, cannot be done by a Connexion of Ideas, Excepting by Knowing ſomething more of Nature, than this Philoſophy is Acquainted with; Nor can it be Said, that *A's* Performing his Promiſe is a Duty, or, what is in Nature Required of him, if Men are only an Higher Order of Brutes, and only Endued with Greater Capacities to Over-Reach and Circumvent One Another; that is, if there is not a Future State, when we ſhall be Accountable to Almighty God for our Actions; which likewiſe ſhould be Demonſtrated in this Syſteme of Moral Good and Evil, of which yet it Takes no Notice.

So Slippery and Uncertain are theſe Arguments for Natural Religion, and ſo Neceſſary is it for us to have a Recourſe to a Revealed.

2d. IT is Said, that there are ſome Ends, which the *Nature* of Things and *Truth* Require us to Aim at, and at which, therefore, if we do not Aim, *Nature* and *Truth* are Denied. If a Man does not Deſire to Prevent Evils, and to be Happy, he Denies both his *own* Nature, and the Nature and Definition of *Happineſs,* to *be* what they *are*; And then Further, willingly to Neglect the *Means,* Leading to any ſuch End, is the Same, as not to Propoſe that End, and muſt Fall under the Same Cenſure; As to the *Means* it is Confeſſed, there is a Difficulty to Know, what *Thoſe* are, and what they are with Reſpect to us, which is Said to Proceed from our Ignorance of the True Natures of Things, of their Operations and Effects in ſuch an Irregular and Diſtempered World, and of thoſe many Incidents, that may Happen, either to Further or Break our Meaſures; It is Farther Said, that our Abilities and Opportunities are not Equal, and that ſome Labour under Diſadvantages Invincible, but ſtill we may Judge, as well as we can, and the Neglect to do this, will be an Omiſſion within the Reach of the Propoſition.

Upon which we ſhall Obſerve; FIRST, that Temporal or Worldly Happineſs and Evils muſt be here Intended, becauſe, whatever the Natures of Things are, their Operations or Effects, or whatever our Diſadvantages are in this World, they cannot Hinder, or Debarr, us from our Purſuits of the Happineſs of the Next; ſince our Eternal Happineſs does not Depend upon the Natures of Things, or any Incidents of this Life, or on the Better or Worſe Circumſtances of it; A Perſon may be Happy Hereafter, by Conforming himſelf only to the Doctrines of Chriſtianity, without Troubling himſelf with the Natures of Things, or the Relations of them; And One, who Lies under Invincible Diſadvantages, as to the Arriving at his *Chief Good* in the Preſent State of Things, may yet be in a Fair Way to Obtain it, in Another and a Future.

SECOND, This therefore is the Aſſertion, That, if a Man does not Deſire to Prevent Temporal Evils, and to be Happy in this World, he Denies both his *own* Nature, and the Nature and Definition of *Happineſs* to *be* what they *are*; From whence it will Follow, that all the Primitive Martyrs and Sufferers for Religion, who, it is

Plain, did not Deſire or Conſult their own Temporal Happineſs, nor were Sollicitous to Prevent Temporal Evils, Acted againſt, and therefore Denied, the *Nature* and *Truth* of Things, and Conſequently, were very Vicious and Profligate Men; Nay Farther, any Good Man, who Prefers the Enduring of Pain to the Loſs of his Vertue, or Contemns Pleaſure, that he may Preſerve it, muſt be Condemned by this Doctrine as a very Wicked and Immoral Perſon, as he Acts againſt *Nature*, and thereby Denies the *Truth* of Things, and the Real Relations of them; For Happineſs is afterwards Defined to be Pleaſure, and Evil, Pain.

THIRD, It is here Taken for Granted, that the *Summum Bonum*, or the *Chief Good*, which a Man ought to Purſue, is Happineſs, or Pleaſure, and the *Chief Evil*, which he ought to Avoid, is Miſery and Pain; Whereas this Philoſophy cannot be Ignorant, or ſhould have Known, that this was the Grand and Principal Diſpute amongſt the Antient Patrons of *Nature* and *Reaſon*, and the ſeveral Sects of them, ſome Affirming, that the *Summum Bonum* Conſiſted in Vertue, Others in Wiſdom, and the *Epicureans* only in Pleaſure, all whoſe Arguments are Admirably Enforced and Explained by *Cicero* in his Books, *De Finibus Bonorum & Malorum*; And when the Stoick *Poſidonius* was Reading a Lecture in Philoſophy, as I Remember, to *Pompey*, under the Greateſt Pain of the Gout, his *Nihil Agis Dolor* Shews the Contempt he had of it.

FOURTH, Let us now Suppoſe, that Temporal, or Eternal, Happineſs is here Meant, no One can be Said by his Facts to Deny both his *own* Nature, and the Nature and Definition of *Happineſs*, unleſs he has by thoſe Facts an *Intention* to do it, an Inſtance of which will Perhaps be Scarcely Found in Human Nature; For can it be Imagined, that any One Adviſedly, and with Deſign, ſhould Aim at his own Miſchief, Contrive his own Miſeries and Misfortunes, and be Bent and Determined upon his own Ruin? Even thoſe, who Put an End to their own Beings here, do it with an Expectation of a Better, or no Life hereafter; And the moſt Immoral and Vicious do not Mean or Intend their own Evil, tho' that may be the Conſequence of their Actions; And therefore, if an *Intention* is Neceſſary to Denominate our Facts, as it moſt Certainly is, ſince no One cannot but Intend his own Happineſs, there can be no Meaſure of Vertue or Vice from ſuch Facts, which are all Deſigned by the Agent for it's own Pleaſure and Satisfaction, either in this World, or Another.

3d. IT is Said, that there are Omiſſions of other Kinds, which will Deſerve to be Annumerated to theſe, by being either *Total*, or *Notorious*, or upon the Score of ſome other *Circumſtance*; and Farther, this Philoſophy goes on to Tell us, that it is Certain, it ſhould not Deny the *Phœniſſæ* of *Euripides* to be an Excellent *Drama*, by not Reading it, nor *Chihil-menar* to be a Rare Piece of Antiquity, by *not* going to See it; But ſhould it, having Leiſure, Health, and Proper Opportunities, Read Nothing, nor make any Inquiries in Order to Improve it's *Mind*, and Attain *ſuch Knowledge*, as may be Uſeful to it, it ſhould then Deny it's *Mind* to *be* what it *is*, and *Uſeful Knowledge* to *be* what it *is*; and it Matters not, what thoſe Studies are, every Man muſt Conſult his own Opportunities and Genius, and Judge for himſelf, as well as he can.

BUT, FIRST, how comes a *Partial* Omiſſion to be leſs Expreſſive of Denying a Thing to be what it is, than a *Total?* And why ſhould the *not* Reading the *Phœniſſæ* of *Euripides*, not Imply a Denial of it's being a Good Tragedy, as well as the *not* Reading any Books at all Imply a Denial of their being Good or Uſeful? This cannot be Explained, and is a Confutation it's ſelf of the Preſent Doctrine.

SECOND, It is Moreover Inexplicable, how an Omiſſion to do a Thing, ſhould be an Evident Denial of that Thing to *be* what it *is*, or how my *not* Improveing my Mind in Knowledge, ſhould be Equivalent to the Aſſerting, that it *is not*, what *it is*; That is, a Rational and Intelligent Being; Since my not Improveing a Rational and Intelligent Being in Knowledge, is not, by any Connexion of Ideas, the Same, as Denying ſuch a Being to be Rational, or Intelligent. On the Contrary, if I, having Leiſure, ſhould Omit all Opportunities of Inſtruction in Matters of Reaſon, Speculation or Learning, and ſhould Devote my Time and my Thoughts to Religion, and to the Adoration and Service of my Creator, to the doing of Good, and to the Performing Acts of Hoſpitality and Benevolence to Mankind, and to the Securing an Intereſt in the Happineſs of another World, I ſhould not much Inform my Underſtanding, or Improve the Knowledge of my Mind, and yet I ſhould not Act by it as an Irrational, or Unintelligent Being, nor ſhould I Affirm it to be ſuch a Being, or Deny it's

Reaſon or Intelligence, but Aſſert it; So little is there in this Fantaſtick Doctrine, and ſo Weak are the Pleas for the Support of it.

Third, It is Told us, that it Matters not, what thoſe Studies are, in which the Mind is to be Employed, which muſt be Left to every Ones Diſcretion, or his Genius and Inclination. How Undefined and Undetermined are the Meaſures in this Philoſophy, by which we ſhould Affirm our Minds to be, what they are? When, if we do but Engage them in any Kind of Study or Contemplation, the Aſſertion is the Same, that they are Rational and Intelligent Beings, altho' it ſhould Happen, as it does in this Delineation of the Religion of Nature, as well as in all the Preſent Syſtemes of Nature it's ſelf, that we are Entertained with nothing elſe in all theſe Studies, but what is, either Precarious, or Ridiculous, or Abſurd, or Irrational; Which muſt be as much a Denial of the Mind's being what it is, a Rational, and Intelligent Being, as any Omiſſion to Inform the Mind can be.

4th. To Prove a *Partial* Omiſſion does not Deny Truth, whereas a *Total* Omiſſion does; It is Said, if I give nothing to this, or that Poor Body, to whom I am under no Particular Obligation, I do not by this Deny them to be Poor, any more than I ſhould Deny a Man to have a Squalid Beard by not Shaving him, to be Naſty by not Waſhing him, or to be Lame by not Taking him on my Back; Many Things it is Said, are here to be Taken into Conſideration, according to the next Propoſition, and Truth Perhaps might be Intrenched upon by doing this; But if I, being of Ability, ſhould never give the Poor any Thing at all, I ſhould then Deny the Condition of the Poor to be what it is, and my Own to be what it is, and thus Truth would be Injured; So again, it is Alledged, If I ſhould not Say my Prayers at ſuch a Certain Hour, or in ſuch a Certain Place and Manner, this would not Imply a Denial of the Exiſtence of God, his Providence, or my Dependence on him; But, if I ſhould never Pray to him, or Notoriouſly Neglect to do it, ſuch a Total or General Omiſſion would be Equivalent, or Nearly ſo, to this Aſſertion, There is no God, who Governs the World, to be Adored; Which, if there is ſuch a Being, muſt be Contrary to Truth; Laſtly, ſhould I find a Man Grievouſly Hurt by ſome Accident, and without Preſent Help like to Periſh, or See his Houſe on Fire, no Body being near to Help; in this Extremity, if I do not give him my Aſſiſtance Immediately, I do not do it at all, which Amounts to a Total Omiſſion; I Deny his Caſe to be what it is, Human Nature to be what it is, and my own Deſires and Expectations, which I am Conſcious to my Self, I ſhould have under the like Miſfortune, to be what they are.

But First, all theſe Inſtances are Far from Proving, that Total Omiſſions Deny Truth, if Partial Ones do not; If the Nature of an Omiſſion is the Formal Denial of that which is Omitted, a Particular or Partial Omiſſion muſt be a Particular or Partial Denial, as well as a Total Omiſſion, a Total Denial; If the Nature of an Omiſſion is not the Formal Denial of that which is Omitted, then a Total Omiſſion does not Infer a Total Denial of it; Thus in the Firſt Inſtance, if I give nothing to this or that Poor Body, I muſt Equally Deny this or that Poor Body to be Poor, as if I give to no Poor Body, I Deny every Poor Body to be Poor; I muſt of Courſe by ſuch Omiſſions Deny both, or Deny neither. So in the Second Inſtance, if I Deny the Exiſtence of a God, his Providence, and my Dependence on him by a Total Omiſſion of his Worſhip, my Particular Omiſſion at any Time to Worſhip him is at that Time a Denial of his Exiſtence; And Indeed, according to this Doctrine, if an Omiſſion Includes the Formality of a Denial, it is Impoſſible for us not to Deny a Man to have a Beard, unleſs we Shave him, *&c.* which is very Abſurd; If it does not Include the Formality of a Denial, then a Total Omiſſion of Charity, or of Devotion, does not Infer or Suppoſe, that I Deny that there are in the World Certain and Real Objects of both.

Second, As in Facts, ſo in Omiſſions, it is Evident, that the Principle and Intention of them is Requiſite to Denominate of what Nature and Complexion they are; Thus, if a Perſon, Acknowledging the Duty of Charity, ſhould Neglect or Refuſe to Relieve the Poor, he does not Deny the Poor to be what they are, but Confeſſes himſelf to be an Ill Man; If another Perſon Thinks he is under no Obligation to any ſuch Duty, he ſtill Affirms the Poor to be what they are, but Proclaims his own Inhumanity, and the Difference betwixt them is, that the One is a Bad

Man in his Practice, tho' his Principle is Good, and the other a Profligate in both. So likewiſe, if any One ſhould Own the Being of a God, but Neglect to Worſhip him, which is the Caſe of Thouſands, it cannot be Preſumed, that his Omiſſion of ſuch Worſhip is a Denial of him, but is an Inſtance of his Madneſs, or his Stupidity; On the Contrary, if any One ſhould Neglect the Worſhip of God, becauſe he Believes there is no ſuch Being, his Omiſſions are ſo many Denials of him, yet not as they are Mere Omiſſions, but as the Deſign and Intention of them makes them ſuch Denials; and the Former, by his Omiſſions, Means nothing Leſs than the Denying of a God, and the Laſt nothing More; The One is Right in his Principle, and Wrong in his Moral, and the Other alike Vicious in Reſpect to Either. The Chief Novelty in this Doctrine ſeems to be in Aſſerting a Greater Authority to Facts, than to Words, as if there was more Truth and Sincerity in the One, than in the Other; whereas Men may as well Deceive by their Actions, as their Words, or Speak their Real Sentiments by them; and both of them can be only Judged of from the Principle and Intention, from which they Proceed, which often Lies Undiſcovered to the Stricteſt and Acuteſt Obſerver, and which therefore we muſt Leave to the Judge of all Hearts, the Great and Final Trier of them.

THIRD, It is Obſervable in the Laſt Inſtance before-Mentioned, of a Man's being Hurt by ſome Accident, or his Houſe being on Fire, it is Said, if I do not give him Immediate Aſſiſtance, I Deny Truth, becauſe, if I do not then Help him, I ſhall not do it *at all*, which therefore is Concluded to be a Total Omiſſion. To what Shifts is this Philoſophy Put to Vindicate it's own Doctrine? For if the not Helping him *at all* Infers a Total Omiſſion, my not Shaving a Man's Beard *at all*, or not Taking a Lame Man upon my Back *at all*, or not Waſhing a Man, who Wants it, *at all*, are to be *Annumerated* to *Total* Omiſſions, and therefore are ſo many Denials of Truth; But it is too Evident, that this *Doctrine* of *Omiſſions* is a very Weak and Infirm One, from what has been already Alledged Concerning it, and more Particularly from it's Shuffling, and it's Precarious Manner of making ſome Omiſſions Denials of Truth, and Others not, according as they are Apparently Vicious, or not Eſteemed to be ſo.

Propoſition the Sixth Explained and Examined.
Mr. Wollaſton's Religion of Nature Delineated. Prop. 6. §. 1.

§. 16. THE Sixth Propoſition is, that in Order to Judge Rightly, what any Thing is, it muſt be Conſidered, not only, what it is in it's Self, or in One Reſpect, but alſo what it may be in any Other Reſpect, which is Capable of being Deny'd by Facts or Practice; And the Whole Deſcription of the Thing ought to be taken in.

THUS, if a Man Steals a Horſe, and Rides away upon him, he may be Said Indeed by Riding him to Uſe him as a *Horſe*, but not as the *Horſe of another Man*, who Gave him no Licenſe to do this; He does not therefore Conſider him, as being what he is, unleſs he Takes in the Reſpect he Bears to his True Owner; But, it is not Neceſſary Perhaps to Conſider, what he is in Reſpect to Colour, Shape, or Age; Becauſe the Thief's Riding away with him, may neither Affirm nor Deny him to be of any Particular Colour, *&c.* upon which Account it is Said, that thoſe, and all thoſe, Properties, Reſpects, and Circumſtances, which may be Contradicted by Practice, are to be Taken into Conſideration; For otherwiſe the Thing to be Conſidered is but Imperfectly Survey'd, and the Whole Compaſs of it being not Taken in, it is Taken not as being what it is, but as what it is *in Part* only, and in other Reſpects Perhaps, as being *what it is not*.

BUT FIRST, in Anſwer to this Propoſition, and the Foregoing Inſtance Alledged upon it, we Say, that there is no Reaſon Aſſigned, why thoſe Reſpects and Relations, which may be Contradicted by Practice, ſhould be only Taken in, to Determine the Nature of Things, unleſs we Originally Suppoſe the Truth of this Doctrine, which is to be Proved, and not Suppoſed; For the Nature of Things, and their Relations, are Fixed and Immutable, but our Actions Deſultory and Uncertain; And therefore the Laſt cannot be a Meaſure or Standard of the Former, and Conſequently we cannot be Said to Act according to, or againſt, the Nature, Truth, or Relations of Things, as we Act according to, or againſt, thoſe Reſpects, which ſo Far as we Know, are Capable of being Affirmed or Denied by our Actions, which is only a Partial Conſideration of ſuch Nature of Things, or their Relations, but as we Act according to, or againſt, the Whole Syſteme of Beings, which is Impoſſible for us to Know, whether we do or not; Becauſe the Real Relations of Things, not Reduced to the Narrow Scantling of our Actions, are Infinite and Boundleſs; And, in the Stile of this Philoſophy, *the Thing to be Conſidered is but Imperfectly Survey'd, and the Whole Compaſs of it not being Taken in, it is Taken, not as being what it is, but as what it is in Part only, and in other Reſpects Perhaps, as being what it is not.*

SECOND, Suppoſing the Thief ſhould, when he Rides away with this Horſe, Uſe him as a Black Horſe, by Throwing a Black Cloth over him, when he is a Grey One, to Conceal his Theft, or ſhould Uſe him as a Young Horſe, when he is an Old One, by Riding him Harder, than is Fit for his Age, to Expedite his Thievery; Here are Facts, which Equally Deny the Relations of Things with the Former, and yet, if it was not another Man's Horſe, it would be Hard to Impute this as a Crime to a Man, or to Hang him for his Clothing, or Quickening, his Steed.

THIRD, However it muſt be Imagined, that, when a Thief Rides away with a Horſe, he does it by Stealth, and in ſuch a Manner, and with ſuch Circumſtances, as to Show, that he does not only Uſe a Horſe, but the Horſe of another Man; And it is to be Preſumed, that we are to Take in, not only the ſeveral Reſpects of the Horſe, but his Rider; And if ſo, the Thief does not only Affirm the Horſe to be a Horſe by his Riding of him, but to be the Horſe of another Man, by his Way and Manner of doing it; and Conſequently, we are to Conclude from this Doctrine, that the Thief Acts according to the Truth, and the Relation of Things, and is therefore a very Vertuous and Moral Perſon.

FOURTH, The Horſe is Suppoſed in this Inſtance to be another Man's; whereas, what Property is, is Unaſcertained here, and is not much better Explained in §. 6. Concerning Truths, Reſpecting Mankind in General, Antecedent to all Human Laws, from every Man having in himſelf a Principle of Individuation, which Diſtinguiſhes and Separates him from all other Men in ſuch a Manner, as may Render him and them Capable of Diſtinct Properties in Things, or, Diſtinct Subjects of Property. Since no One in a Natural State of Things is Intirely Independent of all other Beings in the Univerſe, he Stands in a Thouſand Relations to them, and even according to the Confeſſions of the Preſent Philoſophy. But more Particularly, every One is Dependent upon God, and therefore can have no Property of his Own, nor are his *Life*, or his *Limbs*, *&c.* his own Peculiar; And ſomething like this, in an Inferior and Subordinate Degree, may be Said in Relation to our Parents, Maſters, Guardians, Tutors, or Inſtructors; To whom we Owe many of the Advantages and Improvements of our Minds, which we ſhould have never had without them.

Religion of Nature Delineated· §. 6.

THE Second Inſtance under this Propoſition is, that if a *Rich Man*, being upon a Journey, ſhould be Robbed and Stript, it would be a Second Robbery and Injuſtice Committed upon him, to Take from him Part of his then Character, and to Conſider him, only, as a Rich Man; His Character Completed is a *Rich Man Robbed and Abuſed*, and Indeed at that Time a *Poor* Man and Diſtreſſed, tho' Able to Repay afterwards the Aſſiſtance Lent him; From whence I Suppoſe it is Concluded, that it would be Criminal, and a Denying of Truth, not to Relieve ſuch a *Rich Poor Man*.

FIRST, And ſo it would be, but not upon the Account of a Denial of Truth in this Caſe; For, if you do Relieve him, you Deny him to be a Rich Man, tho' he is One, and if you do not Relieve him, you Deny him to be a Poor One, tho' he is Said to be That too; That is, there is no Action in Nature, which can Aſſert both theſe Truths, that he is a Poor Man, and that he is a Rich Man, but whatever Action it is, which Affirms the One, muſt Deny the Other, and therefore the Acting according to Truth, and the Relations of Things, is here altogether Impoſſible and Impracticable; If it be yet Alledged, that you may Act with him, or by him, as a Poor Man *at that Time*, tho' he is in other Reſpects a Rich One, we do not Take in all the Relations and Reſpects of Things, which yet this Doctrine Informs us, it is Neceſſary for us to do, and that if we do not, we may Affirm Truth in One Reſpect, and Contradict it in another, which is the Preſent Circumſtance.

SECOND, If on the other Hand, we may be Allowed to Act according to the Preſent Reſpect or Relation of Things, without Conſidering any other, and therefore to Relieve a Man, who is Poor at Preſent, tho' in other Reſpects he is Really Rich, we need not Conſider a Horſe, in the Former Inſtance, as another Man's, tho' he Really is ſo, but as a *Poor* Solitary Horſe, without a Rider, and Perhaps not Taken the beſt Care of, which might be Deſired, and therefore, as wanting a better Owner;

Or he may be Conſidered, as a Horſe of little Value to the *Rich* Poſſeſſor, but of great Importance to the Neceſſities of the Poor Thief, *and ſo forth.*

Third, It ſhould be Taken into the Account, what this *Rich Poor Man's* Behaviour was, when he was Robbed and Stript, and what that of the Thieves, who Attacked him; For if they Met him in a Humble and Submiſſive Way, and Repreſented the Great Neceſſities, which they Lay under, and that they muſt be Starved, unleſs they were Relieved, this would Scarcely be any more than a Perſon, Ready to Famiſh, Taking Bread, or Victuals, where he could Meet with it, which, I Think Caſuiſts, and the Laws, Allow to be a Compaſſionate Caſe; And, if a Rich Man Refuſed his Aſſiſtance to ſuch Aggreſſors, he would Deny Truth and theſe Poor, Neceſſitous Men, to be what they were, and the Fault would Lie on his Side, that he did not Relieve them; and, if to this Injuſtice he Added Reproach and Contumely, and Perhaps Shot at them, and Endeavoured to Deprive them of their Lives, he would Deny a Man to be *what he was* by Trying to Murder him; And, if they Took, from him, what he had, and Stript him, they would Affirm, what he was, a very Inhuman Perſon; and therefore, if we Comprehend all theſe Relations in the Account, according to this Doctrine, the Thieves might be very Honeſt Men, and the Rich Man a very Ill One; And to Relieve him might Poſſibly be to Affirm; that he was, what he was not.

The Third Inſtance, under this Propoſition is, that a Man, in giving Aſſiſtance of any Kind to another, ſhould Conſider, what his own Circumſtances are, as well as what the other's are; If they do not Permit him to give it, he does not by his Forbearance Deny the Other to Want it, but if he ſhould Give it, and by that Deny his own or his Family's Circumſtances to be what they are, he would Actually Contradict Truth; And ſince, as it is Said, all Truths are Conſiſtent, nor can any Thing be True any Farther, than as it is Compatible with other Things, that are True, when both Parties are Placed in a Right Light, and the Caſe Properly Stated for a Judgment, the Latter may Indeed be Truly Said to want Aſſiſtance, but not the Aſſiſtance of the Former, any more than a Man, who wants a Guide, may be Said to want a Blind or a Lame Guide.

But First, if a Man, in giving Aſſiſtance of any Kind to another, is to Conſider what his own Circumſtances are, as well as what the other's are, he will not, as in the Former Inſtance, find any Action, which ſhall be Expreſſive of the Truth of both their Circumſtances; For if he Refuſes to give, he Denies the other Perſon's Circumſtances to be what they are, and if he does Give, Suppoſing them Mean, he Denies his own; Since it is Impoſſible to Say, that a Man Contradicts Truth more by Denying his own or his Family's Circumſtances to be what they are, than by Denying thoſe of another Man to be what they are, and ſince our Denials are to be Judged by our Facts. *All Truths,* it is Said, *are Conſiſtent,* and yet it is not ſo here, becauſe the very ſame Fact, which Affirms the Truth of our own Circumſtances, Denies the Truth of another Man's, and Vice Verſâ.

Second, Nor is it Sufficient to Alledge, that nothing can be True any Farther, than as it is Compatible with other Things, that are True, and that, when both Parties are Placed in a Right Light, and the Caſe Properly Stated for a Judgment, the Latter may be Truly Said to want Aſſiſtance, but not the Aſſiſtance of the Former; Becauſe the ſame Fact, which Affirms one Truth, Denies another, and therefore the One is not Compatible with the Other; and as for Stating a Caſe for Judgment, there is no Room left for it, ſince the Fact Declares the Judgment and Sentiment of the Mind, and Conſequently muſt Affirm my own Circumſtances, what they are, and Deny another Man's to be what they are, or, on the Contrary.

The Laſt Inſtance under this Propoſition is, when a Man, *A*, is under ſome Promiſe or Compact to Aſſiſt another, *B*, and at the ſame Time Bound to Conſult his own Happineſs, Provide for his Family, *&c.* and he cannot do Theſe, if he does That, Effectually. For, what muſt *A* do? Here are not Indeed Oppoſite Truths, but there are Truths on Oppoſite Sides; To which this Philoſophy Anſwers, tho' there cannot be two Incompatible Duties, or tho' two Inconſiſtent Acts cannot be both *A*'s Duty at the ſame Time, yet an Obligation, which may be called Mixed, may Ariſe out of thoſe Differing Conſiderations, *A* ſhould Aſſiſt *B*, but ſo, as not to Neglect himſelf and Family, and ſo to take Care of himſelf and Family, as not to forget the other Engagement, *as well and Honeſtly, as he can*; Here the Importance of the Truths on the One Side, and the Other, ſhould be Diligently Compared, and there muſt, in ſuch Caſes, be always ſome Exception or Limitation Underſtood; It is not in Man's Power to Promiſe Abſolutely; He can only Promiſe as One, who may be Diſabled by the Weight and Incumbency of Truths not then Exiſting; In ſhort, when Things are Truly Eſtimated, Perſons Concerned, Times, Places, Ends Intended, and Effects, that Naturally Follow, muſt be Added to them.

To which we Venture to Reply, First, That if *A* is under ſome Promiſe or Compact to Aſſiſt *B*, if he Acts according to Truth, he ought to Aſſiſt him, and if at the ſame Time he is Obliged to Conſult his own Happineſs, and Provide for his Family, which is Inconſiſtent with the Former, if he Acts according to Truth, he ought to do this likewiſe, which yet is Impoſſible, and Shews the Abſurdity of the Preſent Doctrine; For, what muſt *A* do Indeed? When by the Rules of this Philoſophy, his Duty is Impoſſible to be Performed by him.

Second, Nor is it enough to Say, that from Hence a Mixed Obligation Ariſes; How a Mixed Obligation? That he muſt neither Perform his Promiſe Effectually, nor Conſult his own Happineſs, or Provide for his Family, Effectually? Why this would be to Act according to Truth on neither Hands, but to Shuffle, and to Trim Matters ſo, till his Actions on both Sides became a Lie; which ſeems to be Confeſſed here, when this Philoſophy Owns, that in ſuch a Caſe, *A* is not Obliged to Act up to the utmoſt Punctilio and Exactneſs of Truth, but to Act *as well and Honeſtly as he can*; This therefore, it ſeems, is the Celebrated Doctrine of Acting according to Truth, and the Real Relations of Things, that, upon Certain Caſes and Emergencies, we are left at Liberty to Deſert them; And what is Said afterwards of Exceptions, and Limitations, and of being Diſabled by the Weight and Incumbency of Truths not then Exiſting, are only Different Ways of Expreſſion, to Juſtify the ſame Latitudinarian Explanation of it's own Doctrine.

Third, And what is farther Alledged of Perſons Concerned, Times, Places, and *Ends Intended*, being Neceſſary to be Conſidered in a True Eſtimation of Things, is, as we have Shewn, an Entire Confutation of it.

Lastly, If, in Order to Judge Rightly, what any Thing is, we muſt Conſider not only, what it is in it's ſelf, or in One Reſpect, but alſo, what it may be in any other Reſpect, which is Capable of being Denied by Facts or Practice, that is, if we muſt Take in all the Relations of a Thing, under which it Stands to other Beings, the Knowledge of Moral Good and Evil will be an Impoſſible, or ſo Difficult, a Science, that no One, but a Philoſopher, and one of the Subtileſt Kind, will be able to Comprehend it; But one would Think a Religion of Nature, ſhould be a Plain and Simple Buſineſs, Suited to every One's Capacity, and Level to every Man's Reaſon; Unleſs we Imagine, that the Intention of Almighty God was only to Save a few Learned, and Speculative Men and Leave the reſt of the Poor Ignorant World to Shift for themſelves, as well as they could.

We ſhall therefore here Conclude, that Vertue and Vice, or Moral Good and Evil are not to be Eſtimated barely by Men's Facts, or their Actions, but by the Deſigns and Intentions of them likewiſe; and, where thoſe Deſigns and Intentions are to do Good, and to Benefit Mankind, or any Individual Perſon of the whole Species, they are Good and Moral Intentions, and the Actions, which are Derived from them, are ſo too; and where thoſe Deſigns and Intentions are to Prejudice, or to do a Harm or a Miſchief to Mankind, or any Single Individual Perſon of the whole Species, ſuch Intentions are Immoral and Evil, and the Actions or Facts Derived from them; And nothing can be Determined from the Imaginary Conformity or Diſagreement of them to the Truth and Real Relations of Things. Upon which Account it is, that the Chriſtian Religion, which is the Utmoſt Syſteme of Goodneſs to Mankind, and to every Individual of the Species, is the Trueſt Standard of Moral Good and Evil, and which Syſteme, This, nor Any other, Derived from Men's Fancies and Whimſies, which they Call Reaſon, will be ever Able to Overturn.

I do not here Pretend to Affirm, that the Intention is only to be Conſidered, but that it is a Main and Principal Ingredient in every Moral Action, which yet is Rejected from this Philoſophy; The Conformity of our Actions to the Divine Will and Law is likewiſe Expedient to Denominate them Good, and is a Neceſſary Requiſite to the Morality of them.

Propoſitions the Seventh and Eighth Recited and Examined. Mr. Wollaſton's Religion of Nature Delineated. Prop. 7. 8. §. 1.

§. 17. The Seventh Propoſition is, that, when any Act would be Wrong, the Forbearing that Act muſt be Right; Likewiſe, when the Omiſſion of any Thing would be Wrong, the Doing of it, *i. e.* not Omitting it, muſt be Right; Becauſe *Contrariorum Contraria eſt Ratio:* This ſo Manifeſtly Depends upon the Foregoing Doctrine, which has been Examined, that there can be no Occaſion to Conſider it in Particular here.

The Eighth Propoſition is, that Moral Good and Evil are Coincident with Right and Wrong; For that cannot be Good, which is Wrong, nor that Evil, which is Right.

But here it is to be Obſerved, that the Terms of the Queſtion in Diſpute are Changed; And Right and Wrong are Equivocally made to Signify, either Good or Evil, or what is Juſt or Unjuſt, or what is True or Falſe; Or Acting, or not Acting, according to the Nature and the Relations of Things.

First, Therefore if Right and Wrong are Coincident with Good and Evil, that is, Signify Good and Evil; The Reaſon Aſſigned for the Truth of the Propoſition is; For that cannot be Good, which is Evil, nor that Evil, which is Good, which is a very Wiſe, and a moſt Profound Axiom.

Second, If by Right and Wrong is Meant Juſtice or Injuſtice; The Propoſition will not be True, nor the Reaſon Alledged to Prove it; For Moral Good and Evil are not Coincident with Juſtice and Injuſtice, ſince Good and Evil are more Extenſive Terms than Juſt and Unjuſt: a Juſt, Rigid, and Punctual Man may not be a Good, Kind, and Beneficent Man, but a Good Man will at Leaſt be a Juſt Man.

And on the other Hand, an Unjuſt or Unrighteous Perſon may not be an Evil, Malicious, and a Cruel One, but an Evil or Malicious Perſon is at Leaſt an Unjuſt One; So that, tho' we may Say, That cannot be Good, which is Wrong, or Unjuſt, nor That Evil, which is Right or Juſt, yet it does not Follow, that Good and Evil are Coincident with Right and Wrong, Juſtice and Injuſtice; Which Shews the Sophiſtry and Weakneſs of the Reaſon Offered to Evince the Propoſition.

Third, if by Right and Wrong, Truth and Falſehood are Intended, the Reaſoning is much the Same with that under the Preceding Head; Becauſe Good and Evil are more General and Comprehenſive Terms than Truth and Falſehood, as they are, than thoſe of Juſtice or Injuſtice; Truth and Falſehood, as well, as Juſtice and Injuſtice, are at the moſt and the beſt only ſeveral Kinds and Species of Good and Evil, and not the Eſſences of them; Thus, a Man may Speak and Act according to Truth, and yet not be a Good, Kind, and Charitable Man, but a Good and Kind Man will always Speak and Act according to Truth, becauſe he has no Siniſter or Ill Ends, which ſhould Induce him to Deceive; And on the Contrary, he may Speak and Act a Lie, and yet not be an Ill, or Malicious Man, tho' an Ill or a Malicious Man at Leaſt is a Falſe Man and a Liar; So that, again, tho' we may Say, That cannot be Good, which is Wrong, Falſe, or a Lie, nor That Evil, which is Right or True; Yet it does not Follow, that Good and Evil are Coincident with Truth and Falſehood; Which likewiſe Shews the Sophiſtry and Weakneſs of the Reaſon Offered to Evince the Propoſition.

Fourth, If by Right and Wrong is Signified the Acting, or not Acting, according to the Truth and Relations of Things; Which muſt be the Real Senſe of this Propoſition, as it is Agreeable to the Preſent Doctrine; Then the Propoſition will Run thus, that Moral Good and Evil are Coincident with our Acting, or not Acting, according to the Truth and Relations of Things; For that cannot be Good, which Contradicts them, nor that Evil, which Correſponds with them; Which is the very Point in Controverſy, and has been Sufficiently Proved to be a Precarious, Unjuſt, Falſe, and Wrong Aſſertion.

And, as Moral Good and Evil are more Extenſive and General Terms, as we have Shewn, than Juſtice or Injuſtice, Truth or Falſehood, Acting or not Acting according to the Truth and Relations of Things, ſo, as at the Concluſion of the Sixth Propoſition, we here likewiſe Affirm, that the Principle and Intention of doing Good to All is the Foundation of ſuch an Extenſive Morality, and is more Forcibly and Fully Inculcated in the Chriſtian Scheme, than in any other Scheme of Philoſophy or Reaſon, which was ever Propoſed to Mankind.

Propoſitions the Ninth and Tenth and Eleventh Recited and Examined.
Mr. Wollaſton's Religion of Nature Delineated. Prop. 9. 10. 11. §. 1.

§. 18. The Ninth Propoſition is, that every Act of ſuch a Being, as is before Deſcribed, and all thoſe Omiſſions, which Interfere with Truth, *i. e.* Deny any Propoſition to be True, which is True, or Suppoſe any Thing not to be, what it is, in any Regard, are Morally Evil, in ſome Degree or other; The Forbearing ſuch Acts, and the Acting in Oppoſition to ſuch Omiſſions, are Morally Good; And, when any Thing may be either done, or not done, Equally, without the Violation of Truth, that Thing is Indifferent.

This Propoſition is ſo Manifeſtly Derived from the Former Ones, that it is rather a Corollary from them, than a Diſtinct and Independent Propoſition, and therefore we ſhall Say no more Concerning it.

We ſhall only take Notice, that under this Propoſition are Enumerated the ſeveral Chief Opinions of the Antient Philoſophers, Concerning the *Summum Bonum & Malum*, which a Wiſe Man ought to Purſue or Avoid, that is, by which all his Actions ought to be Squared, and to which they ought all to be Ultimately Referred, and by which they might be Diſcriminated and Diſtinguiſhed into Good or Evil; Theſe were Called the *Fines Bonorum & Malorum*, and it is Said, there was ſuch a Variety of Sentiments among the Old Philoſophers Concerning them, that *Varro* Reckons up 288 of them.

However, this Philoſophy Thinks it has Outdone them all, and Propoſes the Acting, or not Acting, according to the Nature and Relations of Things, as a Standard of Vertue and Vice, or of Good and Evil; Which yet is no more, than what One Sect of thoſe Philoſophers has already Offered; Namely, that *Scientia*, or the Truth and Knowledge of Things ſhould be the Ultimate End, or the *Summum Bonum*, at which we ſhould Endeavour to Arrive, and which, as it was Confuted in thoſe Times by Different Sects and Perſuaſions, ſo we have Endeavoured to Shew the Weakneſs of ſuch a Pretenſe, in the Foregoing Arguments againſt it; Beſides which, would not any One in theſe Times Wonder at a Philoſopher among the *Greeks* or *Romans*, who ſhould Affirm that *Science* was the Standard and Meaſure of *Vertue?* And yet this Old, Exploded Plea is Revived, and made to Over-top all the other Criterions of it, with only this Advantage of the Ignorance of it's Preſent Patrons, who are Eminent for Knowing little of the *Science* of Nature, to which yet all our Actions are to be Adapted and made Conformable.

For, First, As to Thoſe, who Reckon nothing to be Good, but what they Call *Honeſtum*, as *Cato* in *Cicero*, this Philoſophy Asks, but what is that *Honeſtum?* To which I Anſwer Briefly, and what is the *Truth and Relations of Things?* But more Fully, the *Honeſtum* was any Action, which Proceeded from a Senſe and Principle of Generoſity and Honour, which would Incite a Man to Prefer the Good of Mankind, or his Country, to any Mean and Paltry Conſiderations of Ambition or Intereſt; Which Falls in with the Character of that *Brave and Excellent Roman*, and is a Juſt Deſcription of his Sentiments and Opinions; And This Agrees with that Definition, which we before Gave of the True Criterion of what is Moral, or Immoral, and which is Exactly Coincident with the Chriſtian Scheme and Standard of Both.

Second, As to Thoſe, who Place all in *Following Nature*, as *Zeno*, this Philoſophy Tells us, if they Mean by that Phraſe, Acting according to the Natures of Things, that is, Treating Things, as being, what they in Nature are, or according to Truth, they Say what is Right; But this does not ſeem to be their Meaning; And if it is only, that a Man muſt Follow his own Nature, ſince his Nature is not Purely Rational, but there is a Part of him, which he has in Common with Brutes, they Appoint him a Guide, which, it is Feared, will Miſlead him, this being, Commonly, more likely to Prevail than the Rational Part; *At the Beſt this Talk is Looſe.*

But 1*ſt*. One would Think this Philoſophy Scarcely Underſtood it's own Quotations, or Imagined, Others could not, Τέλος εἶπε [Ζήνων] τὸ ὁμολογουμένως (*al.* ἀκολούθως) τῇ φύσει ζῆν, ὅπερ ἐστὶ καὶ ἀρετὴν ζῆν. Ἄγει γὰρ πρὸς ταύτην ἡμᾶς ἡ φύσις. *Diog. Laert.* is the Citation. And was it Poſſible for this Philoſophy to Doubt, whether *Following Nature* here Meant the Acting according to the Natures of Things, that is, Treating Things, as being what they are, which if it Meant, it Approves of, or whether it Meant Following our own Nature, as it Conſiſts both of a Rational and Brutal Part, which it Condemns, when it is Impoſſible it ſhould Mean Either? Since *Zeno* does not Trouble himſelf about the Nature or Relations of Things, or our own Natures, but Vertue, and to Live according to Vertue, he Says, is to Live Profeſſedly, or in Conſequence, according to Nature, for Nature Leads us to This; The Τέλος or *Finis* therefore is to Live κατ' ἀρετὴν, which is Said to be to Live τῇ φύσει; And therefore we muſt Know what *Zeno* Meant by Vertue, for That with him was Nature, in Direct Oppoſition to the Preſent Philoſopher, who would make *Zeno* Determine, what Vertue was from either the Nature of Things, or our own Nature; Now it is Plain, what the *Stoicks* Intended by Vertue; Namely, to be Temperate, Chaſte, Sober, and Good, in all Circumſtances of Life, whatever the Nature of Things or our own Beings were, which they did not ſo much Conſider; which alſo Falls in with the *Honeſtum* of *Cato*, who was Addicted to that Sect, and is ſtill Agreeable to the Chriſtian Scheme of Goodneſs and Vertue, and which Knew Little of the Natures and Relations of Things, and yet Perhaps as much, as the Modern Patrons of Them.

2*d*. In Reſpect of the Two Interpretations Pretended from this Paſſage of *Following Nature*, which, we have Shewn, are nothing to the Purpoſe of it, The Firſt, of Following the Natures and Relations of Things, which this Philoſophy Applauds and Eſpouſes, has been Fully Conſidered, and, I am Apt to Think, Refuted; The Second, of Following our own Natures, as Conſiſting of a Rational and a Brutal Part, which it Explodes, we ſhall now Examine; And I would here Ask, and Deſire to be Anſwered, as to the Rational Part, whether it is not, as Reaſon is now Managed, as little to be Followed, as the Brutal; For what is Truth or Reaſon, in Reſpect of Moral Good and Evil? It is not, by the Confeſſions of this Philoſophy, with which I only at Preſent Argue, it is not, what has been hitherto Propoſed by the Antient Philoſophers, or the Modern, and we have Proved, that Truth and Reaſon is not to be Found in the Preſent Syſteme; To what then muſt we have Recourſe, unleſs to the Principle, I have before Mentioned, of a moſt Intenſe Goodneſs to all the World, and which Exactly Agrees, and is Correſpondent with Chriſtianity; Not to Mention, that this Philoſophy, when it Speaks of Right Reaſon afterwards, Tells us; That it Leaves Room for ſo many Diſputes, and *Oppoſite Right-Reaſons*, that nothing can be Settled, while every One Pretends, that *his* Reaſon is Right.

But, which is more, as to the Brutal Part of our Natures, there can be no more Reaſon Aſſigned, why we ſhould not Follow the Natures and the Relations of them, than the Natures and Relations of Beings, which are External to us; For why ſhould it be a more Sacred Matter, or a more Vertuous One, to Conform our ſelves to the Appetites and Propenſities of Beings Diſtinct from us, and their ſeveral Relations in Exiſtence, than to thoſe External Portions of Being, which are United to our ſelves? And is not the Conforming our ſelves to the Real Relations of Things, or of Beings, which have an Intimate Union with our own Minds, as Good and as Moral, as the Conforming our ſelves to the Relations of Things, or of Beings, which are Foreign to us in Nature, and have little or no Affinity with us? And are not the External Parts of Nature in Reſpect of us, as Brutal, as thoſe Parts of it, which are Joined and United to us? I Ask theſe ſeveral Queſtions, that they may be Replied to by this Philoſophy, and it's Patrons, at their Leiſure.

Third, As to Thoſe, who make Right Reaſon to be the Law, by which our Acts are to be Judged, tho' they are Owned to Say ſomething more Particular and Preciſe, yet are Rejected, as to their Sentiments, by this Philoſophy; Becauſe there are ſo many *Oppoſite Right-Reaſons*, that nothing can be Settled from thence.

But here again we muſt Anſwer, that, if we put the whole upon Truth and the Real Relations of Things, there will be as many Oppoſite Truths and Real Relations of Things, as they are Called, as Oppoſite Right-Reaſons; For what is Truth, but Reaſon, or on the Contrary? And, if it is a Difficult Matter to Determine, what Right Reaſon is, it will be no Leſs a One, to Aſcertain, what is Truth, and the Real Relations of Things; Does this Philoſophy in Particular Inform us, which Underſtands nothing of Nature, what Truth or the Real Relations of Things are, by any Inſtances, which have been Alledged by it? Far from it; If not, it is Talking in a General Way, without Telling us, what Truth is, or what the Real Relations of Things are; and therefore we may Say, in it's own Expreſſions, that, *at the Beſt, this Talk is Looſe*.

FOURTH, As to Thoſe, who Contenting themſelves with Superficial and Tranſient Views, Deduce the Difference between Good and Evil, from the *Common Senſe* of Mankind, and Certain Principles, that are Born with us, it is Said, Put the Matter upon a very Infirm Foot; For ſuch Innate Maxims may be only the Impreſſions of Education; And beſides that, the Sentiments of Mankind are not ſo Uniform, and Conſtant, as that we may Safely Truſt ſuch an Important Diſtinction upon them.

1*ſt*. IN Anſwer to which we Say, that thoſe Maxims, which are here Called Innate, cannot Proceed Merely from the Impreſſions of Education, that, as we have before Shewn, there is ſomething Natural to the Mind, which Leads it to ſuch Concluſions, and in Fine, that it is not a *Raſa Tabula*, as ſome Imagine.

2*d*. THE Second Objection againſt *Innate Maxims*, that the Sentiments of Mankind are not Uniform and Conſtant in ſuch Maxims, Equally Obtains againſt Acting according to the *Truth*, or *Nature*, or the *Real Relations of Things*; Becauſe, what is Truth, or Nature, or ſuch Real Relations, will be a Diſpute, and will be Found to be Terms, as Vagrant and Uncertain, as thoſe Innate Maxims.

ON the Other Hand, which we Defend, the Principle of Doing Good, and being Beneficial to All, as Far, and as Much, as Lies in our Power, and Avoiding all Evil and Malicious Intentions, is an Eaſy and a Natural Criterion, of what is Good, or what is Evil, and in which, or the Contrary, Vertue and Vice Conſiſts; No One can Fail to Know, when he Means to do Good to Others, and when he Intends to Injuere, Grieve, Prejudice, Wrong or Oppreſs them; Such Actions therefore, which are Derived from the Former Principle, are Good, Vertuous and Moral; And Thoſe, which Reſult from the Latter, are Immoral, Vicious, and Evil; The Former alſo do not only Agree with the Chriſtian Scheme, but are Conformable to the Infinite Love and Goodneſs of God to his Creatures, and therefore Depend upon the Trueſt Source of all Natural Religion, the Confeſſion of ſuch a Gracious and Beneficent Being; And the Latter are Entirely Oppoſite and Repugnant to Both.

FIFTH, As to Thoſe, who own nothing Good, but Pleaſure or the *Jucundum*, and nothing Evil, but Pain, as *Epicurus* and his Followers; It is Said, they do not Agree in what this Pleaſure is to be Placed, or by what Methods and Actings the moſt of it may be Obtained; As Men have Different Taſtes, Different Degrees of Senſe and Philoſophy, the ſame Thing cannot be Pleaſant to all; And if Particular Actions are to be Proved by this Teſt, the Morality of them will be very Uncertain; The ſame Act may be of One Nature to One Man, and of Another to Another; Beſides, unleſs there be ſome Strong Limitation Added, as a Fence for Vertue, Men will be Apt to Sink into Groſs Voluptuouſneſs, as in Fact the Profeſſors of this Doctrine did, Notwithſtanding all their Talk of Temperance, Vertue, *&c.* and the Bridle will be Uſurped by thoſe Appetites, which it is a Principal Part of all Religion, Natural as well as any *Other*, to Curb and Reſtrain.

BUT 1*ſt*. If we Take this Doctrine Right, it is a True One, that is, if we make the *Jucundum*, or Pleaſure, to Conſiſt in a Rational Enjoyment or in Knowledge, and Doing Good to Others; Which Certainly are the moſt Agreeable, and therefore the moſt Pleaſing and Delightful to a Rational Nature, and is again Coincident with the Truths of Religion, and the Great and Sublime Precepts of Chriſtianity.

2*d*. THE *Truth* and *Relations of Things* will be, as we Said before, as Diſputable a Point, as what is *Pleaſant* and *Agreeable*; And, *if Particular Actions are to be Proved by this Teſt, the Morality of them will be very Uncertain, the ſame Act may be of One Nature to One Man, and of Another to Another.*

3*d*. IT is not *the Deſign of all Religion, Natural, as well as any* Other, *to Curb and Reſtrain our Bodily or Corporeal Appetites*, for this would be to make Religion Conſiſt in Meats, and in Drinks, which are of a Trifling and Inſignificant Nature. Religion has a Higher and Nobler View, to Raiſe our Thoughts to ſomething above this World, and all the Pleaſures and Satiſfactions of it, to make us not only Temperate, Sober, and Chaſte, but Good, Kind, Charitable, and Beneficent to All, and to Aſſure us, that, whatever Afflictions we Meet with in the Performance of our Duty, Almighty God will Reward us in another and a better State, with an *Exceeding Weight of Glory*, for any *Light and Trivial Sufferings*, which we ſhall here Encounter.

Sixth, As to what is afterwards Said, Concerning Vertue being Placed in the Middle betwixt Two Extremes, it is not Worth the While to Conſider it; And as to *Plato's* making Vertue to Conſiſt in ſuch a Likeneſs to God, as we are Capable of, and God to be the Great Exemplar, this Philoſophy Says, it ſhall not Diſpute; tho' it Wants to Know by what Means, we may Attain this Likeneſs, and Thinks it muſt be by the Practice of *Truth*, God being *Truth*, and doing nothing Contrary to it.

1*ſt*. We Say, that it is not Worth the While to Conſider that Doctrine, which Places Vertue betwixt Two Extremes, becauſe Some Vertues may have no Extremes, and Others may Conſiſt in them. For what Extremes has Honeſty and Integrity, Since it has only a Contrary to it, which is Villainy and Knavery? Or, who can be too Extremely Good, Since the Gooder a Man is, the Better he is, and therefore the more Vertuous.

2*d*. We may Attain a Likeneſs to God, more by the Practice of Goodneſs, than Truth, for God is Love or Goodneſs, as well as Truth; and, whereas his Truth, or Knowledge, is Infinite, and Far above our Comprehenſion, his Goodneſs, tho' Immenſe, is yet a Pattern for our Imitation; This *Platonick* Vertue therefore, in Approaching to a Likeneſs to God, Falls in with the Definition, which I Gave before of Morality, and is likewiſe Maintained by the Chriſtian Notions Concerning it.

As to the Propoſitions, which Follow after in this Section, it is not any ways Neceſſary to Examine into them; Becauſe they are Entirely Dependent upon what Foregoes. I ſhall therefore Conclude this Inquiry with Obſerving, that whether this Religion of Nature Delineated is an Effort of Deiſm, or Intended for the Service of a Revealed One, is Perhaps ſomething Difficult to Determine; 'Tis Certain, if what is here Alledged was True, it would not be Significant, with what Deſign it was Propoſed; But, as it Appears to be Falſe, it would be ſome Satisfaction to find, that they are the Errors of an Adverſary, and not of a Friend.

We ſhall not here Object to the Author his Particular Familiarity and Acquaintance with One, who, by many Learned and Judicious Men, was Thought to be no Great Patron of Revelation; Nor yet, that he Relinquiſhed the Orders of the Church, into which he had Entered; Tho' a Truly Pious and Good Chriſtian would Scarcely have done it, Notwithſtanding all the Temporal Riches, or Greatneſs, he ſhould have Acquired; On the Contrary, a Sincere and Humble Chriſtian would have Rejoyced in the Advantage Accrueing to him from thence, by which he might be Rendered more Serviceable to his Sacred Profeſſion, and would have ſooner Reſigned the Latter, than the Former; Of which we have ſome Late and Preſent Inſtances, where Good Men have not Thought their Miniſtery in the Church a Reproach to their Eſtates, or their Fortunes; But this we do not Inſiſt on; Nor any Thing, which is Perſonal.

But, when he makes Vertue and Vice to Conſiſt in Acting or not Acting according to the Truth and Relations of Things, that is, in Natural Knowledge, or in Philoſophy, or an Ignorance of it, he Appears to Sap the very Foundations of a Chriſtian Morality, which Conſiſts in the Imitation of God and our Bleſſed Saviour, in Doing of Good, and in being Loving and Beneficent to Mankind, and he has much leſs Reaſon, as we have Shewn, for ſuch an Aſſertion, than the Chriſtian has for his. It would be a Hard Caſe, if Religion was Put on ſuch a Foot, that no One could be a Good Man, but an Inquirer into the Nature and Reality of Beings, as they Stand in their Infinite Relations to Each Other, and that, no One could be a Vertuous, or a Moral Man, unleſs he was a Philoſopher, or ſome Rich Perſon, who was at Leiſure to Indulge and Riot in his own Speculations. Chriſtianity would Certainly Inform us Wrong, which Tells us, that not *many Wiſe*, not *many Rich*, or *Noble were Called*; And every Poor Peaſant, let him be as Honeſt, and as Innocent, as he will, muſt be of Courſe a Vicious, and Immoral Perſon.

Farther, If we muſt Treat all Things, as they are, we are Obliged in Conſequence of ſuch a Doctrine, to Treat an Enemy not only as *not* a Friend, §. 1. Prop. 11. but as an Enemy; and yet our Saviour and his Religion Inſtructs us, *to Bleſs our Enemies, to do Good to them, who Hate us, and to Pray for thoſe, who Deſpitefully Uſe us, and Perſecute us*, which is not Treating them as they are, but the very Contrary of it; And yet, if we ſhould do this, we are Told, Notwithſtanding the

Rants of ſome Men, we ſhould Cancel the Natural Differences of Things, and Confound Truth with Untruth; which Looks little Better than like a Down-Right Impeachment of the Chriſtian Scheme, and it's Divine Author; If it is not, it is at Leaſt very Incautiouſly Expreſſed, and may be Explained to that Senſe.

Lastly, §. 9. Pag. 211. He Says, I hope, that neither the Doing of this, nor any Thing elſe, Contained in this Delineation, can be the Leaſt Prejudice to *any other True* Religion; Whatever is Immediately Revealed from God, muſt, as well as any Thing elſe, be Treated, as being what it is, which cannot be, if it is not Treated with the Higheſt Regard, *Believed* and *Obeyed*; That therefore, which has been ſo much Inſiſted on by me, and is, as it were, the Burden of my Song, is ſo Far from Undermining *True Revealed* Religion, that it rather Paves the Way for it's Reception.

Here One would Think, if this Philoſophy had been Truly Chriſtian, it ſhould not have Spoke in ſuch General, but in more Plain, Full, and Explicite Terms; Since there is no *Deiſt*, or *Mahometan*, who might not have Said the ſame Thing.

He Hopes, he ſhall not by his Natural Religion do a Prejudice to *any other True* Religion, but whether that *True* Religion is *Moſes*'s, *Chriſt's*, or *Mahomet*'s, is not here Said, but rather left Undetermined; I have Shewn, that ſome of his Notions are Diametrically Oppoſite to the Chriſtian Syſteme, and Conſequently muſt be Prejudicial to it; And if he was Senſible of This, which could Scarcely be Hid from ſo Penetrating a Perſon, he muſt Think Chriſtianity had not a Chance to be in the Number of *any other of his True Religions*, but that it was Directly and Abſolutely Falſe.

He Farther Tells us, that whatever is Revealed from God, ought to be Treated with the Higheſt Regard, that is, to be *Believed* and *Obeyed*; But he is not ſo Good to Inform us, which that *Revelation* is, whether it is the *Chriſtian* One, or whether it may not be the Pretended One to *Mahomet*, or even That of the Goddeſs *Ægeria* to *Numa*; Nothing is Laid Down, as Definitive or Certain, and we are only Told, that *whatever* is *Revealed* from God, ought to be Treated with the Higheſt Regard; And who will Deny a Truth ſo Plain? Or, what Philoſopher would Aſſert One ſo Trifling? When we are not Informed, What this *Revelation* is.

And, when it is Said, that we ought to Treat a Revelation from God with the Higheſt Regard, by *Believing and Obeying it*; What is the Real Conſtruction of ſuch a Short Character of a *Revealed Religion*, (for, which, or what, it is, we are not Suppoſed to Know) unleſs, that we muſt be *Credulous* and *Obey*? That is, the True Senſe of this Paſſage Appears to Amount to this, that, as all this Book has been Spent to Support the Evidences of *Natural Religion* by Inconteſtable Proofs and Reaſons, ſo there is Little to be Said for *Revelation*, Excepting, that, If it is *True*, we are to *Believe* it.

But does not this Philoſophy Know, as the Terms of *Faith* and *Belief* go now, they are only made to Signify the *Probability* of a Doctrine; And Reaſon and Argument are Thought to Infer the *Certainty* of it? If ſo, we Say, the Chriſtian Religion is not only in that Senſe to be *Believed*, but that it has more Evidences for the Truth of it, than This, or *any Other* Syſteme of Natural Religion, which has been hitherto Propoſed to the World; And we muſt Retort the Expreſſion upon the Author, that if this Natural Religion is from God, it ought to be Treated with the Higheſt Regard, *Believed and Obeyed*, but if it Proceeds from Man, as by the Errors it Contains, it moſt Certainly does, we are not Obliged to do Either.

As to what is afterwards Said, that a Natural Religion *Paves* the Way for the Reception of a Revealed, That Certainly cannot be True, till we Know, what that Revelation is; And, if it is the Chriſtian Revelation, it cannot be True, if ſuch a Natural Religion Endeavours either to Overturn or Contradict it; If it does not do it, this Natural Religion Tells us no more than Chriſtianity did before, and therefore might have been, if it Pleaſed, altogether Silent upon the Matter.

Besides in General, the Revelation of God in the Chriſtian Diſpenſation is ſo Exceedingly Superior to any Thing of Human Reaſon, or which Human Reaſon could Imagine, that it cannot be Exacted to the Rules, or the Preſcriptions of it; That the Infinite and Almighty Creator of all Things ſhould Send his Son into the World for our Redemption, that he ſhould Reſign him, as a Sacrifice upon the Croſs, for the Sake of Mankind, is above Human Comprehenſion, and Deſpiſes all the Methods of Reaſon to Account for it; But This is Revelation; And a Revelation, which is Atteſted by Greater Demonſtrations and Proofs, than can be Alledged for any Syſteme of Philoſophy and Natural Religion, which we at Preſent are Acquainted with; And, Suppoſing any One could Prove the Being of a God, or the Immortality of the Soul, or Future Rewards and Puniſhments, beyond the Power of being Refuted; He would be as far from Aſſerting the Chriſtian Scheme, as a Man, who ſhould Prove the Being of a God, and could not Evince his Paternal Care of, and Indulgence towards us, or who ſhould Evince the Soul's Immortality, and could not Demonſtrate the Reſurrection of the Body, or who ſhould Demonſtrate the Rewards and Puniſhments of another Life, but could not Inform us to whom they Belonged, whether to *Jews*, *Turks*, *Infidels*, or *Chriſtians*.

THE Caſe in the Preſent Age ſeems to be This, that Men are Grown very Fond of Philoſophy, and what they Call Reaſon, and would therefore Reduce even the Chriſtian Religion it's ſelf to the Dictates of it; But Chriſtianity, and a Revelation from God, cannot Bend to every Man's Fancy and Humour; It will, in Spite of all the World, be what it is, as much as Nature is, what it is, whatever our Comments may be upon Either; And to Endeavour to make the Chriſtian Revelation of God's Will to Mankind Bow and Submit to any Imaginations, we have Entertained Concerning Nature, or the Suppoſed Relations of Things, is ſomething Like, what we Read of the *Miſſionaries* in *China*, who to Convert thoſe People to Chriſtianity, Mix Their Paganiſm with it. Thus Philoſophick Men are Apt to Take the Chriſtian Religion, not, as it is in it's ſelf, Pure and Uncorrupted, but to Blend Nature and Reaſon with it, if they Mean the Beſt to it; And if the Worſt, to Undervalue and Contemn it; Of the Former Sort, I Take the *Arians*, and *Socinians* to be, and whether *This Philoſopher*, Mr. *Locke*, and *Des-Cartes*, are not in the Number of the Latter, I will not Take upon me to Decide; But would rather Think on the moſt Favourable Part, and, which is the moſt Agreeable to the Chriſtian Candour, that they are only Men, who were Deceived in the Connexion of their Ideas, and Conſequently only *Men Miſtaken*.

HOWEVER, in a Chriſtian Nation, where the Revelation of God to Mankind is Acknowledged by us, One would Wonder, why we ſhould Deſire any Other Religion, than what it Propoſes; It is Suppoſed to be True, what Occaſion then is there for a Natural Religion? If it is not True, why ſhould a Natural Religion Endeavour to Support it? For, as we have Shewn, if Natural Religion was never ſo Fully Proved, it would be of no more Importance to the Truth of the Chiſtian Revelation, than *Mahometiſm* it's ſelf would be, which Acknowledges a God, the Immortality of the Soul, and Future Rewards and Puniſhments.

I ſhall therefore Finiſh this Head of Inquiring into the True Intention of this Religion of Nature Delineated, with One Obſervation, that if the Author of it had been as Scrupulous in *Speaking* of Truth, as he Pretends to be in *Acting* of it, I do not Know, but he had been a much Better Man, and a much Wiſer Philoſopher.

I might Proceed to the other Sections of this Natural Religion; As the Second, Concerning Happineſs; The Third, Concerning Reaſon, and the Ways of Diſcovering Truth; The Fourth, Concerning the Obligations of Imperfect Beings, with Reſpect to their Power of Acting; The Fifth, Concerning Truths Relating to the Deity, Of his Exiſtence, Perfection, Providence, *&c.* The Sixth, Concerning Truths Reſpecting Mankind in General, Antecedent to all Human Laws; The Seventh, Concerning Truths, Reſpecting Particular Societies of Men, or Governments; The Eighth, Concerning Truths Appertaining to Families and Relations; The Ninth, Concerning Truths, belonging to a Private Man, and Reſpecting (Directly) only himſelf. But, as it would be an Endleſs Task to Inquire into every Thing, and as the Principal and Singular Doctrine of this Philoſophy of Acting according to the Truth and Relation of Things has been Examined, I muſt Leave the Diſquiſition to Others, who will Find, as much Weakneſs of Argument in many of the Subſequent Sections, as we have Shewn in the Preſent, I could do it in many Inſtances, but neither my Time, nor the Riſing Number of theſe Sheets, will Permit or Allow it; And I ſhall only Conclude, with my Thanks to my Worthy and Ingenious Friend Mr. *Clarke*, Maſter of the Free-School in *Hull*, for his Sentiments in theſe Matters, which, however the Favourers of this Syſteme may Declaim againſt, Perhaps will find it ſomething Difficult to Anſwer in many Particulars, and which I Received as a Kind Preſent from him, after I had gone thro' theſe Inquiries.

CHAP. VII.

Concerning Ideas from Senſation and Reflection, and that there are Ideas in the Mind Perfectly Immaterial.

§. 1. WHAT We have Alledged in the Foregoing Chapters would be Sufficient to Overthrow this Whole Eſſay of the Mind or Underſtanding, which yet has been Diſcourſed upon the Conceſſion of that Notion, which is Fundamental to the Reſt, and which we ſhall here Examine into, Namely, that all our Ideas Ariſe, and are Derived, from Senſation and Reflection.

The Notion, that all our Ideas are Derived from Senſation and Reflection, next to be Examined. Mr. Locke's Eſſay, Chap. 1, *&c.* Book. 2.

§. 2. EITHER our Ideas are thus Diſtinguiſhed; Firſt, From their being Different Kinds or Sorts of Ideas, which therefore have Diſtinct and Separate Originals; Or Second, From their being Derived from Diſtinct and Separate Originals, which therefore are Different Kinds and Sorts of Ideas; If the Firſt, it cannot be Proved, that there are not ſeveral other Sources or Fountains of our Ideas, beſides Senſation and Reflection; For, when we Perceive the Sun, or Feel Pain, or Hear Sound, or Taſte Fruit, or Smell a Roſe, or a Violet, All theſe are from Senſation, but they are Different Kinds of Ideas, and therefore muſt, according to the Preſent Suppoſed Doctrine, have Diſtinct and Separate Originals; and Conſequently, Senſation in General will not be a True Account of the Original of theſe Various Ideas.

Our Ideas cannot be thus Diſtinguiſhed from their being Different Kinds and Sorts of Ideas, and therefore Derived from Diſtinct Originals.

AGAIN, our Ideas from Senſation in General are Different from thoſe Ideas, which we have of them upon Recollection; Thus the Sun we See, or the Pain we Feel, or the Sound we Hear, *&c.* are Different from the Ideas, which are Produced in our Minds upon the Remembrance of them; The One, as we have Evidenced in the Principles of Philoſophy, being an Actual Perception, whereas the Other is a Picture, or Repreſentation only, of that Perception; And I may here Add, that Memory is a Bringing of Objects or Things, as Near to the Mind as Poſſible, which is done by Thought, Study, and Application, and Forgetfullneſs is a Removal of them at a Diſtance from it, which is done by Oſcitancy, Intemperance, and Idleneſs, and is Anſwerable to, but not the Same with, our Senſation of Objects at a Nearer Diſtance, and our Non-Senſation of them at a Farther; Nor are theſe Ideas of Remembrance Ideas of Reflection, which are Confined to the Operations of our own Minds, as Perception, Judging, Willing, Remembering, *&c.* For, if our Ideas of Memory may be Termed Ideas of Reflection, becauſe Remembering is an Idea of Reflection, by the ſame Reaſon our Ideas of Senſation may be Termed Ideas of Reflection, becauſe Perception is an Idea of Reflection.

ON the other Hand, if the Ideas, we Remember, may be Accounted Ideas of Senſation, becauſe we could never have Raiſed by Recollection thoſe Ideas in our Minds, unleſs our Senſes had Supplied us with their Archetypes, We Say,

THAT by the Same Argument, it is not Proved, that our Ideas of Reflection may not be Equally Derived from Senſation, becauſe it is not Certain, that we could ever have the Ideas of Perception, Remembering, Doubting, and Willing, if we had not been Furniſhed with Ideas from Senſation, about which our Perception, Remembering, Doubting, Willing, *&c.* were Employed. From whence it is Evident, that our Ideas, which we by our Memories Recollect, are either Diſtinct from our Ideas of Senſation and Reflection, or if they are the ſame with our Ideas of Senſaſation, for the ſame Reaſon our Ideas of Reflection may be ſo too, and Conſequently, that there will be either Three Diſtinctions of our Ideas into thoſe of Senſation, Memory, and Reflection, or that they will be all Reſolved into thoſe of Senſation only.

WHAT has been Argued Concerning the Ideas of our Memories, may be Extended to the other Faculties of the Mind; For, as when we Remember any Idea of Senſation, it is Different from our Actual Perception of it, ſo likewiſe, when we Paſs a Judgment upon that Idea, or when we Receive any Appetites or Inclinations from it, the Appearance of ſuch an Idea in the Mind, when Attended with thoſe Circumſtances, muſt be Different from a bare Impreſſion of it, as Different as Perception is from Memory, or That from Judgment, or from Willing, *&c.*

Moreover, if the Original of Ideas is to be Derived from the Diſtinction of them, our Ideas of Reflection, as we have already Conſidered Thoſe of Senſation, would not come under One Head of Reflection; becauſe, as Perceiving, Remembering, Willing, Judging, Doubting, *&c.* are Different Ideas, they would alſo have Different Originals, and therefore they would not be ſo Properly Called Ideas of Reflection, as the Ideas of Remembering, Willing, Judging, which we Actually Experience in our own Minds; For, if becauſe we Really Will, and Perceive, and Judge, and Remember, we may Term theſe Ideas of Reflection, which we Obſerve in our Minds upon Reflection, we may as Juſtly Say, that our Actual Ideas from Senſation, are Ideas of Reflection, becauſe upon Reflection we Perceive them there.

These Words, Senſation and Reflection, therefore ſeem to Mean no more than this, that what Actually Lies in our Minds, as a Perception there, is Senſation, and Conſequently, when we Will, or Remember, it is Equally a Senſation, as when we See or Feel; On the Contrary, all theſe Actual Perceptions may be Reflected upon by the Mind, and Formed into Abſtracted Ideas of Perceiving, Willing, Judging, and Remembering; This is the True Senſe of the Preſent Philoſophy, if it Really Underſtood, what it's ſelf has Propoſed to us.

But, if ſo, ſince the Ideas of Senſation are Reſolved into our Actual Perceptions, and thoſe of Reflection into our Wills; How are Either of theſe the Original of our Ideas? ſince the Firſt are only Ideas, which we in Fact Experience, the Original of which we are Ignorant, and the Laſt, what we our ſelves Pleaſe.

Nor from their being Derived from Diſtinct Originals, and therefore Different Ideas.

§. 3. Having therefore Conſidered our Ideas, as to the Diſtinction of them, and Shewn from thence, that there is no more Reaſon to make two Sources of our Ideas, than a Hundred, we ſhall in the Second Place Examine, whether this Philoſophy can Direct us to any True Fountains of our Ideas, by which we may Diſtinguiſh them.

'Tis Said, Senſation and Reflection are ſuch, we have Shewn, that the Firſt Expreſſes our Real and Actual Perceptions, and the Laſt the Abſtractions of our Wills; whereas our Really Perceiving any Thing, or our Pleaſure in Abſtracting, can neither of them be the Fountains or Sources of our Ideas; For, what we Perceive is only Matter of Fact, without our Knowing, what is the Riſe or Origin of that Experience, which we have from Bodies or our own Minds; and what we Abſtract by our Wills is no Origin of Ideas, unleſs of Chimerical Ones, which may be, as well Thoſe of a Centaur, or a Gryffon, as the Abſtracted Ones of Perception, *&c.* if the Origin of both is Reſigned to the Will and Pleaſure of the Mind.

Besides which, it will be Perfectly Impoſſible to Determine, what are the Origins or Sources of our Ideas, unleſs we Know the Nature of Matter, and our own Minds, neither of which this Philoſophy is Pleaſed to Oblige us with; For till that is Aſcertained, how is it Evident, that our Ideas of Senſation and Reflection too, may not be Originally Derived from Matter? Since if, as the Preſent Eſſay Believes, Almighty God may Impreſs upon a Syſteme of Matter, Fitly Diſpoſed, a Faculty of Thinking, Matter may be Capable of Reflection too; that is, our Wills, by which any Reflection is made, may be the Reſult of Material Influences, as our Senſations are, and therefore may be Reckoned of the ſame Nature and Order with them.

Mr. Locke's Eſſay, P. 310. Edit. 3.

Moreover, without any Reference to this Eſſay, before we are Convinced, what the Nature of Matter is, it will not be Certain, that our Ideas of Reflection, or of our Wills, are Derived from any other Source, than thoſe of Senſation; becauſe our Senſations are Derived from Matter, and it is not Proved, that our Reflections may not be Derived from our Senſations, which if True, Senſe will be the Origin of all our Ideas.

On the Contrary, it will not be Manifeſt and Plain by this Philoſophy, that the Ideas of Senſation may not be Owing to a Faculty in the Mind of Producing thoſe Ideas; ſince, as it can Raiſe Ideas in it's ſelf by Memory, and by Dreams, and in a Frenzy, till this Eſſay can give us an Explication of theſe Phenomenons, or Appearances, in our Minds, it will be in Vain for it to Fix any Opinion, how, or from whence, our Ideas have their Origin, whether from Senſation or Reflection, or Both.

To Sum up, as Briefly as we can, what has been hitherto Said; If the Diſtinction of our Ideas is a Reaſon, why we ſhould Aſcribe them to Different Origins, every Diſtinct Idea, even of Senſation, will have a Peculiar Source belonging to it, from whence it is Derived; In the next Place, Memory will have an Equal Title with Senſation, to be a Diſtinct Fountain of our Ideas, and ſo of the other Faculties of our Minds, as Willing, Judging, *&c.* Third, what we in Fact Experience, and what we Arbitrarily Contrive of our ſelves, are alſo Fountains of our Ideas; That is, in the Firſt Caſe, becauſe we Perceive a Thing, therefore the Perceiving of it is the Riſe and Cauſe of that Perception, and in the Second, becauſe we Contrive and Form Abſtracted Ideas by our own Will and Pleaſure, therefore thoſe Arbitrary Abſtractions are another Origin and Fountain of them, Both which it is Abſurd to Affirm.

If the Diſtinction of Ideas cannot Furniſh us with the True Origin of them, much Leſs can that Philoſophy do it, which neither does Know, nor does Pretend to Know, the Cauſes of Senſation or Reflection.

§. 4. Having Conſidered the Opinion of our Ideas being Derived from Senſation and Reflection, as the Two Fountains or Sources of them, we ſhall now Inquire, whether there are not Ideas in our Minds, which are Perfectly Immaterial, that is, which cannot either be Derived from Senſation, or any Reflections upon thoſe Senſations, which this Philoſophy ſeems to Aim at and Deſign, tho', according to it's Uſual Method, it would make us Believe, that it Means by Reflection, nothing more than Reflecting upon the Operations of our own Minds; But this Double *Entendre* is the very Temper of this Gentleman in his Whole Philoſophy, as it is in his Religion, which, whilſt he Profeſſes in Appearance, he Endeavours to Ruin in Reality, and which ſeems to be the True Character of this Fallacious Perſon.

That there are Ideas in our Minds Perfectly Immaterial, which are neither Derived from Senſation nor Reflection upon ſuch Senſation.

We therefore Say in Proof of This, *1ſt.* That all our Ideas of Reflection, as Thoſe of Willing, Judging, Remembering, *&c.* are not Originally Derived from Senſation, but from the Mind; *2d.* That all the Paſſions and Impulſes of the Mind, in which the Mind, Feels as much as in it's Senſations, (and which therefore may be Called Perceptions, tho' not Ideas, or Particular Images, and Repreſentations of Things) That all of them have no Dependence upon, or Connexion with, our Senſations; *3d.* That even in our Ideas of Memory, and Dreams, there is ſomething Mixes it's ſelf in thoſe Perceptions, which no Senſations ever did, or could, Supply us with; *4th.* That the ſeveral Ideas of Solidity, Space and Figure, Eternity, of Infinity, of God, or of Mathematicks in the General and the Abſtracted Notions of them, are not Owing to our Senſations; *5th.* That the Graces, and the Divine Influences, from the Chriſtian Religion are Superior to any Thing of Senſe, or what can Poſſibly be Derived from it; Each of theſe we ſhall Endeavour to Prove in the Following Sections.

§. 5. First, When we Reflect upon our own Minds, we find Thought, Willing, Judging, Doubting, Remembering, *&c.* to be Properties Exiſting there, which Evidently are Immaterial, for when I Doubt, or Will, or Remember, or Perceive, whatever the Objects of my Doubt, Will, Memory, or Perception, are, the Acts of the Mind in theſe ſeveral Inſtances are as Diſtinct from the Objects, as Memory is from the Thing, which is Remembered, Perception from the Thing Perceived, and ſo of the Reſt; or as Diſtinct as Action is from Paſſion, or the Contrary; Therefore, thus far the Matter ſeems to be Clear.

Firſt, all our Ideas of the Operations of our own Minds are not Derived from a Reflection upon our Senſations.

But then the Queſtion will be Asked us, whether we could have ever Willed, or Doubted, or Perceived, or Remembered, if it had not been for our Senſations; It would not be Neceſſary at Preſent to Anſwer this any other Way, than by Saying, That, if we could not, it is as Certain, that we could never have had our Senſations, if it had not been for our Perception, which Perception is before Proved to be Immaterial, and Conſequently, that Judging, Willing, Remembering, *&c.* which Depend upon Perception, could not Exiſt without it neither.

But we ſhall Farther Add, that Perception may be without Senſation; for tho', by the Union of our Minds with our Bodies, we are Trained up to Think and Believe they are Inſeparable, yet it is Manifeſt, that for the ſame Reaſon, that Senſation could not be without Perception, Perception may be without Senſation; Since, as the Mind is Active in it's Senſations, whilſt the Objects it Perceives are Paſſive, or the Mind, Paſſive, whilſt the Objects are Active upon it, ſo it is moſt Certain, that if there had never been any Action or Paſſion of the Object in Reſpect of the Mind, the Mind, Notwithſtanding that, would ſtill Retain it's own Force, whether it Conſiſted in the being Paſſive or Active, That is, it's own Proper Perception, or Perceptive Faculties, Abſtracted from the External Object Perceived; and the only Inquiry, which will Remain is, whether it can Perceive it's own Action or Paſſion, which it is Evident it may, by the Reflections we Daily make upon our own Thinking, Remembering, Judging, *&c.* in General, which every One may, when he Pleaſes, Perceive.

Moreover, my Perception of my own Thinking, of my Remembering, *&c.* is not a Perception from Senſation, nor can be Derived from it; Not the Firſt, unleſs Thinking and Remembering are Senſations, which no One will Affirm, who will not Say, that when I Think, I See, or Hear, Something, Notwithſtanding that Intenſe Thinking, even Deprives us in a Manner of thoſe Senſations; Not the Laſt, ſince, what I Think of, is not always External Objects, but the Inward Operations of my own Mind; and, that thoſe Operations cannot Subſiſt, without Something being Derived into it from External Objects, cannot be Proved, till it is Firſt Known, wherein the Nature of the Mind, and of Matter Conſiſts; Neither of which this Philoſophy has made one Step towards the Informing us of; Or at Leaſt, till it is Evidenced, that the Mind may not Exerciſe it's ſelf in it's own Operations, without Conſidering it's Senſations, or the Impreſſions of Material Beings, which we have Shewn, It may.

Second, The Ideas of the ſeveral Paſſions in our Minds are not Derived from a Reflection upon our Senſations.

§. 6. Second, As to the ſeveral Paſſions and Impulſes of our Minds, as Courage, Fear, Love, Hatred, Joy, Sorrow, *&c*; as alſo Vehemence and Impetuouſneſs in our Tempers, or Sloth and Deſpondency; Patience and Æquanimity, or Anger and Fury; None of theſe have any Semblance or Appearance of Ideas or Images in them, Drawn from Senſation; All that the Philoſophy of Ideas does, or can, Propoſe to us, is only, that we Receive Certain Repreſentations in our Minds from External Objects, that we have a Power of Reflecting upon, Comparing, Compounding, Abſtracting, and Judging of thoſe Repreſentations, *&c.* Which yet Falls Short of the Powers and Faculties of the Mind, in Conſidering, Viewing, and Comparing it's own Operations, Abſtracted from our Senſes, as we have already Obſerved; But, for ſuch a Philoſophy, which is only Concerned in a Dull and Languid Perception and Compariſon of Ideas, to give an Account of the Great and Sublime Paſſions of our Minds, which are not Expreſſible by the bare Shadows and Images of Things, is as much Impoſſible, as that the Seeing ſeveral Objects in a Looking Glaſs, and Comparing them together, ſhould Furniſh us with the Forces and Impetus's, or the Actions of the Rays of Light, which are Incident, or of the Reflexive Ones of Glaſs, and the Receptive of our own Minds; Betwixt which, and the Plain Repreſentation of ſuch Images to us, there is no Connexion.

If it be yet Said, that the Objects of our ſeveral Paſſions, as Courage, Fear, *&c.* are Material; That will no more Prove, that our Paſſions themſelves are not Originally in the Mind, than becauſe our Senſations are Derived from External Objects, therefore the Perception of them is not in the Mind Diſtinct from Matter.

Mr. Locke's Eſſay, Pag. 310. Edit. 3.

Lastly, If the Mind is Affirmed to be Material, or if it is Suggeſted, that it is not Impoſſible but it may be ſo; Whilſt this Eſſay is ſo far from Knowing, what the Nature of the Mind is, that it is Ignorant of the very Eſſence, and the Eſſential Properties of Matter, as we have Shewn in our Principles of Philoſophy, we can only Think, that ſuch Opinions are Taken up at Random, and that the Mind might as well have been Surmiſed to be a Broomſtick, made of ſome Unknown Wood or Other, as to be Material, that is, of It Knows not What.

Third, Our Ideas of Memory and Dreams, not Merely Owing to a Reflection upon our Senſations.

§. 7. Third, In our Ideas of Memory and Dreams, we Said there was Something Alledged by the Mind, which could not be Owing to our Ideas from Senſation; For it has been already Proved in the Principles of Philoſophy, that at the moſt, they are only Repreſentations or Images of what we Actually Perceive by Senſation; There muſt be therefore ſomething Peculiar in the Mind, which it never Derived from Senſation, if the ſame Idea, which it Received by an Actual Impreſſion from an External Object, upon Remembering it Repreſents in a Different Manner to it's ſelf; and which ſeem to have the Same Relation, as a Shadow has to it's Subſtance; And this is more Evident, when we Conſider Thought in General, for when we Remember, that we Thought at ſuch a Time, a Recollection of it is ſtill Farther Removed from any Idea of Senſation.

As to our Dreams, beſides what we have Said in Reſpect of our Memories, the Ideas of which thoſe of our Dreams ſeem moſt to Reſemble, it may be Added; That, both in the Nature of the Perceptions, and in our Judgments of them, they are Extremely Different from any Thing, which could be had from Senſation. As to the Nature of them, there is Something, which ſeems almoſt to be Inexpreſſible by any Notions, which we have of Material Beings; Thus in our moſt Senſible Dreams, and which we can beſt Remember, when we Awake, we ſeem upon Reflection to have been Converſant in another World of Beings, where every Thing is Quite and Plainly Different from what we Apprehend here, and yet at the Same Time we Think we do not Dream, and the Painting is ſo Lively upon our Minds, that even our Senſations are Dull and Languid in the Compariſon; All which cannot be Derived from, or Owing to our Senſations; And yet all this we in Fact Experience, which can never be Solved by a Grave Philoſophy of Ideas, and the bare Perceiving, and Comparing, and Compounding of them; and which can be no other Ways Accounted for, than by our *Pſycheometria*, and our Doctrine of the Expanſive and Contractive Forces.

Besides which, it is what every One, if he Pleaſes, may Obſerve in our Dreams, as to our Judgments upon the Perceptions, we have in them, that we often, and Indeed Generally, make one Idea to Paſs into another, without Diſtinguiſhing the Propriety of each; Thus we make the Idea of a Man to Change into another of a Brute, or even of an Inanimate Being, and yet we Dream all the While of the Same Individual Perſon; And the Same may be Said of other Beings, as in the Idea of Water, it is many Times Changed into that of a Solid, or of a Viſcous Subſtance, when we Perceive nothing of all this from our Senſations; Alſo the Proportions of Men and Animals are Repreſented to us many Times more Gigantick and Terrible, and at other Times Infinitely more Dwarfiſh, than what are found in Nature; We ſhall not at Preſent Say, what the Reaſon of this is, becauſe, if we could not give any Account of it, we ſhould yet be upon the Level with the Modern Philoſophy, which does not Offer, or Pretend, to do it; Which, Notwithſtanding, may be done by the Above-mentioned Principles; What is to be Affirmed at Preſent is, that if the Perceptions in our Dreams are Owing to our Senſations, they would be Correſpondent to thoſe Senſations, whereas, in the Inſtances Alledged, they are Directly Contrary to them.

§. 8. Fourth, There are ſeveral Ideas, *viz.* of Solidity, Space, or Extenſion, Figure, *&c.* of Duration, Eternity, of Infinity, and of God, which are in a Manner wholly Owing to the Mind; As to the Firſt Three Mentioned, it may Perhaps ſeem a very Improbable Aſſertion, but there is nothing more Evident, than that all theſe are, as they now Lie in Philoſophy, Ideas Formed by the Mind it's ſelf; For what is Solidity? Why Truly it is Received from Touch, it Fills Space, but is Diſtinct from Space, as alſo from Hardneſs, and upon it Depends Impulſe, Reſiſtence, and Protruſion; But, if any One Asks at Length, what Solidity is, this Philoſophy Begs the Favour to be Excuſed from Telling you, only that it is a Simple Idea not to be Defined, and Diſmiſſing you with Contempt, for ſo Impertinent an Inquiry, ſends you to your Senſes to be Informed.

Fourth, the Ideas of Solidity, not Owing Solely to a Reflection upon our Senſations. Mr. Locke's Eſſay, Chap. 4. Book 2.

But, if it is not to be Defined, why is it Defined? Why are we Told, that it is an Idea, which is Received from Touch, which Fills Space, and is Diſtinct from both It and Hardneſs, and on which Depends Impulſe, Reſiſtence, and Protruſion? If That is the Definition of it, it is either an Imperfect and a Wrong One, or, if it were True and Exact, there would be no Occaſion for any farther Inquiry, What Solidity is, nor to be in the leaſt Diſturbed, if any One ſhould Preſume to Ask the Queſtion, what it was.

The Caſe therefore ſeems to be, that this Philoſophy had been Amuſeing it's ſelf for ſeveral Sections together about Solidity, which it did not Comprehend, and at Laſt, being Conſcious to it's own Ignorance, began to be Magiſterial, and to Send it's Readers, if they would not Acquieſce in what it had Pronounced to them, to be Informed, if they Pleaſed, from their Senſes. And, that this is ſo, we ſhall Prove from giving a True Notion of Solidity, and then Examining the ſeveral Parts of this Definition of it. What any One therefore Means by Solidity, or Impenetrability, is only This, That a Cube of Gold, for Inſtance, cannot Enter into another Cube of Gold, and Preſerve the ſame Dimenſions it had before; That a Needle cannot Penetrate into the very Body and Dimenſions of Water, or Quick-Silver, but the Parts of Each muſt Recede, before there can be any Penetration, that is, ſuch a Notion of Solidity, or Impenetrability, only comes to this, that every Being will Preſerve it's own Nature, whilſt it Continues the Same; which is no other than an Axiom, and Equally Extends to all the Beings in the Univerſe, and even to the Mind it's ſelf; for One Thought cannot Croud or Preſs it's ſelf, if we may ſo Call it, into another Thought, and that Thought Continue, what it was; and therefore Thought is Equally Solid and Impenetrable with Matter, as we have already Shewn, Space, Light, Colour, *&c.* to be in our Principles of Philoſophy, according to ſuch a Notion, which is, what is Really Meant by Solidity, or any Thing being Impenetrable.

If what we have Affirmed is True, it is Manifeſt, that Solidity, or Impenetrability, is an Abſtracted Idea of the Mind, and no more belongs to Matter, than to Thought; from whence we may Juſtly Conclude, that Solidity is not Owing to Senſation, ſince it is an Idea Formed by the Mind, and which would be Certain, whether there were any External Objects or not, Namely, that One and the Same Thought is not Two Different Ones, or, that there is no Penetration of them.

Mr. Locke's, Eſſay, Chap. 4. §. 1. Book 2.

Chap. 4. §. 2. Book 2.

Chap. 4. §. 3.

Chap. 4. §. 4.

Chap. 4. §. 5.

From this Notion of Solidity or Impenetrability, which is the Real Senſe of that which is Deſign'd and Intended by thoſe Words, it will be Evident in the Firſt Place, that the Idea of Solidity is not Received by our Touch, ſince ſuch an Idea may be as well Collected from our Thoughts, as our Feeling; in the Second Place, that the Solids of Mathematicians have no Connexion with this Idea, Seeing they are only Portions of Abſtracted Space or Extenſion; Third, that it is no more Intimate or Eſſential to Body, than to the Mind; Fourth, that Solidity, or Impenetrability, does not Fill Space, becauſe any Thing being it's ſelf, is not an Argument that ſuch a Being is Extended, which will be Neceſſary to the Filling of Space, or, which Amounts to the Same, Solidity, or Impenetrability, is an Abſtracted Notion, which therefore cannot Inform us of the Nature of That, which is Solid and Impenetrable, whereas it will be Requiſite to Know That, before we can Affirm the other; Beſides which, as we have already Demonſtrated in the Principles of Philoſophy, Chap. 4. §. 11. Book 1. Abſtracted Space is not Commenſurate to the Dimenſions of Matter, and Conſequently the Conſtipation of it, if that is Meant by it's Solidity, does more than Fill it, that is, the Real Dimenſions of a Cubical Inch of Gold Contains more, than a Cubical Inch of Abſtracted Space; Fifth, that Solidity, or Impenetrability, is not Diſtinct from Space, if it Fills it, and is Commenſurate with it; becauſe one Space can no more Penetrate into another Space, and Continue the Same, than Body can Penetrate into Body; nor can Body Penetrate into Space, ſince it only Exiſts in it's own Dimenſions, and not in Space, which we have Proved, is no other than an Abſtracted Idea; Sixth, that Solidity, or Impenetrability, is Indeed Different from Hardneſs, for it is very Plain, that a Soft or a Fluid Body is as much it's ſelf, as a Hard One is; Seventh, that Impulſe, Reſiſtence, and Protruſion, do not Depend upon Solidity, or Impenetrability, for the Hindering any Foreign Being from Encroaching on it's Own, whilſt it Continues the Same, Appertains to Thought, to Space, to Light, Heat, Colour, *&c.* and to all Beings whatever, as well as to the Dimenſions of Body, and is no Proof, or Evidence, what the Nature of that Being is, which yet muſt be Known and Underſtood, before it can be Said, that Impulſe Reſiſtence and Protruſion Depend upon it; For we may as Juſtly Pronounce from this Notion of Impenetrability, that Impulſe, Reſiſtence, and Protruſion Depend upon Thought, Space, Light, *&c.* as upon Matter's Preſerving it's own Dimenſions, whilſt they Continue the Same; Moreover, Solidity, or Impenetrability, more Properly Depends upon Reſiſtence, than the Latter upon the Former, for how are the Dimenſions of Bodies Impenetrable, but by a Reſiſtence to a Penetration?

We muſt alſo Say, in Full Confutation of this Doctrine, that it is not Evident, that Bodies may not be Penetrable in that Senſe, which this Philoſophy ſeems to Think they are not, that is, in their Dimenſions being Conſtipated and Condenſed in the Same Abſtracted Space; Thus, altho' a Cubical Inch of Water, cannot Enter into a Cubical Inch of Water, and That Body Preſerve it's Nature, which is an Axiom, yet it is not Manifeſt, that by Certain Forces in Being, Water may not be Transformed into another Kind of Body, which ſhall Equally Fill Abſtracted Space, and, Notwithſtanding that, be Double, or Treble, or Quadruple, the Dimenſions it had before, according to the *Geometria Solidorum ſive Materiæ*, and therefore, in ſuch a Caſe, Water may Admit of a Penetration of Dimenſions, and be as little Obſtinate in Preſerving it's own Nature, as Thought, or Heat, or Light, is, which, we are Senſible, are Capable of a Continual Change and Alteration.

Chap. 4. §. 6.

FROM all which it is Plain, what we at Firſt Aſſerted, that this Philoſophy had been Gravely Diſcourſing about Solidity, which at Length, upon a Conſciouſneſs of it's own Ignorance, it was Forced Ingenuouſly to Confeſs, it did not Underſtand.

Abſtracted Ideas in General, as Extenſion, Space, Figure, &c. *not Owing Purely to a Reflection upon our Senſations.* See §. 8. at the Beginning.

§. 9. WHAT we have Alledged Concerning Solidity, that it is an Abſtracted Idea Formed by the Mind, is Certain alſo of Extenſion or Space, of Figure, and the Like; And the only Queſtion, which Remains, is, whether our Abſtracted Ideas are not Owing to Senſation, by which is Meant, either the Objects of thoſe Abſtractions, or the Abſtractions themſelves; If the Firſt, that will not Entitle them to the being Ideas of Senſation, ſince there is an Abſtraction, as well as an Object of it, which Enters into all our Abſtracted Ideas; If the Laſt, ſince our Abſtractions are Performed by the Will and Pleaſure of the Mind, it is Evident, our Abſtracted Ideas are not Owing to Senſation, unleſs the Will of the Mind is the Same with the Senſation of it.

BESIDES which, the very Objects of our Abſtractions are Different from our Ideas of Senſation; For, that the Dimenſions of One Body do not Penetrate into thoſe of Another, is not Seen or Felt, but Conceived.

FARTHER, when we Abſtract our Thoughts, our Wills, or Memories, *&c.* they are ſtill Leſs Owing to Senſation, that is, even the very Objects of our Abſtractions.

IF yet it be Laſtly Said, that our Senſations are Firſt, and all the Operations or Abſtractions of the Mind Proceed Originally from External Impreſſions, That cannot be Known, till it is Firſt Shewn, what the Nature of the Mind and Matter is, and that the Mind is Poſſeſſed Originally of nothing, but what it Derives from Matter; For, unleſs that is done, it will not be Certain, how our Senſations themſelves are Performed, and that there is not as much Owing Primarily to the Mind, which Receives thoſe Senſations, as to Matter which Impreſſes them, and if ſo, that there is not as much Reflection in our Senſations, as there is in the Ideas, which are Said to be Deduced from that Source and Fountain.

SINCE therefore from what Foregoes, there may be Reflection in Senſation, and ſince there may be alſo Reflection without Senſation, it cannot be Proved, that there may not be Ideas in our Minds, which are not Owing to Senſation, Excepting it is Firſt Evidenced, that the Actions or Reflections of our own Minds are not as Natural to them, as any Actions or Impreſſions upon them from External Agents.

The Idea of Eternity not Derived from Mere Reflection upon our Senſations. See §. 8. at the Beginning.

§. 10. BUT the Ideas of Eternity, of Infinity, and of God, are ſtill more Removed from our Senſations, than the Abſtracted Ideas we Mentioned; As to That, of Eternity, it is Certain, it may be as well Inferred and Collected from our own Thought and Exiſtence, as from the Senſations of any Beings External to them; For we can have no more Reaſon to Say, that any External Being Exiſted from Eternity, than that our Thinking One did; and if we Affirm, that there muſt have always Exiſted Something, becauſe Nothing could not Produce Something, ſuch an Argument will be Equally Valid from Thought, or from our Mind, as from Matter or Body; But if it is Said, our own Thought, and the Exiſtence of the Mind, is Derived from Senſation, it muſt be Enquired, what is Meant by Senſation; If by That is Intended an External Impreſſion, without the Mind to Receive it, there is no Senſation in the Mind at all; If what is Signified by it, is, that the Mind is Matter, and the Actions of External Objects upon it are no other than the Signatures of a Seal upon Wax, it will be Neceſſary to Prove the Mind is Matter; The Contrary of which we have Evidenced in our *Pſycheometria*; If neither of theſe are Affirmed, the only Thing which Remains is, that Thought, or the Mind, is not Derived from Senſation, but is a Being Peculiar and Diſtinct from Matter, or any Senſations from it.

Nor the Idea of Infinity. See. §. 8. at the Beginning, and Eſſay, Chap. 17. Book 2.

§. 11. THE next Idea Aforementioned is That of Infinity, which is Said to be Derived into the Mind by it's Actual Perceptions of what is Finite, as the Obvious Portions of Extenſion, which Affect our Senſes, and the Ordinary Periods of Succeſſion, whereby we Meaſure Time and Duration, and Repeating without Bounds thoſe Ideas, for by this Means we find, we have no more Reaſon to Stop at one Addition of theſe Finite Quantities, than another; It is alſo Said, that the Infinity of the Eſſence of Almighty God is not to be Conſidered here, unleſs in Reſpect of his Duration and Ubiquity, and that, what other Notions we have of his Infinite Power, Wiſdom, and Goodneſs, are only Figurative, and are Properly Inexhauſtible and Incomprehenſible, *&c.*

We Say, There are Conſequently only Three Things, which are to be Conſidered in this Doctrine; Firſt, whether Extenſion and Duration are the only Ideas, by which we Know, what is Finite; Second, whether Repeating ſuch Ideas by the Mind is Able to give us a Notion of what is Infinite; Third, whether the Notion of an Infinite Wiſdom, Power, and Goodneſs, are only Figurative, and not Real, Expreſſions of them.

Extenſion and Duration, not the only Ideas, by which we Know, what is Finite.

As to the Firſt, Namely, whether Extenſion and Duration are the only Ideas, by which we Know what is Finite, which this Philoſophy Affirms, it is ſo Apparently Confuted by Numbers of Inſtances, that it would be almoſt Needleſs to Diſprove it; For, whenever we Experience, in Particular, any Thought, or Paſſion, or any Idea, or Knowledge, or Wiſdom, or Power, whatſoever, in our own Minds, or when, in General, upon Reflection, we find, that our Minds themſelves Exiſt, we See they are Finite and Determined Perceptions, and therefore may as Juſtly give us an Idea of what Finite is, as any Portions of Duration or Extenſion, we ſhall Pleaſe to Conſider.

The Powers and Faculties of our Minds Furniſh us with the Same Idea.

Nor, is it Sufficient to Alledge, that Finite Properly belongs to Extenſion and Duration, as having Stated Limits, and which are Confined within Fixed and Certain Terms or Boundaries; For, if that were a Definition of what is Finite, There would be no Finite Number; Since That has not the like Kind of Determination or Preſcription of it's Limits with Extenſion or Duration, nor has even Duration the Same with Extenſion; the Firſt is Incluſive of ſo many Units which it Contains, and Excluſive of all Others, the Second of Moments of Time which Paſſes, and the Laſt of Parts Extended; But, as Unit in Number is not a Moment in Duration, nor a Moment in Duration a Part in Extenſion, and yet each of theſe Furniſhes us with the Idea of Finite, ſo likewiſe, tho' a Thought, or a Paſſion, or a Power in the Mind, or the Mind it's ſelf, are Different from a Unit, a Moment, or an Extended Part, as they alſo Differ amongſt themſelves, yet are they Incluſive of what they are, and Excluſive of any Thing elſe beſides, and therefore may as Juſtly be Termed and Denominated Finite, as any Portion of Number, Time, or Extenſion can be.

Infinity as Properly and Really belongs to the Wiſdom, Power, and Goodneſs of God, as to Extenſion and Duration.

If this Philoſophy has Given us but a Lame and Imperfect Account, of what is Finite, it's Method of Supplying us with the Idea of Infinite is ſtill more Exceptionable, which Leads us to the Second Inquiry, whether a Repetition of the Finite Ideas of Duration and Extenſion is Capable of Impreſſing a Notion upon us, of what is Infinite; It is Plain in the Firſt Place, that it cannot do it in General, ſince the Finite Ideas of Duration and Extenſion, however Repeated, cannot Produce an Infinity of Thought, Wiſdom, Power, *&c.* and which therefore this Philoſophy is Pleaſed to Reject from being Properly Infinite, and to make them only Figuratively and Incomprehenſibly ſo; which is Suitable to it's whole Procedure, in giving Arbitrary Definitions of Things, of Certainty before, and of Finite here, and then Telling us, what ought to be Accounted Certain or Finite, and what not, according as it Squares with ſuch Arbitrary Definitions; On the Contrary, if we have Proved, that Thought or Knowledge, or any of the Powers or Ideas of our Minds may be as Juſtly Termed Finite, as any Portions of Duration and Extenſion, Infinity, without any Figure or Incomprehenſibleneſs, will as Truly, and with as Great a Propriety, belong to the One, as to the Other.

For if it be Said, that Infinite Thought, or any other Power of the Mind, is an Abſurdity, 'tis Owned, if we Mean the Abſtracted Idea of Thought, or of any Power of the Mind in General; and in the Same Abſtracted Senſe, Duration, or Extenſion, are not Infinite; for, whether Finite or Infinite, it is in Abſtraction neither More nor Leſs than Duration and Extenſion, ſince neither of them in General can be Finite or Infinite; If we Conſider Indeed Particular Portions of Duration or Extenſion, they may be Termed Finite, and accordingly, if we Conſider a Particular Force of Thought, of Willing, or Judging, they may be Finite likewiſe, and Granting the Notion, which this Philoſophy Propoſes of Repeating Finite Ideas, ſuch Determinate Forces of Thought, Willing, or Judging, *&c.* are as Capable of being Repeated to Infinity, as Space or Duration is.

What we Intend by the Forces of Thinking, Willing, Judging, *&c.* is, that even in our Ordinary and Common Senſations, as of Colour, or Sound, or Taſte, or Smell, or Feeling, there may be a Thouſand Different Forces in the Mind it's ſelf, by which ſeveral Minds Perceive theſe Senſations with ſo many Different Degrees of Force; as ſome are more Quick-Sighted, or have a Sharper Ear, or a more Delicate Palate, or an Acuter Noſe, or a more Senſible Feeling, than others have in ſeveral Various and Intermediate Degrees of Senſation; If this is Said to be Owing to the Organs of Senſation, and not to the Mind it's ſelf, That we have Shewn to be Impoſſible in our *Pſycheometria*, or Eſſay upon the Mind; If not, the Contrary is not Proved.

In the Mean Time, if the Mind is Capable of Different Forces in it's Senſations, this Philoſophy, I Believe, will Own, that it is ſo in it's Reflections and Judgments, or it's Volitions upon them; For, if the Mind has a Peculiar Force, by which it Sees, or Hears, or Feels, or Taſtes, or Smells Differently, it will be Certain, that it's Reflections upon thoſe Forces will be Anſwerably Various, whilſt the Power and Energy of the Mind, whatever it is, is Diverſify'd, and Conſequently, that it's Judgments, Volitions, *&c.* are ſo.

It has been hitherto Said, that our Notion of Infinity cannot in General be Confined to the Repetition of the Ideas of any Certain Portions of Duration or Extenſion, becauſe there may be Finite Thought, Willing, Judging, *&c.* which if they were Repeated, according to the Different Forces of Different Minds, might be Infinite.

Our Idea of Infinity cannot be Collected from the Repetition of a Finite, or the Negation of it.

But we muſt Farther Affirm, that if Infinity was Applicable only to Duration and Extenſion, yet a Repetition of the Finite Portions of it could never Produce our Idea of Infinite; ſince any Determined Repetition of them would but Produce a Greater Finite, and an Actual and Infinite Repetition of them is both Impoſſible to a Finite Mind, and at the ſame Time as Inconceivable, as an Actual Idea of Infinite, (which this Philoſophy Denies, and Affirms our Idea of Infinite to be only Negative of what is Finite;) For an Infinite Repetition of a Finite Idea is Equally Unintelligible with one Repetition, or one Impreſſion of an Infinite Idea.

Besides which, that our Idea of Infinite is Negative only, is Surmiſed upon this Opinion, that we muſt have Expreſs Images or Repreſentations in our Minds of what we Perceive and Know, it being a Neceſſary Doctrine to Certainty from the Connexion of Ideas; Thus, when it is Said, our Idea of Infinite is Negative of what is Finite, it is Suppoſed, that the Actual Idea, we have of any Thing, is no other than Finite, and that, what we have Beyond That, is only a Conception of ſomething more, which, when we have Taken in to our Former Idea, will ſtill make a Finite One.

In Anſwer to which, we ſhall Say, that if it is Neceſſary to Certainty from the Connexion of Ideas, that thoſe Ideas ſhould be Expreſs and Determined in the Mind, Either our Idea of Infinity is Expreſs and Determined, and therefore not a Negative One, ſince to Deny in General, that any Finite is an Infinite, is not to Affirm Expreſly what is Infinite; Or, if our Idea of Infinity is not Poſitive and Determined, we cannot be Certain of any Thing Concerning it, and therefore, it would be no Great Matter, whether Infinity of Wiſdom, and Power, were Figurative or not, when Applied to Almighty God, ſince this Philoſophy has Conſulted ſo well for the Proper Infinity of Duration and Extenſion, that at the Laſt, we ſhall Know nothing of Either; Which Shews the Admirable Contrivance of it, whilſt in one Part we are Soothed with an Infallible Demonſtration of the Being of a God, and, in another, are in Effect Plainly and Roundly Told, that Infinite Wiſdom, and Infinite Power, is an Improper and Incomprehenſible Way of Speaking, and that when we Aſſert it, even, of Space and Duration, there is nothing Certain Concerning it.

If therefore we have any Idea at all of Infinite, it is Manifeſt, it cannot be Collected from any Repetition of our Ideas of Finite, which only would Produce a Greater Finite; nor from a Negation of what is Finite, ſince That would be Affirming nothing; For, where we Deny, that any Finite Idea of Space or Duration, be it Greater or Smaller it Matters not, is Infinite, we do not in the Leaſt Say, what is ſo, no more, than, if we Said an Inch was Leſs than a Foot, we ſhould be Imagined to Determine what a Foot was, All that is more than an Inch being Wholly out of our Notice.

Therefore not from Senſation, and Reflection.

From whence we may Juſtly Conclude, that this Philoſophy does not Give us an Account, how our Idea of Infinity is Derived from Senſation; ſince a Greater Finite can only Poſſibly Ariſe from a Repetition of a Finite Portion of Space or Duration, with which our Senſes Supply us.

To which we may Add, that the Portions of Either, which we Receive in our Senſations, may be Extremely Different, and Conſequently, Seeing the Mind has an Equal Power in Multiplying in the One Caſe, as it has in the Other, One Univerſal and Infinite Space or Duration will be Greater than Another, according to the Different Meaſures, which we Repeat; If the Meaſures are Certain and Fixed, what are they? If an Inch or a Yard, why are they more Entitled to be the Foundations of our Repetitions, than a Foot or an Ell? If an Indiviſible Point in Time or Extenſion, how are they Owing to our Senſations?

That we may have an Idea of an Actual Infinite.

Lastly, That we may have an Idea of an Actual Infinite is Plain from the Nature of Incommenſurables, in which there is no Common Point, be it never ſo Small, which will Meaſure them; Thus the $\sqrt{2}$, is Incommenſurable with Unit, or the Diagonal with the Side of a Square, and yet we have as Expreſs a Notion or Idea of the Root of Two, as we have of Unit; For, tho' we do not Know by Extraction of the Root, what the Effable Number is, (nor do we Know of how many Parts Unit Conſiſts,) it is not from thence to be Concluded, that we have not a Notion or Idea of $\sqrt{2}$, ſince we can Reaſon and Argue Concerning it, with as much Clearneſs and Diſtinction, as if we had Entered into the ſeveral Parts, of which it is made up; Thus we can Affirm, that $\sqrt{2}$, Multiply'd into it's ſelf, is Equal to 2, that, if an Extraction is made, it is Actually Infinite and Inexhauſtible, that if it is Compared with the $\sqrt{3}$, it is in Power as 2 to 3, *&c.* and we can Say no more of Unit or a 100, for each of them may Conſiſt of an Infinite Number of Parts, whoſe Expreſſions we are Ignorant of, but this we are Certain of Concerning them, that Unit Multiply'd into Unit, or 100 into 100, will Produce Unit, or 10000, *&c.*

Whatever Notions therefore we can Fully and Perfectly Reaſon upon with the Utmoſt Certainty and Diſtinction, they may be Juſtly Termed Abſolute and Actual Ideas of our Minds; If ſo, this is Manifeſt in General, that we can Argue and Reaſon upon Infinites, as Clearly, as we can upon Finites; Thus we are as Sure, that an Infinite is not any the Greateſt Finite, as we are, that 1 is not 2; that an Infinite is never to be Exhauſted by any Diviſion, or Reached by any Multiplication; that it is One and Similar in it's ſelf, and the Like; and the ſeveral Propoſitions, which may be Formed and Evinced upon ſuch a Notion, may be Found in the Excellent Mr. *Raphſon*, in his Eſſay upon an Abſolute and Infinite Space.

Raphſon de Spatio Reali ſive Infinito.

If thoſe cannot be Accounted Poſitive and Actual Ideas in our Minds, upon which we can Reaſon with Juſtneſs and Accuracy, but thoſe only, which are Determined Images and Repreſentations of Things, and which Lie in the Mind, as Pictures there, which this Philoſophy ſeems to Imagine, we can no more Comprehend, what a 100 or a 1000 is, or what a Chiliagone, or a Billiagone is, that is, a Figure of a Million of Millions of Sides, than we can Conceive Infinity; ſince an Infinite Number, or a Figure of an Infinite Number of Sides, is not Leſs Expreſſed in the Mind, than a 100, or a 1000 is, or the Polygones we Mentioned; for no one ever Saw 100 or a 1000 Diſtinct Units in his Mind at Once, no more than all the Sides of thoſe Polygones; And Conſequently, both Finite and Infinite here, are no other than Diſtinct Ideas Lodged in the Mind, upon which we can Reaſon, and have Peculiar Conceptions of, Proper to each, and by which they are Diſtinguiſhed from all Others, and not Images or Expreſs Characters of thoſe Numbers, or Figures, which we Conſider.

Therefore ſuch an Idea not from Senſation and Reflection.

If this is ſo, then our Idea of Infinity, ſince it is far from being Finite, cannot be Derived from our Senſations, or Reflections, but muſt be Owing to the Boundleſs Nature of the Mind, which Forms an Idea Superior to any Thing, which is Finite, and can Reaſon upon it with as much Perſpicuity, as it can upon thoſe Finite Ideas, which it Receives from Senſation or Reflection.

Nor That of Eternity.

Another Idea is That of Eternity, which we Conſidered Above, which cannot be Derived from Senſation or Reflection, and for the ſame Reaſons likewiſe, which are Alledged before, in Reſpect of Infinity; Since Eternity is nothing elſe but an Infinite Duration.

Nor the Idea of God. See §. 8. at the Beginning.

§. 12. The Laſt Idea is that of God, the Demonſtration of which we have already Examined, and ſhall here only Inquire, according to what was Propoſed in the Third Place, §. 11. Paragraph 2*d.* whether the Notions of an Infinite Wiſdom, Power, and Goodneſs, are only Figurative, and not Real Expreſſions of them; we have Proved, that our Finite Ideas of Power, Wiſdom, and Goodneſs, may be as Properly Called Finite, as thoſe of Extenſion and Duration, and Conſequently, if the Repetition of ſuch Ideas is Neceſſary only to the making them Infinite, that Infinity, without any Metaphor or Figure, may be as Properly Affirmed of Wiſdom, Power, and Goodneſs, as of Duration and Extenſion; but we have farther Shewn, that there is ſomething more, which is Requiſite to the Idea of Infinity, than a bare Repetition of what is Finite, and Conſequently, that our Ideas of Infinite Wiſdom, Power, and Goodneſs, are not Derived from our Repetitions of any Finite Ideas of them, but from the Nature of the Mind, which is Able to Conceive Ideas Superior to any Thing, which it has from it's own Senſations and Reflections, or any Repetitions of them; what that Nature is, we have already Endeavoured to Explain in our Eſſay upon the Mind, and which, we are fully Satisfied, muſt be Derived from Almighty God; It is Sufficient, if it is Fact, as it Certainly is, that we have Ideas, which no Senſe, or Reflection upon that Senſe, could Supply us with, and how we come by them, has been a Former Inquiry.

Unneceſſary to Inquire into what is Advanced, Concerning the Being of a God, by other Philoſophers.

§. 13. After what we have Said, Concerning the Demonſtrations of an Almighty Being from the Connexion of Ideas, and the Principles, upon which it Proceeds, Contained in this Eſſay upon Human Underſtanding, and *Des-Cartes*'s Meditations, which, we have Endeavoured to Evince, are Falſe in the Chief Parts of them, it will be Unneceſſary to Prove, that all other Demonſtrations of a God are Inſufficient, which Depend upon the Corpuſcular Philoſophy, or Sir *Iſaac Newton's* Mathematick Improvement of it, that is, which Depend upon the Suppoſition of a Vacuum, as Dr. *Bentley's*, Dr. *Cheyne's* and Others; Seeing, if the Corpuſcular Principles, and Conſequently a Vacuum, are Erroneous, which we have Shewn in our Principles of Philoſophy to be ſo, it will be Impoſſible to Maintain any Demonſtrations of a God from Principles, which are not Juſtifiable in themſelves.

The Immortality of the Soul not Proved by this Philoſophy.

§. 14. That which we are therefore Farther in this Chapter to Conſider, is, whether the Immortality of the Soul can be Proved from the Preſent Philoſophy; which it is Impoſſible it ſhould be, when it neither Knows the Nature of Matter, or of Motion, or of the Mind, and only Depends upon the Idea of Matter and Motion, which the Syſteme of a Similar or Homogeneous One has given us, and which is Confuted in our Philoſophy, and upon the Elaborate Eſſay on the Mind, which is Reſolved, as we have Shewn, into Terms, and Artificial Names of Things, without Underſtanding even ſo much, as what is Meant by them.

CHAP. VIII.

Concerning Identity and Diverſity.

What Identity and Diverſity is, according to this Syſteme of Ideas
Mr. Locke's Eſſay, Chap. 27. Book 2. Edit. 3.

§. 1. After Having Conſidered the Ideas of Infinity, Solidity, and the Reſt, we ſhall Conclude this Inquiry into the Syſteme of Ideas, with ſome Obſervations upon the Notions, which have been Entertained in it, Concerning Identity and Diverſity, and Concerning Enthuſiaſm; The Former of which are Said to Conſiſt in the Mind's Comparing any Real Being, as Exiſting at any Determined Time and Place with it's ſelf, Exiſting at another Time, upon which it Forms it's Conceptions of Identity and Diverſity; for, when we See any Thing, to be in any Place, in any Inſtant of Time, we are Sure, be it what it will, that it is that very Thing, and not another, which at that ſame Time Exiſts in another Place, how Like and Undiſtinguiſhable ſoever it may be in all other Reſpects; And in this, it is Said, Identity Conſiſts; when the Ideas, it is Attributed to, Vary not at all from what they were that Moment, wherein we Conſider their Former Exiſtence, and to which we Compare the Preſent; What is afterwards Said, is to the Same Purpoſe.

The Whole of which, is only this, that a Thing may be Properly Said to be the Same, when in one Inſtant of Time, in which it is Conſidered, and in one Place it is that very Thing, and not Another, Notwithſtanding the Cautions of this Philoſophy.

The Objections againſt ſuch a Notion.

§. 2. But First, If Identity Conſiſts in this, it is ſo Confined, and Limited an Opinion of it, that any Thing, tho' it ſhould not be Varied, cannot be Called the Same in any Different Intervals of Time, and Conſequently the ſame Undiſtinguiſhed Atom of Solid Matter, Exiſting in Different Times, and in the ſame Place, muſt be Different, which is Impoſſible.

Second, Whatever is Diſcourſed in the Following Sections, Concerning the Identity of Vegetables, of a Brute, a Man, or a Perſon, is all Falſe; ſince the Identity of thoſe ſeveral Subſtances is not Determined from being, what they are, in one Inſtant of Time, but from other Methods; Agreeable to what is afterwards Explained in this Chapter.

Third, according to this Notion of Identity, Two Undiſtinguiſhed Solid Atoms of Matter, which Exiſt in the Same Time, but in Different Places, have not any Identity belonging to them; and yet it is not Explained, why in this Theory of Identity, Two Undiſtinguiſhed Atoms of Matter may not as well be Called the Same in Reſpect of their being to all Appearances ſo in the ſame Inſtant of Time, as One Portion of Matter ſhould be Deemed to be the Same, which is to all Appearances ſo, tho' Exiſting in Two Differing Inſtants of Time; Unleſs the Identity of Place has a Greater Prerogative in Determining, what the Identity of a Thing in General is, than the Identity of Time has.

Identity as before Explained, an Abſtracted Notion.

§. 3. From what Foregoes, it is Plain, that Identity is, in theſe Logicks, a General and Abſtracted Notion, which is ſo far from Extending to all the Particular Beings, which we Call the Same, that it cannot ſo much as Solve the Identity of one Atom of Matter, which is Suppoſed to be Unvaried, whether in Reſpect of Time, or Place, or it's own Exiſtence; Much Leſs will it ſerve us in the Identity of a Vegetable, a Brute, a Man, or a Perſon, which will be afterwards Conſidered.

What the Identity of every Particular Being is, ought to be Determined, and not a General Notion Propoſed of it.

§. 4. On the Contrary, a General Notion of Identity ought not to be Firſt Defined and Determined in our Minds, and then all other Beings, in Reſpect of their Identity, Submitted to that Notion, but the Method we ought to Purſue, is to Conſider all Kinds of Beings, which we Call the Same, and then Pronounce, whether they are ſo, and wherein the Identity of each Particular Being Conſiſts, and afterwards Compare thoſe Identities, and Examine wherein they Agree, and from thence Form, if any muſt be Formed, our Idea, or Notion, of Identity in General.

The Diſtinction of Beings, whoſe Identity this Philoſophy Conſiders.

§. 5. As to what therefore Reſpects the Identity of Things, ſince they are Infinitely Various, we ſhall only Conſider thoſe, which are Pointed at and Marked out to us in this Philoſophy, Namely, the Identity of Time, Place, an Individual or Single Atom of Matter, a Subſtance, Vegetables, Animals, a Man, and a Perſon.

The Identity of Time not Explained.

§. 6. First, in Reſpect of the Identity of Time, That cannot be Known, unleſs by Telling us, what Time is, and the Meaſure of it; If it is the Succeſſion of Ideas, by which we Determine the Limits of Time, and the Certain Boundaries of any Portion of it, which we Conſider, 'tis a Falſe and Unequal Standard of it; ſince the Succeſſion of Men's Ideas are as Various, as any Different Velocities of Bodies whatſoever; If it is not, what other Meaſure of Time is there? Not from Pendulums, which are Irregular; Not from the Revolutions of the Heavenly Bodies, which are ſo likewiſe; If therefore there is any Identity of Time, it muſt be Fixed from our own Abſtracted Conceptions of it, Namely, that Time is ſome Certain Succeſſion of the Inſtants of it, which Agrees to every Man's Succeſſion of Ideas; But ſince That is not True, that there is ſuch a Succeſſion of the Inſtants of Time, which will Agree to every Man's Succeſſion of Ideas, Identity of Time is either Nothing, or is an Abſtracted Idea.

If the Identity of Time is an Abſtracted Idea, from the Particular Succeſſions of every Man's Thoughts, or the Motions of any External Beings, Then, Firſt the Succeſſion of Ideas cannot be the Meaſure of Real Duration; And Second, the Identity of Time can only Conſiſt in the Abſtraction of it from all Particular Succeſſions of Ideas or Motion, and in Conſidering it only, as any other Abſtracted Extenſion, as Length, Separated from Breadth or Thickneſs, which is no more than an Arbitrary Creature of the Mind.

THE Identity of Time therefore, if there is any, muſt Conſiſt in the Mind's Abſtraction of Duration from every Thing, which may Cauſe any Alteration in it's Succeſſion, and therefore a Unit or Inſtant of Time is it's ſelf, and no other; Not becauſe it is not Different, if Applied to Particular Beings, in Reſpect of Thinking or Motion, but becauſe all Beings are Thrown out of the Queſtion, and the Mind Conſiders barely the Abſtracted Idea of Time, and the Portions of it.

Nor That of Place.

§. 7. THE next Sort of Identity to be Examined into, is that of Place, which as has been before Proved in our Principles of Philoſophy, is no more than an Abſtracted Idea, and therefore the Identity of Place Conſiſts in that Abſtraction, which is made by the Mind of Extenſion in General, in Excluſion of all other Properties Inherent in it.

Nor That of Matter.

§. 8. IF therefore the Identity of Time and Place are either Nothing, or Abſtractions of the Mind, of what Significance is it, to Determine the Identity of a Thing, as of any Solid Portion of Matter, by them? But this is not all, for which Way, Abſtracted from Time and Place, is a Solid Portion of Matter Conceived to be the Same? Unleſs from Conſidering, or Suppoſing, that all Matter is Similar and Homogeneous, which is, according to what has been Shewn in the Principles of Philoſophy, no other than an Abſtracted Idea.

WHAT therefore the Real Identity of Matter is, muſt be Computed from the Nature of it, as much, as That of Vegetables, or of an Animal, or of any other Subſtance; And therefore the Identity of Subſtances, of God, of Spirits, or of Corporeal Beings, muſt be Conſidered in the ſame Manner; that is, we are to Know the Natures of theſe ſeveral Subſtances, before we can Determine Concerning the Identity of them, or which is the Same, before we can Know, that a Thing is, what it is.

Nor That of God, or of Subſtances. Mr. Locke's Eſſay, Chap. 27. Book 2. §. 2.

§. 9. As to the Being of a God, we have already Conſidered the Notions, which this Syſteme of Ideas has Received, and Entertained Concerning it, and therefore, ſince it is not Acquainted with the Nature of any ſuch Being, it can as little Determine Concerning the Identity of it; And for the ſame Reaſon, it cannot Preſcribe to us the Identity of Finite Intelligences, or of Bodies; Becauſe it does not Know what they are, and therefore cannot Inform us what it is, to be, what they are, in which Conſiſts Identity.

Nor That of Vegetables. Mr. Locke, Chap. 27. §. 4.

§. 10. FROM what has been before Said, it is Evident, that the *Principium Individuationis* is as little Accounted for, as Identity is, we ſhall therefore next Proceed, to the Identity of Vegetables; In Order to Aſcertain which, this Philoſopher Deſires us to Conſider, wherein an Oak Differs from a Maſs of Matter; which ſeems to him to be in this, that the One is only the Coheſion of Particles of Matter any how United, the other ſuch a Diſpoſition of them, as Conſtitutes the Parts of an Oak, and ſuch an Organization of thoſe Parts, as is Fit to Receive and Diſtribute Nouriſhment, ſo as to Continue and Frame the Wood, Bark, and Leaves, *&c.* of an Oak, in which Conſiſts the Vegetable Life.

IN Anſwer to which we muſt Firſt Say, that it is an Idle Amuſement to Tell us, what is the Diſtinction betwixt a Solid Particle of Matter, and ſeveral of them Arranged together, ſo as to have an Organization, Fit to Receive and Diſtribute Nouriſhment, *&c.* when this Perſon neither Knows what Matter is, nor what ſuch an Organization is.

SECOND, Granting both, this Philoſopher, if we may Call him ſo, does not, nor can Shew, that there is any Identity in Vegetables, unleſs he Relinquiſhes his Notion of Determining the Identity of a Thing, by the Time and Place, in which it Exiſts; ſince there may be ſeveral Different Things, as to Time and Place, which are Requiſite to Maintaining the Life of the Vegetable, whilſt the Identity of the Vegetable Continues the Same.

THIRD, as to the Identity of the Life in the Vegetable, if the Organization of it is not Known, and what is Required to Support the Life of it, it will not be Certain in what the Identity of that Life Conſiſts, becauſe it may not be the ſame Life; It may in Certain Caſes, Flag and Revive, as a Fire does, according to the Fuel Adminiſtred to it, and other Circumſtances of Blowing, under which that Fire is Fomented; And therefore, as the Fire will not be in all Caſes the Same, tho' there is a Fire Continued, ſo the Vegetable Life may not be, tho' it is likewiſe Continually Kept up and Preſerved.

Nor That of an Animal, or a Man.

§. 11. In Reſpect of the Identity of an Animal, or of a Man, the ſame Thing may be Said, Namely, that there is no Aſcertaining the Identity of Either, without Underſtanding the Real Nature, in which they Conſiſt, that is, it is Impoſſible for us to Know they are, what they are, till we Know, what they are.

Nor That of a Perſon.

§. 12. The laſt Thing to be Conſidered, is Perſonal Identity, which is Affirmed to be Reſolved into Self-Conſciouſneſs, and is Properly, what a Man Calls his Self, and from Hence it is Suppoſed, that a Man may be Himſelf after the Reſurrection, without his having the Same Numerical Body; We ſhall therefore Conſider this Notion more Particularly, becauſe Religion ſeems in ſome Meaſure to be Affected with it.

We ſhall then First Say, that we cannot Underſtand, how a Perſon is here made Diſtinct from the Man; For the Identity of a Man, is Said to be Different from the Identity of a Perſon, whereas it is Impoſſible, unleſs we Name Things at Pleaſure, that the Same Man ſhould not be the Same Perſon, or Vice Verſâ.

Second, If a Man is to be Called Himſelf from his Self-Conſciouſneſs, Nothing of his Body belongs to him, as he is That Self; Whereas, it cannot be Imagined, that the whole Syſteme of Mind and Body muſt not Enter into the Definition of what a Man's Self is, unleſs we Diſcard one Half of our Selves from our Selves; For, where there is that Intimate Union betwixt the Soul and Body, that if the One is Affected, the other is, it would be a very Abſurd way of Conſidering Self, to Suppoſe it only Conſiſts in one Part of that Compounded and United Being.

Third, To Suppoſe Self to Conſiſt in Self-Conſciouſneſs, is to Imagine, that we ſhall not be our Selves in any Different Period of our Lives; For let a Man Extend his Conſciouſneſs, as far as it is Poſſible for him to do, by Recollecting all the Thoughts and Actions, which he is Conſcious have Paſſed, and do belong to him, and let him Join them with the Preſent Thoughts he has, he will not yet be Able to make the very ſame Collection Perhaps in any other Stage or Period of his Life, ſome Things will be Forgotten, and others Remembered, which were not Put into his Former Collection; and Conſequently the Identity of a Perſon cannot Conſiſt in Self-Conſciouſneſs, where Self-Conſciouſneſs it's ſelf is Continually Diverſify'd and Varied, unleſs Identity Conſiſts in Diverſity.

Fourth, Some Perſons have Thought they were Made of Glaſs, of Butter, *&c.* as *Mallbranche* Tells us, of which they were then Perhaps as Conſcious, as they could be of any Thing elſe; wherein then does the Identity of a Mad Perſon Conſiſt? If in Self-Conſciouſneſs; will that Mad Perſon be the Same, as when he is Sober, and is Conſcious of no ſuch Thought, nor Perhaps, Knows, that he ever was? It will be Said, if a Mad Perſon is not the Same with a Sober, or with One in his Senſes, why does the Identity of a Mad Perſon Conſiſt in Self-Conſciouſneſs, and the Identity of a Sober Perſon likewiſe, when Self-Conſciouſneſs in this Caſe is as Different, as Identity and Diverſity can be? that is, if the Identity of a Perſon Conſiſts in Self-Conſciouſneſs, which in this Circumſtance is Acknowledged to be Different, Perſonal Identity Conſiſts in Perſonal Diverſity.

Fifth, In Self-Conſciouſneſs of what I now do Know, that I have Formerly Thought or Acted, or of what I now Think or Act, the Memory in the Firſt Caſe is Intirely Concerned, and in the Laſt, my Actual Perception, and Conſequently, to Self-Conſciouſneſs the Faculties of the Mind are Requiſite, which, being Various and Different in the ſeveral Circumſtances of Time and Occaſion, and in the Vivacity of them, cannot be a True Meaſure of what Self-Conſciouſneſs is, and in Conſequence, None of the Identity of a Perſon, which is Suppoſed to Conſiſt in Self-Conſciouſneſs.

Sixth, If this could be Granted, that Self-Conſciouſneſs Depends upon Memory and Actual Perception, and that theſe did not Enter into the Account of Self-Conſciouſneſs, in Reſpect of what the Mind either Really Perceived or Recollected, yet, if the Nature of thoſe Faculties of the Mind are not Known, we cannot either make the Identity of a Perſon to Conſiſt in Self-Conſciouſneſs, or Self-Conſciouſneſs in the Nature of thoſe Faculties; and therefore the Identity of a Perſon in the Deity, or in Humanity, cannot be Determined, but by Knowing the Nature of Each.

Lastly, the Identity therefore of a Perſon can be only Known, by Conſidering what the Deity, or any Particular Man is in Reſpect of his Soul and his Body Jointly; If this Philoſophy can Tell us the Nature of Either (which it Diſclaims the Knowledge of) ſomething might be done towards the Informing us, what the Identity of God, a Man, or a Perſon is, which can be no otherways Aſcertained, than by Knowing more of the Deity, a Soul and a Body, than what this Syſteme of Ideas does, or can Pretend to.

For if there is any Thing of Identity in God, a Perſon, or a Man, it muſt be Determined, by what the Real and Intrinſick Qualities of that Infinite Being, a Man or a Perſon are, and the Latter, both in Reſpect of his Body and Mind, which will make him the Same Man and Perſon, whether Mad, or in his Senſes, whether Well, or in a Feaver, or in any other Differing Circumſtances of his Life.

This may be done, becauſe we have already done it, and upon the ſame Principles we Dare Aſſure the World of the Reſurrection of the ſame Body; All which will be Impoſſible, unleſs by Conſidering and Knowing the Nature of the Mind, and of that Organized Matter, which is United to it, nothing of which the Preſent Philoſophy is Acquainted with; But we have likewiſe Conſidered this Doctrine of Identity in another Place, and therefore ſhall not here Enlarge.

CHAP. IX.

Concerning Enthuſiaſm.

§. 1. HAVING Conſidered the Advances made by this Philoſophy, in Favour of Reaſon, and Shewn the Abſurdity of it's Pretenſes, we ſhall Finiſh our Preſent Inquiry with it's Chapter of Enthuſiaſm, and Examine, whether it is not as Erroneous and Falſe in it's Derogateing from Religion, as it has been Proved to be Extravagant and Partial in it's Bounties and Conceſſions to Reaſon.

§. 2. It is therefore in the Firſt Place Told us, that if we are Willing to Find out what Truth is, our Minds muſt be Prepared with a Love of it, that we are to Value and Eſteem it for it's own Sake, and therefore ſhould never give our Aſſent to any Propoſition beyond the Evidence, we have for it; for, whatever Credit or Authority, which we give to any Propoſition more than it Receives from the Principles and Proofs it Supports it's ſelf upon, is Owing to our Inclinations that Way, Derived from our Paſſions or Intereſts.

The Intention of Examining into this Doctrine of Enthuſiaſm. Mr. Locke's Eſſay, Book 4. Chap. 19. §. 1 Edit. 3. *Truth is to be Highly Valued and Eſteemed, and the only Queſtion is, what it is, and where it is to be Found.*

It is Agreed; ſince Truth is That, which we have Endeavoured Unpaſſionately and Impartially to Follow in our whole Reſearch of it, both in the Demonſtration of Religion, in the Principles of Philoſophy, and in this Inquiry into the Syſteme of Ideas; But then we muſt be Allowed to Say, that this Eſſay has little Reaſon to be ſo Strenuous an Advocate of That, with which it Appears to have ſo Small an Acquaintance.

For, it is not Sufficient to make General Declamations, and Ingenious Harangues upon Truth, and the Excellency of it; ſince no Wiſe or Thinking Man can have any other than an Intimate Value and Reſpect for it; Every One will be Ready to Embrace it, when they Know, where it lies, what it is, and which Way we may Attain it; But That's the Queſtion; This Philoſophy Suppoſes, it Conſiſts in the Connexion of Ideas, which if we ſhould Deny, and at the ſame Time Diſprove, as we have already done, all the Fine Eloquence, which is Spent upon Truth, that is, in Reality upon the Delicate Notions, which this Philoſophy has Eſpouſed, will in Effect be no more than a Panegyrick upon Falſehood; if any other Truth is Meant, than what this Eſſay Endeavours to Eſtabliſh for ſuch, I am Certain and am Conſcious by my own Experience, that the Patrons of theſe Ideas, and Perhaps the Author of them himſelf, if now Alive, would not Speak Half ſuch Kind Things of it.

It is Farther Said, that we ſhould never give an Aſſent to a Propoſition, which ſhould Reach Farther than the Proofs or Evidences we have for it; But then, what the Nature of the ſeveral Proofs or Evidences are, which ought to Convince us, we are not Informed, unleſs the Connexion of Ideas will Satisfy us, which, we have at Large Shewn, is Fallible, or if not, that it is not the only Meaſure of Truth, and that if it were, this Philoſophy does not Underſtand, what is Meant by it; If therefore it is Perfectly Ignorant of the Nature of the Evidences, by which we may be Fully Aſſured and Convinced of any Thing, it cannot but be Superlatively Wiſe, when it Tells us, that our Aſſent to a Propoſition ſhould Reach no Farther than the Evidences, by which it is Supported, that is, than the Evidences, which it Knows Nothing of.

No more Tyrannizing, or Aſſuming in Religion, than there is in Philoſophy and Reaſon. Mr. Locke's, Eſſay, Chap. 19. §. 2.

§. 3. THE next Thing we are Led to in this Chapter is, the Aſſuming an Authority of Dictating to Others, and a Forwardneſs to Preſcribe to their Opinions, which is Said to be a Conſtant Concomitant of a Biaſs, and a Corruption of our Judgments; For how, it Continues, almoſt can it be otherwiſe, but that he ſhould be Ready to Impoſe on Others Belief, who has already Impoſed on his Own, who Tyrannizes over his own Mind, and Uſurps the Prerogative, that belongs to Truth alone, which is to Command Aſſent by, and in Proportion to, that Evidence, which it Carries with it?

WHO theſe Men are, who Aſſume an Authority of Dictating to Others, who are Corrupt and Biaſſed in their Judgments, who, after they have Impoſed a Belief upon their own Minds, Require the ſame of other Men, and Uſurp the Prerogative which belongs to Truth of Commanding Aſſent, it is not Difficult to Gueſs; For it Perhaps would have been a Civility done them, if the Teachers of the Chriſtian Religion in General had been Excepted out of this Character, and Chriſtianity it's ſelf Excuſed from being an Enthuſiaſm; If that is the Caſe, the Senſe, as Innocent and as Juſt as it may Appear, will Really be Reſolved into theſe Two Impious and Falſe Affirmations.

FIRST, That Thoſe, who Aſſume and Take upon themſelves to Require from other Men an Aſſent to the Chriſtian Faith or Belief, Uſurp a Prerogative, which only belongs to Truth, or to the Connexion of Ideas, or which is the Same, which only belongs to this Wiſe Aſſertor and Dictator of them.

SECOND, That there is more Real Truth in the Connexion of Ideas, that is, in this Eſſay on Human Underſtanding, than there is in the Scriptures themſelves, which Inculcate and Teach that Chriſtian Belief, and that Men ought to Reſign their Aſſents up to the Former, rather than to the Latter.

As to the Firſt, we ſhall only Say, that the Compariſon Inſinuated betwixt the Author of this Sort of Truth, and the Apoſtles of Chriſt, who did Require, and Inſiſt upon, a Belief in Chriſtianity, and therefore in this Perſon's Senſe, Aſſumed a Prerogative, which only belonged to Truth, or his Ideas, or betwixt himſelf, and even thoſe Good and Excellent Perſons, who ſtill Affirm the ſame Faith, and are Ready to Vindicate it with the Hazard, or with the Loſs, of their Lives, is Ridiculous; Since he has not given us the ſame Proofs of his Ingenuity and Sincerity, in any Reſpect, which they have done.

AND as to the Second, we are Aſſured, that there is more Truth in any One Page of the Scriptures, than there is to be Found in his whole Eſſay; if any Thing we have Said in Demonſtration of the Firſt, or any Thing we have Alledged in Confutation of the Laſt, is True and Certain; of which we have at Preſent as Full a Conviction, as he had in the moſt Demonſtrative Part of it, the Proof of a God, which it is much to be Queſtioned, whether he himſelf was Conſcious, or Thought, was One.

Concerning Enthuſiaſm, being a Third Ground of Aſſent. See §. 3. Chap. 19.

§. 4. A Third Ground of Aſſent is Enthuſiaſm, which is Said to be as much Relied on, as *Faith* or *Reaſon*, by which Words Compared with §. 1. and 2. of this 19 Chap. it will be Farther Evident, that we have not Injured this Philoſophy, when we Said, in the Preceding Section, that, by the Aſſuming an Authority in Matters of Belief, was Really Meant the Preaching of the Chriſtian Faith; Since there are only Three Grounds Affirmed, upon which Men Generally Aſſent, either Reaſon, Faith, or this Third of Enthuſiaſm; the Firſt of which was Recommended in the Firſt Section, the Second Exploded in the Next, and in the Preſent Enthuſiaſm is Propoſed to be Conſidered.

MOREOVER, where can Men be Imagined to Tyrannize and Aſſume according to the Harſh Language of this *Gentleman*, or in the Trueſt, where are Men ſo much Concerned to Recommend with all the Zeal and Authority, which is Owing to a True Faith, a Belief to Others, Excepting in Chriſtianity? For in Common Matters, the Liberty of Aſſenting, or not, upon another's Belief, is Left at Large, and there is no Aſſuming, or Authority, Uſed to Enforce it.

LASTLY, Faith and Reaſon, or Belief and Certainty, are always made Uſe of to Signify the Religion we Profeſs, and the Connexion of Ideas Propoſed by this Eſſay, betwixt the Evidences of which, we have Abundantly Proved, there is no Compariſon.

Revelation or Faith, not Reducible to Reaſon. See §. 4. Chap. 19.

§. 5. What we have Said, is ſtill Farther Confirmed in the Section here Cited; For what was Called Faith and Reaſon before, is now Termed Reaſon and Revelation; therefore, as the Firſt Section of this Chapter was Spent in a Commendation of Truth or Reaſon, the Second in Deſpiſing the Aſſuming upon Faith or Belief, the Third, in Telling us, there was only another Ground of Aſſent, which was Enthuſiaſm, ſo the Fourth is Contrived, to Reduce Faith or Revelation to Reaſon, and when that is done, this Philoſophy with Abundance of Satisfaction Proceeds to Confute Enthuſiaſm.

It is therefore to be Conſidered, whether Revelation, or Faith, can be Reduced to Reaſon; It is Affirmed it may, becauſe *Reaſon* is Natural *Revelation*, whereby the Eternal Father of Light, and Fountain of all Knowledge, Communicates to Mankind that Portion of Truth, which he has Laid within the Reach of their Natural Faculties; *Revelation* is Natural *Reaſon*, Enlarged by a New Set of Diſcoveries Communicated to, (He would Say, by) God Immediately, which Reaſon Vouches the Truth of, by the Teſtimony and Proofs it gives, that they come from God; So that he that Takes away Reaſon, to make Way for Revelation, Puts out the Light of both, and does much what the Same, as if he ſhould Perſuade a Man to Put out his Eyes, the better to Receive the Remote Light of an Inviſible Star by a Teleſcope.

Is therefore Reaſon Natural Revelation? Which Way? How is it Proved? We are Ready to Own, and to Evince, that a True Knowledge of Nature is One Revelation of Almighty God to us, and that ſuch a Revelation is in ſome Degree Correſpondent to that, which we have from the Chriſtian Religion; that is, as the Firſt gives us an Impreſſion of an Almighty Being, ſo the Laſt Advances that Notion to the utmoſt Heighth and Perfection; But a Real Knowledge of Nature cannot be Meant by Reaſon in this Philoſophy, ſince it is not Acquainted with one Syllable of it, which cannot Tell us what the Nature of the Mind or of Matter is, nor even Furniſh us with any Account of the Properties of either, whether of Solidity, Colour, or Indeed any Affections of Bodies in the one Caſe, or of Willing, Judging, Remembering, or Thinking in the other, unleſs that in Matter of Fact, we have all theſe Perceptions; but which Way, or in what Manner we come by them, it is as Ignorant, as if it had never Conſidered one Word of Either.

The True Knowledge of Nature cannot therefore be Meant by *Reaſon* here, ſince the Preſent Philoſophy of Ideas Knows nothing of it; we are by Conſequence next to Inquire, what it does Mean or Intend by *Reaſon*, which is Manifeſt from the whole Tenor of it, Namely, the *Connexion* of *Ideas*; we have already Shewn, it does not Underſtand, what the *Connexion* of *Ideas* is, or if it did, that there is no Compariſon betwixt the Evidences from the Modern *Connexion* of *Ideas*, and the Evidences of *Matter* of *Fact*, by which the Chriſtian Religion or *Revelation* is Proved; which Evidences the Author of this Philoſophy of Ideas did not give himſelf the Trouble to Inquire into.

Let it yet be, as this Eſſay Suppoſes, that *Reaſon* Conſiſts in the *Connexion* of *Ideas*, and let *Revelation* be, what we Actually Find in the Scriptures, that our Bleſſed Saviour was Born of a Virgin, that he Died for the Redemption of Mankind, that he Roſe again, and Aſcended into Heaven, which is the Leaſt, that the Scriptures Recount of him, beſides his being Really Man, and Truly God, how is *Reaſon* Natural *Revelation*, or the Contrary? Unleſs all this Matter of Fact can be Reduced to the *Connexion* of *Ideas*, that is, unleſs we can be as Fully Aſſured, that our Saviour was Born of a Virgin, *&c.* by the *Connexion* of *Ideas*, as we can be, that the Three Angles of a Triangle are Equal to Two Right Ones, Suppoſing that Propoſition Abſolutely True, which is ſo only upon Certain Abſtractions and Limitations.

It is Notwithſtanding Affirmed, that *Reaſon* is Natural *Revelation*, whereby the Eternal Father of Light, and Fountain of all Knowledge, Communicates to Mankind that Portion of Truth, which he has Laid within the Reach of their Natural Faculties: What *Reaſon*, and what *Revelation* is, we have already Endeavoured to Explain, and if that Explanation is Juſt, the Senſe of the Preſent Philoſophy Amounts to This; That *Reaſon*, or the *Connexion* of the *Ideas* of our Minds, is Natural *Chriſtianity*, which is Abſurd, or if not, Depends upon *Fact*, and therefore is not, nor can be, Exacted to the *Connexion* of *Ideas*.

If by *Revelation* is Meant, not the *Chriſtian Religion*, but *Inſpired Reaſon*, and that Portion of it, which Almighty God has Alloted to every Man; we cannot be of the Opinion, that the Eternal Father of Light, and Fountain of all Knowledge, as this Philoſopher is Pleaſed here to Term Almighty God, does Communicate this Reaſon to us by an Immediate Influence or Inſpiration.

First, Becauſe this would be Aſſerting a Philoſophick Enthuſiaſm, whilſt it is Declaiming againſt a Religious One, and Patronizing a *Light within* Derived from the Eternal Father of Light, whilſt it Endeavours to Confute ſuch Notions.

Second, Becauſe, according to the Notions of this Author, Inſpirations can only belong to Religion in thoſe Caſes, where Reaſon cannot Aſſiſt us; For, As this Philoſophy Argues in another Place againſt the Idea of God being Innate or Inſpired, from the Ability there is in every One, by his Rational Faculties to Collect ſuch an Idea, and Tells us, that we may as well Affirm Colours to be Innate, tho' we have Eyes given us to Receive thoſe Ideas, which would be, upon that Suppoſition, of no Uſe to us; So we may as Juſtly Say, that what Lies *within the Reach of our Rational Faculties*, cannot be Thought or Imagined to be Inſpired, or Derived into our Minds by the *Eternal Father of Light* from any Immediate Impreſſion, ſince there can be no more Reaſon Aſſigned, why we may not Collect or Infer what Lies within the *Reach* of our *Natural Faculties*, without an Inſpiration from Almighty God, than why we may not have an Idea of him from our Reaſon, or a Perception of Colours from our Sight, without any ſuch Inſpiration, which the Preſent Philoſophy Owns we may.

Third, In General to Think of an *Inſpired Reaſon*, becauſe Almighty God Endues us with our Rational Faculties, is a moſt Egregious Abſurdity; ſince by the ſame Argument, our Eyes, Legs, and Hands are Inſpired, all which we Owe to the Great and Wiſe Framer of them.

Moreover, if Bare *Reaſon* was an *Inſpiration* from Almighty God, *Ariſtotle*, *Tully* in his Philoſophical Tracts, and even *Demoſthenes* in his Orations, which are Certainly Writ with the Greateſt Force of Reaſon and Senſe, that it is Poſſible for a Lover and a Patriot of his Country to Poſſeſs againſt all the Factions Raiſed by *Philip* to it's Ruin, Theſe, and a Thouſand more, at this Rate, muſt be Inſpired.

Fourth, If this is Granted, all the ſeveral Diſputes Recounted by *Cicero* in his Book *De Finibus Bonorum & Malorum*, Concerning the *Summum Bonum*, or the Chief End, which a Man, or a Rational Creature, ought to Purſue in his Actions, and the Chief Evil, which he ought to Avoid, will be all Inſpirations, tho' they are as Contradictory to each Other, as any Sentiments can well be, and Almighty God will be Introduced, as an Inſpirer of theſe Contradictions.

Lastly, if *Reaſon*, or the *Connexion* of *Ideas*, is an *Inſpiration* from Almighty God, this Philoſopher ſeems to have had as Small a *Portion* of that Inſpiration, as any Philoſopher who has Writ, and much Leſs than either *Socrates*, *Plato*, *Ariſtotle*, or *Tully*; Does not all the World Know, that in Matters, which are Capable of the Stricteſt Demonſtration, we are Deceived? Will Almighty God therefore Seduce us into Error, which he muſt do, if Reaſon is an Inſpiration from him? Did even *Euclid*, or *Archimedes*, or *Apollonius*, or Sir *Iſaac Newton* Demonſtrate, what they Think they have, by Inſpiration, or by the Natural Force of their Minds? If the Firſt, we ſhall not for the Future Look upon theſe Geometers, as Plain Men, but as Inſpired, which will be a greater Conceſſion, than what has been hitherto Allowed them, and which if we Grant to Them, we are not yet Willing to Give, however *Deſirous* he may be of it, to One, who is even an Irrational Being in the Compariſon; If the Laſt is Affirmed, that their Demonſtrations were Owing to the Force of their own Minds, that is, if *Euclid*, *Archimedes*, *Apollonius*, or Sir *Iſaac Newton*, were not Inſpired, when they Writ, we may with the greateſt Eaſe and Facility be Led to Think, that Mr. *Locke* was not.

Letter to Mr. Molyneux.

It has been hitherto Affirmed, that *Reaſon* is Natural *Revelation*, which has been Conſidered, the next Aſſertion is, that *Revelation* is Natural *Reaſon*, Enlarged by a New Set of Diſcoveries, Communicated to (or rather) by God Immediately, which Reaſon Vouches the Truth of by the Teſtimony and Proofs it Gives, that they come from God.

To which it is Anſwered, that, if by *Revelation* is Meant *Reaſon Inſpired*, that has been already Sufficiently Confuted, or if it were not, it would only come to this, that *Reaſon Inſpired* was *Natural Reaſon*, which is a Contradiction; if by *Revelation* the *Chriſtian Religion* is Intended, and is Said to be *Natural Reaſon*, Enlarged by a New Set of Diſcoveries Communicated by God Immediately, which Reaſon Vouches the Truth of by the Teſtimony and Proofs it Gives, that they come from God, there are ſeveral Things to be Objected.

First, that the *Chriſtian* Religion is not *Natural* Reaſon or Religion, Enlarged by a New Set of Diſcoveries, unleſs this Philoſophy, or any other, can Tell us what *Natural* Religion is, which it is Impoſſible it ſhould do, whilſt it neither Knows the Nature of the Mind, or of Matter, and Conſequently cannot Inform us, what the Real Principles of Natural Religion are; if not, it is Evident, that Chriſtianity cannot be Said to be an Improvement of That, of which this Philoſophy is Entirely Ignorant.

Second, If Natural Religion was Known and Aſcertained, there would be yet no Connexion betwixt it, and the Myſteries and Facts of the Chriſtian Religion, where the Union of the God-Head with our Humanity, the Reſurrection and Aſcenſion of our Lord, and all the Miracles Performed by him, are Confeſſedly above the Powers of Nature, and are the Reſult of the Abſolute Will and Pleaſure of God.

Third, This *New Set of Diſcoveries*, (as a *Revelation* of the *Chriſtian Religion* from God is with Great Modeſty Called by this Philoſophy,) is not Vouched by Reaſon; 'tis True, we have Abundant Evidences to Convince us of the Truth of Chriſtianity, from Hiſtory and Fact, but ſuch Evidences, as we have Shewn, are Perfectly Different from thoſe, which *Reaſon* Vouches, or, which is the Same, the *Connexion* of the *Ideas* of our own Minds; that is, Chriſtianity is no Rational Science, to which the Writer of this Eſſay would in his Abundant Wiſdom Reduce it, unleſs to be Born of a Virgin, to be a Man and to be United to God, to Work Miracles, Riſe from the Dead, and Aſcend into Heaven, are all Rational and Agreeable to the Common Courſe and Tenor of Things.

Lastly, if yet Chriſtianity was a Rational Science, why this Writer in Particular ſhould Endeavour to make it One, is Unaccountable, who ſeems to Underſtand as Little, what *Reaſon*, as what *Religion* is; Had he been Maſter Indeed of the Utmoſt Force and Accuracy of Thinking, had he been Acquainted with the Foundations and Principles of all the Rational Sciences, of Mathematicks, and in them of Fluxions, Algebra, and Geometry, of Philoſophy in it's ſeveral Parts and Diſtinctions, of Aſtronomy, and the Laws of Attraction and Gravitation in the Planets, had he been Poſſeſſed of all this Wonderful Knowledge, there might have ſome Reaſon Appeared for his Reducing ſo Rebellious a Doctrine to all the Rational Sciences of this World, as that of Chriſtianity, it ſeems is, to his Own; but when it is not Evident, that he Underſtood any Thing of this, and Knew, Perhaps, Little more in Mathematicks, than that the Three Angles of a Triangle were Equal to Two Right Ones, or that the Square of the Hypotenuſe was Equal to the Squares of the Sides, when in Philoſophy and Aſtronomy he ſeems to be a Perfect Novice, and in his own Syſteme of Ideas Abſurd thro' the Whole, it is Inconceivable, with what Deſign he ſhould be ſo Fond of Reaſon, which had been ſo Unkind a Friend to him, unleſs that he Undervalued Religion only, becauſe he Underſtood it Leſs.

To Shew us however, that he had ſome Notion of Aſtronomy, we have the Ingenious Compariſon of putting out our Eyes, the better to Receive the Remote Light of an Inviſible Star by a Teleſcope.

See Beginning of this Section.

The Reaſon given by this Philoſophy of the Riſe of Enthuſiaſm, and the Confutation of it. Mr. Locke's Eſſay, § 15. Chap. 19.

§. 6. Having with great Satisfaction of Mind Reduced Chriſtianity, and Obliged it to Say nothing more than what the *Inſpired Reaſon* of this Philoſophy ſhall Dictate to it, it Proceeds to give us a Reaſon of the Riſe of Enthuſiaſm; Which in Short is, that it is an Eaſier Way for Men to Eſtabliſh their Opinions, and Regulate their Conduct, than the Tedious, and not always Succeſsfull, Labour of Strict Reaſoning; Hence it is Said, that we See in all Ages Men, in whom Melancholy has Mixed with Devotion, or whoſe Conceit of themſelves has Raiſed them into an Opinion of a greater Familiarity with God, have often Flattered themſelves with a Perſuaſion of an Immediate Intercourſe with the Deity; It is Farther Told us, that God cannot be Denied to be Able to Enlighten the Underſtanding by a Ray Darted into the

Mind Immediately from the Fountain of Light; This they, that is, theſe Religious People, Underſtand he has Promiſed to do, and who then, it is Said, have ſo Good a Title to Expect it, as thoſe, who are Peculiarly Choſe by him, and Depend on him?

But in the Firſt Place, the Riſe of Enthuſiaſm cannot be Owing to Men's Lazineſs, or their Unwillingneſs to take Pains in Conſidering, nor yet to a Deſire of Eſtabliſhing Opinions for Authentick and Sacred; This Indeed is the True and Juſt Character of the Preſent Philoſophy, which by it's own Confeſſion, was Compoſed Oſcitantly and Lazily, at Leiſure Hours, and for the Diverſion of the Author; and when it was Conſcious, that it had too Little of Truth and Diligence in it's Inquiries to Recommend it, it took Care to Eſtabliſh the Opinions it Affirmed by a Pretenſe, that they were Rational Inſpirations, and Proceeded from an Underſtanding, Enlightened by a Ray Darted into the Mind Immediately from the Fountain of Light; Notwithſtanding, that, whoever Looks over this Eſſay with any Kind of Impartiallity, will Find, if Things were ſo, Almighty God would Appear to be but a Fallible Inſpirer; The Riſe of Enthuſiaſm is therefore made to be only Agreeable to that Picture, this Philoſopher had in his Mind of his own Enthuſiaſm, and which Suppoſes Men not to Lie under any Convictions of their being Inſpired, but to Pretend it for their Eaſe and Authority, which is much the Caſe with this Eſſay.

Since therefore the Riſe of a Real Enthuſiaſm cannot be Owing to that, which is an Affected One, and in Pretenſe, for ſuch an Enthuſiaſm is only the Reſult of Cunning, Artifice, or Deſign, and is neither Agreeable to any True Convictions in Religion or Philoſophy, both which we Hope it is Poſſible for Men to have, however the Preſent Philoſopher might be Unacquainted with them; If then this cannot be a Cauſe of Enthuſiaſm, Seeing Enthuſiaſm is as Settled a Conviction, and as Firm a Perſuaſion of God's Reall Influence upon the Mind, as any other, which Proceeds from the Evidences of Reaſon; for thoſe, who Take themſelves to be thus Inſpired by God, are as Certain, and as Fully Aſſured in their own Breaſts, that they are ſo, as that they Think any Thing is True, and it is as Firm a Principle, upon which they Act, and Shape the Conduct of their Lives, as that of Reaſon is to other Men; They are Ready to Die in, and for the Sake of, theſe Convictions, and Conſequently, ſuch Notions are Derived from another Source, than what this Philoſophy Apprehends, of Deſign or Affectation; Since this therefore is in Fact and Experience ſo, it will become us to Inquire, with Greater Care and Exactneſs, than has been here done, from whence it is, and what is the Nature of it.

The Nature of Enthuſiaſm Conſidered, and Confuted. §. 6. and 7. Chap. 19.

As to the Nature of Enthuſiaſm, this Philoſophy is Equally to Seek, whilſt it only makes it to Conſiſt in any Groundleſs Opinions, which come to Settle themſelves Strongly upon our Fancies, and from thence is Termed an Illumination from the Spirit of God, or in any Odd Action we find in our ſelves a Strong Inclination to do, the Impulſe to which is Reſolved into a Supernatural Call or Direction.

This is Said to be Properly Enthuſiaſm, which, tho' Founded neither on Reaſon, nor Divine Revelation, but Riſing from the Conceits of a Warmed or Over-Weening Brain, Works yet, where it once gets Footing, more Powerfully on the Perſuaſions and Actions of Men, than either of thoſe Two, or both together, Men being moſt Forwardly Obedient to the Impulſes they Receive from themſelves, and the Whole Man is Sure to Act the moſt Vigorouſly, where the Whole Man is Carried by a Natural Motion; for Strong Conceit, like a New Principle, Carries all Eaſily with it, when got above Common Senſe; and Freed from all Reſtraint of Reaſon and Check of Reflection, it is Heightened into a Divine Authority in Concurrence with our own Temper and Inclination.

We cannot but Obſerve upon this Account of Enthuſiaſm in the Firſt Place, that it is here Owned to Work as Powerfully on the Perſuaſions and Actions of Men, as Reaſon or Revelation, or both put together; Whatever therefore was Said before, Concerning the Riſe of Enthuſiaſm, is Contradicted in the Preſent Account of it; For, if Men Act in Enthuſiaſm upon as full Perſuaſions and Convictions as they do in Reaſon and Revelation, Enthuſiaſm cannot Proceed from any Artifice or Cunning in the Enthuſiaſtick Perſon, by which he may Hope to Eſtabliſh the Authority of his Opinions, with the Greater Eaſe and Facility, and without any Laborious Inquiry into Things.

SECOND, we may Affirm, that if Enthuſiaſm is neither Founded on Reaſon nor Revelation, the Firſt Containing the Principles of the Preſent Eſſay, and the Laſt thoſe, we Hope, of Chriſtianity, it is Evident, there is ſomething in Enthuſiaſm, which is not Reducible to a Philoſophy of Ideas, which Depends upon thoſe of Senſation, or Reflection only.

THIRD, In the Explaining the Nature of Enthuſiaſm it's ſelf, which we will, if this Philoſophy Pleaſes, or if it will not, Call now by another Name, to Wit, Inſpiration, for what is Meant by Enthuſiaſm, is the Immediate Influx of the Deity, and the Communications of his Spirit, into the Mind of Man; and therefore, tho' the *Light within*, is in Grimace and Pretenſe only Oppoſed, yet the Real Intention ſeems to be Levelled againſt any Influence of God upon us; In the Explaining therefore the Nature of Enthuſiaſm, there is Abundance of Oratory Uſed, to make us Believe it is only a Heat of Fancy, and a Fulneſs of our ſelves, and upon that Hypotheſis, the Whole Man is Sure to Act, *&c.*

Is therefore Inſpiration in General, or the Influence of God upon the Mind, a Heat of Fancy only, and a Fulneſs of our ſelves? If it is ſo, we are Ready with the Firſt to Subſcribe to the Rational Inſpirations of this Eſſay, and on the Contrary to Acknowledge, that the Grace and Aſſiſtances of the Spirit of God, which all Good Men have hitherto Owned, is a Wild and Enthuſiaſtick Pretenſe; that our Publick Prayers to the Almighty for his Aid and Direction in our Affairs and Concerns, or for his Influences and Spirit to Aſſiſt us in our Progreſs towards Holineſs and Vertue, are Ridiculous and Vain; and that the Whole Liturgy of the Church, and the Supplications and Addreſſes to God thro' Whole the Chriſtian World, are nothing elſe than Mere Pageantry and Amuſement. For This, in Effect, is the Senſe and Affirmations of the Preſent Philoſophy, which ſeems very Ambitious to Demoliſh all that is Divine, and Great, in Religion, in Order to Eſtabliſh it's own Enthuſiaſm, and Vend it's *New Set* of *Diſcoveries*, under the Specious Character of being an Inſpired Syſteme. Happy Enthuſiaſt! Whoſe Opinions are *Founded neither on* Reaſon, *nor* Divine Revelation, *but Riſing from the Conceits of a Warmed or Over-Weening Brain, Work yet more Powerfully on the Perſuaſions of this* Gentleman, *than either of* Thoſe Two, *or* Both together, *for Strong Conceit, like a New Principle, Carries all Eaſily with it, when got above* Common Senſe; *it is Heightened into a* Divine Authority, *in Concurrence with his own* Temper and Inclination.

BUT Suppoſing That, which is Generally Taken for Grace, Inſpiration, or the Influence of God upon Human Minds, is only a Heat of Fancy, or Fullneſs of our ſelves; which Way is it Evidenced or Proved? For ſo Important a Point, as this, would Deſerve ſome Sort of Connexion of Ideas, or other, to Explain it to us; Eſpecially, ſince this Philoſophy has ſo Entire a Command of Uniting, Compounding, Abſtracting, and Connecting, theſe Imaginary Pictures in the Mind at Pleaſure, and yet there is not one Idea Connected to Aſſure us of it; and the only Method which is Taken to *Convince* the *Thing* is, by *Naming* it *Enthuſiaſm*; For by that Means it is Thought, we may be Eaſily brought to Acknowledge it to be nothing elſe, but the Fancies and Illuſions of our own Minds; And from whence, as before, it is Evident, that the Chief Artifice of this Philoſophy is Placed, in Playing with Terms, and Abuſing us by the ſame Names Affixed to Things of a Different Nature; For, if an Influence, or an Immediate Inſpiration, from God is Enthuſiaſm, and That a Wild Fancy of our own Begetting, an Influence from Almighty God is ſo; But, if a Real Influence from the Deity is not Enthuſiaſm, or if Enthuſiaſm is not an Extravagant Fancy, all which it has hitherto Offered, is to no Purpoſe; and yet, theſe ſeveral Terms Paſs under one General Name of Enthuſiaſm, without the Leaſt Evidence to Prove they are the Same.

This Doctrine of Enthuſiaſm farther Conſidered and Refuted.
§. 8. Chap. 19. Mr. Locke's Eſſay,

§. 7. THIS Philoſophy Proceeds; Tho' the Odd Opinions and Extravagant Actions, Enthuſiaſm has Run Men into, were enough to Warn them againſt this Wrong Principle, ſo Apt to Miſguide them both in their Belief and Conduct, yet the Love of ſomething Extraordinary, the Eaſe and Glory it is to be Inſpired, and be above the Common and Natural Ways of Knowledge, ſo Flatters many Men's Lazineſs, Ignorance, and Vanity, that when Once they are Got into this Way of Immediate Revelation, of Illumination without Search, and of Certainty without Proof and without Examination, 'tis a Hard Matter to get them out of it, Reaſon is Loſt upon them, they are above it, they See the Light Infuſed into their Underſtandings, and cannot be Miſtaken; 'tis Clear and Viſible there, like the Light of Bright Sun-Shine, ſhews it's ſelf, and Needs no other Proof, but it's own Evidence, they Feel the Hand of God Moving them within, and the Impulſes of the Spirit, and cannot be Miſtaken in what they Feel; this Light from Heaven is Strong, Clear, and Pure, Carries it's own Demonſtration with it, and we may as Rationally take a Glow Worm to Aſſiſt us to Diſcover the Sun, as to Examine the Celeſtial Ray by our Dim Candle, Reaſon; This is the Way of Talking of theſe Men; They are Sure, becauſe they are Sure, and their Perſuaſions are Right, only becauſe they are Strong in them.

We are therefore here to Reply as before, that by Enthuſiaſm is Meant, either a Heat of Fancy and Fullneſs of our ſelves, by which we Conceive, we are Inſpired, or elſe the Immediate Influx of God's Spirit upon our Minds; If the Firſt, who does not Know, that Conceit, and Heat of Fancy, has Led Men into a Thouſand Unaccountable Whimſies? Who that Reads this Romantick and Imaginary Syſteme of Ideas without Prejudice, will not Readily Own it? Who will Deny that the Love of ſomething Extraordinary, the Eaſe and Glory it is to have our *Reaſon Inſpired*, and to be above the *Common and Natural Ways of Knowledge and Conſidering Things as they are*, may ſo Flatter ſome Men's *Ignorance*, *Lazineſs*, *and Vanity*, that when Once they are Got into this Way, it will be a Hard Matter to Get them out of it? Who will not Confeſs, that *True Reaſon* is Loſt upon ſuch Men? They are above it, they See the *Creatures* of their own *Imaginations* Clearly and Diſtinctly, and the Various *Connexions* of them; Their *Arbitrary Abſtractions*, and the *Images* they Frame in their own Minds, and which they Shuffle and Combine as they Pleaſe, are Strong, Clear, and Diſtinct, and Carry their own Demonſtration with them, and we may as Rationally Take a Glow Worm to Aſſiſt us to Diſcover the Sun, as to Examine ſuch *Elevated Fancies* by the *Common Rules* of Plain Thinking, according to that which Simple Nature Dictates to us; *This is the Way of Talking of theſe Men*; They are Sure of their Abſtracted Notions, and their Connexions of them, becauſe they are Sure, and their Perſuaſions are *Right*, only becauſe they are *Strong* in them; And is not this I would Ask, a moſt Refined and Accompliſhed Way of Arguing, which, Changing the Terms of Enthuſiaſm, Heat of Fancy, Conceit, *&c.* for thoſe of Reaſon, Connexion, and Compounding and Abſtracting of Ideas, is Equally Valid and Concluſive?

If Enthuſiaſm is not Conceit, Heat of Fancy, and the Like, but the Real Influences of the Deity upon the Mind, the Section Alledged from the Preſent Philoſophy will Evidently Take this Turn, which is not only Impious and Blaſphemous, (a Trifle it may be to ſome of the Patrons of this Philoſophy, if not to the Author of it) but, which they will more Exceedingly Reſent, Abſurd and Irrational; Namely, tho' the Odd Opinions and Extravagant Actions, the Influence, Grace, or Inſpiration of God has Run Men into, were enough to Warn them againſt this Principle, ſo Apt to Miſguide them, both in their Belief and Conduct, yet the Love of ſomething Extraordinary, the Eaſe and Glory it is to be Inſpired, ſo Flatters many Men's Ignorance, Lazineſs, and Vanity, that when once they are Got into this Way of Immediate Revelation, that is, when they are once Really Influenced or Inſpired by God, Reaſon is Loſt upon them, they are above it, they See the Light Infuſed into their Underſtandings, they Feel the Hand of God Moving them within, and the Impulſes of the Spirit, and cannot be Miſtaken in what they Feel.

What *Chriſtian*, *Mahometan*, *Jew*, or even *Heathen*, unleſs an *Atheiſt*, would not Bluſh, not only at the Writing, but almoſt at the Recital of ſuch a Paragraph, which yet ſeems to be the Ultimate Meaning of this Philoſophy.

As to the Blaſphemy and Impiety of it, what can be more Atheiſtical and Profane, than to Tell us, that the Real Inſpiration of God, or the Divine Grace, is a Principle Apt to Miſguide us? Let God be True, may we not Juſtly Say in this Caſe, and every Man a Liar? For, if there is any God, or if this Philoſophy will Really Own One, Truth and Veracity is an Attribute as Eſſential to him, as his Wiſdom or his Power, or as Error and Falſehood to Human Nature, or to this *Gentleman*; Can therefore Veracity it's ſelf Miſguide us? It is both Impious and Abſurd to Affirm it.

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A Second Impiety is the Ridiculous way of Repreſenting ſuch an Operation of God upon Men's Minds, by the Eaſe and Glory of being Ignorant and Inſpired, by Feeling the *Hand* of God Moving them *Within*, and Seeing the *Light Infuſed* into their Underſtandings; For, if there is any Real Grace of God, by which Good Men are upon their Prayers to him Influenced and Directed, this Philoſopher might have Treated ſo Sacred an Emanation of the Deity with a Greater Deference and Regard than he has done, which every one who Reads him, and Conſiders the Air and Stile of his Writing, will find he has, on the Contrary, Endeavoured to Expoſe to Common Contempt and Ridicule, according to what he afterwards Modeſtly in Effect Tells us, that this Light, that is, any Influx from the Deity upon our Minds, is an *Ignis Fatuus*, which Leads us Round in this Circle; It is a Revelation, or it is the Grace and Influence of God, becauſe we Apprehend it to be ſo, and we Apprehend it to be ſo, becauſe it is a Revelation from him.

But, beſides the Atheiſm of what Foregoes, let us now Inquire into the Senſe, and the Connexion of Ideas, which it Alledges, which Fails as we have before Obſerved, in making a Being of Infinite Veracity, Capable of Deceiving us; In the Second Place, tho' Almighty God by his Influences could Deceive us, and this Philoſopher could make an *Ignis Fatuus* of them, how is Eaſe and Glory, Ignorance, Vanity, and *Inſpiration*, Joined with the Idea of an Almighty Being Deceiving us? If he does it by his own Power, and by his Arbitrary Influences upon our Minds, there is neither Eaſe, nor Glory, nor Ignorance, nor Vanity, which has any Thing to do in the Queſtion; Becauſe, upon this Suppoſition, Almighty God may do what he Pleaſes with us, and the Conſulting our own Eaſe or Glory, our being Ignorant or Vain, will Signify nothing, whilſt we are only Paſſive to the Divine Influx.

How Enthuſiaſm may be Diſcovered, according to the Senſe of this Philoſophy, with it's Refutation.
Mr. Locke's Eſſay, §. 10. Chap. 19.

§. 8. This Philoſophy goes on with Enthuſiaſm, and has a Mind to Declare to us, how it may be Diſcovered; 'Tis Said, there is an Internal Light or Impulſe, becauſe Men See or Feel it; But it is Asked, whether this Seeing is the Perception of the Truth of the Propoſition, or of This, that it is a Revelation from God; as alſo, whether this Feeling is a Perception of an Inclination or Fancy to do ſomething, or of the Spirit of God Moving that Inclination, which it is Told us, ought to be Carefully Diſtinguiſhed; Since we may Perceive the Truth of a Propoſition, and yet not Perceive, that, it is an Immediate Revelation from God; Nay we may Perceive, that we came not by this Knowledge in a Natural Way, and ſo may Conclude it Revealed, without Perceiving, that it is a Revelation from God; Becauſe there are Spirits, which, without being Divinely Commiſſioned, may Excite thoſe Ideas in us, and Lay them in ſuch Order before our Minds, that we may Perceive their Connexion, ſo that the Knowledge of any Propoſition coming into our Minds, we Know not how, is not a Perception, that it is from God; Much Leſs is a Strong Perſuaſion, that it is True, a Perception, that it is from God, or that it is ſo much, as True.

Besides, if a Propoſition is True, it will be Certain from it's own Evidence, and therefore Revelation will be Superfluous; and if a Propoſition is not Certainly True, but only what we are Perſuaded is ſo, we do not See, but Believe, which muſt therefore Depend upon Teſtimony; What Teſtimony therefore have we, it is Asked, that God did Reveal any Thing to us, by an Immediate Influence upon our Minds? If we Know not this, we Know nothing; how great ſoever the Aſſurance is, that we are Poſſeſſed with, it is Groundleſs; and whatever Light we Pretend to, it is but Enthuſiaſm; it is an *Ignis Fatuus*, which Leads us Continually Round in this Circle, it is a Revelation, becauſe we Firmly Believe it, and we Believe it, becauſe it is a Revelation. What has been Said of Light in the Mind, and our Seeing of it, may be Applied to Impulſe, and our Feeling of it.

Upon what has been now Propoſed to us from this Philoſophy, we may Obſerve, that Revelation is not Pretended to be Impoſſible; ſince it is Said, there are Spirits, which, without being Divinely Commiſſioned from God, may Reveal Things to us; which if Granted, it will be moſt Certain, that all our Ideas do not Proceed from Senſation and Reflection, becauſe they Poſſibly may be Derived to us from ſome Superior Beings, and Conſequently, this whole Philoſophy is Falſe; If it is not Granted, the Mention of it is only to Evade a Revelation from God, without it's Believing, that any Revelation can be made even from Superior Spirits, at the ſame

Time that it Affirms it, which is, to Say no Worſe of it, a moſt Notorious Hypocriſy and Prevarication.

Be this yet as it will, how can we Diſcover, whether we have any Influences from the Deity or not, by our being Certain of ſeveral Truths and Propoſitions without him? No one ever Affirmed, that *Euclid*, or *Archimedes*, or *Apollonius*, were Inſpired, and yet all this Learned and Wiſe Reaſoning againſt any Influences of God upon Human Minds, is Reſolved into this, that we may Perceive the Truth of a Propoſition in Mathematicks, without an Inſpiration from him; which no one has hitherto Denied, Excepting, that this *Gentleman's* Mathematicks, which do not Reach Far, Namely, that the Three Angles of a Triangle are Equal to Two Right Ones, have not all the Truth in them, which *his* Connexion of Ideas Suppoſes.

On the Contrary, tho' we do not Acknowledge the Mathematicians, and much Leſs this Philoſopher of Ideas, to be Inſpired, yet we cannot but Think, the Firſt Preachers of Chriſtianity were, and that every Good Chriſtian has the Aſſiſtances of the Almighty, is Endued with his Grace, and Partakes of the Influences of the Divine Spirit, as St. *Paul*, St. *Peter*, and the reſt of the Apoſtles, and in this Laſt Age, the Great Authors of our Reformation, beſides all thoſe Excellent Men in the Church of *Rome*, in our Own, and amongſt our Diſſenting Brethren; All whoſe Piety and Goodneſs are Sufficient Proofs, that they do not Derive their Affections, or Ideas, from themſelves, but from God.

We cannot therefore Imagine, tho' we have Conſidered all Ways to find any other Senſe of this Philoſophy, what Farther it can Mean, unleſs, that if *Euclid*, *Archimedes*, or *Apollonius*, were not Inſpired, it is not Certain, that St. *Paul*, or St. *Peter*, or St. *John* were, that is, that becauſe Mathematicks are True, Religion is Falſe; whereas, I Hope it has been Proved in the Demonſtration of Religion, and in the Principles of Philoſophy, and the Geometry of Solids, that Religion is True, and Mathematicks in one Senſe are Falſe.

[illegible] Under-ſtanding may be Diſcovered, according to the Senſe of this Philoſophy, with it's Aſſertion. Mr. Locke's Eſſay, §. 10. Chap. 19.

If any Thing Farther is Intended, it is this, that, tho' we Know a Propoſition is True, we are not Sure it is Revealed, which comes to the Same with That, which we have already Mentioned, that no One will Affirm, that *Euclid*, *Archimedes*, or *Apollonius* were Inſpired; Is it therefore Certain, as was before Said, that St. *Paul*, St. *Peter*, or St. *John*, were not? Or is it any ways Evident, that becauſe, what we can Fully Comprehend by our Reaſon, we do not want any Revelation for, that therefore there is nothing which is Superior to our Reaſon, and which we do want a Revelation for? May we not at Leaſt with as Great Truth Affirm, that Revelation is Above, and Superior to Reaſon, as this *Gentleman*, that Reaſon is Superior to Revelation, by his Endeavouring to Reduce Revelation to it?

Lastly, It may ſtill be Inquired, how we Know, that the Apoſtles, or any Good Men ſince, have had the Influences of God upon their Minds, if it cannot be Diſcovered this Way? To which we ſhall only at Preſent Firſt Anſwer, that, how the Divine Influences are Performed upon the Mind, is a Secret, to which this Searcher into Human Underſtanding has not Arrived, as Perhaps he never had the Happineſs to Experience them; That does not yet Hinder, but that in Fact Men have been Inſpired, and have Intimately Communicated of the Divine Eſſence, as the Apoſtles and Diſciples of our Lord, whom we Mentioned; Second, that the Truth of a Propoſition, which is Collected and Inferred from the Connexion of Ideas, cannot be a Teſt of Revelation, let it come into the Mind by what Way ſoever we Pleaſe; Becauſe Revelation, if there is any ſuch Thing, does Furniſh us according to the Confeſſions of this Philoſophy, with a *New Set of Diſcoveries*, and which therefore are not Reducible to what the bare Faculties of the Mind can Reveal to us.

The only Queſtion therefore which Remains, is, whether Almighty God does Impart himſelf to Human Minds or not, and by what Methods; Neither of which is Entered into by this Philoſophy, which only Tells us, that Truth in it's Senſe of it, or the Connexion of Ideas, does not want any Inſpiration; which, as it is Certain it does not, becauſe what we do Arbitrarily of our ſelves, cannot be an Impreſſion from God, ſo this Philoſophy, which is a Syſteme of Ideas of it's own Making and Compoſition, cannot Proceed from any *Inſpired* Reaſon; And by the ſame Argument, that our own Arbitrary Compoſitions are not an Impreſſion from God, it will be Concluded, that Revelation, or the Will and Impreſſion of God, is not Reaſon, or the Acts and Volitions of our own Minds, upon which our Abſtractions of Ideas, and the Connexion of them, Depend, and Conſequently our Reaſon, which is Formed from Both.

Mr. Locke's Eſſay, §. 11. Chap. 19.

After this Deſcant upon the Diſcovery of Enthuſiaſm, which Signifies nothing to the Matter, ſince it Affirms, in Truth, no more than that *Euclid*, or *Archimedes*, or *Apollonius*, were not Inſpired, which no One ever Imagined they were, we are at Length Told, that the whole Queſtion is Reſolved into This, how it ſhall be Known, that any Propoſition in our Minds is a Truth Infuſed by God, that is, how we are Certain it is a Revelation.

But may we not Ask this Philoſopher of Ideas, what is Truth? For, if it is That, which is Thought to be ſo by a Connexion of Ideas, or a Mathematick and Abſtracted Reaſoning, Contrived and Formed in our own Minds, there is no Queſtion at all, whether it comes from God, or how we Know it does, becauſe it is Maniſeſtly Derived from our ſelves; and we may as Juſtly Say, that we Walk, or Eat, or Drink by Inſpiration, as that we Reaſon in this Senſe of it; Since the One is only the Action of our Bodies, or of our Senſes, and the other of our Minds.

If therefore Reaſon, or the Connexion of the Ideas of our own Minds, is this Truth, as it is Perfectly Human, what is Divine and is Suppoſed to Proceed from God, cannot be Exacted to it, unleſs we will Affirm, that the Thoughts of God are our Thoughts, and his Ways, our Ways; The very Notion of a Deity is that of a Being, Infinitely Different from That of our Minds, and for that Reaſon, if there is any Influx of God upon us, the Thoughts and Conceptions, with which he Inſpires us, muſt be Suitable to the Nature of the Being which Inſpires, and Conſequently, cannot be Reduced to the Perceptions of the Being, which is Inſpired, or to any Connexion of them; Becauſe this would be to make Almighty God in no Reſpect Different from our ſelves; And as the Revelation cannot be Submitted to our Rules or Methods of Thinking, without the Greateſt Abſurdity, ſo neither can we come to Know, whether any Thing is Revelation, by the Help of our Reaſon, not even in Chriſtianity it's Self; For we have already Shewn, that the Scriptures, and the Inſpirations therein Contained, do not Depend upon the Evidences from Reaſon, or what is always Meant by it in this Philoſophy, the Connexion of Ideas, which is only Employed in the Compariſon of Abſtracted Ones, and is therefore an Evidence Peculiar to Mathematicks and Philoſophy, or to thoſe Sciences, which are Properly Termed Rational, and which is Perfectly Diſtinct from the Convictions, which the Mind Receives from Teſtimony, and Matter of Fact, according to what has been at Large Evinced, and on which the Evidences of Chriſtianity are Founded.

The Problem Reſolved, which has Occaſioned ſo much Difficulty to this Philoſophy, how we may be Certain, that we have an Immediate Revelation from God.

§. 9. It will be therefore Laſtly Inquired, which way we may be Certain, that we have an Immediate Revelation from God, ſince it cannot be Known by Conſidering the Rationalneſs, or the Truth, of the Propoſition Revealed; Nor yet can we Say, that we are Inſpired by God from any Strong Impulſe, which we find in our ſelves, or from any Firm and Settled Perſuaſion, which we have, that we are ſo; For every Fancy, ſo far is True, every Wild and Extravagant Impetus of the Mind might, if ſo, be Deemed an Inſpiration.

In Anſwer to which Difficulty, we are to Take Notice, that an Influx of the Deity upon Men's Minds is Confeſſed to be Poſſible, that it has in Fact been, that the Apoſtles of our Lord and the ſeveral Prophets of Old were Inſpired, and all the Queſtion is, how they, or any Good and Pious Men ſince, ſhould be Certain of it; If this were not Owned, we might Shew, that the Influences of the Deity upon us are no Ways Inconceivable from what we have Alledged in the Demonſtration of the Chriſtian Religion, in Reſpect of the Divine Afflatus, which we may as Fully Apprehend to be Capable of Impreſſing Notions upon us, and Raiſing or Exciting Ideas in us, as we are Certain, that ſeveral Silent Motions of the Hand or Eye, *&c.* by the Mediation of the Æther, are Productive of Various and Differing Ideas in us; To which we may Add, that there is no Reaſon, which can even be given, why one of our Thoughts Communicates it's ſelf to another, by which they become Mutually Conſcious, which will not Obtain to Prove, that the Deity, if it Pleaſes, may Communicate it's ſelf to us, ſo that we may be Conſcious of his Mind and Intention; Since there is no more Tie, or Neceſſary Connexion, betwixt one Thought, which my Will ſhall Arbitrarily Excite in my Mind, and another, than there is be-

betwixt one Thought, which the Almighty Will and Pleaſure of God ſhall Excite in me, and another, which is Derived from my own Mind, as to the being Conſcious of Either; However this is, it will not be Denied, but that, in Fact, the Deity has Communicated it's ſelf to Men's Minds; If therefore we could not give an Account, how, or in what Manner, this was Performed, it would yet be no greater Objection againſt Inſpirations in Fact, than becauſe we Know not the Reaſon, or Way, by which the Magnet is Attractive of Iron, or Amber of a Hair, or a Glaſs Tube of a Feather, it would therefore be an Objection, whether the Magnet, or Amber, or the Glaſs upon Attrition, have in Fact any ſuch Communicative or Attractive Qualities.

And the ſame may be Said, in another Senſe, Concerning our Certainty of Inſpiration; For, altho we were not Poſſeſſed of any Criterion, by which we could Diſtinguiſh, what was a Revelation from God, and what was not, as the Truth of a Propoſition Viſible from the Connexion of Ideas, or a Firm and Invincible Perſuaſion, are Undoubtedly no ſuch Criterion, it would not from thence Follow, that all Inſpiration, or the Influence of God upon Men's Minds, was nothing elſe but Enthuſiaſm; Since, tho' we could not Tell, how to Diſtinguiſh ſuch Divine Influences, or had no Certain Rule, to which we might Reduce them, they would not be Leſs Real on that Account; Thus a Pious and Good Man, upon his Earneſt Prayers and Addreſſes to Almighty God, may Receive thoſe Returns of his Grace and Spirit from him, which may Enable him to Perform his Duty, with greater Care and Exactneſs, tho' he ſhould not be Able to Deſcribe, how that Aſſiſtance is Communicated to him, or Bring it to any Standard, by which he might Judge of it.

So, in the Rational Sciences, tho' no Criterion of Certainty were Fixed, Truth would not be Leſs it's ſelf, or the Evidences of Things Leſs Convincing; For we may Know, that the ſeveral Propoſitions in *Euclid* are True, upon thoſe Suppoſitions and Abſtractions, which he Aſſumes, whatever the General Meaſure of Truth is, and that 2 and 3 make 5, however Metaphyſical and Speculative our Notions may Prove Concerning the Nature of Truth, and in what it Conſiſts; which Notions may Alter, according to the Different Complexions of Men's Minds, and their Various Acquirements, Studies, or Contemplations; One may Think the Viſible Connexion of Ideas the only Teſt, Another Experimental Knowledge, either from our Senſations or Reflections, *&c.*

It is Sufficient, that the Real Criterion of Truth has been Perhaps as much Diſputed, as a Criterion of Inſpiration can be, and if the One is no Objection againſt Reaſon, the other will be of as Little Force or Importance againſt an Immediate Revelation, I Say an Immediate Revelation, and which I will Venture to take upon me to Defend; For, as we have already Said, tho' this Philoſopher in Pretenſe Argues againſt the *Light within*, which ſome Modern Sectaries Maintain, who ſeem, however Harſh the Compariſon may Appear, to be much the Same to the Primitive Chriſtians, as Apes or Monkeys are to Men, yet it is too Evident and Plain, that the ſame Wiſe Arguments are Alledged to Weaken the Inſpirations of Chriſtianity it's ſelf; Since he at Length Confeſſes, that if even the Apoſtles and Diſciples of our Lord had not ſome Atteſtations to their being Inſpired, beſides their bare Perſuaſions of it, Namely, Miracles, they would have had as little Reaſon to have Depended upon their Deriving any Immediate Influences from God, as any others; But we muſt Say with this *Gentleman's* Leave, that if Men cannot be Satisfied, that they are Influenced by God, unleſs a Miracle is Wrought to Confirm them in the Perſuaſion, there is not One Pious Perſon ſince the Firſt Ages of our Religion, who can be Sure, that he has been Aſſiſted by the Grace of Almighty God, in any of the Actions, or Occurrences, of his Life; his Prayers for it muſt have been Idle and Vain, and the Whole Chriſtian World is under a Groſs Miſtake, to Think, there is any ſuch Grace or Influences of the Deity Derived to him.

Mr. Locke's Eſſay, §. 15. Chap. 19.

Qnakeriſm not Primitive Chriſtianity, but a Burleſque upon it.

We have thereſore thus far Conſidered, what Criterion this Philoſophy Alledges for Inſpiration; Namely, either a Perception in the Mind, that a Propoſition is True, which cannot be a Proof, that it comes from God, or elſe a Strong Inclination and Fancy to Believe, it is an Emanation from him; Neither of which, we Own, can be a Meaſure, by which we may Judge of any Thing Proceeding from a Divine Influence; His Criterion of Reaſon and Truth, to Wit, the Connexion of Ideas, we have alſo before Examined, and Shewn it to be Falſe, and ſince he Plainly Confeſſes, that he is Ignorant of any, by which we can Determine upon Inſpiration, we ſhall Endeavour here to Supply one, tho' it is not, according to what has been Said, any ways Neceſſary for us to do, in Order to Aſcertain the Reality of Inſpiration it's ſelf.

How we are to Diſtinguiſh True Inſpirations from Falſe or Pretended Ones.

'TIS Own'd, that Inſpirations have in Fact been, we have Explained even how it is Poſſible for Inſpirations to be, and what we are now farther to Evince is, the Method, or Way, of Diſtinguiſhing True Inſpirations from Falſe or Pretended Ones.

OF which there only at Preſent ſeem to be Two Ways; either by Conſidering the Nature of that Infinite Being, which is Suppoſed to Inſpire Men; or elſe by Conſidering the Nature of that Religion, which is in Fact Acknowledged to be Inſpired; By the Firſt we may Know, Abſtracted from Miracles, whether the Apoſtles were Inſpired, of which Miracles ſtill are a Collateral Evidence; and by the Laſt, whether Any ſince have been.

THERE is nothing more Rational than to Think, that the Emanations or Influences of Almighty God are Suitable to his own Nature, and if he is a Being of Infinite Knowledge, Goodneſs, Juſtice, and Truth, that what is Derived from him into Others will be of the ſame Form and Reſemblance; Inſpiration therefore cannot Conſiſt either in a Connexion of Ideas, or in an Heat of Fancy, or a Fullneſs of our ſelves, but in the moſt Elevated Notions of Things and a Fore-Knowledge of them, in the utmoſt Love and Goodneſs, in the moſt Exact and Unbiaſſed Juſtice, and in the Greateſt Truth and Sincerity; and where theſe Qualities of the Mind are Reſident, Superior to what we can find in Mere Human Nature, where there is likewiſe ſuch a Syſteme of Knowledge Advanced, which Inculcates all theſe Perfections in the Higheſt Manner, and which all the Wiſdom in this World could never have Arrived to, it Appears to be no Leſs than a Demonſtration, that ſuch Perſons were Inſpired, and did Communicate and Partake of the Divine Eſſence, and of a Real and Certain Influx from it; All which is the Caſe of the Apoſtles and Diſciples of our Lord, according to what has been Evinced in the *Demonſtration* of the *Truth* and *Divinity* of the *Chriſtian Religion.*

MOREOVER, when to theſe Inward Inſpirations of the Mind, we Perceive Miracles to Atteſt them, it is Plain, that at the ſame Time, that it Pleaſed Almighty God to give an Impreſſion, and Strike a Character, as it were, of his Immenſe Wiſdom, Goodneſs, Juſtice, and Truth upon the Inſpired, he alſo Exerted his Infinite Power to Convince the World of it; That there might be no Attribute Wanting, which he did not Fully Explain in that Religion, which he Intended ſhould be the Meaſure of our Lives and Actions; And when farther there is a more than Human Zeal and Fervour, and the utmoſt Energy and Emotion of Soul, which Accompanies ſuch Impreſſions, it is nothing elſe but a Conſequence of the Almighty and Divine Impulſe, which, we may Juſtly Conceive, would Shake and Raiſe all the Faculties of the Mind, and Enforce and Envigorate them to that Degree, which in their own Natural State, they could not Poſſibly Experience; However, as This, without the Other, may be in ſome Meaſure Imitated, ſo we here Mention it, not as a Diſtinguiſhing Mark of Inſpiration, but as the Probable Reſult and Conſequence of it.

FROM this Mark or Character of Inſpiration, we may Know, that the Chriſtian One is the only True and Divine Religion, which has been hitherto Propoſed to Mankind; becauſe the Deity has not made ſo Full and Entire an Impreſſion of it's ſelf upon any Religion as the Chriſtian; For in all others, there is either Cruelty, or Injuſtice, or Folly, or Falſeneſs and Inſincerity, or a Defect of Power, or a Heat of Fancy; Quite Contrary to the Characters we find in the Chriſtian Religion, of the utmoſt Mercy and Clemency in Almighty God, and of Love and Forgiveneſs in us, towards each Other; of the moſt Rigid and Exact Juſtice in Sacrificeing his Son for the Attonement of Sins, and of the Greateſt Truth and Sincerity in the Diſciples of this Religion, ſo far as to Die in the Profeſſion of it; of the Infinite Power of God, in the Working of Miracles, and of a True and Unpaſſionate Zeal and Elevation of Mind in Them, without any *Wild Heats* of *Fancy*, and which was ſuch, as was a Solid Support to the Diſciples of our Lord againſt the Oppoſition of the whole World, amidſt all the Hardſhips and Difficulties, they Encountered.

BUT what is there Like this in *Mahometiſm*, or the *Arian* Scheme, Unleſs Luſt and Ambiton, Turbulency, Cruelty, and Ill-Nature, are the Meek, the Humble, and Juſt Paſſions,

ſions, Derived from Chriſtianity? And wherever we find ſuch Affections, as theſe, Mixed with any Religion, or even with the Natural or Rational One of the Preſent Philoſophy, we may Juſtly Conclude, that it is not Inſpiration, but Enthuſiaſm; becauſe it has a Deep Reliſh of the Violences and Irregularities of Human Nature, but None of the Divine, by which, according to the Foregoing Rule, we ought to Judge of Inſpiration.

THE Second Method of Determining, what is Inſpiration, is by Conſidering the Nature of that Religion, which is Confeſſedly Inſpired, as the Chriſtian Religion is Acknowledged to be, and Knowing, how far thoſe Inſpirations, we Pretend to, are Conformable with it; For, if no Religion can be Inſpired, unleſs the Chriſtian, whatever is Contrary to that Religion, is Impoſſible ſhould be Inſpiration; becauſe Almighty God cannot Inſpire Contradictions; Since therefore Chriſtianity is now a Rule to us, for Judging of Inſpiration, the Criterion is Evident, and every one, who is a Truly Good and Sincere Chriſtian, we may Juſtly Pronounce Inſpired; which may be done Two Ways, either by the Mediate Influences of the Scriptures of our Religion, or from the Immediate Influences of Almighty God himſelf.

As to the Firſt, there can be nothing more Plain and Manifeſt, that if the Scriptures are Inſpired by Almighty God, which even this Rationaliſt will not Diſown, Thoſe, who Take their Notions and Apprehenſions of Things from them, muſt be Inſpired likewiſe; becauſe the *Hagiographa*, or Holy Writings, are an Expreſſion of the Will and Pleaſure of God to us, and therefore the Perſon, who Reads them, and Entirely Conforms his Notions to them, muſt of Neceſſity be Divine and Inſpired alſo, tho' ſuch Notions are Communicated to us by the Mediation of the Scriptures of that Inſpiration.

As to the Second, there is no Difficulty in Apprehending, that if Almighty God can Immediately Communicate himſelf to us, which we have Shewn he may, or which, if we had not, is Acknowledged, and if that Communication of himſelf is only in the Chriſtian Religion, it is not Difficult, I Say, to Apprehend, how he may Inſpire, or Give his Aſſiſtances in that Way, by Earneſt Prayers and Addreſſes to him, which in a Greater Degree he Communicated to the Firſt Chriſtians; So far Removed is the *Grace of God* from *Enthuſiaſm*, or the *Influences of the Deity* from a *Heat of Fancy*, or a *Fullneſs of our Selves*.

IN Concluſion, we cannot but Think and Believe, that, in this Senſe of Inſpiration, and from what Foregoes, that the *Romaniſts*, that the *Mahometans*, and *Arians*, and Thoſe whom they Call *Quakers*, that the *Diſſenters*, and *Anabaptiſts*, and *Independents*, are All in ſome Degree Inſpired; That is, ſo Far as they Agree with the Church of *England*, in the True and Orthodox Chriſtianity, which it Profeſſes; To Wit, That Chriſt was the Saviour of the World, That he was the Son and Meſſiah of God, That he Died,* and was Crucified for the Sins of Mankind; All this even the *Arians* and *Mahometans* do in ſome Meaſure and Senſe Acknowledge; And, as it is Agreeable with Chriſtianity, is a Divine Revelation, or an Inſpiration from God; And What *Arius* or *Mahomet*, and their Followers, the Laſt, as well as the Former of which, was nothing more than a Chriſtian Heretick, Namely, of the Sect of the *Manichees*, What Theſe have Advanced in Contradiction to the Chriſtian Religion is to be Reſigned to the Infirmities, to the Paſſions and Prejudices, of Human Nature, which might Prevail ſo Far, as to Mix themſelves with Revelations from God, and the Operations of the Divine Mind upon Them.

Dr. Prideaux's Life of *Mahomet*.
Dr. *Cave's* Hiſtoria Literaria.
Mr. Wall of Infant Baptiſm.

IF this is ſo, we may Eaſily Conceive, what the Genius and Complexion of the *Light Within* is, which this Philoſophy is ſo Carefull to Examine, which muſt be, in Part, a Revelation or Inſpiration from God, that is, ſo Far as it Agrees with Chriſtianity; and where It or any other Hereſy Departs from it, it is not an Argument againſt the Truth of the Revelation, or Inſpiration from God, which the Chriſtian Religion Beſtows, but is an Inſtance and Conviction of the Frowardneſs, the Obſtinacy, and Perverſeneſs, of our Minds or our Wills, which are not ſo Paſſive and Obedient, as they ſhould be, to the Divine Impreſſions,

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[illegible] Notions and Apprehenſions of Things [illegible] Reaſon of God [illegible] Notions of [illegible] by the Meditation of the [illegible]

[illegible] Communication of himſelf [illegible] Chriſtian Religion [illegible] Aſſiſtance [illegible] Greater Degree [illegible] Communicated to the firſt Chriſtians [illegible]

[illegible] that the [illegible] and [illegible] Chriſtianity [illegible] the Son and Meſſiah of God [illegible] Agreeable with Chriſtianity [illegible] which [illegible] the Chriſtian Religion [illegible] Human Nature [illegible] themſelves with Revelations from God, and [illegible] Them.

[illegible] Revelation or Inſpiration from God [illegible]

[illegible] Religion [illegible] as they ſhould [illegible]

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CHAP. I.

Of Notation, Addition, Subſtraction, Multiplication, Diviſion, and Extraction of the Roots.

Introduction.

§. 1. ALGEBRA Seems to be the Nobleſt Science, which was ever Invented, and to be the Utmoſt Effort of the Human Genius, by which we Arrive at a Solution of thoſe Problems in Arithmetick, and Geometry, which, without it, it would be Impoſſible to Find out; It was Suppoſed to be Known to the Antients, and there are Evident Footſteps of it in *Euclid* himſelf *; But *Diophantus* of *Alexandria* was the Firſt, who Profeſſedly made Uſe of it, in his Numerical Queſtions, the Mathematicians before being Thought to have Concealed this Art, that their Inventions might Appear more Stupendous and Surprizing to the World; But of Late, *Vieta* Firſt, and after him our own Country Man *Harriot*, then *Des-Cartes*, and *Oughtred*, and ſince, the Celebrated Dr. *Wallis*, the Great and Incomparable Sir *Iſaac Newton*, and the Famous Dr. *Halley*, have been the Chief Promoters and Advancers of it; I ſhall only here Endeavour to give as Compendious an Account of it, as is Poſſible, and I Believe I may Say, that in a Few Sheets, I have Comprized ſeveral Folio's; Mr. *Kerſey*'s Algebra, which is Voluminous enough, is Affirmed by Dr. *Wallis*, to be the moſt Complete Syſteme of this Science, and yet I Believe, there is Scarcely any Thing Material in him, which is not here Fully Explained, Beſides what is Added from *Kinkhuyſenius* and Sir *Iſaac Newton*, and Others, and from my own Obſervations upon theſe Authors, and the Shewing, that this Part of Knowledge likewiſe is Submitted to the Doctrine of the Expanſive and Contractive Forces.

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* Thus, for Inſtance, as moſt of the Propoſitions of the 2*d*. Book of his Elements, Particularly the 11*th*. and many Others, ſo Propoſition the 10*th*. of the 4*th*. Book, which is One of the moſt Ingenious in the Whole Elements, may be Eaſily Solved by Algebra; See the Scheme Annexed to that Propoſition, Edit. *Tacquet*; Let AB be $= a$, $AD = x$, $BD = a - x$, and let the Problem be Suppoſed to be Performed or Effected; Therefore the Triangles, CAB, DCB, are Similar Ones; For the Angle $BAC =$ Angle DCB by the Problem, and the Angle at B is Common to both the Triangles, from whence the Angle $ACB =$ Angle CDB; Conſequently $AB : CB = CD :: CD = AD : DB$. That is, $a : x :: x : a - x$, and $xx = aa - ax$, or $xx + ax = aa$; Therefore $xx + ax + \frac{1}{4} aa = \frac{5}{4} aa$, and $x + \frac{1}{2} a = \sqrt{\frac{5}{4} aa}$, or $x = -\frac{1}{2} a + \sqrt{\frac{5}{4} aa}$, as will be afterwards Seen, and which is the very Solution of the 11*th*. Propoſition of the 2*d*. Book before Cited, or is the Rule there Preſcribed for Interſecting a Line *Mediâ & Extremâ ratione*, and is here Required for Expediting the Preſent Problem. See *Sturmius's Matheſ. Enucleat.* Prop. 27. Sect. 2. Lib. 1. Schol. 2. Pag. 131. Edit. 1.

BUT before I Proceed, I ſhall take Notice, that Algebra is either Applicable to Numbers, or to Extenſion; The Method of doing the Firſt ſhould be Punctually Learned, and then we may with greater Facility go on to the Latter, which is not better Explained by any Author, than by *Sturmius*, at the End of his *Matheſis Enucleata*, nor more Finely Exemplified by any, than by Sir *Iſaac Newton* in his *Algebra*, or his *Arithmetica Univerſalis*.

I muſt alſo Subjoin, that there are Two Ways of Teaching this Science, either by Explaining the Abſtracted Rules and Precepts Firſt, of Notation, Addition, Subſtraction, *&c.* and afterwards Applying them to the Solution of Problems; or elſe, by Immediately Beginning with Problems, and Explaining the Rules or Precepts, as they Occur in each Problem.

IF the Firſt is done, Learners Comprehend nothing of the Meaning of a Known, or Unknown Quantity, of a or of x, or of a Privative or Poſitive One, of $+a$ or $-a$; It is all Myſterious to them, and thro' all the Rules, which are given Concerning them, they are Carried on in the Dark, Know not where they are, and, out of Mere Deſperation, Relinquiſh the Purſuit of the Science, which they Think, they ſhall be never Able to Attain; And ſo it is in Reſpect of Equations. This alſo is very Probably the Reaſon, why, tho' *Engliſh* Letters, as a, b, c, *&c.* are as Intelligible, as the *Arabick* Cyphers, 1. 2. 3, *&c.* which every Boy underſtands, Yet Algebra has Fewer Diſciples among thoſe of Advanced Years, than Arithmetick has among thoſe of much leſs Experienced or Judicious Ones; Notwithſtanding it is Manifeſt, that all Arithmetick whatſoever, and Infinitely more, and even all Geometry, in a Manner, is Contained and Included in this moſt Sublime and Divine Science. There may be Indeed another Reaſon given, why Arithmetick Prevails more than Algebra, but it ſhould not be Acknowledged by Rational Men; Namely, that there is ſome Intereſt at the Bottom, and ſome Gain to be Acquired from the One, whereas there may be Poſſibly little but Reproach or Contempt from the Other, at Leaſt, with or among Certain Genius's, and Complexions, who Chooſe to Ridicule, what they themſelves do not Underſtand.

IF the Second is done, it will be Repeating the ſame Rules and Precepts Over and Over, which will be Tedious, and Cauſe as Great a Confuſion in that Way; And therefore, the only True Method, to make Algebra more Generally Studied, ſeems to be to Haſten to Problems, as ſoon as is Poſſible, and to Lay down thoſe General Rules, which are Perfectly Neceſſary to the Underſtanding them, when Propoſed; we ſhall Proceed in the Middle Way and Intermix the Problems with the Abſtracted Rules Given for the Solving of them, both, as it will be more Inſtructive, and more Diverting.

Of Notation.

§. 2. THE Notation of Algebra is the Language of it; a, b, c, d, e, f, *&c.* that is, the Former Letters of the Alphabet, Signify *Known* Quantities, or Numbers, as 1. 2. 3. 100. 500. 25. 64. 1000, or any other Number or Quantity, whatſoever; r, s, t, v, x, z, y, or the Concluding Letters of the Alphabet, Signify *Unknown* Quantities or Numbers; + Signifies *More*, or is a Note of Addition, as $3+4$ Signifies 7; − Signifies *Leſs*, or is a Note of Subſtraction, as $4-3$ Signifies 1; × is a Note, which Signifies the Multiplying of One Number or Quantity into Another, as 3×4 is 12; — with a Cypher above and below it, is a Note of Diviſion, as $\frac{5}{10}$ Signifies 5 Divided by 10, or, which may be thus Expreſſed, 10) 5 in Common Arithmetick; $\sqrt{}$ Signifies the Root, or the Extraction of it, from any Quantity, as the $\sqrt{4}$ is 2, the $\sqrt{9}$ is 3, $\sqrt{16}$ is 4, *&c.* that is, the Root of any Quantity or Number, is that

Quantity or Number, which, Multiply'd into it's ſelf, is that Quantity or Number; Thus the Root of 4 × Root of 4 is 4, that is, the $\sqrt{4}$ is 2, and ſo of the Reſt; And = is a Note or Mark of Equality, as $4 = 4$. And, Suitable to this, in Algebra, $a + b$ Signifies the Addition of a to b; $a - b$ Signifies the Subſtraction of b from a; $a \times b$, or ab, Denotes the Multiplication of One into the Other; and $\frac{a}{b}$ or $b)\,a$ the Diviſion of One by the Other; And the $\sqrt{a}$, or $\sqrt{b}$, Intends the Extraction of the Root of a or b, which $\sqrt{a}$, or $\sqrt{b}$, Multiply'd into it's ſelf, will Produce a, or b; And in Equations, $a = x$ Signifies, that a is Equal to x.

This ſeems to be a Sufficient Account of Notation, and will Explain any other Quantities whatſoever, be they More or Leſs Complexed, and which we ſhall not here Inſert, as Sir *Iſaac Newton* does, *nè Complexione Terminorum Tyrones, quaſi Conterriti, in Limine Hæreant*, as he himſelf Expreſſes it, and yet Inſerts thoſe Algebraick Quantities, which would Affright them.

All, that we ſhall Farther Obſerve upon this Notation is, that here are only Two Sorts of Quantities Received, in the moſt Extenſive Science, which was ever Propoſed in Nature, Namely, the Poſitive and Privative, Expreſſed by + and −; Which is Exactly Correſpondent to what we Affirm, that there are only Two Principles in It, the Expanſive Force, and the Contractive; The Former of which, or +, is Agreeable to the Expanſive Force, and the Latter, or −, to the Contractive.

An Exemplification of this Doctrine by Problem the 1ſt.

§. 3. We ſhall Exemplify this Doctrine, ſo far as we have Gone, by the Firſt Problem, we ſhall Take Notice of, in Numbers.

Probl. 1. A Merchant had a Certain Sum of Money, which is not Known, which he Increaſed one Third every Year, Deducting a Certain Sum, which is Known, and which he Annually Expended on his Family; And at the End of Three Years, he was Twice as Rich, as he was at Firſt; It is Inquired, what that Unknown Sum was, which he Originally had?

Mr. *Kerſey*, and, after him, Sir *Iſaac Newton*, Teaches us a Method of Solving this, and all other Problems, by Barely Changing the Terms in the *Engliſh*, or any other Language, into the *Algebraick*, and which Method, we ſhall for the moſt Part Follow in the Succeeding Problems; Thus,

In Engliſh.	In Algebra.
1. A Merchant has a Certain Sum of Money, which is Unknown.	1. x.
2. Out of which he Expends the Firſt Year a Certain Sum which is Known a.	2. $x - a$
3. The Remainder he Increaſes by One Third.	3. $x - a + \frac{x - a}{3}$, or $\frac{4x - 4a}{3}$.
4. The Second Year he Expends the ſame Known Sum a.	4. $\frac{4x - 4a}{3} - a$, or $\frac{4x - 7a}{3}$
5. The Remainder he Increaſes by one Third.	5. $\frac{4x - 7a}{3} + \frac{4x - 7a}{9}$, or $\frac{16x - 28a}{9}$
6. The Third Year he Expends the ſame Known Sum a.	6. $\frac{16x - 28a}{9} - a$, or $\frac{16x - 37a}{9}$
7. The Remainder he Increaſes by one Third.	7. $\frac{16x - 37a}{9} + \frac{16x - 37a}{27}$, or $\frac{64x - 148a}{27}$
8. And he is Twice as Rich, as he was at the Beginning.	8. $\frac{64x - 148a}{27} = 2x$

Explanation. 1ſt. This Laſt Step is Called an Equation from the Note of Equality, which Lies betwixt the Two Quantities; and every Problem muſt be Reduced to an Equation, before it can be Solved, and all the Unknown Quantities muſt be Placed on the One Side of it and the Known Ones on the Other, in Order to Determine, what the Unknown Quantity is, by the Known.

Thus, both Sides of the Equation Multiplied by 27, makes $64x - 148a = 54x$; Subſtract $54x$ from both Sides of it, and the Equation will be $10x - 148a = 0$; Add $148a$ to both Sides, and the Equation will be $10x = 148a$; Divide both Sides of it by 10, and the Equation will be $x = \frac{148a}{10}$; All which Depends upon this Plain Axiom, that, if Two Quantities are Equal, Add to, or Subſtract from them, Equal Quantities, or Multiply or Divide them by Equal Quantities, and they will be ſtill Equal. And from hence is Derived that General Rule in Equations, That they will ſtill Continue Such, Let us Tranſpoſe the Terms of them, as we will, to Contrary Sides, if it is done under Contrary Signs; Thus, if $64x - 148a = 54x$, $64x - 148a - 54x = 0$, or $64x = 148a + 54x$, *&c.*

2*d.* The Known Quantity a may Signify any Known Number whatſoever, as 100, 1000, 154, 625, 109, *&c.* Suppoſing the Firſt; Then the Merchant Expends upon his Family every Year 100 *l.* and x the Unknown Sum, which he was Originally Worth, will be $= \frac{14800}{10} l. = 1480 l. =$ to his Gain, at the End of Three Years; For he was then Worth, Double of what he was at Firſt.

3*d.* The Intermediate Steps, from the Second to the Laſt, are made upon the Rule for the Reduction of Quantities of Different Denominations to the ſame Denomination; Which is, by Multiplying the Denominator of the Firſt into the Numerator and Denominator of the Second, and the Denominator of the Second into the Numerator and the Denominator of the Firſt; Thus, $x-a$, or $\frac{x-a}{1}$, $+\frac{x-a}{3}$ is Reduced to $\frac{4x-4a}{3}$, by Multiplying 1 the Denominator of the Firſt into $x-a$ the Numerator of the Second, and into 3 the Denominator of it, which makes, as before $\frac{x-a}{3}$, and by Multiplying 3 the Denominator of the Second into $x-a$ the Numerator of the Firſt, and into 1 the Denominator of it, which makes $\frac{3x-3a}{3} = \frac{x-a}{1}$; Becauſe all Ratios, Multiplied by Equal Quantities, will ſtill be Equal, as $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$, *&c.* But $\frac{x-a}{3} + \frac{3x-3a}{3} = \frac{4x-4a}{3}$; For $\frac{x}{3} + \frac{3x}{3} = \frac{4x}{3}$ and, $-\frac{a}{3} - \frac{3a}{3} = -\frac{4a}{3}$; Or, Add Privative Quantities to Privative, and they will be Privative, as if we Add Poſitive Ones to Poſitive, they will be Poſitive.

What the Analyſis and Syntheſis is in Algebra.

4*th.* This Way of Proceeding is Called the Analytick Method; becauſe it gives us the Analyſis, or Reſolution, of the Problem, and Leads us from the Unknown Quantity to the Known; The Reverſe of which is the Synthetick, or the Syntheſis, which begins with the Quantity Found out by Analyſis, and from thence by a Retrogreſſion Proves the Truth of the Propoſition; Thus in the Foregoing Problem.

If a Merchant is Poſſeſſed of 1480 *l.* and he Increaſes that Sum, every Year, One Third, Subſtracting 100 *l.* which he Expends in his Family, at the End of Three Years, he will be Twice as Rich, as he was at Firſt.

Demonstration. The Merchant's Original Stock is 1480 *l.* he Expends the Firſt Year 100 *l.* and the Remainder is 1380 *l.* which he Increaſes One Third, or with 460 *l.* At the End of the Firſt Year he is therefore Worth 1840 *l.*

The Second Year he likewiſe Expends 100 *l.* and the Remainder is 1740 *l.* which he Increaſes One Third, or with 580 *l.* At the End of the Second Year he is therefore Worth 2320 *l.*

The Third Year he alſo Expends 100 *l.* and the Remainder is 2220 *l.* which he Increaſes One Third, or with 740 *l.* and Conſequently, at the End of the Third Year he will be Worth 2960 *l.* = 2 × 1480 *l.* his Original Stock; and therefore he will be Twice as Rich, as he was at Firſt. *Q. E. D.*

This Synthetick Method the Antients made Uſe of, as being the moſt Beautifull and Agreeable, as well as the moſt Surprizing; And Artfully Kept to themſelves the

Analytick, by which they are Suppoſed to have Found out their Propoſitions, as has been already Intimated.

What a Canon or Rule in every Problem is.

5th. As there is a Syntheſis belonging to every Analyſis, ſo likewiſe there may be a Canon or Rule Formed upon every Equation, which Expreſſes the Problem, and is nothing more than Rendering the *Algebraick* Terms of it into *Engliſh*, or into *that Language*, in which we Write.

Thus, the Preſent Equation is $\frac{148a}{10} = x$; And the Canon Reſulting from thence is, That, If we Take 148 Times the Sum, which the Merchant Annually Expends upon his Family, and Divide it by 10, the Quotient will give the Sum Required, or the Original Stock, which he was Worth. I have been the Longer upon this Problem, that I might Explain to Young Beginners, the Nature, Scope, and Deſign of Algebra, and to make it as Eaſy and Familiar to them, as Poſſible.

Of Addition.

§. 4. The Addition, whether of Integral, or Decimal, or Fractional Numbers, or of Algebraick Quantities, is all Explained by One, and the ſame Principle; Namely, that we muſt always Add Homogeneous Quantities together, or Quantities of the ſame Kind and Denomination; Thus 3 Books + 4 Books = 7 Books; But 3 Books + 4 Pens are not Capable of being Reduced to a Sum, becauſe the Things are Heterogeneous, and of Different Denominations.

1ſt. Hence it is, that in the Addition of Integral or Whole Numbers, the Units are Placed under one another, ſo likewiſe the Tens, the Hundreds, Thouſands, *&c.* and ſo many Tens, as there are in the Units, are Carried to and Counted in the Place of Tens; ſo many Hundreds, as there are in the Tens, are Carried to and Counted in the Place of Hundreds; and ſo many Thouſands, as there are in the Hundreds, are Carried to and Counted in the Place of Thouſands, *&c.*

Thus in the Preſent Example, 9, 3, 0, 5, Stand in the Place of Units, 9, 0, 2, 8, in the Place of Tens, 8, 4, 9, 8, in the Place of Hundreds, 7, 3, 1, in the Place of Thouſands, and 8, 1, in the Place of Ten Thouſands; and 5+0+3+9 = 17, where 7 is ſet Down in the Sum, and 1 Carried to the Place of 10's, and Counted as Unit; 1+8+2+0+9 = 20, where 0 is ſet Down in the Sum, and 2 Carried to the Place of 100's; 2+8+9+4+8 = 31, where 1 is ſet Down in the Sum, and 3 Carried to the Place of 1000's, *&c.*

$$\begin{array}{r} 87899 \\ 13403 \\ 1920 \\ 885 \\ \hline 104107 \\ \hline \end{array}$$

2d. The Caſe is the Same in Decimals, as in the Example Annexed, in which the Integral Numbers are Separated from the Decimal by the Comma, and 9, 0, 2 Stand in the Place of $\frac{1}{10}$, or, of the Tenth Parts of an Unit; 5, 8, 7, in the Place of $\frac{1}{100}$, or, of the Hundredth Parts of an Unit; 3, 0, in the Place of $\frac{1}{1000}$ or of the Thouſandth Parts of an Unit; And 7 in the Place of $\frac{1}{10000}$, or, of the Ten Thouſandth Parts of an Unit; Which are Added together in the ſame Manner, as Whole Numbers are, Carrying ſo many Tens in the Lower Place to the Right Hand, and Counting them as ſo many Units in the Place above to the Left; For $\frac{10}{10000} = \frac{1}{1000}$, and $\frac{10}{1000} = \frac{1}{100}$, and $\frac{10}{100} = \frac{1}{10}$, and $\frac{10}{10} = 1$; And $\frac{3}{10} + \frac{0}{100} + \frac{3}{1000} + \frac{7}{10000} = \frac{3037}{10000}$; becauſe $\frac{3}{10} = \frac{3000}{10000}$, $\frac{0}{100} = \frac{000}{10000}$, and $\frac{3}{1000} = \frac{30}{10000}$; All which Depends upon what was Alledged before, under Article 3. Explanation of Problem 1. That all Ratios, Multiplied by Equal Quantities, will be ſtill Equal.

$$\begin{array}{rr} & 630{,}953 \\ & 51{,}0807 \\ & 305{,}27 \\ \hline \text{Sum is} & 987{,}3037 \\ \hline \end{array}$$

3d. It is the ſame alſo in Fractions, which have the ſame Denomination; For $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$; $\frac{10}{16} + \frac{2}{16} = \frac{12}{16}$; becauſe the 5th. or 16th. Part of an Unit is here the Denomination, or the Kind or Sort of Quantities, which are Added together. But, if the Fractions are of Different Kinds, or Denominations, as $\frac{2}{3}$ and $\frac{3}{4}$, they muſt be Reduced to the Same, according to Article 3. Explanation of Problem 1. that is, to $\frac{8}{12}$ and $\frac{9}{12}$, and then Added together, which will make $\frac{17}{12}$, or 17 Divided by 12.

4th. The Matter is not otherwiſe in Algebra, as in this Firſt Example; x is the Quantity, or Thing, or Denomination, or Sort of Thing, to be Added on the one Hand, and a on the other; So that the Numbers Prefixed are only to be Added, and the Thing, or Denomination of it, to be Annexed to the Sum.

$$\begin{array}{r} 12x + 7a \\ 7x + 9a \\ \hline 19x + 16a \\ \hline \end{array}$$

Thus likewiſe in a Second Example, the $\sqrt{bx}$ is the Denomination or Sort of Thing, to be Added on the one Hand, and the $\sqrt{ab-cx}$ on the other; And let the Quantities be never ſo Complicated, or Perplexed, in Addition they are to be Conſidered only, as ſo many Books, or Pens, or Things, which are of the ſame Kind, to be Added to each other, and $6\sqrt{bx}+9\sqrt{ab-cx}$ may be Imagined to be 6 Books + 9 Pens. Where no Number is Prefixed, Unit is always Suppoſed.

$$\begin{array}{r}
2\sqrt{bx}+\sqrt{ab-cx}\\
3\sqrt{bx}+\sqrt{ab-cx}\\
\sqrt{bx}+7\sqrt{ab-cx}\\
\hline
6\sqrt{bx}+9\sqrt{ab-cx}\\
\hline
\end{array}$$

THE Difficulty therefore in Algebraick Additions will Chiefly Ariſe from the Signs, + and −, made Uſe of in them; Concerning which, the Shorteſt and Plaineſt Rule is, That what + Affirms, − Denies, or what the Firſt Alledges or Brings, the Laſt Annuls and Takes away; If + Carries us 10 Miles Forwards from any Place, − Leads us back to it; If + Lays a Book upon a Table, − Takes it away again; If + makes me Worth ſuch a Sum, − Deprives me of it; If + is the Expanſive Force, as of Heat; − is the Contractive, as of Cold; And, in all Inſtances, theſe Signs are Directly Oppoſite to, and Deſtroy, each other. But − is an Addition to − and + to +, or 2 and 3 Affirmatives will make 5 Affirmatives, and 2 and 3 Negatives will make 5 Negatives; Thus alſo 2 and 3 Expanſive or Contractive Forces, will make 5 Expanſive or Contractive Forces: Or, Addition is Performed by Connecting the Quantities to be Added with their Proper Signs of + and −, and Uniting or Joining together thoſe, which are Capable of it, according to *Oughtred*; The Following Examples will Explain this.

$$\begin{array}{lllll}
 & 17ax-14a+3 & 11bc-7\sqrt{ac} & -4\frac{ax}{b}+6\sqrt{3}+\frac{1}{5} & -6xx+\frac{3}{7}x\\
 & -8ax+4a+2 & 15bc+2\sqrt{ac} & +11\frac{ax}{b}-7\sqrt{3}+\frac{2}{5} & 5x^3+\frac{5}{7}x\\
 & -9ax+7a & & & \\
\hline
\text{Sums} & *\ -3a+5 & 26bc-5\sqrt{ac} & 7\frac{ax}{b}-\sqrt{3}+\frac{3}{5} & 5x^3-6xx+\frac{8}{7}x\\
\hline
\end{array}$$

$$\begin{array}{lrr}
 & -2ayy-4aay+a^3 & 5x^4+2ax^3\\
 & y^3+2ayy-\frac{1}{2}aay & -3x^4-2ax^3+8\frac{1}{4}a^3\sqrt{aa+xx}\\
 & aay+2a^3-\frac{a4}{27} & -2x^4+5bx^3-20a^3\sqrt{aa-xx}\\
 & & -4bx^3-7\frac{1}{4}a^3\sqrt{aa+xx}\\
\hline
\text{Sums} & y^3\ *-3\frac{1}{2}aay+3a3-\frac{a4}{27} & *\ +bx^3+a^3\sqrt{aa+xx}-20a^3\sqrt{aa-xx}
\end{array}$$

WHERE no Sign is Prefixed, + is Suppoſed; The Aſteriſk Denotes a Want or Defect of a Term in the Sum from Certain Quantities in the Addenda, or Addends, Mutually Deſtroying each other.

Probl. the 2d. PROBL. 2. There are Three Unknown Quantities, or Numbers, Required to be Determined, to Wit, What they are, The Sums of each Two of them being Known, or Given.

In Engliſh.	In Algebra.
1. The Three Numbers, or Quantities, Unknown are Required.	$x?\ y?\ z?$
2. The Sums of each Two of them, which are Known, are.	$x+y=a.\ x+z=b.\ y+z=c$
3. What is x by the 2*d*. Step?	$x=a-y$
4. What is z by the ſame Step?	$z=b-x$
5. What is y by the ſame Step?	$y=c-z$
6. Therefore by Comparing the Equations of the Former Steps with each Other.	$x=a-c+z=a-c+b-x$ or $2x=a-c+b$, and $x=\frac{a-c+b}{2}$

7. Alſo, by a Compariſon of the ſaid Equations. | $z=b-x=b-\frac{a+c-b}{2}=\frac{b-a+c}{2}$

8. Laſtly, by a like Compariſon. | $y=c-z=c-\frac{b+a-c}{2}=\frac{c-b+a}{2}$

Explanation. *1ſt.* The *3d. 4th.* and *5th.* Steps are Effected by Subſtracting y, x, and z, from both Sides of the Foregoing Equations, or by Transferring the Quantities to Contrary Sides of them under Contrary Signs, as before Explained, Artic. 1. Explanation of Probl. 1.

The *6th.* is Obtained by Subſtituting the Quantities = to y and z in the Room of them, and the *7th.* and *8th.* by a like Procedure.

2d. a, b, and c, may Signify any Numbers, we ſhall Chooſe, Suppoſing 9, 10, and 13; Therefore $x=\frac{9-13+10}{2}=3$; $z=b-x=10-3=\frac{10-9+13}{2}=7$; $y=c-z=13-7=\frac{13-10+9}{2}=6$.

3d. The Syntheſis is Formed from $x+y=a$; $x+z=b$; $y+z=c$; and from Subſtituting the Known Quantities for x, z and y, the Unknown.

4th. Since a is = to the Sum of the Leaſt Number and the Middle One, $=3+6$; and b is = to the Sum of the Greateſt Number and the Leaſt, $=3+7$; and c is = to the Sum of the Middle Number and the Greateſt, $=6+7$; The Canon for finding out x, or the Leaſt Number, $=\frac{a-c+b}{2}$, will be This; Add the Sum of the Leaſt Number and the Middle One to the Sum of the Greateſt Number and the Leaſt, and Subſtract from this Sum, the Sum of the Middle Number and the Greateſt, and Divide the Whole by 2, it will give the Value of the Leaſt Quantity or Number; And in the ſame Manner Canons are Formed for the Middle Number, and the Greateſt, or for y and z.

Of Subſtraction.

§. 4. As in Addition, ſo in Subſtraction, it is Neceſſary, that the Quantities ſhould be Homogeneous, or of the ſame Kind and Denomination; No One can Subſtract a Line from a Surface, or That from a Solid, or Vice Verſâ; Nor in Numbers can we Subſtract 100 from 10000, unleſs ſo far as 10000 Conſiſts of, and is Reduced to Hundreds.

1ſt. Therefore, Hence it is, that, in Integral Numbers, or in Decimal Ones, Units are Placed under Units, Tens under Tens, and Hundreds under Hundreds, *&c.* and the ſame may be Said of Tenths, Hundredths, *&c.* of which we have the Following Examples, which are only ſo many Inſtances of this Doctrine of Subſtracting Homogeneous Quantities from Homogeneous Ones, that is, Units from Units, Tens from Tens, Tenths from Tenths, *&c.* If the Number is too Small, fromwhich we Subſtract, we muſt Borrow Unit from the Place above, and Repay it in the next Subſtraction.

$$\begin{array}{r} 782579 \\ 63543 \\ \hline 719036 \end{array}$$

2d. In Decimals	547	35,72	46,5003	308,7
	0,63	14,32	3,078	25,74
Remainders are	546,37	21,4	43,4223	282,96

Which Remainders are ſuch, becauſe 3 Hundredths out of 10 Hundredths is 7 Hundredths; 7 Tenths out of 10 Tenths is 3 Tenths, and 10 Tenths, or Unit, out of 70 Tenths, or 7 Units, is 6 Units, in the Firſt Example; And ſo of the Reſt.

3d. In Fractions, which have the ſame Denomination, $\frac{3}{5}-\frac{1}{5}=\frac{2}{5}$, $\frac{10}{16}-\frac{2}{16}=\frac{8}{16}$, becauſe the 5th. or 16th. Part of an Unit is here the Denomination, or Kind, or Sort of Quantities, which are to be Subſtracted from each Other; But if the Fractions are of Different Kinds or Denominations, as $\frac{2}{3}$ and $\frac{3}{4}$, they muſt be Reduced to the ſame, according to Artic. 3. Explanat. Probl. 1. that is, to $\frac{8}{12}$ and $\frac{9}{12}$ and then Subſtracted the one from the Other, and the Reſiduum will be $\frac{1}{12}$.

4th. It is the ſame in Algebra; for x, or a, or bc, or the $\sqrt{ac}$, or x^3, or $\frac{x}{7}$, or xx, or ax, *&c.* are each of them Denominations, or Sorts of Quantitiesor Things, which are the ſame, and therefore may be Subſtracted from each Other; And all the Difficulty, as before,

before in Addition, will be in the Signs + and −; for which there may be this Short Rule, which was there, in Part, Propoſed; That − Denies or Deſtroys + or −, and + Affirms or Supports them; That is, if we Subſtract +, we Take away the Poſitive, and make it −, or a Privation; If we Subſtract −, we Take away the Privative, and make it +, or a Poſition; If we Add +, we Affirm the Poſition; And if we Add −, we Affirm the Privation; Thus −, + = −; −, − = +; +, + = +; +, − = −; Or, Subſtraction is Performed by Connecting the Quantities with all the Signs of the Quantities Subſtracted, Changed into Contrary Ones, and then Uniting Thoſe, which are Capable of it; The Examples Follow, which will Clear This.

$$\begin{array}{lcccc} & 12x+7a & 15bc+2\sqrt{ac} & 5x^3+5\frac{x}{7} & 11\frac{ax}{b}-7\sqrt{3}+\frac{2}{5} \\ & 7x+9a & -11bc+7\sqrt{ac} & 6x^2-3\frac{x}{7} & 4\frac{ax}{b}-6\sqrt{3}-\frac{1}{5} \\ \hline \text{Remainders.} & 5x-2a & 26bc-5\sqrt{ac} & 5x^3-6x^2+8\frac{x}{7} & 7\frac{ax}{b}-\sqrt{3}+\frac{3}{5} \end{array}$$

We ſhall only here Obſerve, in Reſpect of our Doctrine of the Expanſive and Contractive Forces; That the Expanſive Increaſes the Expanſive as +, +, or + Affirmed, = +; And the Contractive the Contractive, as +, −, or − Affirmed, is = −; But the Expanſive Diminiſhes, or Deſtroys, the Contractive, as −, − or − Denied = +, and the Contractive the Expanſive, as −, +, or + Denied = −.

Probl. the 3d. PROBL. 3. Let a Line of *a*, or a Known, Number of Feet be Propoſed to be Divided into *e*, a Known Number of Parts; So that the Exceſs of the Second Part above the Firſt ſhould be *b*, or a Known Number of Feet; of the Third Part above the Firſt, *c*, or a Known Number of Feet; of the Fourth Part above the Firſt, *d*, or a Known Number of Feet, *&c.* The Queſtion is, what is the Firſt Part, *viz.* the Number Unknown? Or, It is Required to Divide a Quantity Known, or Given, into any Number of Parts; So that the Greater Parts may Exceed the Leaſt by Exceſſes or Differences, which are likewiſe Given, or Known.

In Engliſh.	In Algebra.	
1. The Quantity to be Divided, and which is Known is.	a	20
2. It is Divided into any Known Number of Parts.	e	4
3. The Firſt and Leaſt Part is Unknown.	x	2
4. The Second, the Exceſs, *b*, of which above the Firſt is Known.	$x+b$	4
5. The Third, the Exceſs, *c*, of which above the Firſt is alſo Known.	$x+c$	5
6. The Fourth, the Exceſs, *d*, of which above the Firſt is likewiſe Known, *&c.*	$x+d$	9
7. But all the Parts are Equal to the Whole Line.	$ex+b+c+d=a$	
8. Therefore by Changing the Signs of the Equation as before.	$ex=a-b-c-d$	
9. By Dividing both Sides of the Equation by the ſame Quantity.	$x=\frac{a-b-c-d}{e}$	

1ſt. How all theſe ſeveral Steps are made has been already Explained, and they do not want any Farther Comment.

2d. Suppoſing $a = 20$ Feet, $b = 2$ Feet, $c = 3$ Feet, and $d = 7$ Feet, and *e*, ſince 4 Parts are only Conſidered, $= 4$; Therefore $x = \frac{a-b-c-d}{e} = \frac{20-2-3-7}{4}$ Feet $= 2$ Feet; $x+b = 2+2 = 4$ Feet; $x+c = 2+3 = 5$ Feet; $x+d = 2+7 = 9$ Feet; and $2+4+5+9 = 20$ Feet.

3d. The Syntheſis is; If we Take Four Times the Leaſt Number of Four Numbers, and Add the Exceſſes of the ſeveral Numbers above that Leaſt Number, it will Give the Sum of all the Numbers; Thus $4\times\overline{2}+2+3+7=20$.

4th. The Canon for Finding out *x* is; If we take the Sum of all the Numbers, and Subſtract all the Exceſſes, and Divide the Remainder by the Number of the Parts, the Quotient will Give the Firſt and Leaſt Part Required.

PROBL. 4.

Probl. the 4th.

Probl. 4. A Certain Perſon was Deſirous to Diſtribute ſome Money amongſt ſome Poor People, and he Wanted Eight Pence, to Give each of them Three Pence; He therefore Gives each of them Two Pence, and he had Three Pence Over, or to Spare; What was the Number of the Poor?

In Engliſh.	In Algebra.
1. The Number of the Poor People Unknown is.	x.
2. The Perſon, who Diſtributes, has ſo many Three Pences, as there are Poor, Excepting Eight Pence, which is.	$3x-8$ Pence
3. He Gives to Each Two Pence, and there Remain'd in his Hands Three Pence, which is.	$2x+3$ Pence.
4. But what he had in the 2*d*. Step, is Equal to what he had in the 3*d*.	$3x-8=2x+3$ Pence.
5. And by Tranſpoſition of the Quantities to Contrary Sides of the Equation under Contrary Signs, the Number of the Poor was.	$3x-2x=x=3+8=11$.
6. The Sum of Money therefore, which he had, was.	$3x-8=2x+3=25$ Pence.

There is no Occaſion for the Explanation of this Problem, nor of the Canon or Syntheſis of it. And however Low this Sum may be Put for a Diſtribution to Poor People, I would not have it an Example of our Charity to them, but of the Penury or Generoſity of Algebra, which can make the ſame Letters Stand, for as Small, or as Great Sums, as it Pleaſes.

Of Multiplication.

§. 5. Multiplication is nothing more than a Repeated Addition of the ſame Kind, Sort, or Denomination of Quantities; Thus in Integrals $3\times10,=30$, is 10 Added 3 Times to it's ſelf; And in Decimals $3\times\frac{1}{10},=\frac{3}{10}$, is $\frac{1}{10}$ Added Three Times to it's ſelf; So likewiſe in Vulgar Fractions, $2\times\frac{3}{4},=\frac{6}{4}$, is $\frac{3}{4}$ Added Twice to it's ſelf, and $\frac{3}{4}\times\frac{5}{9},=\frac{15}{36}$, is $\frac{5}{9}$ Added Three Fourths to it's ſelf, or, Reduced to the ſame Denomination, $\frac{27}{36}\times\frac{20}{36}=\frac{540}{1296}=\frac{15}{36}$, Signifies, that 20 ſuch Parts of a Unit are Added to it's ſelf, by 27 ſuch Parts of a Unit; And in Algebra $a\times x,=ax$, is x Added to it's ſelf, as often as a Preſcribes; It is the ſame in the Algebraick Signs $+\times+$ makes $+$, becauſe it is $+$ ſo many Times Added together; $+\times-$ makes $-$, becauſe it is $-$ ſo many Times Added together; But $-\times+$ makes $-$, becauſe $-$ Denies ſo many Times the Affirmation or Poſition of $+$; And $-\times-$ makes $+$, becauſe $-$ Denies ſo many Times the Negation or Privation of $-$, and therefore makes the Quantity $+$; And Conſequently all Things will Proceed, as before in Addition, excepting that the Multiplicator is to be Conſidered as the Number, according to which the ſame Quantity, or Multiplicand, is ſo many Times to be Added to it's ſelf; This will Appear in the Following Examples.

1*ſt*. In Integrals the Procedure is, 4×5, then 4×90, and Laſtly 4×700, and Add the Products together; Alſo in the 2*d*. Example, 5×3; 5×40; 5×0; 5×9000; Then 0×3; 0×4, *&c.* afterwards 300×3; 300×40; 300×0; 300×9000; And ſo 2000×3; 2000×40; 2000×0; 2000×9000, and Add the Products together, by Placing the Units under Units, Tens under Tens, Hundreds under Hundreds, *&c.*

Multiplicand	795
Multiplicator	4
	20
Partial Products	36
	28
Whole Product	3180

Multiplicand	9043
Multiplicator	2305
	45215
	0000
Partial Products	27129
	18086
Whole Product	20844115

2*d*. In Decimals, the Multiplication is the ſame, as in Integrals, Excepting, that there muſt be as many Decimal Places Cut off in the Product by a Comma, or Mark of Diſtinction, as there are ſuch Decimal Places in the Multiplicator and Multiplicand, in Order to Preſerve the Value of the ſeveral Places; for thus 100×100=10000 or 4 Places in Integrals, Beſides the Unit.

Multiplicands	72,4		50,18		3,9025
Multiplicators	29		2,75		0,0132
	6516		25090	Partial Products	78050
Partial Products	1448	Partial Products	35126		117075
Whole Product	2099,6		10036		39025
		Whole Product	137,9950,	Whole Product,	0,05151300

3*d.* So likewiſe in Common Fractions, Multiply the Numerators into the Numerators, and the Denominators into the Denominators; thus $\frac{2}{5}\times\frac{3}{7}=\frac{6}{35}$; $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$, &c. for the Reaſon before Alledged.

4*th.* THUS alſo in Radical Quantities of the ſame Denomination, we muſt Multiply the Quantities under the Radical Sign, and Prefix the ſame Sign, Multiplication being only a Repeated Addition; $\sqrt{3}\times\sqrt{5}=\sqrt{15}$; $\sqrt{ab}\times\sqrt{cd}=\sqrt{abcd}$; $\sqrt{^3,5ayy}\times\sqrt{^3,7ayz}=\sqrt{^3,35a^2y^3z}$, &c.

5*th.* IN Algebraick Quantities, which Conſiſt of ſeveral Parts, we are to Multiply all the Parts of the One, into all the Parts of the Other in the ſame Way and Manner, as is done in Numbers, always Obſerving to Multiply the Numbers into the Numbers the Species, or Letters, into the Species, or Letters, the Signs into the Signs, the Roots into the Roots, the Numerators into the Numerators, and the Denominators into the Denominators; The Examples Follow.

$$\begin{array}{lr}
\text{Multiplicand} & aa+2ac-bc \\
\text{Multiplicator} & a-b \\ \hline
 & -aab-2abc+bbc \\
 & a^3+2aac-abc \\ \hline
\text{Product} & a^3+2aac-aab-3abc+bbc
\end{array}
\qquad
\begin{array}{lr}
\text{Multiplicand} & a+b \\
\text{Multiplicator} & a+b \\ \hline
 & ab+bb \\
 & aa+ab \\ \hline
\text{Product} & aa+2ab+bb
\end{array}$$

$$\begin{array}{lr}
\text{Multiplicand} & a+b \\
\text{Multiplicator} & a-b \\ \hline
 & -ab-bb \\
 & aa+ab \\ \hline
\text{Product} & aa\quad *-bb
\end{array}
\qquad
\begin{array}{lr}
\text{Multiplicand} & yy+2ay-\frac{1}{2}aa \\
\text{Multiplicator} & yy-2ay+aa \\ \hline
 & aayy+2a^3y-\frac{1}{2}a^4 \\
 & -2ay^3-4aayy+a^3y \\
 & y^4+2ay^3-\frac{1}{2}aayy \\ \hline
\text{Product} & y^4*-3\frac{1}{2}aayy+3a^3y-\frac{1}{2}a^4
\end{array}$$

$$\begin{array}{lr}
\text{Multiplicand} & 2\frac{ax}{c}-\sqrt{\frac{a3}{c}} \\
\text{Multiplicator} & 3a+\sqrt{\frac{abb}{c}} \\ \hline
 & 2\frac{ax}{c}\sqrt{\frac{abb}{c}}-\frac{aab}{c} \\
 & \frac{6aax}{c}-3a\sqrt{\frac{a3}{c}} \\ \hline
\text{Product} & \frac{6aax}{c}-3a\sqrt{\frac{a3}{c}}+\frac{2ax}{c}\sqrt{\frac{abb}{c}}-\frac{aab}{c}
\end{array}$$

IN which Examples, the Juxta-Poſition of the Letters always Signifies the Multiplication of the Quantities Expreſſed by them, $a^3=aaa=a\times a\times a$; $a^2y^2=aayy=a\times a\times y\times y$, &c. but $3a=a+a+a$; $2a=a+a$, &c. and Denotes the Addition of a, or the Multiplication of 3, or 2, $\times a$, $=3a$, or $2a$.

Probl. the 5th. PROBL. 5. The Sum of Two Numbers being Given, a, and the Difference of the Squares, b, it is Required to Find the Numbers. A Square is any Quantity or Number Multiplied into it's ſelf, as is before Said.

In Engliſh.	In Algebra.
1. Two Unknown Numbers are Sought, whoſe Sum is Known or Given.	$\left.\begin{array}{l} x \\ a-x \end{array}\right\vert = a,$
2. Their Squares are.	$xx.\ aa-2ax+xx.$
3. The Difference of the Squares.	$aa-2ax+xx-xx=aa-2ax.$
4. Which Difference is Equal to b.	$aa-2ax=b.$
5. Or by Tranſpoſition of the Terms, and Dividing both Sides by $2a$.	$aa-b=2ax$, or $x=\dfrac{aa-b}{2a}=\frac{1}{2}a-\dfrac{b}{2a}.$

EXAMPLE. $a=8.\ b=16.\ x=\frac{64-16}{16}=4-\frac{16}{16}=3.\ a-x=5$; The Canon and Syntheſis are Eaſy.

Probl. the 6th.

PROBL. 6. If Two Letter-Carriers *A* and *B*, who are at 59 Miles Diſtance, ſhould ſet out in the Morning to Meet each Other, and *A* ſhould go 7 Miles in 2 Hours, and *B* 8 Miles in 3 Hours, but *B* ſhould ſet out an Hour Later than *A*, the Queſtion is, how many Miles *A* will have Gone, before he Meets *B*.

SOLUTION. 1. The Length of the Whole Way Performed by both the Letter-Carriers, *A* and *B*.	59 Miles.
2. The Length of the Way, which *A* Performs, before he Meets *B*.	x Miles.
3. The Length of the Way, which *B* Performs.	$59-x$ Miles.
4. *A* Diſpatches 7 Miles in 2 Hours, and therefore in Proportion will Diſpatch his Whole Journey in	$\frac{2x}{7}$ Hours; Becauſe $\overset{\text{Mil.}}{7} : \overset{\text{Hour.}}{2} :: \overset{\text{Mil.}}{x} : \overset{\text{Hour.}}{\frac{2x}{7}}$.
5. *B* Diſpatches 8 Miles in 3 Hours, and therefore in Proportion will Diſpatch his Whole Journey in	$\frac{177-3x}{8}$ Hours; Becauſe $\overset{\text{Mil.}}{8} : \overset{\text{Hour.}}{3} :: \overset{\text{Mil.}}{59-x} : \overset{\text{Hour.}}{\frac{59-x\times 3}{8}}$.
6. But *B* Sets out one Hour Later than *A*, and therefore the Number of Hours, which *B* Travels, will be Equal to the Number of Hours, by the 4th. and 5th Steps, which *A* Travels, Abating or Subſtracting One.	$\frac{2x}{7}-1=\frac{177-3x}{8}$. Hours.
7. And by Tranſpoſition of the Terms.	$\frac{2x}{7}=\frac{177-3x}{8}+1$.
8. By Multiplying both Sides by 8.	$\frac{16x}{7}=177-3x+8$.
9. Multiply both Sides of the Equation by 7.	$16x=1295-21x$.
10. By Tranſpoſition of the Terms.	$37x=1295$.
11. Divide both Sides of the Equation by 37, and the Journey of *A* will be.	$x=35$ Miles.
And the Journey of *B*.	$59-x=59-35=24$ Miles.
12. And *A* Travelled 10 Hours, and *B* 9, For, by the 4th. 5th. 6th. and 11th. Steps.	$\frac{2x}{7}=\frac{2\times 35}{7}=10$ Hours. And, $\frac{2x}{7}-1=9=\frac{72}{8}$ $=\frac{177-3x}{8}$ Hours.
13. *A* Travelled at the Rate of 7 Miles in 2 Hours, and *B* of 8 Miles in 3 Hours, for *A* Travelled 5 Times 2 Hours, and *B* 3 Times 3 Hours. But,	$7\times 5=35$ Miles. $3\times 8=24$ Miles.

The Canon and Syntheſis are Eaſy, and the 4th. and 5th. Steps are Explained by the Doctrine of Proportions, or the Golden Rule, or the Rule of Three, which is, That, if there are Four Numbers, which are Proportional, as $2 : 4 :: 3 : 6$, The Two Extremes, Multiplied into each other, will be Equal to the Product of the Two Middle Ones; and therefore, if the Product of the Two Middle Ones is Divided by the Firſt Proportional, the Quotient will give the Fourth, or Laſt; Thus $2\times 6=4\times 3$, and therefore $\frac{4\times 3}{2}=6$. It muſt be Obſerved here, that : :: : or . : : . are Notes of Quantities being Proportional.

The Same Problem Propoſed in more General Terms. The Celerities of Two Mobile's, or Moveables, *A* and *B*, Proceeding in the ſame Courſe, being Known, or Given, together with the Interval, or Diſtance, of the Times and Places, from which they begin their Motion, it is Required to Determine the Meta, or Boundary, or Point, in which they will Meet.

Solution. 1. The Celerities of the Two Moveables, *A* and *B*, are Known, or Given, that is, *A* Paſſes thro' the Space c, in the Time f; And *B* thro' the Space d, in the Time g.

Space. Time.
$c.$ $f.$ of *A*.
Space. Time.
$d.$ $g.$ of *B*.

2. The Interval or Diſtance of the Places, from which they begin their Motion. $e.$

3. The Interval or Diſtance of the Times, from which they begin their Motion. $h.$

4. Cas. 1. If *A* and *B* Tend the ſame Way in their Motion, and *A* is the Fartheſt Diſtant from the Meta at the Beginning of ſuch Motion, That Diſtance, being Unknown, may be Called. $x.$

5. Therefore the Diſtance of *B* from the Meta will be. $x-e.$

6. And Conſequently, the Time, in which *A* will Paſs thro' it's Whole Space x, will be by Proportion. $\frac{fx}{c}$; Becauſe (Space. Time. Whole Space. Whole Time.) $c : f :: x : \frac{fx}{c}.$

7. And the Time, in which *B* will Paſs thro' it's Whole Space $x-e$, will be. $\frac{gx-ge}{d}$; Becauſe (Space. Time. Whole Space. Whole Time.) $d : g :: x-e : \frac{gx-ge}{d}.$

8. If therefore *B* Begins Firſt to Move, ſince h is the Interval, or Diſtance, of the Times, let h be Added to the Shorter Time, and the Times will be Equal, that is.

$$\frac{fx}{c}+h=\frac{gx-ge}{d}.$$

1. Multiply both Sides of the Equation by c, and then by d, and it will be.

$$fx+ch=\frac{cgx-cge}{d}, \text{ and } dfx+dch=cgx-cge.$$

2. Tranſpoſe the Quantities under Different Signs, and it will be.

$$cgx-dfx=dch+cge.$$

3. Divide both Sides of the Equation by $cg-df$, and the Numerator and Denominator by c, and it will be.

$$x=\frac{dch+cge}{cg-df}=\frac{dh+ge}{g-\frac{d}{c}f}.$$

9. If *A* Begins Firſt to Move. Then, Repeat the 3 Steps as before, i. e.

$$\frac{gx-ge}{d}+h=\frac{fx}{c}.$$

1. Multiply both Sides by c and d, it will be.

$$cgx-cge+cdh=fdx.$$

2. Tranſpoſe the Terms, it will be.

$$fdx-cgx=cdh-cge.$$

3. Divide both Sides of the Equation by $fd-cg$, and the Numerator and Denominator by c, and it will be.

$$x=\frac{cdh-cge}{fd-cg}=\frac{dh-ge}{\frac{d}{c}f-g}.$$

Example. 1. If *B* Begins Firſt to Move, Suppoſing the Sun to Proceed, every Day, One Degree in the Ecliptick, and the Moon Thirteen, and that the Sun at a Certain Time is in the Beginning of Cancer, and that, Three Days after, the Moon is in the Beginning of Aries, the Place of their Following Conjunction is Required.

1. Let the Moon be *A*, and the Sun *B*; and ſince they both Tend the ſame Way, and *B* or the Sun Begins to Move Firſt, for *A*'s, or the Moon's, Motion does not Begin till Three Days after, the Diſtance of *A*, or the Moon, from the Meta, or Place of Conjunction, will be by the 4th. and 8th. Steps.

$$x=\frac{cge+dch}{cg-df}=\frac{ge+dh}{g-\frac{d}{c}f}.$$

2. By the 1ſt. 2d. and 3d. Steps and by the Preſent Example.

$$\overset{\text{Degr.}}{=13};\ \overset{\text{Day}}{f=1};\ \overset{\text{Degr.}}{d=1};\ \overset{\text{Day}}{g=1};\ \overset{\text{Degr.}}{e=90};\ \overset{\text{Days}}{h=3}.$$

3. And therefore by Subſtituting the Known Numbers inſtead of the Letters.

$$x=\frac{cge+dch}{cg-df}=\frac{13\times1\times90+13\times1\times3}{13\times1-1\times1}=\frac{ge+dh}{g-\frac{d}{c}f}$$

$$=\frac{1\times90+1\times3}{1-\frac{1}{13}}=\frac{1170+39}{13-1}=\frac{1209}{12}=\overset{\text{Degr.}}{100\tfrac{3}{4}}.$$

4. Lay $100\frac{3}{4}$ Degrees upon the Ecliptick, from the Beginning of Aries, for that will be the Whole Journey of the Moon, before it Meets, or is in Conjunction, with the Sun, and the Meta will be at $10\frac{3}{4}$ Degrees of Cancer; For it is 90 Degrees from the Beginning of Aries to the Beginning of Cancer, and $10\frac{3}{4}$ beyond, but $90+10\frac{3}{4}=100\frac{3}{4}$ Degrees.

5. The Sun's Progreſſion therefore in

$$\overset{\text{Days}}{3}+\overset{\text{Days}}{7}+\overset{\text{Day}}{\tfrac{3}{4}}$$

was

$$\overset{\text{Degr.}}{3}+\overset{\text{Degr.}}{7}+\overset{\text{Degr.}}{\tfrac{3}{4}}=\overset{\text{Degr.}}{10\tfrac{3}{4}}.$$

The Moon's Progreſſion in the ſame Time.

$$\overset{\text{Degr.}}{0}+\overset{\text{Degr.}}{7\times13},=91,+\overset{\text{Degr.}}{9\tfrac{3}{4}}=\overset{\text{Degr.}}{100\tfrac{3}{4}}.$$

EXAMPLE 2. If *A* Begins Firſt to Move, Suppoſing the Sun Proceeds every Day One Degree in the Eclipticк, and the Moon Thirteen, and the Moon at any Certain Time ſhould be in the Beginning of Aries, and after Three Days, the Sun ſhould be in the Beginning of Cancer, the Place of the Following Conjunction is Required.

1. Let the Moon be *A*, and the Sun *B*, as before, and ſince they both Tend the ſame Way, and *A*, or the Moon, begins to Move Firſt, the Length of the Moon's Journey, or it's Diſtance from the Meta, or Place of Conjunction, will be, by the 4th. and 9th. Steps.

$$x=\frac{cdh-cge}{fd-cg}=\frac{dh-ge}{\frac{d}{c}f-g}.$$

2. By the 1ſt. 2d. and 3d. Steps, and by the Preſent Example, as before, there being only a Change of the Signs.

$$\overset{\text{Degr.}}{c=13};\ \overset{\text{Day}}{f=1};$$
$$\overset{\text{Degr.}}{d=1};\ \overset{\text{Day}}{g=1};$$
$$\overset{\text{Degr.}}{e=90};\ \overset{\text{Days}}{h=3}.$$

3. And therefore by Subſtituting the Known Numbers in the Room of the Letters.

$$x=\frac{13\times1\times3-13\times1\times90}{1\times1-13\times1}$$

$$=\frac{39-1170}{1-13}=\frac{-1131}{-12}=94\tfrac{1}{4}\text{ Degrees.}$$

4. Lay $94\frac{1}{4}$ Degrees upon the Ecliptick from the Beginning of Aries, For that will be the Whole Journey of the Moon, before it comes to the Meta, or it's Conjunction with the Sun, and the Meta will Fall, at $4\frac{1}{4}$ Degrees of Cancer, Becauſe, from the Beginning of Aries, to the Beginning of Cancer, is 90 Degrees.

$$90+4\tfrac{1}{4}=94\tfrac{1}{4}\text{ Degrees.}$$

5. The Moon's Progreſſion therefore in

$$3+4+\tfrac{1}{4}\text{ Days.}$$

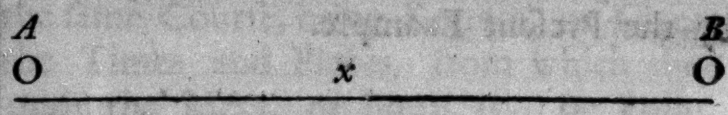
was

$$3\times13+4\times13+\tfrac{1}{4}\times13=94\tfrac{1}{4}\text{ Degrees.}$$

The Sun's Progreſſion in the ſame Time was

$$0+4+\tfrac{1}{4}\text{ Degrees.}$$

CAS. 2. If the Two Mobile's, or Moveables, do not Tend the Same, but Contrary Ways, that is, Meet each Other in *x*.



1. Let *Ax*, or the Diſtance of *A* from the Meta *x*, which is Unknown, be. $x.$

2. Let the Whole Line *AB*, or the Diſtance of *A* from *B*, which is Known, or Given, be. $e.$

3. Therefore the Diſtance of *B* from *x* will be. $e-x.$

4. The Celerities of the Two Moveables *A* and *B* are Known or Given, Namely, *A* Paſſes thro' the Space *c*, in the Time *f*, and *B* thro' the Space *d*, in the Time *g*, as before.

Space. $c.$ Time. $f.$ of *A*.
Space. $d.$ Time. $g.$ of *B*.

5. The Interval or Diſtance of the Times, from which they Begin their Motion, is as before. $h.$

6. The Time, in which *A* Performs it's Space or Journey, *x*, is. $\frac{fx}{c}$; for, $c : x :: f : \frac{fx}{c}$.

7. The Time, in which *B* Performs it's Space or Journey, $e-x$, is. $\frac{ge-gx}{d}$; for $d : g :: e-x : \frac{ge-gx}{d}$.

8. To the Leſſer of which Times Add *h* the Interval of them, and they will become Equal; That is, if *B* Begins Firſt to Move, the Time of *A* will be the Leaſt, to which Add *h*, and they will be Equal. $\frac{fx}{c}+h=\frac{ge-gx}{d}$.

And by Multiplying, Tranſpoſing, and Dividing, as before.

$dfx+cdh=cge-cgx$, and,
$dfx+cgx=cge-cdh$, or,
$x=\frac{cge-cdh}{cg+df}$.

9. If *A* Begins Firſt to Move, the Time of *B* will be the Leaſt, to which Add *h*, and they will be Equal. $\frac{ge-gx}{d}+h=\frac{fx}{c}$.

And by Multiplying, Tranſpoſing, and Dividing, as before.

$cge-cgx+cdh=dfx$, or,
$dfx+cgx=cge+cdh$, and,
$x=\frac{cge+cdh}{cg+df}$.

EXAMPLE. 1. If *A* Begins to Move Firſt, the Firſt Example is That of the Two Letter-Carriers, and is Exactly the Same, as before Set Down; In which,

1. Since they are Two Mobile's, which Tend Contrary Ways, or Meet each other, and *A* Begins to Move Firſt, his Journey will be, according to the 9th. Step. $x=\frac{cge+cdh}{cg+df}$.

2. And by Subſtituting the Known Numbers in the Room of the Letters, which Expreſs the Known Quantities.

$c=7$; $f=2$; $d=8$; $g=3$; $e=59$; $h=1$;

The Journey of *A* is.

$$x=\frac{7\times3\times59+7\times8\times1}{7\times3+8\times2}$$
$$=\frac{1239+56}{21+16}=\frac{1295}{37}=35 \text{ Miles.}$$

The Journey of *B*. $e-x=59-35=24$ Miles.

3. The Progreſſion of *A* in the Time $1+9$ Hours.
was in Length. $3\frac{1}{2}+31\frac{1}{2}=35$ Miles.

The

The Progreſſion of *B* in Length, in the ſame Time, was	$0+24=3\times 8$ Miles.
Example 2. If *B* Begins to Move Firſt.	
1. The Journey of *A* will be.	$x=\frac{cge-cdh}{cg+df}$.
2. And by Subſtitution of the Known Numbers.	$x=\frac{7\times 3\times 59-7\times 8\times 1}{7\times 3+8\times 2}=\frac{1183}{37}=31\frac{36}{37}$ Miles.
And the Journey of *B*.	$e-x=59-31\frac{36}{37}=27\frac{1}{37}$ Miles.
3. The Progreſſion of *A* will be in Time.	$1+9+\frac{5}{37}$ Hours.
in Length, in that Time.	$0+31\frac{1}{2}+\frac{17\frac{1}{2}}{37}$ Miles.
Becauſe	$9\times 3\frac{1}{2}=31\frac{1}{2}$; and $\overset{\text{Mil.}}{3\frac{1}{2}}:\overset{\text{Hour.}}{1}::\overset{\text{Mil.}}{\frac{17\frac{1}{2}}{37}}:\overset{\text{Hour.}}{\frac{5}{37}}$.
The Progreſſion of *B* in the ſame Time.	$2\frac{2}{3}+24+\frac{40}{111}$ Miles.
Becauſe	$2\frac{2}{3}=\overset{\text{Mil.}}{\frac{8}{3}}:\overset{\text{Hour.}}{1}::\overset{\text{Mil.}}{\frac{40}{111}}.\overset{\text{Hour.}}{\frac{5}{37}}$.
But	$\frac{40}{111}=\frac{1}{3}+\frac{1}{37}$ Miles.

I ſhall Conclude this Problem with the Canon, Belonging to this Laſt Example of it; Which will Shew, into what Prodigious and Stupendous Depths Algebra Leads us, which yet is only a Fourth Caſe of the General Problem, and into which it would be Impoſſible for the Greateſt Genius amongſt Men to Penetrate, tho' Endued with the Utmoſt Sagacity; And which Canon is no more than Tranſlating the Algebraick Expreſſions of *x*, in the Laſt Inſtance, into Thoſe of our own Language.

Canon. If there are Two Mobilia, or Moveables, *A* and *B*, which Tend in their Motion to each other, and we Multiply the Space, which *A* Performs in a Time Given, into the Time, in which *B* Performs a Space Given, and we farther Multiply that Product into the Interval or Diſtance of Places, from which they Begin their Motion; And from this Laſt Product, if we Subſtract Another, which is Formed from Multiplying the Space, which *A* Performs in a Time Given, into the Space, which *B* Performs in a Time Given, and That likewiſe into the Interval of Times, from which they Begin their Motion, and then Divide the Difference of the ſaid Products by the Sum, which Ariſes from Multiplying the Space, which *A* Performs in a Given Time, into the Time, in which *B* Performs a Given Space, Added to the Product, which is Made from Multiplying the Space which *B* Performs in a Given Time, into the Time, in which *A* Performs a Given Space; The Quotient will Exhibit the Length of the Line, or Space, which *A* will Perform, or Paſs thro', before it Meets with *B*; If *B* Begins it's Motion Firſt. I have Said, that no Human, tho' the moſt Exalted Genius, could ever Poſſibly Arrive at ſuch a Concluſion, without the Aſſiſtance of Algebra; Angelick Natures yet may Comprehend all this at One View. From whence we may in ſome Meaſure Form an Imagination, how Vaſt and Aſtoniſhing the Divine Knowledge muſt be, which Contemplates at Once all the Mutual Relations, Proportions, and Dependences of Things, in Univerſal Being; That is, Ten Millions of Millions of Millions more, than are Comprized or Contained in the Preſent Theorem.

Of Diviſion.

§. 6. As Multiplication is a Repeated Addition of the ſame Quantity, or Number, according to the Preſcription of the Multiplicator; So Diviſion is a Repeated Subſtraction of the ſame Quantity, or Number, according to the Preſcription of the Quotient. Or, Diviſion is Performed by Inquiring, how many Times the Diviſor is Contained in the Dividend, and Writing down ſo many Units in the Quotient; and this is to be done, as often, as the Diviſor can be Subſtracted from it; Thus in Integrals $\frac{12}{3}=4$, or 3) 12 (4, where the Quotient 4 Shews, that 3 the Diviſor may be 4 Times Subſtracted from, or is 4 Times Contained in 12 the Dividend; So likewiſe 300) 1200 (4, Shews, that 300 is 4 Times Contained in, or may be Subſtracted from 1200, *&c.* In Decimals $\frac{3}{10}$) $\frac{12}{10}$ (4, or $\frac{3}{100}$) $\frac{12}{100}$ (4, *&c.* Declares or Expreſſes the ſame of the Tenth, or the Hundreth Parts of a Unit, *&c.* which was before done of Units, or Hundreds, *&c.* Likewiſe in Vulgar Fractions of the ſame Denomination, it is the ſame,

ſame as in Decimals, $\frac{2}{5})\ \frac{12}{5}\ (4$, Explains, that $\frac{2}{5}$ are 4 Times Contained in, or may be Subſtracted from $\frac{12}{5}$; and in thoſe of Different Denominations, $\frac{2}{5})\ \frac{6}{35}\ (\frac{3}{7}$ Shews, that the Numerator 2 may be 3 Times Subſtracted from 6, and the Denominator 5, 7 Times from 35, that is, that $\frac{2}{5}$ may be Subſtracted $\frac{3}{7}$ from $\frac{6}{35}$. Nor is it otherwiſe in Algebraick Quantities $\frac{ax}{a}=x$, or $a)\ ax\ (x$, Signifies, that a is x Times Contained in, and therefore may be x Times Subſtracted from ax, for $ax-ax=0$. It is alike in the Algebraick Signs; $\frac{+}{+}=+$, or $+)\ +\ (+$, that is, + Divided by + makes + in the Quotient; Becauſe it Denotes, that + in the Diviſor is ſo many Times Poſitively Subſtracted in the Quotient from + in the Dividend. $\frac{-}{-}=+$, or $-)\ -\ (+$, that is, − Divided by − makes + in the Quotient; Becauſe it Shews, that − in the Diviſor is ſo many Times Poſitively Subſtracted in the Quotient from − in the Dividend. $\frac{-}{+}=-$, or $+)\ -\ (-$, that is, − Divided by + makes − in the Quotient; Becauſe it Signifies, that + in the Diviſor is ſo many Times Negatively Subſtracted in the Quotient from − in the Dividend, or that − in the Dividend Contains + in the Diviſor Negatively ſo many Times. Laſtly $\frac{+}{-}=-$, or $-)\ +\ (-$, that is, + Divided by − makes − in the Quotient; Becauſe it Means, that − in the Diviſor is ſo many Times Negatively Subſtracted in the Quotient from + in the Dividend, or that + in the Dividend Contains − in the Diviſor Negatively ſo many Times.

FROM whence, Compared with what goes before in the Multiplication of theſe Signs, will Reſult this Short and General Rule in Reſpect of them; That Multiply or Divide with like Signs, and the Product or Quotient will be +, if with unlike Signs, −; For $+\times+$, or $-\times-$, or $\frac{+}{+}$, or $\frac{-}{-}$, $=+$; And $+\times-$, or $-\times+$, or $\frac{+}{-}$, or, $\frac{-}{+}=-$; Or, which is the ſame, $+\times+$, or $-\times-$, or $+)\ +$, or $-)\ -$, $=+$; And $+\times-$, or $-\times+$, or $-)\ +$, or $+)\ -$, $=-$; The Examples Follow.

1ſt. IN Integrals and Decimals; How many Times 7 in 37? Anſwer, 5 Times. Write 5 in the Quotient, which Multiply into 7 the Diviſor, and Subſtract the Product 35, from the Dividend 37, and there Remains 2, to which bring down 1 in the Dividend, and Add it to 2 or 20, and then Proceed as before. And ſo in the Second Example, in which after a Diviſion is made of the whole Number, there Remains 14 to be Divided by 23, which may either be Expreſſed by a Vulgar Fraction $\frac{14}{23}$, or by a Decimal 6086, &c. by an Addition of ſo many Cyphers to the Dividend, and Continuing the Diviſion, and then Cutting off ſo many Decimal Places in the Quotient, as there are Cyphers Added in the Dividend, becauſe the Diviſor is a whole Number.

```
7) 371 (53
   35
   ---
    21
    21
   ---
     0
```

```
23)4798,0000(208,14/23=,6086&c.
   46
   ---
    19
    00
    ---
    198
    184
    ----
     140
     138
     ---
       20
       00
      ---
       200
       184
       ---
        160
```

THUS likewiſe, - - - - - - - - In which Firſt Example of Decimals, Diviſion is Performed Exactly, as in the Second or in Integrals; Excepting, that after Diviſion, there muſt be ſo many Decimal Places Cut off by a Comma, or ſome other Note of Diſtinction, which together, with thoſe in the Diviſor, may be Equal to the Number of Decimal Places in the Dividend; For there are 4 Decimal Places Expreſſed in the Dividend, and 2 Underſtood, by ſo many

```
46,1) 3,5218 (0,07639,        9043) 20844115 (2305
      3227                          18086
      ----                          -----
       2948                          27581
       2766                          27129
       ----                          -----
        1820                          45215
        1383                          45215
        ----                          -----
         4370                             0
```

many Cyphers being Annexed in the Operation, and there is 1 Decimal Place in the Diviſor, and Conſequently there muſt be 5 Decimal Places in the Quotient; The Reaſon is, becauſe the Diviſor and Quotient in Diviſion, Anſwer to the Multiplicator and Multiplicand in Multiplication, and the Dividend to the Product; But it was before Shewn, in Order to Preſerve the Places, that there muſt be ſo many Decimals in the Product, as there are in the Multiplicator and Multiplicand, and therefore there muſt be ſo many in the Diviſor and Quotient, as there are in the Dividend; For Diviſion Reſolves, what Multiplication Compounds;

```
Alſo, 72,4)2099,6(29   50,18)137,995(2,75   0,0132)0,051513(3,
          1448               10036                  396
          ----               -----                  ----
           6,16               37635                 1191
           6516               35126                 1188
           ----               -----                 ----
             0                 2,090                  33
                               25090                  00
                               -----                 ---
                                 0                    330
                                                      264
                                                      ---
                                                       66
                                                       66
                                                       --
                                                        0
```

In all which Inſtances or Examples of Decimals, Diviſion is Performed in the ſame Manner, as in Integrals; Excepting, as before, that after Diviſion, there muſt be ſo many Decimal Places Cut off in the Quotient, which together with thoſe in the Diviſor, ſhall be Equal to the Number of Decimal Places in the Dividend; Thus in the First Inſtance, there is 1 Decimal Place in the Diviſor and Quotient, = to 1 in the Dividend; In the Second Inſtance, there are 2 in the Diviſor, and 2 in the Quotient = to 3 Expreſſed in the Dividend + 1 Underſtood by Annexing a Cypher to it in the Operation; In the Third Inſtance, there are 4 in the Diviſor, and 4 in the Quotient, = to 6, Expreſſed in the Dividend, and 2 Underſtood by Annexing 2 Cyphers to it in the Operation.

2. In Common or Vulgar Fractions Divide, for the ſame Reaſon, the Numerator by the Numerator, and the Denominator by the Denominator; Thus $\frac{2}{5})\ \frac{6}{35}\ (=\frac{3}{7}$; $\frac{a}{b})\ \frac{ac}{bd}\ (=\frac{c}{d}$; $\frac{3acy}{2bb})-\frac{21accy3}{8b5}(=-\frac{7cyy}{4b3}$; In which, and in all other Caſes, the Signs are to be Divided by the Signs, the Numbers by the Numbers, and the Species, or Letters, by the Species, or Letters, the Roots by the Roots, the Numerators by the Numerators, and the Denominators, by the Denominators.

Thus likewiſe $\frac{c}{d})\ \frac{a}{b}\ (=\frac{ad}{bc}$; and $\frac{5}{4})\ \frac{3}{7}\ (=\frac{12}{35}$; Becauſe theſe Kind of Fractions, which are not Reducible by the Former Method to any other Terms, are Firſt to be Reduced to the ſame Denomination, by Multiplying 7 the Denominator of the Dividend into 5 and 4, the Numerator and Denominator of the Diviſor, and 4 the Denominator of the Diviſor, into 3 and 7 the Numerator and Denominator of the Dividend, and by which $\frac{5}{4}$ becomes = to $\frac{35}{28}$, and $\frac{3}{7}$ = to $\frac{12}{28}$, for Ratios Multiplied by Equal Quantities, as was before Said, are ſtill Equal; And afterwards the Numerator of the Dividend 12 is to be Divided by the Numerator of the Diviſor 35, according to the Principles before Explained; and the ſame is the Caſe of $\frac{c}{d})\ \frac{a}{b}\ (=\frac{ad}{bc}$; Which alſo is Applicable to any other Quantities, however to Appearance Intricate and Involved.

3*d.* In Radicals, $\sqrt{3})\ \sqrt{15}\ (=\sqrt{5}$; $\sqrt{cd})\ \sqrt{abcd}\ (=\sqrt{ab}$; $\sqrt{^3},\ 5ayy)\ \sqrt{^3},35aay^3z\ (=\sqrt{^3},7ayz$; $\frac{\sqrt{a^3}}{c})\ \frac{\sqrt{a^4bb}}{cc}\ (=\frac{\sqrt{abb}}{c}$ which is Proved by what Foregoes in this Section, and by Multiplication in the Former, which Compounds, what Diviſion Reſolves; Thus the $\sqrt{3}\times\sqrt{5}=\sqrt{15}$; $\sqrt{cd}\times\sqrt{ab}=\sqrt{abcd}$, &c.

4*th.* In Algebraick Quantities, which Conſiſt of ſeveral Parts or Members, we are to Divide or Subſtract all the Parts of the One, by or from all the Parts of the Other, if Poſſible till there is no Remainder, Obſerving the Rule Given under the

Head of Vulgar Fractions, Article 2*d.* of Dividing the Signs by the Signs, the Radical Notes by the Radical Notes, the Numbers by the Numbers, the Numerators by the Numerators, the Denominators by the Denominators, and the Species by the Species; In which Laſt, as the Juxta-Poſition of the Letters Signified the Multiplication of them, ſo the Separation of them from each other, Signifies the Diviſion of them, that is, as $aaa = a \times a \times a$, ſo, $a)\ aaa\ (= aa = \frac{aaa}{a}$, *&c.*

It muſt be Farther Remarked, that the Terms of the Dividend and the Diviſor are to be Ranked and Diſpoſed, according to the Dimenſions of ſome one Letter in both Quantities, Placing the Higheſt Dimenſion Firſt, and the Loweſt Laſt, and Suitably in the Intermediate Places, as in Numbers 1000, 100, 10 and 1 Succeed each other, Thus,

$$\begin{array}{l} a)\ aa+3ax-xx\ \left(=a+3x-\frac{xx}{a}\right. \\ \underline{\quad aa\quad} \\ \quad 0+3ax \\ \quad \underline{+3ax} \\ \qquad 0-xx \\ \qquad \underline{-xx} \\ \qquad\quad 0 \end{array} \qquad \begin{array}{l} a-b)\ a^3+2aac \qquad (=aa+2ac-bc \\ \qquad \underline{-aab-3abc+bbc} \\ \underline{a^3-aab} \\ 0+2aac-3abc \\ \underline{\quad 2aac-2abc} \\ \qquad 0-abc+bbc \\ \qquad \underline{-abc+bbc} \\ \qquad\quad 0 \end{array}$$

In which Inſtances, the Way of Proceeding in the Firſt, where the Diviſor Conſiſts of a Single Member only, has been already Explained; For it is no more than Dividing aa, then $3ax$, and afterwards $-xx$, by a, and Multiplying each Quotient into the Diviſor, and Subſtracting the Product from the Dividend; In the Second, where the Diviſor Conſiſts of Two, or More Members, the Letter or Species a, according to which the Dimenſions are Ranged, a^3, a^2, a^1, a^0, is made Uſe of as the Diviſor, and the Quotient is Multiplied into both, or the ſeveral Terms of it, and the Products Subſtracted from the Dividend; Which is Agreeable to what is Practiced in Numbers; As 23) 4798 (208, *&c.* where we Divide 47 by 20, and the Quotient is 2, which yet is to be Multiplied into 23, and the Product 46 to be Subſtracted from the Dividend 47, and ſo of the Reſt, according to the Example before Alledged; To Explain this Farther, the Following Examples or Inſtances are Annexed.

$$\begin{array}{l} \text{As, } -b+a)\ cbb\begin{matrix}-3ac\\-aa\end{matrix}\cdot b\begin{matrix}+a^3\\+2aac\end{matrix}\ \left(-cb\begin{matrix}+2ac\\+aa\end{matrix}\right. \\ \qquad\qquad \overline{\underline{cbb-acb\qquad}} \\ \qquad\qquad\ 0\begin{matrix}-2ac\\-aa\end{matrix}b\begin{matrix}+a^3\\+2aac\end{matrix} \\ \qquad\qquad \overline{\quad\ \begin{matrix}-2ac\\-aa\end{matrix}b\begin{matrix}+2aac\\+a^3\end{matrix}} \\ \qquad\qquad \overline{\qquad\quad 0\qquad 0\qquad} \end{array}$$

$$\begin{array}{l} \text{So, } \begin{matrix}yy-aa\\-cc\end{matrix})\ y^6\begin{matrix}+aa\\-2cc\end{matrix}y^4\begin{matrix}-a^4\\+c^4\end{matrix}yy\begin{matrix}-a^6\\-2a^4cc\\-aac^4\end{matrix}\ \left(y^4\begin{matrix}+2aa\\-cc\end{matrix}yy\begin{matrix}+a^4\\+aacc\end{matrix}\right. \\ \qquad \overline{y^6\begin{matrix}-aa\\-cc\end{matrix}y^4\qquad} \\ \qquad 0\begin{matrix}+2aa\\-cc\end{matrix}y^4\begin{matrix}-a^4\\+c^4\end{matrix}yy \\ \qquad \overline{\ \begin{matrix}+2aa\\-cc\end{matrix}y^4\begin{matrix}-2a^4\\-2aacc\\+aacc\\+c^4\end{matrix}yy} \\ \qquad \overline{\qquad 0\qquad\qquad} \end{array}$$

$$\begin{array}{lll}
0 & +a^4 & -a^6 \\
 & +aacc^{yy} & -2a^4cc \\
 & & -aac^4 \\ \hline
 & +a^4 & -a^6 \\
 & +aacc^{yy} & -a^4cc \\
 & & -a^4cc \\
 & & -aac^4 \\ \hline
 & 0 & 0
\end{array}$$

Thus alſo, $a+b)\ aa*-bb\ (a-b$

$$\begin{array}{lll}
 & aa & +ab \\ \hline
 & 0 & -ab-bb \\
 & & -ab-bb \\ \hline
 & & 0 \quad 0
\end{array}$$

Likewiſe, $yy-2ay+aa)\ y^4*-3\tfrac{1}{2}aayy+3a^3y-\tfrac{1}{2}a^4\ (yy+2ay-\tfrac{1}{2}aa$

$$\begin{array}{l}
y^4-2ay^3+aayy \\ \hline
0\ +2ay^3-4\tfrac{1}{2}aayy \\
\quad +2ay^3-4aayy+2a^3y \\ \hline
\qquad 0\ -\tfrac{1}{2}aayy+a^3y-\tfrac{1}{2}a4 \\
\qquad\quad -\tfrac{1}{2}aayy+a^3y-\tfrac{1}{2}a4 \\ \hline
\qquad\qquad 0 \quad 0 \quad 0
\end{array}$$

Laſtly, $aa+ab\sqrt{2}+bb)\ a^4 * * * +b4\ (aa-ab\sqrt{2}+bb$

$$\begin{array}{l}
a^4+a^3b\sqrt{2}+aabb \\ \hline
0-a^3b\sqrt{2}-aabb \\
\quad -a^3b\sqrt{2}-2aabb-ab^3\sqrt{2} \\ \hline
\quad 0 \qquad +aabb+ab^3\sqrt{2}+b^4 \\
\qquad\qquad +aabb+ab^3\sqrt{2}+b4 \\ \hline
\qquad\qquad 0 \quad 0 \quad 0
\end{array}$$

PROBL. 7. The Power or Force of any Agent being Given, to Find out, how many of ſuch Kind of Agents will Produce a Given Effect *a* in a Given Time *b*. *Probl. the 7th.*

1. The Power of any Agent Given is to Produce a Given Effect *c* in a Time Given *d*.	$\overset{\text{Effect.}}{c}.\ \overset{\text{Time.}}{d}.$ of 1 Agent.
2. A Certain Unknown Number of ſuch Agents or *x*, will Produce a Given Effect *a*, in a Time Given *b*.	$\overset{\text{Effect.}}{a}.\ \overset{\text{Time.}}{b}.$ of x Agents.
3. The Effect therefore of the ſaid Agent in the Time *b* will be by Proportion.	$\frac{bc}{d}$. becauſe $\overset{\text{Time.}}{d}.\ \overset{\text{Eff.}}{c} :: \overset{\text{Time.}}{b}.\ \overset{\text{Eff.}}{\frac{bc}{d}}$.
4. The Effect of *x* ſuch Agents will be likewiſe by Proportion.	$\frac{ad}{bc}$. becauſe. $\overset{\text{Eff.}}{\frac{bc}{d}}.\ \overset{\text{Eff.}}{a} :: \overset{\text{Ag.}}{1}.\ \overset{\text{Ag.}}{x}=\frac{ad}{bc}$.
EXAMPLE, If a Clerk, or a Scrivener, can in 8 Days Write out 15 Folio Leaves or Pages, how many ſuch Clerks, or Scriveners, are Required to Write out 405 ſuch Folio Leaves or Pages in 9 Days? Anſw. 24. For,	$a=405,\ c=15,\ d=8,\ b=9.$ $\frac{ad}{bc}=\frac{405\times 8}{9\times 15}=\frac{3240}{135}=24.$
And, that This is a True Solution is thus Proved by Proportion. Becauſe	$\overset{\text{Days.}}{8}.\ \overset{\text{Fol.}}{15} :: \overset{\text{Days.}}{9}.\ \overset{\text{Fol.}}{16\tfrac{7}{8}}$; and, $\overset{\text{Ag.}}{1}.\ \overset{\text{Fol.}}{16\tfrac{7}{8}} :: \overset{\text{Ag.}}{24}.\ \overset{\text{Fol.}}{405}.$

PROBL. 8. The Powers or Forces of ſeveral Agents being Given, to Determine the Unknown Time *x*, in which Together and Jointly they will Produce the Given Effect *d*. *Probl. the 8th.*

1. Let the ſeveral Agents be *A*, *B*, *C*, and let *A* Produce it's Effect *a* in the Time *e*.

$\overset{\text{Effect.}}{a}.\quad \overset{\text{Time.}}{e}.$ of A the Agent.

2. *B* Produces it's Effect *b* in the Time *f*.

$\overset{\text{Effect.}}{b}.\quad \overset{\text{Time.}}{f}.$ of B the Agent.

3. *C* Produces it's Effect *c* in the Time *g*.

$\overset{\text{Effect.}}{c}.\quad \overset{\text{Time.}}{g}.$ of C the Agent.

4. *A B*, *C*, Produce their Effect *d* in the Time *x*.

$\overset{\text{Effect.}}{d}.\quad \overset{\text{Time.}}{x}.$ of A, B, C, the Agents.

5. The Whole Effect therefore of *A* will be by Proportion.

$\frac{ax}{e}$. because, $\overset{\text{Time.}}{e} : \overset{\text{Eff.}}{a} :: \overset{\text{Time.}}{x} : \overset{\text{Eff.}}{\frac{ax}{e}}$ of A.

6. The Whole Effect of *B* by Proportion is.

$\frac{bx}{f}$. because, $\overset{\text{Time.}}{f} : \overset{\text{Eff.}}{b} :: \overset{\text{Time.}}{x} : \overset{\text{Eff.}}{\frac{bx}{f}}$ of B.

7. The Whole Effect of *C* for the ſame Reaſon will be.

$\frac{cx}{g}$. because, $\overset{\text{Time.}}{g} : \overset{\text{Eff.}}{c} :: \overset{\text{Time.}}{x} : \overset{\text{Eff.}}{\frac{cx}{g}}$ of C.

8. But by the Problem, the Effects of *A*, *B*, *C*, are Equal to *d*. That is,

$\frac{ax}{e} + \frac{bx}{f} + \frac{cx}{g} = d$ Effect.

9. And by Dividing both Sides of the Equation by the ſame Quantity.

$x = \frac{d}{\frac{a}{e} + \frac{b}{f} + \frac{c}{g}}$ Time.

Example. Three Day-Labourers, *A*, *B*, *C*, can each of them Perform a Certain Work or Task Given in a Time Given;

A can Perform it once in Three Weeks.
B Thrice in Eight Weeks.
C Five Times in Twelve Weeks.

The Queſtion is, in what Time, they can all of them together Perform the ſame Work or Task? Anſw. in 6 Days and $5\frac{1}{3}$ Hours; For,

$a=1$, $b=3$, $c=5$, $d=1$, $e=3$, $f=8$, $g=12$;

$x = \frac{d}{\frac{a}{e} + \frac{b}{f} + \frac{c}{g}} = \frac{1}{\frac{1}{3} + \frac{3}{8} + \frac{5}{12}}$ Weeks,

$= \frac{1}{\frac{324}{288}} = \frac{288}{324} = \frac{8}{9}$ of a Week; But

$\frac{1}{9}$ of a Week $= 18\frac{2}{3}$ Hours, and
$\frac{8}{9}$ of a Week $= 144\frac{16}{3}$ Hours,
$=6$ Days $5\frac{1}{3}$ Hours.

And that this Solution is True, is Proved by the 5th. and 9th. Steps, and by the Preſent Example, ſince the Whole Effect of *A* is.

$\frac{ax}{e} = \frac{8}{27}$; for, $\overset{\text{Weeks.}}{3}.\ \overset{\text{Eff.}}{1}. :: \overset{\text{Weeks.}}{\frac{8}{9}}.\ \overset{\text{Eff.}}{\frac{8}{27}} = \frac{8}{27}$ Effect.

The Whole Effect of *B* by the 6th. and 9th. Steps, and by the Preſent Example.

$\frac{bx}{f} = \frac{9}{27}$; for, $\overset{\text{Weeks.}}{8}.\ \overset{\text{Eff.}}{3}. :: \overset{\text{Weeks.}}{\frac{8}{9}}.\ \overset{\text{Eff.}}{\frac{24}{72}} = \frac{9}{27}$ Effect.

The Whole Effect of *C* by the 7th. and 9th. Steps, and by the Preſent Example.

$\frac{cx}{g} = \frac{10}{27}$; for, $\overset{\text{Weeks.}}{12}.\ \overset{\text{Eff.}}{5} :: \overset{\text{Weeks.}}{\frac{8}{9}}.\ \overset{\text{Eff.}}{\frac{40}{108}} = \frac{10}{27}$ Effect.

But all theſe Effects ought to be Equal to *d*, or 1. Which they are, For

$\frac{8}{27} + \frac{9}{27} + \frac{10}{27} = \frac{27}{27} = 1$ Effect.

Probl. the 9th. Probl. 9. To Compound the Diſſimilar Mixtures of Two or more Subſtances or Things ſo, that the Subſtances or Things, which are Mixed, ſhould Obtain a Given Reaſon amongſt each other.

1. The Three Subſtances, or Things Mixed, are.

$A.\ B.\ C.$

2. The Quantities or Proportions of the Subſtances, or Things Mixed, in the Firſt Diſſimilar Mixture are

$dA + eB + fC$.

3. The Quantities or Proportions of them in the Second Diſſimilar Mixture.	$gA+hB+kC.$
4. The Quantities or Proportions of them in the Third Mixture.	$lA+mB+nC.$
5. Let the Mixture, which ought, or is Required, to be Compounded from theſe Three Mixtures or Maſſes be a Known Quantity.	$pA+qB+rC.$
6. Let *x*, *y*, *z*, be the Unknown Numbers, into which if the Given Mixtures are ſeverally Multiplied, The Product will Exhibit the Compoſition or Mixture Required.	$\left.\begin{array}{l} xdA+xeB+xfC+ \\ ygA+yhB+ykC+ \\ zlA+zmB+znC \end{array}\right\rvert = pA+qB+rC.$
7. By Taking the Quantities, which are Multiplied into *A*, *B*, and *C*.	$dx+gy+lz=p;\ ex+hy+mz=q;\ fx+ky+nz=r.$
8. By Tranſpoſing the Terms, and Dividing both Sides of the Equation by the ſame Quantity.	$x=\frac{p-gy-lz}{d}=\frac{q-hy-mz}{e}=\frac{r-ky-nz}{f}.$
9. By Reduction.	$\frac{p-gy-lz}{d}=\frac{q-hy-mz}{e}$ and $\frac{q-hy-mz}{e}=\frac{r-ky-nz}{f}$ Make $\frac{ep-dq+dmz-elz}{eg-dh},=y,=\frac{fq-er+enz-fmz}{fh-ek}.$
10. By Abbreviation, and Putting α for *ep−dq*, β for *dm−el*, γ for *eg−dh*, δ for *fq−er*, ζ for *en−fm*, and θ for *fh−ek*.	$\frac{\alpha+\beta z}{\gamma}=\frac{\delta+\zeta z,}{\theta}$ and $z=\frac{\theta\alpha-\gamma\delta}{\gamma\zeta-\beta\theta}.$
11. *z* being Found.	$\frac{\alpha+\beta z}{\gamma}=y,$
12. And *y* and *z* being Found.	$\frac{p-gy-lz}{d}=x,$ which is therefore Known.

Example. If there are Three Mixtures of Metals Molten together, a Pound of the Firſt of which Contains 12 Ounces of Silver, 1 Ounce of Braſs, and 3 Ounces of Tin; a Pound of the Second Contains 1 Ounce of Silver, 12 Ounces of Braſs, and 3 Ounces of Tin; and a Pound of the Third Contains 0 Ounces of Silver, 14 Ounces of Braſs, and 2 Ounces of Tin; It is Required to Compound theſe Mixtures ſo, that a Pound of the New Compoſition ſhall Contain 4 Ounces of Silver, 9 Ounces of Braſs, and 3 Ounces of Tin.

1. $d, e, f, =12, 1, 3;\ g, h, k, =1, 12, 3;\ l, m, n, =0, 14, 2;\ p, q, r, =4, 9, 3.$
2. Therefore $\alpha=ep-dq=1\times4, -12\times9=-104.$ 3. $\beta=dm-el=12\times14, -1\times0=168.$
4. $\gamma=eg-dh=1\times1, -12\times12=-143.$ 5. $\delta=fq-er=3\times9, -1\times3=24.$
6. $\zeta=en-fm=1\times2, -3\times14=-40.$ 7. $\theta=fh-ek=3\times12, -1\times3=33.$
8. Conſequently $z=\frac{\theta\alpha-\gamma\delta}{\gamma\zeta-\beta\theta}=\frac{33\times-104,-, -143\times24}{-143\times-40,-, 168\times33}=0.$
9. Therefore $y=\frac{\alpha+\beta z}{\gamma}=\frac{-104+168\times0}{-143}=\frac{8}{11}.$
10. And $x=\frac{p-gy-lz}{d}=\frac{4-1\times\frac{8}{11}-0\times0}{12}=\frac{3}{11}.=\frac{q-hy-mz}{e}=\frac{r-ky-nz}{f}.$

From whence it Follows, that if we Take *x*, or $\frac{3}{11}$ths, of a Pound of the Firſt Mixture, and *y*, or $\frac{8}{11}$ths, of a Pound of the Second, and *z*, or 0, of the Third, we ſhall have a Pound of the Compoſition Required; For $\frac{3}{11}+\frac{8}{11}$ths. of a Pound$=1$ Pound; And that this will Solve the Queſtion, is Apparent from hence, becauſe, let *A* be Silver, *B* Braſs, and *C* Tin;

Then $dxA+gyA+lzA=pA$, or $12\times\frac{3}{11}+1\times\frac{8}{11}+0=4$ Ounces of Silver.
Alſo, $exB+hyB+mzB=qB$, or $1\times\frac{3}{11}+12\times\frac{8}{11}+0=9$ Ounces of Braſs.
Laſtly, $fxC+kyC+nzC=rC$, or $3\times\frac{3}{11}+3\times\frac{8}{11}+0=3$ Ounces of Tin.

Probl. the 10th. PROBL. 10. The Prices of ſeveral Mixtures of the ſame Things being Given, and the Proportions of the Things, which are Mixed, it is Required to Determine from thence the Price of Each.

1. The Things which are Mixed are.	$A.\ B.\ C.$
2. The Price of the Firſt Mixture with the Proportions of the Mixed is.	$dA+gB+lC=p.$
3. The Price of the Second Mixture with their Proportions is.	$eA+hB+mC=q.$
4. The Price of the Third Mixture.	$fA+kB+nC=r.$
5. Subſtitute inſtead of *A*, *B*, *C*, the Things, which are Mixed, their Unknown Prices, which are Required, *x*, *y*, *z*, then.	$dx+gy+lz=p, ex+hy+mz=q, fx+ky+nz=r.$
6. Proceeding therefore, as in the Foregoing Problem.	$\frac{\theta\alpha-\gamma\delta}{\gamma\zeta-\beta\theta}=z$; $\frac{\alpha+\beta z}{\gamma}=y$; and $\frac{p-gy-lz}{d}=x.$

EXAMPLE. A Certain Perſon Buys *d* or 40 Buſhels of *A* or Wheat, *g* or 24 Buſhels of *B* or Barley, and *l* or 20 Buſhels of *C* or Oats for *p* or 15 Pounds 12 Shillings; He afterwards Buys *e* or 26 Buſhels of Wheat, *h* or 30 Buſhels of Barley, and *m* or 50 Buſhels of Oats for *q* or 16 Pounds; and at a Third Time he Buys *f* or 24 Buſhels of Wheat, *k* or 120 Buſhels of Barley, and *n* or 100 Buſhels of Oats for *r* or 34 Pounds; The Queſtion is, what was the Price of each Buſhel of Wheat, Barley, and Oats? Anſw. A Buſhel of Wheat was 5 Shillings, of Barley 3 Shillings, and of Oats 2 Shillings.

1. $d, g, l, =40, 24, 20$; $e, h, m, =26, 30, 50$; $f, k, n=24, 120, 100$; $p, q, r=15\frac{3}{5}, 16, 34$;
2. $\alpha=ep-dq=26\times15\frac{3}{5}, -40\times16=-234\frac{2}{5}$; 3. $\beta=dm-el=40\times50, -26\times20=1480$;
4. $\gamma=eg-dh=26\times24, -40\times30=-576$; 5. $\delta=fq-er=24\times16, -26\times34=-500$;
6. $\zeta=en-fm=26\times100, -24\times50=1400$; 7. $\theta=fh-ek=24\times30, -26\times120=-2400$;
8. Therefore $z=\frac{\theta\alpha-\gamma\delta}{\gamma\zeta-\beta\theta}=\frac{562560-288000}{-806400+3552000}=\frac{274560}{2745600}=\frac{1}{10}$ of a Pound $=2$ Shills.
9. Conſequently $y=\frac{\alpha+\beta z}{\gamma}=\frac{-234\frac{2}{5}+148}{-576}=\frac{3}{20}$ of a Pound $=3$ Shills.
10. And $x=\frac{p-gy-lz}{d}=\frac{15\frac{3}{5}-\frac{18}{5}-2}{40}=\frac{1}{4}$ of a Pound $=5$ Shills.

Probl. the 11th. PROBL. 11. The Specifick Gravities of the Mixture, and of the Things which are Mixed, being Given, it is Required to find out the Proportion betwixt one another of the Things, which are Mixed.

1. Let the Specifick Gravity of the Mixture, $A+B$, be.	$e.$
2. The Specifick Gravity of *A*.	$a.$
3. That of *B* let it be alſo.	$b.$
4. Since the Abſolute Gravity, or Weight is Compounded or Conſiſts of the Specifick Gravity Multiplied into the Bulk, the Abſolute Weight of *A* will be.	$aA.$
5. The Abſolute Weight of *B* will be alſo.	$bB.$
6. And the Abſolute Weight of the Mixture $A+B$.	$eA+eB.$
7. Since therefore the Abſolute Gravity or Weight of the Parts muſt be Equal to the Abſolute Gravity or Weight of the Whole.	$aA+bB=eA+eB$
8. And by Tranſpoſition of the Terms.	$aA-eA=eB-bB$
9. And by Reſolving the Equation into it's Proportionals.	$e-b.\ a-e :: A.\ B.$

Example. Let the Specifick Gravity of Gold or *A*, be as 19, or *a*, of Silver or *B*, as $10\frac{1}{3}$, or *b*, and of their Mixture, or $A+B$, or Hiero's Crown, as 17, or *e*; and it is Required to Know, how much Gold and Silver was Uſed in ſuch a Mixture;

1. *A* and *B* are the Bulks of Gold and Silver, but $e-b.\ a-e :: A.\ B.$ or $17-10\frac{1}{3}=\frac{51-31}{3}=\frac{20}{3}.\ 19-17=2 :: 10.\ 3 :: A.\ B$; Therefore there were Ten Parts of Gold in the ſaid Crown for 3 of Silver.

2. Since $e-b$, and $a-e$ are as the Bulks of Gold and Silver in the Crown, or as *A* and *B*, Multiply *a* the Specifick Gravity of Gold $\times e-b$, and *b* the Specifick Gravity of Silver into $a-e$, and the Products will give the Proportion of the Weight of the Gold in the Crown to the Weight of Silver in the Crown; But $e-b.\ a-e :: 10.\ 3$, therefore $a\times e-b=19\times 10=190$, and $b\times a-e=10\frac{1}{3}\times 3=31$, and the Weight of the Gold will be to the Weight of the Silver in the Crown as 190. 31.

3. Therefore the Weight of the Crown will be to the Weight of the Silver, as $190+31=221.\ 31$.

Probl. the 12th.

Probl. 12. Suppoſing *a* Cows or Oxen can Graze or Feed up a Meadow *b* in a Certain Time *c*, and *d* Cows or Oxen can Graze a Meadow *e*, which is Equally Good, in the Time *f*; and the Graſs Grows Uniformly, it is Inquired, how many Cows or Oxen will Graze a like Meadow *g* in the Time *h*?

1. If *a* Oxen Graze a Meadow *b* in the Time *c*, then by Proportion $\frac{e}{b}a$ Oxen in the ſame Time *c* will Graze the Meadow *e*, becauſe by Proportion.

$$\overset{\text{Mead.}}{b}.\ \overset{\text{Ox.}}{a} :: \overset{\text{Mead.}}{e}.\ \overset{\text{Ox.}}{\frac{ea}{b}}$$

2. And in the Time *f*, $\frac{ec}{bf}a$ Oxen will Graze the ſaid Meadow *e*, Suppoſing the Graſs does not Grow after the Time *c*, becauſe by Proportion.

$$\overset{\text{Time}}{f}.\ \overset{\text{Time}}{c} :: \overset{\text{Ox.}}{\frac{ea}{b}}.\ \overset{\text{Ox.}}{\frac{eca}{bf}}$$

3. And in the Time *h*, $\frac{ec}{bh}a$ Oxen will Graze the Meadow *e*, Suppoſing the Graſs does not Grow after the Time *c*, becauſe by Proportion.

$$\overset{\text{Time}}{h}.\ \overset{\text{Time}}{c} :: \overset{\text{Ox.}}{\frac{ea}{b}}.\ \overset{\text{Ox.}}{\frac{eca}{bh}}$$

4. But ſince upon the Account of the Graſs's Growing, *d* Oxen, in the Time *f*, only Graze the Meadow *e*, therefore that Increaſe of the Graſs in the Meadow *e* in the Time $f-c$ will be ſuch, as to be Sufficient to Feed $d-\frac{eca}{bf}$ Oxen in the Time *f*, that is, ſuch as will be Sufficient to Feed $\frac{df}{h}-\frac{eca}{bh}$ Oxen in the Time *h*; for by Proportion. $\overset{\text{Time}}{h} : \overset{\text{Time}}{f} :: \overset{\text{Oxen}}{d-\frac{eca}{bf}} : \overset{\text{Oxen}}{\frac{df}{h}-\frac{eca}{bh}}$.

5. And in the Time $h-c$ the Increaſe of the Graſs in the Meadow *e* will be ſuch as to be Sufficient to Feed $\frac{h-c}{f-c}\times\frac{df}{h}-\frac{eca}{bh}$ Oxen, becauſe by Proportion.

$$\overset{\text{Time}}{f-c} : \overset{\text{Time}}{h-c} :: \overset{\text{Oxen}}{\frac{df}{h}-\frac{eca}{bh}} : \overset{\text{Oxen}}{\frac{bdfh-ecah-bdcf+aecc}{bfh-bch}}.$$

6. Add this Increment of Oxen in the 5th. Step to $\frac{aec}{bh}$ Oxen, in the 3d. Step, and the Sum will Give the Number of Oxen, for the Feeding of which the Meadow *e* is Sufficient for the Time *h*, That is, $\frac{bdfh-ecah-bdcf+ecfa.}{bfh-bch}$

7. And by Proportion, the Meadow *g* will be Sufficient to Feed $\frac{bdfgh-ecagh-bdcgf+ecfga}{befh-bceh}$ Oxen, in the ſame Time *h*.

$$\text{Becauſe } \overset{\text{Mead.}}{e} : \overset{\text{Mead.}}{g} :: \overset{\text{Oxen}}{\frac{bdfh-ecah-bdcf+ecfa}{bfh-bch}} : \overset{\text{Oxen}}{\frac{bdfgh-ecagh-bdcgf+ecfga}{befh-bceh}}.$$

EXAMPLE. If 12 Cows, or Oxen, can Graze or Feed up $3\frac{1}{3}$ Acres of a Meadow in 4 Weeks, and 21 Oxen can Graze 10 Acres of a like Meadow in 9 Weeks, it is Inquired, how many Oxen will Graze 36 Acres in 18 Weeks? The Anſwer is 36.

1. $a, b, c, d, e, f, g, h = 12, 3\frac{1}{3}, 4, 21, 10, 9, 36, 18$, Reſpectively.

2. Therefore, by Subſtituting the Numbers for the Letters, it will Follow, That $\frac{bdfgh-ecagh-bdcgf+ecfga}{befh-bceh}$ is Equal to 36.

3. Or this Problem may be Traced thro' all the Foregoing Proportions, and the Solution of it Given by Putting the Numbers, inſtead of the Letters, in the ſeveral Steps of the Preceding Proceſs; Which is done in Sir *Iſ. Newton*'s Algebra.

Probl. the 13*th.*

PROBL. 13. The Magnitudes, or Bulks, and the Motions being Given, of Two Spherical Bodies, Moving in the ſame Right Line, and Meeting each other, it is Required to Determine their Motions, after Reflection.

The Reſolution of this Problem Depends upon theſe Conditions, that both the Bodies ſhould Suffer ſo much by Reaction, as Each Acts upon the Other, and that they ſhould Recede from each Other with the ſame Celerity after Reflexion, with which they Approached each Other before it.

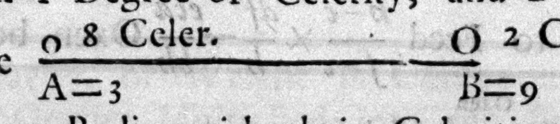
We have already Given an Account of this Problem in what we have Writ upon the Mechanical Philoſophy, and therefore ſhall not Repeat that Trouble here, and only Subjoin an Example or Two.

1. It has been there Shewn, that if the Celerities of *A* and *B* are *a*, *b*, and the Bodies *A*, and *B* Tend in their Motion the ſame Way, the Celerity of *A* after Reflexion will be $\frac{aA-aB+2bB}{A+B}$, and the Celerity of *B* after Reflexion will be $\frac{2aA-bA+bB}{A+B}$.

2. But, if they Meet each Other, then the Sign of *b* is every where to be Changed, and the Celerity of *A* after Reflexion will be $\frac{aA-aB-2bB}{A+B}$, and the Celerity of *B* after it, $\frac{2aA+bA-bB}{A+B}$.

And, if Either of theſe Quantities, or the Motion of *A* in the Former Inſtance, Happens to be Negative, it Shews, that the Motion after Reflexion Tends a Contrary Way to that of *A* before Reflexion.

EXAMPLE 1*ſt*. If Two Homogeneous Bodies *A* of 3 Pounds, and with 8 Degrees of Celerity, and *B* of 9 Pounds with 2 Degrees of Celerity, Tend the ſame Way, Then, $A, a, B, b, = 3, 8, 9, 2$, Reſpectively; From whence $\frac{aA-aB+2bB}{A+B}$ = Celerity of $A=-1$, and $\frac{2aA-bA+bB}{A+B}$ = Celerity of $B=5$; And therefore *A* after Reflexion will Recoil or Recede with 1 Degree of Celerity, and *B* will Proceed on with 5. Thus in the Preſent Scheme



EXAMPLE. 2*d*. If the Same Bodies with their Celerities Meet each other, *A* will Reſult with 7 Degrees of Celerity, and *B* will Proceed Forwards with 3.

And, if the Bulks, or *A* and *B*, are Equal, *A* will Proceed with 1 Degree of Celerity, and *B* with 2.

Of the Extraction of the Roots.

§. 7. As Diviſion is only, which we have Shewn, a Subſtraction, according to the Preſcription of the Quotient, ſo the Extraction of the Root is only a Particular Kind of Diviſion; in which the Diviſor is not Given, but Sought and Inquired;

Thus, if we Extract the Square Root of 16, we want the Diviſor, which Multiplied into the Quotient, ſhall Produce 16, Namely 4, that is, which being Subſtracted from 16 the Dividend, according to the Preſcription of the Quotient ſhall Exhauſt it; From whence it is Manifeſt, that the Rules given before in Diviſion, Concerning the Signs and the Species or Letters, or whatever elſe, Obtain here, ſince the Operation is the ſame, Excepting that the Diviſor is here wanting, which before was given, and therefore we ſhall Proceed to Examples; But here we ſhall Begin with Algebraick Quantities, as being the Foundation of the Rule in Arithmetick for theſe Extractions, and which it would have been Hard to have Found out without them.

1ſt. In Algebraick Quantities, let the Root of $aa+2ab+bb$ be Required to be Extracted; that is, we want to Know, what that Diviſor is, which Subſtracted according to the Preſcription of the Quotient will Exhauſt the Dividend, which we find to be $a+b$; For $a)\ aa+2ab+bb\ (a$, That is, $\frac{aa}{a}=a$, and $a\times a$ Subſtracted from $aa=0$;

$$\begin{array}{lll} & \underline{aa} & \\ 2a) & 0 & +2ab+bb\ (b, \text{ That is, } \dfrac{2ab}{2a}=b; \\ & & \underline{+2ab+bb} \\ & & \quad 0 \end{array}$$

But $b\times 2a$ and $b\times b$ Subſtracted $=0$.

From whence this General Rule for the Extraction of the Square Roots of Quantities or Numbers is Derived; That we ſhould find That Diviſor and Quotient, which Multiplied into each other ſhall Produce the Firſt Part of a Binomial Dividend, That we ſhould then Double That Quotient for a New Diviſor, and Multiply the Second Quotient into the Laſt Diviſor, and into it's Self; Thus a the Firſt Diviſor $=a$ the Firſt Quotient; $2a$ the Second Diviſor $=2a$, the Double of the Firſt Quotient; And $2ab+bb=b$ the Laſt Quotient $\times 2a$, the Double of the Firſt Diviſor, and $\times b$ the Laſt Quotient.

And this is the Standing Method for all Extractions of the Square Root whatſoever, ſince all Numbers, or Quantities, may be Conſidered as Binomials, or as Conſiſting of Two Portions or Members; Thus, $aa+2ab+bb$, may be Divided into aa, and $2ab+bb$; 144 may be Divided into 100 and 44; And the ſame may be Said of any other Quantities, or Numbers, which are therefore Reduced to $aa+2ab+bb$, whoſe Root, by what Foregoes, is $a+b$.

But before we Proceed to any more Inſtances of Binomials, we ſhall give ſome Examples of the Extraction of the Root of Simple Algebraick Quantities; Thus,

$$\sqrt{aa}=a;\ \sqrt{aacc}=ac;\ \sqrt{9aacc}=3ac;\ \sqrt{49a^4xx}=7aax;\ \sqrt{\frac{a^4}{cc}}=\frac{\sqrt{a^4}}{\sqrt{cc}}=\frac{aa}{c};\ \sqrt{\frac{a^4bb}{cc}}=\frac{aab}{c};$$

$$\sqrt{\frac{9aazz}{25bb}}=\frac{3az}{5b};\ 3c\sqrt{\frac{9aazz}{25bb}}=3c\frac{3az}{5b}=\frac{9acz}{5b};\ \frac{a+3x}{c}\sqrt{\frac{4bbx^4}{81aa}}=\frac{a+3x}{c}\times\frac{2bxx}{9a}=\frac{2abxx+6bx^3}{9ac}.$$

And in the Extractions of the Cube Root, $\sqrt{}^3,\frac{8b^6}{27a^3}=\frac{2bb}{3a}$ and of the Bi-Square Root $\sqrt{}^4, aabb=\sqrt{ab}$, &c.

In Binomials or Compound Algebraick Quantities, Agreeable to the Preceding Rule, the Extractions are, as Follow.

Example 1ſt.

$$\begin{array}{ll} a=aa) & a^4+6a^3b+5aabb\surd 12ab^3+4b^4 \quad (aa+3ab-2bb \\ & \underline{a^4}=aa\times aa. \\ 2a=2aa) & 0\ +6a^3b \\ & \quad \underline{+6a^3b+9aabb}=3ab\times\overline{2aa+3ab},=b\times\overline{2a+b} \\ 2a=2aa+6ab) & 0\ -4aabb-12ab^3+4b^4 \\ & \quad \underline{-4aabb-12ab^3+4b^4}=-2bb\times\overline{2aa+6ab-2bb},=b\times\overline{2a+b} \\ & \quad\quad 0 \quad\quad 0 \quad\quad 0 \end{array}$$

Example 2d.

$$\begin{array}{ll} a=x) & xx-ax+\frac{1}{4}aa \quad (x-\frac{1}{2}a \\ & \underline{xx}=x\times x \\ 2a=2x) & 0\ -ax+\frac{1}{4}aa \\ & \quad \underline{-ax+\frac{1}{4}aa}=-\frac{1}{2}a\times\overline{2x-\frac{1}{2}a}=b\times\overline{2a+b} \\ & \quad\quad 0 \quad\quad 0 \end{array}$$

Example 3d.

$$\begin{array}{llll} a=3xx) & 9x^4\begin{array}{l}-24aa\\+12bb\end{array} xx & \begin{array}{l}+16a^4\\-16aabb\\+4b^4\end{array} & \left(3xx\begin{array}{l}-4aa\\+2bb\end{array}\right. \\ & \underline{9x^4}=3xx\times 3xx=a\times a & & \\ & 0 & & \end{array}$$

$2a=6xx)\quad 0\ \begin{matrix}-24aa\\+12bb\end{matrix}\,xx\ \begin{matrix}+16a^4\\-16aabb\\+4b^4\end{matrix}$

$\begin{matrix}-24aa\\+12bb\end{matrix}\,xx\ \begin{matrix}+16a4\\-16aabb\\+4b^4\end{matrix}\ =-4aa\times\overline{\begin{matrix}6xx-4aa\\+2bb\end{matrix}}=b\times\overline{\begin{matrix}2a+b\\+2bb\end{matrix}}$

$\qquad 0 \qquad 0$

4th. Example. $a=yy)\quad y^4+4y^3 * -8y+4 \qquad (yy+2y-2$

$y^4=yy\times yy=a\times a$

$2a=2yy\quad 0+4y^3$

$+4y\,3+4yy=2y\times\overline{2yy+2y}=b\times\overline{2a+b}$

$2a=2yy+4y)\quad 0-4yy-8y+4$

$-4yy-8y+4=-2\times\overline{2yy+4y-2}=b\times\overline{2a+b}$

$\qquad 0 \qquad 0$

IN all which Examples, the Firſt Diviſor is the Root of the Firſt Quantity, and every Succeeding One the Double of the Root already Extracted; And the Firſt Subtrahend is the Firſt Diviſor × the Firſt Quotient, and every Succeeding One the Laſt Quotient × the Diviſor, + that Laſt Quotient; or the Subtrahend may be always the Squares of the Quotient or Root Obtained.

IN the Extraction of the Cube-Root, the Rule Differs from the Former, according to the Difference of the Algebraick Binomial.

$aa)\quad a^3+3aab+3abb+b^3 \qquad (a+b$

$a^3=a\times a\times a$

$3aa)\quad 0+3aab+3abb+b^3$

$+3aab+3abb+b^3=b\times\overline{3aa+3ab+bb}$

$\qquad 0 \qquad 0 \qquad 0$

THAT is, the Firſt Diviſor is the Square of the Firſt Cubical Quantity, and every Succeeding One the Triple of the Square of the Root already Extracted; And the Firſt Subtrahend is the Firſt Quotient, Multiplied Cubically, and every Succeeding One, the Laſt Quotient × Three Times the Square of the Firſt, + Three Times the Rectangle of the Firſt Quotient into the Laſt, + the Square of the Laſt Quotient; Or the Subtrahends may be always the Cubes of the Quotient or Root Obtained; And from hence the Cube-Root of $z^6+6z^5-40z^3+96z-64=zz+2z-4$.

AND by the ſame Method, the Roots of any Higher Powers may be Extracted, that is, by Forming the Rule for Extraction from the Fourth, Fifth, Sixth, or Seventh Power of the Binomial $a+b$, and ſo of the Reſt *in Infinitum*, as will be Shewn Preſently.

2*d.* IN Integrals and Decimals, the Procedure is the ſame, as in Algebraick Binomials; In Extracting the Square-Roots of which, the Cyphers, or Figures, are Diſtinguiſhed into Parts or Members Conſiſting of Two ſuch Cyphers or Figures, Beginning from the Right Hand, and Going on to the Left; As in the Following Examples.

1ſt. Example. $a=300)\quad 9{\cdot}98{\cdot}56 \quad (316$

$9=300\times300=a\times a$

$2a=600)\quad 098$

$61=10\times\overline{600+10}=b\times\overline{2a+b}$

$2a=620)\quad 37{\cdot}56$

$37{\cdot}56=6\times\overline{620+6}=b\times\overline{2a+b}$

$\quad 0\ 0$

$a=4000)$

2d. *Example.*

$a=4000)$ 22·17·87·91,00·00·00 (4709,4363, &c.
$16=4000\times4000=a\times a$
$2a=8000)$ $\overline{6·17}$
$609=\overline{8000+700}\times700=\overline{2a+b}\times b$
$2a=9400)$ 8·87·91
$84681=\overline{9400+9}\times9=\overline{2a+b}\times b$
$2a=94180)$ 4110,00
$376736=\overline{94180+4}\times4=\overline{2a+b}\times b$
$2a=941880)$ 34264,00
$2825649=\overline{941880+3}\times3=\overline{2a+b}\times b$
$2a=9418860)$ 600751,00
$56513196=\overline{9418860+6}\times6=\overline{2a+b}\times b$
$2a=94188720)$ 356190400
$282566169=\overline{94188720+3}\times3=\overline{2a+b}\times b$
73624231 &c.

When the Root is Extracted, half Way, or Beyond, the Remaining Figures or Cyphers may be Found out by Diviſion only.

$a=100)$ - 3·29·76 (181,59 &c.
1
$2a=200)$ 229
$224=\overline{200+80}\times80=\overline{2a+b}\times b$
$2a=360)$ 576
$361=\overline{360+1}\times1=\overline{2a+b}\times b$
$2a=362)$ 215,0
1810 *3258
3400 142, &c.
*

I have hitherto Extracted the Root according to the Real Value of the Places, to ſhew the Correſpondence of the Numeral with the Algebraick Extractions; The Following Examples Expreſs the ſame Operation in a Conciſer Manner, but Depend upon the Foregoing Reaſoning.

32·97·,6 &c. (57,4247, &c.
25
797
749
4860
4576
1148) 284 (247

0,99·85·6 (0,999279, &c.
81
1885
1701
18460
17901
1998) 559 (279

In the Extraction of the Cube-Roots or the Roots of any higher Powers, the Rules are likewiſe Derived from the Algebraick Quantities; a is the Firſt Diviſor in Extracting the Root of the Square or Second Power, and $2a$ the Second Diviſor; $aa=a^2$ is the Firſt Diviſor in Extracting the Root of the Third Power, or of the Cube, and $3aa$, or $3a^2$ the Second Diviſor; a^3 is the Firſt Diviſor of the Fourth Power, and $4a^3$ the Second Diviſor; a^4 is the Firſt Diviſor in the Fifth Power, and $5a^4$ the Second Diviſor, &c. So that the Firſt Diviſor is always the Power Immediately below That, whoſe Root is Extracted, and the Second Diviſor is the Same Power Multiplied into the Index of the Power, whoſe Root is Sought or Extracted.

And in Numbers, If it is a Cube, or the Third Power, every Three Places are to be Diſtinguiſhed by Points, or Commas; If the Fourth Power, every Four Places; If the Fifth Power, every Five Places, *&c.* Beginning or Counting from Unit; And the Subtrahends are the Squares, or Cubes, or Biſquares, or Surſolids, *&c.* of the Quotients, or Roots, Found out or Obtained; Thus in the Cube, and in the Quadrato-Cube, orthe Fifth Power.

	13·312·053	(237			
Subſtract Cube	8			364·30826	(32,5
Diviſor 12)	53	(4 or 3		243	
Subſtract Cube	12167		405)	1213	(2
Diviſor 1587)	11450	(7		33554432	
Subſtract Cube	13312053		5242880)	2876388,0	(5
Remains	0 0				

WHEN the Quadrato-Quadratick Root, or $\sqrt{}^{4}a^{4}$, is to be Extracted, we muſt Twice Extract the Square Root; For $\sqrt{}^{2}a^{4}=aa$ and $\sqrt{}^{2}aa=a$; And when the Cubo-Cubick Root is to be Extracted, or $\sqrt{}^{6}a^{6}$, we muſt Extract the Cube-Root, and then the Square Root of that Root; For $\sqrt{}^{3},a^{6}=a^{2}$, and $\sqrt{}^{2},a^{2}=a$; And the Same may be Said of other Powers, whoſe Indices or Exponents, are Compound Numbers, and may be Reſolved into Simple Ones.

CHAP. II.

Of the Reduction of Fractions and Radicals to the Leaſt Terms, or to the Same Denomination, as alſo of Radicals to more Simple Radicals by the Extraction of the Roots; And likewiſe Of the Form of Equations, the Diſpoſition of Them, and the Extermination of the Unknown Quantities out of Them.

Of the Reduction of Fractions to the Leaſt Terms.

§. 1. FRACTIONS are Reduced to the Leaſt Terms, by Dividing the Numerators and Denominators, by the Greateſt Common Diviſor; Thus $\frac{aac}{bc}=\frac{aa}{b}$ the Greateſt Common Diviſor being c, the Numerator aac, and the Denominator bc. The Reaſon of which is, becauſe Ratio's Divided or Multiplied by Equal Quantities, are ſtill Equal; $\frac{4}{12}=\frac{1}{3}$ where 4 is the Greateſt Common Diviſor; $\frac{203}{667}=\frac{7}{23}$, where the Greateſt Common Diviſor is 29; $\frac{203aac}{667bc}=\frac{7aa}{23b}$, where the Diviſor is $29c$; $\frac{6a^3-9acc}{6aa+3ac}=\frac{2aa-3cc}{2a+c}$, the Diviſor $3a$; $\frac{a^3-aab+abb-b^3}{aa-ab}=\frac{aa+bb}{a}$, the Greateſt Common Diviſor $a-b$;

BY this Way, the Terms after Multiplication or Diviſion may be often Abridged; Thus $\frac{2ab^3}{3ccd}\times\frac{9acc}{bdd}$ or Divided by $\frac{bdd}{9acc}$ is $=\frac{18aabcc^3}{3bccd^3}$, by Reduction $=\frac{6aabb}{d^3}$.

BUT it is better before the Operation to Divide the Terms by the Greateſt Common Diviſor; which if not done, muſt be done afterwards; that is, $2ab^3$ and bdd by b, and $3ccd$ and $9acc$ by $3cc$; from whence will Ariſe $\frac{2abb}{d}\times\frac{3a}{dd}$, or Divided by $\frac{dd}{3a}=\frac{6aabb}{d^3}$, as before.

So likewiſe $\frac{aa}{c}\times\frac{c}{b}=\frac{aa}{1}\times\frac{1}{b}=\frac{aa}{b}$; and $\frac{aa}{c}$ Divided by $\frac{b}{c}=\frac{aa}{b}$; and $\frac{a^3-axx}{xx}\times\frac{cx}{aa+ax}=\frac{a-x}{x}\times\frac{c}{1}=\frac{ac}{x}-c$; and 28 Divided by $\frac{7}{3}=4$ Divided by $\frac{1}{3}=12$.

UNDER this Head it has been Thought Requiſite by Algebraiſts to Conſider, which Way we may Find out the ſeveral Diviſors of any Quantity, that from thence we may Know, which is the Greateſt Common Diviſor belonging to any Two Quantities Propoſed; But being of little Uſe to Beginners in this Science, for whom this Abſtract of it is Intended, I ſhall in a Manner wholly Omit that Doctrine, tho' there may be an Eaſy Account Given of it, and which Way it was Invented; As in $x^3-xx-10x+6$, for x Subſtitute 1, 0, −1, Separately and Diſtinctly, and the Numbers, Ariſing from ſuch a Subſtitution, will be −4, 6, 14, which, with all their Diviſors, are to be Placed over againſt the Arithmetical Progreſſion 1, 0, −1, according to the Scheme Subjoined.

THEN, ſince x^3, the Higheſt Term, is Diviſible by no Number Excepting Unit, it is Inquired amongſt the Diviſors for a Progreſſion, whoſe Terms only Differ by a Unit, and Decreaſe from the Uppermoſt to the Lowermoſt in like Manner as 1, 0, −1; of which Kind of Progreſſion there is only One Found amongſt the Diviſors, Namely 4, 3, 2, of which Numbers we Take 3, which ſtands over againſt 0, and Try a Diviſion by $x+3$, which Succeeds, and Produces in the Quotient $xx-4x+2$.

1	−4	1. 2. 4	+4
0	6	1. 2. 3. 6	+3
−1	14	1. 2. 7. 14	+2

Of the Reduction of Fractions to the ſame Denomination.

§. 2. FRACTIONS are Reduced to the ſame Denomination by Multiplying the Terms of Both by the Denominator of Each; Thus in $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b}\times d$ and $\frac{c}{d}\times b$, $=\frac{ad}{bd}$ and $\frac{bc}{bd}$, and the Common Denominator is bd; The Reaſon of which Procedure is, becauſe, as we have before Said, Ratios, Reſpects, or Relations Multiplied by Equal Quantities, will be ſtill Equal; So likewiſe a, or $\frac{a}{1}$, and $\frac{ab}{c}=\frac{ac}{c}$ and $\frac{ab}{c}$;

BUT, where the Denominators have a Common Diviſor, it is Sufficient to Multiply Alternately by the Quotients, Thus $\frac{a^3}{bc}$ and $\frac{a^3}{bd}=\frac{a^3d}{bcd}$ and $\frac{a^3c}{bcd}$, the Quotients being c aud d, and the Common Diviſor b.

THIS Reduction of Fractions is Chiefly Serviceable, or of Uſe, in the Addition and Subſtraction of them, becauſe Things cannot be Added or Subſtracted, unleſs they are of the ſame Sort or Kind; Thus $\frac{a}{b}+\frac{c}{d}$, by Reduction $=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$; and $a+\frac{ab}{c}=\frac{ac+ab}{c}$; and $\frac{a^3}{bc}-\frac{a^3}{bd}=\frac{a^3d-a^3c}{bcd}=\frac{d-c}{bcd}a^3$; and $\frac{c^4+x^4}{cc-xx}-cc-xx=\frac{2x^4}{cc-xx}$; And in Numbers $\frac{2}{3}+\frac{5}{7}=\frac{14}{21}+\frac{15}{21}=\frac{14+15}{21}=\frac{29}{21}$; Alſo, $\frac{11}{6}-\frac{3}{4}=\frac{22}{12}-\frac{9}{12}=\frac{13}{12}$; Likewiſe $\frac{3}{4}-\frac{5}{12}=\frac{9}{12}-\frac{5}{12}=\frac{4}{12}=\frac{1}{3}$; And $3\frac{4}{7}=\frac{3}{1}+\frac{4}{7}=\frac{21}{7}+\frac{4}{7}=\frac{25}{7}$; And $25\frac{1}{2}=\frac{51}{2}$.

WHERE there are ſeveral Fractions of Different Denominations to be Added together, they are to be United One after Another; Thus, in $\frac{aa}{x}-a+\frac{2xx}{3a}-\frac{ax}{a-x}$; Firſt $\frac{aa}{x}-a=\frac{aa-ax}{x}$; Second, $\frac{aa-ax}{x}+\frac{2xx}{3a}=\frac{3a^3-3aax+2x^3}{3ax}$; Third to this laſt Fraction Add $-\frac{ax}{a-x}$, and the Sum will be $\frac{3a^4-6a^3x+2ax^3-2x^4}{3aax-3axx}$; And in Numbers, as $3\frac{4}{7}-\frac{2}{3}$; Firſt $3\frac{4}{7}=\frac{25}{7}$, $=\frac{75}{21}$; Second $\frac{2}{3}=\frac{14}{21}$; Therefore Third, The whole Fraction $=\frac{61}{21}$.

Of the Reduction of Radicals to the Leaſt Terms.

§. 3. WHERE the Root of the Whole cannot be Extracted, Extract the Root of a Diviſor; Thus, the $\sqrt{aabc}=a\sqrt{bc}$, by Extracting the Root of the Diviſor aa; Alſo the $\sqrt{48}=4\sqrt{3}$, where 16 is the Diviſor; And the $\sqrt{48aabc}=4a\sqrt{3bc}$, where $16aa$ is the Diviſor; Becauſe $4a\times\sqrt{3bc}=\sqrt{16aa}\times\sqrt{3bc}=\sqrt{48aabc}$, by the Multiplication of Radicals before Explained, and ſo of the Reſt; So likewiſe $\sqrt{a^3b-4aabb+4ab^3}=\frac{a-2b}{c}\sqrt{ab}$, where $\frac{aa-4ab+4bb}{cc}$ is the Diviſor; $\sqrt{\frac{aaoomm}{ppzz}+\frac{4aam^3}{pzz}}=\frac{am}{pz}\,{}^{cc}\sqrt{oo+4mp}$, where $\frac{aamm}{ppzz}$ is the Diviſor; So $6\sqrt{\frac{75}{98}}=\frac{30}{7}\sqrt{\frac{3}{2}}$, where $\frac{25}{49}$ is the Diviſor, and $\frac{30}{7}\sqrt{\frac{3}{2}}=\frac{30}{7}\sqrt{\frac{6}{4}}=\frac{15}{7}\sqrt{6}$, where $\frac{1}{4}$ is the Diviſor; Thus $a\sqrt{\frac{b}{a}}$ or $a\sqrt{\frac{ab}{aa}}=\sqrt{ab}$, where $\frac{1}{aa}$ is the Diviſor; $\sqrt[3]{8a^3b+16a^4}=2a\sqrt[3]{b+2a}$, and $8a^3$ is the Diviſor, whoſe Cube-Root is to be Extracted; Alſo $\sqrt[4]{a^3x}=\sqrt{a}\times\sqrt[4]{ax}$, where aa is the Diviſor, and the Bi-

Square-Root is Extracted, or $=a\sqrt{}^{4},\frac{x}{a}$, where a^{4} is the Diviſor, and the Bi-Square-Root is Extracted; And $\sqrt{}^{6},a^{7}x5=a\sqrt{}^{6},ax5$, where a^{6} is the Diviſor, and the Cubo-Cube Root is Extracted, or $=ax\sqrt{}^{6},\frac{a}{x}$, where $a^{6}x^{6}$ is the Diviſor, or $=\sqrt{ax}\times\sqrt{}^{3},aax$, where $a^{3}x^{3}$ is the Diviſor. This Reduction of Radicals is not only Convenient for the Abridgment, but for the Addition and Subſtraction of them, which cannot be Performed, unleſs they have the ſame Denomination; Thus $\sqrt{48}+\sqrt{75}$, by Reduction, $=4\sqrt{3}+5\sqrt{3}$, by Addition $=9\sqrt{3}$; $\sqrt{48}-\sqrt{\frac{16}{27}}$, by Reduction, $=4\sqrt{3}-\frac{4}{9}\sqrt{3}$, for $\frac{16}{27}\times\frac{3}{3}=\frac{48}{81}=\frac{16\times3}{81\times1}$; But by Subſtraction $4-\frac{4}{9}\sqrt{3}=\frac{32}{9}\sqrt{3}$; Likewiſe, $\sqrt{\frac{4ab^{3}}{cc}}+\sqrt{\frac{a^{3}b-4aabb+4ab^{3}}{cc}}=\frac{2b}{c}\sqrt{ab}+\frac{a-2b}{c}\sqrt{ab}=\frac{a}{c}\sqrt{ab}$; And $\sqrt{}^{3},\overline{8a^{3}b+16a^{4}}-\sqrt{}^{3},\overline{b^{4}+2ab^{3}}=2a\sqrt{}^{3},\overline{b+2a}-b\sqrt{}^{3},\overline{b+2a}=\overline{2a-b}\sqrt{}^{3},\overline{b+2a}$.

Of the Reduction of Radicals to the ſame Denomination.

§. 4. When there is a Multiplication, or Diviſion, to be made in Radicals of a Different Denomination, they are to be Reduced to the ſame Denomination, which is done, by Prefixing the Radical Sign, whoſe Index is the Leaſt Number, which their Indices will Divide without a Remainder, and Multiplying the Quantities Annexed ſo many Times into themſelves, Abating One, as that Index is Increaſed; Thus $\sqrt{}^{2},ax\times\sqrt{}^{3},aax=\sqrt{}^{6},a^{3}x^{3}\times\sqrt{}^{6},a^{4}xx$ without a Remainder, and the Exponents of $a^{3}x^{3}$ the Quantities Annexed under the $\sqrt{}^{6}$, Namely 3, 3, are a Unit Leſs than 4, by which the Index of the Root is Increaſed from 2 to 6; So in $\sqrt{}^{3},aax$, the Index is Increaſed from 3 to 6, and the Exponents 2, 2, of aa^{2}, x^{2} under the $\sqrt{}^{6}$ are a Unit Leſs than 3; After which Reduction to the ſame Denomination, the Quantities under the Radical Signs are Multiplied into each other, and the Radical Signs Prefixed, as is before Preſcribed in the Multiplication of Radicals; That is, $\sqrt{}^{6},a^{3}x^{3}\times\sqrt{}^{6},a^{4}xx=\sqrt{}^{6},a^{7}x^{5}$; For the ſame Reaſon, it is ſaid $\sqrt{}^{2},a\times\sqrt{}^{4},ax=\sqrt{}^{4},aa\times\sqrt{}^{4},ax=\sqrt{}^{4},a^{3}x$; But here the $\sqrt{}^{2}$, is Increaſed to $\sqrt{}^{4}$, and the Exponent of a under $\sqrt{}^{4}$ ſhould be $2-1=1$, that is, it ſhould be the $\sqrt{}^{4},a$, whereas it is $\sqrt{}^{4},a^{2}$, where the Exponent is 2.

Sir Iſ. Newton's Algebra.

A New Reduction of Radicals to the ſame Denomination.

Perhaps therefore the Following Rule will be more General and Complete, as well as more Natural; Multiply the Radicals Given according to the Preſcription of the Index of the Radical Required, to whoſe Denomination they are to be Reduced, and to the Products Prefix the Index of the ſaid Radical; Or, the Dimenſions of the Quantities are to be Increaſed in Proportion to the Increaſe of the Indices of the Roots; Thus, Suppoſing, as before, $\sqrt{ax}$, and $\sqrt{}^{3},aax$ are to be Reduced to $\sqrt{}^{6}$, Multiply $\sqrt{ax}$, $\sqrt{}^{3},aax$, 6 Times into themſelves, and Prefix $\sqrt{}^{6}$, and they will be $=\sqrt{}^{6},a^{3}x^{3}$, $\sqrt{}^{6},a^{4}xx$; And $\sqrt{ax}\times\sqrt{ax}\times\sqrt{ax}=ax\sqrt{ax}$, and $\sqrt{ax}=\sqrt{}^{3},\overline{ax\sqrt{ax}}$, and $\sqrt{}^{3},\overline{ax\sqrt{ax}}\times\sqrt{}^{3},aax=\sqrt{}^{3},a^{3}x^{2}\sqrt{ax}=\sqrt{}^{6},a7x^{5}$.

Thus the $\sqrt{}^{2},a^{2}=\sqrt{}^{3},a^{3}=\sqrt{}^{4},a^{4}=\sqrt{}^{5},a^{5}=\sqrt{}^{6},a^{6}=\sqrt{}^{7},a^{7}$, &c; Alſo $\sqrt{}^{2}a\times\sqrt{}^{4},ax=\sqrt{}^{4},aa\times\sqrt{}^{4},ax=\sqrt{}^{4},a^{3}x$; Becauſe $\sqrt{a}\times\sqrt{a}\times\sqrt{a}\times\sqrt{a}=a^{2}$ to which Prefix $\sqrt{}^{4}$, for the Reaſon above, which makes $\sqrt{}^{4},a^{2}=\sqrt{a}$; So the $\sqrt{6}\times\sqrt{}^{4},\frac{5}{6}=\sqrt{}^{4},36\times\sqrt{}^{4},\frac{5}{6}=\sqrt{}^{4},30$; Becauſe $\sqrt{6}\times\sqrt{6}\times\sqrt{6}\times\sqrt{6}=36$, and $\sqrt{6}=\sqrt{}^{4},36$ and ſo of the Reſt; Again, $a\sqrt{bc}=\sqrt{aa}\times\sqrt{bc}=\sqrt{aabc}$, becauſe $a=\sqrt{a}\times\sqrt{a}=\sqrt{}^{2},a^{2}$; And $4a\sqrt{3bc}=\sqrt{16aa}\times\sqrt{3bc}=\sqrt{48aabc}$, becauſe $4a=\sqrt{4a}\times\sqrt{4a}=\sqrt{16aa}$; Thus, $2a\sqrt{}^{3}\overline{b+2a}=\sqrt{}^{3},8a^{3}\times\sqrt{}^{3},\overline{b+2a}=\sqrt{}^{3},\overline{8a^{3}b+16a^{4}}$; And $\frac{\sqrt{ac}}{b}=\frac{\sqrt{ac}}{\sqrt{bb}}=\sqrt{\frac{ac}{bb}}$; And $\frac{6abb}{\sqrt{18ab^{3}}}=\frac{\sqrt{36aab^{4}}}{\sqrt{18ab^{3}}}=\sqrt{2ab}$; And ſo of others.

Of the Reduction of Radicals to more Simple Ones by the Extraction of the Roots.

§. 5. The Roots of Quantities, which Conſiſt of Integrals, and Quadratick Radicals are Thus Extracted.

Let A Expreſs the Greater Part of any Quantity, and B the Leſs, and $\frac{A+\sqrt{AA-BB}}{2}$ will be = to the Square of the Greater Part of the Root, and $\frac{A-\sqrt{AA-BB}}{2}=$ to the Square of the Leſſer Part, which is to be Added to the Greater with the Sign Prefixed to B; Thus, if the Quantity is $3+\sqrt{8}$; $A=3$, $B=\sqrt{8}$, and $\sqrt{AA-BB}=1$; Therefore the Square of the Greater Part of the Root will be $=\frac{3+1}{2}=2$, and the Square of the Leſſer will be $\frac{3-1}{2}=1$, and Conſequently the Root will be $=1+\sqrt{2}$; If the Quantity is $\sqrt{32}-\sqrt{24}$, $A=\sqrt{32}$, $B=\sqrt{24}$, and $\sqrt{AA-BB}=\sqrt{8}$; From whence $\frac{\sqrt{32}+\sqrt{8}}{2}$ and $\frac{\sqrt{32}-\sqrt{8}}{2}=3\sqrt{2}$ and $\sqrt{2}$, = to the Squares of the Parts of the Root, and the Root therefore $=\sqrt{}^4,18-\sqrt{}^4,2$; Thus in Algebraick Quantities, if $aa+2x\sqrt{aa-xx}$ is the Quantity to be Extracted, $A=aa$, $B=2x\sqrt{aa-xx}$, $AA-BB=a^4-4aaxx+4x^4$, and the Root of it $=aa-2xx$; The Square of one Part of the Root therefore $=aa-xx$, and of the other xx, and the Root will be $x+\sqrt{aa-xx}$.

How the foregoing Rule was found out by Algebraiſts.

OTHER Examples might be Produced from Algebraiſts, but we ſhall rather chooſe to ſhow, how the foregoing Rule might be found out from the Precepts of this Science, Suppoſing the Extraction of the Root of Affected Quadratick Equations is Known, which will be afterwards Explained; Let a be the Integral Quantity or Number, and b the Surd, the $\sqrt{a+b}$ is Required; Let $\sqrt{x}+\sqrt{y}$ be $=\sqrt{a+b}$, therefore $x+y+2\sqrt{xy}=a+b$; or $x+y=a$, and $2\sqrt{xy}=b$; From whence $x=a-y$, and $4xy=bb$, or $x=\frac{bb}{4y}=a-y$, or $bb=4ya-4yy$, or $\frac{bb}{4}=ya-yy$, and $yy-ya=-\frac{bb}{4}$, and by Completing the Square, and Extracting the Root of the Equation, $yy-ya+\frac{1}{4}aa=\frac{aa-bb}{4}$ and $y-\frac{1}{2}a=\frac{\sqrt{aa-bb}}{4}$, and $y=\frac{1}{2}a+\frac{\sqrt{aa-bb}}{4}=\frac{a+\sqrt{aa-bb}}{2}$; But $x=a-y=a-\frac{1}{2}a-\frac{\sqrt{aa-bb}}{4}=\frac{a-\sqrt{aa-bb}}{2}$; And therefore a, as is Preſcribed in the Rule, is to be the Greater Quantity or Number, and b the Leſs, for otherwiſe the $\sqrt{aa-bb}$ would be an Impoſſible Root; And theſe are the Expreſſions before ſet down Changing a into A, and b into B.

Of the Form of Equations.

§. 6. EQUATIONS are Reduced to the following Forms, according to the higheſt Dimenſions or Powers of the unknown Quantity x, the Reaſon of which is taken from the Conſtruction of the ſeveral Dimenſions or Powers from a Binomial, which Conſiſts of a Known Quantity p, and an Unknown x.

THUS let us take $x+p$ for the Root of the ſeveral Powers and Dimenſions.

THE Square or Second Power therefore will be $=xx+2px+pp$.

THE Cube or Third Power will be alſo $=x^3+3x^2p+3xp^2+p^3$.

THE Bi-Square or Fourth Power likewiſe $=x^4+4x^3p+6x^2p^2+4xp^3+p^4$, &c.

IN the Firſt Power or Dimenſion $x+p$ Stands, as it does, without any Variation.

IN the Second, ſince $2p=p$, and $pp=q$, $xx+2px+pp=xx+px+q$.

IN the Third, ſince $3p=p$, $3p^2=q$, and $p^3=r$, $x^3+3x^2p+3xp^2+p^3=x^3+px^2+qx+r$.

IN the Fourth, $4p=p$, $6p^2=q$, $4p^3=r$, and $p^4=s$, makes it $x^4+px^3+qx^2+rx+s$.

AFTER the Forms of the Dimenſions, or Powers, are thus Eſtabliſhed, The Deſign of Algebra being to Find out the Unknown Quantities from the Known, which are Given, they are caſt into the Model Annexed.

$$\begin{array}{lcl} x=p & & x-p=0 \\ x^2=px+q & & x^2-px-q=0 \\ x^3=px^2+qx+r & \text{Or} & x^3-px^2-qx-r=0 \\ x^4=px^3+qx^2+rx+s & & x^4-px^3-qx^2-rx-s=0 \end{array}$$

FOR the Principal Difficulty in Algebra is to Find out x the Unknown Quantity by the Intervention of the Known Ones p, q, r, s, &c. and therefore they are Thrown on one Side of the Equation, and x and it's Powers on the Other; Or elſe the whole Compounded Quantities of Known and Unknown are made $=0$, in order to Try, if we can Inveſtigate the Unknown that Way; ſince + on one Side of the Equation makes − on the Other, as we have Shewn.

FROM whence it is Manifeſt, that the chief Deſign of Algebraiſts muſt be to Exterminate, and Extricate themſelves from, theſe Unknown Quantities, which are Involved with the Known

Known by all the Methods, they can Think of and Deviſe; Thus, where $x=p$, the Buſineſs is Accompliſhed, and ſo it would be if x^2 was $=p+q$, or $x^3=p+q+r$, &c. But ſince $x^2=px+q$ and $x^3=px^2+qx+r$, the Problem is to Diſengage the Unknown Quantities x in the one Equation, and x^2 and x in the Other, from the Known p, q.

SOMETIMES the Intermediate Terms are Wanting, as in $x^3=*qx+r$, $x^4=*px^3+s$, from which Defect of the Terms the Equation often becomes more Simple, and is likewiſe Depreſſed, as it may Happen, to a Lower Degree or Power; and the Lower That is, the Eaſier is the Equation to be Reſolved; becauſe it is Implicated and Involved with fewer Unknown Quantities, as is Apparent from the Forms above; Thus $x^4=qxx+s$, by Subſtituting $y=xx$, is $yy=qy+s$, an Equation of the Second Power or Dimenſion, and if y is Found, xx will be Known.

IT muſt be farther Obſerved, that as to the Signs Prefixed to the Terms, they may be Infinitely Diverſified and Varied.

Of the Ordering and Diſpoſition of Equations.

§. 7. ALL Equations are to be Ordered and Diſpoſed ſo, as to fall in with One of the Aboveſaid Forms, Preparatory, or Previous, to their Reſolution, which Kind of Reduction of Equations is Comprehended by Algebraiſts in the Following Rules.

Rule 1. RULE 1. If there are any Quantities, which Deſtroy one Another, or which will by Addition or Subſtraction Unite into One, the Equation will be ſo far Abridged.

THUS, if $5b-3a+2x=5a+3x$, Subſtract $2x$ from, and Add $3a$ to, both Sides of the Equation, and it will be $5b=8a+x$; If $\frac{2ab+bx}{a}-b=a+b$, ſince $\frac{2ab}{a}-b=b$, it will be $\frac{bx}{a}=a$.

HENCE it is, that if Quantities are Transferred to Oppoſite Sides of the Equation under Contrary Signs, the Equation will ſtill Subſiſt, that is, the Quantities on both Sides will ſtill be Equal; For, if $5b=8a+x$ in the Former Example, Subſtract $8a$ from both Sides of the Equation, or, which is the ſame Transfer it to the Oppoſite Side under a Contrary Sign, and it will be $5b-8a=x$ or $p=x$; Again, if $aa-3ay=ab-bb+by$, Transfer the Quantities Unknown to One Side of the Equation and the Known to the Other under Contrary Signs, which is nothing but Adding or Subſtracting the ſame Quantities to, or from, both Sides of the Equation, and it will be $aa-ab+bb=3ay+by$; And if we Divide both Sides of the Equation by the ſame Quantity $3a+b$, it will be $\frac{aa-ab+bb}{3a+b}=y$, or $p=x$.

If Laſtly, $abx+a^3-aax=abb-2abx-x3$, Transfer the Terms to Oppoſite Sides of the Equation under Contrary Signs, ſo that the Unknown Quantities ſhould be as much as can be, on the One Side of the Equation, and the Known on the Other, and it will be, $x^3=\begin{matrix}aa\\-3ab\end{matrix}x\begin{matrix}-a3\\+abb\end{matrix}$ or $x^3=*qx+r$.

Rule 2. RULE 2. If we find all the Terms of an Equation Multiplied by any Quantity, we are to Divide them by That Quantity, and if they are Divided by any Quantity, they are to be Multiplied by it.

If $15bb=24ab+3bx$, Divide all the Terms by $3b$, and it will be, $5b=8a+x$; Alſo, if $\frac{b3}{ac}-\frac{bbx}{cc}=\frac{xx}{c}$ Multiply all the Terms by c and it will be $\frac{b^3}{a}-\frac{bbx}{c}=xx$, or $xx=-px+q$.

Rule 3. RULE 3. If there is any Fraction, which can be no farther Reduced, in the Denominator of which is Involved the Unknown Quantity, or the Letter, according to whoſe Dimenſions the Equation is to be Ranged, all the Terms of the Equation are to be Multiplied by that Denominator or by ſome Diviſor of it; Thus, if $\frac{ax}{a-x}+b=x$, Multiply $a-x$ into both Sides of the Equation, and it will be, $ax+ab-bx=ax-xx$, or $ab-bx=-xx$, or $xx=bx-ab$, or $xx=px-q$; So if $\frac{a^3-abb}{2cy-cc}=y-c$, Multiply by the Denominator $2cy-cc$, or by it's Diviſor $2y-c$, that y or the Unknown Quantity may be Cleared from the Denominator, and it will be, $\frac{a^3-abb}{c}=2yy-3cy+cc$,

$+cc$, and by Tranſpoſition $\frac{a^3-abb}{c}-cc+3cy=2yy$, or, $xx=px\pm q$; In like Manner, if $\frac{aa}{x}-a=x$, Multiply by x, and it will be $aa-ax=xx=-px+q$; Alſo if $\frac{aabb}{cxx}=\frac{xx}{a+b-x}$, Multiply all the Terms firſt by xx, and then by $a+b-x$, and it will be $\frac{a^3bb+aab^3-aabbx}{c}=x^4=-px+q$.

Rule. 4. If the Unknown Quantity, or the Letter, according to whoſe Dimenſions the Equation is Ranged or Diſpoſed, is Involved with a Surd or a Radical, which is Incapable of any farther Reduction, all the other Terms are to be Transferred to the Oppoſite Side of the Equation under Contrary Signs, and both Sides to be Multiplied Quadratically, or Cubically, *&c.* as the Radical, or Surd, is a Square-Root or a Cubick, *&c.* Thus, if $\sqrt{aa-ax}+a=x$, by Tranſpoſition $\sqrt{aa-ax}=x-a$, and by Squaring both Sides, $aa-ax=xx-2ax+aa$, or, $0=xx-ax$, or, $ax=xx$, and $a=x=p$; Likewiſe, if $\sqrt[3]{,aax+2axx-x^3}-a+x=0$, by Tranſpoſition $\sqrt[3]{,aax+2axx-x^3}=a-x$, and by Cubeing both Sides, $aax+2axx-x^3=a^3-3aax+3axx-x^3$, or, $xx=4ax-aa$; and, if $y=\sqrt{ay+yy-a\sqrt{ay-yy}}$, by Squaring both Sides, $yy=ay+yy-a\sqrt{ay-yy}$, and by Tranſpoſition $a\sqrt{ay-yy}=ay$, or $y=\sqrt{ay-yy}$, and by Squaring both Sides, $yy=ay-yy$, and again by Tranſpoſition, $2yy=ay$, or $2y=a$, or $x=p$. *Rule 4.*

Rule. 5. If the greateſt Dimenſion of the Unknown Quantity, or of any Letter, be Multiplied by a Known, the whole Equation is to be Divided by it; $2y=a$ makes $y=\frac{1}{2}a$, or $x=p$; $\frac{bx}{a}=a$, if we Divide by $\frac{b}{a}$, becomes $x=\frac{aa}{b}=p$; and $\begin{matrix}2ac\\-cc\end{matrix}x^3\begin{matrix}+a^3\\+aac\end{matrix}xx\begin{matrix}-2a^3c\\+aacc\end{matrix}x-a^3cc=0$, Divided by $2ac-cc$, will be $\frac{x^3\begin{matrix}+a^3\\+aac\end{matrix}xx\begin{matrix}-2a^3c\\+aacc\end{matrix}x-a^3cc}{2ac-cc}=0$, or, $x^3+\frac{a^3+aac}{2ac-cc}xx-aax-\frac{a^3c}{2a-c}=0$, or, $x^3+pxx-qx-r=0$. *Rule 5.*

Rule. 6. This Kind of Reduction is ſometimes Performed by Dividing the Equation by ſome Compound Quantity; Thus $y^3=\mp\frac{2c}{b}yy+3bcy-bbc$ will be Reduced to $yy=-2cy+bc$, for by Tranſpoſition $y^3\pm\frac{2c}{b}yy-3bcy+bbc=0$, and by Dividing by $y-b$, $yy+2cy-bc=0$; or, $x^3\pm pxx-qx+r=0$ is Reduced to $xx+px-q=0$; But the Invention of ſuch Diviſors is very Difficult, and Wants a farther Explanation. *Rule 6.*

Rule. 7. Sometimes, as in all Affected Quadratick Equations, where the Unknown Quantity is Involved with the Known, all which come under the Second Form of $x^2=px+q$, the Reduction may be Performed by the Extraction of the Root of both Sides of the Equation; for which there is this General Rule; Place all the Unknown Quantities on one Side of the Equation, and the Known on the other, as in the Preſent Equation, $x^2-px=q$; Add the Square of the Semi-Coefficient, or of Half the Known Quantity Involved with the Unknown, Namely p, to each Side of the Equation, and there will Reſult. *Rule 7.*

1*ſt.* $x^2-px+\frac{1}{4}pp=\frac{1}{4}pp+q$, the Former of which will be an Exact Square, whoſe Root $x-\frac{1}{2}p=\pm\sqrt{\frac{1}{4}pp+q}$, becauſe $+\times+$ or $-\times-$ will Produce the ſame Square; and therefore $x=\frac{1}{2}p\pm\sqrt{\frac{1}{4}pp+q}$.

2*d.* Thus alſo, if $xx=px-q$; Then $xx-px=-q$; and $xx-px+\frac{1}{4}pp=\frac{1}{4}pp-q$; and $x-\frac{1}{2}p=\pm\sqrt{\frac{1}{4}pp-q}$; and $x=\frac{1}{2}p\pm\sqrt{\frac{1}{4}pp-q}$.

3*d.* If $xx=-px+q$; Then $xx+px=q$, and $xx+px+\frac{1}{4}pp=\frac{1}{4}pp+q$; and $x+\frac{1}{2}p=\pm\sqrt{\frac{1}{4}pp+q}$, and $x=-\frac{1}{2}p\pm\sqrt{\frac{1}{4}pp+q}$.

4*th.* If $xx=-px-q$; Then $xx+px=-q$; and $xx+px+\frac{1}{4}pp=\frac{1}{4}pp-q$; and $x+\frac{1}{2}p=\pm\sqrt{\frac{1}{4}pp-q}$, and $x=-\frac{1}{2}p\pm\sqrt{\frac{1}{4}pp-q}$.

These are the only four Caſes, which can happen in Equations of the ſecond Form, and are Reſolved alike by Completeing the Square on the one Hand, by the

Addition of the Square of the Semi-Coefficient, which therefore muſt be Added on the other, and ſeems to have been Induſtriouſly Concealed by the preſent Famous Algebraiſt, whoſe Rules and Examples, we have hitherto made Uſe of, as the moſt Significant and Compendious. And from hence is Formed that General Expreſſion for Extracting the Roots of theſe Kind of Equations; That, if $xx = . px . q$, $x = . \frac{1}{2}p \pm \sqrt{\frac{1}{4}pp . q}$, in which $\frac{1}{2}p$ and q in the Latter Equation obtain the ſame Signs with p and q in the Former, and $\frac{1}{4}pp$ is always Affirmative or Poſitive; But the Way of Comeing to this Univerſal Theorem is ſo Indirect and Oblique, in the Algebra here Referred to, that it gives a Suſpicion, that it had not a Mind to Impart the whole Secret.

The Inſtances, to which this Theorem is Applied, and of which we ſhall afterwards give many more, are, if $yy = \frac{2xxy}{a} + xx$, and we are to Extract the Root y; Therefore $\frac{2xx}{a} = p$, and $xx = q$; $\frac{xx}{a} = \frac{1}{2}p$, and $\frac{x^4}{aa} + xx = \frac{1}{4}pp . q$; or $y = \frac{xx}{a} \pm \sqrt{\frac{x4}{aa} + xx}$; Alſo, if $yy = ay - 2cy + aa - cc$, $a - 2c = p$ and $aa - cc = q$, and $y = \frac{1}{2}a - c \pm \sqrt{\frac{5}{4}aa - ac}$; Likewiſe, if $x^4 = -aaxx + ab^3$, $xx = -\frac{1}{2}aa \pm \sqrt{\frac{1}{4}a^4 + ab^3}$, and $x = \sqrt{-\frac{1}{2}aa \pm \sqrt{\frac{1}{4}a^4 + ab^3}}$.

I ſhall only farther Obſerve, that here is a Certain and Determined way of finding out two Values of x, or the unknown Quantity, in Equations of the ſecond Form, by adding the Square of the Semi-coefficient to each Side of the Equation; But it is not from hence Manifeſt, that there may not be ſtill other Values of x, which we have no Method of Arriving to; And This perhaps may be One of the Defects in Algebra; but Another and the Principal is, That we have Nothing like the ſame Certain Rule for finding out x, or the Unknown Quantity, in Equations of an Higher Form, as in $x^3 = px^2 + qx + r$, &c. ſo as to Complete the Cube $x^3 - px^2 - qx = r$; And tho' a great Deal is done in Algebra towards finding out the Unknown Quantities in it, yet it ſhews, what a vaſt Field of Knowledge, notwithſtanding, Remains undiſcovered in that Science, and how Infinitely it falls ſhort of an Abſolute Perfection, which is ſtill Sufficient to Exerciſe the Genius's of many Ages to Accompliſh; The Third, Fourth, and the ſtill Higher Powers, lying very much in the Dark to us, and are ſcarcely better Reſolved than by the Tentative Methods, which will be afterwards Explained.

Of the Extermination of the Unknown Quantities out of Equations, and of Surd or Irrational Ones.

§. 8. When there are ſeveral Equations, which Expreſs the Nature of the Queſtion to be Reſolved, and there is a Variety of Unknown Quantities Involved in them, ſuch Equations are to be taken Two at once, if there are more than Two, and to be ſo Connected together, that One of the Unknown Quantities ſhould be Exterminated in every Operation, and a New Equation Produced; Thus, if $2x = y + 5$ and $x = y + 2$, it will be $2x - x = y + 5 - y - 2$, or $x = 3$; And it is to be Obſerved, that by every Equation one Unknown Quantity may be Exterminated, and therefore if there are ſo many Equations Included in the Terms of the Queſtion or Problem, as there are Unknown Quantities, All may at length be Reduced to One, in which there will Remain only one Unknown Quantity, as in the Inſtance Above; But, if there are more Unknown Quantities by One, than there are Equations in the Queſtion, there will Remain in the laſt Equation Two Unknown Quantities; if there are more Unknown Quantities by Two, than there are Equations in the Queſtion, there will Remain Three Unknown Quantities in the Laſt, and ſo on; Thus, according to the former Inſtance, If $2x = y + z$ and $x = y + 2$, $2x - x = y + z - y - 2$, or $x = z - 2$; If $2x = y + z$, and $x = y + s$, $2x - x = y + z - y - s$, or $x = z - s$, &c. Sometimes Two or More Unknown Quantities may be Exterminated by Two Equations only; As, if $ax - by = ab - az$, and $bx + by = bb + az$, Therefore by Addition of Each Side of the Equations $ax - by + bx + by = ab - az + bb + az$, or $ax + bx = ab + bb$; But ſuch Caſes, as Theſe, do Evince, that, either there is ſome Fault Latent in Stating the Queſtion, or, that the Calculation upon it is Erroneous, or Unskilfull.

The Method, by which one Unknown Quantity may be Exterminated in Every Equation is Comprehended by Algebraiſts in the following Rules.

Rule 1.

Rul. 1. When the Quantity to be Exterminated is of one Dimenſion only in Both the Equations, Both it's Values are to be Sought, which are to Form a New Equation; Thus, if $a + x = b + y$, and $2x + y = 3b$; and y is to be Exterminated, it will be, $a + x - b = y$, and $3b - 2x = y$; and therefore $a + x - b = 3b - 2x$, or $x = \frac{4b - a}{3}$; So, if

$2x = y$,

$2x=y$, and $5+x=y$, it will be, $2x=5+x$, or $x=5$; if $ax-2by=ab$, and $xy=bb$, it will be $\frac{ax-ab}{2b}(=y)=\frac{bb}{x}$, or $xx-bx-\frac{2b^3}{a}=0$; again, if $\frac{bbx-aby}{a}=ab+xy$, and $bx+\frac{ayy}{c}=2an$, and x is to be Exterminated, it will be $\frac{aby+aab}{bb-ay}(=x)=\frac{2aac-ayy}{bc}$, and by Reduction, $y^3-\frac{bb}{a}yy-\frac{2aac-bbc}{a}y+bbc=0$; Laſtly, if $x+y-z=0$ and $ay=xz$, and z is to be Exterminated, it will be $x+y(=z)=\frac{ay}{x}$, or $xx+xy=ay$.

THIS alſo may be done by Subſtracting one of the Values of the Unknown Quantity from the other, and making the Reſiduum, or Remainder, Equal to 0; ſo, in the firſt Example, Subſtract $3b-2x$ from $a+x-b$, and there will Remain $a+3x-4b=0$, or $x=\frac{4b-a}{3}$.

RUL. 2. When in one of the Equations at leaſt, the Quantity to be Exterminated is of one Dimenſion only, it's Value is to be Sought in that Equation, and then to be Subſtituted in the other Equation in the Place of the Quantity to be Exterminated; Thus, if $xyy=b^3$, and $xx+yy=by-ax$, and x is to be Exterminated; by the firſt Equation $x=\frac{b^3}{yy}$ which Subſtitute in the Second for x; and then $\frac{b^6}{y^4}+yy=by-\frac{ab^3}{yy}$, and by Reduction $y^6-by^5+ab^3yy+b^6=0$; likewiſe, if $ayy+aay=z^3$ and $yz-ay=az$, and y is to be Exterminated, in the Second Equation $y=\frac{az}{z-a}$, which Subſtitute in the firſt Equation for y, and then $\frac{a^3zz}{zz-2az+aa}+\frac{a^3z}{z-a}=z^3$, and by Reduction $z^4-2az^3+aazz-2a^3z+a^4=0$; in like Manner, if $\frac{xy}{c}=z$, and $cy+zx=cc$, and z is to be Exterminated, Subſtitute $\frac{xy}{c}$, in the firſt Equation, in the Place of z in the ſecond, and $cy+\frac{xxy}{c}=cc$. *Rule 2.*

Uſe and Exerciſe in theſe Matters will Furniſh us with more Contracted Methods of Proceeding; as, if $ax=\frac{bbx-b^3}{z}$ and $x=\frac{az}{x-b}$, Multiply Equals by Equals and the Products will be Equal, or $axx=abb$, or $x=b$; But ſuch Caſes are Particular, and therefore cannot fall under any General Rule or Preſcription.

RUL. 3. When the Quantity to be Exterminated is in Neither of the Equations of one Dimenſion only, the Value of it's higheſt Power is to be ſought in Both; and, if thoſe Powers are not the ſame, the Equation of the Leſſer Power is to be Multiplied by the Quantity to be Exterminated, or by the Square, Cube, &c. of it, ſo as to make it of the ſame Power with the other Equation; and then the Values of the ſaid Powers are to be Formed into an Equation, in which the greateſt Power or Dimenſion of the Quantity to be Exterminated will be Diminiſhed, and by Repeating the ſame Operation, the ſaid Quantity at length will be Entirely taken away. *Rule 3.*

THUS, if $xx+5x=3yy$, and $2xy-3xx=4$, and x is to be Exterminated; from the firſt Equation $xx=-5x+3yy$ and from the Second $xx=\frac{2xy-4}{3}$, therefore $3yy-5x=\frac{2xy-4}{3}$, and by Reduction $9yy-15x=2xy-4$, or $x=\frac{9yy+4}{2y+15}$, which Value of x Subſtitute in the Room of it in one of the former Equations, as in $xx+5x=3yy$; From whence $\frac{81y^4+72yy+16}{4yy+60y+225}+\frac{45yy+20}{2y+15}=3yy$; or, by Multiplying by $4yy+60y+225$, $81y^4+72yy+16+90y^3+40y+675yy+300=12y^4+180y^3+675yy$, or, $69y4-90y^3+72yy+40y+316=0$.

Likewise, if $y^3=xyy+3x$, and $yy=xx-xy-3$, and y is to be Exterminated, Multiply the laſt Equation by y, and it will be $y^3=xxy-xyy-3y$, of the ſame Dimenſions with the former; and therefore $xyy+3x=xxy-xyy-3y$, where the Equation is Depreſſed to two Dimenſions of y; Conſequently from this Equation and the Simpleſt of the former, $yy=xx-xy-3$, by which Means y may be entirely taken away, if we Proceed as in the foregoing Example.

There are other Methods, by which the ſaid Extermination of the Unknown Quantity may be Performed, and ſometimes more Expeditiouſly; Thus, if $yy=\frac{2xxy}{a}+xx$, and $yy=2xy+\frac{x^4}{aa}$, and y is to be Exterminated; Extract the Root of both the Equations, by the General Theorem above, of $x=.\frac{1}{2}p\pm\sqrt{\frac{1}{4}pp.q}$, and it will be $y=\frac{xx}{a}+\sqrt{\frac{x^4}{aa}+xx}$, and $y=x+\sqrt{\frac{x^4}{aa}+xx}$; therefore $\frac{xx}{a}+\sqrt{\frac{x^4}{aa}+xx}=x+\sqrt{\frac{x^4}{aa}+xx}$, and $\frac{xx}{a}=x$, or $xx=ax$ and $x=a$.

Also, if $x+y+\frac{yy}{x}=20$, and $xx+yy+\frac{y^4}{xx}=140$, and x is to be Exterminated, Subſtract y from each Side of the firſt Equation, and it will be $x+\frac{yy}{x}=20-y$, which Squared makes $xx+2yy+\frac{y^4}{xx}=400-40y+yy$; Subſtract yy from each Side of this laſt Equation, and there will Remain $xx+yy+\frac{y^4}{xx}=400-40y$; therefore $400-40y=140$, or $y=6\frac{1}{2}$; and in moſt other Equations, the Labour may be Contracted.

But, if the Quantity to be Exterminated is of many Dimenſions, it will often Require a very laborious Calculus to do it, which Labour may be much Diminiſhed by the following Examples of Operation.

Example 1. If $axx+bx+c=0$, and $fxx+gx+h=0$, upon the Extermination of x, it will be $\overline{ah-bg-2cf}\times ah:+\overline{bh-cg}\times bf:+\overline{agg+cff}\times c=0$.

Example 2. If $ax^3+bxx+cx+d=0$; and $fxx+gx+h=0$; upon the Extermination of x, it will be.

$\overline{ah-bg-2cf}\times ahh:+\overline{bh-cg-2df}\times bfh:+\overline{ch-dg}\times\overline{agg+cff}:+\overline{3agh+bgg+dff}\times df=0$.

Example 3. If $ax^4+bx^3+cxx+dx+e=0$, and $fxx+gx+h=0$, upon the Extermination of x, it will be.

$\overline{ah-bg-2cf}\times ah^3:+\overline{bh-cg-2df}\times bfhh:+\overline{agg+cff}\times\overline{chh-dgh+egg-2efh}:+\overline{3agh+bgg+dff}$
$\times dfh:+\overline{2ahh+3bgh-dfg+eff}\times eff:-\overline{bg-2ah}\times efgg=0$.

Example 4. If $ax^3+bxx+cx+d=0$, and $fx^3+gxx+hx+k=0$, upon the Extermination of x it will be.

$\overline{ah-bg-2cf}\times\overline{aahh-achk}:+\overline{ak+bh-cg-2df}\times bdfh:\overline{-ak+bh+2cg+3df}\times aakk:+\overline{cdh-ddg}$
$\overline{-cck+2bak}\times\overline{agg+cff}:+\overline{3agh+bgg+dff-3afk}\times ddf:-\overline{3ak-bh+cg+df}\times bcfk:+\overline{bk-2dg}\times bbfk:$
$-\overline{bbk-3adh-cdf}\times agk=0$.

These Examples, which are Standards, or Rules, for other Equations of the like Kind, are firſt to be Explained, and we are then to ſhew the Application of them.

Example 1ſt. Explained.

First, If $ax^2+bx+c=0$, $x^2=\frac{-bx-c}{a}$; and if $fx^2+gx+h=0$, $x^2=\frac{-gx-h}{f}$; therefore $\frac{bx+c}{a}=\frac{gx+h}{f}$, and by Reduction $bfx-gax=ah-fc$, and $x=\frac{ah-fc}{bf-ga}$; Conſequently $axx+bx+c=\frac{a^2h^2-2ahfc+f^2c^2}{f^2b^2-2fbag+a^2g^2}\times a:+\frac{ah-fc}{bf-ga}\times b:+c=0$; and by Reduction $\overline{a^2h^2-2ahfc+f^2c^2}\times a:+\overline{abbf-bf^2c-ga^2h+gafc}\times b:+\overline{f^2b^2-2fbag+a^2g^2}\times c=0$; or $a^3h^2-2a^2hfc+af^2c^2:+abb^2f-b^2f^2c-bga^2h+bgafc:+cf^2b-2cfbag+ca^2g^2=0$; Take away $-b^2f^2c+cf^2b^2$, and $+bgafc-cfbag$, which Deſtroy each other, and Divide the Remaining Equation by a, it will be, $a^2h^2-2ahfc+f^2c^2:+bb^2f-bgah:-cfbg+cag^2=0$, or $\overline{ah-2cf-bg}\times ah:+\overline{bh-cg}\times fb:+\overline{agg+cff}\times c=0$; And in the ſame Manner may the Second, Third, and Fourth Examples be Explained.

Examples 2d. 3d. 4th. in the ſame Manner Explained.

Second, As to the Application of them it is thus; if $xx+5x-3yy=0$, and $3xx-2xy+4=0$, and x is to be Exterminated; according to the firſt Rule, for $1, 5$ $0-3yy$; $3, -2y$, and 4, we Subſtitute a, b, c; f, g, and h; from whence the following Equation will be Derived, Namely, $\overline{4+10y+18yy}\times4:+\overline{20-6y^3}\times15:+\overline{4yy-27yy}\times-3yy=0$; or, $16+40y+72yy+300-90y^3+69y^4=0$.

Likewise, if $y^3-xyy-3x=0$, and $yy+xy-xx+3=0$, and y is to be Exterminated; according to the ſecond Rule, for $1, -x$, $0, -3x$; 1, $x, -xx+3$, and y we Subſtitute a, b, c, d; f, g, h, and x Reſpectively, and the Equation will be $\overline{3-xx+xx}\times\overline{9-6xx+x^4}:\overline{-3x+x^3+6x}\times\overline{-3x+x^3}:+3xx\times xx:\overline{+9x-3x^3-x^3-3x}\times-3x=0$; or $27-18xx+3x^4$, $-9xx+x^6$, $+3x^4$, $-18x^2+12x^4=x^6+18x^4-45xx+27=0$; The Reaſon of theſe Subſtitutions is Manifeſt from the Preceding Examples being Rules or Standards for all other Equations of the ſame Kind.

The Rules, which have been hitherto Given, Concern the Extermination of one Unknown Quantity from two Equations; but if ſeveral Unknown Quantities are to be Exterminated from ſeveral Equations, it is to be done Gradually; Thus, if $ax=yz$, $x+y=z$, and $5x=y+3z$, and y is the Quantity Inquired after; Firſt x or z (Suppoſing x) is to be Exterminated by Subſtituting it's Value $\frac{yz}{a}$, found out from the Firſt Equation, into the Second and Third, from whence $\frac{yz}{a}+y=z$, and $\frac{5yz}{a}=y+3z$; and then we Proceed to Exterminate z by the foregoing Rules; Namely, $yz+ay=az$, and $5yz=ay+3az$, or $ay=az-yz$, and $z=\frac{ay}{a-y}$ in the firſt Equation, and $ay=5yz-3az$, and $z=\frac{ay}{5y-3a}$ in the Laſt; Therefore $\frac{ay}{a-y}=\frac{ay}{5y-3a}$, or $a-y=5y-3a$, and $6y=4a$, or $y=\frac{2}{3}a$.

To this Head belong Surd Quantities, which may be alſo Exterminated from Equations, by Suppoſing them Equal to Certain Letters; Thus, if $\sqrt{ay}-\sqrt{aa-ay}=2a+\sqrt[3]{ayy}$, Suppoſe $\sqrt{ay}=t$, $\sqrt{aa-ay}=v$, and $\sqrt[3]{ayy}=x$, it will be, $t-v=2a+x$, $tt=ay$, $vv=aa-ay$, and $x^3=ayy$, and by Exterminating t, v, and x from the Equations, one after another, there will at Length an Equation Reſult, which will be Clear of all Incommenſurables, or Irrational Quantities; The Procedure is, as before.

Of the Extermination of Surds.

CHAP. III.

Concerning the Solution of Problems, and Reducing them to Equations.

§. 1. ALGEBRAISTS Inform us, that Beginners in theſe Studies ſhould firſt Exerciſe themſelves in the Abſtracted Rules before Delivered, and then Proceed to Try their Strength in the Reſolving any Queſtion, Propoſed, into the ſeveral Equations, which Appertain to it; But it ſeems to be a Truer Method of Instruction for Reaſons already Alledged, to Come to Problems, as ſoon as Poſſible, and afterwards to Recur to the Abſtracted Rules for a more Complete Perfection in this Great and Divine Science. 'Tis True, theſe Rules muſt be ſometime Learned, if we are Willing to make any Kind of Proficiency in this Sort of Knowledge; but if we are Deſirous to Conſult the Genius of Beginners in it, we ſhall find the beſt Way to Humour That, is to Lead them to Problems from the firſt Step we Take with them in the Instructing of them; and therefore it is, that we have Intermixed Problems with the Rules, which are Given for their Solution. And what I have done in this Caſe, has been Dictated to me from my own Experience in theſe Matters; For tho', by Reading Dr. *Wallis*, Mr. *Harriot*, *Deſcartes*, *Schooten*, *Florimond de Beaune*, *Huddenius*, *Kerſey*, and Others, I was very Early, and when an Undergraduate in this Univerſity, Acquainted with the Abſtracted Rules of Algebra, yet it was all an Abſtraction, and a Kind of Myſterious One, till I afterwards Knew to what Purpoſe and Deſign thoſe Abſtractions were Intended. Does any One

That Problems are to be Haſtened to, as ſoon as Poſſible, and the Abſtracted Rules to be more Punctually Learned afterwards.

One Teach the Greek or the Latin Language by Grammatical Rules only, without Applying thoſe Rules at the ſame Time to the Underſtanding of Authors, who Write in thoſe Languages? If any One does, I Suppoſe he will find his Scholars to know the Rules by Rote, or Memory, but Perfectly Unapprized of the Meaning of them; It is Confeſſed by Algebraiſts, that Algebra is a Particular Kind of Language, and therefore at the ſame Time, that we Learn the Language, we ſhould be Informed for what Intention and Purpoſe it is Learned.

What the Algebraiſt is Chiefly to Aim at in the Solving of Problems, or Queſtions, Propoſed.

§. 2. What we have Said in the former Section Reſpects the Way and Method, which ſhould be Uſed in the Teaching of Algebra; The Next Thing is to Know, how we ſhould, upon any Queſtion or Problem Propoſed, Expreſs all the Circumſtances and Conditions of it by ſo many Equations; In Order to which, we are Firſt to Examine, whether the Propoſitions, or Sentences, in which the Problem, or Queſtion, is Put, are Capable of being Reduced to Algebraick Terms, or to the Language of Algebra, in the ſame Manner as we Signify our Notions or Conceptions, by Hebrew, Greek, or Latin Characters; If this can be done, as it may be in Queſtions, or Problems, which Concern Numbers and Abſtracted Quantities, we are then, if it is found Neceſſary, to Denominate the Known, and Unknown Quantities, and to Expreſs the Senſe, and Meaning, of the Problem or Queſtion in an Analytick or Algebraick Stile; and the Conditions of the Problem or Queſtion thus Expreſſed will give as many Equations, as are Requiſite to the Solution of it; But it may ſometimes Happen, that the Problem or Queſtion is ſo Stated, that it may ſeem Difficult to Tranſlate it into the Algebraick Language; If yet we make ſome few Alterations, and Attend to the Senſe, rather than the Sound of the Words, the Verſion will be Eaſy; So every Language has it's Particular Idiom or Propriety, and if we Tranſlate out of One into Another, we muſt Conſider the Senſe, and not the Words, to make an Accurate and Juſt Tranſlation; It muſt be farther Obſerved, that the fewer Unknown Quantities we make Uſe of in any Problem or Queſtion, the more Expedite and Artificial will be the Solution.

Problem the 14th. Propoſed and Solved.

§. 3. Probl. 14. It is Propoſed to Find out Three Unknown Numbers, the Conditions Given of which are, That they are Continually Proportional, That the Sum of them is Equal to 20, and the Sum of the Squares of them Equal to 140; What are the Numbers?

1. There are Three Unknown Numbers Sought.	$x, y, z?$ or $x, y, \frac{yy}{x}?$
2. They are Continually Proportional.	$x.y::y.z$, or $z=\frac{yy}{x}$.
3. The Sum of all of them is Equal to 20.	$x+y+\frac{yy}{x}=20$.
4. The Sum of their Squares is Equal to 140.	$xx+yy+\frac{y^4}{xx}=140$.

This Problem is Solved § 8. Rule 3. of the Preceding Chapter.

Or otherwiſe, by Reduction, $xx\substack{+y\\-20}x+yy=0$, and $x^4\substack{+yy\\-140}xx+y^4=0$, and let x be Exterminated; Therefore according to Example the 3*d*. Rule the 3*d*. of the forecited Chapter, for *a*, *b*, *c*, *d*, *e*; and *f*, *g*, *h*; Subſtitute 1, 0, $yy-140$, 0, y^4; and 1, $y-20$, yy, and the Equation will be $\overline{-yy+280}\times y^6:\overline{+2yy-40y+260}\times\overline{260y^4-40y^5}:+3y^4\times y^4:-2yy\times y^6\overline{-40y^5+400y^4}=0$, or $1600y^6-20800y^5+67600y^4=0$, or $1600y^2-20800y+67600=0$; or $y^2=13y-42\frac{1}{4}$, and by Extraction of the Root, by the General Theorem of $x=.\frac{1}{2}p\pm\sqrt{\frac{1}{4}pp.q}$, $y=6\frac{1}{2}\pm\sqrt{42\frac{1}{4}-42\frac{1}{4}}=6\frac{1}{2}$; or, $1600y^6-20800y^5+67600y^4=1600y^6-20800y^5+10400y^5$, and $67600y^4=10400y^5$, or, $1600y^6=10400y^5$, in both which Caſes $y=6\frac{1}{2}=\frac{67600}{10400}=\frac{10400}{1600}$; When y is found out, it's Value, to Wit, $6\frac{1}{2}$, is to be Subſtituted, in the Stead of it, in the foregoing Equation, Namely, $xx\substack{+y\\-20}x+yy=0$, from whence will Ariſe the following $xx-13\frac{1}{2}x+42\frac{1}{4}=0$, and by Extraction of the Root of the Equation, $x=6\frac{3}{4}\pm\sqrt{3\frac{5}{16}}$, of which the Firſt is the Greateſt Term, and the Laſt the Leaſt; For x in the Queſtion Ambiguouſly Repreſenting Either of the Extreme Terms, upon that Account Admits of a Double Value, Either of which may be x, and the Other $\frac{yy}{x}$.

ANOTHER Way of Solving this Problem is by Extracting the Root of the Equation, According to the General Theorem, before Explained, of $x = .\tfrac{1}{2}p \pm \sqrt{\tfrac{1}{4}pp \cdot q}$. Thus $xx = \begin{array}{l}20\\-y\end{array} x - yy$, and $x = 10 - \tfrac{1}{2}y \pm \sqrt{100 - 10y - \tfrac{3}{4}yy}$, Square the Three Numbers and the Sum of them will be $400 - 40y = 140$ by the Problem, or $400 - 140 = 40y$, or $y = 6\tfrac{1}{2}$ as before; Substitute $6\tfrac{1}{2}$ for y in the Values of x, and the Greateſt Number or Proportional will be $6\tfrac{3}{4} + \sqrt{3\tfrac{5}{16}}$ and the Leaſt $6\tfrac{3}{4} - \sqrt{3\tfrac{5}{16}}$; Which Numbers, it is Manifeſt, will Anſwer the Conditions of the Queſtion, or Problem; For Firſt, they are Continually Proportional; ſince $6\tfrac{3}{4} - \sqrt{3\tfrac{5}{16}} . 6\tfrac{1}{2} :: 6\tfrac{1}{2} . 6\tfrac{3}{4} + \sqrt{3\tfrac{5}{16}}$; Second, $6\tfrac{3}{4} - \sqrt{3\tfrac{5}{16}} + 6\tfrac{3}{4} + \sqrt{3\tfrac{5}{16}} + 6\tfrac{1}{2} = 20$; Third, the ſame Numbers Squared will be Found Equal to 140.

§. 4. PROBLEM 15. To Find out Four Numbers, which are Continually Proportional, of which the Sum of the Two Middle Ones is Equal to 12, and the Sum of the Two Extreme Ones Equal to 20. *Problem the 15th.*

1. THE Second Number is Suppoſed to be. x.
2. THEREFORE the Third Number is by the Problem, which makes the Second and Third Number Equal to 12. $12 - x$; becauſe $x + 12 - x = 12$.
3. THE Firſt Number therefore, becauſe they are by the Problem Continually Proportional, will be. $\frac{xx}{12-x}$; for $\frac{xx}{12-x} . x :: x . 12 - x$.
4. THE Fourth Number for the ſame Reaſon will be. $\frac{144 - 24x + xx}{x}$; for $x . 12 - x :: 12 - x . \frac{144 - 24x + xx}{x}$.
5. BUT by the Problem the Firſt and Fourth Numbers are Equal to 20 or. $\frac{xx}{12-x} + \frac{144 - 24x + xx}{x} = 20$.
6. AND by Reduction and Extraction, the Second Number will be. $xx = 12x - 30\tfrac{6}{7}$, or, $x = 6 + \sqrt{5\tfrac{1}{7}}$.
7. THEREFORE by the Second Step the Third Number will be, $12 - x = 6 - \sqrt{5\tfrac{1}{7}}$.
8. THEREFORE by the Third Step the Firſt Number will be. $\frac{xx}{12-x} = \frac{41\frac{1}{7} + 12\sqrt{5\frac{1}{7}}}{6 - \sqrt{5\frac{1}{7}}}$.
9. AND by the Fourth Step the Fourth Number will be. $\frac{144 - 24x + xx}{x} = \frac{41\frac{1}{7} - 12\sqrt{5\frac{1}{7}}}{6 + \sqrt{5\frac{1}{7}}}$.

WHICH Numbers Solve all the Conditions of the Problem; For 1ſt. They are Continually Proportional, ſince $\frac{41\frac{1}{7} + 12\sqrt{5\frac{1}{7}}}{6 - \sqrt{5\frac{1}{7}}} . 6 + \sqrt{5\tfrac{1}{7}} :: 6 + \sqrt{5\tfrac{1}{7}} . 6 - \sqrt{5\tfrac{1}{7}}$, and $6 + \sqrt{5\tfrac{1}{7}} . 6 - \sqrt{5\tfrac{1}{7}} :: 6 - \sqrt{5\tfrac{1}{7}} . \frac{41\frac{1}{7} - 12\sqrt{5\frac{1}{7}}}{6 + \sqrt{5\frac{1}{7}}}$; 2*d*. The Sum of the Middle Numbers, $6 + \sqrt{5\tfrac{1}{7}} + 6 - \sqrt{5\tfrac{1}{7}} = 12$; 3*d*. The Sum of the Extreme Numbers $\frac{41\frac{1}{7} + 12\sqrt{5\frac{1}{7}}}{6 - \sqrt{5\frac{1}{7}}} + \frac{41\frac{1}{7} - 12\sqrt{5\frac{1}{7}}}{6 + \sqrt{5\frac{1}{7}}}$ $= \frac{246\frac{6}{7} + 113\frac{1}{7}\sqrt{5\frac{1}{7}} + 60\frac{12}{7} + 306\frac{18}{7} - 113\frac{1}{7}\sqrt{5\frac{1}{7}}}{36 - 5\frac{1}{7}} = \frac{4320}{7}$ Divided by $\frac{216}{7} = \frac{4320}{216} = 20$.

§. 5. PROBLEM 16. To Find out four Numbers, which are Continually Proportional, whoſe Sum a is Given, and the Sum of the Squares b. *Problem the 16th.*

IT is here Told us by Algebraiſts, that, altho' it is Uſual to Inquire Immediately after the Quantities Wanted or Deſired, that is, the Unknown Ones, yet if there Happen to be Two, which are Ambiguous, or which are Endued with Conditions, which are in all Reſpects Alike, as in the Preſent Problem, the Two Middle and the Two Extreme Numbers of the Four Proportionals are, it will be better to Inquire after other Quantities, which are not Ambiguous, and by which the Ambiguous may be Determined, as for Inſtance, the Sum, or Difference, or Rectangle of Them.

1. Let us now Suppoſe the Sum of the Two Middle Numbers Equal to an Unknown Quantity. s.

2. That the Rectangle of them is Equal to Another Unknown Quantity. r.

3. The Sum of the Extreme Terms will be the Sum of the Middle Terms Subſtracted from the Sum Given of all the Terms, or. $a-s$.

4. The Rectangle of the Extreme Terms will be Equal to the Rectangle of the Middle Terms from the Nature of Proportion, and therefore Equal likewiſe to. r.

5. Suppoſe the Firſt and Second Numbers to be. x. y.

6. Therefore the Third Number will be the Second Subſtracted from the Sum of the Two Middle Ones, as if 16, 8, 4, and 2 are the Numbers, $4=8+4-8$ or. $s-y$.

7. And the Fourth Number will be the Sum of the Two Middle Numbers, + the Firſt Number, Subſtracted from the Sum of all of them; Thus, $16+8+4+2-8-4-16=2$, or. $a-s-x$.

8. The Rectangle under the Middle Terms will be. $sy-yy=r$.

9. And by Reduction and Extraction. $y=\frac{1}{2}s\pm\sqrt{\frac{1}{4}ss-r}$.

10. The Rectangle under the Extreme Terms will be. $ax-sx-xx=r$.

And by Reduction and Extraction.

$$x=\frac{a-s}{2}\pm\sqrt{\frac{ss-2as+aa-r}{4}}.$$

11. The Sum of the Squares of theſe Four Numbers will by the Problem be Equal to b, or. $2ss-2as+aa-4r=b$, and $r=\frac{1}{2}ss-\frac{1}{2}as+\frac{1}{4}aa-\frac{1}{4}b$.

12. Subſtitute for r it's Value in the Former Step in the Equations of x and y, Steps the Ninth and Tenth, and the Two Middle Terms, Namely, the Second and Third will be. $\frac{1}{2}s\pm\sqrt{\frac{1}{4}b-\frac{1}{4}ss+\frac{1}{2}as-\frac{1}{4}aa}$.

And the Two Extreme Ones, Namely the Firſt and Fourth. $\frac{a-s}{2}\pm\sqrt{\frac{1}{4}b-\frac{1}{4}ss}$.

13. But s is ſtill Unknown, therefore by Subſtituting p, and q for the Roots of the Laſt Step, the Second and Third will be. $\frac{1}{2}s\pm p$.

And the Firſt and Fourth will be. $\frac{a-s}{2}\pm q$.

14. But by a Remaining Condition of the Problem, which has not been yet Satisfy'd, the Rectangle under the Second and Fourth Terms is Equal to the Square of the Third, or.

$$\frac{as-ss}{4}-\frac{1}{2}qs+\frac{pa-ps}{2}-pq=\frac{1}{4}ss-ps+pp.$$

15. Also by another Condition of the Problem not yet Conſidered, the Rectangle under the Firſt and Third Terms is Equal to the Square of the Second, or.

$$\frac{as-ss}{4}+\frac{1}{2}qs\,\frac{-pa+ps}{2}-pq=\frac{1}{4}ss+ps+pp.$$

16. Subſtract the Equation of the Fourteenth Step from that of the Fifteenth, and it will be. $qs-pa+ps=2ps$, or, $qs=pa+ps$.

17. Subſtitute the Roots above in the Twelfth Step for p and q in the Thirteenth, and.

$$s\sqrt{\tfrac{1}{4}b-\tfrac{1}{4}ss}=\overline{a+s}\sqrt{\tfrac{1}{4}b-\tfrac{1}{4}ss+\tfrac{1}{2}as-\tfrac{1}{4}aa}.$$

18. And by Squaring both Sides of the Equation and Extracting the Root.

$$ss=-\frac{b}{a}s+\tfrac{1}{2}aa-\tfrac{1}{2}b,\ \text{and}\ s=-\frac{b}{2a}\pm\sqrt{\frac{bb}{4aa}+\tfrac{1}{2}aa-\tfrac{1}{2}b}.$$

Let a be 30, and b 340, and let us take the Poſitive Value of s.

THEREFORE $s=-\frac{17}{3}+\sqrt{\frac{2809}{9}}=-\frac{17}{3}+\frac{53}{3}=\frac{36}{3}=12$.

THE Firſt Term $=\frac{30-12}{2}+\sqrt{85-36}=9+7=16$; The Fourth Term $=9-7=2$; The Second Term $=6+\sqrt{85-36+180-225}=6+\sqrt{4}=8$; The Third Term $=6-\sqrt{4}$; Conſequently, 16. 8. 4. 2 will Anſwer all the Conditions of the Problem; For Firſt, They are Continually Proportional, and therefore $16\times4=8\times8$, and $8\times2=4\times4$; Second, $16+8+4+2=30$, the Sum of the Proportionals; Third, $256+64+16+4=340$, the Sum of the Squares of ſuch Proportionals.

WE ſhall now Proceed to Give ſome farther Problems from Algebraiſts to Exerciſe the Beginners in this Science, which Terminate in Simple, or Simple Quadratick, or Simple Cubick Equations, which they ſhould be firſt Practiced in, and Algebra would be then an Eaſy Science to Them.

Problems the 17th. 18th. and 19th.

§. 6. PROBL. 17. There are Two Numbers whoſe Sum is 26, or b, and their Difference, to Wit, the Exceſs of the Greater above the Leſſer is 8, or c; What are the Numbers?

PROBL. 18. There are Two Numbers, whoſe Sum is 40, or b, and the Greater Number hath ſuch a Proportion to the Leſſer, as 3 to 2, or as r to s; What are the Numbers?

PROBL. 19. There are Two Numbers, whoſe Sum is 7, or b, and the Difference of their Squares is 21, or d; What are the Numbers?

SOLUTION of PROBL. 17.

Greater Number. x.
Leſſer Number. $b-x$.
Difference. $2x-b$.
By the Problem. $2x-b=c$.
By Reduction. $x=\frac{1}{2}b+\frac{1}{2}c$.
And. $b-x=\frac{1}{2}b-\frac{1}{2}c$.

$$\begin{array}{l|l} b=26 & \frac{1}{2}b+\frac{1}{2}c=13+4=17. \\ c=8 & \frac{1}{2}b-\frac{1}{2}c=13-4=9. \end{array}$$

$17+9=26$.
$17-9=8$.

Or.

Leſſer Number. x.
Greater Number. $b-x$.
Difference. $b-2x$.
By the Problem. $b-2x=c$.
By Reduction. $x=\frac{1}{2}b-\frac{1}{2}c$.
And. $b-x=\frac{1}{2}b+\frac{1}{2}c$.

SOLUTION of PROBL. 18.

Greater Number. x.
Leſſer Number. $\frac{sx}{r}$.
Becauſe. $r.\ s :: x.\ \frac{sx}{r}$.
Sum of Both. $x+\frac{sx}{r}$.
By the Problem. $x+\frac{sx}{r}=b$.
By Reduction. $rx+sx=rb$.
And $x=\frac{rb}{r+s}$; And $\frac{sx}{r}=\frac{sb}{r+s}$.

$$\begin{array}{l|l} b=40 & \frac{rb}{r+s}=\frac{120}{5}=24. \\ r=3 & \\ s=2 & \frac{sb}{r+s}=\frac{80}{5}=16. \end{array}$$

$24.\ 16 :: 3.\ 2$; $24+16=40$.

SOLUTION of PROBL. 19.

Greater Number. x.
Leſſer Number. $b-x$.
Square of the Greater. xx.
Sq. of the Leſſer $xx-2bx+bb$.
Difference of Sq^rs. $2bx-bb$.
By the Probl. $2bx-bb=d$.
By Reduction. $x=\frac{bb+d}{2b}$.
Alſo. $b-x=b-\frac{bb-d}{2b}$,
$=\frac{2bb-bb-d}{2b}=\frac{bb-d}{2b}$.

$$\begin{array}{l|l} b=7 & \frac{bb+d}{2b}=\frac{70}{14}=5. \\ d=21 & \frac{bb-d}{2b}=\frac{28}{14}=2. \end{array}$$

$5+2=7$; $25-4=21$.

These Problems End in Simple Equations.

THESE Problems End in what are Termed Simple Equations, where the Unknown Quantity x is of One Dimenſion only, and is on One Side of the Equation, and all the Known Quantities are on the Other.

Problems the 20th. 21ſt. and 22d.

§. 7. PROBL. 20. A Certain Perſon being Ask'd, what was the Age of every One of his Four Sons, Anſwer'd, the Eldeſt was 4, or b, Years Elder than the Second, the Second was 4, or b, Years Elder than the Third, the Third was Four Years Elder than the Fourth or Youngeſt, and the Double of the Youngeſt Son's Age was Equal to the Age of the Eldeſt; What was the Age of Each Son?

PROBL. 21. A Merchant began to Trade with a Certain Number of Pounds, by his Firſt Voyage he Doubled that Stock, by his Second he Loſt 1200*l.* or b, by his Third he Doubled his Remaining Stock, by his Fourth he Loſt again 1200*l.* or b, and then he had no Money Left; How many Pounds did the Merchant begin to Trade with?

Probl. 22. A Gentleman Hired a Servant for a Year for 120*s*, or *c*, together with a Livery Cloak, Valued at a Certain Number of Shillings; but when $\frac{7}{12}$, or *d*, Parts of the Year were Expired, the Maſter Falling out with his Servant, put him away, and gives him the Cloak with 50*s*, or *f*, and ſo the Servant Received full Satisfaction for the Time of his Service; How many Shillings was the Cloak Valued at?

Solution of Probl. 20.

Age of the Eldeſt. x.
Of the Second Son. $x-b$.
Of the Third. $x-2b$.
Of the Fourth. $x-3b$.
By the Problem. $2x-6b=x$.
By Reduction. $x=6b$.
$x=4\times6=24$.
$x-b=20$.
$x-2b=16$.
$x-3b=12$.
And $2\times12=24$.

Or,

Age of the Youngeſt. x.
Of the Third Son. $x+b$.
Of the Second. $x+2b$.
Of the Firſt. $x+3b$.
By the Problem. $2x=x+3b$.
By Reduction. $x=3b$.
$x=3\times4=12$.
$x+b=16$.
$x+2b=20$.
$x+3b=24$.
And $2\times12=24$.

Solution of Probl. 21.

Pounds the Merchant Began with. x.
Pounds at the End of the Firſt Voyage. $2x$.
Pounds at the End of the Second Voyage. $2x-b$.
Pounds at the End of the Third Voyage. $4x-2b$.
Pounds at the End of the Fourth Voyage. $4x-3b$.
By the Problem. $4x-3b=0$.
By Reduction. $x=\frac{3}{4}b$.
$b=1200l.$ $x=900l.$
$2x=1800l.$
$2x-b=600l.$
$4x-2b=1200l.$
$4x-3b=3600l.-3600l.=0$.

Solution of Probl. 22.

Number of Shillings the Cloak Valued at. x.
Due of the Cloak to the Servant for *d* Parts of the Year, which he Served. dx.
Becauſe. $\overset{\text{Yr.}}{1}.\overset{\text{Sh.}}{x}::\overset{\text{Yr.}}{d}.\overset{\text{Sh.}}{dx}$.
Due of the Money-Wages *c* for *d* Parts of the Year, which he Served. cd.
Becauſe. $\overset{\text{Yr.}}{1}.\overset{\text{Sh.}}{c}::\overset{\text{Yr.}}{d}.\overset{\text{Sh.}}{cd}$.
By the Probl. $x+f=dx+cd$.
By Reduction $x-dx=cd-f$,
Or. $dx-x=f-cd$.
And. $x=\frac{cd-f}{1-d}$, Or, $\frac{f-cd}{d-1}$.

$$\left.\begin{array}{l} c=120 \\ d=\frac{7}{12} \\ f=50 \end{array}\right| \frac{cd-f}{1-d}=\frac{70-50}{\frac{5}{12}}$$

$=48$ Shillings.
$\frac{f-cd}{d-1}=\frac{-20}{-\frac{5}{12}}=+48$ Shillings.
$48s+50s=98s$.
$\frac{7}{12}$ of $48s+\frac{7}{12}$ of $120s=98s$.
h. e. $7s\times4s+7s\times10s=98s$.

These Three Problems likewiſe End in Simple Equations.

Problems 23d. 24th. 25th. and 26th.

§. 8. **Problem 23.** One being Asked, What a Clock it was, Anſwered, That the Time Then Paſſed from Noon was Equal to $\frac{33}{40}$, or *b*, Parts of the Time Remaining until Midnight; What was the Preſent Hour, Suppoſing the Time between Noon and Midnight, to be Divided into 12, or *c*, Equal Hours?

Problem 24. A General of an Army, having ſet his Soldiers in a Square Battel, there Happened to be 500, or *b*, Soldiers to Spare; But to Increaſe the Square, ſo as that it's Side might Conſiſt of 1, or *c*, Soldiers more than the Side of the Former Square, there would be 29, or *d*, Soldiers wanting; How many Soldiers had the General in his Army?

Problem 25. Two Perſons *A* and *B* Diſcourſe of their Money in this Manner, *A* ſaith, if *B* would give him a Crown, or *c*, then *A* ſhould have as many Crowns as *B* had left, but *B* ſaith, if *A* would give him a Crown, then *B* ſhould have Twice as many Crowns as *A* had left; How many Crowns had each Perſon?

Problem 26. Two Men Diſcourſing of their Money, *A* ſaid to *B*, give me *c*, or 4, Shillings of yours, and I ſhall have *e*, or 3, Times as much as you; *B* ſaid, give me *d*, or 6, Shillings of yours, and we ſhall have both Alike; How much had Each of Them?

SOLUTION of PROBL. 23.

The Time Sought after Noon. x.

The Time from thence till Midnight. $c-x$.

b Parts of that Time. $bc-bx$.

By the Problem. $x=bc-bx$.

By Reduction. $x+bx=bc$.

And. $x=\frac{bc}{1+b}$

$b=\frac{33}{40}$, $c=12$ | $\frac{bc}{1+b}=\frac{\frac{396}{40}}{\frac{73}{40}}=5\frac{31}{73}$

12 Hours $=\frac{876}{73}$; $5\frac{31}{73}=\frac{396}{73}$

$\frac{876}{73}-\frac{396}{73}=\frac{480}{73}$ of an Hour the Time Remaining till Midnight; $\frac{12}{73}=40th.$ Part of that Time, and 33 of ſuch Parts $=\frac{396}{73}=5\frac{31}{73}$, the Time Sought after Noon.

Or.

The Time till Midnight. x.

The Time Sought after Noon. $c-x$.

b Parts of the Firſt Time. bx.

By the Problem. $c-x=bx$.

By Reduction. $bx+x=c$.

And. $x=\frac{c}{1+b}$.

$=\frac{480}{73}$, as before, $=6\frac{42}{73}$ Hours;

But, $5\frac{31}{73}+6\frac{42}{73}$ Hours. $=12$ Hours.

SOLUTION of PROBL. 24.

The Side of the firſt Square. x.

The Number of Soldiers in the Firſt Square. xx.

The Number of Soldiers in the whole Army. $xx+b$.

The Side of the Second Square. $x+c$.

The Square. $xx+2cx+cc$.

By the Problem.

$xx+2cx+cc-d=xx+b$.

By Reduction.

$2cx=b-cc+d$, or $x=\frac{b+d}{2c}-\frac{1}{2}c$.

Therefore. $xx+b=\frac{bb+2bd+dd}{4cc}+\frac{1}{4}cc+\frac{1}{2}b-\frac{1}{2}d$.

$b=500$, $c=1$, $d=29$ | $xx+b=\frac{250000+29000+841}{4}$

$+\frac{1}{4}+250-14\frac{1}{2}$

$=69960\frac{1}{4}+\frac{1}{4}+250-14\frac{1}{2}$

$=70196$, Soldiers.

$x=264$, and, $xx=69696$.

And $xx+b=69696+500$ $=70196$.

$264+1=265$; The Square, $=70225$, and $70225-29=$ 70196 Soldiers.

That is, $x+c$ is the Side of the Second Square; and $\overline{x+c}|^{q}-d=xx+b$, As was Required in the Problem; So that all the Conditions of it being Satisfied, the Solution of it is Juſt.

SOLUTION of PROBL. 25, 26.

Number of Crowns which A had. x.

Number of Crowns, which B had after he had given c Crowns, to A. $x+c$.

Number of Crowns which B had, at Firſt. $x+2c$.

Number of Crowns which B had after A had given him One. $x+3c$.

And the Number of Crowns, which A had, after he had given One to B. $x-c$.

But by the Problem. $x+3c$ $=2x-2c$.

By Reduction. $x=5c$.

And. $x+2c=7c$.

$c=1$ Crown; The Proof,

$5+1=7-1=6$ Crowns.

$7+1=\overline{5-1}\times 2=8$ Crowns.

Probl. 26. Number of Shillings, which A had. x.

Number of Shillings, which B had, after he had given c Shillings to A. $\frac{1}{e}x+\frac{1}{e}c$.

Number of Shillings, which B had at Firſt. $\frac{1}{e}x+\frac{1}{e}c+c$.

Number of Shillings, which B had after A had given him d Shillings. $\frac{1}{e}x+\frac{1}{e}c+c+d$.

But by the Probl. $x-d$ $=\frac{1}{e}x+\frac{1}{e}c+c+d$.

And. $x-\frac{1}{e}x=\frac{1}{e}c+c+2d$.

Or. $\frac{e-1}{e}x=\frac{1}{e}c+c+2d$.

And. $x=\frac{c+ec+2ed}{e-1}$.

$c=4$, $e=3$, $d=6$,

$x=\frac{4+3\times 4+2\times 3\times 6}{3-1}=26s.$

$\frac{1}{e}x+\frac{1}{e}c+c=\frac{26}{3}+\frac{4}{3}+4=14s.$

But. $26s+4s=30s.$

$14s-4s=10s.$

Alſo. $14s+6s$ | Or. $26s-6s$ | $=20s.$

Theſe Two Problems and the Two Annexed, Terminate alſo in Simple Equations. The Laſt of which was Propoſed to me by a Friend ſome Years ago, and Formerly Fellow of this College.

§. 9. PROBL. 27. A Vintner having Two Sorts of French Wines, one Sort worth 10*d.* or b, the Quart, and the other 6 *d.* or c, *per* Quart, would have a Mixt Quantity of both Sorts, to Conſiſt of 100 Quarts, or m, that might be worth 7 *d*, or f, *per* Quart; What Quantity of each Sort muſt be taken to make that Mixture?

Problems 26th. 27th. 28th.

Problem 28. A Ciſtern in a Certain Condoit is Supplied with Water by Two Pipes, of ſuch Capacities, or Diameters, that by both their Cocks *A* and *B*, ſet open at once the Ciſtern will be Fill'd in 12, or *b*, Hours, but by the Cock *A* alone in 20, or *c*, Hours. In what Time will the Ciſtern be Fill'd by the Cock *B* alone?

Problem 29. A Shepherd in the Time of War Driving a Flock of Sheep, Fell into the Hands of Three Companies of Soldiers, who Compell'd him to Deliver the Half of his Flock, with Half a Sheep over to the First Company, alſo Half his Remaining Flock, with Half a Sheep over to the Second Company, likewiſe Half the Reſt of his Flock, with Half a Sheep over to the Third Company, which Diviſions the Shepherd Perform'd without Killing a Sheep, and there Remain'd 20, or *b*, Sheep to himſelf; How many had he in his Flock at Firſt?

SOLUTION of PROBL. 27.

Number of Quarts of the better Sort of Wine. x.

Number of Quarts of the Worſer Wine. $m-x$.

The Worth of the Better Sort of Wine at *b per* Quart. bx.

Becauſe $1 . b :: x . bx$.

The Worth of the Worſer Sort of Wine at *c per* Quart. $mc-cx$.

Becauſe $1 . c :: m-x . cm-cx$

But by the Problem.

$mc-cx+bx=fm$.

By Reduction $x=\frac{fm-cm}{b-c}$.

$$\begin{array}{l|l} b=10 & \\ c=6 & x=\frac{100}{4}=25. \\ m=100 & m-x=100-25=75 \\ f=7 & \end{array}$$

$mf=700\,d.=25\times10\,d.+75\times6\,d.=250\,d.+450\,d.$

Canon or Theorem

$b-c . m :: f-c . x$.

Hence the Rule of Alligation Alternate in Vulgar Arithmetick.

Alſo $m-x=m-\frac{fm+cm}{b-c}$

$=\frac{bm-cm-fm+cm}{b-c}$,

$=\frac{bm-fm}{b-c}$; Therefore

$b-c . m :: b-f . m-x$.

SOLUTION of PROBL. 28.

Time Sought. x.

The Part of the Ciſtern Fill'd by the Cock *B* in *b* Hours. $\frac{b}{x}$.

For. $x . 1 :: b . \frac{b}{x}$.

The Part of the Ciſtern Fill'd by the Cock *A* in *b* Hours. $\frac{b}{c}$.

For. $c . 1 :: b . \frac{b}{c}$.

But by the Problem Theſe Two Parts = to the Whole Ciſtern.

That is, $\frac{b}{x}+\frac{b}{c}=1$.

By Reduction. $b+\frac{bx}{c}=x$,

And, $cb+bx=cx$, Or, $x=\frac{cb}{c-b}$

$c=20$, $b=12$,

$x=\frac{240}{8}=30$ Hours.

Canon or Theorem.

$c-b . c :: b . x$.

Proof.

$\frac{b}{x}=\frac{12}{30}$, $\frac{b}{c}=\frac{12}{20}$.

$\frac{12}{30}+\frac{12}{20}=\frac{2}{5}+\frac{3}{5}=\frac{5}{5}=1$.

SOLUTION of PROBL. 29.

Number of Sheep, which the Shepherd had at Firſt. x.

Number of Sheep Delivered to the Firſt Company of Soldiers. $\frac{1}{2}x+\frac{1}{2}$.

The Remainder to him after ſuch Delivery. $\frac{1}{2}x-\frac{1}{2}$,

The Number of Sheep Delivered to the Second Company. $\frac{1}{4}x-\frac{1}{4}+\frac{2}{4}=\frac{1}{4}x+\frac{1}{4}$.

Remainder to him after ſuch Delivery. $\frac{1}{2}x-\frac{1}{2}-\frac{1}{4}x-\frac{1}{4}=\frac{1}{4}x-\frac{3}{4}$.

The Number of Sheep Delivered to the Third Company. $\frac{1}{8}x-\frac{3}{8}+\frac{4}{8}=\frac{1}{8}x+\frac{1}{8}$.

Remainder to him after ſuch Delivery. $\frac{1}{4}x-\frac{3}{4}-\frac{1}{8}x-\frac{1}{8}=\frac{1}{8}x-\frac{7}{8}$.

By the Problem. $\frac{1}{8}x-\frac{7}{8}=b$.

By Reduction. $x=8b+7$.

$b=20$, $x=167$.

1*ſt*. Company had. $83\frac{1}{2}+\frac{1}{2}=84$.

Remain'd to the Shepherd. $167-84=83$.

2*d*. Company had. $41\frac{1}{2}+\frac{1}{2}=42$.

Remain'd to the Shepherd. $83-42=41$.

3*d*. Company had. $20\frac{1}{2}+\frac{1}{2}=21$.

Remain'd to the Shepherd. $41-21=20$.

These Problems alſo End in Simple Equations.

Problems 30th, 31ſt, 32d.

§. 10. **Probl. 30.** Two Merchants *A* and *B* were Copartners in Traffick, the Sum of their Stocks was 300 *l.* or *b*, the Stock of *A* Continu'd in Company 9, or *c*, Months, and the Stock of *B* 11, or *d*, Months, they Gain'd a certain Sum of Money, which they Divided Equally; What was each Merchant's Stock at Firſt?

Problem 31. A Certain Man being Ask'd, how many Years Old he was, Anſwered, if $\frac{1}{20}$, or b, Part of the Number of Years he had Liv'd were Multiplied by $\frac{5}{8}$, or c, Parts of the ſame Number, the Product would give his Age; What was it?

Problem 32. There are Two Numbers, the Greater of which hath ſuch Proportion to the Leſs, as 3 to 2, or as r to s, and the Sum of the ſaid Numbers hath ſuch Proportion to the Sum of their Squares, as 1 to 13, or as b to c; What are the Numbers?

Solution of Probl. 30.

The Stock of *A*, when he Enter'd Partner. x.
The Stock of *B* therefore. $b-x$.
The Stock of *A* Multiplied by the Time it Continu'd in Company, *viz.* by c, Produces. cx.
The Stock of *B* Multiplied by the Time it Continu'd in Company, *viz.* by d, Produces. $db-dx$.
But by the Problem, Becauſe they Divided the Gains Equally. $cx=db-dx$.

By Reduction $x=\frac{db}{c+d}$.

And $b-x=b-\frac{db}{c+d}=\frac{cb}{c+d}$.

$$\left.\begin{array}{l} b=300. \\ c=9. \\ d=11. \end{array}\right| \begin{array}{l} x=\frac{3300}{20}=165\ l. \\ b-x=\frac{2700}{20}=135\ l. \end{array}$$

$135\ l.+165\ l.=300\ l.$

Pounds.	Pounds.	Months.	Months.
135.	165 ::	9.	11.

For $165\times 9=135\times 11=1485$
i. e. 165 *l.* for Nine Months is the ſame, as 135 *l.* for 11 Months.

Canon or Theorem.
$c+d.\ b::d.\ x$ Stock of *A*.
$c+d.\ b::c.\ b-x$ Stock of *B*.

Solution of Probl. 31.

Number of Years, or Age, Sought. x.
By the Problem, b Parts of $x \times x \times c$ Parts of $x \times x$ $=x,=bx\times cx$, Or, $bcxx=x$.
By Reduction. $bcx=1$.
And. $x=\frac{1}{bc}$.

$$\left.\begin{array}{l} b=\frac{1}{20} \\ c=\frac{5}{8} \end{array}\right| \begin{array}{l} x=\frac{1}{\frac{5}{160}}. \\ =\frac{160}{5}=32\ \text{Years.} \end{array}$$

For $1\frac{3}{5}=\frac{1}{20}$ of 32, and $20=\frac{5}{8}$ of 32, and $20\times 1\frac{3}{5}=20+12=32$ Years.

Or,

According to the Problem, $\frac{1}{20}$*th.* Part of his Age, Namely 32 Years, Multiplied into $\frac{5}{8}$*ths.* of it will Give the Whole of his Age; So that, the Conditions of the Queſtion being Satiſfied, the Problem is Fully and Abſolutely Solved, without which no Solutions can be Accurate, or Complete.

Solution of Probl. 32.

Greater Number. x
Leſſer. $\frac{sx}{r}$.
Becauſe $r.\ s::x.\ \frac{sx}{r}$.
Sum of Both. $x+\frac{sx}{r}$.
Sum of the Squares of Both. $xx+\frac{ssxx}{rr}$
By Proportion.
$b.\ c::x+\frac{sx}{r}.\ \frac{crx+csx}{br}$.
By Probl. $\frac{crx+csx}{br}=xx+\frac{ssxx}{rr}$
By Reduction It Follows, That, $\frac{cr+cs}{br}=x+\frac{ssx}{rr}$.
And. $cr+cs=brx+\frac{bssx}{r}$.
And. $crr+csr=brrx+bssx$.
And. $x=\frac{crr+csr}{brr+bss}$.

$$\left.\begin{array}{l} r=3. \\ s=2. \\ b=1. \\ c=13. \end{array}\right| \begin{array}{l} x=\frac{195}{13}=15. \\ \frac{sx}{r}=\frac{30}{3}=10. \end{array}$$

And, that this Solution is Right, is thus Proved, Becauſe.
15. 10 :: 3. 2, And
25. 325 :: 1. 13.

That is, the Two Numbers Required, 15 and 10, are to each Other, as 3 to 2; and the Sum of the Said Numbers is to the Square of Them, as 1 to 13, which were the Conditions Propoſed in the Problem.

Theſe Problems likewiſe End in Simple Equations.

Problems 33d. 34th. 35th

§. 11. **Problem 33.** There are Two Numbers, the Greater of which hath ſuch Proportion to the Leſſer, as 2 to 1, or as r to s, and the Sum of the Squares of the ſaid Numbers is 125 or b; What are the Numbers?

Problem 34. A Merchant Bought a certain Number of Yards of Linnen Cloth, at 12 *d*, or b, *per* Yard, and if the Number of Pence, Paid for all the Cloth, be Multiplied by the Number of Yards Bought, the Product will be 30000, or c; What was the Number of Yards Bought?

Problem 35. Two Merchants *A* and *B* were Copartners, *A* brought in a Certain Number of Pounds, which Continued in Company 4, or c, Months, *B* brought in

100

100, or *b*, Pounds, which Continued in Company ſuch a Time, that if it be Multiplied by the Stock of *A*, it makes 50 or *d*; At the End of their Partnerſhip they had Gained 60 *l.* whereof *A* had 40, or *r*, Pounds for his Share, and *B* the Reſt, *viz.* 20, or *s*, Pounds; What was the Stock, which *A* put in at Firſt, and how many Months did the Stock of *B* Continue in Company?

SOLUTION of PROBL. 33.

Greater Number. x.

Leſſer Number. $\frac{sx}{r}$.

Becauſe. $r.\ s :: x.\ \frac{sx}{r}$.

Sum of their Sq^{rs}. $xx + \frac{ssxx}{rr}$.

But, by the Problem.

$$xx + \frac{ssxx}{rr} = b.$$

By Reduct. $rrxx + ssxx = brr$.

And. $xx = \frac{brr}{rr+ss}$.

And. $x = \sqrt{\frac{brr}{rr+ss}}$.

$b = 125$.
$r = 2$.
$s = 1$.

$$x = \sqrt{\frac{500}{5}} = 10.$$

$$\frac{sx}{r} = \frac{10}{2} = 5.$$

$$= \frac{s}{r}\sqrt{\frac{brr}{rr+ss}} =$$

$$\sqrt{\frac{ssbrr}{rrrr+rrss}} = \sqrt{\frac{ssb}{rr+ss}}$$

But 10. 5 :: 2. 1
And the Squares $= 100 + 25$.
Which were the Conditions Required in the Problem.

SOLUTION of PROBL. 34.

Number of Yards Bought. x.

Number of Pence Paid for the Cloth. bx.

Number of Yards Bought × Number of Pence Paid. bxx.

Which by the Probl. is $= c$ *i. e.* $bxx = c$.

By Reduction. $xx = \frac{c}{b}$

And. $x = \sqrt{\frac{c}{b}}$.

$b = 12$.
$c = 30000$.

$$x = \sqrt{\frac{30000}{12}} = \sqrt{2500}.$$

$= 50$ Yards.
But $50 \times 12 = 600$ *d.* the Price Paid for the Cloth. $= bx$.
And, 600×50 the Number of Yards $= 30000$.
Or; $12 \times 50 \times 50 = bxx = c$.

SOLUTION of PROBL. 35.

Stock of *A*. x.

Stock of *A* into the Time it Continued in Comp. cx.

The Time that the Stock of *B* Continued in Comp. $\frac{d}{x}$

Stock of *B*, *i. e.* b into the Time it Continued in Company, *i. e.* $\frac{d}{x}$. $\frac{bd}{x}$.

According to the Nature of the Rule of Fellowſhip with Time.

Stock *A*. Stock *B*.
$cx.\ \frac{bd}{x}$
Gn. *A* Gn. *B*.
$::\ r.\ s.$

But, by the Probl. $scx = \frac{rbd}{x}$

By Reduction. $scxx = rbd$

And. $xx = \frac{rbd}{sc}$.

And. $x = \sqrt{\frac{rbd}{sc}}$

$r = 40$
$b = 100$
$d = 50$
$s = 20$
$c = 4$

$$x = \sqrt{\frac{200000}{80}} = 50\ l.$$

$$\frac{d}{x} = \frac{50}{50} = 1 \text{ Month.}$$

$50 \times 4 = 200 = cx$,
$100 \times 1 = 100 = x)\ bd$.

St. and Tim. *A*. St. and Tim. *B*. Gn. *A*. Gn. *B*.
And. 200. 100 :: 40. 20.

Theſe Problems End in Simple Quadratick Equations. Problems 36th. 37th. 38th.

Theſe Problems End in Equations, which are Termed Simple Quadratick Equations, where the Unknown Quantity xx is a Square on one Side of the Equation, and all the Quantities are Known on the Other.

§. 12. PROBLEM 36. Certain Noblemen made a Progreſs for their Pleaſure; every Nobleman carried with him the ſame Sum of Pounds; the Number of the Noblemen was Equal to the Number of Servants, which Attended upon each Nobleman; the Number of Pounds, which each Nobleman had, was the Double of the Number of all their Servants, and the Sum of all their Money was 3456 *l*; What was the Number of Noblemen, and how many Pounds and Servants had Each?

PROBLEM 37. A Merchant Bought as many Pounds of Pepper for One Crown, as was half the Number of Crowns he Laid out; Then in Selling the Pepper he Receiv'd for every 25 *l.* of Pepper as many Crowns as he Paid for all the Pepper, and in Concluſion he had 20 Crowns; How many Crowns did he lay out?

PROBLEM 38. There are Two Numbers, the Greater of which hath ſuch Proportion to the Leſſer, as 3. 2, or *r. s*, and the Sum of the Cubes of the Two Numbers is 4375, or *b*; What are the Numbers?

Solution of Probl. 36.

The Number of Noblemen. x.

The Number of Servants, which Attended each Nobleman. x.

The whole Number of Servants. xx.

The Number of Pounds, which each Nobleman had. $2xx$.

The Sum of all their Money. $2xxx$.

Equation. $2xxx=3456$.

Half of which Sum is, $xxx=1728$.

And. $x=\sqrt{^3}.\,1728=12$.

Proof.

The Number of Noblemen. 12.

The Number of Servants Attending each Nobleman. 12.

The whole Number of Servants. 144.

Number of Pounds, which each Nobleman had. 288.

The Sum of all their Money, the Former Number being Multiplied into Twelve. 3456.

Solution of Probl. 37.

Number of Crowns laid out. x.

Number of Pounds of Pepper Bought for one Crown. $\frac{x}{2}$

The whole Quantity of Pepper Bought. $\frac{xx}{2}$

Crown. Crown. Pepper. Pepper.

Becauſe $1.\ x : : \frac{x}{2}.\ \frac{xx}{2}$.

The Number of Crowns, for which all the Pepper was Sold. $\frac{xxx}{50}$

Becauſe $25.\ x : : \frac{xx}{2}.\ \frac{x^3}{50}$.

By the Problem, The Equation will be. $\frac{x^3}{50}=20$.

By Reduction. $x^3=1000$.

Therefore. $x=10$ Crowns.

Quantity of Pepper Bought. $50\,l$.

He therefore Bought $5\,l$. of Pepper for 1 Crown (5 being Half of 10.) or, $\frac{x}{2}$.

For every $25\,l$. of Pepper he Receiv'd 10 Crowns, and therefore for $50\,l$. he Received 20 Crowns, the Sum Expreſſed in the Problem. $\frac{x^3}{50}=20$.

Solution of Probl. 38.

The Greater Number. x.

The Leſſer Number. $\frac{sx}{r}$.

Becauſe $r.\ s : : x\ \frac{sx}{r}$.

The Cube of the Greater Number. xxx.

The Cube of the Leſſer. $\frac{sssxxx}{rrr}$.

Sum of the Cubes $x^3+\frac{s^3x^3}{r^3}$.

Which by the Problem is $=b$.

By Reduction.

$$r^3x^3+s^3x^3=r^3b.$$

And. $x^3=\frac{r^3b}{r^3+s^3}$

Therefore. $x=\sqrt{^3}.\,\frac{r^3b}{r^3+s^3}$

And. $\frac{sx}{r}=\frac{s}{r}\sqrt{^3}.\,\frac{r^3b}{r^3+s^3}$

$=\sqrt{^3}.\,\frac{s^3r^3b}{r^6+r^3s^3}=\sqrt{^3}.\,\frac{s^3b}{r^3+s^3}$

$r=3$.

$s=2$.

$b=4375$.

Therefore. $x=\sqrt{^3}.\,\frac{118125}{35}$

Or, $x=\sqrt{^3}.\,3375=15$.

And. $\frac{sx}{r}=\frac{30}{3}=10$.

But. $15.\ 10 : : 3.\ 2$.

And. $3375+1000=4375$.

These Problems End in Equations, which are Termed Simple Cubick Equations, where the Unknown Quantity xxx is a Cube on one Side of the Equation, and all the Quantities are Known on the Other.

These Problems End in Simple Cubick Equations.

CHAP. IV.

Of Affected Quadratick Equations, and the ſeveral Forms of Them.

Of the ſeveral Forms of Affected Quadratick Equations and their Reſolution.

§. 1. AFFECTED Quadratick Equations are Diſtinguiſhed into their ſeveral Forms by the Variety of the Privative or Poſitive Signs, which are Uſed in Them; Thus, in the firſt Form, in Particular, $xx+6x=55$, or, $xx=-6x+55$, and in General, $xx+cx=b$, or $xx=-cx+b$; In the Second Form, in Particular, $xx-10x=24$, or $xx=10x+24$, and in General, $xx-bx=k$, or $xx=bx+k$; And the ſame may be Said of the Third Form; For in Two Quantities Equal to a Third, there can be only theſe Three Caſes, in Reſpect to the Signs, either $xx+cx=b$, or $xx-cx=b$, or $cx-xx=b$; For the Fourth Form of $-xx-cx=b$, is Impoſſible; Becauſe a Privative Quantity cannot be Equal to a Poſitive; It muſt be alſo Obſerved with Regard to theſe Quadratick Forms, That they may be Continued *in Infinitum*; For $x^8+ex^4=n$, $x^{10}+ax^5=q$, &c. in which Equations a, q, e, n are Known Quantities, and x, x^2, x^3, *&c.* the Unknown; And, the Former being Affected, or Involved, with the Latter is the Reaſon, that they are Termed Affected Equations; As Likewiſe, becauſe $x^2, x^4, x^6, x^8, x^{10}$, *&c.* are Squares of x, x^2, x^3, *&c.* They are upon that Account Named Quadratick.

FIRST FORM.

In Numbers. In Letters.

$$\begin{array}{ll|l|l|l}
1\textit{ſt}. & xx+6x=55, & xx=-6x+55. & xx+cx=b, & xx=-cx+b. \\
2d. & x^4+8xx=48, \quad \text{And,} & x^4=-8x^2+48. \quad \text{Or,} & x^4+dx^2=f, \quad \text{And,} & x^4=-dx^2+f. \\
3d. & x^6+4x^3=837, & x^6=-4x^3+837. & x^6+gx^3=h, & x^6=-gx^3+h. \\
 & \&c. & \&c. & \&c. & \&c.
\end{array}$$

SECOND FORM.

In Numbers. In Letters.

$$\begin{array}{ll|l|l|l}
1\textit{ſt}. & xx-10x=24, & xx=10x+24. & xx-bx=k, & xx=bx+k. \\
2d. & x^4-6xx=27, \quad \text{And,} & x^4=6xx+27. \quad \text{Or,} & x^4-pxx=d, \quad \text{And,} & x^4=pxx+d. \\
3d. & x^6-2x^3=48, & x^6=2x^3+48. & x^6-mx^3=g, & x^6=mx^3+g. \\
 & \&c. & \&c. & \&c. & \&c.
\end{array}$$

THIRD FORM.

In Numbers. In Letters.

$$\begin{array}{ll|l|l|l}
1\textit{ſt}. & 10x-xx=24, & xx=10x-24. & cx-xx=n, & xx=cx-n. \\
2d. & 5xx-x^4=4, \quad \text{And,} & x^4=5xx-4. \quad \text{Or,} & rxx-x^4=s, \quad \text{And,} & x^4=rxx-s. \\
3d. & 9x^3-x^6=8, & x^6=9x^3-8. & dx^3-x^6=t, & x^6=dx^3-t. \\
 & \&c. & \&c. & \&c. & \&c.
\end{array}$$

ALL which Forms may be Reſolved by the General Theorem Demonſtrated Above, *viz.* $x = .\frac{1}{2}p \pm \sqrt{\frac{1}{4}pp\,.\,q}$; And the Problem in all of Them is; What is x?

FIRST FORM.

PROBLEM 1*ſt.* Reſolution of the Firſt Equation of the Firſt Form, Column 4*th.*

By the General Theorem, $x=-\frac{1}{2}c \pm \sqrt{\frac{1}{4}cc+b}$; $\begin{array}{l} c=6 \\ b=55 \end{array}\Big|\, x=-3 \pm \sqrt{9+55} = \Big|\begin{array}{l} 8-3 \\ -8-3 \end{array}\Big|\begin{array}{l} =+5. \\ =-11. \end{array}$

Proof; If $x=5$, $\begin{array}{r} xx=25 \\ 6x=30 \end{array}\Big|=55$; If $x=-11$, $\begin{array}{r} xx=121 \\ 6x=-66 \end{array}\Big|=55.$

PROBLEM 2*d.* Reſolution of the Second Equation of the Firſt Form, Column 4*th.*

By the Theorem, $xx=-\frac{1}{2}d \pm \sqrt{\frac{1}{4}dd+f}$; $\begin{array}{l} d=8 \\ f=48 \end{array}\Bigg|\begin{array}{l} xx=-4 \pm \sqrt{16+48} = \Big|\begin{array}{l} 8-4= \\ -8-4 \end{array}\Big|\begin{array}{l} +4. \\ -12. \end{array} \\ x=\Big|\begin{array}{l} +2 \\ \sqrt{-12} \end{array} \end{array}$

Proof; If $x=2$, $\begin{array}{r} x^4=16 \\ 8xx=32 \end{array}\Big|=48$; If $x=\sqrt{-12}$, $\begin{array}{r} x^4=144 \\ 8xx=-96 \end{array}\Big|=48.$

PROBLEM 3*d.* Reſolution of the Third Equation of the Firſt Form, Column 4*th.*

By the Theorem, $xxx=-\frac{1}{2}g \pm \sqrt{\frac{1}{4}gg+h}$; $\begin{array}{l} g=4 \\ h=837 \end{array}\Big|\, x^3=-2 \pm \sqrt{4+837}=29-2=27;$

Or, $=-29-2=-31.$

Therefore, $x=\sqrt[3]{.-\frac{1}{2}g \pm \sqrt{\frac{1}{4}gg+h}} = \Big|\sqrt[3]{.27}=3.$
Or, $x=\Big|\sqrt[3]{.-31}.$

Proof; If $x=3$, $\begin{array}{r} x^6=729 \\ 4x^3=108 \end{array}\Big|=837$; If $x=\sqrt[3]{.-31}$, $\begin{array}{r} x^6=961 \\ 4x^3=-124 \end{array}\Big|=837.$

SECOND FORM.

PROBL. 4*th.* Reſolution of the Firſt Equation of the Second Form, Column 4*th.*

By the Theorem, $x=\frac{1}{2}b \pm \sqrt{\frac{1}{4}bb+k}$; $\begin{array}{l} b=10 \\ k=24 \end{array}\Big|\, x=5 \pm \sqrt{25+24}=5 \pm 7 = \Big|\begin{array}{l} +12. \\ -2. \end{array}$

Proof; If $x=12$, $\left.\begin{array}{r} xx=144\\ -10x=-120\end{array}\right|=24$; If $x=-2$, $\left.\begin{array}{r} xx=4\\ -10x=20\end{array}\right|=24.$

Probl. *5th.* Reſolution of the Second Equation of the Second Form, Column *4th*,

By the Theorem, $xx=\tfrac{1}{2}p\pm\sqrt{\tfrac{1}{4}pp+d}$; $\left.\begin{array}{r} p=6\\ d=27\end{array}\right| xx=3\pm\sqrt{9+27}=3\pm6=\left|\begin{array}{l} +9\\ -3.\end{array}\right.$

And Conſequently, by Extracting the Root of xx, $x=\sqrt{\tfrac{1}{2}p\pm\sqrt{\tfrac{1}{4}pp+d}}=\left|\begin{array}{l}\sqrt{9}=3.\\ \sqrt{-3}\end{array}\right.$

Proof; If $x=3$, $\left.\begin{array}{r} x^4=81\\ -6xx=-54\end{array}\right|=27$; If $x=\sqrt{-3}$, $\left.\begin{array}{r} x^4=9\\ -6xx=18\end{array}\right|=27.$

Probl. *6th.* Reſolution of the Third Equation of the Second Form, Column *4th*.

By the Theorem, $x^3=\tfrac{1}{2}m\pm\sqrt{\tfrac{1}{4}mm+g}$; $\left.\begin{array}{r} m=2\\ g=48\end{array}\right| x^3=1\pm\sqrt{1+48}=\left|\begin{array}{l} 1+7=8.\\ 1-7=-6\end{array}\right.$

Therefore, $x=\sqrt{^3.\tfrac{1}{2}m\pm\sqrt{\tfrac{1}{4}mm+g}}=\sqrt{^3.8}=2$, Or, $\sqrt{^3.-6}$.

Proof; If $x=2$, $\left.\begin{array}{r} x^6=64\\ -2x^3=-16\end{array}\right|=48$; If $x=\sqrt{^3.-6}$, $\left.\begin{array}{r} x^6=36\\ -2x^3=12\end{array}\right|=48.$

THIRD FORM.

Probl. *7th.* Reſolution of the Firſt Equation of the Third Form, Column *4th*.

By the Theorem, $x=\tfrac{1}{2}c\pm\sqrt{\tfrac{1}{4}cc-n}$; $\left.\begin{array}{r} c=10\\ n=24\end{array}\right| x=5\pm\sqrt{25-24}=5\pm1=\left|\begin{array}{l} 6.\\ 4.\end{array}\right.$

Proof; If $x=6$, $\left.\begin{array}{r} 10x=60\\ -xx=36\end{array}\right|=24$; If $x=4$, $\left.\begin{array}{r} 10x=40\\ -xx=-16\end{array}\right|=24.$

Probl. *8th.* Reſolution of the Second Equation of the Third Form, Column *4th*.

By the Theorem, $xx=\tfrac{1}{2}r\pm\sqrt{\tfrac{1}{4}rr-s}$; $r=5\,|\,xx=2\tfrac{1}{2}\pm\sqrt{6\tfrac{1}{4}-4}=2\tfrac{1}{2}\pm1\tfrac{1}{2}=\left|\begin{array}{l} 4.\\ 1.\end{array}\right.$

And, Therefore, $x=\sqrt{\tfrac{1}{2}r\pm\sqrt{\tfrac{1}{4}rr-s}}$; $s=4\,|\,x=\left|\begin{array}{l}\sqrt{4}=\\ \sqrt{1}=\end{array}\right|\begin{array}{l} 2\\ 1\end{array}$

Proof; If $x=2$, $\left.\begin{array}{r} 5xx=20\\ -x^4=-16\end{array}\right|=4$; If $x=1$, $\left.\begin{array}{r} 5xx=5\\ -x^4=-1\end{array}\right|=4.$

Probl. *9th.* Reſolution of the Third Equation of the Third Form, Column *4th*.

By the Theorem, $x^3=\tfrac{1}{2}d\pm\sqrt{\tfrac{1}{4}dd-t}$; $\left.\begin{array}{r} d=9\\ t=8\end{array}\right| x^3=4\tfrac{1}{2}\pm\sqrt{20\tfrac{1}{4}-8}=4\tfrac{1}{2}\pm3\tfrac{1}{2}=\left|\begin{array}{l} 8.\\ \text{or, } 1.\end{array}\right.$

And, Therefore, $x=\sqrt{^3.\tfrac{1}{2}d\pm\sqrt{\tfrac{1}{4}dd-t}}=\left|\begin{array}{l}\sqrt{^3.8}=\\ \sqrt{^3.1}=\end{array}\right|\begin{array}{l} 2.\\ 1.\end{array}$

Proof; If $x=2$, $\left.\begin{array}{r} 9x^3=72\\ -x^6=-64\end{array}\right|=8$; If $x=1$, $\left.\begin{array}{r} 9x^3=9\\ -x^6=-1\end{array}\right|=8.$

In the Firſt and Second Forms, the Roots are, the One Negative, the Other Poſitive, And in the Third, both Roots Poſitive; That is, where the Higheſt Unknown Power in the Firſt and Third Columns is Poſitive, One Root is Negative, the Other Poſitive, where the Higheſt Power is Negative, both Roots are Poſitive, and become Poſſible or Impoſſible, according as the Quantity, from which the Extraction is made, is Poſitive or Negative, and according to the Nature of the Root, which is to be Extracted; Thus $\sqrt{^2}-8$ is Impoſſible, or $\sqrt{^2.-4}$; Becauſe, no Quantity Multiplied into it's ſelf will Produce a Privative One; But $\sqrt{3}-8$ is Poſſible, becauſe $-2\times-2\times-2=-8$, *&c.*

§. 2. **Probl.** 1. There are Two Numbers, whoſe Difference is 16, or *c*, and the Product of their Multiplication is 36, or *b*; What are the Numbers?

Probl. 2. There are Three Numbers in Continual Proportion, the Difference of the Extremes is 16, or *c*, and the Mean Proportional is 6, or *m*; What are the Extremes?

Probl. 3. There are Two Numbers, whoſe Difference is 15, or *d*, and if the Product of the Multiplication of the ſaid Two Numbers be Divided by 2 or *c*, the Quotient will give the Cube of the Leſſer Number; what are the Numbers?

Problems, which Produce Equations Falling under one of the Abovementioned Forms; 1ſt. 2d. 3d. Problems.

SOLUTION of PROBL. 1.

Greater of the Two Numbers Sought. x.

Leſſer Number. $x-c$.

The Product of the Two Numbers. $xx-cx$.

By the Problem. $xx-cx=b$.

By Reduction. $xx=cx+b$.

By Reſolut. $x=\frac{1}{2}c\pm\sqrt{\frac{1}{4}cc+b}$.

$$\begin{array}{l|l} c=16 & x=8\pm\sqrt{64+36} \\ b=36 & =8\pm10=18 \\ & \text{Or, } -2 \end{array}$$

Difference of x, and $x-c$ is $18-2=16$, or, $-2+2+16=16$.

Product of their Multiplication is, $18\times2=36$,

Or, elſe, $-2\times\overline{-2-16}=36$.

Or,

Leſſer Number. x.

Greater Number. $x+c$.

The Product of the Two Numbers. $xx+cx=b$.

By Reſolut. $x=-\frac{1}{2}c\pm\sqrt{\frac{1}{4}cc+b}$

$=-8\pm10=\left|\begin{array}{l}+2.\\-18.\end{array}\right.$

And. $-18\times\overline{-18+16}=36$.

Difference. 16.

SOLUTION of PROBL. 2.

Leſſer of the Two Extreme Proportionals Sought. x.

Greater. $x+c$.

Product of them. $xx+cx$.

Square of the Middle Proportional. mm.

By the Nature of Proportionals. $xx+cx=mm$.

Becauſe by the Problem, It will be, $x.\ m::m.\ x+c$.

By Reduct. $xx=-cx+mm$.

By Reſolution.

$x=-\frac{1}{2}c\pm\sqrt{\frac{1}{4}cc+mm}$.

$$\begin{array}{l|l} c=16 & x=-8\pm\sqrt{64+36} \\ m=6 & =-8\pm10=2,\text{or},-18. \end{array}$$

Difference of the Extremes, x, $x+c$. 2, and 18, $=16$.

Or, -18, from $-18+16=16$.

and the Mean Proportional 6;

For 2. 6 :: 6. 18, Or,

-18. 6 :: 6. -2.

Since 18 is 3 Times more than 6, as 6 is than 2, and the Multiplication of $-\times-$ is $=+\times+$.

SOLUTION of PROBL. 3.

Leſſer Number Sought. x.

Greater Number. $x+d$.

Product of them. $xx+dx$.

By the Probl. $\frac{xx+dx}{c}=x^3$.

By Reduction.

$\frac{x+d}{c}=xx$, or, $xx=\frac{1}{c}x+\frac{d}{c}$

By Reſolution.

$x=\frac{1}{2c}\pm\sqrt{\frac{1}{4}cc+\frac{d}{c}}$.

$$\begin{array}{l|l} c=2 & x=\frac{1}{4}+\sqrt{\frac{1}{16}+\frac{15}{2}} \\ d=15 & =\frac{1}{4}\pm2\frac{3}{4}=3, \end{array}$$

Or, $-2\frac{1}{2}$.

Greater Number. 18, or, $12\frac{1}{2}$.

Difference $18-3=15$.

Or, $12\frac{1}{2}--2\frac{1}{2}=15$.

$\frac{18\times3}{2}=\frac{54}{2}=27$, the Cube of 3, the Leſſer Number.

Or, $\frac{12\frac{1}{2}\times-2\frac{1}{2}}{2}=-15\frac{5}{8}$

$=$ Cube of $-2\frac{1}{2}$.

Problems 4th. 5th. 6th.

§. 3. PROBL. 4. There are Three Numbers in Geometrical Proportion Continu'd, and Such, that if the Difference between the Sum of the Extremes and Mean be Multiplied by the Sum of the Extremes, the Product will be 112c, or *b*, but if the Said Difference be Multiplied by the Sum of all the Three Proportionals, the Product will be 1456, or *c*; What are the Proportionals?

PROBL. 5. There are Two Numbers, whoſe Sum is 10, or *b*, and the Proportion, which their Difference bears to the Sum of their Squares, is, as 2 to 29, or as *r* to *s*; What are the Numbers?

PROBL. 6. There are Two Numbers, the Product of whoſe Multiplication is, 48, or *p*, and the Difference of their Squares is 28, or *d*; What are the Numbers?

SOLUTION of PROBL. 4.

Difference of the Sum of the Extremes and Mean. x.

By the Problem the Sum of the Extremes. $\frac{b}{x}$.

The Mean Proportional. $\frac{b}{x}-x$.

For thus, in 3. 9. 27.

Since $3+27-9=21$, the Difference berwixt the Sum of the Extremes and Mean $3+27-21=9$, the Mean Proportional.

SOLUTION of PROBL. 5.

Greater Number. x.

Leſſer Number. $b-x$.

Difference. $2x-b$.

Square of the Greater Number. xx.

Square of the Leſſer.

$bb-2bx+xx$.

Sum of the Squares.

$bb-2bx+2xx$.

By the Problem.

$2x-b.\ bb-2bx+2xx::r.\ s.$

Equation will be.

$2sx-sb=rbb-2rbx+2rxx$.

SOLUTION of PROBL. 6.

Greater of the Numbers. x.

The Leſſer will be. $\frac{p}{x}$.

The Square of the Firſt. xx.

Of the other. $\frac{pp}{xx}$.

Difference of the Squares. $xx-\frac{pp}{xx}$.

Which by the Probl. $=d$.

By Reduction.

$-pp+x4=dxx$.

And, $x^4=dxx+pp$.

The Sum of all Three. $\frac{b}{x}+\frac{b}{x}-x.$
$=\frac{2b}{x}-x.$

But by the Probl. $2b-xx=c.$
$xx\times\frac{2b}{x}-x=c.$ Or,

By Reduction. $xx=2b-c.$
And. $x=\sqrt{2b-c}.$

$b=1120$ | $\sqrt{2b-c}=\sqrt{2240-1456}=\sqrt{784}$
$c=1456$ | $=28=x.$

$\frac{b}{x}=\frac{1120}{28}=40$; Sum of the Extremes $\frac{b}{x}, -x=40, -28$
$=12.$ The Mean.

The Extremes are found to be 4 and 36, by a Problem, in which their Sum 40 is Given, and the Mean Proportional 12. *Vid.* Probl. 2*d.*
But, 4. 12 :: 12. 36.
Likewiſe, $28\times40=1120.$
Laſtly, $28\times52=1456.$

By Reduction.
$2rxx=2rbx-rbb+2sx-sb.$
Or, $xx=bx-\frac{1}{2}bb+\frac{s}{r}x-\frac{sb}{2r},$
$=bx+\frac{s}{r}x-\frac{1}{2}bb-\frac{sb}{2r}.$

By Reſolution or Extraction.
$x=\frac{1}{2}b+\frac{s}{2r}\pm\sqrt{\frac{ss}{4rr}-\frac{bb}{4}}.$

$b=10$ | $x=5+7\frac{1}{4}$
$r=2$ | $\pm\sqrt{\frac{841}{16}-\frac{400}{16}}$
$s=29$ | $=12\frac{1}{4}\pm\sqrt{\frac{441}{16}}$
$=12\frac{1}{4}\pm5\frac{1}{4}=17\frac{1}{2},$ Or, 7.

The Greater Number $=7.$
Leſſer $10-7=3.$ Proof; $3+7=10$; and 4. 58 :: 2. 29.
$17\frac{1}{2}$ and $-7\frac{1}{2}$ will, Alſo, Solve this Problem;
For their Sum, $17\frac{1}{2}-7\frac{1}{2}=10.$
Their Difference $=25.$
The Sum of their Squares will be $=362\frac{1}{2}.$
But, 25. $362\frac{1}{2}$ :: 2. 29.

By Reſolution or Extraction, It will be, $xx=\frac{1}{2}d\pm\sqrt{\frac{1}{4}dd+pp}.$
And, $x=\sqrt{\frac{1}{2}d\pm\sqrt{\frac{1}{4}dd+pp}}.$

$d=28$ | $xx=14\pm\sqrt{2500}=64$
$p=48$ | Or, $-36.$

Therefore, $x=8$, or, $\sqrt{-36}.$
The Leſſer Num. $6=\frac{48}{8}=\frac{p}{x}.$
Proof; Their Prod. $6\times8=48.$
Differ. of Sq^rs., $64-36=28.$

Or, this Problem may be Solved, by taking even an Impoſſible Root for x, which is One of it's Values, Namely, $\sqrt{-36}.$
For then the Leſſer Number, $\frac{p}{x}=\frac{48}{\sqrt{-36}}.$
Their Product $\sqrt{-36}\times\frac{48}{\sqrt{-36}}=48.$
And the Difference of their Squares $-36-\frac{2304}{-36}$
$=-36+64=28.$

§. 4. Probl. 7. There are Two Numbers, whoſe Sum is 14, or b, and if the Sum of their Squares be Multiplied by the Sum of their Cubes, the Product is 72800, or c; What are the Numbers?

Problems the 7th. 8th. and 9th.

Probl. 8. There are Two Numbers, the Product of whoſe Multiplication is 20, or b, and the Difference of their Cubes is 61, or d; What are the Numbers?

Probl. 9. A Merchant Bought certain Cloths, and Sells them at $17\frac{1}{4}$ *l.* or b, the Cloth, and then found, that by every 100 *l.* or c, that he had laid out, he Gain'd as many Pounds, as he Paid for One Cloth; What was the firſt Coſt of a Cloth?

Solution of Probl. 7.

One of the Numbers Requir'd. $\frac{1}{2}b+x.$
The other. $\frac{1}{2}b-x.$
Square of the firſt Number is. $xx+bx+\frac{1}{4}bb.$
Square of the Second. $xx-bx+\frac{1}{4}bb.$
Sum of the Squares. $2xx+\frac{1}{2}bb.$
Cube of the Firſt Number.
$x^3+\frac{3}{2}bxx+\frac{3}{4}bbx+\frac{1}{8}b^3.$
Cube of the Second.
$-x^3+\frac{3}{2}bxx-\frac{3}{4}bbx+\frac{1}{8}b^3.$
Sum of the Cubes.
$+3bxx+\frac{1}{4}b^3.$
Sum of the Cubes × Sum of the Squares.
$6bx^4+2b^3x^2+\frac{1}{8}b^5=c$ by the Problem.

Solution of Probl. 8.

Greater Number. $x.$
Leſſer Number. $\frac{b}{x}.$
Cube of the Greater. $xxx.$
Cube of the Leſſer. $\frac{bbb}{xxx}.$
Difference of the Cubes.
$\frac{x^6-b^3}{x^3}.$
By the Problem. $=d.$
By Reduction.
$x^6=dx^3+b^3.$
By Reſolution.
$x^3=\frac{1}{2}d\pm\sqrt{\frac{1}{4}dd+b^3}$
And. $x=\sqrt[3]{\frac{1}{2}d\pm\sqrt{\frac{1}{4}dd+b^3}}.$

$b=20$ | $x^3=30\frac{1}{2}\pm\sqrt{8930\frac{1}{4}}.$
$d=61$ |

Solution of Probl. 9.

Firſt Coſt of a Cloth. $x.$
Gain of one Cloth. $b-x.$
Gain for laying out his Money c. $\frac{cb-cx}{x}.$
Becauſe.
$x.\,b-x :: c.\,\frac{cb-cx}{x}.$
But by the Problem.
$\frac{cb-cx}{x}=x.$
And by Reduction.
$xx=-cx+cb.$
By Extraction or Evolution.
$x=-\frac{1}{2}c\pm\sqrt{\frac{1}{4}cc+cb}.$

$c=100$ | $x=-50\pm\sqrt{4225}$
$b=17\frac{1}{4}$ | $=-50\pm65.$

Therefore. $x=$ | $+15$ *l.*
| -115 *l.*

By Reduction.

$$x^4=-\tfrac{1}{3}b^2x^2-\tfrac{1}{48}b^4+\frac{c}{6b}.$$

By Reſolution or Extraction.

$$x^2=-\tfrac{1}{6}b^2\pm\sqrt{\tfrac{1}{36}b^4-\tfrac{1}{48}b^4+\frac{c}{6b}}=-\tfrac{1}{6}b^2\pm\sqrt{\tfrac{1}{144}b^4+\frac{c}{6b}},\text{ and,}$$

$$x=\sqrt{-\tfrac{1}{6}b^2\pm\sqrt{\tfrac{1}{144}b^4+\frac{c}{6b}}}.$$

$b=14$
$c=72800$
$\Big|\ x^2=-32\tfrac{2}{3}\pm\sqrt{\frac{214221}{189}}=$

$-32\tfrac{2}{3}\pm\sqrt{1133\tfrac{84}{189}}$
$=-32\tfrac{2}{3}\pm 33\tfrac{2}{3}=1$, or, $-65\tfrac{4}{3}$.
$\tfrac{1}{2}b+x=7+1=8$; $\tfrac{1}{2}b-x=7-1$
$=6$; $8+6=14$; 100×728
$=72800$.
If $x=-32\tfrac{2}{3}-33\tfrac{2}{3}=-65\tfrac{4}{3}$.
$7-65\tfrac{4}{3}+7+65\tfrac{4}{3}=14$.
$\overline{7-65\tfrac{4}{3}}\big|^2+\overline{7+65\tfrac{4}{3}}\big|^2\times$
$\overline{7-65\tfrac{4}{3}}\big|^3+\overline{7+65\tfrac{4}{3}}\big|^3$
$=72800$.

$$=30\tfrac{1}{2}\pm 94\tfrac{1}{2}=\begin{matrix}125\\-64\end{matrix}$$

And. $x=\begin{matrix}5\text{ or}\\ \sqrt{3}.-64.\end{matrix}$

So, $\frac{b}{x}=\frac{20}{5}=4$, or, $\frac{20}{\sqrt{3}.-64}$.

Alſo. $x\times\frac{b}{x}=5\times 4=20$.

Likewiſe. $125-64=61$.

Or. $\sqrt{3}.-64\times\frac{20}{\sqrt{3}.-64}=20$.
And the Difference of their Cubes upon Trial will be Found to be, in like Manner, $=61$.

Gain of One Cloth.
$b-x=2\tfrac{1}{4}\,l.$
Gain for laying out every 100 *l.* or *c.*

$$\frac{cb-cx}{x}=\frac{1725-1500}{15}$$

$=15\,l.$ = to the Price of One Cloth.
Alſo, 15 *l.* $2\tfrac{1}{4}$ *l* : : 100*l.* 15 *l.*
Thus, -115*l.* Coſt of One Cloth, makes the Gain $132\tfrac{1}{4}$ *l.* for every 100 *l.* or *c.*
And. $\frac{13225}{-115}=-115=$ to the Price of one Cloth,
But, $-115.\,132\tfrac{1}{4}::100.-115.$

I have here all along Choſen, contrary to the Cuſtom and Practice of Algebraiſts, to Expreſs the Negative, as well as the Poſitive Roots of theſe Equations; That Beginners in this Science may See the Wonderfull Latitude and Extent of it, which can Solve Real Problems by even Chimerical and Imaginary Quantities, if Privative Ones may be Accounted ſuch; Tho' I ſhould rather Think, if the Price of a Cloth is worth -115 *l.* it is, as Poſitive a Circumſtance in a Problem, as if it was worth $+15$ *l.* That is, if a Man has a Cloth for leſs than Nothing by 115 *l*, or has 115 *l.* given him for the Purchaſe of the Cloth, his Gain muſt be in Proportion Greater, than if he Paid 15 *l.* for it, ſince -115 *l.* in this Caſe is to him $+115$ *l*, and $+15$ *l.* is to him -15 *l.* And the Reaſon, that theſe Privative Roots are not taken Notice of by Algebraiſts, is, becauſe in Common Life it ſeldom Happens, that a Man Pays Nothing, or Leſs than Nothing, for what he Purchaſes, which is too Generous a Way of Trading, but lays down Money for what he Buys, which, tho' it is + in the Problem, is − to Him.

I have likewiſe been thus Minute in the Arithmetical Operations, for the Sake of my Pupils, for whom theſe Papers were Originally Intended; But the World not Favouring me, as I could have Deſired, in an Employment, which of all Others, I had the Greateſt Inclination to, the Inſtruction of Young Minds, which were Studious and Good, in Religion, Philoſophy, and Reaſon, I hope the Learned World will forgive my being, Perhaps they will Think, too Particular; Tho' I may Say in the whole, that, Notwithſtanding this, there are more Problems Preſſed in a Leſs Room, and Solved in a Clearer Method, than, I Believe, in any other Algebra, which is Extant. The ſeveral Forms of Equations, the Reſolution of which Takes up in a Certain Famous Algebraiſt, near Twenty Pages in Folio, I have Reduced to Leſs than Two, and in ſome Meaſure the ſame may be Said of the other Problems; The Solutions of which are here Abridged, and Drawn into ſeveral Columns, to the Great Advantage of the Learner, who will by that Means See the whole Proceſs, in a Manner, at One View.

Problems the 10th. 11th. and 12th.

§. 5. Probl. 10. Each of Two Captains, whereof One had a Leſſer Number of Soldiers in his Company by 40, or *b*, than the Other, Diſtributed Equally among the Soldiers of his own Company 1200, or *c*, Crowns, whereby it Happened, that the Soldiers of the Leſſer Company had 5, or *d*, Crowns a Piece more than the Soldiers of the Greater Company; What was the Number of Soldiers in each Company, and how many Crowns did each Soldier Receive?

Probl. 11. Two Merchants Sell Linnen Cloth in this Manner, Each Sells 60, or *b*, Ells, and the firſt Merchant Selling 2, or *c*, Ells leſs for one Pound than the Second, Receives for his 60 Ells 5, or *d*, Pounds more than the Second Merchant for his 60 Ells; How many Ells did each Merchant Sell for 1 Pound?

Probl. 12. Two Societies, whereof the One Exceeds the other by 4, or *b*, Men, Divide Two Equal Sums of Crowns, the Men of the Leſſer Society have 8, or c, Crowns a Piece more than thoſe of the Greater, and the Number of Crowns, which each Society Receives, Exceeds the Number of Men of both Societies by 172, or d; What was the Number of Men in each Society, and the Number of Crowns, which each Society had?

SOLUTION of PROBL. 10.

Number of Soldiers in the Leſſer Company. x.

Number of Soldiers in the Greater Company. $x+b$.

Share of Crowns belonging to each Soldier in the Leſſer. $\frac{c}{x}$.

Share of Crowns belonging to each Soldier in the Greater Company. $\frac{c}{x+b}$.

Add d to the laſt Share, it will be. $\frac{dx+db+c}{x+b}$.

And by the Problem.

$$\frac{dx+db+c}{x+b}=\frac{c}{x}.$$

By Reduction. $dxx+dbx+cx=cx+bc$, and, $dxx+dbx=bc$.

And, $xx=-bx+\frac{bc}{d}$.

By Evolution or Extraction.

$$x=-\tfrac{1}{2}b\pm\sqrt{\tfrac{1}{4}bb+\frac{bc}{d}}.$$

$$\begin{array}{l|l} b=40 & x=-20\pm\sqrt{10000} \\ c=1200 & =-20\pm 100=80. \\ d=5 & \quad\text{Or, }-120. \end{array}$$

$x+b=120$, or, -80. $\frac{c}{x}=15$.

$\frac{c}{x+b}=10$. $120-80=40$.

$80\times 15=120\times 10=1200$.

SOLUTION of PROBL. 11.

Number of Ells, which the Firſt Merchant Sold for 1 *l.* x.

Number of Ells, which the Second Sold for 1 *l.* $x+c$.

Money Received by the Firſt Merchant for his 60, or b, Ells $\frac{b}{x}$.

Becauſe $x.\ 1::b.\ \frac{b}{x}$.

Money Received by the Second Merchant for his 60 or b Ells. $\frac{b}{x+c}$.

Becauſe. $x+c.\ 1::b.\ \frac{b}{x+c}$.

Add d Pounds to the laſt Sum, it will be. $\frac{dx+dc+b}{x+c}$.

But by the Problem.

$$\frac{dx+dc+b}{x+c}=\frac{b}{x}.$$

By Reduction. $dxx+dcx+bx=bx+bc$, and $dxx+dcx=bc$,

And, $xx=-cx+\frac{bc}{d}$.

By Evolution or Extraction.

$$x=-\tfrac{1}{2}c\pm\sqrt{\tfrac{1}{4}cc+\frac{cb}{d}}.$$

$$\begin{array}{l|l} b=60 & x=-1\pm\sqrt{25}=4 \\ c=2 & x+c=6 \quad \text{Or, }-6. \\ d=5 & \end{array}$$

$\frac{b}{x}=15\,l.$ $\frac{b}{x+c}=10\,l.$

$$\text{But,}\quad \underset{\text{Ells.}}{4}.\ 1\,l::\underset{\text{Ells.}}{60}.\ 15\,l.$$

$$\text{And. } \underset{\text{Ells.}}{6}.\ 1\,l::\underset{\text{Ells.}}{60}.\ 10\,l.$$

SOLUTION of PROBL. 12.

Number of Men of the Leſſer Society. x.

Number of the Greater. $x+b$.

Number of Crowns, which were Divided. $2x+b+d$.

The Share of each Perſon in the Leſſer Society. $\frac{2x+b+d}{x}$.

The Share of each Perſon in the Greater Society. $\frac{2x+b+d}{x+b}$.

Add c Crowns to the laſt Sum, it will be. $\frac{2x+b+d+cx+cb}{x+b}$.

But by the Problem.

$$\frac{2x+b+d+cx+cb}{x+b}=\frac{2x+b+d}{x}.$$

By Reduction & Extraction.

$$xx=\frac{-\overline{cb+2b}}{c}x+\frac{bb+bd}{c}.$$

$$x=-\frac{b}{2}+\frac{b}{c}\pm\sqrt{\frac{\tfrac{1}{4}ccbb+cbd+bb}{cc}}.$$

$$\begin{array}{l|l} b=4 & x=-1\tfrac{1}{2}\pm\sqrt{90\tfrac{1}{4}} \\ c=8 & =-1\tfrac{1}{2}\pm 9\tfrac{1}{2}=8. \\ d=172 & \quad\text{Or, }-11 \end{array}$$

$x+b=12$. $2x+b+d=192$.

$\frac{2x+b+d}{x}=24$. $\frac{2x+b+d}{x+b}=16$.

And, $\frac{2x+b+d+cx+cb}{x+b}=24$,

$=\frac{2x+b+d}{x}=24$.

§. 6. PROBL. 13. A Graſier, having Bought Certain Oxen for 270, or b, Pounds, finds, that if he had Paid that Sum for 5, or c, Oxen Fewer, every Ox would have Coſt him $\frac{1}{4}$ *l.* or d, more than he Paid for an Ox; What was the Number of Oxen Bought? *Problems 13th. 14th. and 15th.*

PROBL. 14. A Merchant Buys Linnen Cloths of Two Sorts, *viz.* 90, or b, Ells of one Sort, together with 40, or c, Ells of a Worſer Sort for 42, or d, Pounds, and he finds, that in laying out 1 *l.* upon each Sort, he hath $\frac{1}{3}$, or m, of an Ell more of the Worſer Sort than the Other; What was the Price of an Ell of each Sort?

PROBL. 15. A Merchant, having certain Spices, to Wit, 80 *l.* Weight, or b, of Mace, and 100 *l.* Weight, or c, of Cloves, Sells both Quantities for 65, or d, Pounds in Money, whereby it Happened, that he Sold a Quantity of Mace for 10 *l*, or m, and the like Quantity of Cloves, with 60 *l.* Weight, or n, more of Cloves for 20 *l.* or r; How many Pound Weight of Mace did he Sell for 10 *l*?

SOLUTION of PROBL. 13.

Number of Oxen Bought. x.

Coſt of One Ox. $\frac{b}{x}$.

Becauſe. $x . b :: 1 . \frac{b}{x}$.

Subſtract c from the First Number of Oxen; The Remaining Number is. $x-c$.

Coſt of each of this Number. $\frac{b}{x-c}$.

Becauſe. $x-c . b :: 1 . \frac{b}{x-c}$.

But by the Problem.

$$\frac{b}{x-c}-\frac{b}{x}=d.$$

By Reduction. $dxx=dcx+bc$,

And, $xx=cx+\frac{cb}{d}$.

By Evolution or Extraction.

$$x=\frac{1}{2}c\pm\sqrt{\frac{1}{4}cc+\frac{cb}{d}}.$$

$$\begin{array}{l|l} b=270 & x=2\frac{1}{2}\pm\sqrt{1806\frac{1}{4}} \\ c=5 & =2\frac{1}{2}\pm42\frac{1}{2}=45. \\ d=\frac{3}{4} & \quad\text{Or, } -40. \end{array}$$

Alſo. $\frac{b}{x}=6\,l.$ $x-c=40$.

And. $\frac{b}{x-c}=6\frac{3}{4}\,l.$

But. $6\times45=6\frac{3}{4}\times40=270$.
Moreover. 45. 270 :: 1. 6
Likewiſe. 40. 270 :: 1. $6\frac{3}{4}$.
Laſtly, $6\frac{3}{4}\,l-6\,l=\frac{3}{4}\,l=d$.
So that all the Conditions of the Problem being Satisfied, the Solution is Juſt; -40 will no Leſs Anſwer all the Requiſites of it.

SOLUTION of PROBL. 14.

Number of Ells of the better Sort of Linnen, Bought for 1 *l*. x.
Number of Ells of the worſer Sort Bought for 1 *l*. $x+m$.
Coſt of all the Ells of the better Sort. $\frac{b}{x}$.

Becauſe. $x . 1 :: b . \frac{b}{x}$.

Coſt of all the Ells of the worſer Sort. $\frac{c}{x+m}$.

Becauſe. $x+m . 1 :: c . \frac{c}{x+m}$.

But by the Problem.

$$\frac{b}{x}+\frac{c}{x+m}=d.$$

By Reduction.

$$xx=\frac{c+b-dm}{d}x+\frac{bm}{d}.$$

By Extraction or Evolution.

Suppoſing, $\frac{c+b-dm}{d}=f$

'Twill be, $x=\frac{1}{2}f\pm\sqrt{\frac{1}{4}ff+\frac{bm}{d}}$.

$$\begin{array}{l|l} b=90 & x=1\frac{8}{21}\pm\sqrt{\frac{48552}{18522}} \\ d=42 & =1\frac{8}{21}\pm\sqrt{2\frac{5754}{9261}}. \\ c=40 & \\ m=\frac{1}{3} & =1\frac{8}{21}\pm1\frac{13}{21}=3. \\ f=2\frac{16}{21} & \quad\text{Or, } -\frac{5}{21}. \end{array}$$

$x+m=3\frac{1}{3}$; $\frac{b}{x}=30\,l.$

$\frac{c}{x+m}=12\,l$; $30\,l+12\,l=42\,l.$

Price of the better Linnen *per* Ell. 6 *s.* 8 *d.*
3×6 *s.* 8 *d.* = 1 *l.* 90×6 *s.* 8 *d.* = 30 *l.*
Price of the Worſer 6 *s.*
3×6 *s.* = 18 *s.*
Remains 2 *s.* or $\frac{1}{3}$*d.* of 6 *s.* to make up the Pound, which therefore will Purchaſe $\frac{1}{3}$ of an Ell more at 6 *s. per* Ell. 40×6 *s.* = 12 *l.*

SOLUTION of PROBL. 15.

Number of *l.* Weight of Mace, which the Merchant Sold for 10 *l.* or m. x.
Number of *l.* Weight of Cloves, which he Sold for 20 *l.* or r. $x+n$.
Money, which 80, or b, Pounds of Mace Sold for. $\frac{mb}{x}$.

Becauſe. $x . m :: b . \frac{mb}{x}$.

Money which 100 *l*, or c, Pounds of Cloves Sold for. $\frac{rc}{x+n}$

Becauſe. $x+n . r :: c . \frac{rc}{x+n}$.

But by the Problem.

$$\frac{mb}{x}+\frac{rc}{x+n}=d.$$

By Reduction.
$mbx+mbn+rcx=dxx+dnx.$

And. $xx=\frac{rc+mb-dn}{d}x+\frac{mbn}{d}$.

Suppoſe.

$$\frac{rc+mb-dn}{d}=f.$$

By Extraction.

$$x=\frac{1}{2}f\pm\sqrt{\frac{1}{4}ff+\frac{mbn}{d}}.$$

$$\begin{array}{l|l} b=80 & \\ c=100 & x=-8\frac{30}{65}\pm\sqrt{810+\frac{250}{4225}}. \\ d=65 & \\ m=10 & =-8\frac{30}{65}\pm28\frac{30}{65} \\ n=60 & =20\,l. \\ r=20 & \text{Or, } -36\frac{60}{65}\,l. \\ f=-16\frac{60}{65} & \end{array}$$

Whole Quantity of Mace Sold for 40 *l.* of Cloves for 25 *l*; 40 *l* + 25 *l* = 65 *l*; 20 *l* of Cloves + 60 *l* = 80 *l* Sold for 20 *l.*
Becauſe 100. 25 :: 80. 20.

Problems the 16th. 17th. and 18th.

§. 7. PROBL. 16. Two Merchants entered into Partnerſhip, the First brought in a certain Sum of Pounds, which continu'd in Company 12, or b, Months, and the Second put in 30 *l*, or c, for 17, or d, Months; They Gain'd together $18\frac{3}{4}$ *l*, or m, whereof the first Merchant had 26 *l.* or n, for his Principal and Gain; How many Pounds did the first Merchant bring into the Common Stock?

PROBL. 17. Two Merchants entered into Partnerſhip, the First put in a Certain Number of Pounds for 3, or b, Months, the Second put in 50 *l*, or c, more than the First for 5, or d, Months, they Gain'd together 140 *l*, or m, whereof the First Merchant had ſuch Part, that if 60 *l*, or *n*, be added to it, the Sum will be Equal to the Stock, wherewith he entered Partnerſhip; What was the Stock and Gain of each Merchant?

Probl. 18. A Citizen, having Bought a Houſe for a certain Number of Pounds, Sells it for 64 *l*, or *d*, and finds that his Loſs in 100 *l*, or *c*, was Equal to a Fourth Part, or *m*, of the Money, that he Paid for the Houſe; What Number of Pounds did the Citizen Pay for the Houſe?

Solution of Probl. 16.

Pounds, which the Firſt Merchant brought into the Common Stock. $x.$

Which Multiplied into the Time, *b*, it Continued in Company, makes. $bx.$

Second Merchant's Stock *c*, Multiplied into the Time *d*, it Continued in Company makes. $cd.$

Gain of the firſt Merchant. $\frac{mbx}{bx+cd}$

Becauſe.

$$bx+cd.\ m :: bx.\ \frac{mbx}{bx+cd}.$$

Principal and Gain of the firſt Merchant will be. $\frac{mbx}{bx+cd}+x$

By the Problem. $=n.$

By Reduct. $mbx+bxx+cdx=nbx+cdn,$

And. $xx=\overline{\frac{nb-mb-cd}{b}}x+\frac{cdn}{b}.$

Suppoſe. $\frac{nb-mb-cd}{b}=f.$

By Evolution or Extraction.

$$x=\tfrac{1}{2}f\pm\sqrt{\tfrac{1}{4}ff+\frac{cdn}{b}}.$$

$b=12$

$c=30$

$d=17$

$m=18\frac{3}{4}$

$n=26$

$f=-35\frac{1}{4}$

$x=-17\frac{5}{8}\pm\sqrt{1415\frac{41}{64}}$

$=-17\frac{5}{8}\pm37\frac{5}{8}$

$=20,$ or $-55\frac{2}{8}.$

$20\times12=240$

$30\times17=510$ $=750\,l.$

$750.\ 18\frac{3}{4} :: \begin{cases}240.\ 6\\ 510.\ 12\frac{3}{4}\end{cases}$ $20\,l.+6\,l. = 26\,l.$

Solution of Probl. 17.

Stock of the Firſt Merchant. $x.$

Stock of the Second Merchant. $x+c.$

Stock of the Firſt Merchant *x* into the Time *b* it Continued in Company. $bx.$

Stock of the Second Merchant into the Time *d* it Continued in Company. $dx+dc.$

Firſt Merchant's Gain. $\frac{mbx}{bx+dx+dc}.$

Becauſe. $bx+dx+dc.\ m :: bx.\ \frac{mbx}{bx+dx+dc}.$

But by the Problem.

$$\frac{mbx}{bx+dx+dc}+n=x.$$

By Reduction.

$mbx+nbx+ndx+ndc=bxx+dxx+dcx,$ And

$$xx=\overline{\frac{mb+nb+nd-dc}{b+d}}x+\frac{ndc}{b+d}.$$

By Extraction or Evolution,

Suppoſe, $\frac{mb+nb+nd-dc}{b+d}=f$

Then $x=\tfrac{1}{2}f\pm\sqrt{\tfrac{1}{4}ff+\frac{ndc}{b+d}}.$

$b=3$

$c=50$

$d=5$

$m=140$

$n=60$

$f=81\frac{1}{4}$

$x=40\frac{5}{8}\pm\sqrt{3525\frac{25}{64}}$

$=40\frac{5}{8}\pm59\frac{3}{8}=100$

Or, $-18\frac{6}{8}$

$100\times3=300$

$150\times5=750$ $=1050.$

But, $1050.\ 140 :: \begin{cases}300.\ 40.\\ 750.\ 100.\end{cases}$

And. $40\,l.+60\,l.=100\,l.$

Solution of Probl. 18.

Number of Pounds Paid for the Houſe. $x.$

Whole Loſs in Selling the Houſe. $x-d.$

Loſs *per* Cent. $\frac{cx-cd}{x}.$

Becauſe. $x.\ x-d :: c.\ \frac{cx-cd}{x}.$

But by the Problem.

$$\frac{cx-cd}{x}=mx.$$

By Reduction.

$$cx-cd=mxx.$$

And $xx=\frac{c}{m}x-\frac{cd}{m}.$

By Extraction.

$$x=\frac{c}{2m}\pm\sqrt{\frac{cc}{4mm}-\frac{cd}{m}}$$

$c=100$

$d=64$

$m=\frac{1}{4}$

$x=200\pm\sqrt{14400}$

$=200\pm120=$

$320\,l.$ or $80\,l.$

$320\,l.-64\,l.=256\,l.=x-d.$

4)320(=80, and

320. 256 :: 100. 80,

Or, $3\times80=240.+16=256.$

So likewiſe. If $x=80\,l.$

$80\,l.-64\,l.=16\,l.=x-d.$

4)80(=20, and

80. 16 :: 100. 20.

Or,

$$x.\ x-d :: c.\ \frac{cx-cd}{x}.$$

Therefore all the Conditions of the Queſtion being Anſwered, this Problem alſo is Juſtly Solved, and *x* Admits of Two Poſitive Values; Namely, Either 320 *l*, Or, 80 *l*.

§. 8. Probl. 19. Two Merchants Entered into Partnerſhip, the Sum of their Stocks was 165 *l*. or *b*, the firſt Merchant's Stock continued in Company 12, or *c*, Months, and the Stock of the Second 8, or *d*, Months, they Gained a Certain Sum of Pounds, which together with their Stocks, they Divided between themſelves in ſuch a Manner, that the Firſt Merchant Received 67, or *f*, Pounds for his Stock and Gain, and the Second 126, or *g*, for his Stock and Gain; What was each Merchant's Stock and Gain?

Problems the 19th. and 20th.

PROBL. 20. A Certain Footman *A* Departeth from *London* towards *Lincoln*, and at the ſame Time another Footman *B* Departeth from *Lincoln* towards *London*, each keeping the ſame Road, when they Met, *A* ſaith to *B*, I find that I have Travelled 20, or *c*, Miles more than you, and have gone as many Miles in $6\frac{2}{3}$, or *d*, Days, as you have gone Miles in all, 'Tis true ſaith *B*, I am not ſo Good a Footman as you, but I find that at the End of 15, or *f*, Days hence I ſhall be at *London*, if I Travel as many Miles in every one of thoſe 15 Days, as I have done every Day hitherto; What is the Diſtance of thoſe Two Cities from each other, and how many Miles had each Footman Travelled, when they Met?

SOLUTION of PROBL. 19.

The Firſt Merchant's Stock. x.

Second Merchant's Stock. $b-x$.

Gain of the Firſt Merchant. $f-x$.

Gain of the Second Merchant. $g-b+x$.

But by the Nature of the Rule of Fellowſhip with Time.

$$f-x.\ g-b+x :: cx.\ db-dx.$$

Therefore the Equation will be.

$$cgx-cbx+cxx=dbf-dbx-fdx+dxx.$$

By Reduction.

$$cxx-dxx=cbx-cgx-dbx-fdx+dbf.$$

And. $$xx=\overline{\frac{cb-cg-db-fd}{c-d}}x+\frac{dbf}{c-d}.$$

By Extraction, Suppoſing, $$\frac{cb-cg-db-fd}{c-d}=h$$

It will be, $$x=\tfrac{1}{2}h\pm\sqrt{\tfrac{1}{4}hh+\frac{dbf}{c-d}}.$$

$$\begin{array}{l|l} b=165 & x=-173\tfrac{1}{2}\pm\sqrt{52212\tfrac{1}{4}} \\ c=12 & \\ d=8 & =-173\tfrac{1}{2}\pm228\tfrac{1}{2}=\left|\begin{array}{l}55l.\\ -402l.\end{array}\right. \\ f=67 & \\ g=126 & \text{Alſo.}\quad b-x=110l. \\ h=-347 & \text{And.}\quad 110l.+55l.=165l. \end{array}$$

Likewiſe, $f-x=12l;\ 55l.+12l.=67l.$

So. $g-b+x=16l;\ 110l.+16l;=126l.$

$55\times12=660.\quad 12.$

$110\times8=880.\quad 16.$

And. $\overline{1540.}\quad\overline{28}$ are the Sums of them.

But. $$1540.\ 28::\left|\begin{array}{l}660.\ 12.\\ 880.\ 16.\end{array}\right.$$

That is, the Number of Pounds, which belonged to the Firſt Merchant's Stock, was 55 *l.* which Subſtracted from 165*l.* the Sum of both their Stocks, gives 110*l.* the Second Merchant's Stock; The Gain of the Firſt Merchant will therefore be 12*l*, Gain of the Second 16 *l.*

But $12.\ 16 :: 660.\ 880.$

And by Compoſition.

$12+16.\ 16 :: 660+880.\ 880.$

Or, $16+12.\ 12 :: 880+660.\ 660.$

And by Alternation and Inverſion.

$$1540.\ 28::\left|\begin{array}{l}880.\ 16.\\ 660.\ 12.\end{array}\right.$$

Alſo, -402 *l.* will Solve this Problem.

SOLUTION of PROBL. 20.

Diſtance in Miles betwixt the two Cities. x.

Number of Miles, which *A* had Travelled. $\frac{1}{2}x+\frac{1}{2}c$.

Becauſe *c* being the Difference betwixt the Miles, which *A* and *B* Travelled, *A* will Travel $\frac{1}{2}x+\frac{1}{2}c$, and *B* $\frac{1}{2}x-\frac{1}{2}c$, which together are $=x$.

Number therefore of Miles, which *B* Travelled. $\frac{1}{2}x-\frac{1}{2}c$.

Number of Miles, which *A* Travelled in one Day. $\dfrac{\frac{1}{2}x-\frac{1}{2}c}{d}$.

Becauſe. $$d.\ \tfrac{1}{2}x-\tfrac{1}{2}c :: 1.\ \frac{\frac{1}{2}x-\frac{1}{2}c}{d}.$$

Number of Miles, which *B* Travelled in one Day. $\dfrac{\frac{1}{2}x+\frac{1}{2}c}{f}$.

Becauſe. $$f.\ \tfrac{1}{2}x+\tfrac{1}{2}c :: 1.\ \frac{\frac{1}{2}x+\frac{1}{2}c}{f}.$$

Number of Days, in which *A* Travelled all his Number of Miles. $\dfrac{\frac{1}{2}dx+\frac{1}{2}dc}{\frac{1}{2}x-\frac{1}{2}c}$.

Becauſe. $$\frac{\frac{1}{2}x-\frac{1}{2}c}{d}.\ 1 :: \tfrac{1}{2}x+\tfrac{1}{2}c.\ \frac{\frac{1}{2}dx+\frac{1}{2}dc}{\frac{1}{2}x-\frac{1}{2}c}.$$

Number of Days in which *B* Travelled all his Number of Miles. $\dfrac{\frac{1}{2}fx\ \ \frac{1}{2}fc}{\frac{1}{2}x+\frac{1}{2}c}$.

Becauſe. $$\frac{\frac{1}{2}x+\frac{1}{2}c}{f}.\ 1 :: \tfrac{1}{2}x-\tfrac{1}{2}c.\ \frac{\frac{1}{2}fx-\frac{1}{2}fc}{\frac{1}{2}x+\frac{1}{2}c}.$$

But the Number of Days, in which both had Travelled, was Equal, they Setting out at the ſame Time;

Therefore. $$\frac{\frac{1}{2}dx+\frac{1}{2}dc}{\frac{1}{2}x-\frac{1}{2}c}=\frac{\frac{1}{2}fx-\frac{1}{2}fc}{\frac{1}{2}x+\frac{1}{2}c}$$

By Reduct. $$dxx+2dcx+dcc=fxx-2fcx+fcc.$$

And. $$fxx-dxx=2dcx+2fcx+dcc-fcc.$$

Or. $$xx=\overline{\frac{2dc+2fc}{f-d}}x+\overline{\frac{dcc-fcc}{f-d}}\text{ or }-cc.$$

Suppoſing Therefore, $$\frac{2dc+2fc}{f-d}=g.$$

By Extraction. $$x=\tfrac{1}{2}g\pm\sqrt{\tfrac{1}{4}gg-cc}$$

$$\begin{array}{l|l} c=20 & \\ d=6\frac{2}{3} & x=52\pm\sqrt{2304}=52\pm48=100. \\ f=15 & \qquad\qquad\qquad\quad \text{Or, }4. \\ g=104 & \end{array}$$

The Latter Root Impoſſible, becauſe *A* had Travelled 20 Miles more than *B*, the Former Anſwers the Queſtion. $\frac{1}{2}x+\frac{1}{2}c=60$. $\frac{1}{2}x-\frac{1}{2}c=40$. $60+40=100$. $60-40=20$. *A* went 6 Miles a Day, $6\times6\frac{2}{3}=40$; *B* went 4 Miles a Day, $4\times15=60$; Number of Days, in which *A* and *B* Travelled, 10; $6\times10=60$. $4\times10=40$. Alſo, 4 will Solve the Queſtion, $12-8=4$. $12--8=20$, *&c.*

CHAP. V.

Of the ſeveral Ways of Managing Equations of Higher Powers, and Extracting the Roots of Them, by Stevin, Harriot, Vieta, Kerſey, Cardan, *or* Scipio Ferreus, *as alſo by* Deſcartes, *Sir* Iſaac Newton, *Dr.* Halley, *and Mr.* Raphſon.

Of the Reſolution of Cubick and Biquadratick Equations, and thoſe of Higher Powers, Probl. 1. 2. 3.

§. 1. WHEN we come to Cubick or Biquadratick Equations, or Thoſe of a ſtill Higher Power, the Work is more Difficult and Operoſe, and can Scarcely be done any better Way, than by the Tentative, Examples of which I have here Subjoined from *Stevin.*

PROBL. 1. $x^3+26x=40188$; What is the Number x?
PROBL. 2. $x^4+50x=184638.6801$; What is the Number x?
PROBL. 3. $x^3+20xx=1954$; What is the Number x?

SOLUTION of PROBL. 1.

1ſt. Suppoſe. $x=1$.
Then. $x^3=1$.
And. $26x=26$.
Therefore. $x^3+26x=27$.
This being too little.
2d. Suppoſe. $x=10$.
Then. $x^3=1000$.
And. $26x=260$.
Therefore. $x^3+26x=1260$.
This being too little.
3d. Suppoſe. $x=100$.
Then. $x^3=1000000$.
And. $26x=2600$.
Theref. $x^3+26x=1002600$.
This being too much.
4th. Suppoſe. $x=20$.
Then. $x^3+26x=8520$.
This being too little.
5th. Suppoſe. $x=30$.
Then. $x^3+26x=27780$.
This being too little.
6th. Suppoſe. $x=40$.
Then. $x^3+26x=65040$.
This being too much.
7th. Suppoſe. $x=31, 32, 33$, Succeſſively, which all Proving too little.
8th. Suppoſe. $x=34$.
Then. $x^3+26x=40188$.
The Equation Deſired.

And if the Root falls betwixt Two whole Numbers, Trials are to be made by Increaſing the Root with $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, *&c.* and if that will not do with $\frac{11}{100}$, $\frac{12}{100}$, *&c.*

SOLUTION of PROBL. 2.

1ſt. Suppoſe, $x=1$. and 10. which Proving too little.
2d. Suppoſe, $x=100$, which Proving too Great.
3d. Suppoſe, $x=20$, which Proving too little.
4th. Suppoſe, $x=30$, which Proving too Great.
5th. Suppoſe, $x=21$, which alſo being too Great, and 20, too little.
6th. Therefore Suppoſe $x=20.1$, 20.2, 20.3, 20.4, 20.5, 20.6, which all Proving too little.
7th. Suppoſe, $x=20.7$, which will Produce.
$x^4+50x=184638.6801$.

But if 20.7 had Proved too little, and 20.8 too Great, Then Trialls muſt have been made with 20.71, 20.72, 20.73, *&c.* for $.80=.8$, if 20.7, had been too little, and 20.71 had been too Great, Trialls muſt have been made with 20.701, 20.702, 20.703, *&c.* For. $.710=.71$ which will be Exemplify'd in the Following Problem.

Becauſe, $.710=\frac{710}{1000}$;
And, $.71=\frac{71}{100}$;
But, $\frac{710}{1000}=\frac{71}{100}$.

SOLUTION of PROBL. 3.

1ſt. Suppoſe, $x=1$, which is too little.
2d. Suppoſe, $x=10$, which is too Great.
3d. Suppoſe, $x=2, 3, 4, 5, 6, 7, 8$, which are all too little.
4th. Suppoſe, $x=9$, which is too Great.
5th. Suppoſe, $x=8.1$, 8.2, 8.3, which are all too little, but, 8.4 too Great.
6th. Suppoſe, $x=8.31$, which is alſo too Great.
7th. Suppoſe, $x=8.301$, 8.302, 8.303, 8.304, 8.305, 8.306, 8.307, which are all too little, but 8.308, Anſwers the Equation, which is leſs than 8.310, or 8.31.

If the Value of x, or the Root Required be leſs than Unit, Then the Trial is to be made by .1; If this be too Great, Then with .01; If this be too Great, Then with .001; If .1 is too Great, and .01 is too Little, Then Trial muſt be made with .02, .03, .04, *&c.* For $.10=.1$; If again .04 is Leſs, and .05 is More, than ſhould be, Trial muſt be made with .041, .042, .043, *&c.* For, $.050=.05$, or, $\frac{050}{1000}=\frac{05}{100}$.

Problems 4th. 5th. and 6th.

§. 2. Probl. 4. If $x^3+3240x=269$; What is the Number x?

Probl. 5. If $x^3-22xx+157x=360$; What is the Number x?

Probl. 6. If $3200x-x^3=46577$; What is the Number x?

Solution of Probl. 4.

1*ſt*. Suppoſe, $x=1$. Which is too Great.

2*d*. Suppoſe, $x=.1$, Which is likewiſe too Great.

3*d*. Suppoſe, $x=.01$, Which is too Little.

4*th*. Suppoſe, $x=.02, .03, .04, .05, .06, .07, .08$. Which will be all too little.

5*th*. Suppoſe, $x=.09$, Which will be too Great.

6*th*. Suppoſe, $x=.081, .082$. Which will be too little, but .083 will Solve the Problem, and .083 is Leſs than $.090=.09$.

When all the Terms of the Unknown Part of an Equation are Affirmative, as they have hitherto been, it is Eaſy to know, how to Increaſe or Leſſen the Root of an Equation, in Order to make it at Length give the Juſt Solution; But whereas in the Following Inſtances, Subſtractions are to be made, it is harder to Gueſs, how to allow for thoſe Subſtractions, by which Means the Operation becomes more Intricate and Difficult.

'Tis alſo to be Obſerved in theſe Following Equations, that there may Happen to be as many Poſitive or Affirmative Roots, which will Solve the Equation as there are Units in the Index of the higheſt Unknown Power which will be afterwards Shewn.

Solution of Probl. 5.

1*ſt*. Suppoſe, $x=1$. which will be too little, for then the Abſolute Number is 136.

2*d*. Suppoſe, $x=10$. The Abſolute Number is 370, which is more than the Juſt Abſolute Number 360, and therefore 10 is too Great.

3*d*. Suppoſe, $x=2.\ 3.\ 4.$ which will be all too little, but 5 will Solve the Equation Propoſed, beſides which, there are Two other Roots 8 and 9, which will likewiſe Produce the ſame Equation, and will be found out by a Method after Delivered.

Limits of Equations which do not Exceed Biquadratick Ones, are Taught by *Florimond de Beaune*; $>$ $<$ are with Dr. *Wallis*, and ſome other Mathematicians, Signs or Characters for Greater or Leſs; Thus for Example.

Equation Probl. 6. to be Reſolved, $3200x-x^3=46577$.

Therefore $x^3<3200x$.

$$x^2<3200.$$

$$x<56.5, \&c$$

Again, $3200x-46577=x^3$.

Therefore, $3200x>46577$.

And. $x>\dfrac{46577.}{3200}$

Or. $x>14.5$, &c.

Conſequently x, Value of the Root Sought, is Greater than 14.5 and Leſs than 56.5, which accordingly it is, for by the Following Operation it is either 15.7, or 47.

Solution of Probl. 6.

1*ſt*. Suppoſe, $x=1$. The Abſolute Number will be. $=3199$.

2*d*. Suppoſe, $x=10$. The Abſolute Number will be. $=31000$. Both which will be too little.

3*d*. Suppoſe, $x=100$. The Abſolute Number will be. $=-680000$. Which alſo will be too little; But ſince it is Negative, it Shews, that x muſt be ſome Number Leſs than 100; Becauſe a Poſitive Root, if any, will Neceſſarily be found, before the Abſolute Number becomes Privative.

4*th*. Suppoſe, $x=20$. The Abſolute Number will be. $=56000$. Which is too Great. Therefore it is a Number betwixt 10 and 20.

5*th*. Suppoſe, $x=11, 12$, &c. they will be too little. But 15.7 will nearly Anſwer the Equation.

6*th*. Alſo, becauſe 56000 is Greater, and -680000 is Leſs, than the Abſolute Number, there will be another Root betwixt 20 and 100, which after ſeveral Trials, will be found to be 47, which will Exactly Conſtitute the Equation Propoſed.

I ſhall now Proceed to Shew from our Famous Country-Man and Algebraiſt Mr. *Harriot*, the Father and Founder of our Modern Algebra, and to whom *Des-Cartes* is Indebted for all his Inventions and Improvements in that Science, the Way of Forming Equations, which ſhall have ſeveral Roots Given, Affirmative or Negative.

Concerning the Forming of Equations according to Harriot, which ſhall have ſeveral Roots given Affirmative or Negative. Problems 1ſt. 2d. and 3d.

§. 3. PROBL. 1. To Form an Equation, which ſhall have Two Affirmative Roots.

PROBL. 2. To Form an Equation, which ſhall have One Affirmative, and One Negative Root.

PROBL. 3. To Form an Equation, which ſhall have Two Negative Roots.

SOLUTION of PROBL. 1.

$$\left.\begin{array}{l}x=b\\x=c\end{array}\right| i.\ e. \left|\begin{array}{l}x-b=0.\\x-c=0.\end{array}\right.$$

Therefore by Involution. $xx\begin{array}{l}-bx\\-cx\end{array}+bc=0.$

$$\begin{array}{rl}b=2. & xx-5x+6=0.\\ c=3. & \text{or, } 5x-xx=6.\\ & \text{or, } xx=5x-6.\end{array}$$

And by Evolution.

$$x=2\tfrac{1}{2}\pm\sqrt{6\tfrac{1}{4}-6}=2\tfrac{1}{2}\pm\tfrac{1}{2}.$$

If $x=3$, $\left.\begin{array}{r}5x=15\\-xx=-9\end{array}\right|=6.$

If $x=2$, $\left.\begin{array}{r}5x=10\\-xx=-4\end{array}\right|=6.$

SOLUTION of PROBL. 2.

$$\left.\begin{array}{l}x=b\\x=-c\end{array}\right| i.\ e. \left|\begin{array}{l}x-b=0.\\x+c=0.\end{array}\right.$$

Therefore by Involution

$$xx\begin{array}{l}-bx\\+cx\end{array}-bc=0.$$

$$\left.\begin{array}{r}b=2\\-c=-3\end{array}\right|\begin{array}{l}xx+1x-6=0\\xx+1x=+6\\xx=-1x+6.\end{array}$$

By Evolution.

$$x=-\tfrac{1}{2}\pm\sqrt{6\tfrac{1}{4}}=-\tfrac{1}{2}\pm2\tfrac{1}{2}.$$

If $x=2$, $\left.\begin{array}{r}xx=4\\x=2\end{array}\right|=6.$

If $x=-3$, $\left.\begin{array}{r}xx=9\\x=-3\end{array}\right|=6.$

SOLUTION of PROBL. 3.

$$\left.\begin{array}{l}x=-b\\x=-c\end{array}\right| i.\ e. \left|\begin{array}{l}x+b=0.\\x+c=0.\end{array}\right.$$

Therefore by Involution

$$xx\begin{array}{l}+bx\\+cx\end{array}+bc=0.$$

$$\left.\begin{array}{r}-b=-2\\-c=-3\end{array}\right|\begin{array}{l}xx+5x+6=0.\\xx+5x=-6.\\xx=-5x-6.\end{array}$$

By Evolution.

$$x=-2\tfrac{1}{2}\pm\sqrt{6\tfrac{1}{4}-6}.$$

Or. $x=-2\tfrac{1}{2}\pm\tfrac{1}{2}=\begin{array}{r}-2.\\-3.\end{array}$

If $x=-2$, $\left.\begin{array}{r}xx=4\\5x=-10\end{array}\right|=-6.$

If $x=-3$ $\left.\begin{array}{r}xx=9\\5x=-15\end{array}\right|=-6.$

Problems 4th. 5th. 6th. and 7th.

§. 4. PROBL. 4. To Form an Equation, which ſhall have Three Affirmative Roots.

PROBL. 5. To Form an Equation, which ſhall have Two Affirmative Roots, and One Negative.

PROBL. 6. To Form an Equation, which ſhall have One Affirmative Root, and Two Negative.

PROBL. 7. To Form an Equation, which ſhall have Three Negative Roots.

SOLUTION of PROBL. 4.

$$\left.\begin{array}{l}x=b\\x=c\\x=d\end{array}\right| i.\ e. \left|\begin{array}{l}x-b=0.\\x-c=0.\\x-d=0.\end{array}\right.$$

$$\left.\begin{array}{l}x-b\\\times x-c\end{array}\right|\begin{array}{l}=xx-bx-cx+bc.\\\times x-d\end{array}$$

$$\begin{array}{l}=x^3-bxx+bcx\\\quad -cxx+bdx-bcd=0.\\\quad -dxx+cdx.\end{array}$$

$$\left.\begin{array}{r}b=2\\c=3\\d=4\end{array}\right|\begin{array}{l}\qquad\text{Therefore,}\\x^3-bxx+bcx\\\quad -cxx+bdx\\\quad -dxx+cdx-bcd=0.\end{array}$$

$$=x^3-9xx+26x-24=0.$$

Or, $x^3-9xx+26x=24.$

By Involution, then, The Roots are Evidently 2, 3, and 4.

But, how they are Found by Evolution, or Extraction, by the ſame Certain Method, by which the Roots of Quadratick Equations are Expounded, is the Grand Myſtery in this Science, which

SOLUTION of PROBL. 5.

$$\left.\begin{array}{l}x=b\\x=c\\x=-d\end{array}\right| i.\ e. \left|\begin{array}{l}x-b=0.\\x-c=0.\\x+d=0.\end{array}\right.$$

$$\left.\begin{array}{l}x-b\\\times x-c\end{array}\right|\begin{array}{l}=xx-bx-cx+bc\\\times x+d\end{array}$$

$$\begin{array}{l}=x^3-bx^2+bcx\\\quad -cx^2-bdx+bcd=0.\\\quad +dx^2-cdx\end{array}$$

$$\left.\begin{array}{r}b=2\\c=3\\-d=-4\end{array}\right|\text{Therefore,}$$

$$\begin{array}{l}x^3-bx^2+bcx\\\quad -cx^2-bdx+bcd=0.\\\quad +dx^2-cdx\end{array}$$

$$=x^3-x^2-14x+24=0.$$

And, $x^3-x^2-14x=-24.$

Or, 2, 3, and −4, will be the Roots, but the Difficulty again is to find them out by any Certain Method of Evolution, as in Quadratick Equations; That yet theſe are the Roots, is not only Manifeſt from the Conſtruct-

SOLUTION of PROBL. 6.

$$\left.\begin{array}{l}x=b\\x=-c\\x=-d\end{array}\right| i.\ e. \left|\begin{array}{l}x-b=0.\\x+c=0.\\x+d=0.\end{array}\right.$$

$$\left.\begin{array}{l}x-b\\\times x+c\end{array}\right|\begin{array}{l}=xx-bx+cx-bc,\\\times x+d\end{array}$$

$$\begin{array}{l}=x^3-bxx-bcx.\\\quad +cxx-bdx-bcd=0.\\\quad +dxx+cdx.\end{array}$$

$$\left.\begin{array}{r}b=2\\-c=-3\\-d=-4\end{array}\right|\text{Therefore,}$$

$$\begin{array}{l}x^3-bxx-bcx.\\\quad +cxx-bdx-bcd=0,\\\quad +dxx+cdx.\end{array}$$

$$=x^3+5xx-2x-24=0.$$

Or, $x^3+5xx-2x=24.$

If $x=2$,

Then, $\left|\begin{array}{r}x^3=8\\+5xx=20\\-2x=-4\end{array}\right|=24.$

If $x=-3$,

Then, $\left|\begin{array}{r}x^3=-27\\+5xx=45\\-2x=6\end{array}\right|=24.$

Difficulty, he who Surmounts Deſerves to have his Statue in Gold.

All is pretty much hitherto done by Repeated Trialls, and is Reſigned to the Sagacity of the Operator.

But that 2, 3 and 4 will Anſwer the Equation, being each of them put for x, is from hence Manifeſt.

If $x=2$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=8\\ -9xx=-36\\ +26x=52 \end{array}\right|=24.$$

If $x=3$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=27\\ -9xx=-81\\ +26x=78 \end{array}\right|=24.$$

If $x=4$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=64\\ -9xx=-144\\ +26x=104 \end{array}\right|=24.$$

That is, the Glorious Problem to be Found out is; The Sum of the Roots is Given in the Second Member of the Equation, with their Signs + and −, Annexed, and the Product of them in the Fourth, with it's Sign + or −, in the Third the Product of every Two of them with their Signs + or −; What are the Roots? ion of theſe Equations, but alſo from the following Way of Proof.

If $x=2$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=8\\ -x^2=-4\\ -14x=-28 \end{array}\right|=-24.$$

If $x=3$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=27\\ -x^2=-9\\ -14x=-42 \end{array}\right|=-24.$$

If $x=-4$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=-64\\ -x^2=-16\\ -14x=+56 \end{array}\right|=-24.$$

That is, the Noble Problem to be Found out is; The Sum of the Roots is Given in the Second Member of the Equation, with their Signs + and −, as before, and the Product of them in the Fourth, with it's Sign + or −, in the Third the Product of every Two of them with their Signs + or −; What are the Roots?

And this is an Inſtance of the Infinite Defect in Algebra, that of the Infinite Powers, which there are in it, the Second, or Quadratick, is almoſt only Solved by any Certain Method.

If $x=-4$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=-64\\ +5xx=80\\ -2x=8 \end{array}\right|=24.$$

SOLUTION of PROBL. 7.

$$\left.\begin{array}{l} x=-b\\ x=-c\\ x=-d \end{array}\right|\ i.\ e.\ \left|\begin{array}{l} x+b=0.\\ x+c=0.\\ x+d=0. \end{array}\right.$$

$$\left.\begin{array}{r} x+b\\ \times x+c \end{array}\right|\begin{array}{l} =xx+bx+cx+bc.\\ \times x+d=0 \end{array}$$

$$\begin{array}{l} =x^3+bxx+bcx\\ \quad +cxx+bdx+bcd=0.\\ \quad +dxx+cdx \end{array}$$

$$\left.\begin{array}{r} -b=-2\\ -c=-3\\ -d=-4 \end{array}\right|\ \text{Therefore,}$$

$$\begin{array}{l} x^3+bxx+bcx\\ \quad +cxx+bdx+bcd=0.\\ \quad +dxx+cdx \end{array}$$

$$=x^3+9xx+26x+24=0.$$

Or, $x^3+9xx+26x=-24$.

If $x=-2$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=-8\\ 9xx=36\\ 26x=-52 \end{array}\right|=-24.$$

If $x=-3$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=-27\\ 9xx=81\\ 26x=-78 \end{array}\right|=-24.$$

If $x=-4$,

$$\text{Then,}\ \left|\begin{array}{r} x^3=-64\\ 9xx=144\\ 26x=-104 \end{array}\right|=-24.$$

Problem, as before.

AND we may Proceed in the ſame Way and Method with Biquadratick Equations, or Thoſe of the 4*th*. Power, or 5*th*. 6*th*, &c. And like Problems will Ariſe from thence.

But, tho' all theſe Equations are Form'd by the aboveſaid Roots, it is not Certain, that there may not be other Roots, which would Conſtitute the ſame Equation; Thus tho' 2×6 makes 12, yet 3×4 may do it; and tho' 6 and −6, 24 and −24, are found by theſe Suppoſitions of 2, 3 and 4 being $=x$, there may be other Numbers, or Fractions of Numbers, which, for all we know, may Effect the ſame Thing. However,

1*ſt*. If any of theſe Equations $=0$ be Divided by One of the Roots $=0$, a Quadratick Equation will be Produced $=0$; And if That be again Divided by another Root, it will Produce the Remaining Root $=0$; Thus,

$$\begin{array}{lll} x+2=0) & x^3+9xx+26x+24=0 & (x^2+7x+12=0\\ & \underline{x^3+2xx} & \\ & 0+7xx+14x & \\ & \quad\underline{7xx\qquad\qquad} & \\ & \quad\quad 0+12x & \\ & \quad\quad\ \underline{12x+24} & \\ & \quad\quad\quad 0\quad 0 & \end{array}$$

$$\begin{array}{lll} x+3=0) & x^2+7x+12=0 & (x+4=0\\ & \underline{x^2+3x} & \\ & 0+4x & \\ & \ \underline{4x+12} & \\ & \quad 0\quad 0 & \end{array}$$

2*d*. In Equations thus Formed, and in which all the Terms are Extant, and $=0$, as often as − Follows after +, or + after −, ſo often there is an Affirmative Root, and as often, as Two Signs −, or Two Signs +, ſtand next to one another, ſo often there is a Negative Root; Thus in the Firſt, there are Two Affirmative, in the Second, one Affirmative, and one Negative, *&c.*

3*d*. It is ſaid, *In Equations thus Formed*, that is, where the Number of Roots, Affirmative or Negative, is Equal to the Number of Units in the Index of the Power of the Unknown Quantity; Becauſe ſome Equations are Produced by Multiplying the Root into an Impoſſible Equation; Thus $x-2=0 \times xx-4x+5=0$, Produces an Equation $=x^3-6xx+13x-10=0$, which hath only one Affirmative Root, *viz.* $x=2$, or $x-2=0$, and no other, either Affirmative or Negative, for this Reaſon, that $\overline{xx-4x+5}=0$ is an Impoſſible Equation, ſince $xx=4x-5$, and $x=2\pm\sqrt{\overline{4-5}}$, but the $\sqrt{4-5}$, or $\sqrt{-1}$, is Impoſſible; However, Theſe Two Laſt Roots, tho' Impoſſible, and is a Difficulty, with Algebraiſts, we Say, are Affirmative, and do not Deſtroy the Rule.

4*th*. It is to be Obſerved, that in every Equation, which hath as many Affirmative Roots, as there are Dimenſions in the firſt Term, the Coefficient, or Known Quantity, in the Second Term is Equal to the Sum of all the Affirmative Roots, and the known Quantity in the Third Term is Equal to the Sum of the Products of every Two of the ſaid Roots Multiplied into one another, and the Known Quantity in the Fourth Term, as in Biquadratick Equations, Is Equal to the Sum of the Products of every Three of the ſaid Roots, and ſo on, in Equations of the Fifth Power, and the laſt Term, or the Abſolute Quantity, is Equal to the Product of all the Roots Multiplied into each Other; And the ſame Properties will hold, where there is the ſame Number of Negative Roots, or of Negative and Affirmative, Intermixed with each other, Reſpect being had to the Addition and Subſtraction of + and −.

To Form a Biquadratick Equation.

$$\begin{array}{ll}
 & x^3+bxx+bcx \\
 & \quad +cxx+bdx \\
\text{Cubick Eq.} & \quad +dxx+cdx+bcd=0 \\
\text{Root.} & \times x+f=0 \\
 & =x^4+bx^3+bcx^2+bcdx \\
 & \quad +cx^3+bdx^2+fbcx \\
 & \quad +dx^3+cdx^2+fbdx \\
 & \quad +fx^3+fbx^2+fcdx+fbcd=0 \\
 & \qquad\quad +fcx^2 \\
\text{Biquadrat. Eq.} & \qquad\quad +fdx^2 \\
\text{Root.} & \times x+g=0
\end{array}$$

To Form One of the Fifth Power.

$$\begin{array}{ll}
=x^5+bx^4+bcx^3+bcdx^2+fbcdx & \\
\quad +cx^4+bdx^3+fbcx^2+gbcdx & \\
\quad +dx^4+cdx^3+fbdx^2+gfbcx & \\
\quad +fx^4+fbx^3+fcdx^2+gfbdx & \\
\quad +gx^4+fcx^3+gbcx^2+gfcdx+gfbcd=0 & \\
\qquad\quad +fdx^3+gbdx^2 & \\
\qquad\quad +gbx^3+gcdx^2 & \\
\qquad\quad +gcx^3+gfbx^2 & \\
\qquad\quad +gdx^3+gfcx^2 & \\
\qquad\quad +gfx^3+gfdx^2 & \text{Equat. 5th. Power.} \\
\times x+h=0, \; \&c. & \text{Root.}
\end{array}$$

Which will Form an Equation of the 6*th*. Power, and ſo on in Infinitum; The Signs of theſe Equations alſo may be Infinitely Varied; For every Binomial Root, which is Multiplied into its ſelf may be $x+b$, or $x-b$; If x has Two Values; It may be, to Form a Quadratick Equation, as before, $x-b=0 \times x-c=0$, or, $x-b=0 \times x+c=0$, or, $x+b=0 \times x+c=0$; If x has Three Values, as in Cubick Equations, the Signs may be Diverſified, as in the Problems above; It muſt be alſo Noted, that from this Conſtruction of Equations is Derived, the *Tabella Poteſtatum* of Dr. *Halley*, which he Calls his *Spetulum Analyticum Generale*, Sir *Iſ. Newton*'s Algebra, *Pag.* 336. which I have here Subjoyned; In which, $a+e$, $=x+b$, is Continually Multiplied into it's ſelf.

Tabella Poteſtatum.

$$\begin{array}{llllllll}
 & & s & t & u & w & x & y \\
lz^7 & =la^7 & +7la^6e & +21la^5ee & +35la^4e^3 & +35la^3e^4 & +21la^2e^5 & +7lae^6+le^7 \\
kz^6 & =ka^6 & +6ka^5e & +15ka^4ee & +20ka^3e^3 & +15ka^2e^4 & +\;6ka\;e^5 & +\;ke^6 \\
hz^5 & =ha^5 & +5ha^4e & +10ha^3ee & +10ha^2e^3 & +\;5ha\;e^4 & +\;h\;e^5 & \\
gz^4 & =ga^4 & +4ga^3e & +\;6ga^2ee & +\;4ga\;e^3 & +\;g\;e^4 & & \\
fz^3 & =fa^3 & +3fa^2e & +\;3fa\;ee & +\;f\;e^3 & & & \\
dz^2 & =da^2 & +2da\;e & +\;d\;ee & & & & \\
cz & =ca & +\;c\;e & & & & &
\end{array}$$

§. 5. What

The Iſomœria of Vieta, or to Free an Equation from Fractions, when the Firſt, Second, or Third, or any other Term is Incumbered with them, Problems 1ſt. 2d. 3d. 4th. and 5th.

§. 5. It has been already Shewn, how an Equation may be ſo Reduced, that the Firſt Term may have no Coefficient but Unit, but if after an Equation is ſo Reduced, there happens to be any Fraction in the Second, Third, or any of the Following Terms, ſuch Equation may be Reduced to another, whoſe Terms ſhall be all Integers; Which is called the *Iſomœria* of *Vieta*.

PROBL. 1. $x^3+\frac{3}{2}x=225$. What is the Number x?

PROBL. 2. $x^3+\frac{3}{2}x=\frac{265}{2}$. What is the Number x?

PROBL. 3. $x^3+\frac{3}{2}xx=\frac{325}{2}$. What is the Number x?

SOLUTION of PROBL. 1.

Suppoſing, $y=2x$; $2x$ is taken, becauſe 2 is the Denominator of the Fraction $\frac{3}{2}$.

Therefore, $\frac{y}{2}=x$.

1ſt. Term.

And. $\frac{y^3}{8}=x^3$.

2d. Term.

Multiply $\frac{y}{2}=x$ $\frac{3y}{4}=\frac{3}{2}x$.

$\times\frac{3}{2}$ the Fraction.

1ſt. 2d. 1ſt. 2d.

Add the Two laſt Equations. $\frac{y^3}{8}+\frac{3y}{4}=x^3+\frac{3}{2}x$.

1ſt. 2d.

But. $x^3+\frac{3}{2}x=225$.

1ſt. Term. 2d. Term.

Therefore. $\frac{y^3}{8}+\frac{3y}{4}=225$.

And. $y^3+6y=1800$.

But, y being found, $\frac{1}{2}y=x$.

That is, $y=12$. $x=6$.

SOLUTION of PROBL. 2.

Suppoſing. $y=2x$.

1ſt. Term.

Then. $\frac{y}{2}=x$ and, $\frac{y^3}{8}=x^3$.

2d. Term.

And. $\frac{3}{2}\times\frac{y}{2}=x$ is $\frac{3y}{4}=\frac{3}{2}x$.

Add the Two laſt Equations. $\frac{y^3}{8}+\frac{3y}{4}=x^3+\frac{3}{2}x$.

But. $x^3+\frac{3}{2}x=\frac{265}{2}$.

Therefore. $\frac{y^3}{8}+\frac{3y}{4}=\frac{265}{2}$.

And. $y^3+6y=1060$.

But. $y=2x$. $\frac{1}{2}y=x$.

That is, $y=10$. $x=5$.

SOLUTION of PROBL. 3.

Suppoſing. $y=2x$.

Then, by Diviſion. $\frac{y}{2}=x$.

1ſt. Term.

And, by Multipl. $\frac{y^3}{8}=x^3$.

2d. Term.

But, $\frac{3}{2}\times\frac{yy}{4}=xx$ is $\frac{3yy}{8}=\frac{3}{2}xx$.

Add the Two laſt Equations. $\frac{y^3}{8}+\frac{3yy}{8}=x^3+\frac{3}{2}xx$.

But. $x^3+\frac{3}{2}xx=\frac{325}{2}$.

Therefore. $\frac{y^3}{8}+\frac{3yy}{8}=\frac{325}{2}$.

And. $y^3+3yy=1300$.

But $y=2x$, Or, $\frac{1}{2}y=x$.

That is, $y=10$. $x=5$.

PROBL. 4. $x^3+\frac{11}{12}x=\frac{19}{4}$; What is x?

PROBL. 5. $x^4-10x^3+45\frac{5}{6}xx-104\frac{1}{6}x+89=0$.

SOLUTION of PROBL. 4.

Suppoſing $y=12x$, 12 being Equal to the Denominator of the Fraction in the Second Term. $y=12x$.

Therefore by Dividing by 12, $\frac{y}{12}=x$.

1ſt. Term.

And by Cubing both Sides. $\frac{y^3}{1728}=x^3$.

2d. Term.

Alſo, by Multipl. $\frac{11}{12}\times\frac{y}{12}=x$ is $\frac{11y}{144}=\frac{11}{12}x$.

1ſt. and 2d. 1ſt. and 2d.

Add the Two laſt Equations. $\frac{y^3}{1728}+\frac{11y}{144}=x^3+\frac{11}{12}x$.

1ſt. and 2d.

But, by the Problem, $x^3+\frac{11}{12}x=\frac{19}{4}$.

1ſt. and 2d.

Therefore, by the Equat. $\frac{y^3}{1728}+\frac{11y}{144}=\frac{19}{4}$.

And. $y^3+132y=8208$. $\frac{1}{12}y=x$. $y=18$. $x=1\frac{1}{2}$.

SOLUTION of PROBL. 5.

Suppoſing $y=6x$, 6 being the Denominator of the Fraction, $\frac{5}{6}$.

Therefore by Dividing by 6, $\frac{y}{6}=x$.

And by Squaring both Sides, $\frac{yy}{36}=xx$.

1ſt. Term.

Alſo, by Squaring the Laſt, $\frac{y^4}{1296}=x^4$.

Again, by Cubing, $\frac{y}{6}\times\frac{yy}{36}=\frac{y^3}{216}=x^3$.

2d. Term.

And, by Multiplying by 10, $\frac{10y^3}{216}=10x^3$.

3d. Term.

Likewiſe, by the 2*d*. Step, $45\frac{5}{6}\times\frac{yy}{36}=45\frac{5}{6}xx$.

3d. Term.

And, Conſequently. $\frac{275}{216}yy=45\frac{5}{6}xx$.

Laſtly, it will be, $\overset{\text{4th. Term.}}{104\frac{1}{6}\times\frac{y}{6}=104\frac{1}{6}x}$.

And Therefore, $\overset{\text{4th Term.}}{\frac{625}{36}y=104\frac{1}{6}x}$.

Add all the New Invented Terms together, $\frac{y^4}{1296}-\frac{10y^3}{216}+\frac{275y^2}{216}-\frac{625y}{36}+89=0$.

And Reduced to Integers,

$y^4-60y^3+1650y^2-22500y+115344=0$.

If $y=12$, Then $x=\frac{12}{6}=2$.

If $y=18$, Then $x=\frac{18}{6}=3$.

If an Equation is Propoſed, whoſe *2d. 3d. 4th.* Terms, *&c.* have Fractions with Different Denominators, Reduce thoſe Fractions to the ſame Denominator, and then Proceed as above.

The Reaſon of this Procedure is Plain, becauſe, ſince in the firſt Problem $\frac{y^3}{8}$ is Produced by $\frac{y}{2}\times\frac{y}{2}\times\frac{y}{2}$, if we Multiply 8 into the Second Term, 2 the Diviſor will give a Quotient without a Remainder, that is, without a Fraction, and the ſame will obtain in any Multiple of 8, as in the Caſe Mentioned of 3×8, and Divided by 2, or in any other, which Occurs in the foregoing Examples, or Problems.

§. 6. Mr. *Harriot's* Method of Taking away or Deſtroying the Second Term of a Compound or Affected Equation, (See *Harriot's* Algebra. §. 6. Dr. *Wallis Oper. Mathemat.* Cap. 43. Vol. 2.) is to Divide the Coefficient of the Second Term of an Equation Propoſed by the Index of the Power of the Firſt Term, and if the Signs of the Firſt and Second Terms be unlike, that is, if the One be +, and the other −, Subſtract the Quotient from the Affirmative Root Sought, but if the Signs be like, that is, both + or both − Add the ſaid Quotient to the Affirmative Root; Then make an Equation betwixt the ſaid Sum or Remainder, and ſome Letter Repreſenting an Unknown Quantity, from whence a New Equation will ariſe, wherein the Second Term is Wanting.

To take away the Second Term of a Compound or Affected Equation by Harriot's Method, Problems 1ſt. 2d. 3d. and 4th.

Probl. 1. $xx-6x-72=0$. **Probl.** 2. $xx+6x-216=0$. **Probl.** 3. $x^3-18xx-7x+696=0$.

SOLUTION of PROBL. 1.

Suppoſe, $x-\frac{6}{2}=x-3=y$.
Therefore. $x=y+3$.
The firſt Term $xx=yy+6y+9$.
Second Term.
$-6x=-6y-18$.
Therefore. $yy+6y+9-6y$
$-18=yy-9=xx-6x$.
And $xx-6x-72=yy-9$
$-72=yy-81=0$.
Therefore $yy=81$ in which Equation the Second Term is Wanting, and $y=9$, and
$x=9+3=12$
Or, $-9+3=-6$

Proof. $\left.\begin{array}{r} xx=144 \\ -6x=-72 \end{array}\right| =72$.

By the Canon or Theorem for Reſolving Quadratick Equations.

$xx=6x+72$, and,
$x=3\pm\sqrt{9+72}$
$x=3\pm\sqrt{81}=12$
Or, -6

SOLUTION of PROBL. 2.

Suppoſe, $x+\frac{6}{2}=x+3=y$.
Therefore. $x=y-3$.
Firſt Term. $xx=y^2-6y+9$.
Second Term. $6x=6y-18$.
And, $xx+6x=y^2-6y+9$
$+6y-18=y^2-9$.
But, $xx+6x-216=y^2-9$
$-216=y^2-225=0$.
Or, $y^2=225$, and $y=15$.
Or, -15
And. $x=15-3=12$.
Or, $-3-15=-18$

Proof. $\left.\begin{array}{r} xx=144 \\ 6x=\ 72 \end{array}\right| =216$.

By the Canon or Theorem for Reſolving Quadratick Equations.

$xx=-6x+216$.
$x=-3\pm\sqrt{9+216}$.
$=-3\pm\sqrt{225}$.
$=-3\pm15=12$.
Or, -18.

SOLUTION of PROBL. 3.

Suppoſe, $x-\frac{18}{3}=x-6=y$.
Therefore. $x=y+6$.
Squares. $xx=y^2+12y+36$.
Firſt Term. $x^3=y^3+18y^2$
$+108y+216$.
Second Term. $-18xx$
$=-18yy-216y-648$.
Third Term. $-7x=-7y-42$.
Therefore. x^3-18x^2-7x
$+696=y^3+18y^2+108y$
$+216-18y^2-216y-648$
$-7y-42+696=0$; Or,
$y^3-115y+222=0$, in which Equation the Second Term y^2 is Wanting.
If therefore $y=2$, $x=2+6=8$,
If $y=\sqrt{112}-1$,
$x=\sqrt{112}+5$.
Proof.
$x^3=512$, $-18xx=-1152$,
$-7x=-56$.
Therefore, $512-1152-56$
$+696=1208-1208=0$.

PROBL. 4. $xxxx+6xxx+11xx+6x-100=0$. The Inquiry is, What is x?

SOLUT. PROBL. 4. Suppoſe, $x+\frac{6}{4}=x+\frac{3}{2}=y$. Then, $x=y-\frac{3}{2}$. And, $x^2=y^2-3y+\frac{9}{4}$.

Therefore, $x^3=y^3-\frac{9}{2}y^2+\frac{27}{4}y-\frac{27}{8}$.

1ſt. Term, $x^4=y^4-6y^3+\frac{27}{2}y^2-\frac{27}{2}y+\frac{81}{16}$.

2d. Term, $6x^3=6y^3-\frac{54}{2}y^2+\frac{81}{2}y-\frac{324}{16}$.

3d. Term, $11x^2=\frac{22}{2}y^2-\frac{66}{2}y+\frac{396}{16}$.

4th. Term, $6x=\frac{12}{2}y-\frac{144}{16}$.

5th. Term, $-100=-\frac{1600}{16}$.

In all theſe Problems, or Examples, the Rule therefore is Obſerved, of Dividing the Coefficient of the Second Term by the Index of the Power of the Firſt, for a New Unknown Root, and + and − are Prefixed, as Required.

Therefore, the Equation will be, $x^4+6x^3+11x^2+6x-100=0$.

1ſt. Term, Added to the $=y^4-6y^3+\frac{27}{2}y^2-\frac{27}{2}y+\frac{81}{16}$.

2d. Term, Added to the $+6y^3-\frac{54}{2}y^2+\frac{81}{2}y-\frac{324}{16}$.

3d. Term, Added to the $+\frac{22}{2}y^2-\frac{66}{2}y+\frac{396}{16}$.

4th. Term, Added to the $+\frac{12}{2}y-\frac{144}{16}$.

5th. Term, $-\frac{1600}{16}=0$.

All the Fractions are here in each Term Reduced to the ſame Denomination for the more eaſy Collection of them into one Sum.

Sum of the Terms Added make, $y^4 \; * \; -\frac{5}{2}y^2 \; * \; -99\frac{7}{16}=0$.

But, by Reduction it will be, $y^4=\frac{5}{2}y^2+99\frac{7}{16}$.

And, by Extraction or Evolution, $y^2=\frac{5}{4}\pm\sqrt{\frac{1616}{16}}=1\frac{1}{4}\pm\sqrt{101}$.

And again, by Extraction, $y=\sqrt{1\frac{1}{4}\pm\sqrt{101}}$, $x=\sqrt{1\frac{1}{4}\pm\sqrt{101}}-\frac{3}{2}$.

The Reaſon of this Procedure is, becauſe in Quadratick Equations, the Coefficient being Divided by 2, and the Signs Prefixed according to the Rule, will in the Square $=xx$, certainly Produce a Second Term, whoſe Coefficient is Equal to that in the Given Equation, but with a Contrary Sign, and a New unknown Quantity of the ſame Dimenſions Affixed to it, and the Multiplication of the Second Term, according to the New Value of the Unknown Root, will Produce a like Coefficient with the New Unknown Quantity of a like Dimenſion, but with the ſame Sign with the given Equation, and therefore theſe Terms muſt Neceſſarily Deſtroy each other, and the ſame will hold in the Cube, Bi-ſquare, *&c.* by Dividing the Coefficient by 3, 4, *&c.*

The Method of Reſolving all Manner of Compound or Affected Equations in Numbers, Problems, 1ſt. 2d. and 3d.

§. 8. To Reſolve all Manner of Compound or Affected Equations in Numbers; 1ſt. Let the Equation be made Equal to 0, and if Neceſſary Freed from Fractions.

2d. Then Search out all the Juſt Diviſors to the Laſt Term, that is, the Known Abſolute Number; Connect them Succeſſively by + and − to the Root Sought x, and Divide the Equation by it; If the Diviſion Succeeds, ſo as there is no Remainder, either the Known Part of the Diviſor will be Equal to the Unknown, and therefore will be One Root Sought, or at leaſt the Diviſor will be Equal to 0, and Conſequently will leave an Equation $=0$, which will be One Power leſs than the Former, and by that Means, if it becomes One of thoſe 9 Forms Specified before, may be Reſolved by the Canons there Delivered, but if the Firſt Term Contains Three or more Dimenſions, then Examine it by Diviſion, as before.

3*d.* If this does not Succeed, Take away the Second Term by the Method already Explained, by which Means an Equation will ſometimes Reſult, which will Fall under One of the 9 Forms before Mentioned, and then may be Reſolved by the Canons thereto belonging.

4*th.* If none of theſe Ways prove Effectual, then Find out One Affirmative Root by the Method before Expreſſed, either Exactly or nearly True, Join it to the Root Sought x by $-$, and Divide the Equation, and by the ſame Way Proceed to find out the other Roots, either Exactly, or Nearly, True.

PROBL. 1. $x^3-9x^2+26x=24$; Or, $x^3-9x^2+26x-24=0$; What is x?
PROBL. 2. $x^3-22x^2+157x=360$; Or, $x^3-22x^2+157x-360=0$; What is x?
PROBL. 3. $91x-x^3=330$; Or, $x^3-91x+330=0$; What is x?

SOLUTION of PROBL. 1.

1*ſt.* Search out all the Juſt Diviſors of the Abſolute Number 24, which are 1. 2. 3. 4. 6. 8. 12. 24.

2*d.* Then Try $x-1$, $x+1$, $x-2$, $x+2$, *&c.* and $x-2$ will Divide the Equation without a Remainder.

$x-2=0)x^3-9x^2+26x-24^*$
$x^3-2x^2 \quad *=0$
$0-7x^2$
The Quo- $-7x^2+14x$
tient. $0+12x$
$(x^2-7x+12=0.$ $+12x-24$
$x-2=0$ $\quad 0 \quad 0$
$x=2.$
$x^2-7x+12=0.\ x^2=7x-12.$
$x=3\frac{1}{2}\pm\sqrt{12\frac{1}{4}-12}=3\frac{1}{2}\pm\frac{1}{2}$ $=4$ and 3; Therefore 2, 3, and 4, are the Roots Sought, and which will Solve the Equation.

SOLUTION of PROBL. 2.

1*ſt.* The Juſt Diviſors of 360 are 1. 2. 3. 4. 5. 6. 8. 9. 10. 12. 15. 18. 20. 24. 30. 36. 40. 45. 60. 72. 90. 120. 180. and 360.

2*d.* Examine them in Order $x-1.\ x+1.\ x-2.\ x+2.\ x-3.\ x+3,$ *&c.* $x-5$ will Divide the Equation without a Remainder.

$x-5)x^3-22x^2+157x-360^*$
$=0)\ x^3-5x^2 \quad *=0$
$0-17x^2$ Quot. $(x^2-$
$-17x^2+85x$ $(17x+$
$x-5=0$ $\quad 0+72x$ $(72=0$
$x=5$ $\quad +72x-360.$
$0 \quad 0$
Alſo, $x^2-17x+72=0.$
And. $x^2=17x-72.$
$x=8\frac{1}{2}\pm\sqrt{72\frac{1}{4}-72}=8\frac{1}{2}\pm\frac{1}{2}$ $=9$ and 8. The Roots Sought are therefore 5. 8. and 9.

SOLUTION of PROBL. 3.

1*ſt.* The Diviſors of 330 are 1. 2. 3. 5. 6. 10. 11. 15. 22. 30. 55. 66. 110. 165. and 330.

2*d.* Divide in Order by $x-1$, $x+1$, $x-2$, $x+2$, *&c.* $x-5$ will do our Buſineſs.

$x-5)\ x^3-91x+330=0$
$=0)\ x^3-5x^2$ $\quad (x^2+5x$
$0+5x^2$ $\quad (-66=0$
$+5x^2-25x$
$0-66x$
$-66x+330$
$0 \quad 0$
Therefore, $x^2+5x-66=0$
And, $x^2=-5x+66$
Alſo, $x=-2\frac{1}{2}\pm\sqrt{72\frac{1}{4}}$
That is, $x=-2\frac{1}{2}\pm8\frac{1}{2}=6$
Or, -11
The Roots Sought are therefore 5, 6, and -11, which will all Solve the Equation.

PROBL. 4. SOLUTION.

To Find Two Numbers, whoſe Sum ſhall be 5, and if the Sum of their Squares be Multiplied by the Sum of their Cubes, the Product ſhall be 455.

1*ſt.* One of the Numbers Sought. x.
2*d.* The Other. $5-x$.
3*d.* Square of the Firſt Number. xx.
4*th.* Square of the Second. $xx-10x+25$.
5*th.* Sum of the Squares. $2xx-10x+25$.
6*th.* Cube of the Firſt Number. x^3.
7*th.* Cube of the Second, $-x^3+15x^2-75x+125$.

PROBL. 5. SOLUTION.

To Find Four Numbers in Arithmetical Progreſſion Continued, Such, that their Common Difference may be Unit, and the Product made by their Continual Multiplication may be 100.

Firſt Number. x.
Second. $x+1$.
Third. $x+2$.
Fourth. $x+3$.

The Product of their Continual Multiplication will be $x^4+6x^3+11x^2+6x=100$, by the Problem.

The Diviſors of 100 are 1. 2. 4. 5. 10. 20. 25. 50. and 100.

But if Diviſion be Try'd by

PROBL. 6. SOLUTION.

If, $8x^3+63x^2-x^4-341x=1304$, Or,
If, $x^4-8x^3-63xx+341x+1304=0$, What is the Number x?

The Diviſors of 1304 are 1. 2. 4. 8. 163. 326. and 1304; And, after ſeveral Trials, $x-8=0$ Divides the Equation.

Therefore, $x=8$.

And the Quotient, upon Diviſion, is $x^3-63x-163=0$.

The Diviſors of 163 are only 1 and 163.

But $x-1$, $x+1$, $x-163$, $x+163$, will not Divide the Equation Juſtly without a Fraction, and there being

8th. Sum of the Cubes,

$15x^2-75x+125$.

9th. Sum of the Squares × Sum of the Cubes,

$30x^4-300x^3+1375x^2-3125x+3125=455$,

by the Problem.

10th. And by Subſtracting 455 from each Side of the Equation, and Dividing the Remaining Quantity $=0$ by 30, $x^4-10x^3+45\tfrac{5}{6}x^2-104\tfrac{1}{6}x+89=0$. To take away the Fractions.

11th. Suppoſe $y=6x$, and Proceeding, as before, $y^4-60y^3+1650y^2-22500y+115344=0$.

12th. The Diviſors of 115344 are 1. 2. 3. 4. 6. 8. 9. 12. 18. 24. 27, *&c.* and after Trials made, as before, by Diviſion, $y-12=0$ will Exactly Divide the Equation, and therefore $y=12$, and the Quotient will be $y^3-48y^2+1074y-9612=0$.

13th. The Diviſors of 9612 are 1. 2. 3. 4. 6. 9. 12. 18. 27. 36, *&c.* and after ſeveral Trials $y-18=0$ will Exactly Divide the Equation; Therefore $y=18$, and the Quotient will be $y^2-30y+534=0$, and $y^2=30y-534$;

$x-1.\ x+1.\ x-2.\ x+2$, *&c.* to $x+$ or -100, it will not Succeed; ſince this Way therefore is Ineffectual.

Take away the Second Term by the Method before Preſcribed of putting $x=y-\frac{3}{2}$, which will Reduce the Equation to,

$$y^4-2\tfrac{1}{2}y^2-99\tfrac{7}{16}=0.$$

Or,

$$y^4=2\tfrac{1}{2}y^2+99\tfrac{7}{16}.$$

$$y^2=1\tfrac{1}{4}\pm\sqrt{101}.$$

$$y=\sqrt{1\tfrac{1}{4}\pm\sqrt{101}}.$$

Therefore.

$$x=\sqrt{1\tfrac{1}{4}\pm\sqrt{101}}-\tfrac{3}{2}.$$

$$x+1=\sqrt{1\tfrac{1}{4}\pm\sqrt{101}}-\tfrac{1}{2}.$$

$$x+2=\sqrt{1\tfrac{1}{4}\pm\sqrt{101}}+\tfrac{1}{2}.$$

$$x+3=\sqrt{1\tfrac{1}{4}\pm\sqrt{101}}+\tfrac{3}{2}.$$

Which Multiplied into One another are $=100$.

Reſolution the Second.

First Number. $x-1\tfrac{1}{2}$.
Second. $x-\tfrac{1}{2}$.
Third. $x+\tfrac{1}{2}$.
Fourth. $x+1\tfrac{1}{2}$.

Which being Multiplied into one another, the Product is $=100$ by the Problem, from whence the ſame Equation will Riſe as before, $x^4-2\tfrac{1}{2}x^2,=99\tfrac{7}{16}$ and the Numbers Sought will be the ſame.

no Second Term to be taken away, We are to Search out One Affirmative Value of x by the General Method before Delivered.

Which will be $x=9.0055$ *&c.*

Then Divide the ſaid Cubick Equation $x^3-63x-163=0$ by $x-9.0055$.

And the Quotient (the Remainder, after the Diviſion is Ended, being Neglected) is $xx+9.0055x+18.09903025=0$.

But this Equation having no Affirmative Root, the Two, which can be had are only 8 and 9.0055, *&c.*

If every Term of an Equation has an Index of it's Power, which is an even Number, it may be Reduced to a Lower Denomination, as $x^6-29x^4+244x^2-576=0$ Put $y=x^2$, and it will be Reduced to a Cubick Equation, *viz.* $y^3-29y^2+244y-576=0$.

And, $y=15\pm\sqrt{225-534}$, which is an Impoſſible Root in Extraction, but not in Solving the Equation; That is, being a Negative Root, it is Impoſſible to be Extracted; But $15\pm\sqrt{225-534}$ will be Two Roots more, which will be Poſſible, by what has been Said, in Solving the Equation, and will Equally do it with the Two other Affirmative Ones.

Therefore, in this 1ſt Problem $y=12,\ 18,\ 15\pm\sqrt{-309}$, And $x=2,\ 3,\ 2\tfrac{1}{2}\pm\frac{\sqrt{-309}}{6}$.

Proof 1ſt. $x=2$, $5-x=3$, $2+3=5$, $4+9=13$, $8+27=35$; And $455=13\times35$.
Proof 2*d.* $x=3$, $5-x=2$, $2+3=5$, and $13\times35=455$.

Proof 3*d.* $x=2\tfrac{1}{2}+\frac{\sqrt{-309}}{6}$, $5-x=5-2\tfrac{1}{2}-\frac{\sqrt{-309}}{6}=2\tfrac{1}{2}-\frac{\sqrt{-309}}{6}$; Therefore $2\tfrac{1}{2}+\frac{\sqrt{-309}}{6}+2\tfrac{1}{2}-\frac{\sqrt{-309}}{6}=5$. We muſt here Add ſome other Rules in Algebra; That,

1ſt. The Reduction of a Biquadratick Equation, not Falling under the Three Forms Abovementioned, to a Cubick One, or to Two Quadratick Ones, is Tedious and Troubleſome.

2*d.* It is Seldom Capable of being Reduced into Two Quadratick Ones, and when Reduced to a Cubick, may Happen to be ſuch, as it's Root, or Roots, in Numbers cannot be Perfectly Found out by any Rules hitherto Publiſhed.

3*d.* To Shorten the Various Trialls of the Diviſors of the Laſt Term, it will be well to have Recourſe to the Method of Finding out the Limits, within which the Root, or Roots, of an Equation Fall, and then to make Uſe of no Diviſors but what Stand within thoſe Limits. See Dr. *Wallis*'s Algebra, P. 227. Vol. 2. of his Works;

Alſo Sir *Iſaac Newton's* Algebra, Pag, 276. And *Des-Cartes's* Rule Derived from Mr. *Harriot*, where we ſhall find the Reſolution of a Biquadratick Equation into Two Quadratick Ones Performed in this Manner, *viz.* Let $x^4 - x^3 - 5x^2 + 12x - 6 = 0$.

1ſt. Take away the Second Term by making $v + \frac{1}{4} = x$, and $v^4 - \frac{43}{8}v^2 + \frac{75}{8}v - \frac{851}{256} = 0$.

2d. Clear this laſt Equation of Fractions by making $\frac{1}{4}z = v$, &c.

§. 9. *Cardan's* Two Rules Aſcribed by him to *Scipio Ferreus*, for the Reſolution of Certain Cubick Equations in Numbers, are as Follow.

Cardan's *Two Rules for the Reſolution of Cubick Equations. Problems 1ſt. 2d. and 3d.*

All Cubick Equations after the Second Term is taken away, when there is any, are Reducible to the Following Forms.

PROBL. 1. $x^3 = -px + q$, Or, $x^3 = -6x + 20$; What is x?
PROBL. 2. $x^3 = +px + q$, Or, $x^3 = +6x + 40$; What is x?
PROBL. 3. $x^3 = px - q$, Or, $x^3 = 91x - 330$; What is x?

SOLUTION of PROBL. 1.

1ſt. Suppoſe $x =$ to the Difference of Two Unknown Numbers. $y - z$.

2d. Suppoſe $q = y^3 - z^3$ the Difference of their Cubes.

3d. And $p = 3yz$ the Triple of their Multiplication.

Therefore, $x^3 = y^3 - 3y^2z + 3yz^2 - z^3 = q - px$.

And. $px = 3y^2z - 3yz^2$.

4th. y and z are found out by this Problem.

There are Two Numbers, the Product of whoſe Multiplication is $\frac{1}{3}p$, and the Difference of their Cubes is q; What are the Numbers?

The Greater Number. y.

The Leſſer Number. $\dfrac{\frac{1}{3}p}{y}$.

Cube of the Greater Number. y^3.

Cube of the Leſſer. $\dfrac{\frac{1}{27}p^3}{y^3}$

But. $y^3 - \dfrac{\frac{1}{27}p^3}{y^3} = q.\ \dfrac{y^6 - \frac{1}{27}p^3}{y^3} = q.$ Or, $y^6 - qy^3 = \frac{1}{27}p^3$.

And. $y^6 = qy^3 + \frac{1}{27}p^3$.

Or. $y^3 = \frac{1}{2}q \pm \sqrt{\frac{1}{4}qq + \frac{1}{27}p^3}$.

Or. $y = \sqrt{3.\ \frac{1}{2}q \pm \sqrt{\frac{1}{4}qq + \frac{1}{27}p^3}}$

But. $y^3 - z^3 = q$, And, $z^3 = y^3 - q = -\frac{1}{2}q \pm \sqrt{\frac{1}{4}qq + \frac{1}{27}p^3}$

$z = \sqrt{3.\ -\frac{1}{2}q \pm \sqrt{\frac{1}{4}qq + \frac{1}{27}p^3}}$.

$x = y - z$. Therefore,

$$x = \sqrt{3.\ \tfrac{1}{2}q \pm \sqrt{\tfrac{1}{4}qq + \tfrac{1}{27}p^3}} - \sqrt{3.\ -\tfrac{1}{2}q \pm \sqrt{\tfrac{1}{4}qq + \tfrac{1}{27}p^3}}.$$

$p = 6$
$q = 20$ | $x = \sqrt{3.\ 10 \pm \sqrt{108}}$. $-\sqrt{3.\ -10 \pm \sqrt{108}}$

SOLUTION of PROBL. 2.

1ſt. Suppoſe $x =$ to the Sum of Two Unknown Numbers. $y + z$.

2d. Suppoſe $q =$ to the Sum of their Cubes. $y^3 + z^3$.

3d. And $p = 3yz$ the Triple of their Multiplication.

Therefore, $x^3 = y^3 + 3y^2z + 3yz^2 + z^3 = px + q$.

And, $px = 3y^2z + 3yz^2$.

4th. y and z are found out by this Problem.

There are Two Numbers, the Product of whoſe Multiplication is $\frac{1}{3}p$, and the Sum of their Cubes is q; What are the Numbers?

For one of the Numbers Sought Put y.

z the other Number $= \dfrac{\frac{1}{3}p}{y}$.

The Cube of the Firſt Number is. y^3.

Cube of the Second. $\dfrac{\frac{1}{27}p^3}{y^3}$.

By the Problem.

$$y^3 + \frac{\frac{1}{27}p^3}{y^3} = q.$$

By Reduction.

$$\frac{y^6 + \frac{1}{27}p^3}{y^3} = q.$$

And. $y^6 + \frac{1}{27}p^3 = qy^3$.

Or. $y^6 = qy^3 - \frac{1}{27}p^3$.

And. $y^3 = \frac{1}{2}q \pm \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}$.

Or. $y = \sqrt{3.\ \frac{1}{2}q \pm \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}}$.

But. $q = y^3 + z^3$.

Therefore, $z^3 = q - y^3$.

$= q - \frac{1}{2}q \mp \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}$.

$= \frac{1}{2}q \mp \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}$.

And $z = \sqrt{3.\ \frac{1}{2}q \mp \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}}$.

But. $x = y + z$

SOLUTION of PROBL. 3.

1ſt. Suppoſe, $x = y + z$.

2d. Suppoſe, $-q = y^3 + z^3$.

3d. And, $p = 3yz$.

Therefore, $x^3 = y^3 + 3y^2z + 3yz^2 + z^3 = px - q$.

And, $px = 3y^2z + 3yz^2$.

4th. y and z are Found out by this Problem, if by any Way.

There are Two Numbers, the Product of whoſe Multiplication is $\frac{1}{3}p$, and the Sum of their Cubes is $-q$; What are the Numbers?

By the foregoing Proceſs,

$y^6 + \frac{1}{27}p^3 = -qy^3$.

And. $y^6 = -qy^3 - \frac{1}{27}p^3$.

$y^3 = -\frac{1}{2}q \pm \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}$.

$y = \sqrt{3.\ -\frac{1}{2}q \pm \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}}$.

But $-q = y^3 + z^3$

Therefore, $z^3 = -y^3 - q$.

$= -q + \frac{1}{2}q \mp \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}$.

$z = \sqrt{3.\ -\frac{1}{2}q \mp \sqrt{\frac{1}{4}qq - \frac{1}{27}p^3}}$.

$$x = \sqrt{3.\ -\tfrac{1}{2}q \pm \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}} + \sqrt{3.\ -\tfrac{1}{2}q \mp \sqrt{\tfrac{1}{4}qq - \tfrac{1}{27}p^3}}.$$

$p = 91$.
$q = 330$.

$$x = \sqrt{3.\ -165 \pm \sqrt{-685\tfrac{1}{7}}} + \sqrt{3.\ -165 \mp \sqrt{-685\tfrac{1}{7}}}.$$

Therefore this Equation, and the Roots of it Impoſſible, in Reſpect of any Affirmative Solution; The Firſt becauſe $-q$ is Suppoſed Equal to $y^3 + z^3$, a Negative to a Poſitive Quantity; The Second, becauſe the Root of a Negative Quantity is to be Extracted, which is Suppoſed cannot be done.

$$\sqrt[3]{10\pm\sqrt{108}}=1\pm\sqrt{3}.$$

$$\sqrt[3]{-10\pm\sqrt{108}}=\sqrt{3}-1$$

Or, $-\sqrt{3}-1$

$$\left.\begin{array}{ll}\text{theref. } x= & 1+\sqrt{3}-\sqrt{3}+1\\ \text{Or,} & 1-\sqrt{3}+\sqrt{3}+1\end{array}\right|=2.$$

$$\text{Proof, }\left.\begin{array}{r}x^3=8\\ 6x=12\end{array}\right|=20.$$

$$=\sqrt[3]{\tfrac{1}{2}q\pm\sqrt{\tfrac{1}{4}qq-\tfrac{1}{27}p^3}}.$$

$$+\sqrt[3]{\tfrac{1}{2}q\mp\sqrt{\tfrac{1}{4}qq-\tfrac{1}{27}p^3}}.$$

$$\left.\begin{array}{l}p=6\\ q=40\end{array}\right|\begin{array}{l}x=\sqrt[3]{20\pm\sqrt{392}}\\ +\sqrt[3]{20\mp\sqrt{392}}.\end{array}$$

$$\sqrt[3]{20\pm\sqrt{392}}=2\pm\sqrt{2}$$

$$x=2\pm\sqrt{2}+2\mp\sqrt{2}=4$$

$$\text{Proof. }\left.\begin{array}{r}x^3=64\\ -6x=-24\end{array}\right|=40.$$

Notwithſtanding which, there is no Doubt to be made, but that the Root, tho' Impoſſible in Extraction, is not Impoſſible in Reſpect of the Equation; For in General it muſt be Affirmed, that, where the Reaſoning is Accurate, and Exact from One Step to Another in the Solution of any Problem, it is Impoſſible, that it ſhould Lead us into Falſe or Wrong Concluſions; And therefore all the Roots, which are Found out by ſuch a Reaſoning, tho' Perhaps they are not Capable of being Extracted, yet are True and Real Ones in Reſpect of Solving the Problem Propoſed, and will Anſwer all the Conditions of the Queſtion, as well as the Affirmative Ones will do; Examples of this may be Seen in the Foregoing Problems, and in the Nine Forms of Equations before Specified.

Concerning the Method of Reſolving Problems, wherein many Quantities are Sought by Aſſuming different Letters to Repreſent the ſaid Quantities. Problems the 1ſt. 2d. and 3d.

§. 10. Concerning the Method of Reſolving Problems, wherein many Quantities are Sought by Aſſuming different Letters to Repreſent the Said Quantities.

Hitherto, one Letter has been only made Uſe of to Expreſs the Unknown Quantity; We ſhall now make Uſe of different Ones, being in the Solution of ſeveral Problems, the moſt Eligible; Rule 1ſt. By Exterminating the Unknown Quantities One after Another.

SOLUTION OF PROBL. 1.

A Factor Exchanged 6 French Crowns, and 2 Dollars, for 45 Shillings of Engliſh Money, alſo at another Time he Exchang'd 9 French Crowns, and 5 Dollars (each of theſe being of the ſame Value with the Former) for 76 Shillings; What was the Value of a French Crown and of a Dollar in Engliſh Money?

The Value of a French Crown. x.

The Value of a Dollar. y.

Therefore by the Problem.

$$6x+2y=45.$$

$$9x+5y=76.$$

Firſt, by the Firſt Equation find what x is.

$$6x=45-2y.$$

$$x=\frac{45-2y.}{6}$$

Therefore in the Second Equation.

$$\left.\begin{array}{l}9x\\+5y\end{array}\right|=\frac{405-18y}{6}+5y=76.$$

And, $405-18y+30y=456$

$12y=51.\ y=4\tfrac{1}{4}.$ * * * *

See this Problem Continued' Column the 3d'

SOLUTION OF PROBL. 2.

Three Men had, Each of them, a certain Number of Pounds, the Sum of the Firſt and Second Man's Money was, 5, or b, Pounds, the Sum of the Second and Third Man's Money was 12, or c, Pounds, and the Sum of the Third and Firſt Man's Money was 11, or d, Pounds; How many Pounds had Each?

The Number of Pounds which Each had. $x.\ y.\ z.$

1ſt. and 2d. $x+y=b.$

2d. and 3d. $y+z=c.$

1ſt. and 3d. $x+z=d.$

By the 1ſt. Equation $y=b-x.$

By the 1ſt. and 2d. $b-x+z=c$

By the Foregoing $z=c-b+x.$

Therefore, $x+x+c-b=d.$

And, $2x=d-c+b.$

Or, $x=\tfrac{1}{2}d-\tfrac{1}{2}c+\tfrac{1}{2}b.$

Therefore, $y=\tfrac{1}{2}b-\tfrac{1}{2}d+\tfrac{1}{2}c.$

And, $z=\tfrac{1}{2}d+\tfrac{1}{2}c-\tfrac{1}{2}b.$

$$\begin{array}{ll|l}b=5 & x=2 & 2+3=5.\\ c=12 & y=3 & 3+9=12.\\ d=11 & z=9 & 2+9=11.\end{array}$$

SOLUTION OF PROBL. 3.

Rule 2d.

By Subſtracting One Equation from Another, or Adding One to the Other.

$$\begin{array}{ll|l}\text{From} & x+y=b=12 & \text{What are}\\ \text{Subſtr.} & x-y=c=8 & x \text{ and } y?\end{array}$$

$$2y=b-c=4.$$

$$y=\tfrac{1}{2}b-\tfrac{1}{2}c=2.$$

$$x=\tfrac{1}{2}b+\tfrac{1}{2}c=10.$$

$10+2=12.\ \ 10-2=8.$

Or,

$$\begin{array}{ll|l}\text{To} & x+y=b=12 & \text{What are}\\ \text{Add} & x-y=c=8. & x \text{ and } y?\end{array}$$

$$2x=b+c=20.$$

$$x=\tfrac{1}{2}b+\tfrac{1}{2}c=10.$$

$$y=\tfrac{1}{2}b-\tfrac{1}{2}c=2.$$

* * * * Therefore in the Firſt Equation.

$6x+2y=6x+8\tfrac{1}{2}=45,$

$6x=45-8\tfrac{1}{2}=36\tfrac{1}{2},$

$x=6\tfrac{1}{12}.$

A French Crown, 6 *s.* 1 *d.*

A French Dollar, 4 *s.* 3 *d.*

$$\left.\begin{array}{l}6\times 6\ s.\ 1d.=36\ s.\ 6d.\\ 2\times 4\ s.\ 3d.=8\ s.\ 6d.\end{array}\right|=45\ s.$$

$$\left.\begin{array}{l}9\times 6\ s.\ 1d.=54\ s.\ 9d.\\ 5\times 4\ s.\ 3d.=21\ s.\ 3d.\end{array}\right|=76\ s.$$

SOLUTION of PROBL. 4.

Rule 3d.

By Alternate Multiplications of the Equations.

To find Two Numbers, That, if the Quadruple of the Greater be Increaſed with the Triple of the Leſs, it ſhall make 36, But, if the Triple of the Greater be Leſſened by the Double of the Leſs, the Remainder ſhall be 10.

The Greater Number. x.

The Leſſer. y.

By the Problem. $4x+3y=36$. $3x-2y=10$.

1ſt Equation Multiplied $\times 3$ makes $12x+9y=108$.

2d Equation Multiplied $\times 4$ makes $12x-8y=40$.

Subſtract the Second Equation from the Firſt, *viz.* From $12x+9y=108$.

Subſtract $12x-8y=40$.

There Remains $17y=68$.

And, $y=\dfrac{68}{17}=4$.

Alſo, $3x=10+2y=18$.

And, $x=\dfrac{18}{3}=6$.

Proof.

But, $4\times 6+3\times 4=36$.

And, $3\times 6-2\times 4=10$.

Which therefore are the True Values of x and y, and the Firſt is likewiſe Greater than the Laſt, or 6 Greater than 4, as was Required by the Problem.

SOLUTION of PROBL. 5.

Suppoſing.
$$\begin{array}{l} 2x+3y-2z=50. \\ 5x-2y+5z=240. \\ -x+5y-3z=10. \end{array}$$

What are the Numbers $x.y.z$?

1ſt Equation $\times 5=10x+15y-10z=250$.

2d Equation $\times 2=10x-4y+10z=480$.

Subſtract the Former from the Latter, and it will be $-19y+20z=230$.

3d Equation $\times 5=-5x+25y-15z=50$.

This Added to the *2d* Equation, which, Multiplied into 1, Continues the ſame, gives $+23y-10z=290$.

Then take the Equations found out in which are only Two Unknown Quantities, *viz.*

$$\begin{array}{l} -19y+20z=230. \\ +23y-10z=290. \end{array}$$

1ſt Equation $\times 23=-437y+460z=5290$.

2d Equation $\times 19=437y-190z=5510$.

Add theſe Two laſt Equations together.

And, $270z=10800$,

Or, $z=40$.

$23y-10z=23y-400=290$.

$23y=690$.

$y=30$.

$2x+3y-2z=50$.

$=2x+90-80$.

$2x=50-10=40$.

$x=20$.

$$\left.\begin{array}{l} 2x=40 \\ 3y=90 \\ -2z=-80 \end{array}\right|=50.$$

$$\left.\begin{array}{l} 5x=100 \\ 5z=200 \\ -2y=-60 \end{array}\right|=240.$$

$$\left.\begin{array}{l} -x=-20 \\ 5y=150 \\ -3z=-120 \end{array}\right|=10.$$

SOLUTION of PROBL. 6.

Three Men Diſcourſe of their Monies thus; The Firſt ſaith to the other Two, if ye give me 100 *l.* my Money will be made Equal to both your Remaining Monies.

The Second ſaith to the other Two, if ye give me 100*l.* my Money will be made Equal to the Double of both your Remaining Monies; Laſtly, the Third ſaith to the other Two, if if ye give me 100*l.* my Money will be made Equal to the Triple of both your Remaining Monies; How many Pounds had Each?

The Firſt Man's Money. x.

The Second's. y.

The Third's. z.

Or, the Queſtion may be thus Expreſſed.

If $x+100=y+z-100$,

And $y+100=2x+2z-200$,

And $z+100=3x+3y-300$,

What are the Numbers $x.y.z$?

By Tranſpoſition,

$-x+y+z=200$.

$2x-y+2z=300$.

$3x+3y-z=400$.

$2\times$ *1ſt* Equation $=-2x+2y+2z=400$.

Add this and the *2d* Equation together, which Multiplied by 1, Continues the ſame, Sum is, $=y+4z=700$.

Again, $3\times$ *2d* Equation, $=6x-3y+6z=900$.

And $2\times$ *3d* Equation, $=6x+6y-2z=800$.

Subſtract this Equation from the Former; Remainder is, $-9y+8z=100$.

Take the Two Equations Found, in which are only Two Unknown Numbers. $y+4z=700$. and $-9y+8z=100$. *1ſt* Of theſe Equations $\times 9=9y+36z=6300$. *2d* Of theſe Equations $\times 1=-9y+8z=100$, Add them together, And $44z=6400$. $z=145\frac{5}{11}$. $y+4z=700$. $y=700-580\frac{20}{11}=118\frac{2}{11}$; $-x+y+z=200$. $x=y+z-200=263\frac{7}{11}-200=63\frac{7}{11}$. $100+63\frac{7}{11}=118\frac{2}{11}+145\frac{5}{11}-100$. $100+118\frac{2}{11}=126\frac{14}{11}+290\frac{10}{11}-200$. $100+145\frac{5}{11}=189\frac{21}{11}+354\frac{6}{11}-300$.

It may, Perhaps, by ſome, who are more Advanced in this Science, be Wondered at, that I ſhould now Explain the ſeveral Ways of Reſolving Cubick and Biquadratick Equations before Recounted, when the Preſent Algebraiſts are Suppoſed to have gone much beyond any Thing of this Nature; But I would Deſire them to be Informed, that this was Intended for the Inſtruction of my Pupils, and by Way of Preparation

tion only to their Underſtanding Sir *Iſaac Newton's* Algebra, which was Formed, from what I have here Propoſed, and cannot be well Apprehended without ſome ſuch Previous Explanations; With which I have Intermixed ſome Reflections, and Obſervations upon this Science, not to be found in any other Syſtem of it, which, I would Hope, may Tend, both to the Rendering it more Eaſy, and to the Improving it.

SOLUTION of PROBL. 7.

Problems 7th. 8th. and 9th.

One is Offered 10 Apples for a Penny, and 25 Pears for Two-Pence;
If at thoſe Rates he would lay out $9\frac{1}{2}d.$ to buy 100 Apples and Pears together, How many Apples, and How many Pears ought he to have?

Number of Apples. x.
Of Pears. y.
Coſt of the Number of Apples Bought. $\frac{x}{10}$.

Ap. d. Ap. d.
Becauſe. $10.\ 1 :: x.\ \frac{x}{10}$.

Coſt of the Number of Pears. $\frac{2y}{25}$.

Pears d. Pears d.
Becauſe. $25.\ 2 :: y.\ \frac{2y}{25}$.

But according to the Problem, the Price of Apples and Pears, $\frac{x}{10}+\frac{2y}{25}=9\frac{1}{2}d.$
Number of 'em, $x+y=100$.
By Reduction,
1*ſt* of the Two laſt Equations $=50x+40y=4750$.
2*d* of the Two laſt Equations $\times 50=50x+50y=5000$.
Subſtract the 1*ſt* of theſe Two Equations from the 2*d*, Remains,

$$\left.\begin{array}{r}10y=250.\ y=25.\\ x+y=100.\ x=75.\end{array}\right|=100$$

$$\left.\begin{array}{r}75 \text{ Apples}=7\frac{1}{2}d.\\ 25 \text{ Pears}=2d.\end{array}\right|=9\frac{1}{2}d.$$

For. $\frac{x}{10}d=\frac{75}{10}d=7\frac{1}{2}d.$

And. $\frac{2y}{25}d=\frac{50}{25}d=2d.$

SOLUTION of PROBL. 8.

To Divide 90 into Four ſuch Numbers;
That if the Firſt be Increaſed by 2, the Second Leſſened by 2, the Third Multiplied by 2, and the Fourth Divided by 2,
The Sum, Remainder, Product, and Quotient, may be Equal between themſelves.

The Four Numbers Sought. $u.\ x.\ y.\ z.$
Numbers given, 90 and 2, or, b and d.
Or, in Algebra, the Queſtion will be Thus.

If, $u+x+y+z=b$.
And, $u+d=x-d$
And, $u+d=yd$.
And, $u+d=\frac{z}{d}$

What are the Numbers, $u.\ x.\ y.$ and z?
1*ſt* u is Equal to it's ſelf $u=u$.
By the 2*d* Equation. $x=u+2d$.
By the 3*d* Equat. $y=\frac{u+d}{d}$.
By the 4*th* Equat. $z=ud+dd$.
Therefore, by the foregoing Equations.
$2u+2d+\frac{u+d}{d}+ud+dd=u+x+y+z=b$, by Reduction,

$$u=\frac{bd-d^3-2dd-d}{dd+2d+1}.$$

$$\left.\begin{array}{l}b=90\\ d=2\end{array}\right| u=\frac{162}{9}=18.$$

Therefore, $x=18+4=22$.
And, $y=\frac{18+2}{2}=10$.
And, $z=18\times 2+2\times 2=40$.
But, $18+2=20$.
$22-2=20$.
$10\times 2=20$.
$2)\,40\,(=20$.
And, $18+22+10+40=90$.

SOLUTION of PROBL. 9.

There are Two Numbers, whoſe Sum is Equal to the Difference of their Squares, and if the Sum of the Squares of thoſe Two Numbers be Subſtracted from the Square of their Sum, the Remainder will be 60, or, b.

Number given. b.
Greater Number Sought. x.
Leſſer. y.
The Queſtion in Algebraick Terms will be this.
If $xx-yy=x+y$,
And $xx+2xy+yy-xx-yy=b$.
What are x and y?
2*d* Equation. $2xy=b$,
By Reduction. $xy=\frac{1}{2}b$.
1*ſt* Equation Divided by $x+y$ gives $x-y=1$.
Therefore, $x=y+1$.
And, $yy+y=\frac{1}{2}b$.
Or, $yy=-y+\frac{1}{2}b$.
By Extraction,
$y=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+\frac{1}{2}b}$.
$b=60.\ y=-\frac{1}{2}\pm\sqrt{30\frac{1}{4}}$.
$=-\frac{1}{2}\pm\sqrt{\frac{121}{4}}=-\frac{1}{2}\pm\frac{11}{2}$.
That is, $y=-\frac{1}{2}\pm 5\frac{1}{2}=5$.
Or, -6.

$$\left.\begin{array}{r}x=y+1=5+1\\ \text{Or,}\quad -6+1\end{array}\right|\begin{array}{l}=6.\\ =-5.\end{array}$$

6 the Greater Number, and 5 the Leſs will Solve the Problem.
For, $xx-yy=36-25=x+y=6+5=11$.
And $2xy=b=12\times 5=60$.
Alſo, -5 the Greater Number, and -6 the Leſs, will Solve the Problem.

$$\text{For,}\quad \left.\begin{array}{r}xx=25\\ -yy=-36\end{array}\right|\begin{array}{l}=-11=x+y,\\ =-5-6.\end{array}$$

And $2xy=+60=xx+2xy+yy-xx-yy=25+60+36-25-36$.

I Generally take Notice of Privative, as well as Poſitive Roots, and of Impoſſible Ones, as well as Poſſible, becauſe they are all of them Equally Affirmative and Poſſible in the Solution of Problems, there being no Diſtinction betwixt the One and the Other, in that Reſpect, and that which makes the Diſtinction, as I have Obſerved

ved before, is, that the Poſitive and Poſſible Roots are Serviceable to us in the Buſineſs of the World, and therefore the Other are Neglected; Whereas in Nature, Reaſon, and Philoſophy, and in the Relations of Things, the Privative, and Impoſſible Roots, are as Valuable, as the Other, and will as fully Solve all the Conditions of the Problem, and may be in Reality as Perfectly and Truly Extracted, whatever Algebraiſts may at preſent Imagine; A Poſitive Quantity or, +, being a Sign of an Expanſive Force, and a Privative, or −, of a Contractive, and which are both Equally Poſitive Forces in Nature; They are alſo Signs of Credit, or Debt, of Progreſſion and Regreſſion, *&c.* From whence it Follows, That they are only Signs of Quantities of different Kinds, or Denominations, and therefore, as $\sqrt{+100}=+10$, ſo the $\sqrt{-100}=\sqrt{100}=10$, whoſe Denomination is $-,=-10$; That is, it is the Root of a Quantity, or Sum, which is Contractive, which is Owing, or is Regreſſive, *&c.* as the Other is of a Quantity, or Sum, which is Expanſive, which is Due, or is Progreſſive; Nor in this Caſe, is $-10\times-10=+100$, as is Taught by Algebraiſts, but $=-100$; Becauſe, if we Multiply Quantities of the ſame Kind and Denomination into one Another, they will ſtill Continue of the ſame Kind and Denomination, and will not Change into Different Ones. Notwithſtanding which, $-10\times-10=-100$, where the Negation of the Negation of Quantities of the ſame Kind or Denomination is only Conſidered, as is done in Common Algebra.

SOLUTION OF PROBL. 10.

To Divide a given Number, 14, or b, into Three Continual Proportionals, Such, That If the ſaid given Number be Divided ſeverally by One of the ſaid Three Proportionals, the Sum of the Three Quotients may be Equal to $12\frac{1}{4}$, or d.

The Firſt of the Three Proportionals, x.

The Second or Middle. y.

The Third will be therefore. $\frac{yy}{x}$.

Sum of the Three Proportionals will be by the Problem. $x+y+\frac{yy}{x}=b$.

And by Reduction. $xx+xy+yy=bx$.

Alſo, according to the Problem, it will be. $\frac{b}{x}+\frac{b}{y}+\frac{bx}{yy}=d$.

By Reduction. $by^3+by^2x+byx^2=dy^3x$.

Which Divided by y Leaves. $by^2+byx+bx^2=dy^2x$.

And $b\times yy+xy+xx=bx$ Gives. $by^2+byx+bx^2=bbx$.

Therefore, 1*ſt* Equation $=2d$. $dy^2x=b^2x$.

And by Dividing by x. $dy^2=b^2$.

And by Dividing by d. $y^2=\frac{b^2}{d}$.

$b=14$, $d=12\frac{1}{4}$ | By Extraction, $y=\sqrt{\frac{b^2}{d}}=\sqrt{16}=4$.

But, by a Former Equat. $xx+xy+yy=bx$.

That is, ſince $y=4$, $xx+4x+16=14x$.

And by Tranſpoſition. $10x-xx=16$.

Or, by Reduction. $xx=10x-16$.

And, by Extraction. $x=5\pm\sqrt{25-16}$.

Therefore. $x=5\pm3=8$, Or, 2.

Problems the 10th. and 11th.

SOLUTION OF PROBL. 11.

To Find Three Numbers in Arithmetical Progreſſion, Such, that if the Firſt be Multiplied by 1, the Second by 2, the Third by 3, the Sum of the Products may be 62, and that the Sum of the Squares of the 3 Numbers may make 275.

Numbers Sought, $x.\ y.\ z.$ and let x be the Firſt and Smalleſt Term, $62=b$, $275=d$.

Therefore in Algebra, the Problem will be thus Expreſſed.

If, by Arithmetical Progreſſion, $x+z=y+y$, or $y-x=z-y$, x being the leaſt Term.

And Sum of the Products. $x+2y+3z=b$.

And Sum of the Squares. $xx+yy+zz=d$.

What are the Numbers $x.\ y.\ z$?

By Arithm. Progr. $x+z=2y$, and $\frac{1}{2}x+\frac{1}{2}z=y$.

Their Squares, $\frac{1}{4}xx+\frac{1}{2}xz+\frac{1}{4}zz=yy$.

And ſince, $x+z=2y$, and $x+2y+3z=b$,

Therefore, $2x+4z=b$, and $x+2z=\frac{1}{2}b$.

And $\frac{1}{2}b-2z=x$, and $\frac{1}{4}bb-2bz+4zz=xx$.

And by Subſtituting theſe Two laſt Equations in $\frac{1}{4}xx+\frac{1}{2}xz+\frac{1}{4}zz=yy$, it will be, $\frac{1}{16}bb-\frac{1}{4}bz+\frac{1}{4}zz=yy$; But, $zz=zz$,

Therefore, $\frac{5}{16}bb-2\frac{1}{4}bz+5\frac{1}{4}zz=xx+yy+zz=d$, and $5\frac{1}{4}zz=2\frac{1}{4}bz-\frac{5}{16}bb+d$.

And $zz=\frac{2\frac{1}{4}b}{5\frac{1}{4}}z-\frac{\frac{5}{16}}{5\frac{1}{4}}bb+d$, or $pz-q$.

By Extraction. $z=\frac{1}{2},\frac{2\frac{1}{4}b}{5\frac{1}{4}}\pm\sqrt{\frac{1}{4}pp-q}$

$=13\frac{2}{7}\pm\frac{2}{7}=13\frac{4}{7}$.

Or, 13.

Conſequently, $y=9$, or $8\frac{5}{7}$.

And therefore, $x=5$, or $3\frac{6}{7}$.

But 5. 9. 13 are in Arithmetical Progreſſion.

And, $\frac{yy}{x}=\frac{16}{8}=2$, Or, $\frac{16}{2}=8$.

If, $x=2$. $y=4$, $\frac{yy}{x}=8$. And 2. 4 : : 4. 8.

$\frac{14}{2}=7$. $\frac{14}{4}=3\frac{2}{4}$. $\frac{14}{8}=1\frac{3}{4}$. $7+3\frac{2}{4}+1\frac{3}{4}=12\frac{1}{4}$.

If $x=8$. $y=4$. $\frac{yy}{x}=2$. And, 8. 4 : : 4. 2.

The Products. $5+2\times9+3\times13=62$.
The Squares. $25+81+169=275$.
And the ſame Obtains in the other Numbers.
For $3\frac{6}{7}$. $8\frac{5}{7}$. $13\frac{4}{7}$ are in Arithmetical Progr.
The Products, $1\times3\frac{6}{7}+2\times8\frac{5}{7}+3\times13\frac{4}{7}=62$.
The Squares, $=242+31\frac{3}{7}+1\frac{4}{7}=275$.

Concerning the Reſolution of Queſtions Capable of Innumerable Anſwers.

§. 11. As to the Reſolution of Queſtions, Capable of Innumerable Anſwers, by Monſieur *Bachet* in his Problems, Plaiſans and Delectables, qui ſe font par les Nombres, Printed at *Lyons* 1624, The Account is this; If, after a Problem or Queſtion is Reduced to it's ſeveral Equations by the Methods above, there are as many Equations, as there are Quantities Sought, then a Queſtion has a certain Determinate Number of Anſwers; But, whenſoever a Queſtion affords not as many Equations Independent of One Another, as there are Quantities Sought, it is Capable of Innumerable Anſwers.

Queſtions of which latter Kind are very Pleaſant and Delightful, but ſometimes very hard to be Reſolved, Eſpecially, when all the Anſwers in whole Numbers, that a Queſtion is Capable of, are Deſired.

Propos. 1. Two whole Numbers, Primary between themſelves, being Given, that is, whoſe Common Diviſor is 1, as 12 and 5, 20 and 21, 7 and 3, &c. To Find out Two others, Suppoſing x and y, that if x be Multiplied by the Greater of the Two given Numbers, and to the Product there be Added a given whole Number, the Sum ſhall be Equal to the Product of y Multiplied by the Leſſer of the Two Numbers firſt Given; Moreover, To Find out all the whole Numbers, x and y, which ſhall Produce the ſame Effect.

Probl. 1. Let $9x+6$ be $=7y$; Conſequently, 9 and 7 are the Two Primary Numbers given, whoſe Common Diviſor is 1, the given Number Added is 6, and all the Poſſible Values of x and y, in whole Numbers, are Sought.

Therefore This is very Different from all the Preceding Algebra, where Equations are given, from which may be Deduced all the Unknown Quantities, according to the Methods before Preſcribed, ſince here is only One Equation given, and Two or more Quantities are Sought.

Reſolution.	$\overline{1\times9}+6=15$	$1\times7=7$
	$\overline{2\times9}+6=24$	$2\times7=14$
	$\overline{3\times9}+6=33$	$3\times7=21$
	$\overline{4\times9}+6=42$	$4\times7=28$
	$\overline{5\times9}+6=51$	$5\times7=35$
	$\overline{6\times9}+6=60$	$6\times7=42$
	$\overline{7\times9}+6=69$	

Therefore in the 1*ſt* Column, If $x=1$, $9x+6=15$, If $x=2$, $9x+6=24$, &c. If $x=4$, $9x+6=42$. Thus alſo in the 2*d* Column, If $y=1$, $7y=7$, If $y=2$, $7y=14$, &c. If $y=6$, $7y=42$; Conſequently $x=4$, and $y=6$, will Solve the Equation. Becauſe $\overline{4\times9}+6=42$, and $6\times7=42$.

Having found the Two ſmalleſt Terms in whole Numbers, Namely, $x=4$ and $y=6$.

Then $4+7=11$. $4+\overline{2\times7}=18$. $4+\overline{3\times7}=25$, &c. will any of them Repreſent x.
And $6+9=15$. $6+\overline{2\times9}=24$. $6+\overline{3\times9}=33$, &c. will any of them Repreſent y.
That is, $x+$ any Multiple of the Coefficient of y, and $y+$ any Multiple of That of x.

Becauſe the Equation is $9x+6=7y$; If therefore $x=4$, and $y=6$, will Solve the Equation, $9\times\overline{4+7}+6=7\times\overline{6+9}$, $=105$, will likewiſe do it; And ſo of the Reſt.

Another Way of Solving the foregoing Propoſition.

Probl. 2. $8x+97=5y$. What are all the Poſſible Values of x and y?
Therefore 8 and 5 are Two Primary Numbers, whoſe Common Diviſor is 1; And 97 is the given Number Added.

Reſolution.
1. $8+97=105$. That is, the Coefficient of $x+$ the Given Number.
2. $\frac{105}{5}=21=y$. That is, the Former Sum Divided by the Coefficient of y.
3. Therefore $1=x=8)\,\underline{5y}-97=8)\,\underline{105}-97=8)\,8\,(=1$.

Values of x. 1. $1+5=6$. $1+\overline{2\times5}=11$. $1+\overline{3\times5}=16$. &c.
Values of y. 21. $21+8=29$. $21+\overline{2\times8}=37$. $21+\overline{3\times8}=45$. &c.

PROBL. 3. If $49x+6=13y$. What are all the Poſſible Values of x and y?

Reſolution.

1. $49+6=55$. That is, the Coefficient of x + the Given Number.
2. The Former Sum Divided by the Coefficient of y, *viz.* $\frac{55}{13}$, Produces no whole Number, and therefore not Reſolvible into the 2*d* Problem.
3. But $\frac{55}{13}=4\frac{3}{13}$, therefore $4\times13=52$ is Leſs than 55, and $5\times13=65$ is 10 more, which next Multiple of 13, *viz.* 5×13 is to be taken, with the Exceſs of 10 to be Subſtracted out of it; Therefore $65-10=55$.
4. But 49, Prefixed to x, Divided by $13=3\frac{10}{13}$, Therefore $3\times13=39$, and $39+10=49$.
5. Since then, there is 10 Wanting in the Former Equation to make an Exact Multiple of 13, and in this Latter, 10 is to be Added to do it; If theſe Two Equations are Added together, a Number will be Produced, which is an Exact Multiple of it.

$$\left.\begin{array}{l|l}65-10&=55\\39+10&=49\end{array}\right|=104.$$

6. Therefore $\frac{104}{13}=8=y$, and $\frac{104-6}{49}=2=x$. That + 10, and − 10 ſhould ſo Exactly Deſtroy each other is Accidental, and may not Obtain, unleſs in this Example.

Values of x, 2. $2+13=15$. $2+\overline{2\times13}=28$. $2+\overline{3\times13}=41$, &c.
Values of y, 8. $8+49=57$. $8+\overline{2\times49}=106$. $8+\overline{3\times49}=155$, &c.

PROBL. 4. If $82x+66=13y$. What are all the Poſſible Values of x and y?

Reſolution.

1. $82+66=148$. $\frac{148}{13}=11\frac{5}{13}$, therefore $11\times13=143$ will be Leſs than 148; But $12\times13=156$, and Conſequently, $148=156-8$.
2. Alſo $\frac{82}{13}=6\frac{4}{13}$, Therefore $82=\overline{6\times13}+4=78+4$.
3. Since $4=\frac{1}{2}$ of 8, Multiply $78+4$ by the Denominator of the Fraction 2, Therefore $2\times\overline{78+4}=164=156+8$.
4. $$\left.\begin{array}{l}148=\overline{156-8}\\2\times82=\overline{78+4}\times2=156+8\end{array}\right|\text{Therefore }\left.\begin{array}{r}148\\+2\times82\end{array}\right|=312=\begin{array}{l}156-8.\\+156+8.\end{array}$$
5. $\frac{312}{13}=24=y$. $\frac{312-66}{82}=3=x$.

Values of x. 3. $3+\overline{1\times13}=16$. $3+\overline{2\times13}=29$, &c.
Values of y. 24. $24+\overline{1\times82}=106$. $24+2\times82=188$, &c.

PROBL. 5. If $121x+5=93y$. What are all the Poſſible Values of x and y?

Reſolution.

1. $126=121+5$. $\frac{126}{93}=1\frac{33}{93}$, Therefore 1×93 is Leſs than 126, but $2\times93=186$, and $186-60=126$.
2. $\frac{121}{93}=1\frac{28}{93}$, Therefore, $1\times93=93$, and $93+28=121$.
3. Suppoſe. $93z+60=28s$. $z=?$ $s=?$
4. $93+60=153$. $\frac{153}{28}=5\frac{13}{28}$, Therefore $5\times28=140$ is Leſs than 153, but $6\times28=168$, and $168-15=153$.
5. $\frac{93}{28}=3\frac{9}{28}$, Therefore $3\times28=84$, and $84+9=93$.
6. Suppoſe. $28q+15=9r$; $q=?$ $r=?$
7. $28+15=43$. $\frac{43}{9}=4\frac{7}{9}$; 4×9 too Little, but $5\times9=45$, and $45-2=43$.
8. $\frac{28}{9}=3\frac{1}{9}$, and $3\times9=27$. $27+1=28$. $2\times\overline{27+1}=54+2$.
9. $54+2+45-2=99$.

By Regreſſion to Art. 3. 4. 5.	10. $\frac{99}{9}=11=r.$ $\overline{11\times93}+153=1176.$ $\frac{1176}{28}=42=s.$
Art. 10. 1. and 2. And by the Equation.	11. $\overline{42\times121}+126=5208.$ $\frac{5208}{93}=56=y.$
	$\frac{5208-5}{121}=43=x.$

Values of *x*. $43.$ $43+93=136.$ $43+\overline{2\times93}=229,$ *&c.*

Values of *y*. $56.$ $56+121=177.$ $56+\overline{2\times121}=298,$ *&c.*

Propos. 2. To Divide a Given Number into Three or more Numbers, Such, that if every One of them be Multiplied by a different Number Given, the Sum of the Products may be Equal to a Given Number, But the Sum of thoſe Products muſt Fall between the Two Products made by Multiplying the Given Dividend into the Greateſt and Leaſt of the Given Multiplicators.

Probl. 1. To Divide 24 into Three ſuch whole Numbers, that if the Firſt be Multiplied by 36, the Second by 24, and the Third by 8, the Sum of the Three Products may make 516.

Let the Numbers Required be Repreſented by *x, y, z*.

Reſolution.	1. Therefore the Three whole Numbers.	$x+y+z=24.$
	2. And the Products Required.	$36x+24y+8z=516.$
	3. Firſt Equation Multiplied into 36.	$36x+36y+36z=864.$
	4. Second Equation Subſtracted from the Third makes.	$*+12y+28z=348.$
	5. Fourth Equation by Tranſpoſition makes.	$+12y=348-28z.$
	6. Fifth Equation Divided by 12 makes.	$y=29-\frac{7}{3}z.$
	7. If Inſtead of *y* in the Firſt Equation $29-\frac{7}{3}z$ be Subſtituted, it will be.	$x+29-\frac{7}{3}z+z=24.$
	8. That is, by Tranſpoſition.	$x=24-29+\frac{4}{3}z.$
	9. That is.	$x=\frac{4}{3}z-5.$
	10. Therefore. If $<$, is made to be a Sign of Greater.	$\frac{4}{3}z<5.$
	11. And.	$4z<15$
	12. And.	$z<3\frac{3}{4}.$
	13. By the Sixth Equation. If $>$ is made to be Sign of Leſs.	$\frac{7}{3}z>29.$
	14. Therefore.	$7z>87.$
	15. And.	$z>12\frac{3}{7}.$

The whole Number *z* therefore, to be found, muſt be Greater than 3, but not Greater than 12, for whole Numbers are only Required in this Problem; And, ſince *x* and *y* muſt be whole Numbers likewiſe, *z* muſt be ſuch, that if it be Multiplied into $\frac{7}{3}$ in the Sixth Equation, or into $\frac{4}{3}$ in the Ninth Equation, it muſt Produce a whole Number, which therefore can be no other than 6. 9. and 12, and Conſequently the Numbers will ſtand in the following Order.

	x.	*y.*	*z.*	
x and *y* are found out by the 6*th.* and 9*th.* Equations.	3	15	6	$3+15+6=24.$ $36\times3+24\times15+8\times6=516.$
	7	8	9	$7+8+9=24.$ $36\times7+24\times8+8\times9=516.$
	11	1	12	$11+1+12=24.$ $36\times11+24\times1+8\times12=516.$

Probl. 2. If One would lay out 98 Pence to Buy 40 Birds, Suppoſe Partridges, Larks, and Quails; How many of Each Kind may be Bought, when Partridges are at 3 Pence a Piece, Larks at an Half-Penny a Piece, and Quails at 4 Pence a Piece.

Reſolution.

1. x the Number of Partridges, y the Number of Larks, z the Number of Quails, Therefore, $x+y+z=40$.

The Coſt of the Partridges $3x$, of the Larks $\frac{1}{2}y$, and of the Quails $4z$.

2. Therefore. $3x+\frac{1}{2}y+4z=98$.
3. The firſt Equation Multiplied by 3 makes. $3x+3y+3z=120$.
4. The Second Equation Subſtracted from the Third. $\frac{5}{2}y-z=22$.
5. The Fourth Equation by Tranſpoſition makes. $z=\frac{5}{2}y-22$.
6. Subſtitute the Value of z in the laſt Equation, for z in the Firſt. $x+y+\frac{5}{2}y-22=40$.
7. And by Reduction. $x=62-\frac{7}{2}y$.
8. By the 5*th* Equation, $z=\frac{5}{2}y-22$, Therefore, $\frac{5}{2}y<22$.

Alſo. $5y<44$.

And. $y<8\frac{4}{5}$.

9. By the 7*th* Equation, $x=62-\frac{7}{2}y$, Therefore, $\frac{7}{2}y>62$.

Alſo. $7y>124$.

And. $y>17\frac{5}{7}$.

And ſince the Numbers Required muſt be whole Numbers, y muſt be an Even Number, becauſe 5 or 7 into an Odd Number would Produce an Odd Number, and Divided by 2 would make a Fraction in the 5*th* and 7*th* Equations; y muſt alſo be Greater than 8, and Leſs than 17, betwixt which the even Numbers will be 10. 12. 14. 16. which will Solve the Queſtion.

x and z are found out by the 5*th* and 7*th* Equations.

Partr.	Larks.	Quails.		
x.	y.	z.		
27	10	3	$27+10+3=40$.	$3\times27+\frac{1}{2}\times10+4\times3=98$.
20	12	8	$20+12+8=40$.	$3\times20+\frac{1}{2}\times12+4\times8=98$.
13	14	13	$13+14+13=40$.	$3\times13+\frac{1}{2}\times14+4\times13=98$.
6	16	18	$6+16+18=40$.	$3\times6+\frac{1}{2}\times16+4\times18=98$.

Probl. 3. A Vintner having 3 Sorts of Wines, the Prices of which are 24 Pence, 22 Pence, and 18 Pence *per* Gallon, Deſires to make a Mixture out of them, that may Contain 60 Gallons, in ſuch Manner, that the Total Mixture being Sold at ſome Mean Price *per* Gallon, between 24 Pence and 18 Pence, Suppoſe at 20 Pence, may make the ſame Sum of Money, as all the Particular Quantities of Wine in the Mixture at their own Prices would do; What Quantity of each Sort of Wine is to be taken to make that Mixture?

Number of Gallons of the Firſt Sort of Wine x, of the Second y, of the Third z.

Reſolution.

1. Therefore. $x+y+z=60$.
2. And. $24x+22y+18z=60\times20=1200$.
3. Multiply the Firſt Equation by 24, it will be. $24x+24y+24z=1440$.
4. Subſtract the Second from the Third. $2y+6z=240$.
5. Therefore by Tranſpoſition. $2y=240-6z$.
6. And. $y=120-3z$.
7. Subſtitute the Value of y in the laſt Equation, for y in the Firſt, it will be. $x+120-3z+z=60$.
8. And by Reduction and Tranſpoſition. $x=2z-60$.
9. By the laſt Equation. $2z<60$, and $z<30$.
10. And by the Sixth Equation. $3z>120$, and $z>40$.

x and y are found by the 6*th* and 8*th* Equations.

x.	y.	z.		
2	27	31	$2+27+31=60$.	$24\times2+22\times27+18\times31=1200$.
4	24	32	$4+24+32=60$.	$24\times4+22\times24+18\times32=1200$.
6	21	33	$6+21+33=60$.	$24\times6+22\times21+18\times33=1200$.
8	18	34	$8+18+34=60$.	$24\times8+22\times18+18\times34=1200$.
10	15	35	$10+15+35=60$.	*&c.*
12	12	36		
14	9	37		
16	6	38		
18	3	39		

The Rule of Alligation in Vulgar Arithmetick finds out only one Anſwer, which is the 6*th*, Whereas by the Table Above, there are 9 Anſwers in whole Numbers, and in Fractions there may be Innumerable Ones; For z may be Equal to $30\frac{1}{2}$, *&c.*

Concerning Mr. Raphſon's Analyſis Æquationum.

§. 12. Mr. *Raphſon's Analyſis Æquationum*, or his General Way of Extracting the Roots of Equations, whether Simple or Affected, by the Infinite or Converging Series is Derived from the *Exegetice*, or *Exegeſis Numeroſa* of *Vieta*, and *Harriot*, and to which Series, Thoſe of Sir *Iſ. Newton*, *Lagney*, and Dr. *Halley* very much Correſpond; An Explanation of Mr. *Raphſon's* I have here Subjoined.

Let the Equation propoſed, whoſe Root is to be Extracted be $ba-a^3=c$, in which, according to *Harriot*, a is the Unknown Quantity.

Rul. 1. Divide the Unknown Quantity a into Two Parts g and x, the former of which is Known, the other Unknown, therefore $g+x=a$.

And $g^3+3g^2x+3gx^2+x^3=a^3$. That is, their Cubes will be alſo Equal.

And $bg+bx=ba$. Which Conſtitutes the other Part of the Equation.

$$\left.\begin{array}{r}\text{Therefore, } bg+bx-g^3-3g^2x-3gx^2-x^3\\ \text{Or, } bg-g^3+\overline{b-3g^2}\times x-3gx^2-x^3\end{array}\right|=ba-a^3=c.$$

Rul. 2. Reject from this New formed Equation all thoſe Terms, in which any of the Powers of x are, whether by themſelves, or Involved with the known Quantity, and Retain only thoſe Terms, in which the Simple Quantity x is found, from whence will Ariſe this Equation, which is Named the Converging one; *viz.* $bg-g^3+\overline{b-3g^2}\times x=c$, and $c-bg+g^3=\overline{b-3g^2}\times x$.

Rul. 3. From the foregoing Simple Equation, in which one Unknown Quantity x is only Found, is Derived the following Equation; Namely, $x=\frac{c+g^3-bg}{b-3g^2}$, which is the Theorem Fundamental to this Method, and is alſo Termed a Converging One; Becauſe, tho' it will not at Firſt give the True Value of x, yet by Aſſuming continually New Values of g, the Unknown Quantity x will be Produced nearer to the Truth, by which Means there will be a nearer Approximation in Infinitum to the Quantity or Value Sought $a=g+x$.

Rul. 4. Having found out by the Precedeing Equation the Value of x, ſince a may be Equal to $g+x$ or $g-x$, according to the Signs + or −, Add or Subſtract the Value of x to or from g, and let the Sum or Remainder be called g, therefore this New Value of $g. \pm x$ is now Suppoſed $=a$, and $x=$ to the Quantity ſet down before in Rule 3*d.* putting always the New Value of g for g in that Equation.

So likewiſe by a Third Operation, a New Value of g will ariſe = to the laſt $g\pm x$, and this New Value of $g\pm x$ is now Suppoſed $=a$, and ſo on in Infinitum, but a Third or a Fourth Operation will be in General Sufficient to Anſwer any Queſtion Propoſed, or Exhibit any Root to be Extracted.

Rul. 5. To Find out the firſt Term g, let the Equation be Diſtinguiſhed by Points, that is, both the Power, which is to be Reſolved, and the Coefficients in the Manner Practiced by *Vieta*, *Harriot*, and *Oughtred*, as in the Equation of Problem 22.

. . .   .

$$-a^4+323609,663689a^2+4228931,08508785 2a=22540483202,613561987516,$$

where the Reſolvend is Pointed as a Biquadratick, becauſe it is an Equation of the 4*th.* Power, the Coefficient $\times a$ as a Cubick, becauſe $a\times$ the Cube is = to a Biquadratick, the Coefficient $\times a^2$ as a Square, becauſe $a^2\times$ the Square is = alſo to a Biquadratick; For all the Terms of an Equation are Conſidered, as Conſiſting of the ſame Power, ſince otherwiſe they could not, if they were of Heterogeneous Denominations, be Added to, or Subſtracted from one another.

And after the Equation is thus Pointed, let the Numbers be taken, which belong to every firſt Point, as if the Equation had no more to Examine, as is done in the 8*th.* Problem, and from thence, let the firſt Simple Side or Root be Extracted, to which let there be ſo many Cyphers Added, as there Remain Points in the Reſolvend, as is done in the 5*th.* Problem, and afterwards let the Operation Proceed according to the foregoing Rules; g may be Obtained by an Eaſy Conjecture, or by the Limits of Equations, or by Logarithms, See *Oughtred*'s *Clavis Mathemat. De Æquationum Affectarum Reſolutione in Numeris.*

From Various Quantities being put for g, Various Roots of the ſame Equation may be found out, as in Problem 15, 16.

Rul. 6. Since $x=\frac{c+g^3-bg}{b-3g^2}$, by Rule 3*d*, and $g+x=a$, by Rule 1*ſt*, Therefore, $\frac{g+c+g^3-bg}{b-3g^2}=a$ Converging $=\frac{bg-3g^3+c+g^3-bg}{b-3g^2}=\frac{c-2g^3}{b-3g^2}=a$.

From whence it Follows, if *g* Converging is = to *a* Converging Exactly, the Solution will likewiſe be Exactly True, that is, $a=\frac{c-2a^3}{b-3a^2}$, or, $b-3a^2)\ c-2a^3\ (=a$; For $a\times\overline{b-3a^2}$ and Subſtracted from $c-2a^3$ makes $c-ba+a^3=0$, if $\frac{c-2a^3}{b-3a^2}$ is $=a$ without a Remainder, but $c-ba+a^3$ is $=0$ by the Equation, or $ba-a^3=c$, and Conſequently $g+x=a=g$ Converging.

Demonſtration of this Method.

Let any Quantity be taken as $g=$ firſt Member or Part of the Root; It is Affirmed, that this Quantity, be it what it will, according to the Method before ſet down, does Converge to the True Root; Let the Equation Propoſed be Reſumed, $ba-a^3=c$.

$g+x=a$, or, if *g* is Greater than *a*, $g-x=a$.

Therefore, $bg-g^3+\overline{b-3g^2}\times x-3gx^2-x^3=c$,

And $x=\frac{c+g^3-bg}{b-3g^2}+\frac{3gx^2+x^3}{b-3g^2}$.

Put $t=\frac{c+g^3-bg}{b-3g^2}$, and $m=\frac{3gx^2+x^3}{b-3g^2}$, and there will be Eight Different Caſes, according to the Variation of the Signs + and −; That is, $.x=.t.m$ is Termed an Abſolute Theorem, and the Quantity *x* in this Caſe is Abſolute in Diſtinction from $x=t$, which is Converging.

$+x=+t+m$
$-x=-t-m$
$+x=+t-m$
$-x=-t+m$
$+x=-t+m$
$-x=+t-m$
$+x=-t-m$
$-x=+t+m$

From what has been Premiſed it is Evident.

1*ſt*. That in $x=t+m$ (which is the Caſe of $ba-a^3=c$) the Converging *x* is Leſs than the Abſolute *x* by the Quantity *m*, that is, *t* is Leſs than $t+m$, the Part than the Whole, if therefore the Converging $x=t$ be Added to *g*, 'tis Manifeſt that this New *g*, which Conſiſts of the Former *g* with the Converging *x* or *t* Added to it, is Greater than the Former *g*, by the Addition of *x* or *t*, and that by this Means *g* will be Continually Increaſed, and *m* Diminiſhed in Infinitum, and that after an Infinite Convergency to the True Root, the Difference of the Root found out by Extraction from the Real One, will be Leſs than any Aſſignable Quantity.

2*d*. $x=t-m$, or $aa=c$, therefore $g^2+2gx+x^2=c$, and $\frac{c-g^2}{2g}-\frac{x^2}{2g}=x=t-m$; In this Caſe it is Manifeſt, that the Converging $x=t$ is Greater than the Abſolute $x=t-m$ by the Quantity *m*, and Conſequently, if the Converging *x* is Added to *g*, it will make a Quantity Greater than *a*, and from thence $-gg$ Greater than *c*, by which Means $x=t-m$ will be Changed into the Second Form, $-x=-t-m$ and will Converge by Deſcending or Going from a Higher Quantity to a Lower.

3*d*. By the ſame Way of Reaſoning, all the other Caſes may be Reduced to the Two Firſt, and therefore will all Converge, Namely, all that have $+m$ may be Reduced to the firſt Caſe, and all which have $-m$ to the Second, but the four laſt Caſes are not Poſſible, unleſs, by Changing their Signs, they are Converted into the Four Firſt.

A Second and more Particular Demonſtration of the Foregoing Method.

Propos. 1. Let the Equation Propoſed be $a^3=b$, and let *z* be $=x$ the Abſolute Quantity, and not the Converging, and let any Quantity *g* be taken Greater than *a*, 'tis Affirmed, that the next *g*, which Ariſes, by Proceeding in the Method before Delivered, is always Leſs than the Precedeing, but Greater than *a*, and therefore does Converge to the Truth; $g-z=a$ by the Hypotheſis, and $g^3-3g^2z+3gz^2-z^3=a^3=b$, Therefore, $-3g^2z+3gz^2-z^3=b-g^3$, and $-z+\frac{3gz^2-z_3}{3g^2}=\frac{b-g^3}{3g^2}=-x$ the Converging, Therefore, $-z+\frac{3gz^2-z^3}{3g^2}=-x$, let *g* be Added to both Sides of the Equation, and

$g-z+\frac{3gz^2-z^3}{3g^2}=g-x=$ to a New g, and therefore this New g is Leſs than the Precedeing, but $a+\frac{3gz^2-z^3}{3g^2}=$ to the New g, becauſe $a=g-z$, and therefore this New g is Greater than a, that is, the Whole, than the Part. *Q. E. D.*

Propos. 2. Let the Equation Propoſed be $ba-a^3=c$, and Let g be any Quantity Aſſumed Leſs than a. 'Tis Affirm'd, that the next g which Ariſes from the Method of Proceeding before Delivered, is always Greater than the Precedeing, but Leſs than a, and therefore Converges to the Truth, $g+z=a$ by the Hypotheſis, and $bg-g^3+\overline{b-3g^2}\times z-3gz^2-z^3=ba-a^3=c$, therefore $\overline{b-3g^2}\times z-3gz^2-z^3=c+g^3-bg$, and Conſequently, $z-\frac{3gz^2+z^3}{b-3g^2}=\frac{c+g^3-bg}{b-3g^2}=+x$ the Converging; From whence $z=x+\frac{3gz^2+z^3}{b-3g^2}$ Add g to both Sides of the Equation, and $g+z=g+x+\frac{3gzz+z^3}{b-3g^2}=a$; But the New $g=g+x$ is Greater than the Precedeing g by the Quantity x, but Leſs than a, by the Quantity $\frac{3gzz+z^3}{b-3g^2}$, the Part, than the Whole. *Q. E. D.*

Corol. 1. In Equations, where the Higheſt Power is Negative, if g is Leſs than a, it Aſcends in it's Approach to the True Root.

Corol. 2. In Extracting Pure or Simple Roots, if g is Leſs than a, in the Firſt Operation it Aſcends above the Root, and afterwards Converges by Deſcending.

But does not any one See, who Knows the Nature of Infinites, notwithſtanding theſe Demonſtrations, that Approximations may be Infinitely made to a True Root of the Equation, and yet after all may be Infinitely Diſtant from it? One may Approximate Infinitely to the Real Truth of a Line, but yet fall Infinitely ſhort of the Real Truth of a Surface, a Solid, Surſolid, *&c.* and ſo in Numbers, $\frac{12}{13}$ will come nearer to Unit than $\frac{12}{14}$, or $\frac{12}{15}$, but $\frac{12}{14}$, or $\frac{12}{15}$ Approximate nearer to Unit than $\frac{12}{16}$, or $\frac{12}{17}$, *&c.* How therefore is it Proved, that theſe Approximations are the Neareſt and Juſteſt, which can be made? For an Approximation in General to Infinit, can never Signify any Thing to the finding out what the Real Truth is, where the Infinities are Different and Various.

Problem. 1*ſt.* of Mr. *Raphſon* is the Extraction of the Root of 2, which here Follows.

Let $g+x=a$ Equation $aa=c$
In Numbers $aa=2$.

Theorem $x=\frac{c-gg}{2g}$

$$\begin{array}{l}
g=1 \\
c-gg=1 \\
2g=\underline{2)\ 1.0\ (+.5}=x \\
g=1.5 \\
c-gg=-,25 \\
2g=\underline{3)-,25(-},083=x \\
\quad 1.500 \\
\quad \underline{-.083} \\
g=1.417 \\
c-gg=-,007889 \\
2g=2.83\underline{4)-,007889(-},002783=x \\
\quad 1.417 \\
\quad \underline{-.002783} \\
g=1.414217 \\
c-gg=-,000009723089 \\
2g=2.8284\underline{34)-,000009723089}(-,000003437622=x \\
\quad 1.414217 \\
\quad \underline{-.000003437622} \\
a=1.414213562378
\end{array}$$

Concerning Dr. Halley*'s Method of Finding out the Roots of any Equation whatſoever.*

§. 13. Dr. *Halley*'s Method of Finding out the Roots of any Equations whatſoever, without a Previous Reduction of them, is as Follows.

Mr. *Raphſon* in the Year 1690, Publiſhed his Univerſal *Analyſis* of Equations before Explained, upon which, as it is Probable to Think, Monſieur *de Lagney* Profeſſor of the Mathematicks at *Paris*, Entering into the ſame Argument, Propoſed theſe Two Compendious Rules for an Approximation of the Cube Root, the one Rational, the other Irrational, Namely, $a+\frac{ab}{3aaa+b}$, and the $\sqrt{\frac{1}{4}aa+\frac{b}{3a}}+\frac{1}{2}a$, for the Cube $aaa+b$; And the Root of the Fifth Power, Namely, of a^5+b, he thus Expreſſes, $=\frac{1}{2}a+\sqrt{\sqrt{\frac{1}{4}a^4+\frac{b}{5a}}-\frac{1}{4}aa}$; Which Rules are Demonſtrated from the Geneſis of the Cube, and of the Fifth Power.

Thus, let the Side or Root of a Cube be $a+e$; The Cube, according to the Foregoing Generation of it, will be $aaa+3aae+3aee+eee$; If therefore we Suppoſe aaa to be a Cube Number, the next under any Number given, which is not a Cube, eee will be Leſs than Unit, and the Reſiduum, or Remainder, or b, will be Equal to the other Members of the Cube, $3aae+3aee+eee$, $=3aae+3aee$, becauſe eee is ſo Small, as not to be Worth the Conſideration.

For Inſtance, if $aaa+b=7$, and $a=1$; $aaa+3aae+3aee+eee$, or $aaa+b=1+3e+3ee+eee=7$, if e is Conſidered, as nothing; If $aaa+b=26$, and $a=2$; $aaa+3aae+3aee+eee$, or $aaa+b,=8+12e+6ee+eee=26$, if e is Conſidered as nothing; If $aaa+b=63$, and $a=3$; $aaa+3aae+3aee+eee$, or $aaa+b=27+27e+9ee+eee=63$, if e is Conſidered as nothing, *&c.*

Since alſo aae is much Greater than aee, and ſince b is $=3aae+3aee$, e will be $=\frac{b}{3aa+3ae}$, and therefore Nearly$=\frac{b}{3aa}$; And Conſequently, $\frac{b}{3aa+3ae}=\frac{b}{3aa+\frac{3ab}{3aa}}$ $=\frac{b}{3aa+\frac{b}{a}}=\frac{b}{\frac{3aaa+b}{a}}=\frac{3aaa+b}{a})\frac{b}{1}(=\frac{ab}{3aaa+b}=e$, and therefore the Side or Root of the Cube $a+e=a+\frac{ab}{3aaa+b}$, which is Monſieur *de Lagney*'s Expreſſion of it; If aaa is the Cube Number next above the Number given, the Root or Side of $aaa-b$ will be $=a-\frac{ab}{3aaa-b}$ by a like Reaſoning, as before; For $aaa-3aae+3aee-eee=aaa-b$, and becauſe e is very Small, $-b=-3aae$, and $-e=\frac{-b}{3aa}$ Nearly; But, $\frac{-b}{3aa-3ae}$ Comes ſtill nearer to $-e$, and therefore $\frac{-b}{3aa-\frac{3ab}{3aa}}=-e=\frac{-b}{3aa-\frac{b}{a}}=\frac{3aaa-b}{a})\frac{-b}{1}=\frac{-ab}{3aaa-b}$, and Conſequently, $a-e=a-\frac{ab}{3aaa-b}$; Thus let $aaa-b$ be $=2$, and $a=2$; $\overline{a-e}|^3=8-12e+6ee-eee=2$, If e is Conſidered as Nothing; Let $aaa-b$ be $=9$, and $a=3$; $\overline{a-e}|^3=27-27e+9ee-eee=9$, If e is Conſidered as Nothing; Let $aaa-b$ be $=28$, and $a=4$; $\overline{a-e}|^3=64-48e+12ee-eee=28$, If e is Conſidered as Nothing; Let $aaa-b$ be $=65$, and $a=5$; $\overline{a-e}|^3=125-75e+15ee-eee=65$, If e is Conſidered as Nothing, *&c.*

The Irrational Expreſſion, or Form, of the Cube Root is thus Found out; If $aaa+b$ is the Cube, $3aee+3aae=b$, and $ee+ae=\frac{b}{3a}$, and by Extracting the Root of both Sides of the Equation, $\frac{1}{2}a+e=\sqrt{\frac{1}{4}aa+\frac{b}{3a}}$, and $e=-\frac{1}{2}a+\sqrt{\frac{1}{4}aa+\frac{b}{3a}}$, and $a+e=\frac{1}{2}a+\sqrt{\frac{1}{4}aa+\frac{b}{3a}}$. In the ſame Manner the Root of $aaa-b$ will be $=\frac{1}{2}a+\sqrt{\frac{1}{4}aa-\frac{b}{3a}}$; Or, Or, the First may be Expreſſed by $\frac{1}{2}a+\sqrt{\frac{1}{4}aa+\frac{b-eee}{3a}}$, and the Laſt by $\frac{1}{2}a+\sqrt{\frac{1}{4}aa+\frac{eee-b}{3a}}$.

The Form or Expreſſion for the Root of the Surſolid, or Fifth Power, is thus Inveſtigated $\overline{a+e}|^5=a^5+5a^4e+10a^3ee+10a^2e^3+5ae^4+e^5=a^5+b$, or $b=5a^4e+10a^3e^2+10a^2e^3+5ae^4+e^5=a^5+b$, Or, $b=5a^4e+10a^3e^2+10a^2e^3+5ae^4$, e^5 being Neglected on Account of it's Exceeding Smallneſs; Therefore by Dividing by 5 $\frac{b}{5a}=a^3e+2a^2e^2+2ae^3+e^4$; And by Adding $\frac{1}{4}a^4$ to both Sides of the Equation, $\sqrt{\frac{1}{4}a^4+\frac{b}{5a}}=\sqrt{\frac{1}{4}a^4+a^3e+2a^2e^2+2ae^3+e^4}=\frac{1}{2}aa+ae+ee$; Subſtract $\frac{1}{4}aa$ from both Sides of the Equation, that this Laſt Quantity may be a Perfect Square, and $\frac{1}{2}a+e=\sqrt{\sqrt{\frac{1}{4}a^4+\frac{b}{5a}}-\frac{1}{4}aa}$, and $a+e=\frac{1}{2}a+\sqrt{\sqrt{\frac{1}{4}a^4+\frac{b}{5a}}-\frac{1}{4}aa}=$ to the Root of a^5+b.

And by the ſame Way of Reaſoning $\frac{1}{2}a+\sqrt{\sqrt{\frac{1}{4}a^4-\frac{b}{5a}}-\frac{1}{4}aa}=$ to the Root of a^5-b.

From hence Dr. *Halley* Arrived at his General Method of Reſolving all Equations, of what Power ſoever, by the Following Forms or Expreſſions, both Rational, and Irrational.

$$\sqrt{aa+b}=\sqrt{aa+b},\ \text{Or},\ a+\frac{ab}{2aa+\frac{1}{2}b}$$

$$\sqrt[3]{a^3+b}=\tfrac{1}{2}a+\sqrt{\tfrac{1}{4}aa+\frac{b}{3a}},\ \text{Or},\ a+\frac{ab}{3a^3+\frac{2}{2}b}$$

$$\sqrt[4]{a^4+b}=\tfrac{2}{3}a+\sqrt{\tfrac{1}{9}aa+\frac{b}{6aa}},\ \text{Or},\ a+\frac{ab}{4a^4+\frac{3}{2}b}$$

$$\sqrt[5]{a^5+b}=\tfrac{3}{4}a+\sqrt{\tfrac{1}{16}aa+\frac{b}{10a^3}},\ \text{Or},\ a+\frac{ab}{5a^5+\frac{4}{2}b}$$

$$\sqrt[6]{a^6+b}=\tfrac{4}{5}a+\sqrt{\tfrac{1}{25}aa+\frac{b}{15a^4}},\ \text{Or},\ a+\frac{ab}{6a^6+\frac{5}{2}b}$$

$$\sqrt[7]{a^7+b}=\tfrac{5}{6}a+\sqrt{\tfrac{1}{36}aa+\frac{b}{21a^5}},\ \text{Or},\ a+\frac{ab}{7a^7+\frac{6}{2}b}$$

$$\sqrt[8]{a^8+b}=\tfrac{6}{7}a+\sqrt{\tfrac{1}{49}aa+\frac{b}{28a^6}},\ \text{Or},\ a+\frac{ab}{8a^8+\frac{7}{2}b}$$

$$\sqrt[9]{a^9+b}=\tfrac{7}{8}a+\sqrt{\tfrac{1}{64}aa+\frac{b}{36a^7}},\ \text{Or},\ a+\frac{ab}{9a^9+\frac{8}{2}b}$$

$$\sqrt[10]{a^{10}+b}=\tfrac{8}{9}a+\sqrt{\tfrac{1}{81}aa+\frac{b}{45a^8}},\ \text{Or},\ a+\frac{ab}{10a^{10}+\frac{9}{2}b},\ \&c.$$

If a is Aſſumed Greater than the Root, which is Sought, the Formulæ will be the ſame, Excepting, that all the Signs before b will be Changed from + into −; Thoſe who are Deſirous to Know any Thing more of this Doctrine may Conſult Dr. *Halley* himſelf, and the Inſtances and Examples Alledged for an Explanation of it.

CHAP. VI.

Which is Miſcellaneous, Namely, Concerning the Infinite Series, the Square of the Circle, the Inveſtigation of the Longitude; Of Thunder and Lightning; Of the Rainbow; Of the Vegetable Staticks; A Juſt Ridicule of the Corpuſcularian Scheme, or That of Similar or Homogeneous Matter; Concluſion, by Shewing, how many Hypotheſes there are, by which it is Poſſible for Nature to be Explained by Philoſophers.

Concerning Mr. Cotes's Objections to my Diſſertation on Sir Iſ. Newton's Infinite Series, and my Anſwer to thoſe Objections,

§. 1. IT was many Years ago, and at my Firſt Entrance into theſe Mathematick Speculations, when I drew up a Diſſertation in Latin, Concerning Sir *Iſaac Newton*'s Infinite Series, which I Communicated to my Worthy Friend and Acquaintance Mr. *Cotes*, Late Profeſſor of Aſtronomy and Experimental Philoſophy in this Univerſity, by our Preſent Learned and Good Maſter of *Clare-Hall*, Mr. *Morgan*, whom I have Mentioned with Affection and Eſteem in other Parts of this Inquiry; And which Diſſertation I ſhall, if this Volume will Permit, Annex to theſe Papers, to which I had the Following Objections, and the Anſwer to which Objections is here ſet down.

	Mon.	Lat.	Triang.	Pyr.	Tri. Pyr.	Pyr. Pyr.
Monad.	1	1	1	1	1	1
Later.	1	2	3	4	5	
Triang.	1	3	6	10		
Pyram.	1	4	10			
Triang. Pyram.	1	5				
Pyram. Pyram.	1					

In this Table Suppoſed to be Infinitely Continued, 'tis Evident that the Oblique Columns Exhibit the Unciæ belonging to the ſeveral Powers of a Binomial, if we put T for the Number of the Terms, and ω for the Laſt Term of any of theſe Series of Numbers, the Sums of them will be Reſpectively.

Monad. Later. Triang. Pyram. Tr. Pyr.

$$\frac{T+0}{1}\times\omega.\ \frac{T+1}{2}\times\omega.\ \frac{T+2}{3}\times\omega.\ \frac{T+3}{4}\times\omega.\ \frac{T+4}{5}\times\omega,\ \&c.$$

Let m be a General Repreſentative of the Index of any Power, it will be Eaſy upon theſe Data, to Calculate the General Expreſſions of the Unciæ belonging to that Power, and we ſhall find them to be.

$$1.\ 1\times\frac{m-0}{1}.\ 1\times\frac{m-0}{1}\times\frac{m-1}{2}.\ 1\times\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}.\ 1\times\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4},\ \&c.$$

The Terms of any Power of a Binomial, Setting a-part the Unciæ, are always Continual Proportionals; Thus $P.\ R$ Squared has $P^2.\ PR.\ R^2$, Cubed $P^3.\ P^2R.\ PR^2.\ R^3$, Whereof the Firſt Term is always the Analogous Power of the firſt Number of the Root, and every Precedent Term Multiplied by $\frac{R}{P}$ Generates it's Subſequent; Thus, $\overline{P.R}|^m$ has theſe Terms, $P^m.\ P^m\frac{R}{P}.\ P^m\frac{R^3}{P^3}$, &c. Prefixing therefore the Correſpondent Unciæ found, as above Mentioned, we have $\overline{P+R}|^m = P^m + mP^m\frac{R}{P} + m\times$

$\frac{m-1}{2}P^m\frac{R^2}{P^2}+m\times\frac{m-1}{2}\times\frac{m-2}{3}P^m\frac{R^3}{P^3}$, *&c.* Or to Avoid Fractions in the Canon, Putting $\frac{R}{P}=Q$, and Conſequently $R=PQ$, $\overline{P+PQ}|^m=P^m+mP^mQ+m\times\frac{m-1}{2}P^mQ^2+m\times\frac{m-1}{2}\times\frac{m-2}{3}P^mQ^3$, *&c.*

I Suppoſe the Diſſertator will readily Allow of the Validity of this Canon, whilſt m Remains an Integer, but will Deny, that there can be any ſhew of an Argument, drawn from Analogy to Prove it to be of any Uſe, when it becomes a Fraction, or Indeed, that there can be any Proof of it at all, Preſuming that it is Utterly Falſe in ſuch Circumſtances; I will not here Inſiſt upon the Force of Analogy, ſince he ſeems to be ſo Utterly Averſe to it, but will make Uſe of another way of Proof, which may Perhaps be more Suitable to his Conceptions.

'Tis then Affirmed, that putting $\frac{m}{n}$ for any Fraction.

$\overline{P+PQ}|^{\frac{m}{n}}=P^{\frac{m}{n}}+\frac{m}{n}P^{\frac{m}{n}}Q+\frac{m}{n}\times\frac{m-n}{2n}P^{\frac{m}{n}}Q^2$, *&c.* Let this Series be $=S$.

If $\overline{\overline{P+PQ}|^{\frac{m}{n}}}|^n$ be $=S^n$, then will $\overline{P+PQ}|^{\frac{m}{n}}=S$; But by the Former Canon, which is Allowed of, S^n is Equal to the Following Series.

$$P^m+n\times\frac{m}{n}P^mQ+n\times\frac{m}{n}\times\frac{m-n}{2n}P^mQ^2 \qquad +n\times\frac{m}{n}\times\frac{m-n}{2n}\times\frac{m-2n}{3n}P^mQ^3$$

$$+n\times\frac{m}{n}\times\frac{m}{n},\frac{n-1}{2}P^mQ^2 \qquad +2n\times\frac{m}{n}\times\frac{m}{n}\times\frac{m-n}{2n}\times\frac{n-1}{2}P^mQ^3.$$

$$+n\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{m}{n}\times\frac{m}{n}\times\frac{m}{n}P^mQ^3, \text{ \&c.}$$

Which is $=P^m+mP^mQ+m\times\frac{m-1}{2}P^mQ^2+m\times\frac{m-1}{2}\times\frac{m-2}{3}P^mQ^3$, *&c.* Which is $=\overline{P+PQ}|^m$, which is $=\overline{\overline{P+PQ}|^{\frac{m}{n}}}|^n$ Therefore, *&c.* The Same way of Reaſoning may be made Uſe of for the Admittance of the Abovementioned Canon, even, when m Signifies a Negative Quantity, or any whatſoever.

Answer. The Sums are Reſpectively as above, and Vice Verſâ, thoſe Series whoſe Sums are $\frac{T+0}{1}\times\omega$. $\frac{T+1}{2}\times\omega$, *&c.* are Reſpectively Monadick, Lateral, *&c.* What I would here Add is, which Perhaps Mr. *Cotes* has not Obſerved; Let m Deſign the Index of any Integral Power, which is always found in the Series of Laterals, it may, I believe, be Juſtly Demonſtrated, that the Perpendicular Columns are Produced by this Series $1\times\frac{m+0}{1}\times\frac{m+1}{2}\times\frac{m+2}{3}\times\frac{m+3}{4}$, *&c.* That is 1 is the Character of Monadicks, m of Laterals, $\frac{m^2+m}{2}$ of Triangulars, $\frac{m^3+3m^2+2m}{6}$ of Pyramidals, *&c.* Suppoſing we Subſtitute, as is afterwards done, $m-0$, $m-1$, $m-2$, $m-3$, *&c.* Succeſſively in the ſtead of m; If we take therefore for Example the 5*th* Power, m is $=5$, 1, $\times\frac{m-0}{1}$ will give the 2 Firſt Unciæ; $4=m-1$, the 3*d* Uncia, which is a Triangular, $=1\times\frac{m-1+0}{1}\times\frac{m-1+1}{2}=1\times\frac{m-0}{1}\times\frac{m-1}{2}$; 3 is $=m-2$, therefore the 4*th* Uncia, which is a Pyramidal, is $=1\times\frac{m-2+0}{1}\times\frac{m-2+1}{2}\times\frac{m-2+2}{3}=1\times\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}$; 2 is $=m-3$, and Conſequently the 5*th* Uncia, which is a Triang-Pyramidal $=1\times\frac{m-3+0}{1}$, *&c.*

$=1\times\frac{m-0}{1}\times\frac{m-1}{2}\times\frac{m-2}{3}\times\frac{m-3}{4}$; $1=m-4$, and the *6th* Uncia, a Pyr-Pyramidal, and ſince the next Step gives $m-5=0$, the Series is Finiſhed; But to Return, What Mr. *Cotes* Alledges above, as he Says, is very Eaſy, the Calculation I readily Allow of, and the Demonſtration, upon which it is Founded, but muſt Beg the Favour to make Uſe of the very ſame Proof for that Canon, or Series, which I Oppoſed to Sir *Iſ. Newton*'s in thoſe Papers, which, I hope, He has done me the Kindneſs to Peruſe, which is Exactly the ſame with Sir *Iſ. Newton*'s, when Applied to Integral Powers, but Diſſents from his in Fractional; The Series for the *5th* Power is This, $1\times\frac{m-0}{m-4}\times\frac{m-1}{m-3}\times\frac{m-2}{m-2}\times\frac{m-3}{m-1}\times\frac{m-4}{m-0}\times\frac{m-m}{1}$; Here is nothing in the World different from That of Sir *Iſ. Newton*'s, only This, that, whereas in the Former, the Laterals, which are the Numerators, are Expreſſed by $m-4$, $m-3$, $m-2$, $m-1$, m, the ſame Laterals which are the Nominators have not the ſame Expreſſions, which certainly they ought to have, in the Latter the ſame Laterals have the ſame Expreſſions, which, whether it is not the moſt Reaſonable, I leave to Mr. *Cotes* to Conſider; It were no hard Matter to Prove by another Method, that Fractional Powers, from the very Nature of this Table, cannot Admit of the ſame Rule for finding their Unciæ, with That, which is Uſed for Integrals, but I ſhall Omit it, that I may not be too Tedious, ſince this Series will, I Believe, Sufficiently Evidence it to him; However it will Prove thus much, that whether they are to be found by the ſame Rule, or not, Sir *Iſ. Newton*'s Series for the Unciæ of Fractional Powers cannot be Valid; If they are not, it is Plain, ſince Sir *Iſ. Newton* Uſes the ſame Rule; If they are, then this Latter Rule muſt Produce the ſame Series of Unciæ for Fractional Powers, with the Former, which is Impoſſible, or elſe this Latter Rule is not the ſame with Sir *Iſ. Newton*'s for finding the Unciæ of Integral Powers, which yet we have Proved is.

The Terms of any Power, *&c.*) This I allowed in Integral Powers, but Expreſly Denied, that every Precedent Term Multiplied by $\frac{R}{P}$ Generates it's Subſequent in Fractional Ones; my Words are, *Jam verò, inquam, hanc rationem, quare* x^2, x^3 *Dividatur per* c^2, c^3, *non ad fractas poteſtates pertinere, niſi eodem, quo modo Integræ, componantur, quod fieri nequit.* I there Offered ſome Reaſons, why their Compoſition muſt be very Different, nor is there any Thing ſaid here to give me the leaſt Occaſion of Altering That Opinion.

Vid. Diſſertationem, Infra.

Thus far Mr. *Cotes* has Explained the Canon for Integral Powers, which I do very Willingly Embrace and Admit, I always Acknowledged it Good and Irrefragable, ſo long, as it was Applied to Integral Powers, but cannot yet own it's Uſe in Fractional; As to the Preſent Demonſtration, I come now to Conſider it, the Argument lies Thus.

Mr. *Cotes* Affirms, putting $\frac{m}{n}$ for a Fraction, that $\overline{P+PQ}|^{\frac{m}{n}}$ is $=P^{\frac{m}{n}}+\frac{m}{n}P^{\frac{m}{n}}Q$, &c. which Series he Suppoſes $=S$, Therefore If $\overline{\overline{P+PQ}|^{\frac{m}{n}}}|^{n}=\overline{P+PQ}|^{m}$ be $=S^{n}$, then will $\overline{P+PQ}|^{\frac{m}{n}}$ be $=S$; But, by the Former Canon Allowed of, S^{n} is $=\overline{P+PQ}|^{m}$ therefore $\overline{P+PQ}|^{\frac{m}{n}}=S$; If a Suppoſition of the very Thing to be Proved is a Demonſtration of it, I Conſent that this is a very Firm and Good one; To give a nearer View of this Proof, *e. g.* $P^{\frac{1}{2}}$ is Suppoſed $=S^{1}$, or Affirmed ſo to be; If $P^{1}=S^{2}$, then $P^{\frac{1}{2}}=S^{1}$, but P^{1} is Proved $=S^{2}$, upon the Suppoſition, that $P^{\frac{1}{2}}$ is $=S^{1}$, therefore $P^{\frac{1}{2}}$ is $=S^{1}$; for, if $P^{\frac{1}{2}}$ is not Suppoſed $=S^{1}$, P^{1} can never be Proved to be $=$ to S^{2}, any more than to S^{1}; Or to Reſume the Quantities Propoſed, $\overline{P+PQ}|^{\frac{m}{n}}$ is Said to be $=S$, *&c.* which, if it were not Suppoſed, and Taken for granted, without proving, $\overline{P+PQ}|^{m}$ could never be $=S^{n}$; If it can be, without ſuch a Suppoſition, Namely, of That, which is only to be Demonſtrated, therefore let $\overline{P+PQ}|^{\frac{m}{n}}$ be $=Z$, that is, till it's Proved what it is, an Unknown Quantity, and $\overline{P+PQ}|^{n}=S^{n}$ as before; If $\overline{P+PQ}|^{n}=S^{n}$, then will $\overline{P+PQ}|^{\frac{m}{n}}$ be$=Z$; But $\overline{P+PQ}|^{n}=S^{n}$ by the Former Canon, therefore $\overline{P+PQ}|^{\frac{m}{n}}=Z$; This Mr. *Cotes* will Allow to be a ſtrange Concluſion, which if it were Right, we ſhould be as far to Seek as ever, ſince Z is Affirmed to be Unknown, and yet this is the moſt that can be fairly made of Mr. *Cotes*'s Argument; To Clear this Matter, if I can, a little Farther, That $\overline{P+PQ}|^{\frac{m}{n}}$ is $=S$ is the Propoſition to be Evidenced, or which is the ſame, that $\overline{P+PQ}|^{n}$ is $=S^{n}$; for Mr. *Cotes* therefore to Argue thus, if $\overline{P+PQ}|^{n}$ is $=S^{n}$, then $\overline{P+PQ}|^{\frac{m}{n}}=S$, but $\overline{P+PQ}|^{n}$ is $=S^{n}$, by the former Canon, therefore *&c.* is no more than to Say, if $\overline{P+PQ}|^{\frac{m}{n}}$ is $=S$, then $\overline{P+PQ}|^{\frac{m}{n}}=S$, but it is Told us by the Former Canon, that $\overline{P+PQ}|^{\frac{m}{n}}=S$, therefore it is $=S$, that is, if a Thing is ſo, it is ſo, but it is ſo, therefore it is ſo; What I have Said is Seriouſly no other than what is his Real Demonſtration; Now Mr. *Cotes* will, tho' his Argument does not, Acknowledge, that $\overline{P+PQ}|^{\frac{m}{n}}$ cannot Prove it's ſelf to be $=S$ by the Former Canon, therefore $\overline{P+PQ}|^{n}=S^{n}$, which is the ſame as $\overline{P+PQ}|^{\frac{m}{n}}=S$ cannot Prove it by the aforeſaid Canon, nor Indeed by any Canon in the World, unleſs it be Proved that $\overline{P+PQ}|^{n}$ cannot be $=S$, or that $\overline{P+PQ}|^{\frac{m}{n}}$ muſt be $=S$ à Priori, that is, $\overline{\overline{P+PQ}|^{\frac{m}{n}}}\Big|^{n}=S^{n}$; In Short it is Affirmed, that $\overline{P+PQ}|^{n}$ is $=S$, that is, that the Series $=S$ belongs only to an Integral Power, the Proof againſt this is, that $\overline{P+PQ}|^{n}$ is $=S^{n}$, if we Suppoſe $\overline{P+PQ}|^{\frac{m}{n}}=S$, the very Equation which is Denied; I might give other Confutations of this Objection, for it is Eaſy to make Appear, which I have, I ſhould Think, in my Papers done, that $\frac{m}{n}$ cannot Expreſs a Fraction, or Integer, as we ſhall Pleaſe, or have Occaſion for it, which it muſt do, if $\overline{P+PQ}|^{\frac{m}{n}}$ is $=S$.

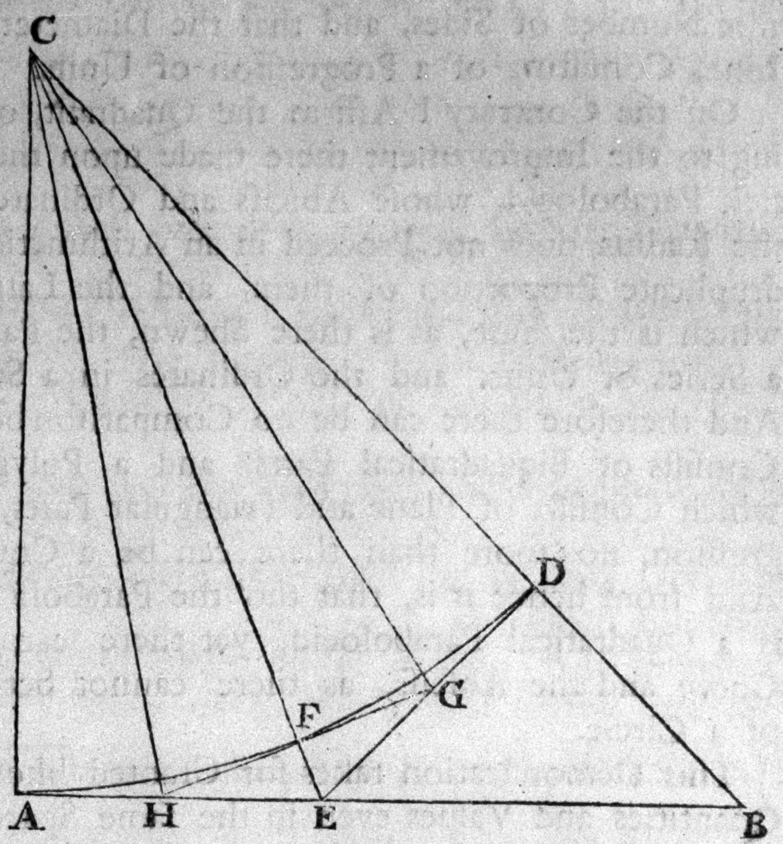
The Reſult of this Converſation between Mr. *Cotes* and me, was, that he would not Enter into any farther Controverſy about this Matter, whether this Proceeded from his Exceeding Modeſty, or his Utter Averſeneſs to Diſpute, which I call Inquiring, or from his Conviction, that his Demonſtration was not Juſt, I ſhall not Take upon me to Determine, but the Report then was by Mr. *Whiſton*, who Knew Nothing of this Kind of Learning, or very Little of it, that he had Entirely Confuted me; Whereas Mr. *Cotes* himſelf could not be a Maſter of this Argument, unleſs he had Read Dr. *Wallis*'s Arithmetick of Infinites, from whom Sir *Iſ. Newton* Derived his Infinite Series, and who yet is now Neglected in that Particular by the Generality of Mathematicians, altho' all their Reaſonings on the Infinite Series Depend upon the Truth or Falſehood of that Arithmetick.

Concerning Mr. Cotes*'s Demonſtration againſt the Square of the Circle.*

§. 2. Mr. Profeſſor *Cotes*'s Demonſtration againſt the Square of the Circle is, as Follows,

Propos. 1. The Square of the Diameter of any Circle, is to the Area of a Regular Polygone of Sixteen Sides Circumſcribed about the ſame, as the Tangent of the Sixteenth Part of the Circumference is to the Quadruple of the Exceſs of it's Secant above the Radius.

Let AD be the 8*th* Part of the Circumference of a Circle Deſcrib'd upon the Center C, with the Radius CA. Draw AB Touching the Circle in A, and Cutting the Radius CD in B, Draw DE Touching the Circle in D, and Cutting the Tangent AB in E, Join C, E, Interſecting the Circle in F; Draw HFG Touching the Circle in F, and Cutting the Tangents AE in H and DE in G; Laſtly, Join CH, CG; Now 'tis Manifeſt, that the Lines AC, and AB are Equal, as alſo the Lines AE, ED, and DB, and the Lines AH, HF, FG and GD, and the Angles ACH, HCF, FCG, GCD; And moreover it is Evident, that the Square of the Diameter is Equal to the Triangle ABC taken Eight Times, and that the Circumſcribed Polygone of Sixteen Sides is Equal to the Triangle AHC taken Thirty Two Times, and that the Tangent of the 16*th* Part of the Circumference is AE, the Secant of the ſame being CE, and the Exceſs of that Secant above the Radius being EF; 'Tis therefore to be Proved, that $8ABC$ is to $32AHC$, or ABC to $4AHC$, as AE to $4EF$.



The Triangle ABC is to the Triangle AHC, as AB to AH, or as AC to FH, or (becauſe the Triangles ACE, FHE are Similar) as AE to EF; Therefore ABC is to $4AHC$, as AE to $4EF$. Q. E. D.

PROPOS. 2. The Tangent of the Sixteenth Part of the Circumference is Greater than the Quintuple of the Exceſs of it's Secant above the Radius; I Say, that AE is Greater than $5EF$. For,

$58CB\times 58CB$, or $3364CBq$, or $6728CAq$ is Greater than $6724CAq$, or, $82CA\times 82CA$.

Therefore $58CB$ is Greater than $82CA$.

And $58CA\times CB$ Greater than $82CAq$.

And $50CA\times CB+8CA\times CB$ Greater than $100CAq-18CAq$.

And $18CAq+8CA\times CB$ Greater than $100CAq-50CA\times CB$.

Whence, $\overline{CB+4CA}\times\overline{CB+4CA}$, or $CBq+16CAq+8CA\times CB$, or $18CAq+8CA\times CB$ is Greater than $100CAq-50CA\times CB$, or $25\times\overline{4CAq-2CA\times CB}$, or $25\times\overline{CBq+2CDq-2CD\times CB}$, or $25\times\overline{CBq+2CDq-2CDq-2CD\times DB}$, or $25\times\overline{CBq-2CD\times DB}$, or $25\times\overline{CDq+DBq}$, or $25\times\overline{CDq+DEq}$, or $25CEq$, or $5CE\times 5CE$.

Therefore $CB+4CA$ is Greater than $5CE$.

And $CB-CA$ Greater than $5CE-5CA$.

And DB, or DE, or AE, Greater than $5EF$. Q. E. D.

PROPOS. 3. The Proportion of the Area of the Circle to the Square of it's Diameter is not That of 4 to 5.

If it be Said, that the Area of the Circle is to the Square of it's Diameter as 4 to 5, then, becauſe the Square of the Diameter is to the Area of a Circumſcrib'd Polygone of 16 Sides, as AE to $4EF$, the Area of the Circle will be to the Area of the Polygone, as AE to $5EF$; And Conſequently, (ſince AE is Greater than $5EF$) the Area of the Circle will be Greater than the Area of the Circumſcrib'd Polygone, that is, a Part will be Greater than the Whole; But a Part is not Greater than the Whole, the Circle is not Greater than the Polygone, therefore the Circle is not to the Polygone, as AE to $5EF$, and Conſequently, it is not to the Square of it's Diameter, as 4 to 5. *Q. E D.*

After I had Publiſhed my Firſt Inquiry into the Principles of the Modern Philoſophy in 1712, and in the Preface to it, and in the *Geometria Solidorum ſive Materia*, had Aſſerted, that the Proportion of the Circle to it's Square, was as 4 to 5, the Ingenious and Learned Profeſſor Above-Named was ſo Kind and Obliging, as to Communicate to me, with great Candour and Friendſhip, the Foregoing Demonſtration againſt it; Tho' if he had been Pleaſed to have Peruſed with Care, either my Preface, or the Geometry Mentioned, he would have found all Demonſtrations Obviated of this Kind; Which Suppoſe, that a Circle is nothing more than a Polygone of an Infinite Number of Sides, and that the Diameter of it is only a Plain and Common Line, Conſiſting of a Progreſſion of Units.

On the Contrary I Affirm the Quadrant, or Fourth Part of a Circle, to be, according to the Improvement there made upon the Arithmetick of Infinites, a Biquadratick Paraboloeid, whoſe Abſciſs and Ordinate are Equal, the Former of which, or the Radius, does not Proceed in an Arithmetical Progreſſion of Units, but in a Quadruplicate Proportion of them, and the Latter in an Arithmetical Progreſſion; Or, which is the ſame, as is there Shewn, the Parts of the Radius or Abſciſs Proceed in a Series of Units, and the Ordinates in a Subquadruplicate Proportion of Them; And therefore there can be no Compariſon betwixt a Circle or a Paraboloeid, which Conſiſts of Biquadratical Parts, and a Polygone of an Infinite Number of Sides, which Conſiſts of Plane and Triangular Parts, which Proceed in an Arithmetical Progreſſion, no more than there can be a Compariſon betwixt a Plane and a Solid; And from hence it is, that tho' the Parabola is Acknowledged to be Squared, which is a Quadratical Paraboloeid, yet there can be no Proportion found betwixt the Curve and the Abſciſs, as there cannot betwixt the Radius and the Circumference of a Circle.

This Demonſtration takes for Granted likewiſe, that there may not be Different Quantities and Values even in the ſame Space, which I have Evidently Proved there may be, both in that Inquiry, and in the Geometry above Cited, and which Kind of Geometry or Mathematicks, tho Perhaps New and Uncommon, muſt be the only Baſis and Foundation of a True Philoſophy; For if it did not take it for Granted, it could not Prove, that a Part was Greater than the Whole, according to my Proportion of 4 to 5, ſince, if the Quantities Included in the Circle are of a Greater Value than Thoſe in the Polygone of an Infinite Number of Sides, there can be no Inference like That, which is here Made, that a Part is Greater than the Whole, becauſe in ſuch a Circumſtance a Part may be Infinitely Greater than a Whole; For thus a Part of Gold is Greater than a Whole of Air, or Water; Since Gold is to Water for Inſtance, as 19 to 1, and therefore $\frac{1}{19}$*th* Part of a Globe of Gold of an Inch Diameter, is Equal in it's Value, Weight, or Matter, to a Whole Globe of Water of the ſame Diameter; But theſe are Notions, which my Worthy and Learned Friend had not Entered into, who had Received the Opinions Embraced, of all Space being alike, and that Matter alſo was Similar and Homogeneous, Purely from Geometrick Abſtractions; Of whom I ſhall only Take this Laſt Farewell, as he is ſince Dead, and as I had a great Eſteem and Value for him, by Telling the World, ſo far as theſe Writings ſhall Reach, that He was One of the moſt Candid, moſt Humble, and Modeſt, as well as One of the moſt Ingenious, Judicious, and Learned of Men, which Perhaps This Age has Produced; His other Writings have abundantly Recommended him to the Philoſophick and Mathematick Part of it, and there will Appear in this Demonſtration againſt what I have Affirmed, and I think Proved, ſuch a Sagacity of Mind, and ſuch a Penetration of Thought, Eſpecially in the ſecond Propoſition, where he Proves, that *AE* is Greater than 5*EF* by that Accurate and Exquiſite Compounding and Dividing of Quantities, which he Uſes, ſo as to come to his Main Concluſion, that it is Deſervedly to be Admired; And all the Difference betwixt him and me is, that we do not Conſider Things in the ſame Way; Upon his Principles of Similar or Abſtracted Space, his Demonſtration is not only Juſt and True, but Exceedingly Fine and Beautifull; But upon my Principles of a Diſſimilar Space, as well as Matter, and that there may be Different Quantities and Values in the ſame Dimenſional Space, or in the ſame Circular Superficies, his Demonſtration does not Affect, what I have Aſſerted, and do Defend, in the Leaſt; He Conſiders a Circle with *Archimedes*, as Conſiſting of an Infinite Number of Small Triangles, whoſe Apices Lie in the Centre, and whoſe Baſes are in the Circumference, and which therefore makes a Plane Figure, I Conſider it, or the Quadrant of it, with Dr. *Wallis* in his Arithmetick of Infinites, as Conſiſting of an Infinite Number of Abſciſſes, the Progreſſion of which I have Shewn will be as 1. 2. 3. 4.

5, *&c.* and of an Infinite Number of Ordinates, the Progreſſion of which will be, as $\surd^4, 1$, $\surd^4, 2$, $\surd^4, 3$, $\surd^4\ 4$, $\surd^4, 5$, *&c.* or which is the ſame, the Abſciſſes will be, as 1^4, 2^4, 3^4, 4^4, 5^4, *&c* and the Ordinates, as 1. 2. 3. 4. 5, *&c.* which will make a Solid Figure, whoſe Proportion will be to it's Square, as 4 to 5. See *Geometria Solidorum*, Pag. 35. And in the ſame Manner is the Parabola Conſidered, and by that Means Squared by *Apollonius* of *Perga* in his Conick Sections; This Way of Conſidering the Circle is alſo Juſtified by Sir *Iſ. Newton's* Endeavours at the Square of it by his Infinite Series, and his Interpolations, Taken from Dr. *Wallis's* Arithmetick of Infinites, and what I have done in this Particular, is only Solving the ſame Problem without any ſuch Infinite Series, or without any ſuch Interpolations by an Improvement upon, or a Correction of, the Aforeſaid Arithmetick of Infinites, and till that Correction or Improvement is Shewn to be Wrong, which I have Propoſed in the *Geometria Solidorum ſive Materiæ*, all Arguments againſt me can be of no Force, or Validity.

Concerning the Inveſtigation of the Longitude.

§. 3. Firſt, it muſt be Obſerved, that if we can Find out the Longitude of a Place by Day from any Point, or Phenomenon Given, as the Sun, we Arrive at an Equal Certainty in theſe Matters, as in the Latitude, ſince that can be only Obtained from Knowing the Elevation of the Pole, which is Equal to it, and which Elevation cannot be Determined, unleſs by Obſervations made in the Night; And therefore, if Clouds Interfere by Day, or by Night, They are no Exceptions to the one Method of Invention, or the other, nor is the Time of Obſerving any Objection to Either; But both of them will be upon the Level in theſe Particulars.

Second, if farther the Longitude may be found out by Night, from any Fixed Star or Phenomenon Given, the Invention of the Longitude will have the Preference to That of the Latitude, tho' not in the Certainty of it's Determination, in which it will be Equal, yet in the Frequency, and Conſtancy of it.

Third, what therefore is to be Done by Mathematicians and Aſtronomers is to Endeavour, if Poſſible, to make the Sun by Day, or a Planet or Fixed Star by Night, as Certain a Point, from whence to Compute the Longitude, as the Pole is One for the Latitude; But This is the Problem, or ſhould be, and This the Inſuperable Difficulty; For how can we make the Sun, or a Star, which by the Earth's Revolution about it's Axis, ſeem Themſelves to Revolve, as Fixed as the Pole?

Fourth, beſides the want of a Fixed Point, from whence to Compute the Longitude, Suppoſing we could have One by Day, as the Sun, or by Night as a Star, or a Planet, in the Equator, yet the ſeveral Places of the Earth, Excepting thoſe which Lie under it, being in Different Parallels to it, Renders this Problem more Unſurmountable than That of the Latitude, which is always Computed upon the Meridian.

Fifth, the Difference therefore of the Two Problems of Inveſtigating the Latitude and Longitude of a Place Proceeds from hence, that the Axis of the Earth is Fixed and Immoveable, ſo that in whatever Meridian we are, we are Conceived to Change our Place only from one Pole to the other upon the ſame Immoveable Axis, in Reſpect of the Latitude of a Place, that is, to Walk, or Sail, or Move, upon the ſame Line from one Point to another; Whereas in the Invention of the Longitude, we are upon a Moveable Line, which Shifts and Changes it's Situation every Minute, as when we are in the Equator, or it's Parallels, which Proceeds from the Revolution of the Earth Round it's Axis.

Sixth, Let us therefore Imagine the Earth to be Quieſcent, and not to Revolve about it's Axis, or which is the ſame, the Heavens and the Celeſtial Bodies not to do ſo, and the Longitude of any Place will be as Eaſily Determined, as the Latitude of it; Becauſe, the Latitude is Suppoſed to be Known, and Conſequently the Inclination of the Equator to the Horizon, which is the Complement of the Latitude to 90 Degrees; Let then an Obſervation be made with a Quadrant Poſited Parallel to the Equator, of the Diſtance of the Place, from the Sun, or any Fixed Star, Suppoſing them Quieſcent, and whoſe Longitudes are Determined and Given, and the Longitude of the Place will be likewiſe Aſcertained, ſince it will be the Diſtance of the Place from the Star or Sun, Obſerved, Added to or Subſtracted from ſuch Longitude.

There are only Two Objections againſt this Procedure, the Firſt is, that it is not the ſame Thing to take an Obſervation from the ſame Fixed Point, when we are in the Equator, and when in Different Parallels to it, to which it is Anſwered, that ſince the Earth is only as a Point in Reſpect of the Infinite Diſtance of the Fixed Stars, and of the Immenſe One of the Sun from it, it is the ſame Thing; For thus it is Acknowledged, that the Horizon, and all it's Parallels, even to the Vertex, will Terminate in the ſame Fixed Stars, which are in the Horizon, upon Account of the Earth's being a Point only, and from hence it is, that the Moon, which is much Nearer to us than the Sun, Appears in the ſame Part of the Heavens, tho' we ſhould Walk ſeveral Miles upon the Earth, and Obſerve it's Situation at Different Intervals, becauſe thoſe Miles, or thoſe Intervals, are only as a Point in Reſpect of the Moon's Diſtance from us; And even in Objects upon the Earth, which are very Remote, it is hard to Draw any Parallel Lines from them, becauſe they Coincide, ſuch Diſtant Objects Uniting, as it were, in a Point

The Second Objection is, that we cannot make any Obſervations with a Quadrant Inclined to the Horizon, ſince thoſe are always made in ſome Vertical Circle, that the Line and Plumbet, which Marks the Degrees of the Quadrant, may Retain it's Perpendicular to it; Let us therefore Suppoſe a Right Situation of the Sphere, in which the Equator, and it's Parallels are Perpendicular to the Horizon, the Equator in ſuch a Caſe will be a Vertical Circle, and Conſequently an Obſervation of our Diſtance in Longitude from the Sun may be Punctually Taken, Allowing the Earth to be Quieſcent, and not to Revolve about it's Axis; If the Equator is now Inclined to the Horizon, let the Quadrant upon any Obſervation be in a Parallel Situation to it, and the Plumbet will ſtill Mark the Degree of Diſtance or Longitude from the Sun, becauſe it will be in rhe Arc of a Circle which Paſſes thro' the ſame Degree of the Quadrant, which it Touched, when the Sphere was in a Right Situation.

Seventh, the only Difficulty, which Remains is, that the Sun by the Earth's Revolution cannot be Conſidered as a Fixed Point; But ſince it Proceeds in the Ecliptick One Degree only in 24 Hours, that Time may be nearly Meaſured by a Sand-Glaſs or an Automaton, but rather by the Firſt, and therefore may be Allowed for in the Computation of the Longitude; Farther, as the Revolution of *Mars* is Performed in 2 Years, of *Jupiter* in 12, and of *Saturn* in 30, they may to all Intents and Purpoſes be Imagined and Deemed as Fixed Points, and therefore will ſerve to Calculate the Longitude from, as well as the Pole for the Latitude; Beſides which, even all the Fixed Stars in this Method may be Eſteemed as ſo many Fixed Points, from which the Longitude may be Determined, if we can Meaſure 24 Hours nearly by an Automaton, or, which ſeems to be much more Certain, by a Sand-Glaſs, or a Water-One.

If I am Miſtaken in Determining the Longitude, I am only Fallible in my Reaſoning, as many Others have been before me, but I am pretty well Satisfied, that this Method of Proceeding is the only Way, that is, to Compute from a Fixed Point in the Heavens, as is done in the Latitude; For it is Impoſſible to Find out the Longitude by Methods, which are Deſultory and Uncertain, as by the Exploſion of Bombs, becauſe the Communication of Light is Uncertain, both on the Account of the Different Force of Gun-Powder, and the Quickneſs of the Light from thence Ariſing, as alſo from the Difference of the Medium, or Air, thro' which the Light is Conveyed; That of the Dipping Needle is alſo Subject to a Thouſand Caſualties, which may Diſturb it's Indications; See the Ingenious Mr. *Graham's* Experiments Concerning it. *Philoſoph. Tranſact.* Numb. 389. 3. Nor can it be better Aſcertained by Pendulums, or Automatons, whoſe Motion is Varied, not only by the Motion of the Ship, but by the Difference of the Mediums, being more or leſs Expanſive and Contractive, in which they Oſcillate; Nor Laſtly, will the Eclipſes of the Satellites of *Jupiter* ſerve the Turn, as is Confeſſed by the Philoſophers Cited, and as is very Evident, becauſe thoſe Eclipſes happen only at Certain Times and Intervalls, and therefore cannot be a Conſtant Direction to thoſe at Sea, who may Want to Know the Longitude of the Place, where they are, every Minute; But, if we could Determine the Longitude in the ſame Way, by ſome Certain and Fixed Point in the Heavens, as the Latitude is by the Elevation of the Pole, the Problem would be Fully and Perfectly Solved, and which I have Endeavoured to do in the Foregoing Eſſay, and which is Submitted to the Judgement of the Aſtronomers, and Thoſe who have a Love and Eſteem for the Mathematick Sciences.

Mr. Whiſton.

Mr. Hugens,

Varenius and Sir Iſ. Newton, Dr. Gregory, Dr. Keill, &c.

Here

How the Variation and Dipping of the Needle may be Accounted for.

I ſhall only here Subjoin in Reſpect to the Variation and Dipping of the Needle, what I have heard, from thoſe who have been Acquainted with theſe Matters in Navigation and in Fact, Namely, that the Variation of the Needle from the North Point to the Eaſt or Weſt is ſuch, that when it Varies to the Eaſt, Mariners are Obliged to Sail ſo much towards the Weſt, as it Varies towards the Eaſt to come Due North, tho' the Needle always Points to the North in the Card, and this Variation is More or Leſs, according as we come Nearer to, or are Farther from the Shore, and which Variation Differs in the Interval of ſeveral Years, even where they are at the ſame Diſtance from the Shore.

The Reaſon of which can be only This, that the Shore Attracts the Card and the Needle by it's Contractive Force, which is Greater than that of Water, and as the Contractive Force of the Shore is Greater or Leſs, ſo is the Needle and Card more or Leſs Attracted to it, * which Contractive Force may be in a Courſe of Years Greater or Leſs, according to the Greater Contractive Force of the Moon, and Leſs Expanſive of the Sun, or the Leſs Contractive Force of the Moon, and the Greater Expanſive of the Sun in their Influences upon it; From whence it is Plain, that the Imaginary Nucleus of Dr. *Halley*, within the Earth, can have no Effect in this Phenomenon.

* For that End of the Needle which Points to the North, will be more Acted upon by the Contractive Force of the Earth, in the Northern Hemiſphere, than That, which Points to the South; Since the Needle, being Parallel to the Horizon, muſt be Nearer to the Earth's Axis in it's Northern, than it's Southern End, in a Northern Hemiſphere, and therefore muſt Act more Strongly towards it, by it's Contractive Force.

From hence alſo it is Probable, that there is a Variation of the Needle in it's Dipping, and that it Dips more in a Shallower Sea, and Leſs in a Deeper, becauſe the Contractive Force of the Earth, by which the Needle Dips, is Greater, than that of Water, which is of an Expanſive Force in the Compariſon; From whence, if this is True, it will be again Manifeſt, that the Dipping Needle can Furniſh us with no Indication of the Longitude.

Here Follow ſome Propoſitions, which may be of Uſe in the Preceding Method of Inveſtigation, by a Point in the Heavens, if the Former is not Juſt.

1ſt. The Meridian of any Place is Found by Prop. 16. Lib. 2. Sect. 4. of Dr. *Gregory*'s Aſtronomy, as likewiſe by Prop. 7. Lib. 2. of *Varenius*'s Geography.

2d. The Altitude of the Pole, and the Situation of the Equinoctial Circle in Reſpect of the Horizon is Determined by Prop. 17. Lib. 2. of Dr. *Gregory's* Aſtronomy.

3d. The Declination of any Sydus, whether Sun or Star, is alſo Known by Prop. 18. Lib. 2. of Dr. *Gregory*'s Aſtronomy.

4th. The Inclination of the Ecliptick to the Equator, and the Ecliptick's Situation, as well as That of the Coluri, the Tropicks, and the Polar Circles in Reſpect of the Equator, is Determined by Prop. 19. Lib. 2. of Dr. *Gregory*'s Aſtronomy.

5th. The Obliquity of the Ecliptick being Given, the Right Aſcenſion and Declination of a Given Point in the Ecliptick, as alſo the Angle, which is Comprehended at the ſame Point betwixt the Meridian and Ecliptick, may be Known by Prop. 20. Lib. 2. of Dr. *Gregory*'s Aſtronomy, and è Converſo the Point of the Ecliptick, to which any of the Foregoing Circumſtances are Agreeable, may be Determined, and therefore the Place of the Sun in the Ecliptick may be Known from Obſervation, by Prop. 18. or Art. 3.

6th. The Revolution of the Earth is in 24 Hours, and therefore there may be Allowed for that Revolution the Sun's Progreſs in the Ecliptick, One Degree, which it makes in the ſame Time; ſo that as the Earth Revolves from Weſt to Eaſt in 24 Hours, it is the ſame Thing, as the Sun's Proceeding in the Ecliptick, One Degree.

7th. As for the Particular Motion or Velocity of the Ship, it can Signify nothing, becauſe if a True Longitude can be Found out, it muſt not Depend upon That, but muſt be Certain, as the Latitude is, whether the Velocity of the Ship be Greater or Leſs.

8th. By Prop, 11. and 6. of *Varenius*'s Geogr. Lib. 2. we may Know, in what Parallel the Ship is, and in what Point of that Parallel to the Equator.

Concerning Thunder and Lightning.

§. 4. I do not Remember, that I have any where taken Notice of that Wonderful Phenomenon of Thunder and Lightning, in Relation to the Doctrine of the Expanſive and Contractive Forces, which ſeems in ſome Meaſure Deſigned by Almighty God, to Strike an Awe and Terror into Men's Minds, and to keep up a Religious Fear and Dread of him in the Hearts of the moſt Obſtinate and moſt Obdurate; For then the Worſt of them, as we may eaſily Imagine, Conceive an Unſpeakable Horror, and would, as it is Reported by *Suetonius* of *Caligula*, by any, the moſt Abject Methods, Skreen themſelves from the Avenging Fury, which they may Juſtly Apprehend from that Good and Righteous Being, they are Conſcious to themſelves, they have Offended; On the Contrary, we may eaſily Believe, a Truly Pious and Religious Man is little Affected with ſuch Paſſions and Emotions of Thought, which may Cauſe Fear or Apprehenſion in him; He Adores the Greatneſs and Majeſty of his Sovereign Preſerver; And is rather Pleaſed and Rejoyced to See and Find himſelf under the Protection of ſo Almighty a Ruler and Governour of all Nature; I Write this, *May* the 21*ſt*. 1726, whilſt Thunder is Rolling about my Room, ſhakes the Chamber I am in, and the very Chair I Sit on, and Lightning Darts in at every Window, and ſeems to fall upon the Table, and on the Papers I now Write, which has given an Handle to this Inquiry into the Preſent Phenomenon; I ſhall not ſtay to Conſider, or to Confute *Des-Cartes*'s Fancifull Notion of Two Clouds Claſhing and Interfering with each Other, and from thence Striking out Lightning, as we are uſed to do Fire from a Couple of Flints, which is Exceedingly Ridiculous and Abſurd; But the Way, which it has Pleaſed Almighty God to Eſtabliſh, for the Production of this Amazing Phenomenon, ſeems to be by Certain Laws of the Expanſive and Contractive Forces, and which Ariſes from a Mutual Conteſt or Conflict betwixt them, till Things are Reduced to an Equilibrium, or Equal Balance, and Then all is Quiet.

Thus in the Year 1725, and in the Beginning of it, there were Scarcely any Rains Fell for ſome Months, when all the Succeeding Summer, we had almoſt Continual Ones every Day; The Former was an Inſtance of the Expanſive Force, and Heat Prevailing, and the Laſt of the Contractive; And there were Storms of Thunder and Lightning betwixt the One and the Other; Juſt before this Tempeſt of Thunder and Lightning, in this Preſent *May* 1726, there had been a great Drought and Heat, which muſt Conſequently Raiſe from the Earth, which was Moiſt before by the Continual Rains, which had Fallen upon it, a great Number of Vapours, or a great Quantity of Water, which in Reſpect of the Sun's Heat, is of a Contractive Force, as the Sun's Heat is of an Expanſive; Theſe Two Forces coming to Contend in the Air, the Contractive Force of the Water makes the Expanſive of the Heat Aſſemble into Flame, and to Dart into Lightning, and the Expanſive on the Contrary Drives and Compreſſes the Moiſt Vapours into Rain or Hail; And where there is a Reſiſtence of One Force to Another, as we have before Shewn, Sound muſt be Produced, which we call Thunder; But at Length the Contractive Prevailing againſt the Expanſive, a Temperate and Cool Air Enſues, or if it does not entirely Prevail, there will be a Sultry Heat, and ſmall Glimmerings, or Languid Flaſhes of Lightning for ſome Time after, which Shews, that the Expanſive is not wholly Subdued by the Contractive.

This Account is in ſome Meaſure Confirmed by Pouring Cold Water upon Fire, or on Hot Melted Lead, allowing for the Difference of the Expanſive and Contractive Force of the One and the Other, in Compariſon with the Subtile Expanſive Force of the Sun Communicated to the Air, and the Subtile Contractive Force of Water Diſſolved into Vapors; For Pour Water in a Certain Degree upon Fire, and it will Emit a Flame and a Sound, as a Drop of Water upon the Flame of a Candle, *&c.* and Pour it upon Hot Melted Lead, and it will Produce an Exploſion.

It is farther Confirmed from what Happened in this Storm of Thunder and Lightning; For about Two Miles from this Place, an Hail Fell at the ſame Time, whoſe Stones were as Large as a Pullet's Egg, and not many Miles farther ſome Barns were entirely Burnt and Conſumed by Lightning, which are Manifeſt and Plain Inſtances of a Conteſt betwixt the Contractive and Expanſive Forces, and which Ended in Rain and a Sultry Heat, a Medium betwixt both, by which it is Evident, that when Extremes are Introduced into Nature, they at Length Settle in nearly an Equal Balance of ſuch Forces, which is the Doctrine I have all along Maintained, in all the other Parts of Nature.

And from Hence it comes to Paſs, that in Winter there is ſeldom any Thunder, becauſe the Contractive Force then Prevails, and there can be no Conteſt betwixt the Expanſive and the Contractive.

Concerning an Appearance of a Rain-Bow.

§. 5. *June* the 2*d*. 1726. I Obſerved a Curious and Spacious Rain-Bow upon *Everton-Heath*, with a Secondary One above it, the Sun was about Two Hours high, from it's Setting, and Shone from *Everton* Church, and the Bow Extended from *Sutton* on the Right Hand, to Sir *George Downing's* on the Left, about Two Miles, with *Potton* Church towards the Right-Hand Extreme of the Bow, and Lying within it; As I Rode towards the Left-Hand Extreme, I ſeemed Apparently to come nearer to it, and if the Diameter of the Bow had been Divided into Six Parts, I ſhould have been about One of thoſe Parts Diſtant from the Left-Hand Extreme, and Five from the Right; as I came nearer the Left-Hand Extreme, it Removed farther from me towards *Gamlingay*, and the Right-Hand Extreme came nearer, till at Length it ſeemed to Fall on *Potton* Church, and Riding ſtill farther towards the Left-Hand Extreme, the ſaid Church was entirely Excluded from the Bow at a Good Diſtance; The Solar Colour or Orange was Outmoſt in the Primary Bow, underneath was Yellow, Green, Blue, and Purple; The Outmoſt Colour of the Secondary Iris was Green, and underneath Blue and Purple, ſo that the Colours were in the ſame Order in the Secondary, as in the Primary, but did not Reach ſo far, the Yellow and Solar Orange being wanting; This does not ſeem ſo fully to Agree with what is Aſſerted by *Des-Cartes*, and other Philoſophers in Relation to this Phenomenon.

§. 6. After I had Printed a great Part of the Preſent Philoſophy of the Expanſive and Contractive Forces, that is, the Six Firſt Books of it, in One of which I had Treated of Vegetation, and had Declared my Opinion to be, that Plants in all Likelihood, had ſome Kind of Senſe and Perception, as being Animals of an Inferior Denomination, which Notion I had Embraced from Reading the Famous Dr. *Grew's* Anatomy of Plants, and *Eraſmus*'s Colloquy, Concerning the Amicitiæ and Inimicitiæ of Them, when at School, but more Eſpecially, from the Analogy of Nature, and a Conſideration of the Infinite Goodneſs, Bounty, and Beneficence of the Author of it; I was Agreeably Surprized with a Book Publiſhed by my Old and moſt Ingenious Friend Mr. *Stephen Hales*, Formerly Fellow of *Benedict College*, which I Think puts that Matter beyond Diſpute in his Vegetable Staticks, where he gives all the Probable Arguments, which can be, to Prove, that there is a Circulation and Expanſion of the Sap, that there is a Perſpiration of Plants, and that the Leaves are Anſwerable to the Lungs in other Animals; And if ſo, what Reaſon is there for us not to Think and Believe, that they have ſome Inferiour Degree of Perception, tho' Perhaps it may be Infinitely below That of a Cockle or an Oyſter? And as for my Doctrine of the Expanſive and Contractive Forces, it is fully Confirmed by what he there Delivers, Concerning the Expanſion of the Sap, and the great Analogy, which there is, between Plants and Animals.

Concerning Mr. Hales*'s Vegetable Staticks.*

§. 7. I ſhall Sum up this Inquiry into the Modern Philoſophy of Similar and Homogeneous Matter, with a Ludicrous, but a True and Real Repreſentation of it, according to what I have Formerly made to my Pupils in my Lectures to Them, which I Intended as ſome Kind of Diverſion to Their more Serious Studies of theſe Matters; For, there are Two Ways of Aſſerting Truth, and Confuting Error; Either by Arguments, which ſhall Explain the Falſe Reaſoning, or by Ridicule, which ſhall fully Expreſs the Senſe of the Errors, which it Propoſes to Explode; The Firſt Method is the Beſt and the Trueſt, or at leaſt the Simpleſt and the Cleareſt; The Laſt ought to be very Cautiouſly Uſed, and never without the Other; Since elſe it may Deviate into Wantoneſs and Wit, without any Solid or Rational Foundation for it's Support, and which can no more Contribute to the Real Knowledge of the Proportions, and Relations of Things, than a Piece of Witticiſm and Romance can Settle the Great and Important Truths of Nature.

A Juſt Ridicule of the Modern Philoſophy of Similar and Homogeneous Matter.

Thus, what I have Argued againſt the Syſtem of a Similar and Homogeneous Matter, on which all the Reſt Depends, Chap. 6, 7, 8, 9, and 10. in my Firſt Part of the Principles of Philoſophy, and Chap. 1. Of the Second Part, Printed in 1712, may be Repreſented in the Following, and a very True Manner; I Preſume to call *Des-Cartes*, the Grand Aſſertor of the Corpuſcularian Scheme, or That of Similar Matter, *Philomuthos*, or a *Lover of Romance*, and I would Humbly beg Leave to Name any Aſſertor of an Heterogeneous and Diſſimilar Matter *Philalethes*, or a *Lover of Truth*; *Philalethes* is Suppoſed to have his Pupils Sitting Round him, and *Philomuthos* is Imagined by his Book to Enter, and *Rohault*, by his, to Follow him, The Former of which comes with a *French* and Vaunting Air, and lays a Piece of *Lead-Ore* upon the Table before them, which Repreſents a Parcel or Portion of Real or Diſſimilar Matter, and which *Philomuthos* calls *Lead*, and is an Image of his Similar or Chimerical Matter.

Philomuth. Shentlemen, do you See That Piece of Lead?

Philaleth. How Monſieur? Lead! By the beſt Eyes, which I am able to Uſe, it ſeems to me to be Lead-Ore, Chap. 6. 7. Par. 1.

Philomuth. No Matter for that, Shentlemen, but do you See that Piece of Lead?

Philaleth. (A little Roughly) What muſt we Renounce our own Eyes and Senſes? Well! Why it may be ſo; Grant it Lead; What then, Monſieur, What will you do with it?

Philomuth. Do with it? Why, Shentlemen, I will Tell you; That very Piece of Lead, For I Say and Pronounce, it to be Lead, notwithſtanding this Man's Impertinence, I will Turn into Gold, Silver, Wood, Stone, into Water, or Fire; Nay, I will make it, if I Pleaſe, into a Diamond, and then again Transform it into a Feather, or any other Subſtance you ſhall Name, Chap. 8. and 9. Part 1.

Philaleth. Very Good! Monſieur; But How?

Philomuth. How? Ignoramus! And beſides that, by what I Perceive, Thou art a very Saucy and Troubleſome J—ck—n—pes, and therefore you may go about your Buſineſs, but as for you, Shentlemen, I will Tell you.

Philaleth. 'Tis well Monſieur, I Thank you for your Hard Names, and your Reproaches, but I muſt beg the Favour, you will Pleaſe to Satisfy me, as well as them; I want to Know, How you do This.

Philomuth. Why then, Shentlemen, I muſt Tell you and all the World, that I can do what I Say; For Firſt, I will Take this Piece of Lead, for it is no more Lead-Ore than it is Gold, and I will Divide it into Ten Millions of Millions of Variouſly Figured Parts, I will Then ſet them to Whirl round upon their Centers, and withal, to Move, Dance, and Play certain Vagaries, in a Promiſcuous Manner, or if I may Uſe, Shentlemen, that Vulgar Expreſſion, Helter-Skelter one amongſt another, by which I ſhall Gain Three Sorts of Matter, which are my Three Elements or Powders, of which all Bodies, or Beings, are Compounded, *Des-Cartes* Princip. §. 51, 52. Par. 3. *Rohault* Cap. 21. Par. 1.

1*ſt.* By theſe Parts Moving and Buſtling This Way and That, I ſhall have my Firſt Element or Powder, which will be the *Intertrimentum*, or the little Corners, or ſmall Angular Nooks, Worn off from Them by this ſame Inteſtine Motion.

2*d.* By this Tumultary Way of doing, I ſhall get my Second Element or Powder, which will Conſiſt of Terſe, Neat, and as Fine Globes, as any One would Deſire to See.

Philaleth. You mean Monſieur, I Suppoſe, as any One would Deſire to See of Things, which are Inviſible.

Deſcartes, and Rohault, in the Places Above-Cited.

Philomuth. Silly! 3*d.* Shentlemen, if there are any Parts, which are Reſolved, neither to be an *Intertrimentum*, nor to be Formed into Globes, why Thoſe I make my Third Element or Powder, and which are of a Thouſand Abnormous Figures, which it is Impoſſible for me to Recount.

Philaleth. I Perceive therefore, Monſieur, you go by Way of Powders; Pray, may they be Thoſe, which they call Pimper-le-Pimp, or what Sort may they be of?

Philomuth. Again! Why thou will't be ſtill Interpoſing thy Nonſenſe and Jargon, I Tell thee, Thou art a D—lt and a S—t.

Philaleth. But I beg, Monſieur, be Pleaſed to Tell us, What Way you may take to Divide this Lead, as you will have it, into all theſe Parts, and after you have Divided it into them, in what Manner do you Proceed to Put them into theſe Various Motions?

Philomuth. Why Truly, Shentlemen, I cannot do That my ſelf, but I know One that can.

Philaleth. I Deſire of all Goodneſs, Monſieur, you will Tell us, who that is?

Philomuth. (Looking very Gravely and Demurely) Is? 'Tis God.

Philaleth. I Suppoſe it is that Infinite Being, Monſieur, which you Endeavoured to Prove from an Idea of him.

Philomuth. (Yawning) Yes.

Philaleth. Why then your Deity and your Philoſophy, I believe, may go together, as Two Imaginary Things of your own Inventing.

Philomuth. Whether I believe a God, or not, that is nothing to you, or theſe Shentlemen, I will Prove, that, let me Divide Matter as I Pleaſe, I will do, what I Told you; I will Turn this Lead into Gold, Silver, Wood, Stone, Fire, Water, into a Diamond, or a Feather.

Philaleth. Good Monſieur, we cannot allow you to Divide Matter, as you think fit, unleſs you can Prove Almighty God has done it, which Being you have not Evinced by your Arguments for One, and as you have been Pleaſed to call me an Ignoramus, a Saucy and Troubleſome Jack——apes, I muſt Requeſt the Favour to Return the ſame Witticiſms and Civilities upon you; And to me you ſeem to take the ſame Method, as the *Tragedians* of Old, who, when they were under a Difficulty, they could not Surmount, Introduced a Deity to Extricate them from it.

Philomuth. You are the One great Engliſh F–l, or the Kn–ve, don't tell me of your Deities; your Sir *Iſ. Newton* is of my Opinion, Excepting, that he will not allow of one of my Elements or Powders, my *Intertrimentum*, but will have all the Parts, of which my Powders Conſiſt, Durable, Solid, and Immutable; That the Deity may not be at the Trouble of Mending Nature every Day, Sir *Iſ. Newt.* Optic. Pag. 343. and 345. *Rohault* Pag. 105. Dr. *Clarke.*

Philaleth. Why really Monſieur, That is one great Part of my Affliction in theſe Matters, that ſo Wiſe and Learned a Man ſhould be ſo Incautiouſly carried into the Sentiments and Whimſies of a *French* Mountebank.

Philomuth. Oh! Oh! Thou moſt *Engliſh* Sc——ndr——l, me have no Patience! Call me Mountebank? Shentlemen, I Tell you, that when theſe Parts of Lead, which are Finer than any Meal, or the moſt Impalpable Powder, are Expoſed to the Air, the Atmoſphere Surrounding them will Compreſs them together, and from thence Proceeds the Coheſion of Matter, Chap. 7. §. 4.

Philaleth. Monſieur, Give me Leave to Inſtruct you, by Asking you one Queſtion or Two, how it comes to Paſs, that the Air, unleſs very Calm Indeed, which would Blow away, or Diſſipate, ſuch a Powder, as you Speak of, ſhould be ſo Kind and Friendly, as to Cement your Leaden One? And whether it would not have been better for you to have Introduced the ſame Deity to put your Lead together again, which you had made Uſe of to Divide it?

Philomuth. Ignorant! But Shentlemen, I muſt Tell you farther, that, if theſe little Parts of Lead are Quieſcent amongſt one another, why there you have Hardneſs, if they are in Motion, I will Undertake for the Fluidity of that Body. Chap. 8.

Philaleth. Methinks, Monſieur, where there is the moſt Action, as in Motion, there ſhould be the moſt Reſiſtence, and therefore the greateſt Hardneſs, and where there is the leaſt Action, as in Quieſcence, there ſhould be the leaſt Reſiſtence, and therefore the greateſt Fluidity; But I do not Preſume to be Peremptory.

Philomuth. Shentlemen, I do not Mind or Regard, what that Obſtinate D–nce, and that C–nc–ted, Pr–t–nding, and Pr–gm—tical C–xc–mb Says, I Tell you, that my Leaden Powder, if the Parts of it are of a certain Kind of Figure, which it is not Neceſſary for me Preciſely to Tell you of what Figure, will become Water, and That Water will Diſſolve Salt, which is nothing elſe than a Maſs of another Kind of my Leaden Powder; Nay I muſt farther Inform you, that if I take another Sort of my ſaid Leaden Powder, and give it's Parts ſtill a different Figure and Shape, I will Engage it will Turn into Spirit of Nitre, or into Aqua Regalis, and will be able to Diſſolve Silver, or Gold, which I likewiſe Make and can Produce to you by another Compoſition of this my Specifick Powder of Lead; I will alſo Take ſome of the ſaid Powder, and will Inject it into certain Bodies, and it ſhall make them Swell and Rarify to a Prodigious Degree, and then by only Squeezing it out of them again, they ſhall become as Denſe and Hard as a Board; By Injecting the ſame Powder into other Bodies, they ſhall put on a Force of Reſiliency, and with another Kind of Management of it, I'll Venture to Affirm, I can make a very Good Handſome Flame Stare you in the Eyes, Chap. 8, 9.

Shentlemen, I Deſire you will not be Amazed, at what I am going to Aſſert to you; I will Raiſe in the Preſence of you all That Individual Piece of Lead, or, which is the ſame, my Powder of it.

Philaleth. Monſieur, Monſieur, Lead Ore!

Philomuth. Well! But I Say it is Lead, I Tell you, Shentlemen, I will Raiſe in the Preſence of you all, That Piece of Lead into a Tree, by Dividing the Parts, as they ſhould and ought to be Divided, or Make it into a Grape, a Pear, a Plum, or an Apple, Chap. 10.

Philaleth. Monſieur, I pray now Gently!

Philomuth. Why thou moſt Abſurd and Stupid Bl—ckh—d, ſince thou Endeavoureſt to Plague and Confound me, I will not only Raiſe That Piece of Lead into a Tree, by Dividing it into ſuch Parts, as I Pleaſe, but I will Throw it into an Horſe or a Man, or any other Animal, according as I Think Fit, and then I would Deſire to know, what will become of your Gently's?

Philaleth. I am Sorry, Monſieur, I ſhould put you into any Paſſion, but I hope you will Excuſe an *Engliſh* Freedom; We are not Uſed here to be Slaves, but to Speak our own Minds in the propereſt Way, which we can; However, ſince this Converſation is ſo Diſagreeable to you, I ſhall here End it, and would only Deſire to Know, Whether this Piece of Lead of yours, by a Juſt and Regular Diviſion of the Parts of it, can give us a Tune upon the Fiddle?

Philomuth. Humph! Why Truly no, Chap. 1. Par. 2.

Philaleth. Very well Monſieur; Perhaps then you and I may be Good Friends for the Future, notwithſtanding all the Hard Names you have Called me; If you will but be ſo Kind, as to Diſcard, as I have done, your Leaden Philoſophy.

Theſe are ſome of the Names and Appellations, I have met with from certain Perſons, the Admirers and Patrons of this Philoſophy, for my Examining only into ſo Ridiculous and Abſurd a Syſteme; Which is the Reaſon of my Introducing them here; Tho' *Des-Cartes,* who was Said to have been a Modeſt Man, would Scarcely have Uſed them himſelf.

But it is in Philoſophy, and in all Kinds of Learning, as it is in the Common Occurrences of Life, that the Slaves or Servants to any Men, or Opinions, are Generally more Inſolent and leſs Decent than their Maſters; And by this I would Deſire to ſhew likewiſe, that if the Corpuſcularians are for Laughing, and not for Arguing, I can fall in with the ſame Method, and that I am not Concerned by whatever Names I am Called, by my own, or any other; For Names ſeem to have no Peculiar Emphaſis in them, which are a mere Compoſition of Letters only, ſince the Real Inquiry will be, what is the Nature, Genius, and Temper of the Man, which I hope will be found to be as Good, and what his Applications and Studies, which I Preſume will Appear to be as Great, as Thoſe of any Perſons, who have Thought, or ſhall Think it their Intereſt, either to Ridicule, or to Reproach him.

Concluſion, On the Ways of Explaining Nature.

§. 8. I ſhall now put an End to my Labours and Fatigues in theſe Studies, which have been Continued for many Years, how Succeſsfully I know not, and muſt Leave to the Deciſion and Determination of Others; I can only Say, that I have Uſed all the Integrity Poſſible in the Purſuit of them, and tho' I Entered a Corpuſcularian into them, I now come out of them under a quite Contrary Conviction, and of which Studies I here take my Final and Laſt Farewell; After I have Subjoined the ſeveral Caſes, under which Nature may be Conſidered and Explained.

Matter muſt be therefore Similar or Diſſimilar, and there muſt be a Plenum, or a Vacuum; A Plenum and a Vacuum it is Evident cannot Subſiſt together, nor a Similar and Diſſimilar Matter; Conſequently there are only Four Caſes Remaining, and therefore there are only Four Poſſible Hypotheſes, which Philoſophers can Eſpouſe, or Maintain.

1*ſt.* That of Similar Matter and a Plenum, which is the *Carteſian* or the *French* Philoſophy, and which are Terms Inconſiſtent and Incompatible in Nature, becauſe, it is Impoſſible the World ſhould be full of Matter, if all Matter was the ſame, ſince then there would be no Difference betwixt Air, Water, or Gold, *&c.* That is, betwixt One Portion of Similar Matter and Another, ſince there are no Vacuities to Cauſe the Difference.

2*d.* That of Similar Matter and a Vacuum, which is that of *Galileus,* Sir *Iſaac Newton,* and our Modern Philoſophers, and is a Mixture of the *French* Derived from *Des-Cartes,* and of the *Italian* Derived from *Galileus,* and is the Old Syſteme of *Epicurus* Compleat, and which Leads us into a Thouſand Abſurdities, as I have Endeavoured to Evince.

3d. That of a Diſſimilar Matter and a Vacuum, which has hitherto had No Patrons, nor Perhaps is likely to have Any; Becauſe if Matter is Acknowledged to be Diſſimilar in it's own Nature, there will be no Occaſion for Vacuities, which are Introduced by Philoſophers to Difference one Portion of Similar Matter from another.

4th. That of a Diſſimilar Matter and a Plenum, which is the Syſteme I have Choſe to Maintain and Defend, and which, as it is Properly my Own, and has hitherto had no Advocates for it, ſo I may Juſtly call it a Philoſophy, which is entirely *Engliſh*, and I hope will in Time Appear to be the moſt *Rational.*

And as I Apprehend this Scheme of Nature to be the moſt Rational, ſo for many Reaſons before Explained, I Believe and Think, it will give us Truer Impreſſions of the Wiſdom, Power, Juſtice, and Goodneſs of the Supreme Being, than any Corpuſcularian, or Atomical, One can do; And Therefore I here End this Philoſophy of the Expanſive and Contractive Forces; Which I heartily Wiſh may Convince the Reaſoners of this Age, How Weak, Frail, and Imperfect, their Preſent Notions of Things are, How Juſt it is for us to Fall into the Sentiments, which I have Propoſed, in Reſpect of Nature and Philoſophy, and What a Value and Eſteem we ought to Entertain for our Chriſtianity, and our Religion, which has been too much Neglected and Diſregarded by Thoſe who have been the moſt Vigorous Patrons of That *Democritian* or *Epicurean* Scheme, which I have here Taken upon me to Refute.

FINIS.

MISCELLANEA
QUÆDAM PHILOSOPHICA,
ATQUE IMPRIMIS,
DISSERTATIO
DE
NOVA INFINITARUM SERIERUM DOCTRINA,
Prout a NEWTONO Aliisque Affirmantur,
In PRÆLECTIONES VARIAS DISTINCTA,
Ad Juventutem CLARENSEM Habitas,

A Roberto GREENE, *A. M.* Aulæ Clarensis Socio.

MISCELLANEA
QUÆDAM PHILOSOPHICA,
SIVE
DISSERTATIONES COMMENTATIONESQUE NONNULLÆ,

I. De Nova Infinitarum Serierum Doctrina, prout a Celeberrimo Newtono Aliiſque Affirmantur, in Variis Prælectionibus ad Clarenſem Juventutem ex Parte habitis, Perſcriptis autem & Exaratis Anno 1705. Cum Adſcitus fueram in hanc Academiam *Octob.* 5. Anno 1695.

II. De Alia Circuli Quadratura, præter Eam, quam in Geometria Solidorum Attulimus; Item, de Problemate Deliaco, ſive Cubi Duplicatione, Anno Circiter 1700.

III. De Sectionibus Conicis Clariſſimi Milneſii.

IV. De Longitudine Imperfecta Quædam, nec niſi Solummodo Inchoata.

V. De Philoſophorum Argumentis ad Vacuum Probandum, deque eorum Calumnia & Infirmitate, Anno Circiter 1701.

VI. De Motu, Gravitatione, & Cohæſione Materiæ, Eaque ex Immediata Dei Actione Profluere, Eodem fere Anno.

VII. De Rationibus Geometricis, Eaſque Animo perinde Competere, qua Percipienti, ac Materiæ, qua Extenſæ, Anno Circiter 1700.

VIII. Medicinæ Collaudatio, *Jun.* 28. 1707. Ad Amiciſſimum mihi Virum *Thomam Hewett* M. D.

IX. Ac Juris Civilis — Eodem Menſe & Anno. Ad Amiciſſimum mihi Virum *Thomam Paske* L. L. D.

X. Gratulatio ad Epiſcopum Elienſem, Reverendiſſimum Virum *Johannem More*, Patronum autem Quondam Chariſſimi Amici *Caroli Morgan*, A. M. Magiſtri hujus Aulæ Optimi, & quadantenus Meum.

XI. Oratio Panegyrica in Divum *Gulielmum*, Anno Circiter 1701. Ad Prænobilem Virum, mihi autem & Amico meo Humaniſſimum Patronum *Thomam Holles*, Ducem Præcellentiſſimum de *Novo-Caſtello*.

I. *Diſſertatio de Nova Infinitarum Serierum Doctrina, prout a Newtono Aliiſque Affirmantur.*

Prælectio Prima.

PLURES Jam Anni abierunt, Doctiſſimi & Optimi Juvenes, cum Noſtram Circuli Quadraturam publici juris fecerim, Quam, paulo poſtquam ex Veſtro Ordine, ſive ex Ephebis, Exceſſiſſem, adinveneram; Quid enim nos poſſet ſuſtinere, ne tantam Gloriam avide amplecteremur, quam facile quis autumaverit, in eum, quicunque inveniſſet, redundaturam? Nam, ſiquid extitit mirabilius, id omnino Dignum Admiratione putabitur, quod ſummi Mathematici Inveſtigarunt, nulli ſunt Aſſecuti, ſiquid Dignum Impenſiori Collaudatione, illud proculdubio, quod Inventum Omnes Admirarentur, Mathematici vero cunctis ſæculis propoſuerunt inveſtigandum; Hujuſmodi ne quicquam; Indocti & Inficeti homines pro Ludibrio & Ineptiis habent, ſi tam Glorioſum Problema Solveretur, Mechanici Homunciones, qui nihil ſapiunt, niſi quod ſuis Commodis Inſervire poſſit, Ogganniunt, Cui Bono? Quid inde accreſcit Humano uſui Fructuoſum? Hoſce quoſcunque Viros ablegamus, alteros ad Inſcitiam ſuam, alteros ad Mentem ſordibus & Lucro in ſuos Quæſtus abreptam; Quod vero ad Viros Doctos & Eruditos attinet, qui Veritatis Studium & Amorem præ ſe ferunt, hæc a me accipiatis, Mei Juvenes, ut qui nollem vos fallere, vel quidpiam vobis proferre, quod non ſit mihi Exploratiſſimum, Nimirum, eos Auctoritate Maximorum Virorum magis niti, quam Ratione; Præſenſi plane, quod mihi obtigit in hac Arena Verſanti, Iſtos, qui ſua Nomina Mathematicis Diſciplinis dediſſent, ſuos potius velle Amicos, quos diligerent, vel Auctores, quos adamarent, Propugnare, quam in ipſam ſcientiam Mathematicam Inquirere vel Perſcrutari; Dominum Newtonum Acceſſi, ut Conſulerem, Orantem, qui Chartulas Perlegeret, ipſis intactis, ne inſpectis certe, rejecit, aggreſſus ſum dein Epiſtola, recuſavit, quid poſthæc arbitremini me putaſſe? Saltem vel contemptum me, vel Problema.

Studia Mathematica ſtatim remiſi, ut ad alia me, non futura forſan uſque adeo ingrata, appellerem, animum plane depoſui de perficienda illa, quam ſuſceperam, Doctrina, Velocitatum, ſive Fluxionum, * Generaliſſima omnium, ni fallor, quæ hactenus comparuere; neque facile, crediderim, me poſſe iſta deturbari animi pertinacia dixerim, an firmitate, donec videam, quid inde Gratiarum ſim conſecuturus, ſi tandem inchoatum opus aliquando ad finem perducerem; Interea, ne ſibi videretur Newtonus ea omnia pernoſſe, quæ poteſt quiſquam proponere, vel quæ ignoraret, opinaretur negligenda eſſe, hæc de ejus Infinitarum ſerierum Doctrina, qua putat ſe quam proxime ad Circuli rationem acceſſiſſe, diſſerere Statuimus, ut ſimul Vobis & Erudito Orbi innoteſcat tum falſitas illius Diſciplinæ, tum periculum, quod imminet veræ Cognitioni, ubi comprehendiſſe eam non aliam ob cauſam videmur, quam quod viri ſumma auctoritate muniti, demonſtrationis perparci, tenebris circumfuſi & denſata caligine, non poſſent ſcilicet, ſi fuiſſet viſum, non ea oſtendiſſe, quæ indicaſſe Solummodo ſat magnum eſſet, ſatiſque præclarum; Utinam tamen Mathematici non amplius paterentur iſtam immanem quorundam Ingeniorum licentiam, ne apud Poſteros perſpicaciores irrideamur, qui, ex libidine certa dominandi in Matheſi, dictitare potius putaremur, quam illam veterem inſiſtere & nimis forſan, ſiqui ita ſentiant, inculcatam demonſtrandi viam, quæ certe Veritati debetur, & rationi erit ſenſiſque humanis tribuenda; Quam Lepida, ſi hoc modo Mathematicas res tractare pergamus, foret iſta olim querela, quam afferunt Hodierni contra Antiquos, *Convincimur*, non *Illuminamur*, cum eouſque proceſſit Matheſis, ut, quum alterum fere præſens ætas ſubduxerit, alterum jam etiam vix uſpiam reperiamus; Nihil Illuſtre, Nihil niſi admodum Obſcurum, vix Quicquam ſatis Explicatum; Ut, ſiquis minus norit Mathematicos, putaverit, vel ſcientiam ſe pudendam ſequi, vel ipſis quidem non nimis perſpectam. Quanta autem, Dii boni, quamque ſingulari benivolentia Walliſium oportet proſequamur, *Tu Decus Omne Tuis*——qui ita univerſam Geometriam declaratam velles, ut appareat, te nihil de iſto tuo Ingenio maximo timuiſſe, nihil de Studiis, quæ ornaſti; ſi Vir, cui Sumus reſponſuri, divinus plane, iiſdem veſtigiis inſtitiſſet, tantum jam partæ gloriæ ſplendorem addidiſſet, quantum ſua ipſius obſcuritas detraxit, & quem, nube amictum, Apollinem

* Geometria Sc. Solidorum ſive Materiæ, quæ a me Excogitata & Inventa erat multo ante Annum 1705, Nempe Anno Circiter 1699, vel 1700.

admirantur omnes, qua tandem Veneratione, quo animi Cultu, dignum eum judicaſſent, vi omnique ſua luce radiantem? Si longior autem nobis dabitur Tranſlatio, hoc, inquam, cedet commodum Animadvertenti ex illa Newtoni Tanti tamque Illuſtris involutione & Tenebris, ut, cum magis poſſit defixis oculis intueri, caligetque minus, tum maculas ejus & labes melius perſentiſcet cernetque.

Ad ipſum Newtonum jam Veniamus; Newtonus hanc habet ſeriem, $\overline{P+PQ}|^{\frac{m}{n}} = P^{\frac{m}{n}} + \frac{m}{n}AQ + \frac{m-n}{2n}BQ + \frac{m-2n}{3n}CQ$, *&c.* quam inſervire ait quarumcunque quantitatum Poteſtatibus quibuſcunque, integris vel fractis, inveniendis, porro & quantitatibus per Poteſtates aut Radices dividendis, *&c.* Quid magis eſſet optandum, quam ut ſe explicatiorem impertiiſſet? Quomodo enim probat? Neſcio; Si juſſeris demonſtret ſaltem, Obmuteſcit; Nonne etiam liceat mihi hanc ſeriem reſpuere abſque operoſa illa & vulgo quidem uſitata demonſtrandi pompa? Reſpuo, ergo falſa eſt; Quid poteſt eſſe præclarius? Hoc modo enim multo faciliorem, quam Veteres, aditum nacti ſumus ad ſcientiæ apicem, cum nihil hic opus ſit, niſi ut addiſcamus audacter affirmare; Intelligat vero interim Newtonus hanc ſuam aſſertionem totidem labefactari argumentis, quot confirmavit ipſe, velim nullis; Si, Inſpiciamus, præceperit, Literas ſuas ad Leibnitium poſteriores, Accedimus; Expoſuit ibi, fateor, vel Enarravit potius, Demonſtravit nequaquam, initia hujus diſciplinæ; Ad exordium iſtius Epiſtolæ hæc affert; In ſerie Curvarum, quarum Baſis ſeu Axis communis ſit x & Ordinatim-applicatæ $\overline{1-x^2}|^{\frac{0}{2}}$. $\overline{1-x^2}|^{\frac{1}{2}}$. $\overline{1-x^2}|^{\frac{2}{2}}$. $\overline{1-x^2}|^{\frac{3}{2}}$. $\overline{1-x^2}|^{\frac{4}{2}}$. $\overline{1-x^2}|^{\frac{5}{2}}$, *&c.* ſi Areæ Alternarum, quæ ſunt x, $x-\frac{1}{3}x^3$, $x-\frac{2}{3}x^3+\frac{1}{5}x^5$, $x-\frac{3}{3}x^3+\frac{3}{5}x^5-\frac{1}{7}x^7$, *&c.* interpolari poſſent, haberemus Areas Intermediarum, quarum prima $\overline{1-x^2}|^{\frac{1}{2}}$ eſt Circulus.

Prælectio Secunda.

Dr. Wallis. Opera Mathemat. Vol. 3. Epiſtol. Collect. Epiſt. 6.

Epiſt. 8.

Imprimis, cur dicantur Areæ a Newtono non ſatis liquet, cum ſint tantum Arearum rationes; ſi enim x æquetur R, vel Radio, quod fieri poteſt, $x-\frac{1}{3}x^3=R-\frac{1}{3}R^3$, & ſi pro Unitate ſumatur Verus Radii Valor, fiet $R^3-\frac{1}{3}R^3=\frac{2}{3}R^3=$ rationi Semiſphæræ ad Cylindrum, cujus altitudo eſt Radius, nemo autem aſſeruerit $\frac{2}{3}R^3$ eſſe reale iſtius figuræ ſpatium, ſi quidem, idem, neceſſe eſt, fateatur æquari Circulum ſemidiametri Quadrato, adeoque perfacile exhiberi ſolutionem Quadraturæ tamdiu deſideratam; Newtonus vero forſan ita ſe velit intelligi, Rationes dicet Arearum ſe indigitaſſe, cum Areas appellitabat.

Inquit porro Newtonus, Quarum prima $\overline{1-x^2}|^{\frac{1}{2}}$ eſt Circulus,] Quæ neque Area eſt, neque Ratio Areæ, ſed Terminus, quem alias vocat Generantem; at nihil hæc moramur, ſufficiat Newtonum monuiſſe, puto Mathematicum, de verbis ejus parum quidem cautis; ea, videlicet, vir ſummus negligit, ad res ſe totum accingit; Videamus,

Inquit præterea, ſi interpolari poſſent, haberemus Areas Intermediarum] Affirmat Newtonus, Walliſium fortaſſis ſecutus, ipſe homuncio nego; Quid rationis allegat Newtonus? Tanti viri auctoritatem habes, nihil amplius poſtulaveris; Quod tamen ad me attinet, fateor me eo magis rationem exigere, quo magis cujuſquam in rebus Mathematicis, quæ neſciunt, auctoritatem imperiumque contemnerem; Interpolationes vero haſce fieri non poſſe alibi ſumus Demonſtraturi, hic Newtono concedimus, at ineptas eſſe dicimus & inutiles Arearum rationibus in expoſita ſerie eruendis, Binis enim de cauſis; Primo, propter errorem, quem adverſus Walliſium probavimus alio in loco, neque opus erit iſto argumento in præſenti; Deinde etiam nihil valent hujuſmodi interpolationes; Quamvis enim media progreſſionalis inter unitatem deſideretur & Indicem Poteſtatis juxta compoſitionem ſimpliciſſimam, cujus radix *e. g.* exquiritur, unitate auctum, ad obtinendam rationem Areæ Radicalis reciprocam, inquam, quamvis illa deſideretur, ubi quantitates ſint in unum terminum compactæ, non tamen ſequitur, ut eodem modo procedamus, cum in plures diſcurrant, quod Newtonus debuiſſet oſtendiſſe antea, quam ad has intercalationes acceſſerat, nos vero poſſumus e contrario; *V. G.* $\overline{x^2}|^2$, juxta compoſitionem ſimpliciſſimam, $=x|^4$, requiratur radicalis areæ ratio, progreſſio erit 1. 3. 5, ratio reciproca ipſius 3 æquabitur $\frac{1}{3}=$ dictæ areæ Rationi; ſumatur jam $\overline{R^2-x^2}|^1$, quæ erit, vel ſaltem latet ejuſdem compoſitio, quoad dignitatem ſimpliciſſima, progreſſio ergo erit pro ratione radicalis areæ reciproca, 1, $1\frac{1}{2}$, 2, adeoque $\frac{2}{3}=$ dictæ areæ rationi, & ratio areæ Radicalis $=$ rationi areæ Quadraticæ, quod fieri nequit; Præterea, Demus R^2-x^2 eſſe compoſitam; invenienda erit, quam tamen nemo aggreſſus eſt, prius cujuſmodi ſit compoſitio vel Dignitas ſimpliciſſima, quam rectam poſſimus debitamque progreſſionalem interſerere; nam fruſtra mediam quærimus, ſi ignorentur extremæ; ſit $z|^{y}=\overline{R^{2}-x^{2}}|^{1}$, juxta compoſitionem ſimpliciſſimam, ergo inter unitatem & $y+1$ media progreſſionalis rationem exhibebit quæſitæ reciprocam, neque alia quælibet, quam ſiquis interpolaverit, incognita nec perſpecta y, omnibus peritioribus Geometris gratum fecerit, mihi autem puſillo humilique erit magnus Apollo; Neſcit hanc Quantitatem Newtonus, Audet interpolare; Imò inquiet forſan Newtonus, ſinguli termini, quorum poteſtates innoteſcant, queant intercalari; Pace Tanti Viri nego autem ita poſſe, ut intercalatæ deſideratam exhibeant areæ rationem, & ſi poſſent, Newtonum in his etiam erraſſe Affirmo; Non enim magis habetur ratio areæ Radicalis ipſius $R^{2}-x^{2}$ ex interpolatione ipſius R^{2} & poſtea x^{2}, quam ipſa Radix ejuſdem ex id genus evolutione, nimirum ex $\sqrt{R^{2}}-\sqrt{x^{2}}$; Si placet Newtono, ſit interpolatio ipſius R^{2} unitas, ad rationem Areæ Radicalis x^{2} obtinendam progreſſio erit 1. 2. 3. & $\frac{1}{2}x^{2}$ = dictæ areæ Rationi, unde, $1-\frac{1}{2}x^{2}$ elicietur, quod, niſi dempto Nominatore 3, minime cum Newtoni ſerie conſentit; At quis porro non videt hanc intercalationem non convenire ipſi $\sqrt{R^{2}-x^{2}}$, Sed $\sqrt{R^{2}}-\sqrt{x^{2}}$; Omitto dicere, ipſam R^{2} nullo modo poſſe interpolari, cum inter 1 & $0+1$ non reperiatur progreſſionalis media; Ergo, ſi neque termini ſimul ſumpti, neque ſeparatim, intercalationes præbeant, quæ recte exprimerent Rationem radicalis Areæ, merito pro ineptis liceat rejicere; nec video, quomodo fieri poſſit interpoſitio, niſi terminorum, quibus junctis utaris vel diviſis; Newtonus vero mallet intercalationem adhuc obſcuriorem, ut nec dixeris vel hos vel illos velle interpolatos ita conturbat & confundit omnia; Cujuſmodi autem hæc ſit Interpolatio in ſequenti Prælectione Exponemus.

Prælectio Tertia. Superiori in noſtra Prælectione contra Intercalationes quaſvis ſerierum Infinitarum Generatim Diſputavimus, jam ad eas, quas Newtonus adhibet, aggrediamur; Adoritur illico autem vir Maximus interpolationem Unciarum; Quamobrem? Quid inde emolumenti quæſo, Newtone? Quaſi Uncias cujuſque Termini recte interpolaveris, cum ipſos Terminos falſe.

Ad has interpolandas, inquis, notabam, quod in omnibus primus terminus eſſet x, quodque ſecundi termini $\frac{0}{3}x^{3}$, $\frac{1}{3}x^{3}$, $\frac{2}{3}x^{3}$, $\frac{3}{3}x^{3}$ eſſent in Arithmeticâ progreſſione, & proinde, quod duo primi termini ſerierum intercalandarum deberent eſſe $x-\frac{\frac{1}{2}x^{3}}{3}$, $x-\frac{\frac{3}{2}x^{3}}{3}$, $x-\frac{\frac{5}{2}x^{3}}{3}$, *&c.*

Cum itaque juxta Analogiæ regulam procedat Newtonus, licet maxime ſit fallax, hic præſertim, Petimus, ubique ut ſervetur, vel oſtendat ſaltem, ubi ipſe, & quare, conſiſteret; Quidni enim, cum addenda eſſet $-\frac{0}{3}x^{3}$ ad x, quo fieret interpolatio inter ipſam & $\frac{1}{3}x^{3}$, adjungeremus etiam $+\frac{0}{5}x^{5}$ ad $x-\frac{1}{3}x^{3}$, pariterque ad $x-\frac{0}{3}x^{3}$? Nam æque $x-\frac{0}{3}x^{3}+\frac{0}{5}x^{5}$ exprimet x, ac $x-\frac{0}{3}x^{3}$; unde, quæcunque fuerit regula interpolationis propter tertium terminum, ſive per Arithmeticam progreſſionem, ſive Triangularem, ſive ignotam porro quamlibet, vel Interpolatio & ſeries terminabitur, vel erit impoſſibilis; Nonne fuerit integrum, pari ratione, hoc modo arguere? Equidem, ne videtur hilum intereſſe, niſi dixeris hanc Doctrinam non perinde quibuſdam Mathematicis, ſi placet, hypotheſibus ſtabiliendis velificari. Verum, ut nihil prætermittamus, cum hujuſmodi ſeries in Locum, quem habet ratio areæ in Curvarum ſerie, in Numerum Terminorum, Signa, Dignitates, Numeraleſque Coefficientes, conſumantur, ſpeciatim de ſingulis inſtituetur Oratio, quæ ſi omnia probaverimus male eſſe collecta per interpolationes, contra illas, quemadmodum ſpero, non parum dixiſſe videbimur, ſi unum tamen alterumve, ſatis. De loco Rationis Areæ, patet Coefficientes Numerales terminorum ſecundorum, ductos in Poteſtatum Indices, conſtituere ſemper Numerum, qui, ſi Unitas adjiciatur, definiet locum, quem occupat Areæ Ratio in ſerie Curvarum, *e. g.* $\frac{0}{3}\times 3,+1=1=$ loco x; $\frac{1}{3}\times^{3},+1=2=$ loco $x-\frac{1}{3}x^{3}$; $\frac{2}{3}\times^{3},+1=3=$ loco $x-\frac{2}{3}x^{3}+\frac{1}{5}x^{5}$; $\frac{3}{3}\times 3,+1=4=$ loco $x-\frac{3}{3}x^{3}+\frac{3}{5}x^{5}-\frac{1}{7}x^{7}$, *&c.* ex Analogia itaque $\frac{\frac{1}{2}}{3}\times^{3},+1=1\frac{1}{2}$ debuiſſet oſtendere locum Rationis areæ $\overline{1-x^{2}}|\frac{1}{2}$, locum nempe impoſſibilem, præterquam quod, poſt interpolationem, ſecundum teneret, adeoque eſſet $1\frac{1}{2}=2$. Quod ad Terminorum Numerum attinet, quis non perſpicit eos in quaque Areæ ratione progredi Arithmetice? Ideoque oportuit Numerus Terminorum Rationis Areæ $\overline{1-x^{2}}|\frac{1}{2}$ eſſe $1\frac{1}{2}$, quod fieri nequit; & quoniam in tantum, Newtone, Analogias deperis, des, obſecro, veniam, ſi, quando nollem omnia percurrere, uno neque peracri argumento rem plane graviſſimam conficiam, ex Analogia cum dicam Finitam debere eſſe ſeriem, tu Infinitam aſſumis; Hæc vero levia ſunt, cum tuis tamen, ni fallor, gravioribus in hoc argumento comparanda; qualiaqualia reddimus tibi, Newtone, contempta eſſe agnoſcimus, &, ſi placet, ad ſigna veniemus; hic autem perquam mirum videtur Newtonum Duplicis methodi indigere, ut ſua ſeries $1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}$. *&c.* rite eruatur; Quare enim non conſtituantur Termini omnes, vel per interpolationem, vel multiplicationem continuam? Liquet verſutia, ratio non liquet; Nam, ſi Terminos omnes, per interpolationes ſigillatim Terminorum inſtitutas, exſculpſiſſet, ſtatim ſeriem definiiſſet ad $\frac{0}{3}x^3$ vel $\frac{0}{5}x^5$, nec infinitionis, quo ſe abderet, habuiſſet aſylum, ſi per Multiplicationem, numerum forſan deſideraſſet, unde progreſſionem inchoaret denominatorum; Quoniam, ſi ad interpolationes ſolum animadvertat, inter $\frac{0}{3}$ & $\frac{1}{3}$ erit intercalata quantitas $\frac{\frac{1}{2}}{3}$, & juxta ſuam analogiam debent nominatores binario creſcere; Siquid aliud velit, ei ſumus poſtea reſponſuri; Nominatores impræſentiarum omittimus; Quemadmodum autem quæſo, Newtone, hæc debita, quæ uſurpas, eſſe ſigna innoteſcit? Verum de hiſce ſignis, in Poſtera Prælectione, fuſiori Copia Diſputabimus.

Prælectio Quarta.

De ſignis jam ſumus Diſceptaturi, & ſi per interpolationem partim, partim per continuam multiplicationem, hanc ſeriem perficias, nam pro ſummo tuo in Matheſi imperio, quid velis, potes, Series erit $1-\frac{\frac{1}{2}}{3}$, *&c.* at hic denuo nova oritur difficultas, uſque adeo volupe eſt Newtono ſemper ſe tegi obſcuritate, nam Quantitas $-\frac{1}{2}$ per interpolationem inventa, pro ſua regula $1\times\frac{m-0}{1}\times\frac{m-1}{2}$, *&c.* multiplicari debebit in $\frac{\frac{1}{2}-1}{2}$, propter terminum obtinendum tertium, ſed mavult $+\frac{1}{2}$ aſſumere, aſſumimus etiam, poſito $-\frac{1}{2}$ pro ſecundo termino, & proinde ſeries conſtabit $1-\frac{\frac{1}{2}}{3}-\frac{\frac{1}{8}}{5}+\frac{\frac{1}{16}}{7}$, *&c.* ſi $+\frac{1}{2}$ recuſet, & jam magis placuerit $-\frac{1}{2}$, erit $1-\frac{\frac{1}{2}}{3}+\frac{\frac{1}{8}}{5}-\frac{\frac{1}{16}}{7}$; ſin autem per ſolam interpolationem rem velit procedere, actum eſt de ſeriebus infinitis; ſi per multiplicationem vero continuam, erit ſeries $1+\frac{\frac{1}{2}}{3}-\frac{\frac{1}{8}}{5}+\frac{\frac{1}{16}}{7}$, *&c.* Apponuntur hic Nominatores, multiplicandum inter, contemnendi; ex his, Newtone, quam vis, eligas, ea vel recte inſtituetur, vel non, ſi recte, illam ſeriem incaſſum $1-\frac{\frac{1}{2}}{3}-\frac{\frac{1}{8}}{5}-\frac{\frac{1}{16}}{7}$ profers, nam utræque nequeunt una conſtare, cum ſigna plane fuerint diverſa, ſique denominatores abjiciantur, ut habeantur Termini generantes, in ſeriebus prædictis illi ducti in ſeipſos nunquam reſtituent $1-1$; Dicendum eſt ergo potius, ut omnia poſſint ad amuſſim quadrare, quidvis enim erit aſſerendum, renuendum, ſumendum, modo hoc efficiatur, ut intercalatæ ſeries cum radicum extractionibus conveniant, dicendum eſt ergo, non iſtas ſeries recte inſtitui ad producendam rationem Areæ, fateor, & Ipſe Idem nego $1-\frac{\frac{1}{2}}{3}-\frac{\frac{1}{8}}{5}-\frac{\frac{1}{16}}{7}$, *&c.* recte eſſe inſtitutam; At quærimus nos, an juxta regulas Analogiæ vel Multiplicationis a Newtono præſcriptas non rite eliciantur? Si minus, vix habeam quod dicerem, niſi videri illas Dictis regulis obtemperare, & ſic videntur, ut neque obſcuritas Newtoni & tenebræ ſuadeant falſam eſſe viſionem; Hucuſque diſputavimus de ſignis Newtoni, prout ipſe eorum rationem ſit dignatus explicare, & quamvis non putem Doctiſſimum virum hæc adeo conturbate perplexeque cogitaſſe, ac dediſſe, cum tamen mallet rem conſulto dementare, non refugimus expoſitioni ejus perquam eleganti occurrere; imo inquiet Newtonus, ſervantur prædictæ Regulæ in ſeriebus illis conficiendis, ſi poneretur, ſigna ea ſequi, quæ multiplicatione Coefficientium vel interpolatione inducuntur, id vero dicet non eſſe concedendum; Quam itaque certam regulam immutabilemque, Newtone, affers, qua poſſimus debita ſigna evocare? Nullam plane; Si queam, dabo; In Arearum Rationum ſerie x. $x-\frac{1}{3}x^3$. $x-\frac{2}{3}x^3+\frac{1}{5}x^5$. $x-\frac{3}{3}x^3+\frac{3}{5}x^5-\frac{1}{7}x^7$, Termini ſemper alterni, quoad numerum locorum quos poſſident, pares ſignum ſibi vendicant ei contrarium, quod conflatur Unciæ ex multiplicatione $1\times\frac{m-0}{1}$, *&c.* in Poteſtatibus integris, impares vero alterni ſignum idem; vel verius, cum ſigna non ad coefficientes Numerales pertineant, ſed ad Quantitatis literatæ Dignitates, Unitas, & Poteſtates, quarum indices Unitate minuti juxta Newtonum, vel ſi pro una quavis x ſumatur unitas,

tas, minuti Nihilo, binariorum continent numerum parem, ſignum ſemper poſitivum aſciſcunt, quarum indices, ſic minuti, imparem, reddunt privativum, ergo ex Analogia & ſecundum poſterioris noſtræ Regulæ ſententiam idem in fractis Poteſtatibus evenire concipias, coefficientes nempe Numerales nihil ad ſigna valere, unde erit ſeries $x-\frac{\frac{1}{2}x^3}{3}+\frac{\frac{1}{8}x^5}{5}-\frac{\frac{1}{16}x^7}{7}$, *&c.* Num hæc placet Newtono? Certe neque cum interpolationibus ſuis neque extractionibus radicum congruit; Confugit ad priorem, quæ ſeriem $1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}$, *&c.* in ſeriem deſideratam immutabit $1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}$, *&c.* Cur igitur interpolationem adhibet Newtonus, cum hæc ſeries ſoli Multiplicationi debeatur & ſignorum Analogiæ? Mihi quidem videtur res conſulto perturbaſſe, adeo nihil in his ſeriebus demonſtrat, ne ſignificat quidem, neſcio, an quo magis reconditiorem ipſius Scientiam miremur, an negligamus imperioſam; Intelligat vero Newtonus priorem iſtam regulam labaſcere, niſi ſigna parium locorum contraria, imparium eadem, cum ſignis $\overline{-x^2}|m$ coincidant, id quod profecto accidit, ubi poteſtates fuerint integræ, in fractis autem nequaquam; ita ut, ſi vel Regulæ iſti vel Analogiæ recte obſequi velimus, ſumenda erit ſeries $1.\ \frac{1}{2}.\ \frac{1}{8}.\ \frac{1}{16}$, *&c.* quæ convertetur in $1-\frac{1}{2}+\frac{1}{8}-\frac{1}{16}$ contra quod aſſerit Newtonus; Si reponat Vir ſummus, neque enim vel leviſſima velim præterire, de ſignis non eſſe ſequendam Analogiam; Cur ponitur $\frac{x-\frac{1}{2}x^3}{3}$ nullam aliam ob cauſam ex vera interpolatione & analogia, quam quod − ſemper inveniatur in ſecundo termino? Si Analogiam proſequi debeamus ad ſecundum uſque terminum, non vero prorſus, præclare ſed tamen precario; & quare illud? [ad reliquas intercalandas conſiderabam, quod Denominatores 1. 3. 5. 7. *&c.* erant in Arithmetica progreſſione, *&c.*] Si denique regerat ipſe, omnes Terminos, qui a ſecundo $\frac{\frac{1}{2}x^3}{3}$ dependent, ſe reputare veluti ejuſdem partes, adeoque ſignum inibi præſcriptum reliquis eſſe commune, audio, aſſentiri nequeo; Qua enim ratione duo tantum termini huic ſeriei debentur? Num ex dilectiſſima ſua Analogia patet? Quia, puto, ea Series, cujus hæc eſt radicalium, habet tantum duos, quo nihil dici poteſt abſurdius; vel forſan, quia Secunda Areæ Ratio, in Curvarum ſerie, iſto numero gaudet, ergo in eadem ſerie interpolata, ſecunda obtinet eundem, &, ſi placet, ideo quia, in dicta ſerie, areæ Ratio tertia tres ſibi vendicat terminos, in eadem interpolata $x-\frac{1}{3}x^3$ quæ binarium numerum oſtentat, ternarium etiam terminorum poſſidebit: Dabit mihi Veniam Newtonus, ſi ludere vel ineptias jam videar effutire, cum hujuſmodi argumentis reſpondere velim, nam qui putet rem in Matheſi adeo momentoſam nullis poſſe confirmari, facile ſentiet præciſa oportere minutiſſima; Neque dici poteſt, quod ſolum reſtat, ſi multiplicemus $1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}$, *&c.* Quæ ſeries Unciarum oritur ex Continua Multiplicatione in $-x^3+x^5-x^7$, nempe $+\frac{1}{2}$ in $-x^3$, $-\frac{1}{8}\times+x^5$, $+\frac{1}{16}\times-x^7$ Rem inde confici, quia $+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}$ deſumuntur, quoad ſigna, ex præſcriptione Terminorum $-x^3$, $+x^5$, $-x^7$, *h. e.* ſi minuantur Poteſtates una Dimenſione x, quia $-x^2$ habet $+\frac{1}{2}$ ſibi attributam, ergò $-x^2\times-x^2=+x^4$ obtinet $-\frac{1}{8}$, ut cum ſerie deſiderata $-\frac{1}{8}x^4$, *&c.* conveniret; Omiſſis ſignis, ad Dignitates Terminorum Accedimus, nego enim x^3, x^5, *&c.* ſeriebus hiſce infinitis competere, ſed hæc Dixiſſe impræſentiarum ſufficiat.

Prælectio Quinta. De Signis Infinitarum Serierum hactenus Diſſeruimus, Juvenes mei optimi, de Terminorum Locis, & de Methodo Interpolationis; pertendimus jam ad Dignitates Terminorum, quas Negavimus hiſce ſeriebus Infinitis Competere; Quid inquiet Newtonus? Radicum extractio, tum hæc, tum illa ſuperiora comprobat, quæ inani opera Niteris convellere; Nihil, in Præſenti, nobis cum Radicum extractionibus; Invenit has ſeries Newtonus, priuſquam illa methodus ipſi innotuerit, ut ipſe fatetur; qua ratione igitur tanta extitit inventio, jam inquirimus, vel, an ullo modo ex principiis ipſius hæ percelebres poſſent ſeries derivari; ex principiis ejus dico x^3, x^5, *&c.* hiſce ſeriebus non omninò contingere; Quare enim contingant, nolit diſerte edicere, liceat ergo agere conjectura; crediderim, quoniam in Poteſtatibus integris primus ſemper terminus habet x, ſecundus x^3, tertius x^5, vel $\overline{x^2}|m$, una cum Multiplicatione x in Dignitates, ergo idem velit fractis oriturum; At quid ſi pari ratiocinio dicerem debere fractarum poteſtatum Numeratores, quos affirmat Newtonus eſſe $\frac{1}{2}.\ \frac{1}{8}.\ \frac{1}{16}$, *&c.* de integris deſumi? Aſſentiretur Newtonus? Nullo modo; Quid enim fiet de comprobatione ejus ex radicibus evolutis? Quid de admirabili ſua

Serie $\frac{m-0}{1}\times\frac{m-1}{2}$, *&c.* ad fractas Poteſtates applicata? Ne leviſſimo tamen argumento ſuaſit, quin æque idem eſſet pronunciandum de Indicibus Dignitatum in Poteſtatibus fractis, ac de earundem coefficientibus Numeris; Si ad dictam ſeriem $\frac{m-0}{1}$, *&c.* propter coefficientes inveniendos nos amandarit, nullam iſtiuſmodi vero agnoſcat, qua eruantur Dignitatum Indices; primum, inquam, diſpicere decuit, annon talis reipſa extaret, deinde, ſi mallet neſcire, ne ſuccenſeat, ſi tamen ſuppeditarem; nam regula erit propter dignitates termini cujuſque, incipiendo ab Ultimo, Primumque verſus inſtituendo progreſſionem, $\overline{x^2}|m$, $\overline{x^2}|m-1$, $\overline{x^2}|m-2$, $\overline{x^2}|m-3$, $\overline{x^2}|m-m$, ſi fuerit Poteſtas biquadratica, vel ſi quævis alia poteſtas, ſimiliter, ponendo ſcilicet pro communi altitudine x æqualem Unitatem in areæ ratione, ſin autem id minus concedat Newtonus, qui ſuo jure aſſumit quodlibet, $\overline{x^2}|m-0, \times x$; $\overline{x^2}|m-1, \times x$; $\overline{x^2}|m-2, \times x$; $\overline{x^2}|m-3, \times x$; $\overline{x^2}|m-m, \times x$, & ſic de cæteris; Quid ſi jam Terminorum Radicum exquiramus Dignitates? Aderunt diverſæ plane ab iis, quas Vir ſummus ſtatuit, nimirum $\overline{x^2}|\frac{1}{2}-0, \times x$; $\overline{x^2}|\frac{1}{2}-1, \times x$; $\overline{x^2}|\frac{1}{2}-2, \times x$; & porro in infinitum, quos Infinitos Errores cum correxerit Newtonus, totidem alios infra dabimus corrigendos; Interea, ſi extractio radicum, ſuæ ſeriei, quod ait, conſentanea, hæc omnia diluat, lubenter poſt hæc fatebimur nos fruſtra diſputaſſe; At, ſi vera fuerit & genuina radicum Vulgaris extractio, liceat, Newtone, qui has ſeries prius inveneris, quam radicum extractionem, paululum mirari quorundam hominum Solertiam, qui ad operationem adeo perſpicuam, ac Radicum extractionem, obſcura adeo & anfractuoſa ſemita, ac hujuſmodi interpolationibus, ſummo conatu Geometras, nec parvo molimine, ſatagunt deducere, & poſtea, tantam poſſe artem in quolibet exiſtere, tantumque ingenium, ut tam falſa via & retrograda ad veritatem perveniret, ea nempe ad metam, quæ ducit ad carceres; Si tamen aliam extractionem callidiorem habes, quam hucuſque percrebruit, defendas ſtrenue, tuo more, ſi placet, tenebris & auctoritate, didicimus enim neque hanc neque illas magnopere pertimeſcere; Venimus jam tandem ad Coefficientes Numeros, de quibus cum adhuc ſumus dicturi, quæ omnia diximus, Newtono condonamus; Verum de his in poſtera Prælectione.

Prælectio Sexta.

Conſulas in hac Diſſertatione ab Ipſius Initio Arithmeticam Infinitorum Clariſſimi Walliſii, aliter Hæc Intelligi non Poſſunt.

Ad Coefficientes Numeros in hiſce ſeriebus Infinitis Jam devenimus; hæc vero diſputatio, tum ad Coefficientium Numeratores, tum ad Nominatores pertinebit; Nominatores primum perpendemus, & quoniam Newtono placent denominatores 1. 3. 5. 7. *&c.* Sinat, rogem ſaltem, quamobrem? Si dixerit, quia eoſdem ſibi aſſerunt Integrarum poteſtatum termini; Quid prohibet, cum Newtonus affirmet Nominatores eſſe de integris, quia in poteſtatibus integris iidem inveniuntur, quin & ipſe pronunciem Numeratores etiam inter integros verſari, quia in poteſtatibus integris idem experimur? Sigillatim, Unciæ in poteſtatibus integris eam ſemper rationem habent ad Dignitatum Indices x^3, x^5, x^7, *&c.* quam exprimunt Terminorum Coefficientes Rationes, quæ item exiſtunt inter Numeros integros, ubi Indices dignitatum fuerint porro integri, ergo, pro analogia ſæpius uſurpata, cum Indices Radicalium, x^3, x^5, x^7, *&c.* juxta Newtonum ſint integri, debebunt etiam Unciæ eſſe integræ, quas itaque ſi reducamus, $\frac{\frac{1}{2}}{3}=\frac{1}{6}=\frac{2}{12}$, *&c.* poſito Newtoni Nominatores, in ratione ad Uncias, eſſe veros; Unde hoc ſequeretur parum ejus ſeriei conveniens, ut ſi deponantur Nominatores, quod petit, quo Termini habeantur generantes, nullos iſtiuſmodi conſequamur, quoniam Uncia non eſſet $\frac{1}{2}$, ſed, vel 2, vel numerus aliquis integer; quomodo hæ autem Unciæ approbantur a Radicum extractionibus, videat Newtonus, ipſe quidem Vir hebes & indoctus me profiteor non eſſe Lynceum; Si dicatur in poteſtatibus integris rationem Terminorum Coefficientium exiſtere inter Numeros integros, idem non evenire in fractis; Reſpondeo x, x^3, x^5, *&c.* ſemper haberi in poteſtatibus integris, in fractis vero aliter evenire; Uncias confici ex multiplicatione $1\times\frac{m-0}{1}$, *&c.* in poteſtatibus integris, in fractis non item; ſecundos Terminos procedere in Arithmetica Unitatis progreſſione in poteſtatibus integris, non vero in fractis; denominatores binario Numero creſcere ibi, hic tamen non eadem integra ratione; Nonne agnoſcit Newtonus hæc omnia æque negari poſſe? Iniquum eſt itaque, & perabſurdum, alia arripere, dum contemnit alia, nullo neque exiſtente neque demonſtrato diſcrimine, cur hæc aſſumantur, Newtone, aſperneris illa, niſi ut ingenioſam, ſi ita putes, hypotheſin conficeres; At

Nominatores neque recte colliguntur; nam x^3 rationem habet $\frac{1}{3}$, x^5 rationem $\frac{1}{5}$, *&c.* fateor, ſed nego x^3, x^5, *&c.* ſeriebus infinitis accidere, probavimus enim indices, a primo termino ſi inceperis, eſſe $\overline{x^2}|m-\infty, \times x$; $\overline{x^2}|m-\infty+1, \times x$; $\overline{x^2}|m-\infty+2, \times x$; adeoque Nominatores erunt $m-\infty+0$, $m-\infty+1$, $m-\infty+2$, *&c.* ſi prius ſingulos duxeris in 2, poſteaque adjicias Unitatem. Abjiciantur, inquit Newtonus, Denominatores, & Termini provenient, quibus gignuntur areæ,] Vix fidem habeam, niſi tantus Vir affirmaverit, hæreo tamen, cum ex modo dictis conſtet, ſi *e. g.* $\frac{\frac{1}{2}}{3}$ eſſet vera ratio inter Unciam & Nominatorem, debitum tamen nominatorem eſſe vel 6 vel 12, *&c.* ex Numeris aliquibus majoribus, quam 3, deſumendum, Unciam ex majoribus, quam $\frac{1}{2}$, quod primo impetu tollet harumce ſerierum convenientiam cum Radicum extractionibus; deinde ex præmiſſis porro liquet $\frac{\frac{1}{2}}{3}$ *e. g.* non eſſe rationem inter Unciam, ſi placet, $\frac{1}{2}$, & Nominatorem, qui erit $\frac{1}{2}$. $\frac{1}{2}-\infty+1\times 2, +1$; & denique probaturi ſumus $\frac{1}{2}$, $\frac{1}{8}$, *&c.* non eſſe quas deſideramus Uncias; quæ nihilominus omnia requiruntur, ut, abjectis Nominatoribus, habeantur Termini Generantes, & iſtiuſmodi Generantes, ut Newtoni ſeries poſſint cum Radicum ejus extractionibus congruere. Abjectis, inquit, Denominatoribus provenient Termini Areæ generantes.] Videamus, an ſe melius gerat Newtonus in Poteſtatibus integris, ergo $x-x^3$ erunt Termini Areæ Lateralis Generantes, $x-2x^3+x^5$ Quadraticæ, *&c.* Fateor me vix uſpiam intelligere, quid velit hicce nebuloſus, licet quantus Vir, non quod nimis forſan ardua profert, ſed quod dubia; Quis enim Sciolus in Mathematicis non exiſtimaverit ſe perſentiſcere, quid ſint Termini areæ, vel areæ potius rationis, Generantes? Moneo, ne tamen cognoſcere ſe præfidat, ſi putet eſſe $1-x^2$, $1-2x^2+x^4$, cum Newtonus præcipiat totius Areæ altitudinem, ſi recte, quid velit, reſciverim, ſingulis Terminis eſſe apponendam; Hiſce vero non immoramur, ad Uncias tranſeamus; Imprimis igitur dicimus fractarum poteſtatum Uncias ex qualibet interpoſitione perfici non poſſe, adeo ut fruſtra interpolationibus incumbat Newtonus, neque enim ipſa fieri poteſt debita interpoſitio, quoniam ſi liceret interſerere quaslibet deſideratas Quantitates inter x. $x-\frac{1}{3}x^3$. $x-\frac{2}{3}x^3+\frac{1}{5}x^5$, haberetur item interpolatio inter aggregatos Rationum Arearum Coefficientes $=1\times\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}$, *&c.* cum $1-\frac{1}{3}=\frac{2}{3}$. $1-\frac{2}{3}+\frac{1}{5}=\frac{8}{15}=1\times\frac{2}{3}\times\frac{4}{5}$, *&c.* verum liquido apparet hanc ſeriem non poſſe interpolari, quia non eadem eſt ratio Arithmetica inter 1 & 2, quæ exiſtit inter 2 & 4. Interea non latet nos Newtonum ſeriem excogitaſſe, quam inſervire putat quarumcunque poteſtatum Unciis Indagandis, ni fallor, quia facto tentamine cum Integrarum poteſtatum Unciis, an fortuito, vel certa ratione, non dicit, coincidit, ad quam ſi denique provocarit Newtonus, modo hanc ipſam defendere & tueri ſolam polliceatur, quæ omnia hucuſque attulimus facile reddimus, decedimus de noſtro jure, certaque ratione, non fortuito, etiam eum hanc ſeriem exſculpſiſſe fatebimur, pro recepta illa, poſt Maximum & Celeberrimum Walliſium, Inductionis methodo; Quæ diximus, tamen conceſſis, ne quidem perſtrictim oſtendit Poteſtates fractas eidem Regulæ Serieive obtemperare; De hac autem Unciarum Regula, ſequenti in Noſtra Prælectione, fuſius liberiuſque diſputabimus, huic, ne Veſtram, Optimi Juvenes, patientiam diutius, quam par eſt, detineam, finem jam-jam Impoſituri.

Prælectio Septima.

De Newtonianis Unciis Verba facturi, & de ipſius Regula, quam iis Excogitavit Inveniendis, dabimus inde elici Uncias *e. g.* Quadrati 1. 2. 1, Cubi 1. 3. 3. 1, *&c.* & puto 1. 1. Lateris; Num ergo ſequitur eodem prorſus modo derivari Uncias Radicis Quadraticæ, Cubicæ, *&c*? Si quidem, valeat inepta illa Poteſtatis directæ & reciprocæ diſtinctio, quæ ita, quod velit Newtonus, æquatis legibus obſequuntur, ut revera nullæ forſan quantitates exigant magis diverſas; Dicat, ſi lubitum fuerit, totum, quod intereſt, diſcrimen, in literam m conferri, precario tamen, nulla enim adhuc probatio; Quod ſi m pro arbitrio aſſumi poteſt pro n ſcilicet, vel $\frac{1}{n}$, nimirum, ſive directa, ſit poteſtas, ſeu reciproca, cæteris ubique manentibus, non fuiſſet opus *ad Lem. 2. Lib. 2. Princip. Mathemat.* Caſu ejus quarto, cum regula $maAm-1$ complexa eſſet omnes; Recte tamen, quæ ſemper vel tantillo demonſtrationis contigit felicitas, ſuo poteſtatum generi ſuam regulam aſſignavit; Permittatur ipſi, quam poſtulavit literam m, ſi modo ſiverit, rogarem, quid iſti numeri appenſi velint? Eſto, illos nihil mutari, ſemper autem integros oriri, dum maneat Index poteſtatis integer, an exinde pateat iſtos numeros ſuam tueri debere integritatem, ſi index in fractionem verteretur? Vides, Newtone, quam Matheſis in æternam diſputationem abiret, ſi præcepta tua pro Oſtenſionibus habeantur; Verum, ut apertius tecum agamus, illa tua ſeries, omiſſis, quæ principia petiiſti, reipſa in aliam ſe formam induit, quam hic habeas pro ſexta Poteſtate, nimirum, in $1\times\frac{m-0}{m-5}\times\frac{m-1}{m-4}\times\frac{m-2}{m-3}\times\frac{m-3}{m-2}\times\frac{m-4}{m-1}\times\frac{m-5}{m-0}$, ita ut Numeratorum progreſſio ſit Nominatorum reciproca, & quod de ſexta poteſtate valet, ad quadraticam, cubicam, vel lateralem etiam, pertingere videbitur, ſi progreſſio inſtituatur ad exemplum, adeoque ſeries Unciarum pro Radice Quadratica erit $1\times\frac{\frac{1}{2}-0}{\frac{1}{2}-\infty+1}\times\frac{\frac{1}{2}-1}{\frac{1}{2}-\infty+2}\times\frac{\frac{1}{2}-2}{\frac{1}{2}-\infty+3}\times\frac{\frac{1}{2}-3}{\frac{1}{2}-\infty+4}\times\frac{\frac{1}{2}-4}{\frac{1}{2}-\infty+5}\times\frac{\frac{1}{2}-5}{\frac{1}{2}-\infty+6}\times\frac{\frac{1}{2}-6}{\frac{1}{2}-\infty+7}\times\frac{\frac{1}{2}-7}{\frac{1}{2}-\infty+8}$, &c. ex qua nunquam eruet Newtonus $1.\ \frac{1}{2}.-\frac{1}{8}+\frac{1}{16}-\frac{5}{128}+\frac{7}{256}$, niſi demonſtraverit $\frac{1}{2}$ æqualem eſſe infinito Numerorum aggregato juxta unitatem creſcentium, quo invicem ſe perimant deſtruantque; Unde clarius porro luce pateſcit non oportere Unitates adhiberi in determinandis Poteſtatum reciprocarum Unciis; nam ſecundum prædictam ſeriem $1\times\frac{m-0}{m-5}$, &c. ut poteſtatis cujuſvis Uncias extrahere poſſimus, neceſſe eſt in Dividendis propius ſemper poſteriores quantitates numerales ad literatas priores accedant, in Diviſoribus a ſe invicem magis inter progrediendum removeantur, ſi vero in fractis poteſtatibus Unitates aſciſcamus, plane e contrario accidit; $\frac{1}{2}-0$, $\frac{1}{2}-1$, $\frac{1}{2}-2$, cum debeant appropinquare, latius divagantur, $\frac{1}{2}-2$, $\frac{1}{2}-1$, $\frac{1}{2}-0$, cum receſſiſſe oporterent, ad complexus advolant; Præterea Terminorum Uncias uſque ad mediam elevari cernimus, in Newtoni ſeriebus continue ſemperque minuuntur; Hæccine tandem ſummus Vir & apprime in Mathematicis verſatus? Quid, obſecro, ex his poſſumus colligere, niſi quod malles hic Auctoritati tuæ & Nomini rem totam concredere, rationem qui tuam toties, tantiſque auſis, alias Confirmaſti? Præter hæc, $\overline{1\,1}|^{1}$, uti ipſe etiam affirmat, $=1+1=2$; $\overline{1\,1}|^{2}=1+2+1=4$; $\overline{1\,1}|^{3}=1+3+3+1=8$, &c. & Procedent numeri, ut a, a^2, a^3, &c. vel a^m; ſed m, ſi Newtonum auſcultemus, exprimere debet, fractam æque quantitatem, ac integram quamvis, ſi ſint Dignitatum Indices, ergo $\overline{1\,1}|^{\frac{1}{2}}=\overline{1\,1}|^{m}$ reddet Unciarum aggregatum, quod æquabitur $\sqrt{2}$; Inquiratur itaque primo, an $1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}$, &c. in Infinitum æqualis fuerit 1,414, &c. =, pro radicum vulgata extractione, radici 2; Deinde Numeralium Unciæ Poteſtatum integrarum conficiuntur ex Involutione $1+1$, quarum ſigna eadem erunt quæ Literatarum, ſi annexæ eſſent, dignitatum; Fractarum autem forſan, ſaltem, ut velit Newtonus, ex Evolutione deſerimur a ſignis dignitatum; Ad Newtonum confugimus; Itaque $1+\frac{1}{2}-\frac{1}{8}$, &c. putanda eſt Evolutio; Adeoque jam ſignorum præfigendorum quantitatibus x^2, x^4, &c. habet poteſtatem; Numquid amplius poſtulat Newtonus? Sufficiuntne ea, quæ conceſſimus? Adducamus $\overline{1-1}|^{1}, \overline{1-1}|^{2}, \overline{1-1}|^{3}, = 1-1=0, =1-2+1=0=1-3+3-1=0$, ergo $\overline{1-1}|^{\frac{1}{2}}=0$; Inquiratur itaque, an $1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}$, &c. æquaretur Nihilo, * $=1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}-\frac{1}{32}$, &c. Porro 1, $+\frac{1}{2}\times-x^2$, $-\frac{1}{8}\times x^4$, $+\frac{1}{16}\times-x^6$, &c. efficit $1-\frac{1}{2}-\frac{1}{8}-\frac{1}{16}$, quæ Series aliter eſſet $1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}$, eadem nempe, quam ſtatuit Newtonus eſſe Evolutionem ipſius $1+1$; Hæccine etiam deſideras, Newtone, hæccine item efflagitas? Nimis profecto libertate tibi conceſſa abuteris & Mathematicorum clementia; Petis, ut ne Unciæ tuarum ſerierum ſignis obſequantur dignitatum, Obtines; Poſcis, ut dignitates Unciarum ſigna ſubeant, Impetras; Pœnitet te poſtulatorum, & denique velis, ut modo ſigna Unciarum dignitatibus obtemperent, modo dignitates Unciis, ne forſan altera alterius ægre ferat imperium; Quod ſi omittamus Quantitates literatas, $\overline{1+1}|^{\frac{1}{2}}$ inquis æqualem eſſe $1\times\frac{m-0}{1}\times\frac{m-1}{2}$, &c. Eſto; Quæſo jam abſque ope literarum ex hac ſerie educeres $\overline{1-1}|^{\frac{1}{2}}$; Nonne vides, vel falſam eſſe regulam hanc tuam, vel inconſtantem & mutabilem? De Evolutionibus autem alibi; Hæc vero putes tamen dicta eſſe, etiam alio conceſſo poſtulato, m nempe exprimere poſſe, ſervato debito juſtoque diſcrimine, tum $\frac{m}{1}$ tum $\frac{1}{m}$. Atque adeo denique ad ſeriem venimus Newtoni ineunte hac diſſertatione deſcriptam, nam pergit; [Sic itaque, inquit, in Epiſtola poſteriori cum D. Leibnitio communicanda, innotuit mihi generalis Reductio Radicalium in infinitas ſeries per Regulam, quam poſui initio Epiſtolæ prioris, antequam ſcirem Extractiones Radicum.] Et quidem, ſi recte intellexiſſet ſua ipſius principia, vidiſſet, poſtquam cognoverat Extractiones

Vid. Geometr. Solidor. Sive Mater. Pag. 37. 38.

* Sturmius. Mathes. Enucleat. Pag. 120.

tractiones Radicum, iſtas quas tandem invenerat, fuiſſe falſas; Ita enim abſunt extractiones illæ, quæ ſuam comprobarent regulam, ut eas ipſas tollant potius intercalationes regulamque inde deductam, penituſque pervertant, cum ex jam demonſtratis totæ illæ ſeries ruant, & regula ea, quam profert, ipſo Newtono confitente, dependet ex iis, quæ conati ſumus refellere & reipſa pervulſa eſſe putamus, ni fallit augurium; Ut neque amplius opus eſſet in cæteros caſus, quos intra generale ſuum Theorema concludit, inquirere, cum oſtendimus in certis caſibus, quos æque complecti velit Theorema, Theorema ipſum Titubare; Ne videamur tamen refugiſſe ſuſceptum opus, non Regulam ejus Generalem ad examen etiam revocare gravabimur; Verum de hac Poſtea.

Prælectio Octava.

Tandem Aliquando, Juvenes Ingenui, ad Theorema Newtonianum, ſive Generalem ejus Regulam, Accedimus; Quæ eſt hujuſmodi, $\overline{P+P\mathcal{Q}}|\frac{m}{n} = P\frac{m}{n} + \frac{m}{n}A\mathcal{Q} + \frac{m-n}{2n}B\mathcal{Q} + \frac{m-2n}{3n}C\mathcal{Q} + \frac{m-3n}{4n}D\mathcal{Q}+$, &c. quam ſeriem libet breviter percurrere; Ponatur itaque $P=1$, $n=1$, $\mathcal{Q}=-\frac{x^2}{1}$, adeoque $\overline{P+P\mathcal{Q}}|\frac{m}{1} = 1 \times \frac{m-0}{1} - x^2$ (vel $-\frac{m+0}{1}x^2$) $\times -\frac{m+1}{2}x^2 \times -\frac{m+2}{3}x^2$, &c. $=$ Seriei, contra quam hucuſque diſputavimus; ſi $\mathcal{Q}$ æquetur $-x^3$ vel $-x^4$, &c. progreſſio erit Indicum hic x^3, x^6, x^9, &c. illic x^4, x^8, x^{12}, &c. pro incremento Lateralium in Poteſtatibus Quadraticis, Cubicis, &c. Atque idcirco eadem, quæ antea attulimus, argumenta, tum contra Dimenſiones & Signa, tum Uncias & Coefficientes, Numerumque & Locum Terminorum, pariter contra hanc Seriem robur habent & pondus, & porro, ſi etiam $+x^2$, $+x^3$, $+x^4$ Lateralia deſignent; Nos vero $-x^2$, $-x^3$, &c. ſeligemus præſertim diſcutiendas, propterea quod inter has exiſtimarunt Mathematici verſari Circuli Terminos Generantes; Si itaque ſumatur $\mathcal{Q}=-x^2$, vel $-x^3$, provenire opinatur Newtonus Quantitates, præter Unitatem, omnes Negativas in Poteſtatibus fractis, ob multiplicationem Negativæ $\mathcal{Q} \times A$, vel B, vel C, &c. $\times$ Coefficientem Numeralem, utraque nempe ſimul privativa vel poſitiva exiſtente. At cur, inquam, hæc fiat multiplicatio, cur retineamus $\mathcal{Q}$ immutatam, eademque ſervetur compoſitio, tam cum ſe, quam cum Coefficientibus, in fractis Poteſtatibus perinde, ac in integris? Si ad analogiam recurritur, nos quoque ad analogiam confugimus, ex qua res plane contrarias probavimus, hac utitur Newtonus, quia $\mathcal{Q}=-x^2$ multiplicata in $A = P\frac{m}{n}, \times\frac{m}{n}$, vel $\mathcal{Q}\times$, $B=\frac{m}{n}A\mathcal{Q}, \times\frac{m-n}{2n}$, &c. conſtituit Terminos Seriei $\overline{P+P\mathcal{Q}}|\frac{m}{n}$, ubi n fuerit Unitas, m vero Numerus integer, ergo ubi m ſit fractus, $n=1, -x^2=\mathcal{Q}$ ducta in $\frac{m}{n}$ A, &c. debitos eruet Seriei Terminos, qui ad fractas Poteſtates pertineant & aſſignentur; Nos, ut videtur, non iniquiorem prius Analogiam; Quia in integris $-x^2+x^4-x^6+x^8$, &c. ea ſibi ſigna attrahunt, quæ habentur ex Dignitatum multiplicatione, neque quicquam eo conferunt Coefficientes Numerales, itaque neque in fractis; Atque ibi prorſus latet Newtoni calumnia, quod in integris poteſtatibus Numeralium Coefficientium ſigna Poſitiva non mutantur cum Negativis, adeoque Terminorum ſigna Involutioni ſemper obtemperant Dignitatum, *e. g.* ſit $m=2$, reliquæ P, $\mathcal{Q}$, & n maneant, $\overline{P+P\mathcal{Q}}|\frac{m}{n} = 1q - 2x^2 + x^4$, ubi $\frac{m-n}{2n}$ ſignum æque obtineat Poſitivum, ac $\frac{m}{n}$, quod ſemper etiam accidet, ſi ſit $m-2n$ vel $m-3n$, &c. puta in Poteſtatibus adhuc, quam poſuimus, altioribus, quoniam ſeries in nihilum ante deſinet quam abeat in privationem; unde ſequitur ſigna penes eſſe dignitates, cum ſignum Poſitivum, ſeu in ſe, ſeu Negativum ductum, quod conſtitutum invenerit, reliquerit integrum; In fractis vero poteſtatibus e contra, ſignorum conficiendorum provincia ex parte in Coefficientes Numerales devolvitur, id quod neque Rationi convenit, nec tamen Analogiæ, toties a Newtono ſine ulla probatione aſſumptæ, conſentaneum; Nam ſit $m=1$, $n=2$, $\overline{P+PQ}|^{\frac{m}{n}} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6$, *&c.* ſi Newtonum audiamus, $= 1 + 1 \times \frac{1}{2} \times -x^2$, neque præterea coefficientes habemus Poſitivos, nihilum vero tranſgredimur, rapimur in privationem; pergamus, $-\frac{1}{4} \times + \frac{1}{2} \times + x^4 = -\frac{1}{8}x^4$, in quibus patet $-\frac{1}{4}$, cum debuit Dignitati $\overline{-x^2}|^2$ conſtituendi ſignum poteſtatem tribuiſſe, exturbandi ſibi vendicare imperium, & pariter de cæteris; Facile autem hæc applicarentur ad prædictas ſeries, ubi Q æquatur $-x^3$, $-x^4$, vel $+x^3+x^4$, *&c.* præterimus tamen, ut ad alias feſtinare poſſit Oratio; Ponatur itaque jam $P=c^2$, c^3, c^4, *&c.* $Q = -\frac{x^2}{c^2}$, *&c.* reliquis conſiſtentibus, adeoque $\overline{P+PQ}|^{\frac{m}{n}} = c^2 - x^2$, cujus *e. g.* Dignitatem Quadraticam imprimis perpendamus, *h. e.* ſit $m=2$, $n=1$, ergo $\overline{P+PQ}|^{\frac{m}{n}} = \overline{c^2-x^2}|^{\frac{2}{1}}$ æquatur $+\overline{c^2}|^2 + 2 \times \overline{c^2}|^2 \times -\frac{x^2}{c^2} + \frac{1}{2} \times -2c^2x^2 \times -\frac{x^2}{c^2}$, & tota Series collectim $= c^4 - 2c^2x^2 + x^4$, quæ cum veritate quadrat; Nihil enim hic mutatur, niſi quod ponitur c^2, pro qua antea habebatur Unitas; Intactis cæteris, m exprimat Unitatem, n Numerum binarium, atque omnia conſtant, quæ contra ſeries arceſſimus Superiores, quoniam vero ingeritur c^2, illam præterea Trutinemus, ſed de his jam ſatis.

Newtonus ait $\overline{P+PQ}|^{\frac{m}{n}} = cc^{\frac{1}{2}} + \frac{1}{2} \times c \times -\frac{x^2}{c^2} + \frac{1-2}{4} \times \frac{-x^2}{2c} \times \frac{-x^2}{c^2}$, *&c.* $= c - \frac{x^2}{2c} - \frac{x^4}{8c^3}$; Ad veterem nempe Analogiam retrahimur, *h. e.* quia debita ſeries producitur ex $\overline{P+PQ}|^{\frac{m}{n}}$ per hujuſmodi operationes, eaſdem itaque ſequitur leges $\overline{P+PQ}|^{\frac{n}{m}}$, licet enim Newtonus rem occultare ſatagat, atque indigitare velit tum $\frac{2}{1}$ tum $\frac{1}{2}$ per eandem $\frac{m}{n}$ deſignari, tamen, ſi recta ſervetur proportio, per contrarias literas erunt & reciprocas deſignandæ; ut ſi m ſit Unitatis fractio, *e. g.* $\frac{1}{2}$, $n=1$, integra reciproca erit $\frac{1}{\frac{1}{2}}$, & ſimiliter, quæcunque ponantur poteſtates, fractæ ſemper & integræ notantur reciproce; Fateor $\frac{1}{2}$ vel 2 exprimi poſſe per $\frac{m}{n}$, ſumas utramvis, utramque nego, verum notatione, ut quæ maxime, contraria; ergo, in præſenti, Analogia ſic rectius inſtituetur, quoniam primus terminus ſeriei, $= \overline{c^2-x^2}|^{\frac{2}{1}}$, producitur ex $\frac{c^2}{1} \times \frac{c^2}{1}$, itaque primus ſeriei, $= \overline{c^2-x^2}|^{\frac{1}{2}}$, habebitur ex $c^{\frac{1}{2}} \times c^{\frac{1}{2}} = c$, & cum ſecundus illius naſcatur ex $2 \times c^4 \times -\frac{x^2}{c^2}$, hujus ſecundus forſan debebit eſſe $\frac{1}{2} \times c \times -\frac{x^2}{c^{\frac{1}{2}}}$ & tertius $-\frac{1}{2}c^{\frac{1}{2}}x^2 \times -\frac{2x^2}{c^{\frac{1}{2}}}$, nihil enim dicimus de x^2, quam antea conſideravimus; An hæc fuerit juſta Analogia, parum ſumus ſolliciti, æquiorem ſaltem ea, quam Newtonus attulit, eſſe contendimus; Hoc tamen indubium habemus, Aſſumi a Newtono $\frac{x^2}{c^2}$, ut in Serie $= \overline{P+PQ}|^2$ deſtrui poſſet Quantitas c^2 ſecundi termini $-2c^2x^2$, qui aliter, ex multiplicatione c^4 in ſolam $-2x^2$, evaderet $-2c^4x^2$ contra conſtitutionem Quadraticam; Item c^4 tertii termini x^4; Qui pariter, niſi eſſet c^4 perempta, videretur exire in $+c^4x^4$ ex multiplicatione $-2c^4x^2 \times -\frac{1}{2}x^2$, denuo contra Quadrati indolem; & cum in his ſeriebus propter decrementum Poteſtatum c^2, c^3 c^4, *&c.* quas radices vocamus per Indices Radicum æquabile in ſingulis Terminis ſeriei cujuſcunque poteſtatis $\overline{P+PQ}|$ Cubicæ, Biquadraticæ, *&c.* ab Indicibus expoſitæ Dignitatis ad Nihilum, idem eveniret; Ratio Univerſim patet, cur PQ ſemper diviſerit Newtonus per P; *E. G,* $\overline{c^3-x^3}|^3$, Index $\overline{c^3}|^3 = c^9$ erit 9, qui inde per 6 & 3 devenit ad 0, adeoque ad Seriem debitam eruendam $\overline{c^3}|^3 + \overline{c^3}|^3 \times Q \times 3 + 1 \times 3\overline{c^3}|^3 Q \times Q$, *&c.* Q æquatur continuo $-\frac{x^3}{c^3}$, ut ipſa tollat a ſecundo termino, quam nimiam $A = c^9$ appoſuerat,

Prælectio Nona.

ſuerat, a tertio quam $B=c^6$, quoad Dignitatem c, & prorſus; Jam vero, inquam, hanc rationem, quare x^2 vel x^3 dividatur per c^2, c^3, non ad fractas poteſtates pertinere, niſi eodem, quo modo integræ, componantur, quod fieri nequit; Num aſſeret Newtonus *e. g.* in $\overline{c^2-x^2}|^2$, quoniam decrementum invenitur a c^4 per c^2 ad 0, in ejuſdem cubo a c^6 per c^4 ad c^2 & dein 0, ideo idem eſſe decrementum in radice ejus Quadratica, Cubica, &c? Aſſerat, nunquam comprobabit, adhibet tamen P pro Diviſore, ut iſtiuſmodi decrementum obtineatur; forſan aſſerere velit, quamvis nolit probare; Ego, mallem, niſi exigat, quam non dedit demonſtrationem, in poteſtatibus reciprocis ipſius $\overline{c^2-x^2}|$ Subrogare $c\frac{1}{2}$ pro c^2 diviſore, in $\overline{c^3-x^3}|$ $c\frac{1}{3}$, &c. ne cum juxta analogiam decreſcere debeat *e. g.* a $c=$ primo termino verſus nihilum, quoad dimenſionem, diminui etiam videatur, verſus infinitum a c ad $c-\infty$; Neque tamen vehementer litigarem in his Quantitatibus $c\frac{1}{2}$, $c\frac{1}{3}$, &c. eſſe Dividentes, c^2, c^3 eſſe pernego; Nemo enim ſcienter dixerit eodem prorſus modo decreſcere c^4 in ſerie Radicali ab ipſa c^4, quo eadem c^4 decreſcit in Quadratica, neque enim c^4 reperitur in ſerie Newtoniana; Siquis autem affirmaverit ſeries reciprocas ipſius $\overline{c^2-x^2}|\frac{m}{1}$ a ſuo primo termino eadem minui ratione, qua directæ, ſecundum Newtonum, *e. g.* ut in $\overline{c^2-x^2}|^2$ a c^4 per c^2 ad c^0, ita in $\overline{c^2-x^2}|\frac{1}{2}$ a $c\frac{4}{4}$ per $c\frac{2}{4}$ ad $c\frac{0}{4}=c$, $c\frac{1}{2}$, c^0, perficietur illa ſeries, quam modo ex precaria analogia Newtonianæ conſimili ſtatuimus, Newtonianæ vero ſeriei minime conſentanea; Si decrementum non fiat per dimenſiones ſed fractiones Quantitatum, id quod velit Newtoni ſeries, Cur in primis adhibeatur c^2 in Poteſtatibus integris ad diminuendas dimenſiones, in fractis ad minutias? Deinde, ſi hanc nullam Analogiam concedamus, Cur non ſervetur Analogia in ipſis minutiis? Nam hoc modo, vel erit ſeries $\frac{c^1}{c^0}-\frac{x^2}{2c^2}-\frac{x^4}{8c^4}$ ſive, $\frac{c^1}{c^0}-\frac{x^2}{2c\frac{1}{2}}-\frac{x^4}{8c^1}$; Quid autem hic habetur cum Newtoni ſerie commune? Hæc, minutula ſi placet, Newtono condonemus, veniamus ad alia; Cæteris itaque manentibus advocetur n; Numquid adminiculatur ejus interpolationibus? Vix dicerem; An latius quid Newtonus erraverit, Dubito; n hucuſque ſummo Viro donavimus, hujus erit temporis, ut requiramus; ne tamen moleſtum habeat, quod beneficium detraxerimus, aliud in ſe collatum putet; Hæc, quæ modo oſtendimus ſine demonſtratione petiiſſe Newtonum, reddimus ipſi; Nihil ſuis fractis poteſtatibus deſideratam $\frac{PQ}{P}$ invidemus; Omnia, præter Coefficientes etiam antea dictas Numerales, auctoritati ejus tribuimus; Num medetur n errori, quem in ſerie $1\times\frac{m-0}{1}$, &c. ineſſe probavimus? Nihil minus; Sit m, *e. g.* $=4$, $n=2$, idem ac ſi ponas $\frac{4}{2}=m$, $n=1$; Series Newtoniana erit $1\times\frac{4-0}{2}\times\frac{4-2}{4}\times\frac{4-4}{6}$; Recte quidem; Verum, inquam, aliter ſe rem habere in fractis poteſtatibus; Nam ſit $m=2$, $n=4$, idem ac ſi ponas $m=\frac{2}{4}$, $n=1$, adeoque cum, ut dictum eſt antea, illa ſeries jam memorata æquetur noſtræ $1\times\frac{m-0}{m-n}\times\frac{m-n}{m-0}\times\frac{m-2n}{1}$, hæc ſeries poteſtatis fractæ erit, vel $1\times\frac{\frac{2}{4}-0}{\frac{2}{4}-\infty+1}\times\frac{\frac{2}{4}-1}{\frac{2}{4}-\infty+2}\times\frac{\frac{2}{4}-2}{\frac{2}{4}-\infty+3}\times\frac{\frac{2}{4}-3}{\frac{2}{4}-\infty+4}$, &c. vel $1\times\frac{2-0}{2-\infty+4}\times\frac{2-4}{2-\infty+8}\times\frac{2-8}{2-\infty+12}\times\frac{2-12}{2-\infty+16}$, &c. Atque ita, ſi minus fuerit Newtonus hallucinatus inducendo n, poſtquam admiſerat Unitatem pro Numerante Poteſtatis Indicis in ſeries fractas, ſimiliter prorſus ac in integras, videas tamen errorem ejus hinc nequaquam minui; Aſſero tandem una eademque regula $\overline{P+PQ}|\frac{m}{n}=P\frac{m}{n}+\frac{m}{n}AQ$, &c. diverſas haſce ſeries peragi non poſſe, quod de Coefficientibus, ut prius de iiſdem, pariter & de aliis has ſeries ſpectantibus ſic porro oſtendimus; Nam ſi integræ poteſtatis Index fuerit $\frac{l}{n}=\frac{m}{1}$ erit Index fractæ reſpondentis $\frac{n}{l}=\frac{1}{m}$, & e converſo, ſi index fractæ ſit $\frac{l}{n}=\frac{m}{1}$ integræ reſpondentis erit $\frac{n}{l}=\frac{1}{m}$, ut patet; Ponatur poſterius, valeat

leat autem regula $1\times\frac{m-0}{1}$, *&c.* ad fractas applicata, eandem dico nequaquam integrarum uncias explicare, quoniam enim *m* eſt quantitas fracta & utrobique eadem, eadem prorſus eſſet ſeries pro integrarum Unciis, quam pro fractarum Unciis aſſumpſimus; ſi prius, eadem eſſet ſeries pro fractarum Unciis, quæ Uncias expoſuit Integrarum, illic nempe *e. g.* in Quadrato, perinde ac in radice Quadratica inveniremus ineſſe Uncias $1+\frac{1}{2}-\frac{1}{8}+\frac{1}{16}$, *&c.* hic in radice quadratica, ut in quadrato, 1. 2. 1; Si inverſa Quantitate *m* invertenda ſit regula, quoniam regula Newtoni ſemper ponit *m* ab illa parte in ſerie, qua in indice conſpicitur, repugnantem non habet, Aſſentior; Poſito ergo *m* eſſe quantitatem integram, recte procederet regula quoad integras poteſtates, ſi permutet regulam Newtonus pro fractis inveniendis, a ſua plane deſciverit, nec fieri poteſt, ut $1\times\frac{m-0}{1}\times\frac{m-1}{2}$, *&c.* poſito nempe $m=$ fractæ, æquaretur $1\times\frac{1}{m-0}\times\frac{2}{m-1}$, *&c.* ſi *m* ponamus æqualem eſſe integræ; & viciſſim ſumatur $m=$ fractæ, detur etiam $1\times\frac{m-0}{1}$, *&c.* veras uncias Radicum proferre, integrarum unciis inveniendis erit ſaltem inepta, niſi $1\times\frac{m-0}{1}$, *&c.* ſumpta $m=$ integræ, adæquet $1\times\frac{1}{m-0}$, *&c.* cum poſueris *m* fractæ eſſe æqualem; *e. g.* in Quadrato unciæ ibi viderentur 1. 2. 1. in radice quadratica 1. $\frac{1}{2}$. 1 hic in radice Quadratica 1. $\frac{1}{2}$. $-\frac{1}{8}$. $\frac{1}{16}$, *&c*, in Quadrato 1. 2. -8. $9\frac{3}{5}$, *&c.* Videat autem Newtonus, an comprobatas Quadrati Uncias reſpuere velit, an ſuas Radicum infirmas, & vix ſibi conſtantes. Longum eſſet omnia Maximi Viri exempla percurrere, ſat habemus indicaſſe initia, ejuſque theorema, eſſe falſa, quod ubicunque accidit, quo ulterius procedimus, non raro magis de recta via deflectimur, ſique titubavit Newtonus in ipſo limine, non mirum eſt, ſi interius provectus collabatur, & qui primos & apertiores caſus, quos poſuit, adeo infelici expedit conatu, eundem fruſtra viderimus in perplexioribus elaborantem; Progrediamur ergo, reliquis prætermiſſis ne multi ſimus, & dicimus Newtonum ita egiſſe in his interpolationibus, ut unum ſaltem juvenile exemplum proferam, ſi minus magnificum, declarando tamen quid velim, ni fallor, idoneum, Nempe, quaſi ipſe, ſi inveniſſem regulam generalem, qua expediantur integrorum Numerorum, cum negativorum, tum poſitivorum, Facti, præter methodum Multiplicationis uſitatam, ſola quadam additione, eandem ad fractorum Productos inveniendos accommodarem; nam, ut Quantitatum Radicalium compoſitio eſt ſpecies certa diviſionis, *e. g.* $x|\frac{1}{2}\div x,1)\ 2=(\frac{1}{2}$; Poteſtativarum, multiplicationis; ita fracti Numeri & integri ſua in compoſitione, quæ ibi eſt diviſio, multiplicatio hic, eadem æque regula comprehendentur ac integræ Poteſtates & fractæ; Ergo hæc regula ſtatuatur ad factum quodcunque ex quibuſvis Quantitatibus Numeratis integris obtinendum; Adjicias datarum ſummæ tot Quantitates minores per Unitatem quam altera, Negativas earundem, ſi altera ſit Negativa, ſi Poſitiva, Poſitivas, quot Unitatibus Hæc abeſt a nihilo, cui tandem aggregato apponatur Illius privativa. Exempla habeas, $-3a\times-5b$, Quantitas Numeranda erit *ab*, ad coefficientem autem Numeralem adipiſcendam Quantitas Unitate minor quam altera -3 æquabitur -4, Negativa ejus $+4$ quinquies erit ſumenda, $+20$ effecimus, augeatur amplius quantitate $+3$, -8 minuatur, Factumque conſtat quæſitum $+15$; Jam ponatur $-3\times+5$, -4 repetita 5 vicibus dabit -20, addatur $+3$ atque etiam $+2$, factum recte erit -15; Sumatur $+3\times+5$, 2 ſibi quinquies addita $=10$, $3+5=8$, negativa Quantitatis $+3=-3$, factum ſive totius aggregatum $=15$; Tandem multiplicetur $+3\times-5$, habeatur -2 quinquies $=-10$, $+3-5=-2$, & negativa $+3=-3$, vel $-10-2-3=-15$. E contra $+5\times-3$, -4 ter ſumpta ſiſtit -12, $5-3=2$, negativa $+5=-5$, & $-5+2-12=-15$ Facto deſiderato; Sic $-5\times+3$, ter $-6=-18$, $-5+3=-2$, $--5=+5$, $-18-2+5=-15$; Item, -5×-3, ter $+6=18$, $-5-3=-8$, $--5=+5$, $18-8+5=+15$; Denique $+5\times+3$, ter $4=12$, $5+3=8$, $-+5=-5$, $12+8-5=15$; adeo ut, ſecundum hanc regulam, quatuor variis componatur modis $+15$, totidem porro -15; Unum aut alterum exemplum ſolummodo amplius adhibemus, ut pateat eſſe generalem, -9×-1, $+10$ ſemel $=10$, $-9-1=-10$, $--9=+9$, & $-10+10+9=+9$ Facto quæſito; $-20\times+50$, -21 quinquaginta vicibus $=-1050$, $-20+50=30$, $--20=20$, & $-1050+20+30=-20\times+50$, & univerſim idem obtinet, ubi factores ſint tantum duo, regulaſque, ni fallor, ſtabilire poſſem, quæ ad Plures pertingerent, modo quicquam putarem ad Matheſin conferre; interea nihil Utile aut Concinnum, hinc oriri videtur, niſi quod per ſolam additionem, licet hic Multiplicatione utamur ob brevitatem, queant omnes caſus multiplicationis in negativis etiam quantitatibus probari; nam hæc regula ad diverſam æquationem non ducit, quantitatibus vero peremptis, quæ diſcrepent, relinquit ipſam propoſitam; Si jam, pro Newtoni more, hanc porro ad fractiones applicaremus, vitioſi eſſemus, nam illæ ſuam obtinent plane diſſentientem, *e. g.* $-\frac{3}{4}\times-\frac{2}{5}$, $\frac{3}{4}$ cumulabitur per 5, tum quoad Numeratorem tum Nominatorem, ita ut fractio habeatur $\frac{15}{20}$, apponatur $\frac{14}{0}$ operatione comparanda, 14 nempe ad 15, 0 ad 20, componentium autem ſumma, vulgari methodo conſtituta, erit $-\frac{23}{20}$, unde eruetur $\frac{6}{20}=-\frac{3}{4}\times-\frac{2}{5}$; Inquiet Newtonus ſuas ſeries cum Radicum extractionibus convenire, nos fateri hanc regulam nequaquam cum fractionibus; Principia ejus, ut jam diximus, ſat habuimus conſideraſſe, num poſſent quemvis recte ratiocinantem ad has ſeries deducere; Si interpolationes omittit, rationeſque ejus perobſcuras inde derivatas, cum ſalebras & dumeta reliquit, per campum perſequemur; Itaque pro confeſſo habemus, quod forſan probavimus, non debitam eſſe, quam Newtonus uſurpat, intercalationem, verum, ſi placet, ex illa intercalatione non debita illæ ſeries proveniunt, quæ competunt prorſus radicum extractionibus; Nihilne hinc conſequitur noſtræ cauſæ opportunum? fatetur Newtonus rectam interpolationem quæſitas ſeries æque expreſſuram, atque extractiones Radicum, ergo ſi non debita fuerit interpolatio, cum tamen conveniat cum Radicum extractione, eadem ratione qua falſa eſt interpolatio, ruet etiam extractio, & is abſurde perquam tuetur hanc, qui cætera faſſus deſerit illam; Ne tamen velle videremur tanti viri potius rationes convellere, quam rem ipſam confutare, nihil Newtonum moramur, extractiones ejus accimus, quas cum falſas eſſe collegimus ex eo, quod prædictæ interpolationes non eſſent veræ, jam polliceor, ſi cuiquam dehinc pateat ipſas extractiones eſſe veras & probabiles, me lubenter conceſſurum interpolationes Newtoni non eſſe falſas; Verum de his in ſequenti Prælectione.

Prælectio Decima.

De Extractionibus Newtoni jam ſumus Diſceptaturi, at profecto male accidit Viri ſummi ſeriebus, quod in ipſo limine nequaquam eaſdem exponant quantitates, quas promiſerit vulgata radicum extractio, neque ullo modo hactenus oſtenſo poſſint utræque ad ſimiles etiam revocari; *e. g.* $\overline{c^2-x^2}|^{\frac{1}{2}}$ juxta vulgatam Radicum extractionem erit æqualis ſeriei $c-\frac{x^2}{2a}-\frac{x^4}{4a^2\times2b}-\frac{x^8}{16a^4\times4b^2\times2d}-\frac{x^{16}}{256a^8\times16b^4\times4d^2\times2e}$, &c. poſita nempe $a=c$, $b=c-\frac{x^2}{2a}$, $d=c-\frac{x^2}{2a}-\frac{x^4}{8a^2b}$, $e=c-\frac{x^2}{2a}-\frac{x^4}{8a^2b}-\frac{x^8}{128a^4b^2d}$, &c; at ſeries Newtoni $\overline{c^2-x^2}|^{\frac{1}{2}}$ æquatur $c-\frac{x^2}{2a}-\frac{x^4}{4a^2\times2a}-$&c; Et quidem non diffitetur ipſe Newtonus Methodum Vietæ & Oughtredi huic negotio minus eſſe idoneam; Num velit Newtonus, ut Veterem Extractionem abjiciamus, amplectamur Suam? Si juſſerit ratio, morigeros habet, ſi auctoritas ejus, pervicaces; Dicat enim modo, quemadmodum in Quotiente ex diviſore oriundo, cum numeros tractamus, opus eſt quadam moderatione, quemnam oporteat quotientem ut aſſumamus, ita in caſu præſenti, quam Poſtulet rationem hic obtinere; nam, dum radicem educimus ex numeris, ne privatio inducatur, minuitur vel augetur quotiens ita, ut Rectangulum debitum & quadratum Subſtrahi poſſint abſque ſignorum mutatione, ſi vero idem in hiſce quantitatibus valeret, ne quidem prima Subſtractio peragi poſſet; Pro Quotiente enim $c-\frac{x^2}{2c}$ invento & conceſſo, prima Subſtractio c^2 ex c^2-x^2 nulla foret, quoniam c^2 major eſt quam c^2-x^2, denuo $-x^2+\frac{x^4}{4c^2}$ erit Subtrahenda ex $-x^2$, quod fieri nequiret, & moderandus eſſet Quotiens, ſi iiſdem legibus hæ ſubmitterentur Quantitates, quibus Numeri ſive Quantitates eodem ſigno affectæ, Sc. poſitivo; Eſto tamen, ut velis Newtone, placeatne, Quotientes moderemur, vel pro lubitu, vel ſecundum quamvis Regulam? Si pro lubitu, ſine ipſe mea aliorumque Omnium utar extractione, tu ſeries infinitas tuas uſurpes licet a noſtris diverſas, utræque tamen ſint veræ, & patiaris, quæſo, Matheſin, Rationis duris ſuis Vinculis Compedibuſque ſolutam, laſcivire tandem poſt tot tantaſque anguſtias, & aliquali ſaltem licentia diffluere; ſi ſecundum regulam quamlibet, aſciſco tuam; Fateris tu $\overline{c^2-x^2}|^{\frac{1}{2}}$ eſſe $c-\frac{x^2}{2c}$, atque inſuper haberi reſiduam $-\frac{x^4}{4c^2}$; Nonne ad hæc obtinenda,

tinenda, Omiſſis interpolationibus, hoc modo proceſſeris Extrahendo?

$$\begin{array}{l} c)\ c^2 - x^2\ \left(c - \frac{x^2}{2c}, \&c. \right. \\ \quad\ c^2 \\ 2c)\ \overline{0 - x^2} \\ \qquad -x^2 + \frac{x^4}{4c^2} \\ \qquad \overline{\quad 0 - \frac{x^4}{4c^2}, \&c.} \end{array}$$

Puto; Ponatur itaque jam $\overline{c - \frac{x^2}{2c}}\Big|^2 = b^2$, adeoque $b^2 - \frac{x^4}{4c^2} = c^2 - x^2$, eademque methodo, qua ipſe Extraxit Magnus Geometra Newtonus $c - \frac{x^2}{2c}$, ipſe radicem extraherem ipſius $b^2 - \frac{x^4}{4c^2} = b - \frac{x^4}{4c^2 \times 2b}$ cum reſidua $\frac{-x^8}{16c^4 \times 4b^2}$; Nihil hic ſimile Newtoni ſeriei, ſi juxta ſuam ipſius regulam extractiones inſtituamus; Non, inquit vir Optimus, hæc nimis vulgaria affero; Experiendo inveni has ſeries in ſe invicem ductas Quantitatem evolvendam reponere, minus ſollicitus, an a regulis vulgo receptis eruantur; Num hunc varium Proteum multiplicemque tandem vinctum comprehendimus? Ex falſa ſua interpolatione pridem invenit has ſeries infinitas, jam forſan Fxperiendo; At Experiendo etiam ipſe inveni methodum notam ſatis iſtas quantitates evocare, quæ involutæ reſtituent evolvendam, atque eaſdem inſimul quantitates per totam eſſe infinitionem iis Newtoni prorſus abſimiles; Aut illæ, aut hæ ſunt falſæ; ſi hæ, valeat Radicum extractio ad Radices tamen obtinendas, ſin illæ, de ſua Doctrina videat Newtonus; certe quippiam adhuc in occulto latet, quod tantus Vir his illas prætulerit; utræque Evolvendam redintegrant, alteræ vero neſcio qua via eliciuntur, probata ſpectataque alteræ, his illas anteponit; Cum ſic velit, ſic jubet, imbellis, ne fæmineam dicam, ratio, hac vice, voluntas nempe ejus, vim ſuppleat argumenti; Vulgatas Extractiones, quoniam ita placet Newtono, aſpernamur, amittimus; Suas ipſius contemplemur; Juxta Newtonum ſic cernuntur;

$$\begin{array}{l} \overline{1 - x^2}\Big|^{\frac{1}{2}} = \left(1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6, \&c. \right. \\ \quad 1 \\ \overline{\quad 0 - x^2 \quad} \\ \quad -x^2 + \frac{1}{4}x^4 \\ \overline{\qquad 0 - \frac{1}{4}x^4 \qquad} \\ \qquad -\frac{1}{4}x^4 + \frac{1}{8}x^6 + \frac{1}{64}x^8 \\ \qquad \overline{\qquad 0 - \frac{1}{8}x^6 - \frac{1}{64}x^8}, \&c. \end{array}$$

Verum $1 - \frac{1}{2}x^2 \times 1 - \frac{1}{2}x^2$ Efficit Exceſſum ipſius $\frac{1}{4}x^4$ ſupra $1 - x^2$, & $1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 \times 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4$, Exceſſum ipſius $\frac{1}{8}x^6 + \frac{1}{64}x^8$, & Tertius Exceſſus erit $\frac{1}{16}x^8 + \frac{1}{64}x^{10} + \frac{1}{256}x^{12}$, &c. Adeoque ita abeſt, ut hæ quantitates deſideratam producant, ut quo plures fuerint termini, eo magis producta excedat quæſitam; *e. g.* prima multiplicatio quæſitam conſtituit, ſed apponit ſimul exceſſum $+\frac{x^4}{4}$, ſecunda totum illum exceſſum perimit, verum majorem inducit $+\frac{x^6}{8} + \frac{x^8}{64}$, tertia neque totum hunc deſtruit, relinquit enim $\frac{x^8}{64}$ & majorem adhuc affert nempe $+\frac{x^8}{16} + \frac{x^{10}}{64} + \frac{x^{12}}{256}$, & ſic prorſus; Ita, ut non ſolum augeantur termini, qui debeant evaneſcere, ſed facultas, ſi ita loqui liceat, perimendi factoribus quoque minuatur; Hancce vocat Newtonus Quantitatis evolvendæ reſtitutionem? Mirum eſſet Newtonum in tantum erraſſe, niſi quod mallet forſan ſe haberi ingenioſum in Matheſi, tum cum oportuiſſet intellectum adhibuiſſe; Dicet forte exceſſus reapſe minui, donec evadant tandem in quantitates minores quibuſvis datis, dum majores ſemper tollantur; Eſto, impræſentiarum; Aſſentimur in minores hoſce exceſſus quantitates quibuſvis datis evaſuros, velim jam exceſſus quoad ſingulos terminos minui; At ſecundum hanc ſententiam eo ipſo tempore, quo Termini Exceſſus Ultimi infinite minuuntur infinitus accreſcet ſeriei Terminorum Numerus; Poſt additionem enim Unius Involventis ad Unitatem *e. g.* etiam Involventem, unum terminum videmus in exceſſu exurgere, Involventium duarum, duos, trium, tres, *&c.* ita ut æquatio ſemper habeatur inter terminos Radicis poſt Unitatem, & terminos exceſſus; Verum, ſi jam ponantur utrique termini infiniti, infinitus Numerus Quantitatum minimarum, Terminorum nempe in Exceſſu Ultimo, ut patet ex Infinitorum Arithmetica vel aream exprimet vel ſaltem lineam, pro natura Quantitatis minimæ, adeoque Newtoni ſeries in ſe ducta nunquam reſtituet $1-c^2$, ſed finite protracta exceſſum ſatis Notabilem annectet, vel infinite etiam, nequicquam contemnendum; Si dicat Terminos Ultimi Exceſſus uno impetu in nihilum deturbari, id vix crediderim, vel enim Aliqui Termini Ultimi exceſſus uſque manebunt, vel Termini deſtruendi ſuperioris, & pariter retroacta infinitione ad Primum uſque exceſſum, *e. g.* ſi $+\frac{x^{12}}{256}$ ſit ultimus Terminus Exceſſus ultimi, & ponatur infinite parvus vel nihilo æqualis, ergo $-\frac{x^6}{16}$ erit etiam infinite parvus vel nihilo æqualis, ex quo multiplicato in ſe conflatur $+\frac{x^{12}}{256}$, atque hinc, fateor, Omnes Termini Ultimi exceſſus æquabuntur porro nihilo, nam $\left.\begin{array}{l}1-\frac{x^2}{2}-\frac{x^4}{8}\\ \times 1-\frac{x^2}{2}-\frac{x^4}{8}\end{array}\right| \times \left|\begin{array}{l}0\\ \times 0\end{array}\right. =0$, verum ex multiplicatione ſuperioris *e.g.* $\left.\begin{array}{l}-\frac{x^4}{8}\\ \times -\frac{x^4}{8}\end{array}\right| \times \left|\begin{array}{l}1-x^2\\ \times 1-x^2\end{array}\right.$ reliquus erat exceſſus, qui deberet perimi, quem 0 nullatenus tollit; Si $\frac{x^{12}}{256}$ non ſit infinite parvus, vel nihilo æqualis; Primo reliquus eſt exceſſus, adeoque non reſtituetur Quantitas evolvenda; Deinde $-\frac{x^6}{16}$ neque erit infinite parvus, vel æqualis nihilo, cum Quantitas quæcunque minima nequeat ex nihili involutione produci; $\frac{x^{12}}{256}$ nempe ex $-\frac{x^6}{16}$, contra Hypotheſin, & tandem ſi $-\frac{x^6}{16}$ non ſit infinite parvus $=0$, conſtabit Ultimus exceſſus ex Terminis pæne infinitis, quorum ſinguli æquales erunt Quantitati, quoniam $-\frac{x^6}{16}$ quantitas in ſe multiplicatur & in omnes prægreſſos terminos, qui erunt etiam quantitates, in ſe porro multiplicatos; Reddantur hæc Newtono, Auferamus antea conceſſum; Quomodo enim probat hos Exceſſus vergere ſemper ad minores? Nego fieri poſſe; Nam ſi $\frac{x^4}{4}$ major fuerit quam $\frac{x^6}{8}+\frac{x^8}{64}$, $\frac{x^4}{4}$ a fortiori major erit quam $\frac{x^6}{8}$ *h. e.* $\frac{1}{4}x$ major quam $\frac{1}{8}x^3$, vel in circulo *e. g.* $\frac{1}{4}$ ſinus x major quam $\frac{1}{8}$ ejuſdem ſinus in ſe Cubice ducti, linea quam Solidum, quod nemo aſſeruerit; ergo multo magis repugnat, ut $\frac{x^4}{4}$ ſit major quam $\frac{x^6}{8}+\frac{x^8}{64}$; itidem, ſi $\frac{x^6}{8}+\frac{x^8}{64}>\frac{5x^8}{64}+\frac{x^{10}}{64}+\frac{x^{12}}{256}$ erit etiam $\frac{x^6}{8}+\frac{x^8}{64}>\frac{5x^8}{64}$ & $\frac{x^6}{8}>\frac{4x^8}{64}=\frac{x^8}{16}$, atque adeo $\frac{1}{8}x>\frac{1}{16}x^3$, vel $\frac{1}{4}x>\frac{1}{8}x^3$, ut prius, quod ſemper etiam accidet, ut dimenſio poſterior quoad Indicem augeatur binario propter primorum in quoque exceſſu Terminorum progreſſionem x^4, x^6, x^8, *&c*; Quod ſi jam, apponantur reliqui Exceſſuum termini, Exceſſus videbuntur ita non minui procedendo, ut immane quantum augeantur, præſertim cum ſequentes eodem modo probari poſſint Termini in ſingulis exceſſibus augeri, quo oſtendimus auctos eſſe ſequentes in diverſis *e. g.* $\frac{x^{10}}{64}$ majorem eſſe quam $\frac{5x^8}{64}$, & porro $\frac{x^{12}}{256}$ quam $\frac{x^{10}}{64}$, *&c.*

Atque tandem Newtoni Principia, unde hæc deducebantur, falſa ſunt & tota caduca, nam quæ diximus omnia facile aliquibus mutatis in cæteras Newtoni ſeries erunt convertenda, ſat habemus oſtendiſſe, quomodo reliquæ ejus poſſint convelli.

Nam hinc ſequitur, $\overline{c^2+x^2}|^{\frac{1}{2}}$ non eſſe $=c+\frac{x^2}{2c}-\frac{x^4}{8c^3}+\frac{x^6}{16c^5}$, &c. prout Affirmat Newtonus in Exemplo ſuo primo.

Vid. Epiſtolam Priorem ad Leibnitium.

Item, $\overline{c^5+c^4x-x^5}|^{\frac{1}{5}}$ non eſſe $=c+\frac{c^4x-x^5}{5c^4}-\frac{2c^8x^2+4c^4x^6-2x^{10}}{25c^9}$, &c. prout Aſſerit in ſecundo.

Denuo $N\times\overline{y^3-a^2y}|^{\frac{-1}{3}}$ non eſſe $=N\times:\frac{1}{y}+\frac{a^2}{3y^3}+\frac{a^4}{9y^5}+\frac{7a^6}{81y^7}$, &c. quod Ponit in tertio; Et $\overline{d+e}|^{\frac{4}{3}}$ non eſſe $=d^{\frac{4}{3}}+\frac{4ed^{\frac{1}{3}}}{3}+\frac{2e^2}{9d^{\frac{2}{3}}}-\frac{4e^3}{81d^{\frac{5}{3}}}$, &c. uti Dicit in Exemplo quarto.

Simplices Poteſtates recte elicit, ut prius conceſſimus; de poteſtatibus reciprocis & integris idem erit dicendum, non item de reciprocis & fractis, adeoque $N\times\overline{d+e}|^{\frac{-1}{3}}$ non $=N\times:\frac{1}{d^{\frac{1}{3}}}-\frac{e}{3d^{\frac{4}{3}}}$, &c.

Si itaque regerat Newtonus inanem nos operam navare, cum hæ ſuæ ſeries adeo conſentiant cum Mathematicorum concluſionibus aliunde demonſtratis; reſpondemus, Primo hanc Doctrinam ergo non eſſe, niſi bellam quandam Hypotheſin, nullis nixam argumentis, quod probavimus, ſed fortuito inventam conſtructamque, nec magis Mathematicam, quam Carteſii Particulas ſtriatas, quæ conceſſæ bene multas forſan explicent Magnetis Proprietates, Deinde hic generatim reponimus, poſtea ſi otium detur ad caſus particulares, ſigillatim aggreſſuri, quoad varias ſolutiones, eas Newtoni ſeries recte exhibere, ſed quæ non deſiderabantur, quæ in receſſu erant, eas adhuc latere, *e. g.* in exemplis præcedentibus, Integræ Poteſtates ſive directæ ſive reciprocæ rite ex ejus ſeriebus inſtituuntur, non autem fractæ, quas igitur dedit ſolutiones demonſtratis convenientes videat, ne cadant ſub conceſſis noſtris, quas non hucuſque demonſtratas, ne ſub iis, quæ Falſa eſſe Demonſtravimus; Nam cum Regula, ſive Theorema proponitur generale, quod ex parte verum ſit, cætera plane falſum, prout male apteve applicetur, poteſt idem eſſe mendax, vel nihil eo veracius; Sufficiat autem hæc de Newtoni ſeriebus abſtractis dixiſſe, ad res porro Geometricas & Numerales progrediamur, Numeralem ſcientiam imprimis contemplabimur, cui hæc præcipua commoda Newtoni ſeries attuliſſe perhibentur, ut Radicum extractio in Numeris, & Logarithmorum conſtructio, facilius quidem peragantur & lautius; Jam vero de his Satis.

De Fluxionibus Newtoni.

Newton. Quadratur. Curvar.

Interea, cum Fluxiones ſuas magni æſtimaverit, ægre forſan tulerit, ſi inſalutatas neglectaſque præterirem, quarum rationem & initia duobus modis explicari velit, alter habetur ad Pag. 392. Vol. 2. Walliſii Algebræ, & poſtea fuſius exponitur ab ipſo Newtono in Quadratura ſua Curvarum, alter ad Pag. 250, Newt. Princip. Math. Ad primum aggredimur, ubi non ſatis liquet, cur Quantitates lineares ad velocitates transferantur, cum velocitates potius ſint lineis exprimendæ, quam lineæ velocitatibus certe non lineæ potius velocitatibus, quam velocitates lineis; Quid enim hinc adjuvamur? Fluant modo puncta quævis æqualibus temporibus, conficientque lineas æquales; at vero, hæc tempora æque componuntur ex momentis quam minimis, ac linea punctis, atque idem plane eſt, ſive lineas definiamus æquales ex æquali Numero punctorum ſive tempora æqualia ex momentorum æquatione; adeoque quicquid Newtonus ſibi viſus fuerit de Quantitatibus Mathematicis Inveniſſe, inquam, cum illas ut motu continuo deſcriptas conſiderat, ut ex partibus quam minimis conſtantes ipſum perpendere, fatetur enim ſe quantitates conſiderare, ut quæ æqualibus temporibus creſcentes & creſcendo genitæ pro Velocitate majori, vel minori, qua creſcunt ac generantur, evadunt majores vel minores, *h. e.* earum incrementum menſuratur per velocitatem ſuam quovis tempore dato, ſed neque definiri poſſet accurate æqualitas temporis vel velocitatis ratio, niſi tum velocitas tum tempus quantitates habeantur ex partibus minimis conſtantes ita ut nihil afferat hæc Doctrina adjumenti methodo Indiviſibilium, removetur altius difficultas, ſiquæ ſit, non ſublata eſt, *e. g.* in *Fig.* 1. *CE*, *Ec* ſunt augmenta Quantitatum *AB*, *BC*, at Fluxiones, ſive incrementorum velocitates *AB*, *BC*, ſunt ut ipſa incrementa *CE*, *Ec*, quæ ſi ignorentur, fruſtra eorum rationem quærimus, ſi nota fuerint, rogo an ſint definita quædam? Necne; ſi definita, omnia quæ contra indiviſibilia

Vid. Newton. Quadratur. Curvar. Fig. 1. Tab. 1.

bilia afferuntur, contra hujuſmodi incrementa valent; Nam ſi minimum incrementum definitum ſit, datur indiviſibile, ſi non, nulla eſt cognitio Quantitatis indeterminatæ; Præterea autem minus liquet Fluxiones *AB*, *BC*, eſſe ut *CE*, *Ec*, niſi, *c*, punctum lineæ *Ec*, plane perfecteque coincidat cum puncto Curvæ *Ac*, quod fieri nequit, ſi *Ac*, *bc* fuerint incommenſurabiles; At Problema, quod imprimis erit in hac doctrina corripiendum, eſt Data æquatione quotcunque fluentes Quantitates involvente, invenire Fluxiones; Fluat, inquit Newtonus, Quantitas x uniformiter, erit $x+o$, & Invenienda ſit Fluxio Quantitatis x^n, Quantitas x^n evadet $\overline{x+o}|^n$, id eſt, per methodum ſerierum infinitarum,
Newton. Quadratur. Curvar. Pag. 168. 169. $x^n + nox^{n-1} + \frac{nn-n}{2} o^2 x^{n-2}$, &c. continue multiplicando $\frac{o}{x}$ in ſe; Nihil hic moramur infinitas ſeries, concedimus; Abjectis itaque x^n & x, Augmenta ſunt nox^{n-1}, &c. & o, Sed $o.\ nox^{n-1} :: 1.\ nx^{n-1}$, adeoque $\frac{nn-n}{2} o^2 x^{n-2} +$, &c. in infinitum pro nihilo habentur, *h. e.* evaneſcunt, quod negamus admodum, imo & aſſerimus, ſi cæteræ quantitates coefficientes repudientur, maneatque autem ſolummodo $o^2+o^3+o^4+o^5+o^6$, &c. ita abeſſe a nihilo, ut infinitam porro conficiant quantitatem ipſa x majorem, Nam o eſt minimum augmentum quantitatis x, ita ut ſi o multiplicaretur in ∞, nimirum, in infinitum, ∞o æquaretur x; & pari ratione $\infty o^n = x^n$, vel $o^n =$ Infiniteſimæ Quantitatis x^n *h. e.* $o^n = \frac{x^n}{\infty}$; ſed in ſerie $o^2+o^3+o^4$, &c. $+o\infty$, $o\infty = x$, adeoque o^2+o^3,
Quadratur. Curvar. Pag. 170. 172. 173. &c. $+x$ major erit quam x; Denuo *z. y. x.* poſt momentum temporis (quod ni fallor indiviſibile erit temporis punctum, quamvis Newtonus ſine involucro occultare ſatagat) incrementis ſuis auctæ evadent $z+o\dot z$, $y+o\dot y$, $x+o\dot x$, id quod fatemur, ob rationem vero allegatam pernegamus $x^3+3x^2o\dot x+3xo^2\dot x\dot x+o^3\dot x^3$, eſſe Fluxionem x^3, & pariter de cæteris, cum revera Fluxio x^3 ſit $o^3\dot x^3$, hoc tamen conceſſo, neque reſiduum dividi debet per o, neque minui quantitas o in infinitum; Si objiciatur Paraboloeidea hoc modo quadrari, agnoſcere has me inquam Fluxiones ubi non applicantur ad Quantitates Binomias, cujuſmodi ſunt nulla Parabolarum genera, *e. g.* in Parabola Quadraticali $y^2 = x^1$, vel $x^{\frac{1}{2}} = y$, adeoque in hoc caſu multiplicanda erit Fluxio quantitatis ſimplicis $\dot x \times x^{\frac{1}{2}} = \dot x x^{\frac{1}{2}}$; Fluxio $x = 1\dot x x^{1-1}$, quæ multiplicata in $x^{\frac{1}{2}}$ efficit $1\dot x x^{1-1+\frac{1}{2}}$, adeoque, ſi invenire velim quantitatem fluentem, abduco $\dot x$ & appono $+1$, ut habeatur $1x^{1-1+1} = 1x$, unde obtineo $1x^{1+\frac{1}{2}}$, divido autem per Unitatem, quæ affixa fuit Fluxioni $1\dot x$ & per $\frac{1}{2}$ inſuper propter $x^{\frac{1}{2}}$, quæ non pertinet ad Fluxionem areæ, quoniam eadem eſt, nec ullo augetur incremento, adeoque ſequitur eandem regulam, quam Fluxio ſimplex, ut ad fluentem inveniendam dividatur Fluxio per totum Indicem; At vero horum nihil poteſt ad Quantitates magis compoſitas accommodari, & generatim in omnibus æquationibus, quarum Fluxiones ad ſimplices reduci queant, confirmatur regula $n\dot x x^{n-1}$, in cæteris non item; Longum eſſet & nimis Operoſum omnia problemata, eorumque demonſtrationes, percurrere; ſat habeo principia eſſe falſa, quæ ſi vera non ſint, quibus variæ Mathematicæ propoſitiones convincuntur, nunquam mihi erit exploratum alias adhibitas Demonſtrationes earundem eſſe indubias; Non gravabimur tamen excutere demonſtrationem Subtangentis Parabolæ, quæ ideo vera eſt non ex principiis, quæ confutaſſe videmur, ſed ex eo, quod cum principiis convenit probatioribus; $x = y^2$, cujus Fluxio erit $\dot x = \dot y^2$, ſed $\dot y^2 = 2\dot y$ & propter ſimilia triangula $\frac{\dot x y}{\dot y} =$ Subtangenti $= \frac{2\dot y y}{\dot y} = 2y$, verum quoad longitudinem $y = y^2 = x$, ergo quoad longitudinem ſubtangens $= 2x$. Similiter in Parabola Cubicali $x = y^3$ vel $x^{\frac{1}{3}} = y$, & Fluxio $\dot x \times x^{\frac{1}{3}} =$ Fluxioni areæ, Fluens ergo erit $\frac{3}{4}xy$; In omnibus quantitatibus hujuſmodi, Fluxio eſt ſimplicis quantitatis; Ubi vero Fluxio involvit Dignitates aliquas Nihili in æquatione diverſas, accurata Geometria nunquam ex iſtis Fluxionibus veras eliciet Fluentes, quoniam neque veræ ſunt Fluxiones, & ſi veræ eſſent, non recte quaſvis, minimas licet, quantitates rejicit Geometra, qui affirmaverit errores quam minimos in rebus Mathematicis non eſſe contemnendos, uti ipſe Newtonus aſſerit Pag. 167. Quadrat. Curvar. Verum hanc dimittimus ad alteram probationem accedamus, *a. b. c.*
Newton. Princip. Lib. 2. Sect. 2. Lem. 2. agnoſco eſſe Fluxiones *A. B. C.* Caſ. 1. $A+a \times B+b = AB+aB+Ab+ab$ juxta priorem demonſtrationem, rejiciuntur autem *AB* & *ab*; *AB* velim, cur *ab*, neſcio; cur retinea-

tur potius, reſpuantur autem $aB+Ab$, hæc habeo; quoniam reipſa $Ab=A$, & $Ba=B$, ſed $A+B$ fieri eſſe nequit infiniteſima ipſius AB; reſpui vero debent, quia lineæ ſunt, neque quicquam faciunt ad definiendam areæ Fluxionem, & ſolum ad dirigendam eam, ut cuivis patebit rectangulum contemplanti; ſed $ab=a+b$ propter infiniteſimas a & b, quæ ſi puncta comprehenderent non eſſent infiniteſimæ, ſi non, in ſe invicem ductio eadem erit ac ſibi invicem additio; Jam ſumatur $A+\frac{1}{2}a\times B+\frac{1}{2}b$, de quo ſubducendum erit rectangulum $A-\frac{1}{2}a\times B-\frac{1}{2}b$, quod ſi exceſſus, uti affirmat Newtonus, vera eſſet Fluxio vel $aB+Ab$, æquaretur $aB+Ab+ab$, vel ejuſdem quantitatis Fluxio eſſet diverſa, utrumque autem abſurdum eſt; At vero nec probat Newtonus, quamobrem $\frac{1}{2}a$, $\frac{1}{2}b$ potius quam a, b uteremur, vel quavis Multiplici vel Submultiplici ipſius a & b; Nam non quia $\begin{array}{l}A-0\\ \times B-0\end{array}$ Subducendum erit ex $\begin{array}{l}A+a\\ \times B+b\end{array}$ ideo ſequitur $\begin{array}{l}A-\frac{1}{2}a\\ \times B-\frac{1}{2}b\end{array}$ Subducendum fore ex $\begin{array}{l}A+a-\frac{1}{2}a\\ \times B+b-\frac{1}{2}b\end{array}$ ad eundem exceſſum conſtituendum, verum plane e contra, uti patet; ſin vero, eodem etiam ratiocinio integrum eſſet $\begin{array}{l}A-\frac{1}{3}a \text{ vel } -\frac{1}{4}a \text{ vel } -3a, -4a\\ \times B-\frac{1}{3}b \text{ vel } -\frac{1}{4}b \text{ vel } -3b, -4b\end{array}$ deſumere ex $\begin{array}{l}A+\frac{2}{3}a\\ \times B+\frac{2}{3}b\end{array}$ $\begin{array}{l}\text{vel } +\frac{3}{4}a \text{ vel } -2a \text{ vel} -3a\\ \text{vel } +\frac{3}{4}b \text{ vel } -2b \text{ vel} -3b\end{array}$, *&c.* Quo quid dici poteſt inſulſius? Cæteri caſus facile ex hoc dependent, atque eodem modo refelluntur; Verum hic tandem conſiſtimus, atque interea, dum Adverſarii hanc ſuam Doctrinam tuentur, nos adornamus noſtram, quam, quandocunque propoſuerimus, nullus dubito, quin huic precariæ & vacillanti videbitur eſſe anteferenda, firmis ſaltem ſi nixa fuerit rationibus, nec auctoritatem ſibi aucupet aliunde, quam ex Veritatis demonſtratione ſuæ; Geometriam velim Solidorum.

Prælectio Undecima alia Affert Argumenta contra Newtoni Series Infinitas, & ejus Interpolationes, & de Terminis in Infinitum Evaneſcentibus Diſputat, & de Natura Infiniti.

Duodecima Agit Fuſius de Radicum Newtoni Extractionibus.

Decima Tertia Inquirit in Radicum Extractiones Oughtredi & Raphſoni, cum Numerales, tum Literatas ſive Specioſas.

Decima Quarta in eodem Verſatur Argumento.

Decima Quinta Raphſoni Theoremata de ſeriebus Ipſius Convergentibus Trutinare ſatagit.

Decima Sexta iſtam Inquiſitionem ad Finem Perducit.

Decima Septima Diſſerit de Newtoni Methodo Inveniendi Primi Radicis Termini.

Decima Octava Demonſtrationem contra Infinitas Series Univerſim molitur, & Leibnitii pariter ac Niewentiitii Infiniteſimas, atque etiam Newtoni Methodum Naſcentium & Evaneſcentium Eliminat.

At Nimium, prorſus Nimium, de hiſce Rebus jamdiu Diſputavimus, quæ huc Pertinent, ut Geometriam noſtram Solidorum, ſive Materiæ, amplius Confirmarem, adeoque Circuli inibi Propoſitam Quadraturam, & ut Nova Matheſeos Elementa Aſſererem, quæ nitidius & verius cum Rerum Natura convenirent, in Magnum tum Matheſeos, tum Philoſophiæ, Incrementum. Manum autem de Tabula.

II. *De Alia Circuli Quadratura, præter Eam, quam in Geometria Solidorum Attulimus; Item, de Problemate Deliaco, ſive Cubi Duplicatione.*

Lem. 1. CIRCULUM apud Mathematicos figuram eſſe planam, cujus omnes radii a centro ad Circuitum pertingentes inter ſe æquantur.

Nemo non agnoſcit Circulum ita definiri, fatentur Geometræ, hujus quadraturam omni ævo efflagitarunt Mathematici, eamque ipſam jam etiam exquirimus.

Lem. 2. Baſin Coni eſſe Circulum, ducique poſſe Parabolam in Cono quolibet eodem, cujus Ordinata maxima fuerit æqualis cuicunque, in baſi, Subtenſæ datæ.

Prima pars Lemmatis patet ex Natura Coni & Generatione; Secunda ex eo, quod Diameter Parabolæ eſt alteri Trianguli mediæ baſi inſiſtentis cruri parallela, quæ infimo ſuo puncto videbitur Diametrum baſis, ſummo autem ejuſdem Trianguli crus alterum in ratione æqualitatis peragrare, dum Latera Parabolæ penes erunt Circumferentiam in eadem ratione æqualitatis cum baſis Diametro.

Lem. 3.

Walliſii Arithmet. Infinitorum.

Lem. 3. In Omni figura, cujus Termini paralleli complentes decreſcunt ſemper à Maximo ad Minimum, qui Termini inter hos interjacent, ſe exprimere ſingulos in Maximo.

Nam Oppoſita Latera Parallelogrammi ſunt æqualia, ſed minora etiam Maximo Termino ex Hypotheſi.

Vel, quod perinde rem conficiet, Omnem Fluxionem cujuſvis Lineæ eſſe ſimilarem, ita ut partes intelligantur cum tota homogeneæ.

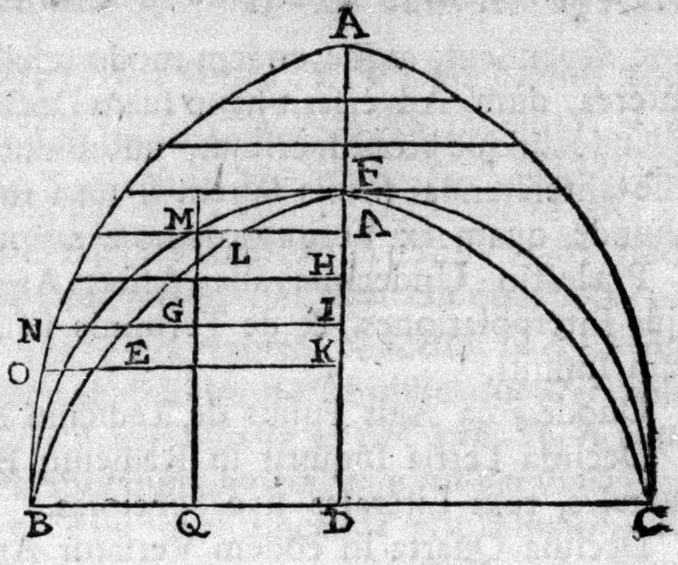
Lem. 4. Sinus Circuli eſſe lineas, vel Poteſtatis cujuſdam latera, aliter enim eſſet Circulus figura Solida contra Lem. 1.

Walliſii Arithmetic. Infinitorum.

Hoc Lemma neceſſe eſt, ut omnes concedant Mathematici, qui ideo in Quantitatum reſiduarum radicibus eruendis tam infinitam operam poſuerunt, ſi itaque quadrata probaverimus rationem habere biquadratorum, eodem ratiocinio horum porro radices erunt inveſtigandæ.

Lem. 5. Radium Circuli, vel Abſciſſam Maximam, in Unitates conſumi, ſive partes, ex quibus conflatur, progredi in ratione ſimplici Arithmetica.

Nam ſinus ſunt ut $\sqrt{R^2-1}$, $\sqrt{R^2-4}$ $\sqrt{R^2-9}$ juxta definitionem Geometrarum.



Exponatur dehinc Parabola *BAC*, cujus Ordinata Maxima, uti in hac figura, ſit æqualis Circuli cujuſcunque Diametro, *e. g. BC*, per Lem. 2. Circa hanc autem tanquam Axem revolutus intelligatur ſemicirculus *BFC*, donec ad angulum conſtiterit *ADF*, vel arbitrarium, vel, ut apertius agamus, poſtea aſſumendum; Id fieri poſſe liquet, quod Coni Triangulum medium ſe in omnes formas induet, neque deſtruetur Conus vel Coniſectio, mutata nimirum ſolummodo Circulorum in Cono ad invicem inclinatione; vel ſi *BFC* replicetur in Parabolam, prout in hac figura cernitur, inde moliente linea *AF* continue protrudetur, dum in idem planum cum plano Parabolæ ex adverſo conciderit; A quolibet autem, quod vocant, Applicationis puncto in Diametro Parabolæ ad lineam *AF* Vertices Parabolæ & Circuli connectentem ducantur Radio *DF* Parallelæ quotvis, concipianturque a tergo Plani *BAC* ita propagari, ut Parallelis in Cono Circulis, cujus hæc eſt Sectio, perficiantur Diametri ſecundum Doctrinam ſectionum Conicarum, adeoque erunt illæ partes adjunctæ Radio *FD* Parallelæ & Æquales, nam $\frac{BD\times DC}{DF}=DF$.

Iiſdem Poſitis, quæ modo in figura *BAC* expreſſimus, cogitemus lineam *AF* puncto jam ſuo *F*, quaſi centro quodam, immobilem, ipſam deorſum agi averſo motu, ſimulque inter urgendum perpetuo verſus centrum Circuli *D* rejiciat Parabolam uſque ad Altitudinem *DF*, cui ergo æquabitur *AD*, & pro Longitudine *AF* erit *ADF* Triangulum ex Scaleno in Iſoſceles effictum vel Æquilaterum; Nam ad Parabolam nihil valet, quocunque tandem modo Linea Verticalis *AF* occurat Abſciſſæ *DA*, ſi ita conus conſtituatur, ut alterum latus medii Trianguli ſit Abſciſſæ Parallelum; eadem enim baſi manente & Coni & Parabolæ, mutatio tantum inducitur eorum Altitudinibus, adeoque, ob clariorem intellectum, Æquilaterum denique Triangulum ſeligemus, cum integrum ſit quodvis ad ejuſdem Circuli Naturam perſcrutandam uſurpare.

Radius itaque Litera R notetur, ſemiordinatam minorem Parabolæ ſignificet p, p^2 exprimat abſciſſam, s minorem ſinum, P vero ſemiordinatam majorem, & r Radii quamlibet partem, S ſinum etiam majorem.

Ex Doctrina ſectionum Conicarum Lineæ inter Verticalem *AF* & Abſciſſam ſive Parabolæ Diametrum *AD* interceptæ, Radioque *FD* Parallelæ, æquantur $\frac{HLq}{R}$, $\frac{GIq}{R}$ ſive $\frac{p^2}{R}$, verum tantundem eſt, ſeu ſeriem $\frac{p^2}{R}$ in Triangulo *ADF* ceperis ab *A* verſus *DF*, ſeu a *D* verſus *AF*, ex natura & notione Æquilateri; partes autem, quæ per ſeriem reim $\frac{p^2}{R}$ a D verſus AF abſcinduntur in radio DF, æquilateræ, ſunt eapropter etiam æquales, adeoque $\frac{p^2}{R}=r$, vel brevius, $\frac{p^2}{R}$ æquantur prædictis lineis, ergo ductis lineis Parallelis AD abſciſſæ, per Lem. 3. $\frac{p^2}{R}=r$, jam R^2-r^2 ex Circuli proprietate $=AMq$, *&c.* $=s^2$, atque $R^2-\frac{p^4}{R^2}=S^2$, deleatur R^2 in $\frac{p^4}{R^2}$, vel quia relatarum quantitates nihil immutant ipſas relationes, nimirum $\frac{p^4}{x}$ eadem eſt in ratione quavis aſſignata, ſeu ponatur x, vel R^2, vel unitas, aut ſaltem, quod apud Mathematicos jamdiu ſolenne eſt, pro Parametro ſive Radio unitatem adhibendo, ac idcirco $R^2-p^4=S^2$, $R^2-P^4=s^2$.

Porto $BD=\surd R$, quoniam DA ex Poſitione Trianguli Æquilateri exæquat R, itaque $BD-o\times BD+o=BDq=R-o$, $BD-EK\times BD+EK=\surd R-P\times\surd R+P=R-P^2$, $BD-LH\times BD+LH=R-p^2=R-\frac{P^2}{R}$ & $R-\frac{p^2}{R}$, propter Parametri rationem nullam; ſi igitur ſumatur $DBF=DBA$ Semi-Parabola nempe Semi-Parabolæ, quarum Vertices F & A, partes p^2 vel $\frac{p^2}{R}$ quæ procederent ab F verſus DA, per Lem. 3. exprimerent totidem æquales in DA a D verſus A, adeoque $R-p^2=AK$ per Proprietatem Parabolæ $=EKq$, vel $R-p^2=P^2$, pariterqne $R-P^2=AH=LHq$, & $R-P^2=p^2$.

Denuo per Lem. 1. $DB=DF$ vel $\surd R=R$, & $R=R^2$, ergo DBF etiam erit Semi-Parabola, cujus Vertex B, Abſciſſa DB, per Lem. autem 3. Semi-Ordinatæ Omnes exprimuntur in DF, ergo ex hypotheſi Æquilateri Singulæ p in ſerie a D verſus F æquantur Singulis p^2 in ſerie a D verſus A; vel, quoniam Fluxio lineæ eſt ſimilaris, partes lineæ $DF=\surd R$ erunt homogeneæ ſive $=\surd r$, quæ reſpondent totidem r in DA, adeoque ubique $p=p^2$ & $p^2=p^4$, tandemque $R-p^2=R^2-p^4$.

Sed in Semi-Paraboloeide Biquadraticali, cujus Abſciſſa æque ac Semi-Ordinata Maxima ſint Radio æquales, vel DF Semi-Ordinata Abſciſſæ $DB=\surd R$, atque proinde $DF=\surd(^4)R$, erunt Rectangula Quadratis Semi-Ordinatarum æqualia $=\surd R^2-\surd p^4=\surd P^4$, $\surd R^2-\surd P^4=\surd p^4$; nam Biquadrata, quibus Abſciſſa eſt R, æquantur Quadratis Semi-Ordinatarum ſibi aſſerentium Abſciſſam $\surd R$, *h. e.* $\surd(^4)R^4$, vel $\surd(^4)R\times\surd(^4)R\times\surd(^4)R\times\surd(^4)R=R$ idem erit ac $\surd R^2$ vel $\surd R\times\surd R=\surd R$, ergo enim $\surd R^4$ vel $\surd R\times\surd R\times\surd R\times\surd R=\surd R^2=\surd R\times\surd R=R$, & $\surd R=\surd(^4)R$, atque eodem plane ratiocinio $\surd(^4)p^8=p^2=r$ idem erit ac $\surd p^4=p$, adeo enim $\surd p^8=p^2$; ſi itaque transferatur Semi-Paraboloeides in BAD ut literis ibi ſignatis utamur, & aſciſcatur ſeries reciproca, quemadmodum antea factum eſt, $\surd R^2-\surd p^4=AK=EKq=\surd P^4$, $\surd R^2-\surd P^4=AH=LHq=\surd p^4$; verum modo demonſtravimus $\surd p^4$ eſſe ut Biquadrata Diametrorum, quarum progreſſio incedit per p^2, vel Parabolæ abciſſas interceptas, & eamobrem $\surd R^2-\surd p^4$, $\surd R^2-\surd P^4$ ſunt ut quadrata Paraboloeidis, cujus Abſciſſa eſt Maxima $\surd R$, vel ut Biquadrata, ſi ponatur R.

Jam $R^2-p^4=S^2$, $R^2-P^4=s^2$, atqui $\surd R^2-\surd p^4=R^2-p^4$, & $\surd R^2-\surd P^4=R^2-P^4$, quoniam $R=\surd R$ & $p^2=p$ vel $p=\surd p$, & $P=\surd P$; ergo $\surd R^2-\surd p^4=S^2$, $\surd R^2-\surd P^4=s^2$, per Lem. autem 5. Radius progreditur ut p^2, ideoque S^2, s^2 ſunt ut Biquadrata $\surd R^2-\surd p^4$, $\surd R^2-\surd P^4$; Extrahatur denique radix Biquadratica earundem Quantitatum per Lem. 4. reſtituaturque directa Progreſſio, tandemque conſtabit deſiderata tamdiu Circuli Quadratura, quæ eadem erit cum Quadratura Paraboloeidis expoſiti, vel Quadrans ad Quadratum Radii ut 4. ad 5. ſed per Lem. 1. & 3. Omnes Radii Radiorumque partes ſunt inter ſe æquales, ac proinde eadem Demonſtratio de cæteris obtinebit, & Circulus integer ad Quadratum Diametri erit ut 16 ad 20. vel ut 4 ad 5. *Q. E. I.*

Coroll. 1. Ergo eadem Proprietas Parabolæ competit, quæ notiſſima eſt in Circulo, Salva dignitate, cujus ſunt compotes Semi-Ordinatæ, quarum nempe quadrata æquantur ſegmentorum Rectangulis, $\surd R+p\times\surd R-p=P^2$ in reciproca Semi-ordinatarum ſerie, $R+p^2$ $(=\surd(^4)R+\surd p)\times R-p^2=S^2$ in ſinuum ordine item inverſo; & viciſſim Biquadrata ſinuum ſunt ad invicem ut Abſciſſæ, perinde atque Quadrata Semi-ordinatarum in Parabolæ; *h. e.* tam hic, quam illic, Semi-ordinatarum & ſinuum quadrata ſunt ut Abſciſſæ, ſi ita Radii deprimantur, ut horum indices ſint ſemper iſtarum ſemiſſis.

Corol. 2. Et hinc demum exhibetur Solutio Problematis Deliaci, ſive Cubi Duplicatio vel Sphæræ; Habeantur enim duæ quævis lineæ datæ, quibus totidem erunt mediæ Proportionales ſtatuendæ; Erigantur datæ a, b, ſibi invicem ad perpendiculum, adeo ut altera ſinum Circuli referat cogniti, cujus altera ſit ſinus verſus, pars reliqua Diametri erit *e. g.* $\frac{a^2}{b}=x$; Super eandem Circuli Diametrum deſcribatur Parabola, quæ itaque abſcindet ex linea a Semi-ordinatam $=p$, inter datas ergo a & b interjacent duæ mediæ proportionales, p, & $pa=$ tertiæ proportionali ab a & p; nam per Corol. 1. $1.a::a.p::p.\frac{b}{a}$, vel $a.p::p.pa::pa.b$; ſed $pa=\surd(^4)b^3=\frac{b}{\surd(^4)b}$.

Corol. 3. Proprietas etiam Parabolæ Bakeriana, ſi jam eſſet requirenda, hinc facile conſequeretur; Nam $L=1.\surd R+p::\surd R-p.R-p^2$ ſive $BQ\times QC=QL$, *h. e.* $BQ.\surd QL::\surd QL.QC$; vel ſi Dimenſio $\surd QL$ in Parabola æquaretur Dimenſioni QL, $BQ.QL::QL.QC$. Eodemque prorſus modo in Circulo $L=1.\surd R^2+p^2::\surd R^2-p^2.\surd R^2-p^4$, vel $1.\surd(^4)R+\surd p::\surd(^4)R-\surd p.\surd R-p$, ex iis quæ nobiſmet ſaltem videmur demonſtraſſe, ſive $BQ\times QC=\surd MQ$, *h. e.* $BQ.\surd(^4)MQ::\surd(^4)MQ.QC$; Quod ſi MQ obtineat Progreſſionem Radii ſeu Unitatis, erit $1.\surd(^4)R^2+\surd p^2::\surd(^4)R^2-\surd p^2.R-p^2$, *h. e.* $CDq+QDq.\surd MQ::\surd MQ.CDq-QDq$, & quidem ſi $\surd(^4)MQ$ æquaretur MQ, quod ſemper in figuris Planis ponitur a Geometris $BQ.MQ::MQ.QC$.

Corol. 4. Videant ergo Mathematici, annon hinc eluceſcat Methodus Æquationes conſtruendi ad Septimam Octavamque poteſtatem aſcendentes, quam, ne in nimiam prolixitatem incidamus, in præſenti omittimus, ſi tempus tulerit, poſthac forſan exhibendam, modo res patiatur, adducendo etiam Parabolam, ut, ea adminiculante, exprimantur radices in Ellipſi, quas ſolius Ellipſeos interſectio partim tantum indigitaverit.

Corol. 5. Propter ea quæ dicta ſunt porro liquet aream Semi-Parabolæ, æquali baſe vel Semi-ordinata Maxima, eſſe ad Circuli Quadrantem, ut 5 ad 6, atque ita ſpatium inter lineam Parabolicam & Peripheriam interceptum ad quadrantem, ut 1 ad 6.

Corol. 6. Item, cum Quadratum Diametri ſit ad Circuli aream, ut Diameter ad quartam circumferentiæ partem, quod vulgo notum eſt, ipſa circumferentia erit ad Diametrum, ut 4 ad $1\frac{1}{4}$, vel ut $3\frac{1}{5}$, adeoque inter arcum & ſinum graduum 30, $=$ Radii Dimidio interceſſerit ratio partium 16 ad 15; Ex his autem convincitur eorum error, qui rationem peripheriæ definire orant ex iniquiſſimo hoc, ut auguror, Poſtulato, omnem figuram Circulo circundatam, Majorem complecti circuitum, quam Circulum, Minorem, inditam; præſertim cum nihil apud Mathematicos fuerit certius, quam inter figuras, quas vocant Iſoperimetras, circulum eſſe maximam; quæ parum videntur conſtanter dici; Interea liceat adnotare Quadraturam Archimedeam non multum a noſtra abludere, illa nimirum eſt ut 11 ad 14, noſtra cum ſit ut 12 ad 15 vel 4, ad 5; Sed $11.14::4.5$. Proxime, cum $55=56$, Proxime.

Corol. 7. Itaque Sphæra ad Cylindrum eandem obtinet proportionem, quam olim, nam ſinus ſunt ut $\surd(^4)1$, $\surd(^4)2$, $\surd(^4)3$, $\surd(^4)R$, adeoque quadrata, ut $\surd 1$, $\surd 2$, $\surd 3$, $\surd R$, vel ad totidem Maxima, ut 2 ad 3.

Corol. 8. Eademque etiam ratione, ſi ponatur infinita Parabolarum, quales prius adhibuimus, totalium, nempe quæ circa directas & tranſverſas æquales Diametros deſcribantur, ſeries continue decreſcentium; erit iſtiuſmodi ſeries ad Cylindroides vel ad totidem Parabolas maximas ut 1 ad 2; Nam figuræ ſimiles ſunt in duplicata ratione homologorum laterum, adeoque ut ſeries Numerorum naturaliter & Arithmetice aſcendentium ad Maximorum ſeriem.

Corol. 9. Et tandem, ut finem Corollariis imponamus, quorum copia eſt uberior ex principiis jam allatis, Hinc ſequitur Lineæ dari poſſe Parabolicæ Rectam aliquam æqualem; Nam $\frac{1}{4}xd^2=$ Circulo, Parabola vero $=\frac{5}{24}xd^2$, ſi ratio peripheriæ ad diametrum jam eſſet ignota; cum autem innoteſcat $\frac{1}{4}xa^2=\frac{1}{4}\times 3\frac{1}{5}d^2$ vel $\frac{1}{4}\times\frac{16}{5}\times 1$; Ignotam tamen x requirimus; Ergo quoniam Parabola, quam uſurpamus, pari ratione vel etiam validiori, quia ad formam Quadrati magis accedit, utpote depreſſior, quam Circulus in triangulum poteſt configurari, cujus Altitudo ſit ſemidiametro æqualis, Baſis lineæ Parabolicæ, neque enim magis repugnat, (ut altitudinem trianguli majorem aſſumamus quam debuiſſemus, quam ſi minorem arceſſiverimus, quod, ut videtur, eſt ſolum diſcrimen inter utraſque ad triangulum conformandas,) Has ob res, inquam, $\frac{5}{6}xd=$ Lineæ Parabolicæ, atque idcirco $\frac{5}{6}xd.4d::2.3$, ſive Linea Parabolica $=2\frac{2}{3}d$, & ad Circumferentiam ut 5 ad 6. nimirum ut Parabola ipſa ad Circulum; Quod ſi ambigitur

tur de Triangulo Parabolæ, ſumatur etiam iſtiuſmodi, quæ circuitione æquetur Circulo, rationeſque ſimiliter definientur, & ſpiſſior ſolummodo linea erit Circularis, Parabolica tenuior; Quæ vero demonſtraſſe videmur de Paraboloeidibus & Parabolis Diametrorum æqualium, facile, ni fallor, ad illa nempe, quorum Diametri ſunt diſſimiles, accommodabuntur, nam ne nimis tuas, Vir ſumme, majores & res arduas interpellem, ad metam properamus.

Scholium. Atque adeo neque Objectionibus immoramur diſſolvendis, quæ ſingulæ, cum noſtra principia linearumque dimenſiones ignorarent Geometræ, ne quidem demonſtrationes, quas allegavimus, poſſint attingere; nihilominus, quia nuperæ Mathematicorum, præter cæteros autem tuæ, illuſtriſſime Newtone, elucubrationes de ſerierum interpolationibus & approximandi methodo, tanta, forſan atque etiam merito, valuerunt auctoritate, ut cum ibi plane conclamatum ſit, deſperandum omnibus hocce Problema viri Docti arbitrentur, ea de cauſa neque negleximus eas ipſas ſeries perpendere, & non modo ſpem facimus, re diligenter & accurate perſpecta, ſed porro Geometris pollicemur Omnium Quantitatum Reſiduarum Binomiarumque Radices quaſcunque extrahi poſſe, quarum ſeries prima eſt æqualium, ea methodo nullus dubitem, quin quicquid in tam abſtruſa ſcientia reſtiterit implicatius vel magis tortuoſum facillime explicaretur; nam ſi hæc ſatis fuerint comprobata, de Hyperbolæ Quadratura etiam nobis conſtat, & de Quantitatum Additiarum evolutione, &, ni fallor, aliarum quarumcunque, Fluxionumque nova Doctrina; Neque hic, ſi Noſtra Mathematicis placeant, conſiſtemus; Nam Matheſin a Lineis & Planis ad Materiam & Corpora, Animumque præterea perreximus Abducere; Quod aliquando Bono cum Deo, ſi vita atque otium Suppeditetur, alterum in Geometria, quam adornamus, Solidorum, alterum in Pſycheometria, quam ſumus aggreſſuri, quemadmodum Spero, dabitur effectum; Siquid vero ſim aſſecutus, quod collineet Præmiſſa Demonſtratio, Scil. Circuli Quadraturam, iſtam omni ævo expetitam, non uberiorem lætitiæ & gaudii fructum ex omnium plauſu ceperim, quam ex tuo Solius, Vir Præclariſſime, limatiſſimo Judicio, Aſſenſioneque tam ſtupendi Ingenii & Doctrinæ Singularis.

Neque demonſtrationes allatæ nimium probant, quaſi eodem modo omnes rationes quaſcunque de Circulo ad Quadratum Diametri affirmare liceret, quamvis ſub certis conditionibus illud profecto verum ſit, quod alias ſumus oſtenſuri; in præſenti autem Quadratura, quam Mathematici tantopere laceſſiverunt, recte intra præſcriptos ab ipſis limites & Concluditur, ni fallor, & Solvitur; Nam, iis obſervatis, nullam aliam patitur Circulus; *E.G.* in triangulo ſemiordinatæ a, b, c, &c. $=$ ſemiordinatis Circuli a, b, c, &c. & abſciſſæ trianguli a, b, c, &c. $=$ abſciſſis circuli a^2, b^2, c^2, &c. *h. e.* $a=\sqrt{a}$, $b=\sqrt{b}$, *&c.* adeoque hoc triangulum poſitum converteretur in Parabolam, nempe, cujus Abſciſſæ forent $=a^2, b^2, c^2$, *&c.* Semiordinatæ a, b, c, &c. vel cujus Abſciſſæ a, b, c, *&c.* ſemiordinatæ $\sqrt{a}, \sqrt{b}, \sqrt{c}$; *&c.* & pariter de aliis figuris ex Æquatione, quæ efficitur Circuli ſinuum & abſciſſarum cum ſemiordinatis & abſciſſis iſtarum figurarum; Nam omnes ex poſitis Diverſis ſe transformatas reperient in Parabolas.

III. *De Conicis Sectionibus Clariſſimi Viri Jacobi Milneſii.*

Prop. XV. Theor. XIV. Pars I. Sectio. I. Sic Enunciatur.

Si Binæ qualibet Rectæ inter ſe Parallelæ, & vel utraque ad eandem Sectionem, vel utraque ad Oppoſitas, vel ſingulæ ad ſingulas, utrinque in Punctis BCFG, *Terminatæ; Vel quarum una, vel utraque Sectionem vel Sectiones oppoſitas contingit in* CG, *ſi opus productæ utrique aſymptoto occurrant in* AD, EH: *Dico rectangula omnia,* $AB\times BD$, $EF\times FH$, $CD\times CA$, $GH\times GE$ *eſſe ſibi invicem æqualia.* Vid. Fig. 39. & 40.

Quæ forſan Generalius ſic Concipi poteſt, & Venuſtius.

Si quæcunque Parallelæ inter Aſymptotos agantur in Eadem Sectione, & a Curva Hyperbolica utcunque bifariam ſecentur, vel ſi Quæcunque Parallelæ inter Curvas Hyberbolicas agantur in oppoſitis Sectionibus & ab Aſymptoto utcunque bifariam ſecentur, Linearum Segmenta in ſe invicem Ducta Conſtituunt Rectangula ubique Æqualia. Vid. Fig. 39, 40, 41, 42, 43, 44. Nam Fig. 39, 41, 43. in Eadem Sectione $AB\times BD=EF\times FH=CD\times CA=GH\times GE$; & Fig. 40, 42, 44. in oppoſitis Sectionibus, $AB\times BD$, ſive, $BA\times AC=EF\times FH$, ſive $FE\times EG=CD\times CA$, ſive $CD\times DB=GH\times GE$ ſive $GH\times HF$.

Subjectum Lemma 3. hoc modo Explicatius Datur.

Cas. 1. $AE=AB+BE$
$ED=AB+EC$
$AC=AB+BE+EC$
$CD=AB$

Ergo; $BE\times EC+AC\times CD=AE\times ED$ Fig. 45.

Cas. 2. $AE=AB+BE$
$ED=AB-CE$
$AC=AB+BE-CE$
$CD=AB$

Ergo; $BE\times EC+AE\times ED=AC\times CD$. Fig. 46.

Cas. 3. $AE=AB+BE$
$ED=CE-AB$
$AC=AB+BE-CE$
$CD=AB$

Ergo; $BE\times EC-AE\times ED=AC\times CD$. Fig. 47.

Prop. XVI. Theor. XV. Sic Enunciatur.

In Hyperbola & Sectionibus oppoſitis, ſi bina qualibet recta, KN, ST, *vel utraque ad eandem Sectionem, vel utraque ad oppoſitas, vel ſingula ad ſingulas, vel altera ad eandem, altera ad oppoſitas utrinque terminatæ, & ſi opus productæ, ſibi mutuo occurrant in* E, *& utrique aſymptoto in* R, V, M, L, *punctis; Dico,* $\left.\begin{matrix}KM\times MN\\ LN\times MN\end{matrix}\right| . RT\times TV :: KE\times EN . SE\times ET$.

Vid. Fig. 50, 51, 52, 53, 54, 55.

Quæ ſic forſan Generalius & Concinnius poſſet Exprimi.

Si Quævis Duæ Lineæ ſibi invicem Occurrant in Certo Puncto E, *Trajiciantur vel Contingantur autem utcunque vel in eadem Sectione vel in Sectionibus Oppoſitis per Duas Curvas Hyperbolicas & Duas Aſymptotos, Rectangula ſub ſegmentis Linearum, quæ Terminantur ab Aſymptotis, Secantur vero vel Diſtinguuntur bifariam a Curva Hyperbolica, Proportionalia erunt Rectangulis ſub ſegmentis Linearum, quæ Terminantur a Curvis Hyperbolicis, ſecantur vero vel Diſtinguuntur a Puncto Concurſus.*

Vid. Fig. 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60.

Nam *Fig.* 51, in Eadem Sectione $LN\times NM . RT\times TV :: KE\times EN . SE\times ET$.

Item in Sectionibus oppoſitis.

Fig. 50. $LN\times NM . RT\times TV :: KE\times EN . SE\times ET$; Ubi LN & NM, æque ac RT & TV Secantur vel Diſtinguuntur a Curva Hyperbolica, Terminantur autem ab Aſymptotis, & KE ac EN, æque ac SE & ET Secantur vel Diſtinguuntur a Puncto Concurſus, Terminantur vero a Curvis Hyperbolicis.

Porro in *Fig.* 52. Eadem Proportio plane obtinet, eodemque modo ad Generalem noſtram Propoſitionem Reducitur.

Fig. 53. Eandem ſortitur Explicationem, uti & *Fig.* 54, 55, 56, 57, 58, 59, & 60. Siquis velit ſingulas Perpendere, & Exigere ad Conditiones, quas modo in Propoſitione Noſtra Præſcripſimus.

Quotcunque igitur Caſus obveniant, a Præmiſſis Solvuntur & Explicantur.

Verumtamen, cum nihil Reſtat in hac Doctrina, niſi ut Oſtendamus, quemadmodum Varios hoſce Caſus adinvenit, qui totidem Figuris Exhibentur, & quomodo cæteros omnes inveſtigare poſſimus; Iſtum etiam Laborem non Refugimus; Singuli enim Caſus facile hac Ratione & Via Innoteſcent; Nempe, ita Lineas *RSETV*, *LKENM*, Fig. 51. in Conicis Sectionibus ſibi Invicem Occurrentes Dividendo, ut utriuſque Partes Sigillatim in Ipſius Diviſione omnes Varios ſitus Obtineant, qui per *ABECD*, Fig. 45. in Superiori Lemmate Deſignantur; Eaſdem autem Lineas ita Componendo, ſive Collocando, ut ſe mutuo Decuſſarent, vel Contingerent, vel Concurrerent, in *E*, Veluti *e. g.* in *Fig.* 51, 50, 53. Obſervandum autem eſt, ſicut in Figuris Lemmatis AB vel $BA=CD$ vel DC, ita RS vel $SR=TV$ vel VT, & LK vel $KL=NM$ vel MN in Sectionum Figuris; Unde neceſſe eſt, ut Quindecim Caſus, Primarii & Principales, vel Poſſibiles vel Impoſſibiles, prout *E* in Diverſum Locum Cedat, Conſtituerentur; Vel enim,

Primo, ubi *Fig.* 45. *ABECD* ſecum Componitur & Collocatur, ita ut Decuſſatio fiat in *E*, Puncto nimirum Utriuſque Lineæ Communi, in Conicis Sectionibus *RSETV* vel *SREVT*, *LKENM* vel *KLEMN*, Fig. 51, & 54. iſtam Compoſitionem Exhibere Cernuntur; Vel, Secundo, Ubi *Fig.* 46. *ABCED* cum ſeipſa Componitur & Collocatur, ita ut fiat Decuſſatio in *E*, Puncto Utriuſque Communi, in Conicis Sectionibus *RSTEV*, *LKNEM*, vel *KLMEN*, Fig. 52. iſtam Compoſitionem Exhibent; Vel,

Tertio, *Fig.* 47. *ABCDE*, ſi Componatur ſive Collocetur cum ſeipſa, ita ut Concurſus ſit in *E*, in Conicis Sectionibus Exprimetur per *RSTVE*, vel *SVRTE*, & *LKNME*, vel *KLMNE*, Fig. 53; Vel,

Quarto, Fig. 48. ſi $A^{B}_{C}ED$ ſecum Componitur, ita ut Decuſſatio fiat in *E*, in Conicis Sectionibus $R^{S}_{T}EV$, $L^{K}_{N}EM$ vel $M^{K}_{N}EL$ Compoſitionem iſtam Significant *Fig.* 59.

Quinto, *Fig.* 49. ſi $A^{B}_{C}DE$ ſecum Componitur ita, ut fiat Concurſus in *E*, in Conicis Sectionibus $R^{S}_{T}VE$, vel *VSTRE*, $L^{K}_{N}ME$ iſtam Compoſitionem fere & magna ex Parte Demonſtrant *Fig.* 55, & 57.

Sextus Caſus eſt, ſi *ABECD* Componatur cum *ABCED* vel *Fig.* 45. cum 46, in Conicis autem Sectionibus *RSETV* cum *LKNEM*, cujus Compoſitionis nullum datur Exemplum.

Septimus Caſus eſt, ſi *ABECD* Componatur cum *ABCDE* vel *Fig.* 45. cum 47, in Conicis autem Sectionibus *RSETV* cum *LKNME* vel *KLMNE* vel *NMLKE*, Fig. 56. & 60.

Octavus eſt Caſus, ſi *ABECD* Componatur cum $A^{B}_{C}ED$ vel *Fig.* 45, cum 48, in Conicis autem Sectionibus *RSETV* cum $L^{K}_{N}EM$, cujus Compoſitionis nullum Extat Exemplum.

Nonus Caſus, ſi *ABECD* Componatur cum $A^{B}_{C}DE$ vel *Fig.* 45, cum 49, in Conicis autem Sectionibus *RSETV* cum $L^{K}_{N}ME$, vel *KLMNE*, cujus Compoſitionis Exemplum ex Parte Habemus in *Fig.* 50.

Decimus Caſus Petitur ex Compoſitione *ABCED* cum *ABCDE* vel *Fig.* 46, cum 47, in Conicis autem Sectionibus ex Compoſitione *RSTEV* cum *LKNME*, cujus nullum habetur Exemplum.

Undecimus Caſus Elicitur ex Compoſitione *ABCED* cum $A^{B}_{C}ED$ vel *Fig.* 46, cum 48, in Conicis autem Sectionibus ex Compoſitione *RSTEV* cum $L^{K}_{N}EM$, cujus nullum Exemplum Comparet.

Duodecimus Caſus Fluit ex Compoſitione *ABCED* cum $A^{B}_{C}DE$ vel *Fig.* 46, cum 49, in Conicis autem Sectionibus ex Compoſitione *RSTEV* cum $L^{K}_{N}ME$, cujus nullum Cernitur Exemplum.

Decimus Tertius Caſus Oritur ex Compoſitione *ABCDE* cum $A^{B}_{C}ED$ vel *Fig.* 47, cum 48, in Conicis autem Sectionibus ex Compoſitione *RSTVE* cum $L^{K}_{N}EM$, cujus Exemplum Nullum Conſtat.

Decimus Quartus Caſus Emanat ex Compoſitione *ABCDE* cum $A^{B}_{C}DE$ vel *Fig.* 47, cum 49, in Conicis autem Sectionibus ex Compoſitione *RSTVE*, vel *VSTRE* cum $K^{K}_{N}ME$ vel $M^{K}_{N}LE$, Quam Compoſitionem Videre eſt *Fig.* 58.

Denique Caſus Decimus Quintus Conficitur ex Compoſitione $A^{B}_{C}ED$ cum $A^{B}_{C}DE$ vel *Fig.* 48, cum 49, in Conicis autem Sectionibus ex Compoſitione $R^{S}_{T}EV$ cum $L^{K}_{N}ME$, cujus Exemplum Deeſt.

Porro Liquet hoſce etiam Caſus Principales & Primarios Variari Poſſe, uti patet in Primo Caſu, item in Quinto & Septimo, Aliiſque.

At vero tamen hos Quindecim Caſus, ſi Rem attentius Trutinare Velimus, in ſex Abire Convenit; Vel enim,

Primo, *E* in utraque Linea Invenitur inter Æquales Partes *AB*, *CD*, Fig. 45. *h. e.* *E* in Sectionum Lineis Conſiſtit inter *RS*, *TV*, & *LK*, *NM*, Fig. 51. & 54. Cas. 1. Vel.

Secundo, *E* in utraque Linea inter *C* & *D* Invenitur, Scilicet in Æqualem Partem *CD* Incidit *Fig.* 46, 48. *h. e.* *E* in Sectionum Lineis Reperitur inter *T*, *V*, & *N*, *M*, Fig. 52, 59. Cas. 2, 4, 11. Vel,

Tertio, Invenitur *E* in utraque Linea ultra *D* vel Æqualem Partem *CD* Fig. 47. 49. *h. e.* in Sectionum Lineis *E* Conſtat ultra *V* & *M*, ſive Æquales Partes *TV*, *NM*, Fig. 53, 55, 57, 58. Cas. 3, 5, 14. Vel,

Quarto, *E* Invenitur in Altera Linea inter Æquales Partes *AB*, *CD*, inter *C* & *D*, vel in Æquali Parte *CD*, in Altera, *Fig.* 45, 46, 48. *h. e.* in Sectionum Lineis *E* inter *RS*, *TV*, & *N*, *M*, Debet Conſpici, Cas. 6, 8. Nam Exemplum Nullum in propatulo eſt, Vel,

Quinto, *E* Invenitur in Altera Linea inter *C* & *D* vel in Æquali Parte *CD*, ultra autem *D*, vel Æqualem Partem *CD*, Excurrit in Altera, *Fig.* 46, 48, 47, 49. *h. e.* in Sectionum Lineis *E* inter *T*, *V*, & ultra *M*, vel inter *N* & *M* & ultra *V* Debet Conjungi Cas. 10, 12, 13, 15. Exemplum enim Nullum Palam eſt, Vel,

Sexto, *E* Invenitur inter Æquales Partes *AB*, *CD*, in Altera Linea, in Altera ultra *D* vel Æqualem Partem *CD* Vagatur, *Fig.* 45, 47, 49. *h. e.* in Sectionum Lineis *E* ſe Oſtentat inter *RS*, *TV* in Altera Linea, ultra *M* in Altera *Fig.* 50, 56, 60. Cas. 7, 9.

Demum hi Sex Caſus in Tres Reſolvuntur, Una Nimirum Linea Poſita; Vel enim *E* Sedem Arripit inter Æquales Partes *AB*, *CD*, Fig. 45. vel inter *C* & *D*, ſive in Æquali Parte *CD*, Fig. 46, 48. vel Ultra *D*, ſive Æqualem Partem *CD*, *Fig.* 47, 49.

In Poſtrema hac Caſuum Enumeratione Quartus & Quintus non Inveniuntur inter Figuras, Quia, ut videtur, fieri nequit in Quarto Caſu, ut *E* inter *C* & *D* Conſiſtat in Altera Linea, ſi in Altera inter *B* & *C* Reperiatur, *Fig.* 45, 46. Nam *Fig.* 45. in Conicis Sectionibus Exhibetur per *RSETV*, *LKENM*, Fig. 51, 54, at vero in Utrovis Caſu Ducere Lineam ad *E*, quæ Figuræ 46. Reſpondeat, non Licet, Quoniam in Sectionibus *E* Interjacet inter Æquales Partes, in *Fig.* 46. autem Incidit in Ipſam Æqualem Partem; In Caſu item Quinto ſuppar eſt Ratio, ubi *E* in Sectionibus inter Æquales Partes Intercedit, in *Fig.* 47. Ultra Æqualem Partem Provehitur.

Sunt itaque, ſi Recte Auguror, Tempus enim mihi Deficit, Quo minus pertinacius in hanc Doctrinam Inquirerem, ſunt Quatuor Solummodo Caſus Poſſibiles Reſpectu Puncti *E*.

Primus Caſus duas Diverſas Poſitiones Patitur, vel enim *E* intra Sectionem *Fig.* 51. vel inter Oppoſitas Sectiones *Fig.* 54. Neceſſe eſt ut Conſtaret, quo Æquales Partes ad Extremas Poſſint Ablegari.

Secundus Caſus Quatuor Variationes Suſcipere Videtur 1. in *Fig.* 52. Ubi Altera Linea Ducitur in Sectione Quavis, Altera Pertendit ad Oppoſitas. 2. In *Fig.* 52. ſi *YX*, *RV*, in Eadem Sectione ſe Invicem Decuſſent in *E*, intra Ipſas Partes Æquales. 3. In *Fig.* 54. ſi Ambæ Lineæ ad Oppoſitas Sectiones Pertingerent & ſe Invicem ſecent intra Ipſas Partes Æquales. 4. In *Fig.* 59. Ubi Contingentes Hyperbolam ſe Invicem Trajiciunt in *E*.

Tertius Caſus Admittit Plures item Variationes. 1. In *Fig.* 53. 2. In *Fig.* 55, & 57, 3. In *Fig.* 58.

Sextus Caſus Variam etiam Poſitionem habet in *Fig.* 50, 56, 60.

Generalis autem Regula propter Linearum Compoſitiones Videtur eſſe Hujuſmodi, Nempe vel Ambas Lineas Pertendere ad Oppoſitas Sectiones, quas vel Secant vel Contingunt, vel Altera Secat, Altera Contingit; Vel Ambas Lineas ad Eandem Sectionem Duci, Quam vel Secant, vel Contingunt, vel Altera Secat, Altera Contingit; Vel Tandem Unam Lineam ad Eandem Sectionem Apponi, quam vel Secat vel Contingit, Alteram inter Oppoſitas, quas item vel Secat, vel Contingit.

Porro Notandum eſt, Omnes Caſus, Quotcunque fuerint, Præmiſſos Exprimi poſſe per *RSETV*, *LKENM*, ubi *E* Medium Locum utrinque Tenet inter Æquales Partes *RS*, *TV*, & *LK*, *NM*; Hoc modo *Fig.* 50. de *RSETV* Patet, & in *LK*, *KE*, *EN*, *NM* ſive *LKENM*, Conſtat *E* Interpoſitam eſſe inter Æquales Partes *LK*, *NM*; Pariter, *Fig.* 52. *RSETV=RS*, *SE*, *ET*, *TV*, & *LKENM=LK*, *KE*, *EN*, *NM*, ubi *E* in Medio ſita eſt inter Æquales Partes *RS*, *TV*, & *LK*, *NM*; Item Fig. 53. *RSETV=RS*, *SE*, *ET*, *TV*, & *LKENM=LK*, *KE*, *EN*, *NM*, ubi *E* Viſitur inter Æquales Partes *RS*, *TV*, & *LK*, *NM*; Idem Dicendum erit de *Fig.* 54, 55, 56, 57, 58, 59, 60. Quod Quivis facile Perſpiciet, ſi Figuras Singulas Oculis modo Dignetur Perſtringere; Ex quo fiet, ut Omnes Varii & Diverſi Caſus ad *Fig.* 45. Exigantur.

Prop. XVII. Theor. XVI. Sic Enunciatur.

In Hyperbola & Sectionibus Oppoſitis, ſi Recta quævis AB *ad eandem vel ad Oppoſitas Sectiones utrinque terminata, & ſi opus Producta; Vel binæ qualibet Rectæ* AB, CD, *inter ſe Parallelæ, & vel utraque ad eandem Sectionem, vel utraque ad Oppoſitas, vel Singulæ ad Singulas utrinque terminatæ, & ſi opus productæ, Binis quibuſcunque* FG, IH *inter ſe Parallelis, & vel utriſque ad eandem Sectionem, vel utriſque ad Oppoſitas, vel Singulis ad Singulas utrinque terminatis, & ſi opus productis, utcunque occurrant, ut in* E, T, *vel* E, T, S, V, *punctis, Erunt Rectangula ex Segmentis ejuſdem Rectæ, vel binarum inter ſe parallelarum, proportionalia Rectangulis ex Segmentis binarum reliquarum, modo Sumantur rectangula a binis communibus punctis facta*, viz.

AE×EB. GE×EF :: *AT×TB. IT×TH* &

AE×EB. GE×EF :: *CV×VD. HV×VI.*

Et ſic de cæteris punctis.

Quæ Propoſitio forſan ſic Generalius Efferri poſſet & Luculentius.

Si inter Sectiones Oppoſitas, vel intra Sectionem Eandem, Concipiatur quaſi Regula Trajecta & ſi opus Producta, v. g. ABDC *Fig.* 61. *& per Lineam quamvis Solitariam, v. g.* GF *vel duas quaſvis Parallelas* v. g. GF, HT *Secetur, Rectangula, quorum Latera ab Altera Parte Definiuntur vel Terminantur Curvis Hyperbolicis, ab Altera Segmenti Puncto, erunt Proportionalia; Nempe Rectangula a Segmentis Laterum Regulæ Facta, Reſpective iis, quæ Formantur a Segmentis Lineæ Solitariæ vel Duarum Parallelarum.*

Cas. 1. *AE×EB. GE×EF* :: *CS×SD. GS×SF*; ubi Regula *ABDC* Linea Solitaria *GSEF* Trajicitur *Fig.* 61, 62, 63, 64. Nam *AEB*, *GEF*, *CSD*, *GSF*, Definiuntur a Curvis Hyperbolicis, in *A*, *B*; *G*, *F*; *C*, *D*; Puncta autem Segmentorum *E*, *S*.

Cas. 2. *AT×TB. HT×TI* :: *CV×VD. HV×VI*; ubi Regula *ABDC* Linea Parallela *HIVT* Secatur; verum *ATB*, *HTI*, *CVD*, *HVI*, Terminantur a Curvis Hyperbolicis in *A*, *B*; *H*, *I*; *C*, *D*; Segmentorum autem Puncta *T*, *V*. Fig. 61, 62, 63, 64. Ponimus enim, uti & in Sequentibus, *ATB*, *HTI*, *CVD*, *HVI* Lineas eſſe Continuas, at Retroactas a Segmentorum Punctis ad Curvam Hyperbolicam.

Cas. 3. *GS×SF. CS×SD* :: *HV×VI. CV×VD*; ubi Regulæ *ABDC* Alterum Latus *CDV* Interſecatur per Duas Parallelas *GF*, *HV*; Nam *GSF*, *CSD*, *HVI*, *CVD*, Includuntur a Curvis Hyperbolicis ad *G*, *F*; *C*, *D*; *H*, *I*; Segmentorum autem Puncta *S*, *V*. Fig. 61, 62, 63, 64.

Cas. 4. *GE×EF. AE×EB* :: *HT×TI. AT×TB*; ubi Regulæ *ABDC* Latus Alterum *ABT* per duas Parallelas *GF*, *HT* in Segmenta Diſtinguitur; Nam *GEF*, *AEB*, *HTI*, *ATB*, Limites ſuos Agnoſcunt in Curvis Hyperbolicis ad *G*, *F*; *A*, *B*; *H*, *I*; Puncta vero Segmentorum Conſtant in *E* & *T*. Fig. 61, 62, 63, 64.

Cas. 2. & 4. Derivantur ex Proportionibus in Propoſitione Allatis; Cas. 3. ad Amuſſim Quadrat cum Cas. 4. uti Palam erit, ſi inter ſe Conferantur; Nam in utroque Caſu *GFTH* Aſſumi poteſt pro Regula, ſicut antea *ABDC* Aſſumpſimus, in Altero autem *AT* Regulam Secabit, & *CV* huic Parallela, in Altero, Quod eodem Recidit; Cum *AT* indeterminate

&

& pro Arbitrio Deſumitur, atque adeo *CV* Demonſtrationi Propoſitionis hujus, æque Inſervit, & Convenit, ac ea, quæ uſurpatur, Nimirum *AT*; Denique Cas. 1. hoc modo Liquet; Nam per Cas. 3.

$GS \times SF.\ CS \times SD :: HV \times VI.\ CV \times VD$. h. e. $CS \times SD.\ GS \times SF :: CV \times VD.\ HV \times VI$.

Sed per Proportiones in Propoſitione Allatas,

$AE \times EB.\ GE \times EF :: CV \times VD.\ HV \times VI$; Ergo $AE \times EB.\ GE \times EF :: CS \times SD.\ GS \times SF$.

Variari, ni fallor, poſſunt Caſus, ut Superius, Adhibendo Punctatas Aſymptotos, & duas Solummodo Lineas ſe invicem Decuſſantes, quarum multifarias Compoſitiones poſtquam invenerimus ex Regulis Antegreſſis, iis Parallelæ, in Propoſitione Præſcriptæ, peragantur ad Diverſas, ſi inde fiat Mutatio Figuræ, Diſtantias, Quod Nempe Accidit in *Fig.* 61. & 62.

Poſſem ad alias Propoſitiones hujuſce Doctrinæ Sectionum Accedere, verum hæc impræſentiarum ſufficiat perſcripſiſſe; ſin autem Eruditiſſimus & Celeberrimus Auctor jam adhuc Vivat, vellem, ut quo Candido Animo hunc meum Commentariolum Impertivi, eodem Benigno Placidoque Amplecti non Dedignaretur.

Multa apud me manent Scripta de Nova Methodo Radices quarundam Æquationum Extrahendi; Quæ Viros Mathematicæ Scientiæ Peritiores forſan ad Ulteriora Molimina in hiſce Studiis poſſet Excitare & Accendere. At his non eſt Locus.

IV. *Imperfecta Quædam de Longitudine, nec niſi Solummodo Inchoata.*

Varen. Geogr. Lib. 3. Cap. 39. 40.

LONGITUDINEM cujuſvis loci ignoti Proponitur inveſtigare, *h. e.* quod ad nautas attinet, curſum navis juxta æquatorem, ejuſve parallelos, inſtitutum per Loxodromias, ſeu Rhombos.

Patet ergo imprimis hujuſce problematis difficultatem in eo verſari, quod meridiani omnes locis omnibus ſint communes, ita ut nullum Meridianum nullius loci dixeris eſſe proprium, id vero efficitur ex telluris motu circa ſuum ipſius axem, per quem unaquæque ejus pars ad ſingulos ordine meridianos appellit, viſitque, abſque ullo, quod adnotari poteſt, diſcrimine,

Quod ſi jam definire poſſemus differentiam temporis, quo eadem ſtella vel Sydus ad diverſos meridianos aggreditur, eorum diſtinctionem ſatis accommodam haberemus, quam differentiam fruſtra quis ex circuitione Telluris exſculpere tentaverit, quia horæ ad ſuos ſemper meridianos relatæ ſunt & appoſitæ, atque adeo nihil ad alios afferunt, quo eorum ſitum agnoſcamus; ſcilicet in hujuſmodi revolutionibus, quamvis reipſa mutantur ſtellarum loci in ratione ſua ad Terram, tamen inter ſe fixæ conſtant & immotæ; unde cum quævis eadem ſtella in duos diverſos meridianos ſuccedit, omnia eadem diſpoſitione eodemque ordine conſpiciuntur, atque eamobrem perinde videtur, ac ſi ſub eodem prorſus meridiano hæſerimus.

Hinc patet, vel oportere punctum in æquatore, uti polus in coluro, immobile ſuſpenſum, cujus acceſſio vel remotio determinaverit Longitudinem, & meridianos diſpunxerit, vel invenienda erit alia temporis menſura, quæ non pendeat ex diurno motu, vel alius aliquis modus, ſi poſſumus excogitare, meridianos dignoſcendi; Sed nullum iſtiuſmodi punctum natura, quaſi ſignum quoddam, nautis expoſuit, vel erexit, nec quidem concipi tale poteſt Tellure movente, quæ enim ſtellas quantumvis fixas & obſtinatas deſerit, figentem animum derelinquet, præter quod inutile foret obſervationibus capiendis; ſiquis tamen certum in quovis Sydere Motum ita conſonum reperiret, ut ejuſmodi ſuppeditaret punctum, gratum faceret Eruditis; de iſta vero inventione forſan eſt deſperandum, aut enim directus in eaſdem partes ille motus ad amuſſim erit cum diurno æquandus, ſic ut dum Terra circumagitur, peragatur ipſe, quo ad eandem terræ partem ſemper dictum Sydus reſpiceret, cujuſmodi vix obtinebimus, aut cognoſcenda erit proportio inter motum diurnum & alium datum, ſic ut diſtantia poſſit innoteſcere ab eodem puncto, quod tantundem ſonat ac invenire menſuram temporis vel motus communem telluris & alicujus corporis diverſi.

Hanc alio modo requiſiverunt Aſtronomi, alio Mechanici, illi ex Jovialibus & Lunæ periodis, hi Automatum vi & auxilio; ſed recurſus ſive reditus cæleſtium iſtorum Luminarium nimis tardi ſunt, quam ut ſatis frequentes obſervationes exhibeant, & automatum motus nimis irregulares, quam ut accuratas & veras; Quid igitur reliquum eſt, niſi ut omni ſpe excuteremur hujus problematis inveniendi? Rem nihilominus tamen adeo momentoſam ne penitus abjiciamus, liceat ſaltem hanc propoſitionem alia, quam quæ percrebruit, methodo trutinare, & videamus, annon fas ſit meridianos, non menſura temporis, verum ſitu & poſitione ſua diſtinguere.

Quæ autem hic deſiderantur ex iis, quæ Vernaculo ſermone Conſcripſimus, erunt Supplenda.

V. *De Philoſophorum Argumentis ad Vacuum Probandum, deque eorum Calumnia & Infirmitate.*

DE Vacuo tam ex diametro pugnantes ſunt opiniones eruditorum hominum tamque inter ſe diſſidentes, varioque adeo ſucceſſu confutatæ & defenſæ, ut exiſtimare velis omnes fere, qui hucuſque ſcripſerunt Philoſophi, vel falſum ſentire, vel quod verum ſit non ſatis tamen exploratum cognitumque proponere; Quot enim, & quam munitiſſima, uti arbitrabantur, experimenta adduxerunt alii ad Vacuum confirmandum, & quoties in ætherem Subtiliſſimum & auras evanuerunt? Nulla interea data explicatione ab iis, qui aliam militiam ſequi maluerunt, quid ſit iſta tenuiſſima materia, aut quomodo etiam corporum dimenſiones poſſet pervadere; Nam & nos, ſi per meatus & foramina pateat iter ſolummodo, & materia hæc craſſa quantumvis minuatur, ſatis diſputaſſe adverſarios lubenter agnoſcimus, ſi res aliter ſe habeat, ne quæſtionem attigerunt; Nam, cum Carteſius tanti fecerit imaginariæ ſuæ triplicis materiæ inventionem, quam Morinum alioſque, ut ipſe ait, de induſtria celavit, tandemque ægre Merſenno ſuo impertiebatur, non ſolum veram triplicem ſed infinitam materiæ diverſitatem, graduſque infinitos, nobiſmet perſuadere velimus ex firmiſſimis & Mathematicis principiis nos inveniſſe; Verum, quoniam noſtram hanc diſputationem prolixiorem fore, & nobis invitis, præſentimus, quicquid hic fuſius eſſet præfandum, ibi conferetur. Atque Lockii imprimis argumenta perpendemus; Quod vero attinet ad diſtinctionem inter Ideas ſpatii & corporis, id nequaquam probat non ſpatium omne ſoliditate vel, inquam, Actione compleri, quia ſpatium concipimus, ubi reipſa corpore vel Actione ſit refertum, neque diffitetur Lockius; Prima igitur ejus probatio ex eo dependet, quòd nemo aſſeruerit, materiam eſſe infinitam; ſolerter admodum & eleganter, precario autem; Aſſerit Quiſpiam; Quid ſequitur? Nihil. Qui enim aliunde nequit Vacuum affirmare, nunquam ſuadebit Materiam vel Actionem non eſſe cum ſpatio infinite diffuſam; Non indignam tamen rem eſſe, ut videtur, putavit, quæ ad Dei Omnipotentiam confugeret; Ea vero conceſſa, non eſſe Vacuum, ſed eſſe poſſe, convincit; Tertia probatio ex Natura motus deprompta eſt; Dividatur itaque, ut velit, corpus ſuum denſiſſimum; Quomodo probatur abſque Vacuo partes ejus non poſſe moveri? Apertiſſime; Nempe, quia partes Solidiſſimæ in Solidiſſimo corpore a motu prohibentur, iis neque competit motus in Tenuiſſima materia; Probatum igitur oportuit non dari in natura Subtiliorem; ſed liquet; ſi dictum enim corpus in partes mittas, quam vis minutas, hujuſmodi autem partibus ſpatium denuo oneretur, in corpus evadet conſtipatiſſimum; Sc. natura quævis tota in ſuas partes conſumitur, poſito ergo materiam non extare niſi Solidiſſimam, omnes ejus partes erunt conſimiles, ergo non datur ſubtilior; Quis vero non videt argumentum in circulo trahi? A partibus enim ad totum ductum ratiocinium ad naturam diverſam ne umbra pertinget; Syllogiſmum ſaltem, certe Metaphyſicum, audiatis, nulla materia ſubtilior habetur, quia nulla eſt niſi denſiſſima; Quomodo autem id? Nimirum, quia nulla eſt ſubtilior.

Lockii Argumenta pro Vacuo.

Videamus jam, an vir Clariſſimus Henricus More felicius rem expediverit; Axiomata ejus ipſi concedimus, damus etiam Cylindrum, revolutumque concipimus; Ubi vero cæteri fere omnes dormitant, nutat & ipſe, & extenſum, per quod tranſit Cylindrus, Ponit eſſe Vacuum; Idem dici poteſt de Curva ejus linea ex telluris & plumbi motu deſcripta, neque colores in Cylindro quos ad ſecundum ejus argumentum imaginationi, ut ait, indulget, vel Subſidii quicquam afferunt, vel Ornamenti. Præter alios autem, qui hanc rem tractarunt, cum majori copia, tum nitidiori demonſtrationis apparatu diſſeruere Raphſonus & Borellus; & quidem ille non inſulſe confirmat in ſpatio pleniſſimo motum, ſi Cyclico-Annularem Excipias, educi non poſſe, neque iſtum ipſum ad veritatis ſimilitudinem propenſiorem videri, quia conatus ad movendum infinitæ quaquaverſum occurreret reſiſtentiæ; Sed quid hæc ad Vacuum? Nam, quia in ſpatio pleniſſimo vel Solidiſſimo omnia

Henrici More Argumenta pro Vacuo.

Raphſon de Spatio Reali Pag. 63. 64. &c.

Raphſoni Argumenta pro Vacuo.

Raphſon. Pag. 42.

nia ſint compreſſiſſima neceſſe eſt, ergo, quoniam in hac ſummæ plenitudinis, vel Soliditatis; conditione langueſcit motus, Vacuum erit neceſſario requirendum. Videat profecto Raphſonus, quod in alium procudit, ne ſibi regeratur, miſera Verborum colluſio, & in re fere Geometrica miſera ἀγεωμετρησία; Ante vero quam tantum interruptæ materiæ hiatum tranſiliiſſet, experiri decuit, annon poſſet in medio conſiſtere, vel adprobare, ſi modo leviter, nihil inter ſummam Plenitudinem vel Craſſitudinem & Vacuum interfuiſſe; Quod ad demonſtrationes ejus de Extenſo intimo ſpectat, recte incedunt, ſeu Solitarium ponatur, ſeu cum denſitate Conjunctum, adeoque nequicquam huc conferre videntur.

Borrelli Argumenta pro Vacuo.

Eximius Borellus non aliter Vacui neceſſitatem aſtruere conatur ex diverſis corporum figuris perpenſis, quæ angulos & interſtitia implerent, atque iis porro, quæ inter ſe commotæ plenitudinem reſpuerint; Sed, quod inique ſemper accidit hujuſmodi argumentis, dum ſumma cautela poris invigilant, nequa materia ſe per illos ſurriperet, deceptis cuſtodibus aliam ſemitam ingreditur, & dimenſionibus, ſi opus fuerit, potita curioſorum oſcitantiam vel labores caſſos eludit ridetque; Unum argumentum ſui, quantum ego vidi, proprium attulit præclarus Philoſophus, in quo negat dimenſiones, quæ ſpatio Vacuo tribuantur, eſſe reales, neque eſſe niſi privationes meras; Quod ſi intellexerat vir eruditus omnes quaſcunque rationes, & præterea nihilum, ſequi & ponere rei exiſtentiam, ad quam ſemper relatio inſtituitur, neque negationem eſſe poſſe ſine poſita quantitate, neque iſta quantitate perempta quicquam de reliqua abſolute potuiſſe affirmari, quam ratio non attigit, perſpexiſſet.

Newtoni Argumenta pro Vacuo.

Huc itaque proceſſimus, nihil firmatum ſpectatumque invenimus, Philoſophi igitur, quicunque a tergo videntur, ſinatis, inſalutati nobis diſcedant, & ad Newtonum, ex quo, ſiquo alio, ſaltem ſanum quippiam deſiderandum eſt, adeamus; In tertio Libro diſerte primo aſſerit Newtonus Vacui eſſe neceſſitatem ex differentia gravitatis Specificæ; At non videt Vir celeberrimus Vacuum in Argumento eſſe probandum, Argumentum contra ſpatium pleniſſimum vel Solidiſſimum procedere, adeoque aſſumit, quod debuit oſtendere, non reperiri materiam Subtiliorem; Probatur, inquit; Nam pondera corporum non pendent ab eorum formis & texturis; Aſſentior; Æther nec prorſus gravitate deſtituitur nec quam pro quantitate materiæ ſuæ minus gravitabit; Forſan fatemur; ſed non differt ab aliis corporibus, niſi in forma materiæ, Ergo—— Enthymema puta; Nam vel latet propoſitio, vel elapſa eſt; Quomodo enim non differt ab aliis corporibus, niſi in forma materiæ? Hoc modo; Non datur Sc. æther, cujus materia eſt ſubtilior, quia quamvis alia eſt Compoſitio, tamen materia eſt eadem in æthere, quæ in reliquis corporibus; Vel, quia eadem eſt, non erit diverſa; Præclare quidem & Argute; Verum, quanta ſit opinionum diſcrepantia intelligatis; Ex hac enim differentia gravitatis Specificæ probaverim Vacuum a Natura depulſum, non niſi in Scholis & Diſputationibus hoſpitari; Ut autem interea huic demonſtrationi ex ipſo ſuccenturiemus, per ſe infirmiori, ad ſecundum librum & experimenta ejus de pyxide & globorum oſcillationibus inſtituta retrahimur; Quantum, ad Pyxidem ponit reſiſtentiam ætheris ſubtiliſſimi, ſiqua ſit, debere in partibus internis pyxidis plenæ eſſe ſenſibilem, quod negamus; Nam non inde ſequetur reipſa eſſe nullam, quia ſenſum relinquit; imo a perceptione remotiſſima eſſe oportebit, cum reſiſtentia mediorum minuatur in ratione ponderis ac denſitatis, ſubtilemque materiam a denſitate ſive craſſitie corporum horumce ponimus infinite diſtare & denſam tamen eſſe contendimus; Ex globorum oſcillationibus nihil amplius auferimus; Nam præterea ex infinita etiam aeris diviſione, quoad magnitudinem Partium, ſubtilem noſtram materiam nunquam exſculpes; quoniam aeris particulæ in æternum diminutæ ſuam ſibi denſitatem tuentur, ea quam quærimus longe craſſiorem; Duas tamen Propoſitiones affert Sagaciſſimus Philoſophus, quibus, uti autumavit, demonſtratis placuit experimenta ejus ſuperſtrui, quas itaque in tranſitu perſtringemus; De propoſitione trigeſima & octava, ſi ſtatuatur globus ejus in canali ſuſpenſus ejuſdem gravitatis ſpecificæ cum fluido incumbenti, nihil probat Newtonus de ſolidorum motu in fluido Subtiliſſimo, niſi ejuſdem ſecum denſitatis, quod vocat, *h. e.* in ſolidiſſimo-ſubtiliſſimo fluido, quod plane quid ſit fateor me non aſſequi; ſi ponatur diverſæ gravitatis ſpecificæ, vel petit Newtonus, ut nulla ſit materia, quæ in ratione ad aliam non reſiſtit, adeoque demonſtrandum reſtat, vel iſtiuſmodi materia conceſſa pariter oſtendat, primo nihilum poſſe in globum ejus gravitare, deinde ex non gravitante motum vi gravitatis impreſſum generari, & denique Actionem & Reactionem inter nihilum & quantitatem eſſe æqualem; Hæc ſi probaverit, quæ omnia pugnant, recte ejus corollaria educit, ſi minus, concidunt; Altera ejus propoſitio, quadrageſima nempe, eſt item merum ſophiſma, non enim valet, quam ponit, proportio, niſi eadem ratione ſit fluidum ſubtiliatum, qua ſolidum ejus Sphæricum augeatur, nimirum, vel in ratione magnitudinis Partium vel denſitatis; Quid autem hinc obtinet? Corporis Sc. infinita denſitate motus amiſſus in dato fluido eſt ad motum ejus totum, ut Corporis data denſitate motus amiſſus in fluido infinite ſubtiliato ad ejus totum; vel, ut effectus dati fluidi in Sphæricum infinite denſius, ſic effectus fluidi infinite tenuioris in aliud Spæricum denſitate datum; *h. e.* ut finitum ad infinitum, ſic finitum ad infinitum; Sed in magnitudine particularum, ut videtur, hæret Newtonus, æquivoceque utitur verbo, *ſubtiliatum*, & quidem ubi iſtiuſmodi ſubtiliatio eſt ſimul impoſſibilis & abſurda; Verum Carteſius, contra quem diſputavit, ejuſque materia ſubtilis, in hunc errorem, ut auguror, inopinantem oppreſſumque ſtudiis Newtonum conjecit, atque ita cæteris ſummi Viri maximis meritis erit condonandus; His nititur experimentum ejus globorum oſcillantium, quibus igitur revulſis corruit & ipſum; Ex hoc tamen experimento, ſi accuratius eſſet exhibitum, demonſtrationem Vacui dependere Vir magnus profitetur, quod magis mirandum eſt, qui tanto alioquin uſus eſt acumine, in tam manifeſta veritatis & clara luce cæcutiret; Hoc enim omnibus Philoſophis prædicimus, ſi experimentis Vacuum, nec ratione, orent elicere, fore ut inutilem operam navaverint, & ſe inane potius ſtudium ſuſcepiſſe, quam inane inveniſſe ſpatium, fatebuntur.

Argumenta quædam contra Vacuum,

Poſthæc, unam tandem aut alteram probationem contra Vacuum, ex abundanti, liceat adducere; Prima ex infinita gravitatum ratione petita eſt, nam compreſſio noſtræ regionis eſt, ut pondus Atmoſphæræ incumbentis, & gravitas reciproce ut quadratum diſtantiæ locorum a centro terræ, quod ſi mille, vel millies mille, ſumantur ſemidiametri terreſtres in altitudinem, in immenſum diminuetur gravitas, atque ergo mediorum denſitates, nunquam tamen evaneſcent, quia altitudo columnæ cum linea denſitatum Logarithmica nunquam coincidet; Neque objici poteſt vacua ubique eſſe interſtitia, neque noſtram ſoliditatem vel Actionem infinitum ſpatium replere, quoniam hæc lex obtinet in quovis puncto Mathematico dato, & noſtra ſoliditas vel Actio rationem hanc gravitatis plane deſtrueret, cum aer ex hac lege ita rarefiat ad altitudinem unius ſemidiametri terreſtris, ut globus, unum digitum latus, impleret omnes Planetarum regiones ad uſque Sphæram Saturni, & longe ultra, adeoque, vel materia ſubtiliſſima ubique erit expanſa, vel rarefactio & condenſatio eadem.

Keill. Introduct. ad Veram Phyſic.

Newton. Princip.

Aliud argumentum ex differentia gravitatis ſpecificæ deſumitur; *E. G.* in æquali mole ſuber eſt ad aurum abryzum, ut unum ad paulo plus quam octoginta & quatuor, ex hypotheſi autem materiam omnem eſſe ſimilarem, nec ſubtiliorem haberi, interſtitia vacua, ſive meatus, erunt in ratione reciproca; ergo, cum in auro abryzo, vel nulli fere meatus compareant, vel tam minuti rarique, ut penitus ab invicem abrumpantur, in ſubere viciſſim materia, vel nulla erit ſenſibilis contra experientiam, vel ita ſparſa & poris interrupta, ut ne in contiguo ulla foret partium continuatio, quod fieri nequit, & quæcunque denique fuerit materiæ forma, hoc ſemper accidet; At fortaſſis non ita videntur vacuitates, ac ſolidæ particulæ, quamvis æque magnæ; Natura enim verecunda nolit ſe nudam ſpectari, & ſe modeſte ſubducit; Atque hic plane mirabile eſt ita oſcitanter, adeo craſſe, uno ore, apertiſque oculis, hucuſque erraſſe Philoſophos, niſi quod nullo modo gravitatem ſpecificam explicari poſſe arbitrarentur, ſi poros & meatus abjecerint; Item per experimentum notiſſimum Florentiæ captum Aqua ex compreſſione validiſſima vix quicquam condenſatur, verum etiam per auri foramina deſudat; Nihil dicimus de partium figura, vel ordine, quæ nunquam, ut videtur, tantum renixum poſſent efficere, ex dicta autem hypotheſi per non iniquam vacui & ſolidi ſpatii moderationem, in partibus Auri ſingulis una Vigeſima fere erit poroſa & pervia, in partibus Aquæ Reciproce; Hæc vero pars Aquæ ſolida, vel ſpatio circundata eſt, vel ſpatium ipſi; Si illa ſpatio, inter expreſſionem globus ſuperficie ſua ſaltem vigecuplo major per ſpatium trudetur vigecuplo minus, quod abſurdum eſt; ſi ſpatium ipſi, partes aquæ ad invicem accedent ex vi illata in ratione fere vigecupla abſque illo ſudore roſcido contra experimentum; In promptu eſſet & argumentum afferre ex ſonis per aerem, & in vacuo, quod vocant, propagatis, cum motus omnis in fluidum impactus divergit a recto tramite in ſpatia immota, pendetque a medii quadam denſitate, ſed omittimus, ne nimis longa nobis trahatur oratio, jam etiam prolixa nimis; poſſemque, ni ita eſſet, eadem, quæ Phyſicis argumentis defendimus, & Mathematicis etiam demonſtrare, ex Paraboloeidium, Spiralium, aliarumque figurarum proprietatibus, eademque opera, ſi otium, & vita integra, noſtra ſtudia conſequatur, perfectam, quæ enim ſpes mihi affulſit, curvarum & circuli quadraturam exhibere, cauſaſque & rationes exponere, cur tot ſummi viri per tot ſæcula

Lockii Tent. tamen.

Wallisii Arithmetica Infinitorum.

in hujuſmodi problematis fruſtra elaborarunt; Interea ex his demonſtratis fluit evidens & genuina explicatio Pellucidi & Opaci, &, ni fallor, nova colorum Theoria; ex his etiam aliquatenus, quod facile eſt oſtendere, eluceſcit magnum illud Leibnitio, Torricellio, aliiſque, in Philoſophia deſideratum, nempe, unde motuum cæleſtium cauſæ, an a motibus ætheris, vel, quod perinde eſt, ab orbibus deferentibus, ſed fluidis, oriantur, item materiæ ex nihilo creatio, gravitatio omnium partium in rerum univerſitate ad omnes; Miranda etiam illa, quæ Newtonus, Halleius, & Archimedes in Arenario, Cæterique de Angulo contactus & materiæ Subtilitate prædicavere, aliaque innumera hactenus, quod ſciam, vel penitus incognita, vel comperta minus, quæ, ſi hujus eſſent temporis, currenti calamo exarentur.

Objectiones contra Allatam Doctrinam.

Duæ tantum reſtant objectiones, quibus breviter reſpondeo; Altera eſt, non concipi poſſe, quomodo materia in materia moveatur; Verum repetendum eſt, nos materiam infinite ſubtilem aſſeruiſſe, atque ita, ad datam quamlibet, rationem habere nihili; nihili autem nulli ſunt effectus, ergo neque reſiſtit, neque gravitat, neque movet, nec movetur, reſpectu corporum, reipſa omnium compos; idem coram accidere videmus, corpora enim quæcunque in terra conſiſtunt in ſe mutuo gravitare deberent, cum tamen ejuſmodi gravitas neutiquam ſentiatur, quoniam gravitas hæc eſt ad gravitatem in terram, ut ſunt ad terram corpora, *h. e.* infinite minor; ſic ob immenſam diſtantiam Aſtronomi parallelas ducunt, ubi lineæ revera ad ſe invicem inclinentur, planetas pro punctis habent, & generatim, ea eſt lex naturæ, vel axioma potius, ut inter comparandum res agant patianturve juxta rationem, quam ad ſe mutuo tuentur, non juxta quid ſint abſolute.

Altera objectio eſt, implicare videri, ut in Solido Solidum poneretur, vel, ut in idem ſpatium plura quaſi denſentur ſpatii volumina; in idem quoad longitudinem & Latitudinem non repugnat, nam quotidie corpus cernimus extenſum in extenſo ſpatio exiſtere, cum in ſpatium ingreditur, vel antea ab ipſo non occupatum, vel ſecum tranſfert; primarii autem loci non moventur; & quid ergo prohibet, quin complicationes Spatii augeantur in infinitum? Sed nimis excurrimus, & dum de pleno pugnamus, materiæ etiam, quam velimus, copia uberior orationi exurgit; ſi vero, quæ diſceptatio tantum aliis faceſſit negotium, ſuppeditavitque meſſem adeo ingentem ſcribentium, eam intra Diſſertationem concluſimus, quod verboſiores fuerimus, vel Philoſophorum incuriæ, & heu nimiis erroribus, vel cautioni debitæ, vel argumenti huic ſaltem novitati, tribuendum judicabitis; Ut autem has Objectiones prorſus, & breviter, Diluam; Ipſe Pono Materiæ Eſſentiam in Actione, vel Vi quadam, Conſiſtere, Diverſas vero Materias Oriri ex Viribus vel Actionibus Diverſis; Nemo itaque non Agnoſcet, vel Vim ſive Actionem Majorem poſſe Minorem Subigere, adeoque in ipſa Movere, vel Majorem ſeu Minorem Vim ſive Actionem poſſe Exiſtere in eodem Spatio, adeoque Dimenſiones Materiæ in iſto ſenſu poſſe Penetrari, vel Solidum in Solido etiam Poni.

VI. *De Motu, Gravitatione, & Cohæſione Materiæ, Eaque ex Immediata Dei Actione Profluere.*

OMNES, qui hactenus de Cohæſione diſputarunt Philoſophi, ita in ſalebris & tanta in obſcuritate rerum implicati fuerunt, ut ſuæ potius ignorationi involucrum prætendere, quod naturæ eſſet detrahendum, quam quippiam ſanum ſpectatumque velle afferre videantur; alii quietem, alii partium ſubtiliorum preſſionem, denſitatem alii confirmant efficere, ut cohæreat materia, ſed nec deſunt, qui ramulis, & neſcio quibus uncis ſalibuſque, navarunt patrocinium, quorum omnium, ſi fuerit opus, facilis eſſet diſſolutio, cum vel rem quæſitam expedire ſatagunt inepta verborum colluſione, vel inani per immenſam materiam indagine, donec tandem defeſſi id ſolum aſſequuntur, ut eorum perſequendi induſtriam magis, quam felicitatem inveſtigandi, fateamur; qui, dum removent tantum difficultatem, non diluunt, perficiunt quidem, ut ea longinquitate proſpectus ſenſibus minuatur, infinita autem quæſtione cumuletur rationi. Nec phyſicas ſedes cauſaſque motus & gravitationis lautius aperuiſſe judicantur; Præclare Newtonus, qui Philoſophus ſit inſtar omnium, Virium quantitates & rationes ex quibuſdam poſitis conditionibus demonſtravit, ſpecies earundem & qualitates non attigit; imo deſerte aſſerit cauſas nondum eſſe cognitas, quibus corporum particulæ ſibi invi-

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tem adhæreſcant, vel ab ſe invicem recedant, Deſperatam tamen eſſe inquiſitionem non putat; in eandem ſententiam profitetur item Vir alius rei philoſophicæ haudquaquam imperitus, Vires, inquit, illæ, & Univerſalis in natura efficacia, per omnia lucent, cognitionem vero noſtram adhuc refugiunt; Itaque omnibus, quas cætera Philoſophantium turba adhibuerunt, explicationibus prætermiſſis; Imprimis Legem hanc Pono Naturæ; Unionis, ſive iſtius Proximitatis, qua res maximam, quæ dari poſſet, eſſentiarum conjunctionem obtinent, paſſiones cum actionibus eſſe reciprocas & æquales in unitis, ſuiſque tandem naturis conſentaneas. Si minus, vel non erit intima, quæ fieri poſſit, eſſentiarum conjunctio, contra Hypotheſin, vel actiones pariter cum eſſentiis ſunt contumaces, immotæ, & incommunicabiles, *h. e.* ſibi ipſis renituntur, vel actio non eſt actio; æquales autem & reciprocæ ſunt, quantum enim altera recipit eſſentia, deperit alteri; cum experientia porro hæc lex ad amuſſim quadrat, hinc enim dependet Lex tertia Newtoni, quam ut ipſe ex multis conſectariis, quæ in Mechanica demonſtratis conveniebant & velificabantur, comprobavit, ita & noſtra ex eo ſtabilitur, quod & illa inde conſequitur atque omnia ejus corollaria, quæ licet ad materiam ſpectent, tamen euſque ſolummodo, in quantum eſt activa, nam quoad ſuam naturam abſtractam, nec premit nec reprimit, ſi nulla enim eſt actio, paſſio intercidit, & vice verſa; neque attinet in quibus ſubjectis actio inhæreat, cum eſſentiæ materiales, quod æque nequeant cum invicem communicari, perinde actiones a ſeipſis arceant, atque diverſiſſimæ quævis *e. g.* cogitantes & extenſæ; hinc itaque præterea in Unione Cogitantis cum Extenſo, incommunicabiles ſunt eſſentiæ, actiones vero reciprocæ non item, ſed plane neceſſariæ; unde profluit animi cum corpore Sympathiæ luculenta explicatio, languente enim & laſſato corpore flacceſſit & defatigatur animus, & viciſſim; hinc animi etiam poſt mortem exiſtentia, phreneſis, paſſiones, Ideæ perceptionis, motus porro forſan cordis, & multa ejuſmodi poſſent derivari. Ergo ex hac lege jam confirmata Deus, ſive Mens Infinita, quia res cogitans eſt ſua natura activa, quia neceſſario cum ſpatio conjuncta eſt infinito, nam ubique exiſtit, vel ſaltem vis mentis infinitæ erit etiam infinita, ubicunque exiſtit, & nihil poteſt intercedere inter iſtam exiſtentiam & ſpatium, quod unionem prohibeat, hinc, inquam, Deus in ſpatium ex neceſſitate ſuæ naturæ agit; verum actio hæc non motum producet, motus enim actualis eſſe nequit, quoniam immobile cum infinito ſpatio reciprocatur, ergo erit Preſſio; Tandemque proponimus, Preſſionem ſaltem eſſe æquabilem, neceſſariam, per omne ſpatium diffuſam, & ad omnes partes circumcirca æqualiter vergentem.

Raphſon de Spatio Reali.

Item Pono pro Naturæ Lege; Partes omni data minores cum omni data majoribus, ſeu cum infiniteſimis, quas vocant, Infinitas earundem, omneſque terminos intermedios, vinculo communi quodam inter ſe contineri, atque invicem ſe neceſſario concludere, ita ut data Infiniteſima Infinitas etiam habeatur, omneſque gradus ſparſi per infinitatem; quod prorſus Axioma eſt; Natura in nihilum non abit, niſi vi velut coacta & quaſi de exiſtentia ingenti molimine & pedetentim depulſa, non ad infinitum ſcandit per ſaltus, ſed multa ſerie tardaque progreſſione emenſa & exhauſta; Infinitum eſt unum in ſe, ut ſi ponatur infinitas *ad intus*, quam appellant, ſimul conſtituetur illa, quæ *ad extra* audit, nam termini ſunt reciproci, nec ſejungi poſſunt; unde pateſcit, cum ſit natura infinite tenuis, *h. e.* craſſitudinis infinite parvæ, ſpatium intimum dico, quæ, neceſſario exiſtit, neceſſitatem item colligi naturæ Solidiſſimæ vel infinite craſſæ, ita ut non magis ſit neceſſaria continuitas ſpatii ex infiniteſimis extenſionis, quam ex craſſitudinis infiniteſimis denſitas, nempe alicubi, nam ad infinitatem hujuſmodi non neceſſe eſt, ut per omne ſpatium extendatur; vel inverſa probatione, dico ſoliditatem vel Spiſſiorem Actionem eſſe infinitam ejuſdem ſpatii, vel Tenuioris Actionis, repetitionem. Si negetur, ſit ſpatium cum corpore, quod ſemper extenſum eſt in eodem loco quo exiſtit, commenſurabile, ita ut utriuſque detur communis menſura, cujus vel eadem erit repetitio vel diverſa; ſi eadem, corpus & ſpatium idem erunt, quod repugnat; ſi diverſa, non poteſt finita eſſe in corpore, modo minima, quæ poſſit, adhibeatur communis menſura, quia ſpatium eſt infinite diviſibile de profunditate, ergo eſt infinite in materia repetitum, ſic ut nulla diviſione conficiatur; *e. g.* concipiamus ſpatium abſque omni profunditate ſua ſolummodo extenſione æquare dimenſionem corporis in ipſo exiſtentis, nempe exponatur globus materiæ, cujus diameter ſit unius pedis vel unciæ, nam eadem valet argumentatio, ſi ſumantur globi quantumvis parvi, vel etiam maximi, huic autem æqualis ſtatuatur globus ſpatii in quo corpus firmatur & recumbit; Si hæc eſſet ultima ſpatii diviſio, ita ut interior adhuc extenſio concipi nequi-

Raphſon de Spatio Reali.

Raphſon. de Spatio Reali. Cohæſio hinc Explicari poteſt.

Item Gravitatio.

Porro & Motus.

Cæteraque fere Omnia Naturæ Phænomena.

ret, vel, uti jam diximus, corpus & ſpatium ſunt æquivoca, vel datur ſoliditas, quæ in ſpatio non exiſtit, vel infinita eſt extenſio quoad profunditatem. At vero porro attentius rem perpendenti neceſſitas Soliditatis in ſpatio infinito innoteſcit ex ipſa natura infinitatis; ſi enim ſpatium aliqua ex parte determinaretur, adeo ut minima habita eſſet tenuitas, ultra quam progredi ſit nefas, infinitas ejus perimitur & evaneſcit, ſed hæc infinitas eſt ipſa materia, quoniam res Infinitas ſibi ex adverſo extenſas coexiſtere repugnat, ſi vero infinita ſit ſoliditas, quoad ſpatii multiplicationem, ſive Actio, eſt, ſua natura, indiviſibilis, immota, atque æterna, ſpatii Affectio; Hinc extenſum datur in extenſo ſine fine; & Mathematicorum poteſtates Biquadraticæ, Cubo-cubicæ, & cæteræ quævis altiſſimæ, quas abſtractas exiſtimarunt, in natura reperiuntur, & materia ſubtilis, licet non Carteſiana; Item datur in natura plenum, item differentia gravitatum ſpecificarum ſine vacuis Interſtitiis contra Newtonum; Ex his, autem, quæ ſupra poſuimus, ſequitur primo, de qua quærimus, dari materiæ cohæſionem a Dei Actione neceſſariam; deinde id quod Acutiſſimus Newtonus magno propoſitionum apparatu oſtendit, Gravitationem eſſe omnium materiæ partium ad omnes, eamque eſſe quantitati materiæ proportionalem; nam per legem primam Preſſio ubique eſt æqualis ſpatii intimi ad omnes plagas quaquaverſum, & per ſecundam habetur ſoliditas vel Actio ſive materia ex infinita vi preſſionum ejuſmodi æqualium, ergo conſtat Gravitatio propoſita; ea vero etiam permanens eſt & neceſſaria rerum affectio ex iis quæ dicta ſunt; Hinc autem porro liquet materiam, ſi ſolitaria eſſet, in ſpatio fore ſuſpenſam, perpetuo ad omnes partes conatu incitatam, abſque ullo tamen abſoluto motu. Si vero jam plures materiæ portiones in ſe invicem agerent, minores quantitates in eam partem, qua fit Preſſio ex majoribus, minus reprimunt, atque ita fortiori Preſſioni ſuccumbant neceſſe eſt, unde oritur motus; magnitudines item corporum eodem modo inter ſe diſcrepant, quo partes ſpatii, quæ, licet ad magnum infinitum pertineant, adeoque nec moveri queant nec dividi, tamen quatenus ſunt partes definitæ utrumque patiuntur; Interea, nec motuum orbitas, nec gravitationum proportiones, nec diverſas leges, quibus materia jam formata ſecundum varias ſuas figuras vel preſſiones, cohærere poſſit, vel fluere diſſoluta, in præſenti non expendimus; quamvis ex multimoda harumce Preſſionum compoſitione, tum Phænomena cæleſtium revolutionum velocitateſque, tum totius fere terreſtris ſyſtematis natura, quod ad figuras & concretiones partium ſpectat, vix dubitem, quin poſſent derivari; Multa ſunt, quæ nuda aſſumptione ex præmiſſis facile eſſet educere; nullam Sc. dari in Natura attractionem, Vim autem Centripetam Newtoni eſſe revera Vim Centrifugam, vel ab aliis corporibus Vim impactam, rationes tamen utriuſque eſſe eaſdem; item æſtus marium neque ex ſolis & Lunæ conjunctis tantum viribus, neque ex acceſſu vel receſſu luminarium, ſed ex diverſa combinatione Preſſionum omnium in Univerſo, ex ſitu & locis differentibus; Vim forſan porro Magneticam majorem eſſe differentia inter Preſſiones omnes Univerſi & Repreſſionem terræ, unde innoteſceret ratio ſoliditatis vel Actionis ejus; ſed hiſce non immoramur. Quod reliquum eſt, ne Quis hæc ita adduci putet, quaſi id mihi ſumam, ut Deus nihil ſua voluntate atque arbitrio in rerum univerſitate exequatur, iniquum enim eſt & perabſurdum, qui tantam molem, etiam infinitam, neceſſaria cogitatione, & velut aliud agens, exerceat, agitet, permoveat, illum nequicquam poſſe conſilio & ratione adminiſtrare, cum nos ipſi quidem, ſummi Archetypi miſera exemplaria, mentis vi ſola noſtrum puſillum mundum vivificantes, & injurias defendere, & adjumenta afferre, aliiſque etiam rebus ſuccurrere & adeſſe, non neceſſario neque inviti, ſed volentes & conſulto, poſſimus.

Hæc a me Scripta fuerunt, cum Newtoni Gravitationem reputarem eſſe omnium Rerum Principium, neque quicquam fere jam etiam Corrigo, niſi quod Aſſeram Preſſionem non forſan eſſe Æquabilem per Totam Rerum Compagem, nec Materiam eſſe Similarem, vel Proportionalem ipſius Ponderi; Quæ Sententia Celeberrima, Originem ducit, uti omnis, ut fertur, Cognitio noſtra, a Pomo; id quod Accepi ab Ingenioſiſſimo & Doctiſſimo Viro, pariter ac Optimo, mihi autem Amiciſſimo, *Martino Folkes* Armigero, Regiæ vero Societatis Socio Meritiſſimo; Quem hic Honoris Cauſa Nomino.

VII. *De*

VII. *De Rationibus Geometricis, Eaſque Animo perinde Competere, qua Percipienti, ac Materiæ, qua Extenſæ.*

MIRARI Subit inter tot celeberrimos viros & naturæ indagatores, quos hæc nupera præcipue, uti antiquioris etiam ævi, umbratilis Philoſophia tulit acerrimos, ne unum extitiſſe, qui, cum rationes Geometricæ tam late pateſcerent, eas fuerit aggreſſus materiæ ab anguſtiis eripere, & quæ corporum ſuperficiebus tam diu innatarunt, in animum capacem ſaltem largumque diffuſas, in ſuam veram & propriam vindicare Libertatem; Fuere, qui ſibi finxerunt Deum ſemper Geometrizantem, quando tamen exiſtimabant res omni ratione carentes juxta ſummam conditas fuiſſe & perfectas, iis, quæ maxima erant præditæ, nullam magnopere vellent accidere, vel minima adminiſtrari; At vero profecto, quorum contentioſæ & petulantes diſputationes ita animum & intellectum in clamorem ſtrepitumque diſſiparunt, ut ex Geometria Arithmeticam etiam eliminarent, ab iſtiuſmodi hominibus nequicquam eſſet ſperandum, qui adeo propinquam cognatamque ſcientiam a ſodalitate rejecerint, aſcituros alienam; quomodo autem ii, qui in tantum hiſce ſæculis, non ita pridem elapſis, Analyticam doctrinam excoluerunt promoveruntque, & intra limites rationum incluſere tam diverſam diſperſamque materiam, cogitationem & animum præterirent, vix ſatis intelligi poteſt: Quod igitur vel ab aliis relictum, vel perſpectum minus, vel non attentatum deſperatumque accepimus, ſi aut noſtra voluntas aſſequetur, aut exſculpet induſtria, cum nihil eſſe queat optatius, quid magis, aut quodnam potius, ſive rerum optimarum cognitioni, ſive Philoſophorum ſtudiis, vel Religioni veritatique, adjumentum gratulemur? Sed objiciet quiſpiam nobis in hanc defenſionem ingreſſuris, apertiſſimis eam nugis impeditam & ineptiis ſeipſam convellere. Quibuſnam enim fidem facere nos poſſe arbitramur, animum ejuſque affectiones, quo nihil reperias ſimplicius, in lineas & figuras iturum, & in quadrata velle areaſque dimitti? Imprimis itaque reponimus infinitam eſſe rem, quæ immane quantum ab extenſione atque adeo à figura abeſt, cujus rationes ad figuram poſſint revocari. Quæſitam ſaltem, ſi minus conſequamur, noſtræ diſputationi brevitatem tranſiliamus, ſi, omnia percurrendo, conemur oſtendere, preſſionem, denſitatem, tempus, velocitatemque, quorum proportiones lineis & angulis veſtitas dedit Newtonus, extenſione vacare; Vis Centripeta autem ita eſt non extenſa, ut neque Animus magis ſpatium reſpuere videatur; ſi enim quocunque demum modo ſpatium multiplices, comminuas, aut commutes, quid inde ex paſſione mera ad actionem conferetur? quoad rationem tamen patitur, capit etiam figuram; Exempla abunde ex Newtono ſi eſſent opus, excerperentur; & quidem generatim, quod fit in ſectionibus Conicis, poſtquam debitas proportionales obtinuerint Mathematici, abſciſſam nempe rectumque latus, ut & ſegmenta diametri in Circulo, figuras abjiciunt, ſic quarumcunque virium vel quantitatum proportiones, ſi nec vires ipſas nec quantitates poſſis, in lineas & figuras licet cumulare, quoniam rationes quædam poſſunt inveniri utriuſque communes. Verum, quid perinde ab extenſione abhorret, atque ipſa non extenſio, quæ niſi recedat, neque iſta prodit? hæc tamen figuris, ſi Walliſium audiamus, poteſt accommodari, & profecto nullus dubitem, quæ radix impoſſibilis vel falſa in Algebra nuncupatur, quin eam aliquando conſtiterit mediam proportionalem in Geometria agnoſcere. Deinde Reſpondemus extenſioni atque ipſis figuris menſuram rationemque competere, non quia vel ſunt extenſio vel figuræ, ſed quoniam aliæ ad alias referri poſſunt & cum ſe invicem comparari; *e. g.* quantitates ſpeciebus deſignatæ, quod eleganti demonſtratione comprobavit Florimondus de Beaune, in eadem, qua ratione corpora & ſpatia, exprimunt etiam lineas, adeoque comparationes in Matheſi ſolæ expenduntur, neque vel lineæ vel ſpatia ingrediuntur in computum, qui idem manet in tam diverſo, qui fieri poteſt, quantitatum tenore; porro enim notationes Algebraicæ non cum figuris quadrant, quaſi neceſſario, ſed fortuito tantum, & quod figuræ iis legibus ſummittuntur, quas iſtæ rationes inferunt & apportant, cum interea ita non valeat ſubjectum cui adjungantur, ut neque dubitaret Franciſcus Van Schooten in reſolutione Problematis, quam artificioſiſſimam vocat, demonſtrationem compingere, ubi quantitates, quas in eam accivit, eſſent omnes incognitæ, & rationes ſolummodo poſitioque innoteſcerent; quod idem perfecit Carteſius in quæſtione Pappi de locis planis, quæ appellant, & Solidis, aliique paſſim Mathematici;

Plato.

Animum Rationes Geometricas Recipere.

Objectio Proponitur.

Prima Reſponſio.

Walliſii Sect. Conic. & Arithmet. Infinitor.

Walliſii Arith. Infinitor.

Reſponſio Secunda.

ſexcentæ

Materia, ſive Actio, ſeu Soliditas ad Matheſin Exigi poteſt.

Item & Perceptio.

Animi Simplicitas vel Extenſionis non huic Matheſi Refragatur.

Intenſio Perceptionis æque eſt Diverſa, ac Extenſio Materiæ.

ſexcentæ vero item ſunt poteſtates, quæ nulla conſtructione hactenus inventa poſſunt ad figuram & lineas aptari, nam ultra quartam neque proceſſit Bakerus, ſuperiores autem non minus rationes Geometricas patiuntur; De angulis contactus & infinite incommenſurabilibus miranda prædicantur, iſti, quos Peletarius & poſt eum Walliſius non eſſe extenſos contra Clavium & Leotaudum defenderunt, ex Leibnitiano differentiali calculo & Speciatim a Newtono probantur infinitas angulorum ſeries inter poſteriores quoſque admittere; quoad incommenſurabiles, quid habet iſta incommenſurabilitas cum extenſione commune? datur tamen Arithmetica ſurdarum, nec rationes ideo evaneſcunt, quia minus figuratæ. Verum de rationibus diſputatum eſt ſatis, & eas ex ſervitute duriſque materiæ vinculis in præclariorem patentioremque animi totiuſque naturæ provinciam, uti ſpes eſt, eduximus; Omnino tamen fatendum eſt, etiamſi rationes haſce jure optimo a materia & extenſione abſtraheremus, eas, niſi cum ipſo conveniant, in animum non recte derivari; nihil autem quod ſciam, ita pertinaciter hiſce relationibus negarunt eruditi, ac ſoliditatem & perceptionem; de ſoliditate in Scholis non ſemel diſputavimus, & oſtendere conati ſumus eam Matheſeωs poſſe legibus obtemperare, jam etiam aggreſſi perceptionem, quibus firmatis & conſtitutis, illud glorioſum εὕρηκα Mathematicis non equidem invideo, miror magis, cum poſthæc, ſi non vana eſt fides, nihil reliquum videatur, quo minus univerſa natura, evoluta, eruta, exuta ex latebris, ante conſpectum, ob ora atque oculos, poſita ſit, appareat, verſetur, niſi otium & penitior Geometriæ & theorematum in ſingulis caſibus inveniendorum ſcientia; quod nollem putetis hoc a me ita eſſe dictum, quaſi quidvis ſolertiæ meæ attribuendum exiſtimarem, neque audeam dicere felicitati plurimum; quid enim, quod inveſtigandum reſtat, non amplectimur? quid non ſectamur, diſpicimus? quid, Dii boni, aliud agimus, at quam non aſſequimur? Interea non poſſumus non mirari Viros Analyticæ peritos nihil de animo ſanum demonſtratumque literatis propinaſſe, cum neque rationem ineptia, ut ex prædictis patet, neque Animi ſimplicitas ex iis, quæ ſumus dicturi, vel religionem ullam, quæ prohibeat, injicere poſſe videretur. Hic itaque tenendum oportet, naturas omnes eſſe reipſa ſimpliciſſimas, nec forſan tam abſtractæ, quam reales & veræ, haberi debebunt, ipſa extenſio non augetur nec diminui poteſt, neque ipſa perceptio, adeoque has naturas neglexerunt Mathematici, id vero attulit mens noſtra finita, ut nihil concipiamus, niſi terminis quibuſdam finibuſque concluſum, quorum ad invicem relatio rationes & ſcientiam proportionalem in mundum produxit, & in tantos evexit tamque ſtupendos veritatis incredibileſque progreſſus; Imo & porro, ſi quælibet finita extenſio vel perceptio cum ſe ſolum comparetur, eſt item ſimpliciſſima, neque compoſita dici poteſt, niſi ad partes ſive ad minores quaſdam quantitates relatio habeatur; Verum etiam in maxima rerum & animi ignoratione, ſi neſciamus animi naturam, an ſimplex ſit, an ex partibus conflata, ratiocinationes inſtituere ex hypotheſi licet, quæ quamvis minus indicarent, quomodo crearetur natura, quomodo creari poſſet, commonſtrant; & quidem non raro, quod videre eſt in fluidis Newtoni, adeo quadrant corollaria cum Phænomenis phyſicis, ut perditiſſimæ fere eſſet dementiæ & pervicaciæ plane inſipientis, ſi aſſenſum abſtineamus, quo minus omnia explicari ſecundum ipſiſſimas naturæ vires & indolem fateremur. Matheſeωs hujus hypotheticæ in animo unum aut alterum exemplum adducere ſufficiat, tum de mentis ſyſtemate abſtracto, tum qua corpori adhæreſcit; Per facultatis autem perceptivæ Intenſionem exprimere velim, qua datis conditionibus certa quælibet perceptio in animum poteſt evocari, adeoque omnes actuales perceptiones potentia complectitur & in eo, quo gradu, in mentem imprimuntur, per Intenſionem vero perceptionis, ipſam perceptionis quantitatem; Jam dico, ſi intellectus in dignitate quavis *e. g.* duplicata, triplicata, ſubduplicata, ſubtriplicata intenſionum perceptionis, dato tempore quoquo modo augeantur, ducanturque in tempora reciproce proportionalia, erunt æquales; & viciſſim, ſi æquales intellectus oriuntur ex iſtiuſmodi multiplicatione, datis temporibus intenſiones innoteſcunt; facilis eſt demonſtratio; item, iiſdem poſitis de incremento intellectuum, ſi quocunque modo intenſiones perceptionis reciproce majores evadant ducanturque intellectus in tempora æqualia, erunt etiam æquales; Ex his ſequitur ætate provectiores, licet craſſiori ingenio, intelligentiores ſunt, quam minus adulti, ſi tamen verſatiles magis & fortiori indole; hinc & dari poſſunt caſus, ubi juvenes porro natu majoribus intelligentia præſtent; hinc itidem ex coætaneis Ephorum prætuleris Theopompo, & committi poteſt, ut cujus pinguior Minerva calcaris indigeat currentem præverteret; ſin aliter fuerint proportiones inſtitutæ, fiet, ut omnia fere, quæ ſunt tum inter hominum, tum brutorum & animalium

malium intellectus, diſcrimina, diverſæ rationes, variique gradus, quos ad ſe invicem obtinent, definirentur; Hæc faciliora ſunt & minoris moliminis. Exempla, quoad animus cum corpore ſit conjunctus, ſunto hujuſmodi; Si vires animi motrices ſint in ratione æqualitatis cum Intenſionibus perceptivæ facultatis, inſtrumenta vero corporis in quæ animus exercetur de reſiſtentia in eadem ratione, perceptio erit, ut facultas perceptiva; Quod ſi fuerint reſiſtentiæ in ratione quavis inæqualitatis, ſunt perceptiones, ut Reſiſtentiæ ab intenſionibus perceptivæ facultatis ſubductæ, pro quantitate perempta ſumpta ſemper æquali poſitiva, ſi reſiſtentia major ſit intenſione, ſi minor vel nulla, ſuppar erit perceptionum ratio; e contra, ſi actiones fuerint in ratione æqualitatis cum Intenſionibus, perceptiones item ſunt in eadem ratione, ſi in ratione inæqualitatis, perceptiones ſunt ut Intenſiones ab Actionibus ſubductæ pro quantitate autem perempta ſumpta æquali poſitiva, ſi Actio major ſit intenſione, ſi minor vel nulla, ſuppar erit perceptionum ratio; Ex his conſequitur, dari poſſe perceptionis facultatem, ubi nulla ſit actualis perceptio, vel actualem cogitationem non eſſe de eſſentia animi; item fieri poſſe, ut perceptio in animo eliciatur abſque interventu materiæ; & ſi corpore animus poſt mortem diſtrahatur, quod neceſſe eſt, eo erit vivacior, quo craſſiora corporis inſtrumenta abjecerit, atque ſimul paſſionibus, cæteris paribus, magis immunis; e contra, fieri poſſe, ut brutorum animi intereant; in actuali autem perceptione, vel præterpropter æqualem, vel minorem, ſemper eſſe reſiſtentiam, quam Vim animi Intenſivam, atque adeo minus ſyſtema corporeum in genere naturam debere minori perceptionis facultati adjungere, unde animalcula minus percipiunt; Quo præterea magis torpore animus laborat, eo eſſe craſſiores ſpiritus animales, vel facultas erit animi debilior; Hinc etiam ſequitur, Muſculos nullum ſibi proprium motum habere, quod Borellus putavit, in magnifico ſuo opere Poſthumo de motu animalium, in quod moribundus Author divinam, quam animam efflavit, inſpiraſſe videatur; item de humana mente judicandum eſſe ſecundum id cujus eſt capax, non ſecundum id quod ſit, & quamvis homines forſan inveniantur, quos ne brutis præſtare affirmaveris, non inde ſequitur reipſa & in ſua natura non brutis illos præcellere; porro contra Lockium identitatem perſonæ non in conſcientia, ſed perceptionis intenſione conſiſtere; hinc denique patet, cur ingenii tarditas plerumque intellectus gravitate & pondere compenſatur; cur Lethargia alii tenentur; cur ex nimio cibo ſopor animum invaſerit; inſania, voluptas, cruciatus, ſomnus, deliquium, cæteræque animi affectiones omnes fere, quas experimur, hinc liquido conſtant, totaque animata natura; Adeoque, ſi nullum aliud argumentum ſuppeteret, quo probetur animi Matheſis, ſat eſſet, poſita ejuſmodi Matheſi, tot tamque varia ſenſatæ naturæ phænomena recte explicari; Quid autem aliud fit in Mathematicis Philoſophiæ principiis Clariſſimi Newtoni? Quid porro in demonſtrationibus Extenſionis? Omnes nempe Geometræ in hac via inſiſtunt, ut ex quibuſdam poſitis, quid deinde conſequeretur, oſtendant, an hypotheſes veræ ſint, necne, parum ſolliciti. Verum nihil etiam magis ſentimus neque in ipſa extenſione, quam quantitatem in animis & perceptione exiſtere; præterquam enim quod quotidie videmus, homines iiſdem fere vitæ inſtitutis præceptiſque utentes, atque pueros admodum, ingenio nimium quantum diſcrepare, cujus nulla reddi poteſt ratio, niſi quod ita ſint nati; bruta adeo ab hominibus diſtant & alia quædam animantia a brutis, ut non ipſa corpora ſenſu carentia perfectius cernantur & verius magnitudine & mole a ſe invicem abeſſe, quam perceptione & animis iſta; Speciatim autem de perceptione, nemo non novit Muſicam ad Matheſin pertinere; Quid vero ſunt ſoni, niſi perceptiones, adeoque quantitatem accipiunt? Nam *e. g.* eadem chorda æquabilis & æqualiter tenſa ſonos edit Longitudinibus reciproce proportionales, ſed perceptiones ſunt ut ſoni, ergo chorda, ſic tenſa, perceptiones in hoc caſu producit Longitudinibus reciproce proportionales; Cæterum, ne ſuppullulanti Orationi concedamus, hic tandem modum ponimus, atque ex iis, quæ prædiximus, Aſſero Rationes Geometricas etiam Animo, qua percipienti, competere, & æque dari poſſe Pſycheometriæ, ac Geometriæ, Elementa.

Matheſis Animi, quoad cum Corpore Connectitur.

Multa hinc Fluunt Conſectaria, in Philoſophia & Medicina.

Datur Quantitas tum in Perceptionis Intenſione, tum in Extenſione Materiæ.

Ergo ſunt Pſycheometriæ, pariter ac Geometriæ, Elementa.

VIII. *Medicinæ Collaudatio, ſive Oratio Habita ante primam Amiciſſimi mihi Viri* Thomæ Hewett A. M. *Diſputationem pro Gradu Doctoratus in Medicina,* Jun. 28. 1707.

QUANTO in Honore ſtetit inter cæteras ingenuas Artes Medicina, quanta benevolentia, quo ſtudio, quam ingenti plauſu, omnes Gentes, Sæcula, Virorumque cuncti ſigillatim Ordines hanc palmariam Scientiam proſecuti ſunt & amplexi, Veſtrum Neminem, Academici, latet, qui illam tantis, neque imprudentes neque inſcii, tamque amplis Dignitatibus exornaſtis; Æſculapium inter Deos Antiqui retulerunt, nec tam propter radios lucemque, quam propter Virtutes Medicas, Veneramur Apollinem; Vos autem, dum iſtam inanem futilemque gloriam indignam eſſe, quam conferrent Sapientes, judicatis, certiſſima eos condecoraſtis laude, cum non Diis, verum Maximis Hominibus, Veſtros Æſculapios velitis annumerari; Maximis inquam, quibus enim Animi facultatibus, qua ingenii vi & acumine, quo rationis momento & gravitate, quibus & ipſi brutis præſtant, cæteris Hominibus præcellitis, iiſdem, quos ad hunc Ampliſſimum Gradum evocatis, Vobiſmetipſis prætuliſtis; Neque eſt, quod miremur Medicos tantam omni ævo nactos fuiſſe Dignitatem; Qui enim ſunt illis, vel ſumma Indole vel Eruditione omnimoda, vel quolibet genere Laudis præſtantiores? Philoſophiam, Matheſin, aliaſque optimarum Artium Diſciplinas, ſi modo ſeorſim extiterint, impenſe admiramur, in Medicina autem ſolum conjunctas non perinde ſuſpicimus, ac ſtupemus, ita ut videatur quaſi Natura velle ſuum Numen affirmare, cum ſui ſtudioſos tam Divina, & velut infinita certa quadam, cognitione remunerarit; Quæcunque, Terra ſuffecerit Mortalibus, promiſerint Sylvæ, generarint Fluvii, Fodinæ etiam ſuppeditarint, Omnia ſunt perſpecta huic Scientiarum Magiſtræ & penitus explorata, Quæ non ſatis eſſe Gloriæ ſuæ ducit has ſuperas Regiones incredibili & ſingulari Virtute ſibi eſſe perdomitas ſubjectaſque, niſi, quod poetæ de Hercule illo perhibent, in ipſius Erebi fauces Plutoniſque Regna aſſerat Victoriam; Neque minus callet Animalium Naturam, mirandamque compagem, quam cum Deus efformarit ſuo ſolius Numine, ipſa ſecundos ſibi vendicat honores, dum poteſt conſervare. Quid tamen minuta hæc & exilia, ſi cum aliis conferantur, commemoremus? Non enim in Agris & Sylvis, in Fluminibus, in Terræ Animantiumque Viſceribus Rimandis verſatur præſertim Medicina; Aggreditur porro ad Maximarum rerum ſcientiam, ſi quidem Animi & Corporis ingenium ſit dignum, quod cognoſcatur; Id vero adeo probe ſibi inveſtigatum, omnibuſque numeris intellectum habet, ut in illud ſingulos Nervos intendi, in eo ſolum deſudari, merito arbitraremini, cum ad alia reſpiciatis, ex iis tantum integram ſuam Laudem nihilominus Gloriamque requirere; At quid majus dici poteſt de Medica Arte, quam cum Deus ipſe ſummam Poteſtatem Sapientiamque demonſtret in Mortalibus ab interitu Vindicandis, Medicina etiam ad Deum ipſum prope accedat Iiſdem ſalutem tribuendo. Hanc præclariſſimam ſcientiam, Doctiſſime Vir, hodierno Die profiteris, quem redditum tandem tuis, ac reſtitutum Academiæ huic, ex Animo gratulamur; Non enim recepiſtis, Academici, Virum leviter hiſce rebus imbutum, aut in earum cortice, vel primo limine, hærentem, ſed Medicum Ingenio perfectum, Induſtria elaboratum, atque omni, cum Doctrina, tum etiam accurata experientia, maximaque ſolertia, inſtructum & abſolutum. Nihil itaque amplius addam, ſi prius monuero, ut te fortiter erigas & accingas, ad defendendam imprimis tuam ipſius opinionem Argumentis, deinde ad ſummi Profeſſoris tela pro Virili retorquenda; Et, quam famam plurimam, Ingenioſiſſime Vir, ex Remediis adhibendis apud Londinenſes tuos jure reportaſti, eandem utinam ex propugnandis aſſerendiſque etiam apud hoſce graviſſimos Viros impræſentiarum Conſequare.

IX. *Collaudatio Juris-Civilis, ſive Oratio habita in Scholis Baccalaureorum Cantabrigiæ* Jun. 25. 1707. *cum* Thomas Paske A. M. *Vir mihi Amiciſſimus diſputavit primo pro Gradu Doctoratus in Jure-Civili.*

CUM tot Videam hiſce in Jure-Civili exercitationibus Viros celeberrimos & ſpectatiſſimos vacare, Locum autem hunc Muſis jucundiſſimum amiciſſimumque adeo frequenti & ingenua Literatorum Juvenum turba decoratum, recreatur mihi paulum Animus ac reficitur, Dolenti Vices tam præclaræ atque admirandæ ſcientiæ; Quis enim, niſi fuerit Reipublicæ imprudens penitus, ignaruſque vitæ communis, & ab omni fori judiciorumque ratione alienus, Quis, inquam, Mortalium non perſpicit, nec ſine aliqua Commiſeratione, tam Veterem Laudatamque Diſciplinam jam tandem, cum apud cunctas fere Gentes ac Societates viguerit, apud Anglos obſoleſcere? Nonne cernimus ubique, municipia, civitates, provincias univerſas, aliis Hodiernis, Lautis quidem & Sapientiſſimis, Legibus regi palam ac gubernari; Nulla non tribunalia occupata ſunt, arrepta, obſeſſa a Viris, licet optimis & doctis, tamen ſuarum ſolummodo forſan Popularium Inſtitutionum peritis, quorum nonnulli minus Modeſti, & in has Scholas, mediusfidius, veſtraſque Eruditas lites, tuamque porro, Digniſſime vir, illuſtrem Cathedram, neſcio an invaderent, niſi harumſce rerum ſua ipſorum Inſcitia, hujus autem Loci Auctoritas, tamque venerandi Cœtus gravitas & exiſtimatio, prohiberent; Rejecta itaque & deturbata a Reipublicæ Gubernaculis, eliminata foro, judiciis exterminata, omni demum patrocinio exuta Dignitatiſque ornamentis, velut Aſtræa quædam, non ficta illa, ſed vera, ex hominum conſortio exacta, ad vos tandem Juris-Civilis Prudentia confugit, imploratque veſtram opem & fidem, ut, de cæteris ſuis Hoſpitiis detruſa & depulſa, hic ſaltem Sedem ſtabilem, certumque in ſeram poſteritatem Domicilium, conſequatur; Hic ſemper Romana Lex floreat inter Optimas Artes, & quæ olim Maxima cum Gloria per totum Terrarum orbem dominata eſt, Gentes vero Populoſque conſtituit ſumma cum æquitate, & in ipſos Reges nobile ſplendidumque propagavit imperium, ſinite, plurima jam confecta ætate, innumeris Laboribus perfuncta, feſſa denique quaſi & laſſata rebus adminiſtrandis, apud vos tandem otio non indecoro deliteſcat; At quis interea tam glorioſam, tantam, tamque potentem rerum Dominatricem has in anguſtias redigi ſatis mirari poteſt, aut dolere redactam? Cui tamen hac ipſa ex ſua humilitate maxime cedet in Gloriam, quod, cum Romani populi Decus, & Robur, quo nihil unquam extitit pulchrius aut præſtabilius, ex rebus humanis exceſſerit, Juſtitia ejus & Prudentia Civilis hodiernum uſque ad Diem etiam fuerit ſuperſtes; unde colligere licet, quanta Vis inſit in Romanis Legibus, cum Romana arma, potentiamque, ſuperarit; Dii boni, quam tuam, Divina Roma, providentiam, ſeveritatem, ſapientiamque ſingularem, in Statutis ſanciendis, conficiendiſque Legibus, haud aſſequimur! Omnia tibi ſunt ſummo jure, ſummaque Religione, cauta & proviſa; Nihil non, inſtituto Præclarum, ordine Venuſtum, oratione Eximium, nec minus ad aliorum Licentiam comprimendam, quam ad tuam ipſius Abſtinentiam, egregiamque Modeſtiam, explicandam Accommodatum; ita, ut vix dici queat, an magis fuerit Conſentaneum iſta tua Decreta alios amplecti, vel iſta te aliis imperare Magnificum; infra quorum ſplendorem dignitatemque tanto magis in ferendis ſuis cæteræ Gentes conciderunt, quanto minus poſſent ad iſtam Urbis Imperioſæ præcellentiſſimam Animi Magnitudinem aſſurgere; Quæcunque Prægrandia, Animoſa, Stupenda, Romani Nominis ſemper fuerunt propria; Attoniti & Mirabundi legimus Triumphos celebratos, extructa Theatra, Maria in terras immiſſa, & adverſas Claſſes media ipſa in Arena Dimicantes; Nullum autem extat monumentum mirabilius hujus excelſæ & generoſæ Indolis, pene dixiſſem, majoris Humana, quam ſuarum Legum abſoluta conſummataque Deſcriptio; in quibus adeo cuncta ſua ſibi conſtant, ut videatur Roma vix, alio conſilio, ſuis Armis Gentes deviciſſe, quam ut Legibus devinciret, & Libertatem iis ademiſſe, non ut ablatam conculcet, ſed remitteret ipſis exornatam. Quod ad te attinet, Doctiſſime Reſpondens, ſentio propemodum his oculis, mehercule, Juſtiniani cineres exultare de patronis tantis, talibuſque ſuis, cum tu, & ſine, ut Lateri tuo adjungam iſtum Amicum tuum Regalem, Virum omni Laude non modo dignum, verum etiam ſuperiorem, cum vos, inquam, tam benigno Animo Succurritis calamitoſæ, & abjectæ fere, Diſciplinæ;

Doctorem Willymott, Collegii Regalis jam Præſidem Digniſſimum.

næ; Erigatis, obteſtor Deos, afflictam, proſtratam, eamque, qua olim pollebant, auctoritatem imperatoriis Legibus in poſterum Vindicetis; in præſenti vero talem, Ornatiſſime Vir, te præbeas impertiaſque toti Academiæ, qualem Omnes Expectamus ex ingenio iſto tuo, quo uſus es maxime, ex caritate autem & gratia, qua apud optimos quoſque ſummopere ac meritiſſime flores, Omnes Vovemus.

X. *Gratulatio ad Epiſcopum Elienſem* Johannem Moore, S. T. P. *Patronum quondam Chariſſimi mei Amici* Caroli Morgan, A. M. *Magiſtri hujus Aulæ Optimi, & quadantenus Patronum Meum.*

QUANTO cum Gaudio accepimus Tuam, Reverende admodum Pater, Evectionem ad Elienſem, quem præter cæteros jure merito es Adeptus Epiſcopatum, vix concipi poteſt, ne vix enarrari; Ita nempe, hos tibi maximos, pariter ac judicio Summæ Reginæ, omniumque conſenſu & comprobatione, apprime debitos honores gratulamur, qua mente, quove animi affectu & ſtudio, hujus ipſius Collegii, horumque, quos incolimus, parietum, Majeſtatem, Decus, Gloriam, amplexaremur; Illud enim vinculum velut inter tuam, Ampliſſime Præſul, atque hujuſce Aulæ dignitatem intercedit, ut quicquid ornamenti præclaram iſtam tuam Doctrinam Pietatemque conſectetur, videatur etiam aliquomodo in ſe redundare; Accedit porro ad Noſtram Lætitiam, non ſolum, quod tanta huic Collegio Gloria contigerit, ut Suos ad Supremum fere Eccleſiæ faſtigium conſpiceret evectos, verum etiam, quod Deus Optimus Maximus id præterea in ipſius felicitate poſuerit, ut Præſentes eoſdem videat Patronos; ut, cum antea nimis longo intervallo remotus in clientelam nos pro Summa tua benevolentia aſciveris, propriori jam amplexu comprehendas, ac foveres; Deo itaque, Imprimis, maximas Gratias habemus, qui ſuo Numine & Providentia, talem, tantumque, huic Collegio conſtituit Fautorem, Deinde impenſe precamur, ut quæ Exiſtimatio ad tam ingentes Honores Veſtram Dominationem perduxerit, Eadem ad Maximos uſque proſequatur.

Cum eſſet Epiſcopus Norvicenſis.

XI. *Oratio Panegyrica in Divum Gulielmum, apud Nonas Novembris* 1701, *ad Prænobilem Virum, mihi autem, & Amico meo, Humaniſſimum Patronum,* Thomam Holles, *Ducem Præcellentiſſimum de Novo-Caſtello.*

A Te Principium; Tibi Deſinet; —— *Virgil. Ecl.* 8.

REDIIT jam iterum ille Dies, Academici, qui huic Genti, Academiæ, Nobis omnibus ſalutem, lætitiam, libertatemque attulit; Dies, tantis, tamque egregiis memorandiſque Deorum beneficiis, præſenti numine, ſolenni fere in terras deſcenſu, inſignitus & ſuperbiens, tot præclaris, ſplendidiſque ſupplicationibus illuſtris, tam vario gaudii apparatu & ornamento cumulatus, ut ipſe quidem tandem veſtibus abjectis pullis, quaſi jam timere deſineret, exultare videatur; Graviſſimum periculum, poſt hominum vero memoriam crudeliſſimum ac maximum, in totam hanc Gentem & Rempubcam deſtinatum, hic dies, ad ſalutem fortunaſque Britanniæ pluſquam fatalis, nobis noſtriſque patribus propulſavit; Quod ita magnum fuit, & omni intricata ratione perplexum, ut neque jam, tam ſalutem conſecuti fuiſſe videamur, quam experti convincimur, non tam miramur, quam ſtupemus, non tam ſentimus, quam cernimus; Utinam vero hic eſſet Conſiſtendum; Nobis, quod potuit, vel inimicorum ira adimere, vel læſorum indignatio abducere, vel ſceleratorum furor detrahere, quod Patria, vel Conſuetudo, vel naſcendi Ratio, vel Religio, charum aut jucundum efficere, Conjuratio hæc, quæ hodierno die patefacta eſt, abſtuliſſet; Quid hominibus eſſe debet majoris rei ſua religione? Illa ita periclitabatur, ut Dii ipſi præſentes, paludati, propriis fere armis & telis, in procinctu, ſuis altaribus adſtarent, propugnarentque; Quid poteſt eſſe libertate dulcius, aut acceptius? Quid patria amabilius? Quid, quam ſua ſibi eſt domus, quam natale ſolum, quam cujuſque fortunæ, quam otium & vitæ tranquillitas, carius aut delectabilius? Quid autem non vobis ex his, Proavi, a Diis etiam oblatum eſt, cum ne ſperare ea, neque enim de illis timere, poteritis? Quid non recepiſtis ex ipſis pene fati faucibus ereptum? Has, has Sedes Muſarum, Academici, quas incolitis, literarumque domicilium, periculi hujus invidia jam fere abſumptas, vobis redditas videtis, reſtitutas, ab interitu vindicatas; ut hi parietes, intra quos continemur, ut videtur, participes eſſe geſtiunt gaudii & lætitiæ omnium communis, qui ornamenta ſua, & auguſtum decus, imo quod ſint, accepta referunt, hujuſce Diei Fauſtitati, Deorum autem immortalium numini & munificientiæ; Ut, quid pronunciare dubitemus, quod de Trojæ Mœnibus perhibent, hos parietes, Deorum opere, Diis etiam merentibus, extrui & ſurrexiſſe? Tantum vero extitit hujuſce diei periculum, tot malis refertum, ut Dii, mehercule, huc in unum Scelus tam multas calamitates contuliſſe videantur, ne non omnem rationem & opem expediendæ noſtræ ſalutis in unum beneficium conferrent; Tantum, quam pene effectum, quam pene illatum fefellimus, eluſimus! Clandeſtinum adverſus Univerſam gentem Angliæ conſilium initur; Rex Optimus, Nobiliſſimorum Plebeiorumque Senatus laudatiſſimus, Regni totius flos & principatus, deſignantur ad Cædem; Conducta eſt Cella; Conjuratores ſacramento rogati; Vis incredibilis Nitroſi Pulveris ſub ipſa Curiæ Contignatione in Cellam coacervatur & adſtruitur, peſti ligna ferrumque circundata; Cuncta recte atque ex animo inſtituuntur, comparantur; Conjuratores nihil prædicant, produnt nihil, Omnes tacent, Altum ſilentium; Poſtero Die convocandus erat ſenatus; Nihil de Conjuratione prolatum, ne ſuſurratur quidem; Acceſſit Carnifex, Cellam ingreditur, Rem perficit; At quo ſe verterent Proavi? Num ad arma confugerent? Ubi vero hoſtes? Comparent nulli. Num promiſſis atque mercede agendum? quibuſcum? qua de re? quam ob cauſam? Non conſtat. Quomodo autem malum exſculpere aggrediantur, quod nec pateat eſſe defoſſum? Latet conjuratio, Obmuteſcit, Elinguis eſt; Imo tot nodis involutam & implicatam Tragœdiam, quem Deum oportuit invocatum, qui ſatis dignus eſſet ad expediendam? Hæc Conjuratio, hæc ipſa, unis literis, illæ vero ad hominem datæ & malum & ſceleri cupientem, ambigua etiam ſcriptura, deprehenſa eſt, patefacta, illuſtrata; Scelus, quo nihil fuit ex Naſcente hoc rerum Ordine improbius, nihil crudelius, nihil cautius, nihil proviſum melius, propiuſve effectum, multorum annorum elaborata lambendo & efformata proles, ſpes & deliciæ Romæ, ſine hominum providentia, ſine ſolertia, ſine ratione conſilii, fractum eſt, irritum fit, in auras diſſipatum. Quod etiam prius periculum irritavit & auxit, jam ornat cumulatque beneficium; ſervamur antequam metuimus, nec Dii ſe iraſci faterentur, priuſquam declaraſſent ſe condonare; Quid vero? Tanta rerum conſentiens, conſpirans, continuata cognatio, quem non coget affirmare, Nos, Rempublicam, Bona, Fortunas, atque hanc celeberrimam veſtram per totum literatum orbem Academiam, integram vobis & incolumem, invito pene & reluctante fato, Dei Optimi Maximi Numine Divino Præſentique, Servari? Neque enim literæ commiſſæ eſſent, neque ad alium hominem delatæ, neque ipſis literis deprehenſa eſſet conjuratio, niſi huic tantæ audaciæ a Diis immortalibus conſilium eſſet ereptum. Quidni etiam illi hanc gentem ſibi charam in tanta pericula adducerent, ut ab iis liberata majori voluptate perfunderetur? Etſi enim, nihil eſt magis optandum, quam intrepida, æquabilis, perpetuaque fortuna Reipublicæ, ſecundo otii & tranquillitatis, ſine ulla turbulenta ratione, curſu, tamen, ſi nobis pacata a terrore, periculiſque ſecura, omnia fuiſſent, incredibili quadam, & pene divina, qua nunc Deorum beneficio fruimur, lætitiæ voluptate caruiſſemus. Quid divinius hominum generi a Diis ſuperis datum eſt, quam ſua cuique Religio? Nobis vero, & propter ejus elegantiam & ſanctitatem, & propter innatam quandam & inſitam noſtris ſuperſtitionem, vita fortuniſque eſt charior; Quid magis delectat, quam ſua quemque patria, quam vita & ſalus? Tamen, non tanta Jucunditate illas a majoribus nobis traditas accepimus, quanta ſervatas & quaſi reſtitutas amplectimur; Ex qua enim re majorem voluptatem capere poſſemus, quam ex felicitate, cujus compotes ſumus, non fortunæ arbitrio, ſed Deorum ſingulari benevolentia, quam ex Religionis & Reipublicæ Diſciplina? Quam Dii, tam mirifice conſervando, ſe comprobare oſtenderint, pro qua, nobis, & hac gente, propugnarint, quaſi ſuas ſedes cælumque defenderent; Nam, ne Dii ipſi profecto, adeo taciturnum, ſui fallax, tam inhumatum, ſepultumque ſcelus, explorare ſuffecerint, niſi, quemadmodum, cum ſuo Cælo timerent, non ſolum in Cæpia & Animalia, ſed etiam ſe in Vectes & Sarmenta induiſſent. Illud nimirum eſt, Nos huic genti, totique populo Anglicano, Exitium conſilio crudeliſſimum, genere perditiſſimum, ratione inferendi inauditum, periculo ſane maximum, defenſum depulſumque hodierno die celebramus; Beneficium quidem ſummum, ſed auditum, ſed acceptum tantum a Majoribus; Fidem habemus, non Oculis intuemur;

Gaudebant de ſua ſalute Proavi, Nos etiam Feſtum dileximus; Nunc vero vivi, videnteſque, hic adſumus, & veneramur; Vidimus enim, Academici, ſub hoc tempus, cum Dii, Diviſque permiſtus Gulielmus, huic iterum genti perituræ ſuo numine atque auxilio præſentes ſuccurrerint; Cum, non domiciliis atque tectis, ſed etiam templis atque delubris ſceleratos ignes denuo illatos, & jam prope ſubjectos, reſtinxerint, gladios in rempublicam eductos repulerint, eorumque aciem a veſtris omnium jugulis retorſerint; Vos Teſtes eſtis, qualem & quam duram ſervitutem, jam in Cervicibus agentem, hic Dies Nobis, noſtriſque Capitibus defendit, qui non poteſtis non reminiſci, cum acerbiſſimo animi ſenſu & dolore, quando auctoritate hujus florentiſſimæ Academiæ projecta, Omnia nutu atque arbitrio gubernarentur Franciſcorum, neſcio quos, perditiſſimorum hominum, atque omni ſcelere Vincentium; Teſtis eſt Oxonium & Collegium Magdalenſe, quæ perduellionis arguitur, quæ erat temporum incommoditas, quod ſe, ſuos mores, & inſtituta, defenderet; Teſtis eſt Londinum, quæ luctu & lachrymis cooperta ſpectabat, cum Reverendi Epiſcopi, homines ſanctiſſimi & integerrimi, ſpreto eſſent jure, invitis legibus, in carcerem detruſi; Teſtis iterum atque ſæpius Londinum, quæ, periculi timore Oppleta, vidit tormenta bellica, totumque civitatis armamentarium, in ſe dirigi, contra tecta urbis, contra templa, & vitas civium; Teſtes ſunt omnes per Regnum & hanc Gentem provinciæ, omnes provinciarum civitates, vici, villæ fere, & tuguria Teſtes, Quæ, vel ſuis immunitatibus & privilegiis orbata, vel ad arbitrium mutatis & in pejus relatis donata, vel iniqua vectigalium irrogatione, alieni militis vi, injuriis, ſcelere, conſumpta, omnia municipali dolore & armis oppreſſa, libertatem amiſſam, perditam, ingemiſcebant, & Rempublicam cum ultimis miſeriis colluctantem; mutum eſt forum, elinguis curia, tacita & fracta civitas; ut ne videremur majoribus noſtris patientia ſcelerum futuri poſteriores, periculo vero multo priores; quo iis pararetur exitium, Rex erat tollendus, quo nobis, ne conſtituendus quidem; ibi ſenatus removendus, hic ne convocatur; tum comparandæ copiæ, nobis bellum inſtructiſſimum, directæ acies, miles inſolens, ante fores, in domo, in penetralibus, ante lares & focos verſatur; Quid vero poſthæc ſuſpicandum eſſet proavis, quod non eſſet nobis timendum? Eadem ferox & effrænata audaxque Religio noſtrum periculum inſpiravit, eadem ſimultas, indignatio eadem, quæ olim tantas tragœdias pene excitaſſet, totamque virorum cladibus cruentaſſet Britanniam; At vero hoc eſſet beneficium, ſi Religio tantum, crudelis licet, ſcelus ſtimulaſſet, nos, omnes miſerias & calamitates, quas hoſtium Furor, quas ſervorum Dominatio & Imperium, quas Gens, parentibus truculenta, nata barbara, inter feras & lupos enutrita, immanitatis diſciplina inſtituta, moribus confirmata, quas Hybernia inferre aut excogitare potuerit, nos, noſtraque Reſpublica, ſubiiſſemus; Imo tamen hoc quidem leve, nobis Victoribus juſſu victorum, Dominis nutu mancipiorum, Viris & Britannis arbitrio abjectæ & vilis militiæ, turpiter, manſuete, ſine gloria, ſine dimicatione, ſine virtute patria, inermis, inertibus, inultis eſſet pereundum: Maximo Opprobrio, Crudelitate maxima, fædiſſima autem Servitute, Magno Gulielmo duce, liberamur! Dies profecto hic tandem, ut videtur, tardius per annum ire pertendit, ſeriuſque ſolito revolvitur, tot, tam magnis, & ſtupendis beneficiis onuſtus graviduſque! Quid enim, Academici, eſt pulchrum aut venuſtum, quid amicum aut jucundum, quid elegans & ſplendidum, quid gratum, quid divinum, quid, quod capeſſere, vel etiam a Diis precari, velimus, quod non hodiernus dies contineat, quod non complectatur, quod non reſtituat nobis, atque etiam deſperatum reducat? Ut neſciamus, mehercule, utrum dies hic, magnum temporum decus & ornamentum, hujus ævi longinqua vetuſtas, ſæcula ipſa conderentur, ut tanta nobis apportent, tamque miranda beneficia, an nos facti, vita, his auris, fruamur, ut tanta experiremur; & videremus, quanto majori cumulo Dii ſuam in nos benevolentiam augere poſſent, cum vitam & patriam ſervarint, quam cum dederint; de quibus difficile erit dictu, an ſint magis celebrandi, qui tam conſpicuum Sibi honorem, aut colendi, qui huic Diei decus & religionem, aut benefici & liberales, qui reipublicæ ſalutem, conciliarint; quæ omnia, adeo mirabilia ſunt, ut nulla terræ pars, non reſpublica, non gens ulla, non natio, paria aut ſimilia re unquam viderit, vel etiam animo ſit conſecuta; ita ut Dii, qui a reliquo mundo hanc noſtram terram pelago & mari ſecernerent & diſtinxerunt, a cæteris gentibus hominum virtute, a multis ingenio, ab omnibus doctrina & pietate, eandem videantur præſentia ſua & beneficiis diſtinxiſſe; & dubitamus quidem, an illi hanc gentem toto orbe diviſam conſtituerint, ut palam fateantur ſe conſervare velle, vel etiam relicto cælo habitare; qui ita præſentes nobis opem & auxilium tulerunt, ut conſpectui noſtro ſe oſtentarent, & ante oculos fere verſarentur; Deos noſtris oculis pene videre poſſemus ultro citroque curſare, noſtrarumque rerum curam peragere; huic genti & populo ſalutem, non afferebant, ſed elaborabant, alios admonendo, excitando alios, his conſilium eripiendo, ferendo illis, providendo adverſus ſiniſtra, ordinando cauſas, quæ dextra eliciant, &, quocunque opus fuerit, adhibendo concinna & idonea; quæ tantarum rerum gubernatio vix potuit eſſe, niſi eſſent adminiſtrata, non ſolum Deorum immortalium nutu & conſilio, verum etiam ſummis eorundem fere curis vigiliiſque. Hoc vero Noviſſimo Beneficio, quid præclarius vel ſplendidius, Cujus gloria tanta eſt, quanta nulli unquam contigit, niſi Diis immortalibus & Divo Gulielmo; quo enim difficilior noſtra erat ſalus, eo accepta erat gratior; eo & major tua gloria, quo beneficium cumulatius. Quid vero majus dici poteſt, quam, Diis etiam inſperantibus, nos a Gulielmo ſervari? Quid glorioſius, quam Gulielmum ſervare tanto otio, tanto ſilentio, ſuo ſolius numine & præſentia? Omnia, quæ confuſa jacebant & indigeſta, quaſi Deo aliquo jubente, nutu ſolius & arbitrio Gulielmi, turbatos extemplo reſtituerunt ordines, & in mirandam rerum pulchritudinem ceſſere. Meruit certe hoc tam divinum & præclarum facinus, ut maximo Deorum beneficio ſacrata eſſet dies, quam ornaturus eras admirando tuo. Hoc ſaltem incommodum tua recipit gloria, quod nimio & abundanti munere oppreſſi & ſtupefacti, memoria ejus carere & ſenſu videmur; quod, tamen & tibi pariet laudem cum omni poſteritate adæquandam, qui ita ſervaſti nos pereuntes, ut appareat te noſtræ ſaluti cum Diis conſuluiſſe ex naturæ tuæ divina quadam benevolentia, ex tua propria animi clementia & miſericordia ſingulari, cum ſtupore noſtro nos neſcire velis, cui vitam & libertatem, quas tu condonaſti, debeamus. Deum itaque Optimum Maximum, imprimis, ſumma Religione, contentiſſimo ardore, pleniſſima animi benevolentia, ſuſpicimus & veneramur, qui ne opinantes, crudeliſſima clade pene obruti, ſolius Dei beneficio & numine erepti, ſæviſſimo periculo, nocturno, inextricabili fere, ſubducimur; nullis precibus vocatus, nam nec timor ullus invaſit, nullis templis extructis, ſine victima, eripuit; Poſtea tibi, Gulielme, maximas, quas poſſumus, & quæ optimo jure tuæ ſunt, gratias habemus, primum, quod beneficium in nos collocaſti, quod nulla erit æternitas abolitura, nulla ætas tacebit, nulla conficiet aut conſumet vetuſtas, deinde, quod noſtras res a te ſervatas & reſtitutas gubernare dignatus es, Gentiſque Anglicanæ ſalute & fortunis, & neceſſitudinem tuam, & hominum dicacium contumelias, poſteriores duxiſti; Vobis poſtremo, Academici, impenſe gratulamur de veſtra omnium ſalute, quam hodierno die conſecuti fuiſtis, de vitis, de fortunis, maxime vero de Religione veſtra & Academia, de illis ipſis ſedibus, quas impletis, quæ omnes, quam pene deploratæ, ſunt a Diis ſapientibus & bonis in veſtram iterum fidem & clientelam commiſſæ. Quod, ſiquis ſpe ducitur ſe poſſe rem noſtram turbare aut evertere, dehinc id ſperare deſiſtat, cum Dii ejus tutelam ſuſceperint; quos enim dedi putabant, defendi intelligunt; quos tradituros ſperabant, vident propugnare; ut vero porro deſperent, Deorum patrocinium, quod dono prius accepimus, in poſterum merebimur, nec recuſabunt gratos defendere, quos tuerentur immeritos. Hoc tamen Deos ſuppliciter & enixe rogamus, noſtra tandem lætitia, qua majorem capere non poſſumus, definiant hujuſce diei celebritatem, nec, quod a tanto naufragio Rempublicam iterum prohiberent, ſuo etiam auxilio percupimus nimium præfidant, ne res noſtræ ſæpius, & duriori manu, tenſæ ſpe citius aliquando diſſilire videantur. Ne diutius vos teneam, hæc gens & reſpublica bis ſub auſpiciis Novembris perfuncta eſt funeſto fatalique exitio; & hic ſpiritus, quem trahimus, non alia conditione ſuppeditari videtur, quam ut Deos, Gulielmum, & hunc ipſum Diem, celebremus; Cujus memoriam, ut dici poteſt Deos ipſos fovere, qui tam ſedulo noſtram ejus & toties implorarint, patriam venerari, vos autem, & hiſce dicendi exercitationibus, & aliis inſtitutis optimis & ſapientibus, caviſſe, ut colant Veſtri benevolentia ſempiterna, ita Nos, quod religioni & officio decet, quod noſtro in vos obſequio, exoptamus in dies vigere, in annos floreſcere, per omne ævum, tempora, æternitatem, fore perennaturam.

FINIS.

MVSEVM
BRITAN
NICVM

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